

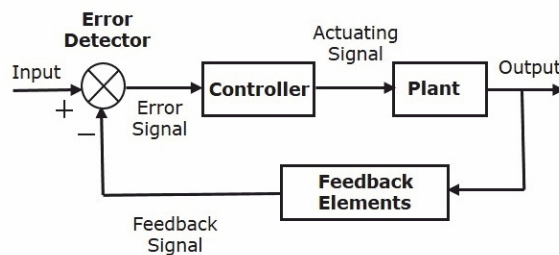


GATE | PSUs

ELECTRICAL ENGINEERING

Control Systems

Text Book: Theory with worked out Examples
and Practice Questions



Chapter 1 Basics of Control Systems

(Solutions for Text Book Practice Questions)

01. Ans: (c)

Sol: $2 \frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 4y(t) = r(t) + 2r(t-1)$

Apply LT on both sides

$$2s^2 Y(s) + 3sY(s) + 4Y(s) = R(s) + 2e^{-s}R(s)$$

$$Y(s)(2s^2 + 3s + 4) = R(s)(1 + 2e^{-s})$$

$$\frac{Y(s)}{R(s)} = \frac{1 + 2e^{-s}}{2s^2 + 3s + 4}$$

02. Ans: (b)

Sol: I.R = $2 \cdot e^{-2t} u(t)$

Output response $c(t) = (1 - e^{-2t}) u(t)$

Input response $r(t) = ?$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s} - \frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$$r(t) = u(t)$$

03. Ans: (b)

Sol: Unit impulse response of unit-feedback control system is given

$$c(t) = t \cdot e^{-t}$$

$$T.F = L(I.R)$$

$$= \frac{1}{(s+1)^2}$$

$$\begin{aligned} \text{Open Loop T.F} &= \frac{\text{Closed Loop T.F}}{1 - \text{Closed Loop T.F}} \\ &= \frac{1}{1 - \frac{1}{(s+1)^2}} = \frac{1}{s^2 + 2s} \end{aligned}$$

04. Ans: (a)

Sol: G changes by 10%

$$\Rightarrow \frac{\Delta G}{G} \times 100 = 10\%$$

$$C_1 = 10\%$$

[$\because$ open loop] whose sensitivity is 100%]

$$\%G \text{ change} = 10\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1 + GH}$$

$$\% \text{ of change in } M = \frac{10\%}{1 + (10)1} = 1\%$$

$$\% \text{ change in } C_2 \text{ by } 1\%$$

05.

Sol:

(i) $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1 + GH}$$

$$S_K^M = \frac{\partial M}{\partial K} \times \frac{K}{M} = 1$$

[$\because$ K is not in the loop $\Rightarrow$ sensitivity is 100%]

(ii) $S_H^M = \frac{\partial M}{\partial H} \times \frac{H}{M} = \frac{\partial}{\partial H} \left(\frac{GK}{1 + GH} \right) \frac{H}{M}$

$$= \left(\frac{GK(-G)}{(1+GH)^2} \right) \left[\frac{H}{GK} \right]$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

06.

Sol: Given data

$$G = 2 \times 10^3, \partial G = 100$$

$$\% \text{ change in } G = \frac{\partial G}{G} \times 100 = 5\%$$

$$\% \text{ change in } M = 0.5\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2 \times 10^3 H}$$

$$1+2 \times 10^3 H = 10$$

$$H = 4.5 \times 10^{-3}$$

07. Ans: (b)

Sol: $K = \frac{\text{output}}{\text{input}} = \frac{c(t)}{r(t)} = \frac{\text{mm}}{^0\text{c}}$

08. Ans: (d)

Sol: Introducing negative feedback in an amplifier results, increases bandwidth.

09. Ans: (a), (b) & (c)

Sol: Negative feedback decreases the gain, increase the bandwidth, reduce sensitivity to parameter variation and more accurate.

10. Ans: (b), (c) & (d)

Sol: Using the transfer function response due to initial conditions [zero input response] can not be obtained.

$L^{-1}[\text{TF}] = \text{IR}$ i.e., inverse laplace transform of the transfer function is the impulse response [IR] of the system.

Chapter 2 Signal Flow Graphs & Block Diagrams

01. Ans: (d)

Sol: No. of loops = 3

Loop1: $-G_1G_3G_4H_1H_2H_3$

Loop2: $-G_3G_4H_1H_2$

Loop3: $-G_4H_1$

No. of Forward paths = 3

Forward Path1: $G_1G_3G_4$

Forward Path 2: $G_2G_3G_4$

Forward Path 3: G_2G_4

$$= \frac{G_1G_3G_4 + G_2G_3G_4 + G_2G_4}{1 + G_1G_3G_4H_1H_2H_3 + G_3G_4H_1H_2 + G_4H_1}$$

02. Ans: (a)

Sol: Number of forward paths = 2

Number of loops = 3

$$\begin{aligned} \frac{Y(s)}{R(s)} &= \frac{\frac{1}{s} \times \frac{1}{s} \times \frac{1}{s} [1-0] + \frac{1}{s}}{1 - \left[\frac{1}{s} \times (-1) \left(\frac{1}{s} \right) (-1) + \frac{1}{s} \times \frac{1}{s} (-1) + \left(\frac{1}{s} \times \frac{1}{s} (-1) \right) \right]} \\ &= \frac{\frac{1}{s^3} + \frac{1}{s}}{1 - \left[\frac{1}{s^2} - \frac{1}{s^2} - \frac{1}{s^2} \right]} = \frac{\frac{1+s^2}{s^3}}{1 + \frac{1}{s^2}} = \frac{1+s^2}{s^3} \times \frac{s^2}{s^2+1} \\ &= \frac{1+s^2}{s} \times \frac{1}{s^2+1} = \frac{1}{s} \end{aligned}$$

03.

Sol: Number of forward paths = 2

Number of loops = 5

Two non touching loops = 4

$$\begin{aligned} TF &= \frac{24[1-(-0.5)] + 10[1-(-3)]}{1 - [-24 - 3 - 4 + (5 \times 2 \times (-1) + (-0.5))] + [30 + 1.5 + 2] + \left(\left(\frac{-1}{2} \right) \times (-24) \right)} \\ &= \frac{76}{88} = \frac{19}{22} \end{aligned}$$

04.

Sol: Number of forward paths = 2

Number of loops = 5

$$T.F = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_1G_2H_1 + G_1G_2G_3 + G_4H_2 + G_1G_4}$$

05. Ans: (c)

Sol: From the network

$$V_o(s) = \frac{1}{sC} I(s) \quad \dots\dots\dots(1)$$

$$-V_i(s) + RI(s) + V_o(s) = 0$$

$$I(s) = \frac{1}{R} V_i(s) + \left(\frac{-1}{R} \right) V_o(s) \quad \dots\dots\dots(2)$$

From SFG

$$V_o(s) = x.I(s) \quad \dots\dots\dots(3)$$

$$I(s) = \frac{1}{R} V_i(s) + y V_o(s) \quad \dots\dots\dots(4)$$

From equ(1) and (3)

$$x = \frac{1}{sC}$$

From equ(2) and (4)

$$y = -\frac{1}{R}$$

06. Ans: (a)

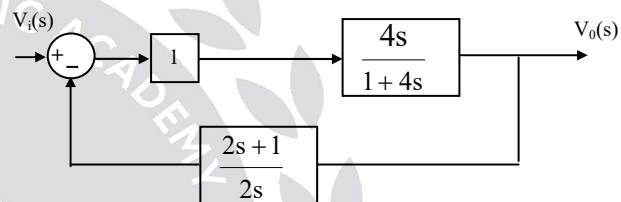
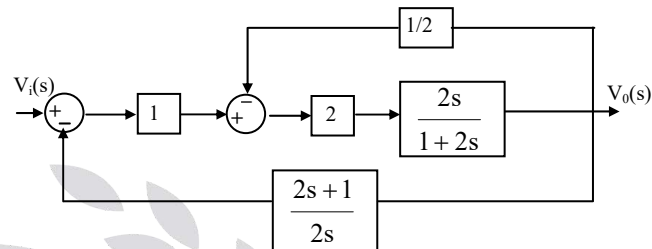
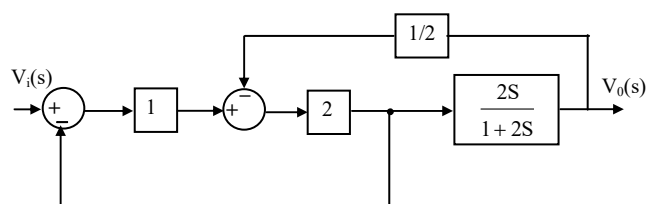
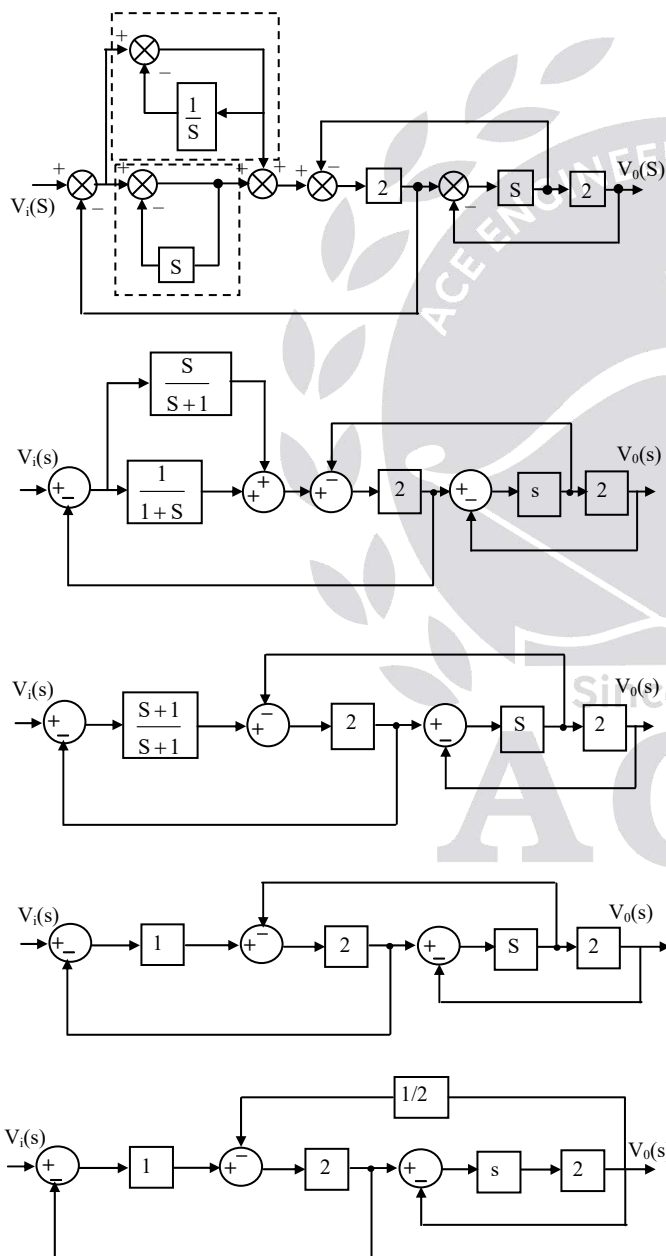
Sol: Use gain formula

$$\text{transfer function} = \frac{G(s)}{1 - \left(G(s) \frac{1}{G(s)} + G(s) \right)}$$

$$= \frac{G(s)}{1 - 1 - G(s)} = -1$$

07.

