



ESE – 2025

MAINS EXAMINATION

QUESTIONS WITH DETAILED SOLUTIONS

MECHANICAL ENGINEERING

(Paper-2)

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MECHANICAL ENGINEERING

ESE MAINS 2025 PAPER – II

Questions with Detailed Solutions

Subject-wise Weightage

S.No.	NAME OF THE SUBJECT	Marks
1	Engineering Mechanics	20
2	Strength of Materials	96
3	Theory of Machines	72
4	Machine Design	72
5	Production Engineering	60
6	Material Science	52
7	Mechatronics & Robotics	64
8	Maintenance Engineering	12
9	IM & OR	32

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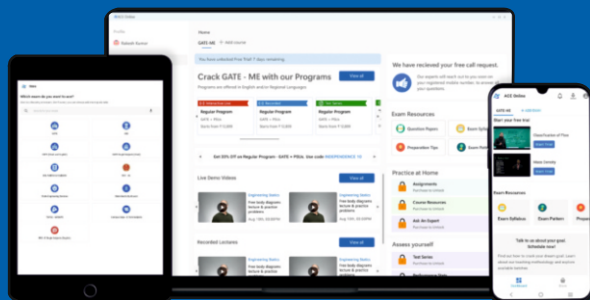
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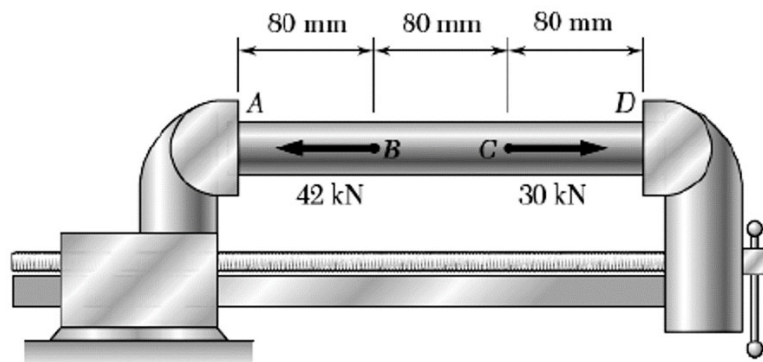
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SECTION – A

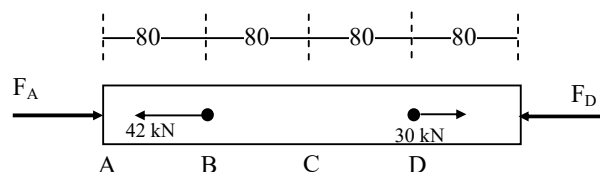
1.(a) A steel tube ($E=200$ GPa) with a 32 mm outer diameter and a 4 mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces as shown in figure are then applied to the tube. After these forces are applied the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine

- the forces exerted by the vise on the tube at A and D.
- the change in length of the portion BC of the tube.



(12 M)

Sol:



- (i) Given $\delta_{AD} = -0.2$ mm, find F_A , F_D = ?

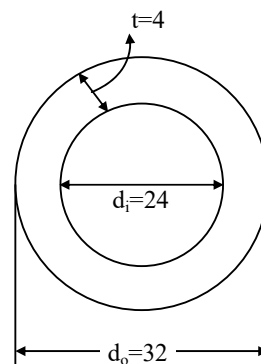
$$A = \frac{\pi}{4} [d_o^2 - d_i^2]$$

$$= \frac{\pi}{4} [32^2 - 24^2]$$

$$= 352 \text{ mm}^2$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= \frac{P_1 \ell_1}{A_1 E_1} + \frac{P_2 \ell_2}{A_2 E_2} + \frac{P_3 \ell_3}{A_3 E_3} \quad \text{----- (1)}$$



Here, $A_1 = A_2 = A_3 = A = 352 \text{ mm}^2$

$\ell_1 = \ell_2 = \ell_3 = 80 \text{ mm}$

$E_1 = E_2 = E_3 = 200 \times 10^3 \frac{\text{N}}{\text{mm}^2}$

$P_1 = -F_A$, $P_2 = 42 - F_A$, $P_3 = 42 - F_A - 30 = 12 - F_A$ (Put in Eq.(1))

$$-0.2 = \frac{80}{352 \times 200 \times 10^3} [-F_A + 42 - F_A + 12 - F_A]$$

$$-176 \text{ kN} = -3F_A + 54$$

$$F_A = 76.66 \text{ kN}$$

To find F_D apply $\Sigma F_x = 0$

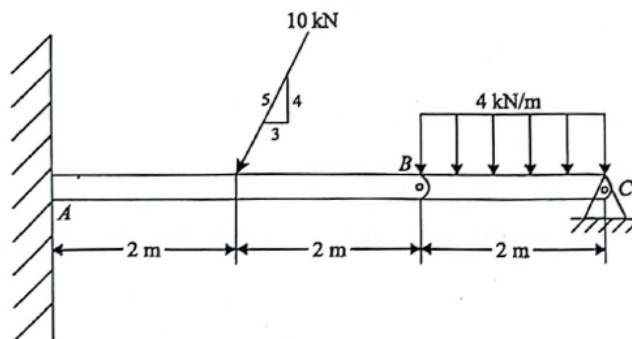
$$42 + F_D = F_A + 30$$

$$F_D = 76.66 + 30 - 42$$

$$F_D = 64.66 \text{ kN}$$

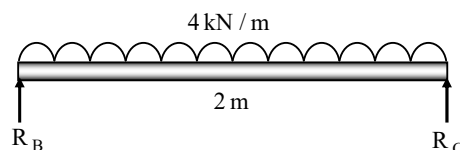
$$\begin{aligned} \text{(ii) Now, } \delta_{BC} &= \frac{P_{BC} \times L_{BC}}{AE} = \frac{[42 - F_A] \times 10^3 \times 80}{352 \times 200 \times 10^3} \\ &= \frac{[42 - 76.66] \times 10^3 \times 80}{352 \times 200 \times 10^3} \\ &= -0.039 \text{ mm} \end{aligned}$$

1.(b) The compound beam shown in figure below is pin connected at B. Determine the components of reaction at its supports. Neglect its weight and thickness.



(12 M)

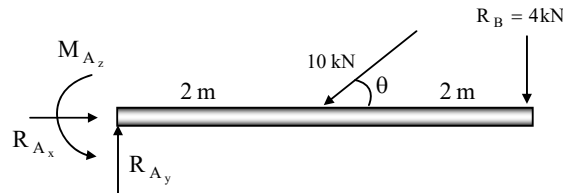
Sol: FBD of BC



As its symmetric,

$$R_B = R_C = \frac{wL}{2} = \frac{4 \times 2}{2} = 4 \text{ kN}$$

FBD of AB



$$\sin \theta = \frac{4}{5}$$

$$\cos \theta = \frac{3}{5}$$

$$\Sigma F_x = 0$$

$$10 \cos \theta = R_{Ax}$$

$$R_{Ax} = 10 \times \frac{3}{5} = 6 \text{ kN}$$

$$\Sigma F_y = 0$$

$$R_{Ay} = 10 \sin \theta + 4$$

$$= 10 \times \frac{4}{5} + 4$$

$$= 12 \text{ kN}$$

$$\Sigma H_A = 0$$

$$10 \sin \theta \times 2 + 4 \times 4 = M_{Az}$$

$$M_{Az} = 10 \times \frac{4}{5} \times 2 + 16 = 32 \text{ kN-m}$$

1.(c) A compound cylinder is formed by shrinking one tube of 100 mm internal diameter and 25 mm wall thickness on to another tube of 100 mm external diameter and 25 mm wall thickness. Both the tubes are made of steel (with $E = 210 \text{ GPa}$) and the shrinkage allowance based on radius is 0.02 mm. Determine the pressure set up at the junction due to shrinkage.

(12 M)

Sol: Given:

$$E = 210 \times 10^3 \text{ MPa}$$

$$\delta_R = 0.02 \text{ mm}$$

Shrinkage pressure = ?

$$\text{Shrinkage allowance} = \frac{r}{E} [\sigma_{H_o} - \sigma_{H_i}] = 0.02 \dots\dots(1)$$

For finding σ_{H_o} and σ_{H_i} we use Lames Equations

$$\Rightarrow \sigma_H = A + \frac{B}{r^2} \quad \sigma_r = A - \frac{B}{r^2} \dots\dots\dots(2)$$

For Inner Tube:

$$\text{At } r = 25 \text{ mm} = 0.025 \Rightarrow \sigma_r = -P_i = 0$$

$$\text{At } r = 50 \text{ mm} = 0.050 \Rightarrow \sigma_r = -P \text{ [Junction pressure]}$$

Put in equation (2)

$$0 = A - \frac{B}{0.025^2} = A - 1600B \Rightarrow A = 1600B$$

$$-P = A - \frac{B}{0.050^2} = A - 400B = 1200B$$

$$\Rightarrow B = \frac{-P}{1200}$$

$$\Rightarrow A = \frac{-4}{3}P$$

So at common radius hoop stress of inner tube

$$\text{At } r = r_i = 0.050$$

$$\sigma_{H_i} = A + \frac{B}{r^2} = \frac{-4}{3}P + \frac{-P}{1200 \times 0.050^2} = \frac{-5}{3}P = -1.67P$$

For outer tube:

$$\text{At } r = 0.050, \sigma_r = -P \text{ and } r = 0.075, \sigma_r = 0$$

$$-P = A - \frac{B}{0.050^2}$$

$$-P = A - 400B \dots\dots\dots(a)$$

$$0 = A - \frac{B}{0.075^2}$$

$$0 = A - 178B$$

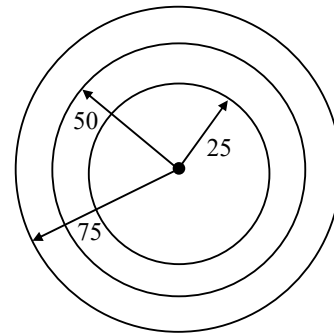
$$A = 178B \dots\dots\dots(b)$$

Solving (a) & (b)

$$-P = 178B - 400B$$

$$P = 222B \Rightarrow B = \frac{P}{222}, A = \frac{178}{222}P$$

At common radius, hoop stress in outer tube is



$$\sigma_{ho} = A + \frac{B}{r_j^2} = \frac{178}{222} P + \frac{P}{222 \times 0.05^2} = 2.60 P$$

Now:

Shrinkage allowance,

$$\delta_R = 0.02 = \frac{r}{E} [\sigma_{h_o} - \sigma_{h_i}]$$

$$0.02 \times 10^{-3} = \frac{0.050}{210 \times 10^9} [2.6 P (-1.67 P)]$$

$$P = 19672131 \frac{N}{m^2} = 19.67 \text{ MPa}$$

- 1.(d) A pair of 20° full depth involute spur gears are in mesh. The larger gear has 48 teeth whereas the pinion has 12 teeth. The module is 10 mm. Determine**
- (i) the reduction in addendum of the gear to avoid interference, and**
- (ii) Contact ratio** **(12 M)**

Sol:

- (i) Given data: $\phi = 20^\circ$, $T = 48$, $t = 12$, $m = 10 \text{ mm}$

$$\text{Gear ratio } G = \frac{T}{t} = \frac{48}{12} = 4, \lambda = \frac{1}{G} = \frac{1}{4} = 0.25$$

$$(a_w)_i = m = 10 \text{ mm}$$

Minimum no. of teeth on wheel to avoid interference,

$$T_{\min} \geq \frac{2f_w}{\sqrt{1 + \lambda(\lambda + 2)\sin^2 \phi} - 1}$$

$$48 \geq \frac{2f_w}{\sqrt{1 + 0.25(0.25 + 2)\sin^2 20^\circ} - 1}$$

$$f_w \leq 0.778$$

$$\text{Addendum of wheel to avoid interference} = (a_w)_m = mf_w = 10 \times 0.778 = 7.78 \text{ mm}$$

$$\begin{aligned} \text{Reduction in addendum} &= (a_w)_i - (a_w)_m \\ &= 10 - 7.78 \\ &= 2.22 \text{ mm} \end{aligned}$$

(ii) $r = \frac{mt}{2} = \frac{10 \times 12}{2} = 60 \text{ mm}$

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$$R = \frac{m.T}{2} = \frac{10 \times 48}{2} = 240 \text{ mm}$$

$$R_A = R + (\text{add})_m = 240 + 7.78 = 247.78 \text{ mm}$$

$$r_A = r + (\text{add})_m = 60 + 7.78 = 67.78 \text{ mm}$$

$$\begin{aligned} \text{Arc of contact} &= \frac{\sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi + \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi}{\cos \phi} \\ &= \frac{\sqrt{247.78^2 - 240^2 \cos^2 20} - 240 \sin 20 + \sqrt{67.78^2 - 60^2 \cos^2 20} - 60 \sin 20}{\cos 20} \\ &= \frac{20.54 + 17.09}{\cos 20} = 40.05 \end{aligned}$$

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{40.05}{\pi \times 10} = 1.27$$

1.(e) A rotating solid shaft of diameter d is under bending moment M and Torque T without any axial load. Determine the equivalent bending moment, M_e , using Maximum Normal Stress Theory (MNST) and Distorsion Energy Theory (DE) and the equivalent twisting moment, T_e , using Maximum Shear Stress Theory (MSST). Effects due to fatigue and stress concentration are to be neglected. (12 M)

Sol: Given data:

Bending moment = m

Torque = T

Z_p = Polar section modulus of solid shaft

$$= \frac{\pi}{16} \cdot d^3$$

Z_H = Horizontal section modulus of solid shaft

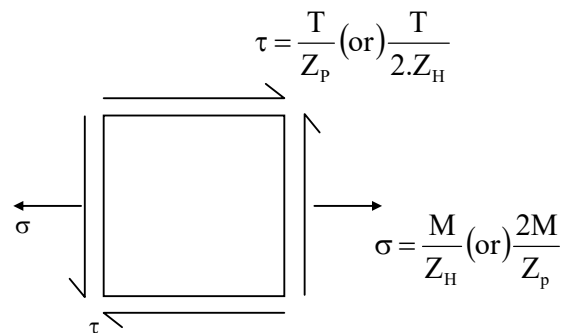
$$= \frac{\pi}{32} \cdot d^3$$

$$Z_p = 2.Z_H$$

When the shaft is subjected to bending and twisting, the extreme position from the bending axis is critical.

