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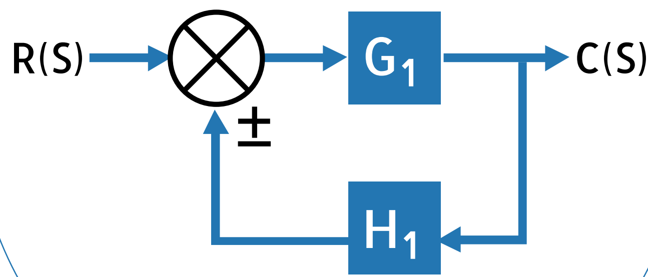


Electronics & Communication Engineering

CONTROL SYSTEMS

Text Book:
Theory with worked out Examples and Practice Questions

Chapter 1 Basics of Control Systems

(Solutions for Text Book Practice Questions)

01. Ans: (c)

Sol: $2\dfrac{d^2y(t)}{dt^2}+3\dfrac{dy(t)}{dt}+4y(t) = r(t) + 2r(t–1)$

Apply LT on both sides

$2s^2\,Y(s) + 3sY(s) + 4Y(s) = R(s) + 2e^{-s}R(s)$

$Y(s)(2s^2 + 3s+4) = R(s)(1+2e^{-s})$

$$\frac{Y(s)}{R(s)} = \frac{1+2e^{-s}}{2s^2+3s+4}$$

02. Ans: (b)

Sol: $I.R = 2.e^{-2t}u(t)$

Output response $c(t) = (1–e^{-2t})\,u(t)$

Input response $r(t) \quad = ?$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s}-\frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$r(t) = u(t)$

03. Ans: (b)

Sol: Unit impulse response of unit-feedback control system is given

$c(t) = t.e^{-t}$

$T.F = L(I.R)$

$$= \frac{1}{(s+1)^2}$$

$$\text{Open Loop T.F} = \frac{\text{Closed Loop T.F}}{1-\text{Closed Loop T.F}}$$

$$= \frac{\frac{1}{(s+1)^2}}{1-\frac{1}{(s+1)^2}} = \frac{1}{s^2+2s}$$

04. Ans: (a)

Sol: G changes by 10%

$$\Rightarrow \frac{\Delta G}{G}\times 100 = 10\%$$

$C_1 = 10\%$

[∵ open loop] whose sensitivity is 100%]

%G change = 10%

$$\frac{\%\text{ of change in M}}{\%\text{ of change in G}} = \frac{1}{1+GH}$$

$$\%\text{ of change in M} = \frac{10\%}{1+(10)1} = 1\%$$

% change in C_2 by 1%

05.

Sol:

(i) $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1+GH}$$

$$S_K^M = \frac{\partial M}{\partial K}\times\frac{K}{M} = 1$$

[∵ K is not in the loop ⇒ sensitivity is 100%]

(ii) $S_H^M = \dfrac{\partial M}{\partial H}\times\dfrac{H}{M} = \dfrac{\partial}{\partial H}\left(\dfrac{GK}{1+GH}\right)\dfrac{H}{M}$

$$= \left(\frac{GK(-G)}{(1+GH)^2}\right)\left[\frac{H}{\frac{GK}{1+GH}}\right]$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

06.

Sol: Given data

$G = 2\times10^3$, $\partial G = 100$

$$\%\text{ change in } G = \frac{\partial G}{G}\times100 = 5\%$$

$$\%\text{ change in } M = 0.5\%$$

$$\frac{\%\text{ of change in } M}{\%\text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2\times10^3 H}$$

$$1+2\times10^3 H = 10$$

$$H = 4.5\times10^{-3}$$

07. Ans: (b)

Sol: $K = \frac{\text{output}}{\text{input}} = \frac{c(t)}{r(t)} = \frac{\text{mm}}{{}^0\text{c}}$

08. Ans: (d)

Sol: Introducing negative feedback in an amplifier results, increases bandwidth.

09. Ans: (a), (b) & (c)

Sol: Negative feedback decreases the gain, increase the bandwidth, reduce sensitivity to parameter variation and more accurate.

10. Ans: (b), (c) & (d)

Sol: Using the transfer function response due to initial conditions [zero input response] can not be obtained.

L^{-1}[TF] = IR i.e., inverse laplace transform of the transfer function is the impulse response [IR] of the system.

Chapter 2 Signal Flow Graphs & Block Diagrams

01. Ans: (d)

Sol: No. of loops = 3

Loop1: $-G_1G_3G_4H_1H_2H_3$

Loop2: $-G_3G_4H_1H_2$

Loop3: $-G_4H_1$

No. of Forward paths = 3

Forward Path1: $G_1G_3G_4$

Forward Path 2: $G_2G_3G_4$

Forward Path 3: G_2G_4

$$= \frac{G_1G_3G_4 + G_2G_3G_4 + G_2G_4}{1 + G_1G_3G_4H_1H_2H_3 + G_3G_4H_1H_2 + G_4H_1}$$

02. Ans: (a)

Sol: Number of forward paths = 2

Number of loops = 3

$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s}\times\frac{1}{s}\times\frac{1}{s}[1-0]+\frac{1}{s}}{1-\left[\frac{1}{s}\times(-1)\left(\frac{1}{s}\right)(-1)+\frac{1}{s}\times\frac{1}{s}(-1)+\left(\frac{1}{s}\times\frac{1}{s}(-1)\right)\right]}$$

$$= \frac{\frac{1}{s^3}+\frac{1}{s}}{1-\left[\frac{1}{s^2}-\frac{1}{s^2}-\frac{1}{s^2}\right]} = \frac{\frac{1+s^2}{s^3}}{1+\frac{1}{s^2}} = \frac{\frac{1+s^2}{s^3}}{\frac{s^2+1}{s^2}}$$

$$= \frac{1+s^2}{s}\times\frac{1}{s^2+1} = \frac{1}{s}$$

03.

Sol: Number of forward paths = 2

Number of loops = 5

Two non touching loops = 4

$$TF = \frac{24[1-(-0.5)]+10[1-(-3)]}{1-[-24-3-4+(5\times2\times(-1)+(-0.5))]+[30+1.5+2]+\left(\left(\frac{-1}{2}\right)\times(-24)\right)}$$

$$= \frac{76}{88} = \frac{19}{22}$$

04.

Sol: Number of forward paths = 2

Number of loops = 5

$$T.F = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_1G_2H_1 + G_1G_2G_3 + G_4H_2 + G_1G_4}$$

05. Ans: (c)

Sol: From the network

$$V_o(s) = \frac{1}{sC}I(s) \quad ....(1)$$

$$-V_i(s) + RI(s) + V_o(s) = 0$$

$$I(s) = \frac{1}{R}V_i(s) + \left(\frac{-1}{R}\right)V_o(s) ....(2)$$

From SFG

$$V_o(s) = x.I(s) \quad ....(3)$$

$$I(s) = \frac{1}{R}V_i(s) + y\,V_o(s) \quad .... (4)$$

From equ(1) and (3)

$$x = \frac{1}{sC}$$

From equ(2) and (4)

$$y = -\frac{1}{R}$$

06. Ans: (a)

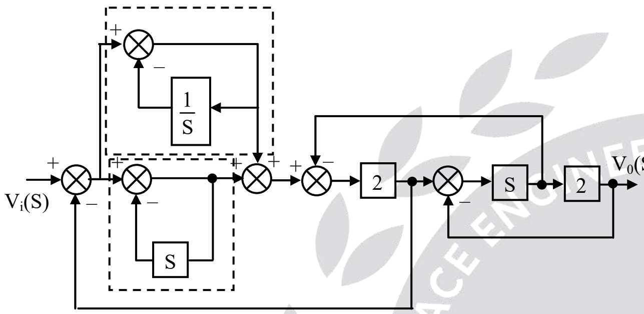
Sol: Use gain formula

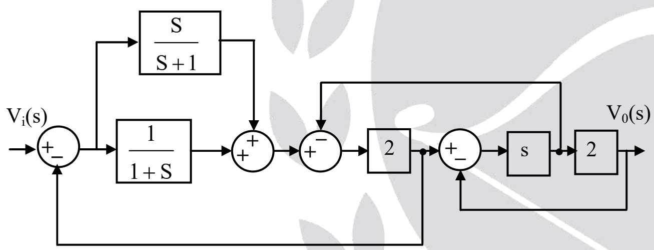
$$\text{transfer function} = \frac{G(s)}{1-\left(G(s)\dfrac{1}{G(s)}+G(s)\right)}$$

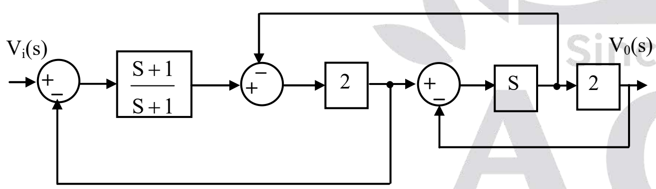
$$= \frac{G(s)}{1-1-G(s)} = -1$$

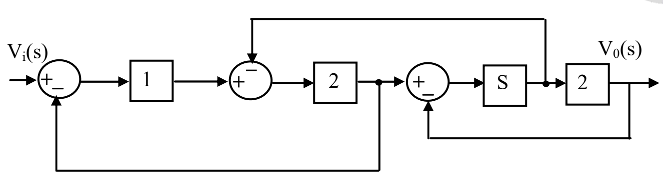
07.

