ACE Engineering Publications

GATE | PSUs

MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

Text Book: Theory with worked out Examples and Practice Questions

Strength of Materials

(Solutions for Text Book Practice Questions)

Chapter 1

SIMPLE STRESSES AND STRAINS

Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

01. Ans: (b)

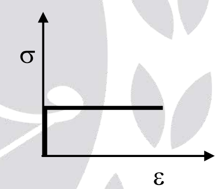
Sol:

- **Ductility:** The property of materials to allow large deformations or large extensions without failure (large plastic zone) is termed as ductility.
- **Brittleness:** A brittle material is one which exhibits a relatively small extensions or deformations prior to fracture. Failure without warning (No plastic zone) i.e. no plastic deformation.
- **Tenacity**: High tensile strength.
- **Creep:** Creep is the gradual increase of plastic strain in a material with time at constant load.
- **Plasticity:** The property by which material undergoes permanent deformation even after removal of load.
- **Endurance limit:** The stress level below which a specimen can withstand cyclic stress indefinitely without failure.
- **Fatigue:** Decreased Resistance of material to repeated reversal of stresses.

02. Ans: (a)

Sol:

- When the material is subjected to stresses, it undergoes to strains. After removal of stress, if the strain is not restored/recovered, then it is called inelastic material.
- For rigid plastic material:



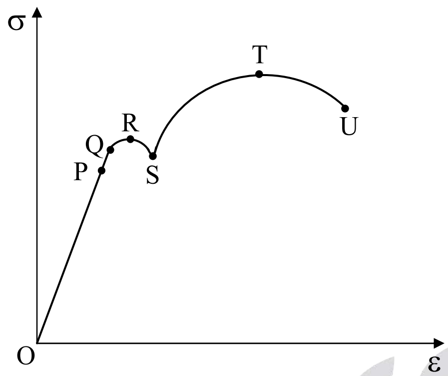
- Any material that can be subjected to large strains before it fractures is called a ductile material. Thus, it has large plastic zone.
- Materials that exhibit little or no yielding before failure are referred as brittle materials. Thus, they have no plastic zone.

03. Ans: (a)

Sol: ***Refer to the solution of Q. No. 01.***

04. Ans: (b)

Sol: The stress-strain diagram for ductile material is shown below.



P – Proportionality limit
Q – Elastic limit
R – Upper yield point
S – Lower yield point
T – Ultimate tensile strength
U – Failure

From above,

OP → Stage I
PS → Stage II
ST → Stage III
TU → Stage IV

05. Ans: (b)

Sol:

- If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is **isotropic** or in other words we can say the isotropy of a material is its characteristics, which gives us the information that the properties are same in the three orthogonal directions x, y and z.
- A material is **homogeneous** if it has the same composition throughout the body. Hence, the elastic properties are the same at every point in the body in a given direction. However, the properties need not to be the same in all the directions for the material.
  Thus, both A and B are false.

06. Ans: (a)

Sol: **Strain hardening** increase in strength after plastic zone by rearrangement of molecules in material.

- **Visco-elastic material** exhibits a mixture of creep as well as elastic after effects at room temperature. Thus their behavior is time dependant

07. Ans: (a)

Sol: ***Refer to the solution of Q. No. 01.***

08. Ans: (a)

Sol: Modulus of elasticity (Young's modulus) of some common materials are as follow:

Material	Young's Modulus (E)
Steel	200 GPa
Cast iron	100 GPa
Aluminum	60 to 70 GPa
Timber	10 GPa
Rubber	0.01 to 0.1 GPa

09. Ans: (a)

Sol: Addition of carbon will increase strength, thereby ductility will decrease.

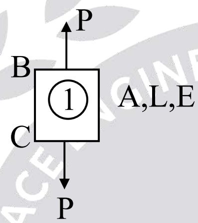
Elastic Constants and Their Relationships

01. Ans (c)

Sol: We know that,

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\therefore \quad \mu = \frac{\Delta D/8}{\frac{PL}{AE}/L}$$



$$\therefore \quad \mu = \frac{\Delta D}{8}\,\frac{AE}{P}$$

$$\therefore \quad 0.25 = \frac{\Delta D}{8}\,\frac{\frac{\pi}{4}(8)^2 \times 10^6}{50000}$$

$$\Rightarrow \quad \Delta D = 1.98 \times 10^{-3} \cong 0.002 \text{ cm}$$

02. Ans: (c)

Sol: We know that,

$$\text{Bulk modulus} = \frac{\delta P}{\delta V/V}$$

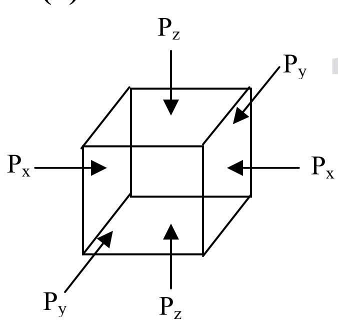
$$\Rightarrow \quad 2.5\times10^5 = \frac{200 \times 20}{\delta V}$$

$$\Rightarrow \quad \delta V = 0.016 \text{ m}^3$$

Linear and Volumetric Changes of Bodies

01. Ans: (d)

Sol:



Let $P_y = P_z = P$

$\varepsilon_y = 0$,

$\varepsilon_z = 0$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu.\frac{\sigma_x}{E}$$

$$\therefore \quad 0 = \frac{(-P)}{E} - \mu\,\frac{(-P)}{E} - \mu\frac{(P_x)}{E}$$

$$\Rightarrow \quad P = \frac{\mu.P_x}{(1-\mu)}$$

02. Ans: (a)

Sol: Given that, $\sigma_c = 4\tau$

Punching force = Shear resistance of plate

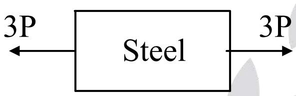
$$\therefore\ \sigma\left(\text{Cross section area}\right) = \tau(\text{surface Area})$$

$$\therefore\ 4\times\tau\times\frac{\pi.D^2}{4} = \tau(\pi.D.t)$$

$\Rightarrow$ D = t = 10 mm

03. Ans: (d)

Sol:



$$\sigma_s = 140\text{ MPa} = \frac{P_s}{A_s}$$

$$\Rightarrow P_S = \frac{140\times 500}{3} \approx 23{,}300\text{ N}$$

$$\sigma_{Al} = 90\text{ MPa} = \frac{P_{A\ell}}{A_{A\ell}}$$

$\Rightarrow$ $P_{Al} = 90 \times 400 = 36{,}000$ N

$$\sigma_B = 100\text{ MPa} = \frac{P_B}{A_B}$$

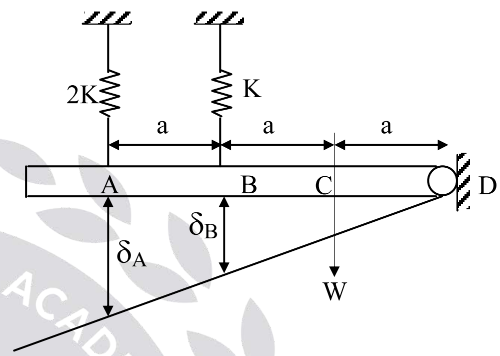
$$\Rightarrow P_B = \frac{100\times 200}{2} = 10{,}000\text{ N}$$

Take minimum value from P_S, A_{Al} and P_B.

