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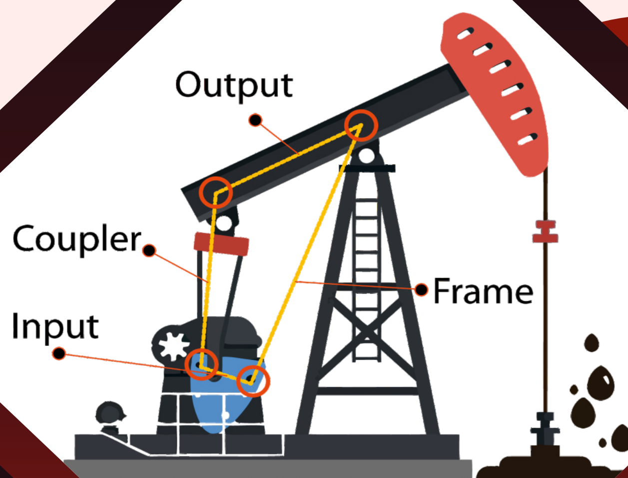


MECHANICAL ENGINEERING

THEORY OF MACHINES & VIBRATIONS

Text Book: Theory with worked out Examples and Practice Questions

Theory of Machines & Vibrations

(Solutions for Text Book Practice Questions)

Chapter 1	**ANALYSIS OF PLANAR MECHANISMS**

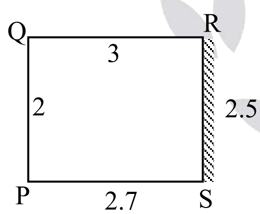
01. Ans: (a, c)

Sol:

- The pair shown has two degree of freedom one is translational (motion along axis of bar and the rotation (rotation about axis). Both motions are independent. Therefore the pair has incomplete constraint.
- Kinematic pair is a joint of two links having relative motion between them. The pair shown form a kinematic pair.

02. Ans: (c)

Sol:



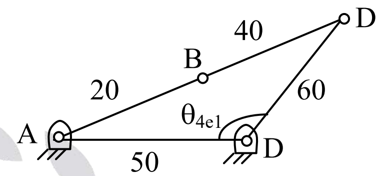
The given dimensions of the linkage satisfies Grashof's condition to get double rocker. We need to fix the link opposite to the shortest link. So by fixing link 'RS' we get double rocker.

03. Ans: (d)

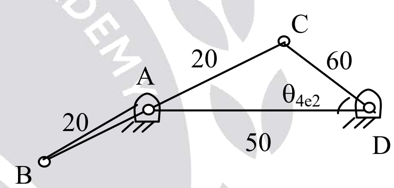
Sol: At toggle position velocity ratio is 'zero' so mechanical advantage is '∞'.

04. Ans: (d)

Sol: The two extreme positions of crank rocker mechanisms are shown below figure.



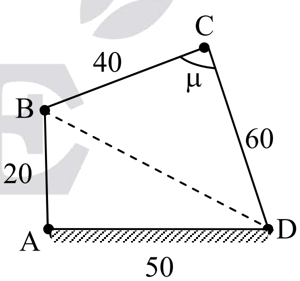
$$\theta_{4e_1} = \cos^{-1}\left(\frac{50^2+60^2-60^2}{2\times 50\times 60}\right) = 65.37^0$$



$$\theta_{4e_2} = \cos^{-1}\left(\frac{60^2+50^2-20^2}{2\times 60\times 50}\right) = 18.19^o$$

05. Ans: (a)

Sol:



Where, μ = Transmission angle

$$BD = \sqrt{20^2+50^2} = 53.85\,\text{cm}$$

By cosine rule

$$\cos\mu = \frac{BC^2+CD^2-BD^2}{2BC\times CD}$$

$$= \frac{40^2 + 60^2 - 53.85^2}{2\times 40\times 60} = 0.479$$

$\mu = 61.37°$

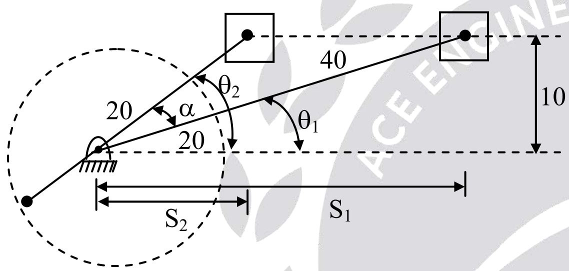
06. Ans: (c)

Sol: Two extreme positions are as shown in figure below.

Let r = radius of crank = 20 cm

l = length of connecting rod = 40 cm

h = 10 cm



Stroke = $S_1 - S_2$

$$S_1 = \sqrt{(\ell + r)^2 - h^2} = \sqrt{60^2 - 10^2} = 59.16\text{cm}$$

$$S_2 = \sqrt{(\ell - r)^2 - h^2} = \sqrt{20^2 - 10^2} = 17.32\text{cm}$$

Stroke = $S_1 - S_2$ = 59.16 – 17.32 = 41.84 cm

07. Ans: (b)

Sol: $\theta_1 = \sin^{-1}\left(\frac{h}{\ell + r}\right) = \sin^{-1}\left(\frac{10}{60}\right) = 9.55°$

$\theta_2 = \sin^{-1}\left(\frac{h}{\ell - r}\right) = \sin^{-1}\left(\frac{10}{20}\right) = 30°$

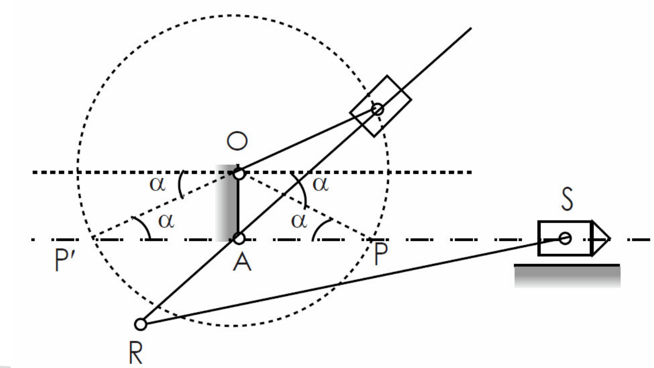
$\alpha = \theta_2 - \theta_1 = 20.41°$

Quick return ratio

$$(\text{QRR}) = \frac{180 + \alpha}{180 - \alpha} = 1.2558$$

08. Ans: 2

Sol:



Whitworth Quick return mechanism

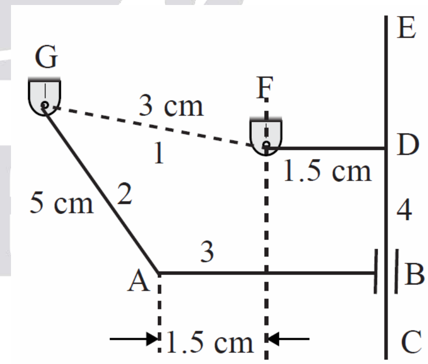
$$\sin\alpha = \frac{\text{fixed link length}}{\text{crank radius}} = \frac{OA}{OP} = \frac{150}{300} = \frac{1}{2}$$

$\alpha = 30°$

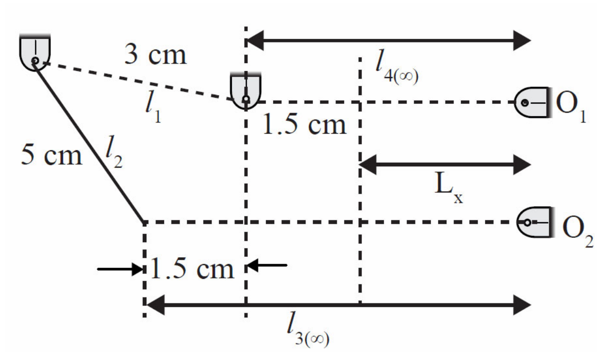
$$\text{QRR} = \frac{180 + 2\alpha}{180 - 2\alpha} = \frac{180 + 2\times 30°}{180 - 2\times 30°} = 2$$

09. Ans: (a)

Sol: The given mechanism is



As we know sliding pair is a special case of turning pair with infinite lengths link. So the equivalent diagram has been drawn below. Since two parallel lines meets at infinite point O_1 and O_2 are same.



$l_1 = 3$ cm

Shortest link, $l_2 = 5$ cm

$l_3 = l_{3(\infty)} = L_\infty + 3 \rightarrow$ longest link

$l_4 = l_{4(\infty)} = L_\infty + 1.5$

For Grashof's rule to satisfy

$l_1 + l_3 \leq l_2 + l_4$

$\Rightarrow 3 + L_\infty + 3 \leq 5 + L_\infty + 1.5$

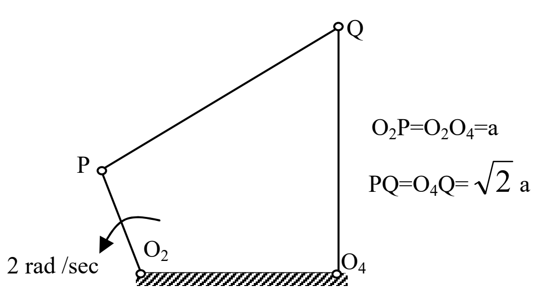
$\Rightarrow 6 \leq 6.5$

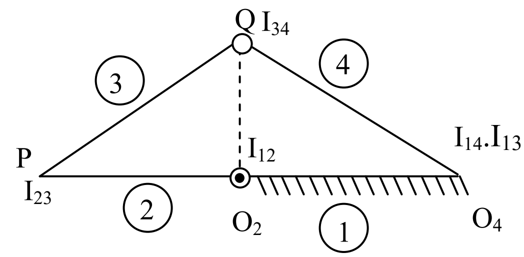
LHS is less than RHS.

Hence, Grashof's rule is satisfied in this mechanism. Since shortest link is fixed. It will be a double crank mechanism.

10. Ans: (c)

Sol: $\angle O_4O_2P = 180^\circ$ sketch the position diagram for the given input angle and identify the Instantaneous Centers.





I_{13} is obtained by joining I_{12} I_{23} and I_{14} I_3

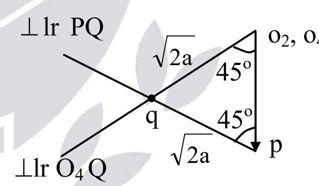
$$\frac{\omega_3}{\omega_2} = \frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \frac{a}{2a}$$

$$\frac{\omega_3}{2} = \frac{1}{2}$$

$\omega_3 = 1$ rad /sec

Alternate Method:

The position diagram is isosceles right angle triangle and the velocity triangle is similar to the position diagram.



Velocity (Diagram)

$V_{qp} = \omega_3 l_3 \Rightarrow \sqrt{2}a = \omega_3 \times \sqrt{2}a$

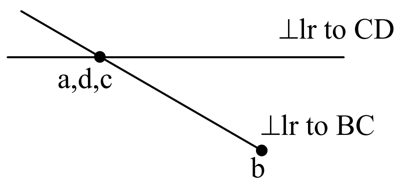
$\omega_3 = 1$

$V_q = l_4\,\omega_4 \Rightarrow \sqrt{2}a = \sqrt{2}a\,\omega_4$

$\Rightarrow \omega_4 = 1$rad/sec

11. Ans: (a)

Sol:



Velocity diagram

$V_C = 0 = dc \times \omega_{CD}$

$\therefore \omega_{CD} = 0$

Note: If input and coupler links are collinear, then output angular velocity will be zero.

12. Ans: (c)

Sol: In a four bar mechanism when input link and output links are parallel then coupler velocity(ω_3) is zero.

$\Rightarrow l_2\, \omega_2 = l_4\, \omega_4$

$l_4 = 2l_2$ (Given)

$\Rightarrow \omega_4 = \omega_2 / 2 = 2/2 = 1$ rad/s

ω_2, ω_4 = angular velocity of input and output link respectively.

Fixed links have zero velocity.

At joint 1, relative velocity between fixed link and input link = 2–0 = 2

Rubbing velocity at joint 1 = Relative velocity × radius of pin = 2×10 = 20 cm/s

At joint 2, rubbing velocity = $(\omega_2+\omega_3) \times r$

$= (2+0)\times 10 = 20$ cm/s

+ve sign means ω_2 and ω_3 are moving in opposite directions.

At joint 3, rubbing velocity = $(\omega_4+\omega_3)\times r$

$= (1+0) \times 10 = 10$ cm/s

At joint 4, rubbing velocity

$= (\omega_4 - 0) \times r$

$= (1 - 0) \times 10 = 10$ cm/s

13. Ans: (d)

Sol: As for the given dimensions the mechanism is in a right angle triangle configuration and the crank AB is perpendicular to the lever CD. The velocity of B is along CD only which is purely sliding component

$\therefore$ Velocity of the slider

$= AB \times \omega_{AB} = 10 \times 250 = 2.5\,m/sec$

14. Ans: (a)

Sol: $QRR = \dfrac{180+2\alpha}{180-2\alpha} = \dfrac{2}{1} \Rightarrow \alpha = 30^o$

$\sin\alpha = \dfrac{OS}{OP} \Rightarrow OS = \dfrac{OP}{2} = 250mm$

15. Ans: (b)

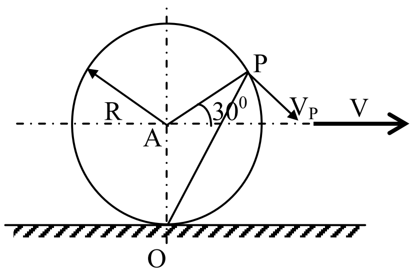
Sol: Maximum speed during forward stroke occurs when PQ is perpendicular to the line of stroke of the tool i. e. PQ, OS & OQ are in straight line

$\Rightarrow V = 250 \times 2 = 750 \times \omega_{PQ}$

$\Rightarrow \omega_{PQ} = \dfrac{2}{3}$

16. Ans: (a)

Sol:



Here 'O' is the instantaneous centre

$V_P = \omega \times OP$

$V_A = R\omega$

In Δ OAP ,

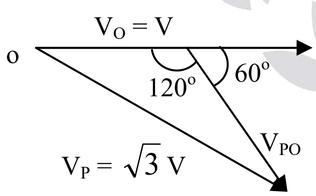
$$\cos 120° = \frac{R^2 + R^2 - OP^2}{2R \times R}$$

$$-0.5 = \frac{2R^2 - OP^2}{2R^2}$$

$OP = \sqrt{3}R$

$V_P = \sqrt{3}R \times \omega = \sqrt{3}V$

OR

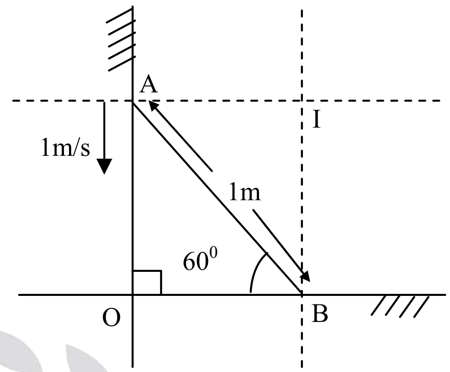


$$V_P = \vec{V}_O + \vec{V}_{PO} = \vec{V} + \overrightarrow{OP} \times \omega$$

$$= \sqrt{V^2 + V^2 + 2V^2 \cos 60} = \sqrt{3}\,V$$

17. Ans: (a)

Sol:



$V_a = 1$ m/s

V_a = Velocity along vertical direction

V_b = Velocity along horizontal direction

So instantaneous center of link AB will be perpendicular to A and B respectively i.e at I

$$IA = OB = \cos\theta = 1 \times \cos 60^0 = \frac{1}{2}\,m$$

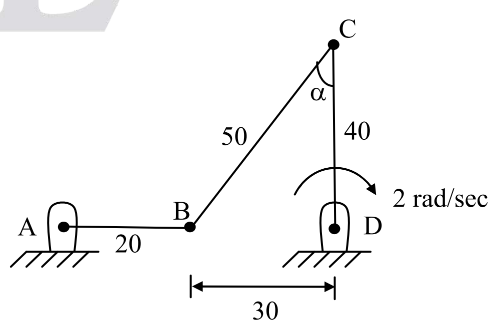
$$IB = OA = \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2}\,m$$

$$V_a = \omega \times IA$$

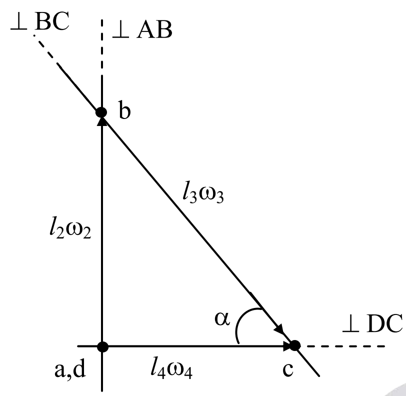
$$\Rightarrow \omega = \frac{V_a}{IA} = \frac{1}{V_2} = 2 \text{ rad/sec}$$

18. Ans: (a)

Sol:



(Position Diagram)



(Velocity Diagram)

Let the angle between BC & CD is α. Same will be the angle between their perpendiculars.

