



GATE | PSUs



Electronics & Communication Engineering

ELECTROMAGNETICS

Text Book: Theory with worked out Examples and Practice Questions

Chapter 1

Static Fields

(Solutions for Text Book Practice Questions)

01. Ans: 1

Sol: $\vec{V} = x \cos^2 y \hat{i} + x^2 e^z \hat{j} + z \sin^2 y \hat{k}$

$$= x \cos^2 y \hat{a}_x + x^2 e^z \hat{a}_y + z \sin^2 y \hat{a}_z$$

From divergence theorem

$$\oint_V \vec{V} \cdot \hat{n} \, ds = \int_V (\nabla \cdot \vec{D}) \, dv \dots\dots\dots 1$$

$$\nabla \cdot \vec{D} = \frac{\partial}{\partial x}(x \cos^2 y) + \frac{\partial}{\partial y}(x^2 e^z) + \frac{\partial}{\partial z}(z \sin^2 y)$$

$$= \cos^2 y + \sin^2 y = 1$$

$$dv = dx dy dz$$

Putting these value in equation 1 we have

$$\oint_V \vec{V} \cdot \hat{n} \, ds = \int_0^1 \int_0^1 \int_0^1 1 \times dx dy dz$$

$$= \int_0^1 dx \int_0^1 dy \int_0^1 dz = 1$$

02. Ans: (c)

Sol: Given $\vec{A} = x y \vec{a}_x + x^2 \vec{a}_y$

Let $I = \oint \vec{A} \cdot d\vec{\ell}$, I is evaluated over the path shown in the Fig., as follows

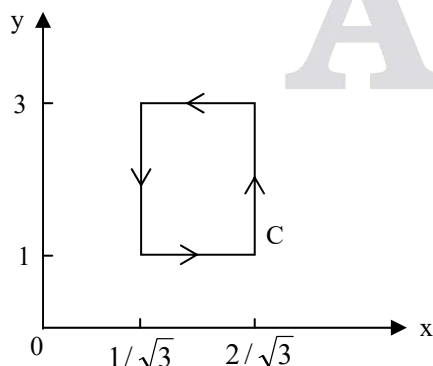


Fig.

$$I = \oint \vec{A} \cdot d\vec{\ell} = \int_{x=1/\sqrt{3}}^{x=2/\sqrt{3}} \vec{A} \cdot \vec{a}_x \, dx, y=1, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$+ \int \vec{A} \cdot d\vec{\ell} = \int_{y=1}^{y=3} \vec{A} \cdot \vec{a}_y \, dy, x = \frac{2}{\sqrt{3}}, y = \text{from } 1 \text{ to } 3$$

$$- \int \vec{A} \cdot d\vec{\ell} = \int_{x=1/\sqrt{3}}^{x=2/\sqrt{3}} \vec{A} \cdot \vec{a}_x \, dx, y=3, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$- \int \vec{A} \cdot d\vec{\ell} = \int_{y=1}^{y=3} \vec{A} \cdot \vec{a}_y \, dy, x=1/\sqrt{3}, y = \text{from } 1 \text{ to } 3$$

$$= \int x y \, dx + \int x^2 \, dy - \int x y \, dx - \int x^2 \, dy$$

$$= y \left. \frac{x^2}{2} \right|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2 y \Big|_1^3 - y \left. \frac{x^2}{2} \right|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2 y \Big|_1^3$$

$$\text{at } y=1 \quad x=2/\sqrt{3} \quad y=3 \quad x=1/\sqrt{3}$$

$$= \frac{1}{2} \left(\frac{4}{3} - \frac{1}{3} \right) + \frac{4}{3} (3-1) - \frac{3}{2} \left(\frac{4}{3} - \frac{1}{3} \right) - \frac{1}{3} (3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\vec{F} = \rho a_\rho + \rho \sin^2 \phi a_\phi - z a_z$
 $= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$

$$\nabla \cdot \vec{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho F_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (F_\phi) + \frac{\partial}{\partial z} (F_z)$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho^2) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho \sin^2 \phi) + \frac{\partial}{\partial z} (-z)$$

$$= 2 + 2 \sin \phi \cos \phi - 1$$

$$= 1 + 2 \sin \phi \cos \phi$$

$$\nabla \cdot \vec{F} \Big|_{\phi=\pi/4} = 2, \quad \nabla \cdot \vec{F} \Big|_{\phi=0} = 1$$

$$\nabla \cdot \vec{F} \Big|_{\phi=\pi/4} = 2 \nabla \cdot \vec{F} \Big|_{\phi=0}$$

04. Ans: (c)

Sol: $\vec{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_z$ $\vec{D} = |\vec{D}| \hat{a}_n$

$$|\vec{D}| = \sqrt{16} = 4$$

$$= \rho_s \hat{a}_n$$

$$\therefore \vec{D} = 4 \left\{ \frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_z}{4} \right\}$$

$$= \rho_s \hat{a}_n \quad \therefore \rho_s = 4 \text{ C/m}^2$$

05. Ans: (d)

Sol: $V = 10y^4 + 20x^3$

$$E = -\nabla V = -60x^2\hat{a}_x - 40y^3\hat{a}_y$$

$$D = \epsilon_0 E = -60x^2\epsilon_0\hat{a}_x - 40y^3\epsilon_0\hat{a}_y$$

$$\nabla \cdot D = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x}(-60x^2\epsilon_0) + \frac{\partial}{\partial y}(-40y^3\epsilon_0)$$

$$= -120x\epsilon_0 - 120y^2\epsilon_0$$

$$\rho_v(\text{at } 2, 0) = -120 \times 2\epsilon_0 - 120 \times 0^2\epsilon_0$$

$$= -240\epsilon_0$$

06. Ans: (d)

Sol: Given

$$V(x, y, z) = 50x^2 + 50y^2 + 50z^2$$

$$\vec{E}(x, y, z) \text{ in free space} = -\text{grad}(V)$$

$$= -\nabla V$$

$$= - \left[\frac{\partial}{\partial x} V \vec{a}_x + \frac{\partial}{\partial y} V \vec{a}_y + \frac{\partial}{\partial z} V \vec{a}_z \right]$$

$$= - \left[100x \vec{a}_x + 100y \vec{a}_y + 100z \vec{a}_z \right] \text{ V/m}$$

$$\vec{E}(1, -1, 1) =$$

$$- \left[100 \vec{a}_x - 100 \vec{a}_y + 100 \vec{a}_z \right] \text{ V/m}$$

$$E(1, -1, 1) = 100 \sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of $\vec{E}$.

$$\vec{a}_E = \frac{\vec{E}(1, -1, 1)}{|\vec{E}(1, -1, 1)|} = \frac{1}{\sqrt{3}} \left[-\vec{a}_x + \vec{a}_y - \vec{a}_z \right]$$

or in i, j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}} [-i + j - k]$

07. Ans: (b)

Sol: For valid B, $\nabla \cdot B = 0$

$$\left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z \right) (x^2 a_x - xy a_y - Kxz a_z) = 0$$

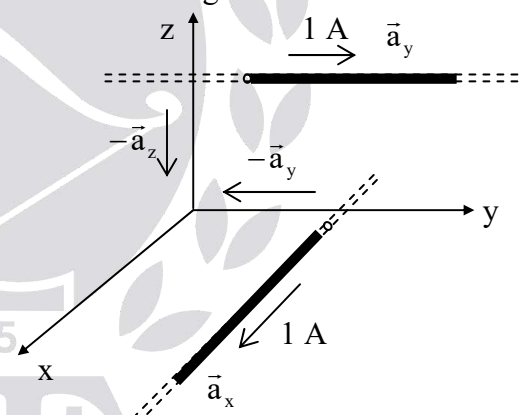
$$2x - x - Kx = 0$$

$$\Rightarrow 2 - 1 - K = 0$$

$$\therefore K = 1$$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the $\vec{a}_y$ direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the $\vec{a}_x$ direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors along the 'x', 'y' and 'z' axes respectively.

