



GATE | PSUs

CIVIL ENGINEERING

Reinforced Cement Concrete

**(Text Book : Theory with worked out Examples
and Practice Questions)**



Reinforced Cement Concrete

(Solutions for Text Book Practice Questions)

01. Material, Workmanship, Inspection and testing

01. Ans: (a), (b), (c), (d)

Sol: Cement requires in total of 23% by weight of water for hydration, this water chemically bounds with the cement compounds and is known as bound water. Some quantity of water is required for cement gel pores. This is about 15% by weight. This water is also known as gel water and is not available for hydration of cement. Hence total water required for complete hydration of water is 38%.

Plasticizers act as deflocculating agents and hence gets adsorb over the cement particles, thereby makes the entrapped water free which modifies the properties of the mix. Dose of plasticizers varies in the range of 0.1 to 0.4% by weight of the cement. Plasticizers usually increases the slump of concrete with a given water content. Plasticizers can reduce the water requirement of a concrete mix for a given workability as a rule of thumb by 10%.

Application of compressive load leads to the development of complex compressive stresses in the specimen due to the restraining effect of the steel plates used over the specimen while testing. This restraining effect is observed due to difference in development of lateral strain in steel plates and concrete specimen. Lateral strain in steel plates is approximately 0.4 times the lateral strain in concrete specimen.

Hence the test results obtained by this test are more than actual. The restraining effect in cylindrical specimen is comparatively less than in cube specimen. In cube specimen the restraining effect is observed over the whole depth but in cylindrical specimen it is limited to the end region. Result obtain by the cylindrical specimen is approximately 0.8 times those obtained by the cubical specimen.

Moist curing aims to keep the concrete as nearly saturated as possible at normal temperature-by continually spraying water, or by 'ponding', or by covering the concrete with a layer of any kind of 'sacking' which is kept wet.

The ingress of curing water into the capillary pores stimulates hydration. This process, in fact, goes on, even after active curing has stopped, by absorption of the moisture in the atmosphere. The period of curing should be as long as conveniently possible in practice. The Code specifies the duration as "atleast seven days from the date of placing of concrete in case of OPC" under normal weather conditions, and at least ten days when dry and hot weather conditions are encountered. When mineral admixtures or blended cements are used, the recommended minimum period is 10 days, which should preferably be extended to 14 days or more.

02. Limit state design method fundamentals

01. Ans: (a), (b), (d)

Sol: The uncertainties due to load and strength in working stress method is taken care by using only one FOS that is applied to strength of the material to get a permissible. By this the strength of the material is underestimated to such an extent that stresses are with in a permissible limit and with in the linear region of the stress strain curve. Non linear strength is completely ignored.

The section sizes produced by working stress method of design are large and have good stiffness therefore perform good under serviceability criteria that is lesser deflections and cracking.

One of The problem with working stress method is that there is not proper utilisation of strength of the material as the strength of the material is underestimated to considerable extent hence the resulting section sizes produced were large more material was required. Hence uneconomical.

Drawbacks of working stress method are taken care by limit state method of design by considering a probabilistic approach for strength and loads(characteristic strength and characteristic load), and applying partial safety factors to both load and strength.

03. Limit State Design- Singly Reinforced Beams

01. Ans: (a)

Sol: For Fe415,

$$\begin{aligned}
 M_{u \text{ limit}} &= \text{Equation (1) with } x_{u \text{ max}} \\
 &= 0.138 f_{ck} b d^2 \\
 &= 0.138 \times 15 \times 200 \times (500)^2 \\
 &= 103.5 \text{ kN-m}
 \end{aligned}$$

02. Ans: (c)

Sol: Balanced (or) limiting percentage of steel (use $x_{u \text{ max}}$)

$$\begin{aligned}
 C &= T \\
 0.36 f_{ck} b x_{u \text{ max}} &= 0.87 f_y A_{st} \\
 0.36 f_{ck} b (0.48d) &= 0.87 \times 415 A_{st} \\
 0.36 \times 15 \times 200 \times 0.48 \times 300 &= 0.87 \times 415 A_{st} \\
 A_{st} &= 430 \text{ mm}^2
 \end{aligned}$$

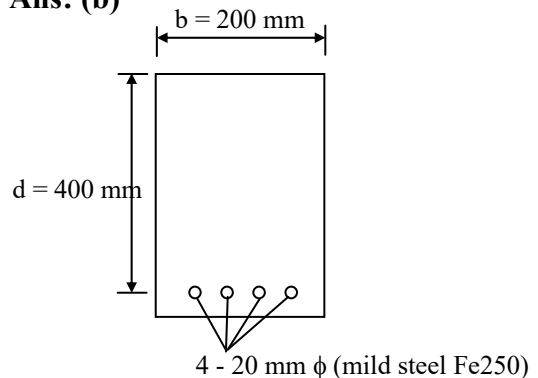
03. Ans: (b)

Sol: $M_u = 138 \times 10^6 \text{ N-mm}$

$$\begin{aligned}
 M_u &= M_{u \text{ limit}} \\
 &= 0.138 \times f_{ck} b d^2 \text{ -- (design as BS)} \\
 138 \times 10^6 &= 0.138 \times 20 \times 200 \times d^2 \\
 d &= 500 \text{ mm}
 \end{aligned}$$

04. Ans: (b)

Sol:



i) $x_{u\max} = 0.53 \times d$
 $= 0.53 \times 400$
 $= 212 \text{ mm}$

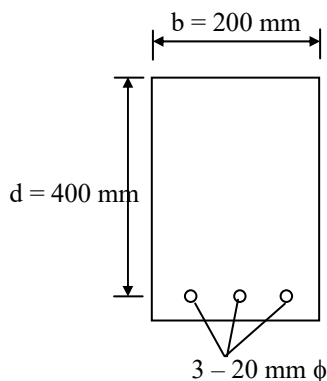
ii) $x_u = ? \quad C = T$
 $0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$
 $0.36 \times 15 \times 200 \times x_u$
 $= 0.87 \times 250 \times 4 \times \left(\frac{\pi}{4} \times 20^2\right)$
 $\Rightarrow 1080x_u = 273318.5$
 $x_u = 253.1 \text{ mm}$

$x_u > x_{u\max} \Rightarrow$ over reinforced section
 Over reinforcement section fails suddenly
 To avoid sudden fail decrease the MR to that of a balanced section

$M_{u\text{ limit}} = 0.148 \times f_{ck} b d^2$
 $= 0.148 \times 15 \times 200 \times 400^2$
 $= 71040000 \text{ N-mm} = 71.04 \text{ kN-m}$
 $\approx 72 \text{ kN-m}$

05. Ans: (d)

Sol:



i) $x_{u\max} = 0.53 \times d$
 $= 0.53 \times 400 = 212 \text{ mm}$

ii) $C = T$
 $0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$
 $0.36 \times 15 \times 200 \times x_u = 0.87 \times 250$
 $\times \left(3 \times \frac{\pi}{4} \times 20^2\right)$

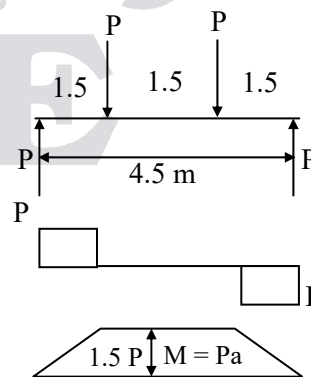
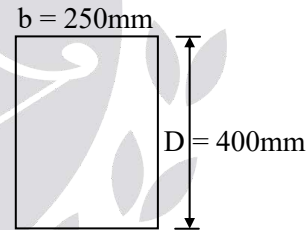
$1080 x_u = 204988.92$
 $x_u = 190 \text{ mm}$

$x_u < x_{\max} \Rightarrow$ Under reinforced section

$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$
 $= 0.36 \times 15 \times 200 \times 190 (400 - 0.42 \times 190)$
 $M_u = 65.7 \text{ kN.m} \approx 66 \text{ kN-m}$

06. Ans: 8.86 kN

Sol:



Homogenous beam

$f_{cr} = 2 \text{ MPa}$

Modulus of rupture/tensile stress of concrete from bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow M = f_{cr} \times z \quad \left[\because z = \frac{bD^2}{6} \right]$$

$$= 2 \left[\frac{250 \times 400^2}{6} \right] = 13.33 \times 10^6 \text{ N-mm}$$

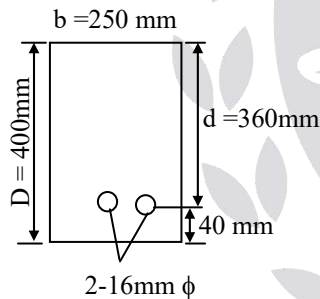
$$M = P \cdot a$$

$$13.3 = P \times 1.5$$

$$P = \frac{13.3}{1.5} = 8.86 \text{ kN}$$

07. Ans: 31.6 kN

Sol:



Reinforced concrete beam

i) $x_{u \max} = 0.48d$
 $= 0.48 \times 360 = 172.8 \text{ mm}$

$$C = T$$

$$0.36f_{ck}bx_u = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 \times x_u$$

$$= 0.87 \times 415 \times \left(2 \times \frac{\pi}{4} \times 16^2 \right)$$

$$1800 x_u = 145186.8$$

$$x_u = 80.65 \text{ mm}$$

$$x_u < x_{\max}$$

\therefore Under reinforced section

$$M.R = 0.36f_{ck} bx_u (d - 0.42x_u)$$

$$= 0.36 \times 20 \times 250 \times 80.65$$

$$(360 - 0.42 \times 80.65)$$

$$M_u = 47.5 \text{ kN-m}$$

$$M_u = P \times a$$

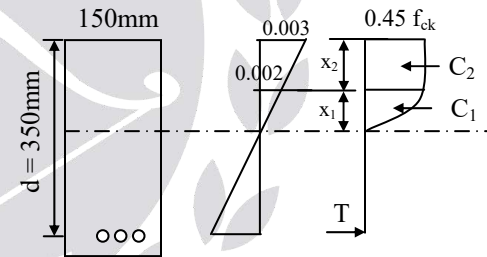
$$47.5 = P \times a$$

$$P = \frac{47.5}{1.5}$$

$$P = 31.6 \text{ kN}$$

08. Ans: 51 kN-m

Sol:



$$x_{u \max} = 0.48 \times d$$

$$= 0.48 \times 350$$

$$= 168 \text{ mm}$$

$$M_{u \text{ limit}} = 0.36f_{ck} b x_{u \max} (d - 0.42 x_{u \max})$$

$$= 0.36 \times 20 \times 150 \times 168 (350 - 0.42 \times 168)$$

$$= 50.70 \times 10^6 \text{ N-m}$$

$$= 51 \text{ kN-m}$$

09. Ans: 503 mm²

Sol: C = T

$$0.36 f_{ck} b x_{u \max} = 0.87 f_y A_{st}$$

$$A_{st} = \frac{0.36 f_{ck} b x_{u \max}}{0.87 \times f_y}$$

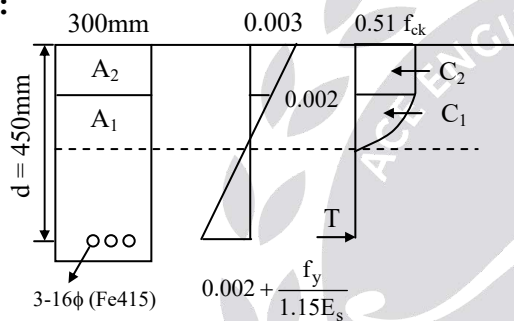
$$= \frac{0.36 \times 20 \times 150 \times 168}{0.87 \times 415}$$

$$= 502.53 \text{ mm}^2$$

$$A_{st} \approx 503 \text{ mm}^2$$

10. Ans: 196 mm

Sol:



$$x_{u \max} = 0.003 \quad \rightarrow (1)$$

$$(d - x_{u \max}) = \left(0.002 + \frac{f_y}{1.15 E_s} \right)$$

$$450 - x_{u \max} = \left(0.002 + \frac{415}{1.1 \times 2 \times 10^5} \right) \quad \rightarrow (2)$$

$$\frac{450 - x_{u \max}}{x_{u \max}} = \frac{0.002 + \frac{415}{1.1 \times 2 \times 10^5}}{0.003}$$

On solving

$$x_{u \max} = 196.04 \text{ mm} = 196 \text{ mm}$$

03. Ans: (b) & (d)

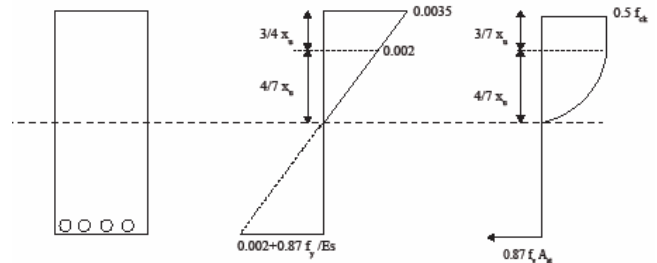
Sol: Section size = 300 × 500 mm

$$A_{st} = 453 \text{ mm}^2$$

$$d = (500 - 40) \text{ mm}$$

$$= 460 \text{ mm}$$

Strength of concrete in compression = 0.5 f_{ck}



From the strain diagram, your a limiting section

$$\frac{x_{u \lim}}{d} = \frac{700}{1100 + 0.87 f_y}$$

For, Fe 415

$$x_{u \lim} = 0.48 \times d$$

$$= 0.48 \times 460 \text{ mm}$$

$$= 220.8 \text{ mm}$$

position of C force on compression side of stress diagram

$$A_1 = 0.5 f_{ck} \times \frac{3}{7} x_u = 0.2142 x_u f_{ck}$$

$$y_1 = \frac{1}{2} \times \frac{3}{7} x_u = \frac{3}{14} x_u$$

$$A_2 = \frac{2}{3} \times 0.5 f_{ck} \times \frac{4}{7} x_u = 0.1904 f_{ck} x_u$$

$$y_2 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u = 0.6248 x_u$$

$$\bar{y} = \frac{0.2142 x_u f_{ck} \times \frac{3}{14} x_u + 0.1904 f_{ck} x_u \times 0.6248 x_u}{0.2142 x_u f_{ck} + 0.1904 f_{ck} x_u}$$

$$\bar{y} = 0.416 x_u$$

$$C. \text{Force} = (0.2142 x_u f_{ck} + 0.1904 x_u f_{ck}) B$$

$$C. \text{Force} = 0.4046 f_{ck} x_u B$$

$$x_{u \lim} = 0.4046 f_{ck} B x_{u \lim} (d - 0.416 x_{u \lim})$$

$$= 0.4046 \times 30 \times 300 \times 220.8 (460 - 0.416 \times 220.8)$$

$$x_{u \lim} = 295.99 \text{ kN-m}$$

04. Limit State Design - Doubly Reinforced Beams

01. Ans: (c)

Sol: BM = 300 kN-m

Concrete, $M_{15} = f_{ck} = 15$

Steel, $f_y = 415$

$f_{sc} = 353.7$ MPa

Effective Cover $d' = 50$ mm

In LSM, we have to use

Factored moment

$$M_u = M \times \gamma_f$$

Use $\gamma_f = 1.5$

$$= 300 \times 1.5 = 450 \text{ kN-m}$$

To calculate $M_{u \text{ limit}}$

$$M_{u \text{ limit}} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 15 \times 350 \times (700)^2$$

$$M_{u \text{ limit}} = 355 \text{ kN-m}$$

$$M_u = 450 \text{ kN-m}$$

$$\therefore M_u > M_{u \text{ limit}}$$

So we need to use 'DRB'

$$M_{u \text{ limit}} = 0.87 f_y A_{st} (d - 0.42 x_{u \text{ max}})$$

$$355 \times 10^6 = 0.87 \times 415 \times A_{st} (700 - 0.42 \times 0.48 \times 700)$$

$$A_{st} = 1759.31 \text{ mm}^2$$

for extra moment we need to provide tensile steel & comp. steel

$$M_u - M_{u \text{ limit}} = 0.87 f_y (d - d') A_{st2}$$

$$(450 - 355) \times 10^6 = 0.87 \times 415 A_{st2} (700 - 50)$$

$$= 234682.5 A_{st2}$$

$$A_{st2} = 404.8 \approx 405 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 2165 \text{ mm}^2$$

Now our purpose is to calculate ' A_{sc} '

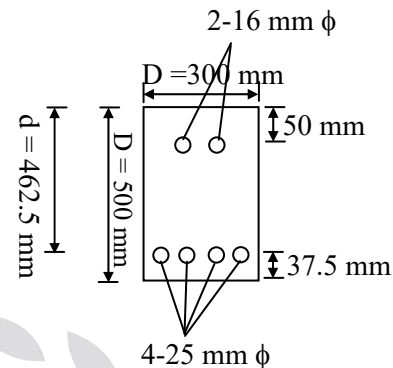
$$M_u - M_{u \text{ limit}} = f_{sc} A_{sc} (d - d')$$

$$(or) f_{sc} A_{sc} = 0.87 f_y A_{st2}$$

$$A_{sc} = 413.2 \text{ mm}^2$$

02. Ans: 271 kN-m

Sol:



$$b = 300 \text{ mm}, D = 500 \text{ mm}, d = 462.5 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2,$$

$$f_{sc} = 0.8566 f_y$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.495 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$\Rightarrow C = T$$

$$\Rightarrow C_1 + C_2 = T$$

$$0.36 \times f_{ck} b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 300 \times x_u + (0.8566 \times 415) \times 402.12$$

$$= 0.87 \times 415 \times 1963.495$$

$$x_u = 209.618 \text{ mm}$$

$$x_{u \text{ max}} = 0.48 \times d$$

$$= 0.48 \times 462.5 = 222 \text{ mm}$$

$$x_u < x_{u \text{ max}}$$

\therefore under reinforced section.

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 300 \times 209.6$$

$$(462.5 - 0.42 \times 209.6) + (0.8566 \times 415)$$

$$\times 402.12 (462.5 - 50)$$

$$= 270.9 \text{ kN-m}$$

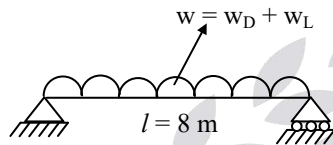
03. Ans: 18.82 kN/m

Sol: Working /line moment,

$$M = \frac{270.9}{1.5} = 180.6 \text{ kN-m}$$

$$\begin{aligned} \text{Self weight of beam, } w_D &= (\gamma_c) b \times D \\ &= (25 \text{ kN/m}^3) \times (0.3 \times 0.5) \end{aligned}$$

$$W = 3.75 \text{ kN/m}$$



$$M = \frac{(w_D + w_L) \times l^2}{8}$$

$$180.6 = \frac{(3.75 + w_L) \times 8^2}{8}$$

$$w_L = 18.825 \text{ kN/m}$$

04. Ans: (a) & (b)

Sol: Statement 1 and 2 are correct.

