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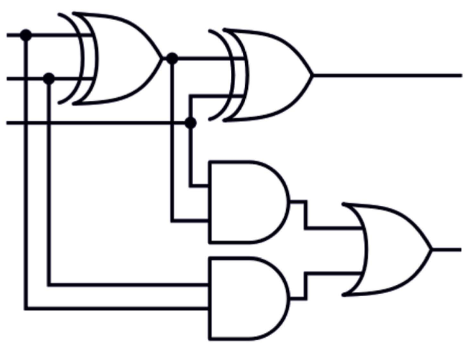
COMPUTER SCIENCE & INFORMATION TECHNOLOGY

Digital Logic

(**Text Book :** Theory with worked out Examples and Practice Questions)



1 Digital Logic

(Solutions for Text Book Practice Questions)

1. Number Systems

01. Ans: (d)

Sol: $135_x + 144_x = 323_x$

$(1\times x^2 + 3 \times x^1 + 5 \times x^0)+(1\times x^2+4\times x^1+ 4\times x^0)$
$= 3x^2 + 2x^1 + 3x^0$

$\Rightarrow x^2+3x+5+x^2+4x+4 = 3x^2+ 2x + 3$

$x^2 - 5x - 6 = 0$

$(x-6)(x+ 1) = 0$ (Base cannot be negative)

Hence x = 6.

(OR)

As per the given number x must be greater than 5. Let us consider x = 6

$(135)_6 = (59)_{10}$

$(144)_6 = (64)_{10}$

$(323)_6 = (123)_{10}$

$(59)_{10} + (64)_{10} = (123)_{10}$

So that x = 6

02. Ans: (a)

Sol: 8-bit representation of

$+127_{10} = 01111111_{(2)}$

1's complement representation of

$-127 = 10000000$.

2's complement representation of

$-127 = 10000001$.

No. of 1's in 2's complement of

$-127 = m = 2$

No. of 1's in 1's complement of

$-127 = n = 1$

$\therefore$ m : n = 2:1

03. Ans: (b) & (d)

Sol: $(14)_{10} = (1110)_2$

+14 = 01110

–14 = 10010

Using sign extension

–14 = 11110010

04. Ans: (c)

Sol: Binary representation of $+(539)_{10}$:

2	539	
2	269	–1
2	134	–1
2	67	–0
2	33	–1
2	16	–1
2	8	–0
2	4	–0
2	2	–0
	1	–0

$(+539)_{10} = (1000011011)_2$
$= (00100\ 0011011)_2$

2's complement $\rightarrow$ 110111100101

Hexadecimal equivalent$\rightarrow (DE5)_H$

05. Ans: 5

Sol: Symbols used in this equation are 0, 1, 2, 3. Hence base or radix can be 4 or higher

$(312)_x = (20)_x\ (13.1)_x$

$3x^2 + 1x +2x^0 = (2x+0)\ (x+3x^0+x^{-1})$

$3x^2+x+2 = (2x)\left(x + 3 + \frac{1}{x}\right)$

$3x^2 + x + 2 = 2x^2 + 6x + 2$

$x^2 - 5x = 0 \Rightarrow x(x-5) = 0$

$x = 0$ (or) $x = 5$

x must be $x > 3$, So x = 5

06. Ans: 3 possible solutions

Sol: $123_5 = x8_y$

$1 \times 5^2 + 2 \times 5^1 + 3 \times 5^0 = x.y^1 + 8 \times y^0$

$25 + 10 + 3 = xy + 8$

$\therefore\ xy = 30$

Possible solutions:

i. $x = 1, y = 30$

ii. $x = 2, y = 15$

iii. $x = 3, y = 10$

3 possible solutions

07. Ans: (b) & (d)

Sol: $[B\ 9\ F.\ AE]_{16}$

$= [1011\ 1001\ 1111\ .\ 1010\ 1110]_2$

Now make 3 bits as single group

Place zero

$$\underbrace{101}_{\downarrow}\ \underbrace{110}_{\downarrow}\ \underbrace{011}_{\downarrow}\ \underbrace{111}_{\downarrow}\ .\ \underbrace{101}_{\downarrow}\ \underbrace{011}_{\downarrow}\ \underbrace{100}_{\downarrow}$$

$$[\ 5\quad 6\quad 3\quad 7\ .\ 5\quad 3\quad 4]_8$$

08. Ans: (c) **09. Ans: (b)**

10. Ans: (b) & (d)

Sol: $[B\quad 9\quad F.\quad A\quad E]_{16}$

$= [1011\ 1001\ 1111.\ 1010\ 1110]_2$

$= [5637.\ 534]_8$

11. Ans: (a), (c) & (d)

Sol: Given Number (N) = 11101

In sign magnitude, MSB is sign and remaining bits are magnitude

$$N = \underbrace{1}_{\text{sign}}\ \underbrace{1101}_{\text{magnitude}}$$

$\Downarrow \qquad \Downarrow$

– [13] So option (A) is correct

and option(B) is wrong

now N = 11101 [Here MSB=1 So Negative number]

