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ESE-2021
(MAINS)

QUESTIONS WITH DETAILED SOLUTIONS

MECHANICAL ENGINEERING

PAPER-II

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MECHANICAL ENGINEERING

ESE _MAINS_2021_PAPER – II

Questions with Detailed Solutions

SUBJECT WISE WEIGHTAGE

S.No.	NAME OF THE SUBJECT	Marks
1	Engineering Mechanics	40
2	Strength of Materials	32
3	Theory of Machines	104
4	Machine Design	64
5	Production Engineering	52
6	Material Science	32
7	Mechatronics & Robotics	92
8	Maintenance Engineering	12
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SECTION – A

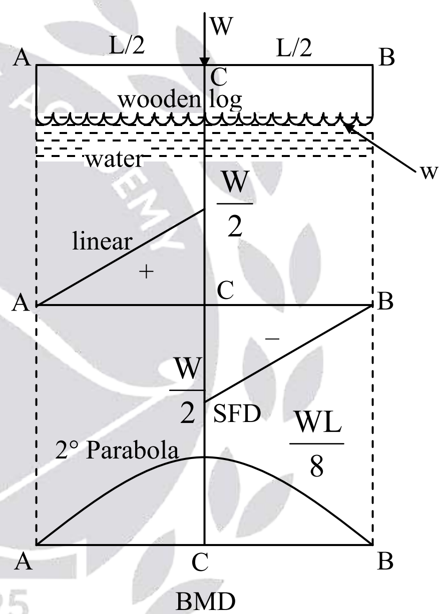
01(a). A light wooden log of length L is floating on water with a concentrated load W acting at the mid-point. Write the equations and draw the diagram for shearing force and bending moment. (12 M)

Sol: Force balance:

$$+\uparrow\ \Sigma F_y = 0$$

$$wL = W$$

$$w = \frac{W}{L}$$



S.F. analysis:

$$S.F_A = 0$$

$$S.F_{C,left} = w \times \frac{L}{2} = \frac{W}{L} \times \frac{L}{2} = \frac{W}{2}$$

$$S.F_{C,right} = \frac{W}{2} - W = -\frac{W}{2}$$

$$S.F_B = 0$$

B.M analysis

$$B.M_A = 0$$

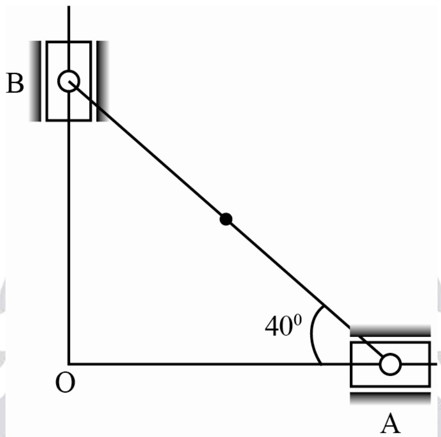
$$B.M_B = 0$$

$$B.M_C = \left(w \times \frac{L}{2}\right)\left(\frac{L}{4}\right)$$

$$= \frac{W}{L} \times \frac{L}{2} \times \frac{L}{4} = \frac{WL}{8}$$

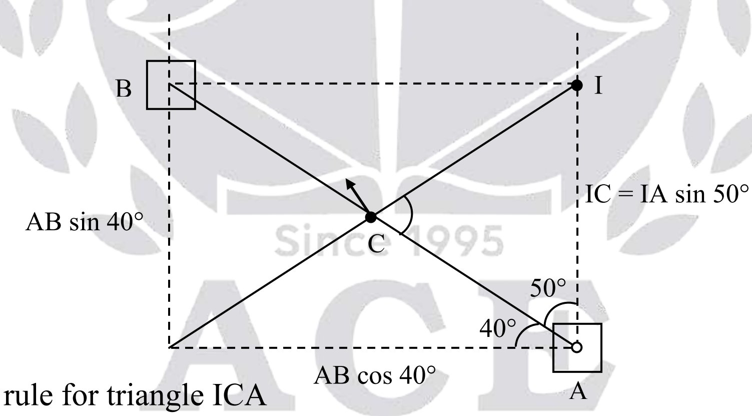
Result: Maximum B.M at C $= \dfrac{WL}{8}$

01(b). The slider A, of an elliptical trammel shown in the figure moves towards O with a velocity of 3 m/s at the instant when AB makes an angle of 40° with the horizontal. Determine the velocity of the mid-point of the link AB at this instant.



(12 M)

Sol: $V_A = 3$ m/sec, $\theta = 40°$, $V_B = ?$



Applying cosine rule for triangle ICA

$$IC^2 = IA^2 + AC^2 - 2 \times IA \times AC \times \cos 50°$$

$$IC^2 = (AB \cos 50°)^2 + \left(\frac{AB}{2}\right)^2 - \left(\frac{2 \times AB^2}{2} \times \cos 50°\right)$$

$$IC = 0.5\ AB$$

$$\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \omega$$

$$\frac{3}{AB\sin 40°} = \frac{V_B}{AB\cos 40°} = \frac{V_C}{0.5\,AB} = \omega$$

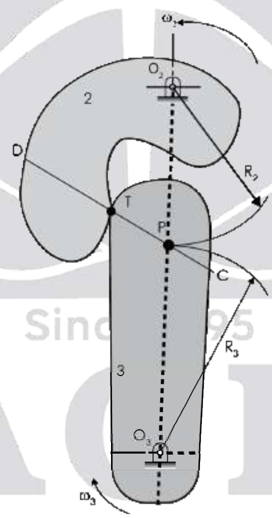
$$\frac{3}{AB\sin 40°} = \frac{V_C}{0.5\,AB} = \omega$$

$\Rightarrow V_C = 2.33$ m/sec

01(c). State and Prove the law of gearing. (12 M)

Sol: Law of gearing:

The law of gearing states that for transmitting constant angular velocity ratio, common normal to the contacting surfaces of mating teeth, at every instantaneous point of contact, must pass through a fixed point on the line centres of the two gears. The fixed point is called the pitch point which divides the line of centres in inverse ratio of the angular velocities of the mating gears.



- From the above diagram, two profiles are in contact at point T; let profile 2 represent the driver and profile 3 represent the driven. The normal to the surfaces CD is called the line of action.
- The normal to the profiles at the point of contact T intersects the line of centers O_2O_3 at the instant center of velocity.

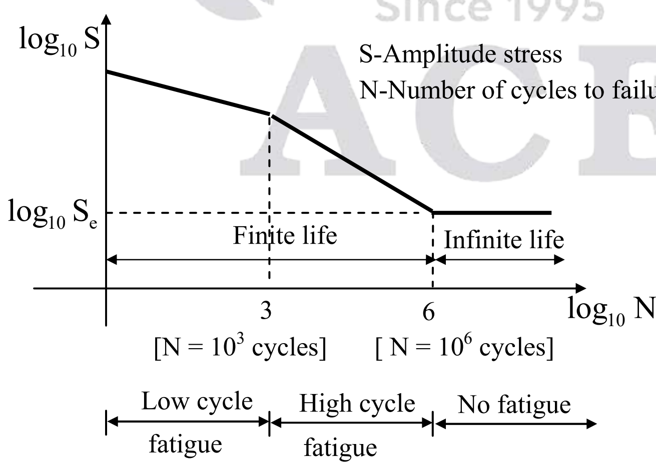
- In gearing, this instant center is generally referred to as the pitch point and usually carries the label P.
- Designating the pitch circle radii of the two gear profiles R_2 and R_3, from the angular velocity ratio theorem, $\frac{\omega_2}{\omega_3} = \frac{R_3}{R_2} = \frac{O_3P}{O_2P}$
- Thus the ratio of angular velocities (ω_2/ω_3) varies inversely as the ratio of distances of point P from centers O_2 and O_3. Clearly, the ratio (ω_2/ω_3) will remain constant as long as the position of point P is fixed along the line O_2O_3.

This leads to following law of gearing.

- In order that a pair of curved surface (tooth profiles) may transmit a constant angular velocity ratio, the shape of contacting tooth profiles must be such that the common normal passes through a fixed point P on the line of centres. The point P divides the line of centres in an inverse proportion as the ratio of angular velocities. The fixed point P is called the pitch point and the line CD (common normal to contacting surfaces) is called the line of action.

01(d). Explain with suitable illustration the S-N curve of a ferrous material and briefly discuss its significance in the design of machine elements. (12 M)

Sol:



- S-N diagram is the graph between amplitude stress and number of cycles to failure under reversed loading. It is obtained from RR-Moore's Test.
- It was identified that the components made of ferrous materials doesn't experience fatigue and does not fail below an amplitude stress of S_e.
- Hence "S_e" plays an important role in the design of components subjected to reversed loading.

Endurance Strength (S_e') :

It is the maximum amplitude stress that the standard specimen used during testing can with stand for a minimum of 10^6 cycles without fatigue failure.

For mild steel

$S_e = 0.5\ S_{ut}$

Let $S = S_3$ when $N = 10^3$ cycles

$\therefore\quad S_3 = 0.9\ S_{ut}$

Corrected Endurance strength (S_e) : It is the maximum amplitude stress that the actual component used in machine can withstand for a minimum of 10^6 cycles without fatigue failure.

$$S_e = K_a \times K_b \times K_c \times K_d \times K_e \times \frac{1}{K_f} \times S_e'$$

Where

K_a = Surface finish factor

K_b = Size factor

K_c = Load factor

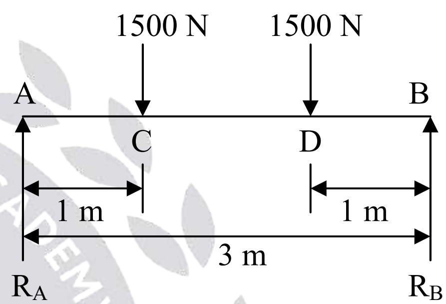
K_d = Temperature factor

K_e = Reliability factor

K_f = Fatigue stress concentration factor.

01(e). A shaft made of mild steel is required to transmit 100 kW at 300 rpm. The supported length of the shaft is 3 metres. It carries two pulleys, each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress to be 60 N/mm², determine the diameter of the shaft. (12 M)

Sol:



Given: P = 100 kW $= 100\times10^3$ W,

N = 300 r.p.m, L = 3 m;

W = 1500 N, $\tau = 60$ N/mm²

We know that the torque transmitted by the shaft.

$$T = \frac{P\times60}{2\pi N} = \frac{100\times10^3\times60}{2\pi\times300} = 3183\ \text{N}-\text{m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. The reaction at each support will be 1500 N. i.e.

$$R_A = R_B = 1500\ \text{N}$$

A little consideration will show that the maximum bending moment lies at each pulley i.e. at C and D.