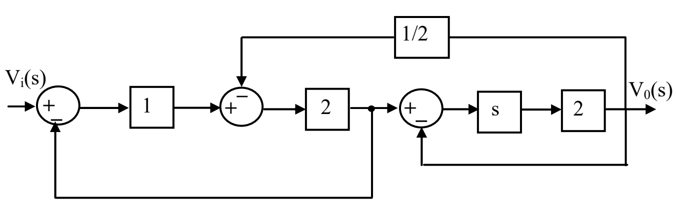
Sol:

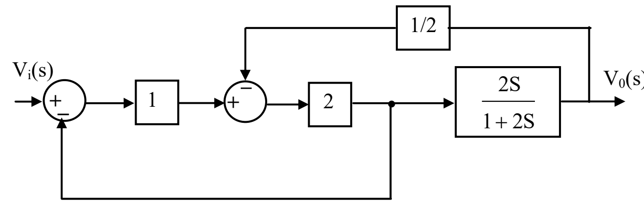


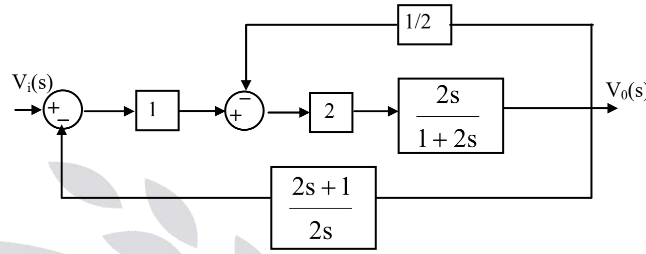


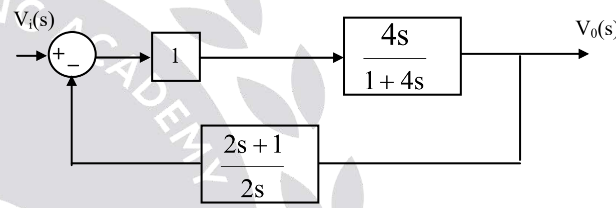












$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \frac{\dfrac{4s}{1+4s}}{1+\dfrac{2(2s+1)}{1+4s}} = \frac{4s}{8s+3}$$

08.

Sol: Apply Mason's Gain formula

$$M = \frac{Y_{out}}{Y_{in}} = \frac{\sum_{k=1}^{N} M_k \Delta_k}{\Delta}$$

No. of forward paths = 2

First forward path gain = $G_1G_2G_3G_4$

Second forward path gain = $G_5G_6G_7G_8$

No. of loops = 4

First loop gain = $-G_2H_2$

Second loop gain = $-G_6H_6$

Third loop gain = $-G_3H_3$

Fourth loop gain = $-G_7H_7$

Non touching loops = 4

Loop gains $\rightarrow G_2H_2G_6H_6$

$\rightarrow G_2H_2G_7H_7$

$\rightarrow G_6H_6G_7H_7$

$\rightarrow G_2H_2G_3H_3$

Transfer function =

$$\frac{G_1G_2G_3G_4(1+G_6H_6+G_7H_7)+G_5G_6G_7G_8(1+G_2H_2+G_3H_3)}{1+G_2H_2+G_3H_3+G_6H_6+G_7H_7+G_2H_2G_6H_6+G_2H_2G_7H_7+G_3H_3G_6H_6+G_3H_3G_7H_7}$$

09. Ans: (a), (b) & (d)

Sol: It is a LTIS, hence $\frac{C}{R}$ can be found

Number of forward paths = 1

Number of loops = 2

Non touching pair = 1

$$\therefore \frac{C}{R}=\frac{(1)}{1-[-1-1]+(-1)(-1)}$$

$$\frac{C}{R}=\frac{1}{4}=0.25$$

10. Ans: (a), (b) & (d)

Sol: $\Rightarrow \frac{Y(s)}{X(s)}=\frac{\frac{1}{(s+1)(s+2)}}{1+\frac{1}{(s+1)(s+2)}}=\frac{1}{s^2+3s+3}$

$$\Rightarrow \frac{Y(s)}{N(s)}=\frac{1-G_{ff}(s)\left(\frac{1}{(s+1)(s+2)}\right)}{1+\frac{1}{(s+1)(s+2)}}=0$$

[Output due to noise is zero]

$G_{ff}(s) = (s + 1)(s + 2)$

$\Rightarrow$ C.E: $s^2 + 3s + 3 = 0$

$\Rightarrow$ Poles locations are $(-3/2 \pm j0.866)$

$\Rightarrow$ System is stable

Chapter 3 Time Response Analysis

01. Ans: (a)

Sol: $\dfrac{C(s)}{R(s)} = \dfrac{1}{1+sT}$, $R(s) = \dfrac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8\left(1-e^{-t/T}\right)$$

$$3.6 = 8\left(1-e^{\frac{-0.32}{T}}\right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

$T = 0.535$ sec

02. Ans: (c)

Sol: $\cos\phi = \xi$

$\cos 60 = 0.5$

$\cos 45 = 0.707$

Poles left side $0.5 \le \xi \le 0.707$

Poles right side $-0.707 \le \xi \le -0.5$

$\therefore 0.5 \le |\xi| \le 0.707$

3 rad/s $\le \omega_n \le 5$ rad/s

03. Ans: (c)

Sol: For R-L-C circuit:

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs}I(s)$$

$$= \frac{1}{Cs}\frac{V_i(s)}{R+Ls+\frac{1}{Cs}}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+LCs^2+1}$$

$$= \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$s^2+\frac{R}{L}s+\frac{1}{LC} = 0$$

$$s^2+2\xi\omega_n s+\omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2}\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}} = 0.5$$

$$M.P = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$= 16.3\% \approx 16\%$

04. Ans: (b)

Sol: $$TF = \frac{8/s(s+2)}{1-\left(\frac{-8\,as}{s(s+2)}-\frac{8}{s(s+2)}\right)}$$

$$= \frac{8}{s(s+2)+8as+8}$$

$$= \frac{8}{s^2+2s+8as+8}$$

$$= \frac{8}{s^2+(2+8a)s+8}$$

$$\omega_n^2 = 8 \Rightarrow \omega_n = 2\sqrt{2}$$

$$2\xi\omega_n = 2+8a$$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

05. Ans: 4 sec

Sol: $T.F = \frac{100}{(s+1)(s+100)} = \frac{100}{s^2+101s+100}$

$\omega_n^2 = 100$

$\omega_n = 10$

$2\xi\omega_n = 101$

$\xi = \frac{101}{20}$

$\xi > 1 \rightarrow$system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$T.F = \frac{100}{100(s+1)} = \frac{1}{s+1}$, Here $\tau = 1$ sec

$\therefore$ Setting time for 2% criterion $= 4\tau$

$= 4$ sec

06.

Sol: $M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$

$= \frac{1.254 - 1.04}{1.04} = 0.2$

$\xi = \sqrt{\frac{(\ln M_p)^2}{(\ln M_p)^2 + \pi^2}}$

$M_p = 0.2$; $\xi = 0.46$

07. Ans: (d)

Sol: Given data: $\omega_n = 2$, $\zeta = 0.5$

Steady state gain = 1

$OLTF = \frac{K_1}{s^2 + as + 2}$ and $H(s) = K_2$

$CLTF = \frac{G(s)}{1+G(s)}$

$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + as + 2 + K_1K_2}$

DC or steady state gain from the TF

$\frac{K_1}{2 + K_1K_2} = 1$

$K_1(1 - K_2) = 2$ .... (1)

CE is $s^2 + as + 2 + K_1K_2 = 0$

$\omega_n = \sqrt{2 + K_1K_2} = 2$

$4 = (2+K_1K_2)$

$K_1K_2 = 2$ .... (2)

Solving equations (1) & (2) we get

$K_1 = 4$, $K_2 = 0.5$

$2\zeta\,\omega_n = a$

$2 \times \frac{1}{2} \times 2 = a$

$a = 2$

08. Ans: (c)

Sol: If R ↑ damping ↑

$\Rightarrow \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

(i) If R↑, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on ξ)

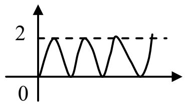
If ξ ↑, C (∞) = remain same

(ii) If $\xi\uparrow$, $\omega_d \downarrow \left(\omega_d = \omega_n\sqrt{1-\xi^2}\right)$

(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow$ 3rd

Statement is false

(iv) If $\xi = 0$



True

$\Rightarrow$ 2 and 4 are correct

09. Ans: A – T, B – S, C- P, D – R, E – Q

Sol:

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

(C) If poles are non-repeated on the jω axis, the step response will have fixed amplitude oscillations.

(D) If the poles are complex & right side of s-plane, response goes to '∞' with damped oscillations.

(E) If the poles are real & right side of s-plane, the step response goes to '∞' without any oscillations.

10.

Sol: (i) Unstable system

$\therefore$ error = ∞

(ii) $G(s) = \dfrac{10(s+1)}{s^2}$

Step → $R(s) = \dfrac{1}{s}$

$k_p = \infty$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+\infty} = 0$$

Parabolic ⇒ $k_a = 10$

$$e_{ss} = \frac{1}{10} = 0.1$$

11.

Sol: $G(s) = 10/s^2$ (marginally stable system)

$\therefore$ Error can't be determined

12.