From Velocity Diagram, $\frac{\ell_2\omega_2}{\ell_4\omega_4} = \tan\alpha$

From Position diagram, $\tan\alpha = \frac{30}{40}$

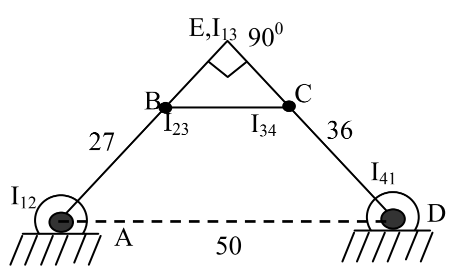
$\therefore\ \omega_2 = \omega_4 \times \frac{\ell_4}{\ell_2} \times \tan\alpha = 2 \times \frac{40}{20} \times \frac{30}{40} = 3$

$\omega_2 = 3$ rad/sec

Note: DC is the rocker (Output link) and AB is the crank (Input link).

19. Ans: (c)

Sol:



I_{13} = Instantaneous center of link 3 with respect to link 1

As AED is a right angle triangle and the sides are being integers so AE = 30 cm and DE = 40 cm

BE = 3 cm and CE = 4 cm

By 'I' center velocity method,

$V_{23} = \omega_2 \times (AB) = \omega_3 \times (BE)$

$$\omega_3 = \frac{1\times 27}{3} = 9\,\text{rad/s}$$

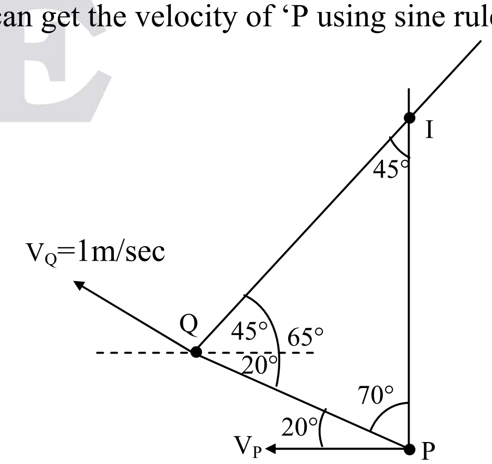
20. Ans: (a)

Sol: Similarly, $V_{34} = \omega_3 \times (EC) = \omega_4 \times (CD)$

$$\omega_4 = \frac{9\times 4}{36} = 1\,\text{rad/s}$$

21. Ans: (d)

Sol: Refer the figure shown below, By knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P using sine rule.



'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

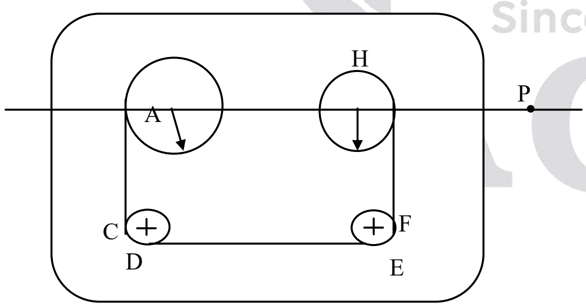
$$V_Q = IQ \times \omega = 1$$

$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1 = 0.9645$$

22. Ans: (c)

Sol: Consider the three bodies the bigger spool (Radius 20), smaller spool (Radius 10) and the frame. They together have three I centers, I centre of big spool with respect to the frame is at its centre A. that of the small spool with respect to the frame is at its centre H. The I centre for the two spools P is to be located.



As for the three centers in line theorem all the three centers should lie on a straight line implies on the line joining of A and H. More over as both the spools are rotating in the same direction, P should lie on the same side of A and H. Also it should be close to the spool running at higher angular velocity. Implies close to H and it is to be on the right of H. Whether P belongs to bigger spool or smaller spool its velocity must be same. As for the radii of the spools and noting that the velocity of the tape is same on both the spools

$$\omega_H = 2\omega_A$$

$$\therefore AP.\omega_A = HP\,\omega_H \quad \text{and}$$

$$AP = AH + HP \Rightarrow HP = AH$$

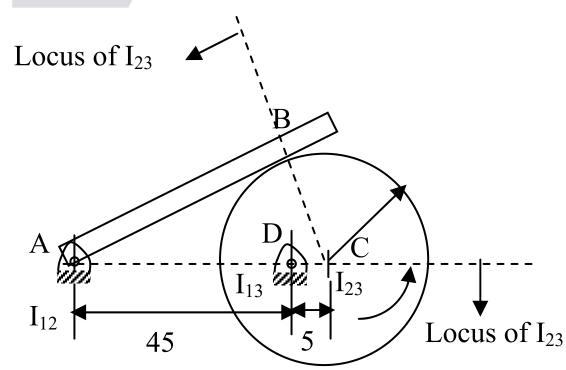
Note:

(i) If two links are rotating in same directions then their Instantaneous centre will never lie in between them. The 'I' center will always close to that link which is having high velocity.

(ii) If two links are rotating in different directions, their 'I' centre will lie in between the line joining the centres of the links.

23. Ans: (b)

Sol: I_{23} should be in the line joining I_{12} and I_{13}. Similarly the link 3 is rolling on link 2.



So the I – Center I_{23} will be on the line perpendicular to the link – 2. (I_{23} lies common normal passing through the contact point)

So the point C is the intersection of these two loci which is the center of the disc.

So $\omega_2(I_{12}, I_{23}) = \omega_3(I_{13}, I_{23})$

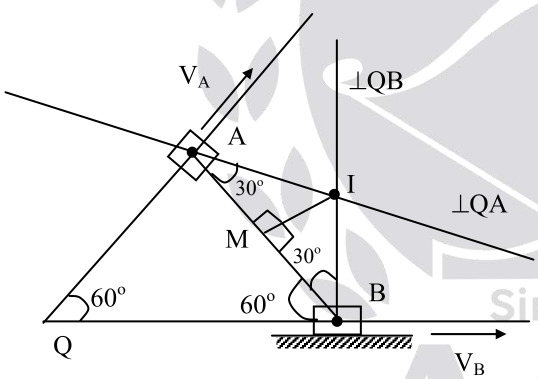
$$\Rightarrow \omega_2 \times 50 = 1 \times 5$$

$$\Rightarrow \omega_2 = 0.1 \text{ rad/sec}$$

24. Ans: 1 (range 0.95 to 1.05)

Sol: Locate the I-centre for the link AB as shown in fig. M is the mid point of AB

Given, V_A = 2 m/sec



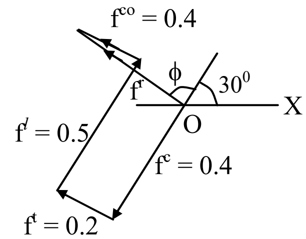
$$V_A = IA.\omega \Rightarrow \omega = \frac{V_A}{IA}$$

$$V_M = IM.\omega = IM\frac{V_A}{IA} = \frac{IM}{IA}.V_A$$

$$= \sin 30^o.V_A = \frac{1}{2}.2 = 1 m/sec$$

25. Ans: (a) & 26. Ans: (b)

Sol:



Centripetal acceleration,

$f^c = r\omega^2 = 0.4$ m/s^2 acts towards the centre

Tangential acceleration, $f^t = r\alpha =$ 0.2 m/s^2 acts perpendicular to the link in the direction of angular acceleration. Linear deceleration = 0.5 m/s^2 acts opposite to velocity of slider

As the link is rotating and sliding so coriolis component of acceleration acts

$f^{co} = 2V\omega = 2 \times 0.2 \times 1 = 0.4$ m/s^2

To get the direction of coriolis acceleration, rotate the velocity vector by 90^0 in the direction of ω.

Resultant acceleration

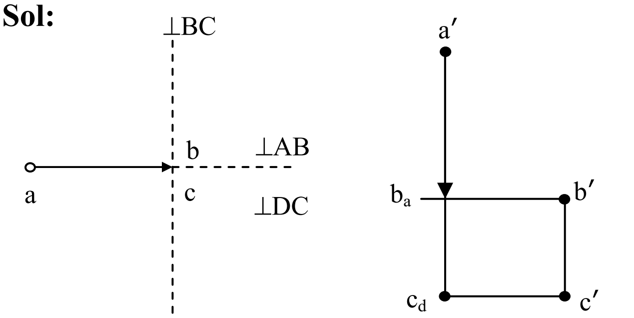
$$= \sqrt{0.6^2 + 0.1^2} = 0.608 \text{ m/sec}^2$$

$$\phi = \tan^{-1}\left(\frac{0.6}{0.1}\right) = 80.5$$

Angle of Resultant vector with reference to OX = 30 + ϕ = 30 + 80.5 = 110.53^0

27. Ans: (a)

Sol:



Velocity Diagram **Acceleration Diagram**

From velocity Diagram, $V_C = V_B$

$l_4\omega_4 = l_2\omega_2$

$25 \times \omega_4 = 50 \times 0.2$

$\Rightarrow \omega_4 = 0.4$ rad/sec

From Acceleration Diagram,

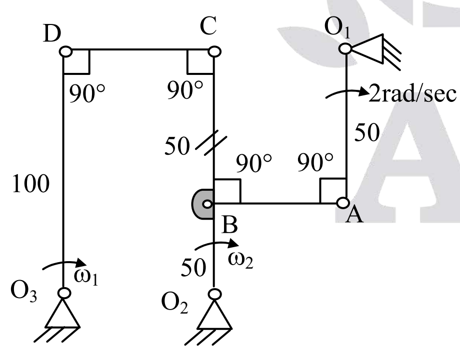
$l_4\alpha_4 = l_2\alpha_2$

$25 \times \alpha_4 = 50 \times 0.1$

$\Rightarrow \alpha_4 = 0.2$ rad/sec^2

28. Ans: (d)

Sol:



As links O_1A and O_2B are parallel then

$V_A = V_B$

$\Rightarrow 50 \times 2 = 50 \times \omega_2$

$\Rightarrow \quad \omega_2 = 2$ rad/sec

As a O_2 C and O_3D are parallel links then

$V_C = V_D$

$\Rightarrow 100 \times 2 = 100 \times \omega_1$

$\Rightarrow \quad \omega_1 = 2$ rad/sec

$V_D = r\omega_1$

$= 100 \times 2 = 200$ mm/sec

$\alpha = 0$ (given), so tangential acceleration a^t

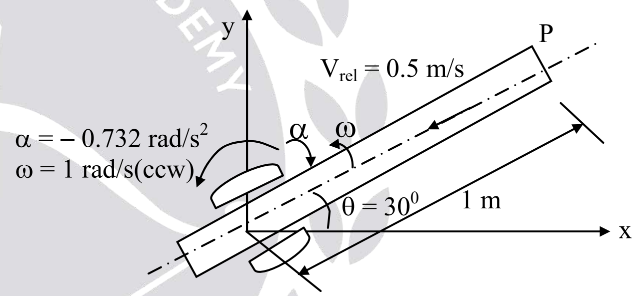
$= r\alpha = 0$

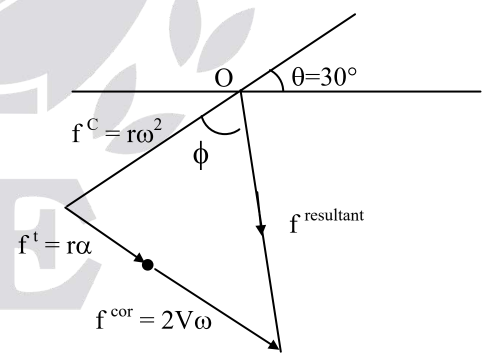
Centripetal acceleration, $f^c = r\omega_1^2$

$= 100 \times (2)^2 = 400$ mm/sec^2

29.

Sol:





Acceleration diagram

Radial relative acceleration, $f^{linear} = 0$

Centripetal acceleration, $f^c = r\omega^2$

$= 1\times1^2 = 1$ m/s^2 (acts towards the center)

Tangential acceleration, $f^t = r\alpha$

$= 1\times0.732 = 0.732$ m/sec^2

Coriolis acceleration, $f^{cor} = 2V\omega$

$$= 2 \times 0.5 \times 1 = 1 \text{ m/sec}^2$$

Resultant acceleration,

$$f^r = \sqrt{1^2 + (1+0.732)^2} = 2 \text{ m/sec}^2$$

$$\phi = \tan^{-1}\left(\frac{1.732}{1}\right) = 60°$$

$$\theta_{reference} = 30 + 180 + 60 = 270^0$$

30. Ans: (d)

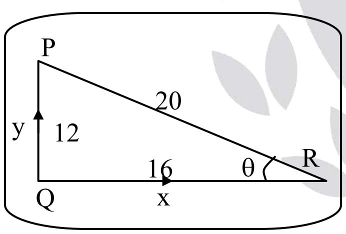
Sol: Angular acceleration of connecting rod is given by

$$a = -\omega^2 \sin\theta\left[\frac{(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}\right]$$

when n = 1, a = 0

31. Ans: (d)

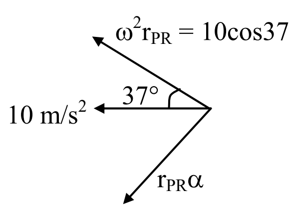
Sol:



Given that, $\vec{a}_{RP} = 10\text{m/s}^2 \angle 180°$

$$\tan\theta = \frac{12}{16} \Rightarrow \theta = 37°$$

Acceleration of R with respect to P is in negative x –direction i.e., along $\overrightarrow{RQ}$



Component of $\vec{a}_R$ along $\overrightarrow{RP}$ is

$$\omega^2 r_{PQ} = 10 \times \cos 37 = 10 \times \frac{16}{20} = 8 \text{ m/s}^2$$

$$\omega^2 = \frac{8}{20} \Rightarrow \omega = \sqrt{\frac{2}{5}}\text{rad/s}$$

Component of $\vec{a}_{RP}$ perpendicular to $\overrightarrow{RP}$ is

$$\alpha\, r_{RP} = 10 \sin 37 = 10 \times \frac{12}{20} = 6 \text{ m/s}^2$$

$$\alpha = \frac{6}{20} = \frac{3}{10}\text{rad/s}^2$$

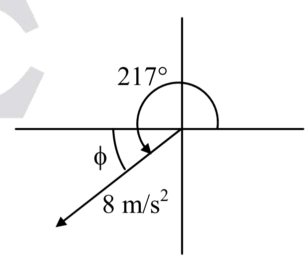
Acceleration $\vec{a}_{RQ}$ is given by

$$\vec{a}_{RQ} = \vec{a}_R - \vec{a}_Q = \vec{a}_R$$

$$a_R = \sqrt{(\omega^2 r_{RQ})^2 + (\alpha r_{RQ})^2}$$

$$= \sqrt{\left(\frac{2}{5}\times 16\right)^2 + \left(\frac{3}{10}\times 16\right)^2} = 8 \text{ m/s}^2$$

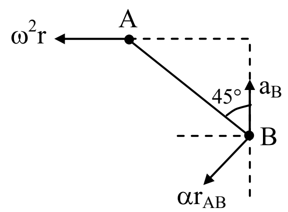
$$\tan\phi = \left(\frac{\alpha r_{RQ}}{\omega^2 r_{RQ}}\right) = \frac{\left(\frac{3}{10}\right)}{\left(\frac{2}{5}\right)} \Rightarrow \phi = 37°$$



∴ Acceleration R is 8 m/s² at 217° from x-axis i.e., $8 \angle 217°$ m/s²

32. Ans: (c)

Sol:



Since the velocity of the point A and B are parallel $\omega_{AB} = 0$.