∴ Maximum bending moment,

$$M = 1500\times1 = 1500\ \text{N}-\text{m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment

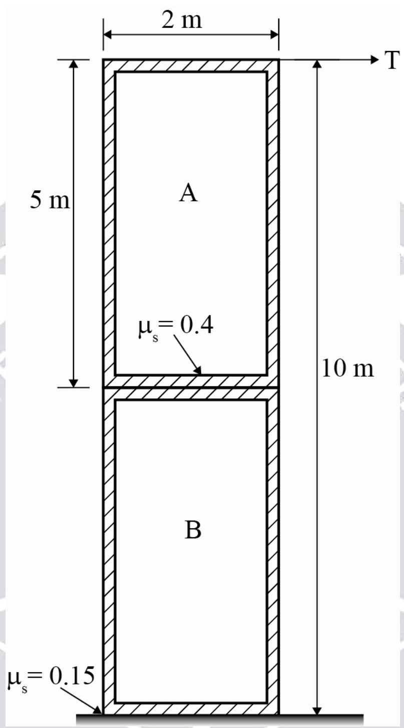
$$T_e = \sqrt{M^2+T^2} = \sqrt{(1500)^2+(3183)^2} = 3519\ \text{N}-\text{m} = 3519\times10^3\ \text{N}-\text{mm}$$

We also know that equivalent twisting moment (T_e),

$$3519\times10^3 = \frac{\pi}{16}\times\tau\times d^3 = \frac{\pi}{16}\times60\times d^3 = 11.8\,d^3$$

$$d^3 = \frac{3519\times10^3}{11.8} = 298\times10^3 \text{ or } d = 66.8 \approx 70 \text{ mm.}$$

02(a). A 500 N crate A rests on a 1000 N crate B. The centres of gravity of the crates are at the geometric centres. The coefficients of static friction between contact surfaces are shown in the diagram. The force T is increased from zero. What is the first action to occur?



(20 M)

Sol:

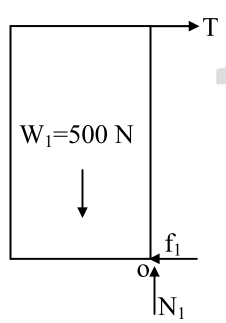


Fig: F.B.D of crate A

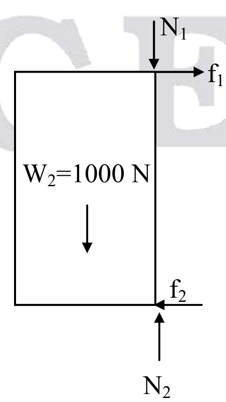


Fig: F.B.D of crate B

Let f_1 be the friction between the two crates and f_2 be the friction between the crate B and surface

$(f_1)_{max} = \mu_s (500)$

$= 0.4 \times 500 = 200$ N

$(f_2)_{max} = \mu_s (1500)$

$= 0.15 \times (1500) = 225$ N

- For slipping to take place between the blocks T = (f_1)max = 200 N
- For slipping to take place between the block and surface the blocks have to start motion.
- For crate A to topple

$$\Sigma M_0 = T \times 5 - W \times 1 = 0$$

$$T = \frac{W}{5} = \frac{500}{5} = 100\text{N}$$

$\because$ This is the least amount of force for which any action is taking place,

Thus for T = 100 N the crate A will topple.

$\therefore$ The first action is the block W_1 impends to topple about its right corner.

02(b). A single cylinder, single acting, 4-stroke engine develops 20 kW at 250 rpm. For the sake of simplicity, the turning moment curve is represented by two isosceles triangles during compression and expansion stroke. The turning moment can be assumed zero for suction and exhaust stroke. The work done by the gases is 3 times more than the work done on the gases. If the flywheel has a mass of 1500 kg and has a radius of gyration of 0.6 m, find the coefficient of fluctuation of speed. (20 M)

Sol: Power = 20 kW

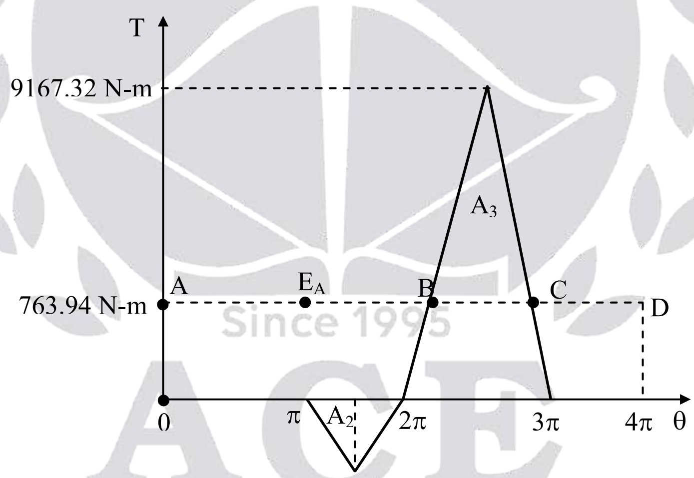
Speed, N = 250 rpm

Mass, m = 1500 kg

Radius of gyration, K = 0.6 m/s

Area of expression stroke = 3 × Area of compression stroke

$A_3 = 3A_2$



Work done per cycle = $\int T d\theta$

$$= A_3 - A_2 = 3A_2 - A_2 = 2A_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Power, $$P = \frac{2\pi NT}{60000}$$

$$20 = \frac{2 \times \pi \times 250 \times T_{mean}}{60000}$$

$$T_{mean} = 763.94 \text{ N-m}$$

Work done per cycle = $T_{mean} \times \theta_{cycle} = 763.94 \times 4\pi = 9600$ N-m …………….. (2)

Eq.(1) = Eq.(2)

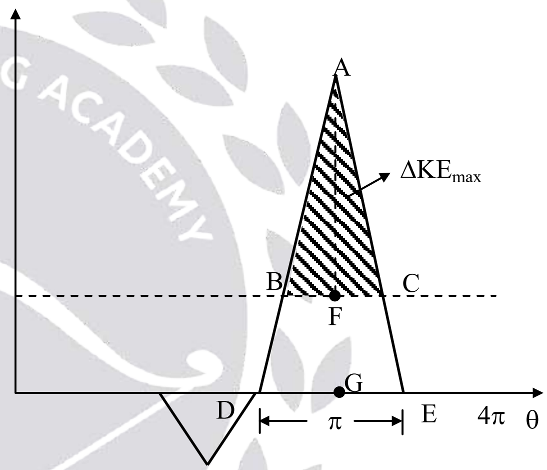
$$2A_2 = 9600$$

$$A_2 = 4800 \text{ N-m}$$

$$A_3 = 3A_2 = 3 \times 4800 = 14400 \text{ N-m}$$

Area of expansion stroke = $\frac{1}{2} \times T_{max} \times \pi = 14400t$

$$T_{max} = 9167.32 \text{ N-m}$$



Fluctuation of energy = Area of ΔABC

From similar triangles

$$\frac{AF}{AG} = \frac{BC}{DE}$$

$$\frac{(9167.32 - 763.94)}{9167.32} = \frac{x}{\pi}$$

x = 2.879 radians

$$\Delta KE_{max} = \frac{1}{2} \times (T_{max} - T_{mean}) \times x$$

$$= \frac{1}{2} \times (9167.32 - 763.94) \times 2.879 \; = 12100.003 \text{ N-m}$$

Coefficient of fluctuation of speed

$$\Delta KE_{max} = I\omega^2 . C_S$$

$$12100 = 1500 \times 0.6^2 \times \left(\frac{2\pi \times 250}{60}\right)^2 C_S$$

$C_S = 0.032 = 3.2\%$

$= \pm\ 1.6\%$

02(c). A single-plate clutch is used to rotate a machine from a shaft rotating at a uniform speed of 300 rpm. Both sides of the clutch are effective, friction lining is of 140 mm inner diameter and 220 mm outer diameter, respectively. The coefficient of friction between friction lining and flywheel surface is 0.28. Assuming uniform wear theory for clutch and maximum intensity of pressure 0.1 MPa, determine the time required to attain full speed by the machine if moment of inertia of the rotating parts is 7.2 kg-m^2. How much energy has been lost during slipping of the clutch? (20 M)

Sol: Given data

Inner diameter, $d_i = 140$ mm, $r_i = 70$ mm

Outer diameter, $d_0 = 220$ mm, $r_0 = 110$ mm

Number of contacting surfaces, n = 2

Coefficient of friction, $(\mu) = 0.28$

Max pressure, $(p_{max}) = 0.1$ MPa

Moment of inertia, (I) = 7.2 kg.m^2

Torque Transmitted according to uniform wear theory

$$T = 2\times\mu\times\pi\times p_{max}\times r_i\,(r_0^2 - r_i^2)$$

$$= 2\times 0.28\times\pi\times 0.1\times 70\,[110^2 - 70^2]$$

$$= 88.67\times 10^3 \text{ N.mm} = 88.67 \text{ N.m}$$

Torque Transmitted, $T = I.\alpha$

$88.67 = 7.2 \times \alpha$

$\alpha = 12.3152$ rad/s

$\omega_2 = \omega_1 + \alpha t$

$\omega_1 = 0$

$\omega_2 = \alpha\, t$

$$\frac{2\pi\times 300}{60} = 12.3152\times t$$

t = 2.5509 sec

$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$$\left(\frac{2\pi\times300}{60}\right)^2 = 2\times12.3152\times\theta$$

$\theta = 40.070$ rad

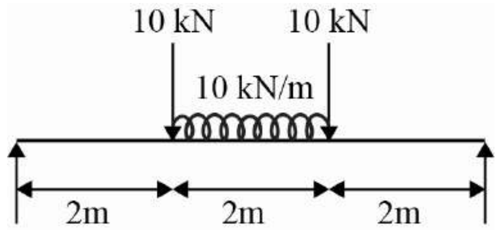
Energy loss = $T \times \theta = 3553.08$ N-m

Time required to attain full speed, $t = 0.784$ sec

Energy lost during slipping

$$E_{lost} = \frac{1}{2}I\left[\omega_2^2 - \omega_1^2\right] = \frac{1}{2}\times7.2\left[\left(\frac{2\pi\times300}{60}\right)^2 - 0^2\right] = 3553\text{ W} = 3.53\text{ kW}$$

03(a). A beam is loaded as shown in the figure.

