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GATE | PSUs

CIVIL ENGINEERING

Engineering Mechanics

Text Book & Work Book: Theory with worked out Examples and Practice Questions)

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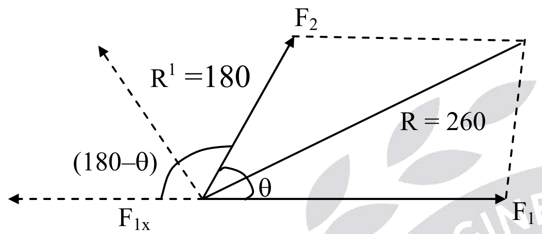
Engineering Mechanics

(Solutions for Text Book Practice Questions)

01. Forces and Moment Systems

01. Ans: (b)

Sol:



Assume $F_1 = 2F_2$ ($F_1 > F_2$)

$F_{1x} = 2F_2$

$$R = \sqrt{F_1^2 + F_2^2 + 4F_2^2 \cos\theta}$$

$$260 = \sqrt{4F_2^2 + F_2^2 + 4F_2^2 \cos\theta}$$

$$260^2 = 5F_2^2 + 4F_2^2 \cos\theta \text{ ------ (1)}$$

$$R^1 = \sqrt{F_{1x}^2 + F_2^2 + 2F_{1x}F_2 \cos\theta}$$

$$180 = \sqrt{4F_2^2 + F_2^2 + 2.F_2.F_2 \cos(180-\theta)}$$

$$180^2 = 5F_2^2 - 4F_2^2 \cos\theta \text{ ------ (2)}$$

$$260^2 = 5F_2^2 + 4F_2^2 \cos\theta$$

$$180^2 = 5F_2^2 - 4F_2^2 \cos\theta$$

$$260^2 + 180^2 = 10F_2^2$$

$\Rightarrow$ $F_2 = 100$N,

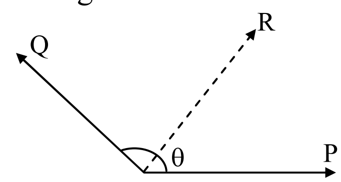
$260^2 = 5(100)^2 + 4(100)^2\cos\theta$

$\Rightarrow \theta = 63.89$

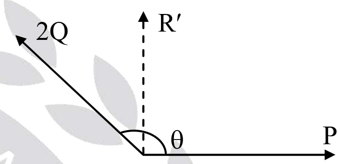
Where θ angle between two forces.

02. Ans: (b)

Sol: Let the angle between the forces be θ



Where, R is the resultant of the two forces.



If Q is doubled i.e., 2Q then resultant (R′) is perpendicular to P.

$$\tan 90 = \frac{2Q\sin\theta}{P + 2Q\cos\theta}$$

$\Rightarrow$ $P + 2Q\cos\theta = 0$

$P = -2Q\cos\theta$ ------(i)

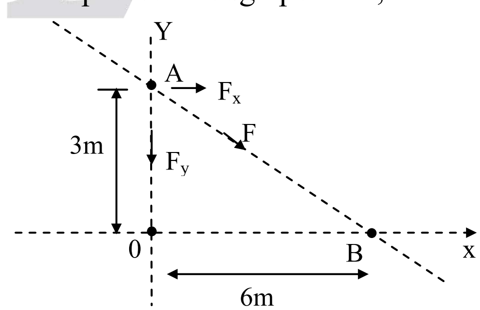
Also, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = Q$ [using eq.(i)]

03. Ans: (b)

Sol: Since moment of F about point A is zero.

$\therefore$ F passes through point A,



$$M_0^F = 180\text{N} - \text{m}$$

$M_B^F = 90\,N - m$

$M_A^F = 0$

$M_0^F = 180 = F_x \times 3 + F_y \times 0$

$F_x = 60N$ ……. (1)

$M_B^F = F_x \times 3 - F_y \times 6 = -90$

$60 \times 3 - 6F_y = -90$

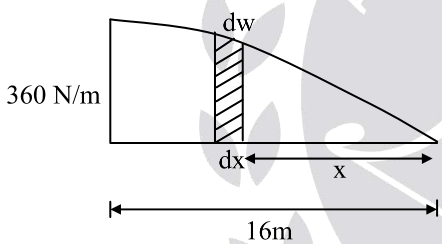
$\Rightarrow F_y = \dfrac{270}{6}$

$F_y = 45N$

$\therefore\ F = \sqrt{F_x^2 + F_y^2} = \sqrt{60^2 + 45^2} = 75$

04. Ans: (a)

Sol:



$$\int_0^w dw = \int_0^{16} w dx$$

$$w = \int_0^{16} 90\sqrt{x}dx = 90\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^{16}$$

$$= 90 \times \frac{2}{3}\left[x^{3/2}\right]_0^{16} = 60\,(16)^{3/2}$$

$w = 3840$ N

The moment due to average force should be equal to the variable force

$R \times d = \Sigma dw \times x$

$$3840 \times d = \int_0^{16} 90\sqrt{x}.dx.x$$

$$= 90\int_0^{15} x^{1.5}dx$$

$$3840d = 90\left[\frac{x^{2.5}}{2.5}\right]_0^{16}$$

$\Rightarrow\ d = 9.6$ m

05. Ans: (c)

Sol: Moment about 'O'

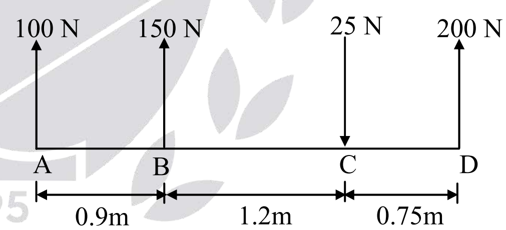
$M_0 = 100\sin 60 \times 3$

$= 300 \times \dfrac{\sqrt{3}}{2} = 150\sqrt{3}$

$= 259.8 \cong 260$ N

06. Ans: (a)

Sol:



$F_R = \Sigma F_y$

$F_R = 100+150-25+200$ (upward force Positive downward force negative)

$R = 425$ N

For equilibrium

$\Sigma M_A = 0$ (since R = resultant)

Let R is acting at a distance of 'd'

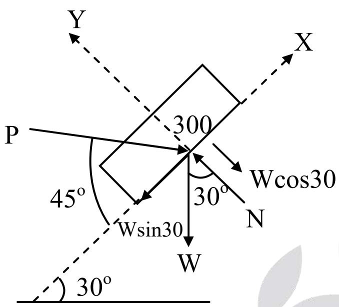
$425 \times d = 150 \times 0.9 + 25 \times 2.1 - 200 \times 2.85$

$\Rightarrow\ d = 1.535$m (from A)

02. Equilibrium of Force Systems

01. Ans: (d)

Sol:



Resolve the forces along the inclined surface

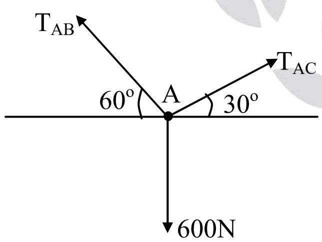
$\sum F_x = 0$

$P\cos45 - W\sin30 = 0$

$P = \dfrac{300\sin30}{\cos45} \Rightarrow P = 212.13$ N

02. Ans: (a)

Sol:



$T_{AB}\cos60° = T_{AC}\cos30°$

$T_{AB} = \sqrt{3}\ T_{AC}$

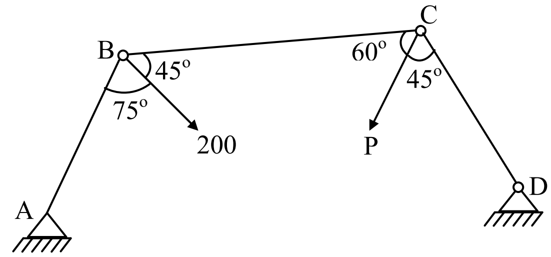
$T_{AB}\sin60° + T_{AC}\sin30° = 600$ N

$\dfrac{3}{2}T_{AC} + \dfrac{1}{2}T_{AC} = 600$

$\Rightarrow\ T_{AB} = 520$ N ; $T_{AC} = 300$ N

03. Ans: (c)

Sol:



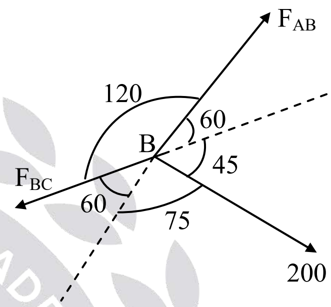


Fig: Free body diagram at 'B'

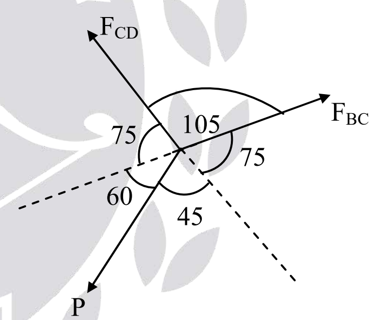


Fig: Free body diagram at 'C'

For Equilibrium of Point 'B'

$$\frac{F_{AB}}{\sin(60+75)} = \frac{F_{BC}}{\sin(60+45)} = \frac{200}{\sin(120)}$$

$F_{BC} = 223.07$ N

From Sine rule at "C".

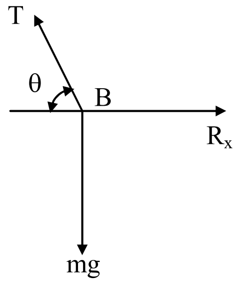
$$\frac{F_{CD}}{\sin(75+45)} = \frac{F_{BC}}{\sin(60+75)} = \frac{P}{\sin105}$$

$$P = \frac{223.07 \times \sin105}{\sin135}$$

$P = 304.71$ N

04. Ans: (d)

Sol:



$$\tan\theta = \frac{125}{275} \Rightarrow \theta = 24.45^0$$

$T\sin\theta = mg.$

$T\sin 24.45 = (35\times 9.81)$

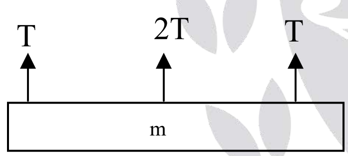
$T = 829.5$ N

$R_x = T\cos 24.45 = 755.4$ N

$R_y = 0$

05. Ans: (c)

Sol:



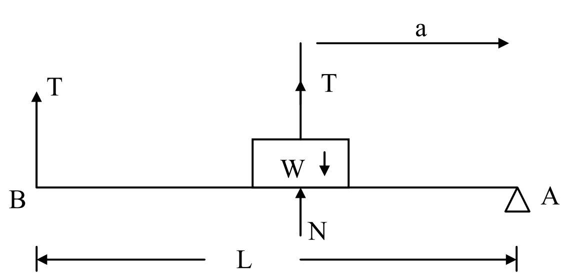
$T+2T+T = mg$

$4T = mg$

$m = 4T/g$

06. Ans: (b)

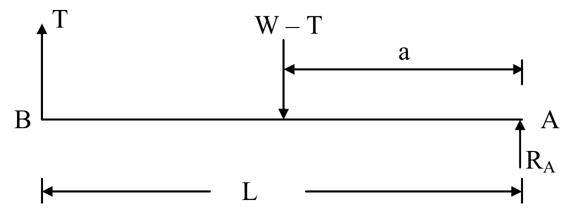
Sol:



For body, $\sum F_y = 0$

$N - W + T = 0$

$\Rightarrow N = W - T$



$\sum F_y = 0$ for entire system

$R_A + T - (W - T) = 0$

$R_A = W - 2T$ ------- (1)

For equilibrium

$\sum M_A = 0$

$T\times L = (W - T)\,a$

$TL = Wa - Ta$

$TL + Ta = Wa$

$T(L + a) = Wa$

$$\Rightarrow T = \frac{Wa}{L+a}$$

T substitute in equation (1)

$$R_A = W - 2\left(\frac{Wa}{L+a}\right)$$

$$= \frac{W(L+a) - 2Wa}{L+a}$$

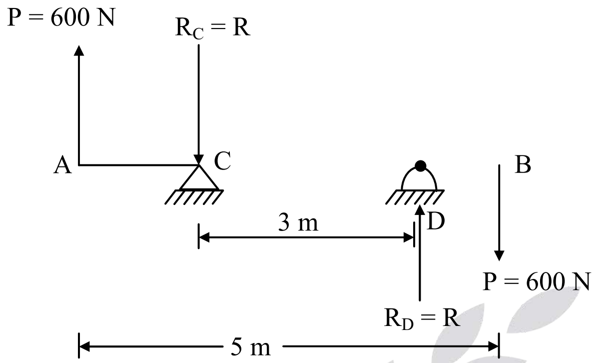
$$= \frac{WL + Wa - 2Wa}{L+a}$$

$$= \frac{WL - Wa}{L+a}$$

$$R_A = \frac{W(L-a)}{L+a}$$

07. Ans: (c)

Sol:



$\sum F_y = 0$

$600 - R_C + R_D - 600 = 0$

$\Rightarrow\ R_C = R_D = R$

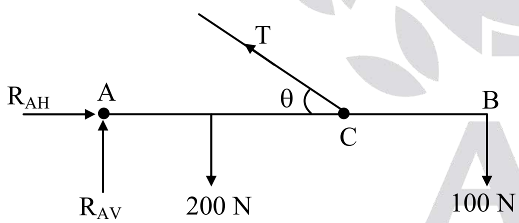
$\sum M = 0$

$600 \times 5 = R \times 3$

$\Rightarrow\ R = 1000\ N = R_C = R_D$

08. Ans: (a)

Sol: F.B.D



$\sum M_A = 0$

$\text{Tan}\theta = \frac{8}{4}$

$\theta = 63.43$

$T\sin\theta \times 4\ (\circlearrowleft) - 200 \times 2\ (\circlearrowright) - 100 \times 6\ (\circlearrowright) = 0$