$\vec{a}_B = \vec{a}_A + \vec{a}_{AB}$

$\vec{a}_B = a_B\hat{j}$

$\vec{a}_A = -\omega^2 r\,\hat{i}$

$\vec{a}_{AB} = -\alpha r_{AB}\sin 45\,\hat{i} - \alpha r_{AB}\cos 45\,\hat{j}$

($\because$ $\omega^2 r_{AB}$ along link AB = 0)

$a_B\hat{j} = -\omega^2 r\hat{i} - (\alpha\hat{i} + \alpha\hat{j}) = -(\omega^2 r + \alpha)\hat{i} - \alpha\hat{j}$

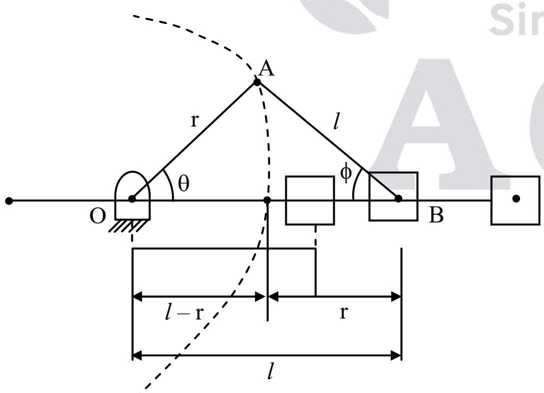
$\omega^2 r + \alpha = 0$

$\alpha = -\omega^2 r$

$a_B = -\alpha = -(-\omega^2 r) = \omega^2 r$

33. Ans: (b) & 34. Ans: (a)

Sol:



$F_P = 2$ kN

$l = 80$ cm $= 0.8$ m

$r = 20$ cm $= 0.2$m

From the triangle OAB

$$\cos\phi = \frac{\ell^2 + \ell^2 - r^2}{2\ell^2}$$

$$= \frac{2\times 80^2 - 20^2}{2\times 80^2} \Rightarrow \phi = 14.36$$

$$\cos\theta = \frac{20^2 + 80^2 - 80^2}{2\times 20\times 80} \Rightarrow \theta = 82.82$$

Thrust connecting rod

$$F_T = \frac{F_P}{\cos\phi} = \frac{2}{\cos 14.36} = 2.065\,\text{kN}$$

Turning moment,

$$T = F_T \times r = \frac{F_P}{\cos\phi}(\sin(\theta+\phi))\times r$$

$$= \frac{2}{\cos 14.36}\times\sin(14.36 + 82.82)\times 0.2$$

= 0.409 kN-m

35. Ans: (b)

Sol: Calculate AB that will be equal to 260 mm

L = 260 mm, P = 160 mm

S = 60 mm, Q = 240 mm

L + S = 320

P + Q = 400

$\therefore$ L+S < P+Q

It is a Grashof's chain

Link adjacent to the shortest link is fixed

$\therefore$ Crank – Rocker Mechanism.

36. Ans: (b)

Sol: $O_2A \parallel O_4B$

Then linear velocity is same at A and B.

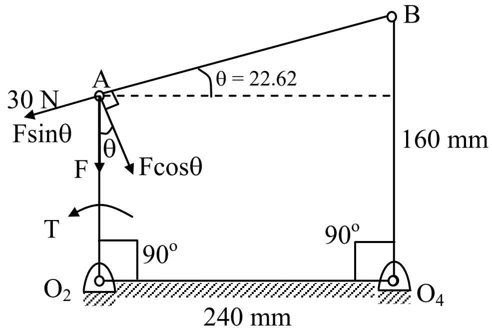
$\therefore$ $\omega_2 \times O_2A = \omega_4 \times O_4B$

$\therefore$ $8\times 60 = \omega_4 \times 160$

$\Rightarrow$ $\omega_4 = 3$ rad/sec

37. Ans: (c)

Sol:



$$\tan\theta = \frac{100}{240} \Rightarrow \theta = 22.62^o$$

As centre of mass falls at O_2

$m\bar{r}\omega^2 = 0 \quad (\because \bar{r} = 0)$

$\alpha = 0$ (Given)

Inertia torque = 0

Since torque on link O_2A is zero, the resultant force at point A must be along O_2A.

$\Rightarrow F\sin 22.62 = 30$

$$\Rightarrow F = \frac{30}{\sin 22.62} = 78\text{ N}$$

The magnitude of the joint reaction at $O_2 = F = 78$ N

38. Ans: (d)

Sol: $I\dfrac{d^2\theta}{dt^2} = T + f(\sin\theta, \cos\theta)$

Where 'T' is applied torque, f is inertia torque which is function of sinθ & cosθ

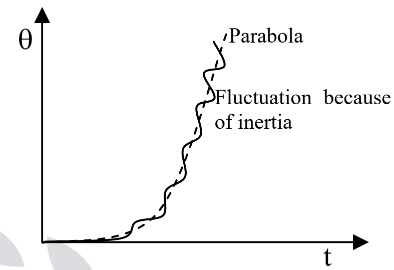
$$\frac{d\theta}{dt} = \frac{T}{I}t + f'(\sin\theta, \cos\theta) + c_1$$

$$\theta = \frac{T}{I}t^2 + c_1 t + f''(\sin\theta, \cos\theta)$$

θ is fluctuating on parabola

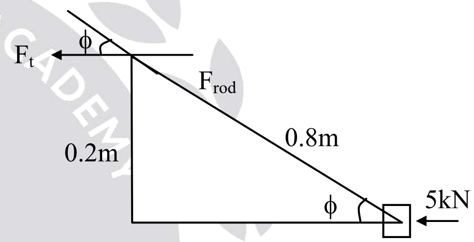
and @ t = 0, θ = 0,

$\dot{\theta}(\text{slope}) = 0$ (because it starts from rest)



39. Ans: 1 (Range 0.9 to 1.1)

Sol:



Given $F_p = 5$ kN

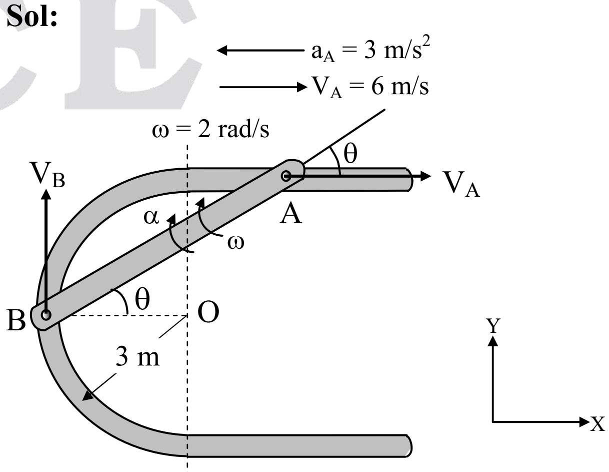
$$F_{rod} = \frac{F_p}{\cos\phi}, \quad F_t = F_{rod}\cos\phi$$

$\therefore F_t = 5$ kN

Turning moment = F_t. r = 5×0.2 = 1 kN-m

40. Ans: (a, d)

Sol:



$\theta = \sin^{-1}\left(\frac{3}{5}\right) \cong 37°$

$\because$ Rod is rigid $\Rightarrow V_A \cos\theta = V_B \cos(90-\theta)$

$\Rightarrow V_B = 8$ m/s

$\omega_{AB} = \frac{6\sin 37 + 8\cos 37}{5} = 2\,\text{rad/s}$

$\vec{a}_B = \vec{a}_{BA} + \vec{a}_A$ .... (i)

$\vec{a}_B = \frac{V_B^2}{3}\hat{i} + a_B^t \hat{j}$ .... (ii)

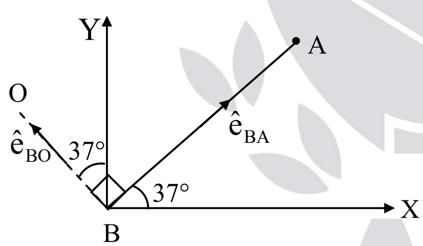
$\left[a_B^t \text{ is tangential component of acceleration}\right]$

[$\hat{e}_{BO}$ is unit vector along BO and $\hat{e}_{BA}$ is unit vector along BA]

$\vec{a}_{BA} = (\vec{a}_{BA})^t + (\vec{a}_{BA})^r$

$\vec{a}_{BA}^t = 5\alpha \hat{e}_{BO}$

$\vec{a}_{BA}^r = \omega^2 \times 5\hat{e}_{BA} = 20\,\hat{e}_{BA}$



$\vec{a}_{BA} = 5\alpha\cos 37\hat{j} - 5\alpha\sin 37\hat{i} + 20\cos 37\hat{i} + 20\sin 37\hat{j}$

$= 4\alpha\hat{j} - 3\alpha\hat{i} + 16\hat{i} + 12\hat{j}$ . ... (iii)

$\vec{a}_A = -3\hat{i}$ .... (iv)

Substituting (iv), (iii), (ii) in (i)

$\frac{64}{3}\hat{i} + a_B^t\hat{j} = 4\alpha\hat{j} - 3\alpha\hat{i} - 3\hat{i} + 16\hat{i} + 12\hat{j}$

$\therefore \frac{64}{3} = 13 - 3\alpha$

$\alpha = -2.78$ rad/s^2

Putting the value of α in eq. (i),

$\vec{a}_B = \vec{a}_{BA} + \vec{a}_A$

$= 4\alpha\hat{j} - 3\alpha\hat{i} + 16\hat{i} + 12\hat{j} - 3\hat{i}$

$= (4 \times -2.78)\,\hat{j} - (3 \times -2.78)\hat{i} + 16\,\hat{i} + 12\,\hat{j} - 3\,\hat{i}$

$= -11.12\,\hat{j} + 8.34\hat{i} + 16\hat{i} + 12\hat{j} - 3\hat{i}$

$= 21.34\,\hat{i} + 0.88\,\hat{j}$

$|a_B| = \sqrt{(21.34)^2 + (0.88)^2} = 21.4$ m/s^2

Chapter 2

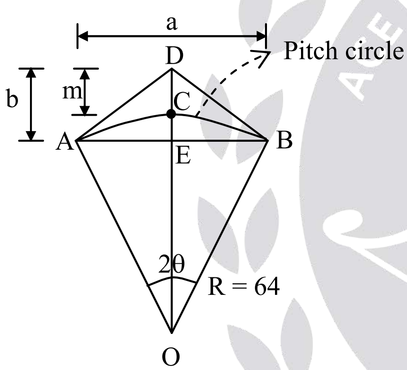
GEAR AND GEAR TRAINS

01. Ans (a)

Sol: Profile between base and root circles is not involute. If tip of a tooth of a mating gear digs into this non-involute portion interference will occur.

02. Ans: (d)

Sol: Angle made by 32 teeth + 32 tooth space = 360°.



$2\theta = \frac{360}{64} = 5.625 \Rightarrow \theta = 2.8125$

$R = \frac{mT}{2} = \frac{4\times32}{2} = 64mm$

$a = R\sin\theta \times 2$

$= 64\times\sin(2.81)\times2 = 6.28$

$OE = R\cos\theta = 64\times\cos(2.8125) = 63.9$ mm

b = addendum+ CE = module +(OC – OE)

$= 4 + (64 - 63.9) = 4.1$

03. Ans: (a)

Sol: When addendum of both gear and pinion are same then interference occurs between tip of the gear tooth and pinion.

04. Ans: Decreases, Increases

05. Ans: (b)

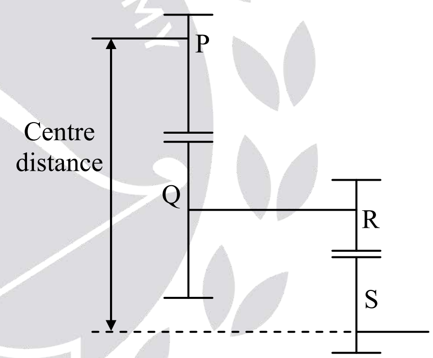
Sol: For same addendum interference is most likely to occur between tip of the gear tooth and pinion i.e., at the beginning of the contact.

06. Ans: (b)

Sol: For two gears are to be meshed, they should have same module and same pressure angle.

07. Ans: (b)

Sol:



Given $T_p = 20$, $T_Q = 40$, $T_R = 15$, $T_S = 20$

Dia of Q = 2 × Dia of R

$m_Q.T_Q = 2m_R.T_R$

Given, module of R = m_R = 2mm

$\Rightarrow m_Q = 2\,m_R \frac{T_R}{T_Q} = 2\times2\times\frac{15}{40} = 1.5\,mm$

$m_P = m_Q = 2mm$

$m_S = m_R = 1.5$ mm

$\text{Radius} = \text{module} \times \frac{\text{No.of teeth}}{2}$

Centre distance between P and S is given by

$R_P + R_Q + R_R + R_T$

$= m_P \frac{T_P}{2} + m_Q \frac{T_Q}{2} + m_R \frac{T_R}{2} + m_S \frac{T_S}{2}$

$= 1.5\left[\frac{40+20}{2}\right] + 2\left[\frac{15+20}{2}\right]$

= 45 + 35 = 80 mm

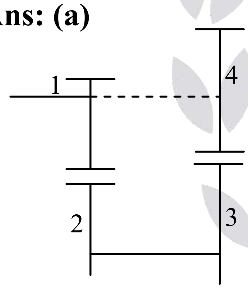
08. Ans: (c)

Sol: $\frac{N_2}{N_6} = \frac{N_3N_5N_6}{N_2N_4N_5} = \frac{N_3N_6}{N_2N_4}$

Wheel 5 is the only Idler gear as the number of teeth on wheel '5' does not appear in the velocity ratio.

09. Ans: (a)

Sol:



$Z_1 = 16$, $Z_3 = 15$, $Z_2 = ?$, $Z_4 = ?$

First stage gear ratio, $G_1 = 4$,

Second stage gear ratio, $G_2 = 3$,

$m_{12} = 3$, $m_{34} = 4$

$Z_2 = 16 \times 4 = 64$

$Z_4 = 15 \times 3 = 45$

10. Ans: (b)

Sol: Centre distance

$= \frac{m_{12}}{2} \times (Z_1 + Z_2) = \frac{m_{34}}{2} \times (Z_3 + Z_4)$

$= \frac{4}{2} \times (15 + 45) = 120\text{mm}$

11. Ans: (a)

Sol: By Analytical Approach

$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{-T_2}{T_1} \times \frac{-T_4}{T_3} = \frac{45}{15} \times \frac{40}{20}$

$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$

12. Ans: (d)

Sol: Data given:

ω_1= 60 rpm (CW, +ve)

ω_4 = –120 rpm [2 times speed of gear -1]

We have, $\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$

$\Rightarrow \frac{60 - \omega_5}{-120 - \omega_5} = 6$, simplifying

$60 - \omega_5 = -720 - 6\omega_5$

$\omega_5 = -156$ rpm CW

$\Rightarrow$ ω_5 = 156 rpm CCW

13. Ans: (c)

Sol: ω_2 = 100 rad/sec(CW+ve),

ω_{arm} = 80 rad/s (CCW) = –80 rad/sec

$\frac{\omega_5 - \omega_a}{\omega_2 - \omega_a} = \frac{-T_2}{T_3} \times \frac{T_4}{T_5}$

$\frac{\omega_5 - (-80)}{100 - (-80)} = \frac{-20}{24} \times \frac{32}{80} = -\frac{1}{3}$

$\Rightarrow \omega_5 = -140$ CW = 140 CCW

14. Ans (c)

Sol: It also rotates one revolution but in opposite direction because of differential gear system

15. Ans: (a)

Sol: r_b = base circle radius,

r_d = dedendum radius,

r = pitch circle radius.

