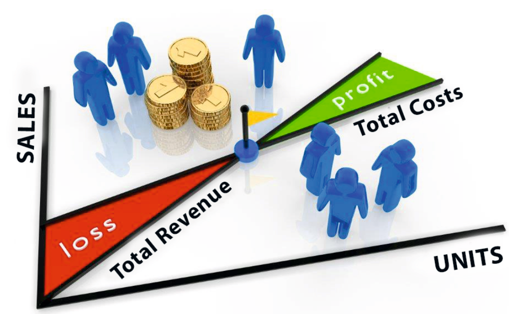
MECHANICAL ENGINEERING

GATE | PSUs

IM & OR

Text Book: Theory with worked out Examples and Practice Questions



IM & OR

(Solutions for Text Book Practice Questions)

Chapter 1	PERT & CPM

01. Ans: (a)

Sol: CPM deals with deterministic time durations.

02. Ans: (a)

Sol: Critical Path :

- It is a longest path consumes maximum amount of resources
- It is the minimum time required to complete the project

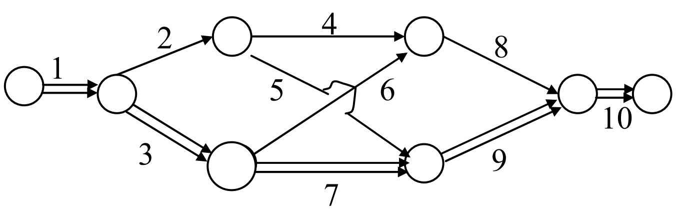
03. Ans: (a)

04. Ans: (a)

Sol: Gantt chart indicates comparison of actual progress with the scheduled progress.

05. Ans: (c)

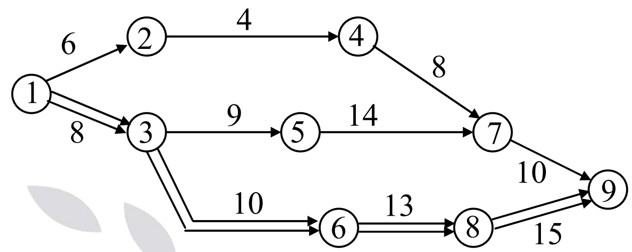
Sol:



Critical path = 1 + 3 + 7 + 9 + 10 = 30 days

06. Ans: (c)

Sol:



Critical path (1-3-6-8-9) = 8 + 10 + 13 + 15

= 46 days

07. Ans: (b)

Sol: Rules for drawing Network diagram:

- Each activity is represented by one and only one arrow in the network.
- No two activities can be identified by the same end events.
- Precedence relationships among all activities must always be maintained.
- No dangling is permitted in a network.
- No Looping (or Cycling) is permitted.

08. Ans: (b)

Sol: **Activity:** Resource consuming and well-defined work element.

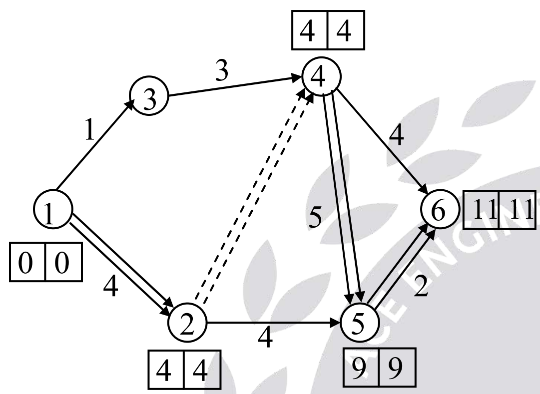
Event: Each event is represented as a node in a network diagram and it does not consume any time or resource.

Dummy Activity: An activity does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.

Float: Permissible delay period for the activity.

09. Ans: (b)

Sol:



10. Ans: (a)

11. Ans: (b)

Sol:

- Beta Distribution is used to decide the expected duration of an activity.
- The expected duration of the project can be described by Normal distribution.

12. Ans: (b)

Sol: T_0 = 8 min, T_m = 10 min, T_p = 14 min,

$$T_e = \frac{T_o + 4T_m + T_p}{6}$$

$$= \frac{8+4\times10+14}{6} = \frac{62}{6} = 10.33 \text{ min}$$

13. Ans: (a)

Sol: Take 4-3, T_e = 6 days

Critical path = 1-2-4-3

= 5 + 14 + 4 = 23 days

$$\sigma_{\text{critical path}} = \sqrt{V_{1-2} + V_{2-4} + V_{4-3}}$$

$$= \sqrt{2^2 + 2.8^2 + 2^2} = 3.979$$

$$z = \frac{\text{Due date} - \text{critical path duration}}{\sigma_{\text{critical path}}}$$

$$z = \frac{27-23}{3.979} = 1.005$$

$\therefore$ P(z) = 0.841

14. Ans: (b)

15. Ans: (c)

Sol: D = 36 days, V = 4 days

$$Z = \frac{36-36}{\sqrt{4}} = 0$$

$\Rightarrow$ P(z) = 50%

16. Ans: (c)

Sol: $\sigma_{cp} = \sqrt{V_{a-b} + V_{b-c} + V_{c-d} + V_{d-e}}$

$$= \sqrt{4+16+4+1} = 5$$

17. Ans: (a)

Sol: The latest that an activity can start from the beginning of the project without causing a delay in the completion of the entire project. It is the maximum time up to which an activity can be delayed to start without effecting the project completion duration time. (LST = LFT – duration).

18. Ans: (c)

Sol: The earliest expected completion time,

Critical path : A-B-C-D-F-E-H

$\Rightarrow 5 + 4 + 8 + 5 + 8 = 30$ days

19. Ans: (d)

Sol: *Critical path :*

1-3-4-6 = 20 days

$$z = \frac{24 - 20}{\sqrt{4}} = \frac{4}{2} = 2$$

$\Rightarrow P(z) = 97.7\%$

20. Ans: (d)

Sol: Variance = $\left(\frac{t_p - t_o}{6}\right)^2$

$$= \left(\frac{22 - 10}{6}\right)^2 = 4$$

21. Ans: (a)

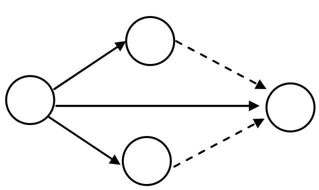
22. Ans: (b)

23. Ans: (a)

24. Ans: (b)

25. Ans: (c)

Sol:

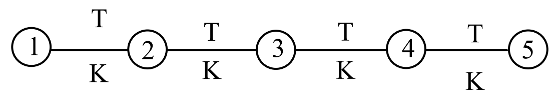


26. Ans: (c)

27. Ans: (b)

28. Ans: (d)

Sol:



Given each activity having time mean duration 'T' and standard deviation 'K'.

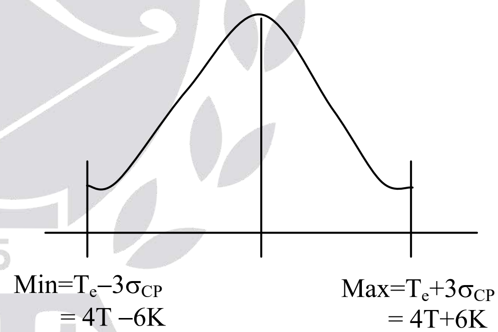
Total time estimate $T_e = 4T$

Variance of the path

$(\Sigma var)_{CP} = R^2 + R^2 + R^2 + R^2$

$= 4R^2$

Standard deviation of CP= $\sqrt{\sum (var)_{CP}}$



$\sigma_{CP} = \sqrt{4K^2}$

$\sigma_{CP} = \pm 2K$

Range of overall project duration likely to be in 4T + 6K and 4T – 6K

i.e., $4T \pm 6K$

Common solutions for Q.29 & Q.30

29. Ans: (b)

30. Ans: (b)

Sol:

Paths	Duration
1-2-4-5 = (AEF)	8+9+6=23
1-2-3-4-5=(ADF)	8+9+6=23
1-3-4-5 (BDF)	6+9+6 = 21
1-4-5 (CF)	16+6=22

∴ Highest time taken paths are AEF and ADF

∴ Critical path's are AEF and ADF

Critical paths are '2'.

Possible cases to crash

A by 1 day that cost = 80

F by 1 day that cost = 130

E and D by 1 day that cost = 20 + 40 = 60

31. Ans: (c)

32. Ans: (c)

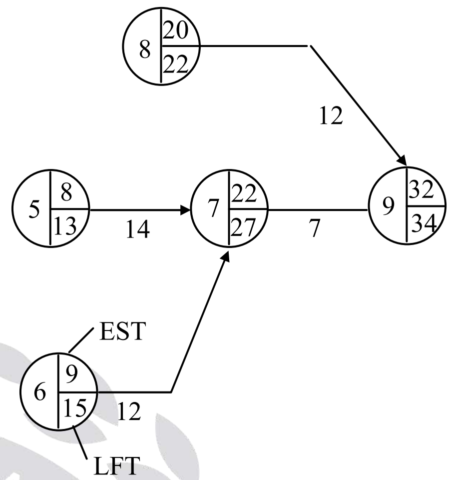
Sol:

Path	Duration
AB	7+5 = 12
CD	6+6 = 12
EF	8+4 = 12

Three critical paths, number of activities to be crashed are 3.

