## GATE I PSUs

## Computer Science \& Information Technology

## DIGITAL LOGIC

## Text Book:

Theory with worked out Examples and Practice Questions

## Digital Logic

## (Solutions for Text Book Practice Questions)

## Chapter

1

## Number Systems

1. Ans: (d)

Sol: $135_{x}+144_{x}=323_{x}$ $\left(1 \times x^{2}+3 \times x^{1}+5 \times x^{0}\right)+\left(1 \times x^{2}+4 \times x^{1}+4 \times x^{0}\right)$ $=3 \mathrm{x}^{2}+2 \mathrm{x}^{1}+3 \mathrm{x}^{0}$
$\Rightarrow \mathrm{x}^{2}+3 \mathrm{x}+5+\mathrm{x}^{2}+4 \mathrm{x}+4=3 \mathrm{x}^{2}+2 \mathrm{x}+3$

$$
x^{2}-5 x-6=0
$$

$(x-6)(x+1)=0 \quad$ (Base cannot be negative)
Hence $x=6$.

> (OR)

As per the given number x must be greater than 5 . Let us consider $x=6$
$(135)_{6}=(59)_{10}$
$(144)_{6}=(64)_{10}$
$(323)_{6}=(123)_{10}$
$(59)_{10}+(64)_{10}=(123)_{10}$
So that $\mathrm{x}=6$

## 02. Ans: (a)

Sol: 8-bit representation of
$+127_{10}=01111111_{(2)}$
1 's complement representation of
$-127=10000000$.
2's complement representation of
$-127=10000001$.
No. of 1's in 2's complement of
$-127=m=2$
No. of 1's in 1's complement of
$-127=n=1$
$\therefore \mathrm{m}: \mathrm{n}=2: 1$
03. Ans: (b) \& (d)

Sol: $(14)_{10}=(1110)_{2}$
$+14=01110$
$-14=10010$
Using sign extension
$-14=11110010$

## 04. Ans: (c)

Sol: Binary representation of $+(539)_{10}$ :

| 2 | 539 |
| :--- | :--- |
| 2 | $269-1$ |
| 2 | $134-1$ |
| 2 | 67 |
| 2 | $63-0$ |
| 2 | $33-1$ |
| 2 | 16 |
| 2 | $8-1$ |
| 2 | 4 |
| 2 | -0 |
| 2 | 2 |

$(+539)_{10}=(1000011011)_{2}$

$$
=(001000011011)_{2}
$$

2's complement $\rightarrow 110111100101$
Hexadecimal equivalent $\rightarrow(\text { DE5 })_{\mathrm{H}}$
05. Ans: 5

Sol: Symbols used in this equation are $0,1,2,3$.
Hence base or radix can be 4 or higher
(312) ${ }_{\mathrm{x}}=(20)_{\mathrm{x}}(13.1)_{\mathrm{x}}$
$3 \mathrm{x}^{2}+1 \mathrm{x}+2 \mathrm{x}^{0}=(2 \mathrm{x}+0)\left(\mathrm{x}+3 \mathrm{x}^{0}+\mathrm{x}^{-1}\right)$
$3 x^{2}+x+2=(2 x)\left(x+3+\frac{1}{x}\right)$
$3 \mathrm{x}^{2}+\mathrm{x}+2=2 \mathrm{x}^{2}+6 \mathrm{x}+2$
$\mathrm{x}^{2}-5 \mathrm{x}=0 \Rightarrow \mathrm{x}(\mathrm{x}-5)=0$
$\mathrm{x}=0$ (or) $\mathrm{x}=5$
$x$ must be $x>3$, So $x=5$
06. Ans: 3 possible solutions

Sol: $123_{5}=\mathrm{x} 8 \mathrm{y}$
$1 \times 5^{2}+2 \times 5^{1}+3 \times 5^{0}=x . y^{1}+8 \times y^{0}$
$25+10+3=x y+8$
$\therefore \mathrm{xy}=30$
Possible solutions:
i. $\mathrm{x}=1, \mathrm{y}=30$
ii. $x=2, y=15$
iii. $x=3, y=10$

3 possible solutions
07. Ans: (b) \& (d)

Sol: $\left[B 9\right.$ F. AE] ${ }_{16}$
$=\left[\begin{array}{lll}10111001 & 1111.10101110\end{array}\right]_{2}$
Now make 3 bits as single group

08. Ans: (c)

Sol: (a) $68=150$

| 001 | 101 | 000 |
| :---: | :---: | :---: |
| 1 | 5 | 0 |

(b) $8 \mathrm{C}=214$

| 010 | 001 | 100 |
| :---: | :---: | :---: |
| 2 | 1 | 4 |

(c) $4 \mathrm{~F}=117$

| 001 | 001 | 111 |
| :---: | :---: | :---: |
| 1 | 1 | 7 |

(d) $5 \mathrm{D}=135$

| 001 | 01 | 1 |
| :---: | :---: | :---: |
| 1 | 3 | 5 |

9. Ans: (b)

Sol: (a)

|  | 75 |
| :---: | :---: |
| $\downarrow$ | $\downarrow$ |
| 111 |  |
|  |  |
|  |  |
|  |  |