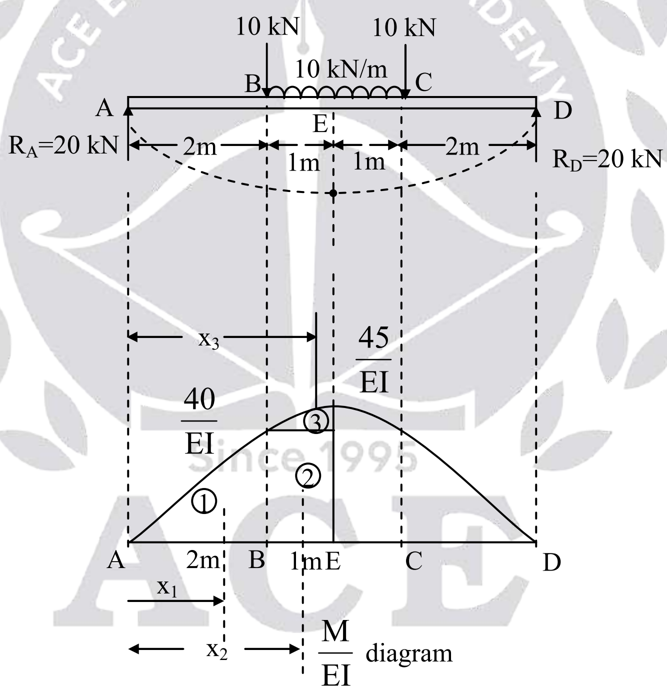


If E = $200 \times 10^6\ kN/m^2$ **and I =** $10^{-4}\ m^4$**, determine**

(i) slope at the ends, and

(ii) maximum deflection of the beam **(20 M)**

Sol:



Due to symmetry

$$R_A = R_D = \frac{\text{Total load}}{2} = \frac{10+10+10\times 2}{2} = 20\,\text{kN}$$

$B.M_A = B.M_D = 0$

$B.M_B = R_A \times 2 = 20 \times 2 = 40$ kN-m

$B.M_C = R_D \times 2 = 20 \times 2 = 40$ kN-m

$B.M_E = 20 \times 3 - 10 \times 1 - (10 \times 1 \times 0.5)$

$= 60 - 10 - 5$

$= 45$ kN-m

Moment Area method: Slope at ends:

$$\theta_A - \theta_E^0 = \frac{A_{[BMD]_E^A}}{EI}$$

$$= \frac{1}{EI}\left[A_1 + A_2 + A_3\right]$$

$$= \frac{1}{200\times10^6\times10^{-4}}\left[\frac{1}{2}\times2\times40+1\times40+\frac{2}{3}\times1\times5\right]$$

$$\theta_A = \frac{83.33}{200\times10^2} = 0.00416 \text{ radians}$$

$\therefore$ Due to symmetry $\theta_A = \theta_D = 0.00416$ radians

Maximum deflection at centre:

$$t_{A/E} = y_C = \frac{A_{[BMD]_E^A}}{EI}.\bar{x}_A$$

$$= \frac{1}{EI}\left[A_1x_1 + A_2x_2 + A_3x_3\right]$$

$$= \frac{1}{200\times10^6\times10^{-4}}\left[\left(\frac{1}{2}\times2\times40\right)\left(\frac{2}{3}\times2\right)+(1\times40)(2+0.5)+\left(\frac{2}{3}\times1\times5\right)\left(2+\frac{5}{8}\times1\right)\right]$$

$$= \frac{1}{200\times10^2}\left[53.33+100+8.75\right]=\frac{162.08}{200\times10^2}$$

$= 0.008104$ m

$\therefore y_E = 0.008104$ m, $y_E = 8.104$ mm

Result: (i) Slope at ends = 0.00416 radian

(ii) Maximum deflection = 0.008104 m

= 8.104 mm

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03(b). A shaft rotates in two bearings A and B, 1.8 m apart and projects 0.45 m beyond A and B. At the extreme ends of the shaft, two pulleys of masses 20 kg and 48 kg are attached with eccentricity of 12.5 mm and 15.5 mm respectively. In the middle of the bearings, another pulley of mass 50 kg is attached with an eccentricity of 15.5 mm.

If the three pulleys have been arranged so as to obtain static balance, determine the dynamic forces produced on the bearings when shaft speed is 300 rpm. (20 M)

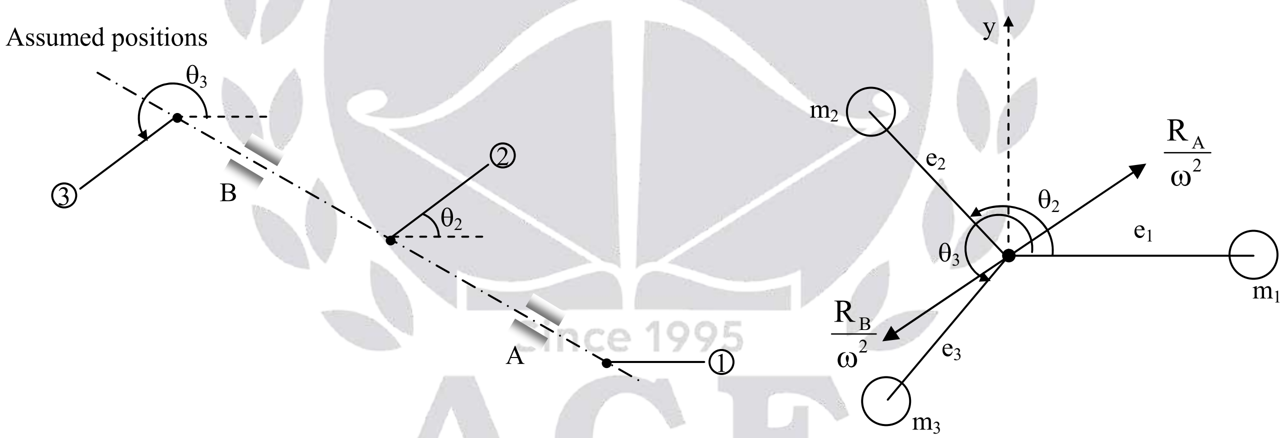
Sol: Given:

$m_1 = 20$ kg; $e_1 = 12.5$ mm; $N = 300$ rpm

$m_2 = 50$ kg; $e_2 = 15.5$ mm

$m_3 = 48$ kg; $e_3 = 15.5$ mm

As system is statistically balance reaction at supports will be equal and opposite.



For system to be statically balance the following condition must be satisfied $\sum F_x = 0$ and $\sum F_y = 0$

$\sum F_x = 0$

$m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 = 0$

$(20 \times 12.5 \cos 0°) + (50 \times 15.5 \cos\theta_2) + (48 \times 15.5 \cos\theta_3) = 0$

$775 \cos\theta_2 + 744 \cos\theta_3 = -250$

$744 \cos\theta_3 = -250 - 775 \cos\theta_2$ ……… (1)

$\sum F_y = 0$

$m_1r_1\sin\theta_1 + m_2r_2\sin\theta_2 + m_3r_3\sin\theta_3 = 0$

$(20 \times 12.5\sin 0°) + (50 \times 15.5\sin\theta_2) + (48 \times 15.5\sin\theta_3) = 0$

$775\sin\theta_2 + 744\sin\theta_3 = 0$

$744\sin\theta_3 = -775\sin\theta_2$ ……… (2)

$\text{Eq.}(1)^2 + \text{Eq.}(2)^2 = 1$

$(744\sin\theta_3)^2 + (744\cos\theta_3)^2 = (-250 - 775\cos\theta_2)^2 + (-775\sin\theta_2)^2$

$744^2 = 775^2[\sin^2\theta_2 + \cos^2\theta_3] + 250^2 + 2(250 \times 775 \times \cos\theta_2)$

$\theta_2 = 106.428°$

$744\cos\theta_3 = -250 - 775\cos 106.428°$

$\cos\theta_3 = -0.0414$

$744\sin\theta_3 = -775\sin 106.428°$

$\sin\theta_3 = -0.9991$

$\therefore$ Position of mass (m_3) will be in 3rd quadrant

$$\tan\theta_3 = \frac{-0.9991}{-0.0414}$$

$\theta_3 = 267.62°$

Taking moment about m_1 and considering only horizontal forces

$\sum M_{1x} = 0$

$$\left(m_2e_2\omega^2\cos\theta_2 \times 1.35\right) + \left(R_{B_x} \times 1.8\right) + \left(m_3e_3\omega^2\cos\theta_3 \times 2.7\right) = 0$$

$$(50\times15.5\times\cos106.428°\times1.35) + (48\times15.5\times\cos267.62\times2.7)\times\omega^2 = -\left(R_{B_x} \times 1.8\right)$$

$$\omega = \left(\frac{2\pi\times300}{60}\right) = 31.41 \text{ rad/sec}$$

$R_{B_x} = -207.768\text{N}$

Taking moment about m_1 and considering only vertical forces

$\sum M_{1y} = 0$

$(m_2 e_2 \omega^2 \sin\theta_2 \times 1.35) + (R_{By} \times 1.8)\ (m_3 e3 \omega^2 \sin\theta_3 \times 2.7) = 0$

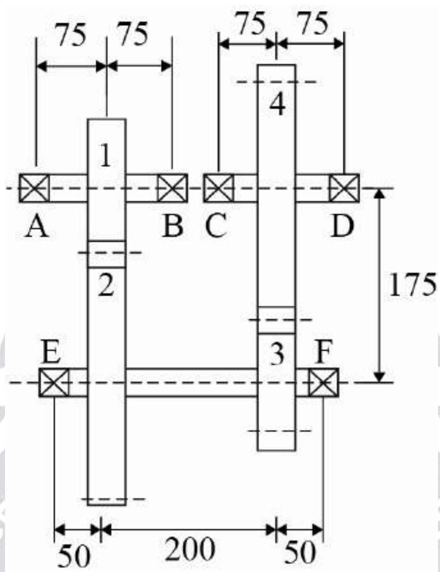
$(50 \times 15.5 \times \sin 106.428 \times 1.35)\omega^2 + (R_{By} \times 1.8) + (48 \times 15.5 \times \sin 267.62 \times 2.7)\omega^2 = 0$

$R_{B_y} = -549.68\text{N}$

$R_B = \sqrt{(R_{Bx})^2 + (R_{By})^2} = \sqrt{(-207.768)^2 + (-549.68)^2} = 587.64\ \text{N}$

Dynamic process produced on the bearings = $R_A = R_B = 587.64$ N

03(c). The layout of a two-stage gear box is shown in the figure.



The number of teeth on the gears are as follows:

$z_1 = 20$ $z_2 = 50$ $z_3 = 20$ $z_4 = 50$

Pinion 1 rotates at 1440 rpm is anti-clockwise direction when observed from the left side and transmits 10 kW power to the gear train. The pressure angle is 20°. Draw a free body diagram of the gear tooth forces and determine the reactions at bearings E and F.

(20 M)

Sol: Given, kW = 10, $n = 1440$ rpm , $\alpha = 20°$

$z_1 = 20$, $z_2 = 50$, $z_3 = 20$, $z_4 = 50$, $a = 175$ mm

Step I Free-body diagram of forces

$$m = \frac{2a}{(z_1 + z_2)} = \frac{2(175)}{(20+50)} = 5 \text{ mm}$$

$d_1' = m\, z_1 = 5(20) = 100$ mm

$d_2' = m\, z_2 = 5(50) = 250$ mm

The forces between gears 1 and 2 are calculated in the following way

$$(M_t)_1 = \frac{60\times10^6(\text{kW})}{2\pi n_1} = \frac{60\times10^6(10)}{2\pi(1440)} = 66314.56 \text{ N-mm}$$

$$P_t = \frac{2(M_t)_1}{d_1'} = \frac{2(66314.56)}{100} = 1326.29 \text{ N}$$

$$P_r = P_t \tan\alpha = 1325.29 \tan(20) = 482.73 \text{ N}$$

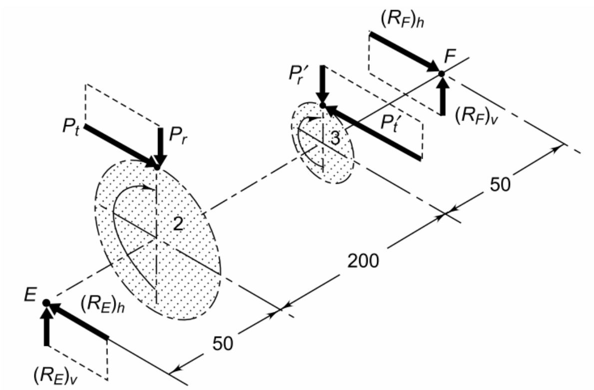
The corresponding forces between gears 3 and 4 are denoted by P_t' and P_r'. Since,

$$P_t \times \frac{d_2'}{2} = P_t' \times \frac{d_3'}{2}$$

$$\therefore\ P_t' = P_t\left(\frac{d_2'}{d_3'}\right) = 1326.29\times\left(\frac{250}{100}\right) = 3315.73 \text{ N}$$

$$P_r' = P_t' \tan\alpha = 3315.73 \tan(20) = 1206.83 \text{ N}$$

The free-body diagram of forces acting on the shaft EF is shown in Fig(i). The pinion -1 is rotating in anti-clockwise direction when observed from the left side of the shaft. Therefore, the gear-2 and the shaft EF are rotating in clockwise direction. Power is transmitted from the gear-1 to the gear-2, and then from gear-3 to the gear-4. Therefore, the gear-2 is driven gear and the gear-3 is the driving gear. The direction of tangential component for the driven gear is the same as that of rotation. Therefore, at the point of contact on the gear-2, the tangential component will act towards the lower right-hand corner. The direction of tangential component for driving gear is opposite to that of rotation. Therefore, at the point of contact on the gear-3, the tangential component will act towards the upper left-hand corner. The radial components will act towards the centres of respective gears.