$\Rightarrow$ P = 10,000 N

04. Ans: (c)

Sol:



From similar triangle

$$\frac{3a}{\delta_A} = \frac{2a}{\delta_B}$$

$3\delta_B = 2\delta_A$ ……. (1)

$$\text{Stiffness } K = \frac{W}{\delta}$$

$$\therefore K_A = \frac{W_A}{\delta_A} \Rightarrow \delta_A = \frac{W_A}{2K}$$

$$\text{Similarly}\qquad \delta_B = \frac{W_B}{K}$$

$$\text{From equation (1)}\quad 3\times\frac{W_B}{K} = 2\times\frac{W_A}{2K}$$

$$\Rightarrow \frac{W_A}{W_B} = 3$$

Thermal/Temperature Stresses

01. Ans: (b)

Sol: Free expansion = Expansion prevented

$$[\ell \alpha t]_s + [\ell \alpha t]_{Al} = \left[\frac{P\ell}{AE}\right]_s + \left[\frac{P\ell}{AE}\right]_{AL}$$

$$11\times10^{-6}\times20 + 24\times10^{-6}\times20$$

$$= \frac{P}{100\times10^3\times200} + \frac{P}{200\times10^3\times70}$$

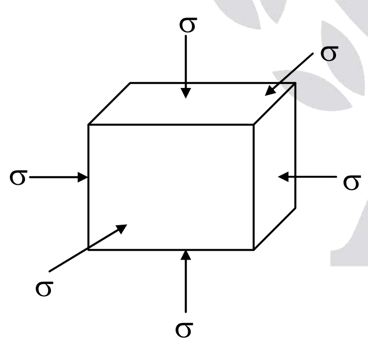
$\Rightarrow$ P = 5.76 kN

$$\sigma_s = \frac{P}{A_s} = \frac{5.76\times10^3}{100} = 57.65\ \text{MPa}$$

$$\sigma_{Al} = \frac{P}{A_{al}} = \frac{5.76\times10^3}{200} = 28.82\ \text{MPa}$$

02. Ans: (a)

Sol:



Strain in X-direction due to temperature,

$$\varepsilon_t = \alpha(\Delta T)$$

Strain in X-direction due to volumetric stress,

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\therefore \quad \varepsilon_x = \frac{-\sigma}{E}(1-2\mu)$$

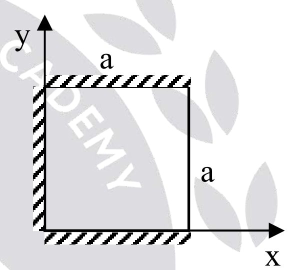
$$\therefore \quad -\sigma = \frac{(\varepsilon_x)(E)}{1-2\mu}$$

$$\therefore \quad -\sigma = \frac{\alpha(\Delta T)E}{(1-2\mu)}$$

$$\Rightarrow \quad \sigma = \frac{-\alpha(\Delta T)E}{1-2\mu}$$

03. Ans: (b)

Sol:



- Free expansion in x direction is aαt.
- Free expansion in y direction is aαt.
- Since there is restriction in y direction expansion doesn't take place. So in lateral direction, increase in expansion due to restriction is **μaαt**.

  Thus, total expansion in x direction is,

  $= a\,\alpha\,t + \mu\,a\,\alpha\,t$

  $= a\,\alpha\,t\,(1+\mu)$

04. Ans: (a, b, d)

Sol:

- Brass and copper bars are in parallel arrangement in composite bar.
- In parallel arrangement load is divided and elongation will be same for both the bars.

$P = P_b + P_c$

$P = A_b\,\sigma_b + A_c\,\sigma_c$

$\delta_b = \delta_c$

$$\Rightarrow \left.\frac{P\ell}{AE}\right|_b = \left.\frac{P\ell}{AE}\right|_{cu}$$

$\therefore\ \ell_b = \ell_c$

$$\therefore\ \frac{\sigma_b}{\sigma_c} = \frac{E_b}{E_c}$$

Hence, a, b, d are correct.

05. Ans: (b, d)

Sol: Elongation produced in prismatic bar due to self weight.

$$\delta\ell = \frac{\gamma\ell^2}{2E}$$

γ = weight density

Now, $\ell \rightarrow 2\ell$

$$\delta\ell' = \frac{\gamma\times(2\ell)^2}{2E} = 4\delta\ell$$

Elongation produced will be 4 times original elongation.

Stress = E × strain

$$\sigma = E\times\frac{\delta\ell}{\ell} = E\times\frac{\gamma\ell}{2E}$$

$$\sigma' = E\times\frac{\gamma 2\ell}{2E}$$

$\sigma' = 2\sigma$

Stress produced will be 2 times maximum stress.

Chapter 2 COMPLEX STRESSES AND STRAINS

01. Ans: (b)

Sol: Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

Maximum shear stress $= \dfrac{\sigma_1 - \sigma_2}{2} = 13$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{18 + (-8)}{2} = 5$$

02. Ans: (b)

Sol: Radius of Mohr's circle, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

$$\therefore\quad 20 = \frac{\sigma_1 - 10}{2}$$

$$\Rightarrow\quad \sigma_1 = 50\ \text{N/mm}^2$$

03. Ans: (b)

Sol: Given data,

$\sigma_x = 150$ MPa, $\sigma_y = -300$ MPa, $\mu = 0.3$

Long dam $\rightarrow$ plane strain member

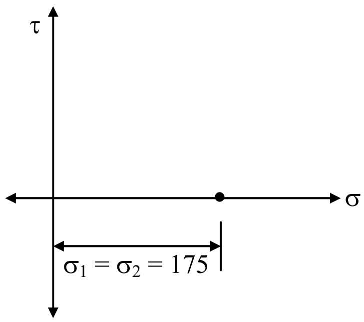
$$\varepsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\therefore\ 0 = \sigma_z - 0.3\times150 + 0.3\times300$$

$$\Rightarrow\ \sigma_z = 45\ \text{MPa}$$

04. Ans: (b)

Sol:



From the above, we can say that Mohr's circle is a point located at 175 MPa on normal stress axis.