(b)

65

$110 \quad 101$
(c)

|  | 37 |  |
| :---: | :---: | :---: |
| $\downarrow$ | $\downarrow$ |  |
| 011 |  | 111 |

(d)

26

010
010
10. Ans: (a), (c) \& (d)

Sol: Given Number (N) = 11101
In sign magnitude, MSB is sign and remaining bits are magnitude
$N=\underbrace{1} \underbrace{1101}$
$\underset{\downarrow}{\operatorname{sign}} \underset{\downarrow}{\downarrow} \underset{\downarrow}{\text { magnitude }}$

- [13] So option (A) is correct
and option(B) is wrong
now $\mathrm{N}=11101$ [Here MSB=1 So Negative number]
2's complement of $\mathrm{N}=-[0011]=[-3]_{10}$
So option (C) is also correct
$\mathrm{N}=11101$ [Her MSB $=1$ so it is Negative number]

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| :---: | :---: | :---: |

1's complement of $\mathrm{N}=-[00010]=(-2)_{10}$
Hence option (D) is also correct
11. Ans: (a)

Sol: Given number $\Rightarrow \mathrm{N}=\left[\begin{array}{lll}7 & 07 & 00\end{array}\right]_{8}$
Number of digits $=5$.
In 6 digits $N=\left[\begin{array}{lll}0 & 70700\end{array}\right]_{8}$
Here radix $=r=8$
For r's [8] complement, write zero bits as it is at LSB, subtract $1^{\text {st }}$ non zero LSB from "r" [Here 8] and remaining digits from r-1 [7]

$$
7778
$$

$\mathrm{N}=\begin{array}{llllll}0 & 7 & 0 & 7 & 0 & 0\end{array}$
$\downarrow \downarrow \downarrow \downarrow \quad \downarrow$ as it is
8 's complement $\mathrm{N}=\left[\begin{array}{llllll}7 & 0 & 7 & 1 & 0 & 0\end{array}\right]_{8}$
12. Ans: (c)

Sol: Given number $\Rightarrow \mathrm{N}=(9900)_{10}$
In 5 digits $\Rightarrow \mathrm{N}=\left[\begin{array}{lllll}0 & 9 & 9 & 0 & 0\end{array}\right]_{10}$
For 10's complement write LSB zero bits as it is then subtract $1^{\text {st }}$ non zero bit from " 10 " \& remaining from"9"

$$
\begin{array}{ccc}
9 & 9(10) \\
\mathrm{N}= & 9 & 9 \\
\hline & 9 & 9
\end{array}
$$

10 's complement $=\begin{array}{lllll}9 & 0 & 1 & 0 & 0\end{array}$

## Chapter

2

## Logic Gates \& Boolean Algebra

1. Ans: (b)

Sol: Truth table of XOR

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{o} / \mathbf{p}$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## Stage 1:

Given one $\mathrm{i} / \mathrm{p}=1$ Always.

$$
\begin{array}{ccc}
1 & X & o / p \\
\hdashline--- & 0 & 1 \\
1 & 1 & 0
\end{array}=\bar{X}
$$

For First XOR gate $\mathrm{o} / \mathrm{p}=\overline{\mathrm{X}}$
Stage 2:


For second XOR gate $\mathrm{o} / \mathrm{p}=1$.
Similarly for third XOR gate $\mathrm{o} / \mathrm{p}=\bar{X}$ \& for fourth $\mathrm{o} / \mathrm{p}=1$
For Even number of XOR gates $\mathrm{o} / \mathrm{p}=1$
For 20 XOR gates cascaded $\mathrm{o} / \mathrm{p}=1$.
02. Ans: 1

Sol: $\mathrm{f}=[\overline{\overline{\mathrm{D}}+\mathrm{A} \overline{\mathrm{B}}+\overline{\mathrm{A}} \mathrm{C}+\mathrm{A} \overline{\mathrm{C}} \mathrm{D}+\overline{\mathrm{A}} \overline{\mathrm{C}} \mathrm{D}]}$
Let $x=\bar{D}+A \bar{B}+\bar{A} C+A \bar{C} D+\bar{A} \bar{C} D[$ thenf $=\bar{x}]$
Simplify $x \Rightarrow$
Simplify $x \Rightarrow$
$\mathrm{x}=\overline{\mathrm{D}}+\mathrm{A} \overline{\mathrm{B}}+\overline{\mathrm{A}} \mathrm{C}+\overline{\mathrm{C}} \mathrm{D} \underbrace{\mathrm{A}+\overline{\mathrm{A}}}_{1}]$
$x=\bar{D}+D \bar{C}+A \bar{B}+\bar{A} C$
$x=[\overline{\mathrm{D}}+(\overline{(\bar{D})} \overline{\mathrm{C}}]+\mathrm{A} \overline{\mathrm{B}}+\overline{\mathrm{A}} \mathrm{C}[\because \mathrm{p}+\overline{\mathrm{p}} q=\mathrm{p}+q]$
$\mathrm{x}=\overline{\mathrm{D}}+\overline{\mathrm{C}}+\overline{\mathrm{A}} \mathrm{C}+\mathrm{A} \overline{\mathrm{B}}$
$\mathrm{x}=\overline{\mathrm{D}}+[\overline{\mathrm{C}}+(\overline{\mathrm{C}}) \overline{\mathrm{A}}]+\mathrm{A} \overline{\mathrm{B}}$
$\mathrm{x}=\overline{\mathrm{D}}+\overline{\mathrm{C}}+\overline{\mathrm{A}}+\mathrm{A} \overline{\mathrm{B}}$
$\mathrm{x}=\overline{\mathrm{D}}+\overline{\mathrm{C}}+[\overline{\mathrm{A}}+(\overline{\mathrm{A}}) \overline{\mathrm{B}}]$
$\mathrm{x}=\overline{\mathrm{D}}+\overline{\mathrm{C}}+\overline{\mathrm{A}}+\overline{\mathrm{B}}$
$\mathrm{x}=\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\overline{\mathrm{D}}$
$\mathrm{f}=\overline{\mathrm{x}}=\overline{\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\overline{\mathrm{D}}}=\overline{\overline{\mathrm{A}}} \cdot \overline{\bar{B}} \cdot \overline{\overline{\mathrm{C}}} \cdot \overline{\overline{\mathrm{D}}}=\mathrm{A} \cdot \mathrm{B} \cdot \mathrm{C} \cdot \mathrm{D}$
Number of minterms are "one"
Ans: 1.
03. Ans: (c)

