

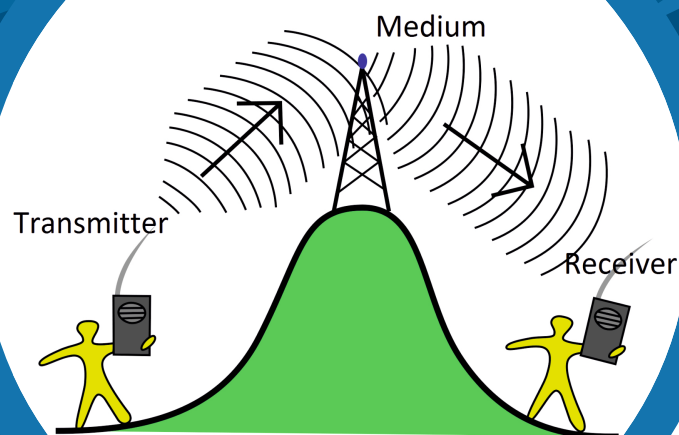


GATE | PSUs

ELECTRONICS & COMMUNICATION ENGINEERING

Communications

**(Text Book : Theory with worked out Examples
and Practice Questions)**



Chapter

1

Introduction

(Solutions for Text Book Practice Questions)

01. Ans: (b)

Sol: We know that

$$e^{-at}u(t) \xleftrightarrow{\text{F.T}} \frac{1}{a + j\omega}$$

$$e^{at}u(-t) \xleftrightarrow{\text{F.T}} \frac{1}{a - j\omega}$$

$$e^{-at}u(t) - e^{at}u(-t) \xleftrightarrow{\text{F.T}} \frac{1}{a + j\omega} - \frac{1}{a - j\omega}$$

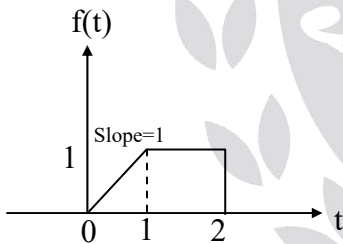
Put $a = 0$

$$u(t) - u(-t) \xleftrightarrow{\text{F.T}} \frac{1}{j\omega} - \frac{1}{-j\omega}$$

$$\text{sgn}(t) \xleftrightarrow{\text{F.T}} \frac{2}{j\omega}$$

02. Ans: (a)

Sol:



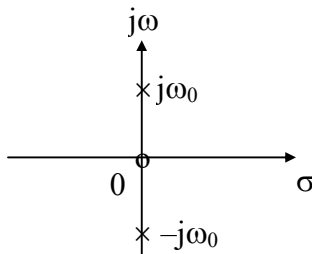
$$f(t) = r(t) - r(t - 1) - u(t - 2)$$

03. Ans: (a)

Sol: The convergence of Fourier transform is along the $j\omega$ -axis in s -plane.

04. Ans: (a)

Sol:

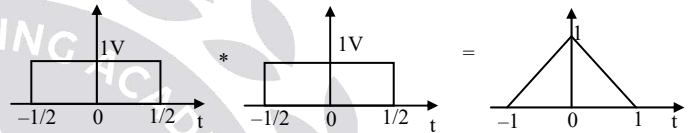


$$F(s) = \frac{s}{s^2 + \omega_0^2} \xleftrightarrow{\text{I.L.T}} f(t) = \cos \omega_0 t$$

$$f(t) = \cos \omega_0 t \xleftrightarrow{\text{I.L.T}} F(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

05. Ans: (d)

Sol:



06. Ans: (c)

Sol: Given $x(t) = e^{-at^2}$

Fourier transform of $x(t)$ is

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(at^2 + j\omega t)} dt$$

$$= e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-\left[\sqrt{at + \frac{j\omega}{2\sqrt{a}}}\right]^2} dt$$

$$\text{Let } p = \sqrt{at} + \frac{j\omega}{2\sqrt{a}}$$

$$dp = \sqrt{a} dt$$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2} dp$$

$$\int_{-\infty}^{\infty} e^{-p^2} dp = \sqrt{\pi}$$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \sqrt{\pi}$$

$$X(\omega) = \sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}$$

07. Ans: (d)

Sol: The EFS expression of a periodic signal $x(t)$

$$\text{is } x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where, ' c_n ' is EFS coefficient.

Apply F.T on both sides

$$X(\omega) = \sum_{n=-\infty}^{\infty} c_n \text{FT}[e^{jn\omega_0 t}]$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$e^{jn\omega_0 t} \leftrightarrow 2\pi\delta(\omega - n\omega_0)$$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

So, it is a train of impulse.

08. Ans: (a)

Sol: $V(j\omega) = e^{-j2\omega}; |\omega| \leq 1$

$$\text{Energy} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |V(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^1 |e^{-j2\omega}|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^1 1 d\omega$$

$$= \frac{2}{2\pi}$$

$$= \frac{1}{\pi}$$

09. Ans: (b)

Sol: Parseval's theorem is used to find the energy of the signal in frequency domain.

$$\therefore \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

10. Ans: (a)

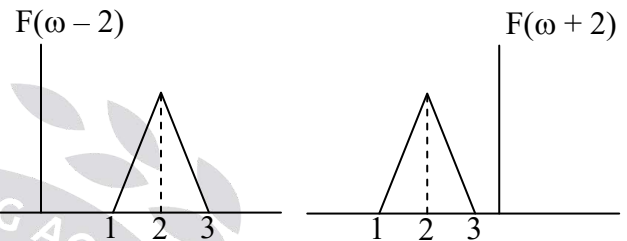
Sol: $f(t) = A.e^{-a|t|} \xleftrightarrow{\text{F.T}} F(j\omega) = \frac{2Aa}{a^2 + \omega^2}$

11. Ans: (d)

Sol: $m(t) = f(t) \cos 2t$

Apply Fourier transform

$$M(f) = \frac{1}{2} [F(\omega - 2) + F(\omega + 2)]$$



12. Ans: (b)

Sol: For band limited signals,

$$S(f) \neq 0; |f| < W$$

$$S(f) = 0; |f| > W$$

13. Ans: (a)

Sol: In a communication system, antenna is used to convert voltage variations to field variation and vice-versa.

14. Ans: (d)

Sol: Hilbert transform of $f(t)$ is

$$\text{H.T}\{f(t)\} = f(t) * \frac{1}{\pi t}$$

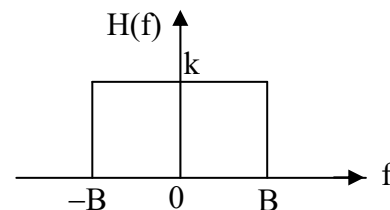
It is in the terms of 't'.

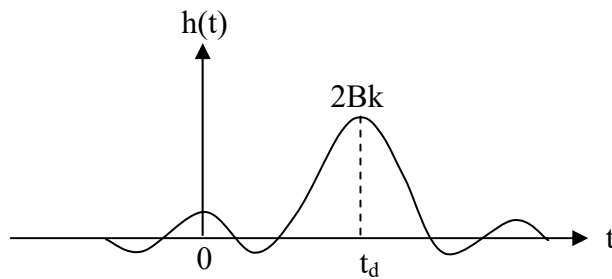
15. Ans: (a)

Sol: For an ideal LPF

$$H(f) = k e^{-j\omega t_0} \quad \text{for } -B < f < B$$

$$h(t) = F^{-1}[H(f)] = 2Bk \text{ sinc } 2B(t - t_d)$$





$h(t) \neq 0$ for $t < 0$

Output exists before input is applied i.e. non-causal, which is physically impossible.

16. Ans: (b)

$$\text{Sol: } \delta(at) = \frac{1}{|a|} \delta(t)$$

$$\delta(2t) = \frac{1}{2} \delta(t)$$

17. Ans: (a)

Sol: By modulation we are translating the low frequency spectrum into high frequency spectrum.

18. Ans: (a)

Sol: We know that

$$P(\text{dBm}) = 10 \log(P \times 10^3)$$

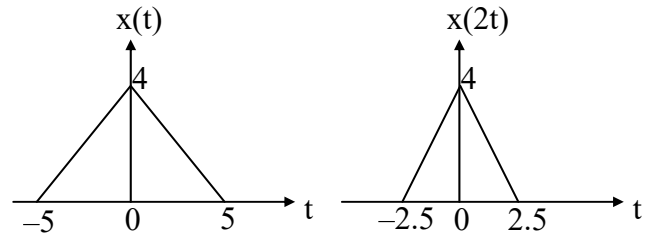
$$-10 = 10 \log(P \times 10^3)$$

$$P \times 10^3 = 10^{-1}$$

$$P = 10^{-4} = 100 \mu\text{W}$$

19. Ans: (a)

Sol: $x(2t)$ means signal time axis is compressed by 2



20. Ans: (b)

Sol: Audio frequency is between 20Hz to 20kHz

21. Ans: (d)

Sol: Telephone channel carries voice. Voice frequency is between 300 Hz to 3500 Hz. So bandwidth is 3200Hz. So we approximately consider 4kHz is the bandwidth requirement of a telephone channel.

22. Ans: (c)

Sol: From the signal spectrum $f_H = 530 \text{ kHz}$,
 $f_L = 50 \text{ kHz}$

$$\text{Bandwidth} = f_H - f_L = 530 \text{ kHz} - 50 \text{ kHz} = 480 \text{ kHz}$$

Chapter 2 Amplitude Modulation

01. Ans: (a)

Sol: $V(t) = A_c \cos \omega_c t + 2 \cos \omega_m t \cdot \cos \omega_c t$.

Comparing this with the AM-DSB-SC signal

$A \cos \omega_c t + m(t) \cdot \cos \omega_c t$, it implies that

$$m(t) = 2 \cos \omega_m t \Rightarrow E_m = 2$$

To implement Envelope detection,

$$A_c \geq E_m$$

$$\therefore (A_c)_{\min} = 2$$

02. Ans: (d)

Sol: $m(t) = (A_c + A_m \cos \omega_m t) \cos \omega_c t$.

$$= A_c \left(1 + \frac{A_m}{A_c} \cos \omega_m t \right) \cos \omega_c t$$

Given

$$A_c = 2A_m$$

$$= A_c \left(1 + \frac{1}{2} \cos \omega_m t \right) \cos \omega_c t$$

$$P_T = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{2} \right], \quad P_s = \frac{A_c^2}{2} \left[\frac{\mu^2}{4} \right]$$

$$\frac{P_T}{P_s} = \frac{1 + \frac{\mu^2}{2}}{\frac{\mu^2}{4}} = \frac{1 + \frac{1}{8}}{\frac{1}{16}} = \frac{9}{8} \times 16$$

$$P_T = 18 P_s$$

03. Ans: (a)

Sol: $m(t) = 2 \cos 2\pi f_1 t + \cos 2\pi f_2 t$

$$C(t) = A_c \cos 2\pi f_c t$$

$$S(t) = [A_c + m(t)] \cos 2\pi f_c t$$

$$S(t) = A_c \left[1 + \frac{1}{A_c} m(t) \right] \cos 2\pi f_c t$$

$$K_a = \frac{1}{A_c}$$

$$A_{m1} = 2, A_{m2} = 1$$

$$\mu_1 = K_a A_{m1} = \frac{2}{A_c}, \mu_2 = K_a A_{m2} = \frac{1}{A_c}$$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\Rightarrow 0.5 = \sqrt{\frac{4}{A_c^2} + \frac{1}{A_c^2}}$$

$$\Rightarrow A_c = \sqrt{20}$$

04. Ans: (c)

Sol: $m(t) = -0.2 + 0.6 \sin \omega_1 t$, $k_a = 1$, $A_c = 100$

$$S(t) = A_c [1 - 0.2 + 0.6 \sin \omega_1 t] \cos \omega_c t$$

$$= 100 [0.8 + 0.6 \sin \omega_1 t] \cos \omega_c t$$

$$V_{\max} = A_c [1 + \mu] = 100 [0.8 + 0.6] = 140 \text{ V}$$

$$V_{\min} = A_c [1 - \mu] = 100 [0.8 - 0.6] = 20 \text{ V}$$

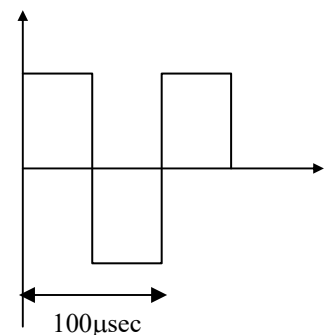
$$= 20 \text{ V to } 140 \text{ V}$$

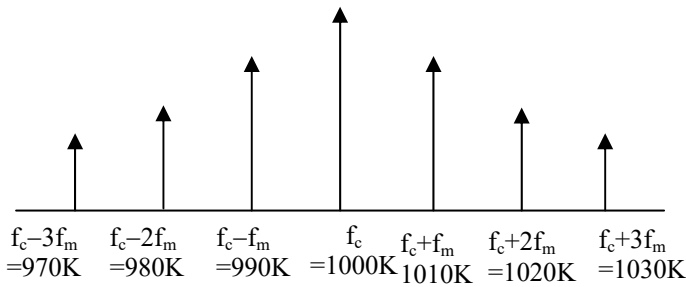
05. Ans: (c)

Sol: $f_c = 1 \text{ MHz} = 1000 \text{ kHz}$

The given $m(t)$ is symmetrical square wave of period $T = 100 \mu\text{sec}$

$$f_m = \frac{1}{T_0} = 10 \text{ kHz}$$

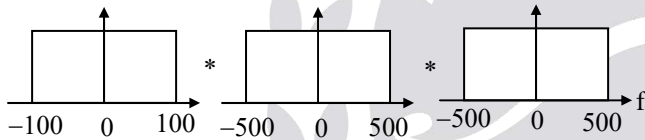




These frequencies 980k, 1020k are not present because the symmetrical square wave it consists of half wave symmetries only odd harmonics are present, even harmonics are dismissed.

06. Ans: (d)

Sol: $m(t) = \text{sinc}(200t)\text{sinc}^2(1000t)$
 $= \text{sinc}(200t)\text{sinc}(1000t)\text{sinc}(1000t)$

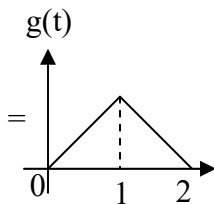
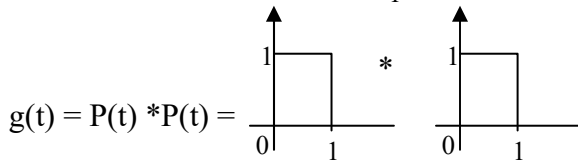
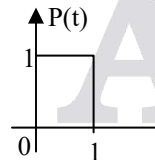


So, highest frequency component in the signal $m(t)$ is $100 + 500 + 500 = 1100$

$BW = 2 \times 1100$
 $BW = 2200 \text{ Hz}$

07. Ans: (a)

Sol: $P(t) = u(t) - u(t-1) \Rightarrow$



$x(t) = 100(P(t) + 0.5g(t))\cos\omega_c t$
 $= 100(1 + 0.5t)\cos\omega_c t$
 $= A_c(1 + K_a m(t))\cos\omega_c t$

$k_a = 0.5, m(t) = t$

$\mu = k_a[m(t)]_{\max}$

$\mu = 0.5 \times 1 = 0.5$

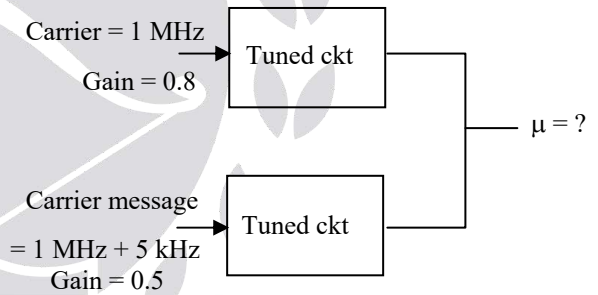
08. Ans: (d)

Sol: $R_L C \leq \frac{\sqrt{1-\mu^2}}{2\pi f_m \mu}$

So it depends on depth of modulation and the highest modulation frequency.

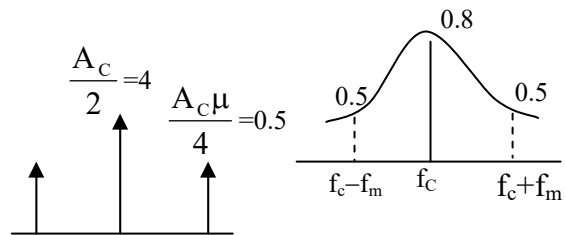
09. Ans: (b)

Sol: $S(t) = 10\cos 2\pi 10^6 t + 8\cos 2\pi 5 \times 10^3 t \cos 2\pi 10^6 t$



$S(t) = 0.8 \times 10 \cos 2\pi 10^6 t$
 $+ 0.5 \times 8 \cos 2\pi 5000 t \cos 2\pi 10^6 t$
 $= 8(1 + \frac{4}{8} \cos 2\pi 5000 t) \cos 2\pi 10^6 t$

$\mu = \frac{4}{8} = \frac{1}{2} = 0.5$



10. Ans: (d)

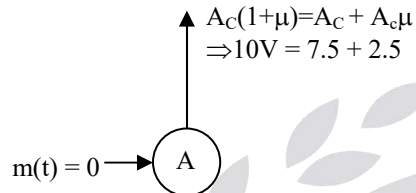
Sol: $A_{\max} = 10V$

$A_{\min} = 5V$

$\mu = 0.1$

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{1}{3} = 0.33$$

$$A_C = \frac{A_{\max} + A_{\min}}{2} = \frac{10 + 5}{2} = 7.5 V$$



Amplitude deviation $A_C\mu = 7.5 \times \frac{1}{3} = 2.5 V$

$\mu_2 = 0.1 \Rightarrow A_{c2}\mu_2 = 2.5$

$A_{c2} = 25 V$

Which must be added to attain = 17.5

11. Ans: (d)
Sol: Modulation index

$\mu = k_a |m(t)|_{\max}$

$$k_a = \frac{2b}{a} = \frac{2(\text{square term coefficient})}{\text{linear term coefficient}}$$

$|m(t)|_{\max} = 1$

$$\mu = 2\left(\frac{b}{a}\right)$$

$$P_{SB} = \frac{1}{2}P_C \Rightarrow P_C \frac{\mu^2}{2} = \frac{1}{2}P_C$$

$$\mu^2 = 1 \Rightarrow \left(2\frac{b}{a}\right)^2 = 1$$

$$\Rightarrow 2\frac{b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

12. Ans: 0.125

Sol: $s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2000\pi t)$

 Here $4\cos(2400\pi t)$ is the carrier signal.

