## GATE I PSUs



## Computer Science \& Information Technology

## THEORY OF COMPUTATION

## Text Book:

Theory with worked out Examples and Practice Questions

# Theory of Computation 

## (Solutions for Text Book Practice Questions)

Chapter
1

## Introduction

1. Ans: (d)

Sol: (a) $\{x \mid x \geq 10$ or $x \leq 5\}$ is infinite set
(b) $\{x \mid x \geq 10$ or $x \leq 100\}$ is infinite set
(c) $\{x \mid x \leq 100$ or $x \geq 200\}$ is infinite set
02. Ans: (b)

Sol:
(a) Set of real numbers between 10 and 100 is uncountable
(b) $\{x \mid x \geq 10$ or $x \leq 100\}$ is finite set. So countable
(c) Set of real numbers between 0 and 1 is uncountable
03. Ans: (d)

Sol: (a) $|\varepsilon|=0$
(b) $|\} \mid=0$
(c) $|\{\varepsilon\}|=1$
04. Ans: (b)

Sol: $\Sigma=\{0,1\}$
$00,01,10,11$ are 2 length strings
05. Ans: (b)

Sol: $\mathrm{w}=\mathrm{abc}$
$\operatorname{Prefix}(w)=\{\varepsilon, a, a b, a b c\}$
06. Ans: (b)

Sol: w = abc
$\operatorname{Suffix}(w)=\{\varepsilon, c, b c, a b c\}$
07. Ans: (d)

Sol: w = abc
Substring $(w)=\{\varepsilon, a, b, c, a b, b c, a b c\}$
08. Ans: (a)

Sol: Language accepted by finite automata is called as Regular language.

## 09. Ans: (d)

Sol: Every recursive language is REL but REL need not be recursive language.

## 10. Ans: (b)

Sol: Every regular grammar is CFG but CFG need not be regular grammar.

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Chapter
2

## Regular Languages

(Finite automata, Regular expression, regular grammar)

## 01. Ans: (a) \& (c)

Sol: Regular Languages are closed under
i) string reversal
ii) intersection with finite sets
02. Ans: (c)

Sol: A minimal DFA that is equivalent to a NFA with n states has atmost $2^{\mathrm{n}}$ states.
03. Ans: (a)

Sol: (a) $(1+01)^{*}(\varepsilon+0)$ generates all strings not containing ' 00 '
(b) $(0+10)^{*}(\varepsilon+1)$ generates invalid string ' 00 '
(c) $(1+01)^{*}$ cannot generate ' 0 '
(d) $(\varepsilon+0)(101)^{*}(\varepsilon+0)$ generates invalid string ' 00 '
04. Ans: (a)

Sol:

05. Ans: (d)

Sol: Given grammar generating all strings ending in ' 00 '
06. Ans: (a)

Sol:

07. Ans: (a)

Sol:

qo: Even a's and Even b's
$\mathrm{q}_{1}$ : Even â's and odd b's
$\mathrm{q}_{2}$ : Odd a's and Even b's
$\mathrm{q}_{3}$ : Odd a's and Odd b's
$\mathrm{q}_{1}$ should be final state.
08. Ans: (b)

Sol: Concatenation of two infinite languages is also infinite. So, infinite languages closed under concatenation.
09. Ans: (c)

Sol: $\left\{\mathrm{wxw}^{\mathrm{R}} \mid \mathrm{x}, \mathrm{w} \in(0+1)^{+}\right\}=0(0+1)^{+} 0+1(0+1)^{+} 1$
$\therefore$ It is regular language
10. Ans: (a)

Sol: (I) NFA with many final states can be converted to NFA with only one final state with the help of $\varepsilon$-moves.
(II) Regular sets are not closed under infinite union
(III) Regular sets are not closed under infinite intersection
(IV) Regular languages are closed under substring operation
$\therefore$ I and IV are correct.

## 11. Ans: (d)

Sol: $\mathrm{r}=(0+1)^{*} 00(0+1)^{*}$
$\mathrm{A} \rightarrow 0 \mathrm{~B}|0 \mathrm{~A}| 1 \mathrm{~A}$
$\mathrm{B} \rightarrow 0 \mathrm{C} \mid 0$
$\mathrm{C} \rightarrow 0 \mathrm{C}|1 \mathrm{C}| 0 \mid 1$
12. Ans: (a)

Sol: $A_{n}=\left\{a^{k} \mid k\right.$ is a multiple of $\left.n\right\}$
For some n ,
$A_{n}$ is regular
Let $\mathrm{n}=5$,
$\mathrm{A}_{\mathrm{n}}=\mathrm{A}_{5}=\left\{\mathrm{a}^{\mathrm{k}} \mid \mathrm{k}\right.$ is multiple of 5$\}$
= regular.