Sol: $f=f_{1} f_{2}+f_{3}$

## 04. Ans: (c)

Sol: Let $\mathrm{x}_{1}=\mathrm{x}_{2}=\mathrm{x}_{3}=\mathrm{x}_{4}$
For all cases options $\mathrm{a}, \mathrm{b}, \mathrm{d}$ not satisfy.
05. Ans: (d)

Sol:
(a) $\bar{A}+\bar{B}$
(b) $\overline{\mathrm{A}+\mathrm{B}}$
(c) $\overline{\overline{\mathrm{A}}}+\overline{\overline{\mathrm{B}}}=\overline{\overline{\mathrm{A}}} \cdot \overline{\overline{\mathrm{B}}}$

$$
=\mathrm{A} . \mathrm{B}
$$

(d) $\overline{\bar{A}} \cdot \overline{\bar{B}}=A+B$
06. Ans: (b)

Sol: $\mathrm{A} \oplus \mathrm{B}=0$
(a)

| A | B | $\mathrm{A} \oplus \mathrm{B}$ |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$\mathrm{A}=\mathrm{B}$
(b) $\overline{\mathrm{A}+\mathrm{B}}=0$

A B A+B $\overline{\mathrm{A}+\mathrm{B}}$
$\begin{array}{llll}0 & 0 & 0 & 1\end{array}$
$\begin{array}{llll}0 & 1 & 1 & 0\end{array}$
$\begin{array}{llll}1 & 0 & 1 & 0\end{array}$
$\begin{array}{llll}1 & 1 & 1 & 0\end{array}$
(c) $\overline{\mathrm{A}} \cdot \mathrm{B}=0$

| $\overline{\mathrm{A}}$ | B | $\overline{\mathrm{A}} \cdot \mathrm{B}$ |
| :---: | :---: | :---: |
| 1 | 0 | 0 |
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| 0 | 1 | 0 |

(d) $\mathrm{A} \oplus \mathrm{B}=1$
$\mathrm{A} \neq 1$ (Refer option a)
07. Ans: (b)

Sol: (a) $\overline{a b+b c+c a+a b c}$
ab.bc.ca.abc
$(\bar{a}+\bar{b}) \cdot(\bar{b}+\bar{c}) \cdot(\bar{c}+\bar{a}) \cdot(\bar{a}+\bar{b}+\bar{c})$
$(\bar{a} \bar{b})+(\bar{a} \bar{c})+\bar{b}+(\bar{b} \bar{c}) \cdot(\bar{c}+\bar{a}) \cdot(\bar{a}+\bar{b}+\bar{c})$
$\bar{a} \bar{b}+\bar{a} \bar{c}+\bar{b}+(1+\bar{c}) \cdot(\bar{c}+\bar{a}) \cdot(\bar{a}+\bar{b}+\bar{c})$
$\overline{\mathrm{a}} \overline{\mathrm{b}}+\overline{\mathrm{a}} \overline{\mathrm{c}}+\overline{\mathrm{b}} \overline{\mathrm{c}}+\overline{\mathrm{b}} \cdot \overline{\mathrm{a}} \cdot(\overline{\mathrm{a}}+\overline{\mathrm{b}}+\overline{\mathrm{c}})$
$\bar{a} \bar{b}+\overline{a c}+\bar{b} \bar{c}+\bar{a} \bar{b}+\bar{a} \bar{b}+\bar{a} \bar{b} \bar{c}$
$\overline{\mathrm{a}} \overline{\mathrm{b}}+\overline{\mathrm{ac}}+\overline{\mathrm{b}} \overline{\mathrm{c}}+\overline{\mathrm{a}} \overline{\mathrm{b}}+\overline{\mathrm{a}}(1+\overline{\mathrm{c}})$
$\overline{\mathrm{a}} \overline{\mathrm{b}}+\overline{\mathrm{b}} \overline{\mathrm{c}}+\overline{\mathrm{c}} \overline{\mathrm{a}}$
(b) $\overline{a b+\bar{a} \bar{b}+\bar{c}}$
$\overline{\mathrm{ab}} \cdot \overline{\mathrm{ab}} . \overline{\mathrm{c}}$
$\overline{\mathrm{a}}+\overline{\mathrm{b}} . \overline{\mathrm{a}}+\overline{\overline{\mathrm{b}}} . \mathrm{c}$
$(\bar{a}+\bar{b}) \cdot(a+b) \cdot c$
$(\bar{a} b+\bar{b} a) . c$
$(\mathrm{A} \oplus \mathrm{B}) \mathrm{C}$
(c) $\overline{a+b c}$
$\overline{\mathrm{a}} . \overline{\bar{b}}$
$\bar{a}(\bar{b}+\bar{c})$
(d) $\overline{(\bar{a}+\bar{b}+\bar{c})} \overline{(a+\bar{b}+\bar{c})} \overline{(\bar{a}+\bar{b}+c)}$
$(\bar{a}+\bar{b}+\bar{c})+(a+\bar{b}+\bar{c}) \cdot(\bar{a}+\bar{b}+c)$
$(\overline{\bar{a}} \overline{\bar{b}} \overline{\bar{b}} \bar{c})+(\overline{\mathrm{a}} . \overline{\bar{b}} \overline{\bar{c}})(\overline{\bar{a}} \overline{\bar{b}} . \bar{c} . \mathrm{c})$
$a b c+\bar{a} b c+a b \bar{c}$
08. Ans: (c)
09. Ans: (a) \& (d)