 $\cos(2000\pi t)$ and $\cos(2000\pi t)$ are the sideband message signals.

$$P_c = \frac{4^2}{2} = 8 W$$

$$P_m = \frac{1}{2} + \frac{1}{2} = 1 W$$

$$\frac{P_m}{P_c} = \frac{1}{8} = 0.125$$

13. Ans: (a, c & d)

Sol: $S_{AM}(t) = 10\cos(2\pi \times 5000t) + 25\cos(2\pi \times 5200t) + 25\cos(2\pi \times 4800t)$

\therefore USB Frequency = 5200 Hz

\therefore LSB Frequency = 4800 Hz

$$\frac{A_c\mu}{2} = 25$$

$$\frac{10 \times \mu}{2} = 25$$

$\therefore \mu = 5$

a, c & d are correct.

NOTE: options are changed for

(a) $\mu = 5$ (b) $\mu = 2.5$

14. Ans: (a & c)

Sol: $S_{AM}(t) = K_1\cos(2\pi \times 5000t) + K_2\cos(2\pi \times 5200t) + K_3\cos(2\pi \times 4800t)$

$c(t) = 10 \cos(2\pi \times 5000t)$

$K_1 = 10 = A_C$

$\mu = 0.5$

$\therefore \frac{A_c\mu}{2} = K_2 = K_3$

$$\frac{10 \times 0.5}{2} = K_2 = K_3$$

$\therefore K_2 = K_3 = 2.5$

a & c are correct.

$f_c + f_m = 5200 \text{ Hz}$

$f_c - f_m = 4800 \text{ Hz}$

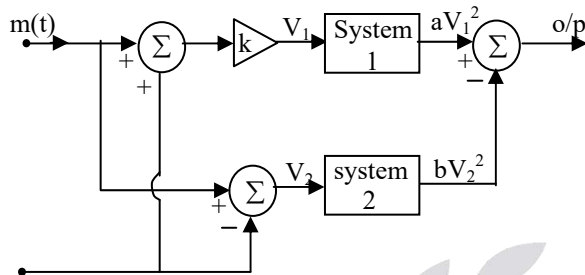
$\therefore 2f_m = 400 \text{ Hz}$

$f_m = 200 \text{ Hz}$

Chapter 3 Sideband Modulation Techniques

01. Ans: (c)

Sol:



$A \cos \omega_c t$

$$V_1 = k [m(t) + c(t)]$$

$$V_2 = [m(t) - c(t)]$$

$$V_0 = aV_1^2 - bV_2^2$$

$$= ak^2[m(t) + c(t)]^2 - b[m(t) - c(t)]^2$$

$$= ak^2 [m^2(t) + c^2(t) + 2m(t)c(t)]$$

$$- b[m^2(t) + c^2(t) - 2m(t)c(t)]$$

$$= [ak^2 - b]m^2(t) + [ak^2 - b]c^2(t)$$

$$+ 2[ak^2 + b][m(t)c(t)]$$

on verification if $k = \sqrt{\frac{b}{a}}$

$$S(t) = 4bm(t)c(t) \rightarrow \text{DSBSC Signal}$$

02. Ans: (d)

Sol: Given $A = 10$

$$m(t) = \cos 1000\pi t$$

$$b = 1$$

B.W = ? and power = ?

$$s(t) = 4b.A \cos 2\pi f_c t \cdot \cos 2\pi (500)t$$

$$= 40 \cdot \cos 2\pi f_c t \cdot \cos 2\pi (500)t$$

$$B.W = 2 f_m$$

$$= 2 (500)$$

$$= 1 \text{ kHz}$$

$$\begin{aligned} \text{Power} &= \frac{A_c^2 A_m^2}{4} \\ &= \frac{1600 \times 1}{4} \\ &= 400 \text{ W} \end{aligned}$$

03. Ans: (c)

Sol: Carrier = $\cos 2\pi (100 \times 10^6)t$

Modulating signal = $\cos(2\pi \times 10^6)t$

Output of Balanced modulator

$$= 0.5[\cos 2\pi (101 \times 10^6)t + \cos 2\pi (99 \times 10^6)t]$$

The Output of HPF is $0.5 \cos 2\pi (101 \times 10^6)t$

Output of the adder is

$$= 0.5 \cos 2\pi (101 \times 10^6)t + \sin 2\pi (100 \times 10^6)t$$

$$= 0.5 \cos 2\pi [(100+1)10^6 t] + \sin 2\pi (100 \times 10^6)t$$

$$= 0.5[\cos 2\pi (100 \times 10^6)t \cdot \cos 2\pi (10^6)t$$

$$- \sin 2\pi (100 \times 10^6)t \cdot \sin 2\pi (10^6)t]$$

$$+ \sin 2\pi (100 \times 10^6)t]$$

$$= 0.5 \cos 2\pi (100 \times 10^6)t \cdot \cos 2\pi (10^6)t$$

$$+ \sin 2\pi (100 \times 10^6)t [1 - 0.5 \sin 2\pi (10^6)t]$$

$$\text{Let } 0.5 \cos 2\pi (10^6)t = r(t) \cos \theta(t)$$

$$1 - 0.5 \sin 2\pi (10^6)t = r(t) \cdot \sin \theta(t)$$

The envelope is

$$r(t) = [0.25 \cos^2 2\pi (10^6)t$$

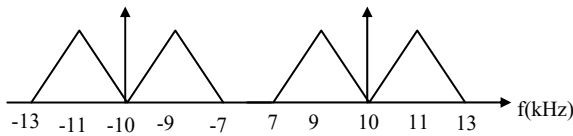
$$+ \{1 - 0.5 \sin 2\pi (10^6)t\}^2]^{1/2}$$

$$= [1.25 - \sin 2\pi (10^6)t]^{1/2}$$

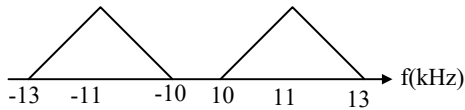
$$= \left[\frac{5}{4} - \sin 2\pi (10^6)t \right]^{1/2}$$

04. Ans: (b)

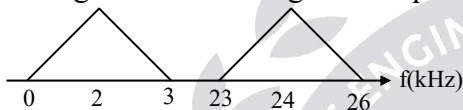
Sol: Output of 1st balanced modulator is



Output of HPF is



The Output of 2nd balanced modulator is consisting of the following +ve frequencies.



Thus, the spectral peaks occur at 2 kHz and 24 kHz.

05. Ans: (c)

Sol: Given

$$f_{m_1} = 100\text{Hz}, f_{m_2} = 200\text{Hz}, f_{m_3} = 400\text{Hz},$$

$$f_c = 100\text{KHz}, f_{c_{Lo}} = 100.02\text{KHz}$$

$$S(t)/T_x = \frac{A_c A_m}{2} [\cos(f_c + f_{m_1})t + \cos(f_c + f_{m_2})t + \cos(f_c + f_{m_3})t]$$

$$S(t)/R_x = [S(t)/T_x] A_c \cos 2\pi f_{c_{Lo}} t$$

$$\Rightarrow \frac{A_c^2 A_m}{4} [\cos(f_c + f_{c_{Lo}} + f_{m_1}) + \cos(f_{m_1} - 20) +$$

$$\cos(f_c + f_{c_{Lo}} + f_{m_2}) + \cos(f_{m_2} - 20) +$$

$$\cos(f_c + f_{c_{Lo}} + f_{m_3}) + \cos(f_{m_3} - 20)]$$

Detector output frequencies:

$$80\text{Hz}, 180\text{Hz}, 380\text{Hz}$$

06. Ans: (b)

Sol: Given

SSB AM is used, LSB is transmitted

$$f_{Lo} = (f_c + 10)$$

$$S(t)/T_x = \frac{A_c A_m}{2} \cos 2\pi [f_c - f_m]t$$

$$S(t)/R_x = \left[\frac{A_c A_m}{2} \cos 2\pi (f_c - f_m)t \right] \cos 2\pi (f_c + 10)t$$

$$\Rightarrow \frac{A_c A_m}{4} [\cos 2\pi (2f_c + 10 - f_m)t + \cos 2\pi (10 + f_m)t]$$

i.e., from 310 Hz to 1010 Hz

07. Ans: (b)

Sol: BW of Basic group = $12 \times 4 = 48$ kHz

BW of super group = $5 \times 48 = 240$ kHz

08. Ans: (d)

Sol: Given 11 voice signals

B.W. of each signals = 3 kHz

Guard Band Width = 1 kHz

Lowest $f_c = 300$ kHz

Highest $f_c =$

$$\Rightarrow f_{c_H} + f_{m_{lost}} = 300\text{kHz} + 11(3\text{kHz}) + 10(1\text{kHz}) = 343 \text{ kHz}$$

$$f_{c_H} = 343 \text{ kHz} - 3 \text{ kHz}$$

$$= 340 \text{ kHz}$$

09. Ans: (b)

Sol: $f_{m1} = 5$ kHz \rightarrow AM

$f_{m2} = 10$ kHz \rightarrow DSB

$f_{m3} = 10$ kHz \rightarrow SSB

$f_{m4} = 2$ kHz \rightarrow SSB

$f_{m5} = 5$ kHz \rightarrow AM

$f_g = 1$ kHz

$$BW = (2f_{m1} + 2f_{m2} + f_{m3} + f_{m4} + 2f_{m5} + 4f_g)$$

$$= 2 \times 5 + 2 \times 10 + 10 + 2 + 2 \times 5 + 4 \times 1$$

$$= 10 + 20 + 10 + 10 + 6$$

$$= 56 \text{ kHz}$$

$$\therefore BW = 56 \text{ kHz}$$

10. Ans: (b & c)

Sol: Power in AM = $P_C + P_{USB} + P_{LSB}$

Power in DSB-SC = $P_{USB} + P_{LSB}$, power in SSB-SC = P_{USB} (or) P_{LSB}

\therefore Power in AM > DSB-SC > VSB = SSB

Option (b) is correct

BW in AM = $2f_{max}$

BW in DSB-SC = $2f_{max}$

BW in SSB-SC = f_{max}

BW in VSB-SC = $f_{max} + \Delta f$

\therefore BW in AM = BW in DSB-SC

> BW of VSB

> BW of SSB

Option (c) is correct

11. Ans: (a, c & d)

Sol: For DSB-SC $\eta = 100\%$

$BW = 2f_{max} = 2 \times 3 \times 10^4 = 60(\text{kHz})$

$S(t) = m(t) c(t)$

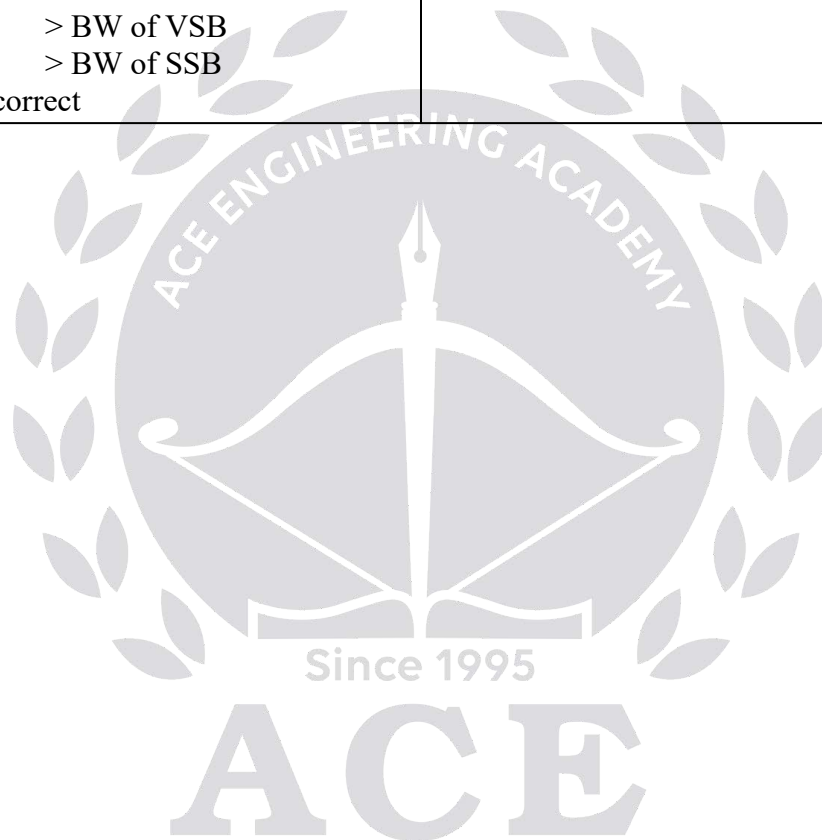
$= 50 \cos(2\pi \times 10^7 t) \cos(2\pi \times 10^4 t)$

$+ 50 \cos(2\pi \times 10^7 t) 5 \cos(5\pi \times 10^4 t)$

$+ 50 \cos(2\pi \times 10^7 t) 4 \cos(6\pi \times 10^4 t)$

$P_t = 26.25(\text{kW})$

(a, c & d are correct)



01. Ans: (a)

Sol: $s(t) = 10 \cos(20\pi t + \pi t^2)$

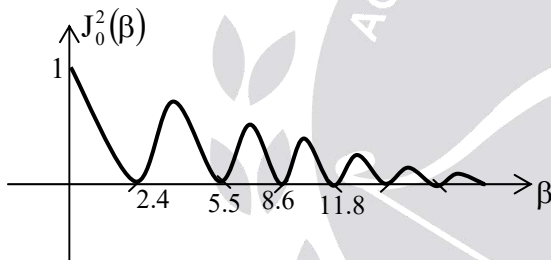
$$f_i = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$f_i = \frac{1}{2\pi} [20\pi + 2\pi t]$$

$$\frac{df_i}{dt} = \frac{1}{2\pi} \times 2\pi \times 1 = 1 \text{ Hz/sec}$$

02. Ans: (d)

Sol: $P_{fc} = \frac{A_c^2 J_0^2(\beta)}{2}$



So, $J_0^2(\beta)$ is decreasing first, becoming zero and then increasing so power is also behave like $J_0^2(\beta)$.