## 13. Ans: (d)

Sol: $L=\left\{a^{m} b^{n} \mid m \geq 1, n \geq 1\right\}=a^{+} b^{+}$is regular.

## 14. Ans: (c)

Sol: DFA accepts L and has m states
It has 2 final states. It implies (m-2) non-final states.
DFA that accepts complement of $L$ also has m states but it has (m-2) final states and 2 non-final states.
15. Ans: (d)

Sol: (a) $0^{*}(1+0)^{*}$; It generates invalid string ' 100 '
(b) 0* 1010* ; It cannot generate valid string ' $\varepsilon$ '
(c) $0^{*} 1^{*} 01^{*}$; It cannot generate valid string ' $\varepsilon$ '
(d) $0^{*}(10+1)^{*}$; It generates all strings not containing ' 100 ' as substring
16. Ans: (a)

Sol: P1: Membership problem for FA is decidable
P2 : Infiniteness problem for CFG is decidable
For P1, CYK algorithm exist
For P2, Dependency graph exist
17. Ans: (b)

Sol: L = set of all binary strings whose last 2 symbols are same.

18. Ans: (a)

Sol: $L=a^{n} b^{n}$ is not regular
It can be proved using Pumping Lemma
L does not satisfy Pumping Lemma
19. Ans: (c)

Sol: It requires 29099 remainders to represent the binary numbers of the given language.
So, 29099 states required.
20. Ans: (d)

Sol: The following problems are decidable for regular languages. Equivalence, Finiteness, Emptiness, infiniteness, totality, containment, Emptiness of complement, Emptiness of intersection, Emptiness of complement of intersection.

## 21. Ans: (a)

Sol: I. $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{2 \mathrm{~m}} \mid \mathrm{n} \geq 0, \mathrm{~m} \geq 0\right\} \Rightarrow$ Regular
II. $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{m}} \mid \mathrm{n}=2 \mathrm{~m}\right\} \Rightarrow$ not regular
III. $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{m}} \mid \mathrm{n} \neq \mathrm{m}\right\} \Rightarrow$ not regular
IV. $\left\{x \subset y \mid x, y \in\{a, b\}^{*}\right\} \Rightarrow$ Regular

So, I \& IV are correct.

## 22. Ans: (c)

Sol: Let $\mathrm{n}=3$
If $\mathrm{w}=\mathrm{abc}$,
Substrings of $w=\{\varepsilon, a, b, c, a b, b c, a b c\}$ non empty substrings of

$$
\mathrm{w}=\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{ab}, \mathrm{bc}, \mathrm{abc}\}
$$

number of substrings of $w$ of length $n$ is

$$
\leq(\Sigma n)+1
$$

number of non empty substrings of $w$ of length $\mathrm{n} \leq(\Sigma \mathrm{n})$.

## 23. Ans: (c)

## Sol:

| $\boldsymbol{\delta}$ | $\mathbf{a}$ | $\mathbf{b}$ |
| :--- | :--- | :--- |
| $\rightarrow \mathrm{A}$ | 3 choices | 3 choices |
| B | 3 | 3 |
| C | 3 | 3 |

$3 \times 3 \times 3 \times \ldots . .6$ times $=3^{6}$ machines possible with ' A ' as initial state.

Final states can be any of subset of $\{A, B$, C
So, $2^{3}$ possible final states combinations.
Total $8 \times 3^{6}$ DFAs.
Number of DFAs with atleast 2 final states $=4 \times 3^{6}$.
24. Ans: (a)

Sol:

$=4$ states
25. Ans: (b)

Sol:

26. Ans: (b)

Sol: (i) $\left.\begin{array}{l}\mathrm{S} \rightarrow \mathrm{aAB} \\ \mathrm{A} \rightarrow \mathrm{b}\end{array}\right\} \mathrm{L}=\phi$
(ii) $\left.\begin{array}{l}\mathrm{S} \rightarrow \mathrm{aA} \mid \mathrm{bB} \\ \mathrm{A} \rightarrow \mathrm{a}\end{array}\right\} \mathrm{L}=\{\mathrm{aa}\}$
(iii) $\left.\begin{array}{rl}\mathrm{S} & \rightarrow \mathrm{aA} \\ \mathrm{B} & \rightarrow \mathrm{aA}\end{array}\right\} \mathrm{L}=\phi$
(iv) $\left.\begin{array}{l}\mathrm{S} \rightarrow \mathrm{aA} \mid \mathrm{bB} \\ \mathrm{B} \rightarrow \mathrm{b}\end{array}\right\} \mathrm{L}=\{\mathrm{bb}\}$
(i) \& (iii) are equivalent.
27. Ans: (c)

