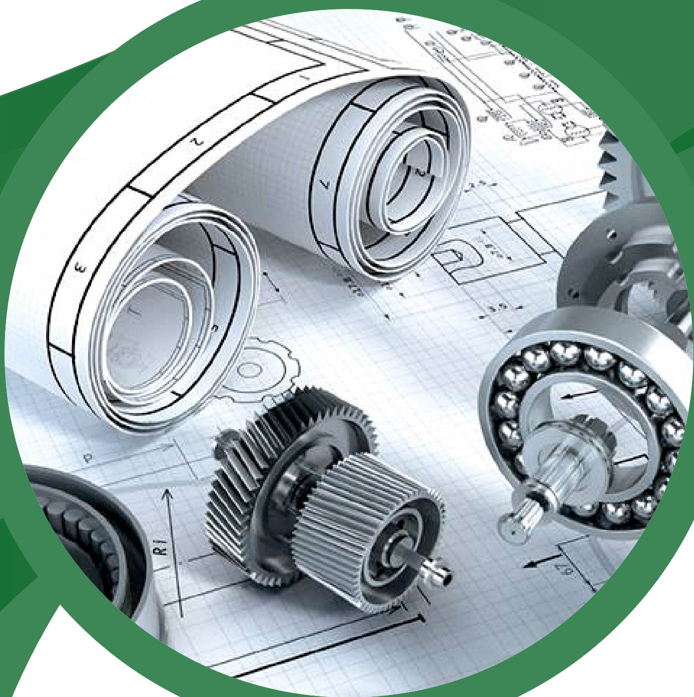
GATE | PSUs

MECHANICAL ENGINEERING

Engineering Mechanics

Text Book & Work Book : Theory with worked out Examples and Practice Questions



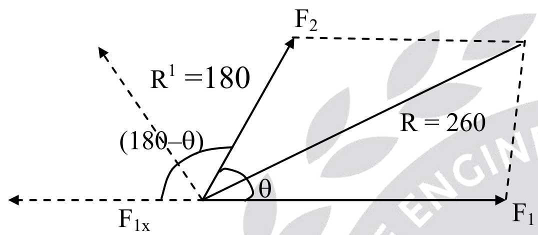
Engineering Mechanics

(Solutions for Text Book Practice Questions)

Chapter 1 Force and Moment Systems

01. Ans: (b)

Sol:



Assume $F_1 = 2F_2$ $(F_1 > F_2)$

$F_{1x} = 2F_2$

$$R = \sqrt{F_1^2 + F_2^2 + 4F_2^2\cos\theta}$$

$$260 = \sqrt{4F_2^2 + F_2^2 + 4F_2^2\cos\theta}$$

$$260^2 = 5F_2^2 + 4F_2^2\cos\theta \text{ ------ (1)}$$

$$R^1 = \sqrt{F_{1x}^2 + F_2^2 + 2F_{1x}F_2\cos\theta}$$

$$180 = \sqrt{4F_2^2 + F_2^2 + 2.F_2.F_2\cos(180-\theta)}$$

$$180^2 = 5F_2^2 - 4F_2^2\cos\theta \text{ ------ (2)}$$

$$260^2 = 5F_2^2 + 4F_2^2\cos\theta$$

$$180^2 = 5F_2^2 - 4F_2^2\cos\theta$$

$$260^2 + 180^2 = 10F_2^2$$

$\Rightarrow$ $F_2 = 100N$,

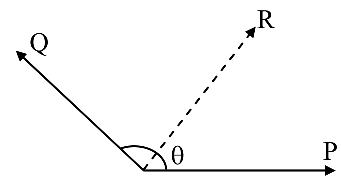
$260^2 = 5(100)^2 + 4(100)^2\cos\theta$

$\Rightarrow \theta = 63.89$

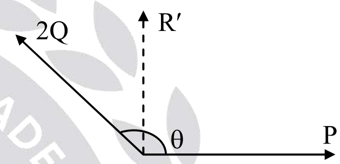
Where θ angle between two forces.

02. Ans: (b)

Sol: Let the angle between the forces be θ



Where, R is the resultant of the two forces.



If Q is doubled i.e., 2Q then resultant (R′) is perpendicular to P.

$$\tan 90 = \frac{2Q\sin\theta}{P + 2Q\cos\theta}$$

$\Rightarrow$ $P + 2Q\cos\theta = 0$

$P = -2Q\cos\theta$ ------(i)

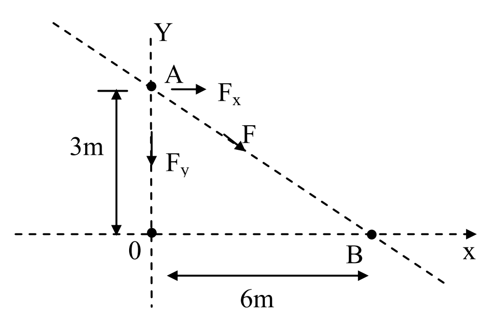
Also, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = Q$ [using eq.(i)]

03. Ans: (b)

Sol: Since moment of F about point A is zero.

$\therefore$ F passes through point A,



$M_0^F = 180N - m$

$M_B^F = 90\,N - m$

$M_A^F = 0$

$M_0^F = 180 = F_x \times 3 + F_y \times 0$

$F_x = 60N$ ……. (1)

$M_B^F = F_x \times 3 - F_y \times 6 = -90$

$60 \times 3 - 6F_y = -90$

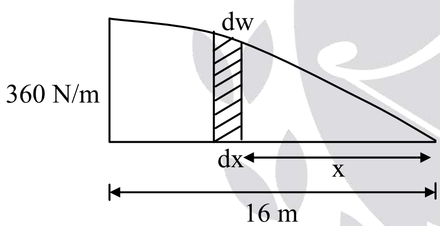
$\Rightarrow F_y = \dfrac{270}{6}$

$F_y = 45N$

$\therefore\ F = \sqrt{F_x^2 + F_y^2} = \sqrt{60^2 + 45^2} = 75$

04. Ans: (a)

Sol:



$$\int_0^w dw = \int_0^{16} w dx$$

$$w = \int_0^{16} 90\sqrt{x}dx = 90\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^{16}$$

$$= 90 \times \frac{2}{3}\left[x^{3/2}\right]_0^{16} = 60\,(16)^{3/2}$$

$w = 3840$ N

The moment due to average force should be equal to the variable force

$R \times d = \Sigma dw \times x$

$$3840 \times d = \int_0^{16} 90\sqrt{x}.dx.x$$

$$= 90\int_0^{15} x^{1.5}dx$$

$$3840d = 90\left[\frac{x^{2.5}}{2.5}\right]_0^{16}$$

$\Rightarrow\ d = 9.6$ m

05. Ans: (c)

Sol: Moment about 'O'

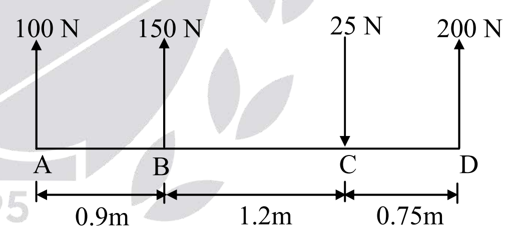
$M_0 = 100\sin 60 \times 3$

$= 300 \times \dfrac{\sqrt{3}}{2} = 150\sqrt{3}$

$= 259.8 \cong 260$ N

06. Ans: (a)

Sol:



$F_R = \Sigma F_y$

$F_R = 100 + 150 - 25 + 200$ (upward force positive and downward force negative)

$R = 425$ N

For equilibrium

$\Sigma M_A = 0$ (since R = resultant)

Let R is acting at a distance of 'd'

$425 \times d = 150 \times 0.9 + 25 \times 2.1 - 200 \times 2.85$

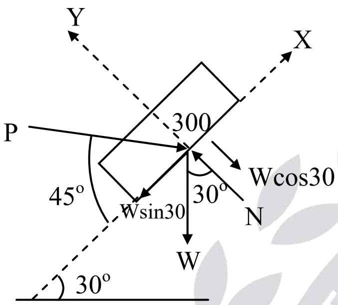
$\Rightarrow\ d = 1.535$m (from A)

Chapter 2

Equilibrium of Force System

01. Ans: (d)

Sol:



Resolve the forces along the inclined surface

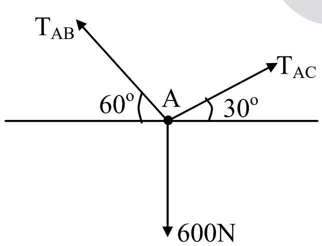
$\sum F_x = 0$

$P\cos 45 - W\sin 30 = 0$

$$P = \frac{300\sin 30}{\cos 45} \Rightarrow P = 212.13\ \text{N}$$

02. Ans: (a)

Sol:



$T_{AB}\cos 60° = T_{AC}\cos 30°$

$T_{AB} = \sqrt{3}\ T_{AC}$

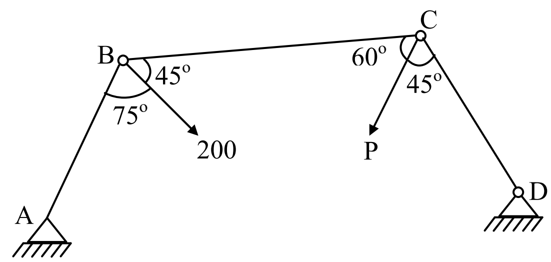
$T_{AB}\sin 60° + T_{AC}\sin 30° = 600$ N

$$\frac{3}{2}T_{AC} + \frac{1}{2}T_{AC} = 600$$

$\Rightarrow T_{AB} = 520$ N ; $T_{AC} = 300$ N

03. Ans: (c)

Sol:



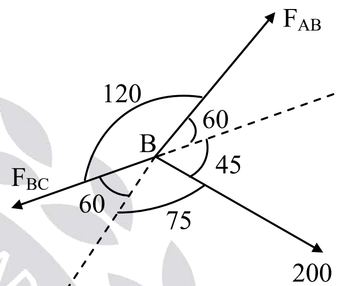


Fig: Free body diagram at 'B'

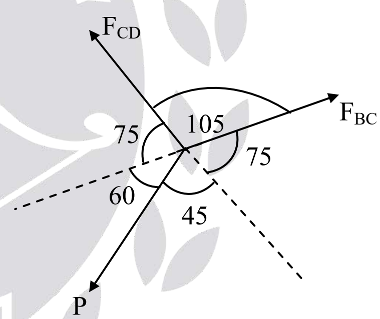


Fig: Free body diagram at 'C'

For Equilibrium of Point 'B'

$$\frac{F_{AB}}{\sin(60+75)} = \frac{F_{BC}}{\sin(60+45)} = \frac{200}{\sin(120)}$$

$F_{BC} = 223.07$ N

From Sine rule at "C".

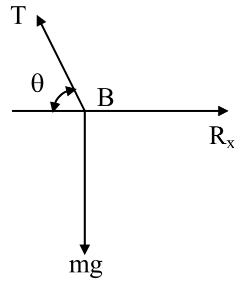
$$\frac{F_{CD}}{\sin(75+45)} = \frac{F_{BC}}{\sin(60+75)} = \frac{P}{\sin 105}$$

$$P = \frac{223.07 \times \sin 105}{\sin 135}$$

P = 304.71 N

04. Ans: (d)

Sol:



$$\tan\theta = \frac{125}{275} \Rightarrow \theta = 24.45^0$$

$T\sin\theta = mg.$

$T\sin 24.45 = (35\times 9.81)$

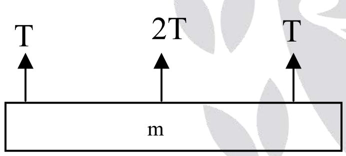
$T = 829.5$ N

$R_x = T\cos 24.45 = 755.4$ N

$R_y = 0$

05. Ans: (c)

Sol:



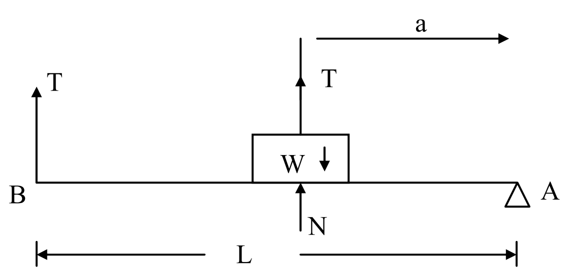
$T+2T+T = mg$

$4T = mg$

$m = 4T/g$

06. Ans: (b)

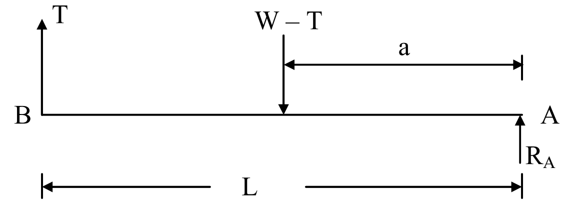
Sol:



For body, $\sum F_y = 0$

$N - W + T = 0$

$\Rightarrow N = W - T$



$\sum F_y = 0$ for entire system

$R_A + T - (W - T) = 0$

$R_A = W - 2T$ ------- (1)

For equilibrium

$\sum M_A = 0$

$T\times L = (W - T)\, a$

$TL = Wa - Ta$

$TL + Ta = Wa$

$T(L + a) = Wa$

$$\Rightarrow T = \frac{Wa}{L+a}$$

T substitute in equation (1)

$$R_A = W - 2\left(\frac{Wa}{L+a}\right)$$

$$= \frac{W(L+a) - 2Wa}{L+a}$$

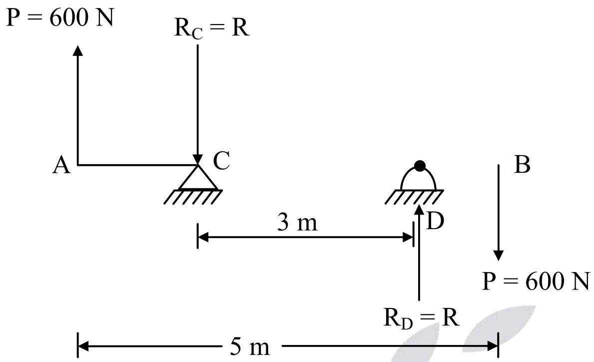
$$= \frac{WL + Wa - 2Wa}{L+a}$$

$$= \frac{WL - Wa}{L+a}$$

$$R_A = \frac{W(L-a)}{L+a}$$

07. Ans: (c)

Sol:



$\sum F_y = 0$

$600 - R_C + R_D - 600 = 0$

$\Rightarrow\ R_C = R_D = R$

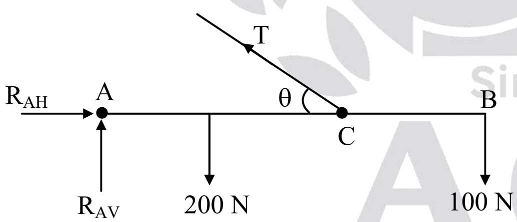
$\sum M = 0$

$600 \times 5 = R \times 3$

$\Rightarrow\ R = 1000\text{ N} = R_C = R_D$

08. Ans: (a)

Sol: F.B.D



$\sum M_A = 0$

$\text{Tan}\theta = \dfrac{8}{4}$

$\theta = 63.43$

$T\sin\theta \times 4\ (\circlearrowleft) - 200 \times 2\ (\circlearrowright) - 100 \times 6\ (\circlearrowright) = 0$