$\therefore$ x and z components of magnetic field are non-zero at the origin.

09. Ans: (a)

Sol: $\nabla \cdot \vec{B} = 0$

A divergence less vector may be a curl of some other vector

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{A} = \vec{B}$$

$$\oint_l \vec{A} \cdot d\vec{l} = \int_s \vec{B} \cdot d\vec{s}$$

$\int_s \vec{B} \cdot d\vec{s}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

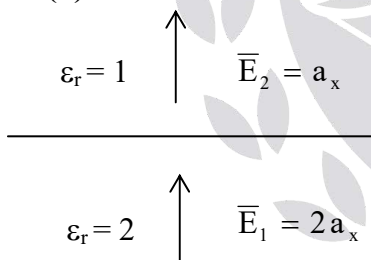
Sol: In general, for an infinite sheet of current density K A/m

$$H = \frac{1}{2} K \times \vec{a}_n$$

$$H = \frac{1}{2} (8\vec{a}_x \times \vec{a}_z) \\ = -4 \vec{a}_y \quad (\because \vec{a}_x \times \vec{a}_z = -\vec{a}_y)$$

11. Ans: (b)

Sol:



$$D_{n_2} - D_{n_1} = \rho_s \rightarrow (a)$$

$$D_{n_2} = \epsilon E_{n_2} = \epsilon_0 a_x$$

$$D_{n_1} = \epsilon_0 2 \times 2 a_x = 4\epsilon_0 a_x$$

From (a)

$$(\epsilon_0 - 4\epsilon_0) a_x = \rho_s \Rightarrow \rho_s = -3\epsilon_0$$

12. Ans: (a)

Sol:

$$\mu_{r_1} = 2 \quad \mu_{r_2} = 1$$

$$z = 0$$

$$B_1 = 1.2\vec{a}_x + 0.8\vec{a}_y + 0.4\vec{a}_z$$

$$B_{n_1} = 0.4\vec{a}_z$$

(Since $z = 0$ has normal component a_x)

$$B_{t_1} = 1.2\vec{a}_x + 0.8\vec{a}_y$$

We know magnetic flux density is continuous

$$B_{n_1} = B_{n_2}$$

$$B_{n_2} = 0.4\vec{a}_z$$

Surface charge, $\bar{k} = 0$

$$H_{t_2} - H_{t_1} = 0$$

$$H_{t_2} = H_{t_1}$$

$$\mu_1 B_{t_2} = \mu_2 B_{t_1}$$

$$B_{t_2} = \frac{1}{2} (1.2\vec{a}_x + 0.8\vec{a}_y)$$

$$B_2 = B_{t_2} + B_{n_2} = 0.6\vec{a}_x + 0.4\vec{a}_y + 0.4\vec{a}_z$$

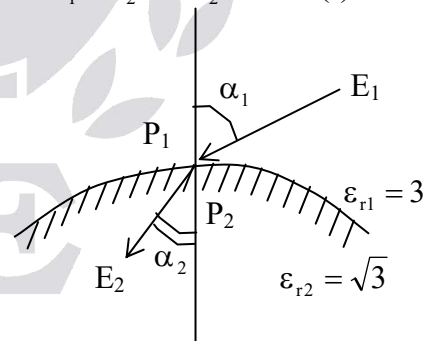
$$\mu_0 \mu_{r_2} H_2 = 0.6\vec{a}_x + 0.4\vec{a}_y + 0.4\vec{a}_z$$

$$H_2 = \frac{1}{\mu_0} [0.6\vec{a}_x + 0.4\vec{a}_y + 0.4\vec{a}_z] \text{ A/m}$$

13. Ans: (b)

Sol: Tangential components of electric fields are continuous ($E_{t_1} = E_{t_2}$)

$$E_1 \sin \alpha_1 = E_2 \sin \alpha_2 \text{ --- (1)}$$



Normal component of electric flux densities are continuous across a charge free interface

$$D_{n_1} = D_{n_2}$$

$$3E_1 \cos \alpha_1 = \sqrt{3}E_2 \cos \alpha_2 \text{ --- (2)}$$

$$\alpha_1 = 60^\circ$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\tan \alpha_1}{3} = \frac{\tan \alpha_2}{\sqrt{3}} \Rightarrow \tan \alpha_2 = 1$$

$$\alpha_2 = 45^\circ$$

Chapter 2 Maxwell's Equations & EM Waves

Identify polarization of following (Page number 71 in Volume –I booklet)

01. $\vec{E} = 20\sin(\omega t - \beta x)\hat{a}_y \text{ V/m}$

Sol: At $x = 0$

$$\vec{E} = 20\sin(\omega t)\hat{a}_y \text{ V/m}$$

Let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 0$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 20\hat{a}_y$$

$$\theta = \pi \Rightarrow \vec{E} = 0$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -20\hat{a}_y$$

$$\theta = 2\pi \Rightarrow \vec{E} = 0$$

i.e., linear polarization and also vertical polarization with respect to $\hat{x}$ -axis

02. $\vec{H} = 45\cos(\omega t - \beta z)\hat{a}_x \text{ A/m}$

Sol: This is linear polarization

03. $\vec{E} = 20\sin(\omega t - \beta z)\hat{a}_x + 30\sin(\omega t - \beta z)\hat{a}_y$

Sol: phase difference between $\hat{a}_x$ component and $\hat{a}_y$ component is 0°

So that it is linear polarization

Note: for phase difference 0° & 180° , irrespective of their amplitudes it must be in linear polarization.

04. $\vec{E} = 55\cos(\omega t - \beta z)\hat{a}_x + 55\sin(\omega t - \beta z)\hat{a}_y$

Sol: Phase difference between $\hat{a}_x$ component and

$$\hat{a}_y \text{ component is } \frac{\pi}{2}$$

Amplitudes are same.

So it is circular polarization

at $z = 0$ and let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 55\hat{a}_x + 0\hat{a}_y$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 0\hat{a}_x + 55\hat{a}_y$$

It is CCW direction i.e. RHCP

05. $\vec{E} = 40\sin(\omega t - \beta y)\hat{a}_x + 50\cos(\omega t - \beta y)\hat{a}_z$

Sol: Phase difference = $\frac{\pi}{2}$

Amplitudes = not same

So it is elliptical polarization. To decide direction of rotation follow below procedure.

At $y = 0$, and Let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 0\hat{a}_x + 50\hat{a}_z$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 40\hat{a}_x + 0\hat{a}_z$$

$$\theta = \pi \Rightarrow \vec{E} = 0\hat{a}_x - 50\hat{a}_z$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -40\hat{a}_x + 0\hat{a}_z$$

It is Anti clock wise direction i.e., Right Hand Elliptical Polarization.

06.

Sol: $\vec{E} = \text{Re}\{[\hat{a}_x + j\hat{a}_y]e^{j(\omega t - \beta z)}\}$

$$\vec{E} = \text{Re}\left[(\cos(\omega t - \beta z) + j\sin(\omega t - \beta z))\hat{a}_x + j(\cos(\omega t - \beta z) + j^2\sin(\omega t - \beta z))\hat{a}_y\right]$$

$$\vec{E} = (\cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y)$$

Magnitudes of amplitudes are same, phase difference is $\frac{\pi}{2}$; So it is circular

polarization. Now we proceed to decide direction of rotation.

