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CIVIL ENGINEERING

FLUID MECHANICS

Text Book: Theory with worked out Examples and Practice Questions

Fluid Mechanics

(Solutions for Text Book Practice Questions)

Chapter 1 Properties of Fluids

01. Ans: (c)

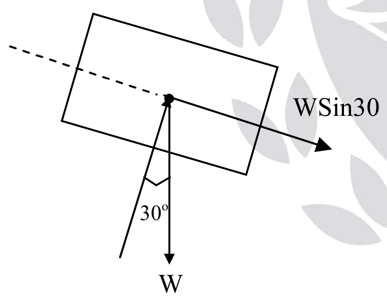
Sol: For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).

02. Ans: 100

Sol: $\tau = \frac{\mu V}{h} = \frac{0.2 \times 1.5}{3 \times 10^{-3}} = 100 \text{ N/m}^2$

03. Ans: 1

Sol:



$$F = \tau \times A$$

$$W \sin 30 = \frac{\mu A V}{h}$$

$$\frac{100}{2} = \frac{1 \times 0.1 \times V}{2 \times 10^{-3}}$$

$$V = 1\text{m/s}$$

Common data Q. 04 & 05

04. Ans: (c)

Sol: $D_1 = 100$ mm ,

$D_2 = 106$ mm

$$\text{Radial clearance, } h = \frac{D_2 - D_1}{2}$$

$$= \frac{106 - 100}{2} = 3\text{mm}$$

L = 150 mm = 0.15 m

$\mu = 0.2$ pa.s

N = 240 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$

$\omega = 8\pi$

$$\tau = \frac{\mu\omega r}{h} = \frac{0.2 \times 8\pi \times 50 \times 10^{-3}}{3 \times 10^{-3}}$$

$$= 83.77\text{N/m}^2$$

05. Ans: (b)

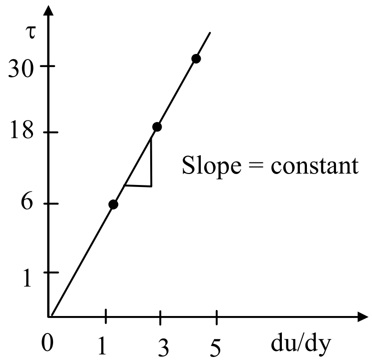
Sol: $\text{Power, } P = \frac{2\pi\omega^2 \mu L r^3}{h}$

$$= \frac{2\pi \times (8\pi)^2 \times 0.2 \times 0.15 \times (0.05)^3}{3 \times 10^{-3}}$$

$= 4.96$ Watt

06. Ans: (c)

Sol:



$\therefore$ Newtonian fluid

07. Ans: (a)

Sol: $\tau = \mu \dfrac{du}{dy}$

$u = 3\sin(5\pi y)$

$\dfrac{du}{dy} = 3\cos(5\pi y)\times 5\pi = 15\pi\cos(5\pi y)$

$\tau\big|_{y=0.05} = \mu \dfrac{du}{dy}\Big|_{y=0.05}$

$= 0.5\times 15\pi\cos(5\pi\times 0.05)$

$= 0.5\times 15\pi\times\cos\left(\dfrac{\pi}{4}\right) = 0.5\times 15\pi\times\dfrac{1}{\sqrt{2}}$

$= 7.5\times 3.14\times 0.707 \approx 16.6\text{N/m}^2$

08. Ans: (d)

Sol:

- Ideal fluid $\rightarrow$ Shear stress is zero.
- Newtonian fluid $\rightarrow$ Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid $\rightarrow$ Shear stress does not vary linearly with the rate of strain.
- Bingham plastic $\rightarrow$ Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

09. Ans: (b)

Sol: $V = 0.01\text{ m}^3$

$\beta = 0.75\times 10^{-9}\text{ m}^2/\text{N}$

$dP = 2\times 10^7\text{ N/m}^2$

$K = \dfrac{1}{\beta} = \dfrac{1}{0.75\times 10^{-9}} = \dfrac{4}{3}\times 10^9$

$K = \dfrac{-dP}{dV/V}$

$dV = \dfrac{-2\times 10^7\times 10^{-2}\times 3}{4\times 10^9} = -1.5\times 10^{-4}$

10. Ans: 320 Pa

Sol: $\Delta P = \dfrac{8\sigma}{D} = \dfrac{8\times 0.04}{1\times 10^{-3}} = \dfrac{32\times 10^{-2}}{10^{-3}}$

$\Delta P = 320\text{ N/m}^2$

11. Ans: (a, d)

Sol: Given data: S.G = 0.8 and

$\nu = 2$ centistokes $= 2\times 10^{-6}\text{ m}^2/\text{s}$

Mass density, $\rho = (\text{S.G})\times \rho_{\text{water at }4^\circ\text{C}}$

$= 0.8\times 10^3 = 800\text{ kg/m}^3$

Dynamic viscosity, $\mu = \rho\times\nu$

$\mu = 800\times 2\times 10^{-6}$ Pa.s

$= 16\times 10^{-4}$ Pa.s

$= 1.6$ centipoise

Chapter 2 Pressure Measurement & Fluid Statics

01. Ans: (a)

Sol: 1 millibar = $10^{-3}\times10^{5}$ = 100 N/m^2

One mm of Hg = $13.6\times10^{3}\times9.81\times1\times10^{-3}$

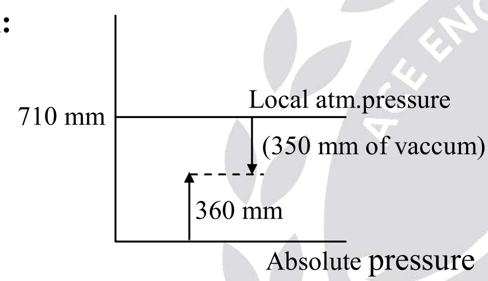
= 133.416 N/m^2

1 N/mm^2 = 1×10^{6} N/m^2

1 kgf/cm^2 = 9.81×10^{4} N/m^2

02. Ans: (b)

Sol:



03. Ans: (c)

Sol: Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure P_A and P_B are same.

04. Ans: (b)

Sol:

- The manometer shown in Fig.1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.
- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.

05. Ans: 2.2

Sol: h_p in terms of oil

$s_o h_o = s_m h_m$

$0.85\times h_0 = 13.6\times0.1$

$h_0 = 1.6$m

$h_p = 0.6+1.6$

$\Rightarrow$ $h_p = 2.2$m of oil

(or) $P_p - \gamma_{oil}\times0.6 - \gamma_{Hg}\times0.1 = P_{atm}$

$$\frac{P_p - P_{atm}}{\gamma_{oil}} = \left(\frac{\gamma_{Hg}}{\gamma_{oil}}\times0.1+0.6\right)$$

$$=\frac{13.6}{0.85}\times0.1+0.6 = 2.2 \text{ m of oil}$$

Gauge pressure of P in terms of m of oil

= 2.2 m of oil

06. Ans: (b)

Sol: $$h_M - \frac{s_w}{s_0}h_{w_1} = h_N - \frac{s_w h_{w_2}}{s_0} - h_0$$

$$h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3$$

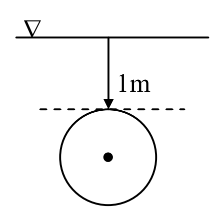
$$h_M - h_N = -13.843 \text{ cm of oil}$$

07. Ans: 2.125

Sol: $$h_p = \bar{h} + \frac{I}{A\bar{h}}$$

$$= 2 + \frac{\pi D^4\times4}{64\times D^2\times2\times\pi}$$

$$= 2 + \frac{2^2\times4}{64\times2} = 2.125\text{m}$$



08. Ans: 10

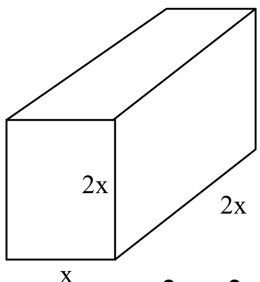
Sol: $F = \rho g \bar{h} A$

$$= 9810 \times 1.625 \times \frac{\pi}{4}\left(1.2^2 - 0.8^2\right)$$

$F = 10$ kN

09. Ans: 1

Sol:



$F_{bottom} = \rho g \times 2x \times 2x \times x$

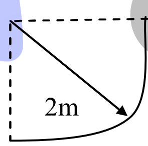
$F_V = \rho g x \times 2x \times 2x$

$$\frac{F_B}{F_V} = 1$$

10. Ans: 10

Sol: $F_V = x \times \pi$

$$F_V = \rho g V = 1000 \times 10 \times \frac{\pi \times 2^2}{4}$$



$F_V = 10\pi$ kN

$\therefore\ x = 10$

11. Ans: (d)

Sol: $F_{net} = F_{H1} - F_{H2}$

$$F_{H1} = \gamma \times \frac{D}{2} \times D \times 1 = \frac{\gamma D^2}{2}$$

$$F_{H2} = \gamma \times \frac{D}{4} \times \frac{D}{2} \times 1 = \frac{\gamma D^2}{8}$$

$$= \gamma D^2\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\gamma D^2}{8}$$

12. Ans: 2

Sol: Let P be the absolute pressure of fluid f3 at mid-height level of the tank. Starting from the open limb of the manometer (where pressure = P_{atm}) we write :

$$P_{atm} + \gamma \times 1.2 - 2\,\gamma \times 0.2 - 0.5\,\gamma \times \left(0.6 + \frac{h}{2}\right) = P$$

or $P - P_{atm} = P_{gauge}$

$$= \gamma(1.2 - 2 \times 0.2 - 0.5 \times 0.6 - 0.5 \times \frac{h}{2})$$

For P_{gauge} to be zero, we have,

$$\gamma(1.2 - 0.4 - 0.3 - 0.25\,h) = 0$$

or $$h = \frac{0.5}{0.25} = 2$$

13. Ans: (a, c)

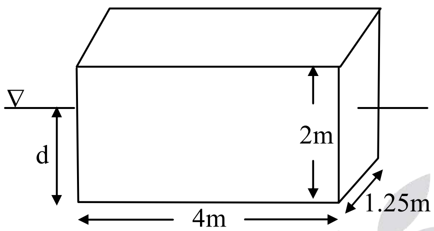
Sol: The limitations of piezometer are :

- It can't measure gas pressure.
- It can't measure high pressure.

Chapter 3
Buoyancy and Metacentric Height

01. Ans: (d)

Sol:



F_B = weight of body

$$\rho_b g V_b = \rho_f g V_f d$$

$$640\times4\times2\times1.25 = 1025\times(4\times1.25\times d)$$

$$d = 1.248\text{m}$$

$$V_{fd} = 1.248\times4\times1.25$$

$$V_{fd} = 6.24\text{m}^3$$

02. Ans: (c)

Sol: Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

$$4\pi r^2 = 6a^2$$

$$\frac{2\pi}{3} = \left(\frac{a}{r}\right)^2$$

$$F_{b,s} \propto V_s$$

$$= \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\frac{\pi r^3}{\left(r\sqrt{\frac{2\pi}{3}}\right)^3}$$

$$= \frac{4}{3}\frac{\pi r^3}{\left(\sqrt{\frac{2\pi}{3}}\times\sqrt{\frac{2\pi}{3}}r^3\right)} = \sqrt{\frac{6}{\pi}}$$

03. Ans: 4.76

Sol: $F_B = F_{B,Hg} + F_{B,W}$

$W_B = F_B$

$$\rho_b g\forall_b = \rho_{Hg} g\forall_{Hg} + \rho_w g\forall_w$$

$$\rho_b\forall_b = \rho_{Hg}\forall_{Hg} + \rho_w\forall_w$$

$$S\times\forall_b = S_{Hg}\forall_{Hg} + S_w\forall_w$$

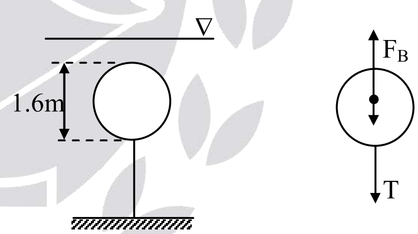
$$7.6\times10^3 = 13.6\times10^2(10-x) + 10^2\times x$$

$$-6000 = -1260x$$

$$x = 4.76 \text{ cm}$$

04. Ans: 11

Sol:



$$F_B = W + T$$

$$W = F_B - T$$

$$= \rho_f g V_{fd} - T$$

$$= 10^3\times9.81\times\frac{4}{3}\pi(0.8)^3 - \left(10\times10^3\right)$$

$$= 21 - 10$$

$$W = 11 \text{ kN}$$

05. Ans: 1.375

Sol: $W_{water} = 5N$

$W_{oil} = 7N$

$S = 0.85$

W – Weight in air

$F_{B1} = W - 5$

$F_{B2} = W - 7$

$W - 5 = \rho_1 g V_{fd}$.....(1)

$W - 7 = \rho_2 g V_{fd}$.....(2)

$V_{fd} = V_b$

$W - 5 = \rho_1 g V_b$

$W - 7 = \rho_2 g V_b$

$2 = (\rho_1 - \rho_2) g V_b$

$$V_b = \frac{2}{(1000-850)9.81}$$

$V_b = 1.3591\times10^{-3} m^3$

$W = 5+(9810\times1.3591\times10^{-3})$

$W = 18.33N$

$W = \rho_b\, g\, V_b$

$$\frac{18.33}{9.81\times1.3591\times10^{-3}} = \rho_b$$

$\rho_b = 1375.05\ kg/m^3$

$S_b = 1.375$

06. Ans: (d)

Sol: For a floating body to be stable, metacentre should be above its center of gravity. Mathematically GM > 0.