Sol: $e_{ss} = \dfrac{1}{11}$, $R(s) = \dfrac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$$k_p = \underset{s\to 0}{\text{Lt}}\, G(s)$$

$$10 = \underset{s\to 0}{\text{Lt}}\, G(s)$$

$k = 10$

$R(s) = \dfrac{1}{s^2}$ (ramp)

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

$\Rightarrow e_{ss} = 0.1$

13. Ans: (a)

Sol: $T(s) = \dfrac{(s-2)}{(s-1)\ (s+2)^2}$ (unstable system)

14. Ans: (b)

Sol: Given data: r(t) = 400tu(t) rad/sec

Steady state error $=10^\circ$

i.e., $e_{ss} = \dfrac{\pi}{180^\circ}(10^\circ)$ radians

$G(s) = \dfrac{20K}{s(1+0.1s)}$ and $H(s) = 1$

$r(t) = 400tu(t) \Rightarrow 400/s^2$

$$\text{Error } (e_{ss}) = \frac{A}{K_V} = \frac{400}{K_V}$$

$$K_V = \lim_{s\to 0} s\,G(s)$$

$$K_V = \lim_{s\to 0} s\frac{20K}{s(1+0.1s)}$$

$$K_V = 20K$$

$$e_{ss} = \frac{400}{20K}$$

$$e_{ss} = \frac{20}{K} = \frac{\pi}{18}$$

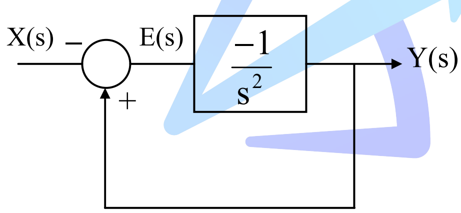
$$K = 114.5$$

15. Ans: (d)

Sol: $$\frac{d^2y}{dt^2} = -e(t)$$

$$s^2\,Y(s) = -\,E(s)$$

$$x(t) = t\,u(t) \Rightarrow X(s) = \frac{1}{s^2}$$



$$Y(s) = \frac{-1}{s^2}E(s)$$

$$\frac{Y(s)}{E(s)} = \frac{-1}{s^2}$$

$$\frac{E(s)}{X(s)} = \frac{-1}{1+\frac{1}{s^2}}$$

$$E(s) = \frac{-s^2}{1+s^2}X(s)$$

$$= \frac{-s^2}{1+s^2}\times\frac{1}{s^2} = \frac{-1}{1+s^2}$$

$$= L^{-1}\left[\frac{-1}{1+s^2}\right] = -\sin\,t$$

16. Ans: (a)

Sol: $e_{ss} = 0.1$ for step input

For pulse input = 10

time = 1 sec

error is function of input

$t \to \infty$ input = 0

$\therefore$ Error = zero

17. Ans: (c)

Sol: $$\frac{C(s)}{R(s)} = \frac{\frac{100}{(s+1)(s+5)}}{1+\frac{100\times 0.2}{(s+1)(s+5)}}$$

$$= \frac{100}{(s+1)(s+5)+20}$$

$$= \frac{100}{s^2+6s+5+20}$$

$$= \frac{100}{s^2+6s+25}$$

$\omega_n^2 = 25,\ \omega_n = 5$

$2\xi\omega_n = 6$

$$\xi = \frac{6}{10} = \frac{3}{5}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= 5\times\frac{4}{5} = 4\text{ rad/sec}$$

18. Ans: (c)

Sol: $$f(t) = \frac{Md^2x}{dt^2} + B\frac{dx}{dt} + Kx(t)$$

Applying Laplace transform on both sides, with zero initial conditions

$F(s) = Ms^2X(s)+BsX(s) + KX(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Characteristic equation is $Ms^2 + Bs + K = 0$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0$$

Compare with $s^2 + 2\zeta\,\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = \frac{B}{M}$$

$$\xi = \frac{B}{2\sqrt{MK}} \qquad \omega_n = \sqrt{\frac{K}{M}}$$

$$\text{Time constant } T = \frac{1}{\zeta\omega_n}$$

$$= \frac{1}{B}\times 2M$$

$$T = \frac{2M}{B}$$

Hence, statements (2 & 3) are correct

19. Ans: (c)

Sol: type 1 system has a infinite positional error constant.

20. Ans: (a)

Sol: Given $G(s) = \frac{1}{s(1+s)(s+2)}, H(s) = 1$.

It is type-I system

Positional error constant $k_p = \underset{s\to 0}{Lt}\ G(s)H(s)$

$$k_p = \underset{s\to 0}{Lt}\ \frac{1}{s(1+s)(s+2)}$$

$$= \infty$$

Steady state error due to step input

$$= \frac{1}{1+k_p} = 0$$

21.

Sol Open loop T/F $G(s) = \frac{A}{s(s+P)}$

$$\text{C.L T/F} = \frac{A}{s^2 + sP + A}$$

$$\omega_n = \sqrt{A}$$

Setting time = $4/\xi\omega_n = 4$

$$2\xi\omega_n = P \qquad \therefore \frac{4}{P/2} = 4$$

$$\xi\omega_n = P/2 \qquad \Rightarrow P = \frac{8}{4} = 2$$

$$e^{\frac{-\pi\xi}{\sqrt{1+\xi^2}}} = 0.1 \Rightarrow \frac{\pi\xi}{\sqrt{1-\xi^2}} = \ell n 10$$

$$= 2.3$$

$$\Rightarrow \frac{\xi^2}{1-\xi^2} = 0.5373$$

$$\Rightarrow 1.5373\,\xi^2 = 0.5373$$

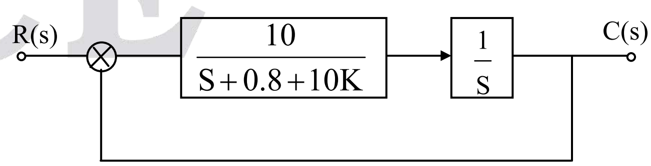
$$\xi = 0.59$$

$$\xi\omega_n = 1$$

$$\Rightarrow \omega_n = 1.694 \Rightarrow A = \omega_n^2 = 2.861$$

22.

Sol:



$$\frac{C(s)}{R(s)} = \frac{10}{s(s+0.8+10K)+10}$$

$$= \frac{10}{s^2 + s(0.8+10K)10}$$

$$\omega_n = \sqrt{10} \qquad 2\xi\omega_n = 0.8 + 10\,K$$

$$\Rightarrow 2\times\frac{1}{2}\times\sqrt{10} = 0.8+10K$$

$$\Rightarrow K = 0.236$$

$$t_r = \frac{\pi-\phi}{\omega_d} = \frac{\pi-\cos^{-1}(\xi)}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi-\pi/3}{2.88} = 0.764\,\text{sec}$$

$$t_p = \frac{\pi}{\omega_d} = 1.147\,\text{sec}$$

$$\%Mp = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.163\times 100 = 16.3\%$$

$$t_s\ (\text{for } 2\%) = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times\sqrt{10}} = 2.52\,\text{sec}$$

23. Ans: (a), (c) & (d)

Sol: CLTF $\Rightarrow \frac{C(s)}{R(s)} = \frac{3k}{2s+1+3k}$

$\Rightarrow$ CL pole $s = -\left(\frac{1+3k}{2}\right)$

$\Rightarrow$ time constant $\tau = \left(\frac{2}{1+3k}\right)$

If $k = 3 \Rightarrow \tau = 0.2$ sec

If $k > 3 \Rightarrow \tau < 0.2$ sec

If $k = 3 \Rightarrow \tau = 0.2$ sec $\Rightarrow$ BW $= \frac{1}{\tau}$ rad/sec

$$BW = \frac{1}{0.2} = 5\ \text{rad/sec}$$

24. Ans: (a), (c) & (d)

Sol: $\Rightarrow$As poles moves toward left side, the system time constant is decreases and system is more relative stable.

$\Rightarrow$ Damping ratio increases & percentage of peak overshoot decreases.

$\Rightarrow$ Damped oscillations (ω_d) is constant. Hence peak time is constant.

25. Ans: (a), (b) & (d)

Sol: Roots are $(-2 \pm j2\sqrt{3})$ complex

$0 < \zeta < 1$ – under damped system

Natural frequency $= \sqrt{16} = 4$ rad/sec

Damping ratio $\zeta = \frac{4}{2(4)} = 0.5$

Under damped system has damped oscillations .

26. Ans: (b) & (c)

Sol: OLTF $= \frac{20}{s+2}$, $H(s) = 1$

$$\text{CLTF} = \frac{\frac{20}{s+2}}{1+\frac{20}{s+2}} = \frac{20}{s+22}$$

$$\text{DC gain} = \frac{20}{22} = \frac{10}{11}$$

Steady state error to a unit step input $= \left(1-\frac{20}{22}\right)$ which is non zero

27. Ans: (b) & (d)

Sol: In OLTF two poles are at the origin

$\therefore$ It is type '2'

$$CE = 1 + \frac{10(s+1)^4}{s^2(s+2)} = 0\ \text{, 4 roots it has}$$

$\therefore$ 4^{th} order system

Type 2 system error to step and ramp input $s = 0$

$$k_a = \underset{s\to 0}{Lt}\, s^2 G(s) = \frac{10}{2} = 5$$

Error $= \frac{1}{5} = 0.2$ to a parabolic input

Chapter 4 Stability

01.

Sol: $CE = s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7	
s^4	4(1)	8(2)	4(1)	
s^3	6(1)	6(1)	0	
s^2	1	1	0	$\rightarrow$ Row of AE
s^1	0(2)	0	0	$\rightarrow$ Row of zero
s^0	1			

No. of AE roots = 2
No. of sign changes
Below AE = 0
No. of RHP = 0
No. of LHP = 0
No. of $j\omega p$ = 2

No. of CE roots = 5
No. of sign changes
in 1st column = 0
$\therefore$ No .of RHP = 0
No. of $j\omega p$ = 2
$\Rightarrow$ No .of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$s = \pm 1\, j = \pm j\omega_n$

$\omega_n = 1$ rad/ sec

Oscillating frequency $\omega_n = 1$ rad/sec

02.

Sol: (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

$+\,s^5$	1	1	1
$+\,s^4$	1	1	1
$+\,s^3$	0(2)	0(1)	0
$+\,s^2$	$\frac{1}{2}$	1	
$(1) - s^1$	-3	0	
$(2) + s^0$	1		

$AE\ (1) = s^4 + s^2 + 1 = 0$

$$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$$

$$\Rightarrow 2s^3 + s = 0$$

AE

No. of sign changes below AE = 2
No. of AE roots = 4
No .of RHP = 2
No .of LHP = 2
No. of $j\omega p$ = 0

CE

No. of sign changes in 1st column = 2
No. of CE roots = 5
No. of RHP = 2
No. of LHP = 3
No. of $j\omega p$ = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

s^6	1	2	-1	-2
s^5	2(1)	0	$-2(-1)$	0
s^4	2(1)	+0	$-2(-1)$	0
s^3	0(4)	0	0	0
s^2	$0(\varepsilon)$	-1	0	0
s^1	$4/\varepsilon$			
$-s^0$	-1			

$AE = s^4 - 1 = 0$

$$\frac{dAE}{ds} = 4s^3 + 0 = 0$$

CE	AE
No. of CE roots = 6	No. of AE roots = 4
No. of sign changes in the 1st column= 1	No. of sign changes below AE = 1
No .of RHP = 1	No. of RHP = 1
No .of LHP = 3	No. of $j\omega p$ = 2
No. of $j\omega p$ = 2	No. of LHP = 1

03.