33. Ans: (c)

Sol:

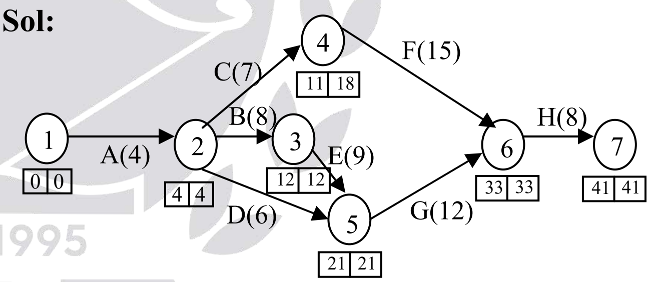


$(\text{Total Float})_{6\text{-}7} = 27 - 9 - 12 = 6$

$(\text{Free float})_{6\text{-}7} = 28 - 9 - 12 = 1$

34. Ans: (a-7, b-41)

Sol:



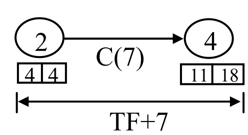
Path **duration**

1-2-4-6-7 = 4 + 7 + 15 + 8 = 34

1-2-3-5-6-7 = 4 + 8 + 9 + 12 + 8

= **41 (days) (critical path)**

1-2-5-6-7 = 4 + 6 + 12 + 8 = 30



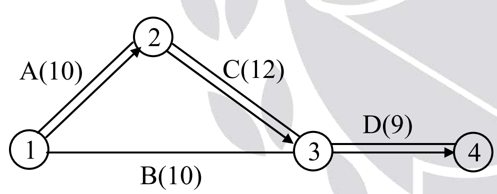
TF + 7 = 18 – 4

⇒ TF = 14 – 7 = **7**

35. Ans: (31 days, $\sqrt{6}$)

Sol:

Activity	Time estimated $T_e = \dfrac{T_o + 4T_m + T_p}{6}$	Standard deviation $\sigma = \dfrac{T_p - T_o}{6}$
A	$\dfrac{5+4\times10+15}{6}=10$	$\dfrac{15-5}{6}=\dfrac{5}{3}$
B	$\dfrac{2+4\times5+8}{6}=5$	$\dfrac{8-2}{6}=1$
C	$\dfrac{10+4\times12+14}{6}=12$	$\dfrac{14-10}{6}=\dfrac{2}{3}$
D	$\dfrac{6+4\times8+16}{6}=9$	$\dfrac{16-6}{6}=\dfrac{5}{3}$



Critical path :

1-2-3-4 = 10 + 12 + 9 = 31 days

$$\sigma_{cp} = \sqrt{V_{1-2} + V_{2-3} + V_{3-4}}$$

$$= \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{6}$$

36 Ans: (a, d)

Sol: In a PERT network of project scheduling the activities in the network follows beta distribution and expected duration of project follows normal distribution.

37 Ans: (c, d)

Sol: In a project-crashing, crash only critical activities (activity on critical path, those activities with zero slack).

Crash activities with the lowest crashing cost per unit of time first until the desired project duration is achieved.

38 Ans: (b, d)

Sol: Linked bar chart can show the inter-dependencies among the project activities

In a PERT network, each activity will have fixed duration, however critical path will have standard deviation.

Chapter 2

Network Models

01. Ans: (c)

Sol:

d_{ij} $\rightarrow$ "Distance from any node i to next node j"

s_j $\rightarrow$ "Denotes shortest path from node P to any node j".

$d_{ij} = d_{QG}$ (Adjacent nodes)

$d_{ij} = d_{RG}$ (Adjacent from node R to G)

$S_j = S_Q$ (Shortest path from node P to node Q)

$S_j = S_R$ (Shortest path from node P to node R)

We can go from P to G via Q or via R.

P to G via Q

$$S_G = S_Q + d_{QG}$$

P to G via R.

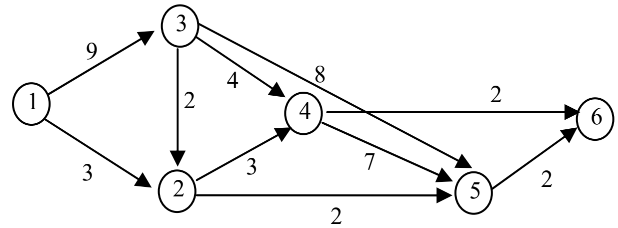
$$S_G = S_R + d_{RG}$$

Optimum answer is minimum above two answers.

$S_G = \text{MIN}\,[S_Q + d_{QG}\,;\; S_R + d_{RG}]$

02. Ans: (c)

Sol:



Path	Cost
1-3-4-6	9+4+2 = 15
1-3-2-4-6	9+2+3+2 = 16
1-3-4-5-6	9+4+7+2 = 22
1-3-2-5-6	9+2+2+2 =15
1-3-2-4-5-6	9+2+3+7+2 = 23
1-2-4-6	3+3+2 = 8
1-2-5-6	3+2+2 = 7
1-2-4-5-6	3+3+7+2 = 15
1-3-5-6	9+8+2 = 19

From the given statement, we got shortest path (least total cost) is 1-2-5-6 and a path which does not have 1-2, 2-5, 5-6 activities should be considered.

The next path which does not have the above activities is 1-3-4-6 = 15

and 1-3-2-4-6 = 16.

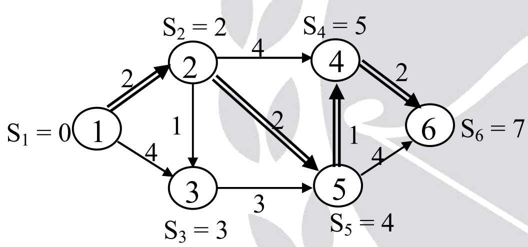
$\therefore$ In this second least total cost is 15.

03. Ans: 7

Sol:

Path	Arc length
1-2-4-6	8
1-2-5-4-6	7
1-2-5-6	8
1-2-3-5-4-6	9
1-3-5-4-6	10
1-3-5-6	11

Shortest path length from node 1 to node 6 is 7.



Chapter 3 Linear Programming

01. Ans: (d)

Sol: A restriction on the resources available to a firm (stated in the form of an inequality or an equation) is called constraint.

02. Ans: (d)

03. Ans: (c)

04. Ans: (d)

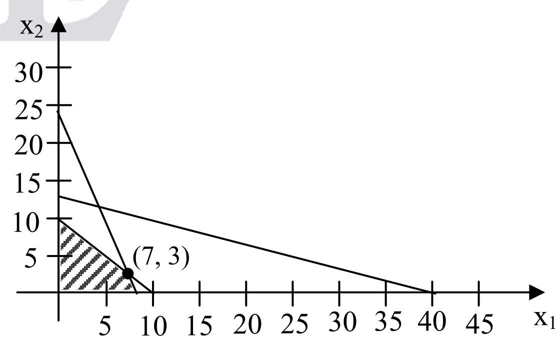
Sol: The theory of LP states that the optimal solution must lie at one of the corner points.

05. Ans: (b)

Sol: The feasible region of a linear programming problem is convex. The value of the decision variables, which maximize or minimize the objective function, is located on the extreme point of the convex set formed by the feasible solutions.

06. Ans: (a)

Sol:



$Z(7, 3) = 2\times7 + 5\times3 = 29$

07. Ans: (a)

Sol: $Z_{max} = x+2y$,

Subjected to

$4y - 4x \geq -1$……… (1)

$5x + y \geq -10$……….. (2)

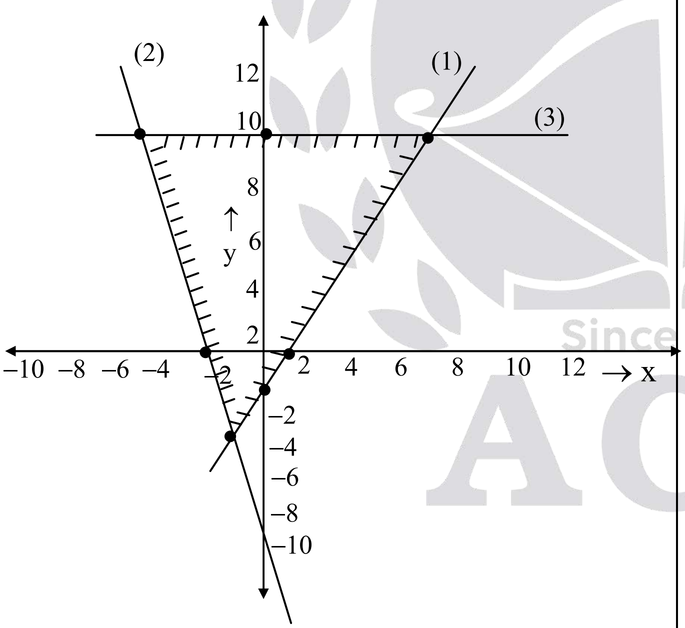
$y \leq 10$…………… (3)

x and y are unrestricted in sign

$$(1) \Rightarrow \frac{x}{\left(\frac{1}{4}\right)} + \frac{y}{\left(\frac{-1}{4}\right)} \leq 1$$

$$(2) \Rightarrow \frac{x}{(-2)} + \frac{y}{(-10)} \leq 1$$

$$(3) \Rightarrow \frac{y}{10} \leq 1$$



Only one value gives max value, then solution is unique.

