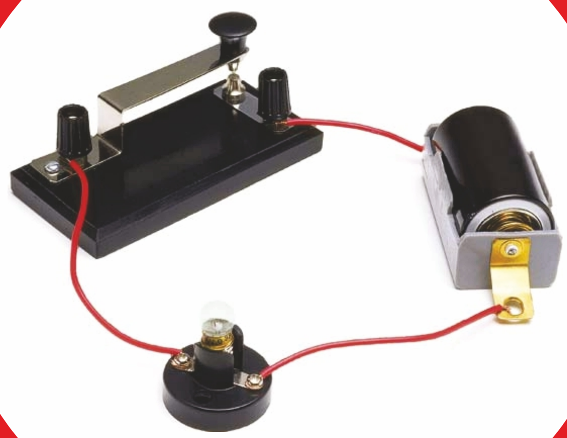
ACE

GATE | PSUs

ELECTRICAL ENGINEERING

Electric Circuits

Text Book: Theory with worked out Examples and Practice Questions



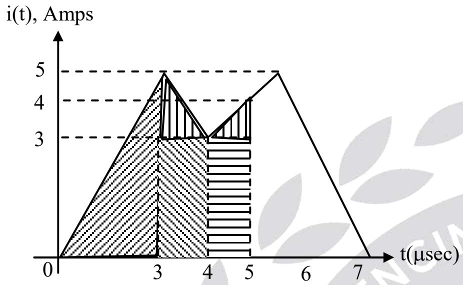
1. Basic Concepts

01. Ans: (c)

Sol: We know that;

$$i(t) = \frac{dq(t)}{dt}$$

$$dq(t) = i(t).dt$$



$$q = \int_0^{5\mu\sec} i(t)dt = \text{Area under i(t) upto 5 μsec}$$

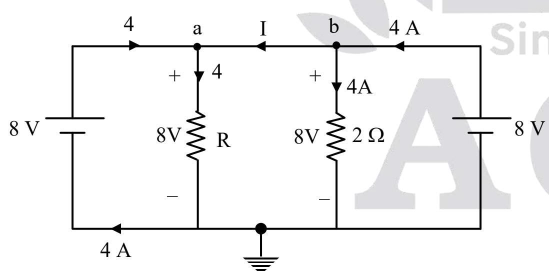
$$q = q_1| + q_2| + q_3|$$

$$= \left(\frac{1}{2}\times3\times5\right) + \left(\frac{1}{2}\times1\times2+(1\times3)\right) + \left(\frac{1}{2}\times1\times1+(1\times3)\right)$$

$$q = 15\mu C$$

02. Ans: (a)

Sol:



Applying KCL at node 'b'

$$I + 4 = 4$$

$$\Rightarrow \quad I = 0A$$

And $\quad \frac{8}{R} = 4$

$$\Rightarrow \quad R = 2\Omega$$

03. Ans: (a)

Sol: The energy stored by the inductor (1Ω, 2H) upto first 6 sec:

$$E_{\text{stored upto 6 sec}} = \int_0^6 P_L dt = \int_0^6 v_L(t) i_L(t) dt$$

$$= \int \left(L \frac{di(t)}{dt}.i(t) \right) dt$$

$$= \int_0^2 \left(2\left[\frac{d}{dt}(3t)\right]\times 3t \right) dt + \int_2^4 \left(2\left[\frac{d}{dt}(6)\right]\times 6 \right) dt$$

$$+ \int_4^6 \left(2\left[\frac{d}{dt}(-3t+18)\right]\times(-3t+18) \right) dt$$

$$= \int_0^2 18t\,dt + \int_2^4 0\,dt + \int_4^6 \left(-6[-3t+18]\right) dt$$

$$= 36 + 0 - 36 = 0\text{ J}$$

(or)

$$E_{\text{stored upto 6sec}} = E_L|_{t=6\sec}$$

$$= \frac{1}{2} L \left(i(t)|_{t=6}\right)^2$$

$$= \frac{1}{2}\times 2\times 0^2 = 0\text{ J}$$

04. Ans: (d)

Sol: The energy absorbed by the inductor (1Ω, 2H) upto first 6sec:

$$E_{\text{absorbed}} = E_{\text{dissipated}} + E_{\text{stored}}$$

Energy is dissipated in the resistor

$$E_{\text{dissipated}} = \int P_R dt = \int (i(t))^2 R\,dt$$

$$= \int_0^2 (3t)^2 \times 1dt + \int_2^4 (6)^2 \times 1dt + \int_4^6 (-3t+18)^2 \times 1dt$$

$$= \int_0^2 9t^2\,dt + \int_2^4 36dt + \int_4^6 (9t^2 + 324 - 108t)dt$$

$$= 24 + 72 + 24$$

$$= 120\text{J}$$

$$\therefore E_{\text{dissipated}} = 120\text{ J}$$

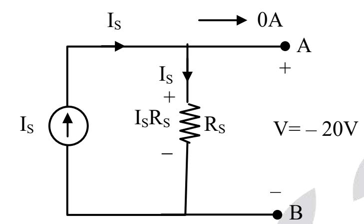
And $E_{\text{stored upto 6 sec}} = 0\,J$

$\therefore\ E_{absorbed} = E_{dissipated} + E_{stored}$

$\Rightarrow E_{absorbed} = 120J + 0J = 120J$

05. Ans: (a)

Sol: Point (–20, 0) $\Rightarrow$ V = – 20V and I = 0A

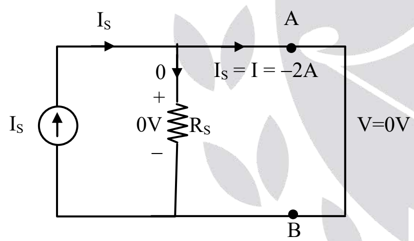


By KVL $\Rightarrow I_S R_S - V = 0$

$\Rightarrow I_S R_S + 20 = 0$

$\Rightarrow I_S R_S = -20V$ ……….. (1)

Point: (0, –2) $\Rightarrow$ V = 0V and I = –2A

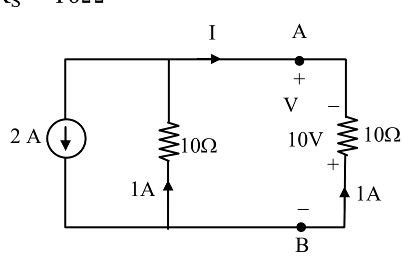


$I_s = I$

$\Rightarrow I_s = -2A$

Substituting I_s in eq. (1)

$R_S = 10\Omega$

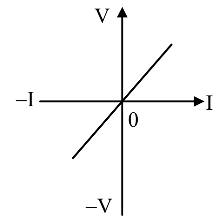


From the diagram;

I = –1A and V = –10V

06. Ans: (a)

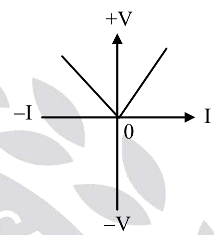
Sol:



* linear
  * Passive
  * bilateral

07. Ans: (b)

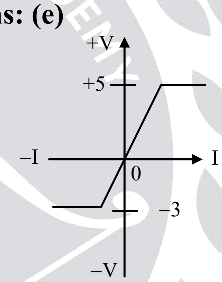
Sol:



* Non linear
* Active
* Unilateral

08. Ans: (e)

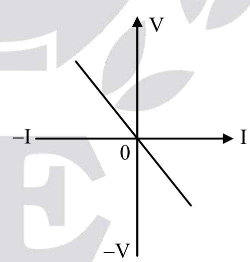
Sol:



* Non linear
* Passive
* Unilateral

09. Ans: (c)

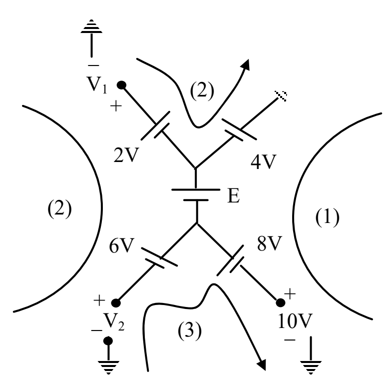
Sol:



* Linear
* Active
* Bilateral

10.

Sol:



(1) By KVL $\Rightarrow + 10 + 8 + E + 4 = 0$

$$E = -22V$$

(2) By KVL $\Rightarrow + V_1 - 2 + 4 = 0$

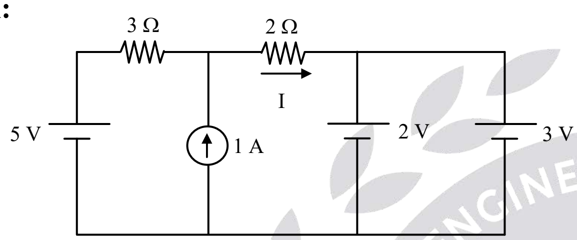
$$V_1 = -2V$$

(3) By KVL $\Rightarrow + V_2 + 6 - 8 - 10 = 0$

$$V_2 = 12V$$

11. Ans: (d)

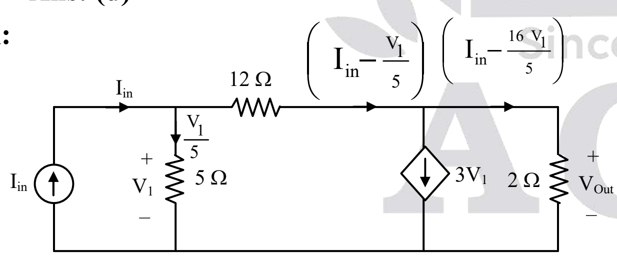
Sol:



Here the 2V voltage source and 3V voltage source are in parallel which violates the KVL. Hence such circuit does not exist. (But practical voltage sources will have some internal resistance so that when two unequal voltage sources are connected in parallel current can flow and such a circuit may exist).

12. Ans: (d)

Sol:



Applying KVL,

$$-V_1 + 12\left(I_{in} - \frac{V_1}{5}\right) + 2\left(I_{in} - \frac{16V_1}{5}\right) = 0$$

$$-V_1 + 12I_{in} - \frac{12V_1}{5} + 2I_{in} - \frac{32V_1}{5} = 0$$

$$14I_{in} = \frac{49}{5}V_1$$

$$\Rightarrow V_1 = \frac{70}{49}I_{in} \text{ …….. (1)}$$

$$\therefore V_{out} = 2\left(I_{in} - \frac{16V_1}{5}\right) \text{ ……. (2)}$$

Substitute equation (1) in equation (2)

$$V_{out} = 2\left(I_{in} - \frac{16}{5} \times \frac{70}{49}I_{in}\right)$$

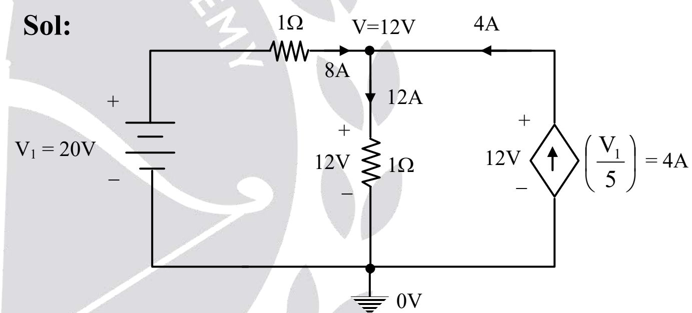
$$= 2\left(\frac{-25}{7}\right)I_{in}$$

$$= \frac{-50}{7}I_{in}$$

$$\therefore V_{out} = -7.143\, I_{in}$$

13. Ans: (c)

Sol:



By nodal $\Rightarrow$

$V - 20 + V - 4 = 0$

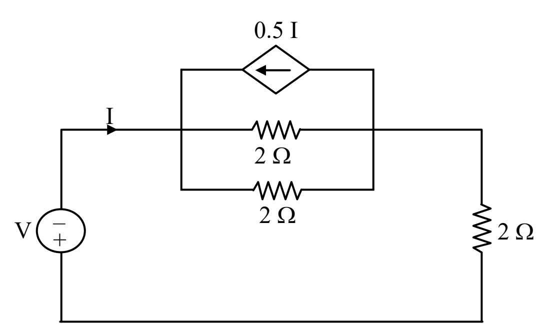
$$V = 12 \text{ volts}$$

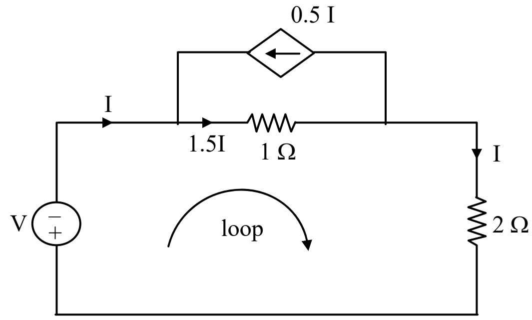
Power delivered by the dependent source is

$P_{del} = (12 \times 4) = 48$ watts

14. Ans: (d)

Sol:





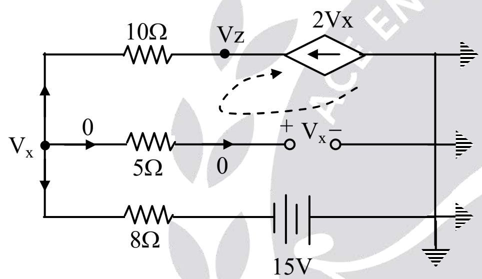
Applying KVL,

$\Rightarrow V + 1.5I + 2I = 0$

$\Rightarrow V = -3.5\ I$

15. Ans: (c)

Sol:



By using Nodal Analysis

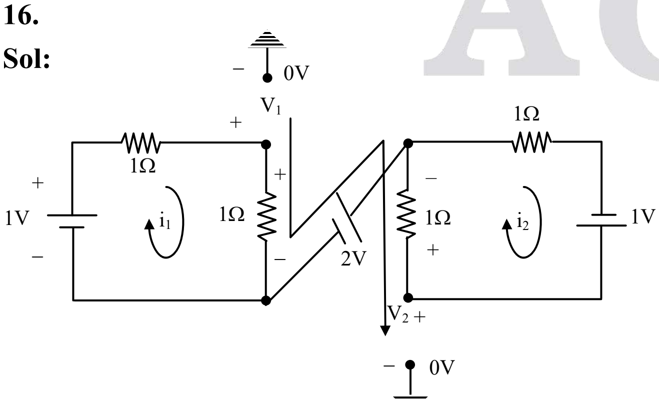
$$\frac{V_x + 15}{8} - 2V_x = 0 \Rightarrow V_x = 1\ V$$

By using nodal Analysis at V_z node

$$\frac{V_z + 15}{18} - 2 = 0 \Rightarrow V_z = +21V$$

16.

Sol:



By KVL $\Rightarrow 1 - i_1 - i_1 = 0$

$i_1 = 0.5A$

By KVL $\Rightarrow - i_2 - i_2 + 1 = 0$

$i_2 = 0.5A$

By KVL $\Rightarrow V_1 - 0.5 + 2 + 0.5 - V_2 = 0$

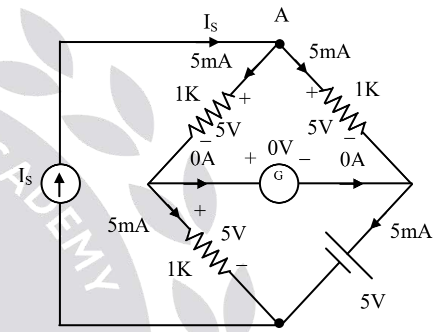
$V_2 = V_1 + 2\ V$

17.

Sol: As the bridge is balanced; voltage across (G) is "0V".

By KCL at node "A" $\Rightarrow - I_s + 5mA + 5mA = 0$

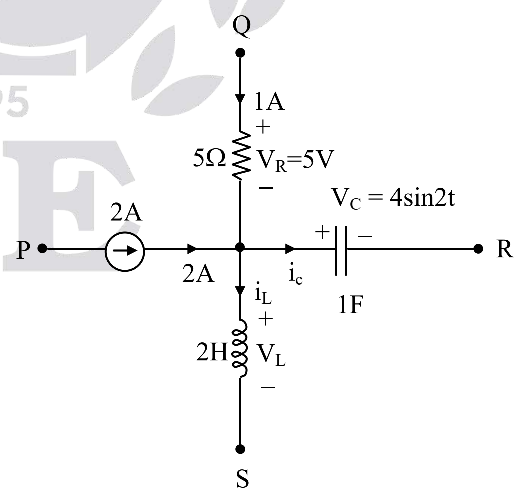
$I_S = 10mA$



18.

Sol: Given data:

$V_R = 5V$ and $V_C = 4\sin 2t$ then $V_L = ?$



$$i_c = \frac{CdV_c}{dt} = \frac{d}{dt}(4\sin 2t) = 8\cos 2t$$

By KCL; $-1 - 2 + i_L + i_c = 0$

$i_L = 3 - 8\cos 2t$

We know that;

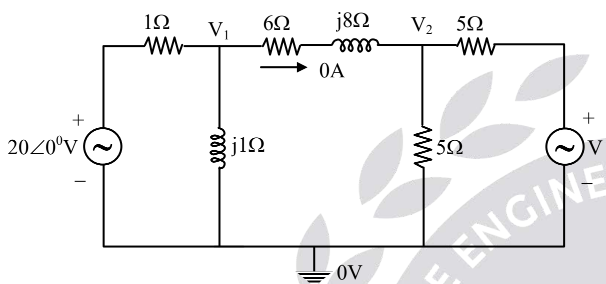
$$V_L = L\frac{di_L}{dt} = 2\frac{d}{dt}(3 - 8\cos 2t)$$

$$= 2(-8)(-2)\sin 2t$$

$$V_L = 32\sin 2t \text{ volt}$$

19.

Sol: V = ? If power dissipated in 6Ω resistor is zero.



$P_{6\Omega} = 0$ W (Given)

$\Rightarrow i_{6\Omega}^2 .6 = 0$

$\Rightarrow i_{6\Omega} = 0 \; (V_{6\Omega} = 0)$

$$\frac{V_1 - V_2}{6 + j8} = 0;\; V_1 = V_2$$

By Nodal ⇒

$$\frac{V_1 - 20\angle 0^0}{1} + \frac{V_1}{j1} + 0 = 0$$

$$V_1 = 10\sqrt{2}\angle 45^0 = V_2$$

By Nodal ⇒

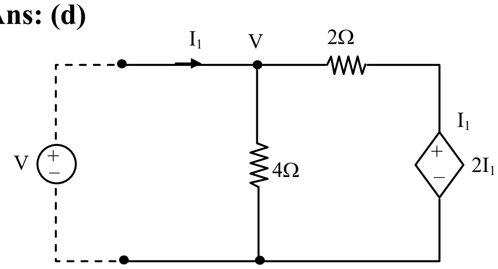
$$0 + \frac{V_2}{5} + \frac{V_2 - V}{5} = 0$$

$$V = 2V_2 = 2(10\sqrt{2}\angle 45^0)$$

$$\therefore V = 20\sqrt{2}\angle 45^0$$

20. Ans: (d)

Sol:



Note: Since no independent source in the network, the network is said to be unenergised, so called a DEAD network".

The behavior of this network is a load resistor behavior.

By Nodal ⇒

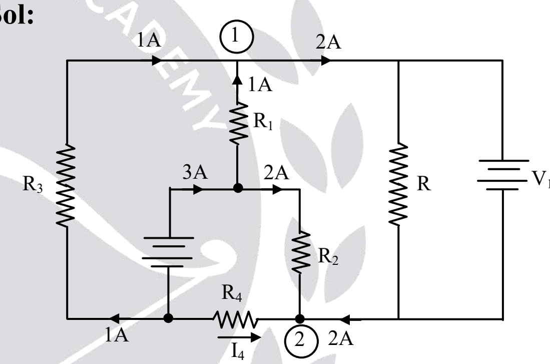
$$-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$$3V = 8I_1$$

$$R_{eq} = \frac{V}{I_1} = \frac{8}{3}\Omega$$

21. Ans: (a)

Sol:



Apply KCL at Node – 1,

$I = I_{R1} + I_{R3} = 1 + 1 = 2A$

Apply KCL at Node-2,

$I_4 = -I_2 - I = -2 - 2 = -4A$

22.

Sol:

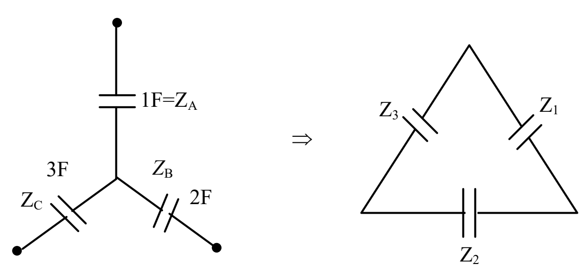


Fig.1

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A Z_B}{Z_C}\right)$$

$$= \frac{1}{s} + \frac{1}{2s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{2s}\right)}{\left(\frac{1}{3s}\right)}$$

$$Z_1 = \frac{1}{s\left(\frac{1}{3}\right)} ; \qquad C = \frac{1}{3}\text{F}$$

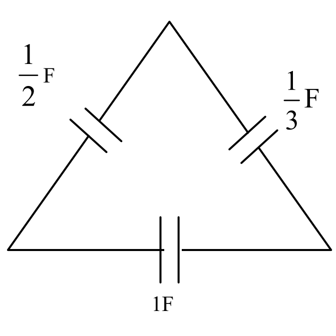
$$Z_2 = Z_B + Z_C + \frac{Z_B\,Z_C}{Z_A} = \frac{1}{2s} + \frac{1}{3s} + \frac{\left(\frac{1}{2s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{s}\right)}$$

$$Z_2 = \frac{1}{S(1)} \ ; C = 1\text{F}$$

$$Z_3 = Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$$

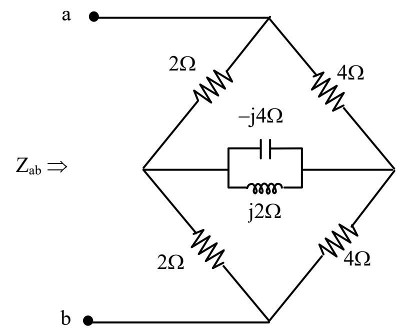
$$= \frac{1}{s} + \frac{1}{3s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{2s}\right)}$$

$$Z_3 = \frac{1}{s\left(\frac{1}{2}\right)} \ ; C = \frac{1}{2}\text{F}$$

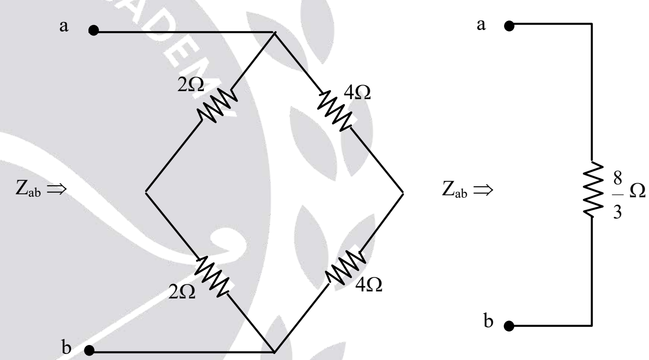


23.

Sol: $Z_{ab} = ?$



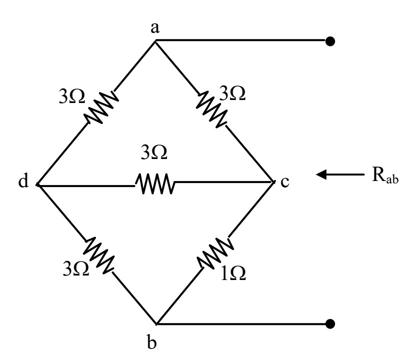
Since 2 × 4 = 4 × 2; the given bridge is balanced one, therefore the current through the middle branch is zero. The bridge acts as below:



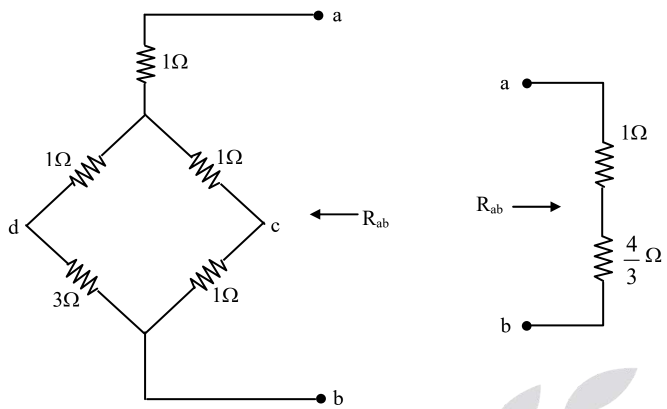
$$Z_{ab} = \frac{4\times 8}{4+8} = \frac{8}{3}\,\Omega$$

24.

Sol: Redraw the circuit diagram as shown below:



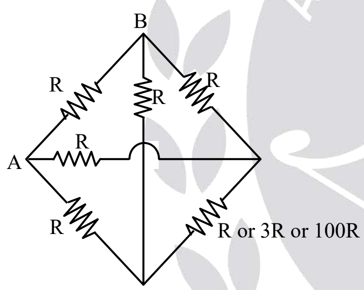
Using Δ to star transformation:

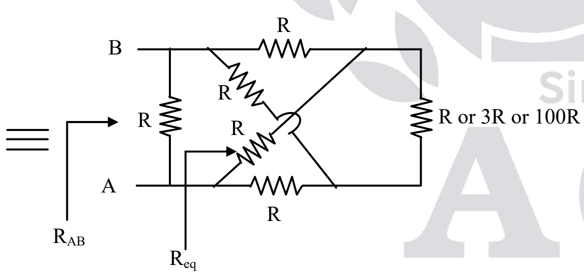


$$\therefore\ R_{ab} = 1 + \frac{4}{3} = \frac{7}{3}\,\Omega$$

25.

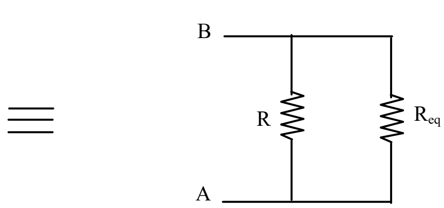
Sol: On redrawing the circuit diagram





As bridge is balanced, $R_{eq} = R$

So $R_{AB} = R \| R_{eq} = R \| R = R/2$



26. Ans: (b)

Sol: The equivalent capacitance across a, b is calculated by simplifying the bridge circuit as shown in Fig. .1 $[\because C = 0.1\mu F]$

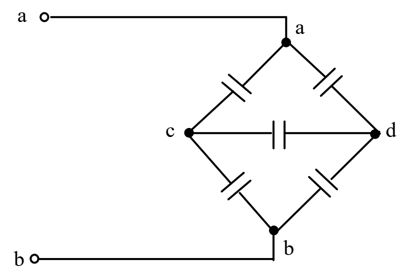
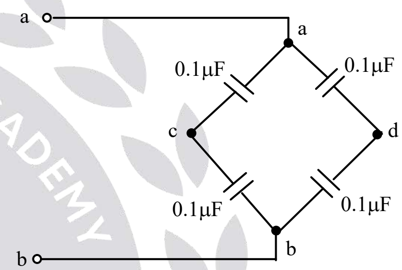
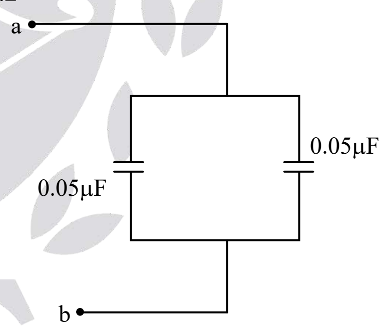


Fig. 1



$$= \frac{0.1 \times 0.1}{0.2} = 0.05\mu F$$

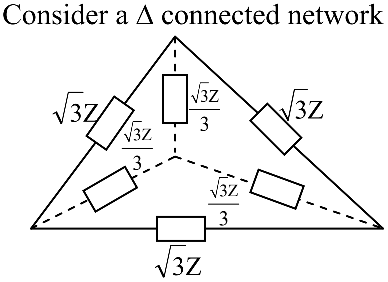


$C_{ab} = 0.1\ \mu F$

Note: The bridge is balanced and the answer is easy to get.

