## GATE IPSUs

## FLUID MECHANICS \& TURBOMACHINERY

## Text Book :

Theory with worked out Examples and Practice Questions

# Fluid Mechanics \& Turbomachinery 

(Solutions for Text Book Practice Questions)

## Chapter <br> 1 <br> Properties of Fluids

1. Ans: (c)

Sol: For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).
02. Ans: 100

Sol: $\quad \tau=\frac{\mu \mathrm{V}}{\mathrm{h}}=\frac{0.2 \times 1.5}{3 \times 10^{-3}}=100 \mathrm{~N} / \mathrm{m}^{2}$
03. Ans: 1

Sol:


$$
\mathrm{F}=\tau \times \mathrm{A}
$$

$W \sin 30=\frac{\mu \mathrm{AV}}{\mathrm{h}}$
$\frac{100}{2}=\frac{1 \times 0.1 \times \mathrm{V}}{2 \times 10^{-3}}$

$$
\mathrm{V}=1 \mathrm{~m} / \mathrm{s}
$$

## Common data Q. 04 \& 05

4. Ans: (c)

Sol: $\mathrm{D}_{1}=100 \mathrm{~mm}, \quad \mathrm{D}_{2}=106 \mathrm{~mm}$
Radial clearance, $\mathrm{h}=\frac{\mathrm{D}_{2}-\mathrm{D}_{1}}{2}$

$$
=\frac{106-100}{2}=3 \mathrm{~mm}
$$

$\mathrm{L}=150 \mathrm{~mm}=0.15 \mathrm{~m}$

$$
\mu=0.2 \text { pa.s }
$$

$$
\mathrm{N}=240 \mathrm{rpm}
$$

$$
\omega=\frac{2 \pi \mathrm{~N}}{60}=\frac{2 \pi \times 240}{60}
$$

$$
\omega=8 \pi
$$

$$
\tau=\frac{\mu \omega \mathrm{r}}{\mathrm{~h}}=\frac{0.2 \times 8 \pi \times 50 \times 10^{-3}}{3 \times 10^{-3}}
$$

$$
=83.77 \mathrm{~N} / \mathrm{m}^{2}
$$

5. Ans: (b)

Sol: Power, $\mathrm{P}=\frac{2 \pi \omega^{2} \mu \mathrm{Lr}^{3}}{\mathrm{~h}}$

$$
\begin{aligned}
& =\frac{2 \pi \times(8 \pi)^{2} \times 0.2 \times 0.15 \times(0.05)^{3}}{3 \times 10^{-3}} \\
& =4.96 \mathrm{Watt}
\end{aligned}
$$

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6. Ans: (c)

Sol:

$\therefore$ Newtonian fluid
07. Ans: (a)

Sol: $\quad \tau=\mu \frac{\mathrm{du}}{\mathrm{dy}}$

$$
u=3 \sin (5 \pi y)
$$

$$
\frac{\mathrm{du}}{\mathrm{dy}}=3 \cos (5 \pi y) \times 5 \pi=15 \pi \cos (5 \pi y)
$$

$$
\left.\tau\right|_{\mathrm{y}=0.05}=\left.\mu \frac{\mathrm{du}}{\mathrm{dy}}\right|_{\mathrm{y}=0.05}
$$

$$
=0.5 \times 15 \pi \cos (5 \pi \times 0.05)
$$

$$
=0.5 \times 15 \pi \times \cos \left(\frac{\pi}{4}\right)=0.5 \times 15 \pi \times \frac{1}{\sqrt{2}}
$$

$$
=7.5 \times 3.14 \times 0.707 \approx 16.6 \mathrm{~N} / \mathrm{m}^{2}
$$

## 08. Ans: (d)

Sol:

- $\quad$ Ideal fluid $\rightarrow \quad$ Shear stress is zero.
- Newtonian fluid $\rightarrow$ Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid $\rightarrow$ Shear stress does not vary linearly with the rate of strain.
- Bingham plastic $\rightarrow$ Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

9. Ans: (b)

Sol: $\mathrm{V}=0.01 \mathrm{~m}^{3}$
$\beta=0.75 \times 10^{-9} \mathrm{~m}^{2} / \mathrm{N}$
$\mathrm{dP}=2 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}$
$\mathrm{K}=\frac{1}{\beta}=\frac{1}{0.75 \times 10^{-9}}=\frac{4}{3} \times 10^{9}$
$K=\frac{-d P}{d V / V}$
$d V=\frac{-2 \times 10^{7} \times 10^{-2} \times 3}{4 \times 10^{9}}=-1.5 \times 10^{-4}$
10. Ans: 320 Pa

Sol: $\Delta \mathrm{P}=\frac{8 \sigma}{\mathrm{D}}=\frac{8 \times 0.04}{1 \times 10^{-3}}=\frac{32 \times 10^{-2}}{10^{-3}}$
$\Delta \mathrm{P}=320 \mathrm{~N} / \mathrm{m}^{2}$
11. Ans: (a, d)

Sol: Given data: S.G $=0.8$ and

$$
v=2 \text { centistokes }=2 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}
$$

Mass density, $\rho=(\mathrm{S} . \mathrm{G}) \times \rho_{\text {water at } 4^{\circ} \mathrm{C}}$

$$
=0.8 \times 10^{3}=800 \mathrm{~kg} / \mathrm{m}^{3}
$$

Dynamic viscosity, $\mu=\rho \times v$

$$
\begin{aligned}
\mu & =800 \times 2 \times 10^{-6} \mathrm{~Pa} . \mathrm{s} \\
& =16 \times 10^{-4} \mathrm{~Pa} . \mathrm{s} \\
& =1.6 \text { centipoise }
\end{aligned}
$$

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## Chapter <br> 2 <br> Pressure Measurement \& Fluid Statics

1. Ans: (a)

Sol: 1 millibar $=10^{-3} \times 10^{5}=100 \mathrm{~N} / \mathrm{m}^{2}$

$$
\text { One } \begin{aligned}
\text { mm of } \mathrm{Hg} & =13.6 \times 10^{3} \times 9.81 \times 1 \times 10^{-3} \\
& =133.416 \mathrm{~N} / \mathrm{m}^{2} \\
1 \mathrm{~N} / \mathrm{mm}^{2} & =1 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2} \\
1 \mathrm{kgf} / \mathrm{cm}^{2} & =9.81 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

2. Ans: (b)

Sol:


Absolute pressure
03. Ans: (c)

Sol: Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure $\mathrm{P}_{\mathrm{A}}$ and $\mathrm{P}_{\mathrm{B}}$ are same.
04. Ans: (b)

Sol:

- The manometer shown in Fig. 1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.
- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.

5. Ans: 2.2

Sol: $\mathrm{h}_{\mathrm{p}}$ in terms of oil

$$
\begin{aligned}
14 \mathrm{~s}_{\mathrm{o}} \mathrm{~h}_{\mathrm{o}} & =\mathrm{s}_{\mathrm{m}} \mathrm{~h}_{\mathrm{m}} \\
0.85 \times \mathrm{h}_{0} & =13.6 \times 0.1 \\
\mathrm{~h}_{0} & =1.6 \mathrm{~m} \\
\mathrm{~h}_{\mathrm{p}} & =0.6+1.6 \\
\Rightarrow \quad \mathrm{~h}_{\mathrm{p}} & =2.2 \mathrm{~m} \text { of oil }
\end{aligned}
$$

(or) $\mathrm{P}_{\mathrm{p}}-\gamma_{\text {oil }} \times 0.6-\gamma_{\mathrm{Hg}} \times 0.1=\mathrm{P}_{\text {atm }}$

$$
\begin{aligned}
\frac{\mathrm{P}_{\mathrm{p}}-\mathrm{P}_{\mathrm{atm}}}{\gamma_{\text {oil }}} & =\left(\frac{\gamma_{\mathrm{Hg}}}{\gamma_{\text {oil }}} \times 0.1+0.6\right) \\
& =\frac{13.6}{0.85} \times 0.1+0.6=2.2 \mathrm{~m} \text { of oil }
\end{aligned}
$$

Gauge pressure of P in terms of m of oil

$$
=2.2 \mathrm{~m} \text { of oil }
$$

6. Ans: (b)

Sol: $\mathrm{h}_{\mathrm{M}}-\frac{\mathrm{s}_{\mathrm{w}}}{\mathrm{s}_{0}} \mathrm{~h}_{\mathrm{w}_{1}}=\mathrm{h}_{\mathrm{N}}-\frac{\mathrm{s}_{\mathrm{w}} \mathrm{h}_{\mathrm{w}_{2}}}{\mathrm{~s}_{0}}-\mathrm{h}_{0}$

$$
\begin{aligned}
& \mathrm{h}_{\mathrm{M}}-\mathrm{h}_{\mathrm{N}}=\frac{9}{0.83}-\frac{18}{0.83}-3 \\
& \mathrm{~h}_{\mathrm{M}}-\mathrm{h}_{\mathrm{N}}=-13.843 \mathrm{~cm} \text { of oil }
\end{aligned}
$$

## 07. Ans: 2.125

Sol:

$$
\begin{aligned}
\mathrm{h}_{\mathrm{P}} & =\overline{\mathrm{h}}+\frac{\mathrm{I}}{\mathrm{~A} \mathrm{\bar{h}}} \\
& =2+\frac{\pi \mathrm{D}^{4} \times 4}{64 \times \mathrm{D}^{2} \times 2 \times \pi} \\
& =2+\frac{2^{2} \times 4}{64 \times 2}=2.125 \mathrm{~m}
\end{aligned}
$$


08. Ans: 10

Sol: $\quad \mathrm{F}=\rho \mathrm{gh} A$

$$
=9810 \times 1.625 \times \frac{\pi}{4}\left(1.2^{2}-0.8^{2}\right)
$$

$\mathrm{F}=10 \mathrm{kN}$
09. Ans: 1

Sol:

$\mathrm{F}_{\text {bottom }}=\rho \mathrm{g} \times 2 \mathrm{x} \times 2 \mathrm{x} \times \mathrm{x}$
$F_{V}=\rho g x \times 2 x \times 2 x$

$$
\frac{\mathrm{F}_{\mathrm{B}}}{\mathrm{~F}_{\mathrm{V}}}=1
$$

10. Ans: 10

Sol: $\quad F_{V}=x \times \pi$

$$
\begin{aligned}
\mathrm{F}_{\mathrm{V}} & =\rho \mathrm{gV}=1000 \times 10 \times \frac{\pi \times 2^{2}}{4} \\
\mathrm{~F}_{\mathrm{V}} & =10 \pi \mathrm{kN} \\
\therefore \mathrm{x} & =10
\end{aligned}
$$


11. Ans: (d)

Sol: $\mathrm{F}_{\text {net }}=\mathrm{F}_{\mathrm{H} 1}-\mathrm{F}_{\mathrm{H} 2}$

$$
\begin{aligned}
\mathrm{F}_{\mathrm{H} 1} & =\gamma \times \frac{\mathrm{D}}{2} \times \mathrm{D} \times 1=\frac{\gamma \mathrm{D}^{2}}{2} \\
\mathrm{~F}_{\mathrm{H} 2} & =\gamma \times \frac{\mathrm{D}}{4} \times \frac{\mathrm{D}}{2} \times 1=\frac{\gamma \mathrm{D}^{2}}{8} \\
& =\gamma \mathrm{D}^{2}\left(\frac{1}{2}-\frac{1}{8}\right)=\frac{3 \gamma \mathrm{D}^{2}}{8}
\end{aligned}
$$

12. Ans: 2

Sol: Let P be the absolute pressure of fluid f 3 at mid-height level of the tank. Starting from the open limb of the manometer (where pressure $=P_{a t m}$ ) we write :
$\mathrm{P}_{\mathrm{atm}}+\gamma \times 1.2-2 \gamma \times 0.2-0.5 \gamma \times\left(0.6+\frac{\mathrm{h}}{2}\right)=\mathrm{P}$
or $\mathrm{P}-\mathrm{P}_{\text {atm }}=\mathrm{P}_{\text {gauge }}$

$$
=\gamma\left(1.2-2 \times 0.2-0.5 \times 0.6-0.5 \times \frac{\mathrm{h}}{2}\right)
$$

For $\mathrm{P}_{\text {gauge }}$ to be zero, we have,

$$
\gamma(1.2-0.4-0.3-0.25 \mathrm{~h})=0
$$

or $\quad \mathrm{h}=\frac{0.5}{0.25}=2$
13. Ans: (a, c)

Sol: The limitations of piezometer are :

- It can't measure gas pressure.
- It can't measure high pressure.

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## Chapter <br> 3 <br> Buoyancy and Metacentric Height

1. Ans: (d)

Sol:


$$
\begin{aligned}
\mathrm{F}_{\mathrm{B}} & =\text { weight of body } \\
\rho_{\mathrm{b}} \mathrm{~V}_{\mathrm{b}} & =\rho_{\mathrm{f}} \mathrm{~g} \mathrm{~V}_{\mathrm{f}} \mathrm{~d} \\
640 \times 4 \times 2 \times 1.25 & =1025 \times(4 \times 1.25 \times \mathrm{d}) \\
\mathrm{d} & =1.248 \mathrm{~m} \\
\mathrm{~V}_{\mathrm{fd}} & =1.248 \times 4 \times 1.25 \\
\mathrm{~V}_{\mathrm{fd}} & =6.24 \mathrm{~m}^{3}
\end{aligned}
$$

2. Ans: (c)

Sol: Surface area of cube $=6 a^{2}$
Surface area of sphere $=4 \pi r^{2}$
$4 \pi r^{2}=6 a^{2}$

$$
\frac{2 \pi}{3}=\left(\frac{\mathrm{a}}{\mathrm{r}}\right)^{2}
$$

$$
\mathrm{F}_{\mathrm{b}, \mathrm{~s}} \propto \mathrm{~V}_{\mathrm{s}}
$$

$$
=\frac{\frac{4}{3} \pi r^{3}}{\mathrm{a}^{3}}=\frac{4}{3} \frac{\pi r^{3}}{\left(r \sqrt{\frac{2 \pi}{3}}\right)^{3}}
$$

$$
=\frac{4}{3} \frac{\pi r^{3}}{\left(\sqrt{\frac{2 \pi}{3}} \times \sqrt{\frac{2 \pi}{3}} r^{3}\right)}=\sqrt{\frac{6}{\pi}}
$$

3. Ans: 4.76

Sol: $\quad F_{B}=F_{B, H g}+F_{B, W}$
$W_{B}=F_{B}$


$$
\begin{aligned}
& \rho_{\mathrm{b}} \mathrm{~g} \forall_{\mathrm{b}}=\rho_{\mathrm{Hg}} \mathrm{~g} \forall_{\mathrm{Hg}}+\rho_{\mathrm{w}} \mathrm{~g} \forall_{\mathrm{w}} \\
& \rho_{\mathrm{b}} \forall_{\mathrm{b}}=\rho_{\mathrm{Hg}} \forall_{\mathrm{Hg}}+\rho_{\mathrm{w}} \forall_{\mathrm{w}} \\
& \mathrm{~S} \times \forall_{\mathrm{b}}=\mathrm{S}_{\mathrm{Hg}} \forall_{\mathrm{Hg}}+\mathrm{S}_{\mathrm{w}} \forall_{\mathrm{w}} \\
& 7.6 \times 10^{3}=13.6 \times 10^{2}(10-\mathrm{x})+10^{2} \times \mathrm{x} \\
& -6000=-1260 \mathrm{x} \\
& \quad \mathrm{x}=4.76 \mathrm{~cm}
\end{aligned}
$$

4. Ans: 11

Sol:

## $\nabla$



$$
\begin{aligned}
\mathrm{F}_{\mathrm{B}} & =\mathrm{W}+\mathrm{T} \\
\mathrm{~W} & =\mathrm{F}_{\mathrm{B}}-\mathrm{T} \\
& =\rho_{\mathrm{f}} \mathrm{~g} \mathrm{~V}_{\mathrm{fd}}-\mathrm{T} \\
& =10^{3} \times 9.81 \times \frac{4}{3} \pi(0.8)^{3}-\left(10 \times 10^{3}\right) \\
& =21-10 \\
\mathrm{~W} & =11 \mathrm{kN}
\end{aligned}
$$

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5. Ans: 1.375

Sol: $W_{\text {water }}=5 \mathrm{~N}$

$$
\begin{aligned}
\mathrm{W}_{\text {oil }} & =7 \mathrm{~N} \\
\mathrm{~S} & =0.85
\end{aligned}
$$

W - Weight in air

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{B} 1}=\mathrm{W}-5 \\
& \mathrm{~F}_{\mathrm{B} 2}=\mathrm{W}-7
\end{aligned}
$$

$$
\begin{equation*}
\mathrm{W}-5=\rho_{1} \mathrm{~g} \mathrm{~V}_{\mathrm{fd}} \tag{1}
\end{equation*}
$$

$\mathrm{W}-7=\rho_{2} \mathrm{gV}_{\mathrm{fd}} \ldots .$. (2)
$\mathrm{V}_{\mathrm{fd}}=\mathrm{V}_{\mathrm{b}}$
$\mathrm{W}-5=\rho_{1} \mathrm{gV}_{\mathrm{b}}$
$\frac{\mathrm{W}-7=\rho_{2} g V_{\mathrm{b}}}{2=\left(\rho_{1}-\rho_{2}\right) \mathrm{gV} \mathrm{b}_{\mathrm{b}}}$
$\mathrm{V}_{\mathrm{b}}=\frac{2}{(1000-850) 9.81}$
$\mathrm{V}_{\mathrm{b}}=1.3591 \times 10^{-3} \mathrm{~m}^{3}$
$\mathrm{W}=5+\left(9810 \times 1.3591 \times 10^{-3}\right)$
$\mathrm{W}=18.33 \mathrm{~N}$
$\mathrm{W}=\rho_{\mathrm{b}} \mathrm{g} \mathrm{V}_{\mathrm{b}}$
$\frac{18.33}{9.81 \times 1.3591 \times 10^{-3}}=\rho_{b}$
$\rho_{\mathrm{b}}=1375.05 \mathrm{~kg} / \mathrm{m}^{3}$
$\mathrm{S}_{\mathrm{b}}=1.375$