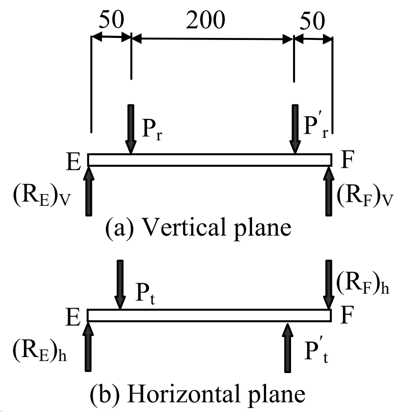


Fig. (i) ***Fig. (ii)***

Step II Reactions at bearings E and F

The forces acting in vertical and horizontal planes are shown in Fig (ii).

Considering vertical forces and taking moments about the bearing E,

$$P_r \times 50 + P'_r \times 250 = (R_F)_v \times 300$$

$$482.73 \times 50 + 1206.83 \times 250 = (R_F)_v \times 300$$

$\therefore$ $(R_F)_V = 1086.15$ N ………. (i)

$$P_r + P'_r = (R_F)_V + (R_E)_V$$

$$482.73 + 1206.83 = 1086.15 + (R_F)_V$$

$\therefore$ $(R_F)_V = 603.41$ N ………… (ii)

Considering horizontal forces and taking moments about the bearing E,

$$P'_t \times 250 - P_t \times 50 = (R_F)_h \times 300$$

$$3315.73 \times 250 - 1326.29 \times 50 = (R_F)_h \times 300$$

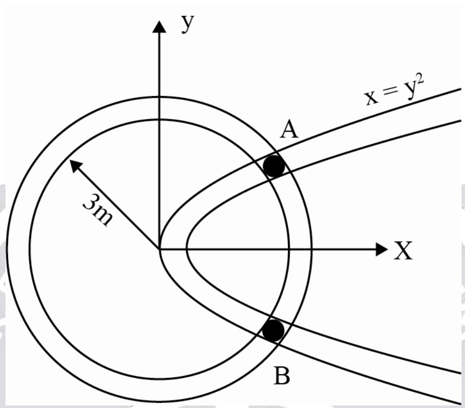
$\therefore$ $(R_F)_h = 2542.06$ N ………. (iii)

$$(R_F)_h + P_t = (R_E)_h + P'_t$$

$$2542.06 + 1326.29 = (R_E)_h + 3315.73$$

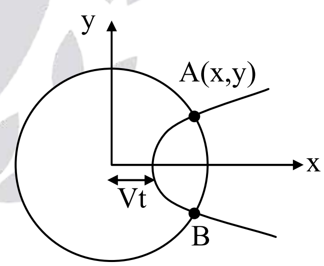
$\therefore$ $(R_E)_h = 552.62$ N ………. (iv)

04(a). Particles A and B are confined to always be in a circular groove of radius 3 m. At the same time, these particles must also be in a slot that has the shape of a parabola. The slot is shown at time t = 0 with equation $x = y^2$. If the slot moves to the right at a constant speed of 1.5 m/s, what are the speed and rate of change of speed of the particles towards each other at t = 1 s? (20 M)



Sol: Equation of slot at any time t,

$$\mathbf{x - Vt = y^2} \quad \ldots\ldots\ldots\ldots\ldots (A)$$



After t = 1 sec, equation of slot will be

$$x - 1.5\times1 = y^2$$

$$\Rightarrow \quad x - 1.5 = y^2 \quad \ldots\ldots\ldots(i)$$

Also, $x^2 + y^2 = 3^2$ ....(ii) [As particle lies on circle]

From (i) and (ii)

$$x = 2.78 \quad \text{and} \quad y = 1.13 \quad \text{(For particle A)}$$

Differentiating equation (ii)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$xv_x + yv_y = 0 \quad \ldots\ldots\ldots(iii)$$

Differentiating equation (A)

$$\frac{dx}{dt} - V = 2y\frac{dy}{dt}$$

$v_x - 1.5 = 2y\, v_y$………(iv)

Solving (iii) and (iv) using x = 2.78 and y = 1.13

we get, $V_x = 0.229$ m/s,

$V_y = -0.56$ m/s

[Negative sign shows that particle A is moving towards negative y direction]

$$V = \sqrt{V_x^2 + V_y^2} = 0.605\,\text{m/s}$$

The speed of both the particle is 0.605 m/s.

Their relative speed towards each other, $v_{rel} = 2v_y = 2\times 0.56 = 1.12$ m/s

Differentiating equation (iii) w.r.t time

$$x\frac{dV_x}{dt} + V_x^2 + y\frac{dV_y}{dt} + V_y^2 = 0$$

$2.78a_x + 0.052 + 1.13\, a_y + 0.3136 = 0$

$2.78a_x + 1.13\, a_y = -0.3656$ …………(v)

Differentiating equation (iv) w.r.t time

$a_x = 2\,[\,ya_y + v_y^2]$

$a_x - 2.26\,a_y = 0.6272$ …………(vi)

Solving (v) and (vi)

$a_x = -0.016\,\text{m/s}^2$,

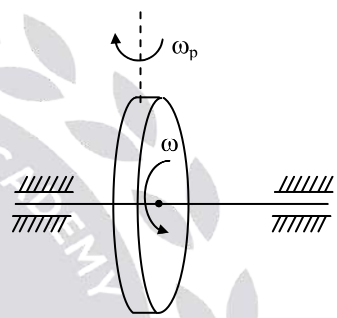
$a_y = -0.28\,\text{m/s}^2$

$$a_{net} = \sqrt{a_x^2 + a_y^2} = 0.28\,\text{m/s}^2$$

The rate of change of speed towards each other $= 2\times a_y = 0.56\,\text{m/s}^2$

04(b). A uniform disc of radius of gyration 60 mm and a mass of 4 kg is mounted centrally on a horizontal axle of 80 mm length between the bearings. It spins at 800 rpm CCW when viewed from the right side bearing. The axle precesses about the vertical axis at 50 rpm in the clockwise direction when viewed from the top. Determine the reaction at each bearing due to the mass and gyroscopic effect. (20 M)

Sol: Radium of gyration (k) = 60 mm



Mass (m) = 4 kg

N = 800 rpm

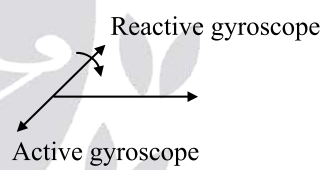
N_p = 50 rpm

Moment of Inertia = m k^2

$$= 4 \times (0.06)^2$$

$$= 0.0144 \text{ kg- m}^2$$

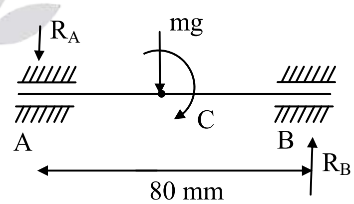
Gyroscopic couple = I. $\omega\omega_p$



$$= 0.0144 \times \left(\frac{2\pi \times 800}{60}\right) \times \left(\frac{2\pi \times 50}{60}\right)$$

$$= 6.316 \text{ N-m}$$

Reactions on bearing due to gyroscope



$$R_A = \frac{C}{\ell} = \frac{6.316}{0.08} = 78.95 \text{ N } (\downarrow)$$

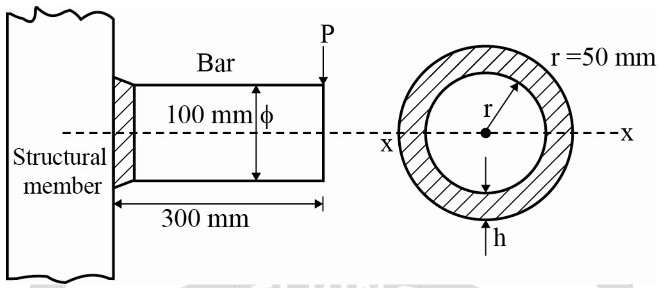
R_B = 78.95 N ($\uparrow$)

Reactions due to weight, $R_A = R_B = \frac{Mg}{2} = \frac{4 \times 9.81}{2} = 19.62$ N

Resultant reactions on A = – 78.95 + 19.62 = – 59.33 N

Resultant reactions on B = 78.95 + 19.62 = 98.57 N

04(c). A solid circular bar of 100 mm diameter is welded to a structural member, as shown in the figure, by a fillet weld all round the bar. Determine the leg dimension of the fillet weld if P = 16 kN and permissible shear stress in weld is 90 N/mm^2.



(20 M)

Sol: Given data

$\tau_{Permissible} = 90$ MPa,

$P = 16$ kN

$t_{min} = 0.707\ h$

Maximum bending moment $(M_{max}) = P \times l$

$$M_{max} = 16 \times 10^3 \times 300 \text{ N.mm}$$

$$= 4.8 \times 10^6 \text{ N.mm}$$

Area moment of inertia, $I = \pi r^3.t_{min}$

Maximum Bending stress $\sigma = \dfrac{M_{max}}{I} y_{max}$

$$= \frac{4.8 \times 10^6}{\pi.r^3.t_{min}}.r$$

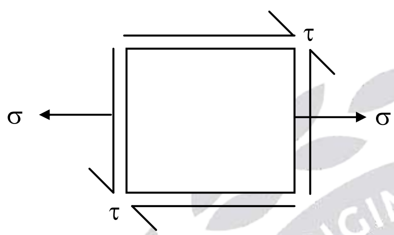
$$\sigma = \frac{4.8 \times 10^6}{\pi \times r^2.t_{min}} = \frac{0.611 \times 10^3}{t_{min}}$$

Maximum shear force, $V_{max} = 16$ kN

Maximum shear stress, $\tau = \dfrac{V_{max}}{A}$

$$\tau = \frac{16 \times 10^3}{2\pi r.t_{min}} = \frac{0.051 \times 10^3}{t_{min}}$$

State of stress of critical particle :



$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{10^3}{t_{min}}\left[\sqrt{\left(\frac{0.611}{2}\right)^2 + 0.051^2}\right]$$

Design condition :

$\tau_{max} = \tau_{permissible}$

$$\frac{10^3}{t_{min}}\sqrt{\left(\frac{0.611}{2}\right)^2 + 0.051^2} = 90$$

$t_{min} = 3.44$ mm

$t_{min} = \dfrac{h}{0.707}$

$h = 4.867$ mm

Size of weld

$\therefore h \simeq 5$ mm

SECTION – B

05(a). A slab of aluminium of dimensions 25 cm × 20 cm × 5 cm is to be cast along with a side cylindrical riser. The riser is not insulated on any surface. If the volume shrinkage of aluminium during solidification is 5%, find

(i) the relationship between diameter and height of cylindrical riser for longest solidification time.