$\Rightarrow T = 279.5\ N$

Now, $\sum F_x = 0$,

$R_{AH} - T\cos\theta = 0$

$R_{AH} = 125\ N$

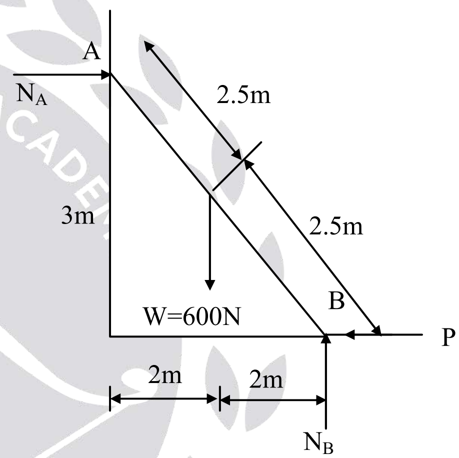
$\sum F_y = 0$

$R_{AV} - 200 - 100 + T\sin\theta = 0$

$\Rightarrow R_{VA} = 50\ N$

09. Ans: 400 N

Sol:



$\sum F_y = 0$

$N_B - W = 0$

$N_B = 600\ N$

$\sum M_A = 0$

$P \times 3 + W \times 2 - N_B \times 4 = 0$

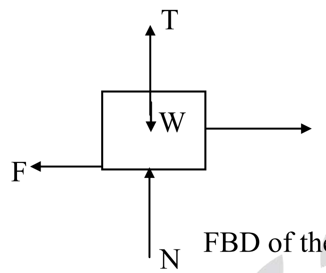
$$P = \frac{4N_B - 2W}{3}$$

$$P = \frac{4 \times 600 - 2 \times 600}{3} = 400N$$

03. Friction

01. Ans: (c)

Sol: The FBD of the above block shown



FBD of the block

$\Sigma Y = 0 \Rightarrow N + T - W = 0$

$N = W - T = 981 - T$

$F = \mu N = 0.2\,(981 - T)$

$\Sigma X = 0 \Rightarrow 100 - F = 0$

$F = 100 = 0.2\,(981 - T)$

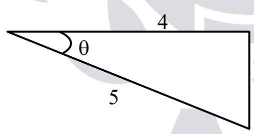
$\Rightarrow T = 481$ N

02. Ans: (c)

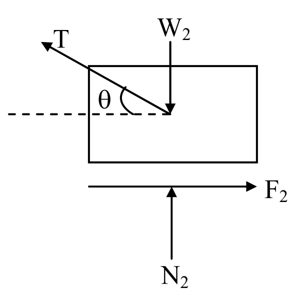
Sol: Given $\tan\theta = \dfrac{3}{4}$

$\sin\theta = 3/5$

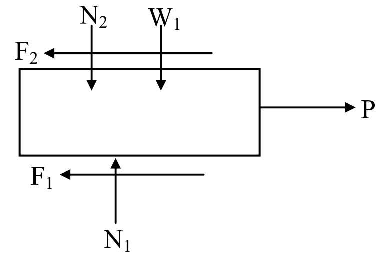
$\cos\theta = 4/5$



Free body diagram for block (2)



Free body diagram for block (1)



From FBD of block (2)

$\Sigma F_x = 0$

$F_2 = T\cos\theta$

$F_2 = \dfrac{4}{5}T = 0.8T$ ------ (1)

$\Sigma F_y = 0$

$N_2 + T\sin\theta - W_2 = 0$

$N_2 = W_2 - T\sin\theta$

$N_2 = 50 - 0.6\,T$

But $F_2 = \mu N_2$

$\Rightarrow F_2 = 0.3(50 - 0.6T)$

$F_2 = 15 - 0.18\,T$ ------ (2)

From (1) & (2)

$0.8T = 15 - 0.18\,T$

$\Rightarrow 0.98T = 15$

$\Rightarrow \quad T = 15.31$ N

$\therefore N_2 = 50 - 0.6T$

$= 50 - 0.6\,(15.31) = 40.81$ N

$F_2 = \mu N_2 = 0.3 \times 40.81 = 12.24$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1 = 40.81 + 200 = 240.81$ N

$F_1 = \mu N_1 \Rightarrow F_1 = 0.3 \times 240.81$

$F_1 = 72.24$ N

$\Sigma F_x = 0$

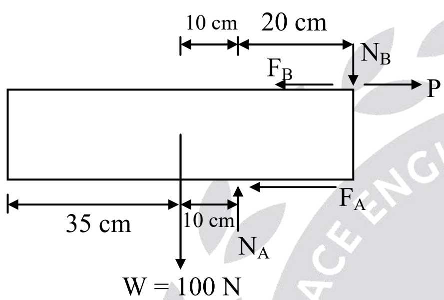
$P - F_1 - F_2 = 0$

$P = F_1 + F_2 = 72.24 + 12.24$

$P = 84.48 \text{ N} \simeq 84.5 \text{ N}$

03. Ans: (b)

Sol: Free Body Diagram



$$F_A = \mu N_A = \frac{1}{3} N_A$$

$$F_B = \mu N_B = \frac{1}{3} N_B$$

$\Sigma M_B = 0$

$-100\times 30(\circlearrowleft) + (N_A \times 20)(\circlearrowleft) + (F_a \times 12)(\circlearrowleft) = 0$

$$-3000 + N_A \times 20 + \frac{1}{3} N_A \times 12 = 0$$

$\Rightarrow N_A = 125 \text{ N}$

$\Sigma F_y = 0$

$N_A - N_B - 100 = 0$

$\Rightarrow N_B = 25 \text{ N}$

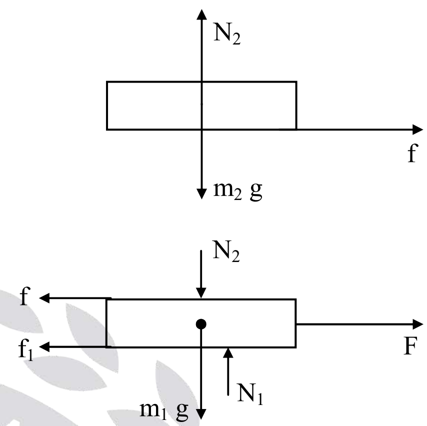
$\Sigma F_x = 0$

$$P = F_A + F_B = \frac{1}{3}(N_A + N_B)$$

$$= \frac{1}{3}(125 + 25) = 50\text{N}$$

04. Ans: (d)

Sol: F.B.D of both the books are shown below.



where, f is the friction between the two books.

f_1 is the friction between the lower book and ground.