Sol: $CE = s^3 + 20\,s^2 + 16s + 16\,K = 0$

s^3	1	16
s^2	20	16K
s^1	$\frac{20(16)-16K}{20}$	0
s^0	16K	

(i) For stability $\frac{20(16)-16K}{20} > 0$

$\Rightarrow 20\,(16) - 16\,K > 0$

$\Rightarrow K < 20$ and $16\,K > 0 \Rightarrow K > 0$

Range of K for stability $0 < K < 20$

(ii) For the system to oscillate with ω_n it must be marginally stable

i.e., s^1 row should be 0

s^2 row should be AE

$$\boxed{\therefore \text{A.E roots} = \pm j\omega_n}$$

$\therefore s^1$ row $\Rightarrow 20\,(16) - 16\,K = 0$

$\Rightarrow K = 20$

AE is $20s^2 + 16\,K = 0$

$20s^2 + 16\,(20) = 0$

$\Rightarrow s = \pm\, j4$

$\omega_n = 4$ rad/sec

04.

Sol: $CE = 1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$s^3 + as^2 + (K+2)\,s + K + 1 = 0$

$s^3 + as^2 + (K+2)\,s + (K+1) = 0$

s^3	1	K + 2
s^2	a	K+ 1
s^1	$\frac{a(K+2)-(K+1)}{a}$	0
s^0	K + 1	

Given,

$\omega_n = 2$

$\Rightarrow s^1$ row $= 0$

s^2 row is A.E

$a\,(K+2) - (K+1) = 0$

$$a = \frac{K+1}{K+2}$$

$AE = as^2 + K + 1 = 0$

$$= \frac{K+1}{K+2}s^2 + K + 1 = 0$$

$$(k+1)\left(\frac{s^2}{k+2} + 1\right) = 0$$

$s^2 + k + 2 = 0$

$s = \pm\, j\sqrt{(k+2)}$

$\omega_n = \sqrt{k+2} = 2$

$k = 2$

$a = \dfrac{k+1}{k+2} = \dfrac{3}{4} = 0.75$

05.

Sol: $s^3 + ks^2 + 9s + 18$

s^3	1	9
s^2	K	18
s^1	$\dfrac{9K-18}{K}$	0
s^0	18	

Given that system is marginally stable,

Hence

s^1 row = 0

$\dfrac{9K-18}{K} = 0$

$9K = 18 \Rightarrow K = 2$

A.E is $9s^2 + 18 = 0$

$Ks^2 + 18 = 0$,

$2s^2 + 18 = 0$

$2s^2 = -18$

$s = \pm j3$

$\therefore\ \omega_n = 3$ rad/sec.

06. Ans: (d)

Sol: Given transfer function $G(s) = \dfrac{k}{(s^2+1)^2}$

Characteristic equation $1 - G(s).H(s) = 0$

$1 - \dfrac{k}{(s^2+1)^2} = 0$

$s^4 + 2s^2 + 1 - k = 0$ ….. (1)

RH criteria

s^4	1	2	1–K
s^3	4	4	-
s^2	1	1–K	
s^1	4K		
s^o	1–K		

$AE = s^4 + 2s^2 + 1 - K$

$\dfrac{d}{ds}(AE) = 4s^3 + 4s$

1–K > 0 no poles are on RHS plane and LHS plane.

All poles are on $j\omega$- axis

$\therefore\ 0 < K < 1$ system marginally stable

07. Ans: (d)

Sol: Assertion: FALSE

Let the TF= s. "s" is the differentiator

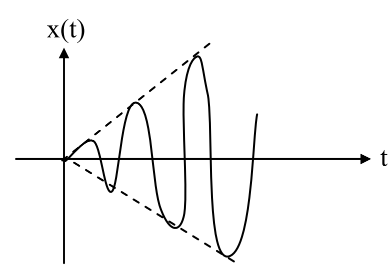
Impulse response $L^{-1}[TF] = L^{-1}[s] = \delta'(t)$

$\underset{t\to\infty}{Lt}\ \delta'(t) = 0$

$\therefore$ It is BIBO stable

Reason: True

$x(t) = t \sin t$



$\underset{t\to\infty}{Lt}\ x(t) = \underset{t\to\infty}{Lt}\ t\sin t$ is unbounded

08. Ans: (a)

Sol: Assertion: TRUE

If feedback is not properly utilized the closed loop system may become unstable.

Reason: True

Feedback changes the location of poles

Let $G(s) = \frac{-2}{s+1}$ $H(s) = 1$

Open loop pole s = –1 (stable)

$$CLTF = \frac{\frac{-2}{s+1}}{1+\frac{-2}{s+1}} = \frac{-2}{s-1}$$

Closed loop pole is at s = 1 (unstable)

$\therefore$ After applying the feedback no more system is open loop. It becomes closed loop system. Hence poles are affected.

09. Ans: (a) & (d)

Sol: RH tabulation:

s^5	1	5	4
s^4	–3	–7	20
s^3	$\frac{8}{3}$	$\frac{32}{3}$	0
s^2	5	20	0
s^1	0(10)	0	0
s^0	20	0	0

$AE = 5s^2 + 20 = 0$

$\frac{dAE}{ds} = 10s = 0$

AE roots = $s = \pm j2$

Two sign changes

$\therefore$ No. of $j\omega$ axis roots = 2

No. of left hand root = 1 (real)

10. Ans: (a), (c) & (d)

Sol: $C.E = 1 + \frac{k}{s(s+4)(s+5)} = 0$

$s^3 + 9s^2 + 20s + k = 0$

s^3	1	20
s^2	9	k
s^1	$\frac{180-k}{9}$	
s^0	k	

$180 - k > 0$

$k < 180$ and

$k > 0$

$\therefore$ Range of k for stability $0 < k < 180$

$k > 180$; Two sign changes in the 1^{st} column

$\therefore$ Number of right half of s-plane poles = 2

k = 180 marginally stable

$\therefore$ Two poles are on the imaginary axis

k < 180 stable

$\therefore$ All the three poles are in the left half of s-plane

Chapter 5 Root Locus Diagram

01. Ans: (a)

Sol: $s_1 = -1 + j\sqrt{3}$

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$s_1 = -1 + j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-1 + j\sqrt{3} + 2\right)^3}$$

$$= \frac{K}{\left(1 + j\sqrt{3}\right)^3}$$

$$= -3\tan^{-1}(\sqrt{3})$$

$$= -180°$$

It is odd multiples of 180°, Hence s_1 lies on Root locus

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-3 - j\sqrt{3} + 2\right)^3}$$

$$= \frac{K}{\left(-1 - j\sqrt{3}\right)^3}$$

$$= -3\,[180° + 60°] = -720°$$

It is not odd multiples of 180°, Hence s_2 is not lies on Root locus.

02. Ans: (a)

Sol: Over damped - roots are real & unequal $\Rightarrow 0 < k < 4$

(b) $k = 4$ roots are real & equal

$\Rightarrow$ Critically damped $\xi = 1$

(c) $k > 4 \Rightarrow$ roots are complex

$0 < \xi < 1 \Rightarrow$ under damped

03. Ans: (a)

Sol: Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3-0}{3-0} = -1$$

centroid = (–1, 0)

04. Ans: (b)

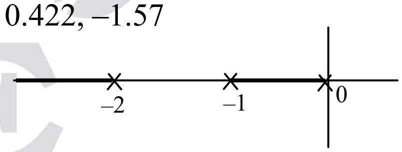
Sol: Break point $= \dfrac{dK}{ds} = 0$

$$\frac{d}{ds}\left(G_1(s).H_1(s)\right) = 0$$

$$\frac{d}{ds}\left[s(s+1)(s+2)\right] = 0$$

$3s^2 + 6s + 2 = 0$

$s = -0.422, -1.57$



But $s = -1.57$ do not lie on root locus

So, $s = -0.422$ is valid break point.

Point of intersection wrt $j\omega$-axis

$$s^3 + 3s^2 + 2s + k = 0$$

s^3	1	2
s^2	3	k
s^1	$\dfrac{6-k}{3}$	0
s^0	k	

As s^1 Row = 0

$k = 6$

$3s^2 + 6 = 0$

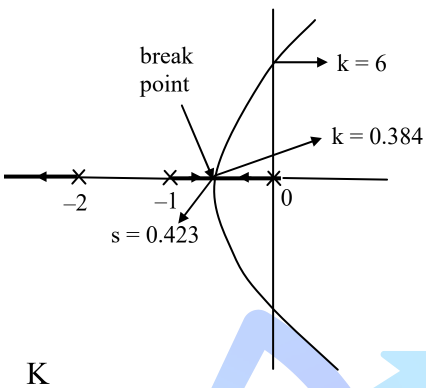
$s^2 = -2$

$s = \pm j\sqrt{2}$

point of inter section: $s = \pm\ j\sqrt{2}$

05. Ans: (b)

Sol:



$$\frac{K}{s(s+1)(s+2)}$$

substitute s = –0.423 and apply the magnitude criteria.

$$\left|\frac{K}{(-0.423)(-0.423+1)(-0.423+2)}\right| = 1$$

K = 0.354

when the roots are complex conjugate then the system response is under damped.

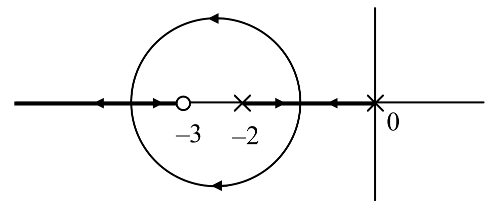
From K > 0.384 to K < 6 roots are complex conjugate then system to be under damped the values of k is 0.384 < K < 6.

06. Ans: (c)

Sol: If the roots are lies on the real axis then system exhibits the non-oscillatory response. from $K \geq 0$ to $K \leq 0.384$ roots lies on the real axis. Hence for $0 \leq K \leq 0.384$ system exhibits the non-oscillatory response.