Sol: $(x+y)(x+\bar{y})+\overline{\bar{x}}+x \bar{y}$ using Distributive Law
$F=(x+y \cdot \bar{y})+(\overline{\bar{x}}+\bar{y})=x+x y=x$
10. Ans: (b) \& (c)

Sol: Let B $\odot$ B [Here no. of $\mathrm{XOR}=1 \Rightarrow$ odd] $\bar{B} \bar{B}+B \cdot B=\bar{B}+B=1$
Let $B \odot B \odot B$ [Here no. of $X O R=2 \Rightarrow$ even] $=1 \odot B=\overline{1} \cdot \bar{B}+1 B=B=$ remain same as given variable
11. Ans: (a), (b) \& (c)

Sol: (a) Consensus theorem rule

$$
\mathrm{AB}+\overline{\mathrm{A}} \mathrm{C}+\mathrm{BC}=\mathrm{AB}+\overline{\mathrm{A}} \mathrm{C} \rightarrow \text { Correct }
$$

(b) R.H.S $(\mathrm{Y}+\mathrm{Z})(\mathrm{X}+\overline{\mathrm{Y}})=\mathrm{XY}+\overline{\mathrm{Y}} \mathrm{Z}+\mathrm{XZ}$ By using Consensus theorem we can write
$Y . X+\bar{Y} Z+X Z=Y X+\bar{Y} Z=L . H . S$
(b) Also correct
(c) $\overline{\overline{\mathrm{A}}} \overline{\overline{\mathrm{B}}} \overline{\overline{\mathrm{C}}}=\overline{\overline{\mathrm{A}}}+\overline{\overline{\mathrm{B}}}+\overline{\overline{\mathrm{C}}}=\mathrm{A}+\mathrm{B}+\mathrm{C} \Rightarrow$ Correct
(d) Simplification of
$(\mathrm{P}+\overline{\mathrm{Q}})(\overline{\mathrm{Q}}+\mathrm{R}+\overline{\mathrm{S}})(\overline{\mathrm{P}}+\overline{\mathrm{Q}}+\mathrm{S})$ is $\overline{\mathrm{Q}}+\mathrm{PRS}$ but given $\overline{\mathrm{Q}}+\mathrm{P} \overline{\mathrm{R}} \mathrm{S}$
12. Ans: (a), (b) \& (d)

Sol: Given $\mathrm{C}=\mathrm{A} * \mathrm{~B}$ but $\mathrm{A} * \mathrm{~B}=\mathrm{AB}+\overline{\mathrm{A}} \overline{\mathrm{B}}$
So $C=A * B=A B+\bar{A} \bar{B}=A \odot B$.
Now $\bar{C}=\overline{A B+\bar{A} \bar{B}}=\overline{\mathrm{A} \odot \mathrm{B}} \quad=\mathrm{A} \oplus \mathrm{B}$

$$
\begin{equation*}
=\mathrm{A} \overline{\mathrm{~B}}+\overline{\mathrm{A}} \mathrm{~B} . \tag{2}
\end{equation*}
$$