03. Ans: (a)

Sol: In an FM signal, adjacent spectral components will get separated by

$$f_m = 5 \text{ kHz}$$

$$\text{Since } BW = 2(\Delta f + f_m) = 1 \text{ MHz} \\ = 1000 \times 10^3$$

$$\Delta f + f_m = 500 \text{ kHz}$$

$$\Delta f = 495 \text{ kHz}$$

The n^{th} order non-linearity makes the carrier frequency and frequency deviation increased by n -fold, with the base-band signal frequency (f_m) left unchanged since $n = 3$,

$$\therefore (\Delta f)_{\text{New}} = 1485 \text{ kHz} \quad \&$$

$$(f_c)_{\text{New}} = 300 \text{ MHz}$$

$$\text{New BW} = 2(1485 + 5) \times 10^3$$

$$= 2.98 \text{ MHz}$$

$$= 3 \text{ MHz}$$

04. Ans: (d)

Sol: $S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t$

$$\Delta f = 3(2f_m) = 12 \text{ kHz}$$

$$\beta = \frac{\Delta f}{f_m} = 6$$

$$\therefore S(t) = \sum_{n=-\infty}^{\infty} 5 \cdot J_n(6) \cos 2\pi(f_c + nf_m)t$$

$$f_c = 1000 \text{ kHz}, f_m = 2 \text{ kHz}$$

$$= \cos 2\pi(1008 \times 10^3)t$$

$$= \cos 2\pi(1000 + 4 \times 2) \times 10^3 t$$

i.e., $n = 4$

The required coefficient is $5 \cdot J_4(6)$

05. Ans: (c)

Sol: $2\pi f_m = 4\pi \cdot 10^3$

$$\Rightarrow f_m = 2 \text{ kHz}$$

$$J_0(\beta) = 0 \text{ at } \beta = 2.4$$

$$\beta = \frac{k_f A_m}{f_m} \Rightarrow 2.4 = \frac{k_f \times 2}{2 \text{ kHz}}$$

$$k_f = 2.4 \text{ kHz/V}$$

$$\text{at } \beta = 5.5$$

$$5.5 = \frac{2.4k \times 2}{f_m}$$

$$\Rightarrow f_m = 872.72 \text{ Hz}$$

06. Ans: (c)

Sol: $\beta = 6$

$$J_0(6) = 0.1506 ; J_3(6) = 0.1148$$

$$J_1(6) = 0.2767 ; J_4(6) = 0.3576$$

$$J_2(6) = 0.2429 ;$$

$$\frac{P_{f_c \pm 4f_m}}{P_T} = ? \quad P_T = \frac{A_c^2}{2R}$$

$$P_{f_c \pm 4f_m} = \frac{A_c^2}{R} \left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_3^2(\beta) + J_4^2(\beta) \right]$$

$$P_{f_c \pm 4f_m} = \frac{A_c^2}{R} \left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_4^2(\beta) \right]$$

$$\frac{P_{f_c \pm 4f_m}}{P_T} = \frac{0.2879}{1/2} = 0.5759 = 57.6 \%$$

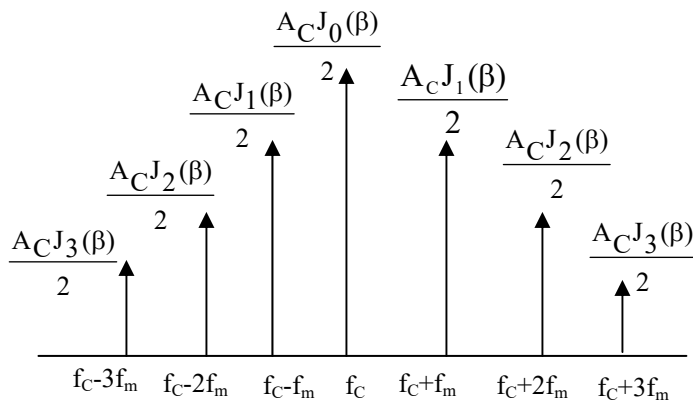
07. Ans: (c)

Sol: $m(t) = 10\cos 20\pi t$

$$f_m = 10 \text{ Hz}$$

inserting correct signal and frequency

$$\beta = \frac{k_f A_m}{f_m} = \frac{5 \times 10}{10} = 5$$



From f_c to $f_c + 4f_m$ pass through ideal BPF

Powers in these frequency components

$$P = \frac{A_c^2}{2R} J_0^2(\beta) + 2 \frac{A_c^2}{2R} J_1^2(\beta) + 2 \frac{A_c^2}{2R} J_2^2(\beta)$$

$$+ 2 \frac{A_c^2}{2R} J_3^2(\beta) + 2 \frac{A_c^2}{1R} J_4^2(\beta)$$

$$= \frac{A_c^2}{2R} \left[(-0.178)^2 + 2(-0.328)^2 + 2(0.049)^2 \right]$$

$$+ 2(0.365)^2 + 2(0.391)^2$$

$$= 41.17 \text{ Watts}$$

08. Ans: (d)

Sol: $P_t = \frac{A_c^2}{2R} (R = 1\Omega)$

$$= \frac{100}{2} = 50 \text{ W}$$

$$\% \text{ Power} = \frac{\text{Power in components}}{\text{total power}} \times 100$$

$$= \frac{41.17}{50} \times 100$$

$$= 82.35\%$$

09. Ans: (d)

Sol: In frequency modulation the spectrum contains $f_c \pm nf_1 \pm mf_2$, where n & $m = 0, 1, 2, 3, \dots$

10. Ans: (c)

Sol: Given $f_c = 1\text{MHz}$

$$f_{\max} = f_c + k_f A_m$$

$$k_p = 2\pi k_f$$

$$k_f = \frac{k_p}{2\pi} = \frac{\pi}{2\pi}$$

$$= \frac{1}{2}$$

$$\begin{aligned}
 &= \left(10^6 + \frac{1}{2} \times 10^5\right) = (10^6 + 0.5 \times 10^5) \\
 &= (10^6 + 5 \times 10^4) \\
 &= (10^3 + 50) 10^3 \\
 &= (10^3 + 50) \text{ k} \\
 &= 1050 \text{ kHz.}
 \end{aligned}$$

$$f_{\min} = f_c - k_f A_m$$

$$\begin{aligned}
 &= \left(10^6 - \frac{1}{2} \times 10^5\right) \\
 &= (10^6 - 0.5 \times 10^5) \\
 &= (10^6 - 5 \times 10^4) \\
 &= (10^3 - 50) 10^3 \\
 &= (10^3 - 50) \text{ k} \\
 &= 950 \text{ kHz}
 \end{aligned}$$

11. Ans: (d)

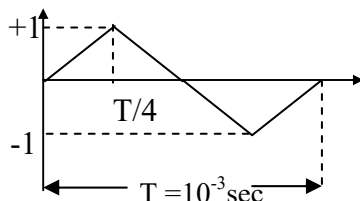
$$\text{Sol: } \beta = \frac{\Delta f}{f_m}$$

$$\Delta \phi = \frac{\Delta f}{f_m}$$

$$\begin{aligned}
 \Delta f &= \Delta \phi f_m \\
 &= k_p A_m f_m
 \end{aligned}$$

12. Ans: (c)

Sol: Given



$$f_c = 100 \times 10^3 \text{ Hz}$$

$$k_f = 10 \times 10^3 \text{ Hz}$$

$$m(t)|_{\max} = +1, m(t)|_{\min} = -1$$

$$\begin{aligned}
 f_i &= f_c \pm \Delta f \\
 &= f_c \pm k_f A_m \\
 &= 100 \times 10^3 \pm 10 \times 10^3 \text{ (m(t))} \\
 &= 110 \text{ kHz \& 90 kHz}
 \end{aligned}$$

13. Ans: (c)

$$\text{Sol: } S(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

$$\begin{aligned}
 f_i &= \frac{1}{2\pi} \frac{d}{dt} \theta_i(t) \\
 &= \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + k_p m(t))
 \end{aligned}$$

$$= f_c + \frac{1}{2\pi} k_p \frac{d}{dt} m(t)$$

$$f_{\max} = f_c + \frac{k_p}{2\pi} \frac{1}{\left(\frac{10^{-3}}{4}\right)} = f_c + \frac{k_p}{2\pi} \times 4 \times 10^3$$

$$= 100 \text{ kHz} + \frac{\pi}{2\pi} \times 4 \times 10^3$$

$$= 102 \text{ kHz}$$

$$f_{\min} = f_c - k_p \frac{1}{\left(\frac{10^{-3}}{4}\right)}$$

$$= f_c - 2 \text{ kHz}$$

$$f_{\min} = 98 \text{ kHz}$$

14. Ans: (c)

Sol: Given,

$$S(t) = A_c \cos(\theta_i(t))$$

$$= A_c \cos(\omega_c t + \phi(t))$$

$$m(t) = \cos(\omega_m t)$$

$$f_i(t) = f_c + 2\pi k(f_m)^2 \cos \omega_m t$$

$$f_i = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = \int 2\pi f_i(t) dt$$

$$\theta_i(t) = \int 2\pi [f_c + 2\pi k(f_m)^2 \cos \omega_m t] dt$$

$$\theta_i(t) = 2\pi f_c t + (2\pi f_m)^2 k \frac{\cos \omega_m t}{\omega_m t}$$

$$\theta_i(t) = \omega_c t + \omega_m k \sin \omega_m t$$

15. **Ans: (b)**

Sol: $\Delta f_{\max} = K_f |m(t)|_{\max}$

$$= \frac{100}{2\pi} \times [10]$$

$$\Delta f_{\max} = \left(\frac{500}{\pi}\right) \text{Hz}$$

16. **Ans: (b)**

Sol: Given that

$$s(t) = \cos[\omega_c t + 2\pi m(t)] \text{volts}$$

$$f_i = \frac{1}{2\pi} \frac{d}{dt} [\omega_c t + 2\pi m(t)]$$

$$= \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + 2\pi m(t)]$$

$$f_i = f_c + \frac{d}{dt} [m(t)]$$

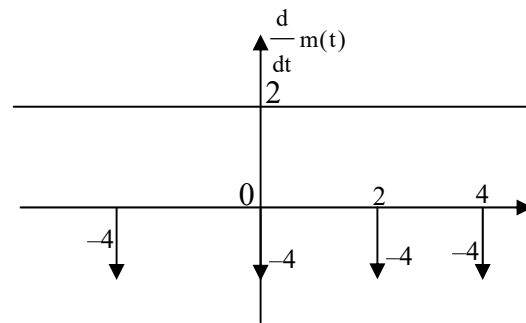
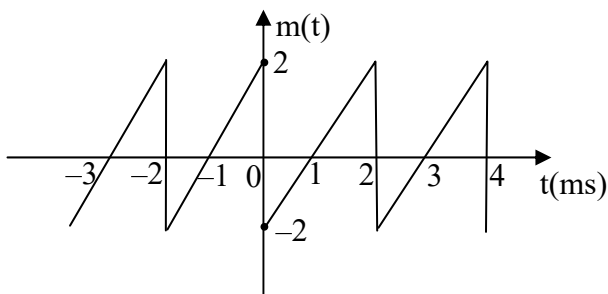
we know that $f_i = f_c + k_f m(t)$

$$\text{Here } k_f m(t) = \frac{d}{dt} [m(t)]$$

$$\Delta f = \max\{k_f m(t)\}$$

$$\Delta f = \max\left[\frac{d}{dt} m(t)\right]$$

$$\Delta f = 2\text{kHz}$$



17. **Ans: (a)**

Sol: $\beta_p = k_p \max[|m(t)|] = 1.5 \times 2 = 3$

$$\beta_f = \frac{k_f \max[|m(t)|]}{f_m}$$

$$= \frac{3000 \times 2}{1000}$$

$$= 6$$

18. **Ans: (a)**

Sol: Using Carson's rule we obtain

$$BW_{PM} = 2(\beta_p + 1)f_m = 8 \times 1000 = 8000\text{Hz}$$

$$BW_{FM} = 2(\beta_f + 1)f_m = 14 \times 1000 = 14000\text{Hz}$$

19. **Ans: 70 kHz**

Sol: $s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$

$$f_i = f_c + \frac{k_p}{2\pi} \frac{d}{dt} x(t)$$

$$= 20\text{k} + \frac{5}{2\pi} \times 5 \frac{d}{dt} (\sin 4\pi 10^3 t - 10\pi \cos 2\pi 10^3 t)$$

$$= 20\text{k} + \frac{25}{2\pi} \times \left[\cos(4\pi 10^3 t - 10\pi \cos 2\pi 10^3 t) \right]$$

$$\left[(4\pi 10^3 + 10\pi \sin 2\pi 10^3 t \times 2\pi 10^3) \right]$$

$$f_{i(t=0.5\text{ms})} = 20\text{k} + \frac{25}{2\pi} \times \cos(4\pi + 10\pi) \times 4\pi \times 10^3$$

$$= 20\text{k} + \frac{25}{2\pi} \times 4\pi \times 10^3$$

$$= 20\text{k} + 50\text{k}$$

$$f_{i(t=0.5\text{ms})} = 70\text{kHz}$$

20. Ans: (a, b & c)

Sol: $s(t) = 100\cos[2\pi \times 10^7 t + 10\sin(8\pi \times 10^3 t)]$

$$\Delta f_i(t) = \frac{1}{2\pi} \frac{d}{dt} [10\sin(8\pi \times 10^3 t)]$$

$$\Delta f_i(t) = 40 \times 10^3 \cos[8\pi \times 10^3 t]$$

$$\Delta f_{\max} = 40(\text{kHz})$$

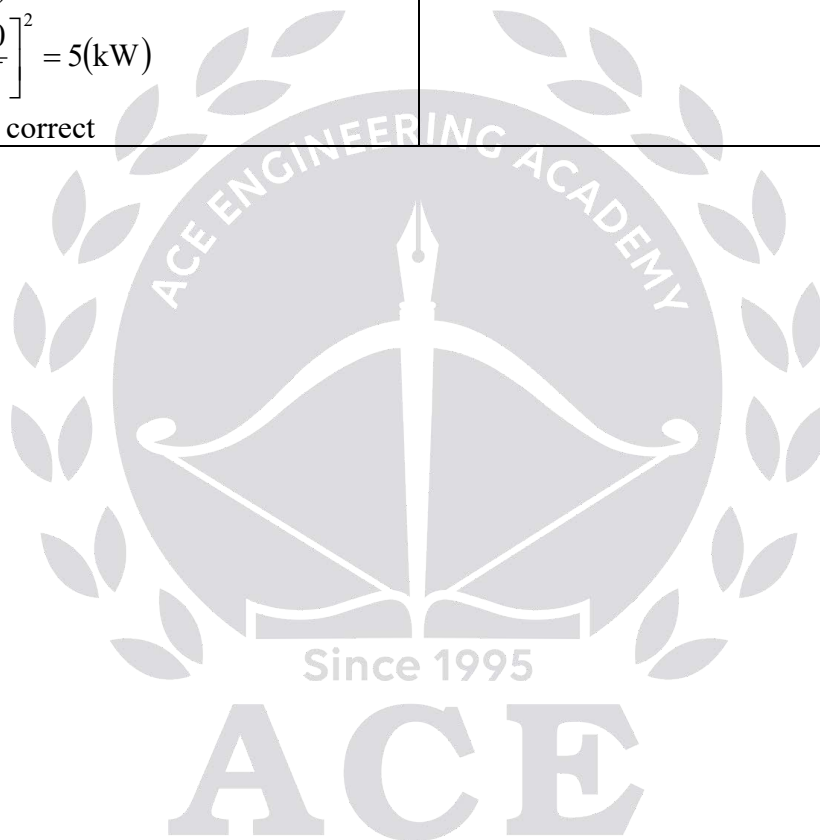
$$\beta = \frac{\Delta f_{\max}}{f_{\max}} = \frac{40 \times 10^3}{4 \times 10^3} = 10$$

$$\text{BW} = 2[\beta + 1] f_{\max} = 2 [10 + 1] 4 \times 10^3 \\ = 88(\text{kHz})$$

$$P_T = P_C = \left[\frac{100}{\sqrt{2}} \right]^2 = 5(\text{kW})$$

\therefore a, b & c are correct

21. Ans: (a, c & d)



01. Ans: (d)

Sol: The image channel selectivity of super heterodyne receiver depends upon Pre selector and RF amplifier only.

02. Ans: (b)

Sol: The image (second) channel selectivity of a super heterodyne communication receiver is determined by the pre selector and RF amplifier.

03. Ans: (d)

Sol: Given $f_s = 4$ to 10 MHz

$$IF = 1.8 \text{ MHz}$$

$$f_{si} = ?$$

$$f_{si} = f_s + 2 \times IF$$

$$= 7.6 \text{ MHz to } 13.6 \text{ MHz}$$

04. Ans: (a)

Sol: Image frequency $f_{si} = f_s + 2 \times IF$
 $= 700 \times 10^3 + 2(450 \times 10^3)$
 $= 1600 \text{ kHz}$

Local oscillator frequency, $f_l = f_s + IF$

$$(f_l)_{\max} = (f_s)_{\max} + IF = 1650 + 450$$

$$= 2100 \text{ kHz}$$

$$(f_l)_{\min} = (f_s)_{\min} + IF = 550 + 450$$

$$= 1000 \text{ kHz}$$

$$R = \frac{C_{\max}}{C_{\min}} = \left(\frac{f_{l\max}}{f_{l\min}} \right)^2 = \left(\frac{2100}{1000} \right)^2 = 4.41$$

05. Ans: (a)

Sol: $f_s(\text{range}) = 88 - 108 \text{ MHz}$

Given condition $f_{IF} < f_{LO}$, $f_{si} > 108 \text{ MHz}$

$$f_{si} = f_s + 2 \times IF$$

$$f_{si} > 108 \text{ MHz}$$

$$f_s + 2IF > 108 \text{ MHz}$$

$$88 \text{ MHz} + 2 \times IF > 108 \text{ MHz}$$

$$IF > 10 \text{ MHz}$$

Among the given options $IF = 10.7 \text{ MHz}$

06. Ans: (a)

Sol: Range of variation in local oscillator frequency is

$$f_{L\min} = f_{s\min} + IF$$

$$= 88 + 10.7$$

$$f_{L\min} = 98.7 \text{ MHz}$$

$$f_{L\max} = f_{s\max} + IF$$

$$= 108 + 10.7$$

$$f_{L\max} = 118.7 \text{ MHz}$$

07. Ans: 5

Sol: $f_s = 58 \text{ MHz} - 68 \text{ MHz}$

When $f_s = 58 \text{ MHz}$

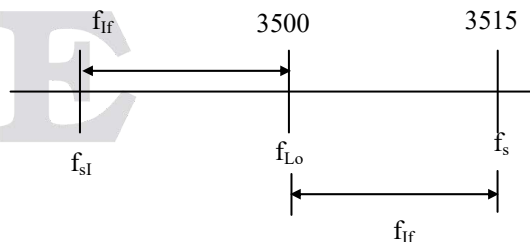
$$f_{si} = f_s + 2IF > 68 \text{ MHz}$$

$$2IF > 10 \text{ MHz}$$

$$IF \geq 5 \text{ MHz}$$

08. Ans: 3485 MHz

Sol:



$$f_{If} = 15 \text{ MHz}$$

$$f_{Lo} = 3500 \text{ MHz}$$

$$f_s - f_{Lo} = f_{IF}$$

$$f_s = f_{Lo} + f_{IF} = 3515 \text{ MHz}$$

$$f_{si} = \text{image frequency} = f_s - 2 f_{IF}$$

$$= 3515 - 2 \times 15$$

$$= 3485 \text{ MHz}$$

09. Ans: (a, b & c)

$$\text{Sol: } \rightarrow f_{IM} = f_s + 2f_{IF} = 555 \times 10^3 + 2(455 \times 10^3) \\ = 1465 \text{ kHz}$$

$$\rightarrow f_{IF} = f_{i_o} - f_s = 1010 \times 10^3 - 555 \times 10^3 \\ = 455 \times 10^3 \text{ Hz}$$

$$\rightarrow \text{IRR} = \sqrt{1 + Q^2 \rho^2} = 113$$

$$Q = 50$$

$$\rho = \frac{f_{IM}}{f_s} - \frac{f_s}{f_{IM}} = \frac{1465}{555} - \frac{555}{1465}$$

\therefore a, b & c are correct.