For the complete profile to be invoulte,

$r_b = r_d$

$r_d = r - 1$ module

$$r = \frac{mT}{2} = \frac{16\times5}{2} = 40 \text{ mm}$$

$\therefore r_b = r_d = 40 - 1 \times 5 = 35$ mm

$r_b = r\cos\phi \Rightarrow \phi \simeq 29^o$

16. Ans: – 3.33 N-m

Sol:
$$\frac{\omega_s - \omega_a}{\omega_p - \omega_a} = \frac{-Z_p}{Z_s}$$

$$\Rightarrow \frac{0-10}{\omega_p - 10} = \frac{-20}{40}$$

$\Rightarrow \omega_p = 30$ rad/sec

By assuming no losses in power transmission

$T_p \times \omega_p + T_s \times \omega_s + T_a \times \omega_a = 0$

$\Rightarrow T_p \times 30 + T_s \times 0 + 5 \times 10 = 0$

$$\Rightarrow T_p = \frac{-50}{30} = -1.67 \text{ N-m},$$

$T_p + T_s + T_a = 0$

$\Rightarrow -1.67 + T_s + 5 = 0$

$\Rightarrow T_s = -3.33$ N-m

17. Ans: (a)

Sol: Train value = speed ratio

18. Ans: (d)

Sol: $T_S + 2\,T_P = T_A$ -----(1)

$$\frac{N_A - N_a}{N_P - N_a} = \frac{T_P}{T_A} \quad \text{------(2)}$$

and
$$\frac{N_P - N_S}{N_S - N_G} = -\frac{T_S}{T_P} \quad \text{------(3)}$$

From (2) and (3)

$$\frac{N_A - N_a}{N_S - N_a} = -\frac{T_B}{T_A}$$

$$\Rightarrow \frac{300-180}{0-180} = -\frac{80}{T_A}$$

$\therefore\ T_A = 120$

$80 + 2\,T_P = 120$

$\Rightarrow T_P = 20$

19. Ans: (a, b, c, d)

Sol:

- Bevel gear is used for connecting two non-parallel or, intersecting but coplanar shafts.
- Spur gear is used for connecting two parallel and coplanar shafts with teeth parallel to the axis of the gear wheel.
- Mitre gear is used for connecting two shafts whose axes are mutually perpendicular to each other.
- Helical gear is used for connecting two parallel and coplanar shafts with teeth inclined to the axis of the gear wheel.

Chapter 3

FLYWHEELS

01.

Sol: Given

$P = 80\ kW = 80\times10^3\ W = 80{,}000W$

$\Delta E = 0.9$ Per cycle

$N = 300\,rpm$

$C_S = 0.02$

$\omega = \frac{2\pi N}{60} = \frac{2\pi\times30}{60} = 31.41 rad/s$

$\rho = 7500\ kg/m^3$

$\sigma_c = 6\ MN/m^2$

$\sigma_c = \rho V^2 = \rho R^2\omega^2$

$R = \sqrt{\frac{\sigma_C}{\rho\omega^2}} = \sqrt{\frac{6\times10^6}{7500\times31.41^2}}$

$R = 0.9$ m

$D = 2R = 1.8m$

$N = 300rpm = 5rps \rightarrow 0.2$ Sec/rev

1 cycle = 2 revolution $(\because 4\,\text{stroke engine})$

$= 0.4\,sec$

Energy developed per cycle

$= 0.4\times80 = 32\ kJ$

$\Delta E = E\,\text{per cycle}\times0.9$

$= 32\times10^3\times0.9 = 28800\ J$

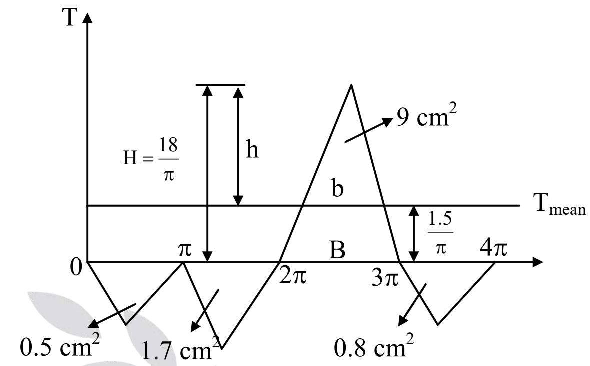
$\Delta E = I\omega^2C_S$

$I = \frac{\Delta E}{\omega^2 C_S}$

$I = 1459.58\ kg\text{-}m^2$

02.

Sol:



Given: 1 cm^2 = 1400 J

Assume on x-axis 1 cm = 1 radian and on y-axis 1 cm = 1400 N-m

$a_1 = -0.5\ cm^2$

$a_2 = -1.7\ cm^2$

$a_3 = 9\ cm^2$

$a_4 = -0.8\ cm^2$

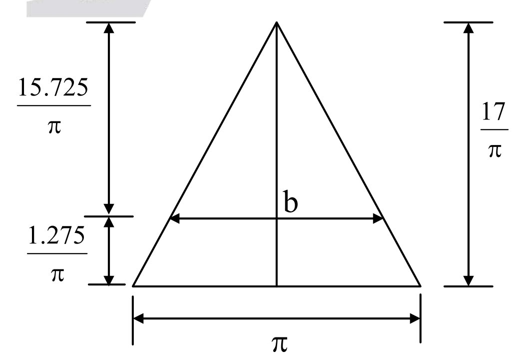
Work done per cycle $= -a_1 - a_2 + a_3 - a_4$

$= -0.5 - 1.7 + 9 - 0.8$

$= 6\ cm^2$

Mean torque $T_m = \frac{\text{Workdone per cycle}}{4\pi}$

$= \frac{6}{4\pi} = \frac{1.5}{\pi}\,cm$



Area of the triangle (expansion)

$$= \frac{1}{2}\times\pi\times H = 9$$

$H = 18/\pi$

Area above the mean torque line

$$\Delta E = \frac{1}{2}\times b\times h$$

From the similar triangles ,

$$\frac{b}{B} = \frac{h}{H} \Rightarrow b = \frac{16.5}{18}\times\pi$$

$$\Delta E = \frac{1}{2}\times b\times\frac{16.5}{\pi}$$

$$= \frac{1}{2}\times\frac{16.5}{18}\times\frac{16.5}{\pi} = 7.56\text{ cm}^2$$

$\Delta E = 7.56 \times 1400 = 10587$ N-m

$N_1 = 102$ rpm, $N_2 = 98$ rpm,

$$\omega_1 = \frac{2\pi N_1}{60} = 10.68\text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = 10.26\text{ rad/s}$$

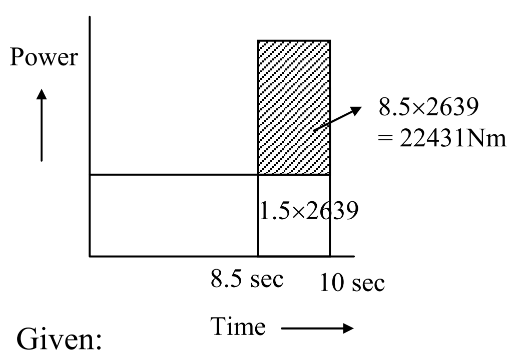
$$\Delta E = \frac{1}{2}\times I\times(\omega_1^2 - \omega_2^2)$$

$$I = \frac{2\times\Delta E}{(\omega_1^2-\omega_2^2)} = \frac{2\times 10587}{10.68^2 - 10.26^2}$$

$I = 2405.6$ kg-m^2

03.

Sol:



Given:

$d = 40$ mm, $t = 30$ mm

$E_1 = 7$ N-m/mm^2, $S = 100$ mm

$V = 25$ m/s, $V_1 - V_2 = 3\%V$, $C_S = 0.03$

$A = \pi dt = \pi \times 40 \times 30$

$= 3769.9 = 3770$ mm^2

Since the energy required to punch the hole is 7 Nm/mm^2 of sheared area, therefore the Total energy required for punching one hole $= 7 \times \pi dt = 26390$ N-m

Also the time required to punch a hole is 10 sec, therefore power of the motor required $= \frac{26390}{10} = 2639$ Watt

The stroke of the punch is 100 mm and it punches one hole in every 10 seconds.

Total punch travel = 200 mm

(up stroke + down stroke)

Velocity of punch = (200/10) = 20 mm/s

Actual punching time = 30/20 = 1.5 sec

Energy supplied by the motor in 1.5 sec is

$E_2 = 2639 \times 1.5 = 3958.5 = 3959$ N-m

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy

$\Delta E = E_1 - E_2$

$= 26390 - 3959 = 22431$ N-m

Coefficient of fluctuation of speed

$$C_S = \frac{V_1 - V_2}{V} = 0.03$$

We know that maximum fluctuation of energy (ΔE)

$22431 = m V^2 C_S = m (25)^2 (0.03)$

$m = 1196$ kg

04. Ans: 4.27

Sol: $I = mk^2 = 200 \times 0.4^2 = 32$ kg-m^2

$$\omega_1 = \frac{2\pi \times 400}{60} = 41.86 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi \times 280}{60} = 26.16 \text{ rad/s}$$

$$\text{Energy released} = \frac{1}{2} I\left(\omega_1^2 - \omega_2^2\right) = 17086.6 \text{ J}$$

$$\text{Total machining time} = \frac{60}{5} = 12 \text{ sec}$$

$$\text{Power of motor} = \frac{17086.6}{12-8} = 4.27 \text{ kW}$$

05. Ans: (d)

Sol: Work done = –0.5+1–2+25–0.8+0.5

= 23.2 cm^2

Work done per cycle = 23.2×100 = 2320

$(\because 1\text{cm}^2 = 100\text{N}-\text{m})$

$$T_{mean} = \frac{\text{W.D per cycle}}{4\pi}$$

$$= \frac{2320}{4\pi} = \frac{580}{\pi} \text{ N}-\text{m}$$

Suction = 0 to π,

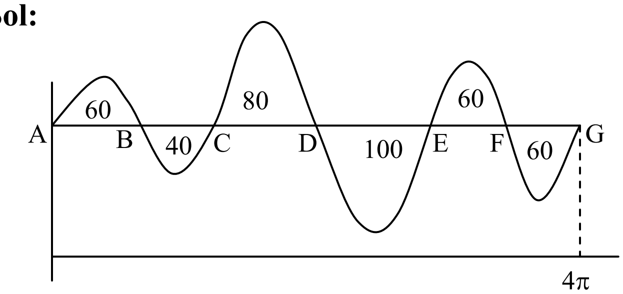
Compression = π to 2π

Expansion = 2π to 3π,

Exhaust = 3π to 4π

06. Ans: (c)

Sol:



$E_A = E$

$E_B = E + 60$

$E_C = E + 60 - 40 = E + 20$

$E_D = E + 20 + 80 = E + 100 = E_{max}$

$E_E = E + 100 - 100 = E$

$E_F = E + 60$

$E_G = E + 60 - 60 = E_{min}$

$\therefore$ R > P > Q > S

07. Ans: (a)

Sol: Let the cycle time = t

Actual punching time = t/4

W = energy developed per cycle

Energy required in actual punching

= 3W/4

During 3t/4 time, energy consumed = W/4

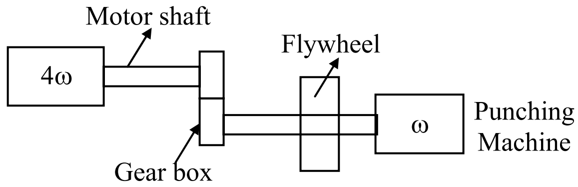
$$E_{max} = \frac{3W}{4}, \; E_{min} = \frac{E}{4}$$

$$\Delta E = E_{max} - E_{min} = \frac{E}{2}$$

$$\frac{\Delta E}{E} = 0.5$$

08. Ans: (c)

Sol:



$C_s = 0.032$

Gear ratio = 4

$I\omega'^2 \times C_s' = I\omega^2 C_s$

$$C_s' = C_s\left(\frac{\omega}{\omega'}\right)^2 = \frac{C_s \times \omega^2}{16\,\omega^2} = \frac{C_s}{16}$$

$= 0.0032 / 16 = 0.002$

(by taking moment of Inertia, I = constant).

Thus, if the flywheel is shifted from machine shaft to motor shaft when the fluctuation of energy (ΔE) is same, then coefficient of fluctuation of speed decreases by 0.2% times.

09. Ans: 0.5625

Sol: The flywheel is considered as two parts $\frac{m}{2}$ as rim type with Radius R and $\frac{m}{2}$ as disk type with Radius $\frac{R}{2}$

$$I_{Rim} = \frac{m}{2}R^2,$$

$$I_{disk} = \frac{1}{2}\times\frac{m}{2}\times\left(\frac{R}{2}\right)^2 = \frac{mR^2}{16}$$

$$I = \frac{mR^2}{2} + \frac{mR^2}{16}$$

$$= \frac{9}{16}mR^2$$

$= 0.5625\ mR^2$

$\therefore\ \alpha = 0.5625$

10. Ans: 104.71

Sol: N = 100 rpm

$$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi}(10000 + 1000\sin 2\theta - 1200\cos 2\theta)d\theta$$

$$= \frac{1}{\pi}\left[10000\theta - 500\cos 2\theta - 600\sin 2\theta\right]_0^{\pi}$$

= 10000 Nm

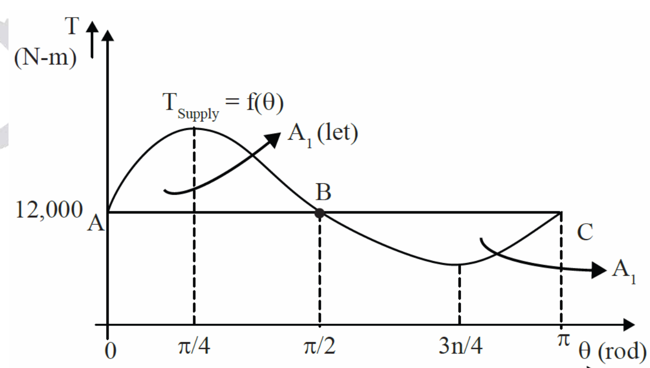
$$\text{Power} = \frac{2\pi NT}{60}$$

$$= \frac{2\times\pi\times 100\times 10000}{60} = 104719.75\ \text{W}$$

P = 104.719 kW

11. Ans: 570

Sol:



$T(\theta) = 12000 + 2500\sin 2\theta$ N-m

$N_m = 200$ rpm

$C_s = \pm 0.5\ \%$, $I = ?$

$C_s = 1\% = 0.01$

$T_{resisting}$ = constant (given)

Therefore, $T_{resisting} = T_{mean} = 12000$ N-m

Let,

Energy of flywheel at point A = E_A

Energy of flywheel at point B = E_B

$= E_A + A_1 = E_{max}$

Energy of flywheel at point C = E_C

$= E_A + A_1 - A_1$

$= E_A = E_{min}$

$(\Delta E)_{max} = E_{max} - E_{min}$

$= E_A - E_C = A_1$

$$A_1 = \int_{\theta_A}^{\theta_B} (T_{supply} - T_{mean}) d\theta$$

$$(\Delta E)_{max} = \int_0^{\pi/2} \{(12000 + 2500 \sin 2\theta) - 12000\} d\theta$$

$$(\Delta E)_{max} = \int_0^{\pi/2} 2500 \sin 2\theta . d\theta$$

$$= 2500 \int_0^{\pi/2} \sin 2\theta . d\theta$$

$$= 2500 \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$= 1250\ [-\cos\pi + \cos 0]$

$= 1250\ [-(-1) + 1] = 2500$ J

$$(\Delta E)_{max} = I\, \omega_m^2\, C_s$$

$$\Rightarrow 2500 = I \times \left(\frac{2\pi \times 200}{60}\right)^2 \times 0.01$$

$\Rightarrow I = 569.93$ kg-m^2

Chapter 4 GOVERNOR

01. Ans: (a)

Sol: As the governor runs at constant speed, net force on the sleeve is zero.

02. Ans: (d)

Sol: At equilibrium speed, friction at the sleeve is zero.