## Verification

Option (a) R. H. S $\Rightarrow \mathrm{B} * \mathrm{C}=\mathrm{BC}+\overline{\mathrm{B}} \overline{\mathrm{C}}$
Sub equations (1) \& (2)
$B * C=B[A B+\bar{A} B]+\overline{\mathrm{B}}[A \bar{B}+\overline{\mathrm{A}} B]$

$$
=\mathrm{AB}+0+\mathrm{A} \overline{\mathrm{~B}}+0
$$

$=\mathrm{A}$; hence option (a) is True
Now option (b)
$\mathrm{A} * \mathrm{C}=\overline{\mathrm{A}} \overline{\mathrm{C}}+\mathrm{AC}$

$$
=\overline{\mathrm{A}}[\mathrm{~A} \overline{\mathrm{~B}}+\overline{\mathrm{A}} \mathrm{~B}]+\mathrm{A}[\overline{\mathrm{~A}} \overline{\mathrm{~B}}+\mathrm{AB}]
$$

$$
=0+\overline{\mathrm{A}} \mathrm{~B}+\mathrm{AB}=\mathrm{B}[\overline{\mathrm{~A}}+\mathrm{A}]=\mathrm{B}
$$

hence option (b) is True
Now A * B * C
$=[A * B]^{*} C[$ But given $A * B=C]$
$=\mathrm{C} * \mathrm{C}$
$=\overline{\mathrm{C}} \overline{\mathrm{C}}+\mathrm{C} \cdot \mathrm{C}=\overline{\mathrm{C}}+\mathrm{C}=1$
So option (d) $\Rightarrow A * B * C=1$ is also True.


1. Ans: (b)

Sol:

02. Ans: (b)

Sol:


$$
\mathrm{f}=\overline{\mathrm{b}} \overline{\mathrm{~d}}+\overline{\mathrm{b}} \overline{\mathrm{c}}
$$

3. 

Sol:



SOP: x y + y w
POS: $y(x+w)$
04. Ans: (a)

Sol: For n-variable Boolean expression,
Maximum number of minterms $=2^{n}$
Maximum number of implicants $=2^{\text {n }}$
Maximum number of

$$
\begin{aligned}
\text { Essential prime implicants } & =\frac{2^{\mathrm{n}}}{2} \\
& =2^{\mathrm{n}-1}
\end{aligned}
$$

5. Ans: (c)

Sol:

$\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\overline{\mathrm{A}} \overline{\mathrm{C}}+\mathrm{BC}$
06. Ans: 3

Sol: $\bar{w} \bar{z}+\bar{w} x \bar{y}+\bar{x} y \bar{z}$


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| :---: | :---: | :---: |

7. Ans: (c)

Sol:

$\mathrm{x}_{2} \mathrm{X}_{4}+\overline{\mathrm{x}}_{3} \mathrm{x}_{1}+\overline{\mathrm{x}}_{4} \overline{\mathrm{x}}_{2}$
08. Ans: (a)

Sol:

09. Ans: (a)

Sol:

| 00 |  | 01 | 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 | 0 | 10 |
| 01 | 0 | 0 | 0 | 1 |
| 11 | 1 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 0 |
|  |  |  |  |  |

E.P.I $=4$
10. Ans: (b) \& (d)

Sol: Given $\mathrm{f}=\Pi \mathrm{M}[1,3,4,6,9,11,12,14]+\mathrm{d}[2,7,8,13]$


$$
\mathrm{f}_{\mathrm{sop}}=\frac{\mathrm{B} \cdot \overline{\mathrm{~B}}}{0}+\mathrm{BD}+\overline{\mathrm{B}} \overline{\mathrm{D}}+\underset{0}{\frac{\mathrm{~B}}{\mathrm{~B}} \cdot \mathrm{D}} \overline{\mathrm{D}}^{\overline{\mathrm{D}}} \mathrm{~J}_{\mathrm{BD}}+\overline{\mathrm{B}} \overline{\mathrm{D}}
$$

## So option (c) is wrong

$$
\left.\begin{array}{l}
f_{\text {sop }}=B \cdot D+\bar{B} \bar{D} \\
f_{\text {pos }}=[B+\bar{D}[\bar{B}+D]]
\end{array}\right] \begin{aligned}
& \text { Both cases var iables A,C } \\
& \text { are absent so Independent } \\
& \text { of "2" variables }
\end{aligned}
$$

Hence option (b) is correct


## Chapter

4 Combinational Circuits

1. Ans: (d)

Sol: Let the output of first MUX is " $\mathrm{F}_{1}$ "
$\mathrm{F}_{1}=\mathrm{AI}_{0}+\mathrm{AI}_{1}$
Where A is selection line, $\mathrm{I}_{0}, \mathrm{I}_{1}=\mathrm{MUX}$ Inputs
$\mathrm{F}_{1}=\overline{\mathrm{S}}_{1} \cdot \mathrm{~W}+\mathrm{S}_{1} \cdot \overline{\mathrm{~W}}=\mathrm{S}_{1} \oplus \mathrm{~W}$
Output of second MUX is
$\mathrm{F}=\overline{\mathrm{A}} \cdot \mathrm{I}_{0}+\mathrm{A} \cdot \mathrm{I}_{1}$
$\mathrm{F}=\overline{\mathrm{S}}_{2} \cdot \mathrm{~F}_{1}+\mathrm{S}_{2} \cdot \overline{\mathrm{~F}}_{1}$
$\mathrm{F}=\mathrm{S}_{2} \oplus \mathrm{~F}_{1}$
But $\mathrm{F}_{1}=\mathrm{S}_{1} \oplus \mathrm{~W}$
$\mathrm{F}=\mathrm{S}_{2} \oplus \mathrm{~S}_{1} \oplus \mathrm{~W}$
i.e., $\mathrm{F}=\mathrm{W} \oplus \mathrm{S}_{1} \oplus \mathrm{~S}_{2}$
02. Ans: 19.2