Sol: $\mathrm{L}=(\mathrm{a}+\mathrm{ba})^{*} \mathrm{~b}(\mathrm{a}+\mathrm{b})^{*}$
strings of length $\leq 3$ :
b, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb
Number of strings $=11$
28. Ans: (b)

Sol: $\mathrm{r}=\left(0^{*}+(10)^{*}\right)^{*}=(0+10)^{*}$
$\mathrm{s}=\left(0^{*}+10\right)^{*}$
$\therefore \mathrm{L}(\mathrm{r})=\mathrm{L}(\mathrm{s})$
29. Ans: (d)

Sol: The following sets are countable sets.

1) Set of regular sets
2) Set of CFLs
3) Set of Turing Machines

The set of real numbers is uncountable
The set of formal languages is uncountable.
30. Ans: (a)

Sol:

b
2 Equivalence classes.
31. Ans: (c)

Sol: $\mathrm{L}=\left((01)^{*} 0^{*}\right)^{*}$
$\left.\begin{array}{l}\mathrm{h}(\mathrm{a})=0 \\ \mathrm{~h}(\mathrm{~b})=01\end{array}\right\} \Rightarrow \begin{aligned} & \mathrm{h}^{-1}(0)=\mathrm{a} \\ & \mathrm{h}^{-1}(01)=\mathrm{b}\end{aligned}$
$h^{-1}(L)=\left(b^{*} a^{*}\right)^{*}=(a+b)^{*}$
32. Ans: (a)

Sol: $L_{1}=a^{*} b$
$\mathrm{L}_{2}=\mathrm{ab} *$

$$
\begin{aligned}
\mathrm{L}_{1} / \mathrm{L}_{2}=\mathrm{a} * \mathrm{~b} / \mathrm{ab}^{*} & =\left\{\mathrm{a}^{*} \mathrm{~b} / \mathrm{ab}, \mathrm{a} * \mathrm{~b} / \mathrm{a}, \ldots\right\} \\
& =\left\{\mathrm{a}^{*}, \phi, \ldots\right\} \\
& =\mathrm{a}^{*}
\end{aligned}
$$

33. Ans: (d)

Sol: (a) $\mathrm{L}\left(\mathrm{r}^{*}\right) \supset \mathrm{L}\left(\mathrm{r}^{+}\right)$
(b) $\mathrm{L}\left((\mathrm{r}+\mathrm{s})^{*}\right) \supset \mathrm{L}\left(\mathrm{r}^{*}+\mathrm{s}^{*}\right)$
(c) $\mathrm{L}\left((\mathrm{r}+\mathrm{s})^{*}\right) \supset \mathrm{L}\left((\mathrm{rs})^{*}\right)$
(d) $\mathrm{L}\left(\mathrm{r}^{*}\right)=\mathrm{L}\left(\left(\mathrm{r}^{+}\right)^{*}\right)$

## 34. Ans: (b)

Sol: Arden's lemma cannot be applied to NFA with $\varepsilon$ moves.

Arden's lemma applied to both DFA and NFA without $\varepsilon$ moves.

## 35. Ans: (d)

Sol: Logic circuits, neural sets, toy's behavior can be modeled with regular sets.
36. Ans: (a)

Sol: $\mathrm{L}=(0+1)^{*} 00$


## 37. Ans: (c)

Sol:

|  | 0 | 1 |
| :---: | :---: | :---: |
| $\rightarrow \mathrm{q}_{0}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{1}$ |
| $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ | $\mathrm{q}_{3}$ |
| $\mathrm{q}_{2}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{0}$ |
| $\mathrm{q}_{3}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ |
| $\mathrm{q}_{4}$ | $\mathrm{q}_{3}$ | $\mathrm{q}_{4}$ |

$=5$ states
38. Ans: (a)

Sol: $3^{\text {rd }}$ symbol from ending is ' 1 ',
$\Downarrow$
DFA has $2^{3}$ states.
39. Ans: (a)

Sol: $L=\left\{a^{i} b^{j} \mid i<100, j<=10000\right\}$

$$
=\left\{\varepsilon, \mathrm{a}, \mathrm{~b}, \ldots, \mathrm{a}^{99} \mathrm{~b}^{10000}\right\}
$$

L is finite set
40. Ans: (a)

Sol: $\mathrm{L}=(0+1)^{*} 0001(0+1) *$
DFA accepts L with 5 states
DFA that accepts complement of L also requires 5 states.
DFA that accepts complement of L.