(ii) the minimum volume of riser required to compensate the shrinkage volume of casting. (Assume volume of riser = 3 × shrinkage volume of casting) (12 M)

Sol:

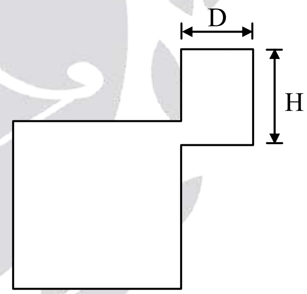
(i) For Side Riser

Here both top and bottom surfaces are exposed to heat transfer

$$V = \frac{\pi}{4}D^2H \quad \text{-----------(1)}$$

$$H = \frac{4V}{\pi D^2} \quad \text{-----------(2)}$$

$$A_S = \pi DH + \frac{\pi}{4}\times D^2 \times 2 \quad \text{-----------(3)}$$



As $t = k\left(\frac{V}{A}\right)^2$

$\therefore$ t can be maximized by minimizing A.

$$\frac{dA}{dD} = 0$$

Substituting eq.(2) in eq.(3)

$$A_S = \frac{\pi}{2}D^2 + \pi D \times \frac{4V}{\pi D^2} = \frac{\pi}{2}\times D^2 + \frac{4V}{D}$$

$$\frac{dA}{dD} = 0$$

$$\frac{\pi}{2}\times 2D + 4V\left(\frac{-1}{D^2}\right) = 0$$

$$= \pi D - \frac{4}{D^2}\times\frac{\pi}{4}D^2H \qquad \text{from (1)}$$

$\Rightarrow \pi D - \pi H = 0$

$\therefore D = H$

(ii) Given: Volume of casting (V_c) = $25\times 20\times 5 = 2500\ \text{cm}^3$

Shrinkage volume $(V_s) = \frac{5}{100}\times 2500 = 125\ \text{cm}^3$

Volume of riser $(V_r) = 3\times V_s = 3\times 125 = 375\ \text{cm}^3$

$$\frac{\pi}{4}D^2H = 375$$

$$\frac{\pi}{4}D^3 = 375 \qquad [\text{as } H = D]$$

$$D^3 = \frac{4\times 375}{\pi}$$

$D = 7.815$ cm

$$M_r = \frac{D}{6} = 1.3\ \text{cm}$$

Case (i): $M_c = \left(\frac{V}{A}\right)_c = \frac{2500}{2(25\times 20 + 25\times 5 + 20\times 5)} = 1.72$

$$\frac{D}{6} \geq M_c$$

$$\frac{7.815}{6} \geq 1.72$$

$1.30 \geq 1.72$ (Not satisfied)

Case (ii): $\frac{D}{6} = M_c$

$\frac{D'}{6} = 1.72 \Rightarrow D' = 10.32$ cm

For side riser, $D' = H' = 10.32$ cm

05(b). To minimize the total processing time, arrange the sequence of operations of the jobs on the following five machines using Johnson's rule. The processing times (in minutes) of different jobs on individual machines are given below. Also find the total idle time on each machine.

Jobs	M/C 1	M/C 2	M/C 3	M/C 4	M/C 5
A	10	8	5	6	12
B	9	9	7	8	13
C	8	7	8	9	11
D	11	6	6	5	9

(12 M)

Sol: Max $\{t_{2j}\}$ (or) max$\{t_{3j}\}$ (or) max $\{t_{4j}\} \le$ min$\{t_{1j}\}$ (or) min$\{t_{5j}\}$

9 (or) 8 (or) 9 $\le$ 8 (or) 9

Condition is satisfied, hence we create two virtual machines (Machine G and Machine H)

Job	m/c G ($m_1+m_2+m_3+m_4$)	m/c H ($m_2+m_3+m_4+m_5$)
A	10+8+5+6 = 29	8+5+6+12=31
B	9+9+7+8 = 33	9+7+8+13 = 37
C	8+7+8+9 = 32	7+8+9+11 = 35
D	11+6+6+5 = 28	6+6+5+9 = 26

Machine G Machine H

A	C	B	D

Job	Machine 1			Machine 2			Machine 3			Machine 4			Machine 5		
	In	PT	Out	In	PT	Out	In	PT	Out	In	PT	Out	In	PT	Out
A	0	10	10	10	8	18	18	5	23	23	6	29	29	12	41
C	10	8	18	18	7	25	25	8	33	33	9	42	42	11	53
B	18	9	27	27	9	36	36	7	43	43	8	51	53	13	66
D	27	11	**(38)**	38	6	**(44)**	44	6	**(50)**	51	5	**(56)**	66	9	**(75)**

Minimum make-span (or) Minimum total processing time = 75

Idle time calculation:

Idle time of $m_1 = 38 - [10+8+9+11] = 0$

Idle time of $m_2 = 44 - [8+7+9+6] = 14$ min

Idle time of $m_3 = 50 - [5+8+7+6] = 24$ min

Idle time of $m_4 = 56 - [6 + 9 + 8 + 5] = 28$ min

Idle time of $m_5 = 75 - [12 + 11 + 13 + 9] = 30$ min

05(c). What are the basic types of corrosion? Briefly describe any one of them. (12 M)

Sol: The basic types of corrosion:

- Uniform corrosion.
- Galvanic corrosion, concentration cells, water line attack
- Pitting.
- Dezincification, Dealloying (selective leaching)
- Atmospheric corrosion.
- Erosion corrosion
- Fretting
- Crevice corrosion, cavitation
- Stress corrosion, intergranular and transgranular corrosion, hydrogen cracking and embrittlement
- Corrosion fatigue.

Stress corrosion cracking (SCC) refers to failure under simultaneous presence of a corrosive medium and tensile stress. Two classic examples of SCC are caustic embrittlement of steels occurring in riveted boilers of steam-driven locomotives and season cracking of brasses observed in brass cartridge cases due to ammonia in environment. Stress cracking of different alloys does occur depending on the type of corrosive environment. Stainless steels crack in chloride atmosphere. Major variables influencing SCC include solution composition, metal/alloy composition and structure, stress and temperature. Crack morphology for SCC failures consists of brittle fracture and inter (or) trans-granular cracking could be observed. Higher stresses decrease time before crack initiation. Tensile stresses of sufficient threshold levels are involved (applied, residual or thermal stresses).

05(d). For an electronic component, the failure density function is defined as

$$f(t) = \begin{cases} 0.002e^{-0.002t} & t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the reliability of the component at 346.6 hours. Also determine the MTTF of the same component. (12 M)

Sol: Failure density function

$$f(t) = \begin{cases} 0.002e^{-0.002t} & t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Reliability} = R(t) = \int_t^\infty f(t)dt = \int_t^\infty 0.002e^{-0.002t}dt$$

$$= \left[\frac{0.002e^{-0.002t}}{-0.002}\right]_t^\infty = \left[-e^{-0.002t}\right]_t^\infty$$

$$= -\left[0 - e^{-0.002t}\right] = e^{-0.002t}$$

Reliability of the component at 346.6 hours is

$$R(346.6) = e^{-0.002(346.6)} = 0.49997 \approx 0.5$$

Mean time to failure = MTTF $= \int_0^\infty R(t)dt$

$$\int_0^\infty e^{-0.002t}dt = \left.\frac{e^{-0.002t}}{-0.002}\right]_0^\infty$$

$$= \frac{-1}{0.002}[0-1] = \frac{1}{0.002} = 500$$

05(e). What kind of motors are used in paper-feed and print-head-positioning motors in printers and plotters? Justify your answer(s) with reason. Show a schematic diagram of a two-phase permanent magnet stepper motor. (12 M)

Sol: The stepper motor is a special type of synchronous motor which is designed to rotate through a specific angle (called a step) for each electrical pulse received by its control unit. Typical step sizes are 7.5°, 15° or larger.

The stepper motor is used *in digitally controlled position control systems in open loop mode*. The input command is in the form of a train of pulses to turn a shaft through a specified angle. There are two advantages in using stepper motors.

One is their compatibility with digital systems and *secondly* no sensors are needed for position and speed sensing as these are directly obtained by counting input pulses and periodic counting if speed information is needed.

Stepper motors have a wide range of applications:

- Paper feed motors in typewriters and printers.
- Positioning of print heads.
- Pens in XY-plotters, recording heads in computer disk drives.
- In positioning of work tables.
- Tools in numerically controlled machining equipment.

The range of applications of these motors is increasing as these motors are becoming available in larger power ratings and with reducing cost.

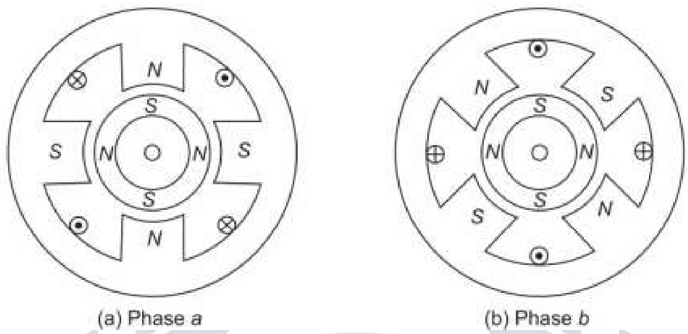


Fig: Permanent-magnet stepper motor

Permanent-Magnet Motor:

Figure shows the phases (stacks) of a two-phase, 4-pole permanent magnet stepper motor. *The rotor is made of ferrite or rare-earth material which is permanently magnetized.* The stator stack of phase *b* is staggered from that of phase a by an angle of 90° elect. When the phase *a* is excited, the rotor is aligned as shown in Fig. (a). If now the phase b is also excited, the effective stator poles shift counterclockwise by $22\frac{1}{2}^\circ$ [Fig. (b)] causing the rotor to move accordingly. If the phase '*a*' is now de-energized (with *b* still energized), the rotor will move another step of $22\frac{1}{2}^\circ$. The reversal of phase a winding current will produce a further forward movement of $22\frac{1}{2}^\circ$, and so on. It is easy to visualize as to how the direction of movement can be reversed.

06(a). During an orthogonal cutting of a metal, the following data were observed:

Chip thickness = 0.5 mm; width of cut = 3 mm; depth of cut = 0.3 mm; feed rate =0.3 mm/rev; cutting force = 1200 N, feed thrust force = 300 N; cutting speed = 3 m/sec; rake angle = 10°

Calculate :

(i) Shear force on the shear plane,

(ii) Coefficient of friction on chip-tool interface and friction angle, and

(iii) Percentage of total energy that goes into overcoming friction at chip-tool interface.