Sol: F.A using H.A

$\mathrm{t}_{\mathrm{AND}}=1.2 \mu \mathrm{~s} ; \mathrm{t}_{\mathrm{OR}}=1.2 \mu \mathrm{sec}$
$\mathrm{XOR}=2 \times 1.2=2.4 \mu \mathrm{sec}$
Time for addition $=$ XOR $1+$ XOR $2=2.4+2.4$ $=4.8 \mu \mathrm{sec}$
Time for carry $=$ XOR1+AND2+OR

$$
=2.4+1.2+1.2=4.8 \mu \mathrm{sec}
$$

for $n$-bit total time required

$$
=(\mathrm{n}-1) \mathrm{tc}+\mathrm{Max}[\mathrm{tc}, \mathrm{ts}]
$$

$\Rightarrow$ for 4 bit
$(4-1) \times 4.8+$ Max [4.8, 4.8]
$3 \times 4.8+4.8=19.2 \mu \mathrm{sec}$
03. Ans: 6

Sol: T $=0 \rightarrow$ NOR $\rightarrow$ MUX $1 \rightarrow$ MUX 2
$2 \mathrm{~ns} \quad 1.5 \mathrm{~ns} \quad 1.5 \mathrm{~ns}$
Delay $=2 \mathrm{~ns}+1.5 \mathrm{~ns}+1.5 \mathrm{~ns}=5 \mathrm{~ns}$
$\mathrm{T}=1 \rightarrow$ NOT $\rightarrow$ MUX $1 \rightarrow$ NOR $\rightarrow$ MUX 2

$$
1 \mathrm{~ns} \quad 1.5 \mathrm{~ns} \quad 2 \mathrm{~ns} \quad 1.5 \mathrm{~ns}
$$

Delay $=1 \mathrm{~ns}+1.5 \mathrm{~ns}+2 \mathrm{~ns}+1.5 \mathrm{~ns}=6 \mathrm{~ns}$
Hence, the maximum delay of the circuit is 6 ns
04. Ans: 195

Sol: Given $\mathrm{t}_{\text {carry }}=12 \mathrm{~ns}$

$$
\begin{aligned}
\mathrm{t}_{\text {sum }} & =15 \mathrm{~ns} \\
\mathrm{n} & =16 \mathrm{bit}
\end{aligned}
$$

Time required $=(16-1) \times 12+\max (15,12)$

$$
15 \times 12+15=195 \mathrm{nsec}
$$

5. Ans: (b), (c) \& (d)

Sol: For " n " bit number addition we require

$$
\begin{aligned}
&=" n " F . A(o r) 1 \text { H.A, (n-1) F.A (or) } \\
&(2 n-1) \text { H.A, (n-1) OR Gates }
\end{aligned}
$$

6. Ans: (a) \& (c)

Sol: Output $=\mathrm{Y}=\sum \mathrm{m}(0,1,2,6)+\overline{\mathrm{C}}$


$$
\begin{aligned}
& \mathrm{Y}=\overline{\mathrm{A}} \overline{\mathrm{~B}}+\mathrm{B} \overline{\mathrm{C}}+\overline{\mathrm{C}} \\
= & \overline{\mathrm{A}} \overline{\mathrm{~B}}+\overline{\mathrm{C}}[\mathrm{~B}+1]=\overline{\mathrm{C}}+\overline{\mathrm{A}} \overline{\mathrm{~B}} \rightarrow \text { Option (c) } \\
\Rightarrow & \mathrm{Y}=\overline{\mathrm{C}}+\overline{\mathrm{A}} \overline{\mathrm{~B}}=(\overline{\mathrm{C}}+\overline{\mathrm{A}})(\overline{\mathrm{C}}+\overline{\mathrm{B}})=(\overline{\mathrm{A}}+\overline{\mathrm{C}})(\overline{\mathrm{B}}+\overline{\mathrm{C}}) \\
\rightarrow & \text { Option (a) }
\end{aligned}
$$

7. Ans: (b)

Sol: In n-bit magnitude comparator the number of combinations for
$\mathrm{A}>\mathrm{B}$ (or) $\mathrm{A}<\mathrm{B}=\frac{2^{2 \mathrm{n}}-2^{\mathrm{n}}}{2}$
Here given 8 bit. So, $n=8$
So total number of combinations for

$$
\begin{aligned}
A>B & =\frac{2^{2 \times 8}-2^{8}}{2}=\frac{2^{16}-2^{8}}{2}=\frac{2^{8} \cdot 2^{8}-2^{8}}{2} \\
& =\frac{2^{8} \cdot\left[2^{8}-1\right]}{2}=255 \times 2^{7} .
\end{aligned}
$$