$\Rightarrow T = 279.5\text{ N}$

Now, $\sum F_x = 0$,

$R_{AH} - T\cos\theta = 0$

$R_{AH} = 125\text{ N}$

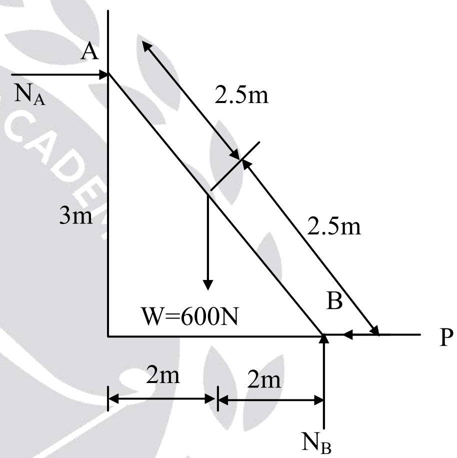
$\sum F_y = 0$

$R_{AV} - 200 - 100 + T\sin\theta = 0$

$\Rightarrow R_{VA} = 50\text{ N}$

09. Ans: 400 N

Sol:



$\sum F_y = 0$

$N_B - W = 0$

$N_B = 600\text{ N}$

$\sum M_A = 0$

$P \times 3 + W \times 2 - N_B \times 4 = 0$

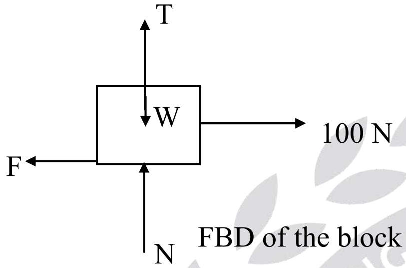
$P = \dfrac{4N_B - 2W}{3}$

$P = \dfrac{4 \times 600 - 2 \times 600}{3} = 400\text{N}$

Chapter 3 Friction

01. Ans: (c)

Sol: The FBD of the above block shown



FBD of the block

$\Sigma Y = 0 \Rightarrow N + T - W = 0$

$N = W - T = 981 - T$

$F = \mu N = 0.2\,(981 - T)$

$\Sigma X = 0 \Rightarrow 100 - F = 0$

$F = 100 = 0.2\,(981 - T)$

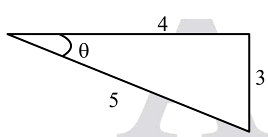
$\Rightarrow T = 481$ N

02. Ans: (c)

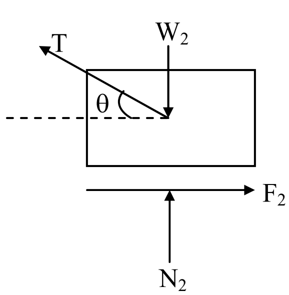
Sol: Given $\tan\theta = \dfrac{3}{4}$

$\sin\theta = 3/5$

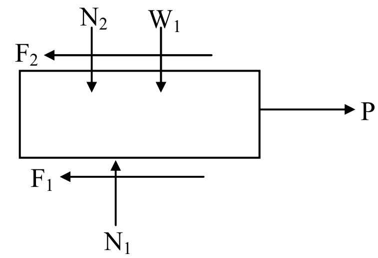
$\cos\theta = 4/5$



Free body diagram for block (2)



Free body diagram for block (1)



From FBD of block (2)

$\Sigma F_x = 0$

$F_2 = T\cos\theta$

$F_2 = \dfrac{4}{5}T = 0.8T$ ------ (1)

$\Sigma F_y = 0$

$N_2 + T\sin\theta - W_2 = 0$

$N_2 = W_2 - T\sin\theta$

$N_2 = 50 - 0.6\,T$

But $F_2 = \mu N_2$

$\Rightarrow F_2 = 0.3(50 - 0.6T)$

$F_2 = 15 - 0.18\,T$ ------- (2)

From (1) & (2)

$0.8T = 15 - 0.18\,T$

$\Rightarrow 0.98T = 15$

$\Rightarrow T = 15.31$ N

$\therefore N_2 = 50 - 0.6T$

$= 50 - 0.6\,(15.31) = 40.81$ N

$F_2 = \mu N_2 = 0.3 \times 40.81 = 12.24$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1 = 40.81 + 200 = 240.81$ N

$F_1 = \mu N_1 \Rightarrow F_1 = 0.3 \times 240.81$

$F_1 = 72.24$ N

$\Sigma F_x = 0$

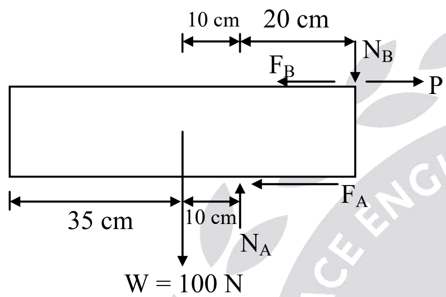
$P - F_1 - F_2 = 0$

$P = F_1 + F_2 = 72.24 + 12.24$

$P = 84.48$ N

03. Ans: (b)

Sol: Free Body Diagram



$F_A = \mu N_A = \frac{1}{3} N_A$

$F_B = \mu N_B = \frac{1}{3} N_B$

$\Sigma M_B = 0$

$-100\times30(\circlearrowleft) + (N_A\times20)(\circlearrowleft) + (F_a \times 12)(\circlearrowleft) = 0$

$-3000 + N_A \times 20 + \frac{1}{3} N_A \times 12 = 0$

$\Rightarrow N_A = 125$ N

$\Sigma F_y = 0$

$N_A - N_B - 100 = 0$

$\Rightarrow N_B = 25$ N

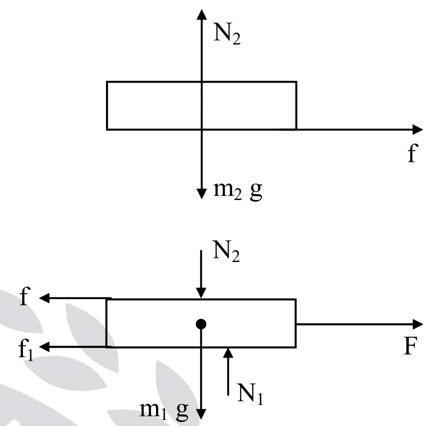
$\Sigma F_x = 0$

$P = F_A + F_B = \frac{1}{3}(N_A + N_B)$

$= \frac{1}{3}(125 + 25) = 50\text{N}$

04. Ans: (d)

Sol: F.B.D of both the books are shown below.



where, f is the friction between the two books.

f_1 is the friction between the lower book and ground.

Now, maximum possible acceleration of upper book.

$$a_{max} = \frac{f_{max}}{m_2} = \frac{\mu m_2 g}{m_2} = \mu \times g$$

$$= 0.3 \times 9.81 = 2.943 \text{ m/s}^2$$

For slip to occur, acceleration (a_1) of lower book. i.e, $a_1 \geq a_{max}$

$$\frac{F - f - f_1}{m_1} \geq 2.943$$

$F - 2.943 - 0.3 \times 2 \times 9.81 \geq 2.943$

[$\because$ $f = f_{max} = 2.943$ and

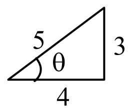
$f_1 = \mu\times(m_1 + m_2)\,g = 0.3 \times 2 \times 9.81$]

$F \geq 11.77$ N

$F_{min} = 11.77$ N

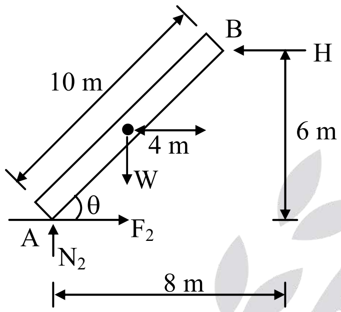
05. Ans: (d)

Sol: $\tan\theta = \dfrac{3}{4} \Rightarrow \sin\theta = \dfrac{3}{5}$

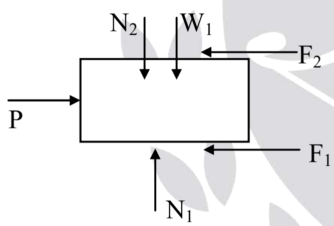


$\cos\theta = \dfrac{4}{5}$

FBD for bar AB (2)



FBD for block (1)



Given W = 280 N , W_1 = 400 N

Now, $\Sigma M_B = 0$

$-W \times 4\,(\circlearrowleft) + N_2 \times 8(\circlearrowleft) - F_2 \times 6\,(\circlearrowleft) = 0$

$-280 \times 4 + N_2 \times 8 - \mu N_2 \times 6 = 0$

$\Rightarrow N_2 = 200$ N

But, $F_2 = \mu N_2 = 0.4 \times 200 = 80$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1$

$= 200 + 400$

$N_1 = 600$ N

But, $F_1 = \mu N_1 = 0.4 \times 600$

$F_1 = 240$ N

$\Sigma F_x = 0$

$P = F_1 + F_2 = 240 + 80$

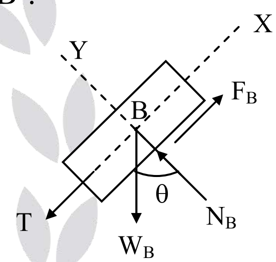
$P = 320$ N

06. Ans: (a)

Sol: Given, $W_A = 200$ N , $\mu_A = 0.2$

$W_B = 300$ N , $\mu_B = 0.5$

FBD for block 'B'.



$\Sigma F_y = 0$

$N_B = W_B\cos\theta$

$N_B = 300\cos\theta$

But, $F_B = \mu N_B = 0.5 \times 300\cos\theta$

$= 150\cos\theta$

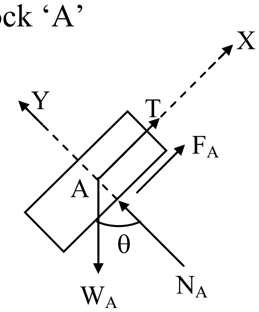
$\Sigma F_x = 0$

$T + W_B\sin\theta - F_B = 0$

$T = F_B - W_B\sin\theta$

$T = 150\cos\theta - 300\sin\theta$ ------ (1)

FBD for block 'A'



$\Sigma F_y = 0$

$N_A - W_A\cos\theta = 0$

$N_A = 200\cos\theta$

$F_A = \mu N_A = 0.2 \times 200\cos\theta$

But, $F_A = 40\cos\theta$

$\Sigma F_x = 0$

$T + F_A - W_A\sin\theta = 0$

$T = W_A\sin\theta - F_A$

$T = 200\sin\theta - 40\cos\theta$

But from equation (1)

$T = 150\cos\theta - 300\sin\theta$

$\therefore 150\cos\theta - 300\sin\theta = 200\sin\theta - 40\cos\theta$

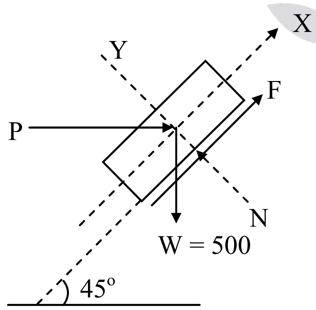
$190\cos\theta = 500\sin\theta$

$\tan\theta = \dfrac{190}{500}$

$\Rightarrow \theta = 20.8^o$

07. Ans: (d)

Sol: FBD for the block



$\Sigma F_y = 0$

$N - W\sin45 - P\sin45 = 0$

$N = \dfrac{500}{\sqrt{2}} + \dfrac{P}{\sqrt{2}}$

But, $F = \mu N = 0.25\left(\dfrac{500}{\sqrt{2}} + \dfrac{P}{\sqrt{2}}\right)$

$\Sigma F_x = 0$

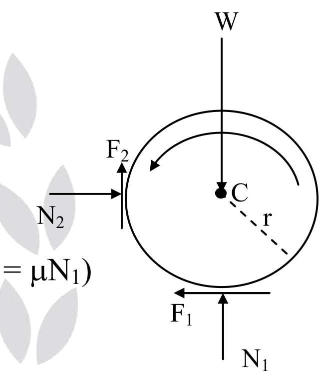
$P\cos45 + F - W\sin45 = 0$

$P\cos45 + 0.25\left(\dfrac{500}{\sqrt{2}} + \dfrac{P}{\sqrt{2}}\right) - 500 \times \dfrac{1}{\sqrt{2}} = 0$

$\Rightarrow P = 300$ N

08. Ans: (a)

Sol: FBD of block



$F_1 = \mu N_1$

$F_2 = \mu N_2$

$\Sigma F_x = 0$

$N_2 - F_1 = 0$

$\Rightarrow N_2 = F_1 \quad (\because F_1 = \mu N_1)$

$N_2 = \mu N_1$

$\Sigma F_y = 0$

$N_1 + F_2 - W = 0$

$N_1 + \mu N_2 - W = 0$

$N_1 + \mu^2 N_1 - W = 0 \qquad (\because N_2 = \mu N_1)$

$N_1(1+\mu^2) = W$

$N_1 = \dfrac{W}{1+\mu^2}$

$N_2 = \dfrac{\mu W}{1+\mu^2}$

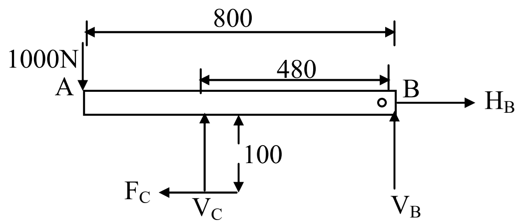
Couple $= (F_1 + F_2) \times r$

$= \mu r(N_1 + N_2)$

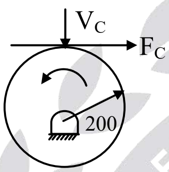
$= \dfrac{\mu r \times W(1+\mu)}{1+\mu^2} \qquad (\because \mu = f)$

09. Ans: 64 N-m

Sol: FBD of shoe bar :



FBD of Drum Brake :



$\sum M_B = 0$

$V_C \times 480 + F_C \times 100 - 1000 \times 800 = 0$

$F_C = \mu V_C = 0.2\ V_C$

$480V_C + 0.2V_C \times 100 = 800000$

$500V_C = 800000$

$V_C = 1600$ N

$F_C = 0.2\ V_C = 0.2\times1600 = 320$ N

$M = 0.2\times F_C = 0.2\times320 = 64$ N-m

10. Ans: (a)

Sol: $\beta = 2\theta$

$\cos\theta = \dfrac{6}{12}$

$\Rightarrow \theta = 60$

$\beta = 360 - 2\theta$

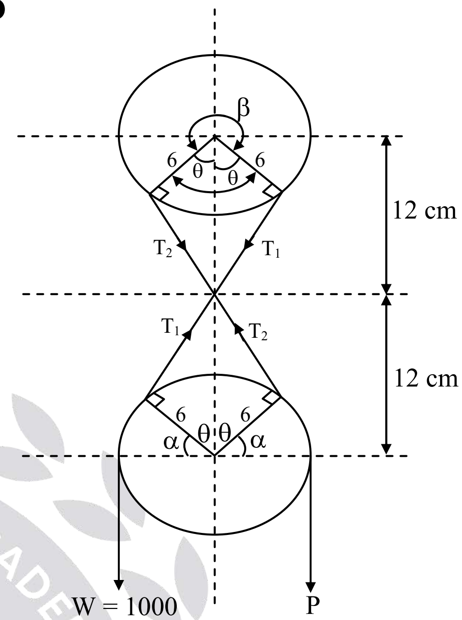
$\beta = 240 = \dfrac{4\pi}{3}$

$2\alpha + 2\theta = 180$

$2\alpha = 180 - 120$

$\alpha = 30 = \dfrac{\pi}{6}$

FBD



(When W moves upwards)