07. Ans: (b)

Sol: $W = F_B$

$\rho_b\, g\, V_b = \rho_f\, g\, V_{fd}$

$\rho_b V_b = \rho_f V_{fd}$

$$0.6\times\frac{\pi}{4}d^2\times 2d = 1\times\frac{\pi}{4}d^2\times x$$

$\Rightarrow x = 1.2d$

$GM = BM - BG$

$$BM = \frac{I}{V} = \frac{\pi d^4}{64\times\frac{\pi}{4}d^2\times1.2d} = \frac{d}{19.2} = 0.052d$$

$BG = d - 0.6d = 0.4d$

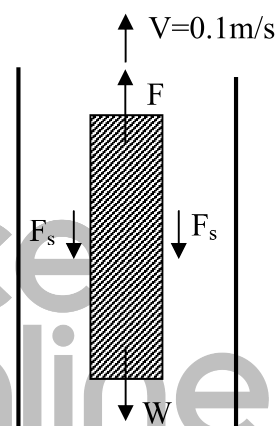
Thus, $GM = 0.052d - 0.4d = -0.348\,d$

$GM < 0$

$\Rightarrow$ Hence, the cylinder is in unstable condition.

08. Ans: 122.475

Sol:



The thickness of the oil layer is same on either side of plate

y = thickness of oil layer

$$= \frac{23.5-1.5}{2} = 11mm$$

Shear stress on one side of the plate

$$\tau = \frac{\mu dU}{dy}$$

F_s = total shear force (considering both sides of the plate)

$$= 2A\times\tau = \frac{2A\mu V}{y}$$

$$= \frac{2\times1.5\times1.5\times2.5\times0.1}{11\times10^{-3}} = 102.2727\ N$$

Weight of plate, W = 50 N

Upward force on submerged plate,

$F_v = \rho g V = 900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3}$

$= 29.7978$ N

Total force required to lift the plate

$= F_s + W - F_v$

$= 102.2727 + 50 - 29.7978$

$= 122.4749$ N

09. Ans: (a, b, c, d)

Sol:

- Passenger ships have less GM than war ships from comfort point of view.
- Lifting a steel ball submerged in water is easier than lifting it when unsubmerged due to buoyant force acting on the ball.
- Apparent weight of a submerged body is always lower than its actual weight due to the force of buoyancy.
- Inverted U-tube manometers are preferred if difference in pressure is small.

Chapter 4 Fluid Kinematics

01. Ans: (b)

Sol: Constant flow rate signifies that the flow is steady.

- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.

Common Data for Questions 02 & 03

02. Ans: 0.94

Sol: $a_{Local} = \frac{\partial V}{\partial t}$

$$= \frac{\partial}{\partial t}\left(2t\left(1-\frac{x}{2L}\right)^2\right)$$

$$= \left(1-\frac{x}{2L}\right)^2 \times 2$$

$$(a_{Local})_{at\ x = 0.5,\ L = 0.8} = 2\left(1-\frac{0.5}{2\times 0.8}\right)^2$$

$$= 2(1-0.3125)^2 = 0.945 \text{ m/sec}^2$$

03. Ans: –13.68

Sol: $$a_{convective} = v.\frac{\partial v}{\partial x} = \left[2t\left[1-\frac{x}{2L}\right]^2\right]\frac{\partial}{\partial x}\left[2t\left(1-\frac{x}{2L}\right)^2\right]$$

$$= \left[2t\left[1-\frac{x}{2L}\right]^2\right]2t\left[2\left(1-\frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$$

At t = 3 sec; x = 0.5 m; L = 0.8 m

$$a_{convective} = 2\times 3\left[1-\frac{0.5}{2\times 0.8}\right]^2 \times 2\times 3\left[2\left(1-\frac{0.5}{2\times 0.8}\right)\right]\left[\frac{-1}{2\times 0.8}\right]$$

$a_{convective} = -14.62$ m/sec^2

$a_{total} = a_{local} + a_{convective} = 0.94 - 14.62$

$= -13.68$ m/sec^2

04. Ans: (d)

Sol: $u = 6xy - 2x^2$

Continuity equation for 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 6y - 4x$$

$$(6y - 4x) + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = (4x - 6y) = 0$$

$$\partial v = (4x - 6y)\,dy$$

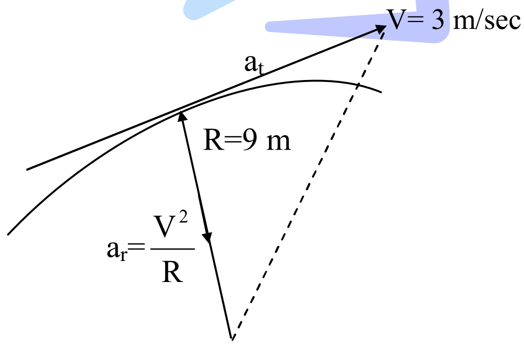
$$v = \int 4x\,dy - \int 6y\,dy$$

$$= 4xy - 3y^2 + c$$

$$= 4xy - 3y^2 + f(x)$$

05. Ans: $\sqrt{2}$ = 1.414

Sol: $\frac{\partial V}{\partial x} = \frac{1}{3}(m/sec/m)$



$$a_r = \frac{V^2}{R} = \frac{(3)^2}{9} = \frac{9}{9} = 1\ m/s^2$$

$$a_t = V\frac{\partial V}{\partial x} = 3 \times \frac{1}{3} = 1\ m/s^2$$

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}\ m/sec^2$$

06. Ans: 13.75

Sol: $a_{t\,(conv)} = V_{avg} \times \frac{dV}{dx}$

$$a_{t\,(conv)} = \left(\frac{2.5 + 3}{2}\right)\left(\frac{3 - 2.5}{0.1}\right) = 2.75 \times 5$$

$$a_{t\,(conv)} = 13.75\ m/s^2$$

07. Ans: 0.3

Sol: $Q = Au$

$$a_{Local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

$$a_{local} = \frac{1}{A}\frac{\partial Q}{\partial t}$$

$$a_{Local} = \left(\frac{1}{0.4 - 0.1x}\right)\frac{\partial Q}{\partial t}$$

$$(a_{Local})_{at\ x=0} = \frac{1}{0.4} \times 0.12 \quad \left(\because \frac{\partial Q}{\partial t} = 0.12\right)$$

$$= 0.3\ m/sec^2$$

08. Ans: (b)

Sol: $\psi = x^2 - y^2$

$$a_{Total} = (a_x)\,\hat{i} + (a_y)\,\hat{j}$$

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 - y^2) = 2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= (2y)(0) + (2x)(2)$$

$$\therefore a_x = 4x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= (2y)\times(2) + (2x)\times(0)$$

$$a_y = 4y$$

$$\therefore a = (4x)\,\hat{i} + (4y)\,\hat{j}$$

09. Ans: (b)

Sol: Given, The stream function for a potential flow field is $\psi = x^2 - y^2$

$\phi = ?$

$$u = \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial\left(x^2 - y^2\right)}{\partial y}$$

$u = 2y$

$$u = -\frac{\partial\phi}{\partial x} = 2y$$

$$\int \partial\phi = -\int 2y\partial x$$

$\phi = -2\,xy + c_1$

Given, ϕ is zero at (0,0)

$\therefore\ c_1 = 0$

$\therefore\ \phi = -2xy$

10. Ans: 4

Sol: Given, 2D – flow field

Velocity, V = 3xi + 4xyj

$u = 3x, \qquad v = 4xy$

$$\omega_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

$$\omega_z = \frac{1}{2}(4y - 0)$$

$$(\omega_Z)_{at(2,2)} = \frac{1}{2}\times 4(2) = 4 \text{ rad/sec}$$

11. Ans: (b)

Sol: Given, $u = 3x$, $\quad v = Cy$, $\quad w = 2$

The shear stress, τ_{xy} is given by

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left[\frac{\partial}{\partial y}(3x) + \frac{\partial}{\partial x}(Cy)\right]$$

$$= \mu\,(0 + 0) = 0$$

12. Ans: (b, c)

Sol: Given: $\vec{V} = x\,\hat{i} - y\,\hat{j}$

Thus, $u = x$ and $v = -y$

$$\frac{\partial u}{\partial x} = 1\ ;\ \frac{\partial u}{\partial y} = 0\ ;\ \frac{\partial v}{\partial x} = 0\ ;\ \frac{\partial v}{\partial y} = -1$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = x\times 1 - y\times 0 = x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = x\times 0 + y\times 1 = y$$

Thus, $\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} = x\,\hat{i} + y\,\hat{j}$

$u = -\dfrac{\partial\psi}{\partial y} = x$; On integration, $\psi = -xy + C$

$u = -\dfrac{\partial\phi}{\partial x} = x$; On integration, $\phi = -\dfrac{x^2}{2} + C$

Chapter 5 Energy Equation and its Applications

01. Ans: (c)

Sol: Applying Bernoulli's equation for ideal fluid

$$\frac{P_1}{\rho g}+Z_1+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+Z_2+\frac{V_2^2}{2g}$$

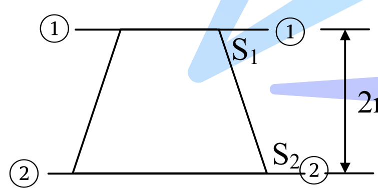
$$\frac{P_1}{\rho g}+\frac{(2)^2}{2g}=\frac{P_2}{\rho g}+\frac{(1)^2}{2g}$$

$$\frac{P_2}{\rho g}-\frac{P_1}{\rho g}=\frac{4}{2g}-\frac{1}{2g}$$

$$\frac{P_2-P_1}{\rho g}=\frac{3}{2g}=\frac{1.5}{g}$$

02. Ans: (c)

Sol:



$$\frac{V_1^2}{2g}=1.27\text{m}\ ,\qquad \frac{P_1}{\rho g}=2.5\text{m}$$

$$\frac{V_2^2}{2g}=0.203\text{m}\ ,\qquad \frac{P_2}{\rho g}=5.407\text{m}$$

$Z_1 = 2$ m , $Z_2 = 0$ m

Total head at (1) – (1)

$$=\frac{V_1^2}{2g}+\frac{P_1}{\rho g}+Z_1$$

$= 1.27 + 2.5 + 2 = 5.77$ m

Total head at (2) – (2)

$$=\frac{V_2^2}{2g}+\frac{P_2}{\rho g}+Z_2$$

$= 0.203 + 5.407 + 0 = 5.61$ m

Loss of head = 5.77 – 5.61 = 0.16 m

∴ Energy at (1) – (1) > Energy at (2) – (2)

∴ Flow takes from higher energy to lower energy

i.e. from (S_1) to (S_2)

Flow takes place from top to bottom.