According to Rankine,

$$\sigma_{\max} \leq S_{yt}$$



$$\frac{\frac{M}{Z_H} + 0}{2} + \sqrt{\left(\frac{\frac{M}{Z_H} - 0}{2}\right)^2 + \left(\frac{T}{2Z_H}\right)^2} \leq s_{yt}$$

$$\frac{1}{Z_H} \cdot \left(\frac{1}{2} \left(M + \sqrt{M^2 + T^2}\right)\right) \leq s_{yt}$$

$$\frac{M_{eq}}{Z_H} \leq s_{yt}$$

∴ According to Rankine

$$M_{eq} = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

According to Vonmises,

$$\sigma_{vm} \leq s_{yt}$$

$$\sqrt{\left(\frac{M}{Z_H}\right)^2 + 3\left(\frac{T}{2Z_H}\right)^2} \leq s_{yt}$$

$$\frac{1}{Z_H} \cdot \sqrt{M^2 + \frac{3}{4}T^2} \leq s_{yt}$$

$$\frac{M_{eq}}{Z_H} \leq s_{yt}$$

According to Vonmises, $M_{eq} = \sqrt{M^2 + \frac{3}{4}T^2}$

According to Tresca,

$$\tau_{max} \leq s_{ys}$$

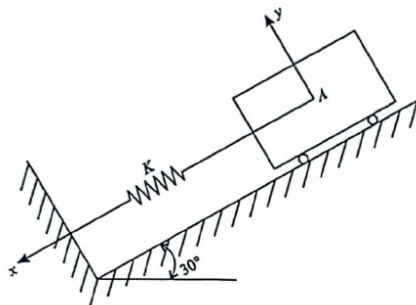
$$\sqrt{\left(\frac{\frac{M}{Z_H} - 0}{2}\right)^2 + \left(\frac{T}{Z_p}\right)^2} \leq s_{ys}$$

$$\frac{1}{Z_p} \cdot \sqrt{M^2 + T^2} \leq s_{ys}$$

$$\frac{T_{eq}}{Z_p} \leq s_{ys}$$

According to Tresca, $T_{eq} = \sqrt{M^2 + T^2}$

2.(a) A cart A shown in figure having a mass of 200 kg is held on an incline so as to just touch the undeformed spring whose constant K is 50 N/mm. If body A is released very slowly, what distance down the incline must A move to reach an equilibrium configuration? If body A is released suddenly, what is its speed when it reaches the aforementioned equilibrium configuration for a slow release ?



(20 M)

Sol: Given

$$m = 200 \text{ kg}$$

$$k = 50 \text{ N/mm} = 50 \times 10^3 \text{ N/m}$$

$$Q = 30^\circ$$

Part -1

Assume x = distance the cart moves down
= Spring deflection

FBD of CART of equilibrium

$$\text{Apply } \Sigma F_x = 0$$

$$200g \sin \theta = Kx$$

$$x = \frac{200 \times 9.81 \times \sin 30}{50000}$$

$$= 0.0196 \text{ m} = 19.6 \text{ mm}$$

Part-2

Released suddenly, find 'v' when it reaches 'x'

Work done by gravity along incline = Spring P.E + [KE] of mass

$$mg \sin \theta x = \frac{1}{2} Kx^2 + \frac{1}{2} mv^2$$

$$200g \sin 30 \times 0.0196 = \frac{1}{2} \times 50000 \times [0.0196]^2 + \frac{1}{2} \times 200 \times v^2$$

$$\Rightarrow v = 0.31 \text{ m/s}$$

2.(b) The torque produced by an engine is given by the expression

$$T = (5000 + 1500 \sin 3\theta) \text{ N-m}$$

where θ (theta) is the angle turned by the crank measured from some datum. The mean engine speed is 300 rpm and the flywheel and other rotating parts attached to the shaft have a mass of 450 kg with radius of gyration 500 mm. Determine

(i) the power of the engine

(ii) percentage fluctuation of speed when the resisting torque is constant

(iii) percentage fluctuation of speed when the resisting torque is $(5000 + 600 \sin \theta)$

(20 M)

Sol: Given : $T = (5000 + 1500 \sin 3\theta) \text{ N-m}$,

$N = 300 \text{ rpm}$

$$\omega = 2\pi \times 300 / 60 = 31.42 \text{ rad/s},$$

$$I = m \times k^2 = 450 \times (0.5)^2 = 112.5 \text{ kg.m}^2$$

(i) Power of the engine:

We know that work done per revolution

$$= \int_0^{2\pi} (5000 + 1500 \sin 3\theta) d\theta = \left[5000\theta - \frac{1500 \cos 3\theta}{3} \right]_0^{2\pi} = 10,000 \pi \text{ N-m}$$

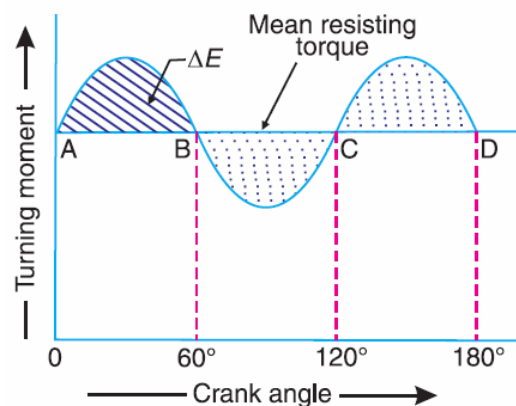
$$T_{\text{mean}} = \frac{10000 \pi}{2\pi} = 5000 \text{ N-m}$$

$$\text{Power} = T_{\text{mean}} \cdot \omega = 5000 \times 31.42 = 157.1 \text{ kW}$$

Let C_s = Maximum or total fluctuation of speed of the flywheel

(ii) When resisting torque is constant:

Since the resisting torque is constant, therefore the torque exerted on the shaft is equal to the mean resisting torque on the flywheel



$$\therefore T = T_{\text{mean}}$$

$$5000 + 1500 \sin 3\theta = 5000$$

$$1500 \sin 3\theta = 0 \text{ or } \sin 3\theta = 0$$

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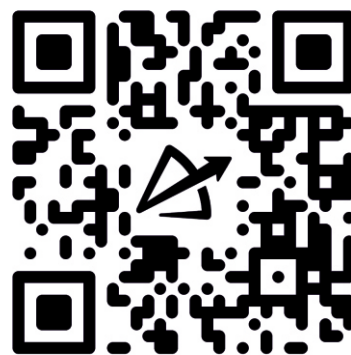


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$$\therefore 3\theta = 0^\circ \text{ or } 180^\circ$$

$$\theta = 0^\circ \text{ or } 60^\circ$$

$\therefore$ Maximum fluctuation of energy,

$$\begin{aligned}\Delta E &= \int_0^{60^\circ} (T - T_{\text{mean}}) d\theta = \int_0^{60^\circ} (5000 + 1500 \sin 3\theta - 5000) d\theta \\ &= \int_0^{60^\circ} 1500 \sin 3\theta d\theta = \left[-\frac{1500 \cos 3\theta}{3} \right]_0^{60^\circ} = 1000 \text{ N-m}\end{aligned}$$

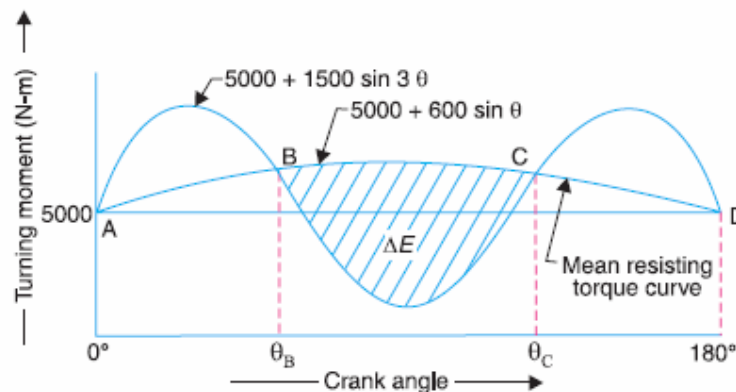
We know that maximum fluctuation of energy (ΔE),

$$1000 = I \omega^2 \cdot C_s = 112.5 \times (31.42)^2 \times C_s$$

$$\therefore C_s = 0.009 \text{ or } 0.9\%$$

(iii) When resisting torque is $(5000 + 600 \sin \theta)$ N-m

The turning moment diagram is shown in figure. Since at points B and C, the torque exerted on the shaft is equal to the mean resisting torque on the flywheel, therefore



$$5000 + 1500 \sin 3\theta = 5000 + 600 \sin \theta \text{ or } 2.5 \sin 3\theta = \sin \theta$$

$$2.5(3 \sin \theta - 4 \sin^3 \theta) = \sin \theta$$

$$3 - 4 \sin^2 \theta = 0.4 \quad (\text{dividing by } 2.5 \sin \theta)$$

$$\sin^2 \theta = \frac{3 - 0.4}{4} = 0.65 \text{ of } \sin \theta = 0.8062$$

$$\therefore \theta = 53.7^\circ \text{ or } 126.3^\circ \text{ i.e. } \theta_B = 53.7^\circ \text{ and } \theta_C = 126.3^\circ$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \int_{53.7^\circ}^{126.3^\circ} [(5000 + 1500 \sin 3\theta) - (5000 + 600 \sin \theta)] d\theta$$

$$= \int_{53.7^\circ}^{126.3^\circ} (1500 \sin 3\theta - 600 \sin \theta) d\theta = \left[-\frac{1500 \cos 3\theta}{3} + 600 \cos \theta \right]_{53.7^\circ}^{126.3^\circ}$$

$$= 1656 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$1656 = I \omega^2 C_s = 112.5 \times (31.42)^2 \times C_s$$

$$\therefore C_s = 0.0037 \text{ or } 0.37 \%$$

2.(c) The following data refers to a pair of spur gears with 20° full depth involute teeth:

Number of teeth on pinion = 24

Number of teeth on gear = 56

Speed of pinion = 1200 rpm

Module = 3 mm

Face width = 30 mm

Both gears are made of steel with an ultimate tensile strength of 600 N/mm^2 . Using the velocity factor to account for the dynamic load and assuming service factor as 1.5, determine

(i) beam strength, and

(ii) rated power that the gears can transmit without bending failure, if the factor of safety is 1.5.

Take Lewis form factor for 24 teeth equal to 0.337, and velocity factor, $C_v = \frac{3}{3+v}$, where v is the pitch line velocity in m/s.