10. Ans: (b & c)

$$\text{Sol: } \rightarrow f_{i_o} - f_s = f_{IF} \\ f_{i_o} = f_{IF} + f_s \\ = 555 \times 10^3 + 1500 \times 10^3 \\ = 2055 \text{ kHz}$$

$$\rightarrow f_{IM} = f_s + 2f_{IF} \\ = 1500 \times 10^3 + 2(555 \times 10^3) \\ = 2610 \text{ kHz}$$

\therefore b & c are correct



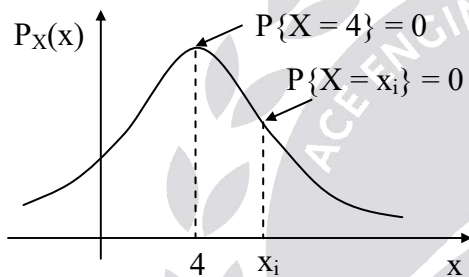
Chapter 6 Random Variables & Noise

01. Ans: (c)

Sol: A continuous Random variable X takes every value in a certain range, the probability that $X = x$, is zero for every x in that range.

Given $P_X(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}}$ is a

continuous Random variable therefore probability of the event $\{X = 4\}$ is zero.



02. Ans: (b)

Sol: Given,

X & Y are two Random Variables

$$Y = \cos \pi x$$

$$f(x) = 1 \quad \frac{-1}{2} < x < \frac{1}{2}$$

$$= 0 \quad \text{else where}$$

$$f(y) = ?$$

$$f(y) = f(x) \left| \frac{dx}{dy} \right|$$

$$x = \frac{1}{\pi} \cos^{-1}(y)$$

$$dx = \frac{1}{\pi} \times \frac{-1}{\sqrt{1-y^2}} dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{\pi \sqrt{1-y^2}}$$

$$f(y) = \frac{1}{\pi \sqrt{1-y^2}}$$

$$\sigma_y^2 = E[y^2] - [E[y]]^2$$

03. Ans: (d)

Sol: The probability density function of the envelope of a sinusoidal plus narrow band noise is Rician.

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right)$$

04. Ans: (a)

Sol: Given,

Differential equation of a system is

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$$

Applying Fourier transform,

$$\Rightarrow Y(f)(1 + jf) = X(f)(jf - 1)$$

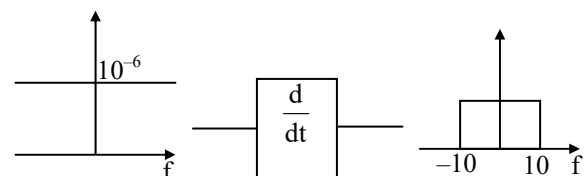
$$\frac{Y(f)}{X(f)} = \frac{-1 + jf}{1 + jf}$$

The transform function of system is a All pass filter

$$\therefore S_y(f) = S_x(f)$$

05. Ans: (a)

Sol:



$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$

$$H(f) = j2\pi f$$

$$|H(f)|^2 = 4\pi^2 f^2$$

$$S_{YY}(f) = 4\pi^2 f^2 S_{XX}(f)$$

The Noise power at the output of the LPF is

$$N_o = \int_{-10}^{10} S_{YY}(f) df$$

$$N_o = \int_{-10}^{10} 4\pi^2 f^2 \times 10^{-6} df$$

$$= 2 \times 4\pi^2 \times 10^{-6} \int_0^{10} f^2 df$$

$$= 2 \times 4\pi^2 \times 10^{-6} \times \frac{10^3}{3}$$

$$\therefore N_o = 0.0263W$$

06. Ans: (a)

Sol: Given,

$$\text{PSD of Noise} = \frac{\eta_0}{2}$$

$$T = 27^\circ C \Rightarrow 300K$$

$$P_n = K.T.B$$

$$\eta_0 = KT$$

$$= 1.38 \times 10^{-23} \times 300$$

$$\text{PSD} = \frac{\eta_0}{2}$$

$$= 1.38 \times 10^{-23} \times 150$$

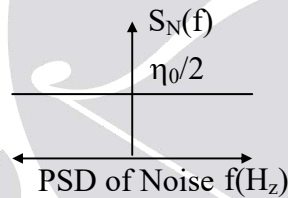
$$= \frac{207}{10^{23}}$$

07. Ans: (b)

Sol: $P_n = K.T.B$

$$= \left(\frac{1}{2} \times 1.38 \times 10^{-23} \times 300 \right) \times 2 \times 10^6 \times 2$$

$$= 8.28 \times 10^{-15} W$$



08. Ans: (b)

$$\text{Sol: } E(X) = \int_{-1}^3 x.p(x) dx = \frac{1}{4} \left[\frac{x^2}{2} \right]_{-1}^3 = 1$$

$$E(X^2) = \int_{-1}^3 x^2 p(x) dx = \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^3 = 7/3$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{3} - 1 = \frac{4}{3}$$

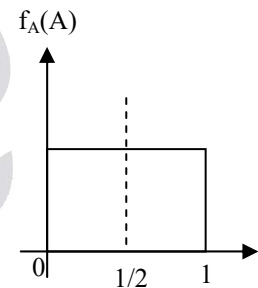
09. Ans: (d)

$$\text{Sol: } R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$$

$$= E[A \cos \omega t_1 A \cos \omega t_2]$$

$$= \cos \omega t_1 \cos \omega t_2 E[A^2] \quad [\because E[A^2] = 1/3]$$

$$= \frac{1}{3} \cos \omega t_1 \cos \omega t_2$$



$$\sigma^2 = \frac{(1)^2}{12} \rightarrow \text{variance}$$

$$E[A^2] = \sigma^2 + [E[A]]^2$$

$$= \frac{1}{12} + \frac{1}{4}$$

$$E[A^2] = \frac{4}{12} = \frac{1}{3}$$

10. Ans: (b)

$$\text{Sol: } R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)]$$

$$\text{Let } t_2 - t_1 = \tau$$

$$E[(A \cos \omega t_1 + B \sin \omega t_1)(B \cos \omega t_2 - A \sin \omega t_2)]$$

$$\therefore E[AB] = E[A] E[B]$$

$$E[AB] = 0$$

$$E[BA] = 0$$

$$E[A^2] = \sigma^2$$

$$E[B^2] = \sigma^2$$

$$\begin{aligned}
 &= \cos\omega t_1 \cdot \cos\omega t_2 E[AB] - \sin\omega t_1 \sin\omega t_2 E[BA] \\
 &\quad - E[A^2] \cos\omega t_1 \sin\omega t_2 \\
 &\quad + E[B^2] \sin\omega t_1 \cos\omega t_2 \\
 &= 0 - 0 - \sigma^2 \cos\omega t_1 \sin\omega t_2 + \sigma^2 \sin\omega t_1 \cos\omega t_2 \\
 &= -\sigma^2 (\cos\omega t_1 \sin\omega t_2 + \sin\omega t_1 \cos\omega t_2) \\
 &= -\sigma^2 \sin\omega (t_2 - t_1) (\because \tau = (t_2 - t_1)) \\
 &= -\sigma^2 \sin\omega \tau
 \end{aligned}$$

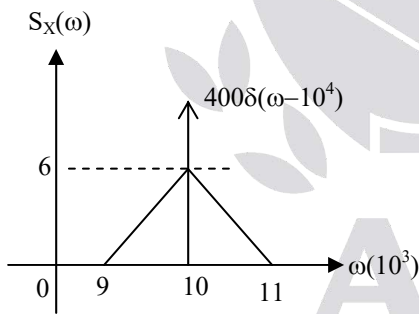
11. Ans: (b)

Sol: X(t) = positive frequencies required
E[X²(t)] and E[X(t)]

$$\begin{aligned}
 E[X^2(t)] &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) d\omega \\
 &= \frac{1}{\pi} \left(400 + \frac{1}{2} (2000) \times 6 \right) = \frac{6400}{\pi}
 \end{aligned}$$

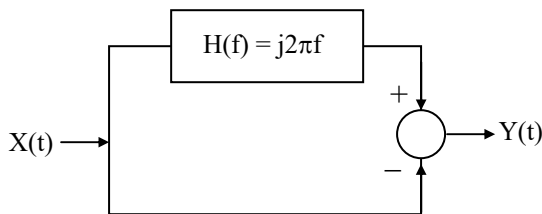
E[X(t)] = 0

[∵ The given function is periodic function]



12. Ans: (a)

Sol:



Overall H(f) = j2πf - 1

R_X(τ) = e^{-πτ²}

Y(t) = X(t) * h(t)

|H(f)|² = (4π²f² + 1)

R_{XX}(T) \xleftrightarrow{FT} S_{XX}(f)

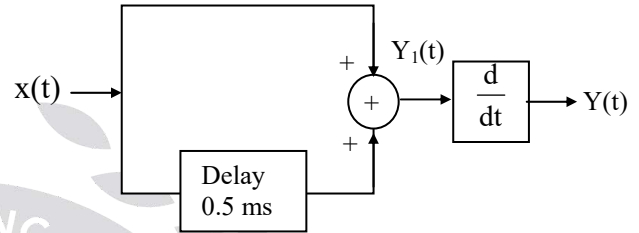
e^{-πτ²} \xleftrightarrow{FT} e^{-πf²}

Normalized Gaussian function

S_{YY}(f) = |H(f)|² S_{XX}(f) = (4π²f² + 1) e^{-πf²}

13. Ans: (d)

Sol:



Y(t) = d/dt (X(t) + X(t - t_d))

Y(f) = j2πf (1 + e^{-j2πft_d}) X(f)

H(f) = Y(f) / X(f)

= j2πf (1 + e^{-j2πft_d})

|H(f)|² = 4cos²πft_d

S_{YY}(f) = |H(f)|² S_{XX}(f) = 4π²f² (2cos(πft_d))² S_{XX}(f)

At S_{YY}(f) = 0

πft_d = (2n+1) * 1 / 2t_d

f = (2n+1) * 1 / (2 * 0.5 * 10⁻³)

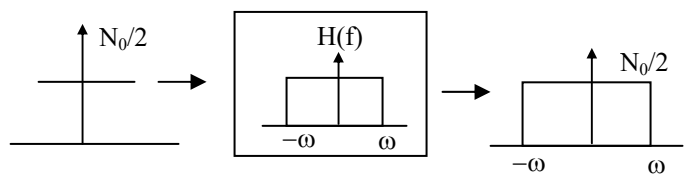
f = (2n+1) 10³

f = (2n+1) f₀

f₀ = 1 kHz

14. Ans: (b)

Sol:



Uncorrelated $\Rightarrow \text{cov}(\tau) \Rightarrow R_{XX}(\tau) - \mu^2 \times (\tau)$
 $\text{cov}(\tau) = R_{XX}(\tau) \Rightarrow R_{n_0}(\tau) = 0$
 $\Rightarrow N\omega_0 \sin(2\omega\tau) = 0, \sin Cx = 0$; x is an integer
 $2\omega\tau = m$
 $\tau = \frac{m}{2\omega}$, integer $m = 1, 2, 3, \dots$

15. Ans: (b)
Sol: We know that,

$$\text{ACF} \xleftrightarrow{\text{F.T}} S_x(f)$$

Taking Inverse Fourier Transform

$$F^{-1}[S_y(t)] = \int_{-\infty}^{\infty} S_y(t) e^{j2\pi f\tau} df$$

$$R_y(\tau) = \int_{-B_0}^{B_0} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2} \left[\frac{e^{j2\pi f\tau}}{j2\pi\tau} \right]_{-B_0}^{B_0}$$

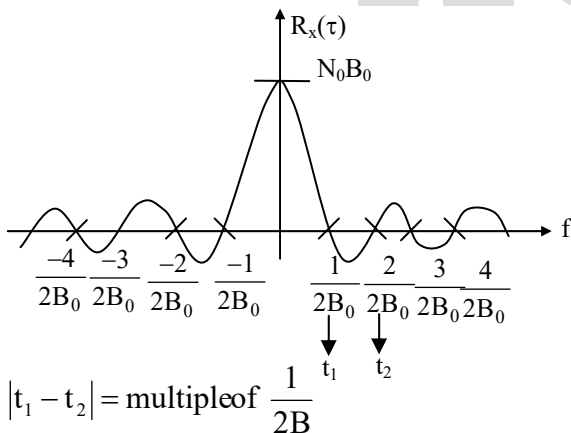
$$= \frac{N_0}{2\pi\tau} \left[\frac{e^{j2\pi B_0\tau} - e^{-j2\pi B_0\tau}}{2j} \right]$$

$$= \frac{N_0}{2\pi\tau} \sin(2\pi B_0\tau)$$

$$= N_0 B_0 \frac{\sin(2\pi B_0\tau)}{2\pi B_0\tau}$$

$$R_y(\tau) = N_0 B_0 \text{sinc}(2B_0\tau)$$

16. Ans: (b)
Sol:



17.

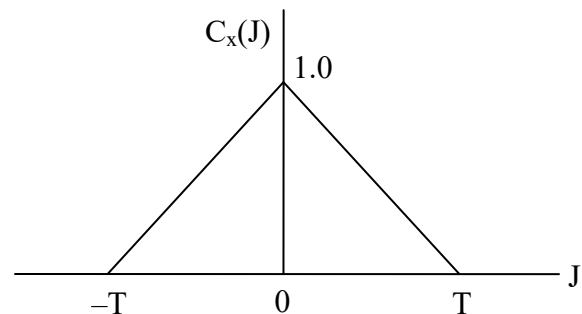
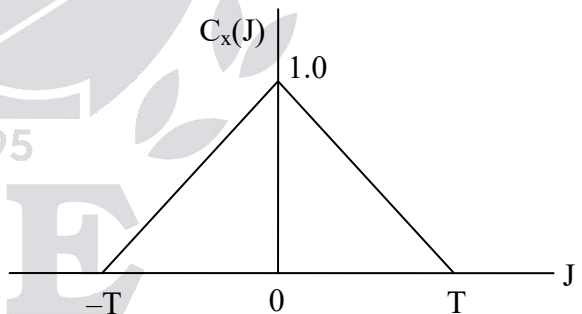
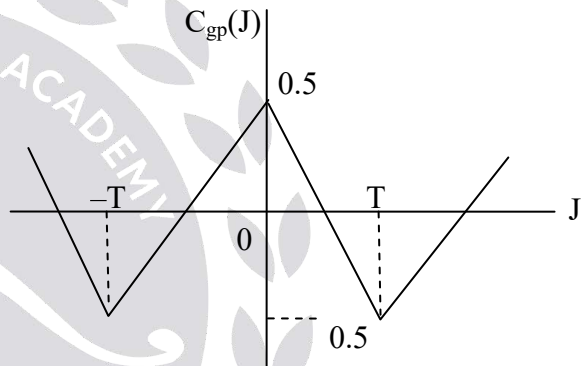
Sol: Since

$$y(t) = g_p(t) + X(t) + \sqrt{3/2}$$

and $g_p(t)$ and $X(t)$ are uncorrelated, then

$$C_Y(\tau) = C_{g_p}(\tau) + C_X(\tau).$$

Where $C_{g_p}(\tau)$ is the auto covariance of the periodic component and $C_X(\tau)$ is the auto covariance of the random component $C_Y(\tau)$ is the plot figure shifted down by $3/2$, removing the DC component $C_{g_p}(\tau)$ and $C_X(\tau)$ are plotted below



Both $g_p(t)$ and $X(t)$ have zero mean, Average

- (a) The power of the periodic component $g_p(t)$ is therefore,

$$\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p^2(t) dt = C_{g_p}(0) = \frac{1}{2}$$

- (b) The average power of the random component $x(t)$ is
 $E[X^2(t)] = C_x(0) = 1$

18.

Sol:

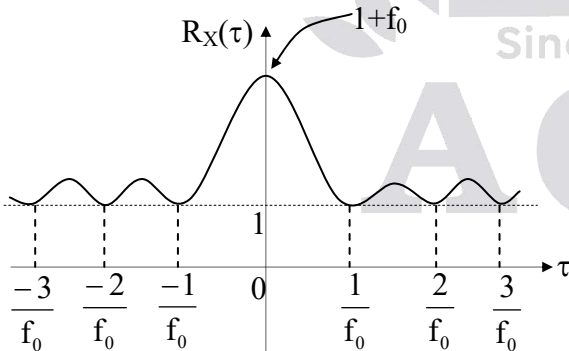
- (a) The power spectral density consists of two components:

- (1) A delta function $\delta(t)$ and the origin, whose inverse Fourier transform is one.
- (2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \text{sinc}^2(f_0\tau)$

Therefore, the autocorrelation function of $X(t)$ is

$$R_X(\tau) = 1 + f_0 \text{sinc}^2(f_0\tau)$$

Which is sketched below:



- (b) Since $R_X(\tau)$ contains a constant component of amplitude 1. It follows that the dc power contained in $X(t)$ is 1.