03. Ans: 1.5

Sol: $A_1=\frac{\pi}{4}d_1^2=\frac{\pi}{4}(0.1)^2=7.85\times10^{-3}\ \text{mm}^2$

$$A_2=\frac{\pi}{4}d_2^2=\frac{\pi}{4}(0.05)^2=1.96\times10^{-3}\ \text{mm}^2$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$Z_1 = Z_2$, it is in horizontal position

Since, at outlet, pressure is atmospheric

$P_2 = 0$

$Q = 100$ lit/sec $= 0.1\ \text{m}^3$/sec

$$V_1=\frac{Q}{A_1}=\frac{0.1}{7.85\times10^{-3}}=12.73\,\text{m/sec}$$

$$V_2=\frac{Q}{A_2}=\frac{0.1}{1.96\times10^{-3}}=51.02\,\text{m/sec}$$

$$\frac{P_{1\text{gauge}}}{\rho_{air}\times g}+\frac{(12.73)^2}{2\times10}=0+\frac{(51.02)^2}{2\times10}$$

$$\frac{P_1}{\rho_{air}.g}=121.53$$

$$P_1=121.53\times\rho_{air}\times g$$

$= 1.51$ kPa

04. Ans: 395

Sol: $Q = 100$ litre/sec $= 0.1\ m^3/sec$

$V_1 = 100$ m/sec; $P_1 = 3 \times 10^5\ N/m^2$

$V_2 = 50$ m/sec; $P_2 = 1 \times 10^5\ N/m^2$

Power (P) = ?

Energy equation :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{3\times10^5}{1000\times10} + \frac{100^2}{2\times10} + 0 = \frac{1\times10^5}{1000\times10} + \frac{50^2}{2\times10} + 0 + h_L$$

$\Rightarrow h_L = 395$ m

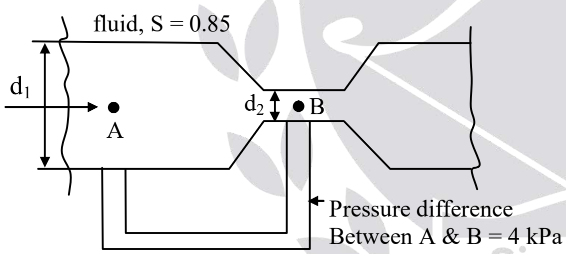
$P = \rho g Q.h_L$

$P = 1000 \times 10 \times 0.10 \times 395$

$P = 395$ kW

05. Ans: 35

Sol:



$d_1 = 300$ mm, $d_2 = 120$ mm

$$Q_{Th} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\Delta P}{w}\right)}$$

$$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.30)^2 = 0.07\,m^2$$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.12)^2 = 0.011\,m^2$$

$\Delta P = 4$ kPa,

$$h = \frac{\Delta P}{w} = \frac{\Delta P}{\rho_f.g}$$

$$= \frac{\Delta P}{s_f\rho_w g} = \frac{4\times10^3}{0.85\times1000\times9.81}$$

$$Q_{Th} = \frac{0.07\times0.011}{\sqrt{(0.07)^2-(0.011)^2}}\sqrt{\frac{2\times9.81\times4\times10^3}{0.85\times1000\times9.81}}$$

$= 0.035\ m^3/sec = 35.15$ ltr/sec

06. Ans: 65

Sol: $h_{stag} = 0.30$ m

$h_{stat} = 0.24$ m

$V = c\sqrt{2gh_{dyna}}$

$V = 1\sqrt{2g(h_{stag} - h_{stat})}$

$= \sqrt{2(9.81)(0.30-0.24)} = 1.085$ m/s

$= 1.085 \times 60 = 65.1$ m/min

07. Ans: 81.5

Sol: $x = 30$ mm,

$g = 10\ m/s^2$

$\rho_{air} = 1.23\ kg/m^3$;

$\rho_{Hg} = 13600\ kg/m^3$

$C = 1$

$V = \sqrt{2gh_D}$

$$h_D = x\left(\frac{S_m}{S} - 1\right)$$

$$h_D = 30\times10^{-3}\left(\frac{13600}{1.23} - 1\right)$$

$h_D = 331.67$ m

$V = 1\times\sqrt{2\times10\times331.67} = 81.5$ m/sec

08. Ans: 140

Sol: $Q_a = C_d \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$

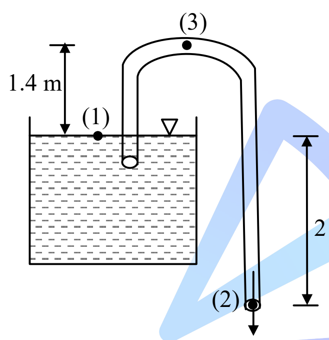
$C_d \propto \dfrac{1}{\sqrt{h}}$

$\dfrac{C_{d_{venturi}}}{C_{d_{orifice}}} = \dfrac{0.95}{0.65} = \sqrt{\dfrac{h_{orifice}}{h_{venturi}}}$

$h_{venturi} = 140$ mm

09. Ans: (b, d)

Sol:



Applying Bernoulli equation between sections (1) & (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

But, $P_1 = 0 = P_2$; $V_1 = 0$;

$Z_1 - Z_2 = 2$ m

So, $0 + 0 + 2 = 0 + \dfrac{V_2^2}{2g} + 0$

$\Rightarrow V_2 = 2\sqrt{g}$ m/s

$$Q = \frac{\pi}{4} d^2 V_2 = \frac{\pi}{4} \times (3 \times 10^{-2})^2 \times 2\sqrt{9.81}$$

$= 4.428 \times 10^{-3}$ m^3/s

Let the point at the summit be denoted by (3).

Then,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

where, $V_3 = V_2 = 2\sqrt{g}$ m/s ;

$Z_3 - Z_1 = 1.4$ m

Thus,

$$\frac{P_3}{\gamma} = -1.4 - \frac{4g}{2g} = -3.4$$

$\Rightarrow \quad P_3 = -3.4 \times 9810$ Pa

$= -33.354$ kPa

Chapter 6 Momentum equation and its Applications

01. Ans: 1600

Sol: S = 0.80

$A = 0.02\ m^2$

V = 10 m/sec

$F = \rho.A.V^2$

$F = 0.80 \times 1000 \times 0.02 \times 10^2$

F = 1600 N

02. Ans: 6000

Sol: $A = 0.015\ m^2$

V = 15 m/sec (Jet velocity)

U = 5 m/sec (Plate velocity)

$F = \rho A\ (V + U)^2$

$F = 1000 \times 0.015\ (15 + 5)^2$

F = 6000 N

03. Ans: 19.6

Sol: V = 100 m/sec (Jet velocity)

U = 50 m/sec (Plate velocity)

d = 0.1 m

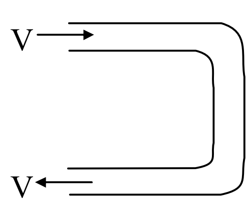
$F = \rho A\ (V - U)^2$

$$F = 1000 \times \frac{\pi}{4} \times 0.1^2 \times (100 - 50)^2$$

F = 19.6 kN

04. Ans: (a)

Sol:



$F_x = \rho aV(\ V_{1x} - V_{2x})$

$= \rho aV(\ V - (-V))$

$= 2\ \rho a\ V^2$

$= 2\times 1000 \times 10^{-4} \times 5^2 = 5\ N$

05. Ans: (d)

Sol: Given, V = 20 m/s,

u = 5 m/s

$F_1 = \rho A(V - u)^2$

Power $(P_1) = F_1 \times u = \rho A(V - u)^2 \times u$

$F_2 = \rho.A.V \times V_r$

$= \rho.A(V).(V-u)$

Power $(P_2) = F_2 \times u = \rho AV(V-u)u$

$$\frac{P_1}{P_2} = \frac{\rho A(V-u)^2 \times u}{\rho AV(V-u) \times u}$$

$$= \frac{V-u}{V} = 1 - \frac{u}{V}$$

$$= 1 - \frac{5}{20} = 0.75$$

06. Ans: 2035

Sol: Given, θ = 30°, $\dot{m}$ = 14 kg/s

$(P_i)_g = 200$ kPa,

$(P_e)_g = 0$

$A_i = 113\times10^{-4}\ m^2$,

$A_e = 7\times10^{-4}\ m^2$

$\rho = 10^3\ kg/m^3$,

$g = 10\ m/s^2$

From the continuity equation :

$\rho A_i\ V_i = 14$

or $V_i = \dfrac{14}{10^3 \times 113 \times 10^{-4}} = 1.24\,m/s$

Similarly, $V_e = \dfrac{14}{10^3 \times 7 \times 10^{-4}} = 20\,m/s$

Let F_x be the force exerted by elbow on water in the +ve x-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write :

$$(P_i)_g A_i + F_x = \dot{m}(V_e \cos 30^o - V_i)$$

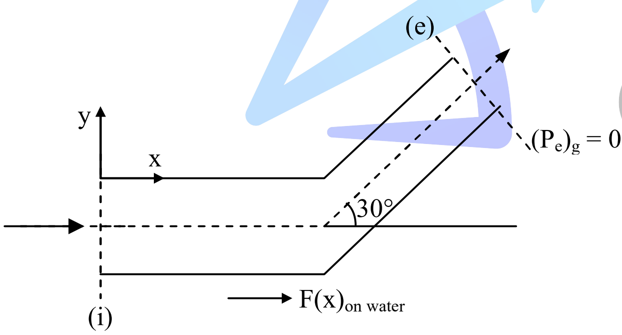
$$F_x = \dot{m}(V_e \cos 30^o - V_i) - (P_i)_g A_i$$

$= 14\,(20\times\cos30° – 1.24) – 200\times10^3\times113\times10^{-4}$

$= 225.13 – 2260$

$= – 2034.87$ N $\approx – 2035$ N

The x-component of water force on elbow is $–F_x$ (as per Newton's third law), i.e., $\cong 2035$ N



07. Ans: (a, d)

Sol: Given:

$d_j = 5$ cm ,

$V_j = 20$ m/s,

$U = 8$ m/s

$F_x = \rho\, A_j\, (V_j – U)\, (V_j – U)$

$= 10^3 \times \dfrac{\pi}{4} \times 0.05^2 \times (20-8)^2$

$= 282.74$ N

Work done per second,

$\dot{W} = F_x \times U$

$= 282.74 \times 8 = 2.262$ kW

Efficiency,

$$\eta = \frac{\dot{W}}{\frac{1}{2}\rho Q \times V_j^2} = \frac{2\dot{W}}{\rho A_j V_j^3} = \frac{8\dot{W}}{\rho \times \pi d_j^2 \times V_j^3}$$

$$= \frac{8 \times 2.262 \times 10^3}{10^3 \times \pi \times (0.05)^2 \times (20)^3}$$

$= 0.288 = 28.8$ %

Chapter 7

Laminar Flow

01. Ans: (d)

Sol: In a pipe, the flow changes from laminar flow to transition flow at Re = 2000. Let V be the average velocity of flow. Then

$$2000 = \frac{V \times 8 \times 10^{-2}}{0.4 \times 10^{-4}} \Rightarrow V = 1\,m/s$$

In laminar flow through a pipe,

$V_{max} = 2 \times V = 2$ m/s

02. Ans: (d)

Sol: The equation $\tau = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{r}{2}\right)$ is valid for laminar as well as turbulent flow through a circular tube.

03. Ans: (d)

Sol: $Q = A.V_{avg}$

$$Q = A.\frac{V_{max}}{2} \qquad (\because V_{max} = 2\,V_{avg})$$

$$Q = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2 \times \frac{1.5}{2}$$

$$= \frac{\pi}{4} \times (0.04)^2 \times 0.75$$

$$= \frac{\pi}{4} \times \frac{4}{100} \times \frac{4}{100} \times \frac{3}{4} = \frac{3\pi}{10000}\ m^3/sec$$

04. Ans: 1.92

Sol: $\rho = 1000$ kg/m^3

$Q = 800$ mm^3/sec $= 800 \times (10^{-3})^3$ m^3/sec

L = 2 m

D = 0.5 mm

$\Delta P = 2$ MPa $= 2 \times 10^6$ Pa

$\mu = ?$

$$\Delta P = \frac{128.\mu QL}{\pi D^4}$$

$$2 \times 10^6 = \frac{128 \times \mu \times 800 \times (10^{-3})^3 \times 2}{\pi (0.5 \times 10^{-3})^4}$$

$\mu = 1.917$ milli Pa – sec

05. Ans: 0.75

Sol: $U_r = U_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$

$$\left[\because \frac{U}{U_{max}} = 1 - \left(\frac{r}{R}\right)^2\right]$$

$$= 1\left(1 - \left(\frac{50}{200}\right)^2\right)$$

$$= 1\left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75\ m/s$$

06. Ans: 0.08

Sol: Given,

$\rho = 0.8 \times 1000 = 800$ kg/m^3

$\mu = 1$ Poise $= 10^{-1}$ N-s/m^2

d = 50 mm = 0.05 m

Velocity = 2 m/s

Reynold's Number, $Re = \frac{\rho VD}{\mu}$

$$= \frac{800 \times 2 \times 0.05}{10^{-1}} = 800$$

$(\because Re < 2000)$

$\therefore$ Flow is laminar,

For laminar, Darcy friction factor

$$f = \frac{64}{Re} = \frac{64}{800} = 0.08$$

07. Ans: 16

Sol: For fully developed laminar flow,

$$h_f = \frac{32\mu VL}{\rho g D^2} \quad (\therefore Q = AV)$$

$$h_f = \frac{32\mu\left(\frac{Q}{A}\right)L}{\rho g D^2} = \frac{32\mu QL}{AD^2 \times \rho g}$$

$$h_f = \frac{32\mu QL}{\frac{\pi}{4}D^2 \times D^2 \times \rho g}$$

$$h_f \propto \frac{1}{D^4}$$

$$h_{f1} D_1^4 = h_{f_2} D_2^4$$

Given, $D_2 = \frac{D_1}{2}$

$$h_{f1} \times D_1^4 = h_{f2} \times \left(\frac{D_1}{2}\right)^4$$

$$h_{f_2} = 16 h_{f_1}$$

$\therefore$ Head loss, increases by 16 times if diameter is halved.