Here

$$\vec{E} = \cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y$$

At $z = 0$ & let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = \hat{a}_x - 0\hat{a}_y$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 0\hat{a}_x - \hat{a}_y$$

$$\theta = \pi \Rightarrow \vec{E} = -\hat{a}_x + 0\hat{a}_y$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = 0\hat{a}_x - \hat{a}_y$$

i.e., we get clock wise rotation i.e.,
Left Hand Circular Polarization

07. not a valid EM wave representation

08.

Sol: $\vec{E} = 5 \cos(\omega t - \beta r) \hat{a}_\theta$

Let $r = 0$ & $\theta = \omega t$

at $\theta = 0 \Rightarrow \vec{E} = 5\hat{a}_\theta$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 0\hat{a}_\theta$$

$$\theta = \pi \Rightarrow \vec{E} = -5\hat{a}_\theta$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = 0\hat{a}_\theta$$

i.e., linear polarization

09.

Sol: $\vec{E} = \text{Im}\{[\hat{a}_x + 2j\hat{a}_z]e^{j(\omega t - \beta y)}\}$
 $= \text{Im}\{[\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_x + [2j\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_z\}$
 $= \sin(\omega t - \beta y)\hat{a}_x + 2\cos(\omega t - \beta y)\hat{a}_z$

Let $y = 0$ & $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 0\hat{a}_x + 2\hat{a}_z$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = \hat{a}_x + 0\hat{a}_z$$

$$\theta = \pi \Rightarrow \vec{E} = 0\hat{a}_x - 2\hat{a}_z$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -\hat{a}_x + 0\hat{a}_z$$

So it is Right Hand Elliptical Polarization

10. $\vec{E} = 20 \sin(\omega t - \beta y) \hat{a}_x + 30 \sin(\omega t - \beta y + 45^\circ) \hat{a}_z$

Sol: let $y = 0$ & $\theta = \omega t$

At $\theta = 0$

$$\Rightarrow \vec{E} = 0\hat{a}_x + 30 \sin 45^\circ \hat{a}_z$$

$$= 0\hat{a}_x + \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 20\hat{a}_x + 30 \sin(135^\circ) \hat{a}_z$

$$= 20\hat{a}_x + \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \pi \Rightarrow \vec{E} = 0\hat{a}_x + 30 \sin(225^\circ) \hat{a}_z$

$$= 0\hat{a}_x - \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -20\hat{a}_x + 30 \sin(315^\circ) \hat{a}_z$

$$= -20\hat{a}_x - \frac{30}{\sqrt{2}} \hat{a}_z$$

Note: $\theta = 62.76^\circ$ is the maximum values direction obtained by

$$\frac{d\vec{E}}{d\theta} = 0 \text{ at } y = 0 \text{ \& } \omega t = \theta$$

$$\text{at } \theta = -\frac{\pi}{4} \Rightarrow \vec{E} = \frac{-20}{\sqrt{2}} \hat{a}_x + 0\hat{a}_z$$

$$\text{at } \theta = \frac{\pi}{4} \Rightarrow \vec{E} = \frac{20}{\sqrt{2}} \hat{a}_x + 30\hat{a}_z$$

So it is RHEP

11. $\vec{E} = 20 \sin(\omega t - \beta z) \hat{a}_x + 20 \sin(\omega t - \beta z + 45^\circ) \hat{a}_y$

Sol: Valid EM wave but polarization can not defined.

This is a valid EM wave representation but it is not satisfy anyone of the polarization principle

Text Book Practice Solutions

01. Ans: (c)

Sol: Given flux $\phi = (t^3 - 2t) \text{mWb}$

$$\text{Magnitude of induced emf } |e'| = \left| \frac{d\phi}{dt} \right|_{t=4 \text{ sec}}$$

$$|e'| = 3t^2 - 2 \Big|_{t=4 \text{ sec}}$$

$$= 3(4)^2 - 2$$

$$= 46 \text{mWb}$$

This 'e' for one turn; but for 100 turns

$$|e| = N|e'| = 100 \times 46 \text{mWb}$$

$$|e| = 4.6 \text{ volts}$$

02. Ans: (d)

Sol: Given,

$$E = 120 \pi \cos(10^6 \pi t - \beta x) \hat{a}_y \text{ V/m}$$

$$H = A \cos(10^6 \pi t - \beta x) \hat{a}_z \text{ A/m}$$

$$\epsilon_r = 8; \mu_r = 2$$

$$\text{We know that, } \frac{E_y}{H_z} = \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$H_z = \frac{E_y}{120\pi \sqrt{\frac{2}{8}}} = \frac{2E_y}{120\pi} = 2A / \text{m}$$

$$H_z = 2 \cos(10^6 \pi t - \beta x) \hat{a}_z \text{ A/m}$$

$$\therefore A = 2$$

$$\beta = \omega \sqrt{\mu \epsilon} = \frac{10^6 \pi \times \sqrt{2 \times 8}}{3 \times 10^8}$$

$$= 0.0418 \text{ rad/m}$$

03. Ans: (b)

Sol: This question relates to normal incidence of a UPW on the air (medium 1) to glass (medium 2) interface as shown in Fig.

Medium, 1	Medium, 2
Air	Glass slab
$n_1 = 1$	$n_2 = 1.5$
$\mu_1 = \mu_0$	$\mu_2 = \mu_0$
$\epsilon_1 = \epsilon_0$	$\epsilon_2 = \epsilon_0 \epsilon_r$

Fig.

If n_1 and n_2 are the refractive indices and v_1 and v_2 are the velocities

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}}$$

$$= \sqrt{\frac{\epsilon_1}{\epsilon_2}} \text{ for } \mu_1 = \mu_2 = \mu_0$$

For $n_1 = 1, n_2 = 1.5$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{1.5} = \frac{2}{3}$$

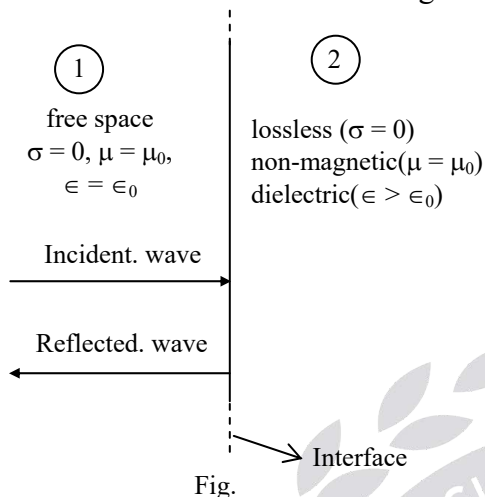
Reflection coefficient,

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1} = -\frac{1}{5}$$

$$\therefore \frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{25} = 4\%$$

04. Ans: (d)

Sol: Normal incidence is shown in Fig.