Statement 3 is wrong. Permissible value for Fe 250 grade of steel when subjected to compression is equal to $0.87 f_y$ for all values of strains. But the permissible values for HYSD bars is required to be found from stress strain curve for their respective values of strains.

Statement 4 is correct.

There is no advantage of using high strength of steel on compression side as compression reinforcement as the permissible stress is relatively low and unrelated to grade of steel. For both Fe 415 and Fe 500 the permissible value in compression is 190 MPa.

05. Limit State Design- Flanged Beams

01. Ans: (c)

Sol:

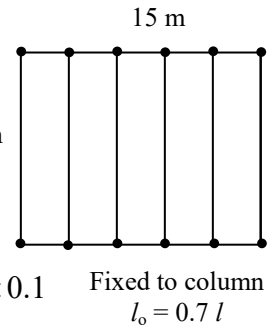
For T-beams,

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

$$= \frac{0.7 \times 10}{6} + 0.25 + 6 \times 0.1$$

$$= 2.01 \text{ m} \not> c = 3 \text{ m}$$

$$\therefore b_f = 2.01 \text{ m}$$



02. Ans: (d)

Sol: L - beam

$$B_f = \frac{l_0}{12} + b_w + 3D_f$$

$$= \frac{10}{12} + 0.25 + 3 \times 0.1$$

$$= 1.38 \text{ m} \not> c = 3 \text{ m}$$

$$\therefore b_f = 1.38 \text{ m}$$

03. Ans: (d)

Sol: $D_f = 100 \text{ mm}$, $b_w = 300 \text{ mm}$, $d = 500 \text{ mm}$,

$$c = 3 \text{ m}, \quad l = 6 \text{ m}, \quad l_0 = 3.6 \text{ m}, \quad b_f = ?$$

$$b_f = \frac{l_0}{6} + b_w + 6D_f \not> c$$

$$= \frac{3.6}{6} + 0.3 + 6 \times 0.1$$

$$= 1.5 \text{ m} \not> c = 3 \text{ m}$$

$$= 1.5 \times 1000 \text{ mm} = 1500 \text{ mm}$$

04. Ans: (a), (d)

Sol: Statement 1 is correct: When the slab is relatively wide, the flexural compressive stress is not uniform over its width. The stress varies from maximum at the web region to progressively at lower values at points farther away from web. The term shear lag is used to explain this concept. The longitudinal stresses at the junction of the web and flange are transmitted through in plane shear to the flange regions. The resulting shear deformations in the flange are maximum at the junction and reduce progressively at regions farther away from the web. Such shear lag behaviour can be easily visualised in the case of a rectangular piece of sponge that is compressed in the middle.

The effective width of flange tends to increase with the span, width and increased flange thickness. It also depends upon the type of loading (concentrated or distributed) and the support conditions. It is seen that the equivalent flange width is less when concentrated load is applied at the midspan of a simply supported beam, compared to the same load when applied as a uniformly distributed beam.

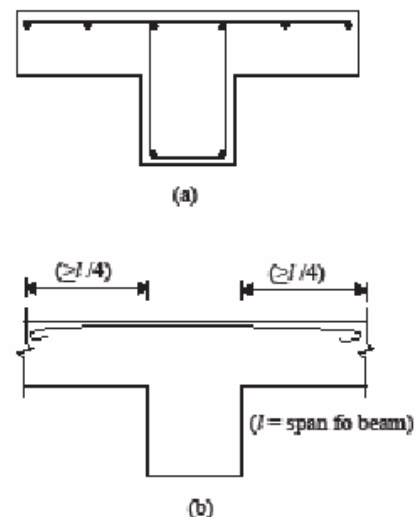
Statement 3 is wrong: It should be noted that the flange is effective only when it is on the compression side that is when the beam is in the sagging mode of flexure (with slab on top). Alternatively if the beam is upturned (inverted T beam) and is subjected to hogging moments, the T beam action is effective, as the flange is under compression.

Statement 4 is correct:

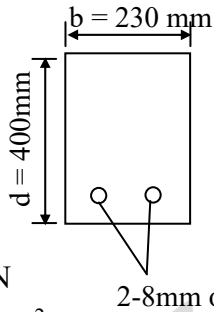
The integral action between the flange and the web is usually ensured by the transverse

bars in the slab and the stirrups in the beam. In the case of isolated flanged beams (as in spandrel beams of staircases), the detailing of reinforcement depicted in Fig. (a) may be adopted. The overhanging portions of the slab should be designed as cantilevers and the reinforcement provided accordingly.

Adequate transverse reinforcement must be provided near the top of the flange. Such reinforcement is usually present in the form of negative moment reinforcement in the continuous slabs which span across and form the flanges of the T-beams. When this is not the case (as in slabs where the main bars run parallel to the beam), the Code (CI. 23.1.1b) specifies that transverse reinforcement should be provided in the flange of the T-beam (or L-beam) as shown in Fig. (b). The area of such steel should be not less than 60 percent of the main area of steel provided at the midspan of the slab, and should extend on either side of the beam to a distance not less than one-fourth of the span of the beam.



Detailing of flanged beams to ensure integral action of slab and beam

06. Limit State of Collapse - Shear
01. Ans: (b)
Sol:


$$V_u = 120 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\text{Main steel, } f_y = 415 \text{ N/mm}^2$$

$$\text{Stirrups, } f_y = 250 \text{ N/mm}^2$$

$$\tau_c = 0.48 \text{ N/mm}^2$$

i) 8mm-2 legged

Stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.53 \text{ mm}^2$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{120 \times 10^3}{400 \times 230}$$

$$= 1.3 \text{ N/mm}^2$$

$$\tau_v \leq \tau_{c \max} - \text{safe in shear}$$

ii) $\tau_v > \tau_c$ - not safe in shear reinforcement

Minimum shear reinforcement is required

$$V_{us} = \frac{(0.87f_y)A_{sv} \times d}{S_v}$$

$$V_{us} = V_u - \tau_c b \cdot d$$

$$= 120 \times 10^3 - 0.48 \times 400 \times 230$$

$$= 75840 \text{ N} = 75.84 \text{ kN}$$

$$75.84 \times 10^3 = \frac{0.87 \times 250 \times 100.53 \times 400}{S_v}$$

$$S_v = 115 \text{ mm c/c}$$

02. Ans: (c)

Sol: $T = 10.90 \text{ kN-m}$

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 120 \times 10^3 + \frac{1.6 \times 10.90 \times 10^6}{230}$$

$$V_e = 196 \text{ kN}$$

Design shear force

$$V_{us} = V_e - \tau_c \cdot b \cdot d$$

$$= 196 \times 10^3 - 0.48 \times 230 \times 400$$

$$V_{us} = 151.84 \times 10^3 \text{ N}$$

$$= 151.84 \text{ kN}$$

03. Ans: (d)

Sol: $b = 230 \text{ m, } d = 450 \text{ mm}$

$$V_u = 50 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$\tau_{c \max} = 2.8 \text{ MPa, } \tau_c = 0.75 \text{ MPa.}$$

$$\tau_v = \frac{V_u}{bd} = \frac{50 \times 10^3}{230 \times 450} = 0.483 \text{ MPa}$$

$$\tau_v < \tau_{c, \max} \text{ safe in shear.}$$

Provide minimum shear reinforcement.

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y}$$

$$A_{sv} = 2 \times \frac{\pi \times 8^2}{4} = 100.53 \text{ mm}^2$$

$$S_v = \frac{100.53 \times 0.87 \times 250}{0.4 \times 230}$$

$$= 237.7 \text{ mm c/c}$$

$$S_v \geq 0.75 d = 0.75 \times 450 = 337.5 \text{ mm}$$

$$S_v \geq 300 \text{ mm}$$

 \therefore Provide spacing of 230 mm c/c

04. Ans: (c)
Sol: $V_u = 100 \text{ kN}$

$$\tau_v = \frac{V_u}{b \times d} = \frac{100 \times 10^3}{230 \times 450} = 0.966$$

 $\tau_v < \tau_{c \text{ max}}$ – shear reinforcement safe

 $\tau_v > \tau_c$ not safe in shear reinforcement

Shear reinforcement is required.

Design shear force for shear reinforcement

$$\begin{aligned} V_{us} &= V_u - \tau_c bd \\ &= 100 \times 10^3 - 0.75 \times 230 \times 450 \\ &= 22.375 \text{ kN} \end{aligned}$$

For vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 250 \times 100.53 \times 450}{22.375 \times 10^3} = 439.75 \text{ mm}$$

Min spacing:

i. 439.75 mm

 ii. $0.75d = 0.75 \times 450 = 337.5 \text{ mm}$

iii. 300 mm

iv. Spacing for min shear reinforcement

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y} \Rightarrow S_v = 237.7 \text{ mm}$$

Provide min spacing of 230 mm c/c.