Now, maximum possible acceleration of upper book.

$$a_{max} = \frac{f_{max}}{m_2} = \frac{\mu m_2 g}{m_2} = \mu \times g$$

$$= 0.3 \times 9.81 = 2.943 \text{ m/s}^2$$

For slip to occur, acceleration (a_1) of lower book. i.e, $a_1 \geq a_{max}$

$$\frac{F - f - f_1}{m_1} \geq 2.943$$

$F - 2.943 - 0.3 \times 2 \times 9.81 \geq 2.943$

$[\because f = f_{max} = 2.943$ and

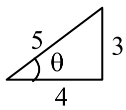
$f_1 = \mu \times (m_1 + m_2) g = 0.3 \times 2 \times 9.81]$

$F \geq 11.77$ N

$F_{min} = 11.77$ N

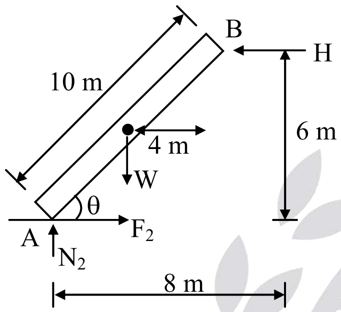
05. Ans: (d)

Sol: $\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$

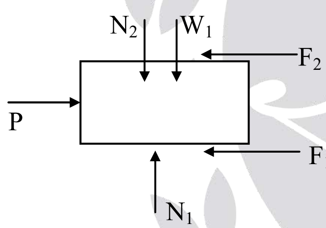


$\cos\theta = \frac{4}{5}$

FBD for bar AB (2)



FBD for block (1)



Given W = 280 N , W_1 = 400 N

Now, $\Sigma M_B = 0$

$-W \times 4\,(\circlearrowleft) + N_2 \times 8(\circlearrowleft) - F_2 \times 6\,(\circlearrowleft) = 0$

$-280 \times 4 + N_2 \times 8 - \mu N_2 \times 6 = 0$

$\Rightarrow N_2 = 200$ N

But, $F_2 = \mu N_2 = 0.4 \times 200 = 80$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1$

$= 200 + 400$

$N_1 = 600$ N

But, $F_1 = \mu N_1 = 0.4 \times 600$

$F_1 = 240$ N

$\Sigma F_x = 0$

$P = F_1 + F_2 = 240 + 80$

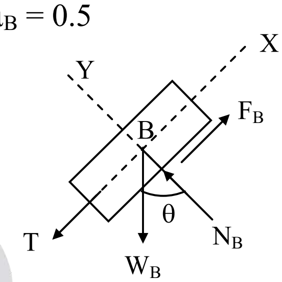
$P = 320$ N

06. Ans: (a)

Sol: Given, $W_A = 200$ N , $\mu_A = 0.2$

$W_B = 300$ N , $\mu_B = 0.5$

FBD for block 'B'.



$\Sigma F_y = 0$

$N_B = W_B\cos\theta$

$N_B = 300\cos\theta$

But, $F_B = \mu N_B = 0.5 \times 300\cos\theta$

$= 150\cos\theta$

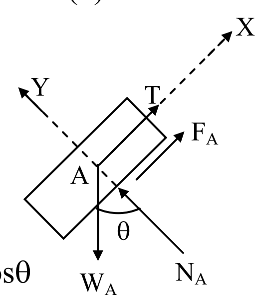
$\Sigma F_x = 0$

$T + W_B\sin\theta - F_B = 0$

$T = F_B - W_B\sin\theta$

$T = 150\cos\theta - 300\sin\theta$ ------ (1)

FBD for block 'A'



$\Sigma F_y = 0$

$N_A - W_A\cos\theta = 0$

$N_A = 200\cos\theta$

$F_A = \mu N_A = 0.2 \times 200\cos\theta$

But, $F_A = 40\cos\theta$

$\Sigma F_x = 0$

$T + F_A - W_A\sin\theta = 0$

$T = W_A\sin\theta - F_A$

$T = 200\sin\theta - 40\cos\theta$

But from equation (1)

$T = 150\cos\theta - 300\sin\theta$

$\therefore 150\cos\theta - 300\sin\theta = 200\sin\theta - 40\cos\theta$

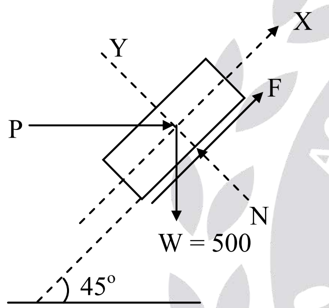
$190\cos\theta = 500\sin\theta$

$$\tan\theta = \frac{190}{500}$$

$\Rightarrow\ \theta = 20.8^{o}$

07. Ans: (d)

Sol: FBD for the block



$\Sigma F_y = 0$

$N - W\sin45 - P\sin45 = 0$

$$N = \frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}$$

$$\text{But, } F = \mu N = 0.25\left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}\right)$$

$\Sigma F_x = 0$

$P\cos45 + F - W\sin45 = 0$

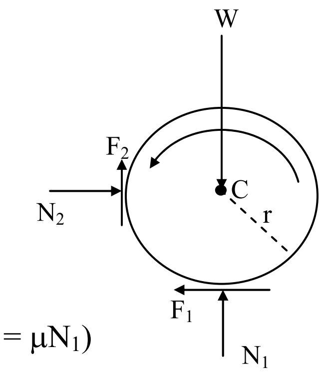
$$P\cos45 + 0.25\left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}\right) - 500 \times \frac{1}{\sqrt{2}} = 0$$

$\Rightarrow$

$P = 300$ N

08. Ans: (a)

Sol: FBD of block



$F_1 = \mu N_1$

$F_2 = \mu N_2$

$\Sigma F_x = 0$

$N_2 - F_1 = 0$

$\Rightarrow N_2 = F_1\ \ (\because F_1 = \mu N_1)$

$N_2 = \mu N_1$

$\Sigma F_y = 0$

$N_1 + F_2 - W = 0$

$N_1 + \mu N_2 - W = 0$

$N_1 + \mu^2 N_1 - W = 0 \qquad (\because N_2 = \mu N_1)$

$N_1(1+\mu^2) = W$

$$N_1 = \frac{W}{1+\mu^2}$$

$$N_2 = \frac{\mu W}{1+\mu^2}$$

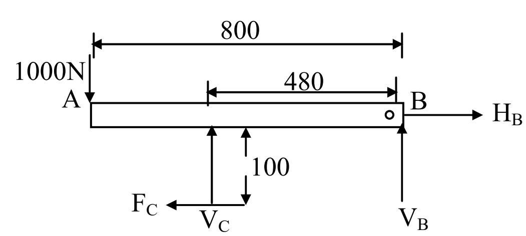
$\text{Couple} = (F_1 + F_2) \times r = \mu r (N_1 + N_2)$

$$= \frac{\mu r \times W(1+\mu)}{1+\mu^2} \qquad (\because \mu = f)$$

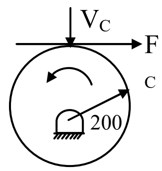
$$= \frac{Wrf(1+\mu)}{1+\mu^2}$$

09. Ans: 64 N-m

Sol: FBD of shoe bar:



FBD of Drum Brake :