(20 M)

Sol: Given data:

Z_p = no. of teeth on pinion = 24

Z_g = no. of teeth on gear = 56

N_p = speed of pinion = 1200 rpm

m = module = 3 mm

b = Face width = 30 mm

S_{ut} = ultimate tensile strength of steel = 600 MPa

n = Factor of safety = 1.5

(i) Beam strength,

As pinion and gear are made of same material pinion is weaker then gear

F_b = Beam strength of pinion

$$= S_b \cdot m \cdot b \cdot Y$$

$$= 600 \times 3 \times 30 \times 0.337$$

$$F_b = 18.2 \times 10^3 \text{ N}$$

(ii) Rated power

V = pitch line velocity

$$= \frac{\pi \cdot d \cdot N}{60}$$

d = pitch circle diameter = $m \cdot Z$

$$= 3 \times 24 = 72 \text{ mm}$$

$$= 0.072 \text{ m}$$

$$V = \frac{\pi \times 0.072 \times 1200}{60} = 4.5 \text{ m/s}$$

$$C_v = \frac{3}{3 + 4.5} = 0.4$$

$\therefore$ For safety

$$\frac{C_s}{C_v} \cdot F_t \leq \frac{F_b}{\text{FOS}}$$

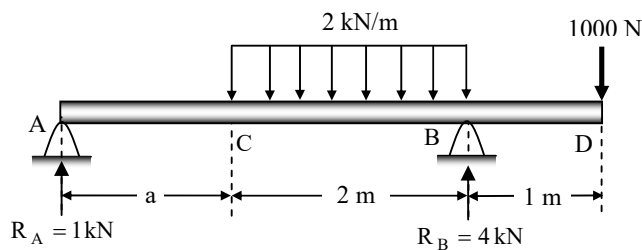
$$\frac{1}{0.4} \times \frac{60 \times P}{2\pi \times 1200} \times 10^3 \times \frac{1}{36} \leq \frac{18.2 \times 10^3}{1.5}$$

Where $F_t = \frac{60P}{2\pi N} \times 10^3 \times \frac{1}{r}$

r = pitch circle radius

$$P \leq 21.96 \text{ kW}$$

3.(a) Find the value of a and draw the bending moment diagram for the beam shown in figure below.



(20 M)

Sol: Take $\Sigma M_c = 0$

$$1 \times a + 2 \times 2 \times 1 + 1 \times 3 = 4 \times 2$$

$$a = 1 \text{ m}$$

From SFD, find zero shear force location 'x' using similar triangle

$$\frac{1}{x} = \frac{3}{2-x}$$

$$2-x = 3x$$

$$4x = 2$$

$$x = 0.5$$

BM at imp locations:

$$\text{At } M_A = 0$$

$$M_D = 0$$

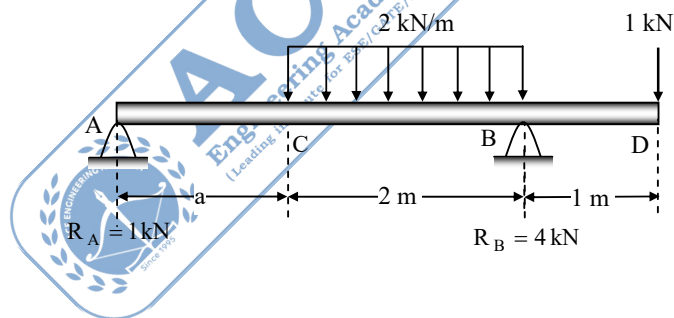
$$M_C = +1 \times 1 = 1 \text{ kN-m}$$

$$M_B = -1 \times 1 = -1 \text{ kN-m}$$

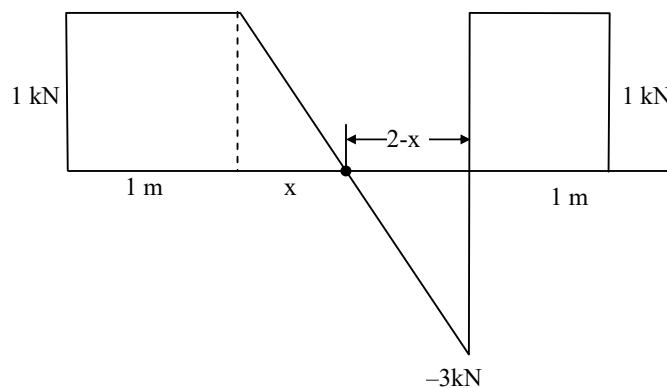
$$M_{\max} \text{ at } 1.5 \text{ m from A}$$

$$= +1 \times 1.5 - 2 \times 0.5 \times 0.25$$

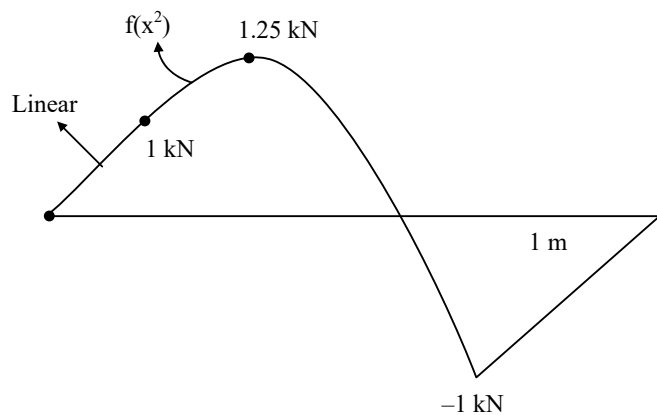
$$= 1.25 \text{ kN-m}$$



SFD



BMD



3.(b)(i)

A torsional pendulum has a natural frequency of 200 cycles/min when vibrating in a vacuum. The mass moment of inertia of the disc is 0.2 kg-m^2 . It is then immersed in oil and its natural frequency is found to be 180 cycles/min. Determine the damping constant.

If the disc, when placed in oil, is given an initial angular displacement of 2° , find its displacement at the end of first cycle.

(10 M)

Sol: $f_n = 200 \frac{\text{cycles}}{\text{min}} = \frac{200}{60} \text{ Hz}$

$$\omega_n = 2\pi \times \frac{200}{60} = 20.94 \text{ rad/s}$$

$$I = 0.2 \text{ kg-m}^2, \quad x_1 = 2$$

$$f_d = 180 \frac{\text{cycles}}{\text{min}} = \frac{180}{60} = 3 \text{ Hz}$$

$$\omega_d = 2\pi \times 3 = 18.85 \text{ rad/s}$$

$$\omega_d = \sqrt{1 - \xi^2} \omega_n$$

$$18.85 = \sqrt{1 - \xi^2} \times 20.94$$

$$\Rightarrow \xi = 0.435$$

$$\xi = \frac{C}{C_c}$$

$$\text{Damping constant } C = \xi C_c = \xi \times 2I \omega_n$$

$$= 0.435 \times 2 \times 0.2 \times 20.94 = 3.64 \frac{\text{Nms}}{\text{rad}}$$

$$\text{Logarithmic decrement} = \ell n \left(\frac{x_1}{x_2} \right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\ell n \left(\frac{2}{x_2} \right) = \frac{2\pi \times 0.435}{\sqrt{1-0.435^2}}$$

$$\Rightarrow x_2 = 0.096^\circ$$

3.(b)(ii)

The natural frequency of vibration of a person is found to be 5.2 Hz while standing on a horizontal floor. Assuming damping to be negligible, determine

- (A) the equivalent stiffness of his body in the vertical direction if the mass of the person is 70 kg.
- (B) the amplitude of vertical displacement of the person if the floor is subjected to a vertical harmonic vibration of frequency 5.3 Hz and amplitude 0.1 m due to an unbalanced rotating machine operating on the floor.

(10 M)

Sol: $f_n = 5.2 \text{ Hz}$,

$m = 70 \text{ kg}$,

$$\omega_n = 2\pi \times 5.2 = 32.67 \text{ rad/s}$$

$$(A) \quad k = m\omega_n^2 = 70 \times (2\pi \times 5.2)^2 = 74724.75 \text{ N/m}$$

(B) $f = 5.3 \text{ Hz}$

$$\omega = 2\pi \times 5.3 = 33.3 \text{ rad/s}$$

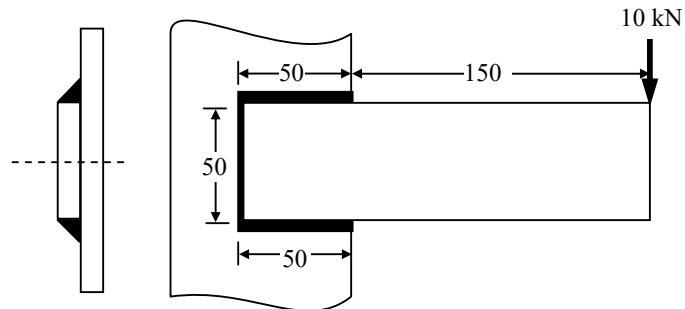
$$y = 0.1 \text{ m}$$

$$\frac{x}{y} = \frac{1}{\left(\frac{\omega}{\omega_n} \right)^2 - 1}$$

$$x = \frac{0.1}{\left(\frac{33.3}{32.67} \right)^2 - 1}$$

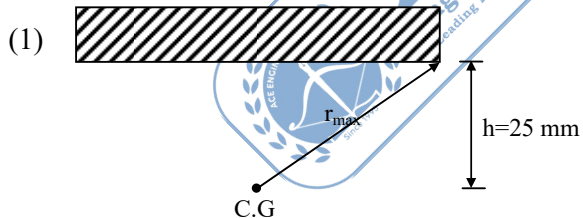
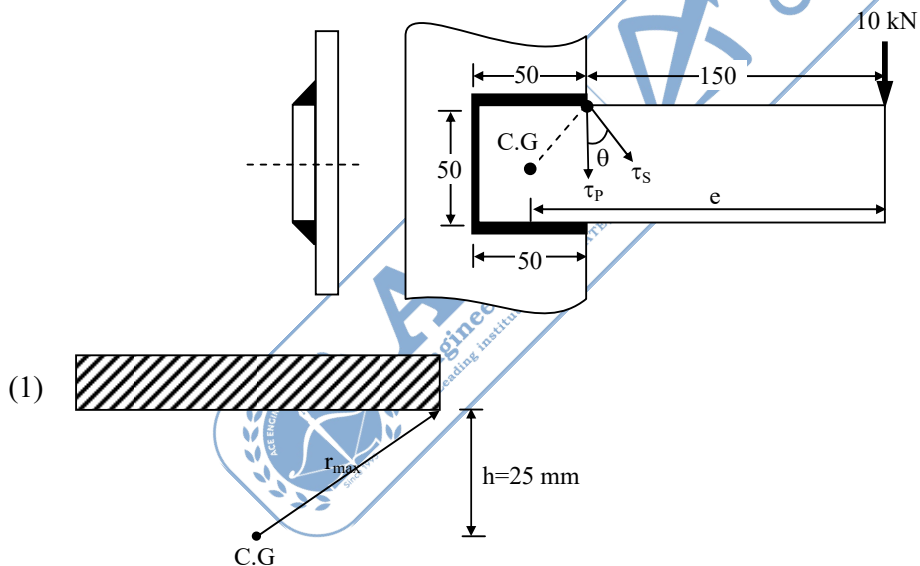
$$x = 2.568 \text{ mm}$$

3.(c) A welded connection of steel plates as shown in figure is subjected to an eccentric force of 10 kN. Assuming static conditions, determine the throat dimension of the welds if the permissible shear stress is limited to 95 MPa



(20 M)

Sol:



$$e = \text{eccentricity in loading} = 150 + 25 = 175 \text{ mm}$$

$$A_1 = A_2 = A = \text{Area of each weld}$$

$$= t \cdot \ell$$

$$\ell = \ell_1 = \ell_2 = 50 \text{ mm}$$

$$A = 50 t \text{ mm}^2$$

$$J = J_1 = J_2$$

= polar moment of inertia of each weld about C G

$$= A \left[\frac{\ell^2}{12} + h^2 \right]$$

$$= 50t \left[\frac{50^2}{12} + 25^2 \right]$$

$$= 41.67t \times 10^3 \text{ mm}^4$$

τ_p = primary shear stress

$$= \frac{10 \times 10^3}{2 \times 50t} = \frac{100}{t} \frac{\text{N}}{\text{mm}^2}$$

τ_s = secondary shear stress

$$= \frac{T}{\Sigma J}, r_{\max} \quad (\because r_{\max} = 25\sqrt{2} \text{ mm})$$

$$= \frac{10 \times 10^3 \times 175}{2 \times 41.67t \times 10^3} \times 25\sqrt{2} = \frac{742.4}{t} \frac{\text{N}}{\text{mm}}$$

$\tau_{\max}$ = maximum possible shear stress

$$= \sqrt{\tau_p^2 + \tau_s^2 + 2\tau_p \tau_s \cos \theta}$$

$$= \frac{1}{t} \sqrt{100^2 + 742.4^2 + 2 \times 100 \times 742.4 \times \cos 45^\circ}$$

$$\tau_{\max} = \frac{816.2}{t} \frac{\text{N}}{\text{mm}^2}$$

Safety :

$$\tau_{\max} \leq \frac{S_{ys}}{\text{FOS}}$$

$$\frac{816.2}{t} \leq 95$$

$$\Rightarrow t \geq 8.6 \text{ mm}$$

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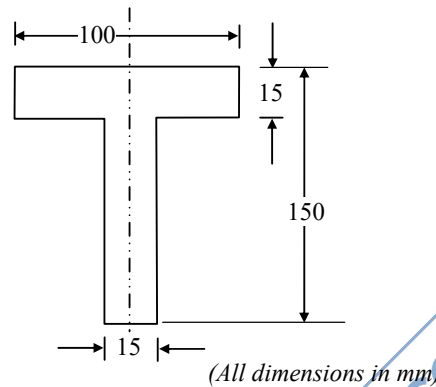