05. Ans: (c)
Sol: $V_u = 150 \text{ kN}$

$$\tau_v = \frac{150 \times 10^3}{230 \times 450} = 1.449 \text{ MPa}$$

 $\tau_v < \tau_{c, \text{max}}$ – safe in shear reinforcement

 $\tau_v > \tau_c \rightarrow$ Shear reinforcement is required.

Design shear force,

$$\begin{aligned} V_{us} &= V_u - \tau_c bd \\ &= 150 \times 10^3 - 0.75 \times 230 \times 450 \\ &= 72.375 \text{ KN} \end{aligned}$$

Shear force taken by bent-up bars.

$$\begin{aligned} V_{us1} &= 0.87 f_y A_{sv} \sin \alpha \\ &= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^\circ \\ &= 102.66 \text{ kN} \end{aligned}$$

$$\nless 0.5 V_{us} = 36.18 \text{ kN}$$

$$\therefore V_{us1} > 0.5 V_{us}$$

As per IS: 456 ; $V_{us1} \nless 0.5 V_{us}$. In this case V_{us1} is exceeding $0.5 V_{us}$. Therefore limit V_{us1} as 36.18 kN, the remaining S.F i.e 36.195 kN should be resisted by vertical stirrups.

Vertical stirrups:

 For $V_{us2} = 36.195 \text{ kN}$

$$36.195 \times 10^3 = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$\begin{aligned} S_v &= \frac{0.87 \times 250 \times \left(2 \times \frac{\pi}{4} \times 8^2 \right) \times 450}{36.195 \times 10^3} \\ &= 271.708 \text{ mm} \end{aligned}$$

Provide minimum center to center spacing of 230 mm c/c

06. Ans: (a)
Sol: Beam -P

$$\tau_{c \text{ max}} = 2.1 \text{ MPa}$$

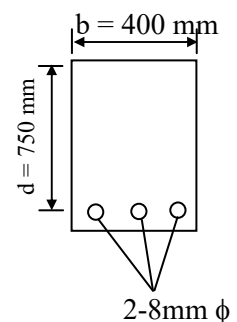
$$f_{ck} = 30 \text{ N/mm}^2$$

$$\tau_c = 0.75 \text{ MPa}$$

$$V_u = 400 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{400 \times 10^3}{750 \times 400}$$

$$\tau_v = 1.33 \text{ N/mm}^2$$



- i) $\tau_v < \tau_{c \max}$ –shear reinforcement safe
 ii) $\tau_v > \tau_c$ Minimum shear reinforcement is required

$$\begin{aligned} V_{us} &= V_u - \tau_c b d \\ &= 400 \times 10^3 - 0.75 \times 400 \times 750 \\ V_{us} &= 175 \text{ kN} \end{aligned}$$

Beam –Q

$$V_u = 750 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{750 \times 10^3}{750 \times 400} = 2.5 \text{ N/mm}^2$$

$$\tau_v > \tau_{c \max}$$

The beam is not safe in shear. It should be revised.

07. Ans: (b), (c), (d)

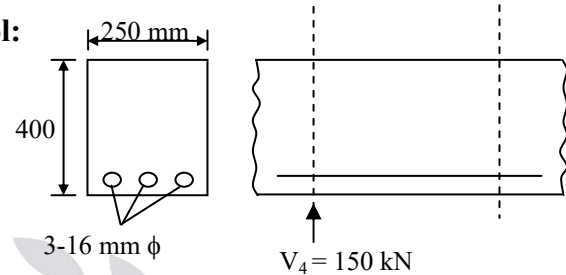
Sol: For Fig 1 the c/s is at a distance ‘d’ from the face of the support. When the support reaction introduces traverser compression in the end region of the member the shear strength of this region is enhanced and inclined cracks do not develop near the face of the support (which is usually the location of maximum shear). In such a case, the code (cl 22.6.2.1) allows a section located at a distance ‘d’ from the face of the support to be treated as critical section. The beam segment between the c/s and the face of the support need to be designed only for shear force at the critical section.

When a heavy load ‘2d’ is introduced from the face of the support, then the face of the support becomes the critical section, as inclined cracks can develop within this region is the shear strength is exceeded.

07. Bond

01. Ans: (c)

Sol:



Flexural bond:

Steel in tension (sagging moment)

$$L_d \geq \frac{M_1}{V_u} + l_0 \rightarrow \text{continuous beam}$$

$$l_0 = 12 \phi = 12 \times 16$$

$$= 192 \text{ mm}$$

$$d = 400 \text{ mm} \left. \vphantom{\begin{matrix} l_0 \\ d \end{matrix}} \right\} \text{Which is greater}$$

$$\text{Take } l_0 = 400 \text{ mm}$$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1} = 870 \text{ mm}$$

$$x_{u, \max} = 0.53 \times 400 = 212 \text{ mm}$$

$$x_u = \frac{0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 15 \times 250}$$

$$= 97.18 \text{ mm}$$

$$x_u < x_{u, \max} \rightarrow \text{Under reinforcement section.}$$

$$M_1 = 0.36 \times 15 \times 250 \times 97.18 (400 - 0.42 \times 97.18)$$

$$= 47.12 \times 10^6 \text{ N-mm}$$

$$L_d \geq \frac{47.12 \times 10^6}{150 \times 10^3} + 400 = 714.15 \text{ mm}$$

$$L_d > 714.15$$

not safe in bond.

02. Ans: (d)

Sol: $\phi = 12\text{mm}$

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 30 \text{ N/mm}^2, \tau_{bd} = 2.4 \text{ MPa}$$

$$L_d = \frac{\phi \sigma_s}{\tau_{bd} \times 4} = \frac{12 \times 0.87 \times 415}{(1.6 \times \tau_{bd}) \times 4} = 282.0703$$

$$L_d = 282.0703 \text{ mm}$$

$$L_d \text{ with } 90^\circ \text{ bend} = 282.0703 - 8\phi$$

$$= 282.0703 - 8 \times 12$$

$$= 186.1 \text{ mm}$$

03. Ans: (d)

Sol: Axially loaded short column

$$\phi = d = 20\text{mm}, \text{ spliced} = 16 \text{ mm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \text{ MPa}$$

$$\left. \begin{array}{l} \text{lap} \leq l_d \\ \leq 24\phi \end{array} \right\} \text{max}$$

Use smaller diameter $\Rightarrow \phi = 16 \text{ mm}$

$$L_d = \frac{\phi \sigma_s}{4 \times \tau_{bd}} = \frac{16 \times 0.87 \times 415}{1.25 \times 4 \times 1.2 \times 1.6} = 601.75 \text{ mm}$$

$$\text{Lap length} \leq L_d = 601.75 \text{ mm}$$

$$\leq 24 \phi = 384 \text{ mm}$$

Use maximum, i.e., 601.75 mm

04. Ans: (d)

Sol: 1) Pull out (bond fail)

$$P_1 = \tau_{bd}[\pi D l]$$

2) Breaking of steel bar

$$P_2 = \sigma_{st} \left[\frac{\pi}{4} \times D^2 \right]$$

} minimum

05. Ans: 46.8

Sol: $f_{ck} = 20 \text{ N/mm}^2$,

$$\tau_{bd} = 1.2 \text{ MPa } \uparrow 60\% - \text{HYSD bars}$$

Steel bar is in tension

$$L_d = \frac{\phi \sigma_s}{4 \times \tau_{bd}} = \frac{\phi \times 360}{4 \times 1.6 \times 1.2} = 46.8\phi$$

06. Ans: 290 mm

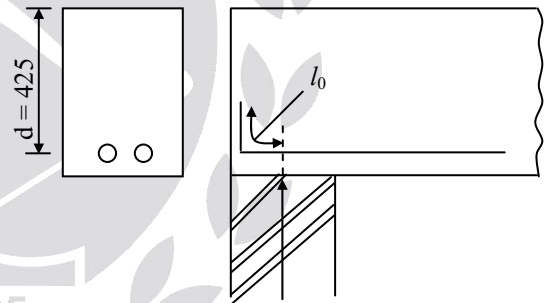
Sol: Given, $V_u = 220 \text{ kN}$

$$A_{st} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$b = 250 \text{ mm}, d = 425 \text{ mm}$$

$$\text{Fe 415, } M_{20}, \tau_{bd} = 1.2 \text{ MPa}$$

$$l_0 = ? \text{ for } 90^\circ \text{ bond}$$



$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 16}{4 \times 1.6 \times 1.2} = 752.1875 \text{ mm}$$

$$L_d (\text{req}) = 752.1875 - 8 \times 16 = 624.1875 \text{ mm}$$

$$x_{u \text{ max}} = 0.48 \times 425 = 204$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 402.12}{0.36 \times 20 \times 250}$$

$$= 80.65 \text{ mm}$$

$x_u < x_{u \max} \rightarrow$ Under reinforced section

$$\begin{aligned} M_1 &= 0.87 f_y A_{st} (d - 0.42 x_u) \\ &= 0.87 \times 415 \times 402.12 (425 - 0.42 \times 80.65) \\ &= 56.78 \times 10^6 \text{ N-mm} \end{aligned}$$

$$L_d = \frac{1.3M_1}{V} + l_0$$

$$624.1875 = 1.3 \times \frac{56.78 \times 10^6}{220 \times 10^3} + l_0$$

$$l_0 = 288.66 \text{ mm}$$

Minimum extension beyond centre of support = 290 mm

07. And: (a), (b)

Sol: As per Clause 26.2.2 the first statement is correct.