## 06. Ans: (d)

Sol: For a floating body to be stable, metacentre should be above its center of gravity. Mathematically GM $>0$.
07. Ans: (b)

Sol: $\quad W=F_{B}$
$\rho_{\mathrm{b}} \mathrm{g} \mathrm{V}_{\mathrm{b}}=\rho_{\mathrm{f}} \mathrm{g} \mathrm{V}_{\mathrm{fd}}$
$\rho_{\mathrm{b}} \mathrm{V}_{\mathrm{b}}=\rho_{\mathrm{f}} \mathrm{V}_{\mathrm{fd}}$
$0.6 \times \frac{\pi}{4} \mathrm{~d}^{2} \times 2 \mathrm{~d}=1 \times \frac{\pi}{4} \mathrm{~d}^{2} \times \mathrm{x}$
$\Rightarrow \mathrm{x}=1.2 \mathrm{~d}$
$\mathrm{GM}=\mathrm{BM}-\mathrm{BG}$
$B M=\frac{I}{V}=\frac{\pi d^{4}}{64 \times \frac{\pi}{4} \mathrm{~d}^{2} \times 1.2 \mathrm{~d}}=\frac{\mathrm{d}}{19.2}=0.052 \mathrm{~d}$
$B G=\mathrm{d}-0.6 \mathrm{~d}=0.4 \mathrm{~d}$
Thus, $\mathrm{GM}=0.052 \mathrm{~d}-0.4 \mathrm{~d}=-0.348 \mathrm{~d}$
$\mathrm{GM}<0$
$\Rightarrow$ Hence, the cylinder is in unstable condition.
08. Ans: 122.475

Sol:


The thickness of the oil layer is same on either side of plate $y=$ thickness of oil layer

$$
=\frac{23.5-1.5}{2}=11 \mathrm{~mm}
$$

Shear stress on one side of the plate

$$
\tau=\frac{\mu \mathrm{dU}}{\mathrm{dy}}
$$

$\mathrm{F}_{\mathrm{s}}=$ total shear force (considering both sides of the plate)

$$
\begin{aligned}
& =2 \mathrm{~A} \times \tau=\frac{2 \mathrm{~A} \mu \mathrm{~V}}{\mathrm{y}} \\
& =\frac{2 \times 1.5 \times 1.5 \times 2.5 \times 0.1}{11 \times 10^{-3}}=102.2727 \mathrm{~N}
\end{aligned}
$$

Weight of plate, $W=50 \mathrm{~N}$
Upward force on submerged plate,

$$
\begin{aligned}
\mathrm{F}_{\mathrm{v}}=\rho \mathrm{gV} & =900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3} \\
& =29.7978 \mathrm{~N}
\end{aligned}
$$

Total force required to lift the plate

$$
\begin{aligned}
& =\mathrm{F}_{\mathrm{s}}+\mathrm{W}-\mathrm{F}_{\mathrm{v}} \\
& =102.2727+50-29.7978 \\
& =122.4749 \mathrm{~N}
\end{aligned}
$$

9. Ans: (a, b, c, d)

## Sol:

- Passenger ships have less GM than war ships from comfort point of view.
- Lifting a steel ball submerged in water is easier than lifting it when unsubmerged due to buoyant force acting on the ball.
- Apparent weight of a submerged body is always lower than its actual weight due to the force of buoyancy.
- Inverted U-tube manometers are preferred if difference in pressure is small.


## Chapter <br> 4 <br> Fluid Kinematics

## 01. Ans: (b)

Sol:

- Constant flow rate signifies that the flow is steady.
- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.


## Common Data for Questions 02 \& 03

2. Ans: 0.94

Sol: $\quad a_{\text {Local }}=\frac{\partial V}{\partial t}$

$$
=\frac{\partial}{\partial t}\left(2 t\left(1-\frac{x}{2 L}\right)^{2}\right)
$$

$$
=\left(1-\frac{\mathrm{x}}{2 \mathrm{~L}}\right)^{2} \times 2
$$

$\left(a_{\text {Local }}\right)_{a t} x=0.5, \mathrm{~L}=0.8=2\left(1-\frac{0.5}{2 \times 0.8}\right)^{2}$

$$
=2(1-0.3125)^{2}=0.945 \mathrm{~m} / \mathrm{sec}^{2}
$$

3. Ans: $\mathbf{- 1 3 . 6 8}$

Sol: $a_{\text {convective }}=v \cdot \frac{\partial v}{\partial x}=\left[2 t\left[1-\frac{x}{2 L}\right]^{2}\right] \frac{\partial}{\partial x}\left[2 t\left(1-\frac{x}{2 L}\right)^{2}\right]$

$$
=\left[2 \mathrm{t}\left[1-\frac{\mathrm{x}}{2 \mathrm{~L}}\right]^{2}\right] 2 \mathrm{t}\left[2\left(1-\frac{\mathrm{x}}{2 \mathrm{~L}}\right)\left(-\frac{1}{2 \mathrm{~L}}\right)\right]
$$

At $\mathrm{t}=3 \mathrm{sec} ; \mathrm{x}=0.5 \mathrm{~m} ; \mathrm{L}=0.8 \mathrm{~m}$

$$
\mathrm{a}_{\text {convective }}=2 \times 3\left[1-\frac{0.5}{2 \times 0.8}\right]^{2} \times 2 \times 3\left[2\left(1-\frac{0.5}{2 \times 0.8}\right)\right]\left[\frac{-1}{2 \times 0.8}\right]
$$

$$
\begin{aligned}
& \mathrm{a}_{\text {convective }}=-14.62 \mathrm{~m} / \mathrm{sec}^{2} \\
& \begin{aligned}
\mathrm{a}_{\text {total }}=\mathrm{a}_{\text {local }}+\mathrm{a}_{\text {convective }} & =0.94-14.62 \\
& =-13.68 \mathrm{~m} / \mathrm{sec}^{2}
\end{aligned}
\end{aligned}
$$

4. Ans: (d)

Sol: $u=6 x y-2 x^{2}$
Continuity equation for 2D flow

$$
\begin{aligned}
& \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \\
& \frac{\partial u}{\partial x}=6 y-4 x \\
& (6 y-4 x)+\frac{\partial v}{\partial y}=0 \\
& \frac{\partial v}{\partial y}=(4 x-6 y)=0 \\
& \partial v=(4 x-6 y) d y \\
& \mathrm{v}=\int 4 \mathrm{xdy}-\int 6 \mathrm{ydy} \\
& =4 x y-3 y^{2}+c \\
& =4 x y-3 y^{2}+f(x)
\end{aligned}
$$

5. Ans: $\sqrt{2}=1.414$

Sol: $\frac{\partial \mathrm{V}}{\partial \mathrm{x}}=\frac{1}{3}(\mathrm{~m} / \mathrm{sec} / \mathrm{m})$

$\mathrm{a}_{\mathrm{r}}=\frac{\mathrm{V}^{2}}{\mathrm{R}}=\frac{(3)^{2}}{9}=\frac{9}{9}=1 \mathrm{~m} / \mathrm{s}^{2}$
$\mathrm{a}_{\mathrm{t}}=\mathrm{V} \frac{\partial \mathrm{V}}{\partial \mathrm{x}}=3 \times \frac{1}{3}=1 \mathrm{~m} / \mathrm{s}^{2}$
$a=\sqrt{\left(a_{r}\right)^{2}+\left(a_{t}\right)^{2}}=\sqrt{(1)^{2}+(1)^{2}}=\sqrt{2} \mathrm{~m} / \sec ^{2}$
06. Ans: 13.75

Sol: $a_{t(\text { conv })}=\mathrm{V}_{\text {avg }} \times \frac{\mathrm{dV}}{\mathrm{dx}}$

$$
\begin{aligned}
& a_{t(\text { conv })}=\left(\frac{2.5+3}{2}\right)\left(\frac{3-2.5}{0.1}\right)=2.75 \times 5 \\
& a_{t(\text { conv })}=13.75 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

7. Ans: 0.3

Sol: $Q=A u$

$$
\begin{aligned}
& \mathrm{a}_{\text {Local }}=\frac{\partial \mathrm{u}}{\partial \mathrm{t}}=\frac{\partial}{\partial \mathrm{t}}\left(\frac{\mathrm{Q}}{\mathrm{~A}}\right) \\
& \mathrm{a}_{\text {local }}=\frac{1}{\mathrm{~A}} \frac{\partial \mathrm{Q}}{\partial \mathrm{t}} \\
& \mathrm{a}_{\text {Local }}=\left(\frac{1}{0.4-0.1 \mathrm{x}}\right) \frac{\partial \mathrm{Q}}{\partial \mathrm{t}} \\
& \begin{aligned}
\left(\mathrm{a}_{\text {Local }}\right)_{\mathrm{at} \mathrm{x}} & =0
\end{aligned} \\
& =\frac{1}{0.4} \times 0.12 \quad\left(\because \frac{\partial \mathrm{Q}}{\partial \mathrm{t}}=0.12\right) \\
& \\
& =0.3 \mathrm{~m} / \mathrm{sec}^{2}
\end{aligned}
$$

8. Ans: (b)

Sol: $\psi=x^{2}-y^{2}$

$$
a_{\text {Total }}=\left(a_{x}\right) \hat{i}+\left(a_{y}\right) \hat{j}
$$

$u=-\frac{\partial \psi}{\partial y}=-\frac{\partial}{\partial y}\left(x^{2}-y^{2}\right)=2 y$

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$$
\begin{aligned}
& \mathrm{v}=\frac{\partial \psi}{\partial \mathrm{x}}=\frac{\partial}{\partial \mathrm{x}}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)=2 \mathrm{x} \\
& \mathrm{a}_{\mathrm{x}}=\mathrm{u} \frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\mathrm{v} \frac{\partial \mathrm{u}}{\partial \mathrm{y}} \\
& \quad=(2 \mathrm{y})(0)+(2 \mathrm{x})(2) \\
& \begin{aligned}
\therefore \mathrm{a}_{\mathrm{x}} & =4 \mathrm{x} \\
\mathrm{a}_{\mathrm{y}} & =\mathrm{u} \frac{\partial \mathrm{v}}{\partial \mathrm{x}}+\mathrm{v} \frac{\partial \mathrm{v}}{\partial \mathrm{y}} \\
& =(2 \mathrm{y}) \times(2)+(2 \mathrm{x}) \times(0) \\
\mathrm{a}_{\mathrm{y}} & =4 \mathrm{y} \\
\therefore \mathrm{a} & =(4 \mathrm{x}) \hat{\mathrm{i}}+(4 \mathrm{y}) \hat{\mathrm{j}}
\end{aligned}
\end{aligned}
$$

## 09. Ans: (b)

Sol: Given, The stream function for a potential flow field is $\psi=x^{2}-y^{2}$
$\phi=$ ?
$u=\frac{-\partial \phi}{\partial \mathrm{x}}=-\frac{\partial \psi}{\partial \mathrm{y}}$
$u=-\frac{\partial \psi}{\partial y}=-\frac{\partial\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)}{\partial \mathrm{y}}$
$\mathrm{u}=2 \mathrm{y}$
$u=-\frac{\partial \phi}{\partial x}=2 y$
$\int \partial \phi=-\int 2 y \partial x$
$\phi=-2 x y+c_{1}$
Given, $\phi$ is zero at $(0,0)$
$\therefore \mathrm{c}_{1}=0$
$\therefore \phi=-2 \mathrm{xy}$

## 10. Ans: 4

Sol: Given, 2D - flow field
Velocity, $V=3 x i+4 x y j$

$$
u=3 x, \quad v=4 x y
$$

$\omega_{\mathrm{z}}=\frac{1}{2}\left(\frac{\mathrm{dv}}{\mathrm{dx}}-\frac{\mathrm{du}}{\mathrm{dy}}\right)$
$\omega_{z}=\frac{1}{2}(4 y-0)$
$\left(\omega_{\mathrm{Z}}\right)_{\mathrm{at}(2,2)}=\frac{1}{2} \times 4(2)=4 \mathrm{rad} / \mathrm{sec}$
11. Ans: (b)

Sol: Given, $u=3 x, \quad v=C y, \quad w=2$
The shear stress, $\tau_{\mathrm{xy}}$ is given by

$$
\begin{aligned}
\tau_{x y} & =\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\mu\left[\frac{\partial}{\partial y}(3 x)+\frac{\partial}{\partial x}(C y)\right] \\
& =\mu(0+0)=0
\end{aligned}
$$

12. Ans: (b, c)

Sol: Given : $\vec{V}=x \hat{i}-y \hat{j}$
Thus, $\mathrm{u}=\mathrm{x}$ and $\mathrm{v}=-\mathrm{y}$

$$
\frac{\partial \mathrm{u}}{\partial \mathrm{x}}=1 ; \frac{\partial \mathrm{u}}{\partial \mathrm{y}}=0 ; \frac{\partial \mathrm{v}}{\partial \mathrm{x}}=0 ; \frac{\partial \mathrm{v}}{\partial \mathrm{y}}=-1
$$

$a_{x}=u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}=x \times 1-y \times 0=x$
$a_{y}=u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}=x \times 0+y \times 1=y$
Thus, $\vec{a}=a_{x} \hat{i}+a_{y} \hat{j}=x \hat{i}+y \hat{j}$
$\mathrm{u}=-\frac{\partial \psi}{\partial \mathrm{y}}=\mathrm{x} ;$ On integration, $\psi=-\mathrm{xy}+\mathrm{C}$
$u=-\frac{\partial \phi}{\partial \mathrm{x}}=\mathrm{x}$; On integration, $\phi=-\frac{\mathrm{x}^{2}}{2}+\mathrm{C}$

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## Chapter <br> 5 <br> Energy Equation and its Applications

1. Ans: (c)

Sol: Applying Bernoulli's equation for ideal fluid
$\frac{P_{1}}{\rho g}+Z_{1}+\frac{V_{1}^{2}}{2 g}=\frac{P_{2}}{\rho g}+Z_{2}+\frac{V_{2}^{2}}{2 g}$
$\frac{P_{1}}{\rho g}+\frac{(2)^{2}}{2 g}=\frac{P_{2}}{\rho g}+\frac{(1)^{2}}{2 g}$
$\frac{P_{2}}{\rho g}-\frac{P_{1}}{\rho g}=\frac{4}{2 g}-\frac{1}{2 g}$
$\frac{P_{2}-P_{1}}{\rho g}=\frac{3}{2 g}=\frac{1.5}{g}$
02. Ans: (c)

Sol:

$\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}=1.27 \mathrm{~m}, \quad \frac{\mathrm{P}_{1}}{\rho \mathrm{~g}}=2.5 \mathrm{~m}$
$\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}=0.203 \mathrm{~m}, \quad \frac{\mathrm{P}_{2}}{\rho \mathrm{~g}}=5.407 \mathrm{~m}$

$$
\mathrm{Z}_{1}=2 \mathrm{~m}, \quad \mathrm{Z}_{2}=0 \mathrm{~m}
$$

Total head at (1) - (1)

$$
\begin{aligned}
& =\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\frac{\mathrm{P}_{1}}{\rho \mathrm{~g}}+\mathrm{Z}_{1} \\
& =1.27+2.5+2=5.77 \mathrm{~m}
\end{aligned}
$$

Total head at (2) - (2)

$$
\begin{aligned}
& =\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\frac{\mathrm{P}_{2}}{\rho \mathrm{~g}}+\mathrm{Z}_{2} \\
& =0.203+5.407+0=5.61 \mathrm{~m}
\end{aligned}
$$

Loss of head $=5.77-5.61=0.16 \mathrm{~m}$
$\therefore$ Energy at (1) - (1) > Energy at (2) - (2)
$\therefore$ Flow takes from higher energy to lower energy
i.e. from $\left(\mathrm{S}_{1}\right)$ to $\left(\mathrm{S}_{2}\right)$