08. Ans: 5.2

Sol: Oil viscosity,

μ = 10 poise = 10×0.1 = 1 N-s/m^2

y = 50 × 10^{-3}m

L = 120 cm = 1.20 m, $\Delta P = 3 \times 10^3$Pa

Width of plate = 0.2 m,

Q = ?

Q = A.V$_{avg}$ = (width of plate×y)V

$$\Delta P = \frac{12\mu VL}{B^2}$$

$$3\times10^3 = \frac{12\times1\times V\times1.20}{(50\times10^{-3})^2}$$

V = 0.52 m/sec

Q = AV$_{avg}$ = (0.2×50×10^{-3}) (0.52)

= 5.2 lit/sec

09. Ans: (a)

Sol: Wall shear stress for flow in a pipe is given by,

$$\tau_o = -\frac{\partial P}{\partial x}\times\frac{R}{2} = \frac{\Delta P}{L}\times\frac{D}{4} = \frac{\Delta P\, D}{4L}$$

10. Ans: 72

Sol: Given, $\rho = 800$ kg/m^3,

$\mu = 0.1$ Pa.s

Flow is through an inclined pipe.

d = 1 × 10^{-2} m,

$V_{av} = 0.1$ m/s, $\theta = 30°$

$$Re = \frac{\rho V_{av} d}{\mu} = \frac{800\times0.1\times1\times10^{-2}}{0.1} = 8$$

$\Rightarrow$ flow is laminar.

Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+h_f$$

But $V_1 = V_2$,

$(Z_2 - Z_1) = 10\sin30° = 5$ m

and $h_f = \frac{32\mu V_{av} L}{\rho g d^2}$

$$\frac{(P_1-P_2)}{\gamma} = (Z_2-Z_1)+\frac{32\mu V_{av}L}{\rho g d^2}$$

$$(P_1-P_2) = \rho g(Z_2-Z_1)+\frac{32\mu V_{av}L}{d^2}$$

$$= 800\times10\times5+\frac{32\times0.1\times0.1\times10}{(1\times10^{-2})^2}$$

$= 40 \times 10^3 + 32\times10^3 = 72$ kPa

11. Ans: (a, b, c, d)

Sol: The following statements regarding laminar flow through pipes are correct.

- Velocity profile is parabolic as given by

$$u = U\left(1 - \frac{r^2}{R^2}\right)$$

- Shear stress, $\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr}$

$$\tau = -\mu \times \left(-\frac{2rU}{R^2}\right) = \frac{2\mu U}{R^2} \times r$$

= Linear profile

- Rate of shear strain profile is also linear.
- Flow is rotational.

Chapter 8

Flow through Pipes

01. Ans: (d)

Sol:

- The Darcy-Weisbash equation for head loss in written as:

$$h_f = \frac{fLV^2}{2gd}$$

where V is the average velocity, f is friction factor, L is the length of pipe and d is the diameter of the pipe.

- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as the nature of pipe surface (smooth or rough)
- For laminar flow, friction factor is a function of Reynolds number.

02. Ans: 481

Sol: Given data,

$\dot{m} = \pi$ kg/s, $d = 5\times10^{-2}$ m,

$\mu = 0.001$ Pa.s , $\rho = 1000$ kg/m^3

$$V_{av} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho\pi d^2} = \frac{4\times\pi}{\rho\pi d^2} = \frac{4}{\rho d^2}$$

$$Re = \frac{\rho V_{av} d}{\mu} = \rho \times \frac{4}{\rho d^2} \times \frac{d}{\mu} = \frac{4}{\mu d}$$

$$= \frac{4}{0.001\times5\times10^{-2}} = 8\times10^4$$

$\Rightarrow$ Flow is turbulent

$$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{(8\times10^4)^{0.25}} = 0.0188$$

$$\Delta P = \rho g \frac{f L V_{av}^2}{2gd} = f\rho L \times \left(\frac{4}{\rho d^2}\right)^2 \times \frac{1}{2d}$$

$$\frac{\Delta P}{L} = f \times \frac{16}{\rho d^5} \times \frac{1}{2} = \frac{8f}{\rho d^5} = \frac{8 \times 0.0188}{10^3 \times (5\times 10^{-2})^5}$$

$$= 481.28 \text{ Pa/m}$$

03. Ans: (a)

Sol: In pipes Net work, series arrangement

$$\therefore h_f = \frac{f \times \ell \times V^2}{2gd} = \frac{f \times \ell \times Q^2}{12.1 \times d^5}$$

$$\frac{h_{f_A}}{h_{f_B}} = \frac{f_A \times \ell_A \times Q_a^2}{12.1 \times d_A^5} \times \frac{12.1 \times d_B^5}{f_B \times \ell_B \times Q_B^2}$$

Given $l_A = l_B$, $f_A = f_B$, $Q_A = Q_B$

$$\frac{h_{f_A}}{h_{f_B}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2 d_B}\right)^5$$

$$= \left(\frac{1}{1.2}\right)^5 = 0.4018 \approx 0.402$$

04. Ans: (a)

Sol: Given, $d_1 = 10$ cm; $d_2 = 20$ cm

$f_1 = f_2$;

$l_1 = l_2 = l$

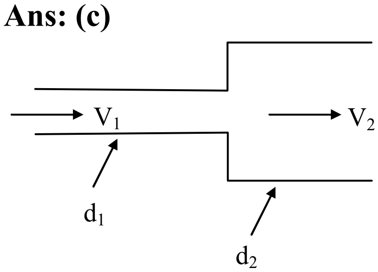
$l_e = l_1 + l_2 = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \Rightarrow \frac{2l}{d_e^5} = \frac{l}{10^5} + \frac{l}{20^5}$$

$\therefore d_e = 11.4$ cm

05. Ans: (c)

Sol:



Given $d_2 = 2d_1$

Losses due to sudden expansion,

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

By continuity equation,

$$Q = A_1V_1 = A_2V_2$$

$$\therefore \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{1}{4}\right)^2$$

$$h_L = \frac{9}{16} \times \frac{V_1^2}{2g}$$

$$\frac{h_L}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

06. Ans: (b)

Sol: Pipes are in parallel

$Q_e = Q_A + Q_B$ ------- (i)

$h_{Le} = h_{L_A} = h_{L_B}$

$L_e = 175$ m

$f_e = 0.015$

$$\frac{f_e L_e Q_e^2}{12.1 D_e^5} = \frac{f_A . L_A Q_A^2}{12.1 D_A^5} = \frac{f_B L_B Q_B^2}{12.1 D_B^5}$$

$$\frac{0.020 \times 150 \times Q_A^2}{12.1 \times (0.1)^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$Q_A = 1.747\ Q_B$ -------(ii)

From (i) $Q_e = 1.747\ Q_B + Q_B$

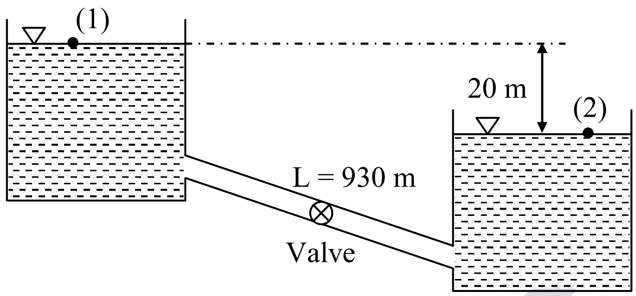
$Q_e = 2.747\ Q_B$ -------(iii)

$$\frac{0.015 \times 175 (2.747 Q_B)^2}{12.1 \times D_e^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$D_e = 116.6$ mm $\simeq 117$ mm

07. Ans: 0.141

Sol:



Given data,

$L = 930$ m , $k_{valve} = 5.5$

$k_{entry} = 0.5$, $d = 0.3$ m

$f = 0.03$, $g = 10$ m/s^2

Applying energy equation for points (1) and (2), we write :

$$\frac{P_1}{\gamma_w}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_w}+\frac{V_2^2}{2g}+Z_2+h_{L,entry}+h_{L,valve}+h_{L,exit}+h_{f,pipe}$$

But $P_1 = P_2 = P_{atm} = 0$

$V_1 = 0 = V_2$

$Z_1 - Z_2 = 20$ m , $k_{exit} = 1$

$$Z_1-Z_2=0.5\frac{V^2}{2g}+5.5\frac{V^2}{2g}+1\times\frac{V^2}{2g}+\frac{fLV^2}{2gd}$$

$$=7\frac{V^2}{2g}+\frac{fLV^2}{2gd}=\frac{V^2}{2g}\left(7+\frac{fL}{d}\right)$$

or $$20=\frac{V^2}{2g}\left[7+\frac{0.03\times 930}{0.3}\right]=100\frac{V^2}{2g}$$

or $$V^2=\frac{20\times 2g}{100}=\frac{20\times 2\times 10}{100}$$

$\Rightarrow$ $V = 2$ m/s

Thus, discharge, $Q=\frac{\pi}{4}\times 0.3^2\times 2$

$= 0.1414$ m^3/s

08. Ans: (c)

Sol: Given data :

Fanning friction factor, $f = m\,Re^{-0.2}$

For turbulent flow through a smooth pipe.

$$\Delta P=\frac{\rho f_{Darcy}LV^2}{2d}=\frac{\rho(4f)LV^2}{2d}$$

$$=\frac{2\rho m Re^{-0.2}LV^2}{d}$$

or $\Delta P \propto V^{-0.2}V^2 \propto V^{1.8}$ (as all other parameters remain constant)

We may thus write :

$$\frac{\Delta P_2}{\Delta P_1}=\left(\frac{V_2}{V_1}\right)^{1.8}=\left(\frac{2}{1}\right)^{1.8}=3.4822$$

or $\Delta P_2 = 3.4822 \times 10 = 34.82$ kPa

09. Ans: (b)

Sol: Given data :

Rectangular duct, L = 10 m,

X-section of duct = 15 cm × 20 cm

Material of duct-Commercial steel,

$\varepsilon = 0.045$ mm

Fluid is air ($\rho = 1.145$ kg/m^3,

$\nu = 1.655\times10^{-5}$ m^2/s)

$V_{av} = 7$ m/s

$$Re=\frac{V_{av}\times D_h}{\nu}$$

where, D_h = Hydraulic diameter

$$= \frac{4 \times \text{Cross sectional area}}{\text{Perimeter}}$$

$$= \frac{4 \times 0.15 \times 0.2}{2(0.15+0.2)} = 0.1714\,\text{m}$$

$$Re = \frac{7 \times 0.1714}{1.655 \times 10^{-5}} = 72495.5$$

$\Rightarrow$ Flow is turbulent.