Given: $E_{\max} = 5 E_{\min}$ in medium 1.

$$\therefore \text{VSWR, } S = \frac{E_{\max}}{E_{\min}} = 5$$

$$|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

Reflection coefficient,

$$K = \frac{E_r}{E_i} = \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1} = \frac{-2}{3}$$

$$-3 \frac{\eta_2}{\eta_1} + 3 = 2 \frac{\eta_2}{\eta_1} + 2$$

$$\therefore \frac{\eta_2}{\eta_1} = \frac{1}{5}, \quad \eta_2 = \frac{1}{5} \eta_1$$

$$\begin{aligned} \eta_1 &= \sqrt{\frac{\mu_0}{\epsilon_0}} \\ &= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9} \\ &= (120\pi) \Omega \end{aligned}$$

$\therefore$ Intrinsic impedance of the dielectric medium, $\eta_2 = \frac{1}{5} \times 120\pi = 24\pi$

05. Ans: (a)

Sol: Given:

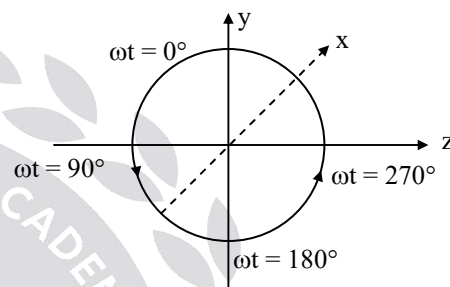
$$\vec{E} = 10(\hat{a}_y + j\hat{a}_z) e^{-j25x} \text{ in free space.}$$

$$\vec{E} = (E_y \hat{a}_y + E_z \hat{a}_z) e^{-j\beta x}$$

$$\beta = 25 = \frac{\omega}{c} \Rightarrow$$

$$\omega = 25c = 25 \times 3 \times 10^8 \text{ rad/s}$$

$$f = 1.19 \text{ GHz} \approx 1.2 \text{ GHz}$$



$$E_y = 10, E_z = j 10$$

E_z leads E_y by 90°

At $x = 0$

$$\text{Let } E_y = 10 \cos(\omega t)$$

$$\text{then } E_z = 10 \cos(\omega t + 90^\circ)$$

A Left Hand screw is to be turned in the direction along the circle as time increases so that the screw moves in the direction of propagation, 'x'.

$\therefore$ The wave is left circularly polarized.

06. Ans: (b)

Sol: $\vec{H} = 0.2 \cos(\omega t - \beta x) \hat{a}_z$

Wave is progressing along +X direction

$\rightarrow (+X)$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

$$\therefore \vec{E} = 0.2\eta \cos(\omega t - \beta x) \hat{a}_y$$

$$\vec{E}_s = 0.2\eta e^{-j\beta x} \hat{a}_y \quad \vec{H}_s = 0.2e^{-j\beta x} \hat{a}_z$$

$$\begin{aligned} \vec{P}_{\text{avg}} &= \frac{1}{2} \vec{E}_s \times \vec{H}_s^* \\ &= \frac{1}{2} (0.2)^2 \eta \hat{a}_x \end{aligned}$$

$$= \frac{1}{2} (0.2)^2 (120\pi) \hat{a}_x \text{ W/m}^2$$

$$x = 1 \text{ plane} \Rightarrow \bar{ds} = dydz \hat{a}_x$$

$$W_{\text{avg}} = \int_s \bar{P}_{\text{avg}} \cdot \bar{ds} \text{ watts}$$

$$= \frac{1}{2} (0.2)^2 (120\pi) \iint dydz$$

$$= \left[\frac{1}{2} ((0.2)^2 (120\pi)) \right] [\pi(5)^2] \times 10^{-4}$$

$$= 0.0592 \text{ Watts}$$

$$= 59.2 \text{ mW} \simeq 60 \text{ mW}$$

07. Ans: (a)

Sol: $P \propto \frac{1}{r^2}$

$$\frac{P_Q}{P_P} = \frac{r_P^2}{r_Q^2} = \left(\frac{R}{2} \right)^2$$

$$\frac{P_Q}{P_P} = \frac{4}{1} = 4:1$$

08. Ans: (b)

Sol: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$

$$\delta \propto \sqrt{\frac{1}{f}} \Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{1.5}{\delta} = \sqrt{\frac{8 \times 10^9}{2 \times 10^9}}$$

$$\delta = \frac{1.5}{2} = 0.75 \mu\text{m}$$

Similarly

$$\frac{1.5}{\delta} = \sqrt{\frac{18 \times 10^9}{2 \times 10^9}} = 3$$

$$\delta = \frac{1.5}{3} = 0.5 \mu\text{m}$$

09. Ans: (b)

Sol: $\frac{\sigma}{\omega\epsilon} = \frac{5}{2 \times \pi \times 25 \times 10^3 \times 80 \times 8.854 \times 10^{-12}}$

$$= 44938.7$$

Since $\frac{\sigma}{\omega\epsilon} \gg 1$ hence sea water is a good conductor

Where attenuation is 90%, transmission is 10%, then $e^{-\alpha x} = 0.1$

Where α is attenuation constant

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2 \times \pi \times 25 \times 10^3 \times 4\pi \times 10^{-7} \times 5}{2}}$$

$$\alpha = 0.7025$$

$$-\alpha x = \ln(0.1)$$

$$-0.7025x = -2.3$$

$$x = 3.27\text{m}$$

10. Ans: (b)

Sol: $\delta = \frac{1}{\alpha} = \frac{1}{2\pi} = 0.159$

11. Ans: (c)

Sol: E is minimum
H is maximum
i.e., 'c' is the option

$$E_{\text{Tan}_1} = E_{\text{Tan}_2} = 0$$

[perfect conductor $E_{\text{Tan}_2} = 0$]

$$H_{\text{Tan}_1} = J_s \times a_n + H_{\text{Tan}_2}$$

$$H_{\text{Tan}_1} = J_s \times a_n$$

[perfect conductor $H_{\text{Tan}_2} = 0$]

12. Ans: (d)

Sol: $\vec{H} = 0.5e^{-0.1x} \cos(10^6 t - 2x) \hat{a}_z \text{ A/m} \rightarrow (+X)$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

Wave frequency = 10^6 radians/s

Phase constant $\beta = 2 \text{ rad/m}$

$$\beta = \frac{2\pi}{\lambda} = 2 \text{ rad/m}$$

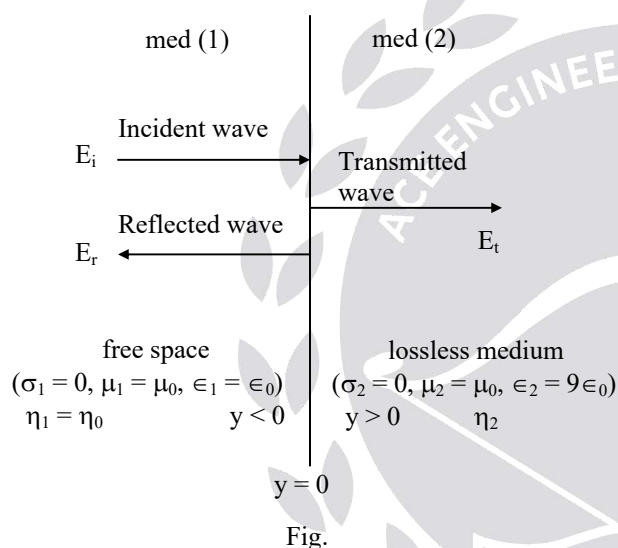
$$\lambda = \pi = 3.14 \text{ m}$$

The wave is traveling along +X direction,
Given wave is polarized along Y.

$\therefore$ It has Y-component of electric field

13. Ans: (a)

Sol: The normal incidence of a plane wave traveling in positive y - direction is shown at the interface $y = 0$ in Fig.