Sol:



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{4s}{1+4s}}{1 + \frac{2(2s+1)}{1+4s}} = \frac{4s}{8s+3}$$

08.

Sol: Apply Mason's Gain formula

$$M = \frac{Y_{out}}{Y_{in}} = \frac{\sum_{k=1}^N M_k \Delta_k}{\Delta}$$

No. of forward paths = 2

First forward path gain = $G_1 G_2 G_3 G_4$

Second forward path gain = $G_5 G_6 G_7 G_8$

No. of loops = 4

First loop gain = $-G_2 H_2$

Second loop gain = $-G_6 H_6$

Third loop gain = $-G_3 H_3$

Fourth loop gain = $-G_7 H_7$

Non touching loops = 4

$$\begin{aligned}\text{Loop gains} &\rightarrow G_2 H_2 G_6 H_6 \\ &\rightarrow G_2 H_2 G_7 H_7 \\ &\rightarrow G_6 H_6 G_7 H_7 \\ &\rightarrow G_2 H_2 G_3 H_3\end{aligned}$$

Transfer function =

$$\frac{G_1 G_2 G_3 G_4 (1 + G_6 H_6 + G_7 H_7) + G_5 G_6 G_7 G_8 (1 + G_2 H_2 + G_3 H_3)}{1 + G_2 H_2 + G_3 H_3 + G_6 H_6 + G_7 H_7 + G_2 H_2 G_6 H_6 + G_2 H_2 G_7 H_7 + G_3 H_3 G_6 H_6 + G_3 H_3 G_7 H_7}$$

09. Ans: (a), (b) & (d)

Sol: It is a LTIS, hence $\frac{C}{R}$ can be found

Number of forward paths = 1

Number of loops = 2

Non touching pair = 1

$$\therefore \frac{C}{R} = \frac{(1)}{1 - [-1 - 1] + (-1)(-1)}$$

$$\frac{C}{R} = \frac{1}{4} = 0.25$$

10. Ans: (a), (b) & (d)

$$\text{Sol: } \Rightarrow \frac{Y(s)}{X(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1 + \frac{1}{(s+1)(s+2)}} = \frac{1}{s^2 + 3s + 3}$$

$$\Rightarrow \frac{Y(s)}{N(s)} = \frac{1 - G_{ff}(s) \left(\frac{1}{(s+1)(s+2)} \right)}{1 + \frac{1}{(s+1)(s+2)}} = 0$$

[Output due to noise is zero]

$$G_{ff}(s) = (s+1)(s+2)$$

$$\Rightarrow \text{C.E: } s^2 + 3s + 3 = 0$$

$$\Rightarrow \text{Poles locations are } (-3/2 \pm j0.866)$$

$\Rightarrow$ System is stable

Chapter 3 Time Response Analysis

01. Ans: (a)

Sol: $\frac{C(s)}{R(s)} = \frac{1}{1+sT}$, $R(s) = \frac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8(1 - e^{-t/T})$$

$$3.6 = 8 \left(1 - e^{\frac{-0.32}{T}} \right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

$$T = 0.535 \text{ sec}$$

02. Ans: (c)

Sol: $\cos \phi = \xi$

$$\cos 60 = 0.5$$

$$\cos 45 = 0.707$$

$$\text{Poles left side } 0.5 \leq \xi \leq 0.707$$

$$\text{Poles right side } -0.707 \leq \xi \leq -0.5$$

$$\therefore 0.5 \leq |\xi| \leq 0.707$$

$$3 \text{ rad/s} \leq \omega_n \leq 5 \text{ rad/s}$$

03. Ans: (c)

Sol: For R-L-C circuit:

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$= \frac{1}{Cs} \frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \quad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2} \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} = 0.5$$

$$M.P = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= 16.3\% \approx 16\%$$

04. Ans: (b)

Sol: $TF = \frac{8/s(s+2)}{1 - \left(\frac{-8as}{s(s+2)} - \frac{8}{s(s+2)} \right)}$

$$= \frac{8}{s(s+2) + 8as + 8}$$

$$= \frac{8}{s^2 + 2s + 8as + 8}$$

$$= \frac{8}{s^2 + (2+8a)s + 8}$$

$$\omega_n^2 = 8 \Rightarrow \omega_n = 2\sqrt{2}$$

$$2\xi\omega_n = 2 + 8a$$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

05. Ans: 4 sec

Sol: T.F = $\frac{100}{(s+1)(s+100)} = \frac{100}{s^2 + 101s + 100}$

$$\omega_n^2 = 100$$

$$\omega_n = 10$$

$$2\xi\omega_n = 101$$

$$\xi = \frac{101}{20}$$

$\xi > 1 \rightarrow$ system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$$T.F = \frac{100}{100(s+1)} = \frac{1}{s+1}, \text{ Here } \tau = 1 \text{ sec}$$

$$\therefore \text{Setting time for 2\% criterion} = 4\tau = 4 \text{ sec}$$

06.

Sol: $M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$

$$= \frac{1.254 - 1.04}{1.04} = 0.2$$

$$\xi = \frac{(\ln M_p)^2}{(\ln M_p)^2 + \pi^2}$$

$$M_p = 0.2 ; \xi = 0.46$$

07. Ans: (d)

Sol: Given data: $\omega_n = 2, \zeta = 0.5$

Steady state gain = 1

$$OLTF = \frac{K_1}{s^2 + as + 2} \text{ and } H(s) = K_2$$

$$CLTF = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + as + 2 + K_1K_2}$$

DC or steady state gain from the TF

$$\frac{K_1}{2 + K_1K_2} = 1$$

$$K_1(1 - K_2) = 2 \quad \dots\dots\dots (1)$$

$$CE \text{ is } s^2 + as + 2 + K_1K_2 = 0$$

$$\omega_n = \sqrt{2 + K_1K_2} = 2$$

$$4 = (2 + K_1K_2)$$

$$K_1K_2 = 2 \quad \dots\dots\dots (2)$$

Solving equations (1) & (2) we get

$$K_1 = 4, \quad K_2 = 0.5$$

$$2\zeta\omega_n = a$$

$$2 \times \frac{1}{2} \times 2 = a$$

$$a = 2$$

08. Ans: (c)

Sol: If $R \uparrow$ damping $\uparrow$

$$\Rightarrow \xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

(i) If $R \uparrow$, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on ξ)

If $\xi \uparrow$, $C(\infty)$ = remain same

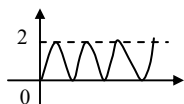
(ii) If $\xi \uparrow$, $\omega_d \downarrow$ ($\omega_d = \omega_n \sqrt{1 - \xi^2}$)

(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow 3^{\text{rd}}$

Statement is false

(iv) If $\xi = 0$

True


 $\Rightarrow 2$ and 4 are correct

09. Ans: A – T, B – S, C- P, D – R, E – Q
Sol:

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

 (C) If poles are non-repeated on the $j\omega$ axis, the step response will have fixed amplitude oscillations.

 (D) If the poles are complex & right side of s-plane, response goes to ' ∞ ' with damped oscillations.

 (E) If the poles are real & right side of s-plane, the step response goes to ' ∞ ' without any oscillations.

10.
Sol: (i) Unstable system

 $\therefore \text{error} = \infty$

(ii) $G(s) = \frac{10(s+1)}{s^2}$

Step $\rightarrow R(s) = \frac{1}{s}$

$k_p = \infty$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+\infty} = 0$$

Parabolic $\Rightarrow k_a = 10$

$$e_{ss} = \frac{1}{10} = 0.1$$

11.
Sol: $G(s) = 10/s^2$ (marginally stable system)

 $\therefore$ Error can't be determined

12.

Sol: $e_{ss} = \frac{1}{11}, R(s) = \frac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$$k_p = \lim_{s \rightarrow 0} s G(s)$$

$$10 = \lim_{s \rightarrow 0} s G(s)$$

$$k = 10$$

$$R(s) = \frac{1}{s^2} \text{ (ramp)}$$

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

$$\Rightarrow e_{ss} = 0.1$$

13. Ans: (a)

Sol: $T(s) = \frac{(s-2)}{(s-1)(s+2)^2}$ (unstable system)