8. Ans: (d)

Sol: As per the definition of MUX, output equation is
$\mathrm{f}=\overline{\mathrm{S}_{1}} \overline{\mathrm{~S}_{0}}\left[\mathrm{I}_{0}\right]+\overline{\mathrm{S}_{1}} \mathrm{~S}_{0}\left[\mathrm{I}_{1}\right] \mathrm{S}_{1} \overline{\mathrm{~S}_{0}}\left[\mathrm{I}_{2}\right]+\mathrm{S}_{1} \mathrm{~S}_{0}\left[\mathrm{I}_{3}\right]$
Here $\mathrm{S}_{1}=\mathrm{P} ; \mathrm{S}_{0}=\mathrm{Q} ; \mathrm{I}_{0}=0 ; \mathrm{I}_{1}=1 ; \mathrm{I}_{2}=\mathrm{R} ; \mathrm{I}_{3}=\overline{\mathrm{R}}$
$\mathrm{f}=\overline{\mathrm{P}} \overline{\mathrm{Q}}[0]+\overline{\mathrm{P}}[1]+\mathrm{P} \overline{\mathrm{Q}}[\mathrm{R}]+\mathrm{PQ}[\overline{\mathrm{R}}]$
$\mathrm{f}=0+\overline{\mathrm{P} Q}[\overline{\mathrm{R}}+\mathrm{R}]+\mathrm{P} \overline{\mathrm{Q} R}+\mathrm{PQ} \overline{\mathrm{R}}$
$\mathrm{f}=\overline{\mathrm{P}} \mathrm{Q} \overline{\mathrm{R}}+\overline{\mathrm{P}} \mathrm{QR}+\mathrm{P} \overline{\mathrm{Q} R}+\mathrm{PQ} \overline{\mathrm{R}}$

$\mathrm{F}=\Sigma \mathrm{m}=[2,3,5,6]$
09. Ans: (b), (c) \& (d)

Sol: For " n " bit number addition we require $=$ "n" F.A (or) 1 H.A, (n-1) F.A (or) (2n-1) H.A, (n-1) OR Gates
10. Ans: (a), (b), (c) \& (d)

Sol: Output of MUX1 is $=F_{1}=\overline{\mathrm{X}} \mathrm{I}_{0}+\mathrm{X}_{1} \mathrm{I}_{1}=$ $\overline{\mathrm{X}}(0)+\mathrm{X}(1) \Rightarrow \mathrm{F}_{1}=\mathrm{X}$
output of MUX2 $=\mathrm{F}_{2}=\overline{\mathrm{Y}} . \mathrm{I}_{0}+\mathrm{Y}_{\mathrm{I}} \mathrm{I}_{1}$
$\mathrm{F}_{2}=\overline{\mathrm{Y}} . \mathrm{X}+\mathrm{YX} \quad\left[\right.$ Here $\left.\mathrm{I}_{1}=\mathrm{F}_{1}=\mathrm{X}\right]$
$\mathrm{F}_{2}=\mathrm{X}[\overline{\mathrm{Y}}+\mathrm{Y}]=\mathrm{X}$
output of MUX3 $=\mathrm{F}=\overline{\mathrm{Z}} . \mathrm{I}_{0}+\mathrm{ZI}_{1}$
$\mathrm{F}=\mathrm{Y} \overline{\mathrm{Z}}+\mathrm{XZ}$ convert into standard form

$$
\begin{aligned}
\mathrm{F} & =(\mathrm{X}+\overline{\mathrm{X}}) \mathrm{Y} \overline{\mathrm{Z}}+\mathrm{X}(\mathrm{Y}+\overline{\mathrm{Y}}) \mathrm{Z} \\
\mathrm{~F} & =\mathrm{XYZ}+\mathrm{X} \overline{\mathrm{Y}} \mathrm{Z}+\mathrm{XY} \overline{\mathrm{Z}}+\overline{\mathrm{X}} \mathrm{Y} \overline{\mathrm{Z}} \\
& =\Sigma \mathrm{m}(2,5,6,7)
\end{aligned}
$$

11. Ans: 3

Sol:

| Decimal | Inputs |  |  | Output |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{Y}$ |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 1 |

Output $\Rightarrow \mathrm{Y}[\mathrm{A}, \mathrm{B}, \mathrm{C}]=\Sigma \mathrm{m}[3,5,6,7]$
simplify using k map


1. Ans: 4

Sol: In the given first loop of states, zero has repeated 3 times. So, minimum 4 number of Flip-flops are needed.
02. Ans: (b)

Sol:

| CLK | Serial in= $\mathrm{B} \oplus \mathrm{C} \oplus \mathrm{D}$ | A B C D |
| :---: | :---: | :---: |
| 0 |  | 1010 |
| 1 | 1 | $\begin{array}{lllll}1 & 1 & 0 & 1\end{array}$ |
| 2 | 0 | $\begin{array}{lllll}0 & 1 & 1 & 0\end{array}$ |
| 3 | 0 | 00011 |
| 4 | 0 | 0001 |
| 5 | 1 | 1000 |
| 6 | 0 | $\begin{array}{llll}0 & 1 & 0 & 0\end{array}$ |
|  |  | 1010 |