Thus, $\sigma_1 = \sigma_2 = 175$ MPa

05. Ans: (c)

Sol: Given that, $\sigma_2 = 0$

$$\therefore \quad \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\therefore \quad \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\therefore \quad \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}$$

$$\therefore \quad \tau^2_{xy} = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

$$\therefore \quad \tau^2_{xy} = \sigma_x . \sigma_y$$

$$\Rightarrow \quad \tau_{xy} = \sqrt{\sigma_x . \sigma_y}$$

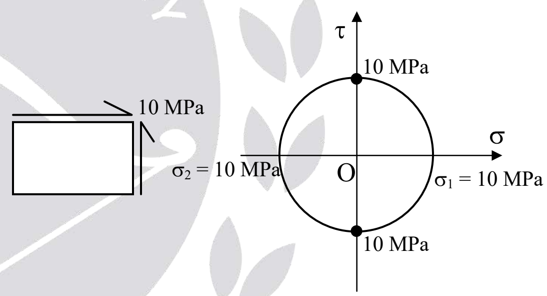
06. Ans: (a, b, d)

Sol:

- Planes on which resultant stress as a result of external loading is purely normal stress i.e., shear stress is zero.
- Such planes are called as principal planes and the corresponding normal stresses are called as principal stresses.
- Principal stress may be maximum or minimum.
- Planes of maximum shear stresses are there in which shear stress is maximum but normal stress is non-zero.

07. Ans: (a, b, c)

Sol:



Diameter of Mohr's circle would be 10 + 10 = 20 MPa

Maximum principal stress = 10 MPa

Minimum principal stress = –10 MPa

Centre of Mohr's circle is at origin.

Maximum shear stress = 10 MPa

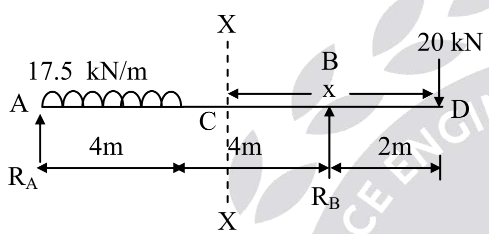
Hence, option (a, b, c) are correct.

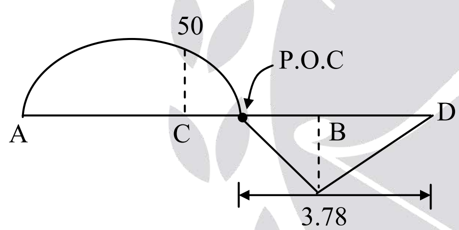
Chapter 3

SHEAR FORCE AND BENDING MOMENT

01. Ans: (b)

Sol: Contra flexure is the point where BM is becoming zero.





Taking moment about A,

$\Sigma M_A = 0$

$\therefore\ 17.5\times4\times\frac{4}{2}+20\times10-R_B\times8=0$

$\therefore\quad R_B = 42.5$ kN

Now, $M_x = -20x + R_B(x-2)$

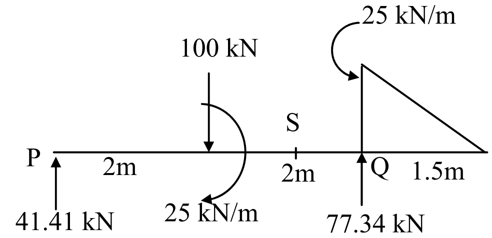
For bending moment be zero $M_x = 0$,

$-20x + 42.5(x-2) = 0$

$\Rightarrow x = 3.78$ m from right i.e. from D.

02. Ans: (b)

Sol:



Take $\Sigma M_P = 0$

$\frac{1}{2}\times25\times1.5\times\left(\frac{1.5}{3}+4\right)-(R_Q\times4)+100\times2+25=0$

$\therefore\quad R_Q = 77.34$ kN

Also, $\Sigma V = 0$

$\therefore\ R_P + R_Q = 100+\frac{1}{2}\times25\times1.5=118.75$ kN

$\therefore\ R_p = 41.41$ kN

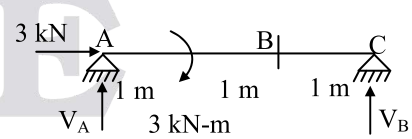
$\Rightarrow$ Shear force at P = 41.41 kN

03. Ans: (c)

Sol: $M_S = R_P(3) + 25 - (100\times1) = 49.2$ kN-m

04. Ans: (c)

Sol:



$-V_B\times3+3=0$

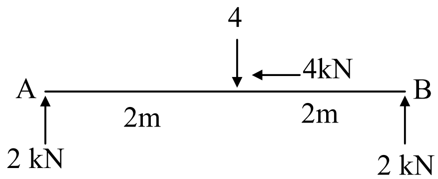
$\therefore\ V_C = 1$ kN

$\therefore$ Bending moment at B,

$\Rightarrow M_B = V_C\times1 = 1$ kN-m

05. Ans: (a)

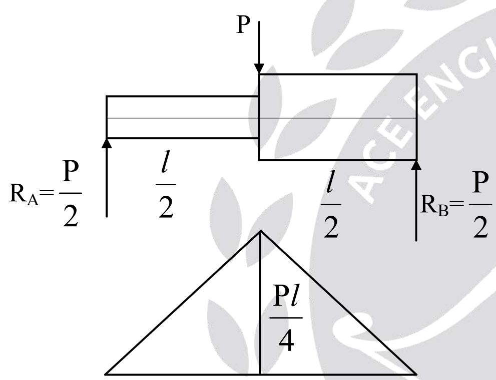
Sol:



Reaction at both the supports are 2 kN and in upward direction.

06. Ans: (c)

Sol:



BMD Diagram

Bending moment at $\frac{l}{2}$ from left is $\frac{Pl}{4}$.

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)

Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

08. Ans: (b, c, d)

Sol:

- Bending moment diagram (BMD) is constant in both the regions with different sign. So only BM is present in the loading diagram.
- BM at 'C' becomes zero from 20 kN-m indicates a concentrated moment and the end A is fixed.

09. Ans: (b, c)

Sol:

- For point load shear force will always be constant.
- There is no change in the shear force diagram due to presence of bending moment at any point.