41. Ans: (a)

Sol: $(00)^{*}+0(00)^{*}+00(000)^{*}$
$(00)^{*}=$ set of all even strings
$0(00)^{*}=$ set of all odd strings
$(00)^{*}+0(00)^{*}=$ set of all strings $=0^{*}$
$\therefore(00)^{*}+0(00)^{*}+00(000)^{*}=0^{*}$
42. Ans: (d)

Sol:
Sol:
$\mathrm{q}_{0}, \mathrm{q}_{2}, \mathrm{q}_{4}$ will be combined

|  | 0 | 1 | 2 |  |
| :---: | :---: | :---: | :---: | :---: |
| $\longrightarrow \mathrm{q}_{0}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ |  |
| $\mathrm{q}_{1}$ | $\mathrm{q}_{3}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{5}$ | same |
| $\longrightarrow \mathrm{q}_{2}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ | $\mathrm{q}_{1}+\mathrm{q}_{3}$ will be |
| $\mathrm{q}_{3}$ | $\mathrm{q}_{3}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{5}$ |  |
| $\longrightarrow \mathrm{q}_{4}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ |  |
| (95) | $\mathrm{q}_{3}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{5}$ |  |

Number of states $=3$
$\left\{\mathrm{q}_{0}, \mathrm{q}_{2}, \mathrm{q}_{4}\right\},\left\{\mathrm{q}_{1}, \mathrm{q}_{3}\right\},\left\{\mathrm{q}_{5}\right\}$
43. Ans: (b)

Sol: i) $\left\{\mathrm{a}^{\mathrm{a}^{\mathrm{n}}} \mid \mathrm{n} \geq 1\right\}$ is not regular
ii) $\mathrm{a}^{\text {prime }}$ is not regular
iii) $\left\{0^{i} 1^{j} \mid i<j<1000\right\}$ is finite. So regular
iv) Complement of $L$ where

$$
\mathrm{L}=(0+1)^{*} 000010101001010010(0+1)^{*}
$$

is also regular
$\therefore$ (iii) \& (iv) are regular sets.

## 44. Ans: (b)

Sol: i) $n^{\text {th }}$ symbol from right end is ' 1 ' $\Rightarrow 2^{\text {n }}$ states
ii) $n^{\text {th }}$ symbol from left end is ' 1 ' $\Rightarrow(n+2)$ states.
$\therefore$ (i) has 64 states (ii) has 7 states.

## 45. Ans: (c)

Sol: $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b}+\mathrm{c})^{*}, \mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})=\mathrm{n}_{\mathrm{c}}(\mathrm{w})\right\}$
L is not regular because symbols have dependency.
46. Ans: (a)

Sol: If $X=r+X s$ and $s$ has no ' $\varepsilon$ ' then $x$ has unique solution otherwise infinite solutions.
47. Ans: (a), (b) \& (d)

Sol: i. $\phi+\varepsilon=\varepsilon$
ii. $\{\varepsilon\}^{*}=\varepsilon=\varepsilon^{*}$
48. Ans: (b) \& (d)

Sol: Option (a) is regular because it is finite language therefore it is regular.
Options (b) \& (d) non regular because it is not satisfying the pumping lemma
49. Ans: (a) \& (d)

Sol: 1.

2.

3.

4.

50. Ans: (a) \& (d)

Sol: Mealy machine does not responds for $\varepsilon$.
Moore machine output depends only on current state

## 51. Ans: (b) \& (d)

Sol: (b) We know that $\mathrm{L}=\Sigma^{*}-\mathrm{L}$
(d)


DFA for $(0+1)^{*} 1=(0+1)^{*} 11^{*}$
Interchange final \& Non-final states


$$
\begin{aligned}
\mathrm{L}= & \left(0+11^{*} 0\right)^{*} \\
\mathrm{~L}= & \left(0+1^{+} 0\right)^{*} \\
\mathrm{~L}= & \left(1^{*} 0\right)^{*} \\
& 0+1^{+} 0=1^{*} 0
\end{aligned}
$$

Chapter
3

## Context Free Languages (CFG, PDA)

1. Ans: (c)

Sol: CFLs are closed under:
i) Finite union
ii) Union
iii) Concatenation
iv) Kleene closure
v) Reversal

CFLs are not closed under:
i) Intersection
ii) Complement
iii) Infinite union

## 02. Ans: (a)

Sol: CFLs are closed under:
i) Finite union
ii) Homomorphism
iii) Inverse Homomorphism
iv) Substitution
v) Reversal
vi) Init
vii) Quotient with regular set.