07. Ans: (a)

Sol:



$$\frac{d}{ds}[G(s).H(s)] = \frac{d}{ds}\left[\frac{k(s+3)}{s(s+2)}\right]$$

$s^2 + 6s + 6 = 0$

break points – 1.27, – 4.73

$$\text{radius} = \frac{4.73 - 1.27}{2} = 1.73$$

center = (–3, 0)

08. Ans: (c)

Sol: $G(s).H(s) = \dfrac{K(s+3)}{s(s+2)}$

$$k\Big|_{s=-4} = \left|\frac{(-4)(-4+2)}{(-4+3)}\right|$$

$$= \left|\frac{(-4)(-2)}{(-1)}\right| = 8$$

09. Ans: (a)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2\pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2\pm 2j$ are two poles

$$\phi_A = 180 - \angle GH\Big|_{S=2\pm 2j}$$

$$GH = \frac{k[s-(2+2j)][s-(2-2j)]}{[s-(-2+2j)][s-(-2-2j)]}$$

$$\angle GH\Big|_{S=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4+4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$

10. Ans: (b)

Sol: $s^2-4s+8 = 0 \Rightarrow s = 2\pm2j$ are two zeroes

$s^2+4s+8 = 0 \Rightarrow s = -2\pm2j$ are two poles

$$\phi_d = 180^o + \angle GH\big|_{S=-2\pm2j}$$

$$\angle GH\big|_{S=-2\pm2j} = \angle\frac{k[s-(2+2j)][s-(2-2j)]}{[s-(-2+2j)][s-(-2-2j)]}\Bigg|_{S=-2\pm2j}$$

$$= \frac{\angle k(-4)(-4+4j)}{\angle 4j}$$

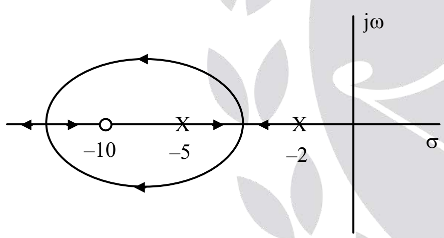
$$= 180^\circ+180^\circ-45^\circ-90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

11. Ans: (d)

Sol: Poles s = –2, –5 ; Zero s = –10



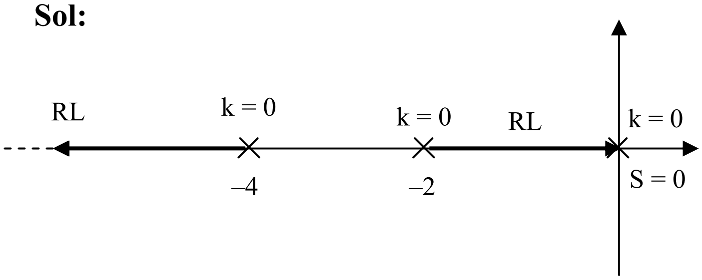
$\therefore$ Breakaway point exist between –2 and –5

12.

Sol: Refer Pg No: 75, Vol-1 Ex: 8

13. Ans: (a), (c) & (d)

Sol:



$$\Rightarrow \text{Centroid } \sigma = \frac{(-2-4)-(0)}{3} = -2$$

$$\Rightarrow \text{Angle of asymptotes } \theta = \frac{(2q+1)180^o}{(p-z)},$$

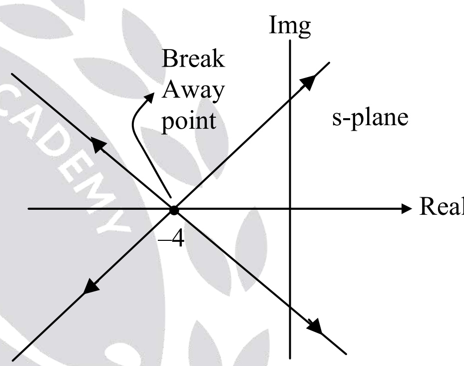
$$q = 0 \Rightarrow \theta = \frac{180^o}{3} = 60^o$$

$$q = 1 \Rightarrow \theta = \frac{3\times180^o}{3} = 180^o$$

$$q = 2 \Rightarrow \theta = \frac{5\times180^o}{3} = 300^o$$

14. Ans: (a) & (b)

Sol: RLD of the system is drawn below



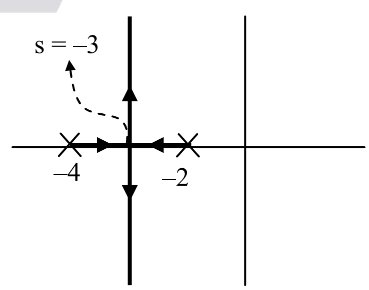
$$\text{Consider } \sigma = \frac{-4-4-4-4}{4-0}$$

$$\sigma = -4$$

All the root loci branches are breaking away at s = –4, hence it is called as a break away point.

15. Ans: (c) & (d)

Sol: RLD of the system is given below



$k\big|_{s=-3} = (1)(1) = 1$

s = –3 is a break in | away point

Chapter 6 Frequency Response Analysis

01. Ans: (c)

Sol: $G(s).H(s) = \dfrac{100}{s(s+4)(s+16)}$

Phase crossover frequency (ω_{pc}):

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{pc}} = -180°$

$-90° - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180°$

$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90°$

$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90°)$

$$\frac{\frac{\omega_{pc}}{4}+\frac{\omega_{pc}}{16}}{1-\frac{\omega_{pc}}{4}.\frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8$ rad/sec

02. Ans: (d)

Sol: $G(s).H(s) = \dfrac{100}{s(s+4)(s+16)}$

Gain margin (G.M) $= \dfrac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$

$$|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} = \frac{100}{\omega_{pc}\sqrt{\omega_{pc}^2+4^2}\sqrt{\omega_{pc}^2+16^2}}$$

$$= \frac{5}{64}$$

$$G.M = \frac{64}{5} = 12.8$$

03. Ans: (c)

Sol: $G(s).H(s) = \dfrac{2e^{-0.5s}}{(s+1)}$

gain crossover frequency,

$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$

$$\frac{2}{\sqrt{\omega_{gc}^2+1}} = 1$$

$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3}$ rad / sec

04. Ans: (b)

Sol: $\omega_{gc} = \sqrt{3}$rad / sec

P.M = $180° + \angle G(j\omega).H(j\omega)/\omega = \omega_{gc}$

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{gc}} = -0.5\,\omega_{gc} - \tan^{-1}(\omega_{gc})$

$= -109.62°$

P.M = 70.35°

05. Ans: (a)

Sol: $M_r = 2.5 = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

$2\xi\sqrt{1-\xi^2} = \dfrac{1}{2.5}$

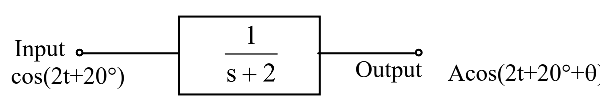
$\xi^4 - \xi^2 + 0.04 = 0$

$\xi^2 = 0.958 \qquad \xi^2 = 0.0417$

$\xi = 0.204 \qquad (M_r > 1)$

06. Ans: (a)

Sol: Closed loop T.F $= \dfrac{1}{s+2}$



$$A = \frac{1}{\sqrt{\omega^2+4}} = \frac{1}{\sqrt{4+4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$\phi = -\tan^{-1}\omega/2$

$= -\tan^{-1}2/2$

$\Rightarrow \phi = -\tan^{-1}(1) = -45°$

$$\text{output} = \frac{1}{2\sqrt{2}}\cos(2t + 20° - 45°)$$

$$= \frac{1}{2\sqrt{2}}\cos(2t - 25°)$$

07. Ans: (c)

Sol: Initial slope = –40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$\therefore$ Part of TF $= G(s)H(s) = \frac{K}{s^2}$

at ω = 0.1

Change in slope = –20 – (–40) = 20°

Part of TF = G(s) H(s) $= \frac{K\left(1+\frac{s}{0.1}\right)}{s^2}$

At ω = 10 slope changed to –60 dB/dec

Change in slope = –60–(–20)

= – 40dB/dec

$$\text{TF }(G(s)H(s)) = \frac{K\left(1+\frac{s}{0.1}\right)}{s^2\left(\frac{s}{10}+1\right)^2}$$

20 log K– 2 (20 log 0.1) = 20 dB

20 log K = 20 – 40

20 log K = –20

K = 0.1

$$G(s)H(s) = \frac{(0.1)\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)^2}$$

$$= \frac{(0.1)\times 10^2(s+0.1)}{(0.1)s^2(s+10)^2}$$

$$G(s)H(s) = \frac{100(s+0.1)}{s^2(s+10)^2}$$

08. Ans: (b)

Sol: $G(s)H(s) = \frac{Ks}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$

12 = 20 log K+ 20 log 0.5

12 = 20logK + (–6)

20 log K = 18 dB = 20 log 2^3

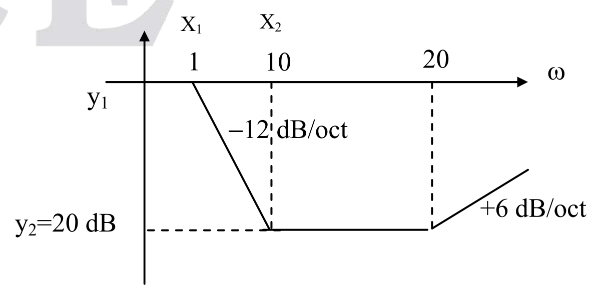
K = 8

$$G(s)H(s) = \frac{8s\times 2\times 10}{(2+s)(10+s)}$$

$$G(s)H(s) = \frac{160s}{(2+s)(10+s)}$$

09. Ans: (b)

Sol:



$$G(s)H(s) = \frac{K\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{20}\right)}{(1+s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40\,\text{dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60\,\text{dB}\Big|_{\omega \le 1}$$