- (c) The mean-square value of $X(t)$ is given by

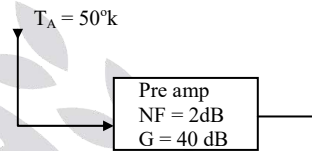
$$E[X^2(t)] = R_X(0) = 1 + f_0$$

The ac power contained in $X(f)$ is therefore equal to f_0 .

- (d) If the sampling rate is f_0/n , where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if $X(t)$ were a Gaussian process.

19. Ans: (a)

Sol:



$$10 \log_{10} NF = 2 \text{ dB}$$

$$\log_{10} NF = 0.2$$

$$NF = 10^{0.2}$$

$$\begin{aligned} \text{Noise temperature} &= (F - 1) T_0 \\ &= (10^{0.2} - 1) 290^\circ \\ &= 169.36 \text{ K} \end{aligned}$$

$$\begin{aligned} \text{Noise power i/p} &= k T_e B \\ &= 1.38 \times 10^{-23} \times (169.36 + 50) \times 12 \times 10^6 \end{aligned}$$

$$\begin{aligned} \text{Noise power at o/p} &= (3.632 \times 10^{-14}) \times 10^4 \\ &= 3.73 \times 10^{-10} \text{ watts} \end{aligned}$$

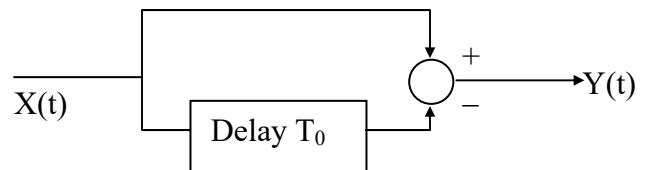
20. Ans: 100 W

Sol:

$$\begin{aligned} E[x^2(t)] &= E[(3V(t) - 8)^2] \\ &= E[9V^2(t) + 64 - 2 \times 3V(t) \times 8] \\ &= E[9V^2(t) + 64 - 48V(t)] \\ &= 9E[V^2(t)] + E[64] - 48E[V(t)] \\ E[V(t)] &= 0, \quad E[V^2(t)] = MS = R(0) = 4e^{-5(0)} = 4, \\ E[\text{constant}] &= \text{constant} \\ E[x^2(t)] &= 9 \times 4 + 64 = 36 + 64 \\ &= 100 \end{aligned}$$

21. Ans: (b)

Sol:



$$Y(t) = X(t) - X(t - T_0)$$

$$\text{ACf of } o/p = R_y(\tau) = E[y(t)Y(t+\tau)]$$

$$R_y(\tau) = E[(X(t) - X(t - T_0)) [X(t + \tau) - X(t + \tau - T_0)]]$$

$$R_y(\tau) = E[(X(t)X(t + \tau) - X(t)X(t + \tau - T_0) - X(t - T_0)X(t + \tau) + X(t - T_0)X(t + \tau - T_0))]$$

$$R_y(\tau) = [R_x(\tau) - R_x(\tau - T_0) - R_x(\tau + T_0) + R_x(\tau)]$$

$$R_y(\tau) = 2R_x(\tau) - R_x(\tau - T_0) - R_x(\tau + T_0)$$

22. Ans: (a, b & c)

$$\text{Sol: } R_{XX}(\tau) = E[x(t)x(t-\tau)]$$

$$= E[20 \cos(\omega_0 t + \theta) 20 \cos(\omega_0 t - \omega_0 \tau + \theta)]$$

$$= \frac{400}{2} E[\cos(2\omega_0 t - \omega_0 \tau + 2\theta)] + \frac{400}{2} E[\cos(\omega_0 \tau)]$$

$$= 200 \cos(\omega_0 \tau)$$

$$S_{XX}(f) = 100 \Delta(f - f_0) + 100 \Delta(f + f_0)$$

$$P = R_{XX}(\tau) = 200 \text{ (Watts)}$$

$$\text{at } \tau = 0$$

\therefore a, b & c are correct

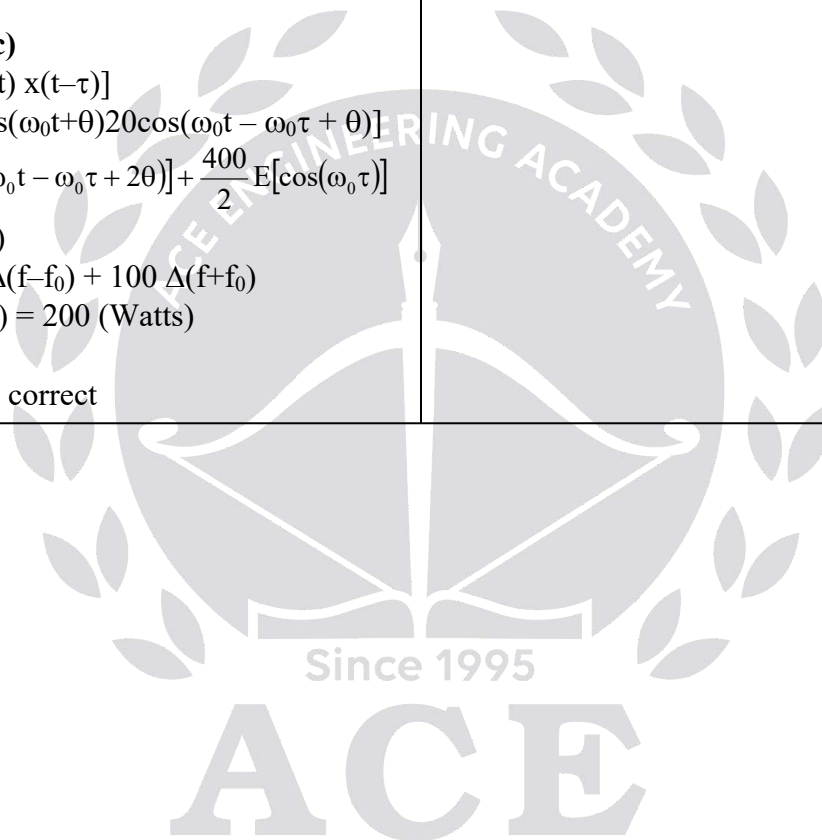
23. Ans: (a & c)

$$\text{Sol: } P = R_{XX}(0) = \int S_{XX}(f) df = \frac{1}{2\pi} \int S_{XX}(\omega) d\omega$$

$$R_{XX}(-\tau) = R_{XX}(\tau)$$

$$S_{XX}(-\omega) = S_{XX}(\omega)$$

Hence a & c are correct.



Chapter 7 Noise in Analog Communication

01. Ans: (d)

Sol: Output of the multiplier

$$= m(t) \cdot \cos \omega_0 t \cos(\omega_0 t + \theta)$$

$$= \frac{m(t)}{2} [\cos(2\omega_0 t + \theta) + \cos \theta]$$

Output of LPF $V_0(t) = \frac{m(t)}{2} \cos \theta$

$$= \frac{1}{2} \cos \theta m(t)$$

Power of o/p signal = $\lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} v_0^2(t) dt$

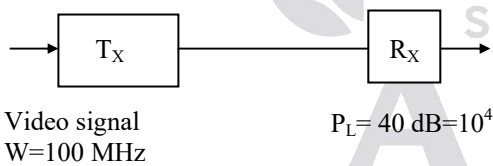
$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} \left(\frac{1}{2} \cos \theta m(t) \right)^2 dt$$

$$= \frac{1}{4} \cos^2 \theta \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} m^2(t) dt \right]$$

$$= \frac{1}{4} \cos^2 \theta P_m$$

02. Ans: (a)

Sol:



$$n_i = n_0$$

$$n_i = n_0 \times W = 10^{-20} \times 100 \times 10^6$$

$$S_i = \frac{P_t}{P_L} = \frac{1 \text{ mw}}{10^4} = 1 \times 10^{-7}$$

$$n_i = 10^{-20} \times 100 \times 10^6$$

$$\frac{S_i}{n_i} = \frac{10^{-7}}{10^{-12}} = 10^5 = 50 \text{ dB}$$

$$\frac{S_i}{n_i} = 50 \text{ dB}$$

03. Ans: (b)

Sol: $\Delta f = 75 \text{ kHz}$

$$f_m = 15 \text{ kHz}$$

$$\left(\frac{S}{N} \right)_0 = 40 \text{ dB} = 10^4$$

$$\text{FOM} = \frac{3}{2} \beta^2; \beta = \frac{\Delta f}{f_m}$$

$$\left(\frac{S_0}{N_0} \right) = \frac{3}{2} \beta^2$$

$$\left(\frac{S}{N} \right)_i = \left(\frac{S}{N} \right)_0 \times \frac{2}{3} \times \frac{1}{\beta^2}$$

$$\left(\frac{S}{N} \right)_{i(\text{dB})} = 24 \text{ dB}$$

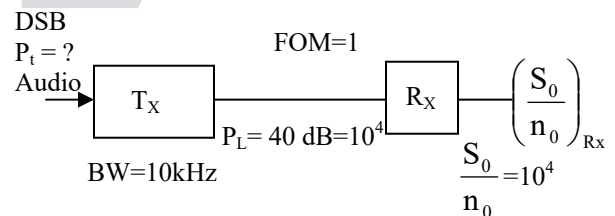
04. Ans: (c)

Sol: $\left(\frac{S}{N} \right)_i = 10 \text{ dB}; \text{FOM} = \frac{1}{3}$

$$\left(\frac{S}{N} \right)_0 = \frac{1}{3} \times 10 = 3.33$$

05. Ans: (a)

Sol:



06. Ans: (a)

Sol: For SSB modulation

$$\Rightarrow \frac{S_0}{N_0} = \frac{S_i}{N_i} = 10^4$$

(Only SSB modulation in one sided $n/2$)

$P_t = ?$

$$\frac{S_i}{n_i} = \frac{S_0}{n_0} = 10^4$$

$$S_i = 10^4 \times 10 \times 10^3 \times 2 \times 10^{-9} \text{ w/Hz}$$

$$S_i = 20 \times 10^{-2}$$

$$(S_i)_{\text{dB}} = (P_t)_{\text{dB}} - (P_L)_{\text{dB}}$$

$$(P_t)_{\text{dB}} = (S_i)_{\text{dB}} + (P_L)_{\text{dB}}$$

$$P_t = S_i P_L = 20 \times 10^{-2} \times 10^4$$

$$P_L = 2 \text{ kW}$$

07. Ans: (c)

Sol: For AM

$$\text{FOM} = \frac{1}{3} \text{ (if } \mu = 1)$$

$$\begin{aligned} \frac{S_0}{N_0} &= \left(\frac{1}{3}\right) \frac{S_i}{N_i} \Rightarrow S_i = 3 \left(\frac{S_0}{N_0}\right) \times N_i \\ &= 3 \times 10^4 \times 2 \times 10^{-9} \times 10 \text{ kHz} = 0.6 \\ \therefore P_t &= S_i \times P_L = 0.6 \times 10^4 = 6 \text{ kW} \end{aligned}$$

08. Ans: (b)

$$\text{Sol: Noise figure} = \frac{(\text{SNR})_{\text{I/P}}}{(\text{SNR})_{\text{O/P}}}$$

$$Nf_{\text{dB}} = \text{SNR}_{\text{i,dB}} - \text{SNR}_{\text{o/p,dB}}$$

$$\begin{aligned} \text{SNR}_{\text{o/p,dB}} &= \text{SNR}_{\text{I/P,dB}} - Nf_{\text{dB}} = 37 - 3 \\ &= 34 \text{ dB} \end{aligned}$$

09. Ans: (a, c & d)

$$\text{Sol: FOM Sinusoidal} = \frac{\mu^2}{2 + \mu^2} = \frac{\frac{1}{4}}{2 + \frac{1}{4}} = \frac{1}{9} = 0.111$$

FOM Triangular

$$= \frac{\mu^2 p_{\text{mn}}}{1 + \mu^2 p_{\text{mn}}} = \frac{\frac{1}{12}}{1 + \frac{1}{12}} = \frac{1}{13} = 0.0769$$

$$\text{Here } P_{\text{mn}} = \frac{1}{3}$$

FOM Square wave

$$= \frac{\mu^2 p_{\text{mn}}}{1 + \mu^2 p_{\text{mn}}} = \frac{\frac{1}{4}}{1 + \frac{1}{4}} = \frac{1}{5} = 0.2$$

$$\text{Here } P_{\text{mn}} = 1$$

$$\text{FOM Square wave (at } \mu=1) = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

a, c & d are correct.

10. Ans: (b & c)

01. Ans: (d)

$$\text{Sol: } \Delta = \frac{V_{\max} - V_{\min}}{2^n}$$

$$\Delta \propto \frac{1}{2^n} ; \frac{\Delta_1}{\Delta_2} = \frac{2^{n_2}}{2^{n_1}}$$

$$\frac{0.1}{\Delta_2} = \frac{2^{n+3}}{2^n}$$

$$\Delta_2 = 0.1 \times \frac{1}{8} \\ = 0.0125$$

02. Ans: (3)

$$\text{Sol: } (BW)_{\text{PCM}} = \frac{n f_s}{2}$$

Where 'n' is the number of bits to encode the signal and $L = 2^n$, where 'L' is the number of quantization levels.

$$L_1 = 4 \Rightarrow n_1 = 2$$

$$L_2 = 64 \Rightarrow n_2 = 6$$

$$\frac{(BW)_2}{(BW)_1} = \frac{n_2}{n_1} = \frac{6}{2} = 3$$

$$(BW)_2 = 3 (BW)_1$$

03. Ans: (c)

Sol: Given,

Two signals are sampled with $f_s = 44100\text{s/sec}$ and each sample contains '16' bits

Due to additional bits there is a 100% overhead.

Out put bit rate =?

$$R_b = n |f_s|$$

$$f_s | = 2f_{s1} = 2 [44100]$$

(∵ two signals sampled simultaneously)

$$n | = 2n$$

(∵ due to overhead by additional bits)

$$R_b = 4 (nf_s) = 2.822\text{Mbps}$$

04. Ans (c)

$$\text{Sol: } \text{Number of bits recorded over an hour} \\ = R_b \times 3600 = 10.16 \text{ G.bits}$$

05. Ans: (c)

$$\text{Sol: } p(t) = \frac{\sin(4\pi W t)}{4\pi W t (1 - 16 W^2 t^2)}$$

$$\text{At } t = \frac{1}{4W} ; P\left(\frac{1}{4W}\right) = \frac{0}{0}$$

Use L-Hospital Rule

$$\begin{aligned} \lim_{t \rightarrow \frac{1}{4W}} p(t) &= \lim_{t \rightarrow \frac{1}{4W}} \frac{4\pi W \cos(4\pi W t)}{4\pi W - 64\pi W^3 (3t^2)} \\ &= \frac{4\pi W (-1)}{4\pi W - 64\pi W^3 \cdot 3 \left(\frac{1}{16W^2}\right)} \\ &= \frac{-4\pi W}{-8\pi W} = 0.5 \end{aligned}$$

06. Ans: 35

Sol: Given bit rate $R_b = 56 \text{ kbps}$, Roll off factor $\alpha = 0.25$

BW required for base band binary PAM system

$$BW = \frac{R_b}{2} [1 + \alpha] = \frac{56}{2} [1 + 0.25] \text{kHz} = 35 \text{kHz}$$

07. Ans: 16

Sol: $R_b = n f_s = 8 \text{ bit/sample} \times 8 \text{ kHz} = 64 \text{ kbps}$

$$\begin{aligned} (B_T)_{\min} &= \frac{R_b}{2 \log_2 M} \\ &= \frac{R_b}{2 \log_2 4} = \frac{R_b}{2 \times 2} \\ &= \frac{R_b}{4} = \frac{64}{4} \\ &= 16 \text{kHz} \end{aligned}$$

08. Ans: (b)

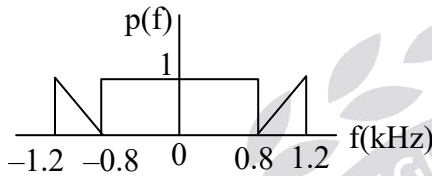
Sol: Given $f_s = 1/T_s = 2k$ symbols/sec

If $P(f) \leftrightarrow p(t)$,
Condition for zero ISI is given by

$$\frac{1}{T_s} \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)T_s$$

$p(0) = \text{area under } P(f)$



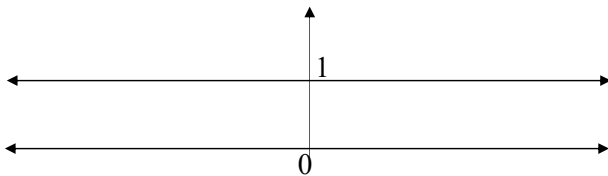
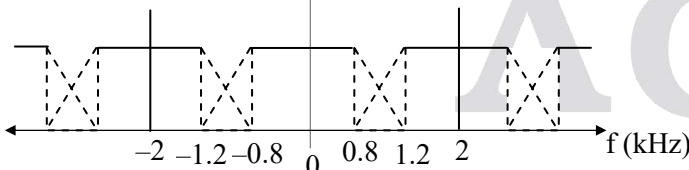
$$\text{Area} = 2 \times \frac{1}{2} (1)(0.4)k + 2 \times 0.8k = 2k$$

$$p(0) T_s = 2k \times \frac{1}{2k} = 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = 1$$