## 03. Ans: (d)

Sol: CFLs are not closed under:
i) Intersection
ii) Intersection with non CFL
iii) Infinite union
04. Ans: (a)

Sol: Decidable problems for CFLs.
i) Emptiness
ii) Finiteness
iii) Non emptiness
iv) Non finiteness (infiniteness)
v) Membership

Following problems are undecidable about CFLs:
i) Equivalence
ii) Containment
iii) Totality
05. Ans: (a)

Sol: i) $\left\{0^{n} 1^{n} \mid n>99\right\}$ is CFL
ii) $\left\{a^{n} b^{n} c^{n} \mid n<990\right\}$ is finite, So CFL
iii) $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{m}} \mathrm{c} \mid \mathrm{m}=l\right.$ or $\left.\mathrm{m}=\mathrm{n}\right\}$ is CFL
iv) $\left\{w w \mid w \in(a+b)^{*}\right.$ and $\left.|\mathrm{w}|<1000\right\}$ is finite,

## so CFL

All languages are CFLs
06. Ans: (a)

Sol: $\mathrm{L}_{1}=\left\{\mathrm{ww} \mid \mathrm{w} \in(0+1)^{*}\right\}$ is not CFL
$\overline{\mathrm{L}}_{1}=\Sigma^{*}-\mathrm{L}_{1}$ is CFL
$L_{2}=\left\{a^{n} b^{n} c^{n} \mid n>1\right\}$ is not CFL
$\overline{\mathrm{L}}_{2}=\Sigma^{*}-\mathrm{L}_{2}$ is CFL.
07. Ans: (b)

Sol: i) $\left\{\mathrm{ww}^{\mathrm{R}} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b})^{*}\right\}$ is CFL but not DCFL
ii) $\left\{w \$ w^{R} \mid w \in(a+b)^{*}\right\}$ is DCFL but not regular
$\therefore$ (ii) accepted by DPDA but (i) accepted by PDA.

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08. Ans: (b)

Sol: i) $\left\{0^{\mathrm{n}} 1^{\mathrm{n}} \mid \mathrm{n}>1\right\}$ is DCFL
ii) $\left\{0^{\mathrm{n}} 1^{2 \mathrm{n}} \mid \mathrm{n}>1\right\} \cup\left\{0^{\mathrm{n}} 1^{\mathrm{n}} \mid \mathrm{n}>10\right\}$ is CFL but not DCFL
$\therefore$ (i) accepted by DPDA and (ii) accepted by PDA.
09. Ans: (c)

## Sol: $\mathrm{S} \rightarrow \mathrm{SS}|\mathrm{a}| \varepsilon$

It is ambiguous CFG.
Every string generated by the grammar has more than one derivation tree.
10. Ans: (a), (b) \& (c)

Sol: $\mathrm{S} \rightarrow \mathrm{a} \mid \mathrm{A}$
$\mathrm{A} \rightarrow \mathrm{a}$
It is ambiguous CFG and has 2 parse trees for string ' $a$ '
For string ' $a$ ', 2 parse trees, 2 LMD's and 2 RMD's are there.

## 11. Ans: (d)

Sol: $\mathrm{L}=\left\{\mathrm{a}^{l} \mathrm{~b}^{\mathrm{m}} \mathrm{c}^{\mathrm{n}} \mid l, \mathrm{~m}, \mathrm{n}>1\right\}$
$\mathrm{L}=\left\{\mathrm{aa}^{+} \mathrm{bb}^{+} \mathrm{cc}^{+}\right\}$
unambiguous CFG that generates L :

$$
\mathrm{S} \rightarrow \mathrm{ABC}
$$

$$
\mathrm{A} \rightarrow \mathrm{aA} \mid \mathrm{aa}
$$

$$
\mathrm{B} \rightarrow \mathrm{bB} \mid \mathrm{bb}
$$

$$
\mathrm{C} \rightarrow \mathrm{cC} \mid \mathrm{cc}
$$

For given L, there exist unambiguous CFG, So $L$ is called as Inherently unambiguous language.