(20 M)

Sol: Given,

$t_2 = 0.5\,\text{mm}$,

W = 3 mm,

d = 0.3 mm,

f = 0.3 mm/rev,

$F_c = 1200$ N,

$F_T = 300$ N,

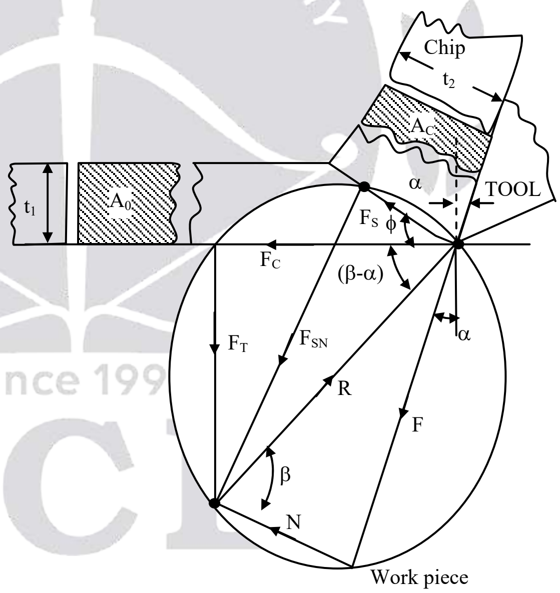
V = 3 m/sec,

$\propto = 10°$.

(i) $F_s = ?$

(ii) $\mu = ?,\ \beta = ?$

(iii) $\%\left(\dfrac{F.V_c}{F_c.V}\right) = ?$



F= friction force

V_c = chip velocity

$t_1 = f = 0.3$ mm ($\because$ no data regarding SCEA)

$\therefore \psi = 0 \rightarrow t_1 = f$

$$r = \frac{t_1}{t_2} = \frac{0.3}{0.5} = 0.6$$

$$\phi = \tan^{-1}\left(\frac{0.6\cos 10^\circ}{1 - 0.6\sin 10^\circ}\right) = 33.41^\circ$$

$$F_s = F_C \cos\phi - F_T \sin\phi$$

$$= 1200\cos 33.41^\circ - 300\sin 33.41^\circ$$

(i) $F_s = 836.5$ N

$$\mu = \tan\beta = \frac{\frac{F_T}{F_c} + \tan\alpha}{1 - \frac{F_T}{F_c}\tan\alpha} = \frac{\frac{300}{1200} + \tan 10^\circ}{1 - \frac{300}{1200}\tan 10^\circ}$$

(ii) $\mu = 0.446$

$$\therefore \beta = \tan^{-1}(\mu) = 24.03^\circ$$

(iii) $F = F_T\cos\alpha + F_c\sin\alpha$

$$= 300\cos 10^\circ + 1200\sin 10^\circ$$

$F = 503.82$ N

$$r = 0.6 = \frac{V_c}{V}$$

$$\%\left(\frac{F.V_c}{F_c.V}\right) = \frac{503.82}{1200} \times 0.6 \quad = 0.252$$

$\therefore 25.2\%$ of total energy goes into overcoming friction at chip-tool interface.

06(b)(i). What are the different structures of ceramic compounds? Explain with neat sketches and examples. (12 M)

Sol: Structures of ceramics compounds :

- Rocksalt structure , Ex: Nacl, Mgo, Feo
- Fluerite & Anti fluorite structure , Ex: Li_2O, CaF_2
- Perovskite structure, Ex: $BaTiO_3$
- Spinel structure, Ex: $MnFe_2O_4$

Rocksalt Structure

- MX Type Compounds based on NaCl or Rocksalt Structure
- Anions (X) form the cation sub lattice with FCC structure.
- Cations (M) fill the octahedral sites.
- 100% occupancy of sites according to the stoichiometry since there will be one octahedral site per anion.
- Radius ratio, $\frac{r_c}{r_a}$ is typically between 0.414 - 0.732 with some exceptions.
- Examples of ceramic materials with such structure as NaCl, MgO, NiO, FeO etc.

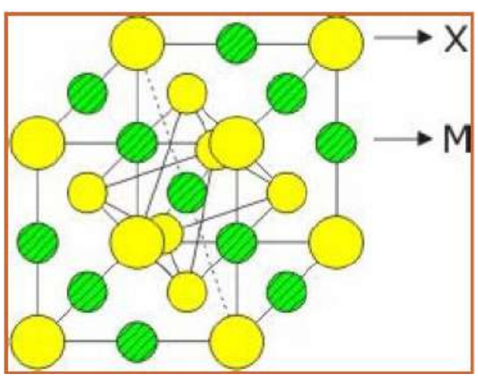


Fig: Schematic of structure of Rocksalt structured compounds

- Lattice type: FCC and motif will be M at 0 0 0 and X at ½ 0 0

  Bond Strength of cations i.e. Na+ = $\text{Bond strength} = \dfrac{\text{Valence of the ion}}{\text{Coordination number of ion}}$

  Valence of anions = Bond strength of cation × coordination of cation $= \dfrac{1}{6} \times 6 = 1$

  Which is the valence of chlorine and hence, proposed packing is appropriate.
- The octahedrons shares at the edges. If there was corner sharing of polyhedra, co-ordination number will be 2 for anions, which won't maintain the stoichiomentry as you can verify using bond-strength relationship.

Antifluorite (A_2X) and Fluorite (AX_2) Structures

Antifluorite

- FCC packing of anions
- All tetrahedral sites filled by cations
- Coordination : Anions: 8, Cations: 4
- Chemical formula: M_2X
- Example: Li_2O, Na_2O, K_2O
- Radius ratio (r_c/r_a): 0.225-0.414

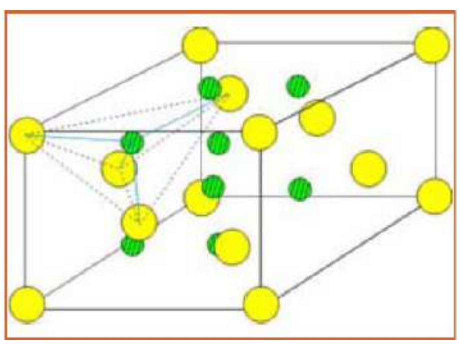


Fig: Antifluorite structure

In this structure in many cases, although $\dfrac{r_c}{r_a}$ ratio predict an octahedral co-ordination, tetrahedral coordination is preferred to fulfill the stoichiometry requirements. In turn, anions are cubic coordinated by cations (CN: 8)

Fluorite Structure (CaF_2 Structure)

- Slightly bigger cations in comparison to other structures
- Example:UO_2, ZrO_2, CaF_2, CeO_2
- Typical representation of the structure appears as if cations make a FCC lattice and anions occupy the tetrahedral sites.

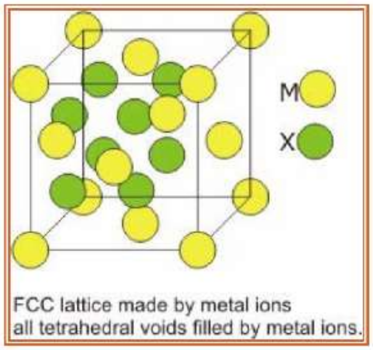


Fig: Fluorite Structure

- While more appropriate Fluorite structure representation is shown below where eight primitive cubic unit cells made by anions are joined together to make a big cube and cations occupy the centers of four of these small cubes in an ordered fashion.

Zinc Blende (MX) Structure

- MX type compounds, also called as sphalerite structured compounds based on a mineral name of sphalerite.
- Mostly oxides and sulphides follow this structure. Examples are ZnO, ZnS, BeO etc.
- Some covalently bonded materials and compounds have similar structure such as GaAs, SiC, BN. You can also visualize diamond also having similar structure with both anion and cation being of same type.
- Typically compounds with tetrahedral co-ordination assume this structure.
- In this structure anions form FCC lattice and cations occupy the tetrahedral interstices.
  - Due to stoichiometry, half of the tetrahedral sites are filled.

- Compounds with radius ratio (γ_c/γ_a): 0.225-0.414 follow this structure with a few exceptions where bonding favours a tetrahedral coordination despite unfavourable radius ratio, especially covalently bonded compounds.
  - Examples:Zn^{2+}-0.06 nm,Be^{2+} - 0.027 nm,O^{2}-0.14 nm, S^{2}-0.184 nm

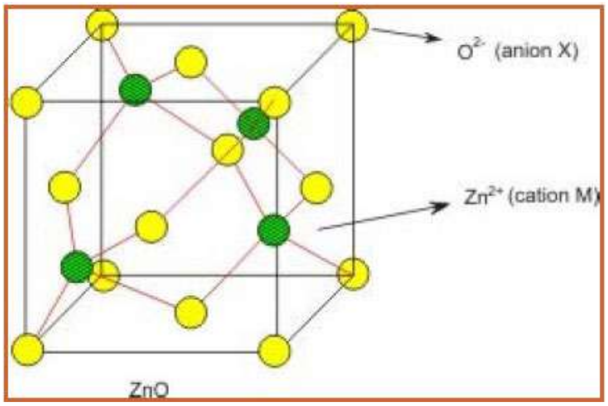


Fig: Zinc Blende or Sphalerite structure

Spinel Structure

- Formulae – $(A^{2}+)(B^{3}+)_2O_4$ or AB_2O_4 or $AO.B_2O_3$
- FCC Packing of anions
- Partial occupancy of both tetrahedral and octahedral sites i.e.1/8th of tetrahedral and ½ of the octahedral sites are occupied.
- A spinel unit-cell is made up of eight FCC cells made by oxygen ions in the configuration 2×2×2, so it is a big structure consisting of 32 oxygen atoms, 8 A atoms and 16 B atoms.
- Depending on how cations occupy different interstices, spinel structure can be Normal or Inverse.

Normal Spinel

- Chemical formula: $(A^{2}+)(B^{3}+)O_4$
- Examples are many aluminates such as $MgAl_2O_4$, $FeAl_2O_4$, $CoAl_2O_4$ and a few ferrites such as $ZnFe_2O_4$ and $CdFe_2O_4$.

- In this structure, all the A^{2+} ions occupy the tetrahedral sites and all the B^{3+} ions occupy the octahedral sites.
- Apply bond strength rule to verify the stoichiometry
- Cations: - A^{2+} - 2/4 ; B^{3+} - 3/6

  Oxygen valence = (2/4×1)+ (3/6×3) = 2

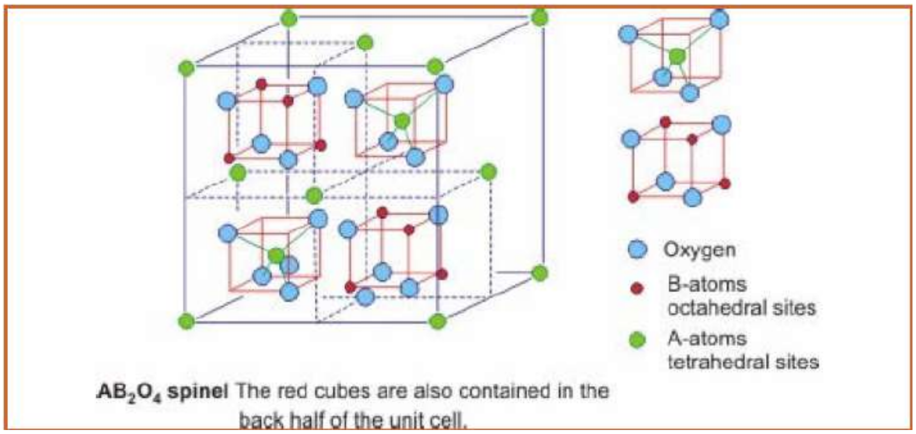


Fig: Schematic of spinel structure

Inverse Spinel $B(AB)O_4$

- Chemical formula: $(A^2+)(B^3+)2O_4$ but can be more conveniently written as $B(AB)O_4$.
- Most ferrite follow this structure such as Fe_3O_4 (or $FeO.Fe_2O_3$), $NiFe_2O_4$, $CoFe_2O_4$ etc.
- In this structure, ½ of the B^3+ ions occupy the tetrahedral sites and remaining ½ B^3+ and all A^{2+} ions occupy the octahedral sites (now you can hopefully make sense of the formula in the previous line).