3. Ans: (b)

Sol:

| J | K | $\mathrm{Q}_{\mathrm{n}}$ | $\overline{\mathrm{Q}}_{\mathrm{n}}$ | $\begin{aligned} & \mathrm{T}=\left(\mathrm{J}+\mathrm{Q}_{\mathrm{n}}\right) \\ & \left(\mathrm{K}+\overline{\mathrm{Q}}_{\mathrm{n}}\right) \end{aligned}$ | $\mathrm{Q}_{\mathrm{n}+1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | $0.1=0$ | $\left.1_{1}\right\}^{0} \mathrm{Q}_{\mathrm{n}}$ |
| 0 | 0 | 1 | 0 | $1.0=0$ |  |
| 0 | 1 | 0 | 1 | $0.1=0$ | $\left.{ }_{0}\right\}_{0}^{0}$ |
| 0 | 1 | 1 | 0 | $1.1=1$ |  |
| 1 | 0 | 0 | 1 | $1.1=1$ | $\left.1_{1}\right\}_{1}^{1}$ |
| 1 | 0 | 1 | 0 | $1.0=0$ |  |
| 1 | 1 | 0 | 1 | $1.1=1$ | $\}^{1} \overline{\mathrm{Q}}_{\mathrm{n}}$ |
| 1 | 1 | 1 | 0 | $1.1=1$ |  |


$\mathrm{T}=\mathrm{J} \overline{\mathrm{Q}_{\mathrm{n}}}+\mathrm{KQ}_{\mathrm{n}}=\left(\mathrm{J}+\mathrm{Q}_{\mathrm{n}}\right)\left(\mathrm{K}+\overline{\mathrm{Q}_{\mathrm{n}}}\right)$

## 04. Ans: (c)

Sol: Asynchronous sequential circuit does not use any clock so it is false.

## 05. Ans: (d)

Sol: Master-slave JK-flip-flop is preferred to an level triggered JK flip-flop. Hence it is false.

## 06. Ans: 3

## Sol:

| $\mathrm{Q}_{2}$ | $\mathrm{Q}_{1}$ | $\mathrm{Q}_{0}$ |  |
| :---: | :---: | :---: | :--- |
| 0 | 0 | 0 | $\rightarrow 0$ |
| 1 | 0 | 0 | $\rightarrow 0$ |
| 0 | 0 | 1 | $\rightarrow 1$ |
| 1 | 0 | 1 | $\rightarrow 1$ |
| 0 | 1 | 0 | $\rightarrow 2$ |
| 1 | 1 | 0 | $\rightarrow 2$ |
| 0 | 1 | 1 | $\rightarrow 3$ |
| 1 | 1 | 1 | $\rightarrow 3$ |

It is $\bmod 8$ counter So, no. of flip flops required is 3 .
07. Ans: (c)

Sol: From circuit we can write
$\mathrm{D}=\mathrm{X} \oplus \mathrm{Q}$
but for $\mathrm{DFF} \Rightarrow \mathrm{Q}_{\mathrm{n}+1}=\mathrm{D}$
$\mathrm{Q}_{\mathrm{n}+1}=\mathrm{D}=\mathrm{X} \oplus \mathrm{Q}$
$\mathrm{Q}_{\mathrm{n}+1}=\overline{\mathrm{X}} \mathrm{Q}+\mathrm{X} \overline{\mathrm{Q}}$
[This is same as T FF output]
T FF output $\mathrm{Q}_{\mathrm{n}+1}=\overline{\mathrm{T}} \mathrm{Q}+\mathrm{TQ}$
Therefore given circuit act as T Flip-Flop
08. Ans: (d)

Sol: $\mathrm{D}_{\mathrm{A}}=\mathrm{Q}_{\mathrm{B}} \odot \mathrm{Q}_{\mathrm{C}} ; \mathrm{D}_{\mathrm{B}}=\mathrm{Q}_{\mathrm{A}} ; \mathrm{D}_{\mathrm{C}}=\mathrm{Q}_{\mathrm{B}}$
Given initially $\mathrm{Q}_{\mathrm{A}}=\mathrm{Q}_{\mathrm{B}}=\mathrm{Q}_{\mathrm{C}}=0$

| $\mathbf{C L O C K}$ | $\mathbf{D}_{\mathbf{A}}$ | $\mathbf{D}_{\mathbf{B}}$ | $\mathbf{D}_{\mathrm{C}}$ | $\mathbf{Q}_{\mathbf{A}}$ | $\mathbf{Q}_{\mathbf{B}}$ | $\mathbf{Q}_{\mathrm{C}}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | - | - | - | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 2 | 1 | 1 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 1 |
| 4 | 1 | 0 | 1 | 1 | 0 | 1 |
| 5 | 0 | 1 | 0 | 0 | 1 | 0 |
| 6 | 0 | 0 | 1 | 0 | 0 | 1 |

The output observed at $\mathrm{Q}_{\mathrm{A}}$ is 0110100 -----
09. Ans: (b) \& (d)

Sol: Total states in ' $n$ ' bit Johnson counter $=2^{n}$ number of used states $=2 \times n$

Here $\mathrm{n}=4$ bits
So, total number of states $=2^{4}=16$
number of used states $=2 \times 4=8$
unused states $=$ Total states - used states
Unused states $=16-8$

$$
=8
$$