08. Ans: (b)

Sol: $Z_{max} = 3x_1+2x_2$

Subjected to

$4x_1+x_2 \leq 60$ ………(1)

$8x_1+x_2 \leq 90$ ………(2)

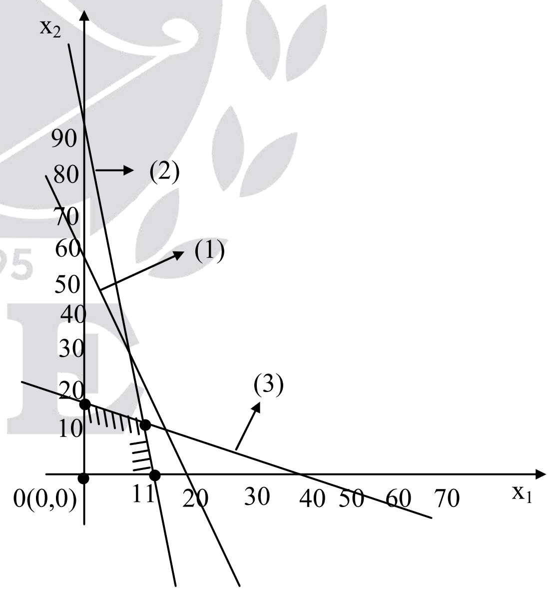
$2x_1+5x_2 \leq 80$ ……… (3)

$x_1, x_2 \geq 0$

$$(1) \Rightarrow \frac{x_1}{15} + \frac{x_2}{60} \leq 1$$

$$(2) \Rightarrow \frac{x_1}{11.25} + \frac{x_2}{90} \leq 1$$

$$(3) \Rightarrow \frac{x_1}{40} + \frac{x_2}{16} \leq 1$$



From the above graph the No. of corner points for feasible solutions are 4

09. Ans: (*)

Sol: Let, P type toys produced = x ,

Q type toys produced = y

	P	**Q**	
Time	1	2	2000
Raw material	1	1	1500
Electric switch	-	1	600
Profit	**3**	**5**	
	x	**y**	

$Z_{max} = 3x + 5y$

$x + 2y \le 2000 \quad ; \frac{x}{2000} + \frac{y}{1000} \le 1$

$x + y \le 1500 \quad ; \frac{x}{1500} + \frac{y}{1500} \le 1$

$y \le 600 \quad ; \frac{y}{600} \le 1$

$x, y \ge 0$

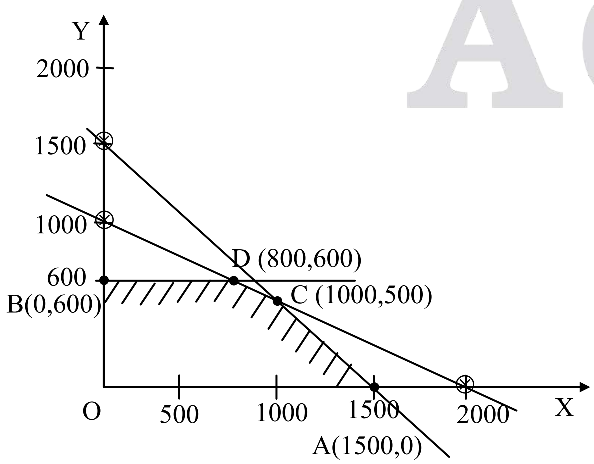
$Z_{max} = 3x + 5y$

$Z_A = 3 \times 1500 + 5 \times 0 = 4500$

$Z_B = 3 \times 0 + 5 \times 600 = 3000$

$Z_C = 3 \times 1000 + 5 \times 500 = 5500$

$Z_D = 3 \times 800 + 5 \times 600 = 5400$



C does not exist in answer.

Hence, Z_{max} is at C, i.e., Z_{max} @ C = 5500

10. Ans: (c)

Sol: $Z_{max} = x_1 + 1.5\,x_2$

Subject to

$2x_1 + 3x_2 \le 6$ ------- (1)

$x_1 + 2x_2 \le 4$ -------- (2)

$x_1 ,\, x_2 \ge 0$

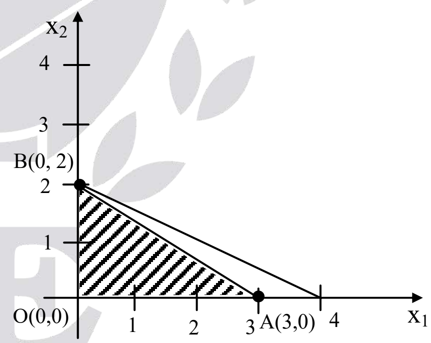
$\frac{x_1}{3} + \frac{x_2}{2} \le 1$

$\frac{x_1}{4} + \frac{x_2}{2} \le 1$

Let, "c" in the intersection of (1) and (2)

Solve (1) & (2) for 'c'.

It follows, $x_1 = \frac{12}{5}; \; x_2 = \frac{2}{5}$



$Z_{max} = x_1 + 1.5x_2$

$Z_0 = 0$

$Z_A = 3 + 1.5 \times 0 = 3$

$Z_B = 3 \times 0 + 1.5 \times 2 = 3$

Problem is having multiple solutions and it is Optimal at (A) and (B).

11. Ans: (a)

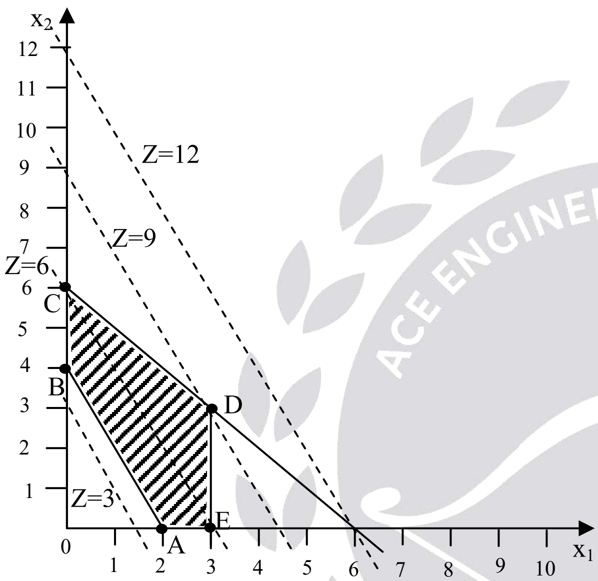
Sol: $Z_{max} = 2x_1 + x_2$

Subjected $x_1 + x_2 \le 6$

$x_1 \le 3$

$2x_1 + x_2 \ge 4$

$x_1 , x_2 \ge 0$



But feasible region is ABCDEA

($\because x_1 , x_2 > 0$)

A(2,0) B(0,4) C(0,6) E(3,0)

D can be obtained by solving

$x_1 \le 3$ & $x_1 + x_2 \le 6$

$\Rightarrow x_1 = 3$ and $x_2 = 3$ and D (3,3)

Z_{max}	A(2,0)	2×2+1×0 = 4
	B (0,4)	0×2+1×4 = 4
	C(0,6)	0×2+1×6 = 6
	E(3,0)	3×2+0×1 = 6
	D(3,3)	3×2+1×3 = 9

$Z_{max} = 9$ at D (3,3)

12. Ans: (d)

13. Ans: (a)

Sol: $Z_{max} = 4 x_1 + 6 x_2 + x_3$

s.t

$2 x_1 - x_2 + 3x_3 \le 5$

$x_1, x_2, x_3 \ge 0$

$2 x_1 - x_2 + 3x_3 + s_1 = 5$

$Z_{max} = 4x_1 + 6x_2 + x_3 + 0 s_1$

$c_j \rightarrow$ s v$\downarrow$	4 x_1	6 x_2	1 x_3	0 s_1	B_0	min Ratio
0 s_1	2	-1	3	1	5	– 5
z_j $c_j - z_j$	0 4	0 6	0 1	0 0	0	

EV

Entering vector exists but leaving vector doesn't exist as minimum ratio column is having negative values. It is a case of unbounded solution space and unbounded optimal solution to problem.

14. Ans: (d)

Sol: Number of zeros in Z row = 4

Number of basic variable = 3

As the number of zeros in Z row is greater than number of basic variable so it has multiple optimal solutions.

15. Ans: (b)

Sol: Solution is optimal; but Number of zeros are greater than the number of basic Variables in $C_j - Z_j$(net evaluation row) hence multiple optimal solutions.

16. Ans: (b)

Sol: If all the elements in the objective row are non-negative incase of maximization, then the solution is said to be optimal.

Here, the solution is optimal, $Z_{max} = 1350$.

17. Ans: (a)

Sol:

- A tie for leaving variable in simplex procedure implies degeneracy.
- If in a basic feasible solution, one of the basic variables takes on a zero value then it is case of degenerate solution

Common Data Solutions

18. Ans: (d) & 19. Ans: (a)

Sol: As the No. of zeros greater than No. of basic variables hence it is a case of multiple solutions or alternate optimal solution exists.

Basic	x_1	x_2	S_1	S_2	S_3	RHS
z	0	0	0	2	0	48
s_1	0	5/3	1	–2/3	0	14
s_3	0	–1/3	0	1/3	1	5
x_1	1	2/3	0	1/3	0	8

From the table gives the optimum $x_2 = 0$,

$x_1 = 8$, $Z_{max} = 48$

Look at the coefficient of the non basic variable in the z-equation of iterations. The coefficient of non basic x_2 is zero, indicating that x_2 can enter the basic solution without changing the value of Z, but causing a change in the values of the variables.

Alternate optimal solution :

Here x_2 is the entering variable.