Flow takes place from top to bottom.
03. Ans: $\mathbf{1 . 5}$

Sol: $\quad \mathrm{A}_{1}=\frac{\pi}{4} \mathrm{~d}_{1}^{2}=\frac{\pi}{4}(0.1)^{2}=7.85 \times 10^{-3} \mathrm{~mm}^{2}$
$\mathrm{A}_{2}=\frac{\pi}{4} \mathrm{~d}_{2}^{2}=\frac{\pi}{4}(0.05)^{2}=1.96 \times 10^{-3} \mathrm{~mm}^{2}$
$\frac{P_{1}}{\rho g}+\frac{V_{b}^{2}}{2 g}+Z_{1}=\frac{P_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+Z_{2}+h_{L}$
$\mathrm{Z}_{1}=\mathrm{Z}_{2}$, it is in horizontal position
Since, at outlet, pressure is atmospheric
$\mathrm{P}_{2}=0$
$\mathrm{Q}=100 \mathrm{lit} / \mathrm{sec}=0.1 \mathrm{~m}^{3} / \mathrm{sec}$
$V_{1}=\frac{Q}{A_{1}}=\frac{0.1}{7.85 \times 10^{-3}}=12.73 \mathrm{~m} / \mathrm{sec}$
$\mathrm{V}_{2}=\frac{\mathrm{Q}}{\mathrm{A}_{2}}=\frac{0.1}{1.96 \times 10^{-3}}=51.02 \mathrm{~m} / \mathrm{sec}$
$\frac{\mathrm{P}_{1 \text { gauge }}}{\rho_{\text {air }} \times \mathrm{g}}+\frac{(12.73)^{2}}{2 \times 10}=0+\frac{(51.02)^{2}}{2 \times 10}$
$\frac{\mathrm{P}_{1}}{\rho_{\text {air }} \cdot g}=121.53$

$$
\mathrm{P}_{1}=121.53 \times \rho_{\text {air }} \times \mathrm{g}
$$

$$
=1.51 \mathrm{kPa}
$$

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## 04. Ans: 395

Sol: $\mathrm{Q}=100 \mathrm{litre} / \mathrm{sec}=0.1 \mathrm{~m}^{3} / \mathrm{sec}$

$$
\begin{array}{ll}
\mathrm{V}_{1}=100 \mathrm{~m} / \mathrm{sec} ; & \mathrm{P}_{1}=3 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \\
\mathrm{~V}_{2}=50 \mathrm{~m} / \mathrm{sec} ; & \mathrm{P}_{2}=1 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
\end{array}
$$

Power ( P ) = ?
Energy equation :

$$
\begin{aligned}
& \frac{\mathrm{P}_{1}}{\rho \mathrm{~g}}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{1}=\frac{\mathrm{P}_{2}}{\rho \mathrm{~g}}+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{2}+\mathrm{h}_{\mathrm{L}} \\
& \frac{3 \times 10^{5}}{1000 \times 10}+\frac{100^{2}}{2 \times 10}+0=\frac{1 \times 10^{5}}{1000 \times 10}+\frac{50^{2}}{2 \times 10}+0+\mathrm{h}_{\mathrm{L}} \\
& \Rightarrow \quad \mathrm{~h}_{\mathrm{L}}=395 \mathrm{~m} \\
& \mathrm{P}=\rho \mathrm{gQ} \cdot \mathrm{~h}_{\mathrm{L}} \\
& \mathrm{P}=1000 \times 10 \times 0.10 \times 395 \\
& \mathrm{P}=395 \mathrm{~kW}
\end{aligned}
$$

5. Ans: 35

## Sol:



$$
\mathrm{d}_{1}=300 \mathrm{~mm}, \mathrm{~d}_{2}=120 \mathrm{~mm}
$$

$$
\begin{aligned}
\mathrm{Q}_{\mathrm{Th}} & =\frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{\sqrt{\mathrm{~A}_{1}^{2}-\mathrm{A}_{2}^{2}}} \sqrt{2 \mathrm{gh}} \\
& =\frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{\sqrt{\mathrm{~A}_{1}^{2}-\mathrm{A}_{2}^{2}}} \sqrt{2 \mathrm{~g}\left(\frac{\Delta \mathrm{P}}{\mathrm{~W}}\right)}
\end{aligned}
$$

$$
\mathrm{A}_{1}=\frac{\pi}{4} \mathrm{~d}_{1}^{2}=\frac{\pi}{4}(0.30)^{2}=0.07 \mathrm{~m}^{2}
$$

$$
\begin{aligned}
\mathrm{A}_{2} & =\frac{\pi}{4} \mathrm{~d}_{2}^{2}=\frac{\pi}{4}(0.12)^{2}=0.011 \mathrm{~m}^{2} \\
\Delta \mathrm{P} & =4 \mathrm{kPa}, \\
\mathrm{~h} & =\frac{\Delta \mathrm{P}}{\mathrm{w}}=\frac{\Delta \mathrm{P}}{\rho_{\mathrm{f}} \cdot \mathrm{~g}} \\
& =\frac{\Delta \mathrm{P}}{\mathrm{~s}_{\mathrm{f}} \rho_{\mathrm{w}} \mathrm{~g}}=\frac{4 \times 10^{3}}{0.85 \times 1000 \times 9.81} \\
\mathrm{Q}_{\text {Th }} & =\frac{0.07 \times 0.011}{\sqrt{(0.07)^{2}-(0.011)^{2}}} \sqrt{\frac{2 \times 9.81 \times 4 \times 10^{3}}{0.85 \times 1000 \times 9.81}} \\
& =0.035 \mathrm{~m}^{3} / \mathrm{sec}=35.15 \mathrm{ltr} / \mathrm{sec}
\end{aligned}
$$

6. Ans: 65

Sol: $\mathrm{h}_{\text {stag }}=0.30 \mathrm{~m}$

$$
\begin{aligned}
& \mathrm{h}_{\text {stat }}=0.24 \mathrm{~m} \\
& \mathrm{~V}
\end{aligned}=\mathrm{c} \sqrt{2 \mathrm{gh}_{\text {dyna }}}, ~ \begin{aligned}
\mathrm{V} & =1 \sqrt{2 \mathrm{~g}\left(\mathrm{~h}_{\text {stag }}-\mathrm{h}_{\text {stat }}\right)} \\
& =\sqrt{2(9.81)(0.30-0.24)}=1.085 \mathrm{~m} / \mathrm{s} \\
& =1.085 \times 60=65.1 \mathrm{~m} / \mathrm{min}
\end{aligned}
$$

7. Ans: 81.5

Sol: $x=30 \mathrm{~mm}$,

$$
\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}
$$

$\rho_{\text {air }}=1.23 \mathrm{~kg} / \mathrm{m}^{3} ; \quad \rho_{\mathrm{Hg}}=13600 \mathrm{~kg} / \mathrm{m}^{3}$
$\mathrm{C}=1$

$$
\begin{aligned}
& \mathrm{V}=\sqrt{2 \mathrm{gh}_{\mathrm{D}}} \\
& \mathrm{~h}_{\mathrm{D}}=\mathrm{x}\left(\frac{\mathrm{~S}_{\mathrm{m}}}{\mathrm{~S}}-1\right) \\
& \mathrm{h}_{\mathrm{D}}=30 \times 10^{-3}\left(\frac{13600}{1.23}-1\right) \\
& \mathrm{h}_{\mathrm{D}}=331.67 \mathrm{~m} \\
& \mathrm{~V}=1 \times \sqrt{2 \times 10 \times 331.67}=81.5 \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

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8. Ans: 140

Sol: $\mathrm{Q}_{\mathrm{a}}=\mathrm{C}_{\mathrm{d}} \frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{\sqrt{\mathrm{~A}_{1}^{2}-\mathrm{A}_{2}^{2}}} \sqrt{2 \mathrm{gh}}$
$\mathrm{C}_{\mathrm{d}} \propto \frac{1}{\sqrt{\mathrm{~h}}}$
$\frac{\mathrm{C}_{\mathrm{d}_{\text {venuri }}}}{\mathrm{C}_{\mathrm{d}_{\text {oifife }}}}=\frac{0.95}{0.65}=\sqrt{\frac{\mathrm{h}_{\text {orifice }}}{\mathrm{h}_{\text {venturi }}}}$
$\mathrm{h}_{\text {venturi }}=140 \mathrm{~mm}$
09. Ans: (b, d)

Sol:


Applying Bernoulli equation between sections (1) \& (2)

$$
\frac{\mathrm{P}_{1}}{\gamma}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{1}=\frac{\mathrm{P}_{2}}{\gamma}+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{2}
$$

But, $\mathrm{P}_{1}=0=\mathrm{P}_{2} ; \mathrm{V}_{1}=0$;

$$
\mathrm{Z}_{1}-\mathrm{Z}_{2}=2 \mathrm{~m}
$$

So, $0+0+2=0+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+0$

$$
\Rightarrow \mathrm{V}_{2}=2 \sqrt{\mathrm{~g}} \mathrm{~m} / \mathrm{s}
$$

$$
\begin{aligned}
\mathrm{Q} & =\frac{\pi}{4} \mathrm{~d}^{2} \mathrm{~V}_{2}=\frac{\pi}{4} \times\left(3 \times 10^{-2}\right)^{2} \times 2 \sqrt{9.81} \\
& =4.428 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Let the point at the summit be denoted by (3).

Then,

$$
\frac{\mathrm{P}_{1}}{\gamma}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{1}=\frac{\mathrm{P}_{3}}{\gamma}+\frac{\mathrm{V}_{3}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{3}
$$

where, $\mathrm{V}_{3}=\mathrm{V}_{2}=2 \sqrt{\mathrm{~g}} \mathrm{~m} / \mathrm{s}$;

$$
\mathrm{Z}_{3}-\mathrm{Z}_{1}=1.4 \mathrm{~m}
$$

Thus,

$$
\begin{aligned}
\frac{\mathrm{P}_{3}}{\gamma} & =-1.4-\frac{4 \mathrm{~g}}{2 \mathrm{~g}}=-3.4 \\
\Rightarrow \quad \mathrm{P}_{3} & =-3.4 \times 9810 \mathrm{~Pa} \\
& =-33.354 \mathrm{kPa}
\end{aligned}
$$

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## Chapter Momentum equation and <br> 6 its Applications

1. Ans: 1600

Sol: $S=0.80$
$\mathrm{A}=0.02 \mathrm{~m}^{2}$
$\mathrm{V}=10 \mathrm{~m} / \mathrm{sec}$
$\mathrm{F}=\rho . \mathrm{A} \cdot \mathrm{V}^{2}$
$\mathrm{F}=0.80 \times 1000 \times 0.02 \times 10^{2}$
$\mathrm{F}=1600 \mathrm{~N}$
02. Ans: 6000

Sol: $\mathrm{A}=0.015 \mathrm{~m}^{2}$
$\mathrm{V}=15 \mathrm{~m} / \mathrm{sec}$ (Jet velocity)
$\mathrm{U}=5 \mathrm{~m} / \mathrm{sec}$ (Plate velocity)
$\mathrm{F}=\rho \mathrm{A}(\mathrm{V}+\mathrm{U})^{2}$
$F=1000 \times 0.015(15+5)^{2}$
$\mathrm{F}=6000 \mathrm{~N}$
03. Ans: 19.6

Sol: $V=100 \mathrm{~m} / \mathrm{sec}$ (Jet velocity)
$\mathrm{U}=50 \mathrm{~m} / \mathrm{sec} \quad$ (Plate velocity)
$\mathrm{d}=0.1 \mathrm{~m}$
$\mathrm{F}=\rho \mathrm{A}(\mathrm{V}-\mathrm{U})^{2}$
$\mathrm{F}=1000 \times \frac{\pi}{4} \times 0.1^{2} \times(100-50)^{2}$
$\mathrm{F}=19.6 \mathrm{kN}$
04. Ans: (a)

Sol:


$$
\begin{aligned}
\mathrm{F}_{\mathrm{x}} & =\rho a \mathrm{~V}\left(\mathrm{~V}_{1 \mathrm{x}}-\mathrm{V}_{2 \mathrm{x}}\right) \\
& =\rho \mathrm{aV}(\mathrm{~V}-(-\mathrm{V})) \\
& =2 \rho a \mathrm{~V}^{2} \\
& =2 \times 1000 \times 10^{-4} \times 5^{2}=5 \mathrm{~N}
\end{aligned}
$$

5. Ans: (d)

Sol: Given, $V=20 \mathrm{~m} / \mathrm{s}$,

$$
\mathrm{u}=5 \mathrm{~m} / \mathrm{s}
$$

$$
\mathrm{F}_{1}=\rho \mathrm{A}(\mathrm{~V}-\mathrm{u})^{2}
$$

Power $\left(P_{1}\right)=F_{1} \times u=\rho A(V-u)^{2} \times u$

$$
\begin{aligned}
\mathrm{F}_{2} & =\rho \cdot \mathrm{A} \cdot \mathrm{~V} \times \mathrm{V}_{\mathrm{r}} \\
& =\rho \cdot \mathrm{A}(\mathrm{~V}) \cdot(\mathrm{V}-\mathrm{u})
\end{aligned}
$$

Power $\left(\mathrm{P}_{2}\right)=\mathrm{F}_{2} \times \mathrm{u}=\rho A V(\mathrm{~V}-\mathrm{u}) \mathrm{u}$

$$
\begin{aligned}
\frac{P_{1}}{P_{2}} & =\frac{\rho A(V-u)^{2} \times u}{\rho A V(V-u) \times u} \\
& =\frac{V-u}{V}=1-\frac{u}{V} \\
& =1-\frac{5}{20}=0.75
\end{aligned}
$$

6. Ans: 2035

Sol: Given, $\theta=30^{\circ}$, $\dot{\mathrm{m}}=14 \mathrm{~kg} / \mathrm{s}$
$\left(\mathrm{P}_{\mathrm{i}}\right)_{\mathrm{g}}=200 \mathrm{kPa}$,
$\left(\mathrm{P}_{\mathrm{e}}\right)_{\mathrm{g}}=0$
$\mathrm{A}_{\mathrm{i}}=113 \times 10^{-4} \mathrm{~m}^{2}$,

$$
\begin{aligned}
& A_{e}=7 \times 10^{-4} \mathrm{~m}^{2} \\
& \rho=10^{3} \mathrm{~kg} / \mathrm{m}^{3}, \\
& \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

From the continuity equation :

$$
\begin{aligned}
\rho \mathrm{A}_{\mathrm{i}} \mathrm{~V}_{\mathrm{i}} & =14 \\
\text { or } \quad \mathrm{V}_{\mathrm{i}} & =\frac{14}{10^{3} \times 113 \times 10^{-4}}=1.24 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Similarly, $\mathrm{V}_{\mathrm{e}}=\frac{14}{10^{3} \times 7 \times 10^{-4}}=20 \mathrm{~m} / \mathrm{s}$
Let $F_{x}$ be the force exerted by elbow on water in the + ve $x$-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write :
$\left(\mathrm{P}_{\mathrm{i}}\right)_{\mathrm{g}} \mathrm{A}_{\mathrm{i}}+\mathrm{F}_{\mathrm{x}}=\dot{\mathrm{m}}\left(\mathrm{V}_{\mathrm{e}} \cos 30^{\circ}-\mathrm{V}_{\mathrm{i}}\right)$
$\mathrm{F}_{\mathrm{x}}=\dot{\mathrm{m}}\left(\mathrm{V}_{\mathrm{e}} \cos 30^{\circ}-\mathrm{V}_{\mathrm{i}}\right)-\left(\mathrm{P}_{\mathrm{i}}\right)_{\mathrm{g}} \mathrm{A}_{\mathrm{i}}$
$=14\left(20 \times \cos 30^{\circ}-1.24\right)-200 \times 10^{3} \times 113 \times 10^{-4}$
$=225.13-2260$
$=-2034.87 \mathrm{~N} \approx-2035 \mathrm{~N}$
The x-component of water force on elbow is $-\mathrm{F}_{\mathrm{x}}$ (as per Newton's third law), i.e., $\cong 2035 \mathrm{~N}$

07. Ans: $(\mathbf{a}, \mathrm{d})$

Sol: Given:

$$
\begin{gathered}
\mathrm{d}_{\mathrm{j}}=5 \mathrm{~cm}, \\
\mathrm{~V}_{\mathrm{j}}=20 \mathrm{~m} / \mathrm{s}, \\
\mathrm{U}=8 \mathrm{~m} / \mathrm{s} \\
\mathrm{~F}_{\mathrm{x}}=\rho \mathrm{A}_{\mathrm{j}}\left(\mathrm{~V}_{\mathrm{j}}-\mathrm{U}\right)\left(\mathrm{V}_{\mathrm{j}}-\mathrm{U}\right) \\
=10^{3} \times \frac{\pi}{4} \times 0.05^{2} \times(20-8)^{2} \\
=282.74 \mathrm{~N}
\end{gathered}
$$

Work done per second,

$$
\begin{aligned}
\dot{\mathrm{W}} & =\mathrm{F}_{\mathrm{x}} \times \mathrm{U} \\
& =282.74 \times 8=2.262 \mathrm{~kW}
\end{aligned}
$$

Efficiency,

$$
\begin{aligned}
\eta & =\frac{\dot{\mathrm{W}}}{\frac{1}{2} \rho \mathrm{Q} \times \mathrm{V}_{\mathrm{j}}^{2}}=\frac{2 \dot{\mathrm{~W}}}{\rho \mathrm{~A}_{\mathrm{j}} \mathrm{~V}_{\mathrm{j}}^{3}}=\frac{8 \dot{\mathrm{~W}}}{\rho \times \pi \mathrm{d}_{\mathrm{j}}^{2} \times \mathrm{V}_{\mathrm{j}}^{3}} \\
& =\frac{8 \times 2.262 \times 10^{3}}{10^{3} \times \pi \times(0.05)^{2} \times(20)^{3}} \\
& =0.288=28.8 \%
\end{aligned}
$$

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## Chapter <br> 7

1. Ans: (d)

Sol: In a pipe, the flow changes from laminar flow to transition flow at $\mathrm{Re}=2000$. Let V be the average velocity of flow. Then

$$
2000=\frac{\mathrm{V} \times 8 \times 10^{-2}}{0.4 \times 10^{-4}} \Rightarrow \mathrm{~V}=1 \mathrm{~m} / \mathrm{s}
$$

In laminar flow through a pipe,

$$
V_{\max }=2 \times V=2 \mathrm{~m} / \mathrm{s}
$$

## 02. Ans: (d)

Sol: The equation $\tau=\left(-\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right)\left(\frac{\mathrm{r}}{2}\right)$ is valid for laminar as well as turbulent flow through a circular tube.