Hence, option (a & d) are WRONG statements.

Chapter 4: CENTRE OF GRAVITY & MOMENT OF INERTIA

01. Ans: (a)

Sol: $\bar{y} = \dfrac{E_1 y_1 + E_2 y_2}{E_1 + E_2}$

$$\Rightarrow \bar{y} = \frac{2E_2\left(h + \frac{h}{2}\right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \quad (\because E_1 = 2E_2)$$

$$\Rightarrow \bar{y} = 1.167h \text{ from base}$$

02. Ans: (b)

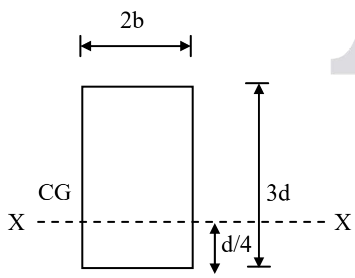
Sol: $\bar{y} = \dfrac{A_1E_1Y_1 + A_2E_2Y_2}{A_1E_1 + A_2E_2}$

$$= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2E_1 + 6a^2(2E_1)}$$

$$= \frac{22.5a^3E_1}{15a^2E_1} = 1.5a$$

03. Ans: 13.875 bd³

Sol:



$$y = \frac{5}{4}d$$

$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3$$

$$\text{M.I about } X - X\big|_{\text{at } d/4 \text{ distance}} = I_G + Ay^2$$

$$= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2$$

$$= \frac{111}{8}bd^3 = 13.875bd^3$$

04. Ans: 6.885×10⁶ mm⁴

Sol:

$$I_x = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12} + Ah^2\right)$$

$$= \frac{60 \times 120^3}{12} - 2\left(\frac{30 \times 30^3}{12} + (30 \times 30) \times 30^2\right)$$

$$= 6.885 \times 10^6 \text{ mm}^4$$

05. Ans: 152146 mm⁴

Sol:

$$I_x = \frac{30 \times 40^3}{12} - \frac{\pi \times 20^4}{64} = 152146 \text{ mm}^4$$

$$I_y = \frac{40 \times 30^3}{12} - \left(\frac{\pi \times 20^4}{64} + 2\left(\frac{\pi}{2} \times 10^2 \times \left(15 - \frac{4 \times 10}{3\pi}\right)^2\right)\right)$$

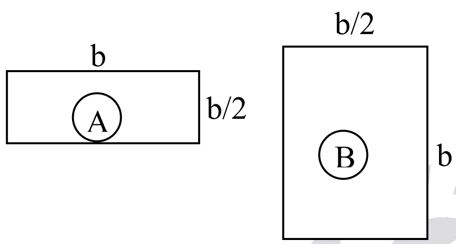
$$= 45801.34 \text{ mm}^4$$

Chapter 5

THEORY OF SIMPLE BENDING

01. Ans: (b)

Sol:



By using flexural formula, $\sigma = \frac{M}{Z}$

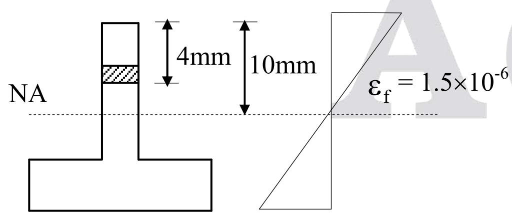
$\therefore\ \sigma \propto \frac{1}{Z}$ $\quad (\because M \text{ is constant})$

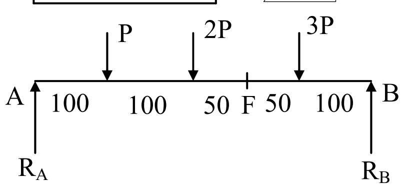
$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\frac{\left(\frac{b}{2}\times b^2\right)}{6}}{\frac{b\times\left(\frac{b}{2}\right)^2}{6}} = 2$$

$\Rightarrow \sigma_A = 2\sigma_B$

02. Ans: (b)

Sol:





$\therefore \sum M_A = 0$

$\therefore\ P\times100 + 2P\times200 + 3P\times300 = R_B\times400$

$\therefore\ R_B = 3.5\ P, \qquad R_A = 2.5\ P$

Take moments about F and moment at F

$M_F = R_B\times150 - 3P\times50 = 375P$

Also, $\frac{M_F}{I} = \frac{\sigma_b}{y_F}$

$$\therefore\ \frac{375P}{2176} = \frac{\left(1.5\times10^{-6}\times200\times10^3\right)}{6}$$

$\Rightarrow \quad P = 0.29\ N$

03. Ans: (b)

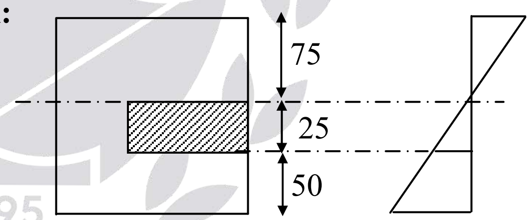
Sol: By using Flexural formula,

$$\frac{E}{R} = \frac{\sigma_b}{y_{max}} \Rightarrow \frac{2\times10^5}{250} = \frac{\sigma_b}{(0.5/2)}$$

$\Rightarrow \sigma_b = 200\ N/mm^2$

04. Ans: (c)

Sol:



By using flexural formula,

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore\ \frac{16\times10^6}{\frac{100\times150^3}{12}} = \frac{f}{25} \Rightarrow f = 14.22\ MPa$$

Now, Force on hatched area

= Average stress × Hatched area

$= \left(\frac{0+14.22}{2}\right)(25\times50) = 8.9\ kN$

05. Ans: (b)

Sol: By using flexural formula, $\dfrac{f_{Tensile}}{y_{top}} = \dfrac{M}{I}$

$$\Rightarrow f_{Tensile} = \frac{0.3\times 3\times 10^6}{3\times 10^6}\times 70$$

(maximum bending stress will be at top fibre so $y_1 = 70$ mm)

$\Rightarrow f_{Tensile} = 21$ N/mm^2 = 21 MN/m^2

06. Ans: (c)

Sol: Given data:

P = 200 N, M = 200 N.m

A = 0.1 m^2, $I = 1.33\times10^{-3}$ m^4

y = 20 mm

Due to direct tensile force P,

$$\sigma_d = \frac{P}{A} = \frac{200}{0.1}$$

$= 2000$ N/m^2 (Tensile)

Due to the moment M,

$$\sigma_b = \frac{M}{I}\times y$$

$$= \frac{200}{1.33\times10^{-3}}\times 20\times10^{-3}$$

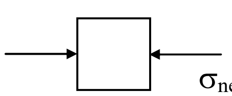
$= 3007.52$ N/m^2 (Compressive)

$\sigma_{net} = \sigma_d - \sigma_b$

$= 2000 - 3007.52$

$= -1007.52$ N/m^2

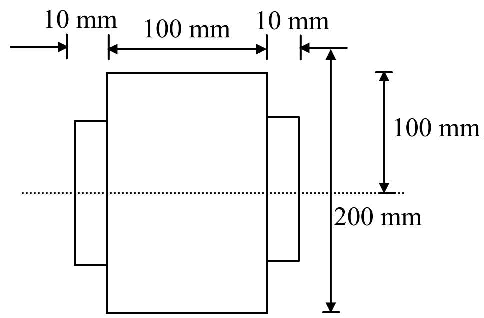
Negative sign indicates compressive stress.