Row	Basic	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
R_1	z	0	0	0	2	0	48	
R_2	s_1	0	5/3	1	–2/3	0	14	14/(5/3)=8.4 → Leaving variable
R_3	s_3	0	–1/3	0	1/3	1	5	–
R_4	x_1	1	2/3	0	1/3	0	8	8/(2/3)=12

Entering variable

Row	**Basic**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**RHS**
R_1	z	0	0	0	2	0	48
$R'_2 = \frac{R_2}{(5/3)}$	x_2	0	1	3/5	–2/5	0	42/5
$R'_3 = R'_3 + \frac{R'_2}{3}$	s_3	0	0	1/5	1/5	1	39/5
$R'_4 = R_4 - \frac{2}{3}R'_2$	x_1	1	0	–3/5	3/5	0	12/5

In the above table $x_1 = \frac{12}{5}$, $x_2 = \frac{42}{5}$, $s_3 = \frac{39}{5}$

20. Ans: (c)

21. Ans: (a)

22. Ans: (c)

Sol: $Z_{min} = 10x_1 + x_2 + 5x_3 + 0S_1$

Dual , $W_{min} = 50y_1$

subjected to

$5y_1 \le 10$, $y_1 \le 2$, $W_{max} = 100$

$3y_1 \le 5$, $y_1 \le 5/3$, $W_{max} = 250/3$

y_1 , $y_2 \ge 0$

$\Rightarrow Z_{max} = 250 / 3$

Common Data for Questions

23. Ans: (c)

Sol: Given, $Z_{max} = 5x_1 + 10x_2 + 8x_3$

Subjected to

$3x_1 + 5x_2 + 2x_3 \le 60 \rightarrow$ Material

$4x_1 + 4x_2 + 4x_3 \le 72 \rightarrow$ Machine hours

$2x_1 + 4x_2 + 5x_3 \le 100 \rightarrow$ Labour hours

$x_1, x_2, x_3 \ge 0$

$3x_1 + 5x_2 + 2x_3 + s_1 = 60$

$4x_1 + 4x_2 + 4x_3 + s_2 = 73$

$2x_1 + 4x_2 + 5x_3 + s_3 = 100$

$Z_{max} = 5x_1 + 10x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$

$C_j \rightarrow$		5	10	8	0	0	0	B_0	Min Ratio
C_B	SV	x_1	x_2	x_3	s_1	s_2	s_3		
10	x_2	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{-1}{6}$	0	8	
8	x_3	$\frac{2}{3}$	0	1	$\frac{-1}{3}$	$\frac{5}{12}$	0	10	
0	S_3	$\frac{-8}{3}$	0	0	$\frac{1}{3}$	$\frac{-17}{12}$	1	18	
Z_j		$\frac{26}{3}$	10	8	$\frac{2}{3}$	$\frac{5}{3}$	0	160	
$C_j - Z_j$		$\frac{-11}{3}$	0	0	$\frac{-2}{3}$	$\frac{-5}{3}$	0		

$\frac{C_j - Z_j}{x_2}$	–11	0	0	–2	10	0	LL=2 UL=10	10–2=8 10+10=20
$\frac{C_j - Z_j}{x_3}$	$\frac{-11}{2}$	0	0	2	–4	0	LL=4 UL=2	8–4=4 8+2=10

In $C_j - Z_j$ row all elements are negatives or zeros, hence the solution is optimal and unique..

Basic variables are:

$x_2 = 8$, $x_3 = 10$, $s_3 = 18$

i.e., production of B = 8 units, C = 10 units

18 labours hours remained unutilized

Non Basic variable

$x_1 = 0$, $s_1 = 0$, $s_2 = 0$

Resource materials and resource machine hours are fully utilized. In ($C_j - Z_j$) row at optimality, the values under s_1, s_2 and s_3 columns represents the shadow prices.

So, If 1 kg material increases, contribution increases by $\frac{2}{3}$.

If 1 kg material decreases, contribution decreases by $\frac{2}{3}$.

If 1 kg material increases, then production B increases by $\frac{1}{3}$ and production C decreases by $\frac{1}{3}$

If m/c hr increases by 1 units, contribution increases by 5/3.

If m/c hr decreases by 1 units, contribution decreases by $\frac{5}{3}$

If m/c hr increases by 1 units, production B decreases by $\frac{1}{6}$ and production increases by $\frac{5}{12}$.

If m/c hr decreases by 1 units, production B increases by $\frac{1}{6}$ and production C decreases by $\frac{5}{12}$

If 1 unit of A produces, contribution decreases by $\frac{11}{3}$, production B decreases by $\frac{1}{3}$, production C decreases by $\frac{2}{3}$.

24. Ans: (a)

Sol: If 3 kg material increases, contribution increases by $3 \times \frac{2}{3}$ = Rs. 2

25. Ans: (a)

Sol: Present profit = 160 $\Rightarrow 160 - \frac{5}{3} \times 12 = 140$/-

26. Ans: (b)

Sol: New production of B

$$= 8 - \left(12 \times \frac{-1}{6}\right) = 8 + \left(12 \times \frac{1}{6}\right)$$

$$= 8 + 2 = 10 \text{ units}$$

27. Ans: (c)

Sol: If materials are increased by 3kgs then the new production of C is $= 10 + \left(3 \times \frac{-1}{3}\right)$

$$= 10 - \left(3 \times \frac{1}{3}\right) = 10 - 1 = 9$$

28. Ans: (a)

Sol: If 1 unit of A produces, contribution decreases by $\frac{11}{3}$

29. Ans: (a)

Sol: If 6 units of A are produced then the new profit is,

$$160 - \left(6 \times \frac{11}{3}\right) = 138$$

30. Ans: (a)

Sol: Production of B , $3 \times \frac{1}{3} = 1$

Production of C, $3 \times \frac{2}{3} = 2$

Common data 35 & 36

31. Ans: (b) , 32. Ans: (b)

Sol: Basic variables

$x_1 = 20$, $x_2 = 10$

Non-basic variables

$s_1 = 0 \Rightarrow$ first constraint is fully consumed.

$s_2 = 0 \Rightarrow$ second constraint is fully consumed.

$x_3 = 0$ (unwanted variable)

	x_1	x_2	x_3	s_1	s_2	RHS
z-row	0	0	2	1	2	110
x_1	1	0	1	1	-1	20
x_2	0	0	0	–1	2	10

	s_1
z-row	1
x_1	1
x_2	–1

If RHS value of 1st constraint increases by 1 unit then

From the table

z increases by 1 unit, x_1 increases by 1 unit, x_2 decreases by 1 unit,

If RHS value of 2nd constraint increases by 1 unit then

	s_2
z-row	2
x_1	-1
x_2	2

From the table

z increases by 2 units, x_1 decreases by 1 unit x_2 decreases by 2 units,

If RHS value of 1st constraint decreases by 10 units then z decreases by 10 units,

The new objective value ,

$$Z_{max} = 110 - 10 = 100$$

33. Ans: (c)

Sol:

	X_1	X_2	S_1	S_2	RHS	Ratio
z-row	–3	**–5**	0	0	0	0
S_1	2	**1**	1	0	2	2/1=2
S_2	3	**2**	0	1	4	4/2=2

Entering variable X_2

Minimum ratio = min(2/1, 4/2) = 2^*

*Tie w.r.t leaving variables S_1 and S_2

Thus it has degenerate solution.

34. Ans: (d)

Sol:

	X_1	X_2	S_1	S_2	RHS
z-row	**–2**	–1	0	0	0
S_1	**–2**	1	1	0	4
S_2	**0**	1	0	1	3

Entering variable X_1

Ratio = Min{4/–2, 3/0}

As there is no least positive ratio, there is no leaving variable which results the problem has unbounded solution.

35. Ans: 28000

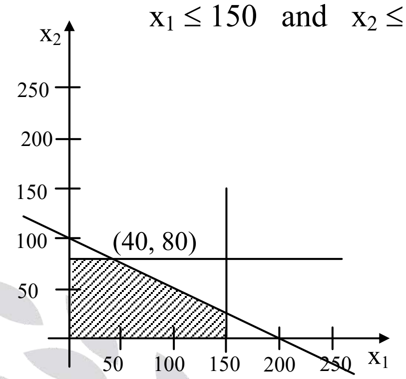
Sol:

Demand	Products		Maximum available
	Chairs (x_1)	**Tables (x_2)**	
Wood	1	2	200
Chairs	1	–	150
Tables	–	1	80
Profit/loss	100	300	

$Z_{max} = 100x_1 + 300x_2$

Subject to $x_1 + 2x_2 \le 200$

$x_1 \le 150$ and $x_2 \le 80$



$Z_{max} = 100 \times 40 + 300 \times 80 = 28000$

36. Ans: 5000

Sol:

Demand	Products		Maximum available
	A (x_1)	**B (x_2)**	
Raw material	1	1	850
Special type of buckle	1	—	500
Ordinary buckle	–	1	700
Time	1	½	500
Profits/unit	10/-	5/-	

Constraints :

x_1 = No. of belts of type 'A'

x_2 = No. of belts of type 'B'

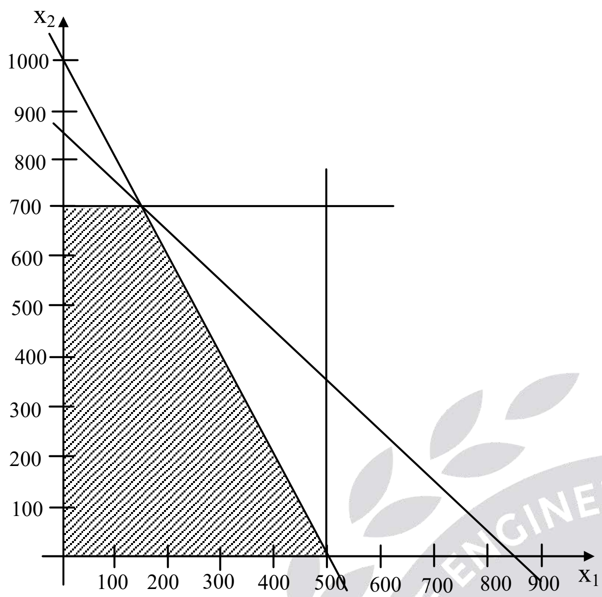
$Z_{max} = 10x_1 + 5x_2$

s.t $x_1 + x_2 \le 850$

$x_1 \le 500$, $x_2 \le 700$

$x_1 + \frac{1}{2}x_2 \le 500$, $x_1, x_2 \ge 0$

$2x_1 + x_2 \le 1000$,



$Z_{max} = (10\times150) + (5\times700) = 5000$ /-

37. Ans: (a, d)

Sol: In a linear programming problem (LPP)

- All decision variables are inter-related and non-negative
- Objective function must be straight line
- There exists proportional relationship between objective and constraints.
- Activities and constraints are finite in number.
- All decision variables should be positive.

38. Ans: (c, d)

Sol: In a standard form of LPP

- All decision variables and RHS values must be non-negative.
- In-equalities are to be eliminated by using slack or surplus variables.