Using Haaland equation to find friction factor,

$$\frac{1}{\sqrt{f}} \simeq -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D_h}{3.7}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\frac{6.9}{72495.5} + \left(\frac{0.045 \times 10^{-3}}{0.1714 \times 3.7}\right)^{1.11}\right]$$

$$= -1.8 \log[9.518 \times 10^{-5} + 2.48 \times 10^{-5}]$$

$$= -1.8 \log(11.998 \times 10^{-5})$$

$$\frac{1}{\sqrt{f}} = 7.058$$

$$f = 0.02$$

The pressure drop in the duct is,

$$\Delta P = \frac{\rho f L V^2}{2 D_h}$$

$$= \frac{1.145 \times 0.02 \times 10 \times 7^2}{2 \times 0.1714} = 32.73\ \text{Pa}$$

The required pumping power will be

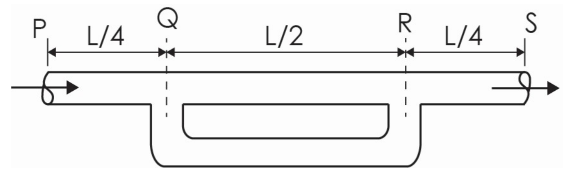
$P_{pumping} = Q\,\Delta P = A\,V_{av} \times \Delta P$

$= (0.15 \times 0.2) \times 7 \times (32.73)$

$= 6.87\ \text{W} \simeq 7\ \text{W}$

10. Ans: 26.5

Sol:



Case I: Without additional pipe,

Let Q be the discharge through the pipe. Then

$$\frac{P_P}{\gamma} + \frac{V_P^2}{2g} + Z_P = \frac{P_S}{\gamma} + \frac{V_S^2}{2g} + Z_S + \frac{f L Q^2}{12.1\, d^5}$$

But $V_P = V_S$

and $Z_P = Z_S$

P_P and P_S are the pressures at sections P and S, respectively.

Thus,

$$\frac{P_P}{\gamma} - \frac{P_S}{\gamma} = \frac{f L Q^2}{12.1 d^5} \quad \text{-------(1)}$$

Case II: When a pipe (L/2) is connected in parallel.

In this case, let Q′ be the total discharge.

$Q_{Q-R} = \dfrac{Q'}{2}$ and $Q_{R\text{-}S} = Q'$

Then,

$$\frac{P'_P}{\gamma} + \frac{V_P'^2}{2g} + Z'_P = \frac{P'_S}{\gamma} + \frac{V_S'^2}{2g} + Z'_S + \frac{f(L/4)Q'^2}{12.1\, d^5} + \frac{f(L/2)(Q'/2)^2}{12.1\, d^5} + \frac{f(L/4)Q'^2}{12.1\, d^5}$$

P_P' and P_S' are the pressures at sections P and S in the second case.

But $V_P' = V_S'$; $Z_P' = Z_S'$

So, $\frac{P'_P}{\gamma} - \frac{P'_S}{\gamma} = \frac{f L Q'^2}{12.1 d^5}\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{4}\right]$

$$= \frac{5}{8} \times \frac{f L Q'^2}{12.1 d^5} \text{ -------(2)}$$

Given that end conditions remain same.

i.e., $\frac{P_P}{\gamma} - \frac{P_S}{\gamma} = \frac{P'_P}{\gamma} - \frac{P'_S}{\gamma}$

Hence, equation (2) becomes,

$$\frac{f L Q^2}{12.1 d^5} = \frac{5}{8}\frac{f L Q'^2}{12.1 d^5} \quad \text{from eq.(1)}$$

or $\left(\frac{Q'}{Q}\right)^2 = \frac{8}{5}$

or $\frac{Q'}{Q} = 1.265$

Hence, percentage increase in discharge is

$$= \frac{Q' - Q}{Q} \times 100$$

$= (1.265 - 1) \times 100$

$= 26.5$ %

11. Ans: 20%

Sol: Since, discharge decrease is associated with increase in friction.

$$\frac{df}{f} = -2 \times \frac{dQ}{Q} = 2\left[-\frac{dQ}{Q}\right]$$

$$= 2 \times 10 = 20\%$$

12. Ans: (c, d)

Sol: Given data:

$H_G = 80$ m, $D = 0.5$ m, $L = 4$ km

$f = 0.02$, $\eta = 0.75$

$$\eta = 0.75 = \frac{H_G - h_f}{H_G} \Rightarrow h_f = H_G (1 - \eta)$$

$h_f = 80 \times (1 - 0.75) = 20$ m

But, $h_f = \frac{f L Q^2}{12.1 \times D^5}$

$$20 = \frac{0.02 \times 4000 \times Q^2}{12.1 \times (0.5)^5}$$

$\Rightarrow Q = 0.3075 \text{ m}^3/\text{s}$

$\therefore P_{act} = \rho g Q H_{net}$

$= 10^3 \times 9.81 \times 0.3075 \times (80 - 20)$

$= 180.995$ kW

Now, $V_j = V_N = \sqrt{2gH_{net}}$

$= \sqrt{2 \times 9.81 \times 60} = 34.31$ m/s

From discharge, we have

$Q = A_N V_N$

$$0.3075 = \frac{\pi}{4} \times d_N^2 \times 34.31$$

$\Rightarrow d_N = 0.1068$ m $= 10.68$ cm

Chapter 9 Elementary Turbulent Flow

01. Ans: (b)

Sol: The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

where V* is the shear velocity.

02. Ans: (d)

Sol: $\Delta P = \rho g h_f$

$$= \frac{\rho f L V^2}{2D} = \frac{\rho g f L Q^2}{12.1 D^5}$$

For Q = constant

$$\Delta P \propto \frac{1}{D^5}$$

or $$\frac{\Delta P_2}{\Delta P_1} = \frac{D_1^5}{D_2^5} = \left(\frac{D_1}{2D_1}\right)^5 = \frac{1}{32}$$

03. Ans: 2.4

Sol: Given: V = 2 m/s

f = 0.02

$V_{max} = ?$

$$V_{max} = V(1 + 1.43\sqrt{f})$$

$$= 2\left(1 + 1.43\sqrt{0.02}\right)$$

$$= 2 \times 1.2 = 2.4 \text{ m/s}$$

04. Ans: (c)

Sol: Given data:

D = 30 cm = 0.3 m

$Re = 10^6$

f = 0.025

Thickness of laminar sub layer, $\delta' = ?$

$$\delta' = \frac{11.6\nu}{V^*}$$

where V^* = shear velocity = $V\sqrt{\frac{f}{8}}$

ν = Kinematic viscosity

$$Re = \frac{V.D}{\nu}$$

$$\therefore \nu = \frac{V.D}{Re}$$

$$\delta' = \frac{11.6 \times \frac{VD}{Re}}{V\sqrt{\frac{f}{8}}}$$

$$\delta' = \frac{11.6 \times D}{Re\sqrt{\frac{f}{8}}}$$

$$= \frac{11.6 \times 0.3}{10^6 \times \sqrt{\frac{0.025}{8}}}$$

$$= 6.22 \times 10^{-5} \text{ m} = 0.0622 \text{ mm}$$

05. Ans: 25

Sol: Given:

L = 100 m; D = 0.1 m

h_L = 10 m; $\tau = ?$

For any type of flow, the shear stress at wall/surface $\tau = \frac{-dP}{dx} \times \frac{R}{2}$

$$\tau = \frac{\rho g h_L}{L} \times \frac{R}{2}$$

$$\tau = \frac{\rho g h_L}{L} \times \frac{D}{4}$$

$$= \frac{1000 \times 9.81 \times 10}{100} \times \frac{0.1}{4}$$

$$= 24.525 \text{ N/m}^2 = 25 \text{ Pa}$$

06. Ans: 0.905

Sol: k = 0.15 mm

$\tau = 4.9\ N/m^2$

ν = 1 centi-stoke

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07\ m/sec$$

ν = 1 centi-stoke

$$= \frac{1}{100}\text{stoke} = \frac{10^{-4}}{100} = 10^{-6}\,m^2/sec$$

$$\frac{k}{\delta'} = \frac{0.15\times10^{-3}}{\left(\frac{11.6\times\nu}{V^*}\right)}$$

$$= \frac{0.15\times10^{-3}}{\frac{11.6\times10^{-6}}{0.07}} = 0.905$$

07. Ans: (a)

Sol: The velocity profile in the laminar sublayer is given as

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

or $\nu = \dfrac{y(V^*)^2}{u}$

where, V* is the shear velocity.

Thus, $\nu = \dfrac{0.5\times10^{-3}\times(0.05)^2}{1.25}$

$= 1\times10^{-6}\ m^2/s$

$= 1\times10^{-2}\ cm^2/s$

08. Ans: 47.74 N/m²

Sol: Given data :

d = 100 mm = 0.1 m

$u_{r=0} = u_{max} = 2$ m/s

Velocity at r = 30 mm = 1.5 m/s

Flow is turbulent.

The velocity profile in turbulent flow is

$$\frac{u_{max} - u}{V^*} = 5.75\log\left(\frac{R}{y}\right)$$

where u is the velocity at y and V* is the shear velocity.

For pipe, y = R – r

= (50 – 30) mm = 20 mm

Thus,

$$\frac{2-1.5}{V^*} = 5.75\log\left(\frac{50}{20}\right) = 2.288$$

or $V^* = \dfrac{0.5}{2.288} = 0.2185\,m/s$

Using the relation,

$V^* = \sqrt{\dfrac{\tau_w}{\rho}}$ or $\tau_w = \rho\,(V^*)^2$

$\tau_w = 10^3 \times (0.2185)^2 = 47.74\ N/m^2$

09. Ans: (a, b)

Sol: The following statements are true for turbulent flow through pipes:

- Velocity profile is logarithmic (in the overlap region) expressed as

$$\frac{u}{u^*} = 2.5\,\ell n\left(\frac{yu^*}{\nu}\right) + 5.0$$

- Surface roughness plays an important role in contributing towards determining head loss.

Chapter 10 Boundary Layer Theory

01. Ans: (c)

Sol: $Re_{Critical} = \frac{U_\infty x_{critical}}{\nu}$

Assume water properties

$$5\times10^5 = \frac{6\times x_{critical}}{1\times10^{-6}}$$

$x_{critical} = 0.08333$ m = 83.33 mm

02. Ans: 1.6

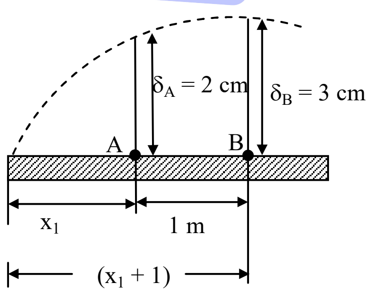
Sol: $\delta \propto \frac{1}{\sqrt{Re}}$ (At given distance 'x')

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{Re_2}{Re_1}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6$$

03. Ans: 80

Sol:



$$\delta \propto \sqrt{x}$$

$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1+1)}}$$

$$x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1+1}}$$

$$\frac{4}{9} = \frac{x_1}{x_1+1}$$

$5x_1 = 4 \Rightarrow x_1 = 80$ cm

04. Ans: 2

Sol: $\tau \propto \frac{1}{\delta}$

$$\tau \propto \frac{1}{\sqrt{x}} \quad \because \delta \propto \sqrt{x}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{4} = 2$$

05. Ans: 3

Sol: $\frac{U}{U_\infty} = \frac{y}{\delta}$

$$\frac{\delta^*}{\theta} = \text{Shape factor} = ?$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \left(1 - \frac{y}{8}\right) dy$$

$$= y - \frac{y^2}{2\delta}\Big|_0^\delta$$

$$= \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \frac{y}{8}\left(1 - \frac{y}{\delta}\right) dy$$

$$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta}\Big|_0^\delta = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\text{Shape factor} = \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$$

06. Ans: 22.6

Sol: Drag force,

$$F_D = \frac{1}{2} C_D.\rho.A_{Proj}.\, U_\infty^2$$

B = 1.5 m, ρ = 1.2 kg/m^3

L = 3.0 m, ν = 0.15 stokes

U_∞ = 2 m/sec

$$Re = \frac{U_\infty L}{\nu} = \frac{2\times3}{0.15\times10^{-4}} = 4\times10^5$$

$$C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{4\times10^5}} = 2.09\times10^{-3}$$

Drag force,

$$F_D = \frac{1}{2}\times2.09\times10^{-3}\times1.2\times(1.5\times3)\times2^2$$

= 22.57 milli-Newton

07. Ans: 1.62

Sol: Given data,

U_∞ = 30 m/s,

ρ = 1.2 kg/m^3

Velocity profile at a distance x from leading edge,

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

δ = 1.5 mm

Mass flow rate of air entering section ab,

$$(\dot{m}_{in})_{ab} = \rho U_\infty(\delta\times1) = \rho U_\infty \delta \text{ kg/s}$$

Mass flow rate of air leaving section cd,

$$(\dot{m}_{out})_{cd} = \rho\int_0^\delta u(dy\times1) = \rho\int_0^\delta U_\infty\left(\frac{y}{\delta}\right)dy$$

$$= \frac{\rho U_\infty}{\delta}\left[\frac{y^2}{2}\right]_0^\delta = \frac{\rho U_\infty \delta}{2}$$

From the law of conservation of mass :

$$(\dot{m}_{in})_{ab} = (\dot{m}_{out})_{cd} + (\dot{m}_{out})_{bc}$$

Hence, $(\dot{m}_{out})_{bc} = (\dot{m}_{in})_{ab} - (\dot{m}_{out})_{cd}$

$$= \rho U_\infty \delta - \frac{\rho U_\infty \delta}{2}$$

$$= \frac{\rho U_\infty \delta}{2}$$

$$= \frac{1.2\times30\times1.5\times10^{-3}}{2}$$

= 27×10^{-3} kg/s

= $27\times10^{-3}\times60$ kg/min

= 1.62 kg/min

08. Ans: (b)

Sol: For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in y-direction, $\frac{\partial u}{\partial y}$ only.