$\sigma_{net} = 1007.52$ N/m^2

07. Ans: 80 MPa

Sol:



Maximum stress in timber = 8 MPa

Modular ratio, m = 20

Stress in timber in steel level,

100 → 8

50 → f_w

$\Rightarrow$ $f_w = 4$ MPa

Maximum stress developed in steel is = $m\cdot f_w$

= 20×4 = 80 MPa

Convert whole structure as a steel structure by using modular ratio.

08. Ans: 2.43 mm

Sol: From figure, $A_1B_1 = l = 3$ m (given)

$$AB = \left(R - \frac{h}{2}\right)\alpha = l - l\alpha t_1 \text{ ------ (1)}$$

$$A_2B_2 = \left(R + \frac{h}{2}\right)\alpha = l + l\alpha t_2 \text{ ------ (2)}$$

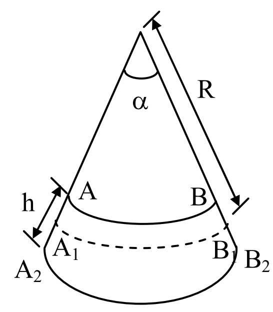
Subtracting above two equations (2) – (1)

$h(\alpha) = l\alpha\,(t_2 - t_1)$

but $A_1B_1 = l = R\alpha$

$$\Rightarrow \alpha = \frac{l}{R}$$

$$\therefore h\left(\frac{l}{R}\right) = l\alpha\,(\Delta T)$$



$$R = \frac{h}{\alpha(\Delta T)}$$

$$= \frac{250}{(1.5\times10^{-5})(72-36)}$$

$R = 462.9$ m

From geometry of circles

$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2}$ {ref. figure in Q.No.02}

$2R\cdot\delta-\delta^2 = \frac{L^2}{4}$ (neglect δ^2)

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8\times462.9} = 2.43 \text{ mm}$$

Shortcut:

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^o - 36^o = 36^o$). Bottom temperature being more, the beam deflects down.

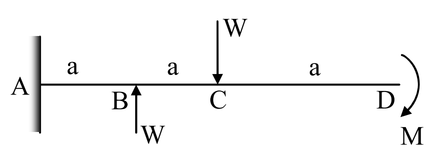
$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h}$$

$$= \frac{1.5\times10^{-5}\times36\times3000^2}{8\times250}$$

$= 2.43$ mm (downward)

09. Ans: (c)

Sol:

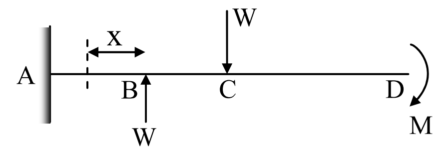


$BM_C = M$

$BM_B = M + Wa$

$BM_A = M + W(2a) - Wa = M + Wa$

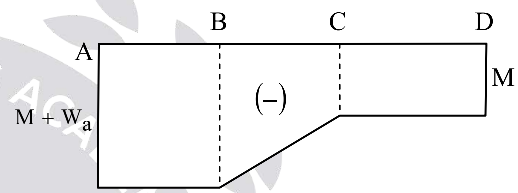
Taking a section between A & B



$M_{xx} = M + W(a+x) - Wx$

$= M + Wa$

So, pure bending theory is valid in constant B.M region.



Note: option (c) is correct.

Chapter 6

SHEAR STRESS DISTRIBUTION IN BEAMS

01. Ans: (a)

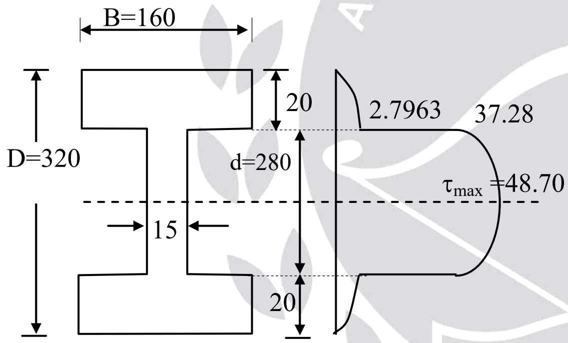
Sol: $\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{f}{b.d}$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$\therefore$ d = 250 mm = 25 cm

02. Ans: 37.3

Sol:



All dimensions are in mm

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12}$$

$= 171.65 \times 10^6$ mm^4

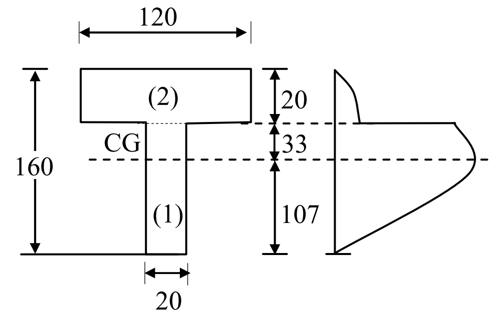
$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (160 \times 20 \times 150)$$

= 37.28 MPa

03. Ans: 61.43 MPa

Sol:



All dimensions are in mm

$I_{NA} = 13 \times 10^6$ mm^4

y_{CG} = 107 mm from base

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

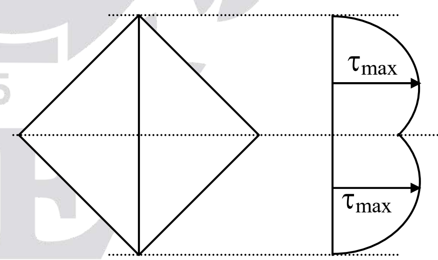
$A\bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5)$

$= 114090$ mm^3

$$\tau_{max} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20} = 61.43 \text{ MPa}$$

04. Ans: (b, c)

Sol:



From the above diagram, the shear force distribution across the section of beam will be zero at top and bottom.