03. Ans: (a)

Sol: $$mr\omega^2 = \frac{r}{h}\left(mg + \frac{Mg(1+k)}{2}\right)$$

$k = 1$

$$\omega^2 = \frac{9.8}{2 \times 0.2}(10 + 2)$$

$\omega = 17.15$ rad/sec

04. Ans: (a)

Sol: $$mr\omega^2 a = \frac{1}{2} \times 200 \times \delta \times a$$

$$\delta = \frac{1 \times 20^2 \times 0.25 \times 2}{200}$$

$= 0.5 \times 2 = 1$ cm

05. Ans: (b)

06. Ans: (a)

Sol: $r_1 = 50$ cm , $F_1 = 600$ N

$F = a + rb$

$600 = a + 50\ b$

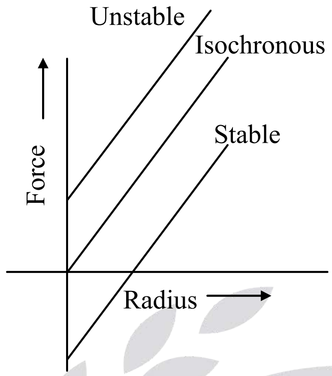
$700 = a + 60\ b$

$10\ b = 100$

$b = 10$ N/cm

$a = 100$ N

$F = 100 + 10\ r$



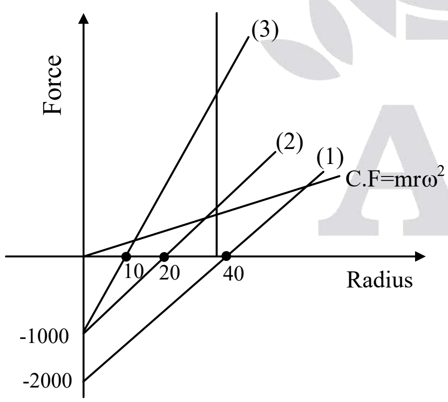
This is unstable governor. It can be isochronous if its initial compression is reduced by 100 N.

07. Ans: (d)

Sol: By increasing the dead weight in a porter governor it becomes more sensitive to speed change.

08. Ans: (a)

Sol:



At radius, $r_1 = F_1 < F_2 < F_3$

$\therefore$ As Controlling force is less suitable 1 is for low speed and 2 for high speed ad 3 is for still high speed.

(1) is active after 40 cm

(2) is active after 20 cm

(3) is active after 10 cm

At given radius above 20

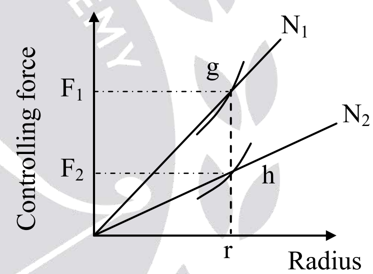
$F_3 > F_2$

$mr\omega_3^2 > mr\omega_2^2$

$\omega_3 > \omega_2$

09. Ans: (c)

Sol: If friction is taken into account, two or more controlling force are obtained as show in figure.



In all, three curves of controlling force are obtained as follows.

(a) for steady run (neglecting friction)

(b) while sleeve moves up (f positive)

(c) while sleeve moves down (f negative)

The vertical intercept *gh* signifies that between the speeds corresponding to *gh*, the radius of the ball does not change while direction of movement of sleeve does. Between speeds N_1 and N_2, the governor is insensitive.

10. Ans: 1063 N, 284 rpm, 250 N, 316 rpm

Sol: Given,

m = 8 kg

F_1 = 1500 N at r_1 = 0.2 m and

F_2 = 887.5 N at r_2 = 0.13 m,

For spring controlled governor, controlling force is given by

$F = a\,r + b$

$1500 = a \times 0.2 + b$

$887.5 = a \times 0.13 + b$

$\therefore\ a = 8750,\ b = -250$

$F = 8750\,r - 250$

At r = 0.15 m,

$F = 8750 \times 0.15 - 250 = 1062.5$ N

So, controlling force, F = 1062.5 m

$F = mr\omega^2$

$1062.5 = 8 \times 0.15\,\omega^2$

$\therefore\quad \omega = 29.76$ rad/s

$$N = \frac{60\,\omega}{2\pi} = 284 \text{ rpm}$$

For isochronous speed

$F = a\,r = 8750 \times 0.15 = 1312.5$ N

$F = mr\omega^2$

$1312.5 = 8 \times 0.5 \times \omega^2$

$\Rightarrow\quad \omega = 33.07$ rad/s

$$N = \frac{60\omega}{2\pi} = 316 \text{ rpm}$$

The increase in tension is 250 N to make the governor isochronous.

11. Ans: (a, d)

Sol:

- A governor is said to be unstable if the radius of rotation falls as the speed increases.
- Spring controlled governors can become isochronous.
- By increasing the initial compression of the spring the mean speed can be increased.
- Isochronisms for a centrifugal governor can be achieved only at the expense of its stability.

Chapter 5

BALANCING

01. Ans: (c)

Sol: Unbalanced force (F_{un}) $\propto m r \omega^2$

Unbalance force is directly proportional to square of speed. At high speed this force is very high. Hence, dynamic balancing becomes necessary at high speeds.

02. Ans: (a)

Sol: Dynamic force = $\frac{W}{g} e \omega^2$

Couple = $\frac{W}{g} e \omega^2 a$

Reaction on each bearing = $\pm \frac{W}{g} e \omega^2 \frac{a}{l}$

Total reaction on bearing

$$= \left(\frac{W}{g} e \omega^2 \frac{a}{l}\right) - \left(\frac{W}{g} e \omega^2 \frac{a}{l}\right) = 0$$

03. Ans: (b)

Sol: Since total dynamic reaction is zero the system is in static balance.

04. Ans: (a)

05. Ans: (b)

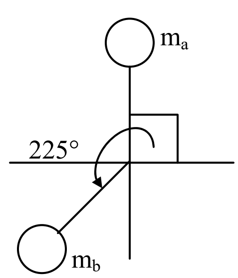
Sol:

$m_a = 5$ kg, $r_a = 20$ cm

$m_b = 6$ kg, $r_b = 20$ cm

$m_c = ?$, $r_c = 20$ cm

$m_d = ?$, $\theta_c = ?$, $\theta_d = ?$



Take reference plane as 'C'

For complete balancing

$\sum mr = 0$ & $\sum mrl = 0$

$2m_d \cos\theta_d - 9\sqrt{2} = 0$

$\Rightarrow m_d \cos\theta_d = 9\sqrt{2}$

$2m_d \sin\theta_d - 5 - 9\sqrt{2} = 0$

$m_d \sin\theta_d = = \frac{1}{2}\left(5 + 9\sqrt{2}\right)$

$$m_d = \sqrt{\left(\frac{9}{\sqrt{2}}\right)^2 + \left[\frac{1}{2}\left(5+9\sqrt{2}\right)\right]^2} = 10.91\text{kg}$$

$$\theta_d = \tan^{-1}\left[\frac{\frac{1}{2}\left(5+9\sqrt{2}\right)}{\frac{9}{\sqrt{2}}}\right] = 54.31^0$$

$= 90 - 54.31 = 35.68$ w.r.t 'A'

$m_c \cos\theta_c + m_d \cos\theta_d - 3\sqrt{2} = 0$

$\Rightarrow m_c \cos\theta_c + 10.91 \cos 54.31 - 3\sqrt{2} = 0$

$m_c \cos\theta_c = -2.122$

$m_c \sin\theta_c + m_d \sin\theta_d - 3\sqrt{2} + 5 = 0$

$m_c \sin\theta_c + 10.91 \sin 54.31 - 3\sqrt{2} + 5 = 0$

$m_c \sin\theta_c = -9.618$

$$m_c = \sqrt{(-2.122)^2 + (-9.618)^2} = 9.85\text{kg}$$

$$\tan\theta_c = \frac{-9.618}{-2.122}$$

$\theta_c = 257.56$ or $257.56 - 90$ w.r.t 'A'

$= 167.56$

S.No	m	(r×20)cm	(*l*×20)cm	θ	mrcosθ	mrsinθ	mr*l*cosθ	mr*l*sinθ
A	5	1	–1	90	0	5	0	–5
B	6	1	3	225	$-3\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-9\sqrt{2}$
C	m_c	1	0	θ_c	$m_c\cos\theta_c$	$m_c\sin\theta_c$	0	0
D	m_d	1	2	θ_d	$m_d\cos\theta_d$	$m_d\sin\theta_d$	$2m_d\cos\theta_d$	$2m_d\sin\theta_d$

Common data Q. 06 & 07

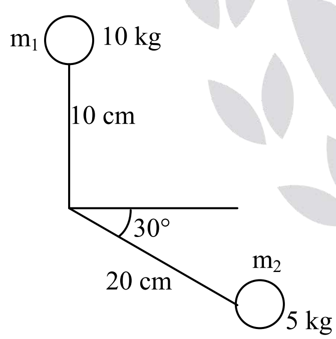
06. Ans: (a)

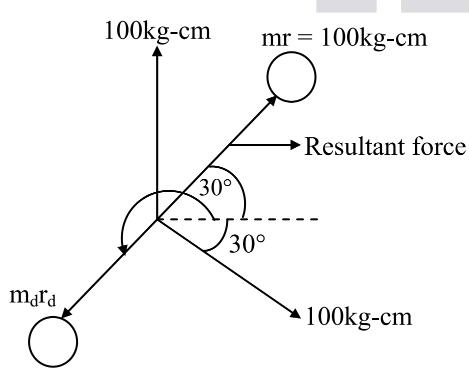
Sol: m_1 = kg , m_2 = 5kg , r_1 = 10cm

r_2 = 20cm, m_d = ?, r_d = 10cm

m_1r_1 = 100 kg cm

m_2r_2 = 100kg cm





Keep the balancing mass m_d at exactly opposite to the resultant force

$\therefore m_dr_d$ = 100kg-cm

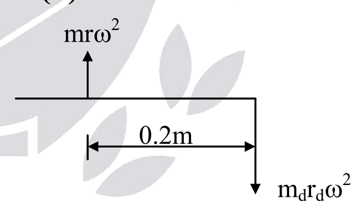
$\Rightarrow m_d\times10$ = 100 kg-cm

m_d = 10kg cm

$\theta_d = 180 + 30 = 210$

07. Ans: (d)

Sol:



mr = 100kg-cm = 1kgm

$N = 600 \text{ rpm} \Rightarrow \omega = \dfrac{2\pi N}{60} = 20\pi \text{rad/s}$

Couple 'C' $= mr\omega^2 \times 0.2 = 1\times(20\pi)^2\times0.2$

$= 789.56$ Nm

Reaction on the bearing

$$= \frac{\text{couple}}{\text{distance between bearing}}$$

$$= \frac{789.56}{0.4} = 1973.92\text{N}$$

08. Ans: (a)

Sol:

Plane	m (kg)	r (m)	L (m) (reference Plane A)	θ	F_x (mrcosθ)	F_y (mrsinθ)	C_x (mr*l*cosθ)	C_y (mr*l*sinθ)
D	2 kg.m		0.3	0	2	0	0.6	0
A	$-m_a$	0.5m	0	θ_a	$-0.5m_a\cos\theta_a$	$-0.5m_a\sin\theta_a$	0	0
B	$-m_b$	0.5m	0.5	θ_b	$-0.5m_b\cos\theta_b$	$-0.5m_b\sin\theta_b$	$-\frac{m_b}{4}\cos\theta_b$	$-\frac{m_b}{4}\sin\theta_b$

$C_x = 0 \Rightarrow \frac{m_b\cos\theta_b}{4} = 0.6$

$C_y = 0 \Rightarrow \frac{m_b\sin\theta_b}{4} = 0$

$\Rightarrow m_b = 2.4\text{kg}, \quad \theta_b = 0$

$\Sigma F_x = 0$

$\Rightarrow 2 - 0.5\, m_a\cos\theta_a - 0.5\, m_b\cos\theta_b = 0$

$\Rightarrow \frac{m_a}{2}\cos\theta_a = 0.8$

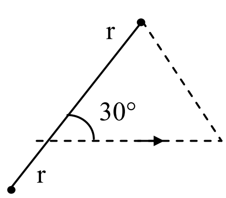
$\Sigma F_y = 0 \Rightarrow \frac{m_a}{2}\sin\theta_a = 0$

$\therefore \theta_a = 0°, \; m_a = 1.6$ kg

(Note: mass is to be removed so that is taken as –ve).

09. Ans: 30 N

Sol:



$m_b = 6$ kg

Crank radius

= stroke/2 = 0.1 m,

ω = 10 rad/sec

Unbalanced force along perpendicular to the line of stroke = $m_b r\omega^2 \sin 30°$

$= 6 \times (0.1) \times (10)^2 \sin 30°$

= 30 N

10. Ans: (b)

Sol:

- Primary unbalanced force = $mr\omega^2\cos\theta$

  At θ = 0° and 180°, Primary force attains maximum.

  Secondary force = $\frac{mr\omega^2}{n}\cos 2\theta$ where n is obliquity ratio. As n > 1, primary force is greater than secondary force.
- Unbalanced force due to reciprocating mass varies in magnitude. It is always along the line of stroke.

11. Ans: (b)

Sol: In balancing of single-cylinder engine, the rotating balance is completely made zero and the reciprocating unbalance is partially reduced.

12. Ans: 2

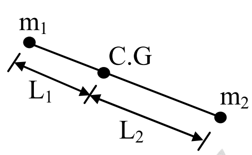
Sol: By symmetric two system is in dynamic balance when

$mea = m_1e_1a_1$

$$m_1 = m\frac{e}{e_1}.\frac{a}{a_1} = 1\times\frac{50}{20}\frac{2}{2.5} = 2\text{kg}$$

13. Ans: (a)

Sol:



$$m_1 = \frac{mL_2}{L_1+L_2} = \frac{100\times 60}{100} = 60\text{kg}$$

$$m_2 = \frac{mL_1}{L_1+L_2} = \frac{100\times 40}{100} = 40\text{kg}$$

$$I = m_1L_1^2 + m_2L_2^2$$

$= 60 \times 40^2 + 40\times 60^2$

$= 240000$ kg cm^2

$= 24$ kg m^2

Chapter 6 CAMS

01. Ans: (d)

Sol: Pressure angle is given by

$$\tan\phi = \frac{\frac{dy(\theta)}{d\theta} - e}{y(\theta) + \sqrt{(r_p)^2 - (e)^2}}$$

where, ϕ is pressure angle ,

θ is angle of rotation of cam

e is eccentricity

r_p is pitch circle radius

y is follower displacement

02. Ans: (d)

Sol: Cycloidal motion

$$y = \frac{h}{2\pi}\left(\frac{2\pi\theta}{\phi} - \sin\left(\frac{2\pi}{\phi}\theta\right)\right)$$

$$\dot{y}_{max} = \frac{2h\omega}{\phi} \qquad \text{------(1)}$$

Simple harmonic motion :

$$\dot{y}_{max} = \left(\frac{\pi}{2}\frac{h\omega}{\phi}\right) \qquad \text{------(2)}$$

Uniform velocity :

$$\dot{y} = \frac{h\omega}{\phi} \qquad \text{------(3)}$$

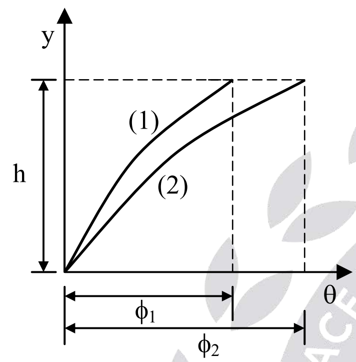
From (1), (2) and (3) we observe that

$$V_{cyclodial} > V_{SHM} > V_{UV}$$

03. Ans: (b)

Sol: $\tan\beta = \dfrac{V_{follower}}{(r_b + y)\omega} = \dfrac{\left(\dfrac{dy}{d\theta}\right)}{(r_b + y)}$

If the lift of follower, h = constant and angle of action ϕ increases.



From the diagram $\left(\dfrac{dy}{d\theta}\right)_1 > \left(\dfrac{dy}{d\theta}\right)_2$

As h = constant, if ϕ increases, $\dfrac{dy}{d\theta}$ decreases. So, ϕ decreases.