For P_{min} calculation,

$W > T_1$

$\dfrac{W}{T_1} = e^{\mu\alpha}$

$T_1 = \dfrac{1000}{e^{\frac{\pi}{6}\times\frac{1}{\pi}}} = 846.48$ N

$\because\ \dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_2 = \dfrac{848.48}{e^{\frac{1}{\pi}\times\frac{4\pi}{3}}} = 223.12$ N

$\dfrac{T_2}{P_{min}} = e^{\mu\alpha}$

$\Rightarrow P_{min} = \dfrac{223.12}{e^{\frac{1}{\pi}\times\frac{\pi}{6}}}$

$P_{min} = 188.86\ \text{N} \approx 189$ N

For P_{max} calculation

$\dfrac{T_1}{W} = e^{\mu\alpha}$

$T_1 = 1000 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$

$T_1 = 1181.36$ N

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = 1181.36 \times e^{\frac{1}{\pi}\times\frac{4\pi}{3}} = 4481.65$ N

$\frac{P_{max}}{T_2} = e^{\mu\alpha}$

$P_{max} = 4481.68 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$

$P_{max} = 5300$ N

11. Ans: (b)

Sol: Given $\mu = 0.2$, $\tan\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

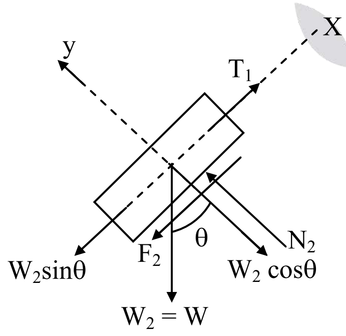


Fig: FBD (1)

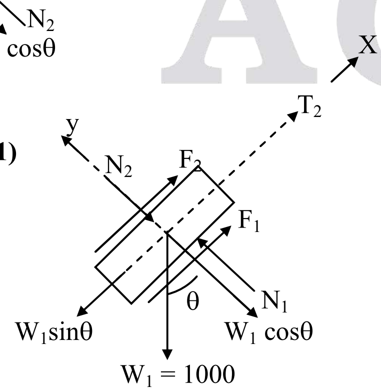


Fig: FBD (2)

From FBD (1)

$\Sigma F_y = 0$

$N_2 - W_2\cos\theta = 0$

$N_2 = W_2\cos\theta = W\times 0.8$

$N_2 = 0.8$ W

$\therefore F_2 = \mu N_2 = 0.2 \times 0.8$ W

$F_2 = 0.16$ W

$\Sigma F_x = 0$

$T_1 - W_2\sin\theta - F_2 = 0$

$T_1 = F_2 + W_2\sin\theta = 0.16\text{ W} + 0.6\text{W}$

$T_1 = 0.76$ W

From FBD (2)

$\Sigma F_y = 0$

$N_2 + W_1\cos\theta = N_1$

$N_1 = N_2 + W_1\cos\theta$

$N_1 = 0.8\text{W} + 1000 \times \frac{4}{5}$

$N_1 = 0.8\text{ W} + 800$

$F_1 = \mu N_1 = 0.2\,(0.8\text{ W} + 800)$

$= 0.16\text{ W} + 160$

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = T_1\, e^{\mu\beta} = 0.76\text{ W}\, e^{0.2\times\pi}$

$T_2 = 1.42$ W

$\Sigma F_x = 0$

$T_2 + F_1 + F_2 = W_1\sin\theta$

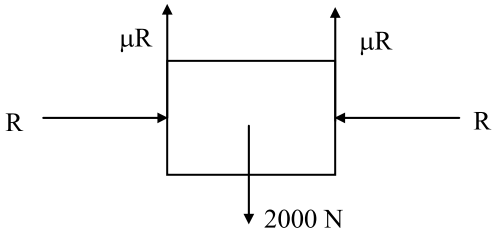
$1.42\text{W} + 0.16\text{W} + 160 + 0.16\text{W} = 1000 \times \frac{3}{5}$

$1.74\text{ W} = 440$

$\Rightarrow W = 252.87$ N

12. Ans: (d)

Sol:



At equilibrium

$2\mu R = 2000$

$\Rightarrow R = \dfrac{2000}{2\times 0.1} = 10{,}000\text{N}$

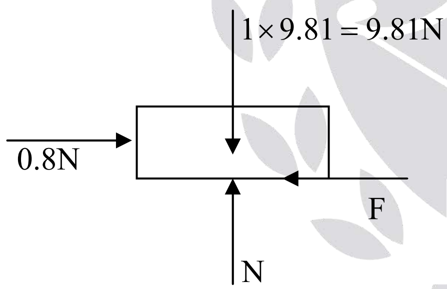
Taking moment about pin

$10{,}000 \times 150 = F \times 300$

$\Rightarrow F = 5000\text{ N}$

13. Ans: (b)

Sol:



$\Sigma Y = 0$

$\Rightarrow N = 9.81\text{ N}$

$F_s = \mu N = 0.1 \times 9.81 = 0.98\text{ N}$

The External force applied = 0.8 N < F_s

$\Rightarrow$ Frictional force = External applied force = 0.8 N

14. Ans: (b)

Sol:

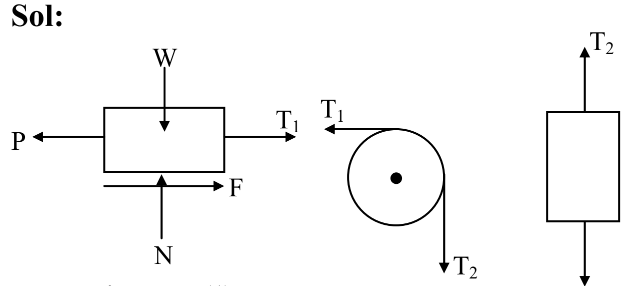


Fig: FBD (1) **Fig: FBD (2)** **Fig: FBD (3)**

From FBD (3)

$\Sigma F_y = 0$

$T_2 - 200 = 0$

$\Rightarrow T_2 = 200$

From FBD (2)

$\dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_1 = T_2 e^{\mu\beta} = 200 \times e^{0.3\times\frac{\pi}{2}}$

$T_1 = 320.39\text{ N}$

From FBD (1)

$\Sigma F_y = 0$

$N - W = 0$

$N = 1000\text{ N}$

$F = \mu N$

$= 0.3 \times 1000$

$F = 300\text{ N}$

$\Sigma F_x = 0,\ T_1 + F - P = 0$

$320.39 + 300 = P$

$\Rightarrow P = 620.39$

$\Rightarrow P = 620.4\text{ N}$

Chapter 4 Kinematics of Particle Rectilinear and Curvilinear Motion

01. Ans: (d)

Sol: $x = 2t^3 + t^2 + 2t$

$$V = \frac{dx}{dt} = 6t^2 + 2t + 2$$

$$a = \frac{dv}{dt} = 12t + 2$$

At $t = 0 \Rightarrow V = 2$ and $a = 2$

02. Ans: (a)

Sol: $V = kx^3 - 4x^2 + 6x$

$V_{\text{at } x = 2 \text{ if } k = 1} = 2^3 - 4(2)^2 + 6(2) = 4$

$$a = \frac{dV}{dt} = k.3x^2\frac{dx}{dt} - 8x\frac{dx}{dt} + 6\frac{dx}{dt}$$

$a = 3x^2(V) - 8x(V) + 6(V)$

$= 3(2)^2\times4 - (8\times2\times4) + 6(4)$

$= 8\ m/s^2$

03. Ans: (d)

Sol: Given, $a = 6\sqrt{V}$

$$\frac{dV}{dt} = 6\sqrt{V}$$

$$\int\frac{dV}{\sqrt{V}} = \int 6\,dt$$

$$2\sqrt{V} = 6t + C_1$$

Given, at $t = 2$ sec, $V = 36$

$\Rightarrow 2\sqrt{36} = 6(2) + C_1$

$\Rightarrow C_1 = 0$

$2\sqrt{V} = 6t$

$V = 9t^2$

But $V = \frac{ds}{dt} = 9t^2$

$$\int ds = \int 9t^2\,dt$$

$S = 3t^3 + C_2$

At, $t = 2$ sec, $S = 30$ m

$\Rightarrow 30 = 3(2)^3 + C_2$

$\Rightarrow C_2 = 6$

$\therefore S = 3t^3 + 6$

At $t = 3$ sec

$S = 3(3)^3 + 6$

$S = 87$ m

04. Ans: (a)

Sol: Given, $a = -8S^{-2}$

$$\Rightarrow \frac{dV}{dt} = \frac{d^2s}{dt^2} = -8s^{-2} = a$$

We know that, $\int V\,dv = \int a\,ds$

$$\frac{V^2}{2} = \int -8s^{-2}\ ds$$

$$\frac{V^2}{2} = \frac{8}{S} + C_1$$

Given, at $S = 4$ m , $V = 2$ m/sec

$$\Rightarrow \frac{2^2}{2} = \frac{8}{4} + C_1$$

$\Rightarrow C_1 = 0$

$$\therefore \frac{V^2}{2} = \frac{8}{S}$$

$$V = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \frac{ds}{dt} = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \int \sqrt{s}\, ds = \int 4\ dt$$

$$\frac{2}{3}s^{3/2} = 4t + C_2$$

At t = 1, S = 4

$$\Rightarrow \frac{2}{3}(4)^{3/2} = 4(1) + C_2$$

$$\Rightarrow C_2 = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\therefore \frac{2}{3}s^{3/2} = 4t + C_2$$

$$\Rightarrow \frac{2}{3}s^{3/2} = 4t + \frac{4}{3}$$

At t = 2 sec

$$\frac{2}{3}s^{3/2} = 4(2) + \frac{4}{3}$$

$$\Rightarrow s = 5.808 \text{ m}$$

$$a = \frac{-8}{s^2} = \frac{-8}{5.808^2} = -0.237 \text{ m/sec}^2$$

05. Ans: (c)

Sol: Given, $a = 4t^2 - 2$

$$\frac{dv}{dt} = 4t^2 - 2$$

$$dv = (4t^2 - 2)\, dt$$

$$v = \frac{4t^3}{3} - 2t + C_1$$

$$\frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$$

$$\int dx = \int\left(\frac{4t^3}{3} - 2t + C_1\right)dt$$

$$x = \frac{4t^4}{3\times 4} - 2.\frac{t^2}{2} + C_1 t + C_2$$

$$x = \frac{t^4}{3} - t^2 + C_1 t + C_2$$

Given condition,

At t = 0, x = –2 m

$$\Rightarrow -2 = C_2$$

At t= 2, x = –20 m

$$\Rightarrow -20 = \frac{2^4}{3} - 2^2 + 4(2) + (-2)$$

$$\Rightarrow C_1 = \frac{-29}{3}$$

$$\therefore x = \frac{t^4}{3} - t^2 - \frac{29}{3}t - 2$$

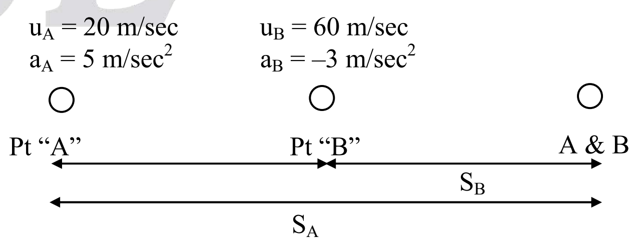
$\therefore$ at t = 4 sec

$$x = \frac{4^4}{3} - 4^2 - \frac{29}{3}(4) - 2$$

$$= 28.67 \text{ m}$$

06. Ans: (b)

Sol:



Let S_A be the distance traveled by "A"

Let S_B be the distance traveled by "B"

$S_A = S_B + 384$

$$u_A t + \frac{1}{2} a_A t^2 = u_B t + \frac{1}{2} a_B t^2 + 384$$

$$20t + \frac{1}{2} 5t^2 = 60t - \frac{1}{2} 3t^2 + 384$$

$4t^2 - 40t - 384 = 0$

$t = 16$ sec (or) $t = -6$ sec

$\therefore t = 16$ sec

07. Ans: (b)

Sol: Take , $y = x^2 - 4x + 100$

Initial velocity, $V_0 = 4\hat{i} - 16\hat{j}$

If V_x is constant

V_y , a_y at x = 16 m

$$V_x = V_{1x} = \frac{dx}{dt} = 4$$

$$V_y = \frac{dy}{dt} = 2x\frac{dx}{dt} - 4\frac{dx}{dt}$$

$(V_y) = 2x\,(4) - 4(4)$

$V_y = 8x - 16$

$(V_y)_{at\ x = 16} = 8\,(16) - 16 = 112$ m/sec

$$a_y = \frac{dV}{dt} = \frac{d}{dt}(2xV_x - 4V_x)$$

($\because V_x$ = constant)

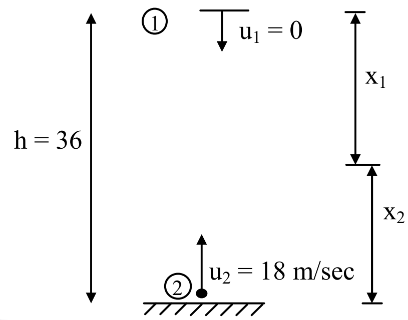
$$= 2V_x\frac{dx}{dt} = 2V_x.\ V_x$$

$a_y = 2V_x^2$

$(a_y)_{x = 16} = 2\times 4^2 = 32$ m/sec^2

08. Ans: (c)

Sol:



Let at distance of "x_1' ball (1) crossed ball (2)

$\therefore x_1 + x_2 = 36$ m

$$x_1 = 0(t) + \frac{1}{2}gt^2 \quad (\because s = ut + \frac{1}{2}at^2)$$

$$x_1 = \frac{1}{2}gt^2 \text{ -------- (1)}$$

$$x_2 = 18(t) - \frac{1}{2}gt^2$$

($\because a = -g$ moving upward)

$x_1 + x_2 = 36$

$$\Rightarrow \frac{1}{2}gt^2 + 18t - \frac{1}{2}gt^2 = 36$$

$\Rightarrow 18\,t = 36$

$\Rightarrow t = 2$ sec

$$\therefore x_1 = \frac{1}{2}(9.81).2^2$$

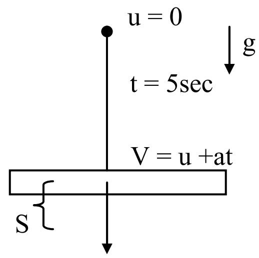
= 19.62 m (from the top)