$\Rightarrow$ 20 log K = 60

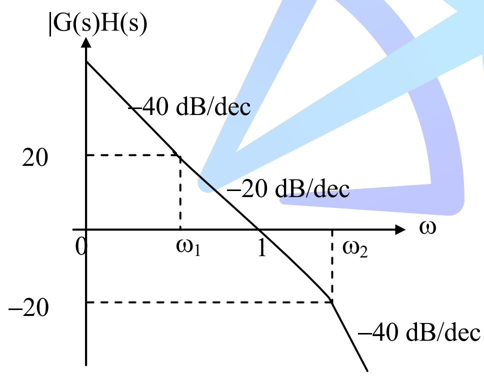
$$K = 10^3$$

$$G(s)H(s) = \frac{10^3 (s+10)^2 (s+20)}{10^2 \times 20 \times (s+1)^2}$$

$$= \frac{(s+10)^2 (s+20)}{2(s+1)^2}$$

10. Ans: (d)

Sol:



ω_1 calculation:

$$\frac{0 - 20}{\log 1 - \log \omega_1}$$

= –20 dB/dec

$\omega_1 = 0.1$

ω_2 calculation:

$$\frac{-20 - 0}{\log \omega_2 - \log 1}$$

= – 20dB/dec

$\omega_2 = 10$

$$G(s)H(s) = \frac{K\left(1 + \frac{s}{0.1}\right)}{s^2\left(1 + \frac{s}{10}\right)}$$

20logK–2 (20 log 0.1) = 20

20 logK = 20 – 40

K = 0.1

$$G(s)H(s) = \frac{0.1 \times \frac{1}{0.1}(0.1 + s)}{s^2 \frac{1}{10}(10 + s)}$$

$$= \frac{10(0.1 + s)}{s^2 (10 + s)}$$

11.

Sol: $\dfrac{200}{s(s+2)} = \dfrac{100}{s\left(1 + \frac{s}{2}\right)}$

$$x = -KT \Rightarrow -(100) \times \frac{1}{2} = x = -50$$

12. Ans: (c)

Sol: For stability (–1, j0) should not be enclosed by the polar plot.

For stability

1 > 0.01 K

$\Rightarrow$ K < 100

13.

Sol: GM = –40 dB

$$20 \log \frac{1}{a} = -40 \Rightarrow a = 10^2$$

POI = 100

14.

Sol: (i) $GM = \frac{1}{0.1} = +10 = 20\,dB$

$PM = 180° - 140° = 40°$

(ii) $PM = 180 - 150° = 30°$

$GM = \frac{1}{0} = \infty$ $\quad$ $POI = 0$

(iii) ω_{PC} does not exist

$GM = \frac{1}{0} = \infty$ $\quad PM = 180° + 0° = 180°$

(iv) ω_{gc} not exist

$\omega_{pc} = \infty$

$GM = \frac{1}{0} = \infty$

$PM = \infty$

(v) $GM = \frac{1}{0.5} = 2$

$PM = 180 - 90$
$= 90^0$

15. Ans: (d)

Sol: For stability (–1, j0) should not be enclosed by the polar plot. In figures (1) & (2) (–1, j0) is not enclosed.

∴Systems represented by (1) & (2) are stable.

16. Ans: (b)

Sol: Open loop system is stable, since the open loop poles are lies in the left half of s-plane ∴ P = 0.

From the plot N = –2 .

No. of encirclements N = P – Z

N = –2, P = 0 (Given)

∴ N = P – Z

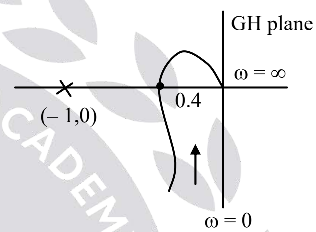
–2 = 0 – Z

Z = 2

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

17. Ans: (c)

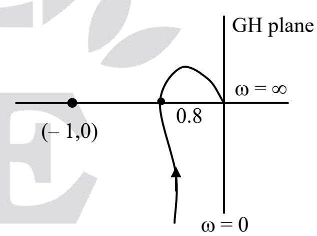
Sol:



$\frac{K_c}{K} = 0.4$ $\quad$ When K = 1

Now, K double, $\frac{K_c}{K} = 0.4$

$K_c = 0.4 \times 2 = 0.8$



Even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on 'ξ'

$\xi \propto \frac{1}{\sqrt{K}}$ as K is increased ξ reduced, then more oscillations.

18. Ans: (a)

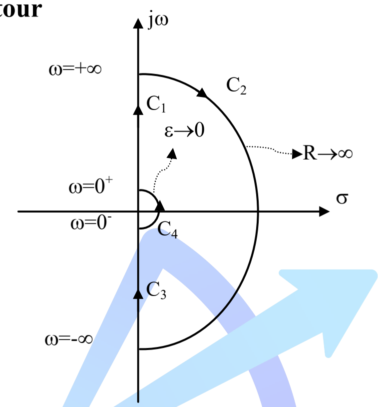
Sol: Given system $G(s) = \dfrac{10(s-12)}{s(s+2)(s+3)}$

It is a non minimum phase system since $s = 12$ is a zero on the right half of s-plane

19.

Sol: Given that $G(s)H(s) = \dfrac{10(s+3)}{s(s-1)}$

s-plane
Nyquist Contour



- Nyquist plot is the mapping of Nyquist contour(s-plane) into G(s)H(s) plane.
- The Nyquist contour in the s-plane enclosing the entire right half of S-plane is shown figure.

The Nyquist Contour has four sections C_1, C_2, C_3 and C_4. These sections are mapped into G(s)H(s) plane .

Mapping of section C_1: It is the positive imaginary axis, therefore sub $s = j\omega$, $(0 \le \omega \le \infty)$ in the TF G(s) H(s), which gives the polar plot

$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

Let $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$$G(j\omega)H(j\omega) = \frac{10\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}} \angle \left\{ \tan^{-1}\left(\frac{\omega}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega)] \right\}$$

At $\omega = 0 \Rightarrow \infty\angle -270^0$

At $\omega = \omega_{pc} = \sqrt{3} \Rightarrow 10\angle -180^0$

At $\omega = \infty \Rightarrow 0\angle -90^0$

point of intersection of the Nyquist plot with respect to negative real axis is calculated below

$$\text{Arg}G(j\omega)H(j\omega) = \arg\frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$= -180^0$ will give the 'ω_{pc}'

Magnitude of $G(j\omega)H(j\omega)$ gives the point of intersection

$$\angle \tan^{-1}\left(\frac{\omega}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega))$$

$$= -180^0 \Big|_{\omega = \omega_{pc}}$$

$$\angle \tan^{-1}\left(\frac{\omega_{pc}}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega_{pc})) = -180^0$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{3}\right) + \tan^{-1}(\omega_{pc}) = 90^0$$

Taking "tan" both the sides

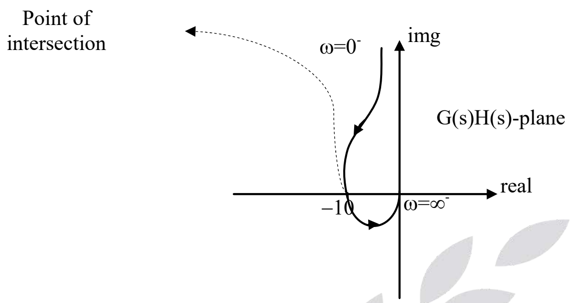
$$\frac{\frac{\omega_{pc}}{3} + \omega_{pc}}{1 - \frac{(\omega_{pc})^2}{3}} = \tan 90^0 = \infty$$

$$1 - \frac{\omega^2_{pc}}{3} = 0$$

$$\omega_{pc} = \sqrt{3} \text{ rad/sec}$$

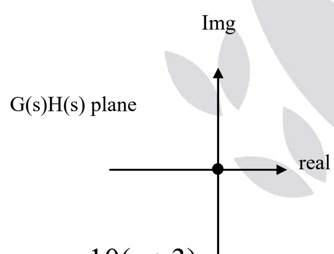
Therefore the point of intersection is

$$\left|G(j\omega)H(j\omega)\right| \text{at } \omega_{pc} = \frac{10\sqrt{\omega_{pc}^{\ 2}+3^2}}{\omega_{pc}\sqrt{1+\omega_{pc}^{\ 2}}} = 10$$



The mapping of the section C_1 is shown figure.

Mapping of section C_2: It is the radius 'R' semicircle, therefore sub $s = \lim\limits_{R\to\infty} Re^{j\theta}$ (θ is from 90^0 to 0^0 to -90^0) in the TF G(s)H(s), which merges to the origin in G(s)H(s) plane.



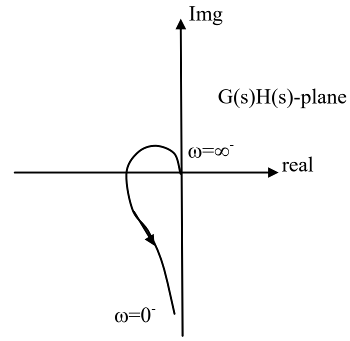
$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{2(Re^{j\theta}+3)}{Re^{j\theta}(Re^{j\theta}-1)} \approx 0$$

The plot is shown in figure.

Mapping of section C_3: It is the negative imaginary axis, therefore sub $s = j\omega$, $(-\infty \le \omega \le 0)$ in the TF G(s)H(s), which gives the mirror image of the polar plot and is symmetrical with respect to the real axis,

The plot is shown in figure.



Mapping of section C_4: It is the radius 'ε' semicircle, therefore sub $s = \lim\limits_{\varepsilon\to 0} \varepsilon e^{j\theta}$ $(-90^o \le \theta \le 90^o)$ in the TF G(s)H(s), which gives clockwise infinite radius semicircle in G(s)H(s) plane.