Chapter 4 Inventory Control

01. Ans: (b)

Sol: $\text{EOQ} = \sqrt{\dfrac{2AS}{CI}}$

$$\text{EOQ}_1 = \sqrt{2}\times\sqrt{\frac{2AS}{CI}}$$

$$\text{EOQ}_1 = \sqrt{2}\times\text{EOQ}$$

02. Ans: (c)

Sol: $\text{EOQ} = \sqrt{\dfrac{2DC_o}{C_c}}$

03. Ans: (b)

Sol: A = 900 unit

S = 100 per order

CI = 2 per unit per year

$$\text{EOQ} = \text{ELS} = \sqrt{\frac{2AS}{CI}}$$

$$= \sqrt{\frac{2\times900\times100}{2}} = 300$$

04. Ans: (c)

Sol: Inventory carrying cost:

It involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

05. Ans: (b)

Sol: At EOQ, Carrying cost = Ordering cost

06. Ans: (d)

Sol: Inventory carrying cost involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

07. Ans: (a)

Sol: $A = 800$, $S = 50/-$,

$C_s = 2$ per unit = CI

$$(\text{TIC})_{EOQ} = \sqrt{2ASCI}$$

$$= \sqrt{2 \times 800 \times 50 \times 2} = 400$$

08. Ans: (c)

Sol: $TC(Q_1) = TC(Q_2)$

$$\frac{kd}{Q_1} + \frac{hQ_1}{2} = \frac{kd}{Q_2} + \frac{hQ_2}{2}$$

$$kd\left(\frac{Q_2 - Q_1}{Q_1 Q_2}\right) = \frac{h}{2}(Q_2 - Q_1)$$

$$\frac{2kd}{h} = Q_1 Q_2$$

$$(Q^*)^2 = Q_1 \times Q_2$$

$$Q^* = \sqrt{Q_1 \times Q_2} = \sqrt{300 \times 600} = 424.264$$

09. Ans: (c)

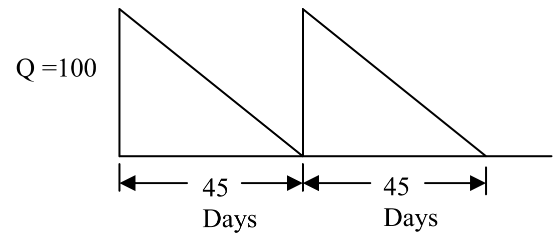
Sol: $$\frac{EOQ_1}{EOQ_2} = \sqrt{\left(\frac{2AS}{CI}\right)_A} \times \sqrt{\left(\frac{CI}{2AS}\right)_B}$$

$$= \sqrt{\left(\frac{2 \times 100 \times 100}{4}\right)} \times \sqrt{\left(\frac{1}{2 \times 400 \times 100}\right)}$$

$(EOQ)_A : (EOQ)_B = 1:4$

10. Ans: (d)

Sol: (No of orders $= \frac{A}{Q} = \frac{12\,\text{months}}{45\,\text{days}} = \frac{12}{1.5} = 8$)



$$TVC = \frac{A}{Q}S + \frac{Q}{2}CI.$$

$$= 8 \times 100 + \frac{100}{2} \times 120 = \text{Rs. } 6800$$

11. Ans: (b)

Sol: Average inventory

$$= \frac{Q}{2} = \frac{6000}{2} = 3000 \text{ per year}$$

$= 250$ per month

12. Ans: (b)

Sol: $P = 1000$, $r = 500$, $Q = 1000$

$$I_{max} = \frac{1000}{1000}(1000 - 500) = 500$$

13. Ans: correct key (c)

Sol: D = 1000 units,

C_0 = Rs.100/order,

C_c = 100/unit/year, C_s = 400/unit/year

$$Q_{max} = EOQ_s \times \frac{C_s}{C_c + C_s}$$

$$= \sqrt{\frac{2DC_0}{C_c}}\sqrt{\frac{C_c + C_s}{C_s}} \times \left(\frac{C_s}{C_c + C_s}\right)$$

$$= \sqrt{\frac{2\times1000\times100}{100}}\sqrt{\frac{100+400}{400}}\times\left(\frac{400}{100+400}\right)$$

= 40 units

14. Ans: (d)

Sol: Re-order level = 1.25[Σx p(x)]

= 1.25 [80×0.2+100×0.25+120×0.3+140×0.25]

= 140 units

Demand	80	100	120	**140**
Probability	0.20	0.25	0.30	**0.25**
Cumulative probability (Service level)	0.2	0.45	0.75	**1.0**

Service Level = 100 %

15. Ans: (b)

16. Ans: (b)

17. Ans: (d)

Sol: C – Class means these class items will have very less consumption values. – least consumption values

$B \rightarrow 300 \times 0.15 = 45$

$F \rightarrow 300 \times 0.1 = 30$

$C \rightarrow 2 \times 200 = 400$

$E \rightarrow 5 \times 0.3 = 1.5$

$J \rightarrow 5 \times 0.2 = 1.0$

$G \rightarrow 10 \times 0.05 = 0.5$

$H \rightarrow 7 \times 0.1 = 0.7$

$\therefore$ G, H items are classified as C class items because they are having least consumption values.

18. Ans: (b)

Sol: In ABC analysis :

Category "A" = Low safety stock

Category "B" = Medium safety stock

Category "C" = High safety stock

19. Ans: (a, b)

Sol: EOQ is the quantity of materials ordered at each order point that minimizes the total annual costs for a material in a fixed order quantity inventory system and it will minimize the total annual cost of ordering and carrying the inventory.

At EOQ,

Annual ordering cost = Annual inventory carrying cost

20. Ans: (a, b, d)

Sol: ABC analysis is a technique of inventory management.

A category includes those inventory which are very important to the organization and consist of 70-80% of inventory value but only 10-20 % of the quantity.

A category item requires very strict control, fixed order quantity and no safety stock.

Chapter 5 Forecasting

01. Ans: (d)

02. Ans: (d)

Sol:

- A simple moving average is a method of computing the average of a specified number of the most recent data values in a series.
- This method assigns equal weight to all observations in the average.
- Greater smoothing effect could be obtained by including more observations in the moving average.

03. Ans: (a)

Sol: 3 period moving avg $= \dfrac{100+99+101}{3}$

$= 100$

4 period moving average

$= \dfrac{102+100+99+101}{4} = 100.5$

5 period moving average

$= \dfrac{99+102+100+99+101}{5} = 100.2$

Arithmetic Mean

$= \dfrac{101+99+102+100+99+101}{6}$

$= 100.33$

04. Ans: (a)

Sol: $D_t = 100$ units , $F_t = 105$ units

$\alpha = 0.2$

$F_{t+1} = 105 + 0.2\,(100 - 105) = 104$

05. Ans: (c)

Sol: $D_t = 105$, $F_t = 97$, $\alpha = 0.4$

$F_{t+1} = 97 + 0.4\,(105 - 97) = 100.2$

06. Ans: (c)

Sol: $F_{t+1} = F_t + a\,(X_t - F_t)$

07. Ans: (c)

Sol: Another form of weighted moving average is the exponential smoothed average. This method keeps a running average of demand and adjusts if for each period in proportion to the difference between the latest actual demand and the latest value of the forecast.

08. Ans: (a)

09. Ans: (b)

Sol:

Period	D_i	F_i	$(D_i - F_i)^2$
14	100	75	625
15	100	87.5	156.25
16.	100	93.75	39.0625
			$\Sigma(D_i - F_i)^2 = 820.31$

$F_{15} = F_{14} + \alpha(D_{14} - F_{14})$

$= 75 + 0.5(100 - 75) = 87.5$

$F_{16} = F_{15} + \alpha(D_{15} - F_{15})$

$= 87.5 + 0.5(100 - 87.5) = 93.75$

$$\text{Mean square error (MSE)} = \frac{\sum (D_i - F_i)^2}{n}$$

$$= \frac{820.31}{3} = 273.13$$

10. Ans: (a)

Sol:

Period	D_i	F_i	$\lvert(D_i - F_i)\rvert$
1	10	9.8	0.2
2	13	12.7	0.3
3	15	15.6	0.6
4	18	18.5	0.5
5	22	21.4	0.6

$\Sigma|D_i - F_i| = 2.2$

11. Ans: (d)

Sol:

m_1 = moving average periods give forecast $F_1(t)$

m_2 = moving average periods give forecast $F_2(t)$

$m_1 > m_2$

$F_1(t)$ is a stable forecast has less variability.

$F_2(t)$ is a sensitive (inflationary) forecast and has high variability.

12. Ans: (d)

Sol: Following are the purposes of long term forecasting :

- To plan for the new unit of production.
- To plan for the long-term financial requirement.
- To make the proper arrangement for training the personal.
- Budgetary allegations are not done in the beginning of a project. So, deciding the purchase program is not the purpose of long term forecasting.

13. Ans: (d)

Sol:

- Time horizon is less for a new product and keeps increasing as the product ages. So, statement (I) is correct.
- Judgemental techniques apply statistical method like random sampling to a small population and extrapolate it on a larger scale. So, statement (II) is correct.
- Low values of smoothing constant result in stable forecast. So statement (3) is correct.

14. Ans: (i) 50, (ii) 52.5, (iii) (42.5, 40)

Sol:

(i) $F_7 = \dfrac{60 + 50 + 40}{3} = 50$

(ii) $F_7 = \dfrac{60 \times 0.5 + 50 \times 0.25 + 40 \times 0.25}{0.5 + 0.25 + 0.25} = 52.5$

(iii) 2 period moving average $= \dfrac{60 + 50}{2} = 55$

4 period moving average

$$= \frac{60 + 50 + 40 + 20}{4} = 42.5$$

5 period moving average

$$= \frac{60 + 50 + 40 + 20 + 30}{5} = 40$$

15. Ans: (114.8 units, 9 periods)

Sol: At $\alpha = 0.2$

$F_{may} = 100 + 0.2\,(200 - 100) = 120$

$F_{june} = 120 + 0.2\,(50 - 120) = 106$

$F_{july} = 106 + 0.2\,(150 - 106) = 114.8$

Time	Demand	Forecast
April	200	100
May	50	120
June	150	106
July	-	114.8

$$\alpha = \frac{2}{n+1}$$

$$n+1 = \frac{2}{\alpha} \Rightarrow n = \frac{2}{0.2} - 1 = 9\,\text{period}$$

16. Ans: (a, b)

Sol: In a time series analysis of forecasting :

- Exponential smoothing of time series data assigns exponentially decreasing weights for newest to oldest observations
- Simple moving average method uses equal weights to the previous data
- Un-stable demand data requires higher value of 'α' and lower value of 'n'

  where, $\alpha = \dfrac{2}{n+1}$

  $$F_{t+1} = F_t + \alpha\,(D_t - F_t)$$

  If smoothening coefficient (α) is 1 then the latest forecast would be equal to the previous period actual demand.