Perovskite (ABO_3) Structure

- ABO_3 type compounds
- Examples are many titanates like $BaTiO_3$, $SrTiO_3$, $PbTiO_3$ etc. which happen to be technologically very useful compounds as we will see in later modules.

- In ABO_3 structured compounds, A ion is twelve fold coordinated by oxygen (like a dodecahedra) and B ion is octahedrally coordinated by oxygen ions.
- Oxygen atoms form an FCC-like (not FCC) cell with atoms missing from the corners which are occupied by A atoms.
- Bond strength check:

Cation: Ba: 2/12 = 1/6 and Ti: 4/6 = 2/3

Oxygen valence = 1/6 x Coordination number by Ba + 2/3 x coordination number by Ti .

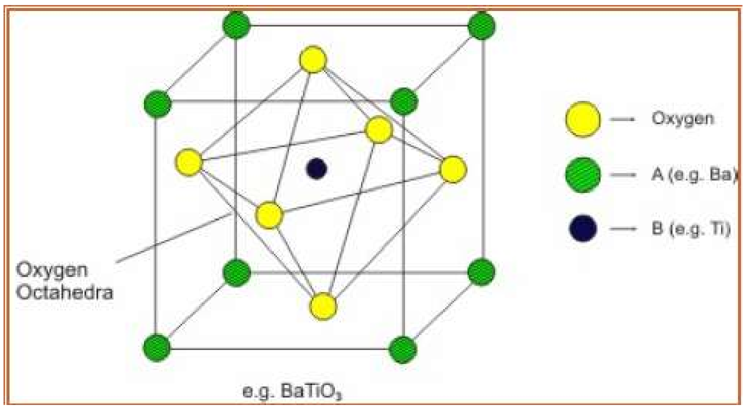


Fig: Perovskite structure

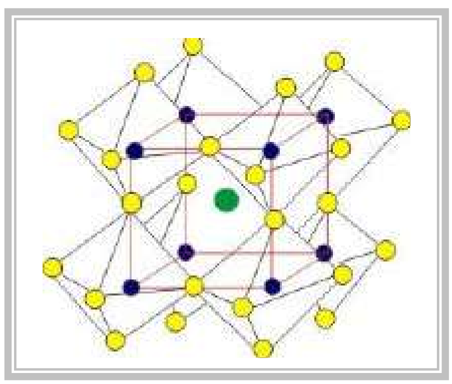


Fig: Polyhedra model of perovskitestructure

06(b)(ii). The lattice constant of a metal with cubic lattice is 2:88 Å. The density of the metal is 7200 kg m^{-3}. Calculate the number of unit cells present in 1 kg of the metal. (8 M)

Sol: Given data,

Lattice constant = a = 2.88A° = 2.88×10^{-10} m

Density = 7200 kg/m^3

Volume of unit cell = $V_{\text{unit cell}} = a^3 = \left(2.88\times10^{-10}\right)^3$

AW = atomic weight, AN = Avagadro's number

$$\text{Density} = \frac{n\times AW}{AN\times V_{\text{unit cell}}}$$

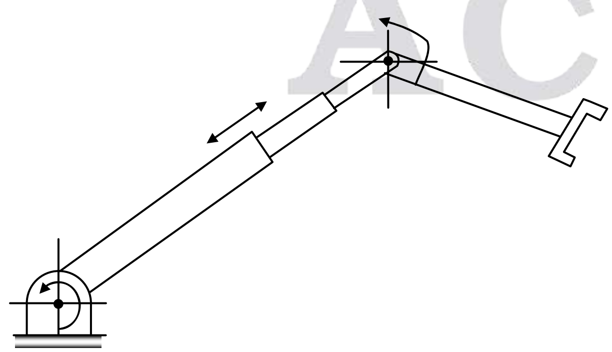
$$\left(\frac{n\times AW}{AN}\right)_{\text{mass of unit cell}} = \rho\times V_{\text{unit cell}}$$

$$= 7200\times\left(2.88\times10^{-10}\right)^3 = 1.719\times10^{-25}\ \text{kg / unit cell}$$

$$\text{Number of unit cell/kg} = \frac{1}{1.7199\times10^{-25}} = 5.8\times10^{24}$$

06(c). Consider the three-link planar manipulator of the figure given below. Derive the forward kinematic equations using the D-H Algorithm.

Assume suitable link and joint parameters.



where,

Link length - a_i

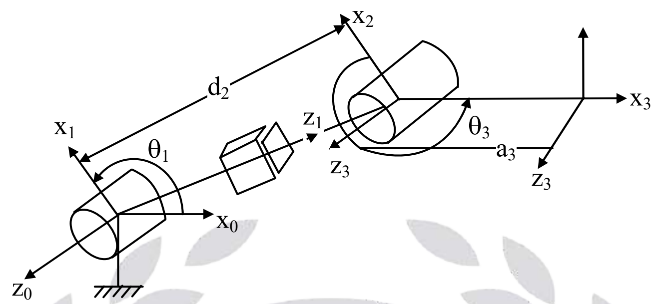
Link twist- α_i

Joint angle - θ_i

Joint displacement – d_i

(20 M)

Sol:



Link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	0	– 90°	d_2	0
3	a_3	0	d_3	θ_3

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \qquad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1A_2A_3 = \begin{bmatrix} c_{13} & -s_{13} & 0 & s_1d_2 + a_3c_{13} \\ s_{13} & c_{13} & 0 & -c_1d_2 + a_3s_{13} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

07(a). The production rates (units/day) of five workers on five different machines are given in the following table. To maximize the total production, assign the workers on specific machines using Hungarian method. Also find the total production per day. (20 M)

		Production Units per day				
		M1	M2	M3	M4	M5
Workers	A	24	37	18	34	34
	B	18	37	20	31	42
	C	26	25	14	37	42
	D	22	25	26	40	50
	E	24	34	30	37	46

Sol: Maximization case:

	Production units per day				
	M_1	M_2	M_3	M_4	M_5
A	24	37	18	34	34
B	18	37	20	31	42
C	26	25	14	37	42
D	22	25	26	40	50
E	24	34	30	37	46

Highest production units / day = 50

Opportunity loss matrix:

	M_1	M_2	M_3	M_4	M_5
A	50-24	50-37	50-18	50-34	50-34
B	50-18	50-37	50-20	50-31	50-42
C	50-26	50-25	50-14	50-37	50-42
D	50-22	50-25	50-26	50-40	50-50
E	50-24	50-34	50-30	50-37	50-46

⇒

	M_1	M_2	M_3	M_4	M_5	Row min
A	26	13	32	16	16	13
B	32	13	30	19	8	8
C	24	25	36	13	8	8
D	28	25	24	10	0	0
E	26	16	20	13	4	4

	M_1	M_2	M_3	M_4	M_5	Row min
A	13	0	19	3	3	13
B	24	5	22	11	0	8
C	16	17	28	5	0	8
D	28	25	24	10	0	0
E	22	12	16	9	0	4
Column min	13	0	16	3	0	

0	0	$\boxed{3}$	0	$\boxed{3}$
11	5	6	8	0
3	17	12	$\boxed{2}$	0
15	25	8	7	0
9	12	0	6	0

⇒

0	0	5	0	$\boxed{5}$
9	$\boxed{3}$	6	6	0
1	15	12	0	$\boxed{0}$
13	23	8	5	0
7	10	0	4	$\boxed{0}$

Solution Matrix:

$\boxed{0}$	0	5	0	8
6	$\boxed{0}$	3	3	0
1	15	12	$\boxed{0}$	3
10	20	5	2	$\boxed{0}$
7	10	$\boxed{0}$	4	3

Production units per day:

	M_1	M_2	M_3	M_4	M_5
A	**24**	37	18	34	34
B	18	**37**	20	31	42
C	26	25	14	**37**	42
D	22	25	26	40	**50**
E	24	34	**30**	37	46

Production units / day:

A – M_1 : 24

B – M_2 : 37

C – M_4 : 37

D – M_5 : 50

E – M_3 : 30

Total : 178

Maximum total production units / day = 178

07(b). In a roll forming process, a 20 mm thick plate is rolled to 16 mm in a four high mill. Determine the coefficient of friction considering it as maximum possible reduction. The diameter of the roll is 480 mm. Also find neutral section, backward and forward slips and maximum pressure. It is given that the value of σ_o = 120 N/mm^2 for hot rolls of mild steel at about 1100°C. (20 M)

Sol: Given,

$H_0 = 20$ mm; $H_1 = 16$ nm

$D = 480$ mm; $\sigma_y = \sigma_0 = 120$ MPa

$\mu = ?$

$H_n = ?$

Backward slip = ?

Forward slip = ?

Maximum pressure = ?

(i) $\mu = ?$

$$\tan\alpha = \sqrt{\frac{\Delta H}{R}} = \sqrt{\frac{4}{240}} = 0.129$$

$\mu = \tan\alpha = 0.129$

$\mu = 0.129$

(ii) $H_n = H_1 + (20 \text{ to } 30\% \text{ of } \Delta H)$

$= 16 + 0.25 \times (\Delta H)$

$= 16 + 0.25 \times (20 - 16)$

$H_n = 17$ mm

(iii) Backward slip = ?

$$\text{Backward slip} = \frac{V - V_0}{V} = 1 - \frac{V_0}{V}$$

$$\frac{H_0}{H_n} = \frac{V}{V_0} \rightarrow \frac{V_0}{V} = \frac{H_n}{H_0}$$

$$= 1 - \frac{H_n}{H_0} = 1 - \frac{17}{20}$$

Backward slip = 0.15 = 15%

(iv) Forward slip = $\frac{V_1 - V}{V} = \frac{V_1}{V} - 1$

$$\frac{H_1}{H_n} = \frac{V}{V_1} \rightarrow \frac{V_1}{V} = \frac{H_n}{H_1}$$

$$= \frac{H_n}{H_1} - 1 = \frac{17}{16} - 1$$

Forward slip = 0.0625 = 6.25%

(v) Maximum pressure = ?

$$P_{max} = \frac{\sigma_y}{n}\left[(n-1)\left(\frac{H_0}{H_n}\right)^n + 1\right]$$

$$n = \frac{2\mu L}{\Delta H} = \frac{2 \times 0.129 \times 30.98}{4}$$

$$L = \sqrt{R\Delta H} = \sqrt{240 \times 4} = 30.98$$

$$n = 1.9984 \simeq 2$$

$$P_{max} = \frac{120}{2}\left[(2-1)\left(\frac{20}{17}\right)^2 + 1\right]$$

$$P_{max} = 143 \text{ N/mm}^2$$

07(c). The kinematic parameters of a 3R planar articulated robot are given below:

link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	0	d_3	θ_3

Determine the kinematic model of the robot using D-H Algorithm and relation between adjacent frames. (20 M)

Sol: Kinematic parameters of 3 R planar articulated robot are given

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	0	d_3	θ_3

Then kinmatic model of robot and relation between adjacent frames as following.

$$ {}_0T^3 = {}_0T^1\ {}_1T^2\ {}_2T^3 $$

Relation between adjacent frames

$$ {}_0T^1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Adjacent frame is rotated rotated by θ_1 w.r.t to z axis and translated by a_i unit.