The actual bond stress distribution is maximum at the point of embedment and decreases gradually to zero at the extreme end but in the design for bond the bond stress is assumed to be constant.

Splices in flexural members should not be at sections where the bending more than 50 percent of the moment of resistance and not more than half of the bars should be spliced at a section.

The development length of each bar of bundled bars shall be that for the individual bar, increased by 10 percent for two bars in contact, 20 percent for three bars in contact and 33 percent for four bars in contact as per clause 26.2.1.2. Such an increase in the development length is warranted because of the reduction in anchorage bond caused by the reduced interface surface between the steel and the surrounding concrete.

08. Limit State of Collapse - Torsion

01. Ans: (d)

Sol: i) size – 300 × 1000 mm

$$V_u = 150 \text{ kN}; \quad M_u = 150 \text{ kN}$$

$$T_u = 30 \text{ kN-m}$$

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 150 \times 10^3 + \frac{1.6 \times 30 \times 10^6}{300} = 310 \text{ kN}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[1 + \frac{D}{b} \right]}{1.7}$$

$$= 150 + \frac{30 \left[1 + \frac{1000}{300} \right]}{1.7}$$

$$= 226.47 \text{ kN-m}$$

02. Ans: (d)

$$b = 300 \text{ mm}, \quad D = 600 \text{ mm}$$

$$V = 100 \text{ kN}, \quad M = 100 \text{ kN-m}$$

$$T = 34 \text{ kN-m}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[1 + \frac{D}{b} \right]}{1.7}$$

$$= 100 + \frac{34 \left[1 + \frac{600}{300} \right]}{1.7}$$

$$= 160 \text{ kN-m}$$

03. Ans: (a)

Sol: $T = 68 \text{ kN-m}$

$$M_{e2} = M_T - M_u$$

If $M_T < M_u$ then no need of A_{sc}

$$M_T = \frac{T_u \left(1 + \frac{D}{b}\right)}{1.7} = \frac{68 \left(1 + \frac{600}{300}\right)}{1.7}$$

$$= 120 \text{ kN-m}$$

$M_T > M_u$ – additional compression steel is required for M_{e2} i.e. $M_{e2} = M_T - M_u$
 $= 120 - 100$
 $= 20 \text{ kN-m}$

04. Ans: (a)

Sol: $b = 500$, $D = 700 \text{ mm}$

$d = 35 \text{ mm}$, $V = 15 \text{ kN}$

$M = 100 \text{ kN-m}$, $T = 10 \text{ kN-m}$

$$\tau_c = 1.5 \text{ MPa}$$

If $\tau_{ve} \neq \tau_c$ ignore torsion

If $\tau_{ve} > \tau_c$ consider torsion for A_{st}

$$V_e = V_u + V_T$$

$$= V_u + 1.6 \frac{T_u}{b}$$

$$= 15 + 1.6 \left(\frac{10}{0.5}\right)$$

$$= 47 \text{ kN}$$

$$\tau_{ve} = \frac{V_e}{b.d} = \frac{47 \times 10^3}{500 \times (700 - 35)} \approx \frac{47}{0.5 \times 0.7}$$

$$= 0.14 \text{ MPa}$$

$$\tau_{ve} < \tau_c$$

\therefore Design BM for A_{st} is M_u only

$$M_u = 100 \text{ kN-m}$$

05. Ans: (d)

Sol: $V = 20 \text{ kN}$, $T = 9 \text{ kN-m}$

$b = 300 \text{ mm}$, $M = 200 \text{ kN-m}$

gross depth = 425 mm

cover = 25 mm

$$V_e = V_u + V_T$$

$$= V_u + 1.6 \frac{T_u}{b} = 20 + 1.6 \left(\frac{9}{0.3}\right) = 68 \text{ kN}$$

06. Ans: (b)

Sol: As $\tau_{ve} < \tau_c$

$$T_u = 0$$

$M_{e1} = M_u = 200 \text{ kN-m}$

A_{st} based on M_u only

07. Ans: (a), (d)

Sol: Equivalent shear force, $V_{ue} = V_u + \frac{1.6T_u}{B}$

$$= 8 + \frac{1.6 \times 6.5}{0.29}$$

$$= 43.86 \text{ kN}$$

Nominal shear stress

$$\tau_v = \frac{V_{ue}}{bd} = \frac{43.86 \times 10^3}{290 \times 500} = 0.302 \text{ N/mm}^2$$

$\tau_c = 0.48 \text{ N/mm}^2 > \tau_v$, hence no shear reinforcement required, but minimum shear reinforcement is provided as per clause 41.3.2, and the beam will be designed for the given factored bending moment i.e. 90 kN-m.

The effect of torque will only be taken in this value when $\tau_v > \tau_c$ as per clause 41.3.3 of IS : 456 : 2000.

side face reinforcement = 0.1% of BD

$$= \frac{0.1}{100} \times 290 \times 500 = 145 \text{ mm}^2$$

$$\text{one each face} = \frac{145 \text{ mm}^2}{2} = 72.5 \text{ mm}^2$$

9. Slabs

01. Ans: (a), (b), (c), (d)

Sol: As per clause B-5.2.1.1 of annexure B IS 456:2000 statement 1 is correct.

Statement 2 is correct. Shear reinforcement is only provided when the edges and corners are restricted from lifting.

Statement 3 is correct as per clause D1.8, D1.9, D1.10 of annexure D.

Statement 4 is correct as per Clause D1.2 of Annexure D.

10. Limit State of Collapse - Compression

01. Ans: (c)

Sol: $b = 300 \text{ mm}$

$d = 600 \text{ mm}$

$f_y = 415 \text{ MPa}$

$f_{ck} = 20 \text{ MPa}$

$$P_u = 0.40 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_{sc} = 0.8\% A_g$$

$$= \frac{0.8}{100} (300 \times 600) = 1440 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= 300 \times 600 - 1440$$

$$= 178560 \text{ mm}^2$$

$$P_u = 0.4 \times 20 \times 178560 + 0.67 \times 415 \times 1440$$

$$P_u = 1829 \text{ kN}$$

02. Ans: (d)

Sol: $d = 300 \text{ mm};$

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2;$

$$P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$A_{sc} = \left(\frac{\pi}{4} \times 300^2 \right) \times \frac{1}{100} = 706.85 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= \left(\frac{\pi}{4} \times 300^2 \right) - 706.85$$

$$= 69978.98 \text{ mm}^2$$

$$P_u = 1.05 (0.4 \times 20 \times 69978.98 + 0.67 \times$$

$$415 \times 706.85)$$

$$= 794.19 \text{ kN}$$

03. Ans: (d)

Sol: $A_g = 300 \times 300 \text{ mm}$

$f_{ck} = 20 \text{ N/mm}^2,$

$A_c = A_g \text{ (neglecting } A_{sc})$

$f_y = 415 \text{ N/mm}^2$

$A_{sc} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63$

$$P_u = 0.4 \times 20 \times 300 \times 300 + 0.67 \times 415 \times 1256.63$$

$$= 1069 \text{ kN}$$

04. Ans: (d)

Sol: $m = \frac{E_{\text{strong}}}{E_{\text{weak}}} = \frac{E_{\text{steel}}}{E_{\text{conc}}}$

compatibility condition for composite
(RCC) members

$\delta_s = \delta_c$

$$\frac{P_s l}{A_s E_s} = \frac{P_c l}{A_c E_c}$$

$$\frac{P_s}{P_c} = \frac{A_s}{A_c} \left(\frac{E_s}{E_c} \right) = \frac{1\% A_c}{A_c} \times 10 = 10\%$$

05. Ans: (b), (c), (d)
Sol:

$$e_{\min} = \frac{\text{un supported length}}{500} + \frac{\text{lateral dimension}}{30}$$

Minor Axis

$$= \frac{5000}{500} + \frac{450}{30}$$

$$= 10 + 15 = 25 \text{ mm}$$

$\neq (e_{\max})$

Major Axis

$$\frac{5000}{500} + \frac{600}{30}$$

$$= 30 \text{ mm}$$

$\neq (e_{\max})$

$e_{\max} = 0.05 \times 450 = 22.5 \text{ mm}$

$e_{\max} = 0.05 \times 600 = 30 \text{ mm}$

For the column to be short axially loaded,
minimum eccentricity cannot be greater than
0.05 times the lateral dimension.