04. Ans: (b)

Sol: L = 4 cm , $\phi = 90^o = \pi/2$ radian ,

$\omega = 2$ rad/sec , $\theta = \dfrac{2}{3}\times 90 = 60°$

$\dfrac{\theta}{\phi} = \dfrac{2}{3}$

$$s(t) = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\phi}\right)$$

$$= 2(1 - \cos 120) = 3\text{cm}$$

$$V(t) = \frac{L}{2}\times\frac{\pi}{\phi}\times\omega\times\sin\left(\frac{\pi\theta}{\phi}\right)$$

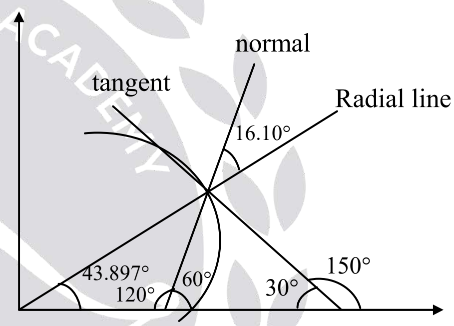
$$= \frac{4}{2}\times 2\times 2\sin(120) = 7\text{cm/s}$$

$$a(t) = \frac{L}{2}\left(\frac{\pi}{\phi}\right)^2\times\omega^2\times\cos\left(\frac{\pi\theta}{\phi}\right)$$

$$= \frac{4}{2}\times 2^2\times 2^2\times\cos(120) = -16\text{cm/sec}^2$$

05. Ans: (b)

Sol:



$x = 15\cos\theta$,

$y = 10 + 5\sin\theta$

$$\tan\phi = \frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\left(\dfrac{dx}{d\theta}\right)} = \frac{5\cos\theta}{-15\sin\theta}$$

at $\theta = 30°$,

$$\tan\phi = \frac{5\times\frac{\sqrt{3}}{2}}{-15\times\frac{1}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow \phi = 150°$$

$$\tan\theta = \frac{y}{x} = \frac{10 + 5\sin\theta}{15\cos\theta} = \frac{10 + 5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

Pressure angle is angle between normal and radial line = 16.10° .

or $x = 15\cos\theta$,

$y = 10 + 5\sin\theta$ at $\theta = 30^\circ$

$$\left(\frac{x}{15}\right)^2 + \left(\frac{y-10}{5}\right)^2 = 1$$

$$x = \frac{15\sqrt{3}}{2}, \quad y = 125$$

$$\frac{2x}{15^2} + \frac{2(y-10)}{5^2}.\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{(y-10)9} = \frac{-15\sqrt{3}}{2\left(\frac{3}{2}\right)\times 9} = \frac{-1}{\sqrt{3}}$$

$$\tan\theta = \frac{-1}{\sqrt{3}}$$

Then normal makes with x-axis

$\tan^{-1}(\sqrt{3}) = 60^\circ$

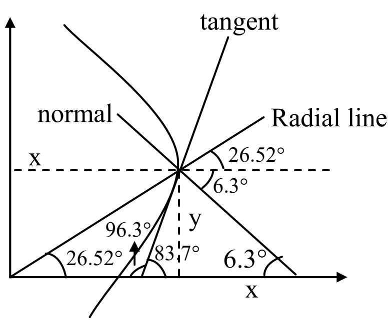
$$\tan\theta = \frac{y}{x} = \frac{10+5\sin\theta}{15\cos\theta} = \frac{10+5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

With follower axis angle made by normal (pressure angle) = 60°–43.897° = 16.10°

06. Ans: (a)

Sol:



Let α be the angle made by the normal to the curve

$$\left(\frac{dy}{dx}\right)_{(4,2)} = 9$$

$$\tan\alpha = \frac{dy}{dx} = 4x - 7$$

At x = 4 & y = 2 ,

$\alpha = \tan^{-1}(9) = 83.7°$

The normal makes an angle

$$= \tan^{-1}\left(\frac{-1}{9}\right) = 6.3° \text{ with x axis}$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.52°$$

Pressure angle is angle between normal and radial line = 26.52 + 6.3 = 32.82°

07. Ans: (b)

Sol: For the highest position the distance between the cam center and follower

= (r + 5) mm

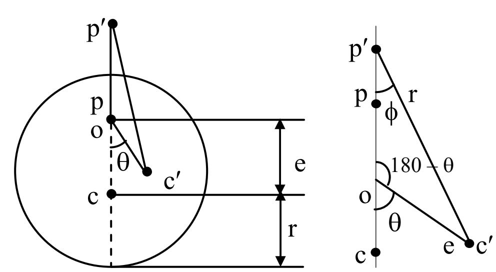
For the lowest position it is (r – 5) mm

So the distance between the two positions

= (r + 5) – (r – 5) = 10 mm

08. Ans: (a)

Sol:



When 'c' move about 'o' through 'θ', point 'p' moves to p′. 'ϕ' is angle between normal drawn at point of contact which always passes through centre of circle and follower axis. So this is pressure angle.

From Δle p′oc′

$$\frac{r}{\sin(\pi-\theta)}=\frac{e}{\sin\phi}$$

$$\sin\phi=\frac{e}{r}\sin\theta$$

ϕ is maximum θ = 90°

$$\sin\phi=\frac{e}{r}$$

Pressure angle s maximum at pitch point

$$\phi=\sin^{-1}\left(\frac{e}{r}\right)=30°$$

09. Ans: 48

Sol: Equation for displacement (for flat-face follower) is given by $y = 4(2\pi\theta - \theta^2)$

Radius of curvature, $r \not< 40$ mm

$$\rho = y + r_b + \frac{d^2y}{d\theta^2}$$

$$\rho_{min} = \left(y + r_b + \frac{d^2y}{d\theta^2}\right)_{min}$$

$$\frac{dy}{d\theta} = 4(2\pi - 2\theta)$$

$$\frac{d^2y}{d\theta^2} = -8$$

$\Rightarrow y_{min} = y(0) = 4[2\pi(0) - (0)^2] = 0$

$\Rightarrow 40 = [0 + (r_b)_{min} - 8]$

$\therefore$ Minimum base radius, $(r_b) = 48$ mm

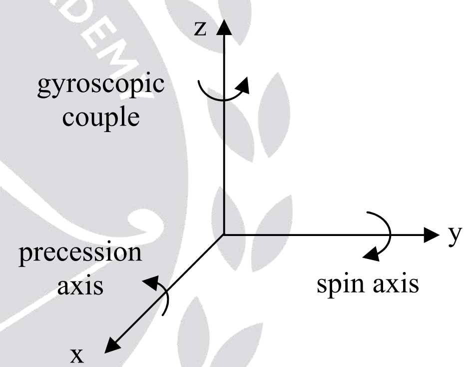
Chapter 7	GYROSCOPE

01. Ans: (c)

Sol: Due to Gyroscopic couple effect and centrifugal force effect the inner wheels tend to leave the ground.

02. Ans: (d)

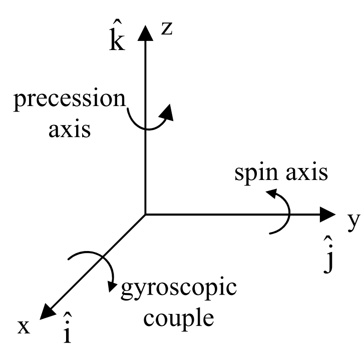
Sol: Pitching is angular motion of ship about transverse axis.



Due to pitching gyroscopic couple acts about vertical axis.

03.

Sol: m = 1000 kg, $r_k = 200$ mm



$I = 1000 \times (0.2)^2 = 40$ kg-m^2

N = 5000 rpm (CCW) looking from stern

$$\omega = \frac{2\pi \times 5000}{60} = 523.6 \text{ rad/s}$$

$$\vec{\omega} = -523.6\,\hat{j}$$

Precession velocity

$$\omega_p = \frac{V}{r} = \frac{25 \times 0.514}{400} = 0.032125 \text{ rad/s}$$

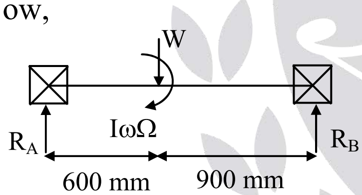
$$\vec{\omega}_p = 0.0312\,\hat{k}$$

Gyroscopic couple = $I(\vec{\omega} \times \vec{\omega}_p)$

$$G = 40(-523.6\,\hat{j} \times 0.032125\,\hat{k})$$

$$= -672.826\ \hat{i} \text{ N-m} = 672.826 \text{ N-m(CW)}$$

Now,



$$R_A + R_B = mg = 9810 \text{ N}$$

$$\sum M_A = 0$$

$$R_B \times 1.5 - 672.826 - 9810 \times 0.6 = 0$$

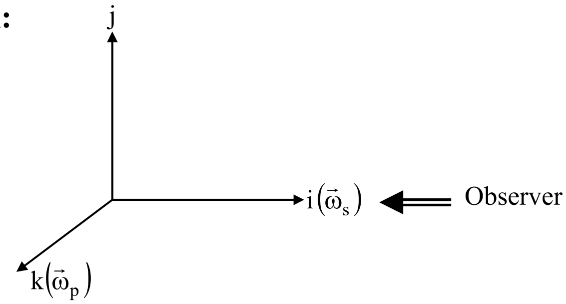
$$R_B = 4372.54 \text{ N}$$

$$R_A = 5437.45 \text{ N}$$

The bow of ship tends to fall because of gyroscopic action.

04. Ans: 1681 N-m

Sol:



k = 220 mm, m = 210 kg

$I = 210 \times (0.22)^2 = 10.164$ kg-m^2

$$\omega_s = \frac{2\pi \times 1800}{60} = 1884.95 \text{ rad/s}$$

$$\omega_p = \frac{1200 \times \frac{5}{18}}{3800} = 0.0877 \text{ rad/s}$$

$M = I\,\omega_s\,\omega_p$

$= 10.164 \times 0.0877 \times 1884.95$

$= 1681$ N-m

$\vec{\omega}_S$ is acting towards nose (positive x-axis)

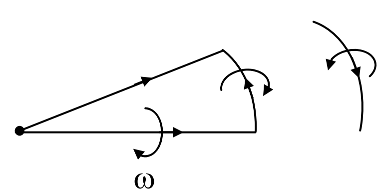
$\vec{\omega}_p$ is out the plane of paper (positive z-axis)

$\vec{\omega} \times \vec{\omega}_p$ is acting towards negative y-axis (right side). So the pilot will turn left side in order to keep aircraft in vertical plane.

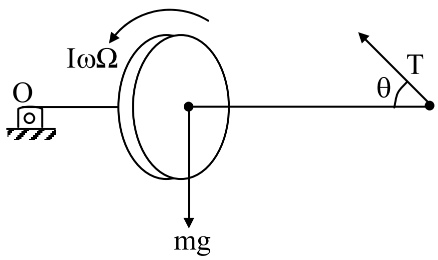
05. Ans:

Sol:

(i)



The gyroscopic couple is = IωΩ



$\sum M_o = 0$

$2a.T\sin\theta + I\omega\Omega = mg \times a$

$$\frac{2a.T.b}{\sqrt{4a^2+b^2}} + \frac{mr^2}{2}\omega\Omega = mg \times a$$

$$T = \frac{\sqrt{4a^2+b^2}}{2ab}\left(mga - \frac{mr^2}{2}\omega\Omega\right)$$

For clockwise rotation of precession

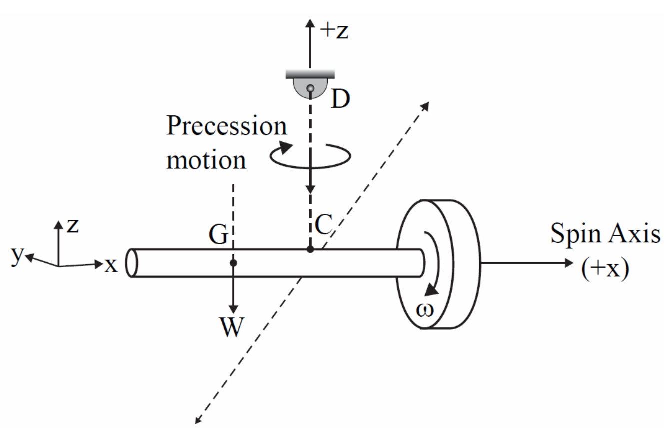
(ii) $\sum M_o = 0$

$2a.T\sin\theta - I\omega\Omega = mg \times a$

$$T = \frac{\left(mga + \frac{1}{2}mr^2\omega\Omega\right)\left(b^2+4a^2\right)^{\frac{1}{2}}}{2ab}$$

06. Ans: (a)

Sol:



The spin vector will chase the couple or torque vector and produces precession motion in system. Hence precession will be –y direction and due to gyroscopic effect the shaft will rotate about negative z-axis.

07. Ans: (a, b, d)

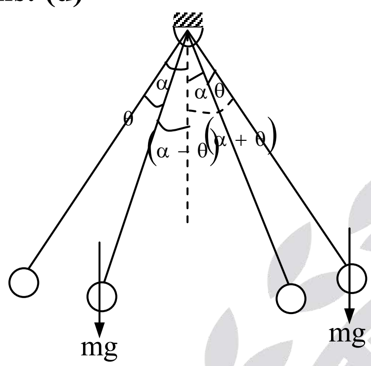
Sol: Gyroscopic couple $= I.(\omega \times \omega_P)$

Chapter 8

MECHANICAL VIBRATIONS

01. Ans: (d)

Sol:



Let the system is displaced by θ from the equilibrium position. It's position will be as shown in figure.

By considering moment equilibrium about the axis of rotation (Hinge)

$$I\ddot{\theta} + mg\ell\sin(\alpha+\theta) - mg\ell\sin(\alpha-\theta) = 0$$

$$I = m\ell^2 + m\ell^2 = 2m\ell^2$$

After simplification

$$2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha\ \sin\theta = 0$$

For small oscillations (θ is small) $\sin\theta = \theta$

$$\therefore 2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha.\theta = 0$$

$$\omega_n = \sqrt{\frac{2mg\ell\cos\alpha}{2m\ell^2}} = \sqrt{\frac{g\cos\alpha}{\ell}}$$

02. Ans: (c)

Sol: Let, V_o is the initial velocity,

'm' is the mass

Equating Impulse = momentum

$$mV_o = 5kN \times 10^{-4}\sec$$

$$= 5\times10^3\times10^{-4} = 0.5\sec$$

$$\therefore V_0 = \frac{0.5}{m} = 0.5\,m/\sec$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10000}{1}} = 100\,rad/\sec$$

When the free vibrations are initiate with initial velocity,

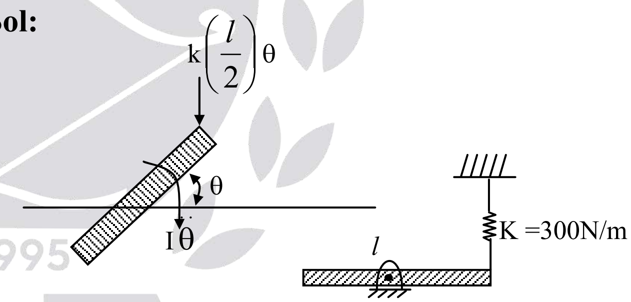
The amplitude

$$X = \frac{V_0}{\omega_n}\ \text{(Initial displacement)}$$

$$\therefore X = \frac{V_0}{\omega_n} = \frac{0.5\times10^3}{100} = 5\,mm$$

03. Ans: (c)

Sol:



By energy method

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}Kx^2 = \text{constant}$$

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}K\times\left(\frac{\ell}{2}\theta\right)^2 = \text{constant}$$

Differentiating w.r.t 't'

$$\frac{dE}{dt} = I\ddot{\theta} + \frac{K}{2}\times\frac{\ell^2}{4}\times 2\theta = 0$$

$$I = \frac{m\ell^2}{12}$$

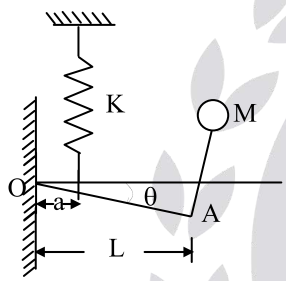
$$\frac{m\ell^2}{12}\ddot{\theta} + \frac{K\ell^2}{4}\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{3K}{m}\theta = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{3K}{m}} = 30\,\text{rad/sec}$$

04. Ans: (a)

Sol:



Assume that in equilibrium position mass M is vertically above 'A'. Consider the displaced position of the system at any instant as shown above figure.

If Δ_{st} is the static extension of the spring in equilibrium position, its total extension in the displaced position is $(\Delta_{st} + a\theta)$.