$\sum M_B = 0$

$V_C \times 480 + F_C \times 100 - 1000 \times 800 = 0$

$F_C = \mu V_C = 0.2\ V_C$

$480V_C + 0.2V_C \times 100 = 800000$

$500V_C = 800000$

$V_C = 1600$ N

$F_C = 0.2\ V_C = 0.2 \times 1600 = 320$ N

$M = 0.2 \times F_C = 0.2 \times 320 = 64$ N-m

10. Ans: (a)

Sol: $\beta = 2\theta$

$\cos\theta = \dfrac{6}{12}$

$\Rightarrow \theta = 60$

$\beta = 360 - 2\theta$

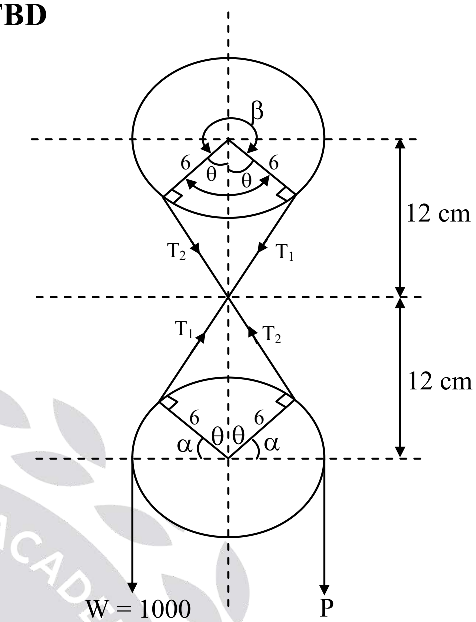
$\beta = 240 = \dfrac{4\pi}{3}$

$2\alpha + 2\theta = 180$

$2\alpha = 180 - 120$

$\alpha = 30 = \dfrac{\pi}{6}$

FBD



(When W moves upwards)

For P_{min} calculation,

$W > T_1$

$\dfrac{W}{T_1} = e^{\mu\alpha}$

$T_1 = \dfrac{1000}{e^{\frac{\pi}{6}\times\frac{1}{\pi}}} = 846.48$ N

$\therefore \dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_2 = \dfrac{848.48}{e^{\frac{1}{\pi}\times\frac{4\pi}{3}}} = 223.12$ N

$\dfrac{T_2}{P_{min}} = e^{\mu\alpha}$

$\Rightarrow P_{min} = \dfrac{223.12}{e^{\frac{1}{\pi}\times\frac{\pi}{6}}}$

$P_{min} = 188.86\text{ N} \approx 189$ N

For P_{max} calculation

$$\frac{T_1}{W} = e^{\mu\alpha}$$

$$T_1 = 1000 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$$

$$T_1 = 1181.36 \text{ N}$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$T_2 = 1181.36 \times e^{\frac{1}{\pi}\times\frac{4\pi}{3}} = 4481.65 \text{ N}$$

$$\frac{P_{max}}{T_2} = e^{\mu\alpha}$$

$$P_{max} = 4481.68 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$$

$$P_{max} = 5300 \text{ N}$$

11. Ans: (b)

Sol: Given $\mu = 0.2$, $\tan\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

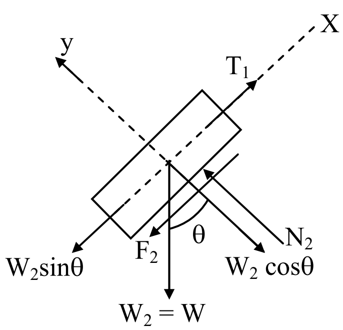


Fig: FBD (1)

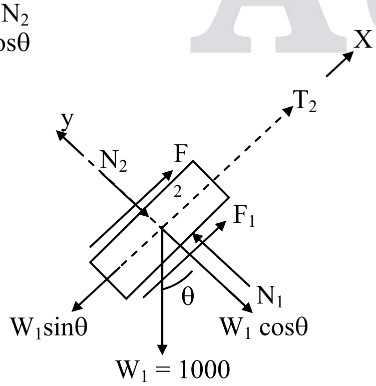


Fig: FBD (2)

From FBD (1)

$\Sigma F_y = 0$

$N_2 - W_2 \cos\theta = 0$

$N_2 = W_2 \cos\theta = W\times 0.8$

$N_2 = 0.8 \text{ W}$

$\therefore F_2 = \mu N_2 = 0.2 \times 0.8 \text{ W}$

$F_2 = 0.16 \text{ W}$

$\Sigma F_x = 0$

$T_1 - W_2\sin\theta - F_2 = 0$

$T_1 = F_2 + W_2\sin\theta = 0.16 \text{ W} + 0.6\text{W}$

$T_1 = 0.76 \text{ W}$

From FBD (2)

$\Sigma F_y = 0$

$N_2 + W_1 \cos\theta = N_1$

$N_1 = N_2 + W_1 \cos\theta$

$N_1 = 0.8\text{W} + 1000 \times \frac{4}{5}$

$N_1 = 0.8 \text{ W} + 800$

$F_1 = \mu N_1 = 0.2\,(0.8 \text{ W} + 800)$

$= 0.16 \text{ W} + 160$

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = T_1 e^{\mu\beta} = 0.76 \text{ W } e^{0.2\times\pi}$

$T_2 = 1.42 \text{ W}$

$\Sigma F_x = 0$

$T_2 + F_1 + F_2 = W_1 \sin\theta$

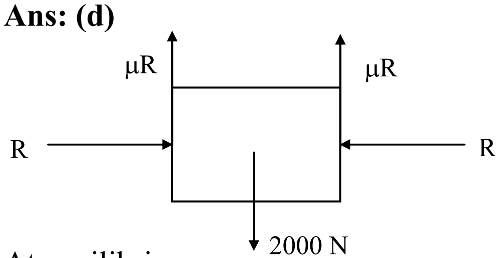
$1.42\text{W} + 0.16\text{W} + 160 + 0.16\text{W} = 1000 \times \frac{3}{5}$

$1.74 \text{ W} = 440$

$\Rightarrow \text{W} = 252.87 \text{ N}$

12. Ans: (d)

Sol:



At equilibrium

$2\mu R = 2000$

$\Rightarrow R = \dfrac{2000}{2\times 0.1} = 10{,}000\text{N}$

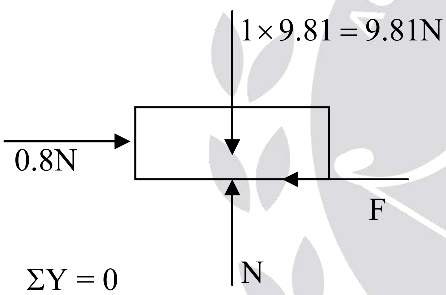
Taking moment about pin

$10{,}000\times 150 = F\times 300$

$\Rightarrow F = 5000\text{ N}$

13. Ans: (b)

Sol:



$\Sigma Y = 0$

$\Rightarrow N = 9.81\text{ N}$

$F_s = \mu N = 0.1\times 9.81 = 0.98\text{ N}$

The External force applied = 0.8 N < F_s

$\Rightarrow$ Frictional force = External applied force = 0.8 N

14. Ans: (b)

Sol:

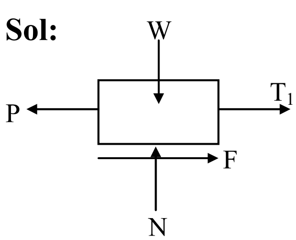


Fig: FBD (1)

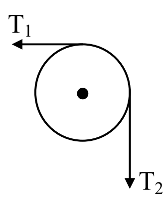


Fig: FBD (2)