14. Ans: (b)
Sol: Given data: $r(t) = 400tu(t)$ rad/sec

 Steady state error $= 10^\circ$

$$\text{i.e., } e_{ss} = \frac{\pi}{180^\circ} (10^\circ) \text{ radians}$$

$$G(s) = \frac{20K}{s(1+0.1s)} \text{ and } H(s) = 1$$

$$r(t) = 400tu(t) \Rightarrow 400/s^2$$

$$\text{Error } (e_{ss}) = \frac{A}{K_v} = \frac{400}{K_v}$$

$$K_V = \lim_{s \rightarrow 0} s G(s)$$

$$K_V = \lim_{s \rightarrow 0} \frac{20K}{s(1+0.1s)}$$

$$K_V = 20K$$

$$e_{ss} = \frac{400}{20K}$$

$$e_{ss} = \frac{20}{K} = \frac{\pi}{18}$$

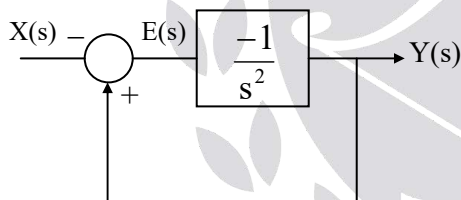
$$K = 114.5$$

15. Ans: (d)

Sol: $\frac{d^2 y}{dt^2} = -e(t)$

$$s^2 Y(s) = -E(s)$$

$$x(t) = t u(t) \Rightarrow X(s) = \frac{1}{s^2}$$



$$Y(s) = \frac{-1}{s^2} E(s)$$

$$\frac{Y(s)}{E(s)} = \frac{-1}{s^2}$$

$$\frac{E(s)}{X(s)} = \frac{-1}{1 + \frac{1}{s^2}}$$

$$E(s) = \frac{-s^2}{1+s^2} X(s)$$

$$= \frac{-s^2}{1+s^2} \times \frac{1}{s^2} = \frac{-1}{1+s^2}$$

$$= L^{-1} \left[\frac{-1}{1+s^2} \right] = -\sin t$$

16. Ans: (a)

Sol: $e_{ss} = 0.1$ for step input

For pulse input = 10

time = 1 sec

error is function of input

$t \rightarrow \infty$ input = 0

$\therefore$ Error = zero

17. Ans: (c)

Sol: $\frac{C(s)}{R(s)} = \frac{100}{(s+1)(s+5)}$

$$1 + \frac{100 \times 0.2}{(s+1)(s+5)}$$

$$= \frac{100}{(s+1)(s+5) + 20}$$

$$= \frac{100}{s^2 + 6s + 5 + 20}$$

$$= \frac{100}{s^2 + 6s + 25}$$

$$\omega_n^2 = 25, \omega_n = 5$$

$$2\xi\omega_n = 6$$

$$\xi = \frac{6}{10} = \frac{3}{5}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 5 \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= 5 \times \frac{4}{5} = 4 \text{ rad/sec}$$

18. Ans: (c)

Sol: $f(t) = \frac{M d^2 x}{dt^2} + B \frac{dx}{dt} + Kx(t)$

Applying Laplace transform on both sides,
with zero initial conditions

$$F(s) = Ms^2 X(s) + BsX(s) + KX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Characteristic equation is $Ms^2 + Bs + K = 0$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0$$

Compare with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = \frac{B}{M}$$

$$\zeta = \frac{B}{2\sqrt{MK}} \quad \omega_n = \sqrt{\frac{K}{M}}$$

$$\begin{aligned} \text{Time constant } T &= \frac{1}{\zeta\omega_n} \\ &= \frac{1}{\frac{B}{2M}} \times 2M \\ T &= \frac{2M}{B} \end{aligned}$$

Hence, statements (2 & 3) are correct

19. Ans: (c)

Sol: type 1 system has a infinite positional error constant.

20. Ans: (a)

Sol: Given $G(s) = \frac{1}{s(1+s)(s+2)}$, $H(s) = 1$.

It is type-I system

Positional error constant $k_p = \lim_{s \rightarrow 0} G(s)H(s)$

$$\begin{aligned} k_p &= \lim_{s \rightarrow 0} \frac{1}{s(1+s)(s+2)} \\ &= \infty \end{aligned}$$

Steady state error due to step input

$$= \frac{1}{1+k_p} = 0$$

21.

Sol Open loop T/F $G(s) = \frac{A}{s(s+P)}$

$$\text{C.L T/F} = \frac{A}{s^2 + sP + A}$$

$$\omega_n = \sqrt{A}$$

$$\text{Setting time} = 4/\xi\omega_n = 4$$

$$2\xi\omega_n = P \quad \therefore \frac{4}{P/2} = 4$$

$$\xi\omega_n = P/2 \quad \Rightarrow P = \frac{8}{4} = 2$$

$$e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1 \Rightarrow \frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln 10$$

$$= 2.3$$

$$\Rightarrow \frac{\xi^2}{1-\xi^2} = 0.5373$$

$$\Rightarrow 1.5373 \xi^2 = 0.5373$$

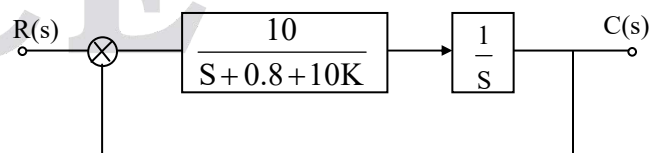
$$\xi = 0.59$$

$$\xi\omega_n = 1$$

$$\Rightarrow \omega_n = 1.694 \Rightarrow A = \omega_n^2 = 2.861$$

22.

Sol:



$$\frac{C(s)}{R(s)} = \frac{10}{s(s+0.8+10K)+10}$$

$$= \frac{10}{s^2 + s(0.8+10K)+10}$$

$$\omega_n = \sqrt{10}$$

$$2\xi\omega_n = 0.8 + 10K$$

$$\Rightarrow 2 \times \frac{1}{2} \times \sqrt{10} = 0.8 + 10K$$

$$\Rightarrow K = 0.236$$

$$t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \cos^{-1}(\xi)}{\omega_n \sqrt{1 - \xi^2}}$$

$$= \frac{\pi - \pi/3}{2.88} = 0.764 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = 1.147 \text{ sec}$$

$$\%Mp = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.163 \times 100 = 16.3\%$$

$$t_s (\text{for } 2\%) = \frac{4}{\xi\omega_n} = \frac{4}{0.5 \times \sqrt{10}} = 2.52 \text{ sec}$$

23. Ans: (a), (c) & (d)

Sol: CLTF $\Rightarrow \frac{C(s)}{R(s)} = \frac{3k}{2s + 1 + 3k}$

$$\Rightarrow \text{CL pole } s = -\left(\frac{1 + 3k}{2}\right)$$

$$\Rightarrow \text{time constant } \tau = \left(\frac{2}{1 + 3k}\right)$$

$$\text{If } k = 3 \Rightarrow \tau = 0.2 \text{ sec}$$

$$\text{If } k > 3 \Rightarrow \tau < 0.2 \text{ sec}$$

$$\text{If } k = 3 \Rightarrow \tau = 0.2 \text{ sec} \Rightarrow BW = \frac{1}{\tau} \text{ rad/sec}$$

$$BW = \frac{1}{0.2} = 5 \text{ rad/sec}$$

24. Ans: (a), (c) & (d)

Sol: $\Rightarrow$ As poles move toward left side, the system time constant decreases and system is more stable.

$\Rightarrow$ Damping ratio increases & percentage of peak overshoot decreases.

$\Rightarrow$ Damped oscillations (ω_d) is constant. Hence peak time is constant.

25. Ans: (a), (b) & (d)

Sol: Roots are $(-2 \pm j2\sqrt{3})$ complex

$0 < \zeta < 1$ – under damped system

Natural frequency $= \sqrt{16} = 4 \text{ rad/sec}$

$$\text{Damping ratio } \zeta = \frac{4}{2(4)} = 0.5$$

Under damped system has damped oscillations.

26. Ans: (b) & (c)

Sol: OLTF $= \frac{20}{s + 2}$, $H(s) = 1$

$$\text{CLTF} = \frac{\frac{20}{s + 2}}{1 + \frac{20}{s + 2}} = \frac{20}{s + 22}$$

$$\text{DC gain} = \frac{20}{22} = \frac{10}{11}$$

Steady state error to a unit step input

$$= \left(1 - \frac{20}{22}\right) \text{ which is non zero}$$

27. Ans: (b) & (d)

Sol: In OLTF two poles are at the origin

$\therefore$ It is type '2'

$$\text{CE} = 1 + \frac{10(s+1)^4}{s^2(s+2)} = 0, \text{ 4 roots it has}$$

$\therefore$ 4th order system

Type 2 system error to step and ramp input $s = 0$

$$k_a = \lim_{s \rightarrow 0} s^2 G(s) = \frac{10}{2} = 5$$

$$\text{Error} = \frac{1}{5} = 0.2 \text{ to a parabolic input}$$

Chapter 4 Stability

01.

Sol: CE = $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7	
s^4	4(1)	8(2)	4(1)	
s^3	6(1)	6(1)	0	
s^2	1	1	0	→ Row of AE
s^1	0(2)	0	0	→ Row of zero
s^0	1			

No. of AE roots = 2

No. of sign changes

Below AE = 0

No. of RHP = 0

No. of LHP = 0

No. of jwp = 2

No. of CE roots = 5

No. of sign changes

in 1st column = 0

∴ No. of RHP = 0

No. of jwp = 2

⇒ No. of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$s = \pm 1j = \pm j\omega_n$

$\omega_n = 1 \text{ rad/sec}$

Oscillating frequency $\omega_n = 1 \text{ rad/sec}$

02.

Sol: (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

$+s^5$	1	1	1
$+s^4$	1	1	1
$+s^3$	0(2)	0(1)	0
$+s^2$	$\frac{1}{2}$	1	
(1) $-s^1$	-3	0	
(2) $+s^0$	1		

AE (1) = $s^4 + s^2 + 1 = 0$

$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$

⇒ $2s^3 + s = 0$

AE

No. of sign changes below
AE = 2

No. of AE roots = 4

No. of RHP = 2

No. of LHP = 2

No. of jwp = 0

CE

No. of sign changes in
1st column = 2

No. of CE roots = 5

No. of RHP = 2

No. of LHP = 3

No. of jwp = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

s^6	1	2	-1	-2
s^5	2(1)	0	-2(-1)	0
s^4	2(1)	+0	-2(-1)	0
s^3	0(4)	0	0	0
s^2	0(ε)	-1	0	0
s^1	4/ε			
$-s^0$	-1			

$$AE = s^4 - 1 = 0$$

$$\frac{dAE}{ds} = 4s^3 + 0 = 0$$

CE	AE
No. of CE roots = 6	No. of AE roots = 4
No. of sign changes in the 1 st column = 1	No. of sign changes below AE = 1
No. of RHP = 1	No. of RHP = 1
No. of LHP = 3	No. of jωp = 2
No. of jωp = 2	No. of LHP = 1

03.