Given: $\vec{E}_i = E_{iz} \vec{a}_z$

where $E_{iz} = 24 \cos(3 \times 10^8 t - \beta y) \text{ V/m}$

$$\omega = 3 \times 10^8 \text{ rad/s}, \beta = \frac{\omega}{v}$$

For free space, $v = v_0 = 3 \times 10^8 \text{ m/s}$

$$\therefore \beta = 1 \text{ rad/m}$$

$$\eta_1 = \eta_0 = \frac{E_{iz}}{H_{ix}}$$

$$\therefore H_{ix} = \frac{E_{iz}}{\eta_0} = \frac{24 \cos(3 \times 10^8 t - \beta y)}{120 \pi}$$

$$\vec{H}_i = H_{ix} \vec{a}_x$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\eta_1 - 1}{\eta_1 + 1}$$

Where $\frac{\eta_1}{\eta_2} = \frac{\sqrt{\mu_1 \epsilon_2}}{\sqrt{\epsilon_1 \mu_2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{9\epsilon_0}{\epsilon_0}} = 3$

$$\therefore \frac{H_r}{H_i} = \frac{3 - 1}{3 + 1} = \frac{1}{2}$$

$$\begin{aligned} \therefore \vec{H}_r &= \frac{1}{2} \frac{24}{120 \pi} \cos(3 \times 10^8 t + 1y) \vec{a}_x \\ &= \frac{1}{10 \pi} \cos(3 \times 10^8 t + 1y) \vec{a}_x \text{ A/m} \end{aligned}$$

Note that $\vec{H}_r$ is reflected wave which travels in negative y direction, which corresponds to $+\beta y$ term with $\beta = 1$ in the expression for $\vec{H}_r$.

14. Ans: (b)

Sol: Brewster's angle $\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$\theta_B = \tan^{-1} \sqrt{\frac{1}{3}} = 30^\circ$$

At this angle there is no reflected wave when wave is parallel polarized.

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\sqrt{\epsilon_1} \sin \theta_i = \sqrt{\epsilon_2} \sin \theta_t$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

$$\sin \theta_t = \sqrt{3} \frac{1}{2} (\theta_i = 30^\circ)$$

$$\theta_t = 60^\circ$$

15. Ans: (d)

Sol: Given that

$$E_t = -2E_r$$

Where

E_t is electric field of transmitted wave

E_r is electric field of reflected wave

$$\frac{E_t}{E_r} = -2$$

If E_i is electric field of incident wave.

$$\text{But } -\frac{2E_r}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\text{and } \frac{E_r}{E_i} = \frac{-\eta_2}{\eta_1 + \eta_2}$$

$$\text{and also } \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\text{so } \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-\eta_2}{\eta_2 + \eta_1}$$

$$\eta_1 = 2\eta_2$$

$$\frac{\eta_1}{\eta_2} = 2 \Rightarrow \sqrt{\frac{\epsilon_2}{\epsilon_1}} = 2 \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$

16. Ans: (a, b, c)

Sol: Given that, $\sigma = 5 \text{ S/m}$

$$\epsilon_r = 1$$

$$E = 250 \sin(10^{10} t) \text{ V/m}$$

We know that conduction current density,

$$J_c = \sigma E$$

Putting the values we get,

$$J_c = 5 \times 250 \sin(10^{10} t)$$

$$J_c = 1250 \sin(10^{10} t) \text{ A/m}^2$$

Displacement current density, $J_D = \frac{\partial D}{\partial t}$

$$J_D = \frac{\partial(\epsilon E)}{\partial t} = \epsilon \frac{\partial E}{\partial t}$$

$$J_D = \epsilon_0 \epsilon_r \frac{\partial}{\partial t} (250 \sin(10^{10} t))$$

$$= \epsilon_0 \times 250 \times 10^{10} \cos(10^{10} t)$$

$$J_D = 22.125 \cos(10^{10} t) \text{ A/m}^2$$

Since given that

$|J_c| = |J_D|$, we have to find the frequency

$$|\sigma E| = |j\omega \epsilon E|$$

$$\omega = \frac{\sigma}{\epsilon}$$

$$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r} = \frac{5}{2\pi \times \frac{1}{36\pi} \times 10^{-9}} = 90 \text{ GHz}$$

Chapter 3 Transmission Lines

01. Ans: (b)

Sol: $Z_{in} = Z_0 \frac{Z_R + jZ_0 \tan \beta \ell}{Z_0 + jZ_R \tan \beta \ell}$

Phase velocity

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{2\pi f}{\beta}$$

$$\beta = \frac{2\pi f}{v_p} = \frac{2 \times \pi \times 10^8}{2 \times 10^8} = \pi$$

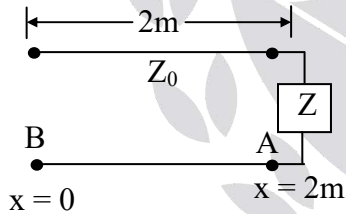
$$\beta \ell = \pi \cdot \ell \Rightarrow \pi \text{ (Given } \ell = 1\text{m)}$$

$$\tan \beta \ell = 0$$

$$Z_{in} = Z_R = (30 - j40)\Omega$$

02. Ans: (a)

Sol:



$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = \frac{C_2}{C_1} e^{j4\beta} \text{ at } (x = 2)$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(0)} \text{ at } (x = 0)$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta(0)}}{\frac{C_2}{C_1} e^{j4\beta}} = e^{-j4\beta}$$

$$v_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\pi}{2}$$

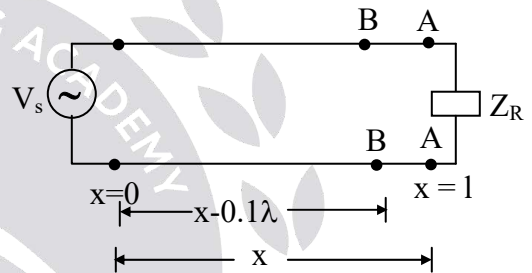
Given $f = 50 \text{ MHz}$

$$v_p = 2 \times 10^8 \text{ m/s}$$

$$\frac{K_B}{K_A} = e^{-j4\left(\frac{\pi}{2}\right)} = e^{-j2\pi} = 1 \text{ (or) } \frac{\Gamma_i}{\Gamma_R} = 1$$

03. Ans: (b)

Sol:



$$V = C_1 e^{-j\beta x} + C_2 e^{+j\beta x}$$

$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = 0.3 e^{-j30^\circ} = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(x-0.1\lambda)}$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta x} e^{-j4\frac{\pi}{\lambda} \cdot 0.1\lambda}}{\frac{C_2}{C_1} e^{2j\beta x}}$$

$$K_B = K_A \cdot e^{-j4\pi} = 0.3 e^{-j30^\circ} e^{-72^\circ} = 0.3 e^{-j102^\circ}$$

Note: In the options $0.3 e^{j102^\circ}$ is given.

But correct answer is $0.3 e^{-j102^\circ}$

04. Ans: (c)

Sol: From the voltage SW pattern,

$$V_{\min} = 1, V_{\max} = 4, \text{VSWR} = S = 4$$

$$Z_0 = R_0 = 50 \Omega$$

Let the resistive load be R_L

For Resistive loads

$$S = \frac{R_L}{R_0} \quad \text{for } R_L > R_0$$

$$= \frac{R_0}{R_L} \quad \text{for } R_0 > R_L$$

$$\therefore R_L = S R_0 = 4 \times 50 = 200 \Omega \quad \text{for } R_L > R_0$$

$$R_L = R_0/S = 50/4 = 12.5 \Omega \quad \text{for } R_0 > R_L$$

As voltage minimum is occurring at the load point, $R_L = 12.5 \Omega$.