## 03. Ans: (d)

Sol: $\mathrm{Q}=\mathrm{A} . \mathrm{V}_{\text {avg }}$

$$
\begin{aligned}
\mathrm{Q} & =\mathrm{A} \cdot \frac{\mathrm{~V}_{\max }}{2} \quad\left(\because \mathrm{~V}_{\max }=2 \mathrm{~V}_{\text {avg }}\right) \\
\mathrm{Q}= & \frac{\pi}{4}\left(\frac{40}{1000}\right)^{2} \times \frac{1.5}{2} \\
& =\frac{\pi}{4} \times(0.04)^{2} \times 0.75 \\
& =\frac{\pi}{4} \times \frac{4}{100} \times \frac{4}{100} \times \frac{3}{4}=\frac{3 \pi}{10000} \mathrm{~m}^{3} / \mathrm{sec}
\end{aligned}
$$

## 04. Ans: 1.92

Sol: $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$
$\mathrm{Q}=800 \mathrm{~mm}^{3} / \mathrm{sec}=800 \times\left(10^{-3}\right)^{3} \mathrm{~m}^{3} / \mathrm{sec}$
$\mathrm{L}=2 \mathrm{~m}$
$\mathrm{D}=0.5 \mathrm{~mm}$

$$
\begin{aligned}
& \Delta \mathrm{P}=2 \mathrm{MPa}=2 \times 10^{6} \mathrm{~Pa} \\
& \begin{array}{l}
\mu=? \\
\Delta \mathrm{P}=\frac{128 . \mu \mathrm{QL}}{\pi \mathrm{D}^{4}} \\
2 \times 10^{6}=\frac{128 \times \mu \times 800 \times\left(10^{-3}\right)^{3} \times 2}{\pi\left(0.5 \times 10^{-3}\right)^{4}} \\
\mu=1.917 \mathrm{milli} \mathrm{~Pa}-\mathrm{sec}
\end{array}
\end{aligned}
$$

5. Ans: 0.75

Sol: $\quad U_{r}=U_{\max }\left(1-\left(\frac{r}{R}\right)^{2}\right)$

$$
\begin{aligned}
& {\left[\because \frac{\mathrm{U}}{\mathrm{U}_{\max }}=1-\left(\frac{\mathrm{r}}{\mathrm{R}}\right)^{2}\right]} \\
& =1\left(1-\left(\frac{50}{200}\right)^{2}\right) \\
& =1\left(1-\frac{1}{4}\right)=\frac{3}{4}=0.75 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

6. Ans: 0.08

Sol: Given,

$$
\begin{aligned}
& \rho=0.8 \times 1000=800 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mu=1 \text { Poise }=10^{-1} \mathrm{~N}-\mathrm{s} / \mathrm{m}^{2} \\
& \mathrm{~d}=50 \mathrm{~mm}=0.05 \mathrm{~m} \\
& \text { Velocity }=2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Reynold's Number, $\operatorname{Re}=\frac{\rho V D}{\mu}$

$$
=\frac{800 \times 2 \times 0.05}{10^{-1}}=800
$$

$$
(\because \operatorname{Re}<2000)
$$

$\therefore$ Flow is laminar,
For laminar, Darcy friction factor

$$
\mathrm{f}=\frac{64}{\mathrm{Re}}=\frac{64}{800}=0.08
$$

## 07. Ans: 16

Sol: For fully developed laminar flow,
$h_{f}=\frac{32 \mu \mathrm{VL}}{\rho g D^{2}}(\therefore \mathrm{Q}=\mathrm{AV})$
$\mathrm{h}_{\mathrm{f}}=\frac{32 \mu\left(\frac{\mathrm{Q}}{\mathrm{A}}\right) \mathrm{L}}{\rho \mathrm{gD}^{2}}=\frac{32 \mu \mathrm{QL}}{\mathrm{AD}^{2} \times \rho \mathrm{g}}$
$\mathrm{h}_{\mathrm{f}}=\frac{32 \mu \mathrm{QL}}{\frac{\pi}{4} \mathrm{D}^{2} \times \mathrm{D}^{2} \times \rho \mathrm{g}}$
$h_{f} \propto \frac{1}{D^{4}}$
$\mathrm{h}_{\mathrm{f} 1} \mathrm{D}_{1}^{4}=\mathrm{h}_{\mathrm{f}_{2}} \mathrm{D}_{2}^{4}$
Given, $\quad D_{2}=\frac{D_{1}}{2}$

$$
\begin{aligned}
\mathrm{h}_{\mathrm{f} 1} \times \mathrm{D}_{1}^{4} & =\mathrm{h}_{\mathrm{f} 2} \times\left(\frac{\mathrm{D}_{1}}{2}\right)^{4} \\
\mathrm{~h}_{\mathrm{f}_{2}} & =16 \mathrm{~h}_{\mathrm{f}_{1}}
\end{aligned}
$$

$\therefore \quad$ Head loss, increases by 16 times if diameter is halved.

## 08. Ans: 5.2

Sol: Oil viscosity,

$$
\begin{aligned}
& \mu=10 \text { poise }=10 \times 0.1=1 \mathrm{~N}-\mathrm{s} / \mathrm{m}^{2} \\
& \mathrm{y}=50 \times 10^{-3} \mathrm{~m} \\
& \mathrm{~L}=120 \mathrm{~cm}=1.20 \mathrm{~m}, \quad \Delta \mathrm{P}=3 \times 10^{3} \mathrm{~Pa}
\end{aligned}
$$

Width of plate $=0.2 \mathrm{~m}$,
$\mathrm{Q}=$ ?
$\mathrm{Q}=\mathrm{A} . \mathrm{V}_{\text {avg }}=($ width of plate $\times \mathrm{y}) \mathrm{V}$
$\Delta \mathrm{P}=\frac{12 \mu \mathrm{VL}}{\mathrm{B}^{2}}$
$3 \times 10^{3}=\frac{12 \times 1 \times \mathrm{V} \times 1.20}{\left(50 \times 10^{-3}\right)^{2}}$
$\mathrm{V}=0.52 \mathrm{~m} / \mathrm{sec}$
$\mathrm{Q}=\mathrm{AV}_{\text {avg }}=\left(0.2 \times 50 \times 10^{-3}\right)(0.52)$

$$
=5.2 \mathrm{lit} / \mathrm{sec}
$$

9. Ans: (a)

Sol: Wall shear stress for flow in a pipe is given by,

$$
\tau_{\mathrm{o}}=-\frac{\partial \mathrm{P}}{\partial \mathrm{x}} \times \frac{\mathrm{R}}{2}=\frac{\Delta \mathrm{P}}{\mathrm{~L}} \times \frac{\mathrm{D}}{4}=\frac{\Delta \mathrm{PD}}{4 \mathrm{~L}}
$$

## 10. Ans: 72

Sol: Given, $\rho=800 \mathrm{~kg} / \mathrm{m}^{3}$,

$$
\mu=0.1 \mathrm{~Pa} . \mathrm{s}
$$

Flow is through an inclined pipe.

$$
\begin{gathered}
\mathrm{d}=1 \times 10^{-2} \mathrm{~m}, \\
\mathrm{~V}_{\mathrm{av}}=0.1 \mathrm{~m} / \mathrm{s}, \quad \theta=30^{\circ} \\
\operatorname{Re}=\frac{\rho V_{\mathrm{av}} \mathrm{~d}}{\mu}=\frac{800 \times 0.1 \times 1 \times 10^{-2}}{0.1}=8
\end{gathered}
$$

$\Rightarrow$ flow is laminar.

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Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,
$\frac{\mathrm{P}_{1}}{\gamma}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{1}=\frac{\mathrm{P}_{2}}{\gamma}+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{2}+\mathrm{h}_{\mathrm{f}}$
But $V_{1}=V_{2}$,
$\left(Z_{2}-Z_{1}\right)=10 \sin 30^{\circ}=5 \mathrm{~m}$
and $\quad h_{f}=\frac{32 \mu \mathrm{~V}_{\mathrm{av}} \mathrm{L}}{\rho \mathrm{gd}^{2}}$

$$
\begin{aligned}
& \frac{\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\gamma}=\left(\mathrm{Z}_{2}-\mathrm{Z}_{1}\right)+\frac{32 \mu \mathrm{~V}_{\mathrm{av}} \mathrm{~L}}{\rho \mathrm{gd}^{2}} \\
& \begin{aligned}
\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right) & =\rho g\left(\mathrm{Z}_{2}-\mathrm{Z}_{1}\right)+\frac{32 \mu \mathrm{~V}_{\mathrm{av}} \mathrm{~L}}{\mathrm{~d}^{2}} \\
& =800 \times 10 \times 5+\frac{32 \times 0.1 \times 0.1 \times 10}{\left(1 \times 10^{-2}\right)^{2}} \\
& =40 \times 10^{3}+32 \times 10^{3}=72 \mathrm{kPa}
\end{aligned}
\end{aligned}
$$

## 11. Ans: (a, b, c, d)

Sol: The following statements regarding laminar flow through pipes are correct.

- Velocity profile is parabolic as given by

$$
\mathrm{u}=\mathrm{U}\left(1-\frac{\mathrm{r}^{2}}{\mathrm{R}^{2}}\right)
$$

- Shear stress, $\tau=\mu \frac{d u}{d y}=-\mu \frac{d u}{d r}$

$$
\begin{aligned}
\tau & =-\mu \times\left(-\frac{2 \mathrm{r} \mathrm{U}}{\mathrm{R}^{2}}\right)=\frac{2 \mu \mathrm{U}}{\mathrm{R}^{2}} \times \mathrm{r} \\
& =\text { Linear profile }
\end{aligned}
$$

- Rate of shear strain profile is also linear.
- Flow is rotational.


## Chapter <br> 8 <br> Flow through Pipes

1. Ans: (d)

Sol:

- The Darcy-Weisbash equation for head loss in written as:

$$
h_{f}=\frac{\mathrm{fLV}^{2}}{2 g d}
$$

where V is the average velocity, f is friction factor, $L$ is the length of pipe and $d$ is the diameter of the pipe.

- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as the nature of pipe surface (smooth or rough)
- For laminar flow, friction factor is a function of Reynolds number.

2. Ans: 481

Sol: Given data,

$$
\begin{aligned}
& \dot{\mathrm{m}}=\pi \mathrm{kg} / \mathrm{s}, \quad \mathrm{~d}=5 \times 10^{-2} \mathrm{~m}, \\
& \mu=0.001 \mathrm{~Pa} . \mathrm{s}, \quad \rho=1000 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mathrm{~V}_{\mathrm{av}}=\frac{\dot{\mathrm{m}}}{\rho \mathrm{~A}}=\frac{4 \dot{\mathrm{~m}}}{\rho \pi \mathrm{~d}^{2}}=\frac{4 \times \pi}{\rho \pi \mathrm{d}^{2}}=\frac{4}{\rho \mathrm{~d}^{2}} \\
& \operatorname{Re}=\frac{\rho V_{\mathrm{av}} \mathrm{~d}}{\mu}=\rho \times \frac{4}{\rho \mathrm{~d}^{2}} \times \frac{\mathrm{d}}{\mu}=\frac{4}{\mu \mathrm{~d}} \\
& =\frac{4}{0.001 \times 5 \times 10^{-2}}=8 \times 10^{4}
\end{aligned}
$$

$\Rightarrow$ Flow is turbulent

$$
\begin{aligned}
& \mathrm{f}=\frac{0.316}{\operatorname{Re}^{0.25}}=\frac{0.316}{\left(8 \times 10^{4}\right)^{0.25}}=0.0188 \\
& \begin{aligned}
& \Delta \mathrm{P}=\rho \mathrm{g} \frac{\mathrm{fLV}}{\mathrm{av}} \\
& 2 \mathrm{gd}=\mathrm{f} \rho \mathrm{~L} \times\left(\frac{4}{\rho d^{2}}\right)^{2} \times \frac{1}{2 \mathrm{~d}} \\
& \begin{aligned}
\frac{\Delta \mathrm{P}}{\mathrm{~L}}=\mathrm{f} \times \frac{16}{\rho \mathrm{~d}^{5}} \times \frac{1}{2} & =\frac{8 \mathrm{f}}{\rho \mathrm{~d}^{5}}=\frac{8 \times 0.0188}{10^{3} \times\left(5 \times 10^{-2}\right)^{5}} \\
& =481.28 \mathrm{~Pa} / \mathrm{m}
\end{aligned}
\end{aligned} . \begin{array}{l} 
\\
\end{array}
\end{aligned}
$$

## 03. Ans: (a)

Sol: In pipes Net work, series arrangement

$$
\begin{aligned}
& \therefore \mathrm{h}_{\mathrm{f}}=\frac{\mathrm{f} \times \ell \times \mathrm{V}^{2}}{2 \mathrm{gd}}=\frac{\mathrm{f} \times \ell \times \mathrm{Q}^{2}}{12.1 \times \mathrm{d}^{5}} \\
& \frac{\mathrm{~h}_{\mathrm{f}_{\mathrm{A}}}}{\mathrm{~h}_{\mathrm{f}_{\mathrm{B}}}}=\frac{\mathrm{f}_{\mathrm{A}} \times \ell_{\mathrm{A}} \times \mathrm{Q}_{\mathrm{a}}^{2}}{12.1 \times \mathrm{d}_{\mathrm{A}}^{5}} \times \frac{12.1 \times \mathrm{d}_{\mathrm{B}}^{5}}{\mathrm{f}_{\mathrm{B}} \times \ell_{\mathrm{B}} \times \mathrm{Q}_{\mathrm{B}}^{2}}
\end{aligned}
$$

Given $l_{\mathrm{A}}=l_{\mathrm{B}}, \mathrm{f}_{\mathrm{A}}=\mathrm{f}_{\mathrm{B}}, \mathrm{Q}_{\mathrm{A}}=\mathrm{Q}_{\mathrm{B}}$

$$
\begin{aligned}
\frac{\mathrm{h}_{\mathrm{f}_{\mathrm{A}}}}{\mathrm{~h}_{\mathrm{f}_{\mathrm{B}}}} & =\left(\frac{\mathrm{d}_{\mathrm{B}}}{\mathrm{~d}_{\mathrm{A}}}\right)^{5}=\left(\frac{\mathrm{d}_{\mathrm{B}}}{1.2 \mathrm{~d}_{\mathrm{B}}}\right)^{5} \\
& =\left(\frac{1}{1.2}\right)^{5}=0.4018 \approx 0.402
\end{aligned}
$$

## 04. Ans: (a)

Sol: Given, $\mathrm{d}_{1}=10 \mathrm{~cm} ; \mathrm{d}_{2}=20 \mathrm{~cm}$
$\mathrm{f}_{1}=\mathrm{f}_{2}$;
$l_{1}=l_{2}=l$
$l_{\mathrm{e}}=l_{1}+l_{2}=2 l$
$\frac{l_{\mathrm{e}}}{\mathrm{d}_{\mathrm{e}}^{5}}=\frac{l_{1}}{\mathrm{~d}_{1}^{5}}+\frac{l_{2}}{\mathrm{~d}_{2}^{5}} \Rightarrow \frac{2 l}{d_{e}^{5}}=\frac{l}{10^{5}}+\frac{l}{20^{5}}$
$\therefore \mathrm{d}_{\mathrm{e}}=11.4 \mathrm{~cm}$

## 05. Ans: (c)

Sol:


Given $\mathrm{d}_{2}=2 \mathrm{~d}_{1}$
Losses due to sudden expansion,

$$
\begin{aligned}
\mathrm{h}_{\mathrm{L}} & =\frac{\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)^{2}}{2 \mathrm{~g}} \\
& =\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}\left(1-\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}\right)^{2}
\end{aligned}
$$

By continuity equation,

$$
\begin{aligned}
& \mathrm{Q}=\mathrm{A}_{1} \mathrm{~V}_{1}=\mathrm{A}_{2} \mathrm{~V}_{2} \\
& \frac{\mathrm{~V}_{2}}{\mathrm{~V}_{1}}=\frac{\mathrm{A}_{1}}{\mathrm{~A}_{2}}=\left(\frac{\mathrm{d}_{1}}{\mathrm{~d}_{2}}\right)^{2}=\left(\frac{1}{2}\right)^{2} \\
& \mathrm{~h}_{\mathrm{L}}=\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}\left(1-\frac{1}{4}\right)^{2} \\
& \mathrm{~h}_{\mathrm{L}}=\frac{9}{16} \times \frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}} \\
& \frac{\mathrm{~h}_{\mathrm{L}}}{\mathrm{~V}_{1}^{2}}=\frac{9}{16}
\end{aligned}
$$

6. Ans: (b)

Sol: Pipes are in parallel

$$
\begin{aligned}
& \mathrm{Q}_{\mathrm{e}}=\mathrm{Q}_{\mathrm{A}}+\mathrm{Q}_{\mathrm{B}} \\
& \mathrm{~h}_{\mathrm{Le}}=\mathrm{h}_{\mathrm{L}_{\mathrm{A}}}=\mathrm{h}_{\mathrm{L}_{\mathrm{B}}} \\
& \mathrm{~L}_{\mathrm{e}}=175 \mathrm{~m} \\
& \mathrm{f}_{\mathrm{e}}=0.015
\end{aligned}
$$