27. Ans: (a)

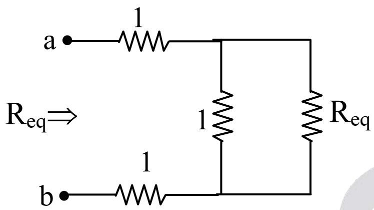
Sol: Consider a Δ connected network



Then each branch of the equivalent Y-connected impedance is $\frac{\sqrt{3}Z}{3} = \frac{Z}{\sqrt{3}}$

28. Ans: (a)

Sol: Network is redrawn as



$$R_{eq} = 1 + 1 + \frac{R_{eq}}{1+R_{eq}}$$

$$= 2 + \frac{R_{eq}}{1+R_{eq}} = \frac{2+2R_{eq}+R_{eq}}{1+R_{eq}}$$

$$R_{eq} + R_{eq}^2 = 2 + 3R_{eq}$$

$$R_{eq}^2 - 2R_{eq} - 2 = 0$$

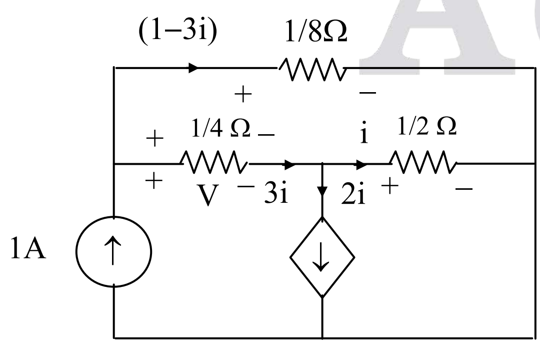
$$R_{eq} = \left(1+\sqrt{3}\right)\Omega$$

29. Ans: (c)

Sol: Applying KCL

$I_{0.25\Omega} = 2i + i = 3i$

$I_{0.125\Omega} = (1 - 3i)$ A



Applying KVL in upper loop.

$$-\frac{(1-3i)}{8} + \frac{i}{2} + \frac{3i}{4} = 0$$

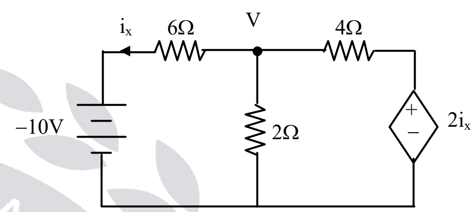
$$\frac{5i}{4} = \frac{1-3i}{8} \Rightarrow 10i = 1 - 3i$$

$$\therefore i = \frac{1}{13}\text{A}$$

$$V = \frac{3i}{4} = \frac{3}{4}\times\frac{1}{13} = \frac{3}{52}\text{V}$$

30. Ans: (a)

Sol:



Applying KCL at Node V

$$\frac{V}{2} + \frac{V-2i_x}{4} + i_x = 0 \;\ldots\ldots\ldots.\; (1)$$

$$i_x = \frac{V+10}{6} \Rightarrow V = 6i_x - 10$$

Put in equation (1), we get

$3i_x - 5 + i_x - 2.5 + i_x = 0$

$5i_x = 7.5$

$i_x = 1.5$A

$V = -1$V

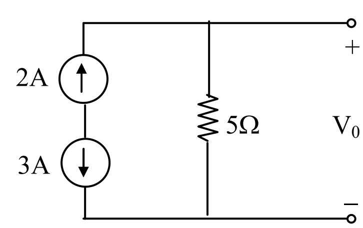
$$I_{\text{dependent souce}} = \frac{V-2i_x}{4} = \frac{-1-3}{4} = -1\text{A}$$

$\therefore$ Power absorbed = ($I_{\text{dependent source}}$) ($2i_x$)

= (–1) (3) = –3W

31. Ans: (d)

Sol: V_0 = ?



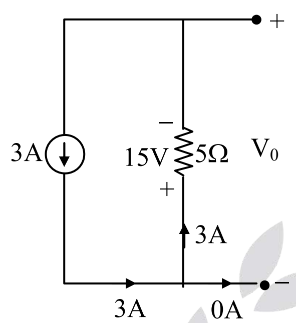
By KCL ⇒ +2 + 3 = 0

+ 5 ≠ 0

Since the violation of KCL in the circuit ; physical connection is not possible and the circuit does not exist.

32. Ans: (b)

Sol: Redraw the given circuit as shown below:



By KVL ⇒

$-15 - V_0 = 0$

$V_0 = -15V$

33. Ans: (d)

Sol: Redraw the circuit diagram as shown below:

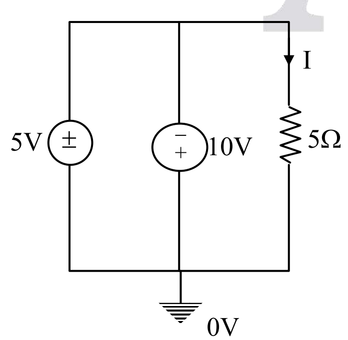
Across any element two different voltages at a time is impossible and hence the circuit does not exist.

Another method:

By KVL ⇒

$5 + 10 = 0$

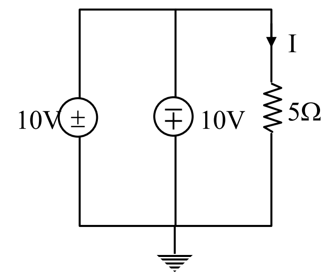
$15 \neq 0$



Since the violation of KVL in the circuit, the physical connection is not possible.

34. Ans: (d)

Sol: Redraw the given circuit as shown below:



By KVL ⇒

$-10 - 10 = 0$

$-20 \neq 0$

Since the violation of KVL in the circuit, the physical connection is not possible.

35. Ans: (b)

Sol: Redraw the given circuit as shown below:

By KVL ⇒

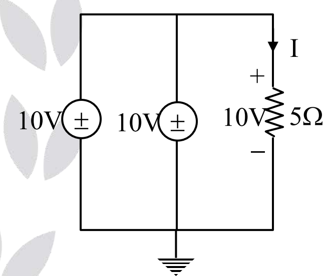
$10 - 10 = 0$

$0 = 0$

KVL is satisfied

$I_{5\Omega} = \dfrac{10}{5} = 2A$

$I_{5\Omega} = 2A$



36. Ans: (d)

Sol:

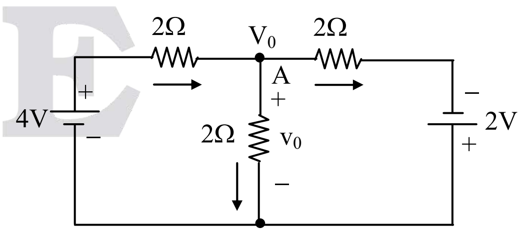


Fig. 1

The diode is forward biased. Assuming that the diode is ideal, the Network is redrawn with node A marked as in Fig. 1.

Apply KCL at node A

$$\frac{4 - v_0}{2} = \frac{v_0}{2} + \frac{v_0 + 2}{2}$$

$$\frac{3v_0}{2} = 1$$

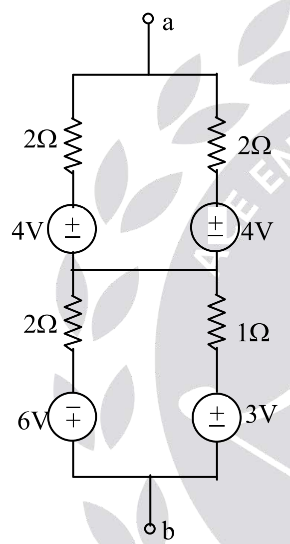
$$v_0 = \frac{2}{3}V$$

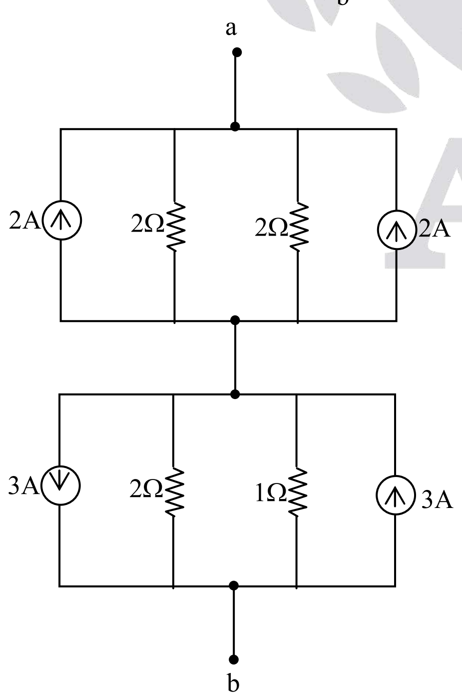
(Here polarity is different what we assume so

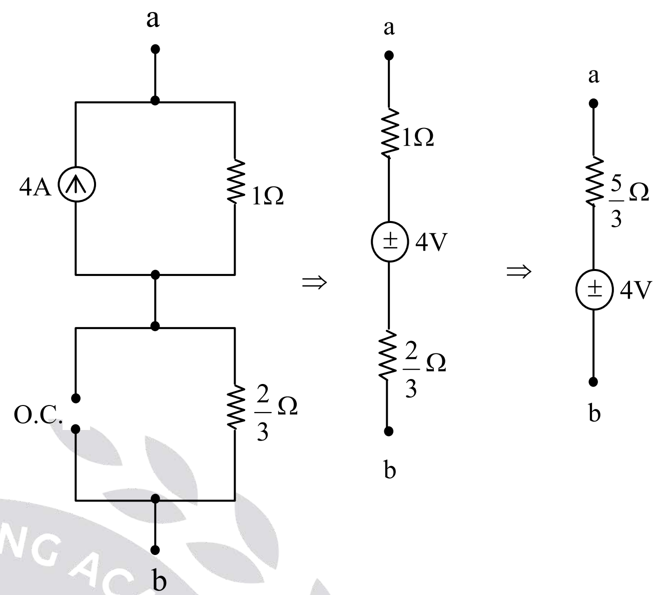
$$V_0 = \frac{-2}{3}V$$

37.

Sol: The actual circuit is

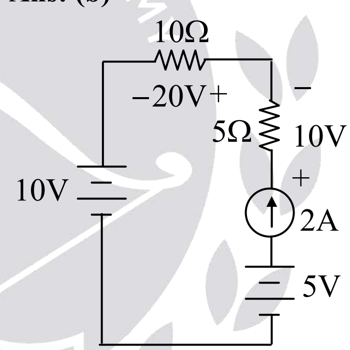






38. Ans: (b)

Sol:



Voltage across 2A = 10 + 20 + 10 – 5

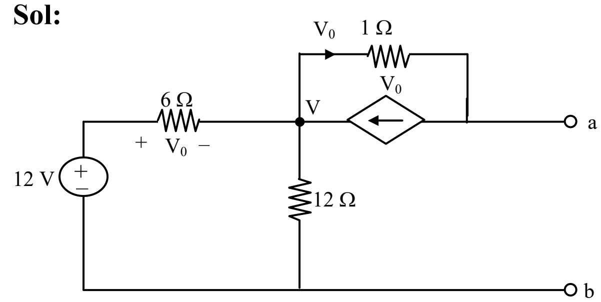
= 35 V

∴ Power supplied = VI

= 35× 2 = 70 W

39. Ans: (d)

Sol:



Applying KCL at node V

$$\frac{V-12}{6}+\frac{V}{12}-V_0+V_0=0$$

$$\Rightarrow \frac{V}{6}+\frac{V}{12}=2 \Rightarrow V = 8V$$

$\therefore V_0 = 4V$

Applying KVL in outer loop

$\Rightarrow -V+1(V_0) +V_{ab} = 0$

$\Rightarrow V_{ab} = V - V_0 = 8 - 4 = 4V$

40.

Sol: By KVL

$\Rightarrow V_i - 6 - 10 = 0$

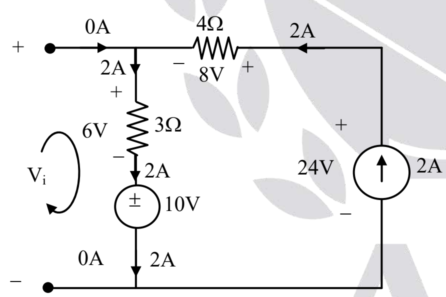
$V_i = 16V$

$P_{4\Omega} = (8 * 2) = 16$watts – absorbed

$P_{2A} = (24 * 2) = 48$ watts delivered

$P_{3\Omega} = (6*2) = 12$ watts – absorbed

$P_{10V} = (10 * 2) = 20$ watts – absorbed



48 = 16 + 12 + 20

48 = 48 W

Since; $P_{del} = P_{abs}$ = 48 watts. Tellegen's Theorem is satisfied.

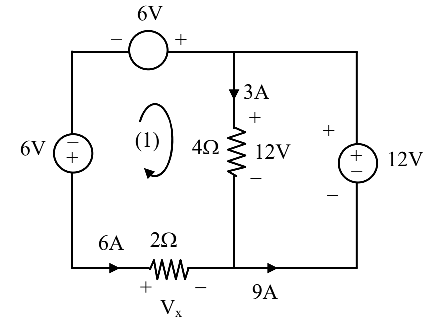
41.

Sol: By KVL in first mesh

$\Rightarrow V_x - 6 + 6 - 12 = 0$

$V_x = 12V$

$P_{12v} = (12 \times 9) = 108$ watts delivered



$P_{4\Omega} = (12 \times 3) = 36$ watts – absorbed

$P_{6V} = (6 \times 6) = 36$ watts – absorbed

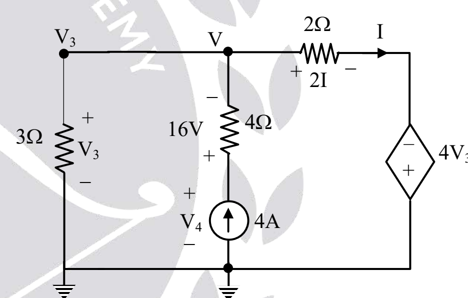
$P_{6V} = (6 \times 6) = 36$ watts – delivered

$P_{2\Omega} = (12\times6) = 72$ watts – absorbed

Since $P_{del} = P_{abs;}$ Tellegen's theorem is satisfied.

42.

Sol:



By Nodal $\Rightarrow$

$$\frac{V}{3}-4+\frac{V+4V_3}{20}=0$$

$$\frac{5V}{6}=4-2V_3 \text{ ………… (1)}$$

By KVL $\Rightarrow$

$V_3 - 2I + 4V_3 = 0$

$5V_3 - 2I = 0$ ………….. (2)

By KVL $\Rightarrow$

$V = V_3$ ……….(3)

Substitute (3) in (1), we get

$$V_3=\frac{24}{17}\ ;\ V_4 = V + 16 = \frac{24}{16}+16=\frac{296}{17}\ V$$

$$V_3=\frac{24}{17}\ \text{Volt and } I=\frac{60}{17}\ A$$

$P_{3\Omega}$ = 0.663W absorbed

$P_{4\Omega}$ = 64W absorbed

P_{4A} = 69.64W delivered

$P_{2\Omega}$ = 24.91W absorbed

P_{4V3} = 19.92Wdelivered

Since $P_{del} = P_{abs}$ = 89.57W ; Tellegen's Theorem is satisfied.

43. Ans: (a, d)

Sol: → For practical voltage source V_S is connected in series with its internal resistance R_S as low as possible. For ideal V.S ⇒ $R_S = 0\ \Omega$

→ For practical current I_S its internal resistance R_S connected in parallel as maximum as possible. For ideal C.S ⇒ $R_S = \infty\ \Omega$

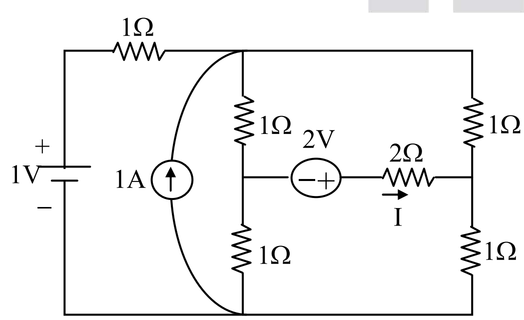
Any element connected with an ideal current source is not effect.

Any element connected in parallel with an ideal voltage source is not effect.

2. Circuit Theorems

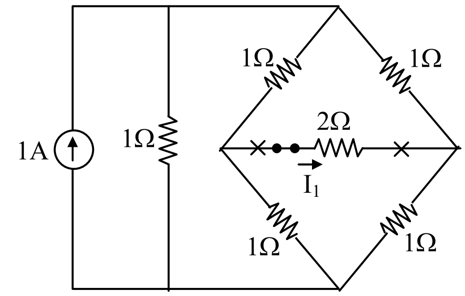
01.

Sol: The current "I" = ?



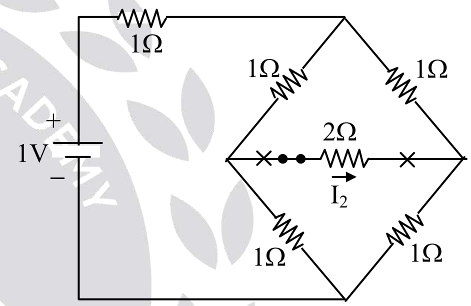
By superposition theorem, treating one independent source at a time.

(a) When 1A current source is acting alone.



Since the bridge is balanced ; $I_1 = 0A$

(b) When 1V voltage source is acting alone

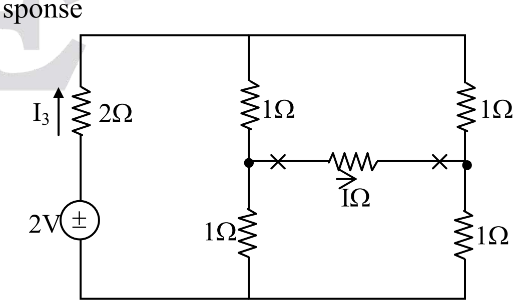


$I_2 = 0A$

Since the bridge is balanced.

(c) When 2V voltage source is acting alone and apply Reciprocity theorem, interchange source 2 volt and 1 Ω positions.

$$\frac{\text{Excitation}}{\text{Response}} = \text{same}$$



$$I_3 = \frac{2}{3} = 0.66A$$

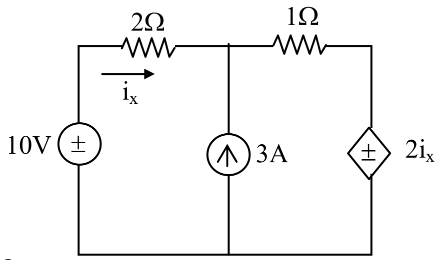
By superposition theorem; $I = I_1 + I_2 + I_3$

$I = 0 + 0 + 0.66A$

$I = 0.66A$

02.

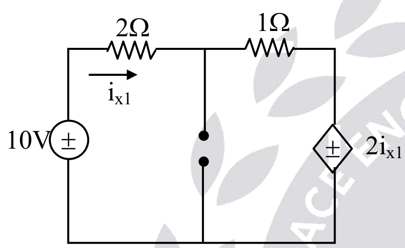
Sol:



$i_x = ?$

By super position theorem; treating only one independent source at a time

(a) When 10V voltage source is acting alone

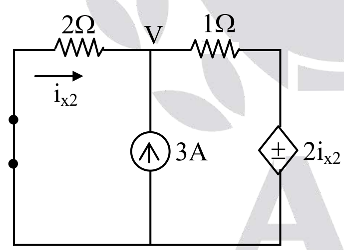


By KVL ⇒

$10 - 2i_{x1} - i_{x1} - 2i_{x1} = 0$

$i_{x1} = 2A$

(b) When 3A current source is acting alone



By Nodal ⇒

$$\frac{V}{2} - 3 + \frac{(V - 2i_{x2})}{1} = 0$$

$3V - 4i_{x2} = 6$ ……….. (1)

And

$$i_{x2} = \frac{0 - V}{2} \Rightarrow V = -2i_{x2} \text{ …..(2)}$$

Put (2) in (1), we get

$$i_{x2} = -\frac{3}{5}\text{A}$$

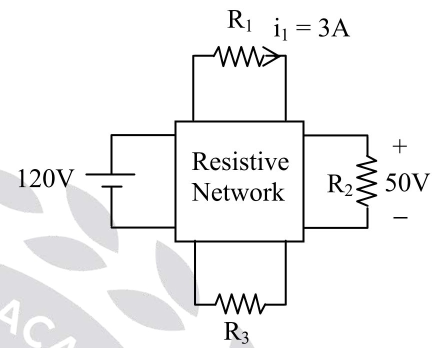
By SPT ;

$$i_x = i_{x1} + i_{x2} = 2 - \frac{3}{5} = \frac{7}{5}$$

$\therefore\ i_x = 1.4A$

03.

Sol:



$P_{R_3} = 60\text{ W}$

For 120 V → $i_1 = 3$ A

For 105 V → $i_1 = \frac{105}{120} \times 3 = 2.625\text{A}$

For 120 V → $V_2 = 50$ V

For 105 V → $V_2 = \frac{105}{120} \times 50 = 43.75\text{ V}$

$V_2 = 120\text{ V} \Rightarrow I^2R_3 = 60\text{ W} \Rightarrow I = \sqrt{\frac{60}{R_3}}$

For $V_S = 105$ V

$$P_3 = \left(\frac{105}{120}\sqrt{\frac{60}{R_3}}\right)^2 \times R_3 = 45.9\text{ W}$$

04. Ans: (b)

Sol: It is a liner network

$\therefore V_x$ can be assumed as function of i_{s1} and i_{s2}

$V_x = Ai_{s_1} + Bi_{s_2}$

$80 = 8A + 12B \quad \rightarrow (1)$

$0 = -8A + 4B \quad \rightarrow (2)$

From equation 1 & 2

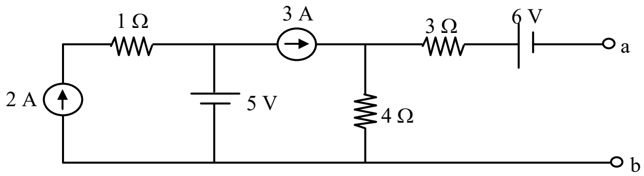
$A = 2.5,\ B = 5$

Now, $V_X = (2.5)(20) + (5)(20)$

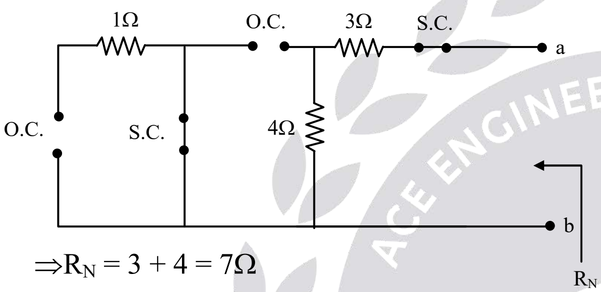
$V_x = 150V$

05. Ans: (c)

Sol:



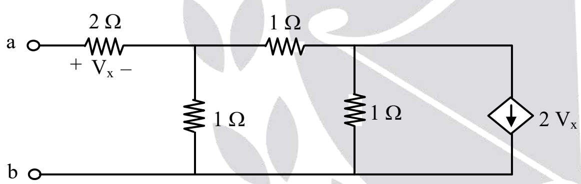
For finding Norton's equivalent resistance independent voltage sources to be short circuited and independent current sources to be open circuited, then the above circuit becomes



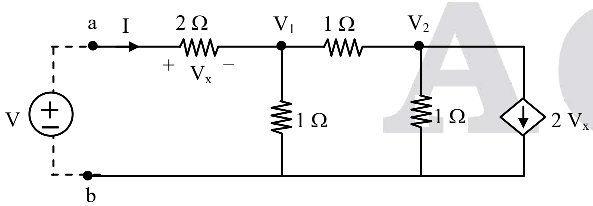
$\Rightarrow R_N = 3 + 4 = 7\Omega$

06. Ans: (b)

Sol:



Excite with a voltage source 'V'



Apply KCL at node V_1

$$-I + \frac{V_1}{1} + \frac{V_1 - V_2}{1}$$

$\Rightarrow 2V_1 - V_2 - I = 0$ ……(1)

Apply KCL at node V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + 2V_x = 0$$

$2V_2 - V_1 + 2V_x = 0$ …… (2)

But from the circuit,

$V_x = 2I$ ……. (3)

Substitute (3) in (2)

$\Rightarrow 2V_2 - V_1 + 4I = 0$

$4V_2 - 2V_1 + 8I = 0$

From (1),

$2V_1 = V_2 + I$

$\therefore\ 4V_2 - (V_2 + I) + 8I = 0$

$\Rightarrow 3V_2 + 7I = 0$

$$\Rightarrow V_2 = -\frac{7I}{3}$$

Substitute (V_2) in (1)

$$2V_1 - \left(-\frac{7I}{3}\right) - I = 0$$

$$2V_1 + \frac{7}{3}I - I = 0 \Rightarrow 2V_1 = \frac{-4I}{3}$$

$$\Rightarrow V_1 = \frac{-2I}{3}$$

$$\therefore V = V_x + V_1 = 2I + \left(-\frac{2I}{3}\right)$$

$$= \frac{4I}{3}$$

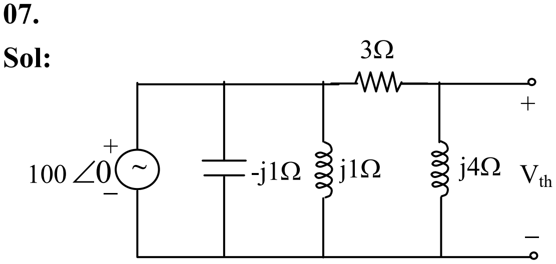
$$\Rightarrow V = \frac{4I}{3}$$

$$\Rightarrow \frac{V}{I} = \frac{4}{3}\Omega$$

$$\Rightarrow R_{eq} = \frac{4}{3}\Omega$$

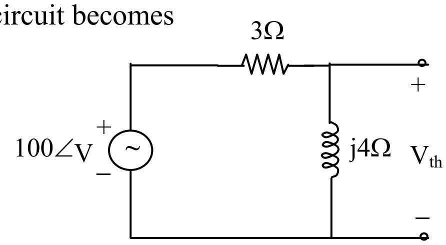
07.

Sol:



Here j1Ω and – j1Ω combination will act as open circuit.

The circuit becomes

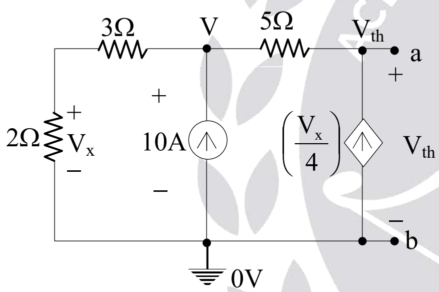


$$\Rightarrow V_{th} = \frac{100\angle 0^\circ \times j4}{3 + j4}$$

$$= 80\angle 36.86^\circ \text{ V}$$

08.

Sol: Thevenin's and Norton's equivalents across a, b.