From the Newton's second law, we have

$$I_0\ddot{\theta} = Mg(L + b\theta) - k(\Delta_{st} + a\theta)a \;...(1)$$

But in the equilibrium position

$MgL = k\Delta_{st}\,a$

Substituting the value in equation (1), we have $I_0\ddot{\theta} = (Mgb - ka^2)\theta$

$$\Rightarrow I_0\ddot{\theta} + (ka^2 - Mgb)\theta = 0$$

$$\omega_n = \sqrt{\frac{ka^2 - Mgb}{I_0}}$$

$$\tau = 2\pi\sqrt{\frac{I_0}{ka^2 - Mgb}}$$

The time period becomes an imaginary quantity if $ka^2 < Mgb$. This makes the system unstable. Thus the system to vibrate the limitation is

$$ka^2 > Mgb$$

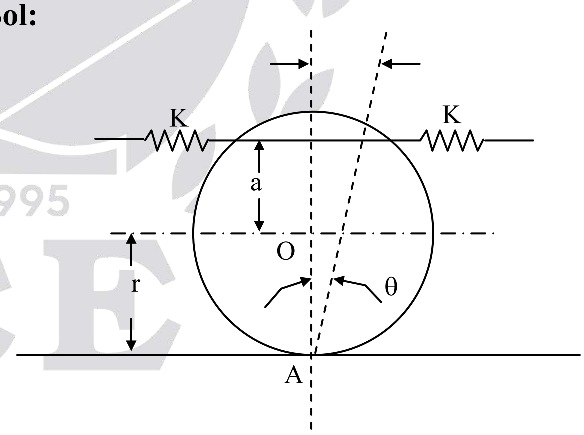
$$b < \frac{ka^2}{Mg}$$

Where W = Mg

05. Ans: (a)

06.

Sol:



$$KE = \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2$$

$$= \frac{3}{4}mr^2\dot{\theta}^2$$

$$PE = \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2 = Kx^2$$

$x = (r + a)\theta$

$\Rightarrow PE = K\{(r + a)\theta\}^2$

$$\frac{d}{dt}KE + \frac{d}{dt}PE = 0$$

Substituting in the above equation

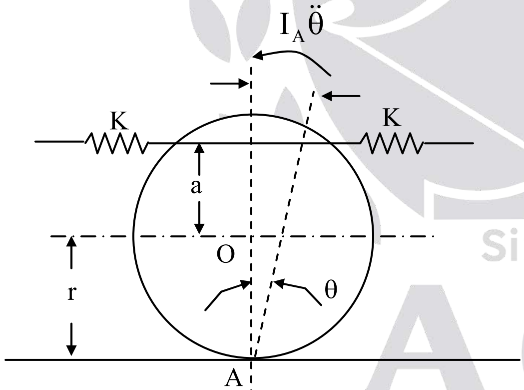
$$\frac{3}{2}mr^2\ddot{\theta} + 2K(r+a)^2\,\theta = 0$$

Natural frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{4K(r+a)^2}{3mr^2}}$$

So $f_n = 47.74$ Hz.

OR



Taking the moment about the instantaneous centre 'A'.

$I_A\ddot{\theta} + 2K\,(r+a)\,\theta\,(r+a) = 0$

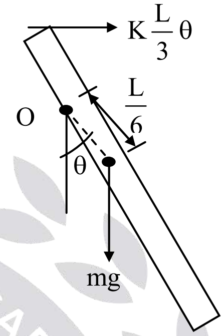
$$I_A = \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

$$\frac{3}{2}mr^2\ddot{\theta} + 2k(r+a)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{2k(r+a)^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$$

07. Ans: (b)

Sol:



By considering the equilibrium about the pivot 'O'

$$I_O\ddot{\theta} + mg\times\frac{L}{6}\sin\theta + K\frac{L}{3}\theta\times\frac{L}{3} = 0$$

$$\frac{mL^2}{9}\ddot{\theta} + \left(mg\times\frac{L}{6} + \frac{KL^2}{9}\right)\theta = 0 \quad (\because \sin\theta \approx \theta)$$

$$\omega_n = \sqrt{\frac{mg\times\frac{L}{6} + \frac{KL^2}{9}}{\frac{mL^2}{9}}} \Rightarrow \omega_n = \sqrt{\frac{3g}{2L} + \frac{K}{m}}$$

08. Ans: (d)

Sol: $X_0 = 10$ cm, $\omega_n = 5$ rad/sec

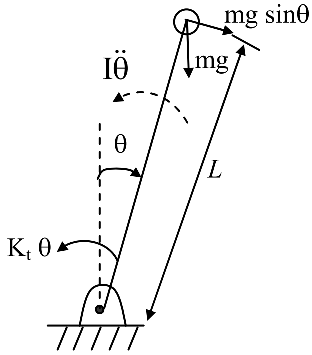
$$X = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

If $v_0 = 0$ then $X = x_0$

$\therefore X = x_0 = 10$cm

09. Ans: (c) & 10. Ans: (c)

Sol:



$I = mL^2$

The equation of motion is

$$mL^2\ddot{\theta} + (k_t - mgL)\theta = 0$$

Inertia torque = mL^2

Restoring torque = $k_t - mgL\sin\theta$

$= (k_t - mgL)\theta$

11. Ans: 0.0658 N.m²

Sol: For a Cantilever beam stiffness,

$$K = \frac{3EI}{\ell^3}$$

Natural frequency,

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3EI}{m\ell^3}}$$

Given $f_n = 100$ Hz

$\Rightarrow \omega_n = 2\pi f_n = 200\pi$

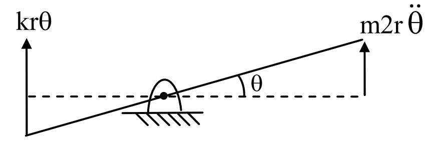
$$200\pi = \sqrt{\frac{3EI}{m\ell^3}}$$

Flexural Rigidity

$$EI = \frac{(200.\pi)^2.m\ell^3}{3} = 0.0658 \text{ N.m}^2$$

12. Ans: (d)

Sol: Free body diagram



Moment equilibrium about hinge

$$m2r\ddot{\theta}.2r + k\theta.r = 0$$

$$4mr^2\ddot{\theta} + kr^2\theta = 0$$

$$\omega_n = \sqrt{\frac{kr^2}{4mr^2}} = \sqrt{\frac{k}{4m}} = \sqrt{\frac{400}{4}}$$

13. Ans: 10 (Range 9.9 to 10.1)

Sol: $KE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2$

$m = 5$ kg, $\theta = \frac{x}{r}$

$$I = \frac{20\times r^2}{2} = 10r^2$$

$$KE = \frac{1}{2}5\dot{x}^2 + \frac{1}{2}10r^2.\frac{\dot{x}^2}{r^2} = \frac{1}{2}(15)\dot{x}^2$$

$\therefore m_{eq} = 15$

$$PE = \frac{1}{2}kx^2$$

$\therefore k_{eq} = k = 1500\text{N/m}$

Natural frequency

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{1500}{15}} = 10\text{rad/sec}$$

14. Ans: (a)

Sol: Take the reference position as 'B'

$$(PE)_A = \frac{1}{2}k\left(\delta^2 + mgx\right)$$

$$(PE)_B = \frac{1}{2}k(\delta + x)^2$$

Change in potential energy:

$$= \frac{1}{2}k(\delta + x)^2 - \frac{1}{2}\times k\times\delta^2 - mgx$$

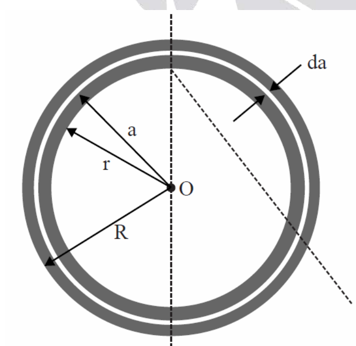
$$= \frac{1}{2}\times k\times x^2 + k\,\delta\,x - mgx$$

$$= \frac{1}{2}\times k\times x^2 + m\,g\,x - mgx \quad [\because k\delta = mg]$$

$$= \frac{1}{2}\times k\,x^2$$

15. Ans: 2.65

Sol:



Considering the differential ring mass of differential ring,

$$dm = \frac{\text{Mass of ring}}{\text{Volume of ring}}(dV)$$

$$dm = \frac{M}{\pi\left(R^2 - r^2\right)t}\times 2\pi a da\times t$$

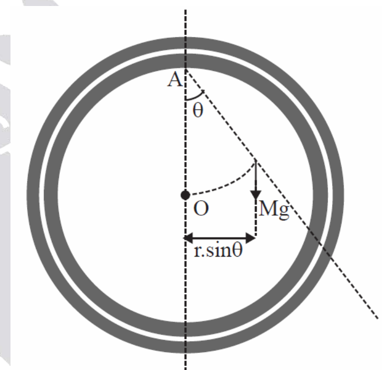
$$dm = \frac{2M\,ada}{\left(R^2 - r^2\right)}$$

$$I_o = \bar{I} = \int a^2\,dm = \int_r^R \frac{2Ma^3\,da}{\left(R^2 - r^2\right)}$$

$$\bar{I} = \frac{2M}{\left(R^2 - r^2\right)}\times\left[\frac{a^4}{4}\right]_r^R = \frac{M}{2}\frac{\left(R^4 - r^4\right)}{\left(R^2 - r^2\right)} = \frac{M}{2}\left(R^2 + r^2\right)$$

$$I = \bar{I} + md^2 = \frac{M}{2}\left(R^2 + r^2\right) + Mr^2 = 3$$

$$= \frac{3M}{4}\left(R^2\right) + Mr^2 = \frac{5MR^2}{4}$$



Taking moments about A

$\Sigma M_A = 0$

$$I\ddot{\theta} + Mg(r\sin\theta) = 0$$

$\sin\theta \cong \theta$

$$\frac{5MR^2}{4}\ddot{\theta} + Mg\left(\frac{R}{\sqrt{2}}\right)\theta = 0$$

$$\ddot{\theta} + \left(\frac{4g}{5\sqrt{2}\,R}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{4g}{5\sqrt{2}\,R}}$$

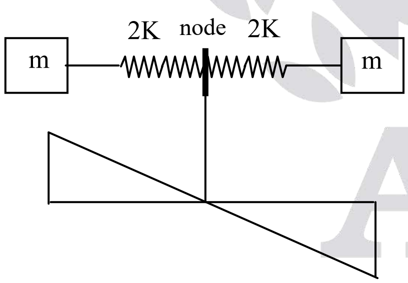
$$T = \frac{2\pi}{\omega_n} = \pi\left(2\times\sqrt{\frac{5\sqrt{2}}{5}}\times\sqrt{\frac{R}{g}}\right)$$

$\beta = 2.65$

16. Ans: (a)

Sol: The given system is a 2 D.O.F one without constraints and exhibits a rigid body motion for which the frequency is zero. The node shape corresponding to the non zero frequency is as shown in figure. As the masses are equal in both sides, the node will be at the middle. By fixing the spring at the node we can separate into two single D.O.F systems and both will have same natural frequency. As the node falls in the middle of the spring, the spring is divided into two equal halves and each will have stiffness of 2K. So the frequency for each system is equal to $\sqrt{\frac{2K}{m}}$.

Hence the frequencies for the given system are 0 and $\sqrt{\frac{2K}{m}}$



Alternative Method:

For the above diagram the equation can be written as

$$m_1\ddot{x}_1 + K(x_1 - x_2) = 0$$

$$m_2\ddot{x} + K(x_2 - x_1) = 0$$

Assuming the solution of the form.

$$x_1 = A_1 \sin\omega t$$

$$x_2 = A_2 \sin\omega t$$

$$\Rightarrow -m_1\omega^2 A_1 + K(A_1 - A_2) = 0$$

$$\Rightarrow -m_2\omega^2 A_2 + K(A_2 - A_1) = 0$$

Amplitude ratio

$$\frac{A_1}{A_2} = \frac{K}{K - m_1\omega^2} = \frac{K - m_2\omega^2}{K}$$

$$\Rightarrow \frac{K}{K - m_1\omega^2} = \frac{K - m_2\omega^2}{K}$$

$$\Rightarrow -K(m_2\omega^2 + m_1\omega^2) + m_1 m_2\omega^4$$

$$\Rightarrow m_1 m_2\left\{\omega^4 - K\omega^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right\} = 0$$

……$(m_1 = m_2 = m)$

Solving this equation we get $\omega_1 = 0$ and

$$\omega_2 = \sqrt{\frac{2K}{m}}$$

17. Ans: (b)

Sol: When the centre of the disk is displaced by x, then the energy of the system is written as

E = Energy of both springs + Translational kinetic energy of disk + Rotational kinetic energy of disk

$$E = \frac{1}{2}\,2k\left(\frac{x}{2}\right)^2 + \frac{1}{2}k(2x)^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

[∵ If centre of disk is displaced by x then spring of stiffness 2k will deflect by x/2]

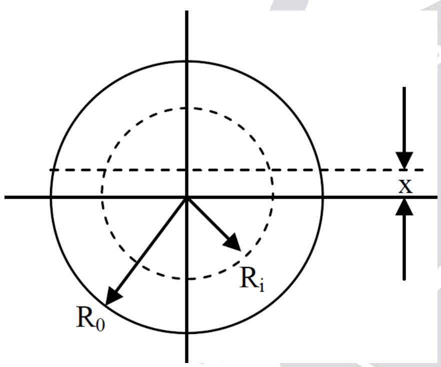
$$E = \frac{9kx^2}{4} + \frac{3}{4}mv^2 \quad \left[I = \frac{mr^2}{2} \;\&\; \omega = \frac{v}{r}\right]$$

$$\frac{dE}{dt} = \frac{9}{4}k.2x\frac{dx}{dt} + \frac{3}{4}m.2v\frac{dv}{dt} = 0$$

$$\Rightarrow a = -\frac{3kx}{m} \quad \Rightarrow \omega = \sqrt{\frac{3k}{m}}$$

18. Ans: 8.66

Sol: When the ball is displaced by small distance 'x' in vertical direction then the displaced volume is changed by $\pi R_o^2 x$ as shown in figure.



This leads to unbalanced buoyancy force of $\rho\pi R_o^2$ xg. The unbalanced buoyancy force tries to restore the ball in equilibrium position.

∴ Restoring force per unit displacement

$$= \frac{\rho\pi R_o^2 x g}{x}$$

i.e., $K = \rho\pi R_o^2$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{\rho\pi R_o^2 g}{\rho_s \times \frac{4}{3}\pi(R_o^3 - R_i^3)}} \quad \text{......(1)}$$

In equilibrium position weight = buoyancy force

$$\rho_s \times \frac{4}{3}\pi(R_o^3 - R_i^3) \times g = \rho \times \frac{4}{3}\pi R_o^3 \times \frac{1}{2} \times g$$

$$\rho_s \times \frac{4}{3}\pi(R_o^3 - R_i^3) \times g = \frac{2}{3}\rho\pi R_o^3 \quad \text{..... (2)}$$

Substitute in equation (1)

$$\omega_n = \sqrt{\frac{\rho\pi R_o^2 \times g}{\frac{2}{3} \times \pi R_o^3}} = \sqrt{\frac{3}{2} \times \frac{g}{R_o}}$$

$$= \sqrt{\frac{3}{2} \times \frac{10}{0.2}} = 8.66 \text{ rad/s}$$

19. Ans: (b)

Sol: In damped free vibrations the oscillatory motion becomes non-oscillatory at critical damping.

Hence critical damping is the smallest damping at which no oscillation occurs in free vibration

20. Ans: (a)

Sol: $\omega_n = 50 \text{ rad/sec} = \sqrt{\frac{5}{m}}$

If mass increases by 4 times

$$\omega_{n_1} = \sqrt{\frac{k}{4m}} = \frac{1}{2} \times \sqrt{\frac{k}{m}} = \frac{50}{2} = 25 \text{ rad/sec}$$

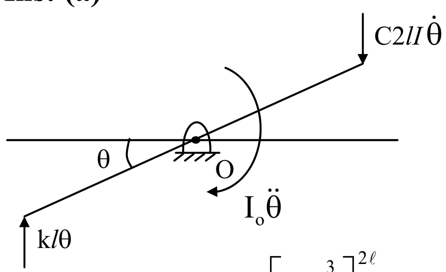
Damped frequency natural frequency,

$$\omega_d = \sqrt{1 - \xi^2} \times \omega_n$$

$$\Rightarrow 20 = \sqrt{1 - \xi^2} \times 25 = 0.6 = 60\%$$

21. Ans: (a)

Sol:



For slender rod, $I_o = \left[\rho \frac{x^3}{3}\right]_{-\ell}^{2\ell}$

$$= \frac{\rho}{3}\times\left(8\ell^3 + \ell^3\right) = \frac{9\rho\ell^3}{3} = 3\rho\ell^3 = m\ell^2$$

Where, $\rho = m/3l$

Considering the equilibrium at hinge 'O'.