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- 4.(a) Find the position of the centroid of a T-section shown in figure below. The flange is 100×15 mm and the web is 135×15 mm. A cantilever of length 3000 mm and of section shown, with flange at the top, carries a load W at the free end. What is the maximum value of W, if the stress in the section is not to exceed 50 N/mm². (20 M)**



Sol: Find centroid:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{135 \times 15 \times \frac{135}{2} + 100 \times 15 \times 142.5}{135 \times 15 + 100 \times 15}$$

$$= 99.41 \text{ mm from bottom}$$

$$= 99.4$$

MOI about centroid axis,

$$\text{MOI} = I = I_{\text{flange}} + I_{\text{web}}$$

$$= \frac{100 \times 15^3}{12} + (100 \times 15) \times (142.5 - 99.4)^2 + \frac{15 \times 135^3}{12} + 15 \times 135 \times (99.4 - 67.5)^2$$

$$= 2814540 + 5136129 = 7950669 = 7.95 \times 10^6 \text{ mm}^4$$

Part-2: If beam fails by bending

$$\sigma_{\max} = \frac{M_{\max} y_{\max}}{I} \leq 50$$

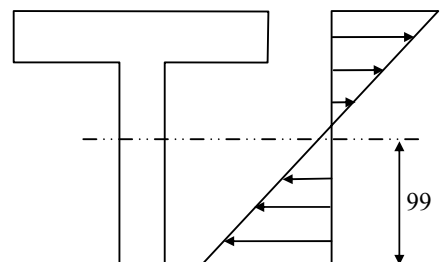
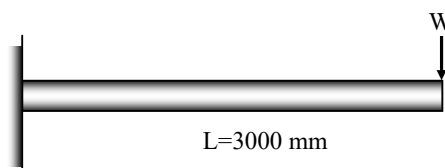
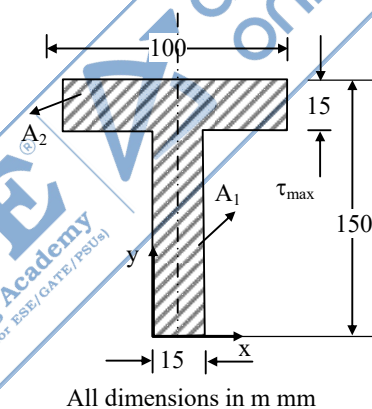
$$M_{\max} = W \times L = 3000 W$$

$$y_{\max} = 99.4$$

$$I = 7.95 \times 10^6 \text{ mm}^4$$

$$\sigma_{\max} = \frac{3000 W \times 99.4}{7.95 \times 10^6} = 50$$

$$W = 1332 \text{ N} = 1.3 \text{ kN}$$



Part-3

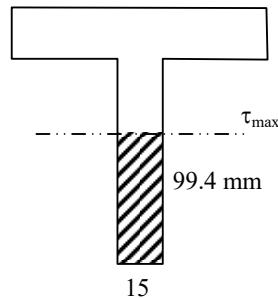
If beam fails by shearing

$$\tau_{\max} = \frac{V_{\max} Q}{Ib} = 50$$

$$\frac{W \times \left[99.4 \times 15 \times \frac{99.4}{2} \right]}{7.95 \times 10^6 \times 15} = 50$$

$$W = 80462 = 80.46 \text{ kN}$$

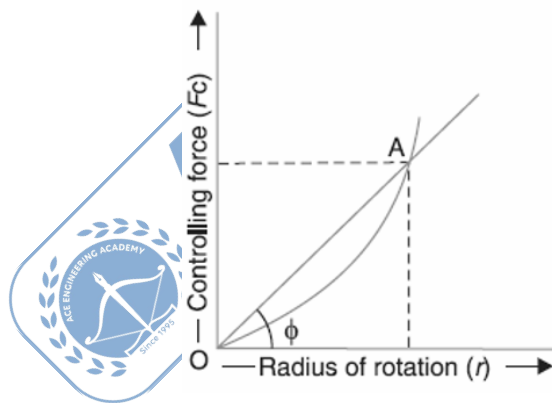
$$W_{\text{safe}} = \min (1.3, 80.46) = 1.3 \text{ kN}$$



4.(b)(i)

What is the controlling force of a governor? Draw a typical controlling force diagram for spring controlled governors and explain how it helps in establishing the stability or instability of a governor. (8 M)

Sol: Controlling Force



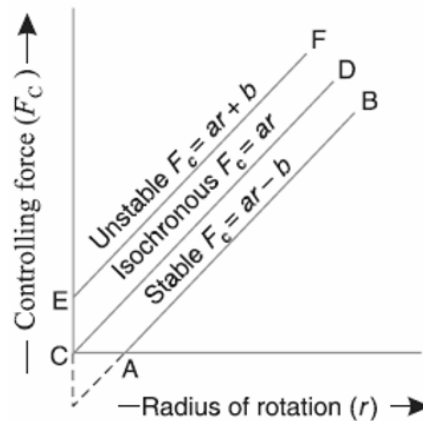
When a body rotates in a circular path, there is an inward radial force or centripetal force acting on it. In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as *controlling force*. It is equal and opposite to the centrifugal reaction.

$$\therefore \text{Controlling force, } F_C = m \cdot \omega^2 \cdot r$$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor).

When the graph between the controlling force (F_C) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as *controlling force diagram*. This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

Controlling Force Diagram for Spring-controlled Governors



The controlling force diagram for the spring controlled governors is a straight line, as shown in Fig. We know that controlling force,

$$F_C = m \cdot \omega^2 \cdot r \quad \text{or} \quad F_C / r = m \cdot \omega^2$$

The following points, for the stability of spring-controlled governors, may be noted:

1. For the governor to be stable, the controlling force (F_C) must increase as the radius of rotation (r) increases, i.e. F_C/r must increase as r increases. Hence the controlling force line AB when produced must intersect the controlling force axis below the origin, as shown in Fig. The relation between the controlling force (F_C) and the radius of rotation (r) for the *stability* of spring controlled governors is given by the following equation

$$F_C = a \cdot r - b \quad \dots\dots (i)$$

where a and b are constants.

2. The value of b in equation (i) may be made either zero or positive by increasing the initial tension of the spring. If b is zero, the controlling force line CD passes through the origin and the governor becomes *isochronous* because F_C / r will remain constant for all radii of rotation. The relation between the controlling force and the radius of rotation, for an *isochronous governor* is, therefore,

$$F_C = a \cdot r \quad \dots\dots (ii)$$

3. If b is greater than zero or positive, then F_C / r decreases as r increases, so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable.

Such a governor is said to be *unstable* and the relation between the controlling force and the radius of rotation is, therefore

$$F_C = a \cdot r + b \quad \dots\dots(iii)$$



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4.(b)(ii)

In case of a Porter governor each arm is 250 mm long and the arms are pivoted on the governor axis. The weight of the sleeve is 300 N and the weight of each ball is 50 N. The radii of rotation of the balls corresponding to minimum and maximum speeds are 130 mm and 170 mm respectively. Determine the range of speed of the governor neglecting friction. If the friction at the sleeve is taken equivalent to 25 N of load at the sleeve, determine how the speed range is modified.

(12 M)

Sol: Weight of each arm (L) = 250 mm

Weight of sleeve = W = 300 N

Weight of flyball = w = 50 N

$$r_1 = 130 \text{ mm}$$

$$r_2 = 170 \text{ mm}$$

$$f = 25 \text{ N}$$

Equilibrium speed of porter governor,

$$\omega^2 = \frac{\left[mg + \frac{Mg}{2}(1+q) \right]}{mh}$$

Upper and lower arms pivoted to axis of rotation and are of same length . So q = 1

$$h_1 = \sqrt{\ell^2 - r_1^2} = \sqrt{250^2 - 130^2} = 213.54 \text{ mm}$$

$$h_2 = \sqrt{\ell^2 - r_2^2} = \sqrt{250^2 - 170^2} = 183.30 \text{ mm}$$

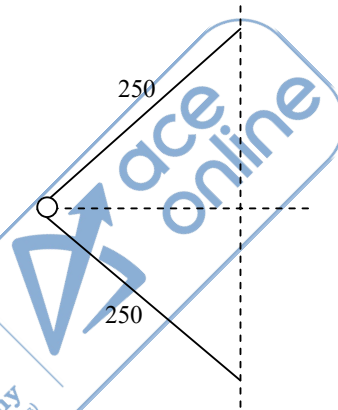
$$\omega_1^2 = \frac{\left(50 + \frac{300}{2}(1+1) \right)}{\left(\frac{50}{9.81} \times 0.21354 \right)} \Rightarrow \omega_1 = 17.93 \text{ rad/sec}$$

$$\omega_2^2 = \frac{\left(50 + \frac{300}{2}(1+1) \right)}{\left(\frac{50}{9.81} \times 0.1833 \right)} \Rightarrow \omega_2 = 19.35 \text{ rad/sec}$$

$$\text{Range of speed} = \omega_2 - \omega_1 = 19.35 - 17.93 = 1.42 \text{ rad/sec}$$

Considering friction:

$$\omega_1^2 = \frac{\left(mg + \frac{Mg-f}{2}(1+q) \right)}{m.h_1} = \frac{\left(50 + \left(\frac{300-25}{2} \right)(2) \right)}{\left(\frac{50}{9.81} \times 0.21354 \right)} = 17.18 \text{ rad/sec}$$



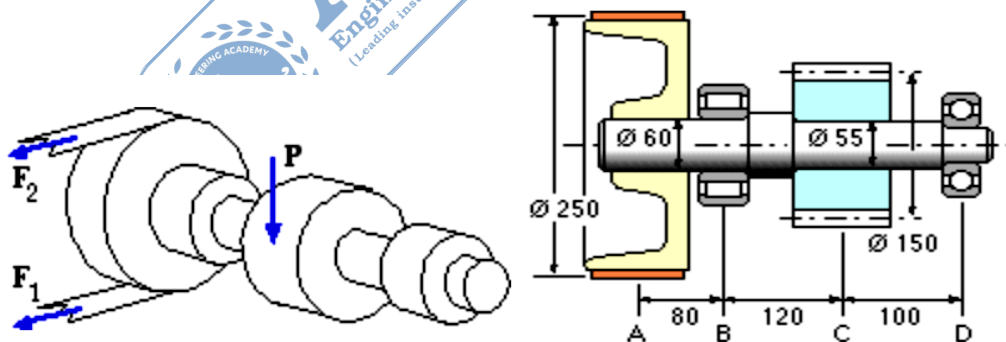
$$\omega_2^2 = \frac{\left(mg + \frac{Mg + f}{2}(1 + q) \right)}{mh_2}$$

$$\omega_2^2 = \frac{\left(50 + \left(\frac{300 + 25}{2} \right)(2) \right)}{\frac{50}{9.81} \times 0.1833}$$

$$\omega_2 = 20.03 \text{ rad/sec}$$

$$\text{Range of speed considering friction} = \omega_2 - \omega_1 = 20.03 - 17.18 = 2.75 \text{ rad/sec}$$

4.(c) The horizontal shaft ABCD is mounted in bearings at B and D as shown in figure. A belt passes around the 250 mm diameter pulley fixed to the shaft at A and a gear pinion of 150 mm pitch diameter is mounted on the shaft at C. Shaft diameters and axial disposition of the components are as sketched. The belt F strand tensions are horizontal and in the ratio $\frac{F_1}{F_2} = 4$, while the vertical reaction on the pinion, P, acts tangentially to the pinion's pitch circle. Ascertain the shaft's safety factor when transferring 20 kW from belt to pinion at a steady 450 rpm, taking the yield strength of the ductile shaft material to be 500 MPa. Fatigue and stress concentrations are to be neglected. All dimensions are in mm. Use Maximum Shear Stress Theory.