By Nodal ⇒

$$\frac{V}{5} - 10 + \frac{V}{5} - \frac{V_{th}}{5} = 0 \quad \text{……….. (1)}$$

$$\frac{V_{th}}{5} - \frac{V}{5} - \frac{V_x}{4} = 0 \quad \text{……….. (2)}$$

$$V_x = \left(\frac{2V}{5}\right) \quad \text{……….. (3)}$$

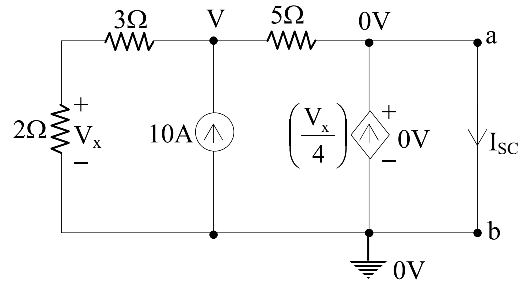
$$\frac{2V}{5} = \left(10 + \frac{V_{th}}{5}\right) \quad \text{………… (4)}$$

$$\frac{V_{th}}{5} = \left(\frac{V}{10} + \frac{V}{5}\right)$$

VDR: $V_x = V \times \dfrac{2}{2+3}$

Solve eq (1) and (2) & (3)

$V_{th} = 150\text{V}, V = 100 \text{ V}$



$$\frac{V}{5} - 10 + \frac{V}{5} = 0$$

$$\frac{2V}{5} = 10$$

$$V = 25\text{V}$$

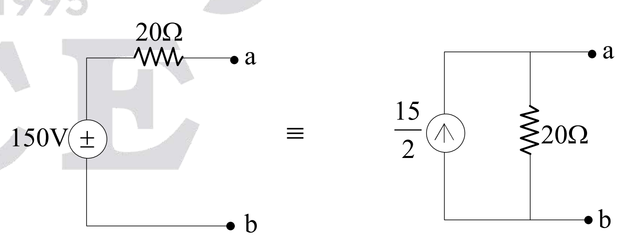
$$V_x = \frac{2V}{5} = \frac{2 \times 25}{5}$$

$$V_x = 10\text{V}, \quad \frac{0 - V}{5} - \frac{V_x}{4} + I_{Sc} = 0$$

$$I_{SC} = \left(\frac{10}{4} + 5\right) = \frac{15}{2}\text{ A}$$

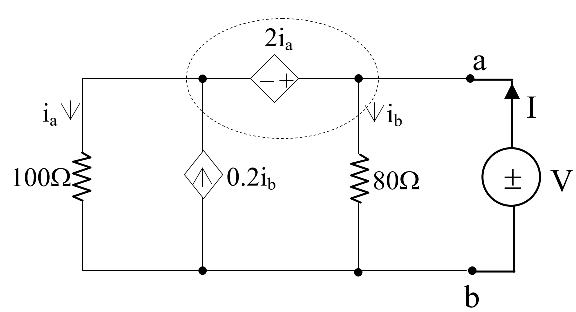
$$I_{SC} = \frac{15}{2}\text{ A}$$

$$R_{th} = \frac{V_{th}}{I_{SC}} = \frac{150}{\frac{15}{2}} = 20\Omega$$



09.

Sol:



Super nodal equation

$\Rightarrow i_a - 0.2i_b + i_b - I = 0$

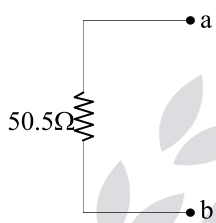
$I = i_a + 0.8i_b$

$V = 80i_b$; $i_b = \dfrac{V}{80}$

- Inside the supernode, always the KVL is written.

By KVL $\Rightarrow$

$100i_a + 2i_a - 80i_b = 0$



$$I = \frac{V}{102} + \frac{0.8 \times V}{80}$$

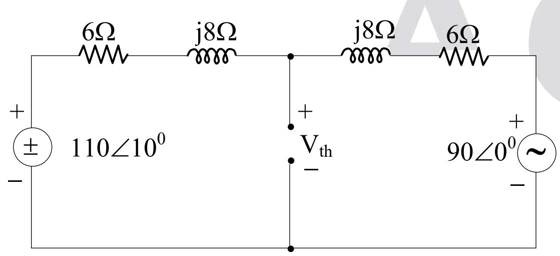
$$\frac{V}{I} = R_L = \frac{1}{\frac{1}{102} + \frac{1}{100}}$$

$= 50.5\Omega.$

$R_L = 50.5\Omega$

10.

Sol: V_{th}:

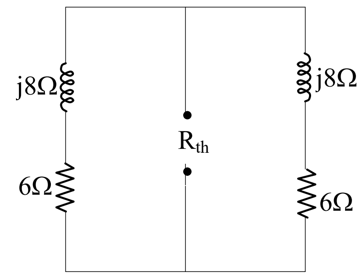


By Nodal $\Rightarrow$

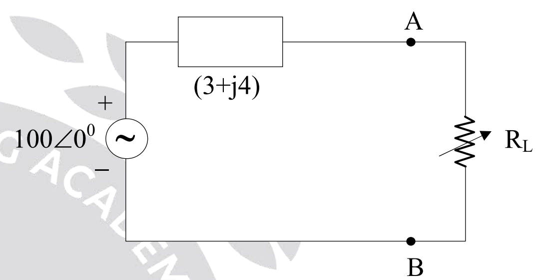
$$\frac{V_{th}}{(6+j8)} - \frac{110\angle 0^0}{(6+j8)} + \frac{V_{th}}{(6+j8)} - \frac{90\angle 0^0}{(6+j8)} = 0$$

$2V_{th} = 200\angle 0^0 \Rightarrow V_{th} = 100\angle 0^0.$

R_{th}:



$R_{th} = (6 + j8)\|(6+j8) \equiv (3 + j4)\Omega$



$R_L = |3+j4| = 5\Omega$

$$I = \frac{100\angle 0^0}{(8 + j4)}$$

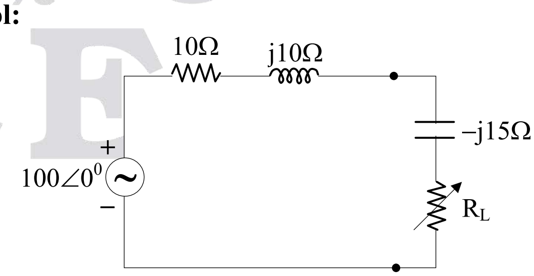
$P = |I|^2 \times R_L$

$P_{max} = 125 \times 5 = 625$ W

$\therefore\ P_{max} = 625$ watts

11.

Sol:



The maximum power delivered to "R_L" is

$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$

Here $R_S = 10\Omega$; $X_S = 10\Omega$ & $X_L = -15\Omega$

$R_L = \sqrt{10^2 + (10-15)^2}$

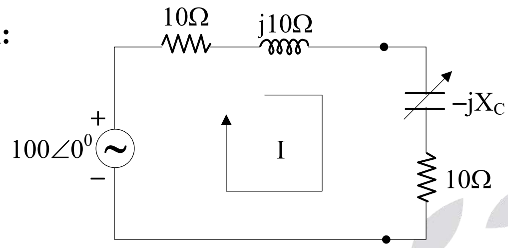
$R_L = 5\sqrt{5}\ \Omega.$

$$I = \frac{100\angle 0^0}{(10 + j10 - j15 + 5\sqrt{5})}$$

$$P_{max} = |I|^2 .5\sqrt{5} = 236W$$

12.

Sol:



The maximum power delivered to 10Ω load resistor is:

$Z_L = 10 - jX_C = 10 + j(-X_C)$

$X_L = -X_C$

So for MPT; $(X_S + X_L) = 0$

$10 - X_C = 0;$

$X_C = 10$

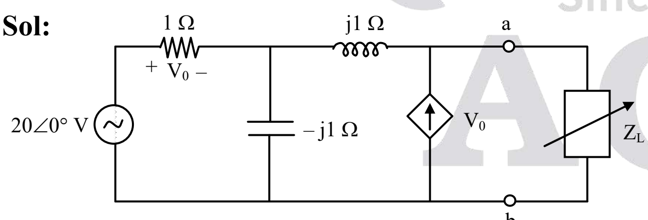
$$I = \frac{100\angle 0^0}{(10 + j10 - j10 + 10)} = 5\angle 0^0$$

$P_{max} = |I|^2 R_L = 5^2(10) = 250W$

$P_{max} = 250$ Watts

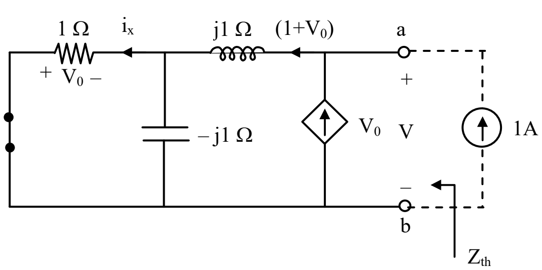
13. Ans: (b)

Sol:



For maximum power delivered to Z_L,

$$Z_L = Z_{th}^*$$



$$i_x = (1 + V_0) \times \frac{-j1}{1 - j1} = (1 + V_0)\ (0.5 - j0.5)$$

But

$V_0 = -i_x$

$= -\ (1+V_0)\ (\ 0.5 - j0.5)$

$(-1-j)\ V_0 = 1 + V_0$

$\Rightarrow V_0\ (-1 - j - 1) = 1$

$$V_0 = \frac{1}{-2 - j} = -0.4 + j0.2$$

Applying KVL

$$+ V_0 - j1(1 + V_0) + V = 0$$

$\Rightarrow V = -V_0 + j1(1+V_0)$

$= 0.4 - j0.2 + j1(0.6+j0.2)$

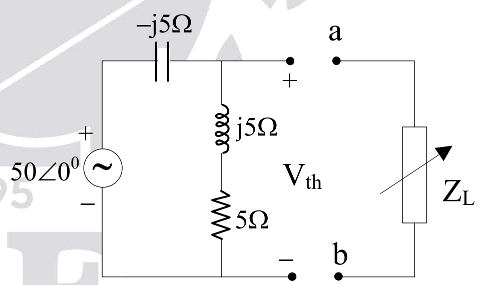
$V = (0.2 + j\ 0.4)V$

$$\therefore Z_{th} = \frac{V}{1} = V = (0.2 + j0.4)\Omega$$

$$\therefore Z_L = Z_{th}^* = (0.2 - j\ 0.4)\ \Omega$$

14.

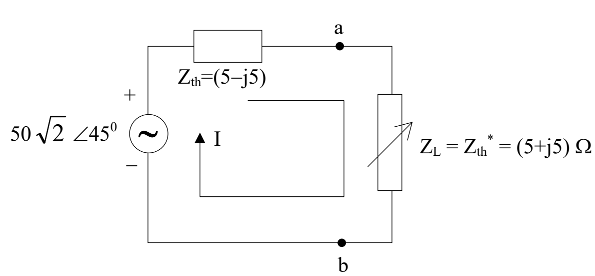
Sol:



The maximum true power delivered to "Z_L" is :

$$V_{th} = \left(\frac{50\angle 0^0}{-j5 + j5 + 5}\right)(j5 + 5) = 50\sqrt{2}\ \angle 45^0$$

$Z_{th} = (-j5)||(5+j5) = (5 - j5)\Omega$



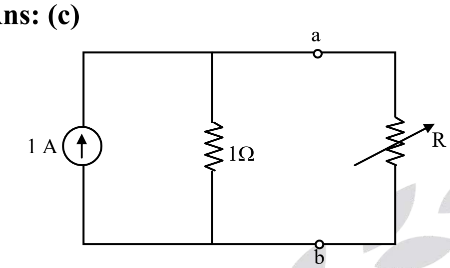
$$I = \frac{50\sqrt{2}\angle 45^0}{(5 - j5 + 5 + j5)} = 5\sqrt{2}\angle 45^0$$

$P = |I|^2 5 = |5\sqrt{2}|^2 .5 = 250$ Watts

$\therefore P_{max} = 250$ watts

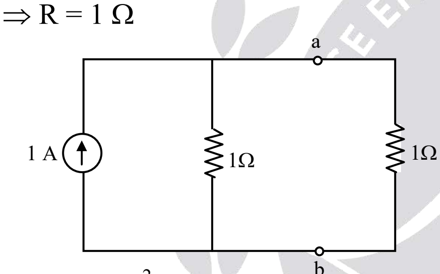
15. Ans: (c)

Sol:



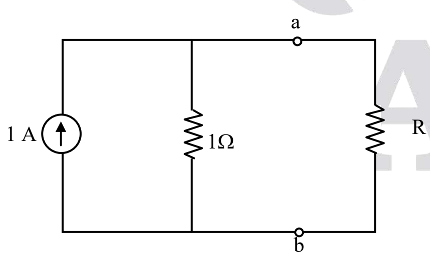
Maximum power will occurs when $R = R_s$

$\Rightarrow R = 1\ \Omega$



$$\therefore P_{max} = \left(\frac{1}{2}\right)^2 \times 1 = \frac{1}{4}\,W$$

$$25\%\text{ of } P_{max} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}\,W$$



current passing through 'R'

$$I = 1 \times \frac{1}{1+R} = \frac{1}{1+R}$$

$$\therefore P = I^2 R = \left(\frac{1}{1+R}\right)^2 R = \frac{1}{16}$$

$\Rightarrow (R+1)^2 = 16R$

$\Rightarrow R^2 + 2R + 1 = 16R$

$\Rightarrow R^2 - 14R + 1 = 0$

$R = 13.9282\Omega$ or 0.072Ω

From the given options $72m\Omega$ is correct

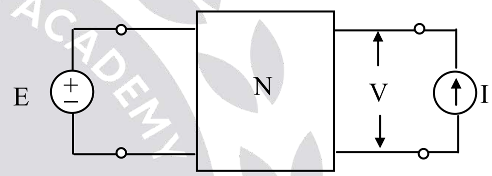
16. The network 'N' shown in figure contains only resistances.

E = 10V and 0V

I = 0A and 2A

V = 3V and 2V respectively.

If E = 100V and I is replaced by R = 2Ω, then determine V.



Sol: For, E = 10V, I = 0A then V = 3V

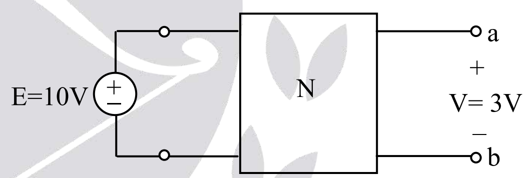


Fig.(b)

$V_{oc} = 3V$ (with respect to terminals a and b)

For, E = 0V, I = 2A then V = 2V

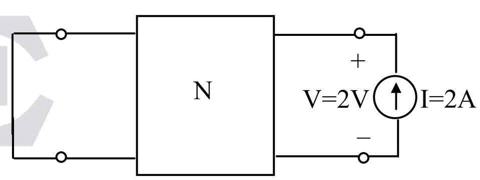
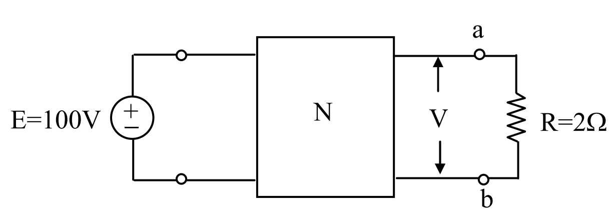


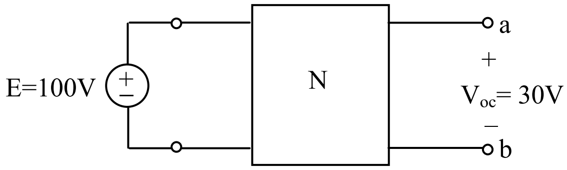
Fig.(c)

Now when E = 100V, and I is replaced by R = 2Ω then V = ?



When E = 100V,

From Fig.(b) using homogeneity principle



For finding Thevenin's resistance across ab independent voltage sources to be short circuited & independent current sources to be open circuited.

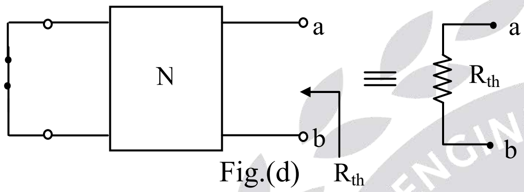
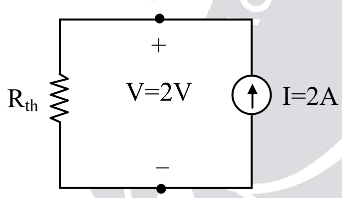


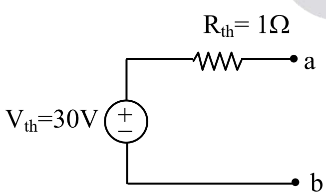
Fig.(d)

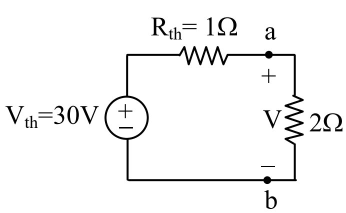
Fig.(c) is the energized version of Fig. (d)



$$\Rightarrow R_{th} = \frac{2}{2} = 1\Omega$$

$\therefore$ With respect to terminals a and b the Thevenin's equivalent becomes.





$$V = 30 \times \frac{2}{2+1} = 20V$$

$\therefore$ V = 20V

17.

Sol: Superposition theorem cannot be applied to fig (b)

Since there is only voltage source given:

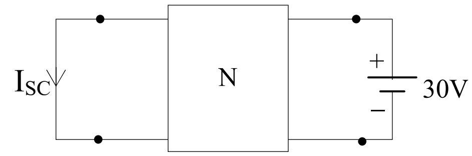
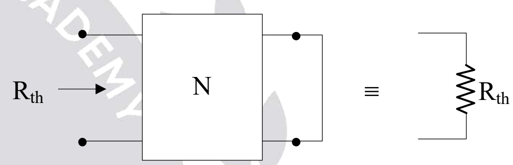


Fig (c)

By homogeneity and Reciprocity principles to fig (a);

I_{SC} = 6A

For R_{th}:



Statement: Fig (a) is the energized version of figure (d)

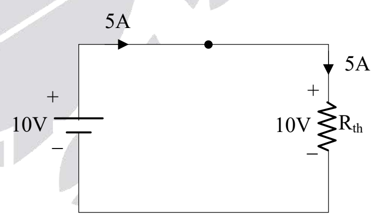


Fig (a)

$10 = R_{th}.\ 5\big|_{\text{by ohm's law}}$

R_{th} = 2Ω.

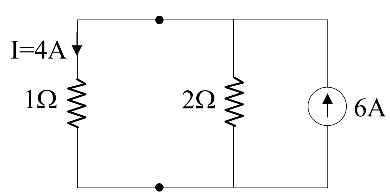


Fig (b)

$$I = \frac{6 \times 2}{(2+1)} = 4A$$

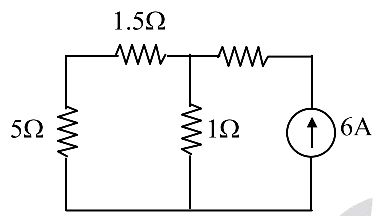
I = 4A

18. Ans: (b)

Sol: $\begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$10 = Z_{11}\,(4) + Z_{12}\,(0)$

$4 = Z_{21}\,(4) + Z_{22}\,(0)$



$$Z_{11} = \frac{10}{4} = 2.5$$

$$Z_{21} = \frac{4}{4} = 1$$

$$I_{5\Omega} = \frac{6\times1}{6.5+1} = \frac{6}{7.5} = 0.8\,\text{A}$$

19. Ans: (b)

Sol:

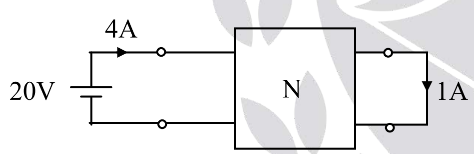


Fig.(a)

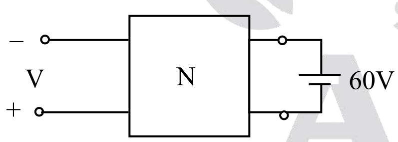
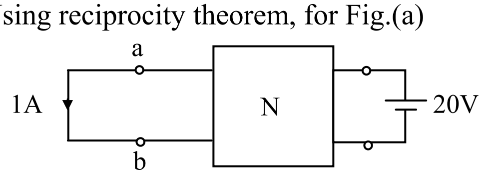


Fig.(b)

Using reciprocity theorem, for Fig.(a)



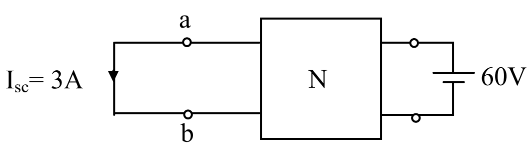


Fig.(c)

Norton's resistance between a and b is

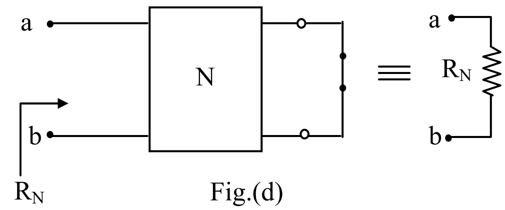
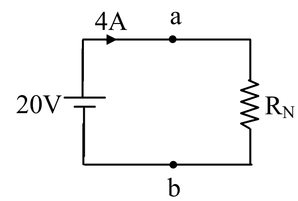


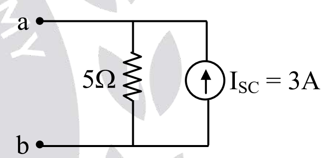
Fig.(d)

Fig.(a) is the energized version of Fig.(d)

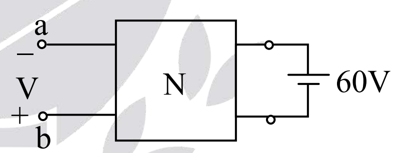


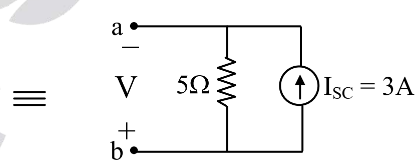
$$\Rightarrow R_N = \frac{20}{4} = 5\Omega$$

With respect to terminals a and b the Norton's equivalent of Fig.(b) is



∴ From Fig.(b)

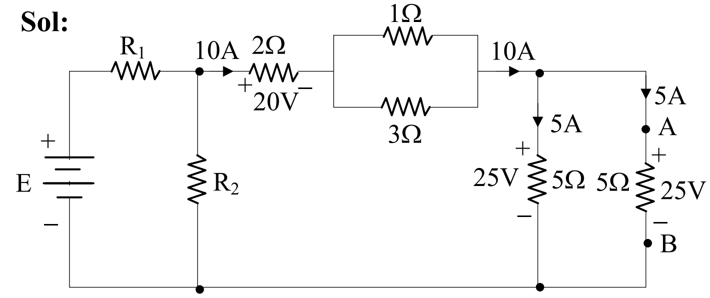


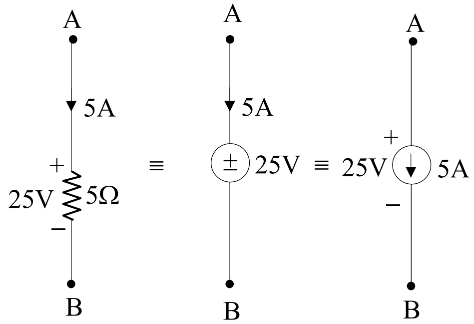


$\Rightarrow V = -\,15V$

20.

Sol:

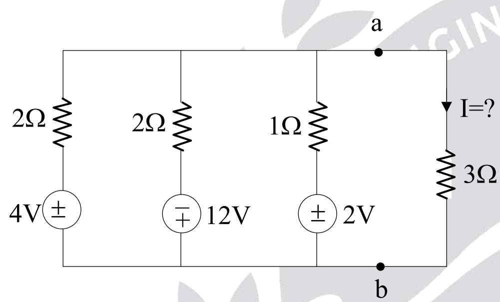




$P_{AB} = P_{5\Omega} = P_{25V} = P_{5A} = 5*25 = 125$ watts (ABSORBED)

21.

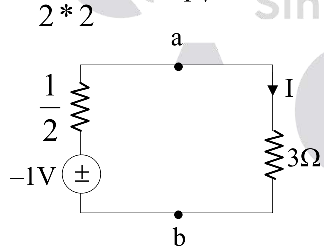
Sol:



By Mill Man's theorem;

$$V' = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$$

$$\equiv \frac{\frac{4}{2} - \frac{12}{2} + \frac{2}{1}}{\left(\frac{1}{2} + \frac{1}{2} + 1\right)} = \frac{4 - 12 + 4}{2 * 2} \equiv -1V$$



$\therefore V' = -1V$

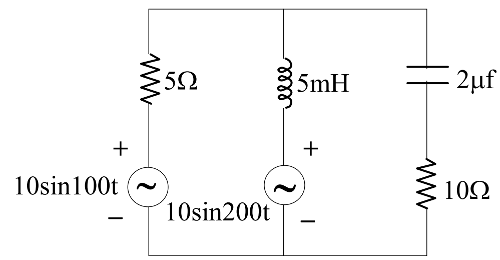
$$\frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{2} + 1 = 2$$

$$\therefore R^1 = \frac{1}{2}\Omega$$

$$I = \frac{-1}{\left(\frac{1}{2} + 3\right)} \Rightarrow I = \frac{-2}{7} A$$

22. Ans: (d)

Sol:



Since the two different frequencies are operating on the network simultaneously; always the super position theorem is used to evaluate the responses since the reactive elements are frequency sensitive

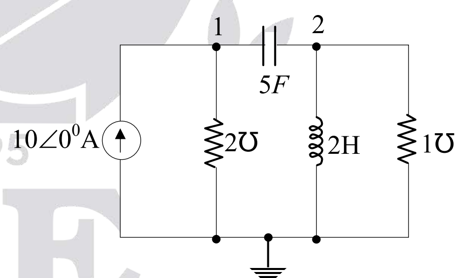
i.e., $Z_L = j\omega L$ and $Z_C = \frac{1}{j\omega c}\Omega$.

23.

Sol: In the above case if both the source are100rad/sec, each then Millman's theorem is more conveniently used.

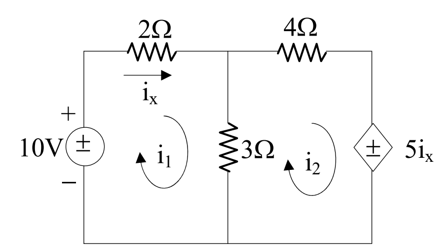
24.

Sol:



25.

Sol:



Nodal equations

$i = GV$

$i_x = i_1$

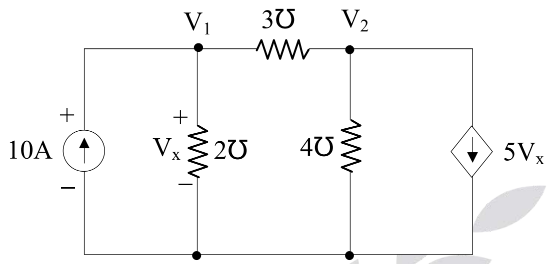
$10 = 2i_1 + 3(i_1 - i_2)$ ………. (1)

$0 = 4i_2 + 2i_x + 3(i_2 - i_1)$ ………. (2)

$V_x = V_1$

$10 = 2V_1 - 3(V_1 - V_2)$ ………….. (3)

$0 = 4V_2 + 2V_x + 3(V_2 - V_1)$ ………. (4)

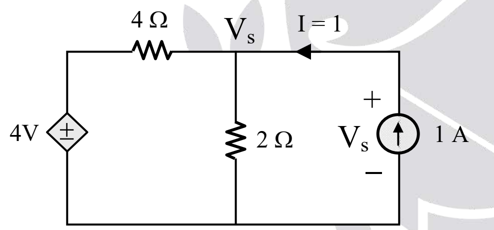


26. (b, c)

Sol: Tellegen's Theorem is applicable to any nonlinear Network.

27. Ans: (c, d)

Sol:



$I = 1 \Rightarrow 4I = 4(1) = 4$ V

$$R_{th} = \frac{V_S}{I}$$

$$\frac{V_S - 4}{4} + \frac{V_S}{2} - 1 = 0$$

$$\frac{3V_S}{4} = 2 \Rightarrow V_S = \frac{8}{3}\text{ V}$$

$$R_{th} = \frac{V_S}{I} = \frac{8}{3}\,\Omega$$

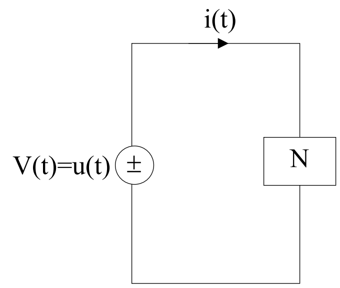
$\because$ There is no independent source, $V_{th} = 0$

$\therefore$ (c, d) are correct.

3. Transient Circuit Analysis

01.