Sol: $CE = s^3 + 20s^2 + 16s + 16K = 0$

$$\begin{array}{c|cc} s^3 & 1 & 16 \\ s^2 & 20 & 16K \\ s^1 & \frac{20(16) - 16K}{20} & 0 \\ s^0 & 16K & \end{array}$$

(i) For stability $\frac{20(16) - 16K}{20} > 0$

$$\Rightarrow 20(16) - 16K > 0$$

$$\Rightarrow K < 20 \text{ and } 16K > 0 \Rightarrow K > 0$$

Range of K for stability $0 < K < 20$

(ii) For the system to oscillate with ω_n it must be marginally stable

i.e., s^1 row should be 0

s^2 row should be AE

$$\therefore \text{A.E roots} = \pm j\omega_n$$

$$\therefore s^1 \text{ row} \Rightarrow 20(16) - 16K = 0$$

$$\Rightarrow K = 20$$

$$\text{AE is } 20s^2 + 16K = 0$$

$$20s^2 + 16(20) = 0$$

$$\Rightarrow s = \pm j4$$

$$\omega_n = 4 \text{ rad/sec}$$

04.

Sol: $CE = 1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$$s^3 + as^2 + (K+2)s + K+1 = 0$$

$$s^3 + as^2 + (K+2)s + (K+1) = 0$$

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & a & K+1 \\ s^1 & \frac{a(K+2) - (K+1)}{a} & 0 \\ s^0 & K+1 & \end{array}$$

Given,

$$\omega_n = 2$$

$$\Rightarrow s^1 \text{ row} = 0$$

s^2 row is A.E

$$a(K+2) - (K+1) = 0$$

$$a = \frac{K+1}{K+2}$$

$$AE = as^2 + K+1 = 0$$

$$= \frac{K+1}{K+2} s^2 + K+1 = 0$$

$$(K+1) \left(\frac{s^2}{K+2} + 1 \right) = 0$$

$$s^2 + K+2 = 0$$

$$s = \pm j\sqrt{K+2}$$

$$\omega_n = \sqrt{k+2} = 2$$

$$k = 2$$

$$a = \frac{k+1}{k+2} = \frac{3}{4} = 0.75$$

05.

Sol: $s^3 + ks^2 + 9s + 18$

s^3	1	9
s^2	K	18
s^1	$\frac{9K-18}{K}$	0
s^0	18	

Given that system is marginally stable,

Hence

$$s^1 \text{ row} = 0$$

$$\frac{9K-18}{K} = 0$$

$$9K = 18 \Rightarrow K = 2$$

$$A.E \text{ is } 9s^2 + 18 = 0$$

$$Ks^2 + 18 = 0,$$

$$2s^2 + 18 = 0$$

$$2s^2 = -18$$

$$s = \pm j3$$

$$\therefore \omega_n = 3 \text{ rad/sec.}$$

06. Ans: (d)

Sol: Given transfer function $G(s) = \frac{k}{(s^2 + 1)^2}$

$$\text{Characteristic equation } 1 - G(s).H(s) = 0$$

$$1 - \frac{k}{(s^2 + 1)^2} = 0$$

$$s^4 + 2s^2 + 1 - k = 0 \dots (1)$$

RH criteria

s^4	1	2	$1-K$
s^3	4	4	-
s^2	1	$1-K$	
s^1	$4K$		
s^0	$1-K$		

$$AE = s^4 + 2s^2 + 1 - K$$

$$\frac{d}{ds}(AE) = 4s^3 + 4s$$

$1-K > 0$ no poles are on RHS plane and LHS plane.

All poles are on $j\omega$ -axis

$\therefore 0 < K < 1$ system marginally stable

07. Ans: (d)

Sol: Assertion: FALSE

Let the TF = s. "s" is the differentiator

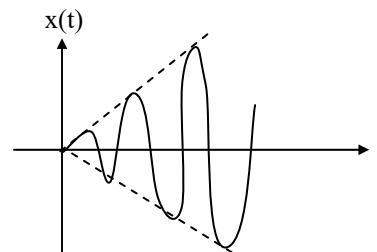
Impulse response $L^{-1}[TF] = L^{-1}[s] = \delta'(t)$

$$\lim_{t \rightarrow \infty} \delta'(t) = 0$$

$\therefore$ It is BIBO stable

Reason: True

$$x(t) = t \sin t$$



$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} t \sin t \text{ is unbounded}$$

08. Ans: (a)

Sol: Assertion: TRUE

If feedback is not properly utilized the closed loop system may become unstable.

Reason: True

Feedback changes the location of poles

$$\text{Let } G(s) = \frac{-2}{s+1} \quad H(s) = 1$$

Open loop pole $s = -1$ (stable)

$$\text{CLTF} = \frac{\frac{-2}{s+1}}{1 + \frac{-2}{s+1}} = \frac{-2}{s-1}$$

Closed loop pole is at $s = 1$ (unstable)

$\therefore$ After applying the feedback no more system is open loop. It becomes closed loop system. Hence poles are affected.

09. Ans: (a) & (d)

Sol: RH tabulation:

s^5	1	5	4
s^4	-3	-7	20
s^3	$\frac{8}{3}$	$\frac{32}{3}$	0
s^2	5	20	0
s^1	0(10)	0	0
s^0	20	0	0

$$AE = 5s^2 + 20 = 0$$

$$\frac{dAE}{ds} = 10s = 0$$

$$AE \text{ roots} = s = \pm j2$$

Two sign changes

$\therefore$ No. of $j\omega$ axis roots = 2

No. of left hand root = 1 (real)

10. Ans: (a), (c) & (d)

$$\text{Sol: C.E} = 1 + \frac{k}{s(s+4)(s+5)} = 0$$

$$s^3 + 9s^2 + 20s + k = 0$$

$$\begin{array}{r|rr} s^3 & 1 & 20 \\ s^2 & 9 & k \\ s^1 & \frac{180-k}{9} & \\ s^0 & k & \end{array}$$

$$180 - k > 0$$

$$k < 180 \text{ and}$$

$$k > 0$$

$\therefore$ Range of k for stability $0 < k < 180$

$k > 180$; Two sign changes in the 1st column

$\therefore$ Number of right half of s -plane poles = 2

$k = 180$ marginally stable

$\therefore$ Two poles are on the imaginary axis

$k < 180$ stable

$\therefore$ All the three poles are in the left half of s -plane

Chapter 5 Root Locus Diagram

01. Ans: (a)

Sol: $s_1 = -1 + j\sqrt{3}$

$$s_2 = -3 - j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$$s_1 = -1 + j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(-1 + j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(1 + j\sqrt{3})^3}$$

$$= -3 \tan^{-1}(\sqrt{3})$$

$$= -180^\circ$$

It is odd multiples of 180° , Hence s_1 lies on Root locus

$$s_2 = -3 - j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(-3 - j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(-1 - j\sqrt{3})^3}$$

$$= -3 [180^\circ + 60^\circ] = -720^\circ$$

It is not odd multiples of 180° , Hence s_2 is not lies on Root locus.

02. Ans: (a)

Sol: Over damped - roots are real & unequal

$$\Rightarrow 0 < k < 4$$

(b) $k = 4$ roots are real & equal

$$\Rightarrow \text{Critically damped } \xi = 1$$

(c) $k > 4 \Rightarrow$ roots are complex

$$0 < \xi < 1 \Rightarrow \text{under damped}$$

03. Ans: (a)

Sol: Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3 - 0}{3 - 0} = -1$$

$$\text{centroid} = (-1, 0)$$

04. Ans: (b)

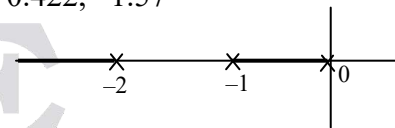
Sol: Break point = $\frac{dK}{ds} = 0$

$$\frac{d}{ds}(G_1(s).H_1(s)) = 0$$

$$\frac{d}{ds}[s(s+1)(s+2)] = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.422, -1.57$$



But $s = -1.57$ do not lie on root locus

So, $s = -0.422$ is valid break point.

Point of intersection wrt $j\omega$ -axis

$$s^3 + 3s^2 + 2s + k = 0$$

$$\begin{array}{r|rr} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & 6-k & 0 \\ s^0 & 3 & k \end{array}$$

As s^1 Row = 0

$$k = 6$$

$$3s^2 + 6 = 0$$

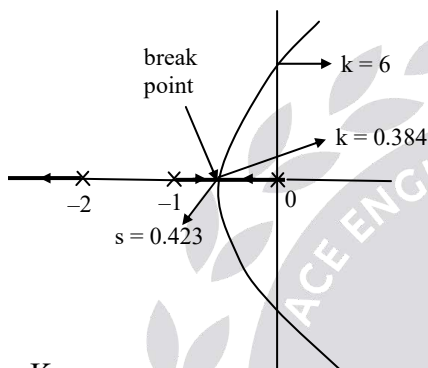
$$s^2 = -2$$

$$s = \pm j\sqrt{2}$$

point of inter section: $s = \pm j\sqrt{2}$

05. Ans: (b)

Sol:



$$\frac{K}{s(s+1)(s+2)}$$

substitute $s = -0.423$ and apply the magnitude criteria.

$$\left| \frac{K}{(-0.423)(-0.423+1)(-0.423+2)} \right| = 1$$

$$K = 0.354$$

when the roots are complex conjugate then the system response is under damped.

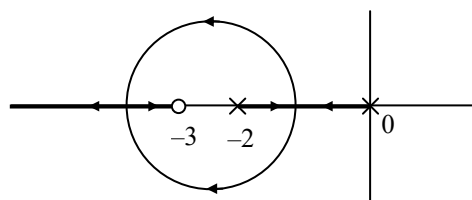
From $K > 0.384$ to $K < 6$ roots are complex conjugate then system to be under damped the values of k is $0.384 < K < 6$.

06. Ans: (c)

Sol: If the roots are lies on the real axis then system exhibits the non-oscillatory response. from $K \geq 0$ to $K \leq 0.384$ roots lies on the real axis. Hence for $0 \leq K \leq 0.384$ system exhibits the non-oscillatory response.