## 12. Ans: (d)

Sol: i) $\left\{a^{p} \mid p\right.$ is prime $\}$ is not regular
ii) $\left\{a^{p} \mid p\right.$ is not prime $\}$ is not regular
iii) $\left\{\mathrm{a}^{2^{n}} \mid \mathrm{n} \geq 1\right\}$ is not regular
iv) $\left\{\mathrm{a}^{\mathrm{n}!} \mid \mathrm{n} \geq 0\right\}$ is not regular

If language over 1 symbol is not regular then it is also not CFL. So all are not CLFs
13. Ans: (c) \& (d)

Sol: i) $\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b})^{*}\right\}=(\mathrm{a}+\mathrm{b})^{*}$ is regular
ii) $\left\{w w \mid w \in(a+b)^{*}\right\}$ is not CFL
iii) $\left\{w w w \mid w \in(a+b)^{*}\right\}$ is not CFL
iv) $\left\{w^{R}{ }^{R} w \mid w \in(a+b)^{*}\right\}$ is not CFL

Only (i) is regular and remaining are not regular.
So, only (i) is CFL and remaining are not CFLs.
14. Ans: (c)

Sol: Decidable problems about CFLs:
i) Emptiness
ii) Infiniteness
iii) Membership
15. Ans: (b)

Sol: Finiteness, Infiniteness, Membership are decidable for CFLs.
16. Ans: (c)

Sol: DCFLs are closed under:
i) Complement
ii) Inverse homomorphism
iii) Intersection with regular set

## 17. Ans: (a)

Sol: DCFLs can be described by LR(k) grammars.
18. Ans: (a)

Sol: $\mathrm{L}=\{1,01, \ldots, 110,0110, \ldots, \ldots\}$
It is neither regular nor CFL.
19. Ans: (a)

Sol: $\mathrm{L}=0^{*} 10^{*} 1$
L is regular, so CFL.
20. Ans: (d)

Sol: In CNF, if length of string is $n$ then derivation length is always $2 \mathrm{n}-1$.
If Derivation length is k then string length is $(k+1) / 2$

## 21. Ans: (a)

Sol: Top down parsing can use PDA.
GNF CFG can be converted to PDA. Such PDA derives a string using LMD.

## 22. Ans: (a)

Sol: If PDA simulated by GNF CFG then the derivation of a string uses LMD.
23. Ans: (b)

## Sol:

i) $L=\left\{w \mid w \in(a+b)^{*}, n_{a}(w)\right.$ is divisible by 3 and $n_{b}(w)$ is divisible by 5$\}$ is regular
ii) $L=\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b})^{*}, \mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})\right\}$ is not regular but CFL
iii) $L=\left\{w \mid w \in(a+b)^{*}, n_{a}(w)=n_{b}(w)\right.$, $\mathrm{n}_{\mathrm{a}}(\mathrm{w})+\mathrm{n}_{\mathrm{b}}(\mathrm{w})$ is divisible by 3$\}$ is not regular but CFL
iv) $L=\left\{w \mid w \in(a+b)^{*}, n_{a}(w) \neq n_{b}(w)\right\}$ is not regular but CFLs.
So, (i) is regular and remaining are CFLs.
24. Ans: (c)

Sol:
i) $\mathrm{L}=(\mathrm{a}+\mathrm{b}+\mathrm{c})^{*}$ is regular
ii) $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b}+\mathrm{c})^{*}, \mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})\right.$ or $\left.\mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{c}}(\mathrm{w})\right\}$ is CFL.
iii) $L=\left\{w \mid w \in(a+b+c)^{*}\right.$, $\left.\mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})+\mathrm{n}_{\mathrm{c}}(\mathrm{w})\right\}$ is CFL.
iv) $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}=\mathrm{b}+\mathrm{c})^{*}, \mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})\right.$, $\left.\mathrm{n}_{\mathrm{a}}(\mathrm{w})=4 \mathrm{n}_{\mathrm{c}}(\mathrm{w})\right\}$ is not CFL.
25. Ans: (a)

Sol: $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d})^{*}, \mathrm{n}_{\mathrm{a}}(\mathrm{w})=\mathrm{n}_{\mathrm{b}}(\mathrm{w})\right.$

$$
\left.=\mathrm{n}_{\mathrm{c}}(\mathrm{w})=\mathrm{n}_{\mathrm{d}}(\mathrm{w})\right\}
$$

L is not CFL but $\overline{\mathrm{L}}$ is CFL
$\mathrm{L}_{1}=\left\{\mathrm{ww} \mid \mathrm{w} \in(\mathrm{a}+\mathrm{b})^{*}\right\}$
$\mathrm{L}_{1}$ is not CFL but $\overline{\mathrm{L}}$ is CFL.