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$\frac{f_{e} L_{e} Q_{e}^{2}}{12.1 D_{e}^{5}}=\frac{f_{A} \cdot L_{A} Q_{A}^{2}}{12.1 D_{A}^{5}}=\frac{f_{B} L_{B} Q_{B}^{2}}{12.1 D_{B}^{5}}$

$$
\begin{align*}
\frac{0.020 \times 150 \times \mathrm{Q}_{A}^{2}}{12.1 \times(0.1)^{5}} & =\frac{0.015 \times 200 \times \mathrm{Q}_{\mathrm{B}}^{2}}{12.1 \times(0.08)^{5}} \\
\mathrm{Q}_{\mathrm{A}} & =1.747 \mathrm{Q}_{\mathrm{B}} \tag{ii}
\end{align*}
$$

From (i) $\quad \mathrm{Q}_{\mathrm{e}}=1.747 \mathrm{Q}_{\mathrm{B}}+\mathrm{Q}_{\mathrm{B}}$

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{e}}=2.747 \mathrm{Q}_{\mathrm{B}} \tag{iii}
\end{equation*}
$$

$$
\frac{0.015 \times 175\left(2.747 \mathrm{Q}_{\mathrm{B}}\right)^{2}}{12.1 \times \mathrm{D}_{\mathrm{e}}^{5}}=\frac{0.015 \times 200 \times \mathrm{Q}_{\mathrm{B}}^{2}}{12.1 \times(0.08)^{5}}
$$

$\mathrm{D}_{\mathrm{e}}=116.6 \mathrm{~mm} \simeq 117 \mathrm{~mm}$

## 07. Ans: 0.141

## Sol:



Given data,

$$
\begin{array}{ll}
\mathrm{L}=930 \mathrm{~m}, & \mathrm{k}_{\text {valve }}=5.5 \\
\mathrm{k}_{\text {entry }}=0.5, & \mathrm{~d}=0.3 \mathrm{~m} \\
\mathrm{f}=0.03, & \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

Applying energy equation for points (1) and (2), we write :

$$
\begin{aligned}
\frac{\mathrm{P}_{1}}{\gamma_{\mathrm{w}}}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{1}=\frac{\mathrm{P}_{2}}{\gamma_{\mathrm{w}}}+ & +\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{2}+\mathrm{h}_{\mathrm{L}, \text { entry }} \\
& +\mathrm{h}_{\mathrm{L}, \text { valve }}+\mathrm{h}_{\mathrm{L}, \text { exit }}+\mathrm{h}_{\mathrm{f}, \text { pipe }}
\end{aligned}
$$

But $\mathrm{P}_{1}=\mathrm{P}_{2}=\mathrm{P}_{\mathrm{atm}}=0$

$$
\mathrm{V}_{1}=0=\mathrm{V}_{2}
$$

$$
\mathrm{Z}_{1}-\mathrm{Z}_{2}=20 \mathrm{~m}, \quad \mathrm{k}_{\mathrm{exit}}=1
$$

$$
\mathrm{Z}_{1}-\mathrm{Z}_{2}=0.5 \frac{\mathrm{~V}^{2}}{2 \mathrm{~g}}+5.5 \frac{\mathrm{~V}^{2}}{2 \mathrm{~g}}+1 \times \frac{\mathrm{V}^{2}}{2 \mathrm{~g}}+\frac{\mathrm{fLV}^{2}}{2 \mathrm{gd}}
$$

$$
=7 \frac{\mathrm{~V}^{2}}{2 \mathrm{~g}}+\frac{\mathrm{fLV}^{2}}{2 \mathrm{gd}}=\frac{\mathrm{V}^{2}}{2 \mathrm{~g}}\left(7+\frac{\mathrm{fL}}{\mathrm{~d}}\right)
$$

$$
\text { or } \quad 20=\frac{\mathrm{V}^{2}}{2 \mathrm{~g}}\left[7+\frac{0.03 \times 930}{0.3}\right]=100 \frac{\mathrm{~V}^{2}}{2 \mathrm{~g}}
$$

$$
\text { or } V^{2}=\frac{20 \times 2 \mathrm{~g}}{100}=\frac{20 \times 2 \times 10}{100}
$$

$$
\Rightarrow \mathrm{V}=2 \mathrm{~m} / \mathrm{s}
$$

Thus, discharge, $\mathrm{Q}=\frac{\pi}{4} \times 0.3^{2} \times 2$

$$
=0.1414 \mathrm{~m}^{3} / \mathrm{s}
$$

8. Ans: (c)

Sol: Given data :
Fanning friction factor, $\mathrm{f}=\mathrm{m} \mathrm{Re}^{-0.2}$
For turbulent flow through a smooth pipe.

$$
\begin{aligned}
\Delta \mathrm{P} & =\frac{\rho \mathrm{f}_{\text {Darcy }} \mathrm{LV}^{2}}{2 \mathrm{~d}}=\frac{\rho(4 \mathrm{f}) \mathrm{LV}^{2}}{2 \mathrm{~d}} \\
& =\frac{2 \rho \mathrm{mRe}^{-0.2}{L V^{2}}^{d}}{}
\end{aligned}
$$

or $\quad \Delta \mathrm{P} \propto \mathrm{V}^{-0.2} \mathrm{~V}^{2} \propto \mathrm{~V}^{1.8} \quad$ (as all other parameters remain constant)

We may thus write :

$$
\frac{\Delta \mathrm{P}_{2}}{\Delta \mathrm{P}_{1}}=\left(\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}\right)^{1.8}=\left(\frac{2}{1}\right)^{1.8}=3.4822
$$

or $\quad \Delta \mathrm{P}_{2}=3.4822 \times 10=34.82 \mathrm{kPa}$

## 09. Ans: (b)

Sol: Given data :
Rectangular duct, $\mathrm{L}=10 \mathrm{~m}$,
X-section of duct $=15 \mathrm{~cm} \times 20 \mathrm{~cm}$
Material of duct-Commercial steel,

$$
\varepsilon=0.045 \mathrm{~mm}
$$

Fluid is air $\left(\rho=1.145 \mathrm{~kg} / \mathrm{m}^{3}\right.$,

$$
\left.v=1.655 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}\right)
$$

$\mathrm{V}_{\mathrm{av}}=7 \mathrm{~m} / \mathrm{s}$
$\operatorname{Re}=\frac{\mathrm{V}_{\mathrm{av}} \times \mathrm{D}_{\mathrm{h}}}{v}$
where, $\mathrm{D}_{\mathrm{h}}=$ Hydraulic diameter

$$
\begin{aligned}
& =\frac{4 \times \text { Crosssec tional area }}{\text { Perimeter }} \\
& =\frac{4 \times 0.15 \times 0.2}{2(0.15+0.2)}=0.1714 \mathrm{~m}
\end{aligned}
$$

$$
\operatorname{Re}=\frac{7 \times 0.1714}{1.655 \times 10^{-5}}=72495.5
$$

$\Rightarrow$ Flow is turbulent.

Using Haaland equation to find friction factor,

$$
\begin{aligned}
\frac{1}{\sqrt{\mathrm{f}}} & \simeq-1.8 \log \left[\frac{6.9}{\operatorname{Re}}+\left(\frac{\varepsilon / \mathrm{D}_{\mathrm{h}}}{3.7}\right)^{1.11}\right] \\
\frac{1}{\sqrt{\mathrm{f}}} & =-1.8 \log \left[\frac{6.9}{72495.5}+\left(\frac{0.045 \times 10^{-3}}{0.1714 \times 3.7}\right)^{1.11}\right] \\
& =-1.8 \log \left[9.518 \times 10^{-5}+2.48 \times 10^{-5}\right] \\
& =-1.8 \log \left(11.998 \times 10^{-5}\right) \\
\frac{1}{\sqrt{\mathrm{f}}} & =7.058 \\
\mathrm{f} & =0.02
\end{aligned}
$$

The pressure drop in the duct is,

$$
\begin{aligned}
\Delta \mathrm{P} & =\frac{\rho f \mathrm{LV}^{2}}{2 \mathrm{D}_{\mathrm{h}}} \\
& =\frac{1.145 \times 0.02 \times 10 \times 7^{2}}{2 \times 0.1714}=32.73 \mathrm{~Pa}
\end{aligned}
$$

The required pumping power will be

$$
\begin{aligned}
\mathrm{P}_{\text {pumping }} & =\mathrm{Q} \Delta \mathrm{P}=\mathrm{A} \mathrm{~V}_{\mathrm{av}} \times \Delta \mathrm{P} \\
& =(0.15 \times 0.2) \times 7 \times(32.73) \\
& =6.87 \mathrm{~W} \simeq 7 \mathrm{~W}
\end{aligned}
$$

10. Ans: 26.5

Sol:


Case I: Without additional pipe,
Let Q be the discharge through the pipe.
Then
$\frac{\mathrm{P}_{\mathrm{P}}}{\gamma}+\frac{\mathrm{V}_{\mathrm{P}}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{\mathrm{P}}=\frac{\mathrm{P}_{\mathrm{S}}}{\gamma}+\frac{\mathrm{V}_{\mathrm{S}}^{2}}{2 \mathrm{~g}}+\mathrm{Z}_{\mathrm{S}}+\frac{\mathrm{fLQ}^{2}}{12.1 \mathrm{~d}^{5}}$
But $\quad V_{P}=V_{S}$
and $\quad Z_{P}=Z_{S}$
$P_{P}$ and $P_{S}$ are the pressures at sections $P$ and S , respectively.

Thus,

$$
\begin{equation*}
\frac{\mathrm{P}_{\mathrm{P}}}{\gamma}-\frac{\mathrm{P}_{\mathrm{S}}}{\gamma}=\frac{\mathrm{fLQ}^{2}}{12.1 \mathrm{~d}^{5}} \tag{1}
\end{equation*}
$$

Case II: When a pipe (L/2) is connected in parallel.
In this case, let $\mathrm{Q}^{\prime}$ be the total discharge.

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$\mathrm{Q}_{\mathrm{Q}-\mathrm{R}}=\frac{\mathrm{Q}^{\prime}}{2}$ and $\mathrm{Q}_{\mathrm{R}-\mathrm{S}}=\mathrm{Q}^{\prime}$
Then,

$$
\begin{aligned}
\frac{\mathrm{P}_{\mathrm{P}}^{\prime}}{\gamma}+\frac{\mathrm{V}_{\mathrm{P}}^{\prime 2}}{2 \mathrm{~g}}+\mathrm{Z}_{\mathrm{P}}^{\prime} & =\frac{\mathrm{P}_{\mathrm{S}}^{\prime}}{\gamma}+\frac{\mathrm{V}_{\mathrm{S}}^{\prime 2}}{2 \mathrm{~g}}+\mathrm{Z}_{\mathrm{S}}^{\prime}+\frac{\mathrm{f}(\mathrm{~L} / 4) \mathrm{Q}^{\prime 2}}{12.1 \mathrm{~d}^{5}} \\
& +\frac{\mathrm{f}(\mathrm{~L} / 2)\left(\mathrm{Q}^{\prime} / 2\right)^{2}}{12.1 \mathrm{~d}^{5}}+\frac{\mathrm{f}(\mathrm{~L} / 4) \mathrm{Q}^{\prime 2}}{12.1 \mathrm{~d}^{5}}
\end{aligned}
$$

$\mathrm{P}_{\mathrm{P}}{ }^{\prime}$ and $\mathrm{P}_{\mathrm{S}^{\prime}}$ are the pressures at sections P and S in the second case.

But $\mathrm{V}_{\mathrm{P}}{ }^{\prime}=\mathrm{V}_{\mathrm{S}^{\prime}} ; \mathrm{Z}_{\mathrm{P}}{ }^{\prime}=\mathrm{Z}_{\mathrm{S}^{\prime}}$
So, $\frac{\mathrm{P}_{\mathrm{P}}^{\prime}}{\gamma}-\frac{\mathrm{P}_{\mathrm{S}}^{\prime}}{\gamma}=\frac{\mathrm{fLQ}^{\prime 2}}{12.1 \mathrm{~d}^{5}}\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{4}\right]$

$$
\begin{equation*}
=\frac{5}{8} \times \frac{\mathrm{fLQ}^{\prime 2}}{12.1 \mathrm{~d}^{5}} \tag{2}
\end{equation*}
$$

Given that end conditions remain same.
i.e., $\frac{\mathrm{P}_{\mathrm{P}}}{\gamma}-\frac{\mathrm{P}_{\mathrm{S}}}{\gamma}=\frac{\mathrm{P}_{\mathrm{P}}^{\prime}}{\gamma}-\frac{\mathrm{P}_{\mathrm{S}}^{\prime}}{\gamma}$

Hence, equation (2) becomes,

$$
\frac{\mathrm{fLQ}^{2}}{12.1 \mathrm{~d}^{5}}=\frac{5 \mathrm{fLQ}^{\prime 2}}{8} \frac{\text { from eq.(1) }}{12.1 \mathrm{~d}^{5}}
$$

or $\left(\frac{\mathrm{Q}^{\prime}}{\mathrm{Q}}\right)^{2}=\frac{8}{5}$
or $\quad \frac{\mathrm{Q}^{\prime}}{\mathrm{Q}}=1.265$
Hence, percentage increase in discharge is

$$
\begin{aligned}
& =\frac{Q^{\prime}-Q}{Q} \times 100 \\
& =(1.265-1) \times 100 \\
& =26.5 \%
\end{aligned}
$$

## 11. Ans: 20\%

Sol: Since, discharge decrease is associated with increase in friction.

$$
\begin{aligned}
\frac{\mathrm{df}}{\mathrm{f}} & =-2 \times \frac{\mathrm{dQ}}{\mathrm{Q}}=2\left[-\frac{\mathrm{dQ}}{\mathrm{Q}}\right] \\
& =2 \times 10=20 \%
\end{aligned}
$$

12. Ans: (c, d)

Sol: Given data:

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{G}}=80 \mathrm{~m}, \mathrm{D}=0.5 \mathrm{~m}, \mathrm{~L}=4 \mathrm{~km} \\
& \mathrm{f}=0.02, \quad \eta=0.75 \\
& \eta=0.75=\frac{\mathrm{H}_{\mathrm{G}}-\mathrm{h}_{\mathrm{f}}}{\mathrm{H}_{\mathrm{G}}} \Rightarrow \mathrm{~h}_{\mathrm{f}}=\mathrm{H}_{\mathrm{G}}(1-\eta) \\
& \mathrm{h}_{\mathrm{f}}=80 \times(1-0.75)=20 \mathrm{~m} \\
& \text { But, } h_{f}=\frac{\mathrm{fLQ}^{2}}{12.1 \times \mathrm{D}^{5}} \\
& 20=\frac{0.02 \times 4000 \times \mathrm{Q}^{2}}{12.1 \times(0.5)^{5}} \\
& \Rightarrow \quad \mathrm{Q}=0.3075 \mathrm{~m}^{3} / \mathrm{s} \\
& \therefore \mathrm{P}_{\text {act }}=\rho \mathrm{g} \text { Q } \mathrm{H}_{\text {net }} \\
& =10^{3} \times 9.81 \times 0.3075 \times(80-20) \\
& =180.995 \mathrm{~kW}
\end{aligned}
$$

Now, $\mathrm{V}_{\mathrm{j}}=\mathrm{V}_{\mathrm{N}}=\sqrt{2 \mathrm{gH}_{\text {net }}}$

$$
=\sqrt{2 \times 9.81 \times 60}=34.31 \mathrm{~m} / \mathrm{s}
$$

From discharge, we have

$$
\begin{aligned}
\mathrm{Q} & =\mathrm{A}_{\mathrm{N}} \mathrm{~V}_{\mathrm{N}} \\
0.3075 & =\frac{\pi}{4} \times \mathrm{d}_{\mathrm{N}}^{2} \times 34.31 \\
\Rightarrow \mathrm{~d}_{\mathrm{N}} & =0.1068 \mathrm{~m}=10.68 \mathrm{~cm}
\end{aligned}
$$

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Chapter
9

## Elementary Turbulent Flow

1. Ans: (b)

Sol: The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by

$$
\frac{\mathrm{u}}{\mathrm{~V}^{*}}=\frac{\mathrm{yV}^{*}}{\mathrm{v}}
$$

where $\mathrm{V}^{*}$ is the shear velocity.
02. Ans: (d)

Sol: $\Delta \mathrm{P}=\rho \mathrm{gh}_{\mathrm{f}}$

$$
=\frac{\rho f \mathrm{LV}^{2}}{2 \mathrm{D}}=\frac{\rho \mathrm{gfL} \mathrm{~L}^{2}}{12.1 \mathrm{D}^{5}}
$$

For $\mathrm{Q}=$ constant

$$
\Delta \mathrm{P} \propto \frac{1}{\mathrm{D}^{5}}
$$

or $\frac{\Delta \mathrm{P}_{2}}{\Delta \mathrm{P}_{1}}=\frac{\mathrm{D}_{1}^{5}}{\mathrm{D}_{2}^{5}}=\left(\frac{\mathrm{D}_{1}}{2 \mathrm{D}_{1}}\right)^{5}=\frac{1}{32}$
03. Ans: 2.4

Sol: Given: $V=2 \mathrm{~m} / \mathrm{s}$

$$
\begin{gathered}
\mathrm{f}=0.02 \\
\mathrm{~V}_{\max }=? \\
\mathrm{~V}_{\max }=\mathrm{V}(1+1.43 \sqrt{\mathrm{f}}) \\
=2(1+1.43 \sqrt{0.02}) \\
=2 \times 1.2=2.4 \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

## 04. Ans: (c)

Sol: Given data:

$$
\begin{aligned}
\mathrm{D} & =30 \mathrm{~cm}=0.3 \mathrm{~m} \\
\operatorname{Re} & =10^{6} \\
\mathrm{f} & =0.025
\end{aligned}
$$