07. Ans: (a)

Sol:



$$\frac{d}{ds} [G(s).H(s)] = \frac{d}{ds} \left[\frac{k(s+3)}{s(s+2)} \right]$$

$$s^2 + 6s + 6 = 0$$

break points $-1.27, -4.73$

$$\text{radius} = \frac{4.73 - 1.27}{2} = 1.73$$

center = $(-3, 0)$

08. Ans: (c)

$$\text{Sol: } G(s).H(s) = \frac{K(s+3)}{s(s+2)}$$

$$k|_{s=-4} = \left| \frac{(-4)(-4+2)}{(-4+3)} \right| = \left| \frac{(-4)(-2)}{(-1)} \right| = 8$$

09. Ans: (a)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_A = 180 - \angle GH|_{s=2+2j}$$

$$GH = \frac{k[s - (2+2j)][s - (2-2j)]}{[s - (-2+2j)][s - (-2-2j)]}$$

$$\angle GH|_{s=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4+4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$$

10. Ans: (b)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_d = 180^\circ + \angle GH|_{s=-2 \pm 2j}$$

$$\angle GH|_{s=-2 \pm 2j} = \angle \frac{k[s - (2 + 2j)][s - (2 - 2j)]}{[s - (-2 + 2j)][s - (-2 - 2j)]}|_{s=-2 \pm 2j}$$

$$= \frac{\angle k(-4)(-4 + 4j)}{\angle 4j}$$

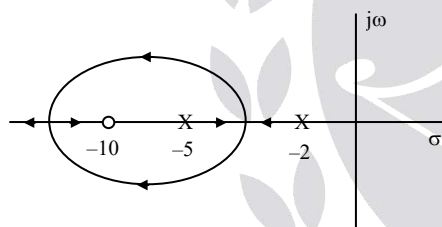
$$= 180^\circ + 180^\circ - 45^\circ - 90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

11. Ans: (d)

Sol: Poles $s = -2, -5$; Zero $s = -10$



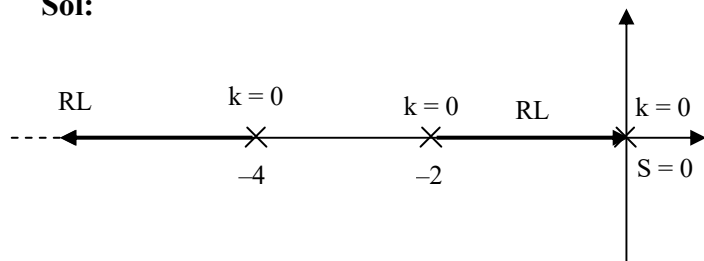
$\therefore$ Breakaway point exist between -2 and -5

12.

Sol: Refer Pg No: 75, Vol-1 Ex: 8

13. Ans: (a), (c) & (d)

Sol:



$$\Rightarrow \text{Centroid } \sigma = \frac{(-2 - 4) - (0)}{3} = -2$$

$$\Rightarrow \text{Angle of asymptotes } \theta = \frac{(2q + 1)180^\circ}{(p - z)},$$

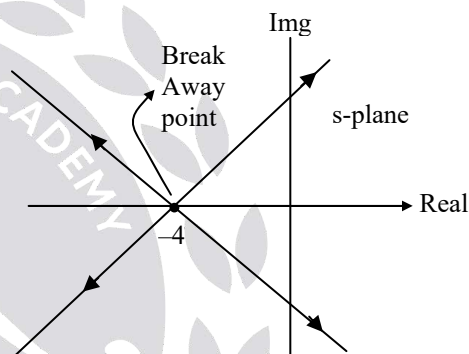
$$q = 0 \Rightarrow \theta = \frac{180^\circ}{3} = 60^\circ$$

$$q = 1 \Rightarrow \theta = \frac{3 \times 180^\circ}{3} = 180^\circ$$

$$q = 2 \Rightarrow \theta = \frac{5 \times 180^\circ}{3} = 300^\circ$$

14. Ans: (a) & (b)

Sol: RLD of the system is drawn below



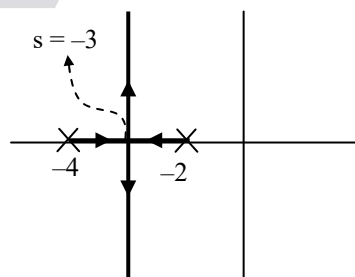
$$\text{Consider } \sigma = \frac{-4 - 4 - 4 - 4}{4 - 0}$$

$$\sigma = -4$$

All the root loci branches are breaking away at $s = -4$, hence it is called as a break away point.

15. Ans: (c) & (d)

Sol: RLD of the system is given below



$$k|_{s=-3} = (1)(1) = 1$$

$s = -3$ is a break in | away point

Chapter 6 Frequency Response Analysis

01. Ans: (c)

Sol: $G(s).H(s) = \frac{100}{s(s+4)(s+16)}$

Phase crossover frequency (ω_{pc}):

$$\angle G(j\omega).H(j\omega) / \omega = \omega_{pc} = -180^\circ$$

$$-90^\circ - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180^\circ$$

$$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90^\circ$$

$$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90^\circ)$$

$$\frac{\frac{\omega_{pc}}{4} + \frac{\omega_{pc}}{16}}{1 - \frac{\omega_{pc}}{4} \cdot \frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8 \text{ rad/sec}$$

02. Ans: (d)

Sol: $G(s).H(s) = \frac{100}{s(s+4)(s+16)}$

$$\text{Gain margin (G.M)} = \frac{1}{|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}}}$$

$$|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} = \frac{100}{\omega_{pc} \sqrt{\omega_{pc}^2 + 4^2} \sqrt{\omega_{pc}^2 + 16^2}}$$

$$= \frac{5}{64}$$

$$\text{G.M} = \frac{64}{5} = 12.8$$

03. Ans: (c)

Sol: $G(s).H(s) = \frac{2e^{-0.5s}}{(s+1)}$

gain crossover frequency,

$$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{2}{\sqrt{\omega_{gc}^2 + 1}} = 1$$

$$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3} \text{ rad/sec}$$

04. Ans: (b)

Sol: $\omega_{gc} = \sqrt{3} \text{ rad/sec}$

$$\text{P.M} = 180^\circ + \angle G(j\omega).H(j\omega) / \omega = \omega_{gc}$$

$$\angle G(j\omega).H(j\omega) / \omega = \omega_{gc} = -0.5 \omega_{gc} - \tan^{-1}(\omega_{gc})$$

$$= -109.62^\circ$$

$$\text{P.M} = 70.35^\circ$$

05. Ans: (a)

Sol: $M_r = 2.5 = \frac{1}{2\xi\sqrt{1-\xi^2}}$

$$2\xi\sqrt{1-\xi^2} = \frac{1}{2.5}$$

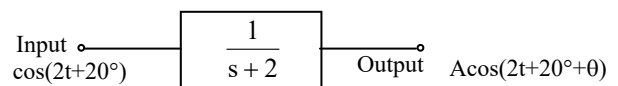
$$\xi^4 - \xi^2 + 0.04 = 0$$

$$\xi^2 = 0.958 \quad \xi^2 = 0.0417$$

$$\xi = 0.204 \quad (M_r > 1)$$

06. Ans: (a)

Sol: Closed loop T.F = $\frac{1}{s+2}$



$$A = \frac{1}{\sqrt{\omega^2 + 4}} = \frac{1}{\sqrt{4 + 4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$$\phi = -\tan^{-1} \omega/2$$

$$= -\tan^{-1} 2/2$$

$$\Rightarrow \phi = -\tan^{-1}(1) = -45^\circ$$

$$\text{output} = \frac{1}{2\sqrt{2}} \cos(2t + 20^\circ - 45^\circ)$$

$$= \frac{1}{2\sqrt{2}} \cos(2t - 25^\circ)$$

07. Ans: (c)

Sol: Initial slope = -40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$$\therefore \text{Part of TF} = G(s)H(s) = \frac{K}{s^2}$$

at $\omega = 0.1$

$$\text{Change in slope} = -20 - (-40) = 20^\circ$$

$$\text{Part of TF} = G(s)H(s) = \frac{K \left(1 + \frac{s}{0.1}\right)}{s^2}$$

At $\omega = 10$ slope changed to -60 dB/dec

$$\text{Change in slope} = -60 - (-20)$$

$$= -40 \text{ dB/dec}$$

$$\text{TF } (G(s)H(s)) = \frac{K \left(1 + \frac{s}{0.1}\right)}{s^2 \left(\frac{s}{10} + 1\right)^2}$$

$$20 \log K - 2(20 \log 0.1) = 20 \text{ dB}$$

$$20 \log K = 20 - 40$$

$$20 \log K = -20$$

$$K = 0.1$$

$$G(s)H(s) = \frac{(0.1) \left(1 + \frac{s}{0.1}\right)}{s^2 \left(1 + \frac{s}{10}\right)^2}$$

$$= \frac{(0.1) \times 10^2 (s + 0.1)}{(0.1)s^2 (s + 10)^2}$$

$$G(s)H(s) = \frac{100(s + 0.1)}{s^2 (s + 10)^2}$$

08. Ans: (b)

$$\text{Sol: } G(s)H(s) = \frac{Ks}{\left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{10}\right)}$$

$$12 = 20 \log K + 20 \log 0.5$$

$$12 = 20 \log K + (-6)$$

$$20 \log K = 18 \text{ dB} = 20 \log 2^3$$

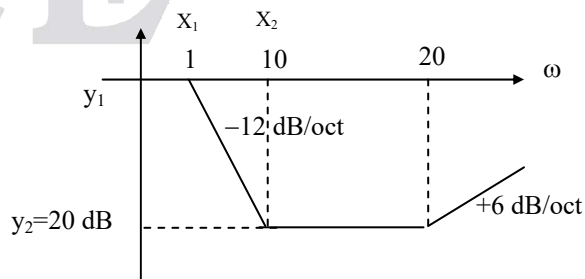
$$K = 8$$

$$G(s)H(s) = \frac{8s \times 2 \times 10}{(2 + s)(10 + s)}$$

$$G(s)H(s) = \frac{160s}{(2 + s)(10 + s)}$$

09. Ans: (b)

Sol:



$$G(s)H(s) = \frac{K \left(1 + \frac{s}{10}\right)^2 \left(1 + \frac{s}{20}\right)}{(1 + s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40 \text{ dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60 \text{ dB} \Big|_{\omega \leq 1}$$

$$\Rightarrow 20 \log K = 60$$

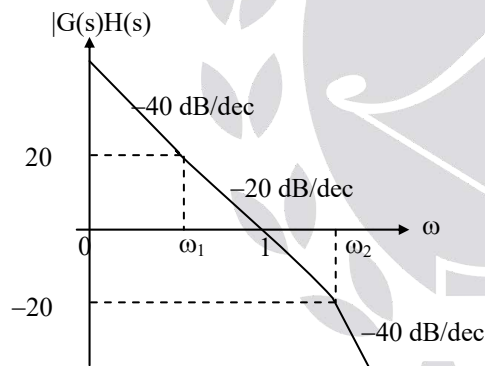
$$K = 10^3$$

$$G(s)H(s) = \frac{10^3 (s+10)^2 (s+20)}{10^2 \times 20 \times (s+1)^2}$$

$$= \frac{(s+10)^2 (s+20)}{2(s+1)^2}$$

10. Ans: (d)

Sol:



ω_1 calculation:

$$\frac{0 - 20}{\log 1 - \log \omega_1}$$

$$= -20 \text{ dB/dec}$$

$$\omega_1 = 0.1$$

ω_2 calculation:

$$\frac{-20 - 0}{\log \omega_2 - \log 1}$$

$$= -20 \text{ dB/dec}$$

$$\omega_2 = 10$$

$$G(s)H(s) = \frac{K \left(1 + \frac{s}{0.1}\right)}{s^2 \left(1 + \frac{s}{10}\right)}$$

$$20 \log K - 2 (20 \log 0.1) = 20$$

$$20 \log K = 20 - 40$$

$$K = 0.1$$

$$G(s)H(s) = \frac{0.1 \times \frac{1}{0.1} (0.1 + s)}{s^2 \frac{1}{10} (10 + s)}$$

$$= \frac{10(0.1 + s)}{s^2 (10 + s)}$$

11.