2's complement of N = – [0011] = $[-3]_{10}$

So option (C) is also correct

N = 11101 [Her MSB =1 so it is Negative number]

1's complement of N = – [00010] = $(-2)_{10}$

Hence option (D) is also correct

12. Ans: (a)

Sol: Given number $\Rightarrow N = [\ 7\ 07\ 00]_8$

Number of digits = 5.

In 6 digits $N = [0\ 7\ 07\ 00]_8$

Here radix = r = 8

For r's [8] complement, write zero bits as it is at LSB, subtract 1st non zero LSB from "r" [Here 8] and remaining digits from r–1 [7]

$$\begin{array}{lcccccc} & 7 & 7 & 7 & 8 & & \\ N= & 0 & 7 & 0 & 7 & 0 & 0 \\ & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \text{ as it is} \end{array}$$

8's complement $N = [7\ \ 0\ \ 7\ \ 1\ \ 0\ \ \ 0]_8$

13. Ans: (c)

Sol: Given number $\Rightarrow N = (9900)_{10}$

In 5 digits $\Rightarrow N = [0\ 9\ 9\ 0\ 0]_{10}$

For 10's complement write LSB zero bits as it is then subtract 1st non zero bit from "10" & remaining from"9"

$$\begin{array}{lccccc} & 9 & 9 & (10) & & \\ N = & \underline{0} & \underline{9} & \underline{9} & \underline{0} & \underline{0} \end{array}$$

10's complement = 9 0 1 0 0

2. Logic Gates & Boolean Algebra

01. Ans: (b)

Sol: Truth table of XOR

A	B	o/p
0	0	0
0	1	1
1	0	1
1	1	0

Stage 1:

Given one i/p = 1 Always.

1 X o/p
----
1 0 1 = $\overline{X}$
1 1 0 = $\overline{X}$

For First XOR gate o/p = $\overline{X}$

Stage 2:

$\overline{X}$ X o/p
----
0 1 1
1 0 1

For second XOR gate o/p = 1.

Similarly for third XOR gate o/p = $\overline{X}$ & for fourth o/p = 1

For Even number of XOR gates o/p = 1

For 20 XOR gates cascaded o/p = 1.

02. Ans: 1

Sol: $f = \overline{[\overline{D} + A\overline{B} + \overline{A}C + A\overline{C}D + \overline{A}\,\overline{C}D]}$

Let $x = \overline{D} + A\overline{B} + \overline{A}C + A\overline{C}D + \overline{A}\,\overline{C}D$ [then $f = \overline{x}$]

Simplify x ⇒

$x = \overline{D} + A\overline{B} + \overline{A}C + \overline{C}D\underbrace{[A + \overline{A}]}_{1}$

$x = \overline{D} + D\overline{C} + A\overline{B} + \overline{A}C$

$x = [\overline{D} + (\overline{\overline{D}})\overline{C}] + A\overline{B} + \overline{A}C$ [$\because p + \overline{p}q = p + q$]

$x = \overline{D} + \overline{C} + \overline{A}C + A\overline{B}$

$x = \overline{D} + [\overline{C} + (\overline{\overline{C}})\overline{A}] + A\overline{B}$

$x = \overline{D} + \overline{C} + \overline{A} + A\overline{B}$

$x = \overline{D} + \overline{C} + [\overline{A} + (\overline{\overline{A}})\overline{B}]$

$x = \overline{D} + \overline{C} + \overline{A} + \overline{B}$

$x = \overline{A} + \overline{B} + \overline{C} + \overline{D}$

$f = \overline{x} = \overline{\overline{A} + \overline{B} + \overline{C} + \overline{D}} = \overline{\overline{A}}.\overline{\overline{B}}.\overline{\overline{C}}.\overline{\overline{D}} = A.B.C.D$

Number of minterms are "one"

Ans: 1.