17. Ans: (a, c, d)

Sol: In exponential smoothing,

- high value of α is high and less for other forecasts. Hence the high value of forecast is only chosen when the nature of demand is not reliable rather unstable.
- lower value of 'α' results more smoothening effect
- lesser 'α' results more smoothening effect
- different - weights have been assigned to the previous data. Exponential smoothing of time series data assigns exponentially decreasing weights for newest to oldest observations.

18. Ans: (a, b)

Sol: In a forecasting process

- Qualitative methods (suitable for long and medium range production planning)
- Quantitative methods (suitable for short range production planning)
- Market survey is well suited for long range production planning (Qualitative methods of forecasting)
- Exponential smoothening is well suited for short range forecasting (Quantitative methods of forecasting)

Chapter 6 Queuing Theory

01. Ans: (a)

Sol: $\lambda = 3$ per day

$\mu = 6$ per day

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{3}{6(6-3)} = \frac{1}{6}\text{day}$$

02. Ans: (c)

Sol: $\lambda = 0.35 \text{ min}^{-1}$,

$\mu = 0.5 \text{ min}^{-1}$

$$P_n = \left[1-\frac{\lambda}{\mu}\right]\left[\frac{\lambda}{\mu}\right]^n$$

$$= \left[1-\frac{0.35}{0.5}\right]\left[\frac{0.35}{0.5}\right]^8 = 0.0173$$

03. Ans: (a)

Sol: $\lambda = 10 \text{ hr}^{-1}$,

$\mu = 15 \text{ hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{15(15-10)} = 1.33$$

04. Ans: (b)

Sol: $\lambda = 4 \text{ hr}^{-1}$, $\mu = \frac{60}{12} = 5 \text{ hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{4^2}{5(5-4)} = \frac{16}{5} = 3.2$$

05. Ans: (b)

Sol: $$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu^2\left(1-\frac{\lambda}{\mu}\right)} = \frac{\rho^2}{(1-\rho)}$$

06. Ans: (d)

Sol: $\lambda = \frac{1}{4} = 0.25\text{min}^{-1}$

$\mu = \frac{1}{3} = 0.33 \text{ min}^1$

$\rho = \frac{\lambda}{\mu} = \frac{0.25}{0.33} = 0.75$

07. Ans: (b)

Sol: $\lambda = \frac{1}{10} = 0.1\text{min}^{-1}$

$\mu = \frac{1}{4} = 0.25\text{min}^{-1}$

System busy $\Rightarrow (\rho) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$

08. Ans: (c)

Sol: $\lambda = 4\text{hr}^{-1}$, $\mu = 6 \text{ hr}^{-1}$

$$P(Q_s \geq 2) = \left(\frac{\lambda}{\mu}\right)^2$$

$$= \left(\frac{4}{6}\right)^2 = \frac{4}{9}$$

09. Ans: (c)

Sol: The cost of providing service in a queuing system increases with decreased mean time in the queue.

10. Ans: (a, b, c)

Sol: In a single server queuing model :

Mean arrival rate (λ) follows Poisson distribution,

Mean arrival time $= \dfrac{1}{\lambda} \Rightarrow$ exponential distribution

Mean service time $= \dfrac{1}{\mu} \Rightarrow$ exponential distribution

$$\rho < 1 \Rightarrow \frac{\lambda}{\mu} < 1 \Rightarrow \lambda < \mu$$

Mean arrival time > Mean service time

11. Ans: (a, d)

Sol: Mean service time,

$$\mu = 6 \text{ minutes} = \frac{60}{6} = 10/\text{hr}$$

$\rho < 1 \Rightarrow \dfrac{\lambda}{\mu} < 1 \Rightarrow \lambda < \mu$ for single server queuing model.

From the given options $\lambda = 4$ /hr (or) 6 /hr

Chapter 7 Sequencing & Scheduling

01. Ans: (a)

Sol: SPT rule

Job	Process time (days)	Completion time
1	4	4
3	5	9
5	6	15
6	8	23
2	9	32
4	10	42
	$\Sigma C_i =$	125

$$\text{Average Flow Time} = \frac{\sum C_i}{n} = \frac{125}{6} = 20.83$$

02. Ans: (a)

Sol: According to SPT rule total inventory cost is minimum.

03. Ans: (d)

Sol: EDD rule can minimize maximum lateness.

The job sequence is **R – P – Q – S**

04. Ans: (d)

Sol: Johnson's rule :

Optimum job sequence **III – I – IV – II**

Do the job 1st if the minimum time happens to be on the machine (M) and do it on the end if .it is on second machine (N). Select either in case of a tie.

05. Ans: (b)

Sol:

Job	**M**			**N**			**Idle**
	In	**PT**	**Out**	**In**	**PT**	**Out**	
III	0	1	1	1	2	3	-
I	1	3	4	4	6	10	1
IV	4	7	11	11	5	16	1
II	11	5	16	16	2	18	-

Total idle time on machine (N) = 3

06. Ans: (a)

Sol: Optimum sequence of jobs

2	3	1	4

07. Ans: (b)

Sol: Optimum sequence is

R	T	S	Q	U	P

Job	**M_1**			**M_2**		
	In	**PT**	**Out**	**In**	**PT**	**Out**
R	0	8	8	8	13	21
T	8	11	19	21	14	35
S	19	27	46	46	20	66
Q	46	32	78	78	19	97
U	78	16	94	97	7	104
P	94	15	109	109	6	115

The optimal make-span time = 115 days

08. Ans: (c)

Sol: Critical Ratio (C.R)

$$\text{C.R} = \frac{(\text{Date required} - \text{Todays Date})}{\text{Days needed to complete job}}$$

$$\text{C.R} = \frac{\text{Available Time}}{\text{Required time}}$$

09. Ans: (a, d)

Sol:

- All jobs are ready for processing
- All processing times are deterministic in nature
- Machine is flexible and can process jobs without any break downs
- No pre-emption i.e the job cannot be unloaded without having completion on a machine
- Travelling times and setup times of jobs are negligible
- Job can be un-loaded from the machine even through it is not getting completed incase of priority rules.

10. Ans: (a, d)

Sol: Sequencing the jobs in the increasing order of their processing time is known as SPT rule.

- It minimizes the mean flow time as well as the number of tardy jobs.
- It minimizes the total inventory (holding) cost.
- It also minimizes the mean tardiness, if all tasks have the same due date.

Chapter 8 Transportation Model

01. Ans: (c)

Sol: A no. of allocations : $m + n - 1$

$\Rightarrow 5 + 3 - 1 = 7$

02. Ans: (a)

Sol: For degeneracy in transportations, number of allocations < $(m + n) - 1$

where m = no. of rows,

n = no. of columns

03. Ans: (b)

Sol: In Transportation problem for solving the initial feasible solution for total cost, Vogel's approximation methods are employed for obtaining solutions which are faster than LPP due to the reduced number of equations for solving.

Optimality is reached using MODI/ U-V method or stepping stone method.

04. Ans: (b)

Sol: It generates the best initial basic feasible solution. This method is the best choice in order to get an optimal solution within minimum number of iterations.

The Vogel's approximation method is also known as the penalty method.

05. Ans: (a)

Sol: No. of allocations = 5

$\therefore$ no. of allocations = $m + n - 1$

$m + n - 1 = 4 + 3 - 1$

$\therefore$ It is a degenerate solution

06. Ans: (a)

Sol:

	1	2	3	4	Supply
A	[10]	[2] 5	[20]	[11] 10	15
B	[12]	[7] 10	[9] 15	[20]	25
C	[5] 5	[14]	[16]	[18] 5	10
Demand	5	15	15	15	50 / 50

Evaluation of empty cells:

Cell (A1) Evaluation = $C_{A1} - C_{A4} + C_{C4} - C_{C1}$

$= 10 - 11 + 18 - 5 = 12$

Cell (A3) Evaluation = $C_{A3} - C_{A2} + C_{B2} - C_{B3}$

$= 20 - 9 + 7 - 2 = 16$

Cell (B1) Evaluation $= 12 - 7 + 2 - 11 + 18 - 4$

$= 10$

Cell (B4) Evaluation $= 20 - 7 + 2 - 11 = 4$

Cell (C2) Evaluation $= 14 - 2 + 11 - 18 = 5$

Cell (C3) Evaluation $= 16 - 9 + 7 - 2 - 18 = 5$

If cell cost evaluation value is '–ve', indicates further unit transportation cost is decreasing and if cost evaluation value is '+ve' indicates further unit transportation cost is increases. If cost evaluation value is zero, unit transportation cost doesn't change.

$\therefore$ As for A3 cell cost evaluation is +16, means that, if we transport goods to A3 the unit transportation cost is increased by 16/-.

Common Data for Questions Q07, Q08 & Q09 :

07. Ans: (b) **08. Ans: (a)**

09. Ans: (b)

Sol:

	1	2	3	4
A	[6]	[1]	[9] 25	[3] 45
B	[11] 30	[5]	[2] 25	[8]
C	[10] 55	[12] 35	[4]	[7]

No. of allocations = 6

R + C – 1 = 6

As No. of allocations = R + C – 1

Hence the problem is not degeneracy case.