as per clause 39.3

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$= 0.4 \times 30 \times \left(450 \times 600 - 6 \times \frac{\pi}{4} \times 12^2 \right)$$

$$+ 0.67 \times 415 \times 6 \times \frac{\pi}{4} \times 12^2$$

$$= 3420.53 \text{ kN}$$

when $e = 0,$

as per clause 39.6

$$P_{uz} = 0.45 f_{ck} A_c \times 0.75 f_y A_{sc}$$

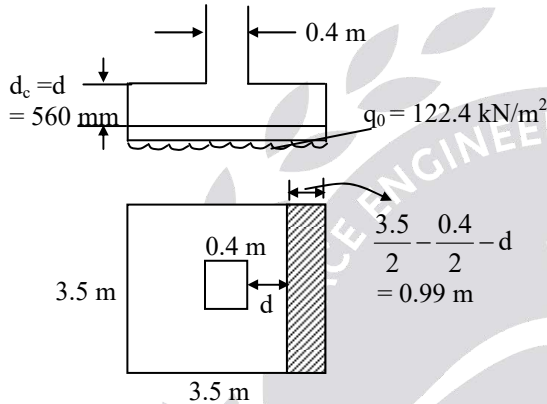
$$= 0.45 \times 30 \times \left(450 \times 600 - 6 \times \frac{\pi}{4} \times 12^2 \right)$$

$$+ 0.75 \times 415 \times 6 \times \frac{\pi}{4} \times 12^2$$

$$P_{uz} = 3847.05 \text{ kN}$$

11. Footings
01. Ans: (b)
Sol: $B = 3.5\text{m}$

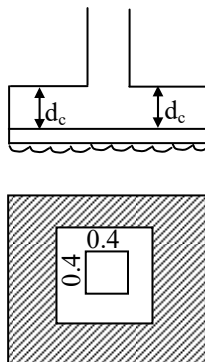
column size = 400 mm

 $d = 560\text{ mm}$
 $q_0 = 122.4\text{ kN/m}^2$


For one way shear

$$\begin{aligned} V_u &= q_0[\text{hatched area}] \\ &= 122.4 [0.99 \times 3.5] \\ &= 425\text{ kN} \end{aligned}$$

$$\begin{aligned} \tau_v &= \frac{V_u}{b \cdot d_c} = \frac{425 \times 10^3}{3500 \times 560} \\ &= 0.22\text{ N/mm}^2 = 0.22\text{ MPa} \end{aligned}$$

02. Ans: (c)
Sol:


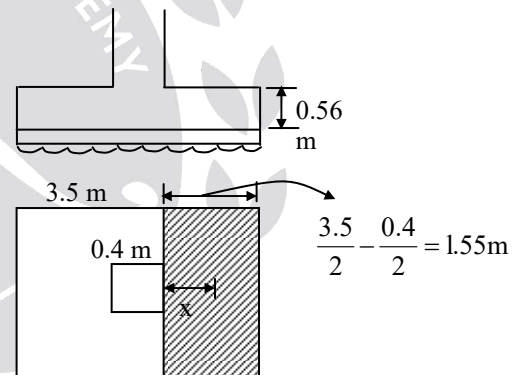
$$B = 0.4 + \frac{0.56}{2} + \frac{0.56}{2} = 0.96$$

$$\begin{aligned} V_u &= q_0[\text{hatched area}] \\ &= 122.4 \times [3.5^2 - 0.96^2] \\ &= 1386\text{ kN} \end{aligned}$$

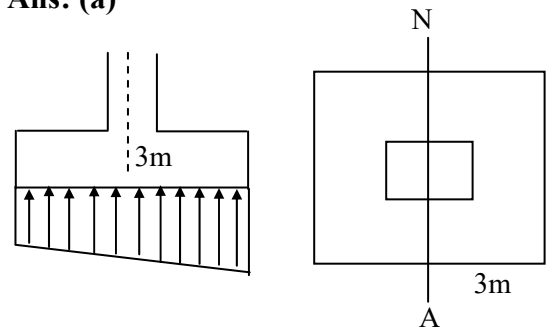
$$\begin{aligned} \tau_v &= \frac{V_u}{pd} = \frac{1386 \times 10^3}{(4 \times 960)(560)} \\ &= 0.64\text{ MPa} \end{aligned}$$

 V_u is more for 2-way

2-way shear is critical

03. Ans: (a)
Sol:


$$\begin{aligned} M_u &= q_0[\text{hatched area} \times \bar{x}] \\ &= 122.4 \left[3.5 \times 1.55 \times \frac{1.55}{2} \right] = 515\text{ kN} \end{aligned}$$

04. Ans: (a)
Sol:


$$\left. \begin{aligned} \sigma_{\max} \\ \sigma_{\min} \end{aligned} \right\} = \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{450}{3 \times 2} \pm \frac{60}{\left(\frac{2 \times 3^2}{6}\right)}$$

$\sigma_{\max} = 95 \text{ kN/m}^2$ compression

$\sigma_{\min} = 55 \text{ kN/m}^2$ compression

As per IS 456 –2000 the assumed pressure distribution below the footing is uniform

05. Ans: (a)

Sol: $l = 2 \text{ m}$; $d = 200 \text{ mm}$

column size = $300 \times 300 \text{ mm}$

$q_0 = 320 \text{ kN}$

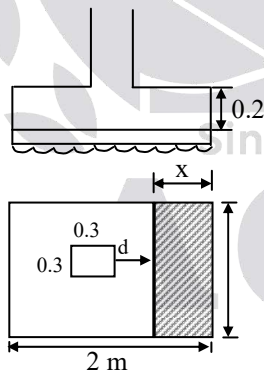
$\tau_v = ?$

$$q_0 = \frac{320}{2 \times 2} = 80 \text{ kN/m}^2$$

$$x = \frac{2}{2} - \frac{0.3}{2} - 0.2$$

$$= 1 - 0.15 - 0.2$$

$$= 0.65$$



One way shear $V_u = q_0$ [hatched area]

$$= 80[0.65 \times 2] = 104 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd_c} = \frac{104 \times 10^3}{2000 \times 200} = 0.26$$

14. Analysis of Prestressed Concrete Members

01. Ans: (b)

Sol: Prestressing force, $P = 2500 \text{ kN}$

Effective span, $l = 10 \text{ m}$

udl on the beam, $w = 40 \text{ kN/m}$

For load balancing

$$P \cdot e = \frac{w l^2}{8}$$

$$(2500)(e) = \frac{(40)(10)^2}{8}$$

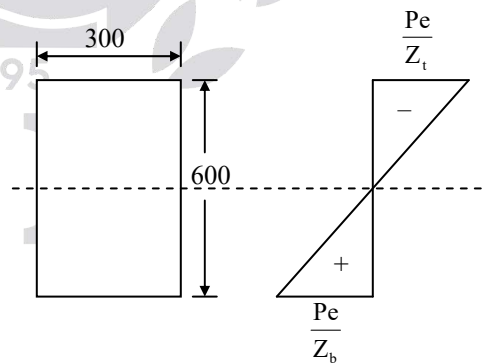
$$e = 0.2 \text{ m} = 200 \text{ mm}$$

02. Ans: (b)

Sol: $\gamma_c = 24 \text{ kN/m}^3$

$\sigma_t = 2 \text{ MPa}$

$\sigma_b = 20 \text{ MPa}$



$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z} \text{----- (1)}$$

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} \text{----- (2)}$$

Adding (1) & (2)

$$20 = \frac{P}{A} + \frac{Pe}{z}$$

$$-2 = \frac{P}{A} - \frac{Pe}{z}$$

$$18 = \frac{2P}{A}$$

$$P = 1620 \text{ kN}$$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{z}$$

$$20 = \frac{1620 \times 10^3}{300 \times 600} + \frac{1620 \times 10^3 \times 6 \times e}{300 \times 600^2}$$

$$e = 122 \text{ mm}$$

$$e \approx 135 \text{ mm}$$

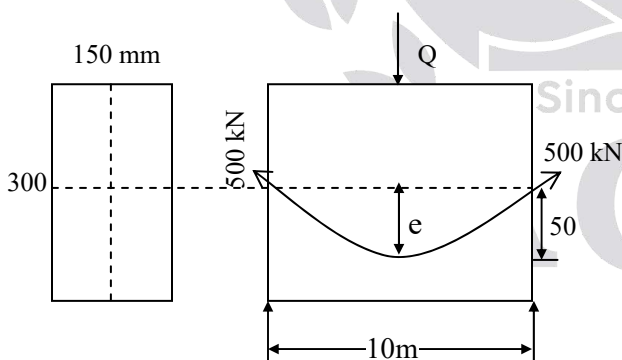
03. Ans: (a)

Sol: $150 \times 300 \text{ mm}$

$l = 10 \text{ m}$, e at support = 0 mm

$e = 50 \text{ mm}$ (center), $P = 500 \text{ kN}$

$Q = ?$ (at center of span)



$$Pe = \frac{Q \times l}{4}$$

$$500 \times \frac{50}{1000} = \frac{Q \times 10}{4}$$

$$100 = Q \times 10$$

$$Q = 10 \text{ kN}$$

04. Ans: (b)

Sol: Self weight

$$\begin{aligned} w_D &= \gamma_c \times b \times D \\ &= (24 \text{ kN/m}^3) \times 0.15 \times 0.3 \\ &= 1.08 \text{ kN/m} \end{aligned}$$

P – line at upper kern point ($\sigma_b = 0$)

$$M_D = \frac{w_D l^2}{8} = \frac{1.08 \times 10^2}{8} = 13.5$$

$$\begin{aligned} \sigma_b = 0 &= \frac{P}{A} + \frac{Pe}{z} - \frac{M_D}{z} - \frac{M_L}{z} \\ &= \frac{500 \times 10^3}{300 \times 150} + \frac{500 \times 10^3 \times 50}{\left(\frac{150 \times 300^2}{6}\right)} - \frac{13.5 \times 10^6}{\left(\frac{150 \times 300^2}{6}\right)} \end{aligned}$$

$$- \frac{M_L}{\left(\frac{150 \times 300^2}{6}\right)}$$

$$0 = 11.11 + 11.11 - 6 - \frac{M_L}{225 \times 10^4}$$

$$M_L = 16.22 \times 225 \times 10^4$$

$$M_L = 36.5 \text{ kN-m,}$$

$$M_L = \frac{Ql}{4}$$

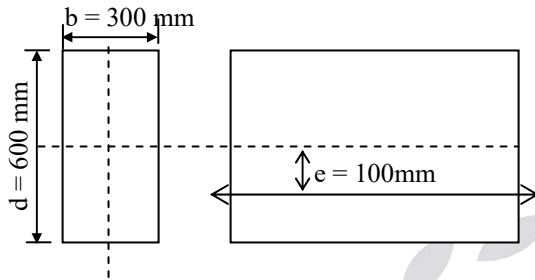
$$36.5 = \frac{Q \times 10}{4}$$

$$146 = Q \times 10$$

$$Q = 14.6 \text{ kN}$$

05. Ans: (c)

Sol: $l = 6 \text{ m}$, $b = 300 \text{ mm}$, $d = 600 \text{ mm}$
 $e = 100 \text{ mm}$, $P = 1000 \text{ kN}$,