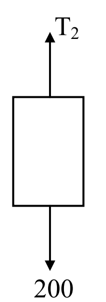


Fig: FBD (3)

From FBD (3)

$\Sigma F_y = 0$

$T_2 - 200 = 0$

$\Rightarrow T_2 = 200$

From FBD (2)

$\dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_1 = T_2\, e^{\mu\beta} = 200\times e^{0.3\times\frac{\pi}{2}}$

$T_1 = 320.39\text{ N}$

From FBD (1)

$\Sigma F_y = 0$

$N - W = 0$

$N = 1000\text{ N}$

$F = \mu N$

$\quad = 0.3\times 1000$

$F = 300\text{ N}$

$\Sigma F_x = 0,\ T_1 + F - P = 0$

$320.39 + 300 = P$

$\Rightarrow P = 620.39$

$\Rightarrow P = 620.4\text{ N}$

04. Kinematics of Particle Rectilinear and Curvilinear Motion

01. Ans: (d)

Sol: $x = 2t^3 + t^2 + 2t$

$V = \dfrac{dx}{dt} = 6t^2 + 2t + 2$

$a = \dfrac{dv}{dt} = 12t + 2$

At $t = 0 \Rightarrow V = 2$ and $a = 2$

02. Ans: (a)

Sol: $V = kx^3 - 4x^2 + 6x$

$V_{\text{at } x=2 \text{ if } k=1} = 2^3 - 4(2)^2 + 6(2) = 4$

$$a = \frac{dV}{dt} = k.3x^2\frac{dx}{dt} - 8x\frac{dx}{dt} + 6\frac{dx}{dt}$$

$a = 3x^2(V) - 8x(V) + 6(V)$

$= 3(2)^2\times4 - (8\times2\times4) + 6(4)$

$= 8 \text{ m/s}^2$

03. Ans: (d)

Sol: Given, $a = 6\sqrt{V}$

$$\frac{dV}{dt} = 6\sqrt{V}$$

$$\int\frac{dV}{\sqrt{V}} = \int 6\,dt$$

$$2\sqrt{V} = 6t + C_1$$

Given, at $t = 2$ sec, $V = 36$

$\Rightarrow 2\sqrt{36} = 6(2) + C_1$

$\Rightarrow C_1 = 0$

$2\sqrt{V} = 6t$

$V = 9t^2$

But $V = \frac{ds}{dt} = 9t^2$

$\int ds = \int 9t^2\,dt$

$S = 3t^3 + C_2$

At, $t = 2$ sec, $S = 30$ m

$\Rightarrow 30 = 3(2)^3 + C_2$

$\Rightarrow C_2 = 6$

$\therefore S = 3t^3 + 6$

At $t = 3$ sec

$S = 3(3)^3 + 6$

$S = 87$ m

04. Ans: (a)

Sol: Given $A = -8S^{-2}$

$$\Rightarrow \frac{dV}{dt} = \frac{d^2s}{dt^2} = -8s^{-2} = a$$

We know that, $\int V\,dv = \int a\,ds$

$$\frac{V^2}{2} = \int -8s^{-2}\,ds$$

$$\frac{V^2}{2} = \frac{8}{S} + C_1$$

Given, at $S = 4$m, $V = 2$ m/sec

$$\Rightarrow \frac{2^2}{2} = \frac{8}{4} + C_1$$

$\Rightarrow C_1 = 0$

$$\therefore \frac{V^2}{2} = \frac{8}{S}$$

$$V = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \frac{ds}{dt} = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \int\sqrt{s}\,ds = \int 4\,dt$$

$$\frac{2}{3}s^{3/2} = 4t + C_2$$

At $t = 1$, $S = 4$

$$\Rightarrow \frac{2}{3}(4)^{3/2} = 4(1) + C_2$$

$$\Rightarrow C_2 = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\therefore \frac{2}{3}s^{3/2} = 4t + C_2$$

$$\Rightarrow \frac{2}{3}s^{3/2} = 4t + \frac{4}{3}$$

At t = 2 sec

$$\frac{2}{3}s^{3/2} = 4(2) + \frac{4}{3}$$

$\Rightarrow$ s = 5.808 m

$$a = \frac{-8}{s^2} = \frac{-8}{5.808^2} = -0.237 \text{ m/sec}^2$$

05. Ans: (c)

Sol: Given, $a = 4t^2 - 2$

$$\frac{dv}{dt} = 4t^2 - 2$$

$$dv = (4t^2 - 2)\,dt$$

$$v = \frac{4t^3}{3} - 2t + C_1$$

$$\frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$$

$$\int dx = \int\left(\frac{4t^3}{3} - 2t + C_1\right)dt$$

$$x = \frac{4t^4}{3\times 4} - 2.\frac{t^2}{2} + C_1 t + C_2$$

$$x = \frac{t^4}{3} - t^2 + C_1 t + C_2$$

Given condition,

At t = 0, x = –2 m

$\Rightarrow -2 = C_2$

At t= 2, x = –20 m

$$\Rightarrow -20 = \frac{2^4}{3} - 2^2 + 4(2) + (-2)$$

$$\Rightarrow C_1 = \frac{-29}{3}$$

$$\therefore x = \frac{t^4}{3} - t^2 - \frac{29}{3}t - 2$$

$\therefore$ at t = 4 sec

$$x = \frac{4^4}{3} - 4^2 - \frac{29}{3}(4) - 2$$

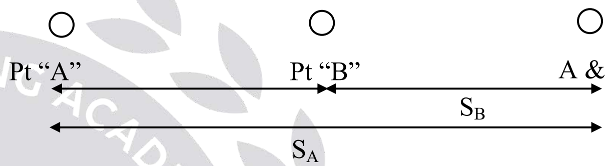
= 28.67 m

06. Ans: (b)

Sol:

u_A = 20 m/sec $\quad$ u_B = 60 m/sec

a_A = 5 m/sec^2 $\quad$ a_B = –3 m/sec^2



Let S_A be the distance traveled by "A"

Let S_B be the distance traveled by "B"

$S_A = S_B + 384$

$$u_A t + \frac{1}{2}a_A t^2 = u_B t + \frac{1}{2}a_B t^2 + 384$$

$$20t + \frac{1}{2}5t^2 = 60t - \frac{1}{2}3t^2 + 384$$

$$4t^2 - 40t - 384 = 0$$

t = 16 sec (or) t = –6 sec

$\therefore$ t = 16 sec

07. Ans: (b)

Sol: Take , $y = x^2 - 4x + 100$

Initial velocity, $V_0 = 4\hat{i} - 16\hat{j}$

If V_x is constant

V_y , a_y at x = 16 m

$$V_x = V_{1x} = \frac{dx}{dt} = 4$$

$$V_y = \frac{dy}{dt} = 2x\frac{dx}{dt} - 4\frac{dx}{dt}$$

$(V_y) = 2x\ (4) - 4(4)$

$V_y = 8x - 16$

$(V_y)_{at\ x = 16} = 8\ (16) - 16 = 112$ m/sec

$$a_y = \frac{dV}{dt} = \frac{d}{dt}(2xV_x - 4V_x)$$

($\because V_x$ = constant)

