ACE

GATE | PSUs

INSTRUMENTATION ENGINEERING

Electricity and Magnetism

(Text Book: **Theory with worked out Examples and Practice Questions**)

Chapter 1 Static Fields & Maxwell's Equations

(Solutions for Text Book Practice Questions)

01. Ans: 1

Sol: $\vec{V} = x\cos^2 y\hat{i} + x^2e^z\hat{j} + z\sin^2 y\hat{k}$

$$= x\cos^2 y\hat{a}_x + x^2e^z\hat{a}_y + z\sin^2 y\hat{a}_z$$

From divergence theorem

$$\oiint \overline{V}.\hat{n}\,ds = \int_v \left(\nabla.\overline{D}\right)dv \;\;.................1$$

$$\nabla.\overline{D} = \frac{\partial}{\partial x}\left(x\cos^2 y\right) + \frac{\partial}{\partial y}\left(x^2e^z\right) + \frac{\partial}{\partial z}\left(z\sin^2 y\right)$$

$$= \cos^2 y + \sin^2 y = 1$$

dv = dxdydz

Putting these value in equation 1 we have

$$\oiint \overline{V}.\hat{n}\,ds = \int_0^1\int_0^1\int_0^1 1\times dx\,dy\,dz$$

$$= \int_0^1 dx\int_0^1 dy\int_0^1 dz = 1$$

02. Ans: (c)

Sol: Given $\vec{A} = xy\,\vec{a}_x + x^2\,\vec{a}_y$

Let $I = \oint \vec{A}.d\vec{\ell}$, I is evaluated over the path shown in the Fig., as follows

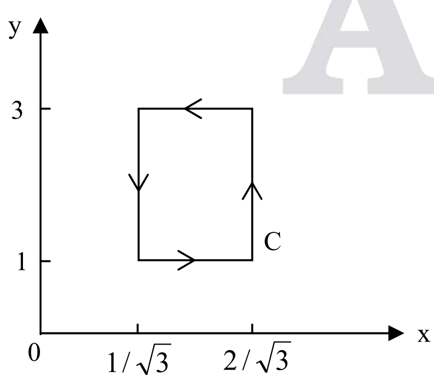


Fig.

$$I = \oint \vec{A}.dx\,\vec{a}_x,\; y = 1,\; x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$+\int \vec{A}.dy\,\vec{a}_y,\;\; x = \frac{2}{\sqrt{3}},\; y = \text{from 1 to 3}$$

$$-\int \vec{A}.dx\,\vec{a}_x,\; y = 3, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$-\int \vec{A}.dy\,\vec{a}_y,\; x = 1/\sqrt{3},\; y = \text{from 1 to 3}$$

$$= \int xy\,dx + \int x^2\,dy - \int xy\,dx - \int x^2\,dy$$

$$= y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2\,y\Big|_1^3 - y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2\,y\Big|_1^3$$

at $y = 1 \quad x = 2/\sqrt{3} \quad\; y = 3 \quad x = 1/\sqrt{3}$

$$= \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) + \frac{4}{3}(3-1) - \frac{3}{2}\left(\frac{4}{3} - \frac{1}{3}\right) - \frac{1}{3}(3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\overline{F} = \rho a_\rho + \rho\sin^2\phi\, a_\phi - za_z$

$$= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$$

$$\nabla.\overline{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho F_\rho\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(F_\phi\right) + \frac{\partial}{\partial z}\left(F_Z\right)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho^2\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\rho\sin^2\phi\right) + \frac{\partial}{\partial z}\left(-z\right)$$

$$= 2 + 2\sin\phi\cos\phi - 1$$

$$= 1 + 2\sin\phi\cos\phi$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2,\;\; \nabla.F\big|_{\phi=0} = 1$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2\nabla.F\big|_{\phi=0}$$

04. Ans: (c)

Sol: $\overline{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_Z \qquad \overline{D} = \left|\overline{D}\right|\overline{a}_n$

$\left|\overline{D}\right| = \sqrt{16} = 4 \qquad\qquad = \rho_s\hat{a}_n$

$$\therefore \overline{D} = 4\left\{\frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_Z}{4}\right\}$$

$$= \rho_s \hat{a}_n \qquad \therefore \rho_s = 4\, C/m^2$$

05. Ans: (d)

Sol: $V = 10y^4 + 20x^3$

$$E = -\nabla V = -60x^2\hat{a}_x - 40y^3\hat{a}_y$$

$$D = \varepsilon_0 E = -60x^2\varepsilon_0\hat{a}_x - 40y^3\varepsilon_0\hat{a}_y$$

$$\nabla . D = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x}(-60x^2\varepsilon_0) + \frac{\partial}{\partial y}(-40y^3\varepsilon_0)$$