The plot is shown below

$$G(\varepsilon e^{j\theta})H(\varepsilon e^{j\theta}) = \frac{10(\varepsilon e^{j\theta}+3)}{\varepsilon e^{j\theta}(\varepsilon e^{j\theta}-1)}$$

$$G(\varepsilon e^{j\theta})H(\varepsilon e^{j\theta}) \approx \frac{10\times 3}{-\varepsilon e^{j\theta}} = \infty\angle 180^0 - \theta$$

When, $\theta = -90^0$ $\quad \infty\angle 270^0$

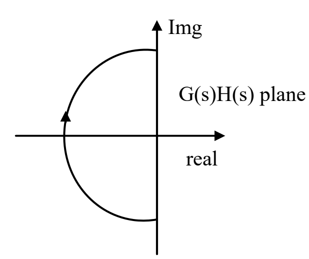
$\theta = -40^0$ $\quad \infty\angle 220^0$

$\theta = 0^0$ $\quad \infty\angle 0^0$

$\theta = 40^0$ $\quad \infty\angle 140^0$

$\theta = 90^0$ $\quad \infty\angle 90^0$

It is clear that the plot is clockwise '∞' radius semicircle centred at the origin



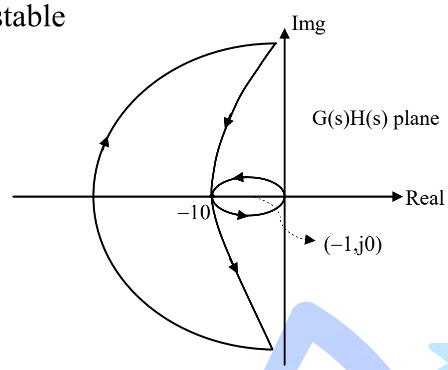
Combining all the above four sections, the Nyquist plot of $G(s)H(s) = \dfrac{10(s+3)}{s(s-1)}$ is shown in figure below

From the plot N = 1

Given that P = 1

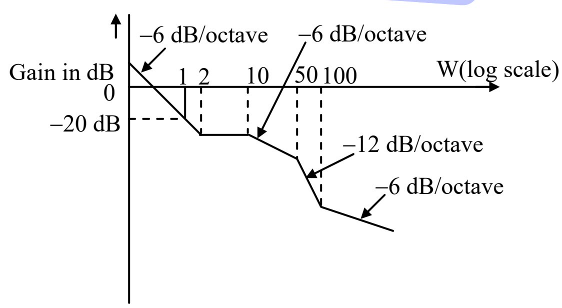
N = P – Z

Z = P – N = 1 – 1 = 0, therefore system is stable



20.

Sol: The given bode plot is shown below.



Initial slope = – 6 db/octave.

i.e., there is one pole at origin (or) one integral term.

portion of transfer function

$$G(s) = \frac{K}{s}$$

At ω = 2 rad/sec, slope is changed to 0dB/ octave.

∴ change in slope

= present slope – previous slope

= 0 – (–6) = 6 dB/octave

∴ There is a real zero at corner frequency $\omega_1 = 2$.

$$(1+sT_1) = \left(1+\frac{s}{\omega_1}\right) = \left(1+\frac{s}{Z}\right)$$

At ω = 10 rad/sec, slope is changed to –6dB/octave.

∴ change in slope = – 6 – 0

= – 6 dB/octave.

∴ There is a real pole at corner frequency $\omega_2 = 2$.

$$\frac{1}{1+sT_2} = \frac{1}{\left(1+\frac{s}{\omega_2}\right)} = \frac{1}{\left(1+\frac{s}{10}\right)}$$

At ω = 50 rad/sec, slope is changed to –12dB/octave.

∴ change in slope = – 12 – (–6)

= – 6 dB/octave

∴ There is a real pole at corner frequency $\omega_3 = 50$ rad/sec.

$$\frac{1}{1+ST_3} = \frac{1}{\left(1+\frac{S}{\omega_3}\right)} = \frac{1}{\left(1+\frac{S}{50}\right)}$$

At ω = 100 rad/sec, the slope changed to –6 dB/octave.

∴ change in slope = – 6 – (–12)

= 6 dB/octave.

∴ There is a real zero at corner frequency $\omega_4 = 100$ rad/sec.

$$\therefore (1+sT_4) = \left(1+\frac{s}{\omega_4}\right) = \left(1+\frac{s}{100}\right)$$

$$\therefore \text{ Transfer function } = \frac{K\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}{s\left(1+\frac{s}{50}\right)\left(1+\frac{s}{10}\right)}$$

$$= \frac{K(s+2)(s+100)}{s(s+50)(s+10)} \frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{50}\cdot\frac{1}{10}}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

In the given bode plot,

at ω = 1rad/sec, Magnitude = –20dB.

$$-20\text{dB} = 20\log K - 20\log\omega + 20\sqrt{1+\left(\frac{\omega}{2}\right)^2} + 20\sqrt{1+\left(\frac{\omega}{100}\right)^2}$$

$$-20\log\sqrt{1+\left(\frac{\omega}{50}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

At ω = 1 rad/sec,

–20 = 20logK – 20 logω/ω = 1 rad/sec

[∵ Remaining values eliminated]

– 20 = 20logK

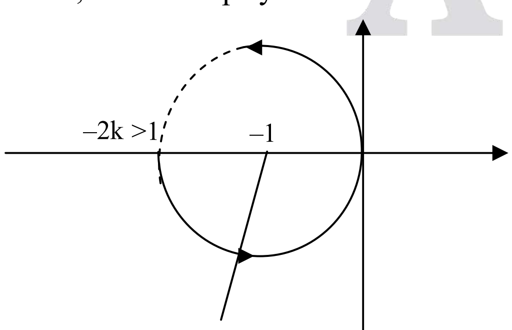
⇒ K = 0.1

∴Transfer function

$$\frac{C(s)}{R(s)} = \frac{0.25(s+2)(s+100)}{s(s+10)(s+50)}$$

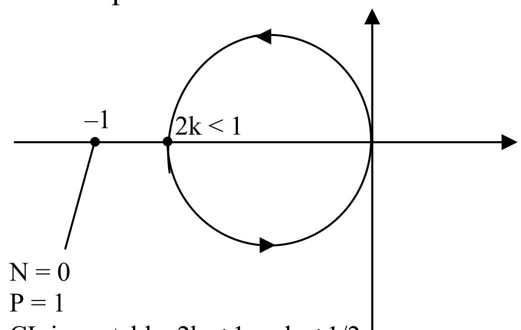
21. Ans: (a) & (d)

Sol: k >1/2, closed loop system is stable.



N = 1
P = 1
CL system is stable
2k > 1 ⇒ k > 1/2

For k < 1/2, one closed loop pole in the right half of s-plane.



N = 0
P = 1
CL is unstable, 2k < 1 ⇒ k < 1/2
N =P–Z ⇒ 0 = 1 – Z ⇒ Z = 1 ⇒ one closed loop Pole in the right half s-plane

22. Ans: (a) & (d)

Sol: ⇒$\omega_{pc} = \infty$. Hence GM = ∞

⇒$\angle\phi|\omega_{gc}$ = –150°, ⇒ PM = 180° + $\angle\phi|\omega_{gc}$

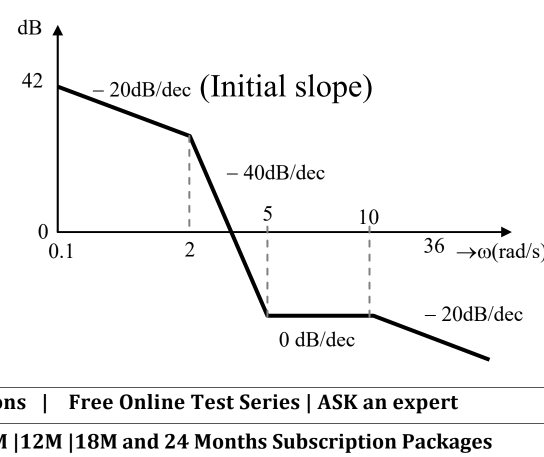
⇒ PM = 180° – 150° = +30° (finite).

23. Ans: (b) & (d)

Sol: $$G(s)H(s) = \frac{10\times 5^2(1+\frac{s}{5})^2}{s\times 2(1+\frac{s}{2})(10)(1+\frac{s}{10})}$$

$$= \frac{12.5(1+\frac{s}{5})^2}{s(1+\frac{s}{2})(1+\frac{s}{10})}$$

$$M\big|_{\omega=0.1} = 20\log 12.5 - 20\log\omega$$
$$= 20\log 12.5 - 20\log 0.1$$
$$\approx 42 \text{ dB}$$



⇒ Slope of the line between 5 rad/sec to 10 rad/sec is 0 dB/dec.

⇒ At high frequency, slope of line is –20 dB/dec.

24. Ans: (b) & (c)

Sol: At any frequency magnitude of the loop transfer function is not unity,

$\therefore$ PM = ∞

System is always stable,

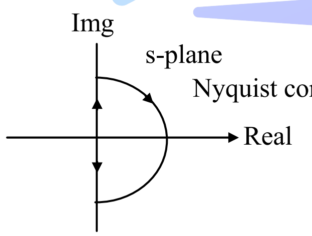
$\therefore$ GM = ∞

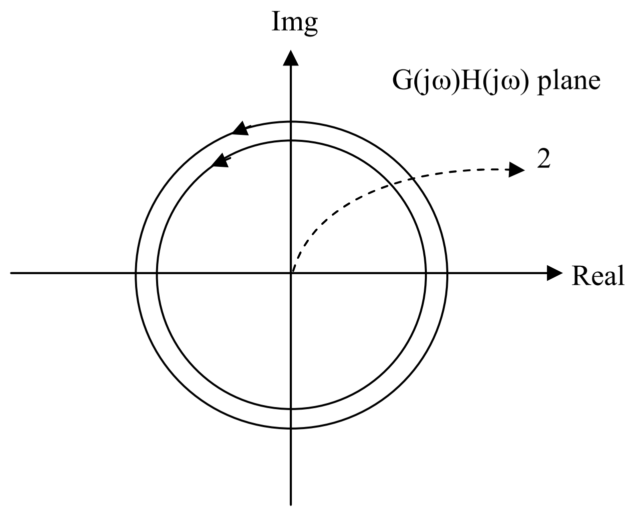
25. Ans: (b) & (c)