$x_2 = 36 - 19.62$

= 16.38 m (from the bottom)

09. Ans: (b)

Sol:



$V = u + at$

$V = 0 + 9.81\ (5)$

$V = 49.05$ m/sec

V = velocity with which stone strike the glass

Velocity loss = 20% of V

$$= \frac{49.05\times 20}{100} = 9.81 \text{ m/sec}$$

$\therefore$ Initial velocity for further movement in glass = 49.05 – 9.81 = 39.24 m/sec

Distance traveled for 1 sec of time is given by

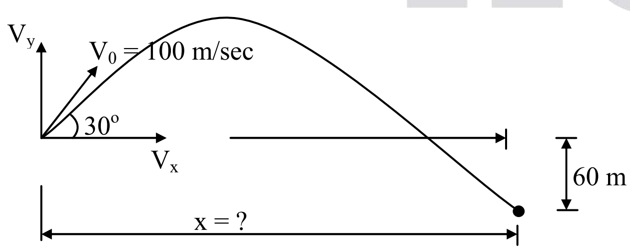
$$S = ut + \frac{1}{2}at^2$$

$$S = 39.24(1) + \frac{1}{2}(9.81)(1)^2$$

$S = 44.145$ m

10. Ans: (a)

Sol:



$a_x = -4$ m/sec^2, $a_y = -20$ m/sec^2

$$V_x = V_0\cos 30 = 100\times\frac{\sqrt{3}}{2} = 86.6 \text{ m/sec}$$

$$V_y = V_0\sin 30 = 100\times\frac{1}{2} = 50 \text{ m/sec}$$

$$y = V_{0y}t + \frac{1}{2}a_y t^2$$

$$-60 = 50t + \frac{1}{2}(-20)t^2$$

$10t^2 - 50t - 60 = 0$

$t = 6$ (or) -1 sec

$\therefore t = 6$ sec

$$x = V_0 t + \frac{1}{2}a_x t^2$$

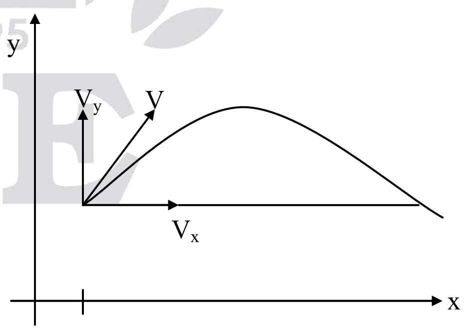
$$x = (86.6\times 6) + \frac{1}{2}(-4)6^2$$

$x = 447.6$ m $\simeq 448$ m

11. Ans: (a)

Sol: Given, V = 20 m/sec

x = 20 m, y = 8.0 m



$V_x = V\cos\theta$, $V_y = V\sin\theta$

$$x = V_x t + \frac{1}{2}at^2 \ (\because a = 0, \text{ along x direction})$$

$x = V\cos\theta\, t$

$20 = 20\cos\theta t$

$$t = \frac{1}{\cos\theta} \text{ ------- (1)}$$

$$y = V_y t - \frac{1}{2}gt^2$$

$$8.0 = V\sin\theta t - \frac{1}{2}gt^2$$

$$8.0 = 20\sin\theta \times \frac{1}{\cos\theta} - \frac{1}{2}\times 9.81 \times \left(\frac{1}{\cos\theta}\right)^2$$

$8 = 20\tan\theta - 4.9\sec^2\theta$

$8 = 20\tan\theta - 4.9\,(1 + \tan^2\theta)$

$4.9\tan^2\theta - 20\tan\theta + 12.9 = 0$

$\tan\theta_1 = 3.28,\ \tan\theta_2 = 0.803$

$\theta_1 = 73.04\ ;\ \theta_2 = 38.76$

12. Ans: (d)

Sol: Range = maximum height

$$\frac{V_0^2\sin 2\theta}{g} = \frac{V_0^2\sin^2\theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2\theta}{2}$$

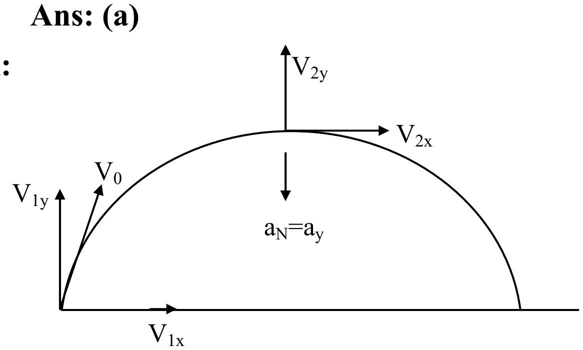
$$\Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$\Rightarrow \tan\theta = 4$

$\therefore\ \theta = \tan^{-1}(4) = 76°$

13. Ans: (a)

Sol:



$V_{1x} = 100 - t^{3/2}$

$V_{2y} = 0 \Rightarrow 100 + 10t - 2t^2 = 0$

$(t-10)(t+5) = 0$

$t = 10$ sec

V_{2x} at $t = 10 \Rightarrow V_{2x} = 100 - 10^{3/2}$

$= 68.37$ m/sec

Radius of curvature, $r = \dfrac{V^2}{a_N}$

Where $a_N = a_y = \left(\dfrac{dV_y}{dt}\right)_{\text{at } t=10\,\text{sec}}$

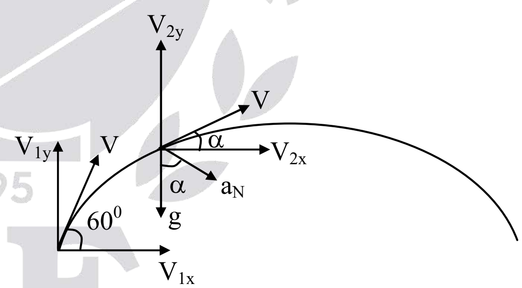
$= (10 - 4t)_{t=10}$

$a_N = -30$ m/sec^2

$$r = \frac{V_{2x}^2}{a_N} = \frac{68.37^2}{30} = 155.8 \text{ m}$$

14. Ans: (a)

Sol:



Given, v = 100 m/sec

$v_{1x} = v\cos 60°$

$= 100 \times 1/2$

$v_{1x} = 50$ m/sec

$v_{1y} = v\sin 60°$

$$= 100 \times \frac{\sqrt{3}}{2}$$

$v_{1y} = 86.6$ m/sec

$v_{2y} = v_{1y} - gt$ (use V = u+at)

$= 86.6 - 9.8(1)$

$v_{2y} = 76.8$ m/sec

$v_{2x} = v_{1x} = 50$ m/sec

$$v_{at\ t=1} = \sqrt{v_{2x}^2 + v_{2y}^2}$$

$$= \sqrt{50^2 + 76.8^2}$$

$= 91.6$ m/sec

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{76.8}{50}\right)$$

$\alpha = 56.9$ rad/sec

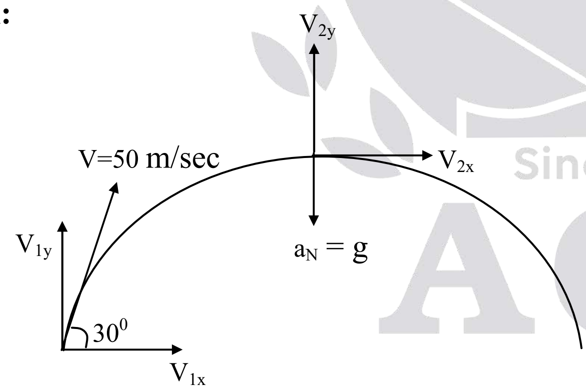
$a_N = g\cos\alpha = 9.81 \times \cos 56.9°$

$= 5.35$ m/sec^2

$$r = \frac{V^2}{a_N} = \frac{91.6^2}{5.35} = 1568.62 \text{ m}$$

15. Ans: (d)

Sol:



$v_{1x} = v\cos 30 = 43.3$ m/sec

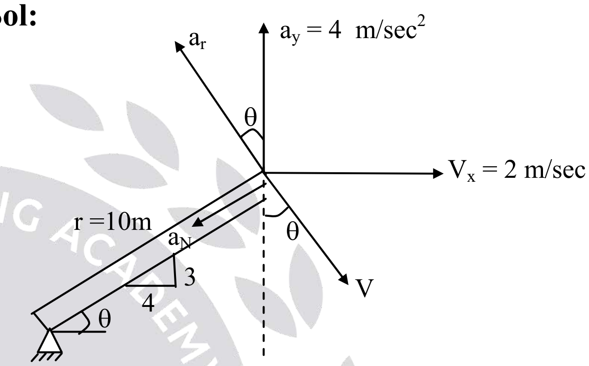
$a_N = g = a$

$$r = \frac{V_{1x}^2}{a_N} = \frac{43.3^2}{9.81} = 191.13 \text{ m}$$

Chapter 5 Kinematics of Rigid Bodies Fixed Axis Rotation and General Plane Motion

01. Ans: (a)

Sol:



$$\tan\theta = \frac{3}{4}$$

$\theta = \text{Tan}^{-1}\ 3/4 = 36.6^0$

$a_y = a_T\cos\theta - a_N\sin\theta$

Note: Velocity will always act in the tangential direction

$V_x = V\sin\theta$

$$V = \frac{2}{\sin 36.6} = 3.33 \text{ m/sec}$$

$$\therefore a_N = \frac{V^2}{r} = \frac{3.33^2}{10}$$

$a_N = 1.111$ m/sec^2

$a_y = a_T\cos\theta - a_N\sin\theta$

$4 = a_T\cos 36.6 - 1.111\sin 36.6$

$\Rightarrow a_T = 5.83$ m/sec^2

$a_T = r\alpha$

$$\alpha = \frac{a_T}{r} = \frac{5.83}{10} = 0.583 \text{ rad/sec}^2$$

02. Ans: (c)

Sol: Given $\omega = 4\sqrt{t}$

$\theta = 2$ radians at $t = 1$sec

$\theta = ?\ \alpha = ?$ at $t = 3$sec

$$\omega = \frac{d\theta}{dt} \Rightarrow \int d\theta = \int \omega dt$$

$$\theta = \int 4\sqrt{t}dt$$

$$\theta = \frac{8}{3}t^{3/2} + c \ldots(1)$$

From given condition, at $t = 1$, $\theta = 2$rad

$$(1) \Rightarrow 2 = \frac{8}{3}(1)^{3/2} + c_1 \Rightarrow c_1 = \frac{-2}{3}$$

$$\therefore\ \theta = \frac{8}{3}t^{3/2} - \frac{2}{3}$$

At $t = 3$ sec, $\theta = \frac{8}{3}(3)^{3/2} - \frac{2}{3}$

$$\theta_{t=3} = 13.18\text{rad}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d(4\sqrt{t})}{dt} = \frac{2}{\sqrt{t}}$$

$$\alpha_{t=3} = \frac{2}{\sqrt{3}} = 1.15\text{rad/sec}^2$$

03. Ans: (b)

Sol: $r = 2$ cm, $\omega = 3$ rad/sec, $a = 30$ cm/s^2

$a_N = r\omega^2 = 2(3)^2 = 18$ cm/sec^2

Since total acceleration $a = \sqrt{a_T^2 + a_N^2}$

$$\Rightarrow a^2 = a_T^2 + a_N^2$$

$$30^2 = a_T^2 + 18^2$$

$a_T = 24$ cm/sec^2

$a_T = r\alpha = 24$

$$\alpha = \frac{24}{2} = 12\text{rad/sec}^2$$

04. Ans: (d)

Sol: Given angular acceleration, $\alpha = \pi$ rad/sec^2

Angular displacement in time t_1 and t_2

$= \pi$ rad $= \theta_2 - \theta_1$

$\omega_{t2} = 2\pi$ rad/sec

$\omega_{t1} = ?$

$$\omega_{t1}^2 - \omega_0^2 = 2\alpha\theta_1$$

$$\omega_{t2}^2 - \omega_0^2 = 2\alpha\theta_2$$

$$\omega_{t2}^2 - \omega_{t1}^2 = 2\alpha(\theta_2 - \theta_1)$$

$$4\pi^2 - \omega_{t1}^2 = 2\pi^2$$

$$\omega_{t1}^2 = 2\pi^2$$

$\omega_{t1} = \pi\sqrt{2}$ rad/s

05. Ans: (c)

Sol: Given retardation

$$\alpha = -3t^2$$

$$\frac{d\omega}{dt} = -3t^2$$

$$\int d\omega = \int -3t^2 dt$$

$$\omega = -t^3 + c_1$$

From given condition at $t = 0$,

$\omega = 27$ rad/sec

$27 = -0^3 + c_1$

$\Rightarrow c_1 = 27$

$\therefore\ \omega = -t^3 + 27$

Wheel stops at $\omega = 0$,

$\Rightarrow 0 = -t^3 + 27$

$\Rightarrow\ t = 3$sec

06. Ans: (c)

Sol: angular speed, $\omega = 5$ rev/sec

$$= 5\times 2\pi \text{ rad/sec}$$

$$\omega = 10\pi \text{ rad/sec}$$

Radius, r = 0.1m

If ω is constant, $d\omega = 0$

$\Rightarrow \alpha = 0 \Rightarrow a_T = 0$ (since $a_T = r\alpha$)

Since $a_T = 0$

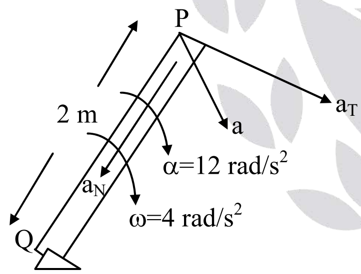
$$a = \sqrt{a_N^2 + a_T^2}$$

$$a = a_N = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$= 0.1\times(10\pi)^2 = 10\pi^2 \text{ m/sec}^2$$

07. Ans: 40

Sol:



Tangential acceleration

$$a_T = r\,\alpha = 2\times 12 = 24 \text{m/s}^2$$

Normal acceleration, $a_N = r\,\omega^2$

$$= 2\times 4^2 = 32 \text{ m/s}^2$$

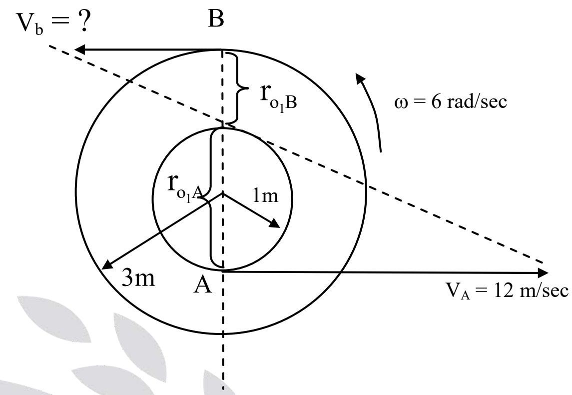
The resultant acceleration

$$a = \sqrt{a_T^2 + a_N^2}$$

$$= \sqrt{24^2 + 32^2} = 40 \text{m/s}^2$$

08. Ans: (b)

Sol:



$$V_A = r_{o_1A}\times\omega$$

$$\Rightarrow 12 = r_{o_1A}\times 6$$

$$r_{o_1A} = 2\text{m}$$

$$4 = 2 + r_{o_1B}$$

$$r_{o_1B} = 2\text{m}$$

$$\therefore\ V_B = r_{o_1B}\times\omega = 2\times 6$$

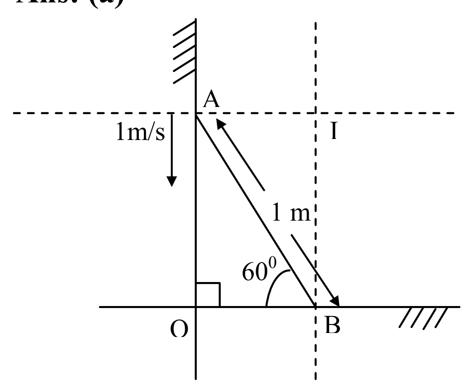
$$V_B = 12 \text{ m/sec}$$

09. Ans: (a)

Sol: Instantaneous centre will have zero velocity because the instantaneous centre is the point of contact between the object and the floor.