Sol: $\frac{200}{s(s+2)} = \frac{100}{s \left(1 + \frac{s}{2}\right)}$

$$x = -KT \Rightarrow -(100) \times \frac{1}{2} = x = -50$$

12. Ans: (c)

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot.

For stability

$$1 > 0.01 K$$

$$\Rightarrow K < 100$$

13.

Sol: $GM = -40 \text{ dB}$

$$20 \log \frac{1}{a} = -40 \Rightarrow a = 10^2$$

$$POI = 100$$

14.

Sol: (i) $GM = \frac{1}{0.1} = +10 = 20 \text{ dB}$

$PM = 180^\circ - 140^\circ = 40^\circ$

(ii) $PM = 180 - 150^\circ = 30^\circ$

$GM = \frac{1}{0} = \infty$ $POI = 0$

(iii) ω_{PC} does not exist

$GM = \frac{1}{0} = \infty$ $PM = 180^\circ + 0^\circ = 180^\circ$

(iv) ω_{gc} not exist

$\omega_{pc} = \infty$

$GM = \frac{1}{0} = \infty$

$PM = \infty$

(v) $GM = \frac{1}{0.5} = 2$

$PM = 180 - 90$
 $= 90^\circ$

15. **Ans: (d)**

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot. In figures (1) & (2) $(-1, j0)$ is not enclosed.

$\therefore$ Systems represented by (1) & (2) are stable.

16. **Ans: (b)**

Sol: Open loop system is stable, since the open loop poles are lies in the left half of s-plane
 $\therefore P = 0$.

From the plot $N = -2$.

No. of encirclements $N = P - Z$

$N = -2, P = 0$ (Given)

$\therefore N = P - Z$

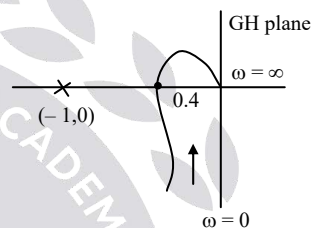
$-2 = 0 - Z$

$Z = 2$

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

17. **Ans: (c)**

Sol:

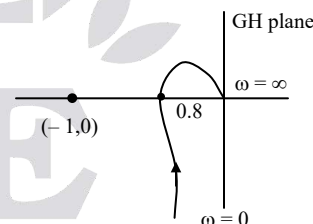


$\frac{K_c}{K} = 0.4$

When $K = 1$

Now, K double, $\frac{K_c}{K} = 0.4$

$K_c = 0.4 \times 2 = 0.8$



Even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on ' ξ '

$\xi \propto \frac{1}{\sqrt{K}}$ as K is increased ξ reduced, then

more oscillations.

18. Ans: (a)

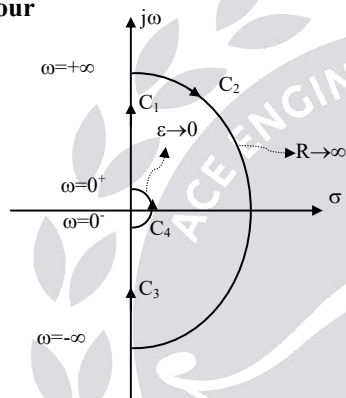
Sol: Given system $G(s) = \frac{10(s-12)}{s(s+2)(s+3)}$

It is a non minimum phase system since $s = 12$ is a zero on the right half of s-plane

19.

Sol: Given that $G(s)H(s) = \frac{10(s+3)}{s(s-1)}$

**s-plane
Nyquist Contour**



- Nyquist plot is the mapping of Nyquist contour(s-plane) into $G(s)H(s)$ plane.
- The Nyquist contour in the s-plane enclosing the entire right half of S-plane is shown figure.

The Nyquist Contour has four sections C_1 , C_2 , C_3 and C_4 . These sections are mapped into $G(s)H(s)$ plane .

Mapping of section C_1 : It is the positive imaginary axis, therefore sub $s = j\omega$, ($0 \leq \omega \leq \infty$) in the TF $G(s)H(s)$, which gives the polar plot

$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

Let $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$$G(j\omega)H(j\omega) = \frac{10\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}} \angle \left\{ \tan^{-1}\left(\frac{\omega}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega)] \right\}$$

At $\omega = 0 \Rightarrow \infty \angle -270^\circ$

At $\omega = \omega_{pc} = \sqrt{3} \Rightarrow 10 \angle -180^\circ$

At $\omega = \infty \Rightarrow 0 \angle -90^\circ$

point of intersection of the Nyquist plot with respect to negative real axis is calculated below

$$\begin{aligned} \text{Arg}G(j\omega)H(j\omega) &= \arg \frac{10(j\omega+3)}{j\omega(j\omega-1)} \\ &= -180^\circ \text{ will give the } \omega_{pc} \end{aligned}$$

Magnitude of $G(j\omega)H(j\omega)$ gives the point of intersection

$$\begin{aligned} \angle \tan^{-1}\left(\frac{\omega}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega)] \\ = -180^\circ \big|_{\omega = \omega_{pc}} \end{aligned}$$

$$\angle \tan^{-1}\left(\frac{\omega_{pc}}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega_{pc})] = -180^\circ$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{3}\right) + \tan^{-1}(\omega_{pc}) = 90^\circ$$

Taking “tan” both the sides

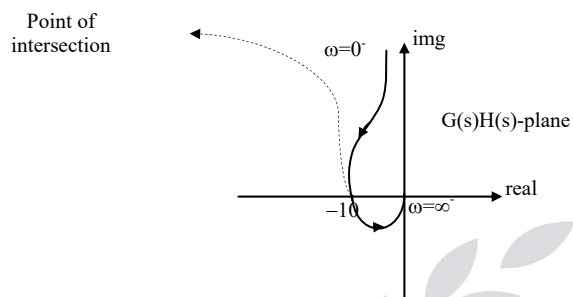
$$\frac{\frac{\omega_{pc}}{3} + \omega_{pc}}{1 - \frac{(\omega_{pc})^2}{9}} = \tan 90^\circ = \infty$$

$$1 - \frac{\omega_{pc}^2}{9} = 0$$

$$\omega_{pc} = \sqrt{3} \text{ rad/sec}$$

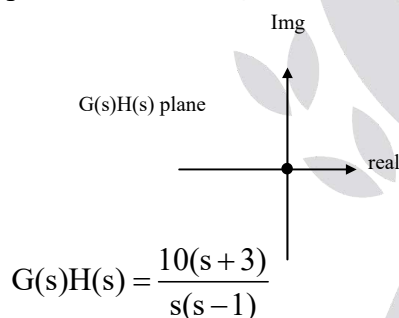
Therefore the point of intersection is

$$|G(j\omega)H(j\omega)| \text{ at } \omega_{pc} = \frac{10\sqrt{\omega_{pc}^2 + 3^2}}{\omega_{pc}\sqrt{1 + \omega_{pc}^2}} = 10$$



The mapping of the section C_1 is shown in figure.

Mapping of section C_2 : It is the radius ' R ' semicircle, therefore sub $s = \lim_{R \rightarrow \infty} R e^{j\theta}$ (θ is from 90° to 0° to -90°) in the TF $G(s)H(s)$, which merges to the origin in $G(s)H(s)$ plane.



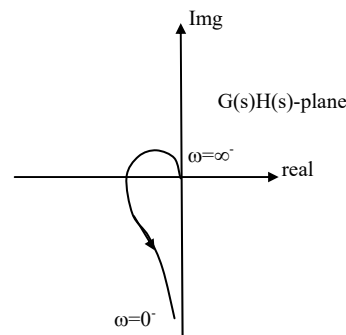
$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

$$G(R e^{j\theta})H(R e^{j\theta}) = \frac{2(R e^{j\theta} + 3)}{R e^{j\theta}(R e^{j\theta} - 1)} \approx 0$$

The plot is shown in figure.

Mapping of section C_3 : It is the negative imaginary axis, therefore sub $s = j\omega$, ($-\infty \leq \omega \leq 0$) in the TF $G(s)H(s)$, which gives the mirror image of the polar plot and is symmetrical with respect to the real axis,

The plot is shown in figure.



Mapping of section C_4 : It is the radius ' ϵ '

semicircle, therefore sub $s = \lim_{\epsilon \rightarrow 0} \epsilon e^{j\theta}$

($-90^\circ \leq \theta \leq 90^\circ$) in the TF $G(s)H(s)$, which gives clockwise infinite radius semicircle in $G(s)H(s)$ plane.