Opportunity cost of cell (i, j) is

$C_{ij} - (U_i + V_j)$

If $C_{ij} - (U_i + V_j) \geq 0 \Rightarrow$ problem is optimal,

Empty cell evaluation (or) Opportunity cost of cells:

$A_1 = -12$, $A_2 = -19$, $B_2 = -8$

$B_4 = 12$, $C_3 = 3$, $C_4 = 12$

From the above as A2 has opportunity cost '–19' indicates unit transportation cost is decreased by 19/-

By forming loop A2, A3, B2, B3 it is observed that to transport minimum quantity is 25 among 25, 30, 35.

$\therefore$ The reduction in the transportation cost is 25 × 19 = 475

10. Ans: (c)

Sol:

	10 –		14 +	
+ 7			12 –	16
5 –	8 +			

By stepping stone method,

Cell evaluation of B – 1 cell

= +7 – 5 + 8 – 10 + 14 – 12

= 2/-

	10 – θ	20 + θ	
+θ		5 – θ	35
20 – θ	10 + θ		

θ = minimum of |10 – θ, 5 – θ, 20 – θ| = 0

θ = 5 units

Increase in cost = 5 × 2 = 10/-

11. Ans: (c)

Sol: To find the number units shifted to A_2 cell.

+θ	20 – θ
15 – θ	25 + θ

θ = minimum value of |15–θ, 20–θ| = 0

θ = 15 units

12. Ans: (b, d)

Sol: In a transportation model

- Vogel's approximation method (VAM) generates best initial basic feasible solution among others.
- least cost method can give an initial solution
- MODI method is efficient method to generate an optimal solution as well as for testing optimality of a transportation solution.
- Vogel's approximation method can give a better initial solution compared to north west corner (NWC) method.

Chapter 9 Assignment Model

01. Ans: (a)

Sol: Let C_{ij} = unit assignment cost

X_{ij} = Decision variable (allocation)

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{J=1}^{n} C_{ij} X_{ij}$$

$$\text{Subject to : } \sum_{i=1}^{n} X_{ij} = 1$$

$$\sum_{j=1}^{n} X_{ij} = 1$$

$X_{ij} = 1$ (when assigned)

$X_{ij} = 0$ (when not assigned)

- Number of decision variables = n^2 (or) m^2
- Number of basic variables = Number of assignments = n (or) m

02. Ans: (c)

03. Ans: (a)

04. Ans: (c)

Sol:

	S_1	S_2	S_3		S_1	S_2	S_3	
P	110	120	130		0	0	0	Column Transaction
Q	115	140	140		0	15	5	
R	125	145	165		0	10	20	
P	0	10	20		5	0	0	
Q	0	25	25	Row Transaction	0	10	0	
R	0	20	40		0	5	15	

$P\text{-}S_2 - 120$

$Q\text{-}S_3 - 140$

$R\text{-}S_1 - 125$

Total = 385

05. Ans: (1-B, 2-D, 3-C, 4-A)

Sol: Step-1:

Take the row minimum of subtract it from all elements of corresponding row.

$$\begin{array}{|cccc|} 1 & 0 & 2 & 3 \\ 0 & 2 & 2 & 1 \\ 8 & 5 & 0 & 1 \\ 0 & 6 & 2 & 4 \end{array}$$

Step – 2 :

Take the column minimum & substract it from all elements of corresponding column.

$$\begin{array}{|cccc|} 1 & 0 & 2 & 2 \\ 0 & 2 & 2 & 0 \\ 8 & 5 & 0 & 0 \\ 0 & 6 & 2 & 3 \end{array}$$

Step – 3 :

Select single zero row or column and assign at the all where zero exists. If there is no single zero row or column. Then use straight line method.

$$\begin{array}{c|cccc|} & A & B & C & D \\ \hline 1 & 1 & \boxed{0} & 2 & 2 \\ 2 & 0 & 2 & 2 & \boxed{0} \\ 3 & 8 & 5 & \boxed{0} & 0 \\ 4 & \boxed{0} & 6 & 2 & 3 \\ \hline \end{array}$$

1 – B : 7

2 – D : 8

3 – C : 2

4 – A : 5

Total cost = 22

06. Ans: (C_1-J_2, C_2-J_1, C_3-J_4, C_4-J_3)

Sol:

$$\begin{array}{c|cccc|} & J_1 & J_2 & J_3 & J_4 \\ \hline C_1 & 10 & \boxed{5} & 15 & 13 \\ C_2 & 3 & 9 & 8 & \boxed{18} \\ C_3 & 10 & 7 & \boxed{2} & 3 \\ C_4 & \boxed{5} & 11 & 7 & 9 \\ \hline \end{array}$$

Step – 1 :

$$\begin{array}{|cccc|} 5 & 0 & 10 & 8 \\ 0 & 6 & 5 & 15 \\ 8 & 5 & 0 & 1 \\ 0 & 6 & 2 & 4 \end{array}$$

Step – 2 :

$$\begin{array}{|cccc|} 5 & 0 & 10 & 7 \\ 0 & 6 & 5 & 14 \\ 8 & 5 & 0 & 0 \\ 0 & 6 & 2 & 3 \end{array}$$

Step – 3 :

$$\begin{array}{|cccc|} 5 & \boxed{0} & 10 & 7 \\ \boxed{0} & 6 & 5 & 14 \\ 8 & 5 & \boxed{0} & 0 \\ 0 & 6 & 2 & 3 \end{array}$$

It may be noted there are no remaining zeroes and row – 4 and column – 4 each has no assignment. Thus optimal solution is not reached at this stage. Therefore, proceed to following important steps.

Step – 4 :

Draw the minimum number of horizontal and vertical lines necessary to cover all zeroes at least once.

Take the above Table

	J_1	J_2	J_3	J_4	
C_1	5	0	10	7	L_2
C_2	0	6	5	14	
C_3	8	5	0	0	L_3
C_4	0	6	2	3	
	L_1				

(i) Mark row – 4 in which there is no assignment

(ii) Mark column 1 which have zeroes in marked column.

(iii) Next mark row 2 because this row contains assignment in marked column 1.

No further rows or columns will be required to mark during this procedure.

(iv) Draw the required lines as follows.

(a) Draw L_1 through marked column 1

(b) Draw L_2 and L_3 through unmarked row (1 and 3)

Step – 5 :

Select the smallest element (2).

Among all the uncovered elements of the above table and substract this value from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines L_1, L_2,and L_3 and leaving the remaining element unchange.

	J_1	J_2	J_3	J_4
C_1	7	$\boxed{0}$	10	7
C_2	$\boxed{0}$	4	3	12
C_3	10	5	0	$\boxed{0}$
C_4	0	4	$\boxed{0}$	5

It may be added that there are no remaining zeroes and every row and column has an assignment.

Since, the no. of assignment = no. of row or column

$\therefore$ The solution is optimal

The pattern of assignment at which job has been assigned to each contractor.

Contractor	Job	Amount (Rs)×1000
C_1	J_2	5
C_2	J_1	3
C_3	J_4	3
C_4	J_3	7
		18×1000 = 18000

Minimum amount = Rs. 18,000/-

07. Ans: (A-J_1, B-J_2, C-J_4, D-J_3, TC=107)

Sol:

	Job 1	Job 2	Job 3	Job 4	
A	20	36	31	27	
B	24	34	45	22	
C	22	45	38	18	
D	37	40	35	28	
A	0	16	11	7	Row Transaction
B	2	12	23	0	
C	4	27	20	0	
D	9	12	7	0	
A	$\boxed{0}$	4	4	7	Column Transaction
B	2	$\boxed{0}$	16	0	
C	4	15	13	$\boxed{0}$	
D	9	0	$\boxed{0}$	0	
	A – J_1 → 20				
	B – J_2 → 34				
	C – J_4 →18				
	D – J_3 → 35				
	107				

08. Ans: (1-A, 2-C, 3-B, 4-Dummy, TC=35)

Sol: Here no. of rows ≠ no. of column

∴ The algorithm is not balanced so add one dummy column.

Operates	**Machine**			
	A	**B**	**C**	**Dummy**
1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

Step – 1:

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Step – 2:

$\boxed{0}$	6	9	0
4	7	$\boxed{0}$	0
26	$\boxed{0}$	9	0
9	10	14	$\boxed{0}$

Here the operator – 4 is assigned to dummy column.

∴ He is the idle worker.

09. Ans: (a, b, c)

Sol: In assignment model,

- Each source has unity supply
- Each destination has unity demand
- Total supply is equal to number of sources
- The total number of allocations in the assignment problem always equal to 'n', which is always less then (2n–1), hence assignment problems always degenerate.

Chapter 10 PPC & Aggregate Planning

01. Ans: (d) **02. Ans: (b)**

03. Ans: (b)

Sol:

Months		Month 1	Month 2	Month 3	Unused capacity	Capacity Available
1	RT	$\boxed{20}$ 90	$\boxed{22}$ 10	$\boxed{24}$	10	100
	OT	$\boxed{24}$	$\boxed{26}$	$\boxed{28}$		20
2	RT		$\boxed{20}$ 100	$\boxed{22}$		100
	OT		$\boxed{24}$ 20	$\boxed{26}$		20
3	RT			$\boxed{20}$ 80		80
	OT			$\boxed{24}$ 30	10	40
	RT / OT	90	130	110		

Level of planned production in overtimes in 3rd period is '30'.