03. Ans: (c)

Sol: $f = f_1 f_2 + f_3$

04. Ans: (c)

Sol: Let $x_1 = x_2 = x_3 = x_4$

For all cases options a, b, d not satisfy.

05. Ans: (d) **06. Ans: (b)**

07. Ans: (b) **08. Ans: (c)**

09. Ans: (a) & (d)

Sol: $(x+y)(x+\overline{y}) + \overline{\overline{x} + x\overline{y}}$

using Distributive Law

F= $(x + y.\overline{y}) + (\overline{\overline{x} + \overline{y}}) = x + xy = x$

10. Ans: (b) & (c)

Sol: Let B ⊙ B [Here no. of XOR=1 ⇒odd]

$\overline{B}\,\overline{B} + B.B = \overline{B} + B = 1$

Let B⊙B⊙B [Here no. of XOR=2 ⇒even]

= 1⊙ B = $\overline{1}.\overline{B} + 1B = B$ = remain same as given variable

11. Ans: (a), (b) & (c)

Sol: (a) Consensus theorem rule

$AB+\overline{A}C+BC = AB+\overline{A}C \rightarrow$ Correct

(b) R.H.S $(Y+Z)(X+\overline{Y}) = XY+\overline{Y}Z+XZ$

By using Consensus theorem we can write

$Y.X+\overline{Y}Z+XZ = YX+\overline{Y}Z = L.H.S$

(b) Also correct

(c) $\overline{\overline{A}\,\overline{B}\,\overline{C}} = \overline{\overline{A}}+\overline{\overline{B}}+\overline{\overline{C}} = A+B+C \Rightarrow$ Correct

(d) Simplification of

$(P+\overline{Q})(\overline{Q}+R+\overline{S})(\overline{P}+\overline{Q}+S)$ is $\overline{Q}+PRS$

but given $\overline{Q}+P\overline{R}S$

12. Ans: (a), (b) & (d)

Sol: Given $C = A * B$ but $A * B = AB+\overline{A}\,\overline{B}$

So $C = A*B = AB+\overline{A}\,\overline{B} = A \odot B$ …….. (1)

Now $\overline{C} = \overline{AB+\overline{A}\,\overline{B}} = \overline{A\odot B} = A\oplus B$

$= A\overline{B}+\overline{A}B$ ………(2)

Verification

Option (a) R. H. S $\Rightarrow B * C = BC+\overline{B}\,\overline{C}$

Sub equations (1) & (2)

$B * C = B[AB+\overline{A}B]+\overline{B}[A\overline{B}+\overline{A}B]$

$= AB+0+A\overline{B}+0$

$= A$; hence option (a) is True

Now option (b)

$A * C = \overline{A}\,\overline{C}+AC$

$= \overline{A}[A\overline{B}+\overline{A}B]+A[\overline{A}\,\overline{B}+AB]$

$= 0+\overline{A}B+AB = B[\overline{A}+A] = B$

hence option (b) is True

Now $A * B * C$

$= [A*B]*C$ [But given $A*B=C$]

$= C * C$

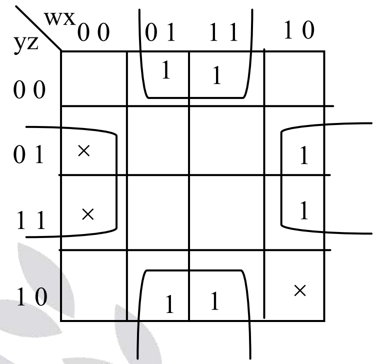
$= \overline{C}\,\overline{C}+C.C = \overline{C}+C = 1$

So option (d) $\Rightarrow A * B * C = 1$ is also True.

3. K-Maps

01. Ans: (b)

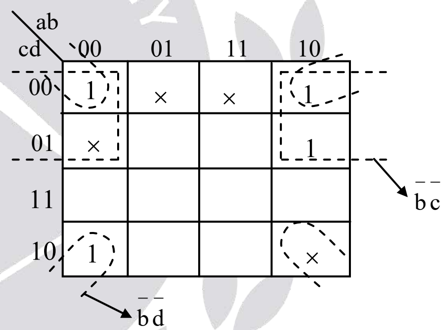
Sol:



$f = \overline{x}z+x\overline{z}$

02. Ans: (b)

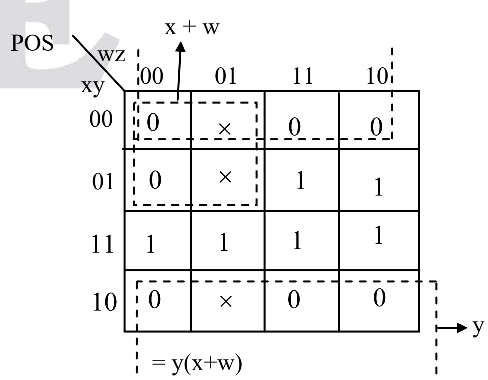
Sol:



$f = \overline{b}\,\overline{d}+\overline{b}\,\overline{c}$

03.