Sol:



$i(t) = e^{-3t}$A for $t > 0$ (given)

Determine the elements & their connection

$$\frac{\text{Response Laplace transform}}{\text{Excitation Laplace transform}} = \text{System}$$

transfer function

$$\text{i.e., } \frac{I(s)}{V(s)} = H(s) = \frac{\frac{1}{(s+3)}}{\frac{1}{s}}$$

$$= \frac{s}{(s+3)} = y(s) = \frac{1}{Z(s)}$$

$$\therefore Z(s) = \left(\frac{s+3}{s}\right)$$

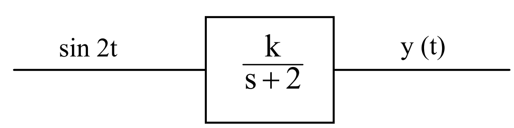
$$= 1 + \frac{1}{s\left(\frac{1}{3}\right)} = R + \frac{1}{SC}$$

$\therefore$ $R = 1\Omega$ and $C = \frac{1}{3}$ F are in series

02. Ans: (c)

Sol: The impulse response of first order system is Ke^{-2t}.

$$\text{So T/F} = L(I.R) = \frac{K}{s+2}$$



$$G(s) = \frac{K}{s+2}$$

$$|G(j\omega)| = \frac{K}{\sqrt{\omega^2 + 2^2}} = \frac{K}{2\sqrt{2}}$$

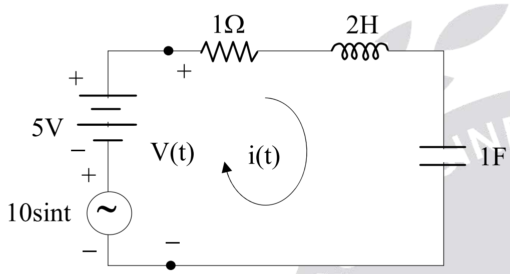
$$\angle G(j\omega) = -\tan^{-1}\frac{\omega}{2} = -\tan^{-1}1 = -\frac{\pi}{4}$$

So steady state response will be

$$y(t) = \frac{K}{2\sqrt{2}}\sin\left(2t - \frac{\pi}{4}\right)$$

03.

Sol:



By KVL $\Rightarrow$ v(t) = (5 + 10sint)volt

Evaluating the system transfer function H(s).

$$\frac{\text{Desired response L.T}}{\text{Excitation response L.T}} = \text{System transfer function}$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{\left(R + SL + \frac{1}{SC}\right)}$$

$$H(s) = \frac{S}{(2s^2 + s + 1)}$$

$$H(j\omega) = \frac{1}{\left(1 + \frac{1}{j\omega} + 2j\omega\right)}$$

II. Evaluating at corresponding ω_s of the input

$H(j\omega)|_{\omega=0} = 0$

$H(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}}\angle{-45^\circ}$

III. $\frac{I(s)}{V(s)} = H(s)$

I(s) = H(s)V(s)

$$i(t) = 0 \times 5 + \frac{1}{\sqrt{2}} \times 10\sin(t - 45°)$$

$i(t) = 7.07\sin(t - 45^\circ)$A

OBS: DC is blocked by capacitor in steady state.

04.

Sol: $\frac{V(s)}{I(s)} = H(s) = Z(s) = \frac{1}{Y(s)} = \frac{1}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}$

$$H(s) = \frac{1}{\left(1 + \frac{1}{s} + s\right)}$$

$$H(j\omega)\Big|_{\omega=1} = \frac{1}{\left(1 + \frac{1}{j} + j\right)} = 1$$

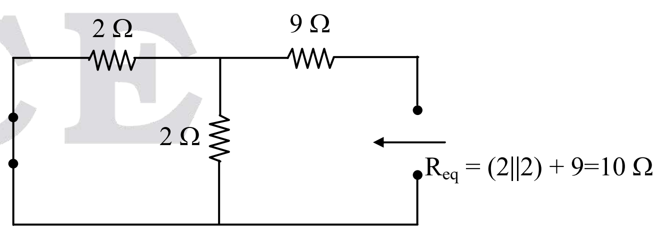
V(s) = I(s) H(s) = sin t

v(t) = sin t volts

05.

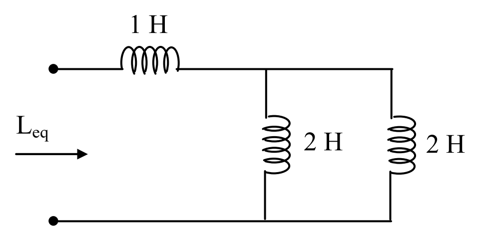
Sol: $\tau = \frac{L_{eq}}{R_{eq}}$

R_{eq} :



$R_{eq} = (2 \parallel 2) + 9 = 10\ \Omega$

L_{eq} :



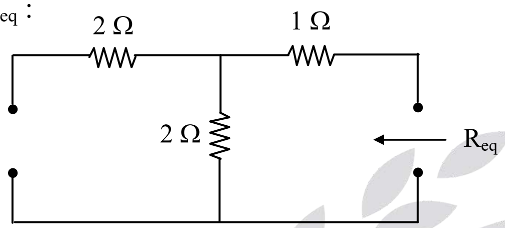
$L_{eq} = (2 \parallel 2) + 1 = 2$ H

$$\therefore \ \tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{10} = 0.2 \text{ sec}$$

06.

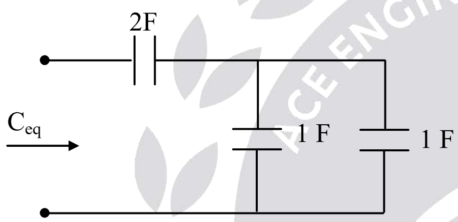
Sol: $\tau = R_{eq} C_{eq}$

R_{eq} :



$R_{eq} = 3\ \Omega$

C_{eq} :



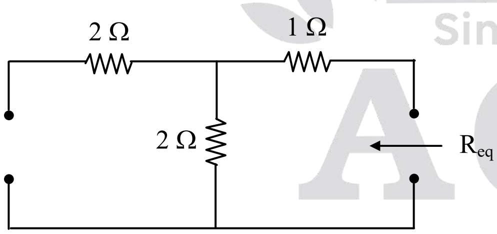
$C_{eq} = 1$ F

$\therefore \ \tau = 3 \times 1 = 3$ sec

07.

Sol: $\tau = R_{eq}$ C

R_{eq}:



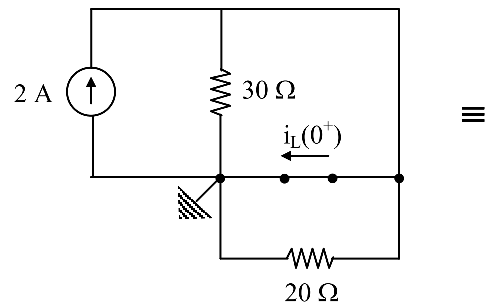
$R_{eq} = 3\ \Omega$

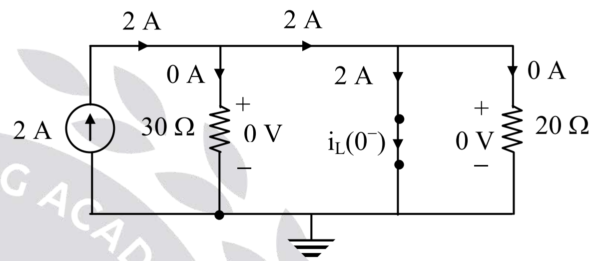
$\therefore \ \tau = 3 \times 1 = 3$ sec

08.

Sol: Let us assume that switch is closed at $t = -\infty$, now we are at $t = 0^-$ instant, still the switch is closed i.e., an infinite amount of time, the independent dc source is connected to the network and hence it is said to be in steady state.

In steady state, the inductor acts as short circuit and nature of the circuit is resistive.





At $t = 0^-$: Steady state: A resistive circuit

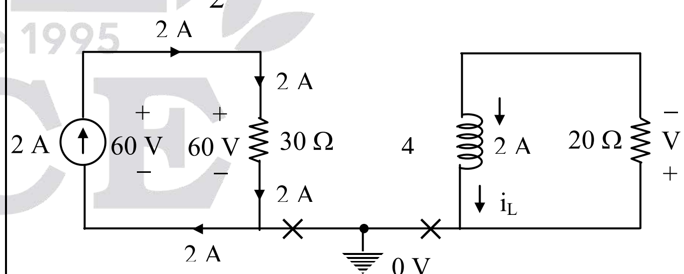
Note: The number of initial conditions to be evaluated at just before the switching action is equal to the number of memory elements present in the network.

(i) $t = 0^-$

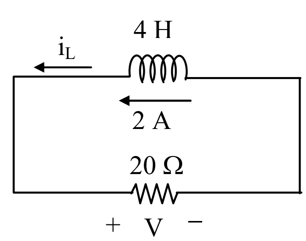
$i_L(0^-) = 2 = i_L(0^+)$

$$E_L(0^-) = \frac{1}{2} L\, i_L^2(0^-)$$

$$= \frac{1}{2} \times 4 \times 2^2 = 8\text{J} = E_L(0^+)$$



For $t \geq 0$

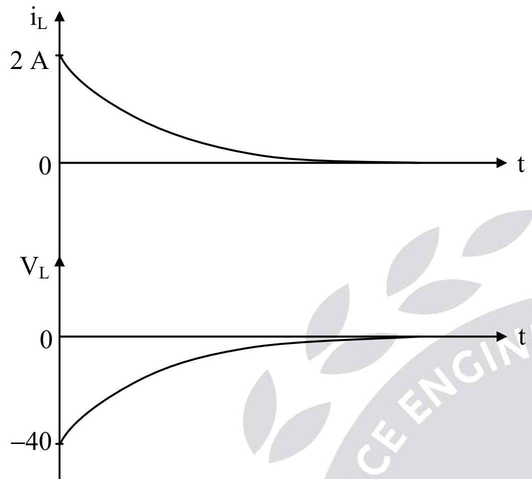


For $t \geq 0$: Source free circuit

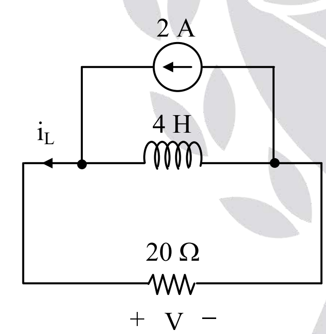
$I_0 = 2\text{ A}\ ;\ \tau = \dfrac{L}{R} = \dfrac{4}{20} = \dfrac{1}{5}$ sec

$i_L = 2\ e^{-5t}$ for $0 \le t \le \infty$

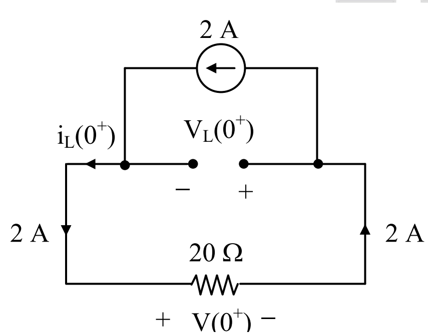
$V_L = L\dfrac{di_L}{dt} = -40\ e^{-5t}$ V for $0 \le t \le \infty$



$t = 5\tau = 5 \times \dfrac{1}{5} = 1$ sec for steady state practically i.e., with in 1 sec the total 8 J stored in the inductor will be delivered to the resistor.



For $t \ge 0$



At $t = 0^+$: Resistive circuit :
Network is in transient state

By KCL:

$-2 + i_L(0^+) = 0$

$i_L(0^+) = 2$ A

$V(0^+) = R\ i_L(0^+)\ |_{\text{By Ohm's law}}$

$V(0^+) = 20\ (2) = 40$ V

By KVL:

$V_L(0^+) + V(0^+) = 0$

$V_L(0^+) = -V(0^+) = -40\text{ V} = V_L(t)\big|_{t=0^+}$

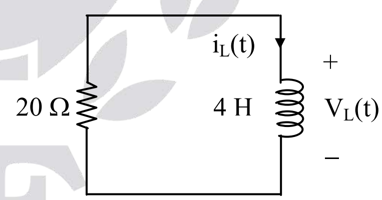
Observations:

$t = 0^-$	$t = 0^+$
$i_L(0^-) = 2$ A	$i_L(0^+) = 2$ A
$i_{20\Omega}(0^-) = 0$ A	$i_{20\Omega}(0^+) = 2$ A
$V_{20\Omega}(0^-) = 0$ V	$V_{20\Omega}(0^+) = 40$ V
$V_L(0^-) = 0$ V	$V_L(0^+) = -40$ V

Conclusion:

To keep the same energy as $t = 0^-$ and to protect the KCL and KVL in the circuit (i.e., to ensure the stability of the network), the inductor voltage, the resistor current and its voltage can change instantaneously i.e., within zero time at $t = 0^+$.

(2)



For $t \ge 0$

$i_L(t) = 2\ e^{-5t}$ A for $0 \le t \le \infty$

$V_L(t) = -40\ e^{-5t}$ V for $0 \le t \le \infty$

Conclusion:

For all the source free circuits, $V_L(t) = -$ve for $t \ge 0$, since the inductor while acting as a temporary source (upto 5τ), it discharges from positive terminal i.e., the current will flow from negative to positive terminals. (This is the must condition required for delivery, by Tellegan's theorem)

(3) $V_L(0^+) = -40$ V

$$V_L(t)\Big|_{t=0^+} = -40\ \text{V}$$

$$L\left.\frac{d\,i_L(t)}{dt}\right|_{t=0^+} = -40$$

$$\left.\frac{d i_L(t)}{dt}\right|_{t=0^+} = -\frac{40}{L} = -\frac{40}{4} = -10\ \text{A/sec}$$

Check :

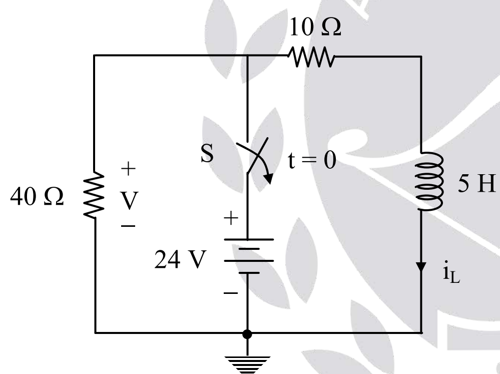
$i_L(t) = 2\,e^{-5t}$ A for $0 \le t \le \infty$

$$\frac{d i_L(t)}{dt} = -10\,e^{-5t}\ \text{A/sec} \quad \text{for } 0 \le t \le \infty$$

$$\left.\frac{d i_L(t)}{dt}\right|_{t=0^+} = -10\ \text{A/sec}$$

09.

Sol:



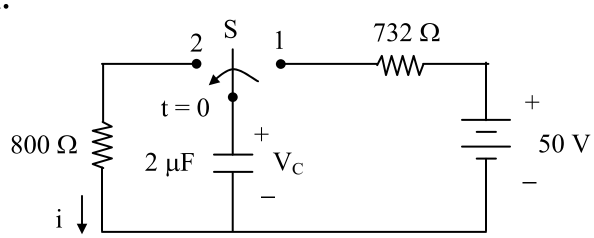
$i_L(0^+) = 2.4$ A

$V(0^+) = -96$ V

$i_L(t) = 2.4\,e^{-10t}$ A for $0 \le t \le \infty$

10.

Sol:



$V_C(0^+) = 50$ V ; $i(0^+) = 62.5$ mA

$$V_C(t) = 50\,e^{-\frac{t}{1.6\times10^{-3}}}\ \text{V} \quad \text{for } t \ge 0$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}}$$

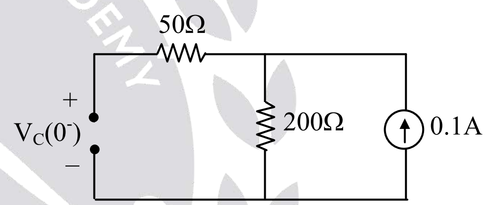
$$= 2\times10^{-6}\,50\,e^{-\frac{t}{1.6\times10^{-3}}} \times \frac{-1}{1.6\times10^{-3}}$$

$$= \frac{100\times10^{-6}}{1.6\times10^{-3}}$$

$$= \frac{1}{16}$$

11.

Sol: Case (i): $t < 0$

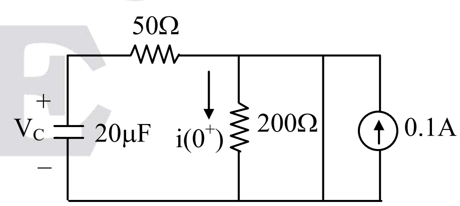


$V_C(0^-) = 20$V & $i(0^-) = 0.1$A

∵ Capacitor never allows sudden changes in voltages

$V_C(0^-) = V_C(0) = V_C(0^+) = 20$V

Case (ii): $t > 0$



To find the time constant $\tau = R_{eq}C$

After switch closed

$R_{eq} = 50\Omega$ $C = 20\mu$F

$i(0^+) = 0$A

$\tau = 50\times20\mu$

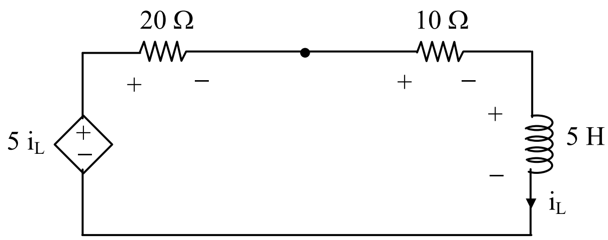
$\tau = 1$msec

$V_C(t) = V_0e^{-t/\tau} = 20e^{-t/1m}$

$V_C(t) = 20e^{-t/1m}$V; $0\le t\le\infty$

12.

Sol: After performing source transformation;



By KVL;

$$5\,i_L - 30\,i_L - 5\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 5\,i_L = 0$$

$(D + 5)\,i_L = 0$

$$i_L(t) = K\,e^{-5t}\ \text{A} \quad \text{for} \quad 0 \le t \le \infty$$

$$\tau = \frac{1}{5}\ \text{sec}$$

13.

Sol: $i_{L_1}(0) = 10\ \text{A}\ ;\ i_{L_2}(0) = 2\ \text{A}$

$$i_{L_1}(t) = I_0\,e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{1}{1} = 1\ \text{sec}$$

$$i_{L_1}(t) = 10\,e^{-t}\ \text{A}$$

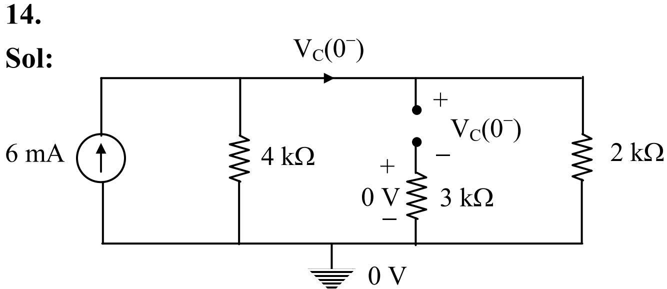
Similarly, $i_{L_2}(t) = I_0\,e^{-\frac{t}{\tau}}$

$$\tau = \frac{L}{R} = 2\ \text{sec}$$

$$i_{L_2}(t) = 20\,e^{-\frac{t}{2}}\ \text{A}$$

14.

Sol:

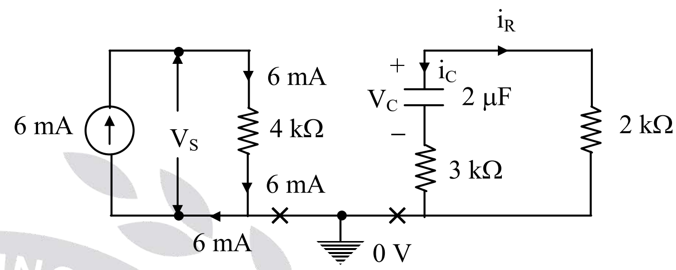


At $t = 0^-$: Steady state: A resistive circuit

By Nodal:

$$-6\ \text{mA} + \frac{V_C(0^-)}{4\text{K}} + \frac{V_C(0^-)}{2\text{K}} = 0$$

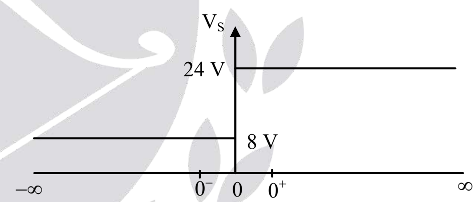
$V_C(0^-) = 8\ \text{V} = V_C(0^+)$



For $t \ge 0$: A source free circuit

$V_s = 6\ \text{m} \times 4\ \text{K} = 24\ \text{V}$

$\tau = R_{eq}\,C = (5\ \text{K})\ 2\ \mu = 10\ \text{m sec}$



$$V_C = 8\,e^{-\frac{t}{10\text{m}}} = 8\,e^{-100t}\ \text{V} \quad \text{for} \quad 0 \le t \le \infty$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}} = -1.6\,e^{-100\,t}\ \text{mA} \quad \text{for} \quad 0 \le t \le \infty$$

By KCL:

$i_C + i_R = 0$

$i_R = -\,i_C = 1.6\,e^{-100\,t}\ \text{mA} \quad \text{for} \quad 0 \le t \le \infty$

Observation:

In all the source free circuit, $i_C(t) = -\text{ve}$ for $t \ge 0$ because the capacitor while acting as a temporary source it discharges from the +ve terminal i.e., current will flow from –ve to +ve terminals.

15.

Sol: By KCL:

$i(t) = i_R(t) + i_L(t)$

$$= \frac{V_R(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$$

$$= \frac{V_S(t)}{10} + i_L(0) + \frac{1}{L}\int_{0}^{t} V_S(t)\,dt$$

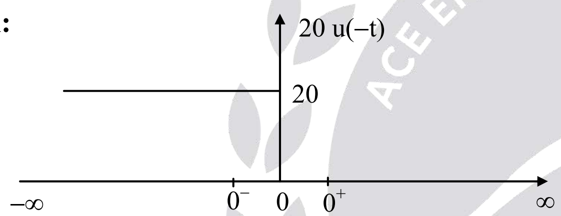
$i(t) = 4t + 5 + 4t^2$

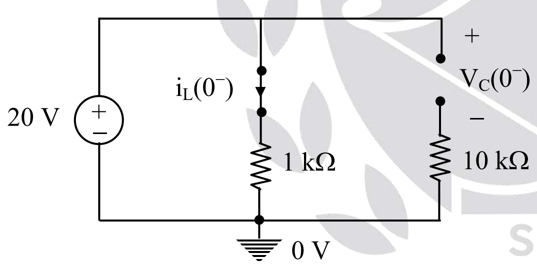
$i(t)|_{t=2\,sec} = 8 + 16 + 5 = 29\text{ A} = 29000\text{ mA}$

16. Ans: (c)

17.

Sol:



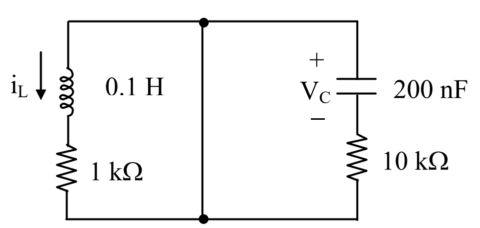


At $t = 0^-$: steady state: A resistive circuit.

(i) $t = 0^-$

$V_C(0^-) = 20\text{ V} = V_C(0^+)$

$$i_L(0^-) = \frac{20}{1K} = 20\text{ mA} = i_L(0^+)$$



For $t \geq 0$: A source free RL & RC circuit

$$\tau = \frac{0.1}{1K} = 100\,\mu\text{sec}$$

$\tau_C = 200 \times 10^{-9} \times 10 \times 10^3 = 2\text{ m sec}$

$$\frac{\tau_C}{\tau_L} = 20 \;;\; \tau_C = 20\,\tau_L$$

Observation:

$\tau_L < \tau_C$; therefore the inductive part of the circuit will achieve steady state quickly i.e., 20 times faster.

$$V_C = 20\,e^{-\frac{t}{\tau_C}}\text{ V} \quad \text{for } 0 \leq t \leq \infty$$

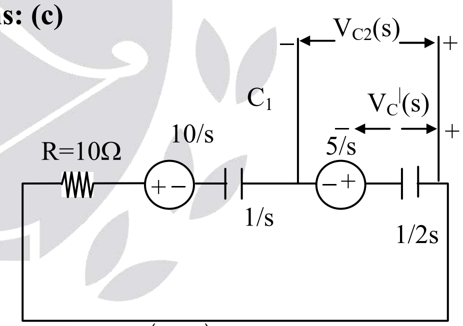
$$i_L = 20\,e^{-\frac{t}{\tau_L}}\text{ mA} \quad \text{for } 0 \leq t \leq \infty$$

$$V_L = L\left.\frac{di_L}{dt}\right|_{\text{By Ohm's law}}$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}}$$

18. Ans: (c)

Sol:



$$V_c^{|}(s) = \frac{\frac{5}{s}\left(\frac{1}{2s}\right)}{R + \frac{1}{s} + \frac{1}{2s}}$$

$$= \frac{\frac{5}{2s^2}}{\frac{2Rs + 2 + 1}{2s}} = \frac{5}{s(2Rs + 3)}$$

$$V_{c_2}(\infty) - V_{c^{|}}^{|}(s) - \frac{5}{s} = 0$$

$$V_c(\infty) = V_c^{|}(s) + \frac{5}{s}$$

$$V_c(\infty) = \underset{s\to 0}{\text{Lt}}\, s.\left[\frac{5}{s(2Rs+3)} + \frac{5}{s}\right] = \frac{5}{3} + 5 = \frac{20}{3}$$

19. Ans: (d)

Sol: at t = 0

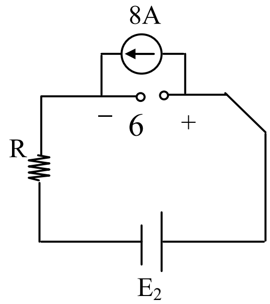
$$L\frac{di(0)}{dt} = V_L(0)$$

$$V_L = 2\times 3 = 6$$

$$V_L = 6V$$

$E_2 + 6 - 8R = 0$

$E_2 = 8R - 6$



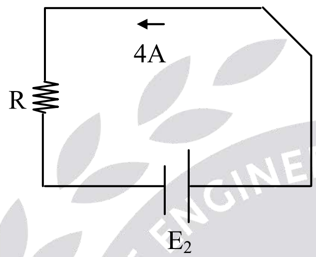
$E_2 - 4R = 0$

$E_2 = 4R$

$8R - 6 = 4R$

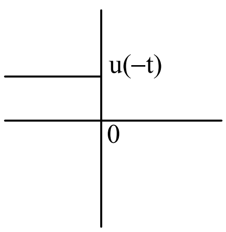
$4R = 6$

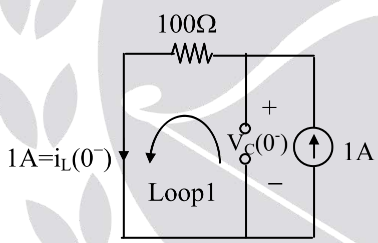
$R = 1.5\Omega$



20. Ans: (d)

Sol: at t < 0





Apply KVL in loop1 $\Rightarrow V_C(0^-) - 100 = 0$

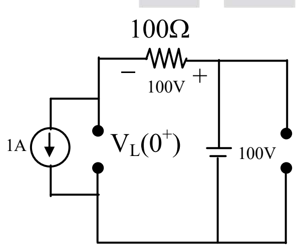
$\Rightarrow V_C(0^-) = 100V$

At $t = 0^+$

$$V_L(0^+) = 0$$

$$L\frac{di(0^+)}{dt} = 0$$

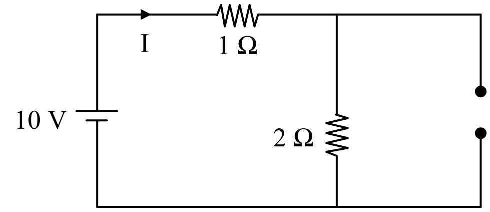
$$\frac{di(0^+)}{dt} = 0$$



21.

Sol: Case -1 at $t = 0^+$

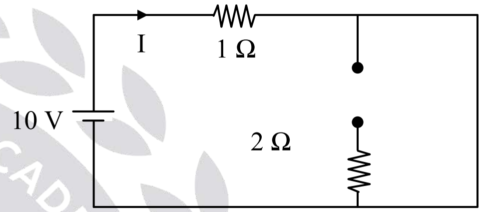
By redrawing the circuit



Current through the battery at $t = 0^+$ is

$$\frac{10}{3}\text{ Amp}$$

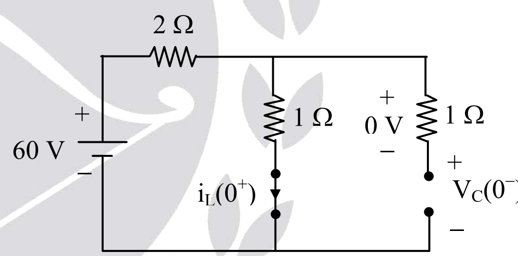
Case -2 at $t = \infty$



Current through the battery at $t = \infty$ is 10 A

22.