Neglecting self weight of the beam

$$\begin{aligned}\sigma_b &= \frac{P}{A} + \frac{Pe}{z} \\ &= \frac{1000 \times 10^3}{300 \times 600} + \frac{1000 \times 10^3 \times 100}{\left(\frac{300 \times (600)^2}{6}\right)} \\ &= 5.55 + 5.55 = 11.11 \text{ MPa}\end{aligned}$$

06. Ans: (b)

Sol: $b = 200 \text{ mm}$, $D = 250 \text{ mm}$
 $A = 500 \text{ mm}^2$, $P = 1000 \text{ MPa}$

$m = 10$

$\varepsilon_s = \varepsilon_c$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_c = \sigma_s \left(\frac{\varepsilon_c}{\varepsilon_s} \right) = \frac{\sigma_s}{m} = \frac{1000}{10}$$

$\sigma_e = 100 \text{ MPa}$

Prestressing force on steel = $\sigma_s \cdot A_s$

$$= 1000 \times 500 = 500 \times 10^3 \text{ N}$$

Compression force in concrete = 500 kN

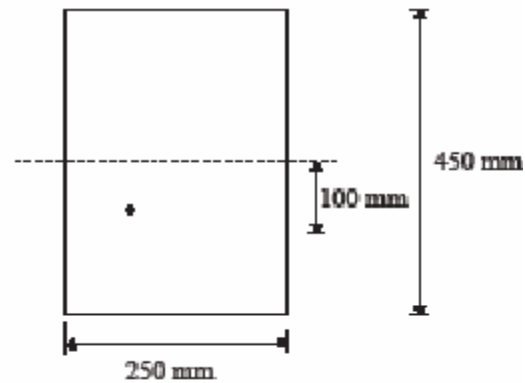
$$= \sigma_c \cdot A_c$$

Compression stress in concrete $\sigma_c = \frac{P_c}{A_c}$

$$= \frac{500 \times 10^3}{200 \times 250} = 10 \text{ MPa}$$

07. Ans: (a), (b), (c)

Sol:



$$\begin{aligned}\text{Prestressing force} &= 1350 \times 412 \\ &= 556.2 \text{ kN}\end{aligned}$$

At transfer stage

The loss due to elastic shortening of concrete is considered transfer stage.

Hence,

$$P_t = 0.95 \times 556.2 \text{ kN}$$

$$P_t = 528.39 \text{ kN}$$

Stresses,

$$\frac{P_t}{A} = \frac{528.39 \times 10^3}{250 \times 450} = 4.69 \text{ MPa}$$

$$\frac{P_t e_y}{I} = \frac{528.39 \times 10^3 \times 450}{\frac{1}{12} \times 250 \times 450^3 \times 2} = 6.26 \text{ MPa}$$

$$\frac{M_d}{I} xy = \frac{2.8 \times 6^2 \times 10^6 \times 450}{8 \times \frac{1}{12} \times 250 \times 450^3 \times 2} = 1.49 \text{ MPa}$$

At top

$$f_t = 4.698 - 6.262 + 1.49 = -0.07 \text{ MPa}$$

At bottom

$$f_b = 4.698 + 6.262 - 1.49 = 9.47 \text{ MPa}$$

(Option (a))

Option (b)

When no loss is considered at initial stage

$$P = 556.2 \text{ kN}$$

$$\therefore \frac{P}{A} = 4.936 \text{ MPa}$$

$$\frac{Pe}{I} y = \frac{6.26}{528.39} \times 556.2 = 6.58 \text{ MPa}$$

$$\frac{M_d}{I} y = 1.49 \text{ MPa}$$

At bottom

$$4.936 + 6.58 - 1.49 = 10.02 \text{ MPa}$$

At final stage

$$P_f = 0.85 \times 556.2 = 472.77 \text{ kN}$$

$$\frac{P_f}{A} = \frac{472.77 \times 10^3}{450 \times 250} = 4.2024$$

$$\frac{P_f e}{I} \times y = \frac{472.77 \times 10^3 \times 100}{\frac{1}{12} \times 250 \times 450^3} \times \frac{450}{2} = 5.6032 \text{ MPa}$$

$$\frac{M_y}{I} \times y = \frac{12.8 \times 6^2 \times 10^6 \times 450}{8 \times \frac{1}{12} \times 250 \times 2 \times 450^3} = 6.8266 \text{ MPa}$$

At top

$$4.2024 - 5.6032 + 6.8266$$

$$= +5.425 \text{ MPa}$$

At Bottom

$$4.2024 + 5.6032 - 6.8266$$

$$= +2.979 \text{ MPa}$$

15. Losses of Prestress

01. Ans: (b)

Sol: $l = 10 \text{ m}$,

$$b = 100 \text{ mm}$$

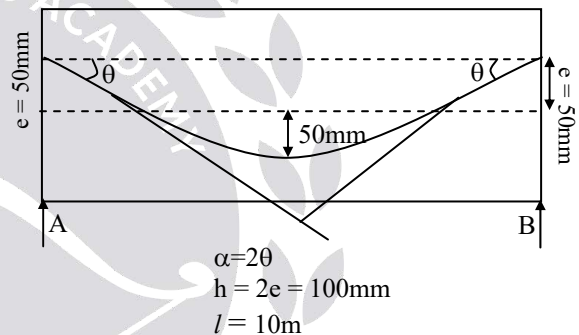
$$D = 300 \text{ mm}$$

$$A = 200 \text{ sq-mm}$$

$$e = 50 \text{ mm}$$

$$\mu = 0.35$$

$$k = 0.0015 \text{ per m}$$



Initial stress in wires = 1200 MPa

Loss of stress in wires = $\sigma(\mu\alpha + kx)$

$$= 1200[0.35 \times \alpha + 0.0015 \times 10]$$

From equation of parabola

$$\theta = \frac{4 \times 0.1}{10} = 0.04 \text{ radians}$$

$$\alpha = 2 \times \theta = 0.08$$

$$\text{Loss} = 1200[0.35 \times 0.08 + 0.0015 \times 10]$$

$$\text{Loss of stress} = 51.6 \text{ MPa}$$

$$\% \text{ loss of stress} = \frac{51.6}{1200} \times 100$$

$$= 4.28 \approx 4.3\%$$

02. Ans: (b)

Sol: Tensioning from both the ends % loss of stress

$$= \frac{\% \text{ loss of stress}}{2} = \frac{4.28}{2} = 2.15$$

03. Ans: (b)

Sol: Straight tendon tensioned from one end

Loss of stress in wires = $\sigma[\mu\alpha + kx]$

($\because \alpha = 0$)

$$1200(0.35 \times (0) + 0.0015 \times 10) = 18 \text{ MPa}$$

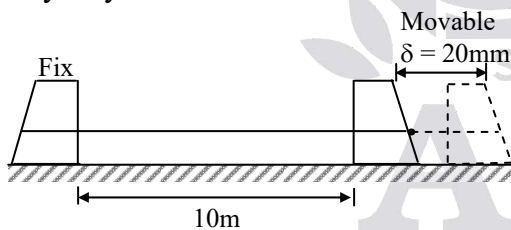
$$\begin{aligned} \% \text{ of loss} &= \frac{18}{1200} \times 100 \\ &= 1.5\% \end{aligned}$$

If tensioned from two ends

$$\frac{\% \text{ of loss}}{2} = \frac{1.5}{2} = 0.75\%$$

04. Ans: (c)

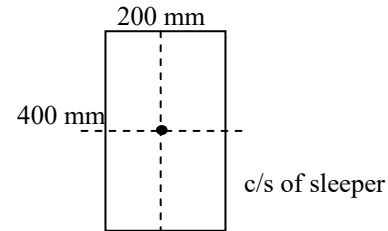
Sol: Hoyer system



$$\delta = \frac{PL}{AE} \quad \left(\text{as } \sigma = \frac{P}{A} \right)$$

Prestress induced in steel wire, $\sigma = \frac{\delta E}{L}$

$$\sigma = \frac{20 \times 2 \times 10^5}{10,000} = 400 \text{ MPa}$$



Eccentricity of Prestress, $e = 0$

Prestressing force in steel wire = $P = \sigma_s \cdot A_s$

$$= 400 \times 500 \text{ mm}^2$$

$$= 200 \text{ kN}$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} (e) = \frac{200 \times 10^3}{200 \times 400} = 2.5 \text{ MPa}$$