(20 M)

Sol: Power transmitted = 20 kW

s_{yt} = yield strength of steel shaft = 500 MPa

d_p = diameter of pulley = 250 mm

d_g = diameter of gear pinion = 150 mm

$$\text{Ratio of tensions} = \frac{F_1}{F_2} = 4$$

$$F_1 = 4.F_2$$

$$\text{Torque transmitted (T)} = \frac{60 \times 20 \times 10^3}{2\pi \times 450}$$

$$T = 424.4 \text{ N-m}$$

Torque transmitted by pulley

$$= (F_1 - F_2) \cdot \frac{dp}{2}$$

F_1 = tight side tension

F_2 = slake side tension

$$424.4 \times 10^3 = (4F_2 - F_2) \cdot \frac{250}{2}$$

$$F_2 = 1.13 \times 10^3 \text{ N}$$

$$F_1 = 4.52 \times 10^3 \text{ N}$$

Torque transmitted by gear

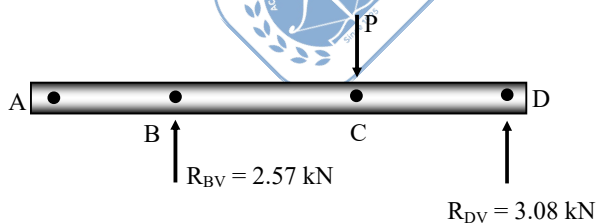
$$F_1 = P \cdot \frac{dg}{2}$$

$$424.4 \times 10^3 = P \times \frac{150}{2}$$

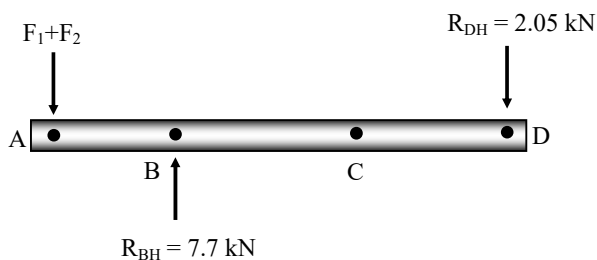
$$\text{Tangential load, } P = 5.66 \times 10^3 \text{ N}$$

F.B.D of shaft:

Vertical:



Horizontal:



Finding bending moment :

$$M_A = 0$$

$$\begin{aligned} M_B &= (F_1 + F_2) \times AB \\ &= (4.52 + 1.13) \times 10^3 \times 0.08 \\ &= 452 \text{ N-m} \end{aligned}$$

$$\begin{aligned} M_C &= \sqrt{(R_{DV})^2 + (R_{DH})^2} \times CD \\ &= \sqrt{(3.08)^2 + (2.05)^2} \times 10^3 \times 0.1 \\ &= 400 \text{ N-m} \end{aligned}$$

$$M_D = 0$$

$$\begin{aligned} \sigma_B &= \text{bending stress at B} \\ &= \frac{32 \times 452 \times 10^3}{\pi \times 60^3} = 21.31 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_C &= \text{Bending stress at C} \\ &= \frac{32 \times 400 \times 10^3}{\pi \times 55^3} = 24.5 \text{ MPa} \end{aligned}$$

$\therefore$ Section C is critical.

According to Tresca:

$$\tau_{\max} \leq \frac{s_{ys}}{n_T}$$

$$\frac{16}{\pi d^3} \sqrt{M^2 + T^2} \leq \frac{s_{ys}}{n_T}$$

$$\frac{16}{\pi d^3} \sqrt{400^2 + 424.4^2 \times 10^3} \leq \frac{500}{n_T}$$

$$n_T = 14$$

SECTION –B

05(a). A manufacturing unit has to supply 4200 unit of a product per year to the customer. The set-up cost per run is Rs. 75. Inventory carrying cost is Rs.1.5 per unit per annum. Shortages are not permitted.

Determine the following:

- (i) Economic order quantity**
- (ii) Optimum number of order per annum**
- (iii) Average annual inventory cost (minimum)**
- (iv) Optimum period of supply per optimum order**

(12 M)

Sol: Annual demand (D) = 4200 units

Carrying cost (C_c) = ₹1.5/unit/year

Setup cost (C_o) = ₹ 75

(i) EOQ

$$\begin{aligned} \text{EOQ} &= \sqrt{\frac{2DC_o}{C_c}} \\ &= \sqrt{\frac{2 \times 4200 \times 75}{1.5}} \\ &= 648 \text{ units} \end{aligned}$$

(ii) Optimum number of orders per annum

$$\begin{aligned} N &= \frac{D}{\text{EOQ}} \\ &= \frac{4200}{648} = 6.48 \end{aligned}$$

(iii) Minimum annual inventory cost (TIC) at

MIN TIC = Total inventory cost at EOQ

$$\begin{aligned} &= \sqrt{2DC_oC_c} \\ &= \sqrt{2 \times 4200 \times 75 \times 1.5} \\ &= 972 \end{aligned}$$

(iv) Optimum cycle time (T)

$$\begin{aligned} T &= \frac{1}{N} = \frac{1}{6.48} = 0.154 \text{ year} \\ &= 1.85 \text{ months} \end{aligned}$$



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5(b). A round specimen of wrought iron of diameter 12.5 mm and gauge length of 100 mm was tested in tension upto fracture. Following observations were obtained:

Load upto yield point = 29.5kN

Maximum load = 44kN

Load at time of fracture = 37kN

Diameter at neck = 9.2 mm

Total extention of specimen = 28.5mm

Calculate:

(i) yield strength

(ii) ultimate strength

(iii) actual breaking stress

(iv) percentage elongation

(v) modulus of resilience at yield point stress

Take Young's modulus $E = 200\text{kN} / \text{mm}^2$

(2×4 + 4 = 12 M)

Sol: Given data:

$$F_{\text{yield}} = 29.5 \text{ kN}, F_{\text{max}} = 44 \text{ kN}, \text{Fracture} = 37 \text{ kN}, d_{\text{neck}} = 9.2 \text{ mm}, \ell_f = 28.5 \text{ mm}$$
$$E = 200 \text{ kN/mm}^2$$

(1) Yield strength:-

$$\sigma_y = \frac{F_y}{A_0} = \frac{29.25 \times 10^3}{\frac{\pi}{4}(12.5)^2} = 238.4 \text{ MPa}$$

(2) Ultimate strength

$$\sigma_{ut} = \frac{F_{ut}}{A_0} = \frac{F_{max}}{A_0} = \frac{37 \times 10^3}{\frac{\pi}{4} (12.5)^2} = 304 \text{ MPa}$$

(3) Actual Breaking stress (True stress at fracture)

$$\sigma_T = \frac{F_{\max}}{A_F} = \frac{37 \times 10^3}{\frac{\pi}{4}(9.2)^2} = 556.59$$

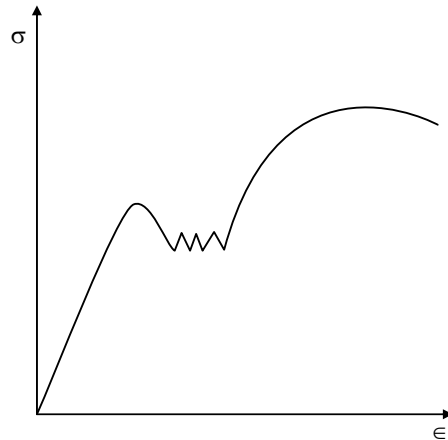
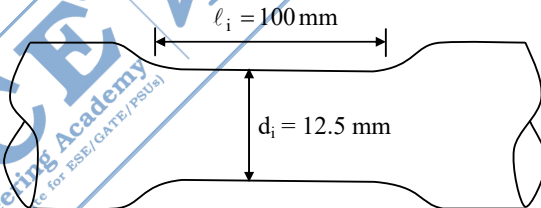
(4) Percentage elongation :

$$\ell_{\text{f}} = 100 + 28.5 = 128.5$$

$$\frac{\delta \ell}{\ell} \times 100 = \frac{\ell_f - \ell_t}{\ell_i} \times 100 = \frac{123.5 - 100}{100} = 28.5$$

(5) Modulus of resilience at yield point stress = $\frac{\sigma_y}{2E}$

$$U_r = \frac{(238.4)^2}{2 \times 200 \times 10^3} = 0.142 \text{ MJ/m}^3$$



5(c). Two systems are represented by its system matrix as given below. Determine the characteristic equation, its roots and establish the stability of each of the system.

$$(i) A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad (ii) A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

(6×2 = 12 M)

Sol: Given: Two systems matrix

Find : Characteristic equation roots stability

System (i):

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Characteristic equation:

$$\det(sI - A) = 0$$

$$sI - A = \begin{bmatrix} s+1 & 0 & 0 \\ 0 & s+2 & 0 \\ 0 & 0 & s+3 \end{bmatrix}$$

$$\det(sI - A) = (s+1)(s+2)(s+3)$$

$$\text{Roots } \lambda_1 = -1, \lambda_2 = -2, \lambda_3 = -3$$

All eigen values have negative real parts, so the system is asymptotically stable.

System (ii):

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

$$\det(sI - A) = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ -1 & 3 & s-3 \end{vmatrix}$$

Cofactor expansion,

$$= s \begin{vmatrix} s & -1 \\ 3 & s-3 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -1 \\ -1 & s-3 \end{vmatrix}$$

Now, compute the 2×2 determinants

$$s(s-3) - (-1)(3) = s(s-3) + 3 = s^2 - 3s + 3$$

$$0(s-3) - (-1)(-1) = -1$$

$$\det(SI - A) = S(S^2 - 3S + 3) + 1 = S^3 - 3S^2 + 3S + 1$$

Roots: The roots are approximately,

$$\lambda_1 = 2.532, \quad \lambda_2 = 0.234 + 1.116i, \quad \lambda_3 = 0.234 - 1.116i,$$

One real positive root and two complex roots with positive real parts: Unstable system.

5(d). A machine component performs a harmonic motion and a vibrometer having a natural frequency of 5 rad/sec and damping ratio = 0.3 is attached to this machine component. If the difference between the maximum and the minimum recorded values is 6 mm, find the amplitude of motion of the vibrating component, when its frequency is 30 rad/sec. (12 M)

Sol: Given data:

Vibrometer natural frequency, $\omega_n = 5 \text{ rad/s}$

Damping ratio = $\xi = 0.3$

Machine frequency = $\omega = 25 \text{ rad/sec}$

Peak – to – peak output = 6 mm

Peak = $x_r = 6/2 = 3 \text{ mm}$

Displacement transmissibility

$$T(\omega) = \frac{(\omega / \omega_n)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$= \frac{\left(\frac{30}{5}\right)^2}{\sqrt{\left(1 - \left(\frac{30}{5}\right)^2\right)^2 + \left(2 \times 0.3 \times \left(\frac{30}{5}\right)\right)^2}} = 1.02317$$

$$\text{Amplitude} = A = \frac{x_r}{T} = \frac{3}{1.02317} = 2.93 \text{ mm}$$

5(e). Determine the inverse of the following transformation matrix:

$$T = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(12 M)

Sol: Given, $T = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 2 \\ 0.369 & 0.819 & 0.439 & 5 \\ -0.766 & 0 & 0.643 & 3 \\ 0 & 0 & 0.643 & 1 \end{bmatrix}$

The transformation matrix is unitary. The inverse simply by transporting the rotating part and calculating the position part by

$$(0.527 \times 2) + (0.369 \times 5) - (0.766 \times 3) = -0.601$$

$$(-0.574 \times 2) - (0.819 \times 5) - (0 \times 3) = -2.947$$

$$(0.628 \times 2) + (0.439 \times 5) + (0.643 \times 3) = -5.38$$

$$\therefore T = \begin{bmatrix} 0.527 & 0.369 & -0.766 & -0.601 \\ -0.574 & 0.819 & 0 & -2.947 \\ 0.628 & 0.439 & 0.643 & -5.38 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

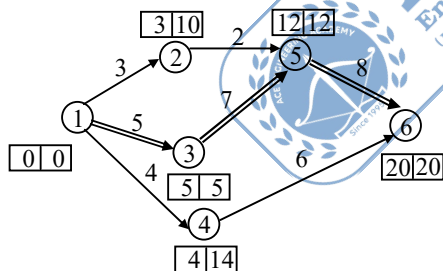
6(a). In an organization, manufacturing of a component is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i-j)	Estimated duration (weeks)		
	Optimistic	Most likely	Pessimistic
1-2	2	2	8
1-3	2	5	8
1-4	3	3	9
2-5	2	2	2
3-5	3	6	15
4-6	3	6	9
5-6	4	7	16

- (i) Draw the network diagram of activities.
- (ii) Find the expected duration and variance for each activity. What is the expected project length.
- (iii) Determine the critical path.
- (iv) Calculate the variance and standard deviation of the project length.
- (v) The earliest and latest expected completion time of each event. (4 × 5 = 20 M)

Sol: Expected duration of activity (t_i) = $\frac{t_0 + 4t_m + t_p}{6}$

Activity	$\frac{t_0 + 4t_m + t_p}{6}$
1-2	$\frac{2 + 4(2) + 8}{6} = 3$
1-3	5
1-4	$\frac{3 + 4(3) + 9}{6} = 4$
2-5	2
3-5	$\frac{3 + 4(6) + 15}{6} = 7$
4-6	6
5-6	$\frac{4 + 4(7) + 16}{6} = 8$