Sol:

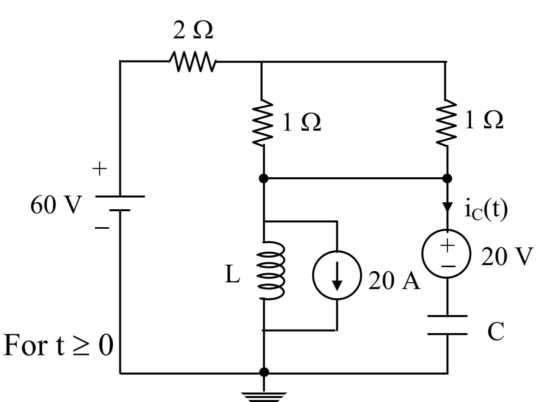


At $t = 0^-$: Steady state: A resistive circuit

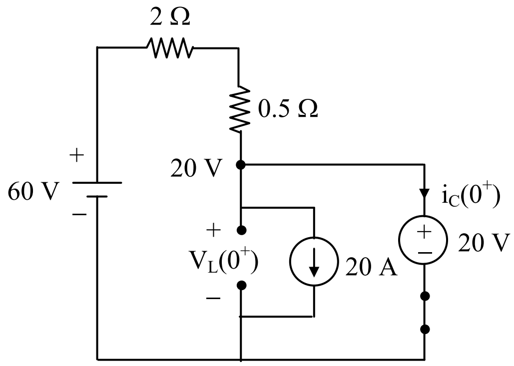
(i) $t = 0^-$:

$$i_L(0^-) = \frac{60}{3} = 20\text{ A} = i_L(0^+)$$

$V_{1\Omega} = 20\text{ V} = V_C(0^-) = V_C(0^+)$



For $t \geq 0$



At $t = 0^+$: A resistive circuit :
Network is in transient state

$V_L(0^+) = 20$ V

Nodal :

$$\frac{20-60}{2.5} + 20 + i_C(0^+) = 0$$

$i_C(0^+) = -4$ A

23.

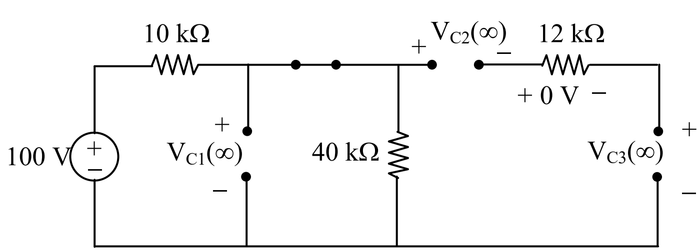
Sol: Repeat the above problem procedure :

$$\left.\frac{d\,i_L(t)}{dt}\right|_{t=0^+} = \frac{V_L(0^+)}{L} = 0 \text{ A/sec}$$

$$\left.\frac{d\,V_C(t)}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = -10^6 \text{ V/sec}$$

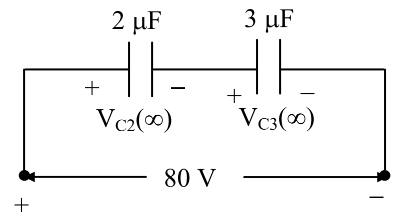
24.

Sol: **Observation:** So, the steady state will occur either at $t = 0^-$ or at $t = \infty$, that depends where we started i.e., connected the source to the network.



At $t = \infty$: Steady state: A Resistive circuit

$$V_{C_1}(\infty) = \frac{100}{50\,K} \times 40\,K = 80 \text{ V}$$

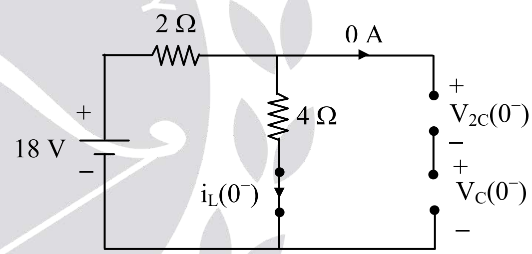


$$V_{C_2}(\infty) = \frac{80 \times 3\,\mu F}{(2+3)\,\mu F} = 48 \text{ V}$$

$$V_{C_3}(\infty) = \frac{80 \times 2\,\mu F}{5\,\mu F} = 32 \text{ V}$$

25.

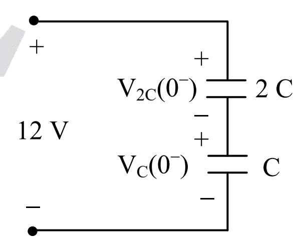
Sol:



At $t = 0^-$: Circuit is in Steady state: Resistive circuit

$i_L(0^-) = 3$ A $= i_L(0^+)$

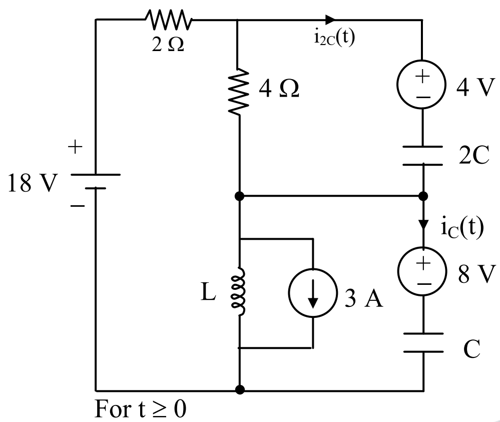
$V_{4\Omega} = 4 \times 3 = 12$ V



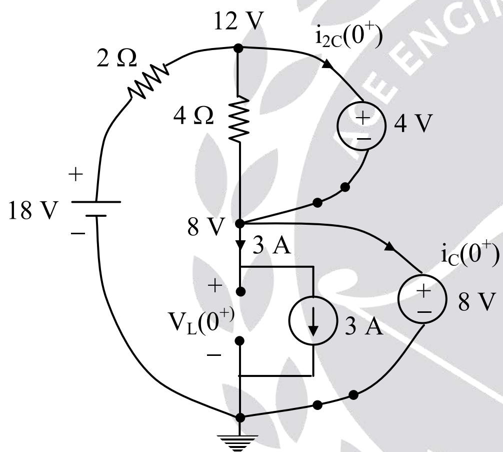
$$V_{2C}(0^-) = \frac{12 \times C}{2C + C}$$

$$= 4 \text{ V} = V_{2C}(0^+)$$

$V_C(0^-) = 8$ V $= V_C(0^+)$



and redrawing the circuit



By Nodal;

$$\frac{12-18}{2}+\frac{12-8}{4}+i_{2C}(0^+) = 0$$

$$\frac{-6}{2}+\frac{4}{4}+i_{2C}(0^+) = 0$$

$i_{2C}(0^+) = 2\text{ A} = i_{2C}(0^-)$

$$\frac{8-12}{4}-i_{2C}(0^+)+3+i_C(0^+) = 0$$

$i_C(0^+) = 0\text{ A} = i_C(0^-)$

26.

Sol: $t = 0^-$ $\quad t = 0^+$ $\quad t = 0^+$

$i_L(0^-) = 5\text{ A}$ $\quad i_L(0^+) = 5\text{ A}$

$$\frac{d\,i_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = 40$$

$i_R(0^-) = -5\text{ A}$ $\qquad i_R(0^+) = -1\text{A}$

$$\frac{d\,i_R(0^+)}{dt} = -40\text{ A/sec}$$

$i_C(0^-) = 0\text{ A}$ $\qquad i_C(0^+) = 4\text{A}$

$$\frac{d\,i_C(0^+)}{dt} = -40\text{ A/sec}$$

$V_L(0^-) = 0\text{ V}$

$V_L(0^+) = 120\text{ V}$

$$\frac{d\,V_L(0^+)}{dt} = 1098\text{ V/sec}$$

$V_R(0^-) = -150\text{ V}$

$V_R(0^+) = -30\text{ V}$

$$\frac{d\,V_R(0^+)}{dt} = -1200\text{ V/sec}$$

$V_C(0^-) = 150\text{ V}$

$V_L(0^+) = 150\text{ V}$

$$\frac{d\,V_C(0^+)}{dt} = 108\text{ V/sec}$$

(i). $t = 0^-$

By KCL ⇒ $i_L(t) + i_R(t) = 0$

$t = 0^-$ ⇒ $i_L(0^-) + i_R(0^-) = 0$

$i_R(0^-) = -5\text{ A}$

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$V_R(0^-) = R\, i_R(0^-) = 30(-5) = -150\text{ V}$

By KVL ⇒ $V_L(t) - V_R(t) - V_C(t) = 0$

$V_C(0^-) = V_L(0^-) - V_R(0^-) = 150\text{ V}$

(ii). At $t = 0^+$

By KCL at 1st node ⇒

$-4 + i_L(t) + i_R(t) = 0$

$-4 + i_L(0^+) + i_R(0^+) = 0$

$i_R(0^+) = -i_L(0^+) + 4$

$i_R(0^+) = -5 + 4 = -1\text{ A}$

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$V_R(0^+) = R\, i_R(0^+)$

$V_R(0^+) = -30$ V

By KVL $\Rightarrow V_L(t) - V_R(t) - V_C(t) = 0$

$V_L(0^+) = V_R(0^+) + V_C(0^+)$

$= 150 - 30 = 120$ V

By KCL at 2nd node;

$-5 + i_C(t) - i_R(t) = 0$

$i_C(0^+) = 4$ A

(iii). $t = 0^+$

By KCL at 1st node $\Rightarrow$

$-4 + i_L(t) + i_R(t) = 0$

$$0 + \frac{d\,i_L(t)}{dt} + \frac{d}{dt} i_R(t) = 0$$

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$$\frac{d}{dt} V_R(t) = R \frac{d}{dt} i_R(t)$$

By KVL $\Rightarrow$

$V_L(t) - V_R(t) - V_C(t) = 0$

$$\frac{d\,V_L(t)}{dt} - \frac{d\,V_R(t)}{dt} - \frac{d\,V_C(t)}{dt} = 0$$

By KCL at node 2:

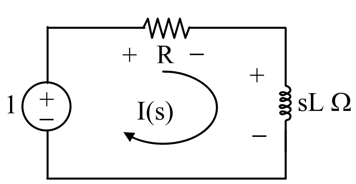
$-5 + i_C(t) - i_R(t) = 0$

$$0 + \frac{d}{dt} i_C(t) - \frac{d}{dt} i_R(t) = 0$$

$$\frac{d}{dt} i_C(0^+) = -(-40) = 40 \text{ A/sec}$$

27.

Sol: Transform the network into Laplace domain



S - domain

$V(s) = Z(s)\, I(s)$

By KVL in S-domain $\Rightarrow$

$1 - R\, I(s) - s L\, I(s) = 0$

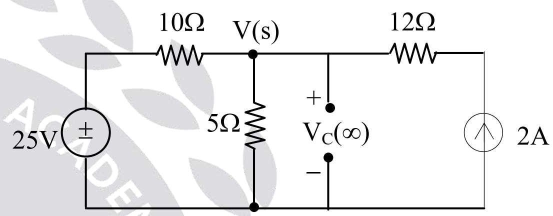
$$I(s) = \frac{1}{L} \frac{1}{\left(s + \frac{R}{L}\right)}$$

$$i(t) = \frac{1}{L} e^{-\frac{R}{L}t} \text{ A} \quad \text{for } t \ge 0$$

28.

Sol: By Time domain approach;

$V_C(0^-) = 5 \times 2 = 10 \text{ V} = V_C(0^+)$



At t =∞: Steady state: A resistive circuit

$$\text{Nodal} \Rightarrow \frac{V_C(\infty) - 25}{10} + \frac{V_C(\infty)}{5} - 2 = 0$$

$V_C(\infty) = 15$ V

$\tau = R_{eq} C = (5 \,||\, 10)\,.\,1 = (10/3)$ sec

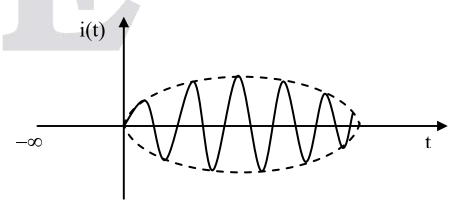
$$V_C = 15 + (10 - 15)\, e^{-\frac{t}{(10/3)}}$$

$V_C = 15 - 5\,e^{-3t/10}$ V for $t \ge 0$

$$i_C = C \frac{dV_C}{dt} = 1.5\, e^{-3t/10} \text{ A} \quad \text{for } t \ge 0$$

29.

Sol:



That is the response is oscillatory in nature

30.

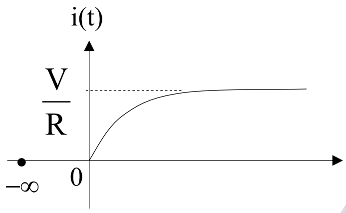
Sol: $i(0^-) = 0 \text{ A} = i(0^+)$

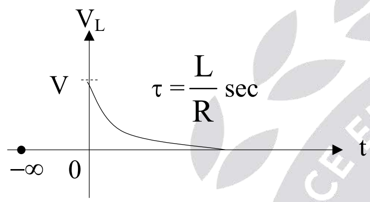
$$i(\infty) = \frac{V}{R} \text{ A}$$

$$\tau = \frac{L}{R}\text{ sec}$$

$$i(t) = \frac{V}{R} + \left(0 - \frac{V}{R}\right) e^{-t/\tau} = \frac{V}{R}(1 - e^{-t/\tau})$$

$$V_L = \frac{L\,di(t)}{dt} = V\ e^{-Rt/L} \text{ for } t \geq 0$$





Exponentially Increasing Response

31.

Sol: $V_C(0^-) = 0 = V_C(0^+)$

$V_C(\infty) = V$

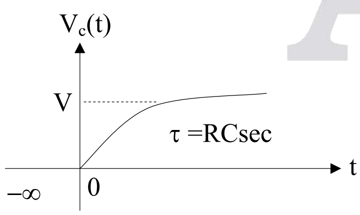
$\tau = RC$

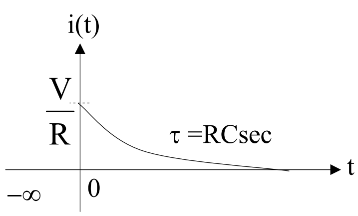
$V_C = V + (0-V)e^{-t/\tau}$

$= V(1-e^{-t/RC})$ for $t \geq 0$

$$ic = C\frac{dv_c}{dt} = \frac{V}{R}e^{-t/RC} \text{ for } t \geq 0$$

$= i(t)$



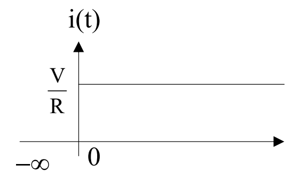


Exponentially Decreasing Response

32.

Sol: It's an RL circuit with $L = 0 \Rightarrow \tau = 0$ sec

$$i(t) = \frac{V}{R},\ \forall t \geq 0 \text{ So, } 5\tau = 0 \text{ sec}$$



i.e., the response is constant

33.

Sol: $i_1 = \dfrac{100u(t) - V_L}{10}$

$$i1 = \left(10u(t) - \frac{1}{100}\frac{di_L}{dt}\right)A$$

Nodal $\Rightarrow$

$$-i_1 + i_L + \frac{V_L - 20i_1}{20} = 0$$

$$-2i_1 + i_L + \frac{1}{200}\frac{di_L}{dt} = 0$$

Substitute i_1;

$$\frac{di_L}{dt} + 40i_L = 800u(t)$$

$$SI_L(s) - i_L(0+) + 40I_L(s) = \frac{800}{s}$$

$i_L(0^-) = 0A = i_L(0^+)$

$$I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

$I_Lt) = 20u(t) - 20e^{-40t}\,u(t)$

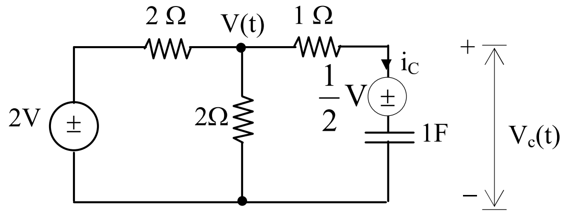
$I_L(t) = 20(1-e^{-40t})\,u(t)$

$$i_l = 10u(t) - \frac{1}{100}d\frac{i_L}{dt}$$

$i_1 = (10-8e^{-40t})\,u(t)$

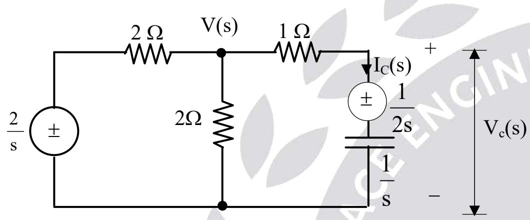
34.

Sol: By Laplace transform approach:



For t≥ 0

Transform the above network into the Laplace domain



For t≥ 0

Nodal ⇒

$$\frac{V(s)-\frac{2}{s}}{2}+\frac{V(s)}{2}+\frac{V(s)-\frac{1}{2s}}{1+\frac{1}{s}}=0$$

$$I_c(s)=\left(\frac{V(s)-\frac{1}{2s}}{1+\frac{1}{s}}\right)$$

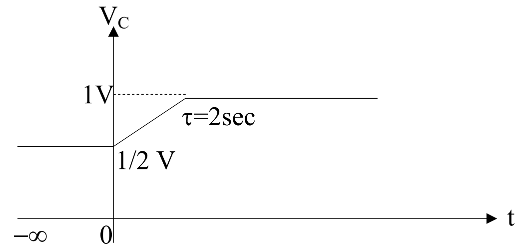
$$\Rightarrow i_c(t)=\frac{1}{4}e^{-\frac{t}{2}}\ \text{A for } t\geq 0$$

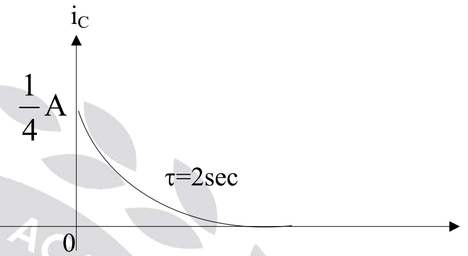
By KVL ⇒

$$V_C(s)-\frac{1}{2s}-\frac{1}{s}I_C(s)=0$$

$$V_C(s)=\frac{1}{2s}+\frac{1}{s}I_C(s)$$

$$v_C(t)=1-\frac{1}{2}e^{-\frac{t}{2}}\ \text{V} \quad \text{for } t\geq 0$$



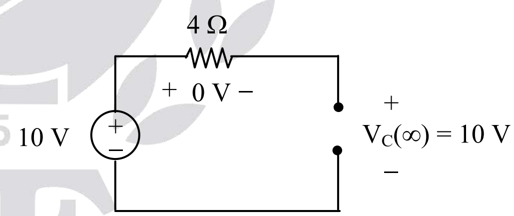


35.

Sol: By Time domain approach ;

$V_C(0) = 6$ V (given)

$V_C(\infty) = 10$ V



At t = ∞ : Steady state : Resistive circuit

$\tau = RC = 8$ sec

$V_C = 10 + (6-10)\,e^{-t/8}$

$V_C = 10 - 4\,e^{-t/8}$

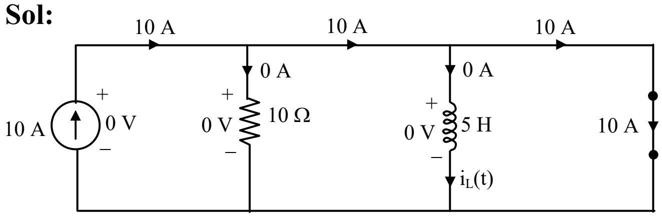
$V_C(0) = 6$ V

$$i_C = C\frac{dV_C}{dt} = e^{-t/8} = i(t)$$

$$E_{4\Omega} = \int_0^\infty (e^{-t/8})^2\ 4\ dt = 16\ \text{J}$$

36.

Sol:

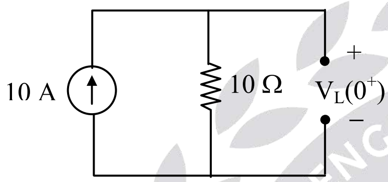


At $t = 0^-$: Network is not in steady state i.e., unenergised

$t = 0^-$:

$i_L(0^-) = 0$ A $= i_L(0^+)$

$V_L(0^+) = 10 \times 10 = 100$ V



At $t = 0^+$: Network is in transient state : A resistive circuit

$i_L(\infty) = 10$ A (since inductor becomes short)

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ sec}$$

$i_L(t) = 10 + (0 - 10)\, e^{-t/\tau}$

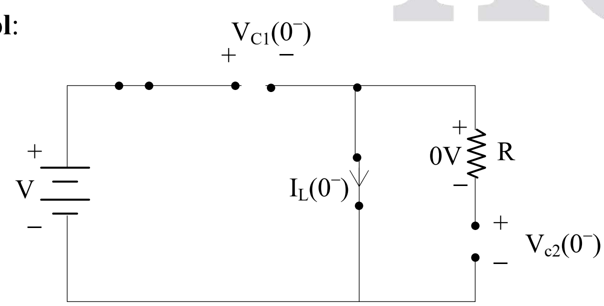
$= 10\,(1 - e^{-t/0.5})$ A for $0 \le t \le \infty$

$$V_L(t) = L\frac{d}{dt} i_L(t) = 100\, e^{-2t} \text{ V for } 0 \le t \le \infty$$

$$E_L\big|_{t=5\tau \text{ or } t=\infty} = \frac{1}{2}Li^2 = \frac{1}{2}\times 5\times 10^2 = 250\text{J}$$

37. Ans: (b)

Sol:



At $t = 0^-$: Steady state: A resistive circuit

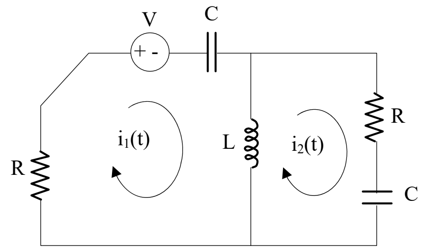
By KVL ⇒

$V - V_{c1}(0^-) = 0$

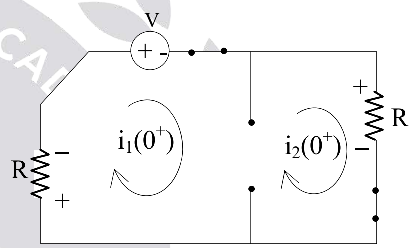
$V_{C1}(0\text{-}) = V = V_{C1}(0^+)$

$V_{C2}(0^-) = 0\text{V} = V_{C2}(0^+)$

$i_L(0^-) = 0\text{A} = i_L(0^+)$



For $t \ge 0$ Fig (a)



At $t = 0^+$: A resistive circuit: Network is in transient state.

$i_1(0^+) = i_2(0^+)$

By KVL ⇒

$-Ri_1(0^+) - V - Ri_1(0^+) = 0$

$$i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$$

OBS: $i_L(t) = i_1(t) \sim i_2(t)$

At $t = 0^+$ ⇒

$i_L(0+) = i_1(0+) \sim i_2(0+)$

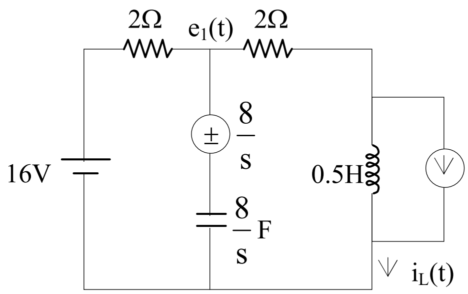
$= 0\text{A}$ ⇒ Inductor: open circuit

38.

Sol: Evaluation of $i_L(t)$ and $e_1(t)$ for $t \ge 0$ by Laplace transform approach.

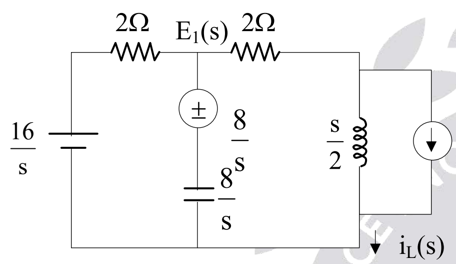
$i_L(0^+) = 6\text{A}$; $i_L(\infty) = 4\text{A}$

$e_1(0^+) = 8\text{V}$; $e_1(\infty) = 8\text{V}$

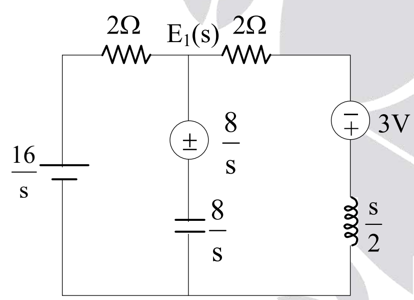


For t ≥ 0

Transform the above network into Laplace domain.



S-domain:



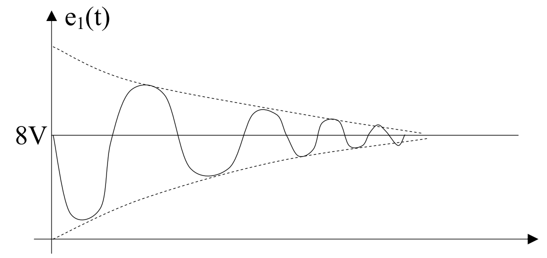
Nodal in S-domain

$$\frac{E_1(s)-16/s}{2}+\frac{E_1(s)-\frac{8}{s}}{\frac{8}{s}}+\frac{E_1(s)+3}{2+\frac{s}{2}}=0$$

$$E_1(s)=\frac{8}{s}\left(\frac{s^2+6s+32}{s^2+8s+32}\right)$$

$$E_1(s)=\frac{8}{s}\left(1-\frac{2s}{(s+4)^2+4^2}\right)$$

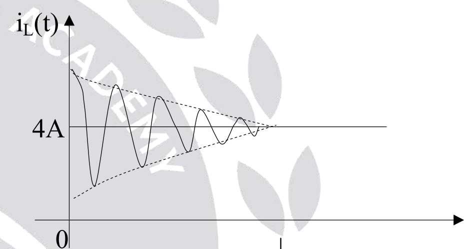
$e_1(t) = 8 - 4e^{-4t}\sin 4t$ V for $t \geq 0$



$$I_L(s)=\frac{E_1(s)+3}{2+\frac{s}{2}}$$

$i_L(t) = 4+2e^{-4t}\cos 4t$ A

for $t \geq 0$ $\omega_n = 4$ rad/sec



OBS: $\tau = \frac{1}{4}\text{sec} = \frac{1}{\xi\omega_n}\Bigg|\ \omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}\times\frac{1}{8}}} = 4$

$$\frac{1}{4}\times\omega_n = \frac{1}{\xi}$$

$$\xi = \frac{4}{\omega_n} = \frac{4}{4} = 1$$

$\xi = 1$ (A critically damped system)

39.

Sol: $\omega t\big|_{t=t_0} = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\omega t_o = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$2\pi(50)t_o = \tan^{-1}\left(\frac{2\pi(50)(0.01)}{5}\right)$$

$$t_o = 32.14\times\frac{\pi}{180^\circ}$$

$t_o = 1.78$ msec.

So, by switching exactly at 1.78msec from the instant voltage becomes zero, the current is free from Transient.

40.

Sol: $\omega t_o + \phi = \tan^{-1}(\omega CR) + \dfrac{\pi}{2}$

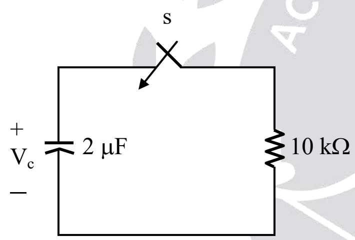
$$2t_o + \frac{\pi}{4} = \tan^{-1}(\omega CR) + \frac{\pi}{2}$$

$$2t_o + \frac{\pi}{4} = \tan^{-1}\left(2\left(\frac{1}{2}\right)(1)\right) + \frac{\pi}{2} = \frac{\pi}{4} + \frac{\pi}{2}$$

$$2t_o = \frac{\pi}{2} \Rightarrow t_o = 0.785\,\text{sec}$$

41. Ans: (b, c, d)

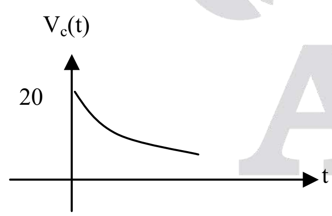
Sol:



(b) $V_c(t) = V_c(0)e^{-t/\tau}$

$V_c = 20e^{-t/\tau} = 20.e^{-t/(1/50)}$

$= 20e^{-50t}$ V



(c) $i_C(t) = C\dfrac{dV_C(t)}{dt}$

$= 2 \times 10^{-6} \times 20\,e^{-50t} \times (-50)$

$i_C(t) = -2e^{-50t}$ mA

(d) $\tau = RC = (10k)\,(2\mu)$

$= 20$ ms

$= \dfrac{20}{1000} = 1/50$ sec

42. Ans: (a, c)

Sol: At $t = 0^+$; $i_1(0) = 2\text{ A} \neq i_2(0) = 1\text{ A}$

So, $2\text{ A} \neq 1\text{ A}$

The given network violates KCL at $t = 0^+$

Constant current applied to inductor the voltage across inductor is impulse.