## 26. Ans: (a) \& (b)

Sol: DTM $\cong$ NTM
Every DCFL has equivalent DTM
Every CFL can have either DPDA and NPDA
Every Recursive Language is REL but vice versa not true
27. Ans: (a) \& (c)

Sol: Decidable for PDAs or CFG includes Membership, infiniteness and emptiness
Any 2 PDAs are not necessary equivalent
Ambiguity problem for CFGs is undecidable
28. Ans: (c) \& (d)

Sol: Complement of CFL is recursive.
Intersection and difference operations are not closed for CFL's
29. Ans: (a) \& (c)

Sol: (a) $\left\{\mathrm{wxw}^{\mathrm{R}} \mid \mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}, \mathrm{x} \in\{\mathrm{a}, \mathrm{b}\}^{+}\right\}$is regular hence it can be DCFL
(b) $\left\{x w w^{R} \mid w \in\{a, b\}^{*}, x \in\{a, b\}\right\}$ is CFL but not DCFL
(c) $\left\{\mathrm{wxw}^{R} \mid \mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}, \mathrm{x} \in \mathrm{T}\right\}$ is odd palindrome hence it is DCFL
(d) $\left\{w w^{R} x \mid w \in\{a, b\}^{*}, x \in\{a, b\}^{*}\right\}$ is CFL
30. Ans: (d)

Sol: (i) $\mathrm{L}_{1}=\left\{1^{\mathrm{n}} 0^{\mathrm{n}} 1^{\mathrm{n}} 0^{\mathrm{n}} / \mathrm{n}>0\right\}$ as there is association all four members it cannot be CFL.
(ii) $\mathrm{L}_{2}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}}\right\} \cup\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{2 \mathrm{n}}\right\}$ it is equivalent $\mathrm{L}_{2}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{k}} / \mathrm{n} \leq \mathrm{k} \leq 2 \mathrm{n}\right\}$ is CFL
31. Ans: (a), (c) \& (d)

Sol: (a), (c) \& (d) we can have PDA therefore they are CFLs.
32. Ans: (a) \& (d)

Sol: (a) \& (d) are true statements because it length of the derivations $k$ and derivation appears as LMD.
33. Ans: (a) \& (b)
Sol:
(a) $S \rightarrow a S b b$
$\mathrm{S} \rightarrow \mathrm{aabb}$
(b) $\mathrm{S} \rightarrow \mathrm{aSbb}$
S $\rightarrow$ aaSAbb
$\mathrm{S} \rightarrow$ aaaAbb
$\mathrm{S} \rightarrow$ aaabBbb
$\mathrm{S} \rightarrow$ aaabbbb

## 34. Ans: (b) \& (d)

Sol: The given grammar generates the odd length palindromes and recognizes by the DPDA. Given grammar is not ambiguous.
35. Ans: (a) \& (b)

Sol: (a) $\left\{a^{n} b^{n}\right\}$ is DCFL and $\left\{a^{n} b^{n}\right\}^{+}$is also DCFL
(b) $\left\{w w^{R}\right\}$ is CFL and $\left\{w w^{R}\right\}^{+}$is also CFL
(c) $\left\{a^{*} b^{*}\right\}$ is regular and $\left\{a^{*} b^{*}\right\}^{+}$is also regular
(d) $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}} \mathrm{c}^{\mathrm{n}} \mid \mathrm{n} \geq 0\right\}$ is CSL but it is not closed under Kleene closure

## Chapter

4
4. Recursive Enumerable Languages
(REG, TM, REL, CSG, LBA, CSL, Undecidability)

1. Ans: (d)

Sol: Turing machine is equivalent to the following:

- TM with single tape
- TM start with blank tape
- TM with 2-way infinite tape
- TM with 2 symbols and blank

2. Ans: (a) \& (c)

Sol: (a) TM with one push down tape and read only is equivalent to push down automata
(b) TM with two push down tapes is equivalent to TM
(c) TM without alphabet is not equivalent to any machine.
03. Ans: (d)

Sol: (a) TM with 4 counters is equivalent to TM
(b) TM with 3 counters is equivalent to TM
(c) TM with 2 counters is equivalent to TM
04. Ans: (d)

Sol: (a) TM with multiple heads $\cong \mathrm{TM}$
(b) Multi dimensional tape $\mathrm{TM} \cong \mathrm{TM}$
(c) n-dimensional tape $\mathrm{TM} \cong \mathrm{TM}$
05. Ans: (a)

Sol: (a) TM that have no link is equivalent to finite automata
(b) TM with 3 pebbles $\cong \mathrm{TM}$
(c) 2-way infinite tape $\mathrm{TM} \cong \mathrm{TM}$
(d) 100000 tape $\mathrm{TM} \cong \mathrm{TM}$
06. Ans: (a) \& (b)

Sol: (a) TM that cannot leave their input is equivalent to LBA
(b) TM that cannot use more than n ! cells on ' $n$ ' length input is not equivalent to TM.
(c) 3-tape TM is equivalent to TM
(d) TM with single symbol alphabet is equivalent to TM
07. Ans: (d)

Sol: The set of partial recursive functions represent the sets computed by turing machines.
08. Ans: (a)