05. Ans: (a)

Sol: Reflection coefficient:

$$\Gamma = \frac{R_L - R_0}{R_L + R_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

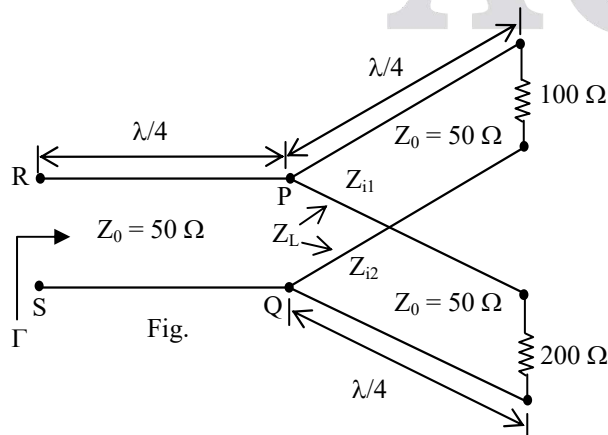
06. Ans: (d)

Sol: The interconnection of TL's is shown in Fig.

$$Z_{i1} = \frac{(50)^2}{100} = 25 \Omega$$

$$Z_{i2} = \frac{(50)^2}{200} = 12.5 \Omega$$

$$Z_L = 25 \parallel 12.5 = \frac{25}{3} \Omega$$



$$\text{Reflection coefficient at PQ} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = -\frac{125}{175} = -\frac{5}{7}$$

$\therefore$ At the input RS,

$$\text{Reflection coefficient, } \Gamma = -\frac{5}{7} e^{-j2\beta\ell}$$

$$\text{As } \beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Gamma = -\frac{5}{7} e^{-j\pi} = \frac{5}{7}$$

07. Ans: (d)

$$\text{Sol: } Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell} \right]$$

i) For a shorted line,

$$Z_L = 0$$

$$\ell = \lambda/8$$

$$\beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{\text{in}} = Z_0 \left[\frac{0 + jZ_0}{Z_0 + 0} \right]$$

$$Z_{\text{in}} = jZ_0$$

ii) For a shorted line means $Z_L = 0$

$$\text{Given that } \ell = \frac{\lambda}{4}$$

$$\beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{0}$$

$$Z_{\text{in}} = \infty$$

iii) Open line means $Z_L = \infty$,

$$\text{Given that } \ell = \frac{\lambda}{2}$$

$$\therefore \beta \ell = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \Rightarrow \tan \pi = 0$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \pi}{Z_0 + jZ_L \tan \pi} \right]$$

$$Z_{in} = Z_L$$

iv) For a matched line of any length

$$Z_L = Z_0$$

$$Z_{in} = Z_0 \left[\frac{Z_0 + jZ_0 \tan \beta \ell}{Z_0 + jZ_0 \tan \beta \ell} \right] = Z_0$$

08. Ans: (c)

Sol: The line is matched as $Z_L = Z_0 = 50 \Omega$ and hence reflected wave is absent.

For the traveling wave, given:

Phase difference for a length of $2 \text{ mm} = \pi/4 \text{ rad}$

Frequency of excitation = 10 GHz

Phase velocity, $v_p = \frac{\omega}{\beta}$

$$\omega = 2\pi \times 10 \times 10^9 \text{ rad/sec}$$

β = Phase-shift per unit length

$$= \frac{\pi}{4 \times 2 \times 10^{-3}} \text{ rad/m}$$

$$v_p = \frac{2\pi \times 10^{10} \times 8}{\pi \times 10^3} = 1.6 \times 10^8 \text{ m/s}$$

09. Ans: (b)

$$\text{Sol: } [S] = \begin{bmatrix} 0.3 \angle 0^\circ & 0.9 \angle 90^\circ \\ 0.9 \angle 90^\circ & 0.2 \angle 0^\circ \end{bmatrix}$$

For reciprocal; $S_{12} = S_{21}$

It is satisfied.

For lossless line $|S_{11}|^2 + |S_{12}|^2 = 1$

$$(0.3)^2 + (0.9)^2 = 0.9 \neq 1$$

$\therefore$ It is a lossy line

10. Ans: (b, c)

Sol: Given:

$$\ell = 2 \text{ m}$$

$$Z_{oc} = -j50 \Omega$$

$$Z_{sc} = j200 \Omega$$

$$Z_0 = \sqrt{Z_{oc} Z_{sc}} = \sqrt{10000}$$

$$Z_0 = 100 \Omega$$

$$\text{Reflection coefficient, } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

When open circuited ($Z_L = \infty$)

$$\Gamma = \frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}}$$

$$\Gamma = 1$$

When short circuit ($Z_L = 0$)

$$\Gamma = \frac{-Z_0}{Z_0} = -1$$

Chapter 4 Waveguides

01. Ans: (b)

Sol: Evanescent modes means no wave propagation.

Dominant mode means, the guide has lowest cut-off frequency.

TM₀₁ and TM₁₀ not possible, the minimum values of m, n for TM are at least 1, 1 respectively.

02. Ans: (a)

Sol: The mode which has lowest cutoff frequency is called dominant mode TE₁₀.

At 4GHz all modes are evanescent.

At 7GHz degenerate modes are possible

TE₁₁ and TM₁₁ are degenerate.

$$f_{c \text{ TE}_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 3 \times 10^{-2}} = 5 \text{ GHz.}$$

At 6 GHz dominant mode will propagate.

At 11 GHz higher order modes are possible

03. Ans: (a)

Sol: Given: In a rectangular WG of cross-section : (a × b)

$$\vec{E} = \frac{\omega \mu}{h^2} \left(\frac{\pi}{a} \right) H_0 \sin \left(\frac{2\pi}{a} x \right) \sin(\omega t - \beta z) \hat{y}$$

The wave is traveling in the z-direction having E_y component only as function of 'x'. As there is no component of

$\vec{E}$ in the direction of propagation, $\vec{a}_z$ the wave is Transverse Electric (TE).

Comparing the 'sin' term in $\vec{E}$ with the general expression: $\sin \left(\frac{m\pi}{a} x \right)$

$$m = 2$$

As there is no function of 'y' in $\vec{E}$, n = 0

∴ The mode of propagation in the WG is TE₂₀

04. Ans: (d)

Sol: Given

$$a = 4.755, b = 2.215,$$

$$f = 12 \text{ GHz, } c = 3 \times 10^8 \text{ m/s}$$

Cut off frequency

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}$$

For TE₁₀, mode

$$f_c = \frac{c}{2a} = 3.15 \text{ GHz}$$

$f > f_c$ (TE₁₀ mode) so it propagates

For TE₂₀ mode

$$f_c (\text{TE}_{20}) = \frac{c}{2} \sqrt{\left(\frac{2}{a} \right)^2} = 2 [f_c (\text{TE}_{10})] = 6.30 \text{ GHz}$$

$f > f_c$ [TE₂₀] so it propagates

For TE₀₁ mode

$$f_c (\text{TE}_{01}) = \frac{c}{2} \sqrt{\frac{1}{b^2}} = \frac{c}{2b} = 6.77 \text{ GHz}$$

∴ $f > f_c$ (TE₀₁) so it propagate

For TE₁₁ mode

$$f_{c[\text{TE}_{11}]} = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 7.47 \text{ GHz}$$

$f > f_c$ (TE₁₁) so it propagate

So, all modes are possible to propagate.