Maximum shear stress does not occur at the neutral axis.

Hence, options (b, c) are correct.

Chapter 7 TORSION

01. Ans: (c)

Sol: Twisting moment = 2 × 0.5 + 1 × 0.5

= 1.5 kN-m

02. Ans: (d)

Sol: $$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{hollow}} = \frac{1}{1-K^4}$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)^4} = \frac{16}{15}$$

03. Ans: 43.27 MPa & 37.5 MPa

Sol: Given D_o = 30 mm , t = 2 mm

∴ D_i = 30 – 4 = 26 mm

We know that $\frac{\tau}{J} = \frac{q}{R}$

$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{max}}{\left(\frac{30}{2}\right)}$$

q_{max} = 43.279 N/mm^2

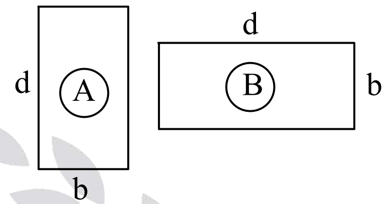
$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{min}}{\left(\frac{26}{2}\right)}$$

q_{min} = 37.5 N/mm^2

Chapter 8 SLOPES AND DEFLECTIONS

01. Ans: (c)

Sol:



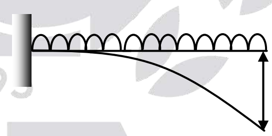
$$y_{max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

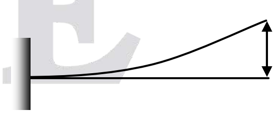
$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

02. Ans: (b)

Sol: Total load W= w*l*



$$y_{max} = \frac{W\ell^3}{8EI} \text{ (Downward)}$$



$$y_{max} = \frac{W\ell^3}{3EI} \text{ (Upward)}$$

$$y_{net} = \downarrow y_{udl} - \uparrow y_w$$

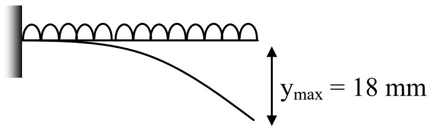
$$\text{Total Net deflection} = \frac{WL^3}{8EI} - \frac{WL^3}{3EI}$$

$$= \frac{-5WL^3}{24EI}$$

(Negative sign indicates upward deflection)

03. Ans: (c)

Sol:



$$\theta_{max} = \frac{wl^3}{6EI} = 0.02 \quad \text{-------(i)}$$

$$y_{max} = \frac{wL^4}{8EI}$$

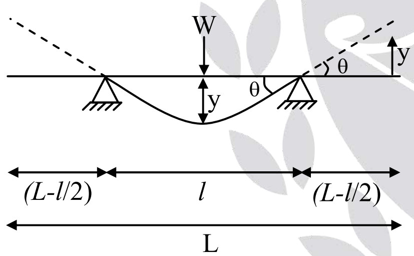
$$\therefore\ 0.018 = \left(\frac{WL^3}{6EI}\right) \times \frac{L \times 6}{8}$$

$$\therefore\ 0.018 = \frac{0.02 \times L \times 6}{8} \qquad [\because \text{ Equation (i)}]$$

$$\Rightarrow \quad L = 1.2 \text{ m}$$

04. Ans: (a)

Sol:



Conditions given

$$\downarrow y = \frac{wl^3}{48EI}$$

$$\theta = \frac{wl^2}{16EI}$$

$$\tan\theta = \frac{y}{(L-\ell)/2}$$

θ is small ⇒ tan θ = θ

$$\therefore\ \theta = \frac{y}{(L-\ell)/2}$$

$$\therefore y = \theta\left(\frac{L-\ell}{2}\right)$$

$$\uparrow y = \theta\left(\frac{L-\ell}{2}\right)$$

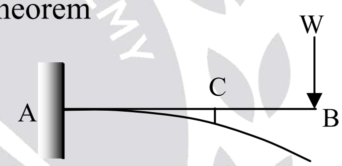
Thus $y\downarrow = y\uparrow$

$$\therefore\ \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-\ell}{2}\right)$$

$$\Rightarrow \quad \frac{L}{\ell} = \frac{5}{3}$$

05. Ans: (c)

Sol: By using Maxwell's law of reciprocals theorem



$$\delta_{C/B} = \delta_{B/C}$$

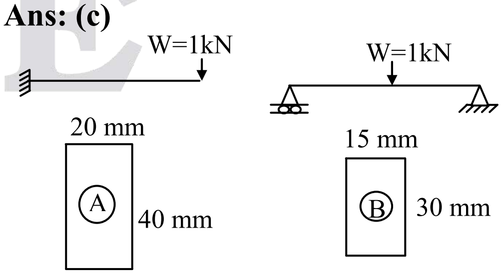
Deflection at 'C' due to unit load at 'B'

= Deflection at 'B' due to unit load at 'C'

As the load becomes half deflection becomes half.

06. Ans: (c)

Sol:

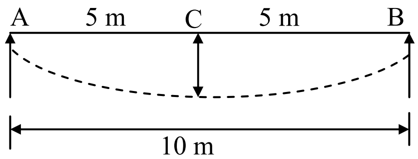


$$y_A = y_B \Rightarrow \left(\frac{wL^3}{3EI}\right)_A = \left(\frac{wL^3}{48EI}\right)_B$$

$$\therefore\ L_B = 400 \text{ mm}$$

07. Ans: 0.05

Sol:



$\therefore$ Curvature, $\dfrac{d^2y}{dx^2} = 0.004$

Integrating with respect to x,

We get, $\dfrac{dy}{dx} = 0.004x$

$$y = \frac{0.004x^2}{2}$$

$$y = 0.002x^2$$

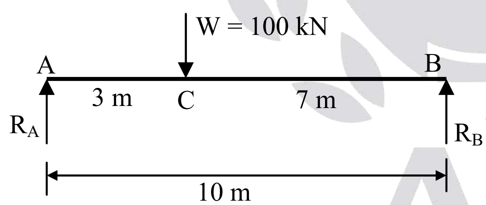
At mid span, x = 5 m

$\therefore$ $y = 0.002\,x^2$

$y = 0.05$ m

08. Ans: (a, b, d)

Sol:



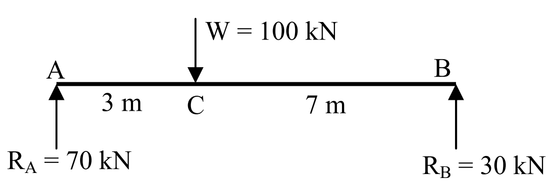
$R_A + R_B = 100$

$M_A = 0$

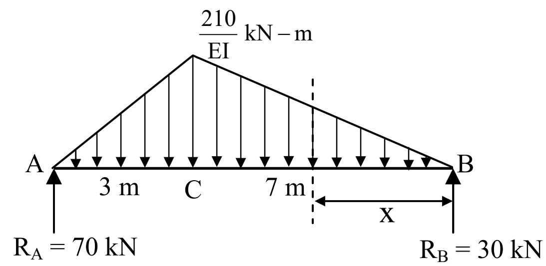
$R_B \times 10 = 100 \times 3$

$R_B = 30$ kN

$R_A = 70$ kN



Using conjugate beam method,



Taking moment about point A,

$$R_B \times 10 = \frac{1}{2}\times 7\times\frac{210}{EI}\left[\left(7-7\times\frac{2}{3}\right)+3\right]+\frac{1}{2}\times 3\times\frac{210}{EI}\left(3\times\frac{2}{3}\right)$$

$$= \frac{105}{EI}\left[\left(7\times\frac{16}{3}\right)+6\right]$$

$$R_B = \frac{455}{EI}\text{ kN}$$

For maximum deflection shear force = 0

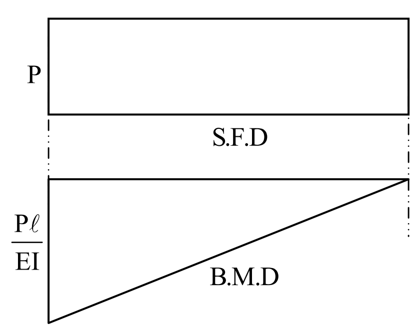
$$(SF)_x = \frac{1}{2}\times x\times\frac{30x}{EI} - \frac{455}{EI} = 0$$

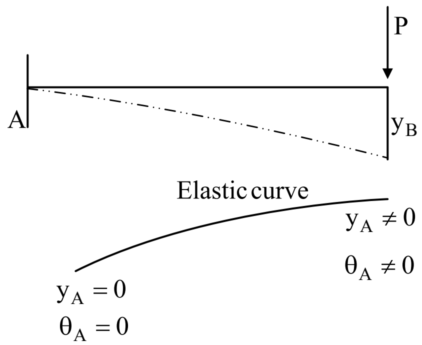
$$15x^2 = 455$$

$\Rightarrow$ x = 5.50 m, which lies between B and C.

09. Ans: (b, c)

Sol: Cantilever beam subjected to a concentrated load at the free end.





From the above diagram bending moment or stress is maximum at fixed end.

From SFD, shear stress is constant along the length of the beam.

Slope of elastic curve is zero at fixed end and maximum at free end.

Hence, option (b, c) are correct.

Chapter 9 THIN PRESSURE VESSELS

01. Ans: (b)

Sol: $\tau_{max} = \sigma_l = \dfrac{\sigma_h - 0}{2} = \dfrac{PD}{4t}$

$$\therefore \tau_{max} = \frac{1.6\times 900}{4\times 12} = 30 \text{ MPa}$$

02. Ans: 2.5 MPa & 2.5 MPa

Sol: Given data:

R = 0.5 m, D = 1m, t = 1mm,

H = 1 m, $\gamma = 10$ kN/m^3, h = 0.5 m

At mid-depth of cylindrical wall (h = 0.5m):

Circumferential (hoop) stress,

$$\sigma_c = \frac{P_{\text{at } h=0.5m}\times D}{4t} = \frac{\gamma h\times D}{4t}$$

$$= \frac{10\times 10^3\times(2\times 0.5)}{4\times 1\times 10^{-3}}$$

$$= 2.5\times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$$

Longitudinal stress at mid-height,

$$\sigma_\ell = \frac{\text{Net weight of the water}}{\text{Cross-section area}}$$

$$= \frac{\gamma\times\text{Volume}}{\pi D\times t}$$

$$= \frac{\gamma\times\frac{\pi}{4}D^2L}{\pi D\times t} = \frac{\gamma\times DL}{4t}$$

$$= \frac{10\times 10^3\times 1\times 1}{4\times 10^{-3}}$$

$$= 2.5\times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$$

03. Ans: (a, b, d)

Sol: The standard formulas for stresses and strains in a thin-walled cylindrical pressure vessel subjected to internal pressure p, with diameter d, thickness t, Young's modulus E, and Poisson's ratio μ are:

Hoop stress (circumferential stress):

$$\sigma_h = \frac{pd}{2t}$$

Longitudinal stress (axial stress):

$$\sigma_L = \frac{pd}{4t}$$

Circumferential strain (ε_h) :

$$\varepsilon_h = \frac{\sigma_h - \mu\sigma_L}{E} = \frac{\frac{pd}{2t} - \mu\frac{pd}{4t}}{E}$$

$$\varepsilon_h = \frac{Pd}{4t\,E}(2-\mu)$$

Longitudinal strain (ε_L) :

$$\varepsilon_h = \frac{1}{E}(\sigma_L - \mu\sigma_h)$$

Hence, options (a, b, d) are correct.

Chapter 10

COLUMNS

01. Ans: (c)

Sol: By using Euler's formula, $P_e = \frac{\pi^2 \times EI}{l_e^2}$

For a given system, $l_e = \frac{l}{2}$

$$\therefore \quad P_e = \frac{4\pi^2 \times EI}{l^2}$$

02. Ans: (b)

Sol: We know that, $P_{cr} = \frac{\pi^2 EI}{\ell_e^2}$

$$\therefore\ P_{cr} \propto \frac{1}{\ell_e^2}$$

$$\therefore\ \frac{P_1}{P_2} = \frac{l_{2e}^2}{l_{1e}^2}$$

$$\therefore\ \frac{P_1}{P_2} = \frac{l^2}{(2l)^2} \Rightarrow P_1 : P_2 = 1 : 4$$

03. Ans: 4

Sol: Euler's crippling load,

$$P = \frac{\pi^2}{l^2}\ EI$$

$$\therefore\ P \propto I$$

$$\Rightarrow \frac{P}{P_o} = \frac{I_{bonded}}{I_{loose}} = \frac{\left[\frac{b(2t)^3}{12}\right]}{2\left[\frac{bt^3}{12}\right]} = 4$$

04. Ans: (c)

Sol: Euler's theory is applicable for axially loaded columns.