The correct option is (b).

09. Ans: 28.5

Sol: Given data,

Flow is over a flat plate.

L = 1 m,

U_∞ = 6 m/s

ν = 0.15 stoke = 0.15×10^{-4} m^2/s

ρ = 1.226 kg/m^3

$$\delta(x) = \frac{3.46x}{\sqrt{Re_x}}$$

Velocity profile is linear.

Using von-Karman momentum integral equation for flat plate.

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2} \quad \text{--------(1)}$$

we can find out τ_w .

From linear velocity profile, $\frac{u}{U_\infty} = \frac{y}{\delta}$, we evaluate first θ, momentum thickness as

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right)dy$$

$$= \left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right)_0^\delta = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\Rightarrow \theta = \frac{\delta}{6} = \frac{1}{6} \times \frac{3.46\,x}{\sqrt{Re_x}}$$

$$= \frac{3.46}{6}\frac{x^{1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}}$$

Differentiating θ w.r.t x, we get :

$$\frac{d\theta}{dx} = \frac{3.46}{6\times 2}\frac{x^{-1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}} = 0.2883\frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0.5\,m} = 0.2883 \times \frac{1}{\sqrt{\frac{6\times 0.5}{0.15\times 10^{-4}}}} = \frac{0.2883}{447.2}$$

-----(2)

From equation (1)

$$\tau_w\big|_{x=0.5m} = \left.\frac{d\theta}{dx}\right|_{x=0.5\,m} \times \rho U_\infty^2$$

$$= \frac{0.2883}{447.2} \times 1.226 \times 6^2$$

$$= 0.02845 \text{ N/m}^2$$

$$\simeq 28.5 \text{ mN/m}^2$$

10. Ans: (a, d)

Sol: Given data:

$\rho = 1.25$ kg/m^3,

$\mu = 1.8\times 10^{-5}$ Pa.s,

$u_\infty = 3$ m/s

Velocity profile: $\frac{u}{u_\infty} = \sin\left(\frac{\pi}{2}\times\frac{y}{\delta}\right)$

K = 4.79

$$\frac{\delta}{x} = \frac{K}{\sqrt{Re_x}} = \frac{4.79}{\sqrt{Re_x}} \quad \text{........ (1)}$$

At x = 0.6 m,

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

$$Re_x = \frac{1.25\times 3\times 0.6}{1.8\times 10^{-5}} = 1.25\times 10^5$$

From eq. (1),

$$\delta\big|_{x=0.6m} = \frac{4.79\times 0.6}{\sqrt{1.25\times 10^5}} = 8.129 \text{ mm}$$

$\tau_o = \mu\left.\frac{du}{dy}\right|_{y=0}$; From given velocity profile,

$$\frac{du}{dy} = u_\infty \times \frac{\pi}{2\delta}\times\cos\left(\frac{\pi}{2}\times\frac{y}{\delta}\right)$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{u_\infty \pi}{2\delta}$$

Thus,

$$\tau_o = \mu \times \frac{u_\infty \pi}{2\delta}$$

$$= 1.8\times 10^{-5}\times\frac{3\pi}{2\times 8.129\times 10^{-3}}$$

$$= 0.01043 \text{ Pa} = 10.43 \text{ milli Pa.}$$

Chapter 11 Force on Submerged Bodies

01. Ans: 8

Sol: Drag power = Drag Force × Velocity

$$P = F_D \times V$$

$$P = C_D \times \frac{\rho A V^2}{2} \times V$$

$$P \propto V^3$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^3$$

$$\frac{P_1}{P_2} = \left(\frac{V}{2V}\right)^3$$

$$P_2 = 8P_1$$

Comparing the above relation with XP,

We get, X = 8

02. Ans: 4.56 m

Sol: $F_D = C_D.\frac{\rho A V^2}{2}$

$$W = 0.8 \times 1.2 \times \frac{\frac{\pi}{4}(D)^2 \times V^2}{2}$$

(Note: A = Normal (or)

projected Area = $\frac{\pi}{4}D^2$)

$$784.8 = 0.8 \times 1.2 \times \frac{\pi}{4}(D)^2 \times \frac{10^2}{2}$$

$\therefore$ D = 4.56 m

03. Ans: 4

Sol: Given data:

l = 0.5 km = 500 m

d = 1.25 cm

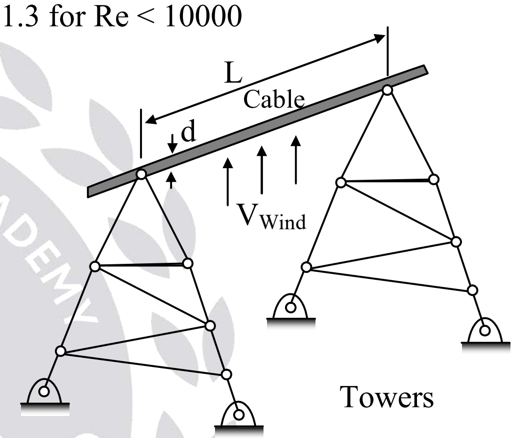
V_{Wind} = 100 km/hr

$\gamma_{Air} = 1.36 \times 9.81 = 13.4\ N/m^3$

$\nu = 1.4 \times 10^{-5}\ m^2/s$

C_D = 1.2 for Re > 10000

C_D = 1.3 for Re < 10000



$$Re = \frac{V.L}{\nu} = \frac{\left(\frac{100 \times 5}{18}\right)(500)}{1.4 \times 10^{-5}}$$

Note: The characteristic dimension for electric power transmission tower wire is "L"

$Re = 992 \times 10^6 > 10{,}000$

$\therefore C_D = 1.2$

$$F_D = C_D \times \frac{\rho A V^2}{2}$$

$$= 1.2 \times \frac{\left(\frac{13.4}{9.81}\right)(L \times d)V^2}{2}$$

$$= \frac{1.2 \times \left(\frac{13.4}{9.81}\right)(500 \times 0.0125)\left(100 \times \frac{5}{18}\right)^2}{2}$$

= 3952.4 N = 4 kN

04. Ans: 0.144 & 0.126

Sol: Given data:

$W_{Kite} = 2.5$ N

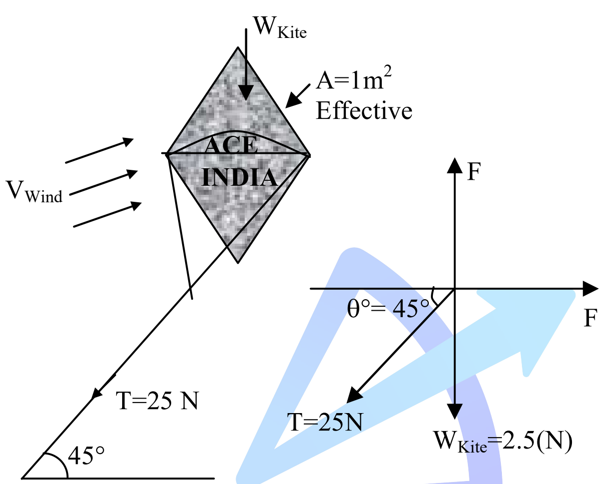
$A = 1\ m^2$

$\theta = 45^o$

$T = 25$ N

$V_{Wind} = 54$ km/hr

$= 54 \times \frac{5}{18} = 15$ m/s



Resolving forces horizontally

$F_D = T\cos45^o$

$C_D \times \frac{\rho AV^2}{2} = 25 \times \cos45^o$

$$\frac{C_D \times \left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 25 \times \frac{1}{\sqrt{2}}$$

$\therefore \quad C_D = 0.126$

Resolving forces vertically

$F_L = W_{Kite} + T\sin45^o$

$\frac{C_L \rho AV^2}{2} = 2.5 + 25\sin45^o$

$$\frac{C_L\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 2.5 + \frac{25}{\sqrt{2}}$$

$\therefore C_L = 0.144$

05. Ans: (a)

Sol: Given data:

$C_{D_2} = 0.75\,C_{D_1}$ (25% reduced)

Drag power = Drag force × Velocity

$P = F_D \times V = \frac{C_D \rho AV^2}{2} \times V$

$P = C_D \times \frac{\rho AV^3}{2}$

Keeping ρ, A and power constant

$C_D V^3$ = constant = C

$$\frac{C_{D_1}}{C_{D_2}} = \left(\frac{V_2}{V_1}\right)^3$$

$$\left(\frac{C_{D_1}}{0.75C_{D_1}}\right)^{1/3} = \frac{V_2}{V_1}$$

$\therefore V_2 = 1.10064 V_1$

% Increase in speed = 10.064%

06. Ans: (c)

Sol: When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid :

Weight of sphere = Buoyant force + Drag force

07. Ans: 0.62

Sol: Given data,

Diameter of dust particle, d = 0.1 mm

Density of dust particle,

$\rho = 2.1\ g/cm^3 = 2100\ kg/m^3$

$\mu_{air} = 1.849\times10^{-5}$ Pa.s,

At suspended position of the dust particle,

$W_{particle} = F_D + F_B$

where F_D is the drag force on the particle and F_B is the buoyancy force.

From Stokes law:

$F_D = 3\pi\mu\ V\ d$

Thus,

$$\frac{4}{3}\times\pi r^3\times\rho\times g = 3\pi\mu Vd + \frac{4}{3}\pi r^3\rho_{air}g$$

or, $\frac{4}{3}\pi r^3 g(\rho-\rho_{air}) = 3\pi\mu_{air}V(2r)$

or $V = \frac{2}{9}r^2 g\left(\frac{\rho-\rho_{air}}{\mu_{air}}\right)$

$$= \frac{2}{9}\times\left(0.05\times10^{-3}\right)^2\times9.81\times\frac{(2100-1.2)}{1.849\times10^{-5}}$$

$= 0.619$ m/s ≈ 0.62 m/s

08. Ans: (b)

Sol: Since the two models M_1 and M_2 have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.

From their shapes, we can say that M_2 reaches the bottom earlier than M_1.

09. Ans: (a)

Sol:

- Drag of object A_1 will be less than that on A_2. There are chances of flow separation on A_2 due to which drag will increase as compared to that on A_1.
- Drag of object B_1 will be more than that of object B_2. Because of rough surface of B_2, the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but C_2 is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on C_2 will be more than that on C_1 because of flow becoming turbulent due to roughness. Hence, drag of object C_2 will be more than that of object C_1.
- Thus, the correct answer is option (a).