$$= -120\, x\varepsilon_0 - 120\, y^2\varepsilon_0$$

$$\rho_v(\text{at } 2, 0) = -120\times 2\varepsilon_0 - 120\times 0^2\,\varepsilon_0$$

$$= -240\,\varepsilon_0$$

06. Ans: (d)

Sol: Given

$$V(x, y, z) = 50\,x^2 + 50\,y^2 + 50\,z^2$$

$$\vec{E}\,(x, y, z) \text{ in free space} = -\text{grad }(V)$$

$$= -\nabla V$$

$$= -\left[\frac{\partial}{\partial x} V\overrightarrow{a_x} + \frac{\partial}{\partial y} V\overrightarrow{a_y} + \frac{\partial}{\partial z} V\overrightarrow{a_z}\right]$$

$$= -\left[100x\,\overrightarrow{a_x} + 100y\,\overrightarrow{a_y} + 100z\,\overrightarrow{a_z}\right] V/m$$

$$\vec{E}\,(1, -1, 1) =$$

$$-\left[100\,\overrightarrow{a_x} - 100\,\overrightarrow{a_y} + 100\,\overrightarrow{a_z}\right] V/m$$

$$E(1,-1,1) = 100\sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of $\vec{E}$.

$$\vec{a}_E = \frac{\vec{E}(1, -1, 1)}{|E(1, -1, 1)|} = \frac{1}{\sqrt{3}}\left[-\overrightarrow{a_x} + \overrightarrow{a_y} - \overrightarrow{a_z}\right]$$

or in i, j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}}\left[-i + j - k\right]$

07. Ans: (b)

Sol: For valid B, $\nabla . B = 0$

$$\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)\left(x^2a_x - xya_y - Kxza_z\right) = 0$$

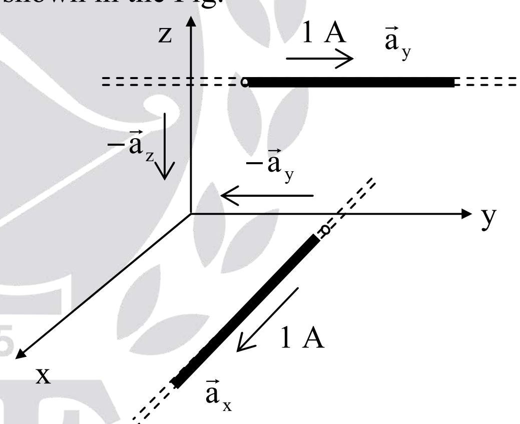
$$2x - x - Kx = 0$$

$$\Rightarrow 2 - 1 - K = 0$$

$$\therefore K = 1$$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the $\vec{a}_y$ direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the $\vec{a}_x$ direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors along the 'x', 'y' and 'z' axes respectively.

$\therefore$ x and z components of magnetic field are non-zero at the origin.

09. Ans: (a)

Sol: $\nabla.\overline{B} = 0$

A divergence less vector may be a curl of some other vector

$\overline{B} = \nabla \times \overline{A}$

$\nabla \times \overline{A} = \overline{B}$

$\oint_l \overline{A}.\overline{dl} = \int_s \overline{B}.\overline{ds}$

$\int_s \overline{B}.\overline{ds}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

Sol: In general, for an infinite sheet of current density K A/m

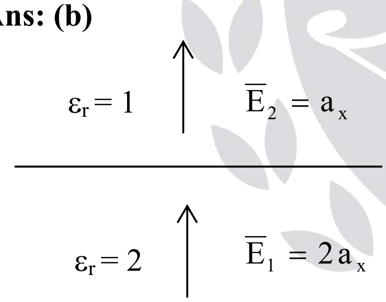
$H = \frac{1}{2} K \times a_n$

$H = \frac{1}{2}(8\bar{a}_x \times \bar{a}_z)$

$= -4\ \bar{a}_y\ (\because \bar{a}_x \times \bar{a}_z = -\bar{a}_y)$

11. Ans: (b)

Sol:



$D_{n_2} - D_{n_1} = \rho_S \rightarrow$ (a)

$D_{n_2} = \in E_{n_2} = \in_0 a_x$

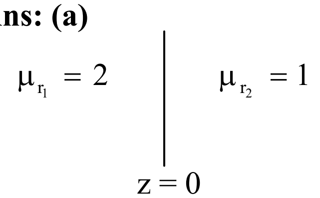
$D_{n_1} = \in_0 2 \times 2a_x = 4\in_0 a_x$

From (a)

$(\in_0 - 4\in_0)\ a_x = \rho_s \Rightarrow \rho_s = -3\in_0$

12. Ans: (a)

Sol:



$B_1 = 1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\vec{a}_z$

$B_{n_1} = 0.4\ \bar{a}_z$

(Since z = 0 has normal component a_x)