Sol:



SOP

xy \ wz	00	01	11	10
00	0	×	0	0
01	0	×	1	1
11	1	1	1	1
10	0	×	0	0

→ wy

→ xy

= xy+yw

SOP: x y + y w

POS: y(x + w)

04. Ans: (a)

Sol: For n-variable Boolean expression,

Maximum number of minterms = 2^n

Maximum number of implicants = 2^n

Maximum number of

Essential prime implicants $= \dfrac{2^n}{2}$

$= 2^{n-1}$

05. Ans: (c)

Sol:

C \ AB	00	01	11	10
0	1	1	0	0
1	0	1	1	0

F(A, B, C) = $\overline{A}\,\overline{C} + BC$

06. Ans: 3

Sol: $\overline{w}\,\overline{z} + \overline{w}\,x\overline{y} + \overline{x}\,y\overline{z}$

wx \ yz	00	01	11	10
00	1			1
01	1	1		1
11				
10				1

07. Ans: (c)

08. Ans: (a)

09. Ans: (a)

10. Ans: (b) & (d)

Sol: Given f = ΠM [1,3,4,6,9,11,12,14]+d[2,7,8,13]

AB \ CD	00	01	11	10
00		0	0	X
01	0		X	0
11	0	X		0
10	X	0	0	

$\left[\overline{B}+D\right] \Leftarrow$

$\Downarrow$

$\left[B+\overline{D}\right]$

$f_{pos} = \left[B+\overline{D}\right]\left[\overline{B}+D\right]$ hence option (d) is correct

$f_{sop} = \underbrace{B.\overline{B}}_{0} + BD + \overline{B}\,\overline{D} + \underbrace{\overline{D}.D}_{0} = BD + \overline{B}\,\overline{D}$

So option (c) is wrong

$\left.\begin{array}{l} f_{sop} = B.D + \overline{B}\,\overline{D} \\ f_{pos}\left[B+\overline{D}\right]\left[\overline{B}+D\right] \end{array}\right\}$ Both cases variables A, C are absent so Independent of "2" variables

Hence option (b) is correct

4. Combinational Circuits

01. Ans: (d)

Sol: Let the output of first MUX is "F_1"

$F_1 = AI_0 + AI_1$

Where A is selection line, I_0, I_1 = MUX Inputs

$F_1 = \overline{S}_1.W + S_1.\overline{W} = S_1 \oplus W$

Output of second MUX is

$F = \overline{A}.I_0 + A.I_1$

$F = \overline{S}_2.F_1 + S_2.\overline{F}_1$

$F = S_2 \oplus F_1$

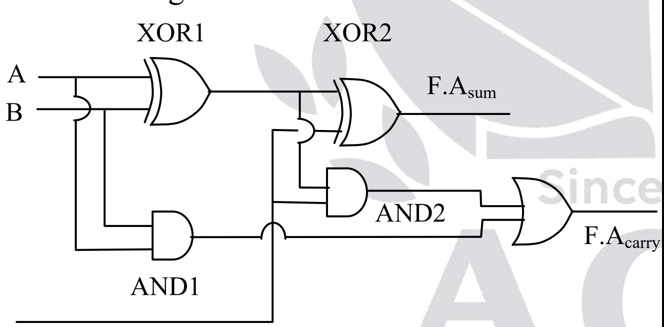
But $F_1 = S_1 \oplus W$

$F = S_2 \oplus S_1 \oplus W$

i.e., $F = W \oplus S_1 \oplus S_2$

02. Ans: 19.2

Sol: F.A using H.A



$t_{AND} = 1.2\mu s$; $t_{OR} = 1.2\mu sec$

XOR = 2×1.2= 2.4 μsec

Time for addition=XOR1 +XOR2 = 2.4+2.4

=4.8μsec

Time for carry = XOR1+AND2+OR

= 2.4+1.2+1.2=4.8μsec

for n-bit total time required

= (n–1) tc+Max[tc, ts]