Chapter 12

Open Channel Flow

02. Ans: (b)

Sol: $Q_1 = 15 m^3/sec$, $y = 1.5$ m

$$S_1 = \frac{1}{1690}, \text{if } S_2 = \frac{1}{1000}$$

Then $Q_2 = ?$

$$Q \propto \sqrt{S}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{\frac{1}{1000}}{\frac{1}{1690}}}$$

$Q_2 = 1.3 \times 15 = 19.5\ m^3/s$

03. Ans: (d)

Sol: $Q = AV$

$$= B \times y \times \frac{1}{n} R^{2/3} S^{1/2}$$

$$= B \times y \times \frac{1}{n} y^{2/3} S^{1/2}$$

$= R \approx y \rightarrow$ For wide rectangular channel $Q \propto y^{5/3}$

$$\frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{\frac{5}{3}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{1.25 y_1}{y_1}\right)^{\frac{5}{3}}$$

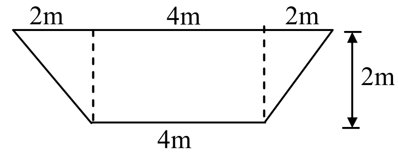
$$\frac{Q_2}{Q_1} = 1.45$$

$Q_2 = 1.45\ Q_1$

It is increased by 45%

05. Ans: 24.33

Sol:



$\tau_{avg} = \gamma_w RS$

$$R = \frac{A}{P}$$

$$A = 2 \times \left(\frac{1}{2} \times 2 \times 2\right) + 4 \times 2$$

$$= 2 \times 2 + 4 \times 2 = 12\ m^2$$

$$P = 4 + 2\sqrt{2^2 + 2^2}$$

$= 9.66$ m

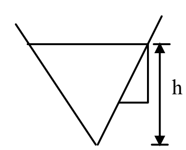
$$R = \frac{12}{9.66} = 1.24\,m$$

$\tau_{avg} = 9810 \times 1.24 \times 0.002$

$= 24.33\ N/m^2$

06. Ans: (d)

Sol: Triangular:



Triangle

P = 2 (Inclined portion)

$$P = 2I = 2h\sqrt{1+m^2} \quad \left(\because I = h\sqrt{1+m^2}\right)$$

$$= 2h\sqrt{1+1^2}$$

$$= 2h\sqrt{2}$$

$$\frac{P}{h} = 2\sqrt{2} = 2.83$$

Trapezoidal: Efficient trapezoidal section is half of the Hexagon for which all sides are equal

Trapezoidal

$I = h\sqrt{1+m^2}$

$P = I = h\sqrt{(1)+\left(\frac{1}{\sqrt{3}}\right)^2} = h(1.15)$

$\frac{P}{h} = 1.15\times 3 = 3.46$ (3 sides are equal)

Rectangular:

$P = b + 2h = 2h + 2h = 4h$ ($b = 2y$)

$\frac{P}{h} = 4$

07. Ans: 0.37

Sol: $A = y\,(b + my)$

$A = \frac{Q}{V} = 4\,m^2$

$4 = \left(b + \frac{y}{\sqrt{3}}\right)y$ ....(I) $\left(\because m = \frac{1}{\sqrt{3}}\right)$

But $b = I$ ($\because$ Efficient trapezoidal section)

$b = y\sqrt{1+m^2}$

$b = \frac{2y}{\sqrt{3}}$ ....(II)

From (I) & (II)

y= 1.519 m

$\therefore\ D = \frac{(b+my)y}{b+2my} = 1.14\,m$

$\therefore F_r = \frac{V}{\sqrt{gD}}$

$F_r = 0.37$

08. Ans: (a)

Sol: Alternate depths

$y_1 = 0.4$ m

$y_2 = 1.6$ m

Specific energy at section =?

$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2g\,y_2^2}$

$0.4 + \frac{q^2}{2\times 9.81\times 0.4^2} = 1.6 + \frac{q^2}{2\times 9.81\times 1.6^2}$

$q^2\left(\frac{1}{3.1392} - \frac{1}{50.22}\right) = 1.6 - 0.4$

$q^2(0.298) = 1.2$

$q^2 = 4.02$

$q = 2\ m^3$ /s/m

$E_1 = y_1 + \frac{q^2}{2gy_1^2}$

$E_1 = 0.4 + \frac{2^2}{2\times 9.81\times 0.4^2}$ =1.68 m

09. Ans: (b)

Sol: Depth = 1.6 m

Specific energy = 2.8 m

$E_1 = \left[y_1 + \frac{v^2}{2g}\right] \Rightarrow 2.8 = 1.6 + \frac{V^2}{2\times 9.81}$

V = 4.85 m/s

$F_r = \frac{V}{\sqrt{gy}}$

$F_r = \frac{4.85}{\sqrt{9.81\times 1.6}} = 1.22 > 1$ (Supercritical)

10. Ans: (c)

Sol: $F_r = 5.2$ (uniform flow)

The ratio of critical depth to normal depth $\frac{y_c}{y_n} = ?$

Note: The given two depths y_c & y_n are not alternate depths as they will have different specific energies.

$$F_r = \frac{V}{\sqrt{gy}} \Rightarrow F_r^2 = \frac{V^2}{gy} = \frac{q^2}{gy^3}\left(\because v = \frac{q}{y}\right)$$

$$\frac{(F_{rn})^2}{(F_{rc})^2} = \frac{q^2}{gy_n^3} \times \frac{gy_c^3}{q^2} = \frac{y_c^3}{y_n^3}$$

$$\frac{y_c^3}{y_n^3} = \frac{(F_{rn})^2}{(F_{rc})^2} \Rightarrow \frac{y_c}{y_n} = \frac{(F_{rn})^{2/3}}{(F_{rc})^{2/3}}$$

$$\frac{y_c}{y_n} = (5.2)^{2/3} = 3$$

11. Ans: (c)

Sol: Rectangular channel

Alternate depths $y_1 = 0.2$, $y_2 = 4$m

$E_1 = E_2$ ($\because$ alternate depths), $F_r = \frac{V}{\sqrt{gD}}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left[1 + \frac{Fr_2^2}{2}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{Fr_2^2}{2}}{1 + \frac{Fr_1^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{4^2}{2}}{1 + \frac{0.2^2}{2}}\right]$$

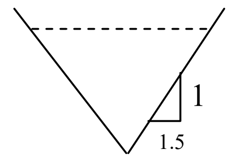
$$\frac{y_1}{y_2} = \left(\frac{2+16}{2+0.04}\right) = 8.8$$

12. Ans: (d)

Sol: Triangular channel

H:V = 1.5:1

Specific energy = 2.5 m



$$E_C = \frac{5}{4}y_c$$

$$\frac{4}{5}E_C = y_c$$

$y_c = 2$m

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} \Rightarrow 2 = \left(\frac{2 \times Q^2}{9.81 \times 1.5^2}\right)^{1/5}$$

Q = 18.79 m^3/sec

13. Ans: 0.47

Sol: $E_1 = E_2 + (\Delta z)$

$$V_1 = \frac{Q}{A_1} = \frac{12}{2.4 \times 2} = 2.5\,\text{m/sec}$$

$A_2 = (b_2 + my_2)y_2 = (1.8 + 1 \times 1.6)\ 1.6$

$= 5.44\ m^2$

$$V_2 = \frac{Q}{A_2} = \frac{12}{5.44} = 2.2\,\text{m/sec}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{(2.5)^2}{2 \times 9.81} = 2.318\,\text{m}$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 1.6 + \frac{2.2^2}{2 \times 9.81} = 1.846\ \text{m}$$

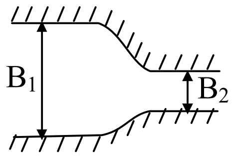
$2.318 = 1.846 + \Delta Z \Rightarrow \Delta Z = 0.47$ m

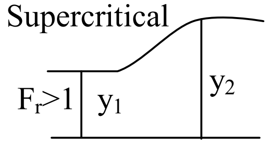
14. Ans: (c)

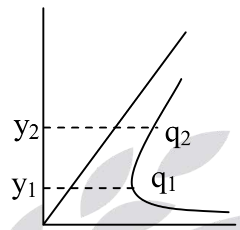
Sol: $F_r > 1$

$B_2 < B_1$

$q_2 > q_1$







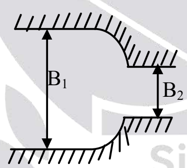
As Potential energy (y) increases then kinetic energy (v) decreases

$\therefore$ 'y' increases and 'v' decreases.

15. Ans: (a)

Sol: $Q = 3m^3/s$

$B_1 = 2m, D = 1.2\ m$



Width reduced to 1.5 m (B_2)

Assume channel bottom as horizontal

$\therefore E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{B_1 y_1} = \frac{3}{2 \times 1.2} = 1.25\ m/sec$$

$$V_2 = \frac{Q}{B_2 y_2} = \frac{3}{1.5 \times y_2} = \frac{2}{y_2}$$

$$1.2 + \frac{(1.25)^2}{2 \times 9.81} = y_2 + \frac{\left(\frac{2}{y_2}\right)^2}{2 \times 9.81}$$

$$1.27 = y_2 + \frac{4}{y_2^2 \times 19.62}$$

$$1.27 = y_2 + \frac{0.2}{y_2^2}$$

$$y_2^2(1.27) = y_2^3 + 0.2$$

$$y_2^3 - 1.27y_2^2 + 0.2 = 0$$

$$y_2 = 1.12\ m$$

$$F_{r_1} = \frac{1.25}{\sqrt{9.81 \times 1.2}}\left[\frac{V}{\sqrt{gD}} < 1\right] = 0.364 < 1$$

Approaching flow is sub critical. If approaching flow is sub critical the level at water falls in the throat portion.

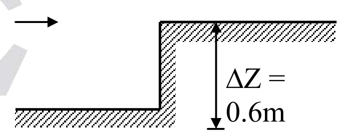
16. Ans: (d)

Sol: Rectangular Channel

$y_1 = 1.2m$; $V_1 = 2.4m/s$

$\Delta Z = 0.6m$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{(2.4)^2}{2 \times 9.81} = 1.49m$$



$Q = 2.4 \times 1.2 = 2.88\ m^3/s/m$

Assuming channel width as constant, the critical depth

$$y_c = \left[\frac{Q^2}{gB^2}\right]^{\frac{1}{3}} = 0.94m$$

Critical specific energy for rectangular channel $E_C = \frac{3}{2}y_c$

$$E_c = \frac{3}{2}(0.94) = 1.41$$

We know for critical flow in the hump portion $E_1 = E_2 + (\Delta Z) = E_C + (\Delta Z)_C$

$\Rightarrow 1.49 = 1.41 + (\Delta Z)_C$

$\therefore (\Delta Z)_C = 0.08\text{m}$

If the hump provided is more than the critical hump height the u/s flow gets affected.

(or)

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{2.4}{\sqrt{9.81\times1.2}} = 0.69<1$$

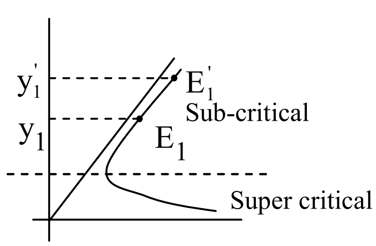
$\Rightarrow$ Hence sub-critical.

If the approaching flow is sub-critical the level of water will fall in the hump portion.

Option (b) is correct if the hump height provided is less than critical hump height.

As the hump height provided is more than critical, the u/s flow gets affected with the increase of the specific energy from E_1 to E_1^1.

In the sub-critical region as the specific energy increases, the level of water rises from y_1 to y_1^1 in the form of a surge.



$$E_1^1 = y_1^1 + \frac{v^{1^1}}{2g}$$

$$E_1^1 = y_1^1 + \frac{q^2}{2gy_1^{1^2}} \quad \ldots (1)$$

Also $E_1^1 = E_c + (\Delta Z)$ provided.

$= 1.41 + 0.6$

$= 2.01\text{m}$

$$\therefore 2.01 = y_1^1 + \frac{2.88^2}{2\times9.81\times y_1^2}$$

Solve by trial & error

for $y_1^1 > 1.2\text{m}$

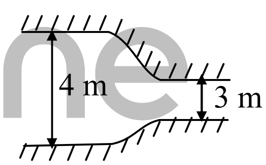
17. Ans: (c)

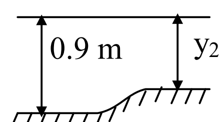
Sol: $B_1 = 4$ m

$B_2 = 3$ m

(U/S) $y_1 = 0.9$ m

$E_1 = E_2 + \Delta Z$





$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta Z$$

$V_1 = V_2$

According to continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2V_2$

$A_1 = A_2$

$B_2y_1 = B_2y_2$

$4\times0.9 = 3\times y_2$

$y_2 = 1.2$ m

$y_1 = y_2 + \Delta Z$

$0.9 = 1.2 + \Delta Z$

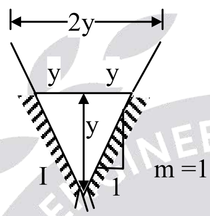
$\Delta Z = -0.3$ m

Negative indicates that the hump assumed is wrong infact it is a drop.