The plot is shown below

$$G(\epsilon e^{j\theta})H(\epsilon e^{j\theta}) = \frac{10(\epsilon e^{j\theta} + 3)}{\epsilon e^{j\theta}(\epsilon e^{j\theta} - 1)}$$

$$G(\epsilon e^{j\theta})H(\epsilon e^{j\theta}) \approx \frac{10 \times 3}{-\epsilon e^{j\theta}} = \infty \angle 180^\circ - \theta$$

$$\text{When, } \theta = -90^\circ \quad \infty \angle 270^\circ$$

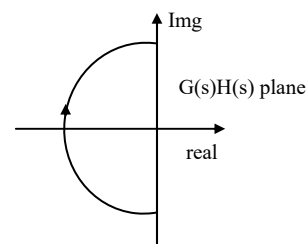
$$\theta = -40^\circ \quad \infty \angle 220^\circ$$

$$\theta = 0^\circ \quad \infty \angle 0^\circ$$

$$\theta = 40^\circ \quad \infty \angle 140^\circ$$

$$\theta = 90^\circ \quad \infty \angle 90^\circ$$

It is clear that the plot is clockwise ' ∞ ' radius semicircle centred at the origin



Combining all the above four sections, the

$$\text{Nyquist plot of } G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

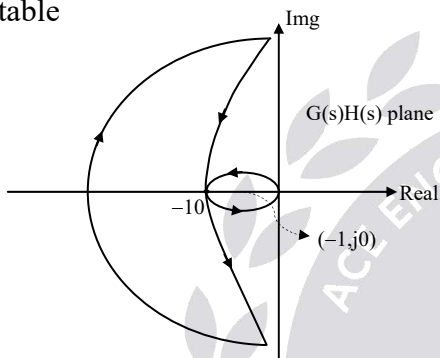
is shown in figure below

From the plot $N = 1$

Given that $P = 1$

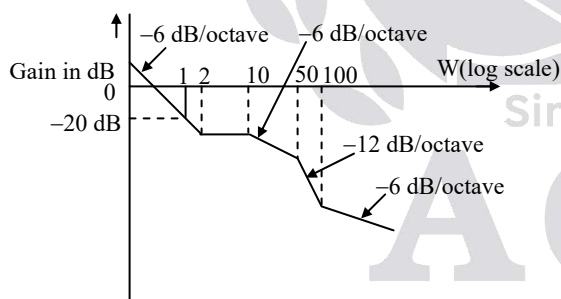
$$N = P - Z$$

$Z = P - N = 1 - 1 = 0$, therefore system is stable



20.

Sol: The given bode plot is shown below.



Initial slope = -6 db/octave.

i.e., there is one pole at origin (or) one integral term.

portion of transfer function

$$G(s) = \frac{K}{s}$$

At $\omega = 2$ rad/sec, slope is changed to 0dB/ octave.

$\therefore$ change in slope

$$= \text{present slope} - \text{previous slope}$$

$$= 0 - (-6) = 6 \text{ dB/octave}$$

$\therefore$ There is a real zero at corner frequency

$$\omega_1 = 2.$$

$$(1 + sT_1) = \left(1 + \frac{s}{\omega_1}\right) = \left(1 + \frac{s}{2}\right)$$

At $\omega = 10$ rad/sec, slope is changed to

-6dB/octave.

$\therefore$ change in slope = -6 - 0

$$= -6 \text{ dB/octave.}$$

$\therefore$ There is a real pole at corner frequency

$$\omega_2 = 2.$$

$$\frac{1}{1 + sT_2} = \frac{1}{\left(1 + \frac{s}{\omega_2}\right)} = \frac{1}{\left(1 + \frac{s}{10}\right)}$$

At $\omega = 50$ rad/sec, slope is changed to

-12dB/octave.

$\therefore$ change in slope = -12 - (-6)

$$= -6 \text{ dB/octave}$$

$\therefore$ There is a real pole at corner frequency

$$\omega_3 = 50 \text{ rad/sec.}$$

$$\frac{1}{1 + sT_3} = \frac{1}{\left(1 + \frac{s}{\omega_3}\right)} = \frac{1}{\left(1 + \frac{s}{50}\right)}$$

At $\omega = 100$ rad/sec, the slope changed to -6 dB/octave.

$\therefore$ change in slope = -6 - (-12)

$$= 6 \text{ dB/octave.}$$

$\therefore$ There is a real zero at corner frequency

$$\omega_4 = 100 \text{ rad/sec.}$$

$$\therefore (1 + sT_4) = \left(1 + \frac{s}{\omega_4}\right) = \left(1 + \frac{s}{100}\right)$$

$$\therefore \text{Transfer function} = \frac{K \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{100}\right)}{s \left(1 + \frac{s}{50}\right) \left(1 + \frac{s}{10}\right)}$$

$$= \frac{K(s+2)(s+100)}{s(s+50)(s+10)} \cdot \frac{1}{50} \cdot \frac{1}{10}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

In the given bode plot,

at $\omega = 1 \text{ rad/sec}$, Magnitude = -20 dB .

$$-20 \text{ dB} = 20 \log K - 20 \log \omega + 20 \sqrt{1 + \left(\frac{\omega}{2}\right)^2} + 20 \sqrt{1 + \left(\frac{\omega}{100}\right)^2}$$

$$-20 \log \sqrt{1 + \left(\frac{\omega}{50}\right)^2} - 20 \log \sqrt{1 + \left(\frac{\omega}{10}\right)^2}$$

At $\omega = 1 \text{ rad/sec}$,

$$-20 = 20 \log K - 20 \log \omega / \omega = 1 \text{ rad/sec}$$

[$\therefore$ Remaining values eliminated]

$$-20 = 20 \log K$$

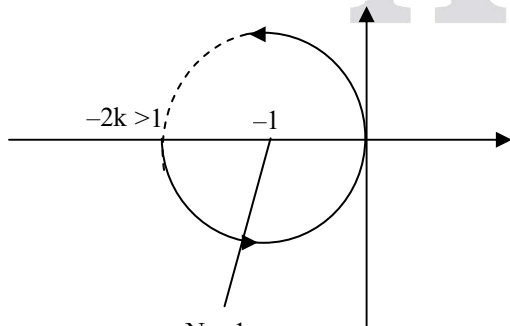
$$\Rightarrow K = 0.1$$

$\therefore$ Transfer function

$$\frac{C(s)}{R(s)} = \frac{0.25(s+2)(s+100)}{s(s+10)(s+50)}$$

21. Ans: (a) & (d)

Sol: $k > 1/2$, closed loop system is stable.



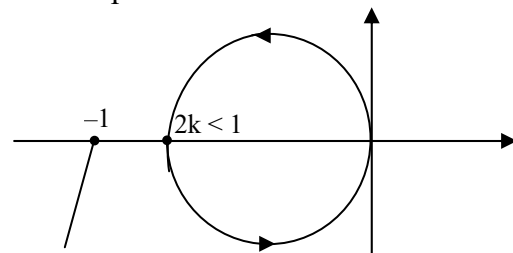
$$N = 1$$

$$P = 1$$

CL system is stable

$$2k > 1 \Rightarrow k > 1/2$$

For $k < 1/2$, one closed loop pole in the right half of s-plane.



$$N = 0$$

$$P = 1$$

CL is unstable, $2k < 1 \Rightarrow k < 1/2$

$N = P - Z \Rightarrow 0 = 1 - Z \Rightarrow Z = 1 \Rightarrow$ one closed loop Pole in the right half s-plane

22. Ans: (a) & (d)

Sol: $\Rightarrow \omega_{pc} = \infty$. Hence $GM = \infty$

$$\Rightarrow \angle \phi |_{\omega_{gc}} = -150^\circ, \Rightarrow PM = 180^\circ + \angle \phi |_{\omega_{gc}}$$

$$\Rightarrow PM = 180^\circ - 150^\circ = +30^\circ \text{ (finite).}$$

23. Ans: (b) & (d)

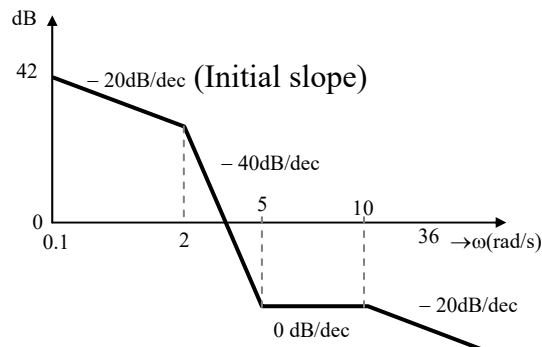
$$\text{Sol: } G(s)H(s) = \frac{10 \times 5^2 (1 + \frac{s}{5})^2}{s \times 2(1 + \frac{s}{2})(10)(1 + \frac{s}{10})}$$

$$= \frac{12.5(1 + \frac{s}{5})^2}{s(1 + \frac{s}{2})(1 + \frac{s}{10})}$$

$$M|_{\omega=0.1} = 20 \log 12.5 - 20 \log \omega$$

$$= 20 \log 12.5 - 20 \log 0.1$$

$$\approx 42 \text{ dB}$$



⇒ Slope of the line between 5 rad/sec to 10 rad/sec is 0 dB/dec.

⇒ At high frequency, slope of line is -20 dB/dec.

24. Ans: (b) & (c)

Sol: At any frequency magnitude of the loop transfer function is not unity,

$$\therefore PM = \infty$$

System is always stable,

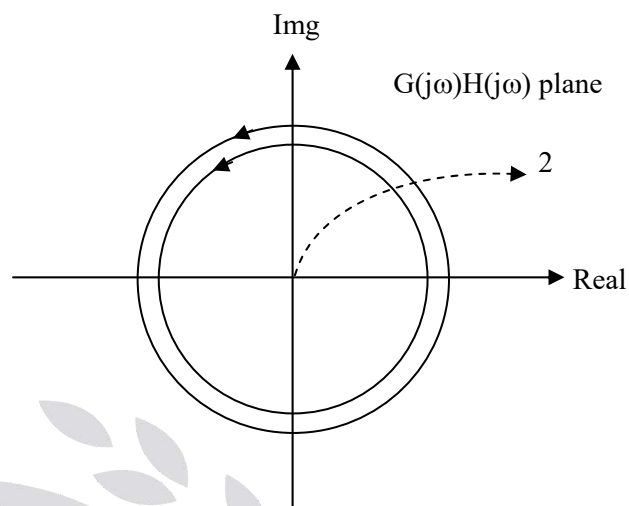
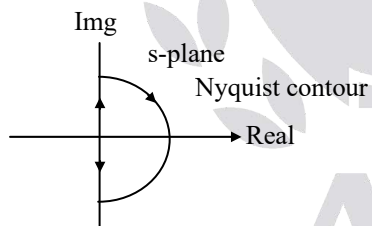
$$\therefore GM = \infty$$