$$_{1}T^{2} = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$_{2}T^{3} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematic model of a robot is

$$_{0}T^{3} = {}_{0}T^{1}\ {}_{1}T^{2}\ {}_{2}T^{3}$$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times {}_{2}T^{3}$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1c_1 + a_2c_{12} \\ s_{123} & c_{123} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

08(a)(i). There are three coordinate frames $o_1\ x_1\ y_1\ z_1$, $o_2\ x_2\ y_2\ z_2$ and $o_3\ x_3\ y_3\ z_3$ and relation between frames are given below:

$$^{1}R_{2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \qquad ^{1}R_{3} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Find the matrix $^{2}R_{3}$ that relates frame {2} and {3}. (10 M)

Sol: There coordinate frames 0,x,y,z, $0_2,x_2,y_2,z_2$, o_3,x_3,y_3,z_3 and relation between frames are given as:

$$^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \frac{-\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & 1/2 \end{bmatrix}$$

$$^1R_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then $^2R_3 = ?$

$\Rightarrow {}^1R_3 = {}^1R_2 . {}^2R_3$

$$^2R_3 = \left[{}^1R_2\right]^T . \left[{}^1R_3\right]$$

$$= {}^2R_1 . 1R_3$$

$$^2R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$

08(a)(ii). The homogeneous coordinates of a point are 1_P = [0 0 10 1]^T. The coordinate frame is translated relative to the fixed {F} coordinate frame, by 6 units along x-axis and –2 units along y-axis. What is the homogeneous transformation matrix that represents the operation? What is the description of point P relative to the reference frame {F} following the translation? (10 M)

Sol: The homogenous coordinates of a point

$${}^1P = \begin{bmatrix}0 & 0 & 10 & 1\end{bmatrix}^T$$

This point is translated by 6 units along x–axis and –2 units along y-axis

Then the homogenous matrix relates above changes are

$$\Rightarrow \begin{bmatrix}1 & 0 & 0 & 6\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow$ Description of point P relative to reference frame (F)

$$\Rightarrow \begin{bmatrix}1 & 0 & 0 & 6\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\\ 10\\ 1\end{bmatrix} = \begin{bmatrix}6\\ -2\\ 10\\ 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}6 & -2 & 10 & 1\end{bmatrix}^T$$

08(b). The demand for an item is observed as 24,000 units per year. The production capacity of the plant is 3,000 units per month. The item cost is ₹ 40 per unit and inventory carrying cost is 12% of the item cost per unit per year. The cost of one set-up is ₹ 480. The shortage cost of one unit is ₹ 240 per year. Supply of the item is non-instantaneous (gradual).

Determine the economic order quantity, optimal number of shortages, production line required for each cycle and the number of set-ups in a year. (20 M)

Sol: Annual demand (D) = 24000 units

Production rate (K) = 3000 units / month

Item cost (C_u) = ₹ 40 / units

Carrying cost (C_c) = 12 % of item cost per annum

= 0.12 × 40 / units / year

Shortage cost (C_s) = ₹ 240 / unit/ year

Setup cost (C_o) = ₹ 480

$$d = \frac{D}{12} = \frac{24000}{12} = 2000\,\text{units / month}$$

$$EBS_s = \sqrt{\frac{2DC_o}{C_c}}\sqrt{\frac{k}{k-d}}\sqrt{\frac{C_c + C_s}{C_s}}$$

$$= \sqrt{\frac{2\times 24000\times 480}{4.8}}\sqrt{\frac{3000}{3000-2000}}\sqrt{\frac{4.8+240}{240}}$$

$$= 2190.89 \times 1.732 \times 1.009$$

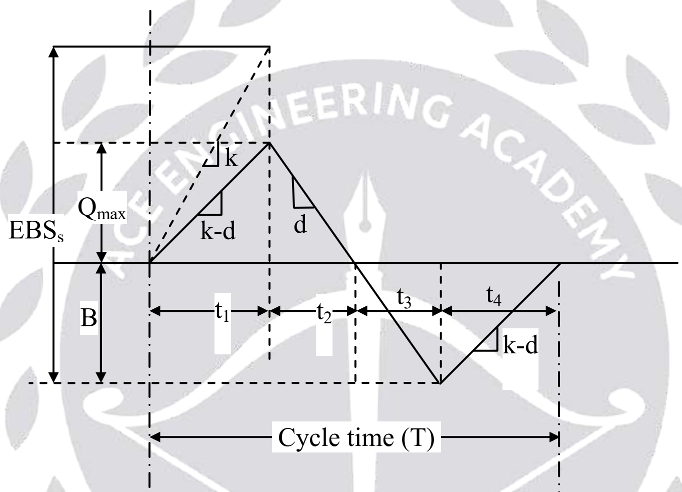
$$= 3832 \text{ units}$$

Maximum inventory (Q_{max}) = $EBS_s\left(\frac{k-d}{k}\right)\left(\frac{C_s}{C_c+C_s}\right)$

$$= 3832\left(\frac{3000-2000}{3000}\right)\left(\frac{240}{4.8+240}\right)$$

= 1252 units

Optimal shortage (B) = $Q_{max} \times \frac{C_c}{C_s} = 1252 \times \frac{4.8}{240} = 25$ units



Total production time/cycle = $t_1 + t_4$

Production time for building 'Q_{max}' = t_1

$$t_1 = \frac{Q_{max}}{k-d} = \frac{1252}{3000-2000} = 1.252 \text{ month}$$

Production time for recovering 'B' $= \frac{B}{k-d} = \frac{25}{3000-2000} = 0.025$ months

Total production time per cycle = 1.252 + 0.025 = 1.27 months

No .of setups / year (N) $= \frac{D}{EBS_s} = \frac{24000}{3832} = 6.26 \approx 7$

Cycle time (T) = $\frac{1}{N} = \frac{1}{6.26}$ year = 1.91 months

Hearty Congratulations to our
GATE - 2021 TOP RANKERS
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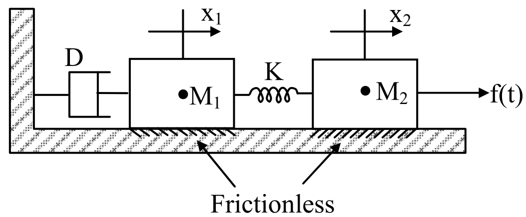
AIR 1ST EE AAKASH DHILL
AIR 1ST ME SUYASH SHARMA
AIR 1ST PI GANESH ADIGAUR
AIR 1ST XE VARUN KAUSHIK
AIR 2nd ME JAY CHAVDA
AIR 2nd PI GOWTHAM GUDIMELLA
AIR 2nd EC M POOJASREE
AIR 2nd ES ANUSH VERMA
AIR 3rd ME MUNISH KUMAR
AIR 3rd ME NUKULA VISWA TEJA
AIR 3rd PI REHAN ANWAR DESAI
AIR 3rd EC MANOJ KUMAR
AIR 3rd CE SHASHIKANT KUMAR
AIR 3rd EE JAYMAL KHUNTI
AIR 4th PI ROHIT SONI
AIR 4th EC P AKHIL
AIR 4th IN DIVAKAR CH.
AIR 4th IN SAURABH JAISWAL
AIR 4th XE JATIN BHANDARI
AIR 4th ES HEMANTH TIWARI
AIR 5th ME ABHISHEKA
AIR 5th EC VAISHNAV KV
AIR 5th CE PRASHANT DWIVEDI
AIR 5th CS PANKAJ LAHKAR
AIR 5th CS S BHATTACHARJEE
AIR 5th PI SUBHANSHU TIWARI
AIR 6th ME ABHISHEK MEWAR
AIR 6th PI AKASH JAISWAL
AIR 6th EC PARAG SAROHA
AIR 6th IN HARSHIT GUPTA
AIR 6th IN RAJU SHARMA
AIR 6th CE TANMAY MAHAJAN
AIR 6th XE MANISH YADAV
AIR 7th ME VATSAL PANCHAL
AIR 7th PI SACHIN DUBOLIYA
AIR 7th PI ATULYA JYOTHI
AIR 8th ME RAJAT GUPTA
AIR 8th ME ROHIT S PATIL
AIR 8th EC ANKUR LAL MEENA
AIR 8th IN KUNAL SAURAV
AIR 8th CE PRANSHU JANGID
AIR 8th CE RAHUL PATI
AIR 8th EE UJJWAL KUMAR
AIR 8th EE HEMANT JINDAL
AIR 9th EC ABHISHEK SINGH
AIR 9th EC ALEESHA ROSE
AIR 9th EC SAI VAMSI DOSAPATI
AIR 9th IN RAMESH KUMAR
AIR 9th CS NIRANJAN NITIN DHOOT
AIR 9th XE RACHIT KUMAR
AIR 10th ME DEEPESH AGARWAL
AIR 10th PI VYOM SHARMA
AIR 10th IN PRAGYA KAUSHIK
AIR 10th CE GOVIND PRASAD B
AIR 10th XE D KUMAR PATIL
AIR 10th XE R SHIVAJI NALE
and many more...
Total 57 Ranks in Top 10
ME : 10
PI : 09
CE : 08
EE : 4
EC : 09
CS : 03
IN : 08
XE : 06

08(c).



Find the state equations in matrix form for the translational mechanical system as shown above. Assume zero initial conditions, where

M_1 = Mass of block 1

M_2 = Mass of block 2

K = Spring stiffness constant

D = Coefficient of viscous friction

x_1 = Displacement of block 1

x_2 = Displacement of block 2 **(20 M)**

Sol: First write the differential equations for the network. (Laplace-transformed equations of motion). Take the inverse Laplace transform of these equations, assuming zero initial conditions and obtain.

$$M_1\frac{d^2x_1}{dt^2}+D\frac{dx_1}{dt}+Kx_1-Kx_2=0 \quad \text{------ (1)}$$

$$-Kx_1+M_2\frac{d^2x_2}{dt^2}+Kx_2=f(t) \quad \text{------ (2)}$$

Now let $\frac{d^2x_1}{dt^2}=\frac{dv_1}{dt}$, and $\frac{d^2x_2}{dt^2}=\frac{dv_2}{dt}$, and then select x_1, v_1, x_2 and v_2 as state variables. Next form two of the state equations by solving equation (1) for $\frac{dv_1}{dt}$ and equation (2) for $\frac{dv_2}{dt}$.

Finally , add $\frac{dx_1}{dt}=v_1$ and $\frac{dx_2}{dt}=v_2$ to complete the set of state equations.

Hence,

$$\frac{dx_1}{dt} = \qquad + v_1$$

$$\frac{dv_1}{dt} = -\frac{K}{M_1}x_1 - \frac{D}{M_1}v_1 + \frac{K}{M_1}x_2$$

$$\frac{dx_2}{dt} = \qquad + v_2$$

$$\frac{dv_2}{dt} = +\frac{K}{M_2}x_1$$

$$-\frac{K}{M_2}x_2 + \frac{1}{M_2}f(t)$$

In vector-matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

where the dot indicates differentiation with respect to time.


Hearty Congratulations to our
ESE - 2020 TOP RANKERS
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AIR 1 PRAKASH JHA E&T
AIR 1 GAURAV KUMAR CE
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AIR 2 PARTH BATRA E&T
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EE 09
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