18. Ans: (a)

Sol: Given :

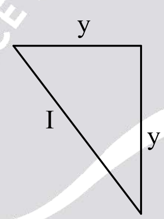
Top width = 2y



Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 2y \times y$

$A = y^2$



Wetted perimeter

$I^2 = \sqrt{y^2 + y^2} = y\sqrt{2}$

(Both sides) total wetted perimeter

(P) $= \sqrt{2}.y + \sqrt{2}.y = 2\sqrt{2}.y$

Hydraulic mean depth

$(R) = \frac{A}{P} = \frac{y^2}{2\sqrt{2}y} = \frac{y}{2\sqrt{2}}$

$y = y_n$(say)

Using Mannings formula

$$Q = A.\frac{1}{n}.(R)^{2/3}.(S)^{1/2}$$

$$0.2 = y_n^2 \frac{1}{0.015}\left[\frac{y_n}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$\frac{1}{y_n^{8/3}} = \frac{1}{0.015 \times 0.2} \times \left[\frac{1}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$y_n^{8/3} = 0.2 \times 0.015 \times \left(2\sqrt{2}\right)^{2/3}\left[\frac{1}{0.001}\right]^{1/2}$$

$(y_n)^{8/3} = 0.189$

$y_n = (0.189)^{3/8}$

$y_n = 0.54$ m

critical depth $(y_c) = \left[\frac{2Q^2}{g}\right]^{1/5}$

(for triangle)

$$y_c = \left[\frac{2 \times 0.2^2}{9.81}\right]^{1/5} = 0.382\text{ m}$$

$y_n > y_c$ (0.54>0.38)

$\therefore$ mild slope

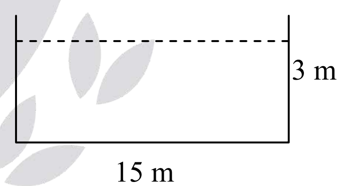
If (actual) depth at flow = 0.4m = y

$Y_n > y > y_c$ [0.54 > 0.4 > 0.38]

$\therefore$ Profile is M_2

19. Ans: 4.36×10^{-5}

Sol:



$\therefore$ Discharge, Q = 29 m^3/sec

Area of rectangular channel, A = 15 × 3 = 45 m^2

Perimeter, P = 15 + 2 × 3 = 21 m

Hydraulic radius, $R = \frac{A}{P} = \frac{45}{21} = 2.142$ m

$\therefore$ The basic differential equation governing the gradually varied flow is

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2T}{gA^3}}$$

$\frac{dy}{dx}$ = Slope of free water surface w.r.t to channel bottom

$$\text{Velocity of flow } V = \frac{Q}{A} = \frac{29}{45}$$

$$= 0.644 \text{ m/sec}$$

∴ By Chezy's equation

$$\text{Velocity, } V = C\sqrt{RS_f}$$

$$0.644 = 65\sqrt{2.142 \times S_f}$$

$$S_f = 4.589 \times 10^{-5}$$

$$S_o = \frac{1}{5000} = 2\times10^{-4}$$

$$\frac{Q^2T}{gA^3} = \frac{29^2 \times 15}{9.81\times 4^3} = 0.0141$$

$$\therefore \frac{dy}{dx} = \frac{2\times10^{-4} - 4.589\times10^{-5}}{1-0.0141}$$

$$= 1.5631 \times 10^{-4}$$

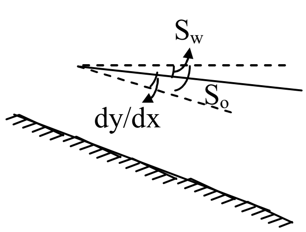
$$\therefore S_o = S_w + \frac{dy}{dx}$$

S_w water surface slope with respect to horizontal

$$S_w = S_o - \frac{dy}{dx}$$

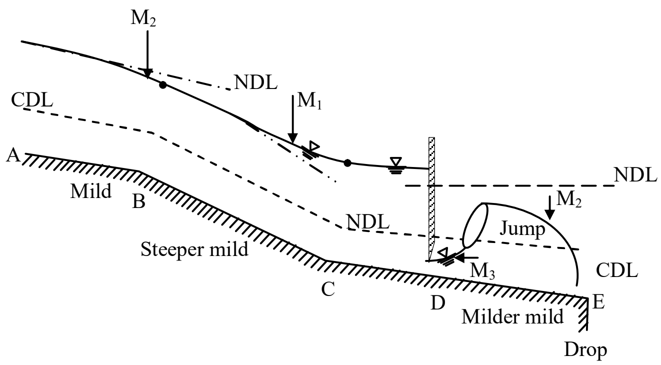
$$= 2 \times 10^{-4} - 1.563 \times 10^{-4}$$

$$S_w = 4.36 \times 10^{-5}$$



20. Ans: (a)

Sol:



22. Ans: 0.74

Sol: Free fall →2nd profile

$$\text{Critical depth, } y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

$$y_c = \left(\frac{2^2}{9.81}\right)^{\frac{1}{3}} = 0.74 \text{ m}$$

$$V = \frac{q}{y_n}$$

$$\frac{2}{y_n} = \frac{1}{n} y_n^{2/3} S^{1/2}$$

$$\frac{2}{y_n} = \frac{1}{0.012} \times y_n^{2/3} (0.0004)^{1/2}$$

y_n = 1.11 m

$y_n > y_c$

Hence the water surface will have a depth equal to y_c

y_c = 0.74 m

23. Ans: (d)

Sol: $q = 2\ m^2/sec$

$y_A = 1.5\ m; y_B = 1.6\ m$

$\Delta E = 0.09$

$$S_o = \frac{1}{2000}$$

$$\overline{S}_f = 0.003$$

$$\Delta x = \frac{\Delta E}{S_o - \overline{S}_f} = \frac{0.09}{\frac{1}{2000} - 0.003} = -36\ m$$

24. Ans: (d)

Sol: Given $q_1 = Q/B = 10\ m^3/s$

$v_1 = 20\ m/s$

$$\therefore y_1 = \frac{q_1}{v_1} = \frac{10}{20} = 0.5\,m$$

We know that relation between y_1 and y_2 for hydraulic jump is

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.81 \times 0.5}} = 9.03$$

$$\therefore \frac{y_2}{0.5} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 \times (9.03)^2}\right]$$

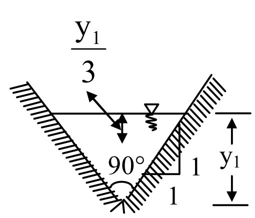
$y_2 = 6.14\ m$

25. Ans: (c)

Sol: $Q = 1\ m^3/s$

$y_1 = 0.5\ m$

$y_2 = ?$



As it is not a rectangular channel, let us work out from fundamentals by equating specific force at the two sections.

$$\left[\frac{Q^2}{gA} + A\bar{z}\right]_1 = \left[\frac{Q^2}{gA} + A\bar{z}\right]_2$$

$$\frac{1^2}{9.81 \times y_1^2} + y_1^2 \times \frac{y_1}{3} = \frac{1^2}{9.81\,y_2^2} + y_2^2 \times \frac{y_2}{3}$$

$$0.449 = \frac{1}{9.81y_2^2} + \frac{y_2^3}{3}$$

$y_2 = 1.02\ m$

26. Ans: (b)

Sol: **Given:**

Head = 5 m = (ΔE)

Froud number = 8.5

Approximate sequent depths =?

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_{r1}^2}\right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8(8.5)^2}\right]$$

$= 11.5\ m$

$y_2 = 11.5\ y_1$

$$\left.\begin{array}{l}\text{(a) } y_2 = 11.5(0.3) = 3.45 \\ \text{(b) } y_2 = 11.5(0.2) = 2.3\,m\end{array}\right\}\text{from options}$$

$y_1 = 0.2$, $y_2 = 2.3\ m$

(or)

$\Delta E = 5\ m$

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\frac{(11.5y_1 - y_1)^3}{4(11.5y_1)y_1} = 5$$

$(10.5y_1)^3 = 230y_1^2$

$1157.625\ y_1 = 230$

$y_1 = 0.2$ m

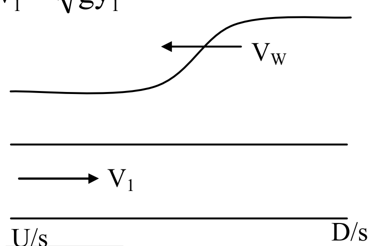
$y_2 = 11.5(0.2)$

$y_2 = 2.3$ m

27. Ans: 1.43

Sol: $y_1 = 1.2$ m

$V_w + V_1 = \sqrt{gy_1}$



$V_1 = \sqrt{9.81 \times 1.2} - 2$

$V_1 = 1.43$ m/s

In this problem if the wave moves downstream the velocity of wave is

$V_w - V_1 = \sqrt{gy_1}$

$V_w = \sqrt{gy_1} + V_1$

$= \sqrt{9.81 \times 1.2} + 2$

$= 5.43$ m/s

Chapter 13 Dimensional Analysis

01. Ans: (c)

Sol: Total number of variables,

n = 8 and m = 3 (M, L & T)

Therefore, number of π's are = 8 – 3 = 5

02. Ans: (b)

Sol: 1. $\frac{T}{\rho D^2 V^2} = \frac{MLT^2}{ML^{-3} \times L^2 \times L^2 \times T^{-2}} = 1$.

→ It is a non-dimensional parameter.

2. $\frac{VD}{\mu} = \frac{LT^{-1} \times L}{ML^{-1}T^{-1}} \neq 1$.

→ It is a dimensional parameter.

3. $\frac{D\omega}{V} = 1$.

→ It is a non-dimensional parameter.

4. $\frac{\rho VD}{\mu} = Re$.

→ It is a non-dimensional parameter.

03. Ans: (b)

Sol: $T = f(l, g)$

Total number of variable,

n = 3, m = 2 (L & T only)

Hence, no. of π terms = 3 – 2 = 1

04. Ans: (c)

Sol:

- Mach Number → Launching of rockets
- Thomas Number → Cavitation flow in soil
- Reynolds Number → Motion of a submarine
- Weber Number → Capillary flow in soil

05. Ans: (b)

Sol: According to Froude's law

$$T_r = \sqrt{L_r}$$

$$\frac{t_m}{t_p} = \sqrt{L_r}$$

$$t_p = \frac{t_m}{\sqrt{L_r}} = \frac{10}{\sqrt{1/25}}$$

$t_p = 50$ min

06. Ans: (a)

Sol: L = 100 m

$V_P = 10\,m/s$,

$$L_r = \frac{1}{25}$$

As viscous parameters are not discussed, follow Froude's law.

According to Froude,

$$V_r = \sqrt{L_r}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{1}{25}}$$

$$V_m = \frac{1}{5} \times 10 = 2\text{ m/s}$$

07. Ans: (d)

Sol: Froude number = Reynolds number.

$\nu_r = 0.0894$

If both gravity & viscous forces are important then

$$\nu_r = (L_r)^{3/2}$$

$$\sqrt[3]{(\nu_r)^2} = L_r$$

$L_r = 1:5$

08. Ans: (c)

Sol: For distorted model according to Froude's law

$$Q_r = L_H L_V^{3/2}$$

$L_H = 1:1000$,

$L_V = 1:100$

$Q_m = 0.1\ m^3/s$

$$Q_r = \frac{1}{1000} \times \left(\frac{1}{100}\right)^{3/2} = \frac{0.1}{Q_p}$$

$Q_P = 10^5\ m^3/s$

09. Ans: (c)

Sol: For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

$$\frac{\rho_w \times V_w \times d_w}{\mu_w} = \frac{\rho_a \times V_a \times d_a}{\mu_a}$$

or $$V_w = \frac{\rho_a}{\rho_w} \times \frac{d_a}{d_w} \times \frac{\mu_w}{\mu_a} \times V_a$$

(where suffixes w and a stand for water and air respectively)

Substituting the values given, we get

$$V_w = \frac{1.2}{10^3} \times \frac{4}{0.1} \times \frac{10^{-3}}{1.8 \times 10^{-5}} \times 1 = \frac{8}{3}\,m/s$$

To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.

i.e, $$\left(\frac{F_D}{\rho A V^2}\right)_P = \left(\frac{F_D}{\rho A V^2}\right)_m$$

Hence, $$(F_D)_p = (F_D)_m \times \frac{\rho_a}{\rho_w} \times \frac{A_a}{A_w} \times \left(\frac{V_a}{V_w}\right)^2$$

$$= 4 \times \frac{1.2}{10^3}\left(\frac{4}{0.1}\right)^2 \times \left(\frac{1}{8/3}\right)^2$$

$= 1.08$ N

10. Ans: 47.9

Sol: Given data,

	Sea water (Prototype testing)	**Fresh water (model testing)**
V	0.5	?
ρ	1025 kg/m^3	10^3 kg/m^3
μ	1.07×10^{-3} Pa.s	1×10^{-3} Pa.s

For dynamic similarity, Re should be same in both testing.

i.e., $$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p}$$

$$V_m = V_P \times \frac{\rho_p}{\rho_m} \times \frac{d_p}{d_m} \times \frac{\mu_m}{\mu_p}$$

$$= 0.5 \times \frac{1025}{10^3} \times 100 \times \frac{10^{-3}}{1.07 \times 10^{-3}}$$

$$= 47.9 \text{ m/s}$$