Thickness of laminar sub layer, $\delta^{\prime}=$ ?

$$
\delta^{\prime}=\frac{11.6 \mathrm{v}}{\mathrm{~V}^{*}}
$$

where $\mathrm{V}^{*}=$ shear velocity $=\mathrm{V} \sqrt{\frac{\mathrm{f}}{8}}$
$v=$ Kinematic viscosity

$$
\begin{aligned}
& \mathrm{Re}=\frac{\mathrm{V} \cdot \mathrm{D}}{v} \\
& \therefore v=\frac{\mathrm{V} \cdot \mathrm{D}}{\mathrm{Re}}
\end{aligned}
$$

$$
\begin{aligned}
\delta^{\prime} & =\frac{11.6 \times \frac{\mathrm{VD}}{\mathrm{Re}}}{\mathrm{~V} \sqrt{\frac{\mathrm{f}}{8}}} \\
\delta^{\prime} & =\frac{11.6 \times \mathrm{D}}{\operatorname{Re} \sqrt{\frac{\mathrm{f}}{8}}} \\
& =\frac{11.6 \times 0.3}{10^{6} \times \sqrt{\frac{0.025}{8}}} \\
& =6.22 \times 10^{-5} \mathrm{~m}=0.0622 \mathrm{~mm}
\end{aligned}
$$

5. Ans: 25

## Sol: Given:

$$
\begin{aligned}
& \mathrm{L}=100 \mathrm{~m} \\
& \mathrm{D}=0.1 \mathrm{~m} \\
& \mathrm{~h}_{\mathrm{L}}=10 \mathrm{~m} \\
& \tau=?
\end{aligned}
$$

For any type of flow, the shear stress at wall/surface $\tau=\frac{-\mathrm{dP}}{\mathrm{dx}} \times \frac{\mathrm{R}}{2}$

$$
\begin{aligned}
\tau & =\frac{\rho \mathrm{gh}_{\mathrm{L}}}{\mathrm{~L}} \times \frac{\mathrm{R}}{2} \\
\tau & =\frac{\rho \mathrm{gh}_{\mathrm{L}}}{\mathrm{~L}} \times \frac{\mathrm{D}}{4} \\
& =\frac{1000 \times 9.81 \times 10}{100} \times \frac{0.1}{4} \\
& =24.525 \mathrm{~N} / \mathrm{m}^{2}=25 \mathrm{~Pa}
\end{aligned}
$$

## 06. Ans: 0.905

Sol: $\mathrm{k}=0.15 \mathrm{~mm}$
$\tau=4.9 \mathrm{~N} / \mathrm{m}^{2}$
$v=1$ centi-stoke
$\mathrm{V}^{*}=\sqrt{\frac{\tau_{\mathrm{o}}}{\rho}}=\sqrt{\frac{4.9}{1000}}=0.07 \mathrm{~m} / \mathrm{sec}$
$v=1$ centi-stoke

$$
=\frac{1}{100} \text { stoke }=\frac{10^{-4}}{100}=10^{-6} \mathrm{~m}^{2} / \mathrm{sec}
$$

$$
\begin{aligned}
\frac{\mathrm{k}}{\delta^{\prime}} & =\frac{0.15 \times 10^{-3}}{\left(\frac{11.6 \times v}{\mathrm{~V}^{*}}\right)} \\
& =\frac{0.15 \times 10^{-3}}{\frac{11.6 \times 10^{-6}}{0.07}}=0.905
\end{aligned}
$$

## 07. Ans: (a)

Sol: The velocity profile in the laminar sublayer is given as

$$
\frac{\mathrm{u}}{\mathrm{~V}^{*}}=\frac{\mathrm{yV}^{*}}{\mathrm{v}}
$$

or $\quad v=\frac{y\left(V^{*}\right)^{2}}{u}$
where, $\mathrm{V}^{*}$ is the shear velocity.
Thus, $v=\frac{0.5 \times 10^{-3} \times(0.05)^{2}}{1.25}$

$$
\begin{aligned}
& =1 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s} \\
& =1 \times 10^{-2} \mathrm{~cm}^{2} / \mathrm{s}
\end{aligned}
$$

8. Ans: $47.74 \mathrm{~N} / \mathrm{m}^{2}$

Sol: Given data :

$$
\begin{aligned}
\mathrm{d} & =100 \mathrm{~mm}=0.1 \mathrm{~m} \\
\mathrm{u}_{\mathrm{r}}=0 & =\mathrm{u}_{\max }=2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Velocity at $\mathrm{r}=30 \mathrm{~mm}=1.5 \mathrm{~m} / \mathrm{s}$
Flow is turbulent.
The velocity profile in turbulent flow is

$$
\frac{\mathrm{u}_{\max }-\mathrm{u}}{\mathrm{~V}^{*}}=5.75 \log \left(\frac{\mathrm{R}}{\mathrm{y}}\right)
$$

where $u$ is the velocity at $y$ and $\mathrm{V}^{*}$ is the shear velocity.

$$
\begin{aligned}
& \text { For pipe, } \begin{aligned}
\mathrm{y} & =\mathrm{R}-\mathrm{r} \\
5 & =(50-30) \mathrm{mm}=20 \mathrm{~mm}
\end{aligned} \\
&=1
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& \frac{2-1.5}{\mathrm{~V}^{*}}=5.75 \log \left(\frac{50}{20}\right)=2.288 \\
& \text { or } \mathrm{V}^{*}=\frac{0.5}{2.288}=0.2185 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Using the relation,

$$
\begin{aligned}
& \mathrm{V}^{*}=\sqrt{\frac{\tau_{\mathrm{w}}}{\rho}} \text { or } \tau_{\mathrm{w}}=\rho\left(\mathrm{V}^{*}\right)^{2} \\
& \tau_{\mathrm{w}}=10^{3} \times(0.2185)^{2}=47.74 \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

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## 09. Ans: (a, b)

Sol: The following statements are true for turbulent flow through pipes:

- Velocity profile is logarithmic (in the overlap region) expressed as

$$
\frac{\mathrm{u}}{\mathrm{u}^{*}}=2.5 \ln \left(\frac{\mathrm{yu}^{*}}{v}\right)+5.0
$$

- Surface roughness plays an important role in contributing towards determining head loss.



## Chapter

10

## Boundary Layer Theory

1. Ans: (c)

Sol: $\operatorname{Re}_{\text {Critical }}=\frac{U_{\infty} X_{\text {critical }}}{v}$
Assume water properties

$$
\begin{aligned}
5 \times 10^{5} & =\frac{6 \times \mathrm{x}_{\text {critical }}}{1 \times 10^{-6}} \\
\mathrm{x}_{\text {critical }} & =0.08333 \mathrm{~m}=83.33 \mathrm{~mm}
\end{aligned}
$$

2. Ans: 1.6

Sol: $\delta \propto \frac{1}{\sqrt{\operatorname{Re}}}$ (At given distance ' x ')

$$
\begin{aligned}
& \frac{\delta_{1}}{\delta_{2}}=\sqrt{\frac{\mathrm{Re}_{2}}{\mathrm{Re}_{1}}} \\
& \frac{\delta_{1}}{\delta_{2}}=\sqrt{\frac{256}{100}}=\frac{16}{10}=1.6
\end{aligned}
$$

3. Ans: 80

Sol:


$$
\delta \propto \sqrt{x}
$$

$$
\frac{\delta_{\mathrm{A}}}{\delta_{\mathrm{B}}}=\sqrt{\frac{\mathrm{x}_{1}}{\left(\mathrm{x}_{1}+1\right)}}
$$

$$
\begin{aligned}
& x=\frac{2}{3}=\sqrt{\frac{x_{1}}{x_{1}+1}} \\
& \frac{4}{9}=\frac{x_{1}}{x_{1}+1} \\
& 5 x_{1}=4 \Rightarrow x_{1}=80 \mathrm{~cm}
\end{aligned}
$$

## 04. Ans: 2

Sol: $\tau \propto \frac{1}{\delta}$

$$
\begin{aligned}
& \tau \propto \frac{1}{\sqrt{\mathrm{x}}} \because \delta \propto \sqrt{\mathrm{x}} \\
& \frac{\tau_{1}}{\tau_{2}}=\sqrt{\frac{\mathrm{x}_{2}}{\mathrm{x}_{1}}} \\
& \frac{\tau_{1}}{\tau_{2}}=\sqrt{4}=2
\end{aligned}
$$

5. Ans: 3

Sol: $\frac{U}{U_{\infty}}=\frac{y}{\delta}$

$$
\frac{\delta^{*}}{\theta}=\text { Shape factor }=?
$$

$$
\delta^{*}=\int_{0}^{\delta}\left(1-\frac{\mathrm{u}}{\mathrm{U}_{\infty}}\right) \mathrm{dy}
$$

$$
=\int_{0}^{\delta}\left(1-\frac{\mathrm{y}}{8}\right) \mathrm{dy}
$$

$$
=\mathrm{y}-\left.\frac{\mathrm{y}^{2}}{2 \delta}\right|_{0} ^{\delta}
$$

$$
=\delta-\frac{\delta}{2}=\frac{\delta}{2}
$$

$$
\begin{aligned}
\theta & =\int_{0}^{\delta} \frac{\mathrm{u}}{\mathrm{U}_{\infty}}\left(1-\frac{\mathrm{u}}{\mathrm{U}_{\infty}}\right) \mathrm{dy} \\
& =\int_{0}^{\delta} \frac{\mathrm{y}}{8}\left(1-\frac{\mathrm{y}}{\delta}\right) \mathrm{dy} \\
& =\frac{\mathrm{y}^{2}}{2 \delta}-\left.\frac{\mathrm{y}^{3}}{3 \delta}\right|_{0} ^{\delta} \\
& =\frac{\delta}{2}-\frac{\delta}{3}=\frac{\delta}{6}
\end{aligned}
$$

Shape factor $=\frac{\delta^{*}}{\theta}=\frac{\delta / 2}{\delta / 6}=3$
06. Ans: 22.6

Sol: Drag force,

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{D}}=\frac{1}{2} \mathrm{C}_{\mathrm{D}} \cdot \rho \cdot \mathrm{~A}_{\text {Proj }} \cdot \mathrm{U}_{\infty}^{2} \\
& \mathrm{~B}=1.5 \mathrm{~m}, \quad \rho=1.2 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mathrm{~L}=3.0 \mathrm{~m}, \quad v=0.15 \mathrm{stokes} \\
& \mathrm{U}_{\infty}=2 \mathrm{~m} / \mathrm{sec} \\
& \mathrm{Re}=\frac{\mathrm{U}_{\infty} \mathrm{L}}{\nu}=\frac{2 \times 3}{0.15 \times 10^{-4}}=4 \times 10^{5} \\
& 5 \\
& \mathrm{C}_{\mathrm{D}}=\frac{1.328}{\sqrt{\operatorname{Re}}}=\frac{1.328}{\sqrt{4 \times 10^{5}}}=2.09 \times 10^{-3}
\end{aligned}
$$

Drag force,

$$
\begin{aligned}
\mathrm{F}_{\mathrm{D}} & =\frac{1}{2} \times 2.09 \times 10^{-3} \times 1.2 \times(1.5 \times 3) \times 2^{2} \\
& =22.57 \text { milli-Newton }
\end{aligned}
$$

7. Ans: 1.62

Sol: Given data,

$$
\begin{aligned}
\mathrm{U}_{\infty} & =30 \mathrm{~m} / \mathrm{s}, \\
\rho & =1.2 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

Velocity profile at a distance x from leading edge,

$$
\begin{aligned}
& \frac{\mathrm{u}}{\mathrm{U}_{\infty}}=\frac{\mathrm{y}}{\delta} \\
& \delta=1.5 \mathrm{~mm}
\end{aligned}
$$

Mass flow rate of air entering section ab ,

$$
\left(\dot{\mathrm{m}}_{\mathrm{in}}\right)_{\mathrm{ab}}=\rho \mathrm{U}_{\infty}(\delta \times 1)=\rho \mathrm{U}_{\infty} \delta \mathrm{kg} / \mathrm{s}
$$

Mass flow rate of air leaving section cd,

$$
\begin{aligned}
\left(\dot{\mathrm{m}}_{\text {out }}\right)_{\mathrm{cd}} & =\rho \int_{0}^{\delta} \mathrm{u}(\mathrm{dy} \times 1)=\rho \int_{0}^{\delta} \mathrm{U}_{\infty}\left(\frac{\mathrm{y}}{\delta}\right) \mathrm{dy} \\
& =\frac{\rho \mathrm{U}_{\infty}}{\delta}\left[\frac{\mathrm{y}^{2}}{2}\right]_{0}^{\delta}=\frac{\rho \mathrm{U}_{\infty} \delta}{2}
\end{aligned}
$$

From the law of conservation of mass :

$$
\left(\dot{\mathrm{m}}_{\mathrm{in}}\right)_{\mathrm{ab}}=\left(\dot{\mathrm{m}}_{\mathrm{out}}\right)_{\mathrm{cd}}+\left(\dot{\mathrm{m}}_{\mathrm{out}}\right)_{\mathrm{bc}}
$$

Hence, $\left(\dot{\mathrm{m}}_{\mathrm{out}}\right)_{\mathrm{bc}}=\left(\dot{\mathrm{m}}_{\mathrm{in}}\right)_{\mathrm{ab}}-\left(\dot{\mathrm{m}}_{\mathrm{out}}\right)_{\mathrm{cd}}$

$$
=\rho \mathrm{U}_{\infty} \delta-\frac{\rho \mathrm{U}_{\infty} \delta}{2}
$$

$$
=\frac{\rho \mathrm{U}_{\infty} \delta}{2}
$$

$$
=\frac{1.2 \times 30 \times 1.5 \times 10^{-3}}{2}
$$

$$
=27 \times 10^{-3} \mathrm{~kg} / \mathrm{s}
$$

$$
=27 \times 10^{-3} \times 60 \mathrm{~kg} / \mathrm{min}
$$

$$
=1.62 \mathrm{~kg} / \mathrm{min}
$$

## 08. Ans: (b)

Sol: For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in $y$-direction, $\frac{\partial u}{\partial y}$ only. The correct option is (b).
09. Ans: 28.5

Sol: Given data,
Flow is over a flat plate.
$\mathrm{L}=1 \mathrm{~m}$,
$\mathrm{U}_{\infty}=6 \mathrm{~m} / \mathrm{s}$
$v=0.15$ stoke $=0.15 \times 10^{-4} \mathrm{~m}^{2} / \mathrm{s}$
$\rho=1.226 \mathrm{~kg} / \mathrm{m}^{3}$
$\delta(\mathrm{x})=\frac{3.46 \mathrm{x}}{\sqrt{\operatorname{Re}_{\mathrm{x}}}}$
Velocity profile is linear.
Using von-Karman momentum integral equation for flat plate.

$$
\begin{equation*}
\frac{\mathrm{d} \theta}{\mathrm{dx}}=\frac{\tau_{\mathrm{w}}}{\rho \mathrm{U}_{\infty}^{2}} \tag{1}
\end{equation*}
$$

we can find out $\tau_{\mathrm{w}}$.
From linear velocity profile, $\frac{u}{U_{\infty}}=\frac{y}{\delta}$, we
evaluate first $\theta$, momentum thickness as

$$
\begin{aligned}
\theta & =\int_{0}^{\delta} \frac{\mathrm{u}}{\mathrm{U}_{\infty}}\left(1-\frac{\mathrm{u}}{\mathrm{U}_{\infty}}\right) \mathrm{dy} \\
& =\int_{0}^{\delta} \frac{\mathrm{y}}{\delta}\left(1-\frac{\mathrm{y}}{\delta}\right) \mathrm{dy}=\int_{0}^{\delta}\left(\frac{\mathrm{y}}{\delta}-\frac{\mathrm{y}^{2}}{\delta^{2}}\right) \mathrm{dy} \\
& =\left(\frac{\mathrm{y}^{2}}{2 \delta}-\frac{\mathrm{y}^{3}}{3 \delta^{2}}\right)_{0}^{\delta}=\frac{\delta}{2}-\frac{\delta}{3}=\frac{\delta}{6} \\
\Rightarrow \theta & =\frac{\delta}{6}=\frac{1}{6} \times \frac{3.46 \mathrm{x}}{\sqrt{\operatorname{Re}_{\mathrm{x}}}} \\
& =\frac{3.46}{6} \frac{\mathrm{x}^{1 / 2}}{\left(\frac{\mathrm{U}_{\infty}}{v}\right)^{1 / 2}}
\end{aligned}
$$

Differentiating $\theta$ w.r.t x , we get

$$
\begin{align*}
& \frac{d \theta}{d x}=\frac{3.46}{6 \times 2} \frac{x^{-1 / 2}}{\left(\frac{U_{\infty}}{v}\right)^{1 / 2}}=0.2883 \frac{1}{\sqrt{\frac{U_{\infty} x}{v}}} \\
& \left.\frac{d \theta}{d x}\right|_{x=0.5 \mathrm{~m}}=0.2883 \times \frac{1}{\sqrt{\frac{6 \times 0.5}{0.15 \times 10^{-4}}}}=\frac{0.2883}{447.2} \tag{2}
\end{align*}
$$

From equation (1)

$$
\begin{aligned}
\left.\tau_{\mathrm{w}}\right|_{\mathrm{x}=0.5 \mathrm{~m}} & =\left.\frac{\mathrm{d} \theta}{\mathrm{dx}}\right|_{\mathrm{x}=0.5 \mathrm{~m}} \times \rho \mathrm{U}_{\infty}^{2} \\
& =\frac{0.2883}{447.2} \times 1.226 \times 6^{2} \\
& =0.02845 \mathrm{~N} / \mathrm{m}^{2} \\
& \simeq 28.5 \mathrm{mN} / \mathrm{m}^{2}
\end{aligned}
$$