The above condition is satisfied by only option (b)

$$\sum_{n=-\infty}^{\infty} P(f - n2k)$$



$$\therefore \sum_{n=-\infty}^{\infty} P(f - n2k) = 1$$

Option (a) is correct if pulse duration is from -1 to $+1$

Option (c) is correct if the transition is from 0.8 to 1.2 , -0.8 to -1.2

Option (d) is correct if the triangular duration is from -2 to $+2$

09. Ans: 200

Sol: $m(t) = \sin 100\pi t + \cos 100\pi t$

$$= \sqrt{2} \cos [100\pi t + \phi]$$

$$\Delta = 0.75 = \frac{V_{\max} - V_{\min}}{L} = \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

$$L = \frac{2\sqrt{2}}{0.75} \approx 4 = 2^n$$

So $n = 2$

$f = 50$ Hz so Nyquist rate = 100

So, the bit rate = $100 \times 2 = 200$ bps

10. Ans: (b)

Sol: Given

$$f_{m_1} = 3.6\text{kHz} \Rightarrow f_{s_1} = 7.2\text{kHz}$$

$$f_{m_2} = f_{m_3} = 1.2\text{kHz} \Rightarrow f_{s_2} = f_{s_3} = 2.4\text{kHz}$$

$$f_s = f_{s_1} + f_{s_2} + f_{s_3}$$

$$= 12\text{kHz}$$

No. of Levels used = 1024

$\Rightarrow n = 10$ bits

\therefore Bit rate = nf_s

$$= 10 \times 12 \text{ kHz}$$

$$= 120 \text{ kbps}$$

11. Ans: (a)

Sol: $(f_s)_{\min} = (f_{s_1})_{\min} + (f_{s_2})_{\min}$

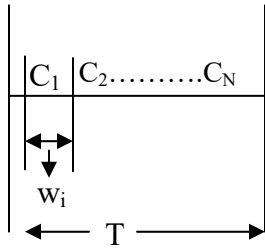
$$+ (f_{s_3})_{\min} + (f_{s_4})_{\min}$$

$$= 200 + 200 + 400 + 800$$

$$= 1600 \text{ Hz}$$

12. Ans: (c)

Sol:



Minimum B.W of TDM is $\sum_{i=1}^N w_i$

13. Ans: (b)

Sol: Number of patients = 10

ECG signal B.W = 100Hz

$$(Q_e)_{\max} \leq (0.25) \% V_{\max}$$

$$\frac{2V_{\max}}{2 \times 2^n} \leq \frac{0.25}{100} V_{\max}$$

$$2^n \geq 400$$

$$n \geq 8.64$$

$$n = 9$$

$$\begin{aligned} \text{Bit rate of transmitted data} &= 10 \times 9 \times 200 \\ &= 18 \text{ kbps} \end{aligned}$$

14. Ans: (a)

Sol: Peak amplitude $\rightarrow A_m$

Peak to peak amplitude A_m

$$\frac{-\Delta}{2} \leq Q_e \leq \frac{\Delta}{2}$$

$$\text{PCM maximum tolerable } \frac{\Delta}{2} = 0.2\% A_m$$

$$\Delta = \frac{\text{Peak to peak}}{L} \Rightarrow \frac{2A/m}{2L} = \frac{0.2}{100} A_m$$

$$(\because \Delta = \frac{2A_m}{L})$$

$$\Rightarrow L = 500$$

$$2^n = 500$$

$$n = 9$$

$$R_b = n(f_s)_{\text{TDM}} + 9$$

$$f_s = R_N + 20\%R_N = R_N + 0.2R_N$$

$$f_s = 1.2R_N = 1.2 \times 2 \times \omega$$

$$f_s = 2.4 \text{ K samples/sec}$$

$$(f_s)_{\text{TDM}} = 5(f_s)$$

$$= 5 \times 2.4 \text{ K}$$

$$= 12 \text{ K sample/sec}$$

$$R_b = (nf_s) + 0.5\%(nf_s)$$

$$= (9 \times 12\text{k}) + \frac{0.5}{100} (9 \times 12\text{k})$$

$$= 108540 \text{ bps}$$

15. Ans: (b)

Sol: To avoid slope over loading, rate of rise of the o/p of the Integrator and rate of rise of the Base band signal should be the same.

$$\therefore \Delta f_s = \text{slope of base band signal}$$

$$\Delta \times 32 \times 10^3 = 125$$

$$\Delta = 2^{-8} \text{ Volts.}$$

16. Ans: (b)

Sol: $x(t) = E_m \sin 2\pi f_m(t)$

$$\frac{\Delta}{T_s} < \left| \frac{dm(t)}{dt} \right| \rightarrow \text{slope overload distortion}$$

takes place

$$\Delta f_s < E_m 2\pi f_m$$

$$\Rightarrow \frac{\Delta f_s}{2\pi} < E_m f_m \quad (\because \Delta = 0.628)$$

$$\Rightarrow \frac{0.628 \times 40\text{K}}{2\pi} < E_m f_m$$

$$f_s = 40 \text{ kHz} \Rightarrow 4 \text{ kHz} < E_m f_m$$

Check for options

(a) $E_m \times f_m = 0.3 \times 8 \text{ K} = 2.4 \text{ kHz}$
($4\text{K} \not< 2.4 \text{ K}$)

(b) $E_m \times f_m = 1.5 \times 4\text{K} = 6 \text{ kHz}$
($4\text{K} < 6 \text{ K}$) correct

(c) $E_m \times f_m = 1.5 \times 2 \text{ K} = 3 \text{ kHz}$
($4\text{K} \not< 3\text{K}$)

(d) $E_m \times f_m = 30 \times 1 \text{ K} = 3 \text{ kHz}$
($4\text{K} \not< 3\text{K}$)

17. Ans: (a)

Sol: Given

$$m(t) = 6 \sin(2\pi \times 10^3 t) + 4 \sin(4\pi \times 10^3 t)$$

$$\Delta = 0.314 \text{ V}$$

$$\text{Maximum slope of } m(t) = \frac{d}{dt}(m(t)) / t = \frac{\pi}{2}$$

$$= 2\pi \times 10^3(6) + 4\pi \times 10^3[4] = 28\pi \times 10^3$$

18. Ans: (c)

Sol: Pulse rate which avoid distortion

$$\Delta f_s = \frac{d}{dt} m(t)$$

$$f_s = \frac{28\pi \times 10^5}{0.314}$$

$$f_s = 280 \times 10^3 \text{ pulses/sec}$$

19. Ans: (a, b & c)

Sol: a. $r_b = (Nn + EB)f_s$

$$r_b = (80 + 5) 5000 = 425(\text{kbps})$$

b. $r_b = Nnf_s$

$$r_b = 10(8+1) 5000 = 450(\text{kbps})$$

c. $r_b = (Nn + EB)f_s$

$$r_b = (80 + 10) 5000 = 450(\text{kbps})$$

d. $r_b = Nnf_s$

$$r_b = 10(8+0.8) 5000 = 440(\text{kbps})$$

\therefore a, b & c are correct

20. The message signal

$$m(t) = \text{Sinc}(400t) \times \text{Sinc}(600t)$$

is sampled then which of the following option is/are correct.

NOTE: options are changed

(a) Nyquist rate = 2 kHz

(b) Nyquist rate = 1 kHz

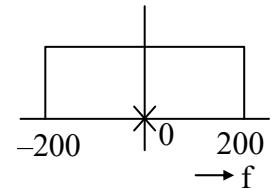
(c) Nyquist interval = 0.5 ms

(d) Nyquist interval = 1 ms

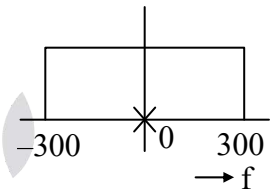
20. Ans: (b & d)

Sol:

$$\text{Sinc}(400t) \xrightarrow{\text{CTFT}}$$



$$\text{Sinc}(600t) \xrightarrow{\text{CTFT}}$$



M(f) frequency will range from -500 to 500 Hz

$$\therefore f_q = 2f_{\text{max}} = 1 \text{ kHz}$$

$$T_{s \text{ MAX}} = \frac{1}{f_q} = 1 \text{ ms}$$

b & d are correct

Chapter 9 Bandpass Data Transmission

01. Ans: (c)

Sol: $(BW)_{BPSK} = 2f_b = 20 \text{ kHz}$
 $(BW)_{QPSK} = f_b = 10 \text{ kHz}$

02. Ans: (b)

Sol: $f_H = 25 \text{ kHz}$; $f_L = 10 \text{ kHz}$

\therefore Center frequency

$$= \left(\frac{25+10}{2} \right) \text{ kHz}$$

$$= 17.5 \text{ kHz}$$

\therefore Frequency offset,

$$\Omega = 2\pi (25 - 17.5) \times 10^3$$

$$= 2\pi (7.5) \times 10^3$$

$$= 15 \times 10^3 \pi \text{ rad/sec.}$$

The two possible FSK signals are orthogonal, if $2\Omega T = n\pi$

$$\Rightarrow 2(15\pi) \times 10^3 \times T = n\pi$$

$$\Rightarrow 30 \times 10^3 \times T = n \text{ (integer)}$$

This is satisfied for, $T = 200 \mu\text{sec.}$

03. Ans: (a)

Sol: $r_b = 8 \text{ kbps}$

Coherent detection

$$\Delta f = \frac{nr_b}{2}$$

Best possible $n = 1$

$$\Delta f = \frac{8K}{2} = 4K$$

To verify the options $\Delta f = 4k$

i.e. $f_{c2} - f_{c1} = 4K$

(a) $20 \text{ K} - 16 \text{ K} = 4 \text{ K}$

(b) $32 \text{ K} - 20 \text{ K} = 12 \text{ K}$

(c) $40 \text{ K} - 20 \text{ K} = 20 \text{ K}$

(d) $40 \text{ K} - 32 \text{ K} = 8 \text{ K}$

04. Ans: (a)

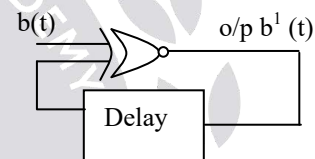
Sol: Non coherent detection of PSK is not possible. So to overcome that, DPSK is implemented. A coherent carrier is not required to be generated at the receiver.

05. Ans: (c)

Sol: In QPSK baud rate = $\frac{\text{bit rate}}{2} = \frac{34}{2}$
 $= 17 \text{ Mbps}$

06. Ans: (d)

Sol:



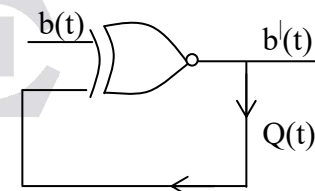
$b(t)$	0	1	0	0	1
$b^1(t)_{\text{(Ref.bit)}}$	0	0	1	0	0
Phase	π	π	0	π	π

07. Ans: (b)

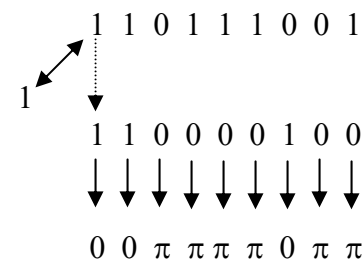
Sol: Given

Bit stream 110 111001

Reference bit = 1



$$b^1(t) = b(t) \odot Q(t)$$



08. Ans: (d)

Sol: $r_b = 1.544 \times 10^6$

$$\alpha = 0.2$$

$$BW = \frac{r_b}{\log_2 M} (1 + \alpha)$$

$$= \frac{1.544 \times 10^6}{2} (1 + 0.2) \quad (\because M = 4)$$

$$BW = 926.4 \times 10^3 \text{ Hz}$$

09. Ans: 0.25

Sol: $BW = 1500 \text{ Hz}$

BW required for M-ary PSK is

$$\frac{R_b [1 + \alpha]}{\log_2 16} = 1500 \text{ Hz}$$

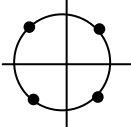
$$\Rightarrow R_b [1 + \alpha] = 1500 \times 4 = 6000$$

$$\Rightarrow (1 + \alpha) = \frac{6000}{4800}$$

$$\text{Roll off factor} \Rightarrow \alpha = \frac{6000}{4800} - 1 = 0.25$$

10. Ans: (b)

Sol:



Here only phase is changing.

From options (b) is the optimum answer.

11. Ans: (b)

Sol: Here 16-points are available in constellation which are varying in both amplitude and phase. So, it 16QAM.

12. Ans: (d)

Sol: $BW = \frac{r_b}{\log_2 M} (1 + \alpha)$

$$36 \times 10^6 = \frac{r_b}{2} (1 + 0.2) (\because M = 4, \text{QPSK})$$

$$r_b = 60 \times 10^6 \text{ bps}$$

NOTE: new question 13th is added in text book

13. Which among the following modulation, schemes consume less bandwidth

- (a) B-PSK (b) Q-PSK
 (c) 64-PSK (d) 64-QAM

13. Ans: (c & d)

Sol: Bandwidth of 64-PSK = $\frac{2r_b}{6} = \frac{r_b}{3}$

Bandwidth 64-QAM = Bandwidth of 64-PSK

14. Ans: (a, b & d)

Sol: M-ary ASK constellation plot will always come on a single line (either x-axis or y-axis).

Chapter 10 Noise in Digital Communication

Noise Ratio

01. Ans: (b)

Sol: Signal to quantization noise ratio only depends on no. of quantization levels (L) and no. of bits per sample(n)

$$\begin{aligned} \text{For sinusoidal input SQNR} &= 1.76+6n \text{ dB} \\ &= 1.76 + 6 \times 12 \\ &= 73.76 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{For uniform distributed signal} &= 6n \text{ dB} \\ &= 6 \times 12 \\ &= 72 \text{ dB} \end{aligned}$$

02. Ans: (a)

Sol: For Bipolar pulses,

$$\text{PSD} = \frac{|P(\omega)|^2}{T_b} \cdot \sin^2\left(\frac{\omega T_b}{2}\right)$$

The zero magnitude occurs for

$$f = n/T_b.$$

$$\begin{aligned} \therefore \text{The width of the major lobe} &= 1/T_b \\ &= f_b \end{aligned}$$

$$\therefore (\text{B.W})_{\min} = f_b$$

$$\begin{aligned} \text{Here, Data rate} &= n f_s \\ &= 8(8 \text{ kHz}) = 64 \text{ kbps} \end{aligned}$$

$$\therefore (\text{B.W})_{\min} = 64 \text{ kHz}$$

03. Ans: (c)

Sol: Since the signal is uniformly distributed,

$$f(x) = \frac{1}{10} \quad \text{for } -5 \leq x \leq 5$$

$$= 0 \quad \text{: else where.}$$

$$\text{Signal Power} = \int_{-5}^5 x^2 f(x) dx = \frac{25}{3} \text{ volts}^2$$

$$\text{Step size} = \frac{V_{p-p}}{L} = \frac{10}{2^8} = 0.039 \text{ V}$$

$$N_q = \frac{\Delta^2}{12} = 0.126 \text{ mW}$$

Signal to noise ratio, SNR in dB is

$$\text{SNR} = 10 \log \left(\frac{\text{signal power}}{\text{Noise power}} \right)$$

$$= 10 \log \left(\frac{25/3}{0.126 \times 10^{-3}} \right)$$

$$= 48 \text{ dB}$$

04. Ans: (b)

Sol: For every one bit increase in data word length, quantization Noise Power becomes $\frac{1}{4}$ th of the original. Hence, Data word

length for n = 9 bits is,

$$\therefore L = 2^n = 2^9 = 512$$

05. Ans: (c)

Sol: $V_{p-p} = -5\text{V to } 5\text{V}$

$$20 \log L = 43.5$$

$$L = 10^{2.175}$$

$$= 149.6$$

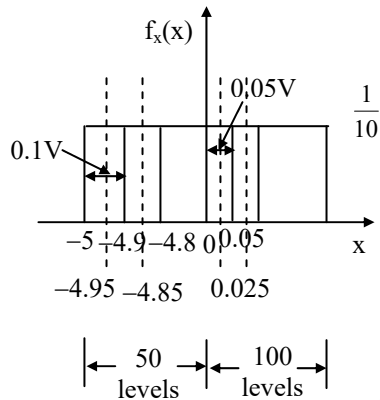
$$\Rightarrow \Delta = \frac{V_H - V_L}{L}$$

$$= \frac{5 - (-5)}{10^{2.175}}$$

$$\Delta = 0.06683$$

06. Ans: (c)

Sol:



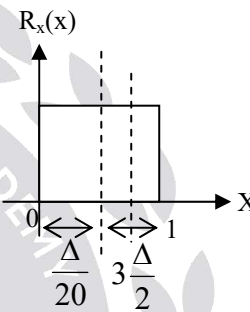
$$\begin{aligned} \text{Signal power } E[X^2] &= \int_{-5}^5 x^2 \left(\frac{1}{10}\right) dx \\ &= \frac{1}{10} \left(\frac{x^3}{3}\right)_{-5}^5 = \frac{1}{30} (250) = \frac{25}{3} \text{ W} \end{aligned}$$