Path	Duration
1-2-5-6	13
1-3-5-6	20
1-4-6	10

Critical path: 1 – 3 – 5 – 6

Expected projected duration = Critical path duration
= 20 days

Variance of project length = V_{CP}
= $V_{1-3} + V_{3-5} + V_{5-6}$

$$= \left(\frac{8-2}{6}\right)^6 + \left(\frac{15-3}{6}\right)^2 + \left(\frac{16-4}{6}\right)^2$$

$$= 9$$

Std. deviation of project length = σ_{cp}

$$= \sqrt{V_{CP}}$$

$$= 3$$

6(b). In a machining operation, under orthogonal cutting condition with a cutting tool of rake angle 12° , the following data were observed :

Vertical component of cutting force = 1600N

Horizontal component of cutting force = 1250N

Chip thickness ratio = 0.25

Cutting speed = 200 m/min

Calculate the following:

(i) Friction force along the rake face

(ii) Normal force on the rake face

(iii) Resultant cutting force

(iv) Shear force along the shear plane

(v) Normal force on the shear plane

(vi) Work done in shear

(vii) Work done in friction

(3 × 6 + 2 = 20 M)

Sol: Rake angle (α_0) = 12°

Vertical component of cutting force (F_c) = 1600 N

Horizontal component of cutting force (F_T) = 1250 N

Chip thickness ratio (r) = 0.25

Cutting speed (V) = 200 m/min

(i) Friction force along the rake face (F)

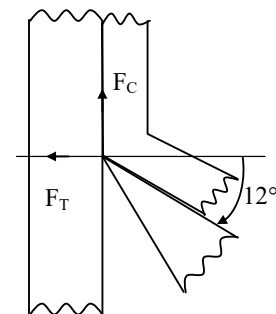
$$F = F_c \sin \alpha_0 + F_T \cos \alpha_0$$

$$= 1600 \sin 12^\circ + 1250 \cos 12^\circ = 1555.3 \text{ N}$$

(ii) Normal force on the rake face (N)

$$N = F_c \cos \alpha_0 - F_T \sin \alpha_0$$

$$= 1600 \cos 12^\circ - 1250 \sin 12^\circ = 1305.14 \text{ N}$$



(iii) Resultant cutting force (R)

$$R = \sqrt{F_c^2 + F_T^2} = \sqrt{1600^2 + 1250^2} = 2030.39 \text{ N}$$

(iv) Shear force along the shear plane (F_s)

$$F_s = F_c \cos \phi - F_T \sin \phi$$

 Shear angle (ϕ)

$$\tan \phi = \frac{r \cos \alpha_0}{1 - r \sin \alpha_0}$$

$$\phi = \tan^{-1} \left(\frac{r \cos \alpha_0}{1 - r \sin \alpha_0} \right)$$

$$= \tan^{-1} \left(\frac{0.25 \cos 12^\circ}{1 - 0.25 \sin 12^\circ} \right)$$

$$\phi = 14.46^\circ$$

$$\therefore F_s = 1600 \cos 14.46 - 1250 \sin 14.46$$

$$= 1237.18 \text{ N}$$

(v) Normal force on the shear plane (N_s)

$$N_s = F_c \sin \phi + F_T \cos \phi$$

$$= 1600 \sin 14.46 + 1250 \cos 14.46$$

$$= 1609.92 \text{ N}$$

(vi) Work done in shear (P_{shear})

$$P_{\text{shear}} = F_s \times V_s$$

$$V_s = \frac{V \cdot \cos \alpha_0}{\cos(\phi + \alpha_0)} = \frac{200 \cos 12^\circ}{\cos(14.46 + 12^\circ)}$$

$$= 195.8 \text{ m/min}$$

$$\therefore P_{\text{shear}} = \frac{1237.18 \times 195.8}{60} = 4037.53 \text{ Watt}$$

(vii) Work done in friction (F_{friction})

$$P_{\text{friction}} = F \times V_c$$

$$V_c = r \cdot V$$

$$= 0.25 \times 200$$

$$= 50 \text{ m/min}$$

$$\therefore P_{\text{friction}} = \frac{1555.3 \times 50}{60} = 1296.08 \text{ Watt}$$



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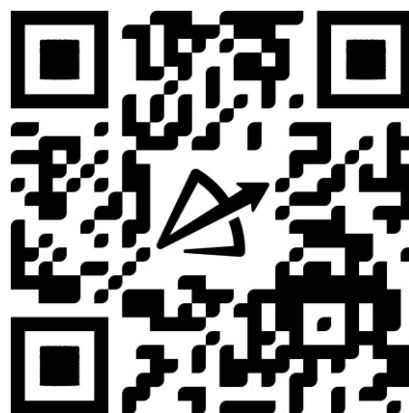
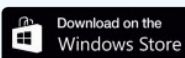
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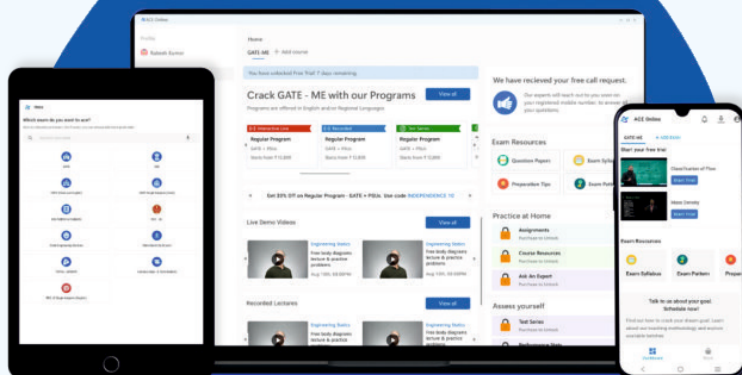
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6(c). Generate the forward kinematic model of the 3 DOF (RRP) Spherical Manipulator Arm, shown in the figure, by

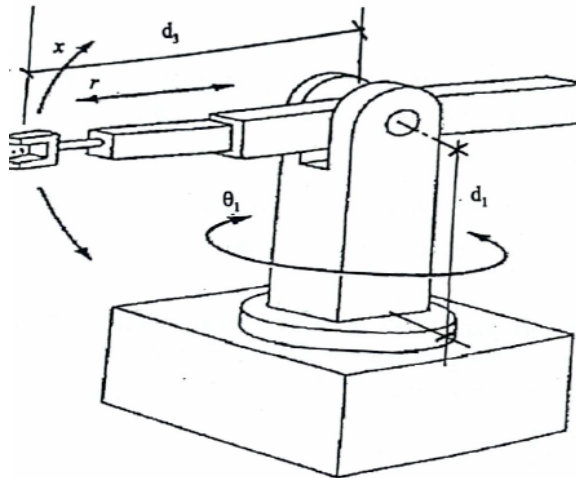
(i) generating and drawing the frames using DH rules

(ii) generating the DH parameters table from the assigned frame

(iii) generating the individual transformation matrices, 0T_1 , 1T_2 and 2T_3

(iv) generating the overall transformation matrix 0T_1

Note: $\{0\}^{\text{th}}$ frame will be at the base of the manipulator.

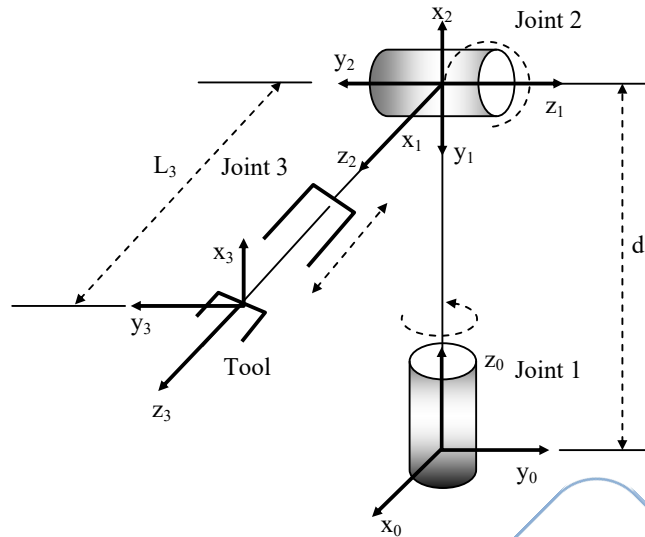


The homogenous transformation matrix ${}^{i-1}T_i$ is given as

$${}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5 × 4 = 20 M)

Sol:



I	a_i	α_i	d_i	θ_i
1	0	-90°	d_1	$\theta_1 = 0^\circ$
2	0	-90°	0	$\theta_2 = -90^\circ$
3	0	0	$d_3 = L_3$	0°

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1 \quad T_2^0 = A_1 A_2$$

$$= \begin{bmatrix} c_1 c_2 & s_1 & -c_1 s_2 & 0 \\ s_1 c_2 & -c_1 & -s_1 s_2 & 0 \\ -s_2 & 0 & -c_2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_1 c_2 & s_1 & -c_1 s_2 & -L_3 c_1 s_2 \\ s_1 c_2 & -c_1 & -s_1 s_2 & -L_3 s_1 s_2 \\ -s_2 & 0 & -c_2 & 3 - L_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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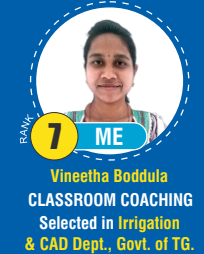
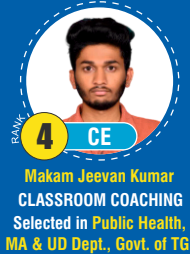
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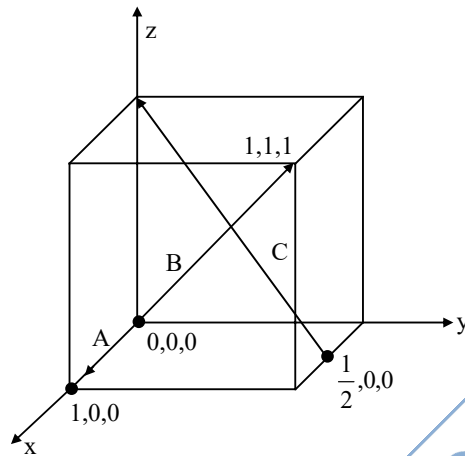
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CE : 434 | EE : 61 | ME : 20

07(a). Explain the meaning of Miller indices in a unit crystal cell. Determine the Miller indices of directions A, B and C in figure below.



(20 M)

Sol: Miller indices are a set of three integers (hkl) that mathematically describe the orientation of a crystal plane within a unitcell. They are determined by reciprocals of plane fractions.

Direction A:

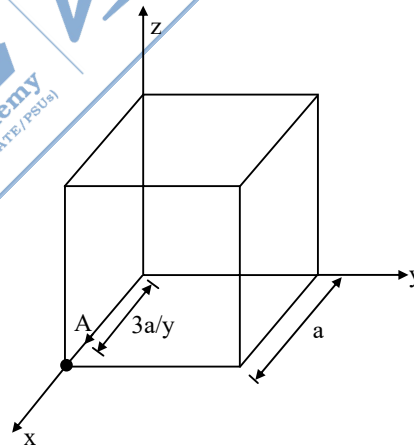
$$\text{Direction fractions} = \left(\frac{3}{4}, 0, 0 \right)$$

Multiply by 4 (to get integers)

$$= 4 \times \frac{3}{4}, 4 \times 0, 4 \times 0$$

$$(3, 0, 0)$$

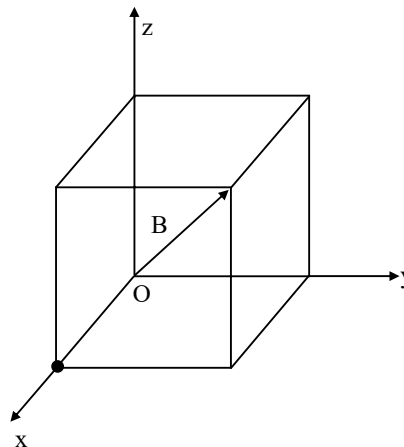
$$\text{Direction indices} = [3 \ 0 \ 0]$$



Direction (B):-

$$\text{Direction fractions} = 1, 1, 1$$

$$\text{Direction indices} [1 \ 1 \ 1]$$

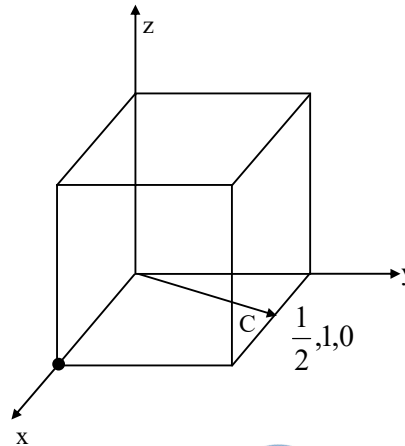


Direction (C):-

$$\text{Direction fractions} = \frac{1}{2}, 1, 0$$

Multiply with 2 (to get integer)

$$\text{Direction indices} = [1 \ 2 \ 0]$$



7.(b)(i)

A shaper machine is used to machine a medium carbon steel workpiece of 225 mm in length and 125 mm in width. The shaper machine is operated at 125 cutting strokes per minute, feed of 0.5 mm per stroke and a depth of cut of 5 mm. The forward stroke is completed in 220°.