⇒for 4 bit

(4–1) ×4.8+ Max [4.8, 4.8]

3×4.8+4.8=19.2μsec

03. Ans: 6

Sol: T = 0 → NOR → MUX 1 → MUX 2

2ns 1.5ns 1.5ns

Delay = 2ns + 1.5ns + 1.5ns = 5ns

T = 1 → NOT → MUX 1→NOR→ MUX 2

1ns 1.5ns 2ns 1.5ns

Delay = 1ns + 1.5ns + 2ns + 1.5ns = 6ns

Hence, the maximum delay of the circuit is 6ns

04. Ans: 195

Sol: Given t_{carry}= 12ns

t_{sum}= 15ns

n= 16 bit

Time required = (16–1) × 12 + max(15, 12)

15×12+15=195nsec.

05. Ans: (b), (c) & (d)

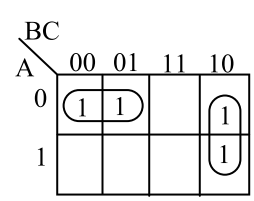
Sol: For "n" bit number addition we require

= "n" F.A (or) 1 H.A, (n–1) F.A (or)

(2n–1) H.A, (n–1) OR Gates

06. Ans: (a) & (c)

Sol: Output =Y= $\sum m(0, 1, 2, 6) + \overline{C}$



$$Y = \overline{A}\,\overline{B} + B\,\overline{C} + \overline{C}$$

$$= \overline{A}\,\overline{B} + \overline{C}[B+1] = \overline{C} + \overline{A}\,\overline{B} \rightarrow \text{Option (c)}$$

$$\Rightarrow Y = \overline{C} + \overline{A}\,\overline{B} = (\overline{C}+\overline{A})(\overline{C}+\overline{B}) = (\overline{A}+\overline{C})(\overline{B}+\overline{C})$$

$\rightarrow$Option (a)

07. Ans: (b)

Sol: In n-bit magnitude comparator the number of combinations for

$$A > B \text{ (or) } A < B = \frac{2^{2n} - 2^n}{2}$$

Here given 8 bit. So, $n = 8$

So total number of combinations for

$$A > B = \frac{2^{2\times 8} - 2^8}{2} = \frac{2^{16} - 2^8}{2} = \frac{2^8.2^8 - 2^8}{2}$$

$$= \frac{2^8.[2^8 - 1]}{2} = 255 \times 2^7.$$

08. Ans: (d)

Sol: As per the definition of MUX, output equation is

$$f = \overline{S_1}\,\overline{S_0}[I_0] + \overline{S_1}S_0[I_1]\ S_1\overline{S_0}[I_2] + S_1S_0[I_3]$$

Here S_1=P; S_0=Q; I_0=0; I_1=1; I_2=R; I_3=$\overline{R}$

$$f = \overline{P}\,\overline{Q}[0] + \overline{P}Q[1] + P\overline{Q}[R] + PQ[\overline{R}]$$

$$f = 0 + \overline{P}Q[\overline{R} + R] + P\overline{Q}R + PQ\overline{R}$$

$$f = \underbrace{\overline{P}Q\overline{R}}_{\substack{0\,1\,0 \\ m_2}} + \underbrace{\overline{P}QR}_{\substack{0\,1\,1 \\ m_3}} + \underbrace{P\overline{Q}R}_{\substack{1\,0\,1 \\ m_5}} + \underbrace{PQ\overline{R}}_{\substack{1\,1\,0 \\ m_6}}$$

$F = \Sigma m = [2, 3, 5, 6]$

09. Ans: (b), (c) & (d)

Sol: For "n" bit number addition we require = "n" F.A (or) 1 H.A, (n–1) F.A (or) (2n–1) H.A, (n–1) OR Gates

10. Ans: (a), (b), (c) & (d)

Sol: Output of MUX1 is $= F_1 = \overline{X}\,I_0 + X.I_1 = \overline{X}(0) + X(1) \Rightarrow F_1 = X$

output of MUX2 $= F_2 = \overline{Y}.I_0 + Y.I_1$

$F_2 = \overline{Y}.X + YX$ [Here $I_1 = F_1 = X$]