4. AC Circuit Analysis

01.

Sol: $I_{avg} = I_{dc} = \dfrac{1}{T}\displaystyle\int_0^T i(t)dt$

$= 3 + 0 + 0 = 3\text{A}$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

$$= \sqrt{3^2 + \left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + 0 + 0 + 0}$$

$= 5\sqrt{2}\text{A}$

02.

Sol: $V_{dc} = V_{avg} = \dfrac{1}{T}\displaystyle\int_0^T V(t)dt = 2\text{V}$

Here the frequencies are same, by doing simplification

$v(t) = 2 - 3\sqrt{2}\,(\cos 10t \times \dfrac{1}{\sqrt{2}} - \sin 10t \times \dfrac{1}{\sqrt{2}})$

$+ 3\cos 10t$

$= 2 + 3\sin 10t$ V

So $V_{rms} = \sqrt{(2)^2 + (\dfrac{3}{\sqrt{2}})^2} = \sqrt{8.5}$ V

03.

Sol: $X_{avg} = X_{dc} = \dfrac{1}{T}\displaystyle\int_0^T x(t)dt = 0$

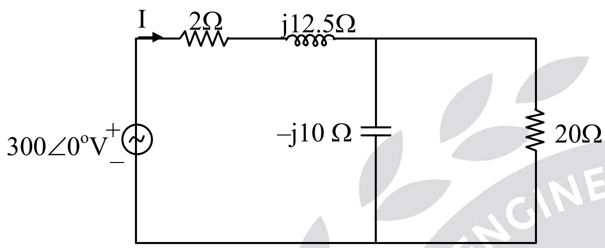
$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \frac{A}{\sqrt{3}}$$

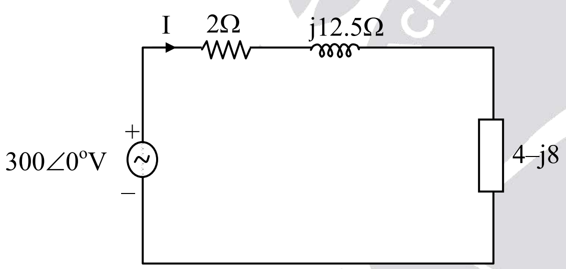
04. Ans: (a)

Sol: For a symmetrical wave (i.e., area of positive half cycle = area of negative half cycle.) The RMS value of full cycle is same as the RMS value of half cycle.

05.

Sol: Complex power, $S = VI^*$





$$\Rightarrow I = \frac{300\angle 0^\circ}{2 + j12.5 + 4 - j8}$$

$\Rightarrow I = 40\angle{-36.86^\circ}$

$\therefore$ Complex power, $S = VI^*$

$= 300\angle 0^\circ \times 40\angle 36.86^\circ$

$= 9600 + j7200$

$\therefore$ Reactive power delivered by the source

$Q = 72000$ VAR

$= 7.2$ KVAR

06.

Sol: $Z = j1 + (1-j1)\|(1 + j2) = 1.4 + j\,0.8$

$$I = \frac{E_1}{Z}\Big|_{\text{By ohm's law}} = \frac{10\angle 20}{1.4 + j8}$$

$$= 6.2017\angle{-9.744^\circ}\ \text{A}$$

$$I_1 = \frac{I(1+j2)}{1 - j1 + 1 + j2}$$

$= 6.2017\angle 27.125^\circ$ A

$$I_2 = \frac{I(1-j1)}{1 - j1 + 1 + j2}$$

$= 3.922\angle{-81.31^\circ}$ A

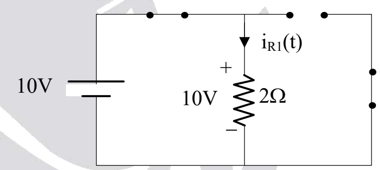
$E_2 = (1-j1)I_1 = 8.7705\angle{-17.875^\circ}$ V

$E_0 = 0.5I_2 = 1.961\angle{-81.31^\circ}$ V

07.

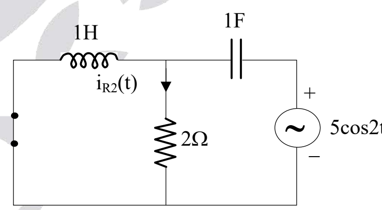
Sol: Since two different frequencies are operating on the network simultaneously always the super position theorem is used to evaluate the response.

By SPT: (i)



Network is in steady state, therefore the network is resistive. $I_{R1}(t) = \frac{10}{2} = 5$A

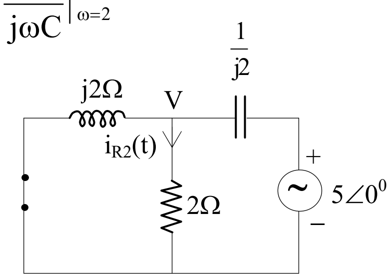
(ii)



Network is in steady state

As impedances of L and C are present because of $\omega = 2$. They are physically present.

$$Z_L = j\omega L;\ Z_c = \frac{1}{j\omega C}\Big|_{\omega=2}$$



Network is in phasor domain

Nodal ⇒

$$\frac{V}{j2}+\frac{V}{2}+\frac{V-5\angle 0^0}{-j0.5}=0$$

$V = 6.32\angle 18.44^0$

$I_{R2} = \dfrac{V}{2} = 3.16\angle 18.44^0 = 3.16\,e^{j18.14^0}$

$i_{R2}(t) = R.P[I_{R2}e^{j2t}]A$

$= 3.16\cos(2t + 18.44^0)$

By super position theorem,

$i_R(t) = i_{R1}(t) + i_{R2}(t)$

$= 5 + 3.16\cos(2t + 18.44^0)A$

08. Ans: (c)

Sol: $\dfrac{1}{s^2+1} - I(s)\left(2+2s+\dfrac{1}{s}\right) = 0$

$$I(s)\left(\frac{2s+2s^2+1}{s}\right)=\frac{1}{s^2+1}$$

$$I(s) + 2s^2I(s) + 2sI(s) = \frac{s}{s^2+1}$$

$$i(t)+\frac{2d^2i}{dt^2}+2\frac{di}{dt}=\cos t$$

$$2\frac{d^2i}{dt^2}+2\frac{di}{dt}+i(t)=\cos t$$

09.

Sol: $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$V = V_R = I.R$

$100 = I.20;\ I = 5A$

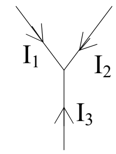
Power factor $= \cos\phi = \dfrac{V_R}{V} = \dfrac{V_R}{V_R} = 1$

So, unity power factor.

10.

Sol: By KCL in phasor - domain

⇒ $-I_1 - I_2 - I_3 = 0$



$I_3 = -(I_1 + I_2)$

$i_1(t) = \cos(\omega t + 90^0)$

$I_1 = 1\angle 90^0 = j1$

$I_2 = 1\angle 0^0 = (1 + j0)$

$I_3 = \sqrt{2}\,\angle\pi + 45^0 = \sqrt{2}\,e^{j(\pi+45)}$

$i_3(t) = \text{Real part}[I_3.e^{j\omega t}]mA$

$= -\sqrt{2}\cos(\omega t + 45^0 + \pi)mA$

$i_3(t) = -\sqrt{2}\cos(\omega t + 45^0)mA$

11.

Sol: $I = \dfrac{V}{R}+\dfrac{V}{Z_L}+\dfrac{V}{Z_C} = 8 - j12 + j18$

$I = 8 + 6j$

$|I| = \sqrt{100} = 10A$

12.

Sol: By KCL ⇒

$-I + I_L + I_C = 0$

$I = I_L + I_C$

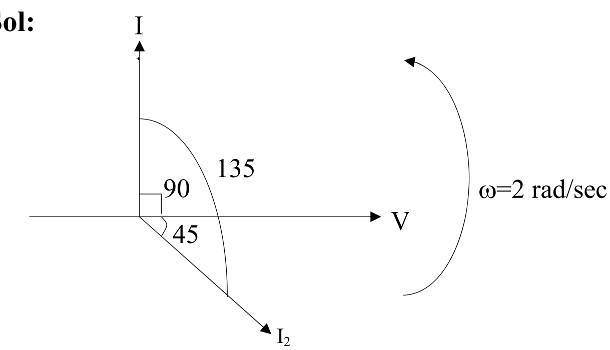
$$I_L = \frac{V}{Z_L}=\frac{V}{j\omega L}=\frac{3\angle 0^\circ}{j(3).\left(\frac{1}{3}\right)}$$

$$I_L = \frac{3\angle 0^0}{j}=\frac{3\angle 0^0}{\angle 90^0} = 3\angle -90^0$$

$I = 3\angle -90^0 + 4\angle 90^0 = -j3 + j4 = j1 = 1\angle 90^0$

13. Ans: (d)

Sol:



$I_1 = I_C = \dfrac{V}{Z_C} = \dfrac{V}{X_C}\angle 90^0$

$I_2 = \dfrac{V}{2+j\omega L} = \dfrac{V}{2+j2} = \dfrac{V}{2\sqrt{2}}\angle 45^0$

Therefore, the phasor I_1 leads I_2 by an angle of 135°.

14.

Sol: $I_2 = \sqrt{I_R^2 + I_C^2} \quad \Rightarrow 10 = \sqrt{I_R^2 + 8^2}$

$I_R = 6A$

$I_1 = I = \sqrt{I_R^2 + (I_L - I_C)^2}$

$10 = \sqrt{6^2 + (I_L - I_C)^2}$

$I_L - I_C = \pm 8A$

$I_L - 8 = \pm 8$

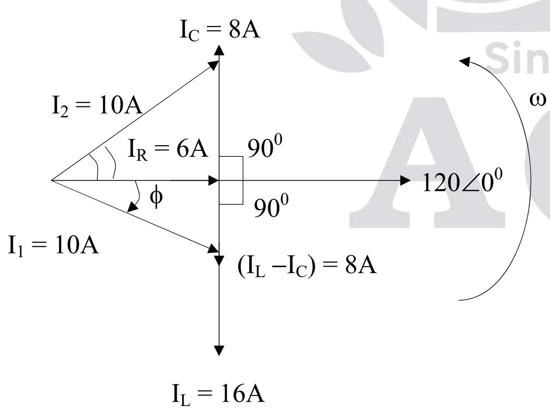
$I_L - 8 = -8$(Not acceptable)

Since $I_L = \dfrac{V}{Z_L} \neq 0.$

$I_L - 8 = 8$

$I_L = 16A$

$I_L > I_C$



I_2 leads $120\angle 0^0$ by $\tan^{-1}\left(\dfrac{8}{6}\right)$

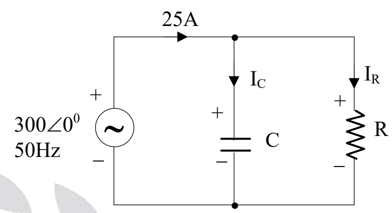
I_1 lags $120\angle 0^0$ by $\tan^{-1}\left(\dfrac{8}{6}\right)$

Power factor $\cos\phi = \dfrac{I_R}{I} = \dfrac{I_R}{I}$

$= \dfrac{6}{10} = 0.6$ (lag)

15.

Sol:



Network is in steady state.

$|I_C| = \left|\dfrac{V}{Z_C}\right| = \left|\dfrac{300\angle 0^0}{(1/j\omega c)}\right| = v\omega c$

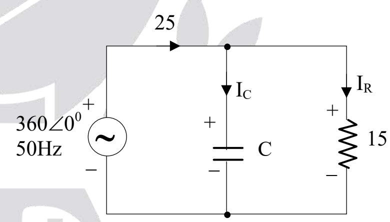
$= 300 \times 2\pi \times 50 \times 159.23 \times 10^{-6}$

$I_C = 15A$

$I = \sqrt{I_R^2 + I_C^2}$

$25 = \sqrt{I_R^2 + 15^2}$

$I_R = 20A$



$V_R = RI_R$|By ohm's law

$300 = R.20$

$R = 15\Omega$

Network is in steady state

$I_R = \dfrac{360}{15} = 24A$

So the required $I_C = \sqrt{25^2 - 24^2}$

$v\omega c = 7$

$360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7$

$f = 19.4Hz$

OBS: $I_C = \dfrac{V}{Z_C}$

$Z_C = \dfrac{1}{j\omega c}\,\Omega$

As $f\downarrow \Rightarrow Z_C\uparrow \Rightarrow I_C\downarrow$

16.

Sol: $P_{5\Omega}$ = 10Watts (Given)

$= P_{avg} = I_{rms}{}^2\, R$

$10 = I_{rms}{}^2.5$

$I_{rms} = \sqrt{2}$ A

Power delivered = Power observed

(By Tellegen's Theorem)

$P_T = I_{rms}{}^2(5 + 10)$

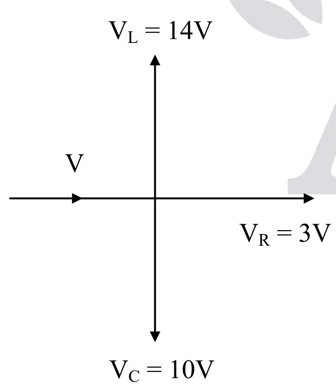
$V_{rms}\, I_{rms}\cos\phi = \left(\sqrt{2}\right)^2 (15)$

$\dfrac{50}{\sqrt{2}}\times\sqrt{2}\,\cos\phi = 2\times 15$

$\cos\phi = 0.6$ (lag)

17. Ans: (d)

Sol:



$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

$$=\sqrt{(3)^2+(14-10)^2}$$

V = 5 V

18.

Sol: $Y = Y_1 + Y_c = \dfrac{1}{Z_L}+\dfrac{1}{Z_C}$

$$= \frac{1}{30\angle 40^0}+\frac{1}{\left(\dfrac{1}{j\omega c}\right)}$$

$$= j\omega c + \frac{1}{30}\angle -40^0$$

$$= j\omega c + \frac{1}{30}(\cos 40^0 - j\sin 40^0)$$

Unit power factor $\Rightarrow$ j-term = 0

$\omega c = \dfrac{\sin 40^0}{30}$

$C = \dfrac{\sin 40^0}{2\pi\times 50\times 30} = 68.1\mu F$

$C = 68.1\mu F$

19. Ans: (b)

Sol: To increase power factor shunt capacitor is to be placed.

VAR supplied by capacitor

$= P(\tan\phi_1 - \tan\phi_2)$

$= 2\times 10^3[\tan(\cos^{-1} 0.65) - \tan(\cos^{-1} 0.95)]$

= 1680 VAR

VAR supplied = $\dfrac{V^2}{X_C} = V^2\omega C = 1680$

$\therefore C = \dfrac{1680}{(115)^2\times 2\pi\times 60} = 337\mu F$

20.

Sol: $Z=\dfrac{V}{I}=\dfrac{160\angle 10^o - 90^o}{5\angle -20^o - 90^o} = 32\angle 30^o$

$\phi = 30^o$ (Inductive)

$V_{rms} = \dfrac{160}{\sqrt{2}}Vj, I_{rms} = \dfrac{5}{\sqrt{2}}$

Real power (P) = $\frac{160}{\sqrt{2}}\times\frac{5}{\sqrt{2}}\times\cos 30^0$

$= 200\sqrt{3}$ W

Reactive power (Q) = $\frac{160}{\sqrt{2}}\times\frac{5}{\sqrt{2}}\times\frac{1}{2}$

= 200 VAR

Complex power = P + jQ = 200($\sqrt{3}$ +j1) VA

21.

Sol: $V = 4\angle 10^o$ and $I = 2\angle -20^o$

Note: When directly phasors are given the magnitudes are taken as rms values since they are measured using rms meters.

$V_{rms} = 4V$ and $I_{rms} = 2A$

$Z = \frac{V}{I} = 2\angle 30^o$; $\phi = 30^o$ (Inductive)

P = $10\sqrt{3}$ W, Q = 10VAR

S = 10($\sqrt{3}$ +j1) VA

22. Ans: (a)

Sol: S = VI*

= $(10\angle 15°)(2\angle 45°)$

= 10 + j17.32

S = P + jQ

P = 10 W   Q = 17.32 VAR

23. Ans: (c)

Sol: $P_R = (I_{rms})^2 \times R$

$I_{rms} = \frac{10}{\sqrt{2}}$

$P_R = \left(\frac{10}{\sqrt{2}}\right)^2 \times 100$

24.

Sol: $P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{240}{\sqrt{2}}\right)^2}{60}$ = 480 Watts

$V = 240\angle 0^0$

$I_R = \frac{V}{R} = \frac{240}{60} = 4A$

$I_L = \frac{V}{Z_L} = \frac{V}{X_L} = \frac{240}{40} = 6A$

$I_C = \frac{V}{Z_C} = \frac{V}{X_C} = \frac{240}{80} = 3A$

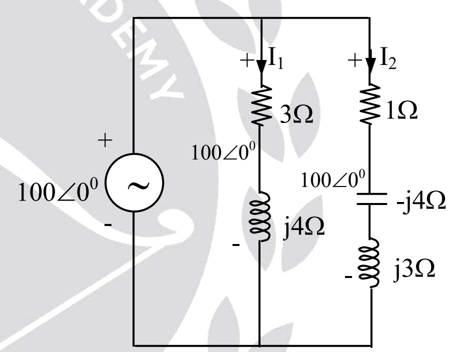
$I_L > I_C$: Inductive nature of the circuit.

$I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{4^2 + 3^2} = 5A$

Power factor = $\frac{I_R}{I} = \frac{4}{5} = 0.8$ (lagging)

25. Ans: (a)

Sol:



NW is in Steady state.

$V = 100\angle 0^0 \Rightarrow V_{rms} = 100V$

$I_1 = \frac{100\angle 0^0}{(3+j4)\Omega} \Rightarrow |I_1| = 20 = I_{1rms}$

$I_2 = \frac{100\angle 0^0}{(1-j1)\Omega} \Rightarrow |I_2| = \frac{100}{\sqrt{2}}A = I_{2rms}$

$P = P_1 + P_2$

$= (I_{1rms})^2.3 + (I_{2rms})^2.1$

$= 20^2.3 + \left(\frac{100}{\sqrt{2}}\right)^2.1$

P = 6200 W

$Q = Q_1 + Q_2$

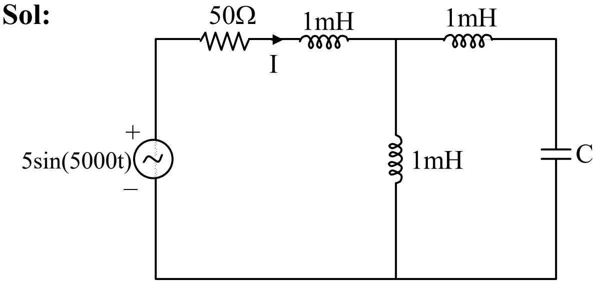
$= (I_{1rms})^2.4 + (I_{2rms})^2.(1)$

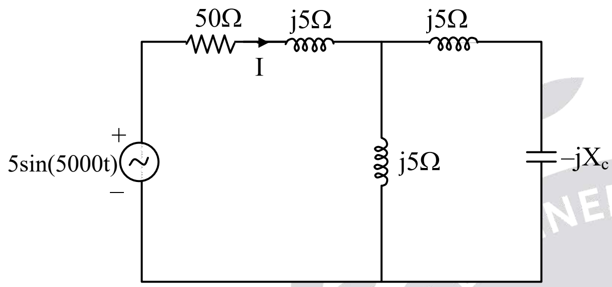
= 3400VAR

So, S = P+jQ = (6200+j3400) VA

26.

Sol:





when I = 0,

⇒ impedance seen by the source should be infinite

$\Rightarrow Z = \infty$

$\therefore Z = (50+j5) + (j5) \parallel j(5 - X_c)$

$$= 50 + j5 + \frac{j5 \times j(5 - X_c)}{j5 + j(5 - X_c)} = \infty$$

$\Rightarrow j\,(10 - X_c) = 0$

$$\Rightarrow X_c = 10 \Rightarrow \frac{1}{\omega c} = 10$$

$$\Rightarrow C = \frac{1}{5000 \times 10} = 20\ \mu F$$

27. Ans: (c)

Sol: $$I_{rms} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$$

$$= \sqrt{25} = 5\,A$$

Power dissipation = $I_{rms}^2\ R$

$= 5^2 \times 10$

$= 250$ W

28.

Sol: $X_C = X_L$

$\Rightarrow \omega = \omega_0$, the circuit is at resonance

$V_C = QV_S\angle{-90^0}$

$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2$$

$$= \frac{1}{\omega_0 cR} = \frac{X_C}{R} = 2$$

$\Rightarrow V_C = 200\angle{-90^0}$

$= -j200V$

29.

Sol: Series RLC circuit

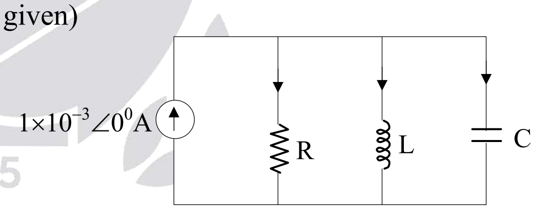
$f = f_L$, PF = $\cos\phi = 0.707$(lead)

$f = f_H$, PF = $\cos\phi = 0.707$(lag)

$f = f_o$, PF = $\cos\phi = 1$

30. Ans: (b)

Sol: Network is in steady state (since no switch is given)



Let I = 1mA

$\omega = \omega_0$(Given)

$\Rightarrow I_R = I$

$I_L = QI\angle{-90^0} = -jQI$

$I_C = QI\angle 90^0 = jQI$

$I_L + I_C = 0$

$|I_R + I_L| = |I - jQI|$

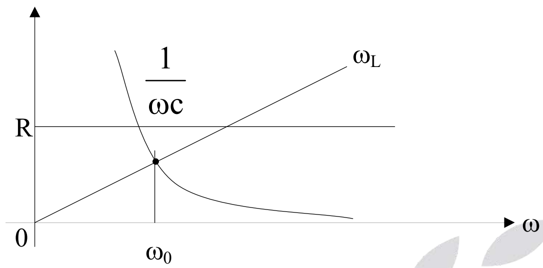
$$= I\sqrt{1+Q^2} > I$$

$|I_R + I_C| = |I + jQI|$

$$= I\sqrt{1+Q^2} > I$$

31. Ans: (c)

Sol: Since; "I" leads voltage, therefore capacitive effect and hence the operating frequency ($f < f_0$)



32.

Sol: $Y = \dfrac{1}{R_L + j\omega L} + \dfrac{1}{R_C - \dfrac{j}{\omega C}}$

$$= \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{R_C + j/\omega c}{R_C^2 + (1/\omega C)^2}$$

$j - \text{term} \Rightarrow 0$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \dfrac{L}{C}}{R_C^2 - \dfrac{L}{C}}} \text{ rad/sec}$$

33.

Sol:

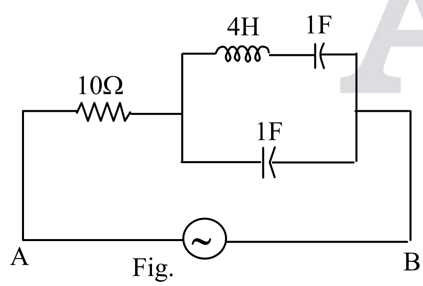


Fig.

The given circuit is shown in Fig.

$Z_{AB} = 10 + Z_1$

where, $Z_1 = \left(\dfrac{-j}{\omega}\right) || \left(j4\omega - \dfrac{j}{\omega}\right)$

$$= \frac{\left(\dfrac{-j}{\omega}\right)\left(j4\omega - \dfrac{j}{\omega}\right)}{\dfrac{-j}{\omega} + j4\omega - \dfrac{j}{\omega}}$$

$$= \frac{4 - \dfrac{1}{\omega^2}}{j4\omega - \dfrac{j2}{\omega}}$$

For circuit to be resonant i.e., $\omega^2 = \dfrac{1}{4}$

$\omega = \dfrac{1}{2} = 0.5$ rad/sec

$\therefore \omega_{resonance} = 0.5$ rad/sec

34.

Sol: (i) $\dfrac{L}{C} = R^2 \Rightarrow$ circuit will resonate for all the frequencies, out of infinite number of frequencies we are selecting one frequency.

i.e., $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{2}$ rad/sec

then $Z = R = 2\Omega$.

$I = \dfrac{V}{Z} = \dfrac{10\angle 0^0}{2} = 5\angle 0^0$

$i(t) = 5\cos\dfrac{t}{2}$ A

$Z_L = j\omega_0 L = j2\Omega$; $Z_C = \dfrac{1}{j\omega_0 c} = -j2\Omega$.

$I_L = \dfrac{I(2 - j2)}{2 + j2 + 2 - j2} = \dfrac{I}{\sqrt{2}}\angle -45^0$

$i_L = \dfrac{5}{\sqrt{2}}\cos\left(\dfrac{t}{2} - 45^0\right)$ A

$i_c = \dfrac{I(2 + j2)}{2 + j2 + 2 - j2} = \dfrac{I}{\sqrt{2}}\angle 45^0$

$i_c = \dfrac{5}{\sqrt{2}}\cos\left(\dfrac{t}{2} + 45^o\right)$ A

$$P_{avg} = I^2_{L(rms)}.R + I^2_{c(rms)}.R$$

$$= \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2.2 + \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2.2$$

= 25 watts

(ii) $\frac{L}{C} \neq R^2$ circuit will resonate at only one frequency.

i.e., at $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4}$ rad/sec

Then $Y = \frac{2R}{R^2 + \frac{L}{C}}$ mho

$$Y = \frac{2(2)}{2^2 + \frac{4}{4}} = \frac{4}{5}\text{ mho}$$

$$Z = \frac{5}{4}\,\Omega$$

$$I = \frac{V}{Z} = \frac{10\angle 0^0}{5/4} = 8\angle 0^0$$

$$i(t) = 8\cos\frac{t}{4}\text{ A}$$

$Z_L = j\omega_0 L = j1\Omega$

$$Z_c = \frac{1}{j\omega_0 C} = -j1\Omega$$

$$I_L = \frac{I(2-j1)}{2+j1+2-j1} = \frac{\sqrt{5}}{4}I.\angle\tan^{-1}\left(\frac{1}{2}\right)$$

$$i_L = \frac{8\sqrt{5}}{4}\cos\left(\frac{t}{4} - \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$I_c = \frac{I(2+j1)}{2+j1+2-j1} = \frac{\sqrt{5}}{4}I\angle\tan^{-1}\left(\frac{1}{2}\right)$$

$$i_c = \frac{8\sqrt{5}}{4}\cos\left(\frac{t}{4} + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$P_{avg} = I^2_{Lrms}.R + I^2_{Crms}R$$

$$= \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2 + \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2$$

= 40 watts

35.

Sol: (i) $Z_{ab} = 2 + (Z_L \parallel Z_C \parallel 2)$

$= 2 + jX_L \| -jX_C \| 2$

$$= \frac{2 + 2X_LX_C(X_LX_C - j2(X_L - X_C))}{(X_LX_C)^2 + 4(X_L - X_C)^2}$$

j–term = 0

$\Rightarrow -2(X_L - X_C) = 0$

$X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.4}} = \frac{1}{4}\text{ rad/sec}$$

At resonance entire current flows through 2Ω only.