$B_{t_1} = 1.2\ \bar{a}_x + 0.8\ \bar{a}_y$

We know magnetic flux density is continuous

$B_{n_1} = B_{n_2}$

$B_{n_2} = 0.4\ \bar{a}_z$

Surface charge, $\overline{k} = 0$

$H_{t_2} - H_{t_1} = 0$

$H_{t_2} = H_{t_1}$

$\mu_1 B_{t_2} = \mu_2 B_{t_1}$

$B_{t_2} = \frac{1}{2}(1.2a_x + 0.8a_y)$

$B_2 = B_{t_2} + B_{n_2} = 0.6\ \bar{a}_x + 0.\bar{4}a_y + 0.4\bar{a}_z$

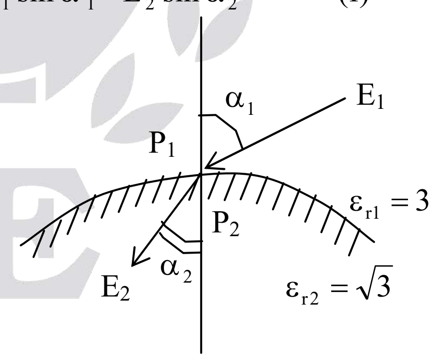
$\mu_0 \mu_{r_2} H_2 = 0.6\ \bar{a}_x + 0.\bar{4}a_y + 0.4\bar{a}_z$

$H_2 = \frac{1}{\mu_0}[0.6\ \bar{a}_x + 0.\bar{4}a_y + 0.4\bar{a}_z]$ A/m

13. Ans: (b)

Sol: Tangential components of electric fields are continuous $(E_{t_1} = E_{t_2})$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$ ----(1)



Normal component of electric flux densities are continuous across a charge free interface

$D_{n_1} = D_{n_2}$

$3E_1 \cos\alpha_1 = \sqrt{3}E_2 \cos\alpha_2$ ----(2)

$\alpha_1 = 60^0$

$\frac{(1)}{(2)} \Rightarrow \frac{\tan\alpha_1}{3} = \frac{\tan\alpha_2}{\sqrt{3}} \Rightarrow \tan\ \alpha_2 = 1$

$\alpha_2 = 45^0$

14. Ans: (c)

Sol: $N = 100$

$\phi = t^3 - 2t$ mWb

According to Faraday's law

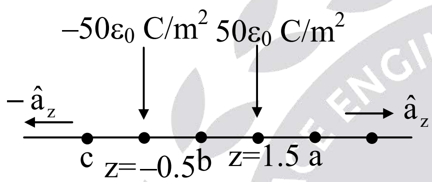
$$E = N\left.\frac{d\phi}{dt}\right|_{t=4\,\text{sec}}$$

$= 100 \times (3t^2 - 2)$ mV $= 4.6$ V

15. Ans: (a, b & d)

Sol:

(a)



Consider a point located at a for which $z > 1.5$ and $z > -0.5$ as shown in figure. At this point $\hat{a}_n = \hat{a}_z$ for both the surface charge densities. Hence

$$\overline{E}(z) = \frac{50\varepsilon_0}{2\varepsilon_0}\hat{a}_z + \left(\frac{-50\varepsilon_0}{2\varepsilon_0}\right)\hat{a}_z = 0\,V/m$$

(b) Consider a point located at b for which $1.5 > z > -0.5$. For surface charge density of $50\varepsilon_0\ C/m^2$, $\hat{a}_N = -\hat{a}_z$ where as for $-50\varepsilon_0 C/m^2$, $\hat{a}_n = \hat{a}_z$

$$\therefore\ \overline{E}(z) = \frac{50\varepsilon_0}{2\varepsilon_0}(-\hat{a}_z) + \left(\frac{-50\varepsilon_0}{2\varepsilon_0}\right)(\hat{a}_z) = -50\hat{a}_z V/m$$

(c) Consider a point located at c for which $z < 1.5$ and $z < -0.5$. At this point $\hat{a}_n = -\hat{a}_z$.

$$\therefore\ \overline{E}(z) = \frac{50\varepsilon_0}{2\varepsilon_0}(-\hat{a}_z) + \left(\frac{-50\varepsilon_0}{2\varepsilon_0}\right)(\hat{a}_z) = 0\,V/m$$

Hence we have $\overline{E} = \begin{cases} 0 & z > 1.5 \\ -50\hat{a}_z & 1.5 > z > -0.5 \\ 0 & z < -0.5 \end{cases}$

(d) In the region $-0.5 < z < 1.5$ $(1.5 > z > -0.5)$ $\overline{E} = -50\hat{a}_z$ and $A = (x, y, z)$ and $B = (x, y, -0.5)$.

$$V_{AB} = -\int_B^A \overline{E}.\overline{dL}$$

$$= -\int_B^A (-50\hat{a}_z).(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= 50\int_{-0.5}^{z} dz = 50[z]_{-0.5}^{z} = 50(z + 0.5)$$

$$= 50z + 25$$