05. Ans: (a)

Sol: Given a = 6cm, b = 4 cm f = 3 GHz

Cut off frequency

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}$$

$$TE_{10}: f_c = \frac{c}{2a} = 2.5 \text{ GHz}$$

$$TE_{01}: f_c = \frac{c}{2b} = 3.75 \text{ GHz}$$

$$TE_{11}: f_c = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 4.50 \text{ GHz}$$

$$TM_{11}: f_c = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 4.50 \text{ GHz}$$

06. Ans: (a)

Sol: $\frac{m\pi}{a} = \frac{2\pi}{a} \Rightarrow m = 2$

$$\frac{n\pi}{b} = \frac{3\pi}{b} \Rightarrow n = 3$$

For TM wave propagating along z-direction

$E_z \neq 0$ and $H_z = 0$

TM₂₃

$$TM_{23} \Rightarrow f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Substitute $c = 3 \times 10^{10}$ cm/sec

$m = 2, a = 6 \text{ cm}$

$n = 3, b = 3 \text{ cm}$

we get $f_c = 15.811 \text{ GHz}$

$$\eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\omega = 10^{12} \Rightarrow f = \frac{10^{12}}{2\pi} = \frac{10^3}{2\pi} \text{ GHz}$$

and $\eta = 120 \pi$. & $f_c = 15.811 \text{ GHz}$

Substitute all the above values and we get

$$\eta_{TM} = 375 \Omega$$

07. Ans: (c)

Sol: $W_{avg} = \frac{1}{4} \frac{E_{yo}^2}{\eta_{TE_{10}}} a.b ; \eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - (\lambda/\lambda_c)^2}}$

$$\eta = 120\pi, \lambda = \frac{c}{f} = \frac{3 \times 10^{10}}{11 \times 10^9} = 2.72 \text{ cm}$$

$$\lambda_c = 2a = 2 \times 2.29 = 4.58 \text{ cm}$$

So we get $\eta_{TE_{10}} = 469.52 \Omega$

Putting all the values

$$\therefore W_{avg} = 31.32 \text{ kW}$$

08. Ans: (a)

Sol: $f_{c_{10}} = \frac{c}{2a} = \frac{3 \times 10^{10}}{2 \times 2} = 7.5 \text{ GHz}$

For $b = a/2$, the next high order mode is TE₀₁ or TE₂₀.

$$\therefore f_{c_{01}} = f_{c_{20}} = \frac{3 \times 10^{10}}{2} = 15 \text{ GHz}$$

So the range of single mode (dominant mode propagation) is

$$7.5 < f < 15 \text{ GHz}$$

09. Ans: (a)

Sol: $\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$

$$f_c = 0.908 \text{ GHz}$$

$$\Rightarrow \lambda_c = \frac{3 \times 10^{10}}{0.908 \times 10^9} = 33.03 \text{ cm}$$

Substitute $\lambda_g = 40 \text{ cm}$, $\lambda_c = 33.03 \text{ cm}$

We get, $\lambda = 25.47 \text{ cm}$

$$\Rightarrow f = \frac{3 \times 10^{10}}{25.47} = 1.18 \text{ GHz}$$

10. Ans: (a)

Sol: $\frac{c}{2a} = 0.908 \text{ GHz}$

$$\Rightarrow a = \frac{3 \times 10^{10}}{2 \times (0.908) \times 10^9}$$

$$= 16.51 \text{ cm}$$

$$\Rightarrow b = \frac{a}{2} = 8.26 \text{ cm}$$

11. Ans: (a)

$$\begin{aligned}\text{Sol: } \bar{\beta} &= \beta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \\ &= \frac{2\pi}{25.47} \sqrt{1 - \left(\frac{0.908}{1.18}\right)^2} \\ &= 0.157 \text{ rad/cm} \\ &= 15.7 \text{ rad/m}\end{aligned}$$

12. Ans: (a, b, c)

$$\text{Sol: } f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 7 \times 10^{-2}} = 2.14 \text{ GHz}$$

Phase velocity,

$$\begin{aligned}v_p &= \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{2.14}{3.5}\right)^2}} \\ &= 3.79 \times 10^8 \text{ m/sec}\end{aligned}$$

$$\lambda_g = \frac{v_p}{f} = \frac{3.79 \times 10^8}{3.5 \times 10^9} = 0.1 \text{ m}$$

$$Z_{TE} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{2.14}{3.5}\right)^2}} = 476 \Omega$$

Chapter 5

Antennas

01. Ans: (c)

Sol: Antenna receives $2 \mu\text{W}$ of power: $P_r = 2 \mu\text{W}$
RMS value of incident E field
 $= 20 \text{ mV/m}$

Power density, P_d

$$= \frac{E^2}{\eta} = \frac{(20 \times 10^{-3})^2}{377} \text{ W/m}^2$$

Effective aperture area, $A_e = \frac{P_r}{P_d}$

$$= \frac{2 \times 10^{-6}}{(20 \times 10^{-3})^2} = \frac{377 \times 2}{400} = 1.885 \text{ m}^2$$

02. Ans: (b)

Sol: Lossless antenna directive gain = $6 \text{ dB} = 4$

Input power to the antenna = 1 mW
for lossless we get 100% efficiency

$$\frac{W_{\text{rad}}}{W_{\text{in}}} = \frac{G_o}{D_o} = 1$$

$$W_{\text{rad}} = W_{\text{in}}$$

$$W_{\text{rad}} = 1 \text{ mW}$$

03. Ans: (c)

Sol: $P_{\text{rad}} = \frac{A_0 \sin^2 \theta}{r^2} \hat{a}_r \text{ W/m}^2$

$$W_{\text{rad}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{A_0 \sin^2 \theta}{r^2} r^2 \sin \theta d\theta d\phi$$

$$= A_0 2\pi \int_{\theta=0}^{\pi} \sin^3 \theta d\theta$$

$$= A_0 2\pi \frac{4}{3}$$

$$W_{\text{rad}} = A_0 \frac{8\pi}{3}$$

$$U = r^2 P_{\text{rad}} = r^2 \frac{A_0 \sin^2 \theta}{r^2} = A_0 \sin^2 \theta$$

$$D_{\text{max}} = \frac{U_{\text{max}}}{W_{\text{rad}}} 4\pi = \frac{|A_0 \sin^2 \theta|_{\text{max}}}{\frac{8\pi}{3} A_0} \times 4\pi$$

$$= \frac{4\pi A_0}{8\pi A_0} \times 3$$

$$= \frac{3}{2} = D_{\text{max}} = 1.5$$

04. Ans: (d)

Sol: Where $W_{\text{rad}} = \iint \bar{P}_{\text{rad}} \cdot d\bar{s}$

$$\bar{P}_{\text{rad}} = \frac{W_{\text{rad}}}{2\pi r^2} \hat{a}_r = \frac{40}{\pi} \hat{a}_r \mu\text{W/m}^2$$

05. Ans: (b)

Sol: $R_{\text{rad}} = 30 \Omega$, $R_l = 10 \Omega$

$$G_D = 4, G_p = ?$$

$$\eta = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_l} = \frac{30}{40} = 0.75$$

$$G_p = \eta G_D$$

$$= 0.75 \times 4 = 3$$

06. Ans: (c)

Sol: $D_g = 30 \text{ dB} = 1000$

$$P_T = 7.5 \text{ kW}$$

$$D_g = \frac{4\pi \times \text{Radiation intensity}}{\text{Radiated Power}}$$

$$D_g = 4\pi \frac{U}{W_{\text{rad}}}$$

$$\therefore U = \frac{7.5 \times 10^3 \times 1000}{4\pi}$$

$$\Rightarrow U = r^2 P_{\text{rad}}$$

P_{rad} : Power density we have to find

P_{rad} at $r = 40 \times 10^3$ m

$$P_{\text{rad}} = \frac{U}{r^2} = \frac{7.5 \times 10^3 \times 1000}{4\pi \times (40 \times 10^3)^2} \text{ W/m}^2$$

07. Ans: (d)

Sol: $W_{\text{rad}} = 10 \text{ kW}$

$$E_{\text{max}} = 120 \text{ mV/m}$$

$$R = 20 \text{ km}$$

$$\eta = 98\%$$

$$P_{\text{rad}} = \frac{E_0^2}{2\eta_0}$$

$$= \frac{(120 \times 10^{-3})^2}{2 \times 120\pi}$$

$$= 1.909 \times 10^{-5}$$

$$U_{\text{max}} = (20 \times 10^3)^2 \times 1.909 \times 10^{-5} = 7636$$

$$D_0 = 4\pi \frac{U_{\text{max}}}{W_{\text{rad}}}$$

$$D_0 = 4\pi \frac{7639.43}{10 \times 10^3} = 9.59$$

$$\eta = \frac{G_0}{D_0} = 0.98$$

$$G_0 = 0.98 \times 9.59 = 9.407$$

08. Ans: 0.21

Sol: Given:

Antenna length, $l = 1 \text{ cm}$

Frequency, $f = 1 \text{ GHz}$

Distance, $r = 100\lambda$

Wave length, $\lambda = \frac{C}{f}$

$$= \frac{3 \times 10^8}{10^9}$$

$$= 30 \text{ cm}$$

$\frac{d\ell}{\lambda} = \frac{1}{30}$, hence the given antenna is

Hertzian dipole.