Quantization Noise power

$$\begin{aligned} &= E[(X - Q(X))^2] \\ &= \int_{-5}^5 [x - Q(x)]^2 f_x(x) dx \\ &= \int_{-5}^{-4.9} [x - (-4.95)]^2 \frac{1}{10} dx \\ &\quad + \int_{-4.9}^{-4.8} [x - (-4.85)]^2 \frac{1}{10} dx + \dots (50 \text{ times}) \\ &\quad + \int_0^{0.05} (x - 0.025)^2 \frac{1}{10} dx \\ &\quad + \int_{0.05}^{0.1} [(x - 0.075)]^2 \frac{1}{10} dx + \dots (100 \text{ times}) \\ &= 50 \int_{-5}^{-4.9} (x + 4.95)^2 \frac{1}{10} dx + 100 \int_0^{0.05} (x - 0.025)^2 \frac{1}{10} dx \\ &= 5 \left[\frac{(x + 4.95)^3}{3} \right]_{-5}^{-4.9} + 10 \left[\frac{(x - 0.025)^3}{3} \right]_0^{0.05} \\ &= \frac{5}{3} [(0.05)^3 + (0.05)^3] + \frac{10}{3} [(0.025)^3 + (0.025)^3] \end{aligned}$$

$$\begin{aligned} &= \frac{5}{3} (125 \times 10^{-6} + 125 \times 10^{-6}) + \frac{10}{3} [(0.025)^3 + (0.025)^3] \\ &= \frac{5}{3} (125 \times 10^{-6} + 125 \times 10^{-6}) + \frac{10}{3} (3.125 \times 10^{-5}) \\ &= \frac{1250}{3} \times 10^{-6} + \frac{312.5}{3} \times 10^{-6} \\ &= 520.83333 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} (\text{SNR})_{\text{dB}} &= 10 \log \left(\frac{25}{3 \times 5.2 \times 10^{-4}} \right) \\ &= 42.04 \text{ dB} \\ &\approx 42 \text{ dB} \end{aligned}$$



07. Ans: (b)

Sol: $E[X - Q(x)]^2$

$$\begin{aligned} &= \int_0^{0.3} (X - 0)^2 (1) dx + \int_{0.3}^1 (X - (0.7))^2 (1) dx \\ &= \left[\frac{x^3}{3} \right]_0^{0.3} + \left[\frac{(x - 0.7)^3}{3} \right]_{0.3}^1 \\ &= \frac{(0.3)^3}{3} + \frac{(0.3)^3}{3} + \frac{(0.4)^3}{3} \\ &= 0.198 \end{aligned}$$

08. Ans: (b)

Sol: Since, all the quantization levels are equiprobable,

$$\int_{-a}^a \frac{1}{4} dx = \frac{1}{3} \Rightarrow a = \frac{2}{3}$$

09. Ans: (a)

$$\text{Sol: } \int_{-2/3}^{2/3} x^2 f(x) dx = \frac{1}{4} \int_{-2/3}^{2/3} x^2 dx = \frac{4}{81}$$

Matched Filter

01. Ans: (d)

Sol: The time domain representation of the o/p of a Matched filter is proportional to Auto correlation function of the i/p signal, except for a time delay

$$R_{ss}(\tau) = \int_0^{10^{-4}} S(t) \cdot S(t + \tau) dt$$

$$= \int_0^{10^{-4}} 10 \sin(2\pi \times 10^6 t) \cdot 10 \sin(2\pi \times 10^6 (t + \tau)) dt$$

$$= 50 \int_0^{10^{-4}} [\cos(2\pi \times 10^6 \tau) - \cos(4\pi \times 10^6 t + 2\pi \times 10^6 \tau)] dt$$

$$= 50 \times 10^{-4} \cos(2\pi \times 10^6 \tau)$$

∴ The Peak is 5mV

02. Ans: (b)

Sol: The matched filter has maximum value of output at $t = T$ is energy of the signal

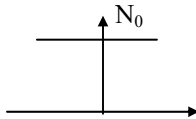
$$\Rightarrow E_s = \int_0^1 A^2 dt + \int_2^3 A^2 (1) dt$$

$$= A^2 + A^2 = 2A^2$$

03. Ans: (d)

Sol: $(SNR)_0 = \frac{E_s}{N_0} = \frac{\frac{B^2}{2} \cdot T}{N}$

$$= \frac{B^2 T}{2N}$$



04. Ans: (b)

Sol: Given,

$$\frac{S_{02}(t)}{N} = \frac{S_{01}(t)}{N} \Rightarrow \frac{2E_{s1}}{N} = \frac{2E_2}{N}$$

$$A^2 T = \frac{B^2}{2} T \Rightarrow A = \frac{B}{\sqrt{2}}$$

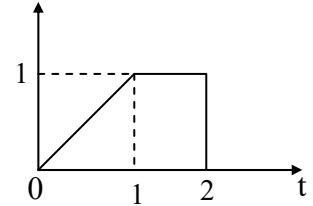
05. Ans: (d)

Sol: Output of the matched filter is maximum which is equal to the energy in the signal

$$E = \int_0^1 1 \cdot t^2 dt + \int_1^2 (1) dt$$

$$= \left[\frac{t^3}{3} \right]_0^1 + 1 \left[t \right]_1^2$$

$$= \frac{1}{3} + 1 = \frac{4}{3}$$



The time instant which occurs the maximum value is its time period $T = 2$

06. Ans: (c)

Sol: Given,

$$H(f) = \frac{1 - e^{-j\omega t}}{j\omega}$$

$$H(f) = \frac{1}{j\omega} - \frac{e^{-j\omega t}}{j\omega}$$

Applying I.F.T

$$h(t) = 0.5(\text{sgn}(t) - \text{sgn}(t - T_0))$$

$$\left(\because F(\text{sgn}(t)) = \frac{2}{j\omega} \right)$$

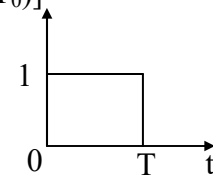
$$= 0.5[2u(t) - 1 - [2u(t - T_0) - 1]]$$

$$= [u(t) - u(t - T_0)]$$

We know that

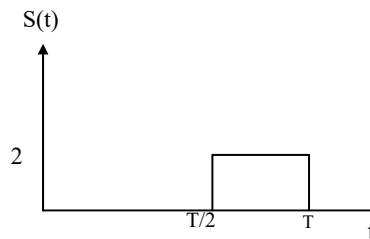
$$h(t) = s^*(t - T)$$

$$\therefore S_i(t)$$



07. Ans: (d)

Sol: The maximum value in the output is energy inside the signal

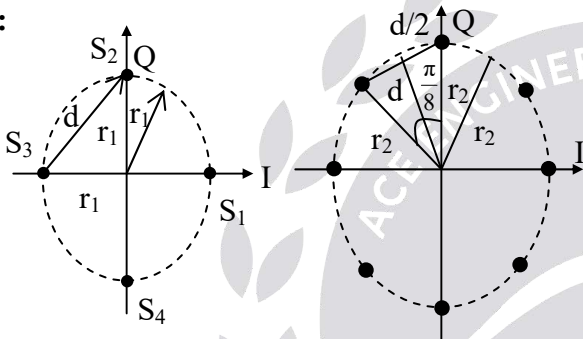


$$\begin{aligned} \Rightarrow S_0(t)_{\max} &= \int_{T/2}^T 2^2 \cdot dt \\ &= 4 \int_{T/2}^T 1 \cdot dt \\ &= 4[T - T/2] \\ &= 2T \end{aligned}$$

Probability of Error

01. Ans: (d)

Sol:



$$\begin{aligned} d &= \sqrt{2} r_1 \\ \Rightarrow r_1 &= \frac{d}{\sqrt{2}} = 0.707 d \\ \Rightarrow r_2 &= \frac{d}{2 \sin \frac{\pi}{8}} = 1.307 d \end{aligned}$$

02. Ans: (d)

Sol: 4-PSK, 8-PSK both have same error probability when both signals have same minimum distance between pairs of signal points.

$$P_e = Q\left(\frac{\sqrt{d_{\min}^2}}{2N_0}\right)$$

$$P_e = 2Q\left(\sqrt{\frac{2E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}\right)$$

Where E_s is the average symbol energy

Given both constellation d_{\min} is same i.e., 'd'

Average Symbol Energy:

$$(E_s)_{4PSK} = \frac{E_{s_1} + E_{s_2} + E_{s_3} + E_{s_4}}{4}$$

Where E_{s_k} is the symbol 'S_k' Energy

= (distance from the origin to the symbol 'S_k')²

$$(E_s)_{4PSK} = \frac{r_1^2 + r_1^2 + r_1^2 + r_1^2}{4} = r_1^2$$

Similarly, For 8 PSK

$$(E_s)_{8PSK} = r_2^2$$

$$\frac{(E_s)_{8PSK}}{(E_s)_{4PSK}} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1.307d}{0.707d}\right)^2$$

In dB,

$$\begin{aligned} (E_s)_{8PSK(\text{dB})} - (E_s)_{4PSK(\text{dB})} &= 10 \log\left(\frac{1.307}{0.707}\right)^2 \\ &= 5.33 \text{ dB} \end{aligned}$$

$$(E_s)_{8PSK} = (E_s)_{4PSK} + 5.33 \text{ dB}$$

8 PSK required additional 5.33 dB

03. Ans: (b)

Sol: Constellation 1:

$$s_1(t) = 0;$$

$$s_2(t) = -\sqrt{2} a \phi_1 + \sqrt{2} a \phi_2$$

$$s_3(t) = -2\sqrt{2} a \phi_1;$$

$$s_4(t) = -\sqrt{2} a \phi_1 - \sqrt{2} a \phi_2$$

$$\text{Energy of } S_1(t) = E_{S1} = 0; E_{S2} = 4a^2;$$

$$E_{S3} = 8a^2; E_{S4} = 4a^2$$

Average Energy of constellation 1

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = 4a^2$$

Constellation 2:

$$s_1(t) = a\phi_1 \Rightarrow E_{S1} = a^2$$

$$s_2(t) = a\phi_2 \Rightarrow E_{S2} = a^2$$

$$s_3(t) = -a.\phi_1 \Rightarrow E_{S3} = a^2$$

$$s_4(t) = -a.\phi_2 \Rightarrow E_{S4} = a^2$$

Average Energy of constellation 2

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = a^2$$

The required Ratio is 4

04. Ans: (a)

Sol: The distance between the two closest points in constellation 1 is $d_1 = 2a$.

The same in constellation 2,

$$d_2 = \sqrt{2} a$$

Since $d_1 > d_2$, Probability of symbol error for constellation 1 is lower

05. Ans: (a)

Sol:
$$S(t) = \sqrt{\frac{2E}{T_b}} \left[\cos(\omega_c t + \frac{2\pi}{m}(i-1)) \right]$$

$$= \sqrt{\frac{2E}{T_b}} \left[\cos\omega_c t \cdot \cos\left(\frac{2\pi}{m}(i-1)\right) - \sin\omega_c t \cdot \sin\frac{2\pi}{m}(i-1) \right]$$

$$= \sqrt{\frac{2}{T_b}} \cos\omega_c t \sqrt{E} \cos\left(\frac{2\pi}{m}(i-1)\right) - \sqrt{\frac{2}{T_b}} \sin\omega_c t \sqrt{E} \sin\frac{2\pi}{m}(i-1)$$

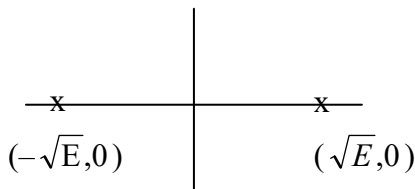
Given binary digital communication $m = 2$

$$\sqrt{\frac{2}{T_b}} \cos\omega_c t \sqrt{E} \cos\pi$$

\therefore basic function = $2 \cos\omega_c t$

$$\Rightarrow T_b = \frac{1}{2}$$

$$2 \cos\omega_c t (\sqrt{E} \cos\pi(i-1)) - [2 \sin\omega_c t] \sqrt{E} \sin\pi(i-1)$$



Distance between two points is:

$$\sqrt{(\sqrt{E} + \sqrt{E})^2 + 0}$$

$$\sqrt{4E} = 2\sqrt{E}$$

Energy of the signal:

$$\int_0^{T_b} (A \cos \omega_c t)^2 dt = \frac{A^2 T_b}{2}$$

$$\Rightarrow d = 2\sqrt{\frac{A^2 T_b}{2}} = 2\sqrt{\frac{A^2 \times T_b}{2}} = A$$

$$(\because T_b = \frac{1}{2}) \quad \therefore d = A$$

06. Ans: (c)

Sol:
$$P_e = Q\left[\sqrt{\frac{E_b}{N_o}}\right]$$

$$E_b = \frac{\alpha^2 T_b}{2} = \frac{\alpha^2}{2R_b}$$

$$\alpha = 4mV, R_b = 500 \text{ kbps},$$

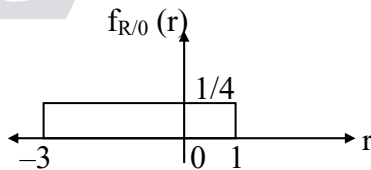
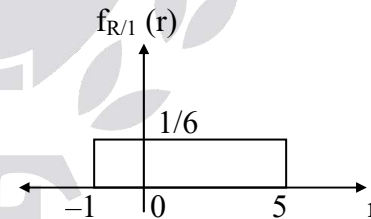
$$N_o = 10^{-12} \text{ W/Hz.}$$

$$\frac{E_b}{N_o} = \frac{16 \times 10^{-6}}{2 \times 500 \times 10^3 \times 10^{-12}} = 16$$

$$P_e = Q[\sqrt{16}] = Q[4]$$

07. Ans: (d)

Sol:

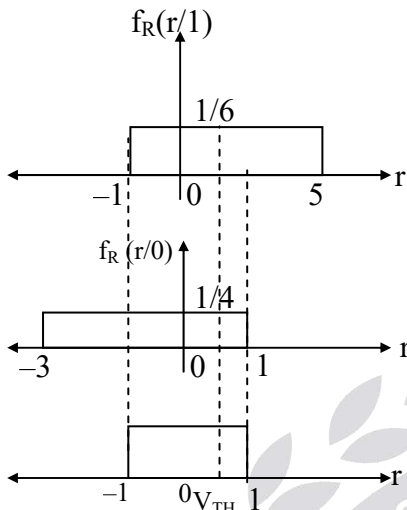


$$P(0) = 1/3; P(1) = 2/3$$

The probability of error of the symbols 0 & 1 are not the same.

\therefore The intersection point of the two pdf's is not the threshold of detection.

Assume the threshold value to be V_{TH}



For minimum error the V_{TH} should lie in the area of intersection of the 2 pdf's.

$$P_{e_1} = \int_{-1}^{V_{TH}} \left(\frac{1}{6}\right) dr = \frac{1}{6}(V_{TH} + 1)$$

$$P_{e_0} = \int_{V_{TH}}^1 \left(\frac{1}{4}\right) dr = \frac{1}{4}(1 - V_{TH})$$

Decision error probability

$$= P_{e_0} P(0) + P_{e_1} P(1)$$

$$= \frac{1}{4}(1 - V_{TH}) \left(\frac{1}{3}\right) + \frac{1}{6}(1 + V_{TH}) \left(\frac{2}{3}\right)$$

$$P_e = \frac{1 - V_{TH}}{12} + \frac{2(1 + V_{TH})}{18}$$

For minimum decision error probability,

$$-1 \leq V_{TH} \leq 1$$

For $V_{TH} = -1$

$$BER = \frac{1 - (-1)}{12} = \frac{1}{6} \text{ (min value)}$$

\therefore Decision error probability = $1/6$

08. Ans: (c)

Sol: The optimum threshold value is

$$\hat{x} = \frac{\sigma^2}{x_1 - x_2} \left[\ln \frac{P(x_2)}{P(x_1)} + \frac{x_1^2 - x_2^2}{2\sigma^2} \right]$$

$$x_1 = 1, x_2 = -1$$

$$P(x_1) = 0.75, \quad P(x_2) = 0.25$$

$$\hat{x} = \frac{\sigma^2}{2} \left[\ln \frac{0.25}{0.75} \right] = -\frac{\sigma^2}{2}$$

So \hat{x} should be strictly negative.

09. Ans: (c)

Sol: $Y = X + Z$

Z is Gaussian RV with mean βx

$$x \in \{-a, +a\}$$

when $\beta = 0$ $E[y] = E[x] + E[z]$

$$E[y] = E[x] = +a$$

$$= a$$

$$BER = Q(a) = 1 \times 10^{-8}$$

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\infty} e^{-\frac{u^2}{2}} du \approx e^{-\frac{v^2}{2}}$$

$$Q(a) = 1 \times 10^{-8} \approx e^{-\frac{a^2}{2}}$$

$$a = 6$$

when $\beta = -0.3$ mean = $6 \times -0.3 = -1.8$

so $E(y) = E(x) + E(z)$

$$= 6 - 1.8 = 4.2$$

$$\text{so } BER = Q(4.2) \approx e^{-\frac{(4.2)^2}{2}}$$

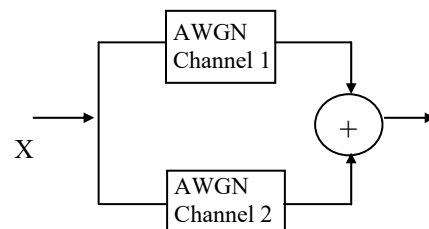
$$\approx 0.0001$$

$$\approx 10^{-4}$$

10. Ans: 1.414

Sol: When the signal is transmitted through a

channel $BER = Q[\sqrt{r}]$.