10. Ans: (a)

Sol:



$V_a = 1$ m/s

V_a = along vertical

V_b = along horizontal

So instantaneous center of V_a and V_b will be perpendicular to A and B respectively

$$IA = OB = l \times \cos\theta = 1 \times \cos 60^0 = \frac{1}{2} m$$

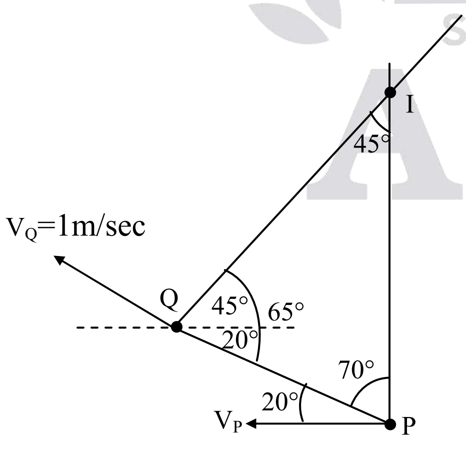
$$IB = OA = l \times \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2} m$$

$$V_a = \omega \times IA$$

$$\Rightarrow \omega = \frac{V_a}{IA} = 2 \text{ rad/sec}$$

11. Ans: (d)

Sol: The velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P using sine rule.



'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

$$V_Q = IQ \times \omega = 1$$

$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1$$

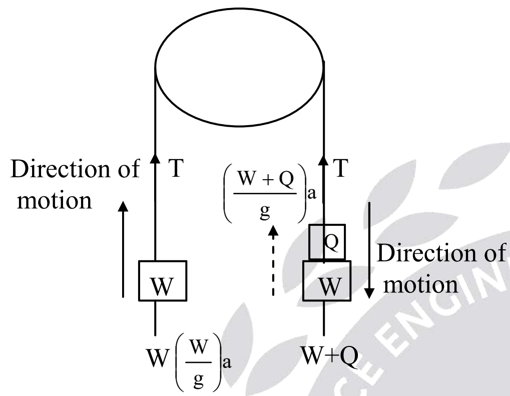
$= 0.9645$ m/s

Chapter 6

Kinetics of Particle and Rigid Bodies

01. Ans: (a)

Sol:



For the left cord,

$\Sigma F_y = 0$

$$T = \left(\frac{W}{g}\right)a + W \text{ ....(1)}$$

For the right cord

$\Sigma F_y = 0$

$$T + \left(\frac{W+Q}{g}\right)a = (W+Q) \ldots(2)$$

From (1) & (2)

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W+Q}{g}\right)a$$

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W}{g}\right)a - \left(\frac{Q}{g}\right)a$$

$$Q - \frac{Qa}{g} = \frac{2Wa}{g}$$

$$Q\left(\frac{g-a}{g}\right) = \frac{2Wa}{g} \Rightarrow Q = \frac{2Wa}{g-a}$$

02. Ans: (b)

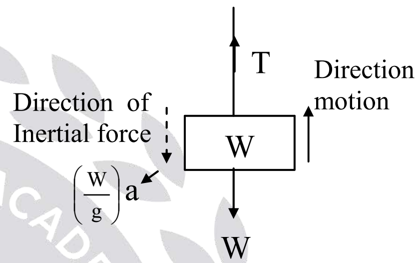
Sol: $u = 0$, $v = 1.828$ m/sec, $S = 1.825$ m,

$v^2 - u^2 = 2as$

$1.828^2 - 0 = 2a \times 1.828$

$$a = \frac{1.828}{2}$$

$a = 0.914$ m/sec^2



For equilibrium, $\Sigma F_y = 0$

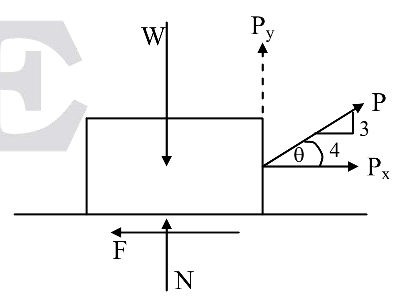
$$T = W + \left(\frac{W}{g}\right)a$$

$$= 4448 + \frac{4448}{9.81} \times 0.194$$

$T = 4862.42$ N

03. Ans: (a)

Sol:



$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}(3/4) = 36.86$$

$$(F_{net})_x = ma$$

$P_x - F = \left(\frac{W}{g}\right)a$

$P\cos 36.86 - F = \left(\frac{W}{g}\right)a$

$0.8P - F = \left(\frac{2224}{g}\right)(0.2g)$

$0.8P - F = 444.8$

$0.8P - F = 444.8 + F$

$P = 556 + 1.25F$ ……(1)

$\Sigma F_y = 0$

$N + P_y - W = 0$

$N = W - P_y$ (since $\mu = \frac{F}{N}$)

$F = \mu N$

$F = \mu\,(W - P_y)$

$\quad = 0.2(2224 - P\sin 36.86)$

$F = 444.8 - 0.12P$ …..(2)

From (1) & (2)

$P = 556 + 1.25(444.8 - 0.12P)$

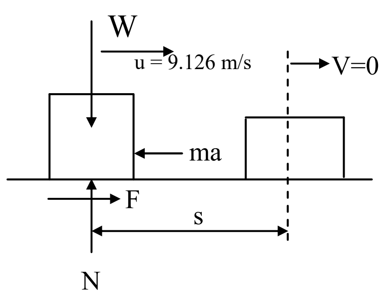
$1.15P = 1112$

$P = 966.95$

$P = 967$ N

04. Ans: (d)

Sol:



From static equilibrium condition

$\Sigma F_y = 0$

$N - W = 0$

$N = W = 44.48$N

From dynamic equilibrium condition

$\Sigma F_x = 0$

$F = ma$

$\mu N = \frac{W}{g}a$

$\mu = \frac{a}{g}$

$a = \mu g$ …….. (1)

Since $v^2 - u^2 = 2as$

$0 - (9.126)^2 = 2(-a) \times 13.689$

$a = 3.042$ m/s^2 …….. (2)

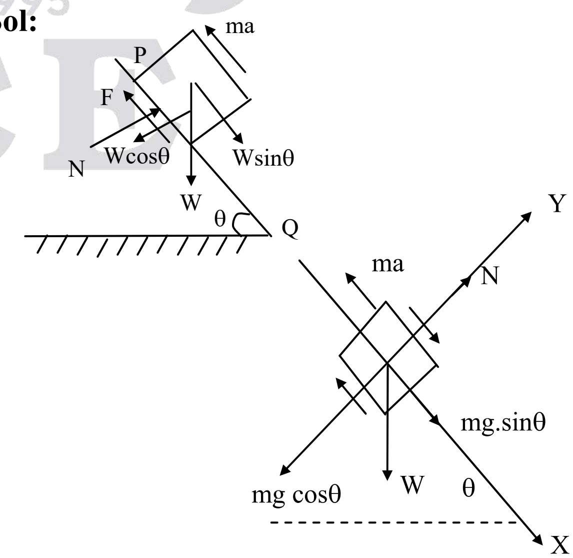
From (1) & (2)

$3.042 = \mu(9.81)$

$\Rightarrow \mu = 0.31$

05. Ans: (a)

Sol:



$\Sigma F_y = 0$ (static equilibrium)

$N - W\cos\theta = 0$

$N = W\cos\theta = mg\cos\theta$

Since $F = \mu N = \mu\, mg\cos\theta$ ....(1)

$\Sigma F_x = 0$ (Dynamic equilibrium)

$F + ma - W\sin\theta = 0$

$F = mg\sin\theta - ma$ ....(2)

From (1) & (2)

$\mu mg\cos\theta = mg\sin\theta - ma$

$\Rightarrow a = g\sin\theta - \mu g\cos\theta$

$\Rightarrow a = g\cos\theta(\tan\theta - \mu)$

Given, PQ = s

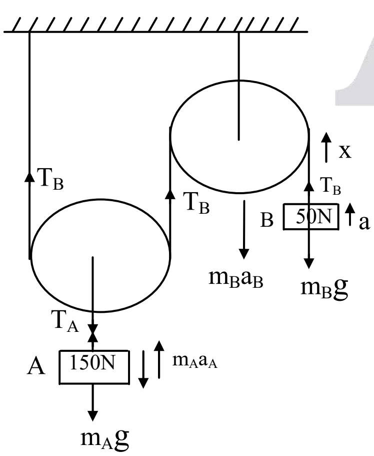
$s = ut + \frac{1}{2}at^2$

$s = 0(t) + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$

$= \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$

06. Ans: (a)

Sol:



$T_A = 2T_B$ ....(1)

Work done by A & B equal

$T_AS_A = T_BS_B$

$2T_BS_A = T_B S_B$

$2S_A = S_B$

$2a_A = a_B$ ....(2)

For 'B' body

$T_B = m_Ba_B + m_Bg$ ....(3)

For 'A' body

$T_A = m_Ag - m_Aa_A$ ....(4)

(2), (3) & (4) sub in (1)

$m_A g - m_A a_A = 2(m_B(2a_A) + m_B g)$

$m_A g - m_A a_A = 4m_B a_A + 2m_B g$

$m_A a_A + 4m_B a_A = m_Ag - 2m_B g$

$$a_A = \frac{m_Ag - 2m_Bg}{m_A + 4m_B}$$

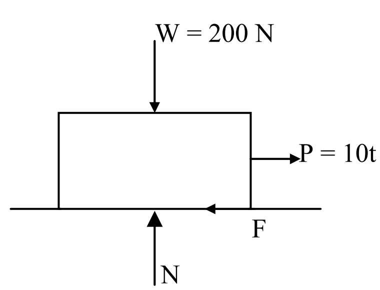
$$= \frac{150 - 2(50)}{\frac{150}{10} + 4\left(\frac{50}{10}\right)}$$

$$= \frac{50}{15 + 20} = \frac{50}{35} = 1.42 \text{ m/s}^2$$

07. Ans: 4.905 m/s

Sol: $\mu_S = 0.4$; $\mu_K = 0.2$

FBD of the block



W.r.t free body diagram of the block:

$F_S = \mu_S N$;

$F_K = \mu_K N$

$\Sigma F_y = 0$

$N - W = 0$

$N = W = 200$ N

Limiting friction or static friction

$(F_S) = 0.4 \times 200 = 80$ N

Kinetic Friction

$(F_K) = 0.2 \times 200 = 40$ N

The block starts moving only when the force, P exceeds static friction, F_S

Thus, under static equilibrium

$\Rightarrow \sum F_x = 0$

$\Rightarrow P - F_S = 0 \Rightarrow 10t = 80$

$$t = \frac{80}{10} = 8 \text{ sec}$$

$\therefore$ The block starts moving only when t >8seconds

During 8 seconds to 10 seconds of time:

According to Newton's second law of motion

Force = mass × acceleration

$$(P - F_K) = m \times \frac{dv}{dt} \Rightarrow (10t - 40) = \frac{200}{9.81} \times \frac{dv}{dt}$$

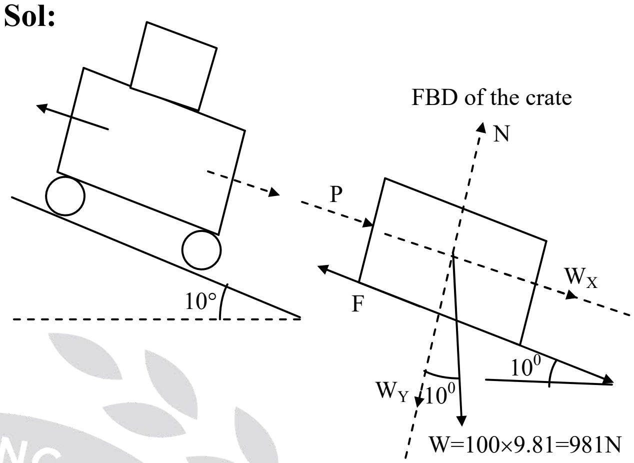
$$\int_8^{10} (10t - 40)dt = \frac{200}{9.81}\int_0^V dv$$

$$\left[5t^2 - 40t\right]_8^{10} = 20.387 \times V \Rightarrow (180 - 80) = 20.387 \times V$$

Velocity (V) = 4.905 m/s

08. Ans: 1.198 m/s^2

Sol:



W.r.t. FBD of the crate:

$W_X = W\sin 10^0 = 981 \times \sin 10^0$

$= 170.34$N

$W_Y = W \cos 10^0 = 981 \times \cos 10^0 = 966.09$ N

$\sum F_Y = 0 \Rightarrow N - W_Y = 0$

$N = W_Y = 966.09$N;

$F = \mu N = 0.3 \times 966.09 = 289.828$ N

$\sum F_X = 0 \Rightarrow P + W_X - F = 0$

$\Rightarrow P + 289.828 - 170.34 = 0$

$P = 119.488$ N

$P = ma = 119.488$ N

$$\Rightarrow a = \frac{119.488}{100} = 1.198 \, m/s^2$$

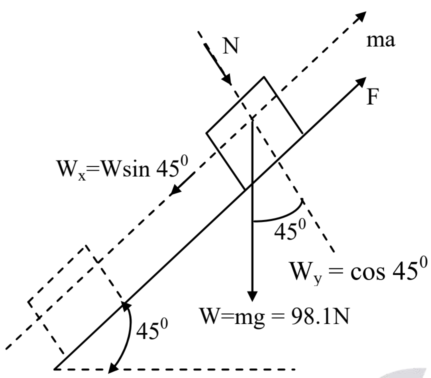
09. Ans: 57.67 m

Sol:

$W_x = W \sin 45$

$= 98.1 \times \sin 45 = 69.367$ N

$W_y = W \cos 45 = 69.367$ N



$\Sigma F_Y = 0$

$N - W_Y = 0$

$N = W_Y = 69.367$ N

$F = \mu_K N = 0.5 \times 69.367 = 34.683$N

$\Sigma F_x = 0$ (Dynamic Equilibrium

D$'$ Alembert principle)