$I_o\ \ddot{\theta} + c2l\dot{\theta}\times 2l + kl\theta\times l = 0$

$\Rightarrow\ ml^2\ddot{\theta} + 4l^2c\ \dot{\theta} + kl^2\theta = 0$

$I_{equivalent} = ml^2,\ C_{eq} = 4l^2c,\ k_{eq} = kl^2$

22. Ans: (b)

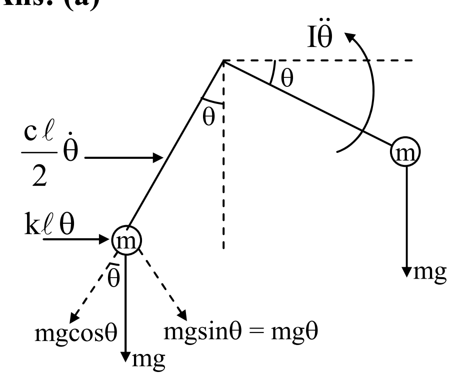
Sol: Damping ratio, $\xi = \frac{c}{c_c} = \frac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}}$

$$= \frac{4\ell^2 c}{2\times\sqrt{k\ell^2\times m\ell^2}}$$

$$= \frac{4\ell^2 c}{2\times\sqrt{mk\ell^4}} = \frac{2c}{\sqrt{km}}$$

23. Ans: (a)

Sol:



$I = m(2l)^2 + ml^2 = 5ml^2$

The equation motion is

$$\left(m\times(2\ell)^2 + m\ell^2\right)\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$= 5m\ell^2\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{k\ell^2 + mg\ell}{5m\ell^2}}$$

$$= \sqrt{\frac{400}{5\times 10}} = 3.162\ \text{rad/s}$$

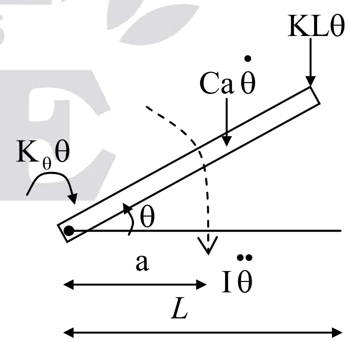
24. Ans: (a)

Sol: $\xi = \frac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}} = \frac{\left(\frac{c\ell^2}{4}\right)}{2\sqrt{(k\ell^2 + mg\ell)\times 5m\ell^2}}$

$$= \frac{\frac{400\times 1^2}{4}}{2\sqrt{(400\times 1^2 + 10\times 9.81\times 1)\times 5\times 10\times 1^2}} = 0.316$$

25. Ans: (a)

Sol:



By moment equilibrium

$I\ddot{\theta} + Ca^2\dot{\theta} + KL^2\theta + K_\theta\theta = 0$

$$\frac{mL^2}{3}\ddot{\theta} + Ca^2\dot{\theta} + \left(KL^2 + K_\theta\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{KL^2 + K_\theta}{mL^2/3}}$$

$$\omega_n = \sqrt{\frac{1500}{0.833}} = 42.26 \text{ rad/sec}$$

26. Ans: (c)

Sol: Refer to the above equilibrium equation

$$C_{eq} = Ca^2$$

$$= 500 \times 0.4^2 = 80 \frac{\text{N}-\text{m}-\text{sec}}{\text{rad}}$$

$\Rightarrow$ C = 80 Nms/rad

Note: For angular co-ordinate

Unit of Equivalent inertia = $\dfrac{\text{N}-\text{m}}{\text{rad/s}^2} = \text{kg}-\text{m}^2$

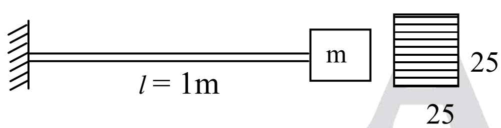
Unit of equivalent damping coefficient = $\dfrac{\text{N}-\text{m}}{\text{rad/s}}$

Unit of equivalent stiffness = N-m/rad

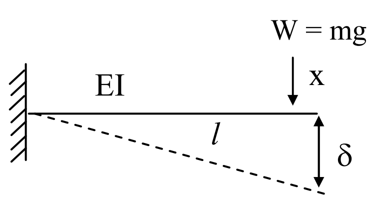
27. Ans: (a)

Sol: Given length of cantilever beam,

l = 1000 mm = 1m, m = 20 kg



Cross section of beam = square



Moment of inertia of the shaft,

$$I = \frac{1}{12} bd^3 = \frac{25 \times (25)^3}{12} = 3.25 \times 10^{-8} \text{ m}^4$$

$E_{steel} = 200 \times 10^9$ Pa

Mass, M = 20kg

Stiffness, $K = \dfrac{3EI}{\ell^3}$

Critical damping coefficient,

$$C_C = 2\sqrt{Km} = 1250 \text{ Ns/m}$$

28. Ans: (c)

Sol: In case of viscous damping drag force is

$$F_D = c\dot{x}$$

where, c = damping coefficient and

x_o = relative velocity.

Thus drag force is directly proportional to relative velocity.

29. Ans: (d)

Sol: x = 10 cm at $\dfrac{\omega}{\omega_n} = 1$;

$\xi = 0.1$

At resonance $x = \dfrac{x_0}{2\xi} = 10$ cm

$\Rightarrow x_0 = 2 \times 0.1 \times 10 = 2$ cm

x_0 = static deflection

At $\dfrac{\omega}{\omega_n} = 0.5$,

$$x = \frac{x_0}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$$

$$x = \frac{2}{\sqrt{\left[1-(0.5)^2\right]^2 + (2\times 0.1\times 0.5)^2}} = 2.64\,\text{cm}$$

30. Ans: (a)

Sol: $m\ddot{x} + Kx = F\cos\omega t$

$m = ?$

K = 3000 N/m,

X = 50 mm = 0.05 m

F = 100 N,

$\omega = 100\,\text{rad/sec}$

$$X = \frac{F}{K - m\omega^2}$$

$$\Rightarrow m = \frac{K}{\omega^2} - \frac{F}{X\omega^2} = 0.1\,\text{kg}$$

31. Ans: (a)

Sol: $\delta = ln\left(\frac{x_1}{x_2}\right) = ln\,2 = 0.693$

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$= \frac{0.693}{\sqrt{4\pi^2 + 0.693^2}} = 0.109$$

$$c = 2\xi\sqrt{k\,m} = 2\times 0.109\times\sqrt{100\times 1}$$

= 2.19 N-sec/m

32. Ans: (b)

Sol: x_{static} = 3 mm, ω = 20 rad/sec

As $\omega > \omega_n$

So, the phase is 180° .

$$-x = \frac{x_{static}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$x = \frac{3}{\sqrt{\left(1-\left(\frac{20}{10}\right)^2\right) + \left(2\times 0.109\times\frac{20}{10}\right)^2}}$$

= 1 mm opposite to F.

33. Ans: (c)

Sol: At resonance, magnification factor = $\frac{1}{2\xi}$

$$\Rightarrow 20 = \frac{1}{2\xi}$$

$$\Rightarrow \xi = \frac{1}{40} = 0.025$$

34. Ans: (c)

Sol: M = 100 kg, m = 20 kg, e = 0.5 mm

K = 85 kN/m, C = 0 or $\xi = 0$

$\omega = 20\pi$ rad/sec

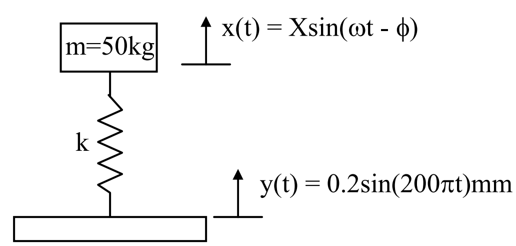
Dynamic amplitude

$$X = \frac{me\omega^2}{\pm\left(k - M\omega^2\right)} = \frac{20\times 5\times 10^{-4}\times(20\pi)^2}{\pm\left(8500 - 100\times(20\pi)^2\right)}$$

$= 1.27\times 10^{-4}$ m

35. Ans:

Sol:



$\omega = 200\pi$ rad/sec, $-X = 0.01$ mm

$Y = 0.2$ mm

$$\frac{X}{Y} = \frac{k}{k - m\omega^2}$$

$$\Rightarrow \frac{-0.01}{0.2} = \frac{k}{k - 50\times(200\pi)^2}$$

$$\Rightarrow \quad k = 939.96 \text{ kN/m}$$

36. Ans: (b)

Sol: $m = 5$ kg, $c = 20$,

$k = 80$, $F = 8$, $\omega = 4$

$$x = \frac{F}{(k - m\omega^2) + (c\omega)^2}$$

$$= \frac{8}{\sqrt{(80 - 5\times 4^2) + (20\times 4)^2}} = 0.1$$

$$\text{Magnification factor} = \frac{x}{x_{static}}$$

$$x_{static} = \frac{F}{k} = \frac{8}{80} = 0.1$$

$$\text{Magnification factor} = \frac{0.1}{0.1} = 1$$

37. Ans: (c)

Sol: Given, $m = 250$ kg, $K = 100,000$ N/m

$N = 3600$ rpm, $\xi = 0.15$

$$\omega_n = \sqrt{\frac{K}{m}} = 20 \text{ rad/sec}$$

$$\omega = \frac{2\pi\times N}{60} = 377 \text{ rad/sec}$$

$$TR = \frac{\sqrt{1 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} = 0.0162$$

38. Ans: (b)

Sol: Transmissibility (T) reduces with increase in damping up to the frequency ratio of $\sqrt{2}$. Beyond $\sqrt{2}$, T increases with increase in damping

39. Ans: 6767.7 N/m

Sol: Given: $f = 60$ Hz, $m = 1$ kg

$$\omega = 2\pi f = 120\pi \text{ rad/sec}$$

Transmissibility ratio, $TR = 0.05$

Damping is negligible, $C = 0$, $K = ?$

We know $TR = \dfrac{K}{K - m\omega^2}$ when $C = 0$

As TR is less than 1 $\Rightarrow \omega/\omega_n >> \sqrt{2}$

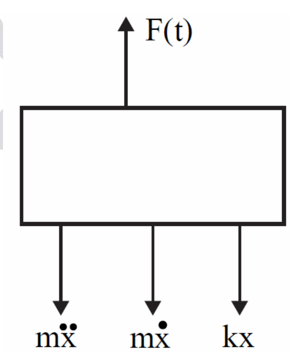
TR is negative

$$\therefore \quad -0.05 = \frac{K}{K - m\omega^2}$$

Solving we get $K = 6767.7$ N/m

40. Ans: (c)

Sol: FBD of mass



$$m\ddot{x} + c\dot{x} + kx = F(t)$$

Solution of differential equation

$x(t) = (C.F) + (P.I)$

Considering condition (P)

If C > 0 and $\omega = \sqrt{k/m}$

For this condition the displacement (x) is given by

[Assume the system to be under damped]

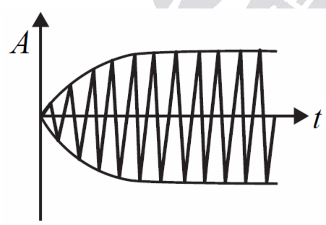
$$x(t) = X_o e^{-\xi\omega_n t}\cos(\omega_d t - \phi) + X\cos(\omega t - \phi)$$

Transient response + steady state response

As $t \rightarrow \infty$ the transient response decays to zero and only steady state response will remain

$$x(t) = X\cos(\omega t - \phi)$$

For this condition the response curve will be



Considering condition (Q)

$c < 0$ and $\omega \neq 0$

The differential equation becomes

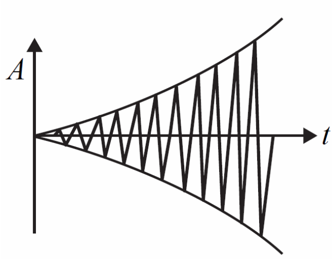
$$m\ddot{x} - c\ddot{x} + kx = F(t)$$

Solution of above differential equation is

$$x(t) = X_o e^{+c_1 t}\cos(\omega_d t - \phi) + X\cos(\omega t - \phi)$$

As $t \rightarrow \infty$ the transient response approaches to ∞ and increases exponentially

The plot will be



Considering condition (R)

$$C = 0\ ,\ \omega = \sqrt{\frac{k}{m}}\ \text{(Resonance)}$$

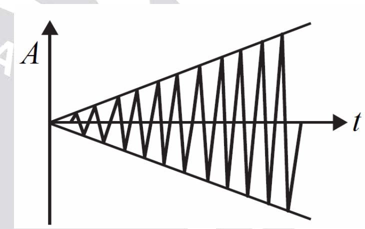
The differential equation is

$$m\ddot{x} + kx = F(t)$$

Solution for above differential equation

$$x(t) = x_o\cos\omega_n t + \frac{\dot{x}_o}{\omega_n}\sin\omega_n t + \underbrace{\frac{x_{static}\,\omega_n t}{2}\sin(\omega_n t)}_{\text{Increases with time linearly}}$$

So the correct plot will be



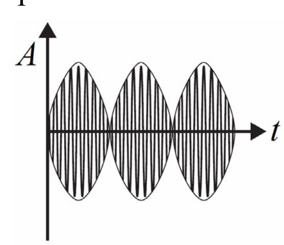
Considering condition (S)

$$c < 0 \text{ and } \omega \cong \sqrt{\frac{k}{m}}$$

If the force frequency is close to, but not exactly equal to, natural frequency of the system, a phenomenon is known as beating. In this kind of vibration the amplitude buils up and then diminishes in a regular pattern. The displacement can be expressed as

$$x(t) = \left(\frac{B}{m}\right)\left[\frac{\cos\omega t - \cos\omega_n t}{\omega_n^2 - \omega}\right]$$

The correct plot will be



41. Ans: (a)

Sol: When a vehicle travels on a rough road whose undulations can be assumed to be sinusoidal, the resonant conditions of the base-excited vibrations, are determined by the Mass of the vehicle, stiffness of the suspension spring, speed of the vehicle, wavelength of the roughness curve.

42. Ans: (b)

Sol: $e = 2\text{mm} = 2\times10^{-3}\text{m}$,

$\omega_n = 10$ rad/s,

$N = 300$ rpm

$$\omega = \frac{2\pi N}{60} = 10\pi\,\text{rad/sec}$$

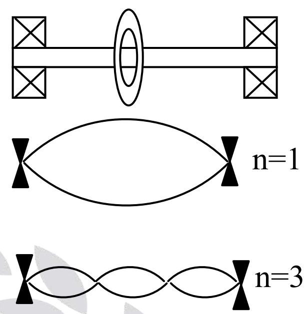
$$X = \frac{me\omega^2}{k - m\omega^2} = \frac{e\omega^2}{\left(\frac{k}{m}\right)^2 - \omega^2} = \frac{e\omega^2}{\omega_n^2 - \omega^2}$$

$$X = \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{2\times10^{-3}\times\left(\frac{10\pi}{10}\right)^2}{\pm\left(1-\left(\frac{10\pi}{10}\right)^2\right)}$$

$= 2.25\times10^{-3}$ m $= 2.25$ mm

43. Ans: (a)

Sol: Number of nodes observed at a frequency of 1800 rpm is 2



n-mode number

The whirling frequency of shaft,

$$f = \frac{\pi}{2}\times n^2\sqrt{\frac{gEI}{WL^4}}$$

For 1st mode frequency, $f_1 = \frac{\pi}{2}\times\sqrt{\frac{gEI}{WL^4}}$

$$f_n = n^2 f_1$$

As there are two nodes present in 3rd mode,

$$f_3 = 3^2 f_1 = 1800\,\text{rpm}$$

$$\therefore f_1 = \frac{1800}{9} = 200\,\text{rpm}$$

$\therefore$ The first critical speed of the shaft = 200 rpm