Sol: $N_{0,0}$ = difference between open loop polar and zero

$N_{0,0} = (2 - 0) = 2$

$N_{0,0} = 2$





Chapter 7 Compensators & Controllers

01. Ans: (a)

Sol: $G_C(s) = (-1)\left(-\frac{Z_2}{Z_1}\right)$

$$= (-1)(-1)\left(\frac{R_2 + \frac{1}{sC}}{R_1}\right)$$

$$G_c(s) = \frac{(100\times 10^3) + \frac{1}{s\times 10^{-6}}}{10^6}$$

$$G_c(s) = \frac{1+0.1s}{s}$$

02. Ans: (c)

Sol: CE $\Rightarrow 1 + G_c(s)\,G_p(s) = 0$

$$= 1 + \frac{1+0.1s}{s}\times\frac{1}{(s+1)(1+0.1s)}$$

$$= 1 + \frac{1+0.1s}{s(s+1)(1+0.1s)} = 0$$

$\Rightarrow s^2 + s + 1 = 0 \Rightarrow \omega_n = 1,$

$$e^{\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]_{\xi=0.5}} = 0.163$$

$M_p = 16.3\%$

03. Ans: (b)

Sol: T.F $= \frac{k(1+0.3s)}{1+0.17s}$

$T = 0.17,\ aT = 0.3 \Rightarrow a = \frac{0.3}{0.17}$

$C = 1\,\mu F$

$T = \frac{R_1R_2}{R_1+R_2}C,\ a = \frac{R_1+R_2}{R_2}$

$$\frac{R_1R_2}{R_1+R_2} = \frac{0.17}{1\times 10^{-6}} = 170000$$

$$\frac{R_1+R_2}{R_2} = 1.764$$

$aT = R_1 C$

$$R_1 = \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6)$$

$= 300\ k\Omega$

Bv

$300\ k + R_2 - 1.76\ R_2 = 0$

$$R_2 = \frac{300}{0.70} = 394.736$$

$= 400\ k\Omega$

04. Ans: (d)

Sol: PD controller improves transient stability and PI controller improves steady state stability. PID controller combines the advantages of the above two controllers.

05.

Sol: For $K_I = 0 \Rightarrow$

$$\frac{C(s)}{R(s)} = \frac{(K_P + K_D s)}{s(s+1) + (K_P + K_D s)}$$

$$= \frac{K_P + K_D s}{s^2 + (1+K_D)s + K_P}$$

$\omega_n = \sqrt{K_P}$

$2\xi\omega_n = 1 + K_D$

$\Rightarrow 2(0.9)\sqrt{K_P} = 1 + K_D$

$\Rightarrow 1.8\sqrt{K_P} = 1 + K_D$ ……… (1)

Dominant time constant $\frac{1}{\xi\omega_n} = 1$

$$\Rightarrow \omega_n = \frac{1}{0.9} = 1.111$$

$K_P = \omega_n^2 = 1.11^2$

$= 1.234$

From eq. (1),

$$\Rightarrow 1.8\times\frac{1}{0.9} = 1 + K_D$$

$\Rightarrow K_D = 1$

06. Ans: (b) & (d)

Sol: Both PD and lead controller improve transient response of the system.

Chapter 8 State Space Analysis

01. Ans: (a)

Sol: $TF = \dfrac{1}{s^2+5s+6}$

$= \dfrac{1}{(s+2)(s+3)}$

$= \dfrac{1}{s+2} + \dfrac{-1}{s+3}$

$\therefore\ A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$C = [1 \quad 1]$

02. Ans: (c)

Sol: Given problem is Controllable canonical form.

(or)

$TF = C[sI - A]^{-1}B + D$

$= [6 \ \ 5 \ \ 1] \begin{bmatrix} s & 1 & 0 \\ 0 & s & 1 \\ -5 & -3 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$= \dfrac{3s^2+15s+18}{s^3+6s^2+3s+5}$

03. Ans: (d)

Sol: $\dfrac{d^2y}{dt^2} + \dfrac{3dy}{dt} + 2y = u(t)$

2^{nd} order system hence two state variables are chosen

Let x_1 (t), x_2 (t) are the state variables

CCF − SSR

Let $x_1(t) = y(t)$ ……….. (1)

$x_2(t) = \dot{y}(t)$ ……….. (2)

Differentiating (1)

$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$ ……….. (3)

$\dot{x}_2(t) = \ddot{y}(t) = u(t) - 3y^1(t) - 2y(t)$

$= u(t) - 3x_2(t) - 2x_1(t)$ ……. (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underset{A}{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u(t)$$

From equation 1. The output equation in matrix form

$y(t) = [1 \quad 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, D = 0$

04. Ans: (b)

Sol: OCF − SSR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$y(t) = [0 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

05. Ans: (c)

Sol: Normal form - SSR

$TF = \dfrac{Y(s)}{G(s)} = \dfrac{1}{s^2+3s+2} = \dfrac{1}{(s+1)(s+2)}$

$\Rightarrow$ Diagonal canonical form

The eigen values are distinct i.e., − 1 & − 2.

$\therefore$ Corresponding normal form is called as diagonal canonical form

DCF - SSR

$\dfrac{Y(s)}{U(s)} = \dfrac{b_1}{s+1} + \dfrac{b_2}{s+2}$

$b_1 = 1, b_2 = -1$

$Y(s) = \underbrace{\frac{b_1}{s+1}}_{x_1} U(s) + \underbrace{\frac{b_2}{s+2}}_{x_2} U(s)$

Let $Y(s) = X_1(s) + X_2(s)$

Where $y(t) = x_1(t) + x_2(t)$ .... (1)

Where $X_1(s) = \frac{b_1}{s+1} U(s)$

$s X_1(s) + X_1(s) = b_1 U(s)$

Take Laplace Inverse

$\dot{x}_1 + x_1 = b_1 u(t)$ .... (2)

$X_2(s) = \frac{b_2}{s+2} U(s)$

$s X_2(s) + 2 X_2(s) = b_2 U(s)$

Laplace Inverse

$\dot{x}_2 + 2x_2 = b_2 u(t)$

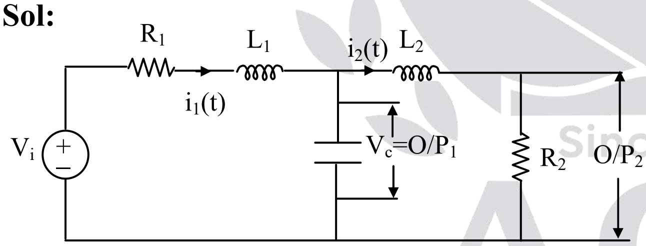
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

From (1) output equation.

$$y(t) = [1\ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

06. Ans: (c)

Sol:



$O/P_1 \Rightarrow y_1 = V_c$

$O/P_2 \Rightarrow y_2 = R_2 i_2$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ i_1 \\ i_2 \end{bmatrix}$$

$y = C X$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}$$

07. Ans: (a)

Sol: $T.F = C[sI-A]^{-1}B + D$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+4 & 1 \\ 3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s^2+5s+1} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+1} \begin{bmatrix} 1 & 0 \end{bmatrix}_{1\times 2} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+1} \begin{bmatrix} s+1 & -1 \end{bmatrix}_{1\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= \frac{1}{s^2+5s+1} [s+1-1]$$

$$= \frac{s}{s^2+5s+1}$$

08. Ans: (c)

Sol: State transition matrix $\phi(t) = L^{-1}[(sI-A)^{-1}]$

$$sI - A = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[[sI-A]^{-1}] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

09. Ans: (b)

Sol: Controllability

$$[M] = [B \quad AB \quad A^2B.. \quad A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$|M| = -1 \neq 0$ (Controllable)

Observability

$$[N] = [C^T \ A^T C^T \ .... \ (A^T)^{n-1} C^T]$$

$$A^TC^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$|N| = 0$ (Not observable)

10. Ans: (c)

Sol: According to Gilberts test the system is controllable and observable.

11. Ans: (c)

Sol: $$\frac{Y(s)}{U(s)} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}$$

at node $\dot{x}_1$

$\dot{x}_1 = -a_1x_1 - a_2x_2 - a_3\,x_3$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

12.

Sol: The given state space equations:

$\dot{X} = X_2$

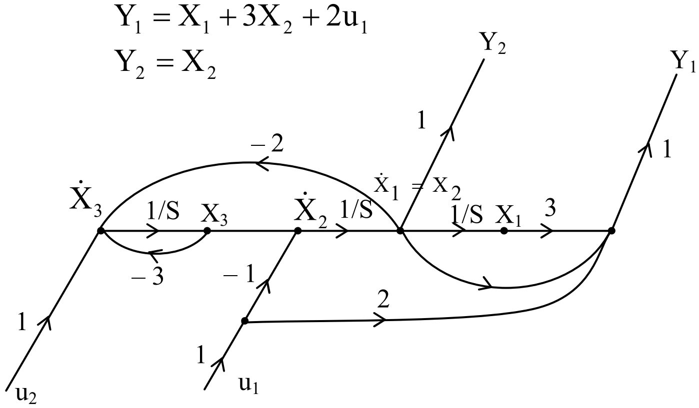
$\dot{X}_2 = X_3 - u_1$

$\dot{X}_3 = -2X_2 - 3X_3 + u_2$

and output equations are :

$Y_1 = X_1 + 3X_2 + 2u_1$

$Y_2 = X_2$



The given state space equations in matrix for

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}_{3\times3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}_{3\times2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2\times3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1}$$

Where A: State matrix
B: Input matrix
C: Output matrix
D: Transition matrix

Characteristic equation

$|sI - A| = 0$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 0 & 2 & S+3 \end{vmatrix} = 0$$

$\Rightarrow s[s(s+3)+2] + 1(0) = 0$

$\Rightarrow s(s^2 + 3s + 2) = 0$

$\Rightarrow s(s+1)(s+2) = 0$

The roots are 0, –1,–2.

13. Ans: (a) & (b)

Sol: (a) $\rightarrow$ state model is in controllable canonical form

(b) $\rightarrow$ state model is in observable canonical form