$W_x - F - ma = 0$

$69.367 - 34.683 - 10 \times a = 0$

$a = 3.468 m/s^2$

$S = ut + \frac{1}{2}at^2$

$\because$ t is unknown we can not use this equation

So use $V^2 - u^2 = 2as$

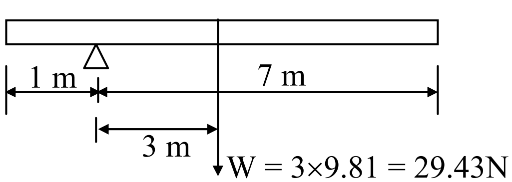
$V = 20 m/s^2$; $u = 0$; $a = 3.468 m/s^2$

$V^2 = 2as$

$$S = \frac{V^2}{2 \times a} = \frac{20^2}{2 \times 3.468} = 57.67\text{m}$$

10. Ans: 2.053 rad/s^2

Sol:



$M = I\alpha$

$M = 29.43 \times 3 = 88.29$N-m

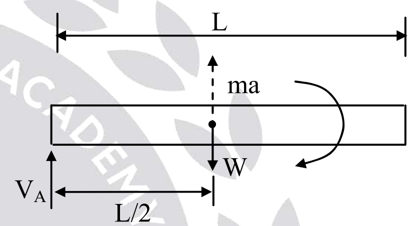
$$I = I_0 + Ad^2 = \frac{m\ell^2}{12} + md^2 = \frac{3 \times 8^2}{12} + 3 \times 3^2$$

$$= 16 + 27 = 43 \text{kg} - \text{m}^2$$

$$\alpha = \frac{M}{I} = \frac{88.29}{43} = 2.053 \text{ rad/s}^2$$

11. Ans: (d)

Sol:



$\Sigma F_y = 0$

$V_A + ma = W$

$V_A = m(g - a)\ldots(1)$

Where, $a = \frac{L}{2}\alpha$

Since, $M = I\alpha$

$$W \times \frac{L}{2} = \left(\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2\right)\alpha$$

$$mg \times \frac{L}{2} = \frac{4mL^2}{12} \times \frac{2a}{L}$$

$$a = \frac{3}{4}g \ldots(2)$$

from (1) & (2)

$$V_A = m\left(g - \frac{3}{4}g\right) = \frac{mg}{4}$$

$$V_A = \frac{W}{4}$$

12. Ans: (d)

Sol: $I = 5 kg.m^2$

$R = 0.25m$

$F = 8\ N$

Mass moment of inertia, $I_x = I_y = \frac{mr^2}{4}$

$I_z = \frac{mr^2}{2}$

$M = I\alpha$

$8\times0.25 = 5\times\alpha$

$\alpha = 0.4$

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\omega^2 - 0^2 = 2(0.4)\times\pi$ (since for half revolution $\theta = \pi$)

$\omega = 1.58$ rad/sec

13. Ans: 4.6 seconds

Sol: $M = 60\ N - m$

$L = 2m,\quad \omega_0 = 0,$

$\omega = 200\ rpm = \frac{200\times2\pi}{60}$

$\omega = 20.94\ \frac{rad}{sec}$

Moment, $M = I\alpha$

$60 = \frac{mL^2}{12}\times\alpha$

$\Rightarrow 60 = \frac{40\times2^2}{12}\times\alpha$

$\alpha = 4.5 rad/sec^2$

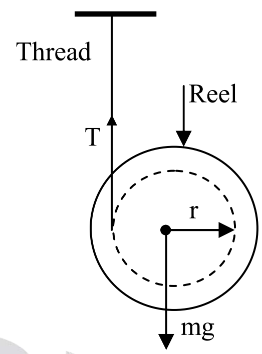
$\omega = \omega_0 + \alpha t$

$20.94 = 4.5t$

$\Rightarrow\quad t = 4.65$ sec

14. Ans: (a)

Sol:



a = linear acceleration,

k = radius of gyration

For vertical translation motion

$mg - T = ma$ ------ (1)

For rotational motion

$T \times r = I\alpha$

$Tr = mk^2\ \alpha = mk^2 \times \frac{a}{r}$

$\Rightarrow T = \frac{mk^2}{r^2}\times a$ ------(2)

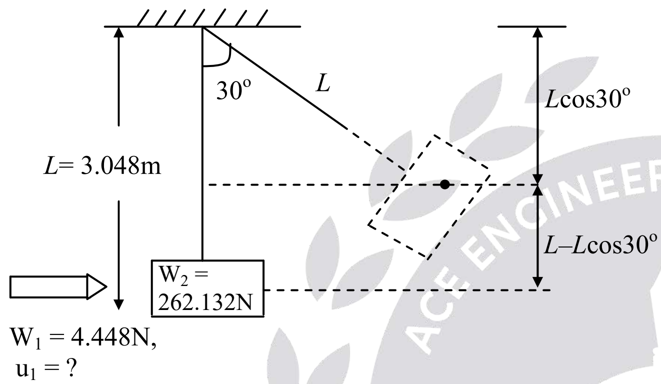
$mg - \frac{mk^2}{r^2}\times a = ma \Rightarrow a = \frac{gr^2}{(k^2 + r^2)}$

Chapter 7

Work-Energy Principle and Impulse Momentum Equation

01. Ans: (a)

Sol:



The loss of KE of shell converted to do the work in lifting the sand box and shell to a height of "$L - L\cos30^o$"

i.e., $Wd = \frac{1}{2}mV^2$

Where $d = L - L\cos30^o$

$= 3.048 - 3.048\times\cos30 = 0.41$ m

$$266.58\times0.41 = \frac{1}{2}\left(\frac{266.58}{9.81}\right)\times V^2$$

$\Rightarrow V = 2.83$ m/sec

Where V is the velocity of block & shell

By momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Where $v_1 = v_2 = V$ & $u_1 = ?$, $u_2 = 0$

$$\frac{4.448}{9.81}\times u_1 = \frac{4.448+262.132}{9.81}\times 2.83$$

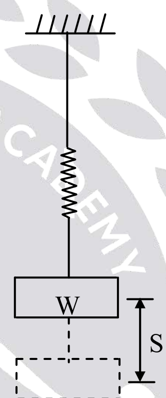
$\Rightarrow u_1 = 169.6$ m/sec

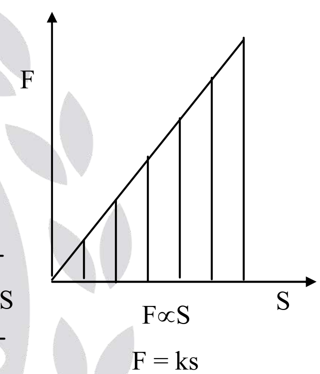
u_1 & u_2 = Initial velocity of shell and block respectively

V_1 & V_2 = Final velocity of block & shell

02. Ans: (b)

Sol:





Strain energy in spring = Area under the force displacement curve.

$$= \frac{1}{2}F\times s = \frac{1}{2}(ks)\times s = \frac{1}{2}ks^2$$

$$\frac{1}{2}ks^2 = \text{Gain of KE}$$

$$\frac{1}{2}ks^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{ks^2}{m} = \frac{ks^2}{w}g$$

$$v = \sqrt{\frac{kg}{w}}.s \qquad \left(\because m = \frac{w}{g}\right)$$

03. Ans: (a)

Sol: Given, m = 2 kg

Position at any time is given as

$$x = t + 5t^2 + 2t^3$$

At t = 0, x = 0,

At t = 3sec,

$x = 3 + 5(3^2) + 2(3^3) = 102\text{m}$

Velocity, $V = \frac{dx}{dt} = 1 + 10t + 6t^2$

Initial velocity i.e., t = 0, is v_i = 1m/s

Final velocity i.e., at t = 3sec,

is $v_f = 1 + 10(3) + 6(3)^2 = 85\text{m/s}$

Work done = change in KE

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2} \times 2\left(85^2 - 1^2\right) = 7224 \text{ J}$$

04. Ans: (a)

Sol: Given force $F = e^{-2x}$

$$\text{Work done} = \int_{x_1}^{x_2} F dx$$

$$= \int_{0.2}^{1.5} e^{-2x} dx = \left[\frac{e^{-2x}}{-2}\right]_{0.2}^{1.5} = 0.31\text{J}$$

05. Ans: (b)

Sol: $F = 4x - 3x^2$

Potential Energy at x = 1.7 = work required to move object from 0 to 1.7m

$$PE = \int_0^{1.7} F dx$$

$$= \int_0^{1.7} \left(4x - 3x^2\right) dx$$

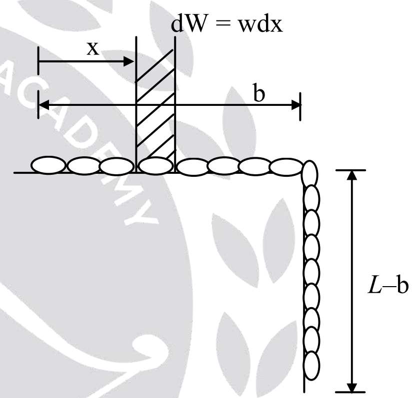
$$= \left[4\left(\frac{x^2}{2}\right) - 3\left(\frac{x^3}{3}\right)\right]_0^{1.7}$$

$$= \left[2x^2 - x^3\right]_0^{1.7}$$

$$= 2(1.7)^2 - (1.7)^3 = 0.867 \text{ J}$$

06. Ans: (c)

Sol:



Where w = weight per unit meter

dw = a small work done in moving small elemental "dx" of chain through a d/s "x"

Work done = change in KE

$$\left(\int_0^b dw \times x\right) + \left(w(L-b) \times b\right) = \frac{1}{2}\left(\frac{wL}{g}\right)v^2$$

$$\int_0^b wdx.x + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + wLb - wb^2 = \frac{1}{2}\frac{wLv^2}{g}$$

$$wLb - \frac{wb^2}{2} = \frac{1}{2}\frac{wLv^2}{g}$$

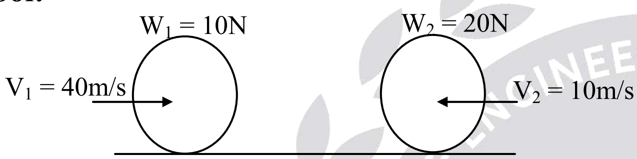
$$b\left(L-\frac{b}{2}\right)=\frac{1}{2}\frac{Lv^2}{g}$$

$$v^2 = 2gb\left(1-\frac{b}{2L}\right)$$

$$v = \sqrt{gb\left(2-\frac{b}{L}\right)}$$

07. Ans: (d)

Sol:



$m_1 = 1\text{kg}$, $m_2 = 2\text{kg}$,(since $g = 10\text{m/sec}^2$)

Velocities before impact

$v_1 = 40$ m/sec, $v_2 = -10$m/s

Velocities after impact

$u_1 = ?$ $u_2 = ?$

Coefficient of restitution e = 0.6

From momentum equation

$$m_1v_1+m_2 v_2 = m_1u_1+m_2u_2$$

$\Rightarrow 1(40) + 2(-10) = 1(u_1) + 2(u_2)$

$\Rightarrow u_1 + 2u_2 = 20$…………………..(1)

$$e = \frac{u_2-u_1}{v_1-v_2} = \frac{\text{relative velocity of Seperation}}{\text{relative velocity of approach}}$$

$$0.6 = \frac{u_2-u_1}{40-(-10)}$$

$\Rightarrow u_2 - u_1 = 30$………………………(2)

From 1 & 2

$u_1 = -13.33$ m/sec

$u_2 = 16.66$ m/sec

08. Ans: (b)

Sol: Given, $m_1 = 3$ kg, $m_2 = 6$ kg

Velocities before impact

$u_1 = 4$ m/s, $u_2 = -1$ m/s

Velocities after impact

$v_1 = 0$m/s , $v_2 = ?$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3(4) + 6(-1) = 3(0) + 6(v_2)$

$\Rightarrow 6 = 6 v_2$

$\Rightarrow v_2 = 1$ m/s

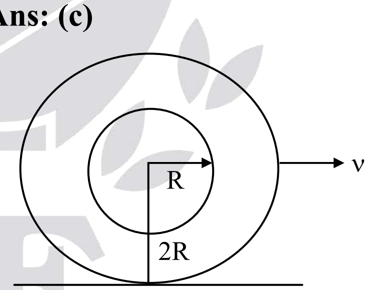
Coefficient of restitution,

$$e = \frac{v_2-v_1}{u_1-u_2}$$

$$e = \frac{1-0}{4-(-1)} = \frac{1}{5}$$

09. Ans: (c)

Sol:



$$KE = \frac{1}{2}mV^2+\frac{1}{2}I\omega^2$$

Where, $\omega = \dfrac{V}{2R}$

$$I = \frac{1}{2}m\left((2R)^2+R^2\right)= \frac{5}{2}mR^2$$

$$\therefore \quad KE = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2$$

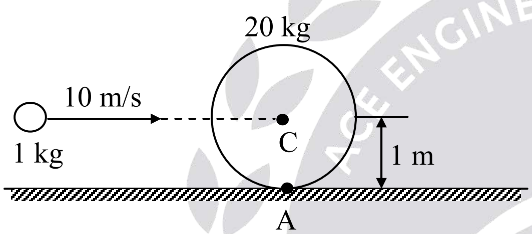
$$= \frac{1}{2}mV^2 + \frac{5}{4}mR^2 \times \frac{V^2}{4R^2}$$

$$= \frac{1}{2}mV^2 + \frac{5}{16}mV^2$$

$$KE = \frac{13mV^2}{16}$$

10. Ans: (a)

Sol:



***Method I* :**

By conservation of linear momentum,we get

$\Rightarrow \; 1\times10 = (20+1)\times V_{cm}$

(where, V_{cm} = velocity of centre of mass)

$\Rightarrow \; V_{cm} = \frac{10}{21} m/s$

Applying angular momentum conservation about an axis passing through the contact point (A) and perpendicular to the plane of paper, we get

$$1\times10\times1 = I_{cm}\omega + 21\times\frac{10}{21}\times1$$

[Angular momentum about any axis passing through A can be written as, $\vec{L}_A = \vec{L}_{cm} + m\left(\vec{r}\times\vec{V}_{cm}\right)$]

$\Rightarrow \omega = 0$ rad/sec

***Method II* :**

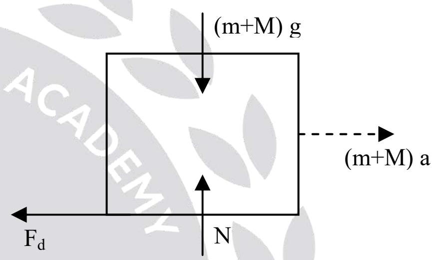
Applying angular momentum conservation about an axis passing through centre of wheel and perpendicular to the plane of paper.

$\therefore \; 0 = I_{cm}\,\omega$

$\Rightarrow \omega = 0$ rad/sec

11. Ans: (a)

Sol:



$m_1 = m \rightarrow$ mass of bullet

$m_2 = M \rightarrow$ mass of block

$u_1 = V \rightarrow$ bullet initial velocity

$u_2 = 0 \rightarrow$ block initial velocity

$v_1 = v_2 = v \rightarrow$ velocity of bullet and block after impact.