RT = Regular time

OT = Over time

04. Ans: (b)

Sol:

Month	Cumulative Production	Cumulative Demand	Inventory		Cost	
			End	Stock out	End inventory	Stock out cost
1	100	80	20	-	40	-
2	180	180	-	-	-	-
3	250	260	-	10	-	100
4	320	300	20	-	40	-
					80	100
				Total	**180**	

05. Ans: (b) **06. Ans: (d)**

07. Ans:

Sol:

Supply from		Demand for					Total Capacity Available (supply)
		Period 1	Period 2	Period 3	Period 4	Un used capacity	
Beginning inventory		^{200} 0	5	10	15	-	200
1	Regular	^{700} 60	65	70	75	0	700
	Overtime	70	75	80	85	300	300
2	Regular		^{500} 60	65	^{200} 70	0	700
	Overtime		70	75	80	300	300
3	Regular			^{200} 60	^{500} 65	0	700
	Overtime			70	^{200} 75	100	300
4	Regular				^{700} 60	0	700
	Overtime				^{300} 70	0	300
		900	500	200	1900	700	4200 4200

Total cost = (700 ×60) + (500 ×60) + (200 ×70) + (200 × 60) + (500 ×65) + (200 ×75)

+ (700 ×60)+ (300 ×70) = Rs 2,08,500/-

08. Ans: (a, b)

Sol: In aggregate production planning :

- MPS is a detailed production schedule and it specifies what end items are to be produced and when.
- MRP is a technique for determining the quantity and timing for the acquisition of dependent demand items needed to satisfy master schedule requirements.
- The most favorable solution is usually a combination of mixed strategy that meets the objectives of the organization in light of its particular circumstances.

Chapter 11 Material Requirement & Planning

01. Ans: (b)

02. Ans: (c)

Sol: Based on master production schedule, a material requirements planning system :

- Creates schedules, identifying the specific parts and materials required to produce end items.
- Determines exact unit numbers needed.
- Determines the dates when orders for those materials should be released, based on lead times.

03. Ans: (d)

Sol: *Refer to the solution of Q.No. 02*

04. Ans: (c)

Sol: MRP has three major input components:

1. Master production Schedule of end items required. It dictates gross or projected requirements for end items to the MRP system.
2. Inventory status file of on-hand and on-order items, lot sizes, lead times etc.
3. Bill of materials (BOM) or Product structure file what components and sub assemblies go into each end product.

05. Ans: (c)

Sol: **Bill of Materials (BOM):** A listing of all components (subassemblies and materials) that go into an assembled item. It frequently includes the part numbers and quantity required per assembly.

Capacity requirements planning (CRP) is a technique for determining what personnel and equipment capacities are needed to meet the production objectives embodied in the master schedule and the material requirements plan.

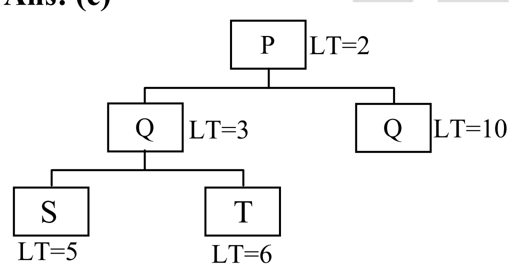
Material Requirement planning (MRP) is a technique for determining the quantity and timing for the acquisition of dependent demand items needed to satisfy master schedule requirements.

Master production Schedule of end items required. It dictates gross or projected requirements for end items to the MRP system

06. Ans: (c) **07. Ans: (b)** **08. Ans: (b)**

09. Ans: (c)

Sol:



Maximum Lead time = 12 weeks

Chapter 12 Break Even Analysis

01. Ans: (c)

Sol: Total fixed cost, TFC = Rs 5000/-

Sales price, SP = Rs 30/-

Variable cost, VC = Rs 20/-

Break even production per month,

$$Q^* = \frac{TFC}{SP - VC} = \frac{5000}{30-20} = 500 \text{ units}$$

02. Ans: (a)

Sol: Total cost = 20 + 3X --------------(1)

Total cost = 50 + X ------------(2)

By solving equ. (1) and (2)

$2X = 30$

$\therefore \quad X = 15$ units

When X = 10 units

$TC_1 = 20 + (3 \times 10) = \text{Rs } 50/-$

$TC_2 = 50 + (1 \times 10) = \text{Rs } 60/-$

Among both, total cost for process is less

So process-1 is choose.

03. Ans: (c)

Sol: In automated assembly there are less labour, so variable cost is less, but fixed is more because machine usage is more. In job shop production, labour is more but machine is less. So variable cost is more and fixed cost is less.

04. Ans: (c)

Sol: TC = Total cost

TC_A = Total cost for jig-A

TC_B = Total for jig-B

$$TC_A = TC_B$$

$$800 + 0.1X = 1200 + 0.08X$$

$$0.02X = 400$$

$$\therefore X = \frac{400}{0.02} = \frac{400}{2}\times 100 = 20{,}000 \text{ units}$$

05. Ans: (d)

Sol: Sales price – Total cost = Profit

$(C_P \times 14000) - (47000 + 14000\times 15) = 23000$

$\therefore C_P = 20$

06. Ans: (b)

07. Ans: (a)

08. Ans: (c)

09. Ans: 1500

Sol:

X	Y
$S_1 = 100$	$S_2 = 120$
$F_1 = 20{,}000$	$F_2 = 8000$
$V_1 = 12$	$V_2 = 40$

$P = q(S - V) - F$

$P_1 = q(100 - 12) - 20{,}000$

$P_2 = q(120 - 40) - 80{,}000$

$P_1 = P_2$

$88q - 20{,}000 = 80q - 80{,}000$

$12000 = 8q$

$\Rightarrow q = 1500$

10. Ans: (b)

11. Ans: (c)

Sol: At breakeven point

Total cost = Total revenue

$FC + VC \times Q = SP \times Q$

$$Q = \frac{FC}{(SP - VC)}$$

FC = 1000/- , VC = 3/- , SP = 4/-

$$Q = \frac{1000}{(4-3)} = 1000 \text{ units}$$

If sales price is increased to 25%

$$SP = 4 + \frac{1}{4}\times 4 = 5/-$$

$$Q^* = \frac{1000}{(5-3)} = 500 \text{ units}$$

$\therefore$ Breakeven quantity decreases by

$$= \frac{100-500}{100}\times 100 = 50\%$$

12. Ans: 225

Sol:

	Standard machine tool	Automatic machine tool
F_1 = F.C.	$\frac{30}{60}\times 200 = \text{Rs.}100$	$2\times 800 =$ Rs.1600 = F_2
V.C	$= \frac{20}{60}\times 200$ = Rs. 73.33	$= \frac{5}{60}\times 800$ = Rs. 66.67

$$q = \frac{1600-100}{73.33-66.67} = 225 \text{ volts}$$

If greater than 225 units then automatic machine tool is economic.

Chapter 13

Lean Manufacturing

01. Ans: (a)

Sol: Lean production requires
- Flexible resources
- Cellular layouts
- Pull system
- Kanbans
- Small lots
- Quick setups
- Uniform production levels
- Quality at the source
- Total Productive Maintenance (TPM)
- Supply chain

02. Ans: (a)

Sol:
- Lean production is an integrated management system that emphasizes the elimination of waste
- Lean production is an integrated set of activities designed to achieve production using minimal inventories of raw materials, work-in-process and finished goods

03. Ans: (d)

Sol: Drawbacks of lean production :
- It is not appropriate for every type of organization. Particularly large variety of low volume production environments.
- Difficult to maintain the discipline of lean production.
- Not the right choice for mass production (or) high-volume repetitive items

04. Ans: (d)

Sol:
- Lean system will have efficient supply chain by means of long-term supplier contracts
- Reduced inventory and improved service quality are the key benefits of lean system
- More product variety and greater flexibility are the key features of lean system

05. Ans: (a)

06. Ans: (b)

Sol: Quality in lean system is based on Kaizen, the Japanese term for "Change for the good of all" or "Continuous improvement".

07. Ans: (d)

Sol: The productivity improvement is not a dimension of lean culture.

08. Ans: (a)

Sol: Time from production finished to customer order delivered is known as delivery lead time.

09. Ans: (b)

Sol: The general definition of SEISO or shine is to clean your workplace and to make it beautiful. In the end, what we are trying to achieve through doing the fourth S is to create an environment that is a clean workplace for us to be within.

10. Ans: (d)

11. Ans: (a, b)

Sol: There are seven wastes in operations :

1. Overproduction
2. Waiting
3. Transporting
4. Processing
5. Inventory
6. Movement
7. Defects

12. Ans: (b, c)

Sol: Work performed on the product which has no value is treated as waste and it should be eliminated

Lean production is an integrated set of activities designed to achieve production using minimal inventories of raw materials, work-in-process and finished goods.

13. Ans: (c)

14. Ans: (a, b)

Sol: Lean production system emphasizes on continuous improvement and standard work practices and pull system instead of push system.

15. Ans: (a)

Sol: Kanban system is meant for maintaining pull system.

16. Ans: (c)

Sol: Daily demand (d) = 250 units

Production lead time (LT) = 1/2 day

Kanban size (C) = 50 units

Safety Stock = 1/4 day

No. of Kanbans,

$$n = \frac{d \times LT + \text{safety stock}}{C}$$

$$= \frac{250 \times 0.5 + (250 \times 0.25)}{50} = 3.75 \approx 4$$

17. Ans: 6

Sol: Daily demand (d) = 500 units

Size of container (C) = 200 units

Safety factor (α) = 20 % = 0.2

Material handling and processing time $(\bar{v})$ = 2 days

No. of Kanbans,

$$n = \frac{d \times LT \times (1+\alpha)}{C}$$

$$= \frac{500 \times 2 \times (1+0.2)}{200} = 6$$

18. Ans: 0.8

Sol: Daily demand (d) = 3000 units

Size of container (C) = 270 units

Waiting time for a container $(\bar{v})$ = 0.8 day

Processing time for a container $(\bar{r})$ = 0.2 day

No. of Kanbans, n = 20

$$n = \frac{d \times (\bar{v} + \bar{r}) \times (1 + \alpha)}{C}$$

$$20 = \frac{3000 \times (0.8 + 0.2) \times (1 + \alpha)}{270}$$

$\Rightarrow \alpha = 0.8$