$F_2 = X[\overline{Y} + Y] = X$

output of MUX3 $= F = \overline{Z}.I_0 + Z\,I_1$

$F = Y\overline{Z} + XZ$ convert into standard form

$F = (X + \overline{X})Y\overline{Z} + X(Y + \overline{Y})Z$

$F = XYZ + X\overline{Y}Z + XY\overline{Z} + \overline{X}Y\overline{Z}$

$= \Sigma m(2, 5, 6, 7)$

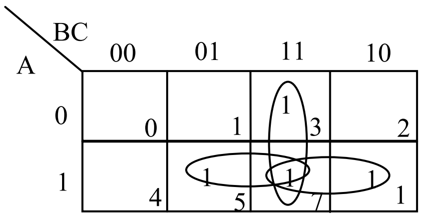
11. Ans: 3

Sol:

Decimal	Inputs A	B	C	Output Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

Output $\Rightarrow$ Y [A, B, C] = Σm [3, 5, 6, 7]

simplify using k = map



$$Y = \underbrace{A\,B}_{1^{st}} + \underbrace{B\,C}_{2^{nd}} + \underbrace{C\,A}_{3^{rd}}$$

Here number of 2 input terms are "3"

5. Sequential Circuits

01. Ans: 4

Sol: In the given first loop of states, zero has repeated 3 times. So, minimum 4 number of Flip-flops are needed.

02. Ans: (b)

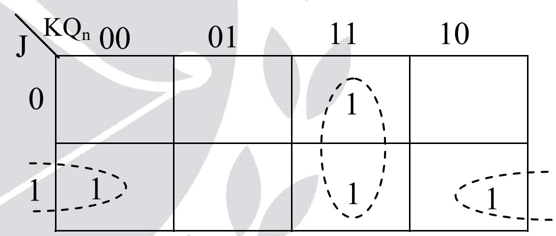
Sol:

CLK	Serial in= $B\oplus C\oplus D$	A B C D
0		1 0 1 0
1	1 →	1 1 0 1
2	0 →	0 1 1 0
3	0 →	0 0 1 1
4	0 →	0 0 0 1
5	1 →	1 0 0 0
6	0 →	0 1 0 0
7	1 →	1 0 1 0

03. Ans: (b)

Sol:

J	K	Q_n	$\overline{Q}_n$	$T = (J+Q_n)(K+\overline{Q}_n)$	Q_{n+1}
0	0	0	1	0.1 = 0	$\left.\begin{matrix}0\\1\end{matrix}\right\} Q_n$
0	0	1	0	1.0 = 0	
0	1	0	1	0.1 = 0	$\left.\begin{matrix}0\\0\end{matrix}\right\} 0$
0	1	1	0	1.1 = 1	
1	0	0	1	1.1 = 1	$\left.\begin{matrix}1\\1\end{matrix}\right\} 1$
1	0	1	0	1.0 = 0	
1	1	0	1	1.1 = 1	$\left.\begin{matrix}1\\0\end{matrix}\right\} \overline{Q}_n$
1	1	1	0	1.1 = 1	



$T = J\,\overline{Q_n} + KQ_n = (J+Q_n)(K+\overline{Q_n})$

04. Ans: (c)

05. Ans: (d)

06. Ans: 3

07. Ans: (c)

Sol: From circuit we can write

$D = X \oplus Q$

but for D FF $\Rightarrow Q_{n+1} = D$

$Q_{n+1} = D = X \oplus Q$

$Q_{n+1} = \overline{X}Q + X\overline{Q}$

[This is same as T FF output]

T FF output $Q_{n+1} = \overline{T}Q + TQ$

Therefore given circuit act as T Flip-Flop

08. Ans: (d)

Sol: $D_A = Q_B \odot Q_C$; $D_B = Q_A$; $D_C = Q_B$

Given initially $Q_A = Q_B = Q_C = 0$

CLOCK	D_A	D_B	D_C	Q_A	Q_B	Q_C
0	-	-	-	0	0	0
1	1	0	0	1	0	0
2	1	1	0	1	1	0
3	0	1	1	0	1	1
4	1	0	1	1	0	1
5	0	1	0	0	1	0
6	0	0	1	0	0	1

The output observed at Q_A is 0110100-----

09. Ans: (b) & (d)

Sol: Total states in 'n' bit Johnson counter = 2^n

number of used states = $2 \times n$

Here n = 4 bits

So, total number of states = $2^4 = 16$

number of used states = $2 \times 4 = 8$

unused states = Total states – used states

Unused states = 16 – 8

= 8