At the input of the receiver signal amplitude is doubled. But when two independent Gaussian Random Variables are added, the resultant random variables is also a Gaussian random. The pdf is the convolution of individual pdf's.

The variance indicates the noise power But the variance is doubled.

Signal power increased by a factor of 4(mean is doubled).

But the noise increases by a factor of 2

So the signal to noise increases by a factor of 2

$$\text{So } b = \sqrt{2} = 1.414$$

$$\text{BER} = Q[\sqrt{2r}] = Q[\sqrt{2}\sqrt{r}] = Q[1.414\sqrt{r}]$$

$$\text{So } b = 1.414$$

11. Ans: (a)

Sol: Probability of error for an AWGN channel for binary transmission is given as

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

$$\text{Where } E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$$

$$\text{Given } s_1(t) = g(t)$$

$$s_2(t) = -g(t)$$

$$E_d = \int_0^T [g(t) - (-g(t))]^2 dt$$

$$= 4 \int_0^T g^2(t) dt$$

$$E_{d,a} = 4 \int_0^1 (1)^2 dt = 4$$

$$E_{d,b} = 4 \left[\int_0^{1/2} (2t)^2 dt + \int_{1/2}^1 (-2t+2)^2 dt \right]$$

$$= \frac{4}{6} + \frac{4}{6} = \frac{4}{3}$$

$$E_{d,c} = 4 \int_0^1 (1-t)^2 dt = \frac{4}{3}$$

$$E_{d,d} = 4 \int_0^1 (t)^2 dt = \frac{4}{3}$$

P_e is minimum when E_d is maximum

E_d of signal (a) is more when compared to E_d of other signals.

\therefore Probability of error is minimum for signal (a).

12. Ans: (b)

$$\begin{aligned} \text{Sol: o/p Noise Power} &= \text{o/p PSD} \times \text{B.W} \\ &= 10^{-20} \times 2 \times 10^6 \\ &= 2 \times 10^{-14} \text{ W} \end{aligned}$$

Since mean square value = Power

$$\frac{2}{\alpha^2} = 2 \times 10^{-14} \Rightarrow \alpha = 10^7$$

13. Ans: (d)

Sol: When a 1 is transmitted:

$$Y_k = a + N_k$$

$$\text{Threshold } Z = \frac{a}{2} = 10^{-6}$$

$$\Rightarrow a = 2 \times 10^{-6}$$

For error to occur, $Y_k < 10^{-6}$

$$2 \times 10^{-6} + N_k < 10^{-6}$$

$$N_k < -10^{-6}$$

$$\therefore P(0/1) = \int_{-\infty}^{-10^{-6}} P(n) dn$$

$$= \int_{-\infty}^{-10^{-6}} (0.5)\alpha e^{-\alpha n} dn, \text{ with } \alpha = 10^7$$

$$= 0.5 \times e^{-10}$$

When a '0' is Transmitted:

$$Y_k = N_k$$

For error to occur, $Y_k > 10^{-6}$

$$\therefore P(1/0) = \int_{10^{-6}}^{\infty} P(n) dn = 0.5 \times e^{-10}$$

Since, both bits are equiprobable, the Probability of bit error

$$= \frac{1}{2} [P(0/1) + P(1/0)]$$

$$= 0.5 \times e^{-10}$$

14. Ans: (a)

Sol: $P(0/1) = P(1/0) = p$
 $\Rightarrow P(1/1) = P(0/0) = 1 - p$
 Reception with error means getting at most one 1.
 $\therefore P(\text{reception with error})$
 $= P(X = 0) + P(X = 1)$
 $= {}^3C_0 (1-p)^0 p^3 + {}^3C_1 (1-p)^1 p^2$
 $= p^3 + 3p^2(1-p)$

15. Ans: (d)

Sol: $p =$ probability of a bit being in error $= 10^{-3}$
 $q =$ probability of the bit not being in error
 $= 1 - p = 1 - 10^{-3}$
 $= 0.999$

- (1) Total number of bits = 10;
 $P_e =$ probability of error
 $= 1 - P(X = 0)$
 $P(X = 0) =$ Probability of no error
 $\therefore P_e = 1 - [{}^{10}C_0 (10^{-3})^0 (1 - 10^{-3})^{10}] = 0.00995$
- (2) Total number of bits = 100
 $P_e = 1 - [{}^{100}C_0 (10^{-3})^0 (1 - 10^{-3})^{100}]$
 $= 0.0952$
- (3) Total number of bits = 1000
 $P_e = 1 - [{}^{1000}C_0 (10^{-3})^0 (0.999)^{1000}]$
 $P_e = 0.632$
- (4) If total number of bits = 10,000
 $= 1 - [({}^{10,000}C_0)(1 - 10^{-3})^0 (0.999)^{10,000}]$
 $= 0.9999$

Conclusion: As the number of bits increases, the probability of error increases and it approaches unity.

16. Ans: (a)

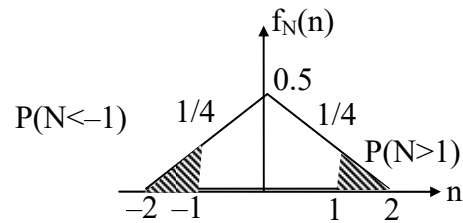
Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

17. Ans: (a)

Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

18. Ans: 0.125

Sol:



$$P(E) = P(x = -1)P\left(\frac{R}{x = -1} > 0\right) + P(x = 1)P\left(\frac{R}{x = +1} < 0\right)$$

$$= 0.5P(x+N > 0) + 0.5P(x+N < 0)$$

$$= 0.5P(-1+N > 0) + 0.5P(1+N < 0)$$

$$= 0.5P(N > 1) + 0.5P(N < -1)$$

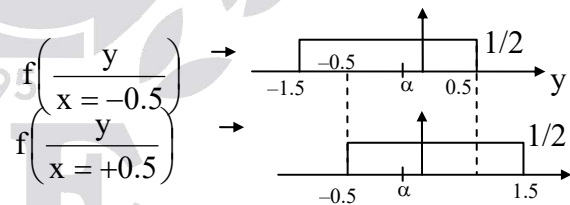
$$= 0.5\left[\frac{1}{2} \cdot \frac{1}{4}(1)\right] + 0.5\left[\frac{1}{2} \cdot \frac{1}{4}\right]$$

$$= \frac{1}{8} = 0.125$$

19. Ans: -0.5

Sol: $x = \{-0.5, 0.5\}$

$P(x = -0.5) = 1/4, P(x = 0.5) = 3/4$



P_e in the overlap region $-0.5 < \alpha < 0.5$

$$P_e = \frac{1}{4} \cdot \frac{1}{2}(0.5 - \alpha) + \frac{3}{4} \cdot \left(\frac{1}{2}\right)(\alpha + 0.5)$$

$$= \frac{0.5}{8} + \frac{1.5}{8} + \left(\frac{3}{8} - \frac{1}{8}\right)\alpha$$

$$= \frac{2}{8} + \frac{2}{8}\alpha$$

$\therefore P_e$ is minimum for $\alpha = -0.5$

20. Ans: (a & c)

Sol: $f_m = 15 \text{ kHz}$

$$f_s = 2f_m = 30 \text{ kHz}$$

$$L = 128$$

$$n = 7 \text{ (Bits/sample)}$$

$$R_b = nf_s = 7 \times 30 \times 10^3 = 210 \text{ (Kbps)}$$

\therefore a & c are correct.

21. Ans: (a & d)

Sol: $s(t)$ occurs at $t = T_b = T(\text{sec})$

$$s(t) = E_{\text{MAX}}\{s(t)\} = \int_0^{\frac{T}{2}} \frac{A^2}{4} dt + \int_{\frac{T}{2}}^T \frac{A^2}{4} dt = \frac{A^2}{4} T$$

\therefore a & d are correct



Chapter 11 Information Theory & Coding

01. Ans: (b)

Sol: Huffman encoder is the most efficient source encoder

$$\begin{array}{cc} 0.5 & \boxed{1} & 0.5 & \boxed{0} \\ 0.25 & \boxed{00} & 0.5 & \boxed{1} \\ 0.25 & \boxed{01} & & \end{array}$$

$$\begin{aligned} \bar{L} &= 1 \times 0.5 + 2 \times 0.25 + 2 \times 0.25 \\ &= 1.5 \text{ bits/symbol} \end{aligned}$$

$$\begin{aligned} \text{Average bit rate} &= 3000 \times 1.5 \\ &= 4500 \text{ bps} \end{aligned}$$

02. Ans: (c)

Sol: Assuming all the 64 levels are equiprobable, $H = \log_2 64 = 6$ bits/pixel

$$\begin{aligned} \text{Total No. of pixels} &= 625 \times 400 \times 400 \\ &= 100 \text{ M pixels/sec} \end{aligned}$$

$$\begin{aligned} \text{Data rate} &= 6 \text{ bits/pixel} \times 100 \times 10^6 \text{ pixel/sec} \\ &= 600 \text{ Mbps} \end{aligned}$$

03. Ans: (b)

$$\text{Sol: } C = B \log \left(1 + \frac{S}{N} \right)$$

$$\text{Since } \frac{S}{N} \gg 1, 1 + \frac{S}{N} \cong \frac{S}{N}$$

$$\therefore C_1 = B \log \frac{S}{N}$$

$$C_2 = B \log \left(2 \cdot \frac{S}{N} \right)$$

$$= B \log 2 + B \log \left(\frac{S}{N} \right) = C_1 + B$$

04. Ans: (b)

Sol: Given

$$B, W = 3 \text{ kHz}$$

$$\text{SNR} = 10 \text{ dB}$$

$$\Rightarrow 10 \log_{10} (\text{SNR}) = 10$$

$$\text{SNR} = 10^1 = 10$$

$$\text{Number of characters} = 128$$

$$\text{Channel capacity} = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$\begin{aligned} &= 3 \times 10^3 \log_2 (1 + 10) \\ &= 10378 \text{ bps} \end{aligned}$$

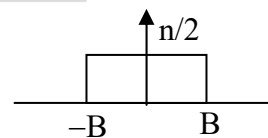
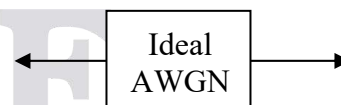
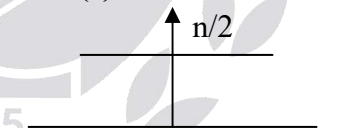
05. Ans: (b)

Sol: Number of characteristics can be sent

$$\text{without any error} = \frac{c}{\log_2 M} = \frac{c}{7} = 1482 \text{ cps}$$

06. Ans: (c)

Sol:



$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$\lim_{B \rightarrow \infty} C = \lim_{B \rightarrow \infty} \frac{S}{n} \times \frac{n}{S} B \log_2 \left(1 + \frac{S}{nB} \right)$$

$$\lim_{B \rightarrow \infty} C = \frac{S}{n} \log_2 e$$

$$(\because \lim_{n \rightarrow \infty} x \log \left(1 + \frac{1}{Q} \right) = \log e)$$

$$\lim_{B \rightarrow \infty} C = 1.44 \frac{S}{n}$$

07. Ans: (b)

$$\begin{aligned} \text{Sol: Max. entropy} &= 512 \times 512 \times \log_2 8 \\ &= 786432 \text{ bits} \end{aligned}$$

08. Ans: (d)

Sol: Maximum entropy of a binary source:

$$H(x) / \text{max} = \log_2 M$$

$$H(x) / \text{max} = \log_2 2 = 1 \text{ bit/symbol}$$

09. Ans: 0.4

$$\begin{aligned} \text{Sol: } P\left(\frac{x=1}{y=0}\right) &= \frac{P(x=1, y=0)}{P(y=0)} \\ &= \frac{P(x=1)P\left(\frac{y=0}{x=1}\right)}{P(x=1)P\left(\frac{y=0}{x=1}\right) + P(x=0)P\left(\frac{y=0}{x=0}\right)} \end{aligned}$$

$$\begin{aligned} &= \frac{0.8 \times \frac{1}{7}}{0.8 \times \frac{1}{7} + 0.2 \times \frac{6}{7}} = 0.4 \end{aligned}$$

10. Ans: (a & d)

11. Ans: (b & c)

$$\text{Sol: } P(x_1) = \frac{1}{3}$$

$$P(x_2) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(y_1) = P(x_1)P\left(\frac{y_1}{x_1}\right) + P(x_2)P\left(\frac{y_1}{x_2}\right)$$

$$P(y_1) = \frac{1}{3}(0.9) + \frac{2}{3}(0.2)$$

$$P(y_1) = \frac{1.3}{3} = 0.433$$

$$P(y_2) = P(x_1)P\left(\frac{y_2}{x_1}\right) + P(x_2)P\left(\frac{y_2}{x_2}\right)$$

$$P(y_2) = \frac{1}{3}[0.1] + \frac{2}{3}[0.8] = \frac{1.7}{3} = 0.5666$$

\therefore b & c are correct

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Chapter 12

Error Correcting Codes

01. Ans: (a & d)

02. Ans: (a, b & c)

Sol:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{ccc} \leftarrow & & \leftarrow \\ I_k & & P \end{array}$$

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{ccc} \leftarrow & & \leftarrow \\ P^T & & I_{n-k} \end{array}$$

Option (a) is correct

$$[C] = [D] [G]$$

$$\therefore [C] = [0 \ 1 \ 1 \ 1 \ 1 \ 0]$$

Option (b) is correct

$$[S] = [r] [H^T] = 0$$

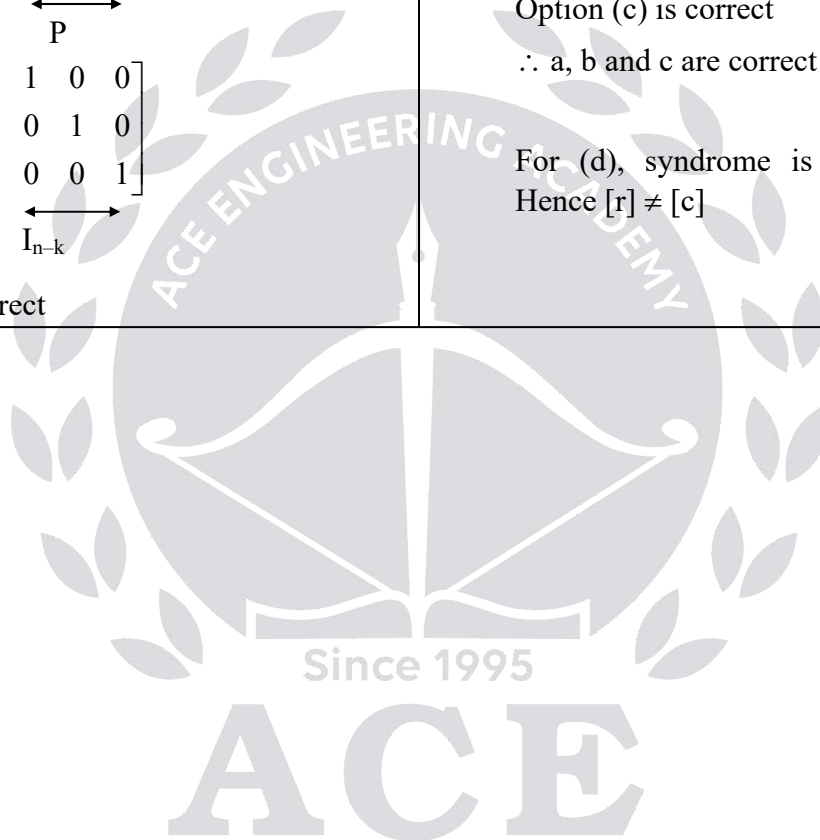
$$\therefore [r] = [C] = [100110]$$

Option (c) is correct

\therefore a, b and c are correct

For (d), syndrome is not equal to zero.

Hence $[r] \neq [c]$



Chapter 13 Optical Fiber Communication

01. Ans: (d)

Sol: $NA = 0.25$

$$n_2 = \frac{C}{V}$$

$$n_2 = \frac{C}{\sqrt{\epsilon_r}}$$

$$n_2 = \sqrt{\epsilon_r} = \sqrt{2.4375}$$

$$n_2 = 1.56$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$(NA)^2 = n_1^2 - n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

$$= \sqrt{(0.25)^2 + 2.4375}$$

$$= \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

02. Ans: (d)

Sol: Number of modes $M = \frac{V^2}{2} \frac{\alpha}{\alpha + 2}$

$$= \frac{1}{2} \left(\frac{2\pi a}{\lambda} (NA)^2 \right) \frac{\alpha}{\alpha + 2}$$

Here a = core radius

λ = wavelength

α = refractive index profile.

03. Ans: (b)

Sol: Power loss = 0.25dB/km

For 100km, the power loss = 100×0.25
= 25 dB

The optical power at 100km

$$= 10 \log 0.1 \times 10^{-3} - 25$$

$$= -65 \text{ dB}$$

In dBm

$$\rightarrow -65 \text{ dB} + 30 = -35 \text{ dBm.}$$

04. Ans: (c)

Sol: Numerical Aperture is used to describes light gathering (or) light collecting ability of an optical fiber.

05. Ans: (c)

Sol: The refractive index of the cladding material should be less than that of the core.

06. Ans: (d)

Sol: Fibers with higher numerical aperture exhibit greater losses and lower bandwidth.

07. Ans: (b & c)

08. Ans: (c)

Sol: Attenuation in optical fibers is mainly caused to absorption, scattering and bending.

\therefore option (c) is correct.