In the far field, the tangential electric field

is given by, $E_\theta = \frac{j\eta I d\ell \sin \theta}{4\pi r}$

$$= \frac{j377 \times 100 \times 10^{-3} \times 2\pi \times 10^{-2} \times 1}{30 \times 10^{-2} \times 4\pi \times 100 \times 30 \times 10^{-2}}$$

$$\therefore |E_\theta| = 0.21 \text{ V/cm}$$

09. Ans: (c)

Sol: Given:

Length of dipole, $\ell = 0.01\lambda$

As it is very small, compared with wavelength, hence it can be approximated to Hertzian dipole

$$R_{\text{rad}} = 80\pi^2 \left(\frac{d\ell}{\lambda} \right)^2$$

$$= 80\pi^2 (0.01)^2$$

$$R_{\text{rad}} = 0.08 \Omega$$

10. Ans: (d)

Sol: $AF = \frac{\sin \frac{n\phi}{2}}{\sin \frac{\phi}{2}}$

take limit

$$\lim_{\frac{n\phi}{2} \rightarrow 0} \frac{\sin \frac{n\phi}{2}}{\frac{n\phi}{2}} \cdot \frac{n\phi}{2} = n$$

$$\lim_{\frac{\phi}{2} \rightarrow 0} \frac{\sin \frac{\phi}{2}}{\frac{\phi}{2}} \cdot \frac{\phi}{2} = n$$

11. Ans: (b)

Sol: In broad side array the BWFN is given by

$$BWFN = \frac{2\lambda}{L} \text{ (rad)}$$

Where, L = length of the array

$$L = (n-1) d$$

Given: n = 9

Spacing, $d = \frac{\lambda}{4}$

$$BWFN = \frac{2\lambda}{(9-1) \frac{\lambda}{4}}$$

$$= \frac{2\lambda}{2\lambda} \times \frac{180}{\pi}$$

$$\therefore BWFN = 57.29^\circ$$

12. Ans: (d)

Sol: The directivity of n-element end fire array is given by

$$D = \frac{4L}{\lambda}$$

Where, L = (n-1)d

$$L \cong nd \quad (\because n = 1000, \text{very large})$$

$$D = \frac{4 \times nd}{\lambda}$$

$$= \frac{4 \times 1000\lambda}{\lambda \times 4}$$

$$\therefore D \cong 1000$$

$$\text{Directivity, (in dB)} = 30$$

13. Ans: 7.78

Sol: Directivity, $D = 4\pi \frac{U_{\max}}{P_{\text{rad}}}$

Given: $U(\theta, \phi) = 2\sin\theta \sin^3\phi$; $0 \leq \theta \leq \pi$,
 $0 \leq \phi \leq \pi$

$$U_{\max} = 2$$

$$P_{\text{rad}} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} 2\sin\theta \sin^3\phi \sin\theta d\theta d\phi$$

$$= 2 \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} \sin^2\theta \sin^3\phi d\theta d\phi$$

$$= 2 \left(\frac{\pi}{2} \right) \left(\frac{4}{3} \right)$$

$$= \frac{4\pi}{3}$$

$$D = 4\pi \times \frac{2}{\left(\frac{4\pi}{3} \right)}$$

$$D = 6$$

$$\text{Directivity, (in dB)} = 10\log 6 = 7.7815$$

14. Ans: 2793

Sol: For Hertzian dipole the directivity, D is given by $D = 1.5$

$$D = \left(\frac{4\pi}{\lambda^2} \right) A_e$$

$$A_e = 1.5 \times \frac{\lambda^2}{4\pi}$$

$$A_e = 0.119 \lambda^2$$

$$\text{Wavelength, } \lambda = \frac{3 \times 10^8}{10^8} = 3\text{m}$$

$$\therefore A_e = 0.119 \times 9$$

$$A_e = 1.074 \text{ m}^2$$

Aperture area of antenna is given by

$$A_e = \frac{P_r}{P}$$

Where, P_r = power received at the antenna load terminals.

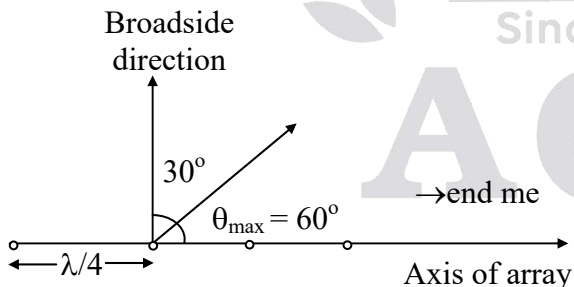
P = power density of incident wave

$$P = \frac{P_r}{A_e} = \frac{3 \times 10^{-6}}{1.074}$$

$$\therefore P = 2.793 \mu\text{W/m}^2 \text{ (or) } 2793 \text{ nW/m}^2$$

15. Ans: (c)

Sol:



Given: No. of elements, $n = 4$

$$\text{Spacing, } d = \frac{\lambda}{4}$$

Direction of main beam (or) principal lobe,

$$\theta_{\max} = 60^\circ$$

Array phase function, ψ is given by

$$\psi = \beta d \cos \theta + \alpha$$

To form a major lobe. $\psi = 0$

$$\alpha = -\beta d \cos \theta_{\max}$$

$$\alpha = -\frac{2\pi}{\lambda} \times \frac{\lambda}{4} \cos 60$$

$$\alpha = -\frac{\pi}{4}$$

The phase shaft between the elements

required is $\alpha = -\frac{\pi}{4}$.

16. Ans: (a, b, c)

Sol:

$$(a) \text{ The wavelength, } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6} = 6\text{m}$$

Hence, the length of half wave dipole is

$$\ell = \frac{\lambda}{2} = \frac{6}{2} = 3\text{m}$$

$$(b) |E_{\phi s}| = \frac{\eta_0 I_0 \cos\left(\frac{\pi}{2} \cos \theta\right)}{2\pi r \sin \theta}$$

$$I_0 = \frac{|E_{\phi s}| 2\pi r \sin \theta}{\eta_0 \cos\left(\frac{\pi}{2} \cos \theta\right)}$$

$$= \frac{10 \times 10^{-6} \times 2\pi \times 500 \times 10^3 \times 1}{120\pi \times 1} = 83.33\text{mA}$$

(c) For half wave dipole antenna, $R_{\text{rad}} = 73\Omega$

$$P_{\text{rad}} = \frac{1}{2} I_0^2 R_{\text{rad}} = \frac{1}{2} (83.33)^2 \times 10^{-6} \times 73 = 253.5\text{mW}$$

So a, b, c is correct