10. Ans: (a, d)

Sol: Given data:

$$
\begin{align*}
& \rho=1.25 \mathrm{~kg} / \mathrm{m}^{3}, \\
& \mu=1.8 \times 10^{-5} \text { Pa.s, } \\
& \mathrm{u}_{\infty}=3 \mathrm{~m} / \mathrm{s} \tag{Since}
\end{align*}
$$

Velocity profile: $\frac{\mathrm{u}}{\mathrm{u}_{\infty}}=\sin \left(\frac{\pi}{2} \times \frac{\mathrm{y}}{\delta}\right)$

$$
\mathrm{K}=4.79
$$

$$
\begin{equation*}
\frac{\delta}{x}=\frac{K}{\sqrt{\operatorname{Re}_{x}}}=\frac{4.79}{\sqrt{\operatorname{Re}_{x}}} \tag{1}
\end{equation*}
$$

At $\mathrm{x}=0.6 \mathrm{~m}$,

$$
\begin{aligned}
& \operatorname{Re}_{\mathrm{x}}=\frac{\rho \mathrm{u}_{\infty} \mathrm{x}}{\mu} \\
& \operatorname{Re}_{\mathrm{x}}=\frac{1.25 \times 3 \times 0.6}{1.8 \times 10^{-5}}=1.25 \times 10^{5}
\end{aligned}
$$

From eq. (1),

$$
\begin{aligned}
& \left.\delta\right|_{x=0.6 \mathrm{~m}}=\frac{4.79 \times 0.6}{\sqrt{1.25 \times 10^{5}}}=8.129 \mathrm{~mm} \\
& \tau_{o}=\left.\mu \frac{d u}{d y}\right|_{y=0} ; \text { From given velocity profile } \\
& \frac{d u}{d y}=u_{\infty} \times \frac{\pi}{2 \delta} \times \cos \left(\frac{\pi}{2} \times \frac{y}{\delta}\right) \\
& \left.\frac{d u}{d y}\right|_{y=0}=\frac{u_{\infty} \pi}{2 \delta}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\tau_{o} & =\mu \times \frac{\mathrm{u}_{\infty} \pi}{2 \delta} \\
& =1.8 \times 10^{-5} \times \frac{3 \pi}{2 \times 8.129 \times 10^{-3}} \\
& =0.01043 \mathrm{~Pa}=10.43 \mathrm{milli} \mathrm{~Pa} .
\end{aligned}
$$

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3. Ans: 4

## Chapter <br> 11 <br> Force on Submerged Bodies

1. Ans: 8

Sol: Drag power $=$ Drag Force $\times$ Velocity

$$
\begin{aligned}
& \mathrm{P}=\mathrm{F}_{\mathrm{D}} \times \mathrm{V} \\
& \mathrm{P}=\mathrm{C}_{\mathrm{D}} \times \frac{\rho A V^{2}}{2} \times \mathrm{V} \\
& \mathrm{P} \propto \mathrm{~V}^{3} \\
& \frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}=\left(\frac{\mathrm{V}_{1}}{\mathrm{~V}_{2}}\right)^{3} \\
& \frac{\mathrm{P}_{1}}{\mathrm{P}_{2}}=\left(\frac{\mathrm{V}}{2 \mathrm{~V}}\right)^{3} \\
& \mathrm{P}_{2}=8 \mathrm{P}_{1}
\end{aligned}
$$

Comparing the above relation with XP , We get, $X=8$
02. Ans: 4.56 m

Sol: $F_{D}=C_{D} \cdot \frac{\rho A V^{2}}{2}$
$\mathrm{W}=0.8 \times 1.2 \times \frac{\frac{\pi}{4}(\mathrm{D})^{2} \times \mathrm{V}^{2}}{2}$
(Note: $\mathrm{A}=$ Normal (or)

$$
\text { projected Area } \left.=\frac{\pi}{4} D^{2}\right)
$$

$784.8=0.8 \times 1.2 \times \frac{\pi}{4}(D)^{2} \times \frac{10^{2}}{2}$
$\therefore \mathrm{D}=4.56 \mathrm{~m}$

Sol: Given data:
$l=0.5 \mathrm{~km}=500 \mathrm{~m}$
$\mathrm{d}=1.25 \mathrm{~cm}$
$\mathrm{V}_{\text {Wind }}=100 \mathrm{~km} / \mathrm{hr}$
$\gamma_{\text {Air }}=1.36 \times 9.81=13.4 \mathrm{~N} / \mathrm{m}^{3}$
$v=1.4 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$
$\mathrm{C}_{\mathrm{D}}=1.2$ for $\mathrm{Re}>10000$
$C_{D}=1.3$ for $\mathrm{Re}<10000$


Note: The characteristic dimension for electric power transmission tower wire is " $L$ "

$$
\operatorname{Re}=992 \times 10^{6}>10,000
$$

$$
\therefore \mathrm{C}_{\mathrm{D}}=1.2
$$

$$
\mathrm{F}_{\mathrm{D}}=\mathrm{C}_{\mathrm{D}} \times \frac{\rho A \mathrm{~V}^{2}}{2}
$$

$$
=1.2 \times \frac{\left(\frac{13.4}{9.81}\right)(\mathrm{L} \times \mathrm{d}) \mathrm{V}^{2}}{2}
$$

| $\mathbf{A C N}$ |  |
| ---: | :--- |
|  | $=\frac{1.2 \times\left(\frac{13.4}{9.81}\right)(500 \times 0.0125)\left(100 \times \frac{5}{18}\right)^{2}}{2}$ |
|  | $=3952.4 \mathrm{~N}$ |
|  | $=4 \mathrm{kN}$ |

## 04. Ans: $0.144 \boldsymbol{\&} \mathbf{0 . 1 2 6}$

## Sol: Given data:

$$
\begin{aligned}
\mathrm{W}_{\text {Kite }} & =2.5 \mathrm{~N} \\
\mathrm{~A} & =1 \mathrm{~m}^{2} \\
\theta & =45^{\circ} \\
\mathrm{T} & =25 \mathrm{~N}
\end{aligned}
$$

$$
\mathrm{V}_{\mathrm{Wind}}=54 \mathrm{~km} / \mathrm{hr}
$$

$$
=54 \times \frac{5}{18}=15 \mathrm{~m} / \mathrm{s}
$$



Resolving forces horizontally

$$
\begin{aligned}
\mathrm{F}_{\mathrm{D}} & =\mathrm{T} \cos 45^{\circ} \\
\mathrm{C}_{\mathrm{D}} \times \frac{\rho A V^{2}}{2} & =25 \times \cos 45^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{C}_{\mathrm{D}} \times\left(\frac{12.2}{9.81}\right)(1)(15)^{2}}{2} & =25 \times \frac{1}{\sqrt{2}} \\
\therefore \quad C_{D} & =0.126
\end{aligned}
$$

Resolving forces vertically

$$
\mathrm{F}_{\mathrm{L}}=\mathrm{W}_{\text {Kite }}+\mathrm{T} \sin 45^{\circ}
$$

$$
\frac{\mathrm{C}_{\mathrm{L}} \rho A V^{2}}{2}=2.5+25 \sin 45^{\circ}
$$

$$
\frac{\mathrm{C}_{\mathrm{L}}\left(\frac{12.2}{9.81}\right)(1)(15)^{2}}{2}=2.5+\frac{25}{\sqrt{2}}
$$

$$
\therefore \mathrm{C}_{\mathrm{L}}=0.144
$$

5. Ans: (a)

Sol: Given data:

$$
\mathrm{C}_{\mathrm{D}_{2}}=0.75 \mathrm{C}_{\mathrm{D}_{1}} \quad(25 \% \text { reduced })
$$

Drag power $=$ Drag force $\times$ Velocity

$$
\begin{aligned}
& P=F_{D} \times V=\frac{C_{D} \rho A V^{2}}{2} \times V \\
& P=C_{D} \times \frac{\rho A V^{3}}{2}
\end{aligned}
$$

Keeping $\rho$, A and power constant

$$
\mathrm{C}_{\mathrm{D}} \mathrm{~V}^{3}=\text { constant }=\mathrm{C}
$$

$$
\frac{\mathrm{C}_{\mathrm{D}_{1}}}{\mathrm{C}_{\mathrm{D}_{2}}}=\left(\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}\right)^{3}
$$

$$
\left(\frac{\mathrm{C}_{\mathrm{D}_{1}}}{0.75 \mathrm{C}_{\mathrm{D}_{1}}}\right)^{1 / 3}=\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}
$$

$$
\therefore \mathrm{V}_{2}=1.10064 \mathrm{~V}_{1}
$$

$\%$ Increase in speed $=10.064 \%$

## 06. Ans: (c)

Sol: When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid :

Weight of sphere $=$ Buoyant force + Drag force

## 07. Ans: 0.62

Sol: Given data,
Diameter of dust particle, $\mathrm{d}=0.1 \mathrm{~mm}$
Density of dust particle,

$$
\begin{aligned}
& \rho=2.1 \mathrm{~g} / \mathrm{cm}^{3}=2100 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mu_{\mathrm{air}}=1.849 \times 10^{-5} \text { Pa.s, }
\end{aligned}
$$

At suspended position of the dust particle,

$$
\mathrm{W}_{\text {particle }}=\mathrm{F}_{\mathrm{D}}+\mathrm{F}_{\mathrm{B}}
$$

where $F_{D}$ is the drag force on the particle and $F_{B}$ is the buoyancy force.

From Stokes law:

$$
\mathrm{F}_{\mathrm{D}}=3 \pi \mu \mathrm{~V} \mathrm{~d}
$$

Thus,

$$
\begin{aligned}
& \quad \frac{4}{3} \times \pi \mathrm{r}^{3} \times \rho \times \mathrm{g}=3 \pi \mu \mathrm{Vd}+\frac{4}{3} \pi \mathrm{r}^{3} \rho_{\text {air }} \mathrm{g} \\
& \text { or, } \frac{4}{3} \pi \mathrm{r}^{3} \mathrm{~g}\left(\rho-\rho_{\text {air }}\right)=3 \pi \mu_{\text {air }} \mathrm{V}(2 \mathrm{r}) \\
& \text { or } \mathrm{V}=\frac{2}{9} \mathrm{r}^{2} \mathrm{~g}\left(\frac{\rho-\rho_{\text {air }}}{\mu_{\text {air }}}\right) \\
& \quad=\frac{2}{9} \times\left(0.05 \times 10^{-3}\right)^{2} \times 9.81 \times \frac{(2100-1.2)}{1.849 \times 10^{-5}} \\
& \quad=0.619 \mathrm{~m} / \mathrm{s} \approx 0.62 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## 08. Ans: (b)

Sol: Since the two models $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.

From their shapes, we can say that $\mathrm{M}_{2}$ reaches the bottom earlier than $M_{1}$.
09. Ans: (a)

Sol:

- Drag of object $\mathrm{A}_{1}$ will be less than that on $A_{2}$. There are chances of flow separation on $A_{2}$ due to which drag will increase as compared to that on $\mathrm{A}_{1}$.
- Drag of object $B_{1}$ will be more than that of object $B_{2}$. Because of rough surface of $B_{2}$, the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but $\mathrm{C}_{2}$ is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on $\mathrm{C}_{2}$ will be more than that on $\mathrm{C}_{1}$ because of flow becoming turbulent due to roughness. Hence, drag of object $\mathrm{C}_{2}$ will be more than that of object $\mathrm{C}_{1}$.
- Thus, the correct answer is option (a).

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## Chapter <br> 12 <br> Dimensional Analysis

1. Ans: (c)

Sol: Total number of variables,
$\mathrm{n}=8$ and $\mathrm{m}=3(\mathrm{M}, \mathrm{L} \& \mathrm{~T})$
Therefore, number of $\pi$ 's are $=8-3=5$
02. Ans: (b)

Sol:

1. $\frac{\mathrm{T}}{\rho \mathrm{D}^{2} \mathrm{~V}^{2}}=\frac{\mathrm{MLT}^{2}}{\mathrm{ML}^{-3} \times \mathrm{L}^{2} \times \mathrm{L}^{2} \times \mathrm{T}^{-2}}=1$.
$\rightarrow$ It is a non-dimensional parameter.
2. $\frac{\mathrm{VD}}{\mu}=\frac{\mathrm{LT}^{-1} \times \mathrm{L}}{\mathrm{ML}^{-1} \mathrm{~T}^{-1}} \neq 1$.
$\rightarrow$ It is a dimensional parameter.
3. $\frac{\mathrm{D} \omega}{\mathrm{V}}=1$.
$\rightarrow$ It is a non-dimensional parameter.
4. $\frac{\rho V \mathrm{D}}{\mu}=\operatorname{Re}$.
$\rightarrow$ It is a non-dimensional parameter.

## 03. Ans: (b)

Sol: $\mathrm{T}=\mathrm{f}(\mathrm{l}, \mathrm{g})$
Total number of variable,

$$
\mathrm{n}=3, \mathrm{~m}=2(\mathrm{~L} \& \mathrm{~T} \text { only })
$$

Hence, no. of $\pi$ terms $=3-2=1$

## 04. Ans: (c)

## Sol:

- Mach Number $\rightarrow$ Launching of rockets
- Thomas Number $\rightarrow$ Cavitation flow in soil
- Reynolds Number $\rightarrow$ Motion of a submarine
- Weber Number $\rightarrow$ Capillary flow in soil

5. Ans: (b)

Sol: According to Froude's law

$$
\begin{aligned}
& T_{\mathrm{r}}=\sqrt{\mathrm{L}_{\mathrm{r}}} \\
& \frac{\mathrm{t}_{\mathrm{m}}}{\mathrm{t}_{\mathrm{p}}}=\sqrt{\mathrm{L}_{\mathrm{r}}} \\
& \mathrm{t}_{\mathrm{p}}=\frac{\mathrm{t}_{\mathrm{m}}}{\sqrt{\mathrm{~L}_{\mathrm{r}}}}=\frac{10}{\sqrt{1 / 25}} \\
& \mathrm{t}_{\mathrm{p}}=50 \mathrm{~min}
\end{aligned}
$$

6. Ans: (a)

Sol: $\mathrm{L}=100 \mathrm{~m}$

$$
V_{P}=10 \mathrm{~m} / \mathrm{s},
$$

$L_{r}=\frac{1}{25}$
As viscous parameters are not discussed, follow Froude's law.

According to Froude,
$\mathrm{V}_{\mathrm{r}}=\sqrt{\mathrm{L}_{\mathrm{r}}}$
$\frac{\mathrm{V}_{\mathrm{m}}}{\mathrm{V}_{\mathrm{p}}}=\sqrt{\frac{1}{25}}$
$\mathrm{V}_{\mathrm{m}}=\frac{1}{5} \times 10=2 \mathrm{~m} / \mathrm{s}$
07. Ans: (d)

Sol: Froude number = Reynolds number.

$$
v_{\mathrm{r}}=0.0894
$$

If both gravity \& viscous forces are important then

$$
\begin{aligned}
v_{\mathrm{r}} & =\left(\mathrm{L}_{\mathrm{r}}\right)^{3 / 2} \\
\sqrt[3]{\left(v_{\mathrm{r}}\right)^{2}} & =\mathrm{L}_{\mathrm{r}} \\
\mathrm{~L}_{\mathrm{r}} & =1: 5
\end{aligned}
$$

8. Ans: (c)

Sol: For distorted model according to Froude's law
$\mathrm{Q}_{\mathrm{r}}=\mathrm{L}_{\mathrm{H}} \mathrm{L}_{\mathrm{V}}^{3 / 2}$
$\mathrm{L}_{\mathrm{H}}=1: 1000$,
$\mathrm{L}_{\mathrm{V}}=1: 100$
$\mathrm{Q}_{\mathrm{m}}=0.1 \mathrm{~m}^{3} / \mathrm{s}$
$\mathrm{Q}_{\mathrm{r}}=\frac{1}{1000} \times\left(\frac{1}{100}\right)^{3 / 2}=\frac{0.1}{\mathrm{Q}_{\mathrm{p}}}$
$\mathrm{Q}_{\mathrm{P}}=10^{5} \mathrm{~m}^{3} / \mathrm{s}$
09. Ans: (c)

Sol: For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

$$
\frac{\rho_{\mathrm{w}} \times \mathrm{V}_{\mathrm{w}} \times \mathrm{d}_{\mathrm{w}}}{\mu_{\mathrm{w}}}=\frac{\rho_{\mathrm{a}} \times \mathrm{V}_{\mathrm{a}} \times \mathrm{d}_{\mathrm{a}}}{\mu_{\mathrm{a}}}
$$

or $\mathrm{V}_{\mathrm{w}}=\frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{w}}} \times \frac{\mathrm{d}_{\mathrm{a}}}{\mathrm{d}_{\mathrm{w}}} \times \frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{a}}} \times \mathrm{V}_{\mathrm{a}}$
(where suffixes $w$ and a stand for water and air respectively)

Substituting the values given, we get
$\mathrm{V}_{\mathrm{w}}=\frac{1.2}{10^{3}} \times \frac{4}{0.1} \times \frac{10^{-3}}{1.8 \times 10^{-5}} \times 1=\frac{8}{3} \mathrm{~m} / \mathrm{s}$
To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.
i.e, $\left(\frac{F_{D}}{\rho A V^{2}}\right)_{P}=\left(\frac{F_{D}}{\rho A V^{2}}\right)_{m}$