(ii) $Z_{ab}\big|_{\omega=\omega_0} = 2 + 2 = 4\Omega$

$X_L = X_C$

(iii) $V_i(t) = V_m\sin\left(\frac{t}{4}\right)V$

$Z = 4\Omega$

$$i(t) = \frac{V_i(t)}{Z} = \frac{V_m}{4}\sin\left(\frac{t}{4}\right) = i_R$$

$$V = 2i_R = \frac{V_m}{2}\sin\left(\frac{t}{4}\right)V = V_C = V_L$$

$$i_C = C\frac{dV_C}{dt} = \frac{V_m}{2}\cos\left(\frac{t}{4}\right)$$

$$i_c = \frac{V_m}{2}\sin\left(\frac{t}{4} + 90^0\right)A$$

$$i_L = \frac{1}{L}\int V_L.dt = \frac{-V_m}{2}\cos\left(\frac{t}{4}\right)$$

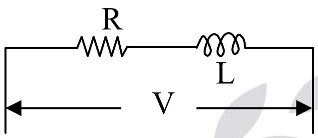
$$i_L = \frac{V_m}{2}\sin\left(\frac{t}{4} - 90^0\right)A$$

OBS: Here $i_L + i_C = 0$

$\Rightarrow$ LC Combination is like an open circuit.

36. Ans: (d)

Sol:



$$Q = \frac{\omega L}{R}$$

$$Q = \frac{2\omega L}{R} = 2 \times \text{orginal} \rightarrow Q - \text{doubled}$$

$$S = V.I \;\; = V.\frac{V}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L}$$

$$S = \frac{V^2}{R^2 + (\omega L)^2} - \frac{V^2.j\omega L}{R^2 + (\omega L)^2}$$

$S = P + jQ$

$$\text{Active power } (P) = \frac{V^2}{R^2 + (\omega L)^2}$$

$$P = \frac{V^2}{R^2(1 + Q^2)}$$

$$P \approx \frac{V^2}{R^2 Q^2}$$

As Q is doubled, P decreases by four times.

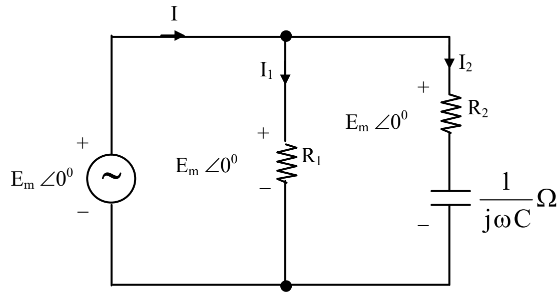
37.

Sol: $Z_C = \dfrac{1}{j\omega C}$

$\omega = 0; Z_C = \infty \;\Rightarrow C:\text{open circuit} \Rightarrow i_2 = 0$

$\omega = \infty; Z_C = 0 \Rightarrow C:\text{Short Circuit} \Rightarrow i_2 = \dfrac{E_m}{R_2}\angle 0^o$

Transform the given network into phasor domain.



Network is in phasor domain.

By KCL in P-d $\Rightarrow$ $I = I_1 + I_2$

$$I_1 = \frac{E_m\angle 0^o}{R_1}$$

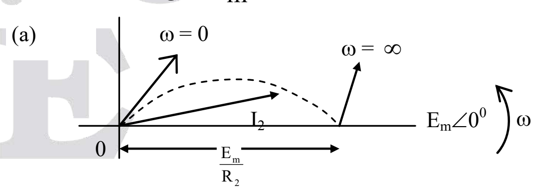
$$I_2 = \frac{E_m\angle 0^o}{R_2 + \frac{1}{j\omega C}} = \frac{E_m\angle 0^o}{R_2 - \frac{j}{\omega C}}$$

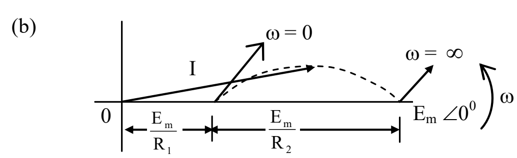
$$I_2 = \frac{E_m\angle \tan^{-1}\left(\frac{1}{\omega CR_2}\right)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)}}$$

$$\omega = \infty \Rightarrow I_2 = \frac{E_m\angle 0^o}{R_2}$$

$\omega = 0 \Rightarrow I_2 = 0A$

$\omega : (0 \text{ and } \infty)$ j the current phasor I_2 will always lead the voltage $E_m\angle 0^o$.

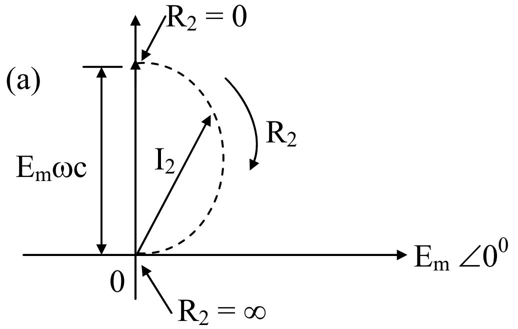


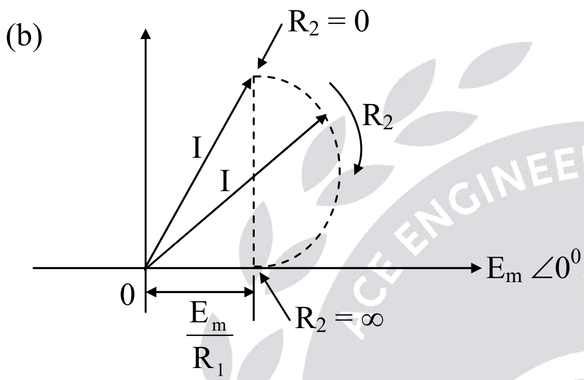


38.

Sol: $R_2 = 0 \Rightarrow I_2 = \dfrac{E_m\angle 0^o}{0 + \frac{1}{j\omega C}} = E_m\omega C\angle 90^o$

$R_2 = \infty \Rightarrow I_2 = 0\,A$





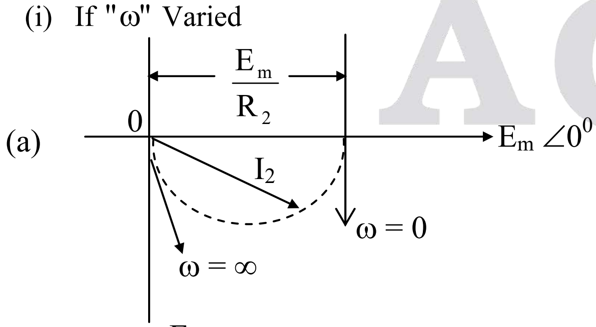
39.

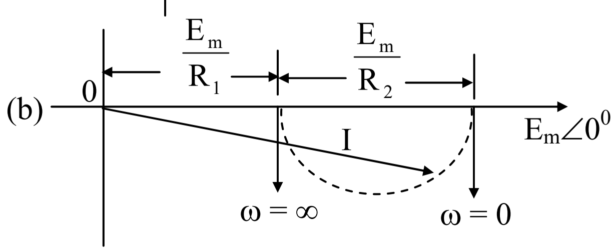
Sol: $I = I_1 + I_2;\; I_1 = \dfrac{E_m\angle 0^o}{R_1}$

$$I_2 = \frac{E_m\angle 0^o}{R_2 + j\omega L}$$

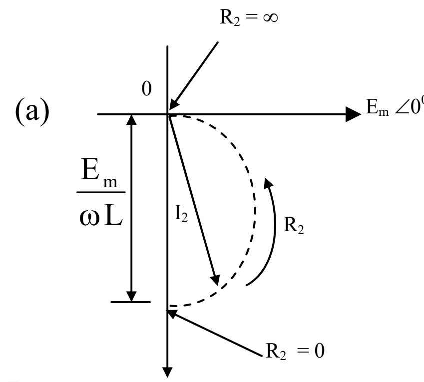
$$= \frac{E_m}{\sqrt{R_2^2 + (WL)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R_2}\right)$$

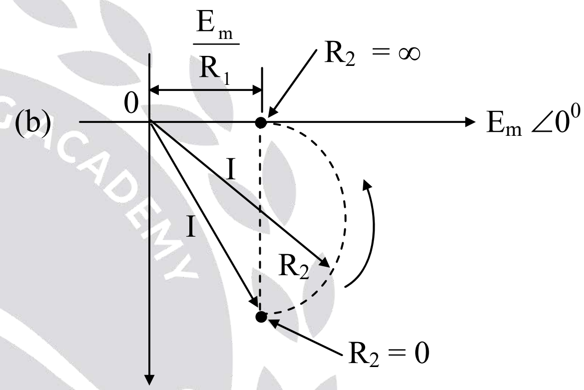
(i) If "ω" Varied





ii. If "R_2" is varied





40. Ans: (a)

Sol: The given circuit is a bridge.

$I_R = 0$ is the bridge is balanced. i.e., $Z_1Z_4 = R_2 R_3$

Where $Z_1 = R_1 + j\omega L_1$,

$$Z_4 = R_4 - \frac{j}{\omega C_4}$$

As $R_2 R_3$ is real, imaginary part of $Z_1 Z_4 = 0$

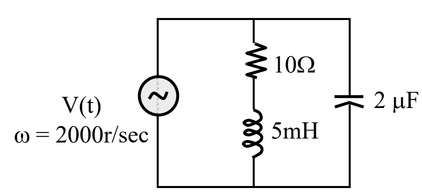
$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \quad \text{or} \quad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

or $Q_1 = Q_4$

Where Q is the Quality factor.

41. Ans: (b, c)

Sol:



$$Y_T = \frac{1}{10 + j\omega(5m)} + j\omega\,(2\mu)$$

$$= \frac{10 - j\omega(5m)}{100 + \omega^2(25\mu)} + j\omega\,(2\mu)$$

$$= \frac{j\omega(5m)}{100 + \omega^2(25\mu)} + j\omega\,(2\mu)$$

$2500 = 100 + \omega^2\,(25\mu)$

$2400 = \omega^2\,(25)\ \mu$

$24 \times 4\ M = \omega^2$

$\omega = 9.8$ rad/sec

(Q) pf of coil = $\frac{R}{Z} = \frac{10}{\sqrt{10^2 + (5m \times 2000)^2}}$

$= \frac{1}{\sqrt{2}} = 0.707$ lag

(R) Q-factor = $\frac{\omega L}{R} = \frac{(2000)(5m)}{10} = 1$

$\therefore$ (b,c) are correct

42. Ans: (a, d)

Sol: $R = 30\ \Omega$, $X_L = 60\ \Omega$, $X_C = 20\ \Omega$

$V(t) = 100\sin 10\omega t$

(a) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$= \tan^{-1}\left(\frac{40}{30}\right) = 53.13°$ lag

(b) p.f = cos 53.13° = 0.6 lag

(c) current is lagging by 53.13°

(d) p.f = cos 53.13° = 0.6 lag

$\therefore$ a, d are correct

5. Magnetic Circuits

01.

Sol: $X_C = 12$ (Given)

$X_{eq} = 12$ (must for series resonance)

So the dot in the second coil at point "Q"

$L_{eq} = L_1 + L_2 - 2M$

$L_{eq} = L_1 + L_2 - 2K\sqrt{L_1 L_2}$

$\omega L_{eq} = \omega L_1 + \omega L_2 - 2K\sqrt{L_1 L_2 \omega.\omega}$

$12 = 8 + 8 - 2K\sqrt{8.8}$

$\Rightarrow K = 0.25$

02.

Sol: $X_C = 14$ (Given)

$X_{Leq} = 14$ (must for series resonance)

So the dot in the 2nd coil at "P"

$L_{eq} = L_1 + L_2 + 2M$

$L_{eq} = L_1 + L_2 + K\sqrt{L_1 L_2}$

$\omega L_{eq} = \omega L_1 + \omega L_2 + 2K\sqrt{\omega L_1 L_2 \omega}$

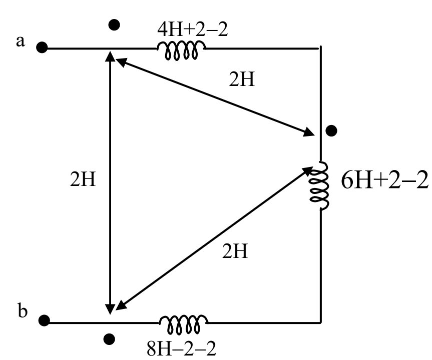
$14 = 2 + 8 + 2K\sqrt{2(8)}$

$\Rightarrow K = 0.5$

03.

Sol: $L_{ab} = 4H+2-2+6H+2-2+8H-2-2$

$L_{ab} = 14H$



04. Ans: (c)

Sol: Impedance seen by the source

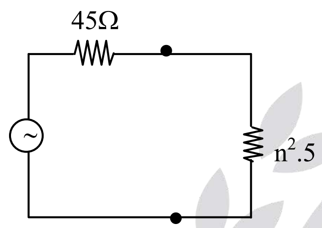
$$Z_s = \frac{Z_L}{16} + (4 - j2)$$

$$= \frac{10\angle 30^o}{16} + (4 - j2)$$

$$= 4.54 - j1.69$$

05.

Sol:



$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 . Z_L$$

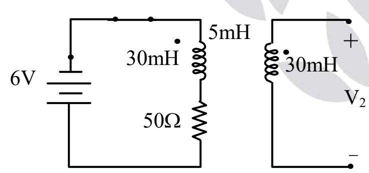
$$R_{in}^{'} = n^2.5$$

For maximum power transfer; $R_L = R_s$

$n^2 5 = 45 \Rightarrow n = 3$

06. Ans: (b)

Sol:



Apply KVL at input loop

$$-6 - 30\times10^{3}\frac{di_1}{dt} + 5\times10^{3}\frac{di_2}{dt} - 50i_1 = 0 \ldots(1)$$

Take Laplace transform

$$-\frac{6}{s} + [-30\times10^{-3}(s) - 50]I_1(s) + 5\times10^{-3}sI_2(s) = 0 \ldots(2)$$

Apply KVL at output loop

$$V_2(s) - 30\times10^{-3}\frac{di_2}{dt} + 5\times10^{-3}\frac{di_1}{dt} = 0$$

Take Laplace transform

$$V_2(s) - 30\times10^{-3}sI_2(s) + 5\times10^{-3}sI_1(s) = 0$$

Substitute $I_2(s) = 0$ in above equation

$V_2 + 5 \times 10^{-3}\, sI_1(s) = 0$ ……… (3)

From equation (2)

$$-\frac{6}{s} + (-30\times10^{-3}(s) + 50)\,I_1(s) = 0$$

$$I_1(s) = \frac{-6}{s\,(30\times10^{-3}(s) + 50)} \quad \text{……… (4)}$$

Substitute eqn (4) in eqn (3)

$$V_2(s) = \frac{-5\times10^{-3}(s)\,(-6)}{s(30\times10^{-3}(s) + 50)}$$

Apply Initial value theorem

$$\underset{s\to\infty}{Lt}\ s\ \frac{-5\times10^{-3}(s)(-6)}{s\,(30\times10^{-3}(s) + 50)}$$

$$v_2(t) = \frac{-5\times10^{-3}\times(-6)}{30\times10^{-3}} = +1$$

07.

Sol: $R_{in}' = \frac{8}{2^2} = 2\Omega$

$R_{in} = 3 + R_{in}' = 3 + 2 = 5\Omega$

$$I_1 = \frac{10\angle 20}{5} = 2\angle 20^0$$

$$\frac{I_1}{I_2} = n = 2 \Rightarrow I_2 = 1\angle 20°A$$

08.

Sol: By the definition of KVL in phasor domain

$V_S – V_0 – V_2 = 0$

$$V_0 = V_S - V_2 = V_S\left(1 - \frac{V_2}{V_S}\right)$$

V= ZI

By KVL

$V_S = j\omega L_1.I_1 + j\omega M\,(0)$

$V_2 = j\omega L_2(0) + j\omega MI_1$

$$V_0 = V_S\left(1 - \frac{M}{L_1}\right)$$

6. Two Port Networks

01.

Sol: The defining equations for open circuit impedance parameters are:

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$[Z] = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix}\Omega$$

02. **Ans: (b)**

Sol: The matrix given is $\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

since $y_{11} \neq y_{22}$

$\Rightarrow$ Asymmetrical, and

$y_{12} \neq y_{21}$ ‘

$\Rightarrow$ Non reciprocal network

03.

Sol: Convert Y to Δ :

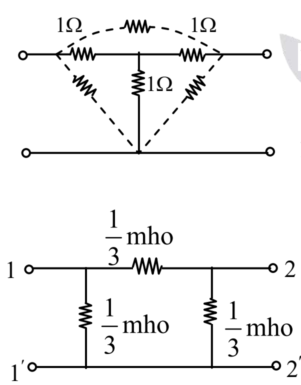


Fig: A

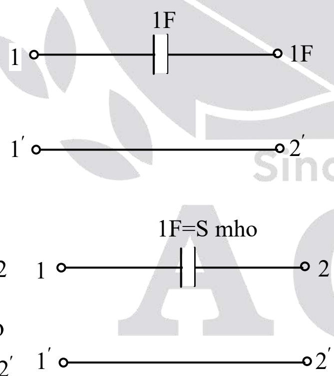


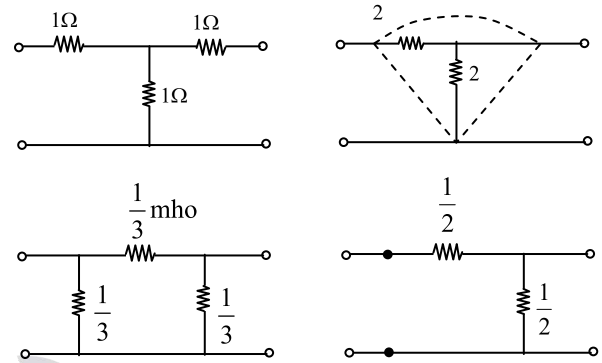
Fig: B

$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad Y_B = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix}$$

$$Y = \begin{bmatrix} S+\frac{2}{3} & -S-\frac{1}{3} \\ -S-\frac{1}{3} & S+\frac{2}{3} \end{bmatrix}\text{mho}$$

04.

Sol:

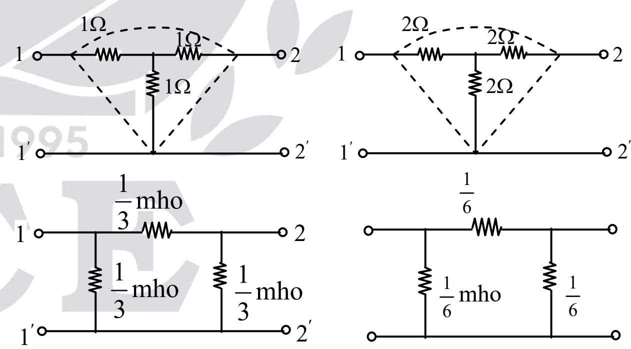


$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad Y_B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{7}{6} & -\frac{5}{6} \\ -\frac{5}{6} & \frac{5}{3} \end{bmatrix}$$

05.

Sol: Convert Y to Δ : Convert Y to Δ :



$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}\text{mho} \qquad Y_B = \begin{bmatrix} \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{2}{6} \end{bmatrix}\text{mho}$$

$$Y = \begin{bmatrix} \frac{6}{6} & -\frac{3}{6} \\ -\frac{3}{6} & \frac{6}{6} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

06.

Sol: $T_1 = T_2 = \begin{bmatrix} 1+\dfrac{1}{-j1} & 1 \\ \dfrac{1}{-j1} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+j & 1 \\ j & 1 \end{bmatrix}$

$T_3 \Rightarrow Z_1 = 1\Omega;\ Z_2 = \infty$

$T_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$T = (T_1)(T_2)(T_3)$

$T = \begin{bmatrix} j3 & 2+j4 \\ -1+j2 & j3 \end{bmatrix}$

07.

Sol: $T_1 : Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$T_1 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$T_2 : Z_1 = 0\ ;\ Z_2 = 2\ \Omega$

$T_2 = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{2} & 1 \end{bmatrix}$

$T = [T_1]\ [T_2]$

$T = \begin{bmatrix} 3.5 & 3 \\ 2 & 2 \end{bmatrix}$

08. Ans: (a)

Sol: For $I_2 = 0$ (O/P open), the Network is shown in Fig.1

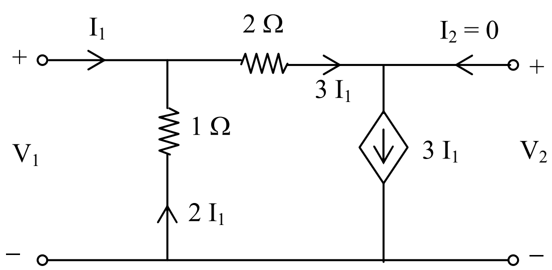


Fig. 1

$V_1 = -2\ I_1$ ………… (1)

$Z_{11} = \dfrac{V_1}{I_1} = -2$

$V_2 = -6\ I_1 + V_1$ …………… (2)

From (1) and (2)

$V_2 = -6\ I_1 - 2\ I_1$

or $V_2 = -8\ I_1$

$Z_{21} = \dfrac{V_2}{I_1} = -8$

For $I_1 = 0$ (I/P open), the network is shown in Fig.2

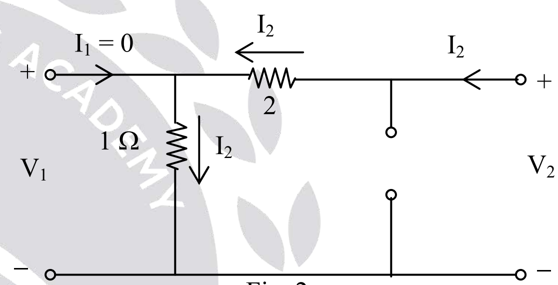


Fig. 2

Note: that the dependent current source with current $3\ I_1$ is open circuited.

$V_1 = 1\ I_2$, $\quad Z_{12} = \dfrac{V_1}{I_2} = 1$

$V_2 = 3\ I_2$, $\quad Z_{22} = \dfrac{V_2}{I_2} = 3$

$[Z] = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$

09.

Sol: By Nodal

$-I_1 + V_1 - 3V_2 + V_1 + 2V_1 - V_2 = 0$

$-I_2 + V_2 + V_2 - 2V_1 = 0$

$Y = \begin{bmatrix} 4 & -4 \\ -3 & 2 \end{bmatrix} \mho$

$[Z] = Y^{-1}$

We can also obtain [g], [h], [T] and $[T]^{-1}$ by re-writing the equations.

10.

Sol: The defining equations for open-circuit impedance parameters are:

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

In this case, the individual Z-parameter matrices get added.

$(Z) = (Z_a) + (Z_b)$

$$[Z] = \begin{bmatrix} 10 & 2 \\ 2 & 7 \end{bmatrix}\Omega$$

11.

Sol: For this case the individual y-parameter matrices get added to give the y-parameter matrix of the overall network.

$Y = Y_a + Y_b$

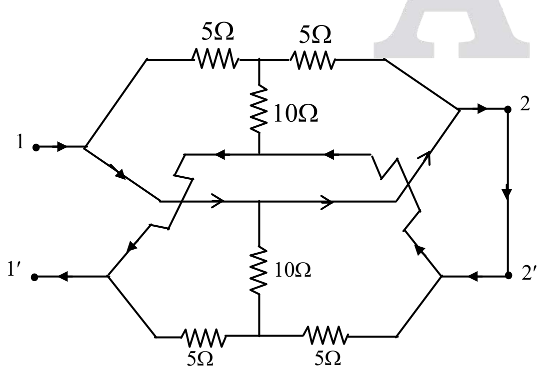
The individual y-parameters also get added

$Y_{11} = Y_{11a} + Y_{11b}$ etc

$$[Y] = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix}\text{mho}$$

12. Ans: (c)

Sol: $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$



$$Y_{11} = \frac{I_1}{0} = \infty$$

13.

Sol: (i). $[T_a] = \begin{bmatrix} 1 + \dfrac{Z_1}{Z_2} & Z_1 \\ \dfrac{1}{Z_2} & 1 \end{bmatrix}$

(ii). $[T_a] = \begin{bmatrix} 1 & Z_1 \\ \dfrac{1}{Z_2} & 1 + \dfrac{Z_1}{Z_2} \end{bmatrix}$

$[T_a]$ and $[T_b]$ are obtained by defining equations for transmission parameters.

14.

Sol: In this case, the individual T-matrices get multiplied

$(T) = (T_1) \times (T_{N1})$

$$(T) = (T_1)(T_{N1}) = \begin{pmatrix} 1 + s/4 & s/2 \\ 1/2 & 1 \end{pmatrix}\begin{pmatrix} 8 & 4 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3s + 8 & 3.5s + 4 \\ 6 & 7 \end{pmatrix}$$

15.

Sol: $Z_{in} = R_{in} = \dfrac{V_1}{I_1} = \dfrac{AV_2 - BI_2}{CV_2 - DI_2} = \dfrac{V_2 - 2I_2}{V_2 - 3I_2}$,

$V_2 = 10(-I_2)$

$Z_{in} = R_{in} = \dfrac{12}{13}\Omega$

16.

Sol: $\left.\dfrac{V_1}{I_1}\right|_{I_2=0} = Z_{11}$

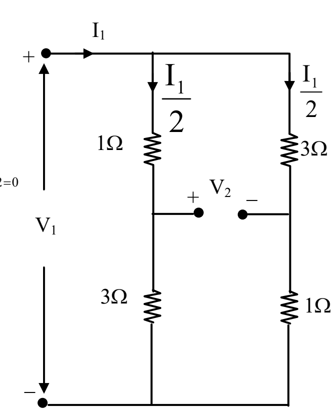
$\Rightarrow\ V_1 = (4 \parallel 4)I_1\,|_{I_2=0}$

$\Rightarrow Z_{11} = 2\Omega$

$V_2 = (4 \parallel 4)I_2\big|_{I_1=0}$

$\Rightarrow Z_{22} = 2\Omega$

By KVL $\Rightarrow$



$$\frac{3I_1}{2} - V_2 - \frac{I_1}{2} = 0$$

$$V_2 = I_1$$

$\Rightarrow Z_{21} = 1\Omega = Z_{12}$

$$Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\Omega$$

$$Y = Z^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}\mho$$

Now [T] parameters;

$V_1 = 2I_1 + I_2$ …….. (1)

$V_2 = I_1 + 2I_2$ ………. (2)

$\Rightarrow I_1 = V_2 - 2I_2$ ……… (3)

Substituting (3) in (1):

$V_1 = 2(V_2 - 2I_2) + I_2 = 2V_2 - 3I_2$ ……(4)

$$T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$T^1 = T^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Now h parameters

$$2I_2 = -I_1 + V_2$$

$I_2 = \frac{-I_1}{I_2} + \frac{V_2}{2}$ ……… (5)

Substitute (5) in (1)

$$V_1 = 2I_1 \frac{-I_1}{2} + \frac{V_2}{2}$$

$V_1 = \frac{3}{2}I_1 + \frac{1}{2}V_2$ ………. (6)

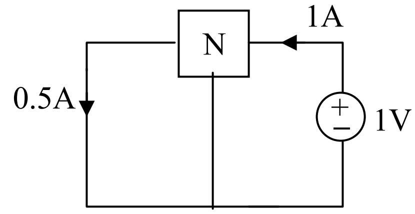
$$h = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

$$g = [h]^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

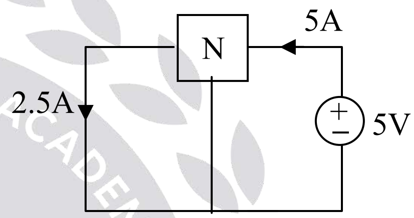
17. Ans: (a)

Sol: $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

Just use reciprocity of fig (a)



Now use Homogeneity



$$\text{So, } Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{5}{5} = 1\,\text{mho}$$

This has noting to do with fig (b) since fig (b) also valid for some specific resistance of 2 Ω at port-1, but Y_{22} , $V_1 = 0$. So S.C port-1

18.