Calculate the percentage of time when the tool is not contacting the workpiece also calculate the total machining time for machining the component.

Assume that approach distance = 30 mm.

(10 M)

Sol: Feed, $f = 0.5$ mm/stroke

Depth of cut, $d = 5$ mm

Forward stroke angle = 220°

Return stroke angle = $360 - 220 = 140^\circ$

$$\% \text{ of time} = \frac{140}{360} = 0.3888 = 38.88\%$$

$$\text{Number of passes} = \frac{\text{W/P width}}{\text{feed}} = \frac{125}{0.5} = 250 \text{ passes}$$

$$\begin{aligned} \text{Stroke length} &= \text{W/P length} + \text{approach length} \\ &= 225 + 30 = 255 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Total machining} &= \frac{\text{No. of passes} \times \text{stroke length}}{\text{Cutting stroke / min} \times \text{W / P length}} \times \frac{360^\circ}{\text{Forward stroke angle}} \\ &= \frac{250 \times 255}{125 \times 255} \times \frac{360}{220} \approx 3.70 \text{ minute} \end{aligned}$$

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CE-98

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7.(b)(ii)

A milling operation is carried out on low carbon steel workpiece. A milling cutter of 125 mm diameter having 10 teeth is operated at 25 m/min to perform the milling operation. The table feed rate is 120 mm/min and depth of cut is 5 mm.

Calculate the following:

(A) Length of the chip in up and down milling operation

(B) Change in path length from up to down milling.

(10 M)

Sol: $D = 125 \text{ mm}$ $z = 10$
 $V = 20 \text{ m/min}$ Feed rate, $k_m = 120 \text{ mm/min}$

Depth of cut, $d = 5 \text{ mm}$

(a) Length of chip, $L_c = 3$

$$L_c = \sqrt{D \cdot d} = \sqrt{125 \times 5}$$

$$L_c = 25 \text{ mm}$$

(B) Change in path length = ?

$$\Delta L = \text{path length} - \text{path length}$$

$$\text{In up milling} \quad \text{in down milling}$$

$$= 25 - 25$$

$$\text{Change in path length, } \Delta L = 0$$

7.(c)(i)

For the given DH parameters table of a manipulation arm; generate the frames as per DH rules. Also give reasons for selection of origins of the frames and the corresponding coordinate axes (X, Y, Z), frame wise, i.e. from frame {0} to {3}.

	d_i	θ_i	a_i	α_i
0T_1	0	θ_1	0	90°
1T_2	0	θ_2	L_2	0
2T_3	0	θ_3	L_3	0

(10 M)

Sol:

Link i	a_i	α_i	d_i	θ_i	q_i	$C\alpha_i$	$S\alpha_i$
1	0	90°	0	θ_1	θ_1	0	1
2	L_2	0	0	θ_2	θ_2	1	0
3	L_3	0	0	θ_3	θ_3	1	0

The link transformation matrices are

$${}^0T_1(\theta_1) = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2(\theta_2) = \begin{bmatrix} C_2 & -S_2 & 0 & L_2 C_2 \\ S_2 & C_2 & 0 & L_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3(\theta_3) = \begin{bmatrix} C_3 & -S_3 & 0 & L_3 C_3 \\ S_3 & C_3 & 0 & L_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix for the endpoint of the arm is , therefore,

$${}^0T_1 = {}^0T_1 {}^1T_2 {}^2T_3 = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & C_1 (L_3 C_{23} + L_2 C_2) \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & S_1 (L_3 C_{23} + L_2 C_2) \\ S_{23} & C_{23} & 0 & L_3 S_{23} + L_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where C_{23} and S_{23} refer to $\cos(\theta_2 + \theta_3)$ and $\sin(\theta_2 + \theta_3)$, respectively.

At the home position, $\theta_1 = \theta_2 = \theta_3 = 0$. Substituting these displacement variable values in equation , the direct kinematic model, the orientation and position of end-of-arm point frame for the home positions is obtained as:

$$T_E = \begin{bmatrix} n_x & o_x & a_x & d_x \\ n_y & o_y & a_y & d_y \\ n_z & o_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & L_2 + L_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From equation , it is observed that in the home position the arm point frame, frame (3), has its x-axis (x_3 -axis) in the same direction as x_0 -axis, y_3 -axis in the z_0 -axis direction, and z_3 -axis in the

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& many more....

negative y_0 -axis direction. The origin of frame (3) is translated by a distance of $(L_2 + L_3)$ in the x_0 -axis direction. This means that if, initially frame (3) is coincident with frame (0), its home position and orientation is obtained by translating the origin by $(L_2 + L_3)$ along x_0 -axis and rotating it by $+90^\circ$ about x_0 -axis. The position and orientation of frame (3) obtained from equation matches with the coordinate system established in Fig., verifying the correctness of the model obtained.

Home position of the articulated arm corresponding to the frame assigned in Fig, that is, $\theta_1 = \theta_2 = \theta_3 = 0$ is drawn in Fig.. An alternate home position can be obtained by adding constant angles to θ_2 and θ_3 . For example, if we added $+90^\circ$ to joint angle θ_2 and -90° to joint angle θ_3 the new home position is drawn in Fig.

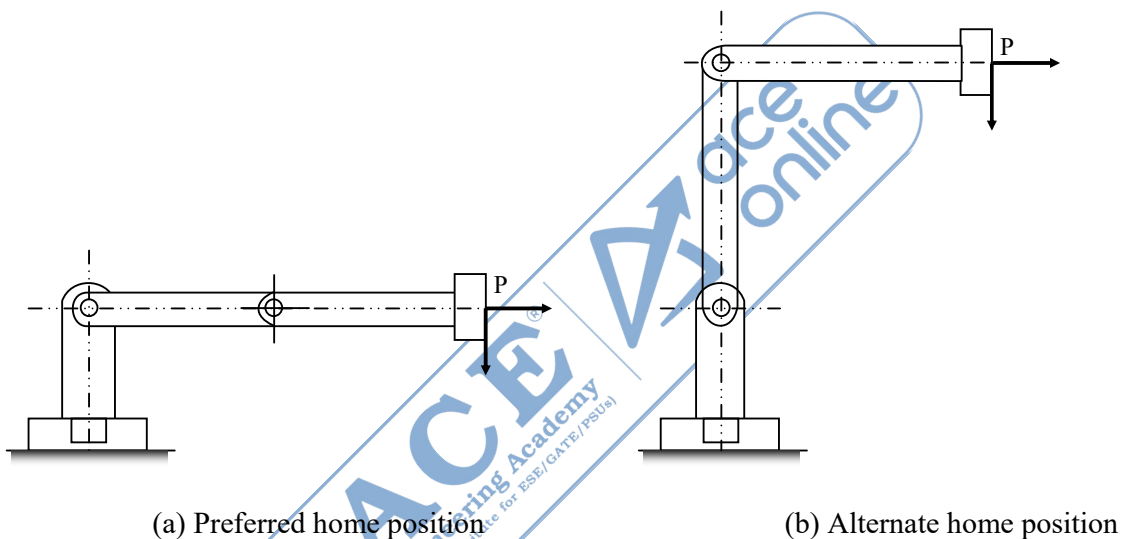


Fig: Two possible home positions for the articulated arm

For this alternate home position of the manipulator the new joint displacements θ'_2 and θ'_3 are defined by adding $+90^\circ$ to joint angle θ_2 and -90° to joint angle θ_3 , respectively.

Frame assignment and the kinematic model formulation for this new home position with displacement variables θ'_1 , θ'_2 , and θ'_3 is left as an exercise for the reader. The joint-link parameters for this home position are tabulated in Table.

Link i	a_i	α_i	d_i	θ_i	q_i	$C\alpha_i$	$S\alpha_i$
1	0	90°	0	θ_1	$\theta'_1 = \theta_1$	0	1
2	L_2	0	0	θ_2	$\theta'_2 = \theta_2 + 90^\circ$	1	0
3	L_3	0	0	θ_3	$\theta'_3 = \theta_3 - 90^\circ$	1	0

7(c) (ii).

The overall transformation matrix of a 3 DOF manipulator arm is given below:

$${}^0T_3 = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & -d_3 \sin \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 & d_3 \cos \theta_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(A) Use inverse kinematic modeling to generate the expressions for the joint parameters θ_1 , d_2 and d_3 .

(B). Determine all possible values of θ_1 , d_2 and d_3 for the above manipulator from the following overall transformation matrix.

$$\begin{bmatrix} 0.866 & 0 & -0.5 & -5 \\ 0.5 & 0 & 0.866 & 8.66 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5×2=10 M)

Sol: Given:

$${}^0T_3 = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & -d_3 \sin \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 & d_3 \cos \theta_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Inverse Kinematics

$$R = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 \\ \sin \theta_1 & 0 & \cos \theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$T_{11} = \cos \theta_1 \quad T_{21} = \sin \theta_1$$

$$\theta_1 = \tan^{-1} \left(\frac{T_{21}}{T_{11}} \right)$$

$$d_2 = T_{34}$$

$$T_{14} = -d_3 \sin \theta_1, \quad T_{24} = d_3 \cos \theta_1$$

$$d_3 = \frac{-T_{14}}{\sin \theta_1} = \frac{T_{24}}{\cos \theta_1}$$

$$(b) {}^0T_3 = \begin{bmatrix} 0.866 & 0 & -0.5 & -5 \\ 0.5 & 0 & 0.866 & 8.66 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{11} = 0.866$$

$$T_{21} = 0.5$$

$$\theta_1 = \tan^{-1}(0.5/0.866) = 30^\circ$$

$$d_2 = T_{34} = 15$$

$$T_{14} = -5 \quad T_{24} = 8.66$$

$$T_{14} = -d_3 \sin \theta_1$$

$$\theta_1 = 30^\circ \Rightarrow \sin \theta_1 = 0.5$$

$$-5 = -d_3(0.5)$$

$$d_3 = 10 \quad (\text{or})$$

$$T_{24} = d_3 \cos \theta_1$$

$$0.866 = d_3(0.866) = 10$$

8.(a)(i)

Explain the 'laws of corrosion'. Give examples of two metals following the law in each case. Oxidation loss on copper surface is 0.05 mm in 15 hours. How much will be the loss in 225 hours?

(10 M)

Sol: Faraday's Law:

It states that the rate of corrosion (mass loss of metal) is directly proportional to the corrosion current—the amount of electric charge flowing due to electrochemical reactions at the metal's surface.

Time-dependent power law

It describes how weight loss or oxide layer growth due to corrosion changes over time, often showing that processes like copper atmospheric corrosion or stainless steel oxidation follow predictable mathematical relationships dependent on both time and reaction dynamics.

Examples of metals following these laws:

- **Faraday's Law:**

Iron (Fe): The rusting of iron (formation of iron oxide) follows Faraday's Law, where the amount of iron corroded is proportional to the measured corrosion current in the environment.

- *Zinc (Zn):* In galvanic cells or as a sacrificial anode, the dissolution of zinc also strictly follows Faraday's Law, with mass loss accurately calculated from the anodic current.