$F_d = \mu N$

$(M+m)a = \mu(M+m)g$

$\Rightarrow a = \mu g$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mV + m(0) = (m + M)V$

$v = \frac{mV}{m+M}$

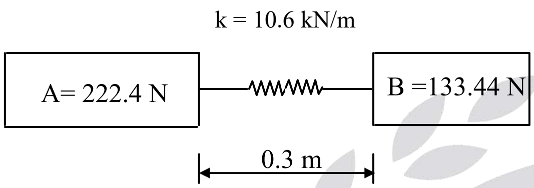
Now from $v^2 - u^2 = 2as$

$$0-\left(\frac{mV}{m+M}\right)^2 = 2\mu gs$$

$$V = \frac{m+M}{m}\sqrt{2\mu gs}$$

12. Ans: (a)

Sol:



$u_A = 0$, $u_B = 0$

From momentum equation

$m_Au_A+m_Bu_B = m_Av_A+m_Bv_B$

$0 = 222.4V_A+133.44V_B$ ………..(1)

$$\frac{1}{2}ks^2 = \frac{1}{2}m_A v_A{}^2 + \frac{1}{2}m_B v_B{}^2$$

$$10.6\times10^3\times0.15^2 = \frac{222.4}{9.81}v_A^2 + \frac{133.44}{9.81}v_B^2$$

………….(2)

From 1 & 2

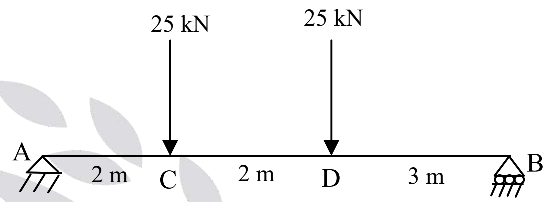
$v_A = -1.98$ m/s ,

$v_B = 3.3$ m/s

Chapter 8 Virtual Work

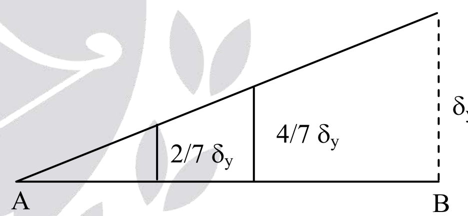
01.

Sol:



Let R_A & R_B be the reactions at support A & B respectively.

Let δ_y displacement be given to the beam at B without giving displacement at 'A'



The corresponding displacement at C & D are $\frac{2}{7}\delta_y$ and $\frac{4}{7}\delta_y$

By virtual work principle,

$$R_A\times0-25\times\frac{2}{7}\delta_y - 25\times\frac{4}{7}\delta_y + R_B\times\delta_y = 0$$

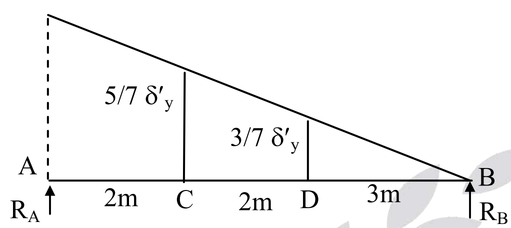
$$\Rightarrow\left(\frac{-150}{7}+R_B\right)\delta_y = 0$$

Since $\delta_y \neq 0$, $R_B-\frac{150}{7}=0$

$$R_B = \frac{150}{7}\text{ kN}$$

Now let us give virtual displacement at A as δ_y',

Therefore corresponding displacement at C & D are $\frac{5}{7}\delta_y'$ & $\frac{3}{7}\delta_y'$



∴ By virtual work principle,

$$R_A \times \delta'_y - 25\times\frac{5}{7}\delta'_y - 25\times\frac{3}{7}\delta'_y + R_B\times 0 = 0$$

$$\left(R_A - \frac{125}{7} - \frac{75}{7}\right)\delta_y' = 0$$

$\delta_y' \neq 0$,

$$R_A - \frac{200}{7} = 0$$

$$R_A = \frac{200}{7}\text{ kN}$$

02. Ans: 750 N

Ans: For equilibrium total virtual work = 0

Let us displace point A by 'dx' the displacement of point B is '3dx'

Work by force P = – Pdx

Work by force 250 N = 250 × 3 dx

250 × 3dx – Pdx = 0

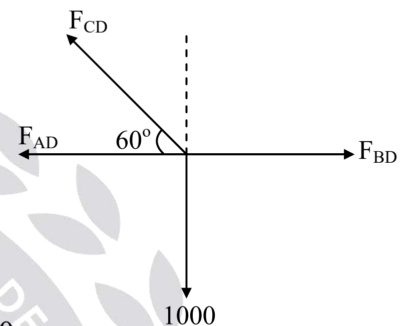
⇒ P = 750 N

Chapter 9

Analysis of Trusses

01. Ans: (b)

Sol: At joint



$\Sigma F_y = 0$

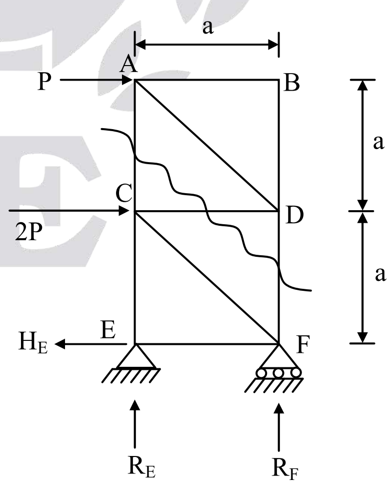
$F_{CD}\sin 60 = 1000$

$$F_{CD} = \frac{1000}{\sin 60}$$

$F_{CD} = 1154$ N

02. Ans: (d)

Sol:



$\Sigma F_x = 0$

$\Rightarrow H_E - P - 2P = 0$

$H_E = 3P$

$\sum F_y = 0$

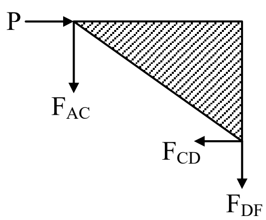
$R_E + R_F = 0$

$\sum M_F = 0$

$P \times 2a + 2P \times a + R_E \times a = 0$

$R_E = -4\ P$ (downward)

$R_F = 4P$ (upward)



$\sum F_x = 0$

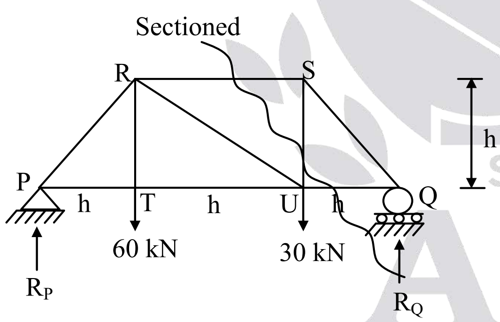
$P - F_{CD} = 0$

$P = F_{CD}$

(Positive indicate CD in tension)

03. Ans: (d)

Sol:

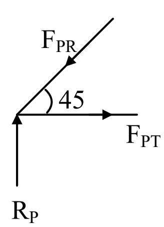


Taking moments about point 'P'

$R_Q \times 3h - 30 \times 2h - 60 \times h = 0$

$R_Q \times 3h = 120\ h$

$R_Q = 40$ kN



$\therefore R_P + R_Q = 60 + 30$

$R_P = 90 - 40$

$R_P = 50$ kN

At joint 'P'

$\sum F_y = 0$

$R_p = F_{PR} \sin 45^o$

$F_{PR} = \dfrac{R_p}{\sin 45}$

$= \dfrac{50}{1/\sqrt{2}}$

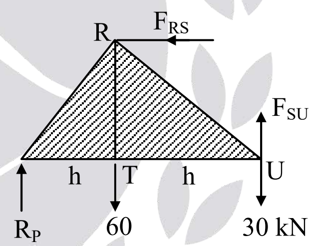
$F_{PR} = 50\sqrt{2}$ (compression)

$\sum F_x = 0$

$F_{PT} = F_{PR} \cos 45$

$F_{PT} = 50\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$F_{PT} = 50$ kN (Tension)



$\sum M_u = 0$

$F_{RS} \times h\ (\circlearrowleft) + 60 \times h\ (\circlearrowleft) - R_P \times 2h (\circlearrowright) = 0$

$F_{RS} \times h + 60\ h - 100\ h = 0$

$F_{RS}\ h = 40\ h$

$F_{RS} = 40$ kN (Compression)

$\sum F_y = 0$

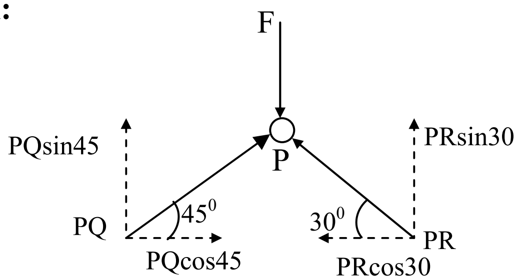
$F_{SU} + R_P - 60 = 0$

$F_{SU} + 50 - 60 - 30 = 0$

$F_{SU} = 40$ kN (Tension)

04. Ans: (b)

Sol:



Force in member PQ considering joint **P**

PQ cos45 = PR cos30

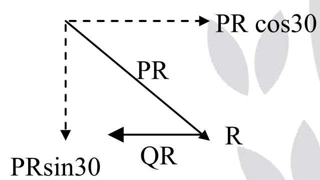
PQ = 1.224 PR

PQ sin45 + PR sin30 = F

$1.224PR \times 0.707 + 0.5PR = F$

PR = 0.732 F

Now, considering joint **R**



$$QR = PR\cos 30 = 0.732F \times \cos 30$$
$$= 0.63F \text{ (Tensile)}$$

05. Ans: (a)

Sol: $\sum F_y = 0 \Rightarrow R_A + R_B = P \times L$

$$\sum M_B = 0 \Rightarrow R_A \times 3L = PL \times \frac{3L}{2}$$

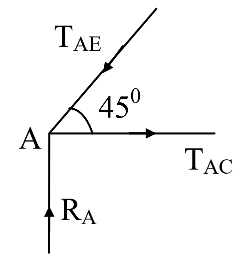
$$\Rightarrow R_A = \frac{PL}{2},\ R_B = \frac{PL}{2}$$

FBD at Point A:

$$\sum F_y = 0$$

$$\Rightarrow T_{AE}\sin 45 = R_A = \frac{PL}{2}$$

$$\Rightarrow T_A = \frac{PL}{\sqrt{2}}$$



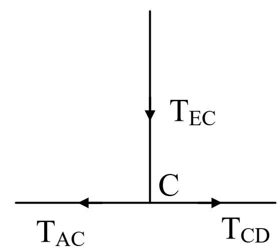
$$\sum F_x = 0 \Rightarrow T_{AC} = T_{AE}\cos 45 = \frac{PL}{2}$$

FBD at Point C:

$$\sum F_y = 0$$

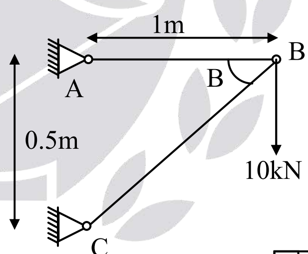
$$\Rightarrow T_{EC} = 0$$

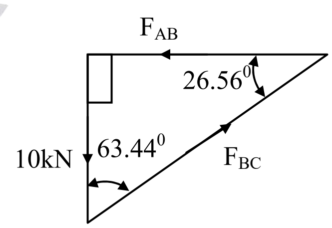
$$T_{AC} = T_{CD} = \frac{PL}{2}$$



06. Ans : 20 kN

Sol:





$$\tan\theta = \frac{0.5}{1.0} \Rightarrow \theta = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.56^0$$

From the Lami's triangle

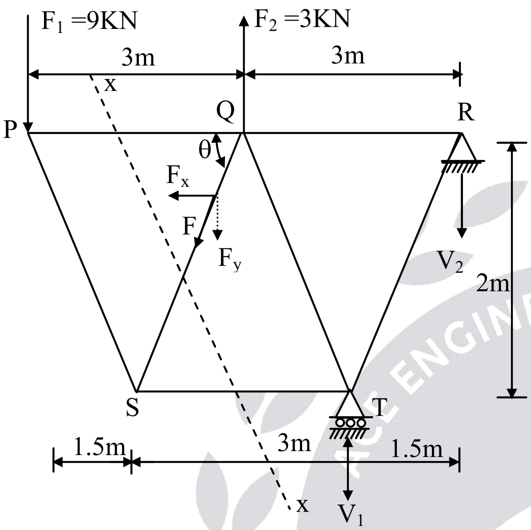
$$\frac{10}{\sin 26.56^0} = \frac{F_{BC}}{\sin 90^0} = \frac{F_{AB}}{\sin 63.44^0}$$

$$F_{AB} = \frac{10}{\sin 26.56} \times \sin 63.44 = 20\,kN$$

$$F_{BC} = \frac{10}{\sin 26.56} \times \sin 90 = 22.36\text{kN}$$

07. Ans: (a)

Sol:



$\sum F_y = 0$

$V_1 + V_2 - 9 + 3 = 0$

$\sum M_R = 0$

$\Rightarrow V_1 \times 1.5 + 3 \times 3 - 9 \times 6 = 0$

$\Rightarrow V_1 = 30$ kN ($\uparrow$)

$V_2 = -30 + 9 - 3 = -24$ kN ($\downarrow$)

Adopting method of sections–section x-x adopted and RHS taken

$$\theta = \tan^{-1}\left(\frac{2.0}{1.5}\right) = 53.13^0$$

$\Sigma F_y = 0$ (W.r.t. RHS of the section x-x)

$V_1 + F_2 - V_2 - F_y = 0$

$\Rightarrow F\sin 53.13 = 30 + 3 - 24$

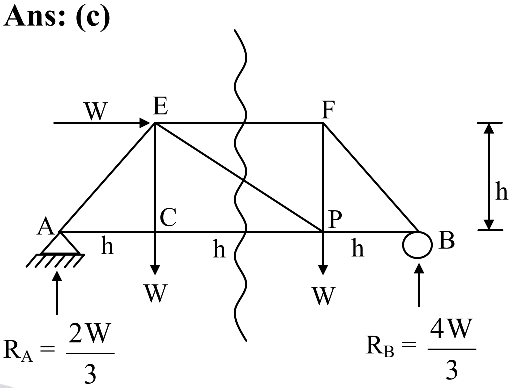
$F = 11.25$ kN (Tension)

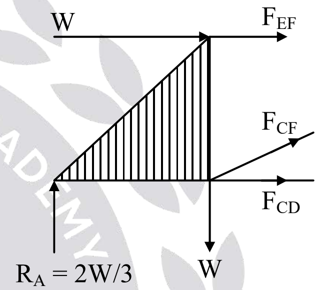
$\therefore$ Force in member

QS = 11.25 kN (Tension)

08. Ans: (c)

Sol:





$\sum M_B = 0$

$W \times h\,(\circlearrowright) - W \times h\,(\circlearrowleft) - W(2h)(\circlearrowleft) + R_A \times 3h(\circlearrowright) = 0$

$Wh - Wh - 2Wh + 3hR_A = 0$

$3hR_A = 2Wh$

$$R_A = \frac{2W}{3}$$

$\therefore R_A + R_B = 2W$

$$R_B = 2W - \frac{2W}{3} = \frac{4W}{3}$$

$\sum F_y = 0$ (at the joint C)

$F_{CF} \sin 45 - W + R_A = 0$

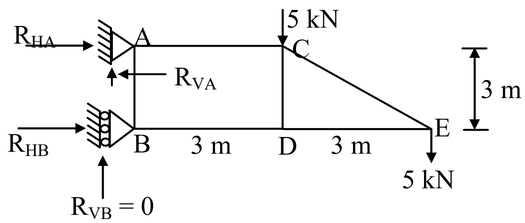
$$F_{CF} \sin 45 - W + \frac{2W}{3} = 0$$

$$F_{CF} \times \frac{1}{\sqrt{2}} = \frac{W}{3}$$

$$\Rightarrow F_{CF} = \frac{W\sqrt{2}}{3}$$

09. Ans: (c)

Sol:



$\sum M_A = 0$

$5\times3\,(\circlearrowright) + 5\times6\,(\circlearrowright) - R_{HB}\times 3 = 0$

$15 + 30 = R_H \times 3$

$R_{HB} = \dfrac{45}{3}$

$R_{HB} = 15$ kN

$\sum F_X = 0$

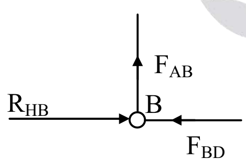
$\therefore R_{HA} + R_{HB} = 0$

$R_{HA} = -R_{HB}$

$R_{HA} = -15$ kN

(Negative indicate R_{HA} is left side)

At joint 'B'



$\sum F_x = 0$

$F_{BD} = 15$ kN

$\sum F_y = 0$

$F_{AB} = 0$

Chapter 10

Power Screw, Belt drive, Lagrange's equation

Power Screw

01. Ans: (b)

Sol: The best threads for a power screw are ball bearing threads.