Force in member AB, $P_{AB} = \frac{F}{\cos 45^o} = \sqrt{2}F$

$$P_{AB} = \frac{\pi^2 EI}{L_e^{\ 2}}$$

$$\therefore \ \sqrt{2}\ F = \frac{\pi^2 EI}{L_e^{\ 2}}$$

$$\Rightarrow F = \frac{\pi^2 EI}{\sqrt{2}L^2}$$

05. Ans: (a)

Sol: Given data:

$L_e = L = 3$ m ,

$\alpha = 12 \times 10^{-6}$ /°C,

$d = 50$ mm $= 0.05$ m

Buckling load, $P_e = \frac{\pi^2 EI}{L_C^2}$

$$\therefore \quad \frac{P_e L}{AE} = L\alpha\Delta T$$

$$\therefore \quad \frac{\pi^2 EI \times L}{L^2 \times AE} = L\alpha\Delta T$$

$$\therefore \quad \frac{\pi^2 \times E \times \frac{\pi}{64} \times d^4 \times L}{L^2 \times \frac{\pi}{4} d^2 \times E} = L\alpha\Delta T$$

$$\therefore \quad \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}}$$

$$\Rightarrow \quad \Delta T = 14.3°C$$

Chapter 11

STRAIN ENERGY

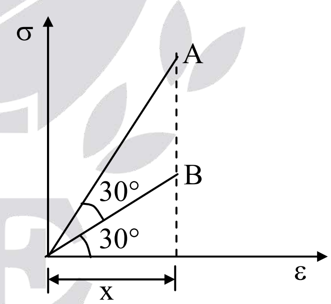
01. Ans: (d)

Sol:

- Slope of the stress-strain curve in the elastic region is called modulus of elasticity.
  For the given curves,
  (Modulus of elasticity)$_A$ > (Modulus of elasticity)$_B$
  $\therefore$ $\mathbf{E_A > E_B}$
- The material for which plastic region is more is stress-strain curve is possessed high ductility. Thus, $\mathbf{D_B > D_A}$.

02. Ans: (b)

Sol:

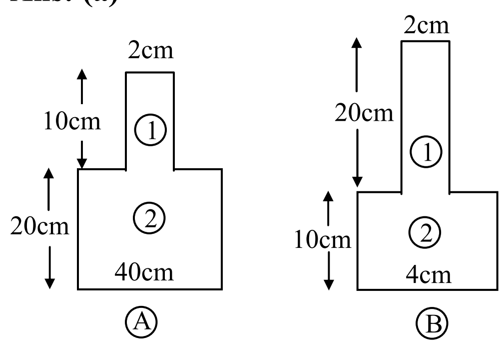


$$\frac{(SE)_A}{(SE)_B} = \frac{\text{Area under curve A}}{\text{Area under curve B}}$$

$$= \frac{\frac{1}{2} \times x \times x \tan 60^o}{\frac{1}{2} \times x \times x \tan 30^o} = \frac{3}{1}$$

03. Ans: (a)

Sol:



$$\frac{U_B}{U_A} = \frac{(V_1+V_2)_B}{(V_1+V_2)_A}$$

$$\therefore \frac{U_B}{U_A} = \frac{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_B}{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_A}$$

$$= \frac{\left[\frac{P^2}{A_1^2}\times A_1\times L_1 + \frac{P^2\times A_2\times L_2}{A_2^2}\right]}{\left[\frac{P^2\times A_1\times L_1}{A_1^2} + \frac{P^2\times A_2\times L_2}{A_2^2}\right]_A}$$

$$\Rightarrow \frac{U_B}{U_A} = \frac{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_B}{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_A} = \frac{7.165}{4.77} = \frac{3}{2}$$

04. Ans: (c)

Sol: A_1 = Modulus of resilience

$A_1 + A_2$ = Modulus of toughness

$$A_1 = \frac{1}{2}\times 0.004\times 70\times 10^6 = 14\times 10^4$$

$$A_2 = \frac{1}{2}\times\left(0.008\times 50\times 10^6\right) + \left(0.008\times 70\times 10^6\right)$$

$$= 76\times 10^4$$

$$A_1 + A_2 = (14+76)\times 10^4 = 90\times 10^4$$

05. Ans: (d)

Sol: Strain energy, $U = \frac{P^2}{2A^2E}.V$

$\therefore\ U \propto P^2$

Due to the application of P_1 and P_2 one after the other

$$(U_1 + U_2) \propto {P_1}^2 + {P_2}^2 \text{ …….. (1)}$$

Due to the application of P_1 and P_2 together at the same time.

$$U \propto (P_1+P_2)^2 \quad \text{……..........(2)}$$

It is obvious that,

$$({P_1}^2 + {P_2}^2) < (P_1+P_2)^2$$

$$\Rightarrow (U_1 + U_2) < U$$

06. Ans: 1.5

Sol: Given data:

$L = 100$ mm,

$G = 80\times 10^3\ \text{N/mm}^2$

$J_1 = \frac{\pi}{32}(50)^4;\quad J_2 = \frac{\pi}{32}(26)^4$

$$U = U_1 + U_2 = \frac{T^2L}{2GJ_1} + \frac{T^2L}{2GJ_2}$$

$\Rightarrow U = 1.5$ N-mm

07. Ans: (a, b)

Sol: Strain energy stored in AB $= \frac{1}{2}\times P\times\delta$

$$= \frac{1}{2}\times P\times\frac{P\ell}{AE}$$

$$= \frac{P^2L}{2AE}$$

Axial deformation of AB $= \frac{PL}{AE}$

Strain energy stored in BC,

$$U = \int_0^{\ell} \frac{M^2 dx}{2EI} \quad (M = Px)$$

$$= \int_0^{\ell} \frac{(Px)^2 dx}{2EI}$$

$$= \frac{P^2 \ell^3}{6EI}$$

The displacement at point B is not equal to $\frac{P\ell^3}{3EI}$, since there is a hinge point C not fixed.

Chapter 12 SHEAR CENTRE

01. Ans: (a)

Sol:

- Shear centre is related to torsion
- On principal plane shear stress is zero
- At fixed end slope is zero.
- Middle third rule is to avoid tension in columns.

02. Ans: (b)

Sol: In case of a thin channel section, if the resultant shear stress does not pass through the shear center, then the bending will occur with torsion.

If the resultant shear stress pass through the shear center, then the bending will occur without torsion.

03. Ans: (a, b, c, d)

Sol: All diagrams are correct representation of shear centre and centre of gravity of various sections.