- Time-dependent power law:

- *Copper (Cu)*: The atmospheric corrosion of copper, such as the development of a green patina, adheres to a time-dependent power law, where the weight loss over time is predictable by such mathematical forms.
- *Austenitic Stainless Steel*: Oxidation at elevated temperatures leads to weight gain due to oxide formation, and the rate follows a similar time-dependent power law involving both diffusion and surface reaction kinetics.
- The oxidation loss on the copper surface is given as 0.05 mm in 15 hours. Assuming the oxidation loss occurs at a constant linear rate, the loss after 225 hours can be calculated by proportional scaling:
- Loss in 15 hours = 0.05 mm Loss rate per hour = $0.05 \text{ mm} / 15 \text{ hours} = 0.00333 \text{ mm/hour}$
- For 225 hours: Loss = $0.00333 \text{ mm/hour} \times 225 \text{ hours} = 0.75 \text{ mm}$
- So, the oxidation loss on the copper surface after 225 hours will be **0.75 mm** if the rate remains constant and linear

8.(a)(ii)

What are the different ‘top-down’ and ‘bottom-up’ methods for the synthesis of nano structured materials. With a sketch briefly explain the mechanical high energy ball milling for the synthesis of nano materials. (10 M)

Sol:

Feature	Top-down Approach	Bottom-up Approach
Starting Material	Bulk solid material	Atoms, molecules, or clusters (liquid or gas state)
Process Type	Subtractive (breaking down)	Additive (building up)
Typical Methods	Mechanical milling, lithography, etching, ablation	Sol-gel, chemical vapor deposition, self-assembly, precipitation
Size & Shape Control	Difficult, leads to heterogeneous distribution	Easier, can achieve homogeneity and finer control
Production Scale	Typically suitable for large scale	More suited for small to moderate scale
Advantages	Simple setups for bulk reduction, integration with micro-fabrication	Better control over nanoscale properties, uniformity, produces advanced nanostructures
Disadvantages	Introduces defects/impurities, high cost, less control, not ideal for soft materials	Can be complex, limited scale, often requires chemical purification

Hearty Congratulations to our students ESE - 2024



Rohit Dhondge



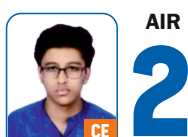
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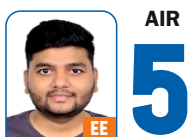
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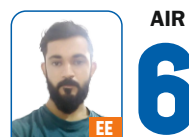
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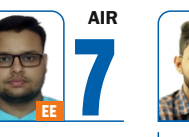
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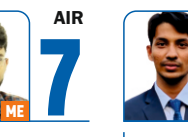
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DINESH KUMAR S



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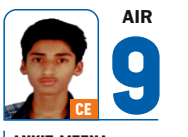
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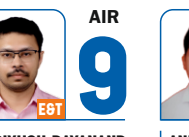
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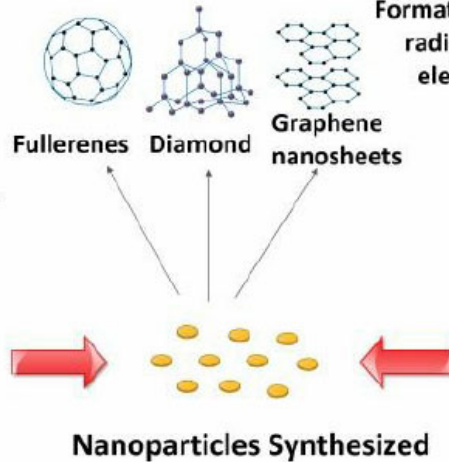
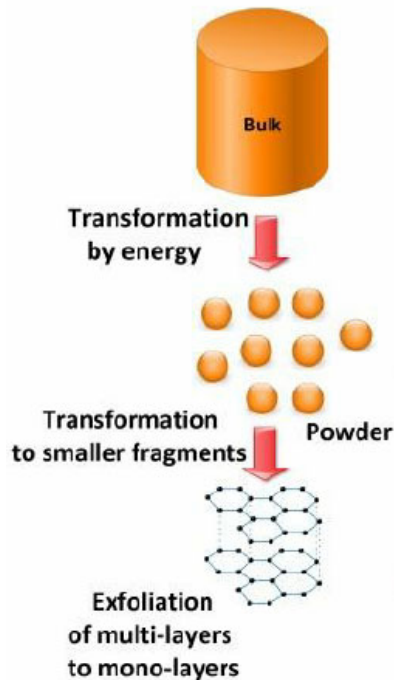
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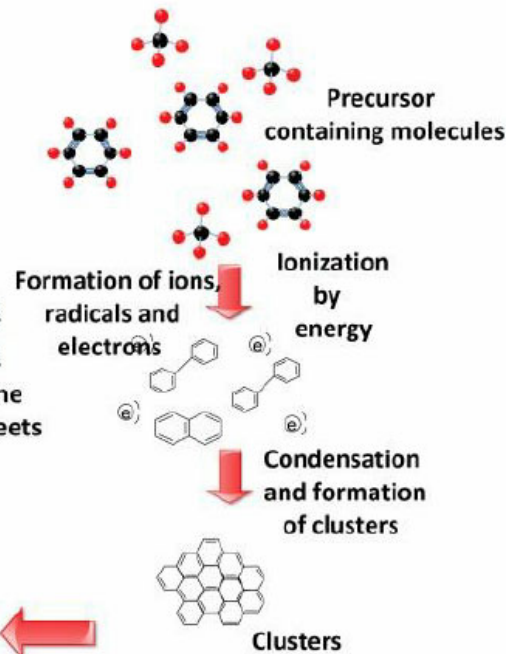
AKSHAY VIDHATE

TOTAL 36 SELECTIONS IN TOP 10 CE: 09 | ME: 10 | EE: 08 | E&T: 09

Top-Down Approach



Bottom-Up Approach



8.(b). Calculate the power required to draw hot-drawn steel wire from 15 mm to 12.5 mm in diameter at 120 m/min. The coefficient of friction between the die and wire is 0.15 and die angle is 5° . Average flow stress for hot-drawn steel is $30 \text{ kgf} / \text{mm}^2$. Also calculate the maximum reduction possible. Assume that back pull = 0.

(20 M)

Sol: Given data:

$$d_0 = 15 \text{ mm}, d_1 = 12.5 \text{ mm}$$

$$V_1 = 120 \text{ m/min} = \frac{120}{60} = 2 \text{ m/sec}$$

$$\mu = 0.15, \alpha = 5^\circ \text{ (Assume semi die angle)}$$

$$\bar{Y}_f = 30 \text{ kgf} / \text{mm}^2 = 30 \times 9.81 \text{ N} / \text{mm}^2$$

$$\text{Power} = P = F_d \times V_1$$

$$F_d = \bar{Y}_f \left(\frac{1+B}{B} \right) \left(1 - \left(\frac{A_1}{A_0} \right)^B \right) \times A_1$$

$$B = \mu \cot \alpha = 0.15 \cot 5^\circ = 1.714$$

$$F_d = 30 \left(\frac{1+1.714}{1.714} \right) \left(1 - \left(\frac{12.5}{15} \right)^{2 \times 1.714} \right) \times 9.81 \times \frac{\pi}{4} (12.5)^2$$

$$= 50,797 \text{ N} = 50.797 \text{ kN}$$

$$\text{Power} = P = F_d \times V_1 = 50.797 \times 2 = 25.4 \text{ kW}$$

Maximum possible reduction:

Condition $\Rightarrow \sigma_d = \text{Flow stress}$

Let us consider average flow stress is equal to flow stress because data insufficient.

$$\sigma_d = \bar{Y}_f$$

$$F_d = \bar{Y}_f \left(\frac{1+B}{B} \right) \left(1 - \left(\frac{A_1}{A_0} \right)^B \right) \times A_1$$

$$\frac{F_d}{A_1} = \bar{Y}_f \left(\frac{1+B}{B} \right) \left(1 - \left(\frac{A_1}{A_0} \right)^B \right)$$

$$\left(\frac{B}{1+B} \right) = 1 - \left(\frac{A_1}{A_0} \right)^B$$

$$\frac{B}{1+B} = 1 - \left(\frac{A_1}{A_0} \right)^B$$

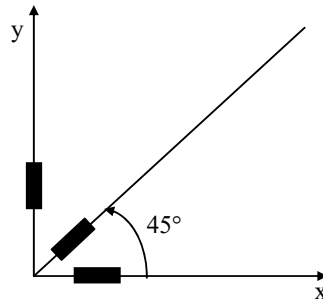
$$\left(\frac{A_1}{A_0} \right)^B = 1 - \frac{B}{1+B} = \frac{1}{1+B}$$

$$\frac{A_1}{A_0} = \left(\frac{1}{1+B} \right)^{\frac{1}{B}} = \left(\frac{1}{1+1.714} \right)^{\frac{1}{1.714}} = 0.631$$

$$\% \text{ Reduction} = \frac{A_0 - A_1}{A_0} = \left(1 - \frac{A_1}{A_0} \right) \times 100$$

$$= (1 - 0.631) \times 100 = 36.84\%$$

8.(c) A 3 element rectangular rosette is used at a certain point on a steel machine part as shown in the figure. Determine the principal strains and principal stresses using analytical expressions, if the measured strains are $\epsilon_0 = -220 \mu/m$, $\epsilon_{45} = 120 \mu/m$ and $\epsilon_{90} = 220 \mu/m$ assuming $E = 200 \text{ GPa}$ and $\nu = 0.3$.



(20 M)

Sol: Given 45° strain rosette

$$\epsilon_0 = -220 \mu \quad \theta_A = 0^\circ$$

$$\epsilon_{45} = 120 \mu \quad \theta_B = 45^\circ$$

$$\epsilon_{90} = 220 \mu \quad \theta_C = 90^\circ$$

$$E = 200 \text{ GPa}$$

⇒ Using strain transformation equation,

$$\epsilon_0 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta_A + \frac{\gamma_{xy}}{2} \sin 2\theta_A$$

Put $\theta_A = 0$

$$\epsilon_0 = \epsilon_x = -220 \mu$$

$$\text{Similarly } \epsilon_{90} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta_C + \frac{\gamma_{xy}}{2} \sin 2\theta_C$$

$$\theta_C = 90^\circ$$

$$\epsilon_{90} = \epsilon_y = 220 \mu$$

$$\epsilon_{45} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2(45^\circ) + \frac{\gamma_{xy}}{2} \sin(2 \times 45^\circ)$$

$$= \frac{\epsilon_x + \epsilon_y + \gamma_{xy}}{2} = 120 \mu$$

$$\gamma_{xy} = 240 - (\epsilon_x + \epsilon_y)$$

$$= 240 - [-220 + 220]$$

$$= 240$$

Now principle strains,

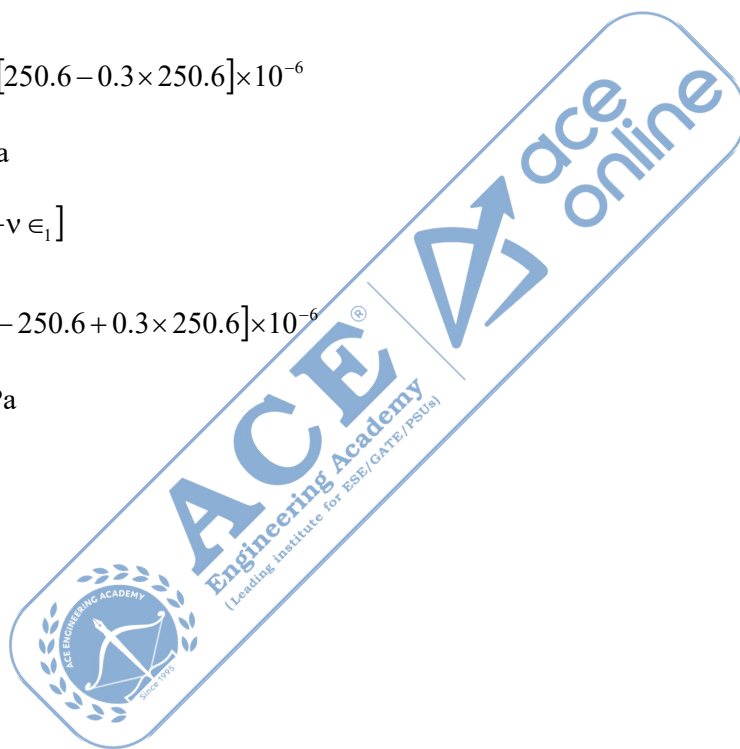
$$\begin{aligned}\epsilon_{1,2} &= \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left[\frac{\epsilon_x - \epsilon_y}{2}\right]^2 + \left[\frac{\gamma_{xy}}{2}\right]^2} \\ &= \frac{-220 + 220}{2} \pm \sqrt{\left[\frac{-220 - 220}{2}\right]^2 + \left[\frac{240}{2}\right]^2}\end{aligned}$$

$$\epsilon_1 = 250.6\mu \quad \text{and} \quad \epsilon_2 = -250.6\mu$$

Relation between principle strains and stress given by hooke's law

$$\begin{aligned}\sigma_1 &= \frac{E}{1 - \nu^2} [\epsilon_1 + \nu \epsilon_2] \\ &= \frac{200 \times 10^3}{1 - 0.3^2} [250.6 - 0.3 \times 250.6] \times 10^{-6} \\ &= 38.55 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_2 &= \frac{E}{1 - \nu^2} [\epsilon_2 + \nu \epsilon_1] \\ &= \frac{200 \times 10^3}{1 - 0.3^2} [-250.6 + 0.3 \times 250.6] \times 10^{-6} \\ &= -38.55 \text{ MPa}\end{aligned}$$



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