02. Ans: (b)

Sol: ACME threads :

- Acme threads can be used to take a load in both directions.
- They are strong, smooth have less wear, and easy to manufacture compared to square thread.

Square thread :

- In this thread, the flanks are perpendicular to the axis of the thread.
- This is used for transmitting motion or power, E.g. Fly presses, Screw jack, vice handles, cross-slide and compound slide, etc.
- The efficiency of a square thread is more than that of trapezoidal threads.
- It can transmit load and power in both directions but It is difficult to manufacture.
- There is no radial or side thrust on the nut.
- The wear on the thread is also a serious problem.

03. Ans: (b)

Sol: Screw is subjected to torque, axial compressive load and bending moment also, sometimes.

Screws are generally made of C30 or C40 steel. As the failure of power screws may lead to serious accident, higher factor of safety of 3 to 5 is taken. Threads may fail due to shear, which can be avoided by using nut of sufficient height. Wear is another possible mode of thread failure as the threads of nut and bolt rub against each other. Nuts are made of softer material than screws so that if at all the failure takes place, nut fails and not the screw, which is the costlier member and is also difficult to replace. Plastic, bronze or copper alloys are used for manufacturing nuts. Plastic is used for low load applications and has good friction and wear properties. Bronze and copper alloys are used for high load applications.

Therefore it is essential to design a power screw for maximum shear stress.

04. Ans: (b)

Sol: Under direct compressive stress,

$$d_c = \sqrt{\frac{4W}{\pi\sigma_c}}$$

Under wear consideration, $d_c = \sqrt{\frac{2W}{P_b\,\psi\,\pi}}$

05. Ans: (c)

Sol: **Self locking screw:**

If friction angle, $\phi \geq$ helix angle, α

Screw is self locking.

i.e., torque required to lower the load is positive.

If $\phi < \alpha$, The screw is over hauling

i.e., torque required to lower the load is negative

For self locking screw,

$$\tan\phi > \tan\alpha$$

$$\mu > \frac{L}{\pi d_m}$$

06. Ans: (c)

Sol:

- A multi-start thread may be used to get larger value of linear displacement per revolution
- A differential screw may be used to get a very small value of linear displacement per revolution.

07. Ans: (c)

Sol:

- A multi-start thread may be used to get larger value of linear displacement per revolution with no guarantee of self locking.
- Multi-start threads are used for transmitting power and generating movement. Because each partial or complete revolution equals more linear travel based on the number of threads, multi-threaded components can

efficiently handle more power. Multi-start threads can also be used for some fastening purposes.

08. Ans: (b)

Sol:

- Multi-start threads are used for transmitting power and generating movement. Because each partial or complete revolution equals more linear travel based on the number of threads, multi-threaded components can efficiently handle more power. Multi-start threads can also be used for some fastening purposes.
- Hence they secure high efficiency.

09. Ans: (d)

Sol: Square thread,

$$\eta = \frac{\text{work output}}{\text{work input}}$$

During one revolution of screw

Work input $= P \times \pi d_m$

Work output = WL

$$\eta = \frac{WL}{P\pi d_m} = \frac{W\tan\alpha}{P} \quad \left[\because \tan\alpha = \frac{L}{\pi d_m}\right]$$

$$\eta = \frac{\text{ideal effort (No friction)}}{\text{Actual effort}}$$

$$\eta = \frac{W\tan\alpha}{W\tan(\phi+\alpha)} = \frac{\tan\alpha}{\tan(\phi+\alpha)}$$

Belt Drives & Wedge

01. Ans: (d)

Sol: $P = \frac{(T_1 - T_2)V}{1000}$ - Flat belt

V= belt (or) rope drive

T_1 & T_2 = Tensions in high and slack side,

V = m/sec, P = kW

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

02. Ans: (c)

Sol: Condition for maximum power transmitted

(i) $T_c = \frac{T_{max}}{3}$

(ii) $T_1 = \frac{2T_{max}}{3}$

(iii) $V = \sqrt{\frac{T_{max}}{3}}$

03. Ans: (b)

Sol: All the stresses produced in a belt are tensile stresses.

04. Ans: (c)

Sol: Power = $(T_1 - T_2)$ V

Due to centrifugal tension,

Total Tension (safe tension):

Total tension on tight side, $T_{t1} = T_1 + T_C$

Total tension on slack side, $T_{t2} = T_2 + T_C$

$T_{t1} - T_{t2} = T_1 - T_2$

Therefore the centrifugal tension has no effect on power transmission.

05. Ans: (a)

Sol: A V-belt marked A-914-50 denotes a standard belt of inside length 914 mm and a pitch length 950 mm.

A belt marked A-914-52 denotes an oversize belt by an amount of (52 – 50) = 2 units of grade number.

06. Ans: (a)

Sol:

- Wire ropes make contact at the bottom of the groove of the pulley.
- V-Belt makes contact at the sides of the groove of the pulley.

07. Ans: (c)

Sol: Let, D = diameter of the pitch circle

T = number of teeth on the sprocket

$$p = D\sin\left(\frac{\theta}{2}\right)$$

We know that, $\theta = \frac{360^\circ}{T}$

$$p = D\sin\left(\frac{360^\circ}{2T}\right) = D\sin\left(\frac{180^\circ}{T}\right)$$

or $D = p\ \text{cosec}\left(\frac{180^\circ}{T}\right)$

08. Ans: (c)

Sol: $\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}}$

09. Ans: (c)

Sol: Maximum tensile stress in belt due to tension, $\sigma = \frac{T_1}{b\,t}$

Due to bending maximum tensile stress occurs on small pulley side 'd'

$$\frac{\sigma_b}{y} = \frac{E}{r} \qquad \left[r = \frac{d}{2}\ ;\ y = \frac{t}{2}\right]$$

$$\sigma_b = \frac{E\,t}{d}$$

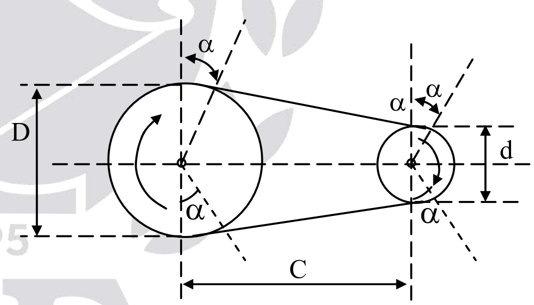
Total maximum stress induced in belt,

$$\sigma_{max} = \sigma + \sigma_b$$

$$\sigma_{max} = \frac{T_1}{bt} + \frac{Et}{d}$$

10. Ans: (a)

Sol:



$$\theta = 180 - 2\alpha$$

$$\sin\alpha = \frac{D_1 - D_2}{2C}$$

$$L_{open} = \pi(R + r) + 2C + \left[\frac{(R - r)^2}{C}\right]$$

$$L_{closed} = \pi(R + r) + 2C + \left[\frac{(R + r)^2}{C}\right]$$

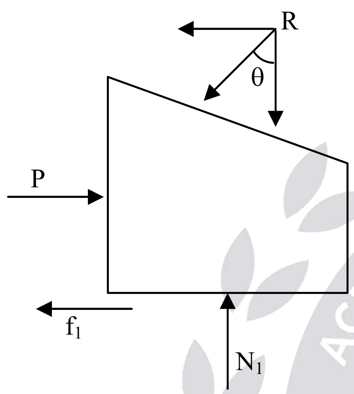
$$L = 2C + \left(\frac{\pi}{2}\right)(D + d) + \left((D - d)^2/4C\right)$$

11. Ans: (d)

Sol: Creep is given by, $\varepsilon = \dfrac{T_1 + T_2}{btE}$

12. Ans: (493.4)

Sol: Given data: g = 9.81 m/s^2, μ = 0.3

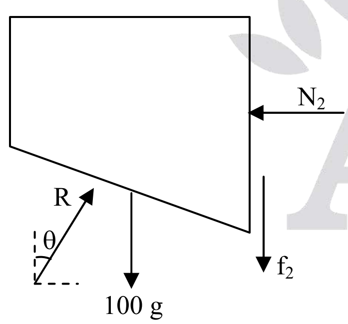


$f_1 = \mu N_1$; $N_1 = R\cos\theta$

$P = f_1 + R\sin\theta$

$P = \mu N_1 + R\sin\theta$

$P = R(\mu\cos\theta + \sin\theta)$ ------- (i)



$f_2 = \mu N_2$; $N_2 = R\sin\theta$

$R\cos\theta = f_2 + 100\,g$

$100\,g = -\mu N_2 + R\cos\theta$

$100\,g = R(\cos\theta - \mu\sin\theta)$ ------- (ii)

Dividing (1) and (2),

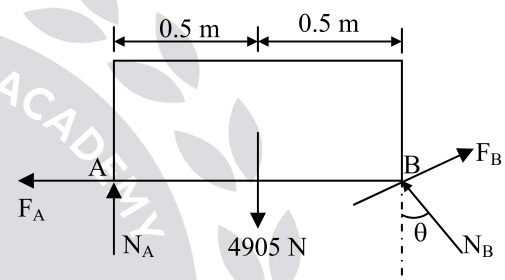
$$\frac{P}{100g} = \frac{R(\mu\cos\theta + \sin\theta)}{R(\cos\theta - \mu\sin\theta)}$$

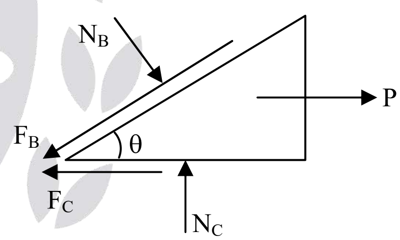
$$P = \frac{\mu\cos(10) + \sin(10)}{\cos(10) - 0.3\times\sin(10)} \times 100(g)$$

$\Rightarrow P = 493.4$ N

13. Ans: (a)

Sol: From free-body diagram of the stone,





$\Sigma M_A = 0$,

$-\ 4905\ (0.5) + (N_B\ \cos 7°)\ (1) + (0.3\ N_B)\ \sin 7°\ (1) = 0$

$N_B = 2383.1$ N

Using this result for the wedge, we have

$\Sigma F_y = 0$;

$N_C - 2383.1\cos 7° - 0.3\ (2383.1)\sin 7° = 0$

$N_C = 2452.5$ N

$\Sigma F_x = 0$;

$2383.1\ \sin 7° - 0.3(2383.1)\ \cos 7° + P - 0.3(2452.5) = 0$

$P = 1154.9$ N = 1.15 kN

LAGRANGE'S EQUATION

01. Ans: (c)

Sol:

(i) PE of spring $= \frac{1}{2}k.x^2$

(ii) K.E of block $= \frac{1}{2} \times M \times x^2$

(iii) KE of rod:

Mass of element, $dm = m \times \frac{dy}{b}$

$$KE = \frac{1}{2}dm \times v^2 = \frac{1}{2} \times dm \times \{v_x^2 + v_y^2\}$$

$$v_x = \dot{x}_1 = \frac{d}{dt}\{x_1\} = \frac{d}{dt}\{x + y\sin\theta\}$$

$$\therefore\ v_x = \dot{x} + y \times \cos\theta \times \dot{\theta}$$

$$v_y = \frac{d}{dt}(y_1) = \frac{d}{dt}(y\cos\theta) = -y\sin\theta \times \dot{\theta}$$

$$\therefore\ \text{KE of rod} = \int \frac{1}{2}.dm \times v^2$$

$$= \int \frac{1}{2} \times m \times \frac{dy}{b} \times \left\{(\dot{x} + y\cos\theta \times \dot{\theta})^2 + (-y\sin\theta \times \dot{\theta})^2\right\}$$

$$= \frac{m}{2b}\int_0^b \{\dot{x}^2 + y^2 \times \dot{\theta}^2 + 2 \times \dot{x} \times y \times \dot{\theta} \times \cos\theta\}\, dy$$

$$= \frac{m}{2b}\int_0^b \left\{\dot{x}^2 \times b + \dot{\theta}^2 \times \frac{b^3}{3} + \dot{x} \times \dot{\theta} \times \cos\theta \times b^2\right\}$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{6}m.\dot{\theta}^2 \times b^2 + \frac{1}{2} \times m.\dot{x}.\dot{\theta} \times \cos\theta \times b$$

$$\text{P.E of rod} = -\frac{1}{2} \times m \times b \times \cos\theta \times g$$

$\therefore$ PE = PE of spring + PE of rod

$$PE = \frac{1}{2}k.x^2 + \left\{-\frac{1}{2} \times m \times g \times b \times \cos\theta\right\}$$

KE = KE of block + KE of rod

$$KE = \frac{1}{2} \times M \times \dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{6}m \times \dot{\theta}^2 \times b^2$$

$$+ \frac{1}{2} \times m \times \dot{x} \times \dot{\theta} \times \cos\theta \times b$$

$\therefore$ Lagrange, L = KE – PE

$$= \frac{1}{2}\{m + M\} \times \dot{x}^2 + \frac{1}{6}m \times \dot{\theta}^2 \times b^2 + \frac{1}{2} \times m \times \dot{x}$$

$$\dot{\theta} \times \cos\theta \times b - \frac{1}{2}k.x^2 + \frac{1}{2} \times m \times b \times \cos\theta \times g$$