25. Ans: (b) & (c)

Sol: $N_{0,0}$ = difference between open loop polar and zero

$$N_{0,0} = (2 - 0) = 2$$

$$N_{0,0} = 2$$



Chapter 7 Compensators & Controllers

01. Ans: (a)

$$\begin{aligned}\text{Sol: } G_C(s) &= (-1) \left(-\frac{Z_2}{Z_1} \right) \\ &= (-1)(-1) \left(\frac{R_2 + \frac{1}{sC}}{R_1} \right) \\ G_C(s) &= \frac{(100 \times 10^3) + \frac{1}{s \times 10^{-6}}}{10^6}\end{aligned}$$

$$G_C(s) = \frac{1 + 0.1s}{s}$$

02. Ans: (c)

$$\begin{aligned}\text{Sol: CE} \Rightarrow 1 + G_C(s) G_P(s) &= 0 \\ &= 1 + \frac{1 + 0.1s}{s} \times \frac{1}{(s+1)(1+0.1s)} \\ &= 1 + \frac{1 + 0.1s}{s(s+1)(1+0.1s)} = 0 \\ \Rightarrow s^2 + s + 1 &= 0 \Rightarrow \omega_n = 1, \\ e^{\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}} \right]}_{\xi=0.5} &= 0.163 \\ M_p &= 16.3\%\end{aligned}$$

03. Ans: (b)

$$\begin{aligned}\text{Sol: T.F} &= \frac{k(1+0.3s)}{1+0.17s} \\ T &= 0.17, aT = 0.3 \Rightarrow a = \frac{0.3}{0.17} \\ C &= 1 \mu F \\ T &= \frac{R_1 R_2}{R_1 + R_2} C, a = \frac{R_1 + R_2}{R_2} \\ \frac{R_1 R_2}{R_1 + R_2} &= \frac{0.17}{1 \times 10^{-6}} = 170000 \\ \frac{R_1 + R_2}{R_2} &= 1.764 \\ aT &= R_1 C \\ R_1 &= \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6)\end{aligned}$$

$$= 300 \text{ k}\Omega$$

$$\begin{aligned}B_v \\ 300 \text{ k} + R_2 - 1.76 R_2 &= 0\end{aligned}$$

$$\begin{aligned}R_2 &= \frac{300}{0.70} = 394.736 \\ &= 400 \text{ k}\Omega\end{aligned}$$

04. Ans: (d)

Sol: PD controller improves transient stability and PI controller improves steady state stability. PID controller combines the advantages of the above two controllers.

05.

$$\begin{aligned}\text{Sol: For } K_I = 0 \Rightarrow \\ \frac{C(s)}{R(s)} &= \frac{(K_P + K_D s)}{s(s+1) + (K_P + K_D s)} \\ &= \frac{K_P + K_D s}{s^2 + (1 + K_D)s + K_P} \\ \omega_n &= \sqrt{K_P} \\ 2\xi\omega_n &= 1 + K_D \\ \Rightarrow 2(0.9) \sqrt{K_P} &= 1 + K_D \\ \Rightarrow 1.8 \sqrt{K_P} &= 1 + K_D \quad \dots\dots\dots (1)\end{aligned}$$

$$\text{Dominant time constant } \frac{1}{\xi\omega_n} = 1$$

$$\begin{aligned}\Rightarrow \omega_n &= \frac{1}{0.9} = 1.111 \\ K_P &= \omega_n^2 = 1.11^2 \\ &= 1.234\end{aligned}$$

From eq. (1),

$$\begin{aligned}\Rightarrow 1.8 \times \frac{1}{0.9} &= 1 + K_D \\ \Rightarrow K_D &= 1\end{aligned}$$

06. Ans: (b) & (d)

Sol: Both PD and lead controller improve transient response of the system.

Chapter 8 State Space Analysis

01. Ans: (a)

Sol: $TF = \frac{1}{s^2 + 5s + 6}$

$$= \frac{1}{(s+2)(s+3)}$$

$$= \frac{1}{s+2} + \frac{-1}{s+3}$$

$$\therefore A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = [1 \quad 1]$$

02. Ans: (c)

Sol: Given problem is Controllable canonical form.

(or)

$$TF = C[sI - A]^{-1}B + D$$

$$= [6 \quad 5 \quad 1] \begin{bmatrix} s & 1 & 0 \\ 0 & s & 1 \\ -5 & -3 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$= \frac{3s^2 + 15s + 18}{s^3 + 6s^2 + 3s + 5}$$

03. Ans: (d)

Sol: $\frac{d^2 y}{dt^2} + \frac{3dy}{dt} + 2y = u(t)$

2nd order system hence two state variables are chosen

Let $x_1(t), x_2(t)$ are the state variables

CCF - SSR

Let $x_1(t) = y(t) \dots \dots \dots (1)$

$x_2(t) = \dot{y}(t) \dots \dots \dots (2)$

Differentiating (1)

$$\dot{x}_1(t) = \dot{y}(t) = x_2(t) \dots \dots \dots (3)$$

$$\dot{x}_2(t) = \ddot{y}(t) = u(t) - 3\dot{y}(t) - 2y(t)$$

$$= u(t) - 3x_2(t) - 2x_1(t) \dots \dots \dots (4)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

A B

From equation 1. The output equation in matrix form

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, D = 0$$

04. Ans: (b)

Sol: OCF - SSR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

05. Ans: (c)

Sol: Normal form - SSR

$$TF = \frac{Y(s)}{G(s)} = \frac{1}{s^2 + 3s + 2} = \frac{1}{(s+1)(s+2)}$$

$\Rightarrow$ Diagonal canonical form

The eigen values are distinct i.e., -1 & -2.

$\therefore$ Corresponding normal form is called as diagonal canonical form

DCF - SSR

$$\frac{Y(s)}{U(s)} = \frac{b_1}{s+1} + \frac{b_2}{s+2}$$

$$b_1 = 1, b_2 = -1$$

$$Y(s) = \underbrace{\frac{b_1}{s+1}}_{x_1} U(s) + \underbrace{\frac{b_2}{s+2}}_{x_2} U(s)$$

Let $Y(s) = X_1(s) + X_2(s)$

Where $y(t) = x_1(t) + x_2(t)$ (1)

Where $X_1(s) = \frac{b_1}{s+1} U(s)$

$s X_1(s) + X_1(s) = b_1 U(s)$

Take Laplace Inverse

$\dot{x}_1 + x_1 = b_1 u(t)$ (2)

$X_2(s) = \frac{b_2}{s+2} U(s)$

$s X_2(s) + 2 X_2(s) = b_2 U(s)$

Laplace Inverse

$\dot{x}_2 + 2x_2 = b_2 u(t)$

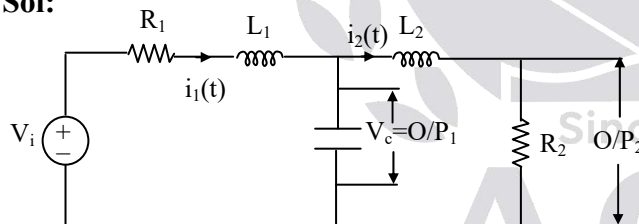
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

From (1) output equation.

$$y(t) = [1 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

06. Ans: (c)

Sol:



$O/P_1 \Rightarrow y_1 = V_c$

$O/P_2 \Rightarrow y_2 = R_2 i_2$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ i_1 \\ i_2 \end{bmatrix}$$

$y = C X$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}$$

07. Ans: (a)

Sol: T.F = $C[sI - A]^{-1}B + D$

$$\begin{aligned} &= [1 \ 0] \begin{bmatrix} s+4 & 1 \\ 3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= [1 \ 0] \frac{1}{s^2 + 5s + 1} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \frac{1}{s^2 + 5s + 1} [1 \ 0]_{1 \times 2} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1} \\ &= \frac{1}{s^2 + 5s + 1} [s+1 \ -1]_{1 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1} \\ &= \frac{1}{s^2 + 5s + 1} [s+1-1] \\ &= \frac{s}{s^2 + 5s + 1} \end{aligned}$$

08. Ans: (c)

Sol: State transition matrix $\phi(t) = L^{-1}[(sI - A)^{-1}]$

$$sI - A = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[[sI - A]^{-1}] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

09. Ans: (b)

Sol: Controllability

$$[M] = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$|M| = -1 \neq 0$ (Controllable)

Observability

$$[N] = [C^T \ A^T C^T \ \dots \ (A^T)^{n-1} C^T]$$

$$A^T C^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$|N| = 0 \text{ (Not observable)}$$

10. Ans: (c)

Sol: According to Gilberts test the system is controllable and observable.

11. Ans: (c)

$$\text{Sol: } \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

at node $\dot{x}_1$

$$\dot{x}_1 = -a_1 x_1 - a_2 x_2 - a_3 x_3$$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

12.

Sol: The given state space equations:

$$\dot{X} = X_2$$

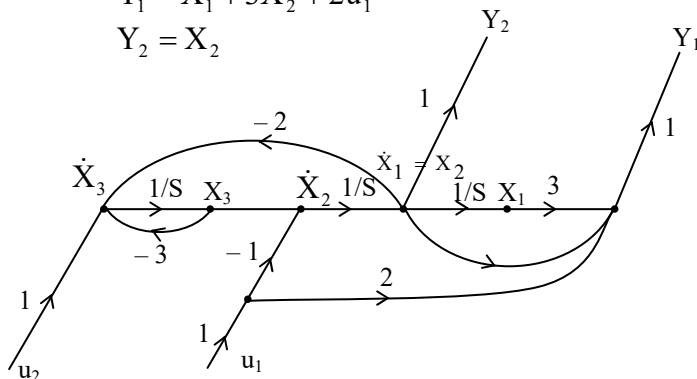
$$\dot{X}_2 = X_3 - u_1$$

$$\dot{X}_3 = -2X_2 - 3X_3 + u_2$$

and output equations are :

$$Y_1 = X_1 + 3X_2 + 2u_1$$

$$Y_2 = X_2$$



The given state space equations in matrix for

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Where A: State matrix

B: Input matrix

C: Output matrix

D: Transition matrix

Characteristic equation

$$|sI - A| = 0$$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 0 & 2 & S+3 \end{vmatrix} = 0$$

$$\Rightarrow s[s(s+3)+2]+1(0)=0$$

$$\Rightarrow s(s^2+3s+2)=0$$

$$\Rightarrow s(s+1)(s+2)=0$$

The roots are 0, -1, -2.

13. Ans: (a) & (b)

Sol: (a) → state model is in controllable canonical form

(b) → state model is in observable canonical form