Sol: $\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \frac{-I_1}{I_2}$

$$\frac{V_2}{V_1} = n$$

$$\Rightarrow V_1 = \frac{1}{n}V_2 - (0)I_2$$

$$\Rightarrow T = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

$$T^1 = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$T^1 = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

Now h-parameters

$$V_1 = (0)I_1 + \frac{1}{n}V_2$$

$$I_2 = \frac{-I_1}{n} + (0)V_2$$

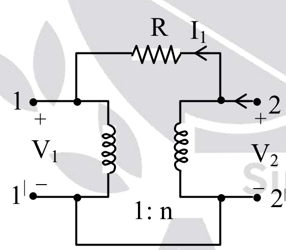
$$g = \begin{bmatrix} 0 & \frac{1}{n} \\ \frac{-1}{n} & 0 \end{bmatrix}$$

$$h = \begin{bmatrix} 0 & -n \\ n & 0 \end{bmatrix}$$

Note: In an ideal transformer, it is impossible to express V_1 and V_2 interms of I_2 and I_2, hence the 'Z' parameters do not exist. Similarly, the y-parameters.

19. Ans: (c)

Sol: $Z_{22} = \left.\frac{V_2}{I_2^1}\right|_{V_1=0}$



$$\frac{V_1}{V_2} = \frac{1}{n} = \frac{I_2}{I_1}$$

$$V_1 = \frac{1}{n}V_2$$

$$\frac{V_2 - V_1}{R} = I_1$$

$$I_2^1 = I_2 + I_1$$

$$\frac{1}{n} = \frac{I_2}{I_1} = \frac{I_2^1 - I_1}{I_1} = \frac{I_2^1}{I_1} - 1$$

$$\frac{I_2^1}{I_1} = \frac{1}{n} + 1 = \frac{1+n}{n}$$

$$I_2^1 = \left(\frac{1+n}{n}\right)I_1$$

$$I_2^1 = \left(\frac{1+n}{n}\right)\left(\frac{V_2 - V_1}{R}\right)$$

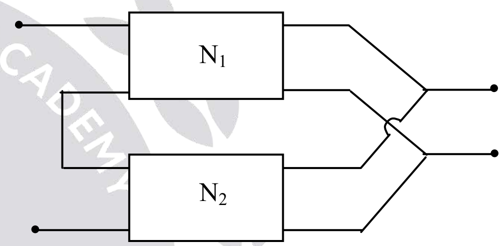
$$I_2^1 = \left(\frac{1+n}{n}\right)\left(\frac{V_2 - \frac{1}{n}V_2}{R}\right)$$

$$\frac{I_2^1}{V_2} = \left(\frac{1+n}{n}\right)\left(\frac{n-1}{nR}\right)$$

$$\frac{V_2}{I_2^1} = \frac{n^2 R}{n^2 - 1}$$

20.

Sol:



For series parallel connection individual h-parameters can be added.

$\therefore$ For network 1, $h_1 = g_1^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

For network 2, $h_2 = g_2^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore h = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

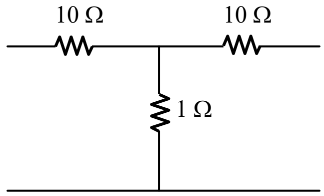
$\therefore$ overall g-parameters,

$$g = h^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$g = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

21. Ans: (a, b)

Sol:



$$[Z] = \begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix}$$

$Z_{22} = 11$, $Z_{12} = 1$

$$[y] = [Z]^{-1} = \frac{1}{121-1}\begin{bmatrix} 11 & -1 \\ -1 & 11 \end{bmatrix}$$

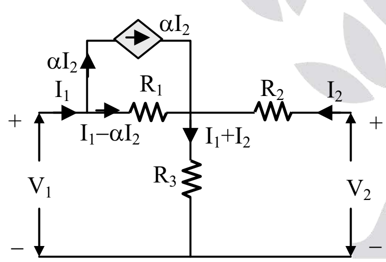
$$= \begin{bmatrix} \frac{11}{120} & \frac{-1}{120} \\ \frac{-1}{120} & \frac{11}{120} \end{bmatrix}$$

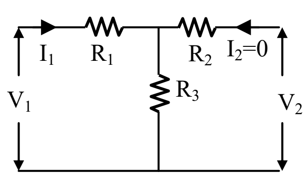
$$Y_{11} = \frac{11}{120}\ \mho,\ Y_{12} = \frac{-1}{120}\ \mho$$

22. Ans: (a, c)

Sol: $V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$





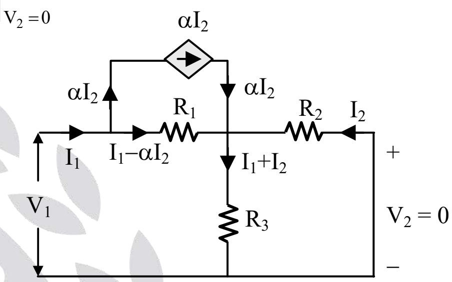
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$V_2 = V_1\frac{R_3}{R_1+R_3}$$

$$\Rightarrow A = \frac{V_1}{V_2} = \frac{R_1+R_3}{R_3}$$

$$\Rightarrow C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{\frac{V_1}{R_1+R_3}}{V_1\frac{R_3}{R_1+R_3}} = \frac{1}{R_3}$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$



$V_1 = (I_1 - \alpha I_2)R_1 - I_2R_2$ ………… (1)

$V_1 = (I_1 - \alpha I_2)R_1 + (I_1 + I_2)R_3$ …… (2)

$I_2R_2 + (I_1 + I_2)R_3 = 0$ ………. (3)

From eq. (3)

$I_2R_2 + I_1R_3 + I_2R_3 = 0$

$$D = \frac{I_1}{-I_2} = \frac{R_2+R_3}{R_3}$$

$$B = \frac{V_1}{-I_2} = \left[\frac{(R_2+R_3)(R_1+R_3)}{R_3} - R_3 + \alpha R_1\right]$$

7. Graph Theory

01. Ans: (c)

Sol: $n > \frac{b}{2} + 1$

Note: Mesh analysis simple when the nodes are more than the meshes.

02. Ans: (c)

Sol: Loops = b – (n–1) ⇒ loops = 5

n = 7 ∴ b = 11

03. Ans: (a)

04.

Sol: Nodal equations required = f-cut sets

$= (n-1) = (10-1) = 9$

Mesh equations required = f-loops

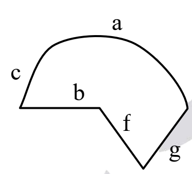
$= b - n + 1 = 17 - 10 + 1 = 8$

So, the number of equations required

= Minimum (Nodal, mesh) = Min(9,8) = 8

05. Ans: (c)

Sol: Not a tree (Because trees are not in closed path)



06. Ans: (a)

07.

Sol: For a complete graph ;

$$b = n_{C_2} \Rightarrow \frac{n(n-1)}{2} = 66$$

$n = 12$

f-cut sets = $(n-1) = 11$

f-loops = $(b-n+1) = 55$

f-loop = f-cutset matrices = $n^{(n-2)}$

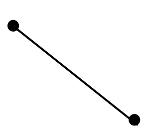
$= 12^{12-2} = 12^{10}$

08. Ans: (a)

Sol: Let N=1

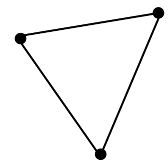
Nodes=1, Branches = 0 ; f-loops = 0

Let N=2



Nodes = 2; Branches = 1; f-loop = 0

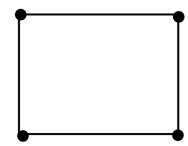
Let N=3



Nodes = 3; Branches = 3; f-loop = 1

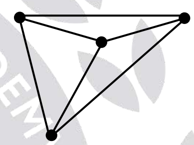
$\Rightarrow$ Links = 1

Let N = 4



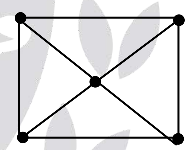
Nodes=4; Branches = 4; f-loops=Links=1

Still N = 4



Branches = 6; f-loops = Links = 3

Let N = 5

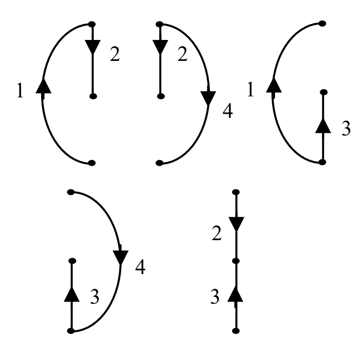


Nodes = 5; Branches = 8; f –loops = Links = 4 etc

Therefore, the graph of this network can have at least "N" branches with one or more closed paths to exist.

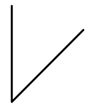
09. Ans: (b)

Sol:

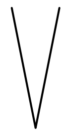


10. Ans: (d)

Sol: (a) 1,2,3,4 $\rightarrow$

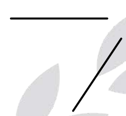


(b) 2,3,4,6 $\rightarrow$



(c) 1,4,5,6 $\rightarrow$

(d) 1,3,4,5 $\rightarrow$



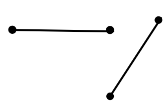
11. Ans: (b)

Sol: $m = b - n + 1 = 8 - 5 + 1 = 4$

12. Ans: (d)

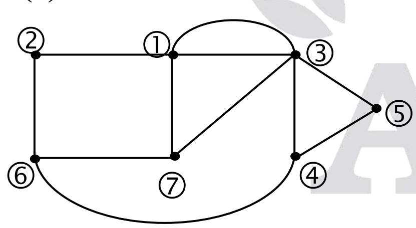
13. Ans: (d)

Sol: The valid cut –set is (1,3,4,6)



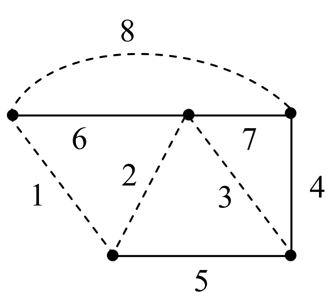
14. Ans: (b)

Sol:



15. Ans: (d)

Sol:



Fundamental loop should consist only one link, therefore option (d) is correct.

8. Passive Filters

01.

Sol:

$$\left.\begin{aligned}\omega = 0 &\Rightarrow V_0 = V_i\\ \omega = \infty &\Rightarrow V_0 = 0\end{aligned}\right\} \Rightarrow \text{Low pass filter}$$

02.

Sol: $\omega = 0 \Rightarrow V_0 = \dfrac{V_i R_2}{R_1 + R_2}$

"V_0" is attenuated $\Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = V_i$

It represents a high pass filter characteristics.

03.

Sol: $H(s) = \dfrac{V_i(s)}{I(s)} = \dfrac{S^2LC + SRC + 1}{SC}$

Put $s = j\omega i = -\dfrac{\omega^2 LC + j\omega RC + 1}{j\omega C}$

$\omega = 0 \Rightarrow H(s) = 0$

$\omega = \infty \Rightarrow H(s) = 0$

It represents band pass filter characteristics

04.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = 0$

It represents Band pass filter characteristics

05.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = V_i$

It represents High Pass filter characteristics.

06.

Sol: $H(s) = \dfrac{1}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 1$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Low pass filter characteristics

07.

Sol: $H(s) = \dfrac{s^2}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 1$

It represents a High pass filter characteristics

08.

Sol: $\omega = 0; V_0 = V_i$

$\omega = \infty; V_0 = 0$

It represents a low pass filter characteristics.

09.

Sol: $\omega = 0 \Rightarrow V_0 = V_{in}$

$\omega = \infty \Rightarrow V_0 = V_{in}$

It represents a Band stop filter or notch filter.

10.

Sol: $H(s) = \dfrac{S}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Band pass filter characteristics

11.

Sol: $H(s) = \dfrac{S^2 + 1}{s^2 + s + 1}$

$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = 1$

It represents a Band stop filter

12.

Sol: $H(s) = \dfrac{1 - s}{1 + s}$

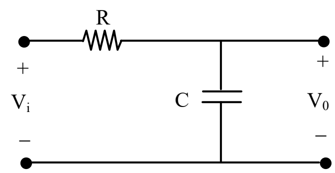
$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = -1 = 1 \angle 180^0$

It represents an All pass filter

13. Ans: (c)

Sol.



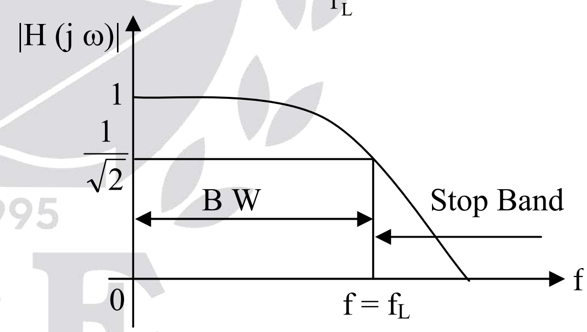
$\omega = 0 \Rightarrow V_0 = V_i$

$\omega = \infty \Rightarrow V_0 = 0$

$$V_0(s) = \left(\frac{V_i(s)}{R + \frac{1}{sc}}\right)\left(\frac{1}{sc}\right)$$

$$\frac{V_0(s)}{V_i(s)} = H(s) = \frac{1}{SscR + 1}$$

$$H(j\omega) = \frac{1}{1 + j\omega cR} = \frac{1}{1 + j\frac{f}{f_L}}$$



Where $f_L = \dfrac{1}{2\pi RC}$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^2}}$$

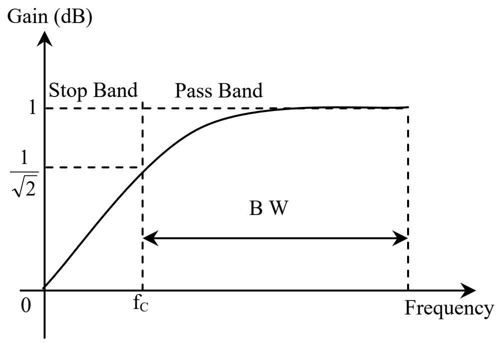
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{f}{f_L}\right)$$

$f = 0 \Rightarrow \phi = 0^0 = \phi_{min}$

$f = f_L \Rightarrow \phi = -45^0 = \phi_{max}$

14. Ans: (b)

Sol:



First order high pass filter = $\frac{s}{1+sT}$

Phase shift = 90 –tanωT

Max. phase shift is at corner frequency

$\omega = \frac{1}{T}$

Max. phase shift = 90 – $\tan^{-1}\omega T$

$= 90 - \tan^{-1}\left(\frac{1}{T}\times T\right)$

= 90 –45

= 45°

15. Ans: (d)

16. Ans: (a)

Sol: Half power of series RC circuit is at t = T (Time constant)

T = RC

Frequency = $\frac{1}{RC}$

17. Ans: (c)

Sol: Magnitude of voltage gain 0.707 is at half power frequency

$\omega = \frac{1}{RC}$

9. Three Phase Circuits

01. Ans: (c)

Sol: $Z_p(\text{star}) = \frac{9\angle 30^\circ \; 9\angle 30^\circ}{27\angle 30^\circ} = 3\angle 30^\circ\,\Omega$

02. Ans: (c)

Sol:

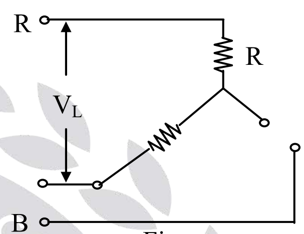


Fig.

Let V_L be the line to line voltage

$V_p = \frac{V_L}{\sqrt{3}}$

Let the total power in star connected load with phase resistance as R be P_1

$P_1 = 3\frac{V_P^2}{R} = 3\frac{V_L^2}{3R} = \frac{V_L^2}{R}$

When one of the phase resistance is removed, the relevant star load is shown in Fig.

Power in this star load

$= P_2 = 2\left(\frac{V_L}{2}\right)^2 \frac{1}{R} = \frac{V_L^2}{2R}$

$\therefore \frac{P_2}{P_1} = 50\ \%$

03. Ans: (d)

Sol: $I_n = 15\angle 0^\circ + 15\angle -120^\circ + 15\angle -240^\circ = 0$

04. Ans: (a)

05.

Sol: The circuit is redrawn with switch open as shown in Fig.1

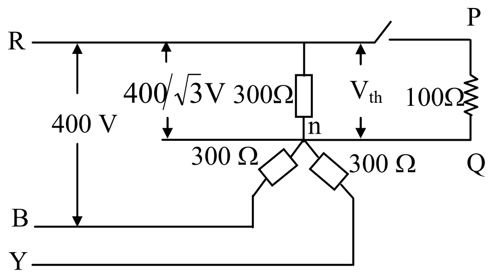


Fig.1

Open circuit voltage, when the switch is open = Thevenin voltage

Phase voltage, $V_{Rn} = \dfrac{400}{\sqrt{3}}$ V

To find Thevenin's equivalent impedance short circuit the voltage sources (Fig. 2 & 3)

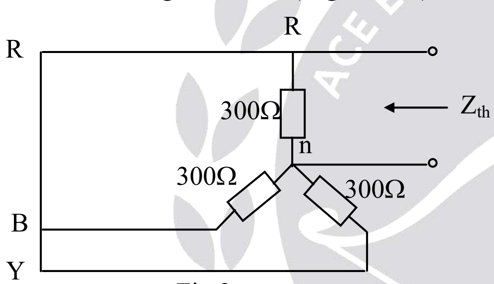


Fig.2

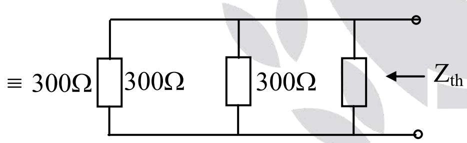


Fig.3

$\therefore Z_{th} = \dfrac{300}{3} = 100\ \Omega$

$\therefore$ Thevenin's equivalent circuit across R, n is shown in Fig. 4 with the switch closed and 100 Ω load across P, Q

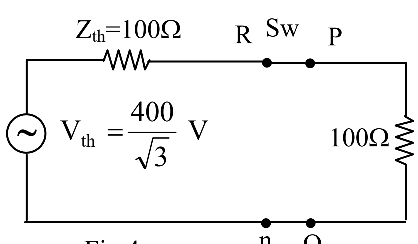


Fig.4

$\therefore$ RMS value of voltage across 100 Ω resistor

$= \dfrac{400}{2\sqrt{3}}\ \text{V} = 115.5\ \text{V}$

06.

Sol:

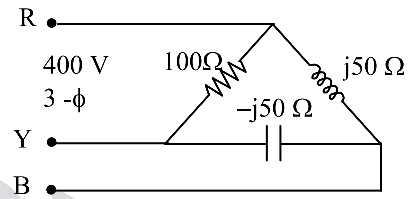


Fig.1

The unbalanced load is shown in Fig. 1. Power is consumed only in 100 Ω resistor.

Power consumed in the delta connected unbalanced load shown in Fig.1 is given by

$$P_1 = \frac{V_{ph}^2}{R} = \frac{(400)^2}{100} = 1600\ \text{W}$$

The star connected load with 'R_x' in each phase is shown in Fig.2.

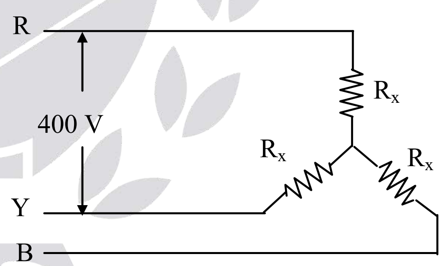


Fig.2

Power consumed in balanced star connected load as in Fig.2 is

$$P_2 = 3 \times \left[\frac{\left(\frac{400}{\sqrt{3}}\right)^2}{R_x}\right] = \frac{400^2}{R_x}$$

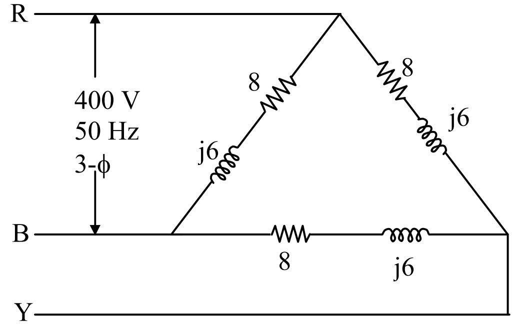
But given $P_1 = P_2$

$\therefore 1600 = \dfrac{400^2}{R_x}$

$\therefore R_x = \dfrac{400 \times 400}{1600} = 100\ \Omega$

07. Ans: (b)

Sol:



Power factor angle of load (ϕ)

$$= \tan^{-1}\left(\frac{6}{8}\right) = 36.86^\circ$$

Active power consumed by the delta connected balanced load as in Fig. is

$P = 3 \times V_{ph} \times I_{ph} \times \cos\phi$

$$= 3 \times 400 \times \frac{400}{\sqrt{8^2+6^2}} \times \cos 36.86 = 38400 \text{ W}$$

Reactive power consumed by the delta connected load is

$Q_1 = 3 \times V_{ph} \times I_{ph} \times \sin\phi$

$$= 3 \times 400 \times \frac{400}{\sqrt{8^2+6^2}} \times \sin 36.86$$

$= 28800$ VAR

Active power consumption remains same even after capacitor bank is connected Reactive power consumed by the delta connected load at a power factor of 0.9

$$Q_2 = \frac{P}{0.9} \times \sin(\cos^{-1} 0.9)$$

$$= \frac{38400}{0.9} \times \sin 25.84$$

$= 18597.96$ VAR

$\therefore$ $Q_2 = 18597.96$ VAR

$\therefore$ Reactive power supplied by star connected capacitor bank $= Q_1 - Q_2$

$= 28800 - 18597.96$

$= 10202.04$

$\cong 10.2$ kVAR

08. Ans: (d)

Sol: The rating of star connected load is given as $12\sqrt{3}$ kVA, 0.8 p.f (lag)

Active power consumed by the load,

$P = 12\sqrt{3} \times 0.8 \times 10^3$

$= 16.627$ kW

Reactive power consumed by the load

$= 12\sqrt{3} \times \sin(\cos^{-1} 0.8) \times 10^3$

$Q_1 = 12.47$ kVAR

Reactive power consumed by the load at unity power factor is

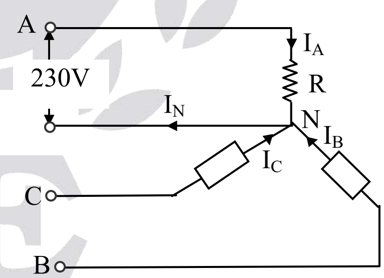
$$Q_2 = \frac{P}{(1)} \times \sin(\cos^{-1} 1) = 0$$

$\therefore$kVAR to be supplied by the delta connected capacitor bank $= Q_1 - Q_2$

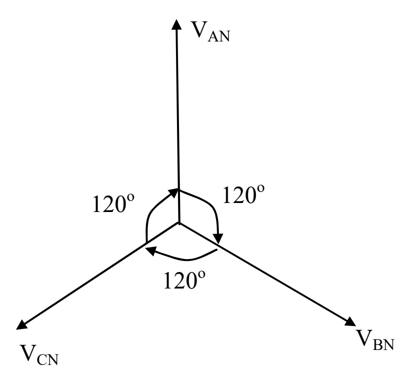
$Q_C = 12.47$ kVAR

09. Ans: (b)

Sol:



Assume V_{AN} as reference



$V_{AN} = 230\angle 0^o$

$V_{BN} = 230\angle -120^o$

$V_{CN} = 230\angle +120^o$

$$\frac{V_{AN}^2}{R} = 4000 \Rightarrow R = \frac{230^2}{4000} = 13.225\Omega$$

$$I_A = \frac{V_{AN}}{R} = \frac{230}{13.225} = 17.3913A$$

$\therefore I_A = 17.3913\angle 0^o$ A

Given neutral current $I_N = 0$

$\Rightarrow I_A + I_B + I_C = 0$

$\Rightarrow I_B + I_C = -(I_A)$

$I_B + I_C = -17.3913$

$$\Rightarrow \frac{V_{BN}}{Z_B} + \frac{V_{CN}}{Z_C} = -17.3913$$

$$\Rightarrow \frac{230\angle -120^o}{Z_B} + \frac{230\angle +120^o}{Z_C} = -17.3913$$

$$\Rightarrow \frac{230\angle -120^o}{Z_B} + \frac{230\angle +120^o}{Z_C} = 17.3913\angle 180^o \text{ A}$$

ssume that pure capacitor in phase B and pure inductor in phase C we will get

$$I_B + I_C = \frac{230\angle -120^o}{X_C\angle -90^o} + \frac{230\angle +120^o}{X_L\angle 90^o}$$

$$= \frac{230\angle -30^o}{X_C} + \frac{230\angle +30^o}{X_L}$$

When we add the two phasors I_B and I_C. with angles -30^o and $+30^o$ we will get the resultant vector with the angle between -30^o and $+30^o$

But,

$I_B + I_C$ should be equal to $17.3913\angle 180^o$

Which has angle of 180^o

$\therefore$ We have taken wrong assumption

$\therefore$ Now take pure inductor in phase B and pure capacitor in phase C we will get

$$I_B + I_C = \frac{230\angle -120^o}{X_L\angle 90^o} + \frac{230\angle +120^o}{X_C\angle -90^o}$$

$$= \frac{230\angle -210^o}{X_L} + \frac{230\angle +210^o}{X_C}$$

$$= \frac{230}{(2\pi\times 50\times L)}\angle -210^o + \frac{230}{\left(\frac{1}{(2\pi\times 50\times C)}\right)}\angle +210^o$$

$$= \frac{0.7321}{L}\angle -210^o + 72256.63\times C\ \angle +210^o$$

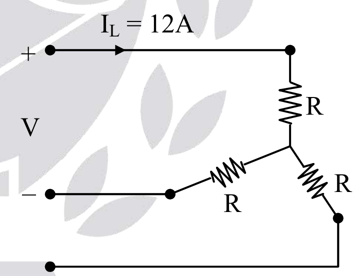
$\therefore$ From the given options by substituting L = 72.95 mH and C = 139.02 μF we will get $I_B + I_C \simeq 17.3913\angle 180^o$

L = 72.95mH in phase B and C = 139.02 μF in phase C should be placed.

10. Ans: (c)

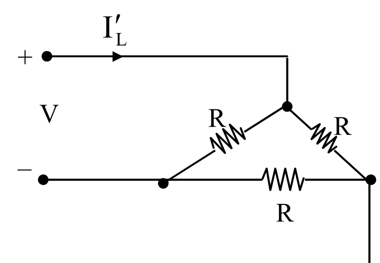
11. Ans: (d)

Sol:

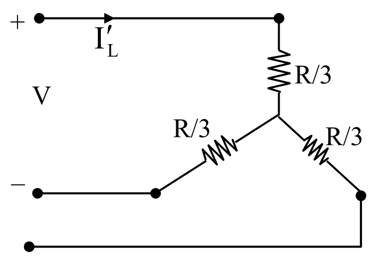


$$\Rightarrow I_L = \frac{\left(\frac{V}{\sqrt{3}}\right)}{R} \Rightarrow \frac{V}{\sqrt{3}R} = 12A$$

Now if the same resistances are connected in delta across the same supply



Transforming Δ into equivalent Y

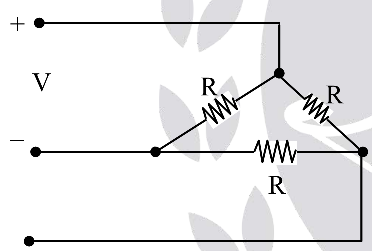


$$\Rightarrow I'_L = \frac{\left(\frac{V}{\sqrt{3}}\right)}{\left(\frac{R}{3}\right)} = \frac{3V}{\sqrt{3}R}$$

$$= 3(12) = 36 \text{ A}$$

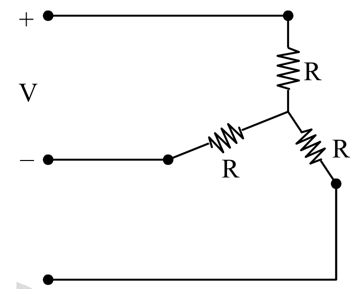
12. Ans: (b)

Sol: Assume the resistances are equal



$$\Rightarrow P_{\text{absorbed}\,\Delta} = 3\frac{V^2}{R} = 60\,\text{kW} \;\ldots\ldots(1)$$

Now, if the resistors are connected in star,



$$\Rightarrow P_{\text{absorbed Y}} = 3\frac{\left(\frac{V}{\sqrt{3}}\right)^2}{R} = 3\times\frac{V^2}{3R} = \frac{V^2}{R}$$

From equation(1), $\Rightarrow \frac{V^2}{R} = 20\,\text{kW}$

$$\therefore P_{\text{absorbed Y}} = \frac{V^2}{R} = 20\,\text{kW}$$