Loss due to elastic shortening

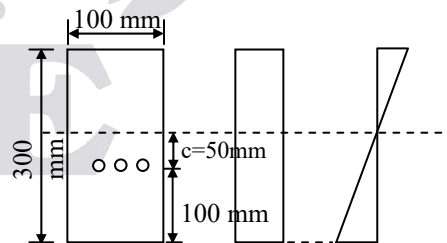
$$= m \times f_c = \left(\frac{E_s}{E_c} \right) f_c$$

$$\sigma = \left(\frac{200,000}{20,000} \right) \times 2.5 = 25 \text{ MPa}$$

$$\% \text{ loss of Prestress} = \frac{25}{400} \times 100 = 6.25\%$$

05. Ans: (d)

Sol:



$$f_c = \frac{P}{A} + \frac{P}{I} (e)^2$$

Initial stress in steel wire = 1200 MPa

Prestressing force in each steel wire

$$P = \sigma_s \cdot A_s$$

$$P = 1200 \times 50 = 60 \text{ kN}$$

$$f_c = \frac{60 \times 10^3}{100 \times 300} + \frac{60 \times 10^3}{\left(\frac{100 \times 300^3}{12}\right)} \times (50)^2$$

$$f_c = 2.66 \text{ MPa}$$

Simultaneous tensioning = loss of prestress is zero

06. Ans: (a)

Sol: Successive tensioning of the 3 cables

$$= \frac{n(n-1)}{2} (m.f_c)$$

$$= \frac{3(3-1)}{2} (6 \times 2.66)$$

$$= 48.0 \text{ MPa}$$

$$\% \text{ of loss} = \frac{48.0}{1200} \times 100 = 4\%$$

(or) For pretensioning system

$$\text{Loss} = n(m \times f_c)$$

$$= 3(6 \times 2.66) = 48.0 \text{ MPa}$$

07. Ans: (c)

Sol: Anchorage slip = 3 mm

$$l = 30 \text{ m}, \sigma = 1200 \text{ MPa}$$

$$E = 2.1 \times 10^5 \text{ MPa}$$

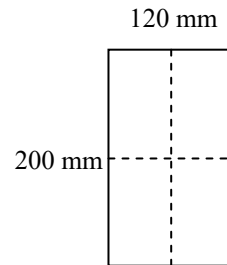
$$E = \frac{\delta E}{l} = \frac{3 \times 2.1 \times 10^5}{30 \times 10^3}$$

$$\sigma = 21 \text{ MPa}$$

$$\% \text{ of loss} = \frac{21}{1200} \times 100 = 1.73\%$$

08. Ans: (b)

Sol:



$$P = 150 \text{ kN}, e = 20 \text{ mm}$$

$$A = 187.5 \text{ mm}^2$$

$$E_s = 2.1 \times 10^5 \text{ MPa}$$

$$E_c = 3.0 \times 10^4 \text{ MPa}$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} . e$$

$$= \frac{150 \times 10^3}{187.5} + \frac{150 \times 10^3 \times 20^2}{\left(\frac{120 \times 200^3}{12}\right)}$$

$$= 800 + 0.75 \text{ MPa}$$

$$f_c = 800.75 \text{ MPa}$$

loss due to elastic shortening = m.f.c

$$= \left(\frac{E_s}{E_c}\right) . f_c$$

$$= \frac{2.1 \times 10^5}{3.0 \times 10^4} \times 0.75 = 4.9$$

Percentage loss in the prestressing steel due to elastic deformation

$$= \frac{4.9}{800.75} \times 100$$

$$= 6.12\%$$

09. Ans: (c)

$$\text{Sol: } \varepsilon = \varepsilon_{\text{shrink}} + \varepsilon_{\text{creep}} \\ = 0.0008$$

Loss of prestress on steel = $\varepsilon \times E_s$

$$= 0.0008 \times 200 \times 10^3$$

$$= 160 \text{ MPa}$$

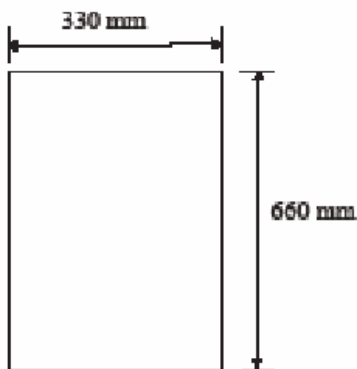
Stress remaining after loss = Initial stress -

Loss

$$= 200 - 160 = 40 \text{ MPa}$$

10. Ans: (a), (b), (d)

Sol:



Given:

span = 8 m

Area of the tendons, $A_s = 1200 \text{ mm}^2$

prestressing force, $P = 1650 \text{ kN}$

Total load = 52 kN/m

Eccentricity, $e = 220 \text{ mm}$

$E_c = 350000 \text{ N/mm}^2$

Let Q_1 = end rotation due to prestressing force only

when a straight profile is provided.

$$Q_1 = \frac{PeL}{2E_c I_c} = \frac{1650 \times 10^3 \times 220 \times 8000}{2 \times 35000 \times \frac{1}{12} \times 330 \times 660^3}$$

$$Q_1 = 524 \times 10^{-3} \text{ radians}$$

Q_2 = rotation at the ends due to the given load.

$$Q_2 = \frac{wL^3}{24E_c I_c} = \frac{52 \times 10^3 \times 8 \times 8000^2 \times 12}{24 \times 35000 \times 330 \times 660^3} \\ = 4 \times 10^{-3} \text{ radians}$$

Q = net rotation at the ends

$$= Q_1 - Q_2$$

$$= (5.24 - 4) \times 10^{-3}$$

$$= 1.24 \times 10^{-3} \text{ radians (hogging)}$$

If the beam is hogging due to net rotation at the ends there will be loss of prestress at the ends.

$$\text{strain lost} = \frac{2eQ}{\ell} = \frac{2 \times 220 \times 1.24 \times 10^{-3}}{8000}$$

$$\text{Prestress lost} = \frac{2 \times 220 \times 1.24 \times 10^{-3}}{8000} \times 2.1 \times 10^5 \\ = 14.32 \text{ N/mm}^2$$

$$\text{Initial prestress} = P_0 = \frac{P}{A_s} = \frac{1650 \times 10^3}{1200} \\ = 1375 \text{ N/mm}^2$$

$$\% \text{ prestress loss } \Delta P\% = \frac{\Delta P\%}{P_0} \times 100 = \frac{14.32}{1375} \times 100 \\ = 1.04\%$$

If a straight profile is replaced by a parabolic profile so that loss is nullified then,

Q_1 = rotation due parabolic profile at the ends.

Q_2 = rotation due to external loading at the ends

$$Q_1 = Q_2$$

$$\frac{PeL}{3E_c I_c} = \frac{wl^3}{24E_c I_c}$$

$$\frac{1620 \times 10^3 \times e \times 8000}{3 \times 35000 \times \frac{1}{12} \times 330 \times 660^3} = 4 \times 10^{-3}$$

$$e = 256.2 \text{ mm}$$

16. Cement**41. Ans: (b) & (d)**

Sol: The presence of Excess magnesia (MgO) in cement makes the cement unsound and expansive that is it has more tendency towards volume change and formation of cracks. Alumina is responsible for quick setting of the cement and if it is in excess lowers the strength.

High alumina cement is not a type of Portland cement. The raw materials used for the manufacture of high alumina cement are limestone (ore of lime) and bauxite (ore of alumina). It is not a quick setting cement. It has a high initial setting time about 30 mins and a less final setting time of about 5 hours. It attains strength in 24 hours and has a high early strength, high heat of hydration, and resistance to chemical attack.

17. Aggregates**01. Ans: (a), (c), (d)**

Sol: Statement (a) is correct as per Clause 5.3.3.1 Concrete mix made from rounded aggregates do produce a workable mix but the development of bond is poor as interlocking between the particles is less, thus unsuitable for high strength concrete.

Very sharp and rough aggregates particles or flaky and elongated require more fine material to produce a workable concrete as their surface area is more. Accordingly, the water requirement and therefore the cement content increases.

Aggregates made from crushed stones higher compressive strength due to development of stronger aggregate mortar bond.

18. Concrete

01. Ans: (a), (b), (c), (d)

Sol: Maximum strength for the mix will only be achieved at a water cement ratio at which minimum capillary cavities will be formed and that water cement ratio is 0.4. It may be noted that for complete hydration of cement under controlled conditions the water requirement is about 38%. When it is decreased less than 0.4 there is improper consistency and workability of concrete resulting in honeycomb structure.

At water cement ratio greater than 0.6, the increase in volume of hydrated products will not be able to occupy the space already filled with water. Hence porosity increases and strength decreases.

Concrete compacted by vibrator displays higher strength even up to a water cement ratio of 0.3. On vibration concrete mix can get fluidized and internal friction between the aggregate particles reduces resulting in entrapped air to rise to the surface. On losing entrapped air concrete gets denser. Vibrations do not affect the strength but in turn increase the strength of concrete with lesser water for a given cement content.

At low water cement ratio with proper compaction capillary cavities will be minimum and hence permeability will be lower.

19. Cement Mortar

01. Ans: (b), (d)

Sol: Sand in mortar does not impart strength but helps in readjustment of strength, which can be achieved by increasing or decreasing its proportion.

Use of sand in mortar helps in reducing the shrinkage of binding material, thereby reducing the tendency of development of cracks in it.

Sand used for mortar mix preferably be well graded as the voids produced will be less and the mortar will be more workable.

Sand in mortar should be free from moisture absorbing chemicals from the atmosphere like alkalis as if present it would lead to the presence of efflorescence.