Hence, $\left(F_{D}\right)_{p}=\left(F_{D}\right)_{m} \times \frac{\rho_{a}}{\rho_{w}} \times \frac{A_{a}}{A_{w}} \times\left(\frac{V_{a}}{V_{w}}\right)^{2}$
$=4 \times \frac{1.2}{10^{3}} \times\left(\frac{4}{0.1}\right)^{2} \times\left(\frac{1}{8 / 3}\right)^{2}$
$=1.08 \mathrm{~N}$
10. Ans: 47.9

Sol: Given data,

|  | Sea water <br> (Prototype testing) | Fresh water <br> (model testing) |
| :---: | :---: | :---: |
| V | 0.5 | $?$ |
| $\rho$ | $1025 \mathrm{~kg} / \mathrm{m}^{3}$ | $10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ |
| $\mu$ | $1.07 \times 10^{-3} \mathrm{~Pa} . \mathrm{s}$ | $1 \times 10^{-3} \mathrm{~Pa} . \mathrm{s}$ |

For dynamic similarity, Re should be same in both testing.

$$
\begin{aligned}
\text { i.e., } & \frac{\rho_{\mathrm{m}} \mathrm{~V}_{\mathrm{m}} \mathrm{~d}_{\mathrm{m}}}{\mu_{\mathrm{m}}}=\frac{\rho_{\mathrm{p}} \mathrm{~V}_{\mathrm{p}} \mathrm{~d}_{\mathrm{p}}}{\mu_{\mathrm{p}}} \\
\mathrm{~V}_{\mathrm{m}} & =\mathrm{V}_{\mathrm{P}} \times \frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{m}}} \times \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{~d}_{\mathrm{m}}} \times \frac{\mu_{\mathrm{m}}}{\mu_{\mathrm{p}}} \\
= & 0.5 \times \frac{1025}{10^{3}} \times 100 \times \frac{10^{-3}}{1.07 \times 10^{-3}} \\
& =47.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Chapter <br> 13 Compressible Fluid Flow

1. Ans: (b)

Sol: Shock wave is an irreversible process. Entropy change across shock wave is always greater than zero, not nearly equal to zero All other statements given are correct.
02. Ans: (c)

Sol: Semi - angle of a Mach come is given by

$$
\begin{aligned}
& \sin \alpha=\frac{1}{M} \\
& \text { Or, } \alpha=\sin ^{-1}\left(\frac{1}{M}\right)
\end{aligned}
$$

## 03. Ans: (d)

Sol: Shock - boundary layer interaction in a convergent - divergent nozzle takes place in the divergent portion where the flow can be supersonic.

## 04. Ans: (d)

Sol: Across normal shock, the stagnation temperature remains constant.
$\mathrm{T}_{0}=$ constant,
Thus, $\mathrm{T}_{1}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{c}_{\mathrm{p}}}=\mathrm{T}_{2}+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{c}_{\mathrm{p}}}=\mathrm{T}_{0}$

As $\mathrm{V}_{2}$ decreases across normal shock, the temperature $\left(T_{2}\right)$ has to increase for $T_{0}$ to remain constant.

All other statements given are correct.
Note: As given above, statement 4 is wrong.
Thus from process of elimination, option (d) is the correct answer.
05. Ans: (a)

Sol: The correct matching is;

$$
\begin{aligned}
& \frac{\mathrm{T}^{*}}{\mathrm{~T}_{0}}=\frac{2}{\gamma+1} \\
& \frac{\mathrm{P}^{*}}{\mathrm{P}_{0}}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \\
& \frac{\rho^{*}}{\rho_{0}}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \\
& \frac{\mathrm{~S}^{*}}{\mathrm{~S}_{0}}=1 \quad \text { (Isentropic flow) }
\end{aligned}
$$

6. Ans: (d)

Sol: In isentropic flow, stagnation pressure, stagnation temperature and stagnation density would remain constant throughout the flow.
07. Ans: (a)

Sol: In a normal shock in a gas

- the upstream flow is supersonic
- the downstream flow is subsonic.

8. Ans: (d)

Sol: The following statements are correct:

- Mach number is equal to one at a point where the entropy is maximum whether it is Rayleigh or Fanno line .
- A normal shock can never appear in subsonic flow.
- The downstream Mach number across a normal shock is always less than one.
- The stagnation pressure across a normal shock decreases.

9. Ans: (b)

Sol: $\sin \alpha=\frac{1}{M}$

$$
\begin{aligned}
& \sin ^{2} \alpha=\frac{1}{\mathrm{M}^{2}} \\
& 1-\cos ^{2} \alpha=\frac{1}{\mathrm{M}^{2}} \\
& \cos ^{2} \alpha=\frac{\mathrm{M}^{2}-1}{\mathrm{M}^{2}} \\
& \cos \alpha=\frac{\sqrt{\mathrm{M}^{2}-1}}{\mathrm{M}} \\
& \Rightarrow \alpha=\cos ^{-1}\left(\frac{\sqrt{\mathrm{M}^{2}-1}}{\mathrm{M}}\right)
\end{aligned}
$$

## 10. Ans: (c)

Sol: Given data:
At $\mathrm{T}_{1}=15^{\circ} \mathrm{C}$,

$$
\mathrm{V}_{1}=400 \mathrm{~km} / \mathrm{hr}=400 \times \frac{5}{18} \mathrm{~m} / \mathrm{s}
$$

At $\mathrm{T}_{2}=-25^{\circ} \mathrm{C}, \mathrm{V}_{2}=$ ? for $\mathrm{M}_{1}=\mathrm{M}_{2}$

$$
\mathrm{M}_{1}=\frac{\mathrm{V}_{1}}{\mathrm{C}_{1}}=\frac{400 \times 5}{18 \times \sqrt{\mathrm{kR}(273+15)}}=\frac{400 \times 5}{18 \times \sqrt{\mathrm{kR} \times 288}}
$$

$$
M_{2}=M_{1}: \frac{V_{2}}{\sqrt{\mathrm{kR}(273-25)}}=\frac{400}{\sqrt{\mathrm{kR} \times 288}}
$$

Where $\mathrm{V}_{2}$ is in $\mathrm{km} / \mathrm{hr}$
$\mathrm{V}_{2}=400 \times \sqrt{\frac{248}{288}} \mathrm{~km} / \mathrm{hr}=371.2 \mathrm{~km} / \mathrm{hr}$
11. Ans: (d)

Sol: Given data : T $=273+15^{\circ}=288 \mathrm{k}$
Mach angle, $\alpha=30^{\circ}$
$\sin \alpha=\frac{1}{M}=\frac{C}{V}=\frac{\sqrt{\mathrm{kRT}}}{\mathrm{V}}$
$\mathrm{V}=\frac{\sqrt{\mathrm{kRT}}}{\sin \alpha}=\frac{\sqrt{1.4 \times 287 \times 288}}{\sin 30^{\circ}}=680.35 \mathrm{~m} / \mathrm{s}$
12. Ans: (b)

Sol: For a normal shock,
$\mathrm{M}_{1}($ upstream Mach number $)=1.68$
$\gamma=1.4, \mathrm{M}_{2}($ downstream Mach number $)=$ ?
We know that for a normal shock,
$M_{2}^{2}=\frac{M_{1}^{2}+\frac{2}{(\gamma-1)}}{2 M_{1}^{2}\left(\frac{\gamma}{\gamma-1}\right)-1}$
$=\frac{1.68^{2}+\frac{2}{(1.4-1)}}{2 \times 1.68^{2} \times\left(\frac{1.4}{0.4}\right)-1}=\frac{1.68^{2}+5}{2 \times 1.68^{2} \times 3.5-1}$ $=0.417$
$\Rightarrow \mathrm{M}_{2}=0.646$
Note: Answer key is (b).


## 13. Ans: (a)

Sol: Across a normal shock wave

- mach number decreases
- static pressure increases
- stagnation pressure decreases
- static temperature increases

14. Ans: (b)

Sol: A normal shock wave

- is irreversible
- is not isentropic
- can occur in diverging part of converging -diverging nozzle.

15. Ans: (c)

Sol: For Fanno line shown in the figure,


- subsonic flow proceeds along PQ not along PQR
- supersonic flow proceeds along RQ not along PQR
- subsonic flow proceed along PQ and supersonic flow proceeds along RQ.


## 16. Ans: (a)

Sol: For Rayleigh line, the temperature is maximum at $\frac{1}{\sqrt{\gamma}}$ while heating a subsonic flow.

## 17. Ans: (b)

Sol: Given data:
Rayleigh flow

$$
\begin{array}{ll}
\mathrm{P}_{1}=2 \mathrm{bar}, & \mathrm{~T}_{1}=60^{\circ} \mathrm{C} \\
\mathrm{~d}=60 \mathrm{~mm}, & \mathrm{~V}_{1}=40 \mathrm{~m} / \mathrm{s}
\end{array}
$$

From the Rayleigh line, it is known that at maximum entropy, $\mathrm{M}=1$

## 18. Ans: (c)

Sol: For Fanno flow in a constant area duct with supersonic flow at inlet and choking condition achieved, if the pipe length is reduced, then the flow will be still supersonic.
19. Ans: (b)

Sol: Refer to the figure given below, where the increase or decrease in thermodynamic properties in a divergent passage for $\mathrm{M}>1$ at entry is shown.


Thus, for supersonic flow entry to a diverging passage, pressure and density will decrease along the passage.

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20. Ans: (b)

Sol: Reyleigh line flow is a flow in a constant area duct without friction but with heat transfer.
21. Ans: (c)

Sol: Fanno line flow is a flow in a constant area duct with friction but in the absence of heat transfer and work.

## Chapter <br> 14 <br> Turbomachinery

1. Ans: 1000

Sol: $\mathrm{T}=$ Moment of momentum of water in a turbine $=$ Torque developed $=15915 \mathrm{~N}-\mathrm{m}$ Speed $(N)=600 \mathrm{rpm}$
Power developed $=\frac{2 \pi \mathrm{NT}}{60}$

$$
\begin{aligned}
& =\frac{2 \times \pi \times 600 \times 15915}{60} \\
& =1000 \times 10^{3} \mathrm{~W}=1000 \mathrm{~kW}
\end{aligned}
$$

2. Ans: 4000

Sol:

$$
\begin{aligned}
& \mathrm{Q}=50 \mathrm{~m}^{3} / \mathrm{sec}, \quad \mathrm{H}=7.5 \mathrm{~m} \\
& \eta_{\text {Turbine }}=0.8 \\
& \eta_{\text {Turbine }}=\frac{\mathrm{P}_{\text {shaft }}}{\mathrm{P}_{\text {water }}}=\frac{\mathrm{P}_{\text {shaft }}}{\rho g \mathrm{~g}\left(\mathrm{H}-\mathrm{h}_{\mathrm{f}}\right)} \\
& 0.8=\frac{\mathrm{P}_{\text {shaft }}}{1000 \times 9.81 \times 50(7.5-0)} \\
& \mathrm{P}_{\text {shaft }}=2943 \times 10^{3} \mathrm{~W}=2943 \mathrm{~kW} \\
& =\frac{2943}{0.736} \mathrm{HP}=4000 \mathrm{HP}
\end{aligned}
$$

3. Ans: 1

Sol: We know that

$$
\mathrm{U}=\frac{\pi \mathrm{DN}}{60}=\mathrm{k}_{\mathrm{u}} \cdot \sqrt{2 \mathrm{gH}}
$$

where $\mathrm{D}=$ diameter of wheel

$$
\mathrm{N}=\text { speed of turbine }=600 \mathrm{rpm}
$$

$\mathrm{H}=$ Head available of Pelton wheel turbine

$$
=300 \mathrm{~m}
$$

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| :--- | :--- | :--- |

$$
\begin{aligned}
\therefore \frac{\pi \times \mathrm{D} \times 600}{60} & =0.41 \sqrt{2 \times 9.81 \times 300} \\
\mathrm{D} & =1.0 \mathrm{~m}
\end{aligned}
$$

## 04. Ans: (b)

Sol: Specific speed of turbine is expressed as :

$$
\begin{aligned}
\mathrm{N}_{\mathrm{s}} & =\frac{\mathrm{N} \sqrt{\mathrm{P}}}{\mathrm{H}^{5 / 4}}=\frac{\mathrm{T}^{-1} \sqrt{\mathrm{FLT}^{-1}}}{\mathrm{~L}^{5 / 4}} \\
& =\mathrm{F}^{\frac{1}{2}} \mathrm{~L}^{\frac{1}{2}-\frac{5}{4}} \mathrm{~T}^{-1-\frac{1}{2}} \\
& =\mathrm{F}^{1 / 2} \mathrm{~L}^{-3 / 4} \mathrm{~T}^{-3 / 2}
\end{aligned}
$$

## 05. Ans: (b)

Sol: $\mathrm{P}=8.1 \mathrm{MW}=8100 \mathrm{~kW}$
$\mathrm{H}=81 \mathrm{~m} \quad ; \quad \mathrm{N}=540 \mathrm{rpm}$
Specific speed $N_{S}=\frac{N \sqrt{P}}{(H)^{5 / 4}}$

$$
=\frac{540 \times \sqrt{8100}}{(81)^{5 / 4}}=\frac{540 \times 90}{243}=200
$$

$60<\mathrm{N}_{\mathrm{S}}<300$ (Francis Turbine)
06. Ans: (a)

Sol: The specific speed is lowest for Pelton wheel and highest for Kaplan turbine. $\mathrm{N}_{\mathrm{s}}$ for Francis turbine lies between those of Pelton wheel and Kaplan turbine.
07. Ans: $(\mathbf{a}, \mathrm{c}, \mathrm{d})$

Sol:

- Only the tangential component of absolute velocity is considered into the estimation of theoretical head of a turbo machine. Hence, statement (a) is correct.
- A high head turbine has a low value of specific speed. Hence, statement (b) is wrong.
- For the same power, a turbo machine running at high specific speed will be small in size. Hence, statement (c) is correct.
- Pelton wheel is the tangential flow turbine whereas the Propeller and Kaplan turbines are axial flow units. Hence, statement (d) is correct.

8. Ans: (a)

Sol: $u=\frac{\pi D N}{60}$,
But $u \propto \sqrt{\mathrm{H}}$
Hence, for a given scale ratio.

$$
\mathrm{N} \propto \mathrm{H}^{1 / 2}
$$

9. Ans: (d)

Sol: Caviation in any flow passage will occur, if the local pressure at any point in the flow passage falls below the vapour pressure corresponding to the operating temperature.
10. Ans: (d)

Sol: Cavitation in a reaction turbine may occur at inlet to draft tube. It is expected that the pressure at inlet to draft tube may fall below the vapour pressure.

## 11. Ans: 1000

Sol: Given $\mathrm{N}_{\mathrm{p}}=500 \mathrm{rpm}$

$$
\frac{D_{m}}{D_{p}}=\frac{1}{2}
$$

We know that

$$
\left(\frac{\mathrm{ND}}{\sqrt{\mathrm{H}}}\right)_{\mathrm{m}}=\left(\frac{\mathrm{ND}}{\sqrt{\mathrm{H}}}\right)_{\mathrm{P}}
$$

Given H is constant

$$
\begin{aligned}
& \therefore \frac{\mathrm{N}_{\mathrm{m}}}{\mathrm{~N}_{\mathrm{p}}}=\frac{\mathrm{D}_{\mathrm{p}}}{\mathrm{D}_{\mathrm{m}}} \Rightarrow \frac{\mathrm{~N}_{\mathrm{m}}}{500}=2 \\
& \Rightarrow \mathrm{~N}_{\mathrm{m}}=1000 \mathrm{rpm}
\end{aligned}
$$

12. Ans: 73

Sol: Given $\mathrm{P}_{1}=100 \mathrm{~kW}$

$$
\mathrm{H}_{1}=100 \mathrm{~m} \quad \text { and } \quad \mathrm{H}_{2}=81 \mathrm{~m}
$$

We know that

$$
\begin{aligned}
\left(\frac{\mathrm{P}}{(\mathrm{H})^{3 / 2}}\right)_{1} & =\left(\frac{\mathrm{P}}{(\mathrm{H})^{3 / 2}}\right)_{2} \\
\therefore \frac{100}{(100)^{3 / 2}} & =\frac{\mathrm{P}_{2}}{(81)^{3 / 2}} \\
\mathrm{P}_{2} & =72.9 \mathrm{~kW} \simeq 73 \mathrm{~kW}
\end{aligned}
$$

New power developed by same turbine $=73 \mathrm{~kW}$

## 13. Ans: (b)

Sol: Given data :
$\mathrm{D}_{\text {runer }}=\mathrm{D}_{\text {tip }}=3 \mathrm{~m}$,
$\mathrm{D}_{\text {hub }}=\frac{1}{3} \times \mathrm{D}_{\text {runner }}=1 \mathrm{~m}$,
Velocity of flow, $\mathrm{V}_{\mathrm{f}}=5 \mathrm{~m} / \mathrm{s}$,

$$
\mathrm{u}=40 \mathrm{~m} / \mathrm{s}
$$

Discharge through the runner is,

$$
\begin{aligned}
\mathrm{Q} & =\frac{\pi}{4}\left(\mathrm{D}_{\text {tip }}^{2}-\mathrm{D}_{\text {hub }}^{2}\right) \times \mathrm{V}_{\mathrm{f}} \\
& =\frac{\pi}{4}\left(3^{2}-1^{2}\right) \times 5=31.4 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