Sol: (a) Turing machines are equivalent to C programs.
(b) TMs that always halt are equivalent to halting C programs.
(c) Halting C programs not equivalent to turing machines
(d) $\mathrm{C}++$ programs are equivalent to turing machines.
09. Ans: (c)

Sol: Set of turing machines is logically equivalent to set of LISP programs.
10. Ans: (b)

Sol: Class of halting turing machines is equivalent to class of halting prolog programs
$\therefore$ The class of prolog programs describes a richer set of functions.
11. Ans: (a)

Sol: The class of an assembly programs is equivalent to class of all functions computed by turing machines.
12. Ans: (a)

Sol: Set of regular languages and set of recursive languages are closed under intersection and complement.
13. Ans: (c)

Sol:

- Non-deterministic TM is equivalent to deterministic TM
- Non-deterministic halting TM is equivalent to deterministic halting TM.

14. Ans: (d)

Sol: Universal TM is equivalent to TM.
15. Ans: (a)

Sol: $\mathrm{L}=$ Set of regular expressions
$\overline{\mathrm{L}}=\phi$
L is REL and $\overline{\mathrm{L}}$ is also REL
So, L is recursive language.
16. Ans: (a)

Sol: Algorithms $\cong$ Procedures $\cong$ TMs

## 17. Ans: (a)

Sol: Hyper computer is equivalent to TM.
TM can accept non-regular.
18. Ans: (b)

Sol: TM head restricted to input accepts CSL

## 19. Ans: (b)

Sol: Type 0 grammar is equivalent to turing machine.
20. Ans: (c)

Sol: Type 1 grammar is equivalent to linear bounded automata.

## 21. Ans: (a) \& (d)

Sol: $\mathrm{L}=\left\{\mathrm{wwwwwww} / \mathrm{w} \in(\mathrm{a}+\mathrm{b}+\mathrm{c})^{*}\right\}$
L is CSL but not CFL
$\mathrm{So}, \mathrm{L}$ is also recursive language

## 22. Ans: (a) \& (d)

Sol: $L=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}!} \mathrm{c}^{(\mathrm{n}!)!} \mid \mathrm{n}>1\right\}$
L is CSL but not CFL
$\mathrm{So}, \mathrm{L}$ is also recursive language
So (a) \& (d) are false
23. Ans: (d)

Sol: $L=\left\{w w^{R} / w \in(a+b)^{*}\right\}$
L is CFL but not regular

## 24. Ans: (d)

Sol: $(0+1+\ldots-+n+A+B+\ldots+F) * 1(0+1+$ $\ldots . .+9+\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}+\mathrm{E}+\mathrm{F})^{*}$
It is regular language
25. Ans: (c)

Sol: $L=a^{47^{n}}$
L is CSL

## 26. Ans: (d)

Sol: Recursive languages are closed under union, intersection, complement, reversal and concatenation.

Recursive languages are not closed under substitution, homomorphism, quotient and subset.

## 27. Ans: (d)

## Sol:

- Regular sets are closed under finite union, intersection, complement, homomorphism, inverse homomorphism and reversal.
- Containment, equivalence, emptiness, totality problems are decidable for regular sets.


## 28. Ans: (d)

Sol: The following problems are undecidable for CFL's

1. Equivalence
2. Totality
3. Containment

## 29. Ans: (c)

Sol: The following problems are undecidable for CSL's

1. Finiteness
2. Emptiness
3. Totality $\left(\Sigma^{*}\right)$
4. Equivalence
5. Containment
6. Ans: (d)

Sol: Undecidable problems for recursive sets:

1. Emptiness
2. Infiniteness
3. Regularity
4. Equivalence
5. Containment

Membership problem is decidable for recursive sets
31. Ans: (a), (b) \&(d)

Sol: Given TM accepts only 2 strings of length one $\mathrm{L}=\{0,1\}$
32. Ans: (a), (b) \& (c)

Sol: All conversions are possible other than convert NPDA to DPDA.
33. Ans: (c) \& (d)

Sol: $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}} \mathrm{c}^{\mathrm{n}} \mid \mathrm{n} \geq 1\right\}$
L is CSL and it can be defined by DTM in polynomial space
$\therefore \mathrm{L}$ is in CSL and Recursive Languages
34. Ans: (a) \& (c)

Sol: Equality of DPDA is decidable and can be decided in polynomial time
So on the same argument (c) and (a) are true.
35. Ans: (a) \& (d)

Sol: CSL is accepted by LBA and LBA is a TM with finite read and write tape-bounded so it won't be accepted by DFA even if we add any no. of states because the tape of DFA is read only.
Again regular expression always generates regular language