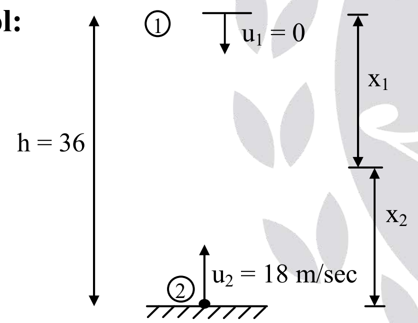
$$= 2V_x\frac{dx}{dt} = 2V_x.\ V_x$$

$$a_y = 2V_x^2$$

$(a_y)_{\ x = 16} = 2\times4^2 = 32$ m/sec^2

08. Ans: (c)

Sol:



Let at distance of "x_1' ball (1) crossed ball (2)

$\therefore x_1 + x_2 = 36$

$$x_1 = 0(t) + \frac{1}{2}gt^2 \quad (\because s = ut + \frac{1}{2}at^2)$$

$$x_1 = \frac{1}{2}gt^2 \text{ -------- (1)}$$

$$x_2 = 18(t) - \frac{1}{2}gt^2$$

($\because a = -g$ moving upward)

$x_1 + x_2 = 36$

$$\Rightarrow \frac{1}{2}gt^2 + 18t - \frac{1}{2}gt^2 = 36$$

$\Rightarrow 18\ t = 36$

$\Rightarrow t = 2$ sec

$$\therefore x_1 = \frac{1}{2}(9.81).2^2 = 19.62 \text{ m (from the top)}$$

$x_2 = 36 - 19.62$

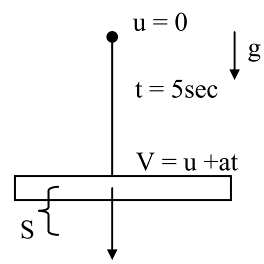
$= 16.38$ m (from the bottom)

09. Ans: (b)

Sol: $V = u + at$

$V = 0 + 9.81\ (5)$

$V = 49.05$ m/sec



V = velocity with which stone strike the glass

Velocity loss = 20% of V

$$= \frac{49.05\times20}{100} = 9.81 \text{ m/sec}$$

$\therefore$ Initial velocity for further movement in glass = 49.05 – 9.81 = 39.24 m/sec

Distance traveled for 1 sec of time is given by

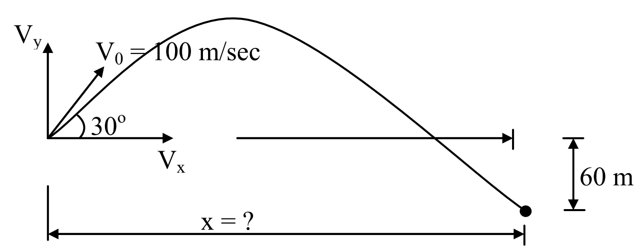
$$S = ut + \frac{1}{2}at^2$$

$$S = 39.24(1) + \frac{1}{2}(9.81)(1)^2$$

$S = 44.145$ m

10. Ans: (a)

Sol:



$a_x = -4$ m/sec^2 , $a_y = -20$ m/sec^2

$V_x = V_0 \cos 30 = 100\times\frac{\sqrt{3}}{2} = 86.6$ m/sec

$V_y = V_0 \sin 30 = 100\times\frac{1}{2} = 50$ m/sec

$$y = V_{oy}t + \frac{1}{2}a_y t^2$$

$$-60 = 50t + \frac{1}{2}(-20)t^2$$

$10t^2 - 50t - 60 = 0$

$t = 6$ (or) -1 sec

$\therefore t = 6$ sec

$$x = V_0 t + \frac{1}{2}a_x t^2$$

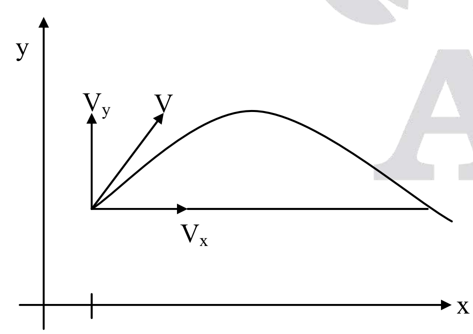
$$x = (86.6 \times 6) + \frac{1}{2}(-4)6^2$$

$x = 447.6 \text{ m} \simeq 448$ m

11. Ans: (a)

Sol: Given, V = 20 m/sec

x = 20 m, y = 8.0 m



$V_x = V\cos\theta$, $V_y = V\sin\theta$

$x = V_x t + \frac{1}{2}at^2$ ($\because$ a = 0 along x direction)

$x = V\cos\theta\, t$

$20 = 20 \cos\theta t$

$$t = \frac{1}{\cos\theta} \text{ ------- (1)}$$

$$y = V_y t - \frac{1}{2}gt^2$$

$$8.0 = V\sin\theta t - \frac{1}{2}gt^2$$

$$8.0 = 20\sin\theta\times\frac{1}{\cos\theta} - \frac{1}{2}\times 9.81\times\left(\frac{1}{\cos\theta}\right)^2$$

$8 = 20\tan\theta - 4.9\sec^2\theta$

$8 = 20\tan\theta - 4.9(1+\tan^2\theta)$

$4.9\tan^2\theta - 20\tan\theta + 12.9 = 0$

$\tan\theta_1 = 3.28$, $\tan\theta_2 = 0.803$

$\theta_1 = 73.04$; $\theta_2 = 38.76$

12. Ans: (d)

Sol: Range = maximum height

$$\frac{V_0^2\sin 2\theta}{g} = \frac{V_0^2\sin^2\theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2\theta}{2}$$

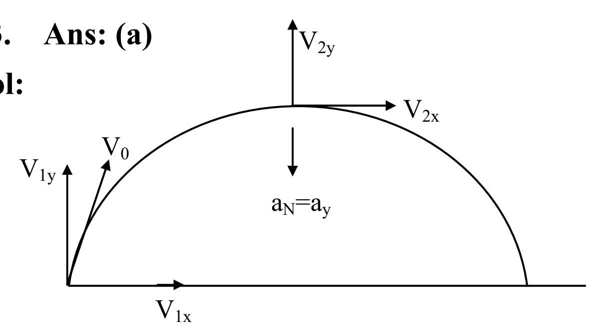
$$\Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$\Rightarrow \tan\theta = 4$

$\therefore \theta = \tan^{-1}(4) = 76°$

13. Ans: (a)

Sol:



$V_{1x} = 100 – t^{3/2}$

$V_{2y} = 0 \Rightarrow 100 + 10t – 2t^2 = 0$

$(t–10)(t+5) = 0$

$t = 10$ sec

V_{2x} at $t = 10 \Rightarrow V_{2x} = 100 – 10^{3/2}$

$= 68.37$ m/sec

Radius of curvature, $r = \dfrac{V^2}{a_N}$

Where $a_N = a_y = \left(\dfrac{dV_y}{dt}\right)_{at\ t=10\,sec}$

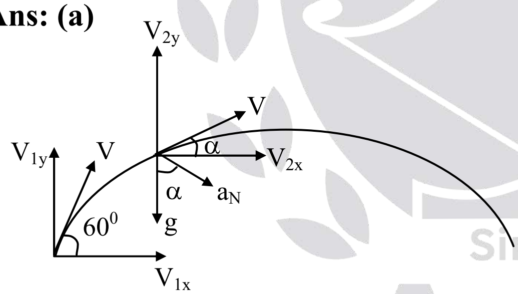
$= (10 – 4t)_{t=10}$

$a_N = –30$ m/sec^2

$$r = \frac{V_{2x}^2}{a_N} = \frac{68.37^2}{30} = 155.8 \text{ m}$$

14. Ans: (a)

Sol:



Given, v = 100 m/sec

$v_{1x} = v\cos60^0 = 100\times1/2$

$v_{1x} = 50$ m/sec

$v_{1y} = v \sin60 = 100\times\dfrac{\sqrt{3}}{2}$

$v_{1y} = 86.6$ m/sec

$v_{2y} = v_{1y} – gt$ (use $V = u+at$)

$= 86.6 – 9.8(1)$

$v_{2y} = 76.8$ m/sec

$v_{2x} = v_{1x} = 50$ m/sec

$$v_{at\ t=1} = \sqrt{v_{2x}^2 + v_{2y}^2}$$

$$= \sqrt{50^2 + 76.8^2}$$

$= 91.6$m/sec.

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \text{Tan}^{-1}\left(\frac{76.8}{50}\right)$$

$\alpha = 56.9$

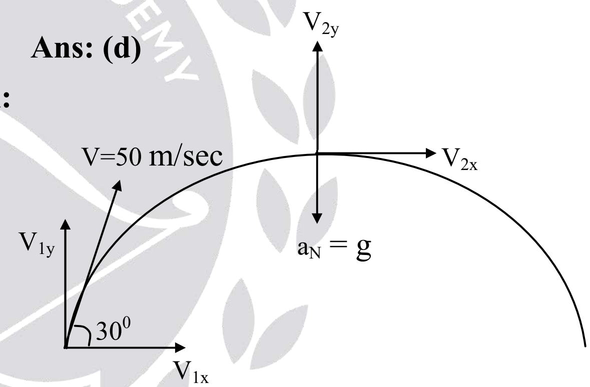
$a_N = g\cos\alpha = 9.81\times\cos56.9$

$= 5.35$m/sec^2

$$r = \frac{V^2}{a_N} = \frac{91.6^2}{5.35} = 1568.62 \text{ m}$$

15. Ans: (d)

Sol:



$v_{1x} = v \cos30 = 43.3$ m/sec

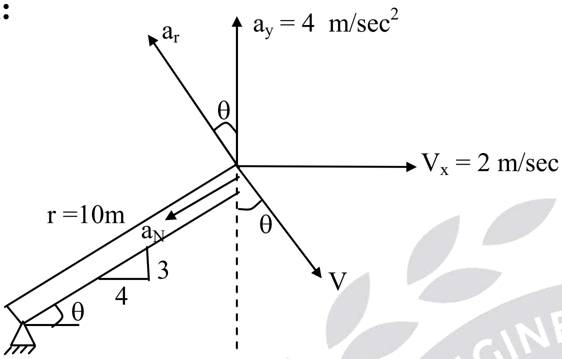
$a_N = g = a$

$$r = \frac{V_{1x}^2}{a_N} = \frac{43.3^2}{9.81} = 191.13 \text{ m}$$

05. Kinematics of Rigid Bodies Fixed Axis Rotation and General Plane Motion

01. Ans: (a)

Sol:



$\tan\theta = \dfrac{3}{4}$

$\theta = \text{Tan}^{-1}\, 3/4 = 36.6^0$

$a_y = a_T\cos\theta - a_N\sin\theta$

Note: Velocity will always act in the tangential direction

$V_x = V\sin\theta$

$V = \dfrac{2}{\sin 36.6} = 3.33$ m/sec

$\therefore\ a_N = \dfrac{V^2}{r} = \dfrac{3.33^2}{10}$

$a_N = 1.111$ m/sec^2

$a_y = a_T\cos\theta - a_N\sin\theta$

$4 = a_T\cos 36.6 - 1.111\sin 36.6$

$\Rightarrow a_T = 5.83$ m/sec^2

$a_T = r\alpha$

$\alpha = \dfrac{a_T}{r} = \dfrac{5.83}{10}$

$= 0.583$ rad/sec^2

02. Ans: (c)

Sol: Given $\omega = 4\sqrt{t}$

$\theta = 2$ radians at t = 1sec

$\theta = ?\ \alpha = ?$ at t = 3sec

$\omega = \dfrac{d\theta}{dt} \Rightarrow \int d\theta = \int \omega dt$

$\theta = \int 4\sqrt{t}dt$

$\theta = \dfrac{8}{3}t^{3/2} + c \ldots(1)$

From given condition, at t = 1, θ = 2rad

$(1) \Rightarrow 2 = \dfrac{8}{3}(1)^{3/2} + c_1 \Rightarrow c_1 = \dfrac{-2}{3}$

$\therefore\ \theta = \dfrac{8}{3}t^{3/2} - \dfrac{2}{3}$

At t = 3 sec , $\theta = \dfrac{8}{3}(3)^{3/2} - \dfrac{2}{3}$

$\theta_{t=3} = 13.18$rad

$\alpha = \dfrac{d\omega}{dt} = \dfrac{d(4\sqrt{t})}{dt} = \dfrac{2}{\sqrt{t}}$

$\alpha_{t=3} = \dfrac{2}{\sqrt{3}} = 1.15\text{rad/sec}^2$

03. Ans: (b)

Sol: r = 2 cm, $\omega = 3$ rad/sec , a = 30 cm/s^2

$a_N = r\omega^2 = 2(3)^2 = 18$ cm/sec^2

Since total acceleration $a = \sqrt{a_T^2 + a_N^2}$

$\Rightarrow a^2 = a_T^2 + a_N^2$

$30^2 = a_T^2 + 18^2$

$a_T = 24$ cm/sec^2

$a_T = r\alpha = 24$

$\alpha = \dfrac{24}{2} = 12\text{rad/sec}^2$

04. Ans: (d)

Sol: Given angular acceleration, $\alpha = \pi$ rad/sec^2

Angular displacement in time t_1 and t_2

$= \pi \text{ rad} = \theta_2 - \theta_1$

$\omega_{t2} = 2\pi$ rad/sec

$\omega_{t1} = ?$

$$\omega_{t1}^2 - \omega_0^2 = 2\alpha\theta_1$$

$$\omega_{t2}^2 - \omega_0^2 = 2\alpha\theta_2$$

$$\omega_{t2}^2 - \omega_{t1}^2 = 2\alpha(\theta_2 - \theta_1)$$

$$4\pi^2 - \omega_{t1}^2 = 2\pi^2$$

$$\omega_{t1}^2 = 2\pi^2$$

$$\omega_{t1} = \pi\sqrt{2}$$

05. Ans: (c)

Sol: Given retardation

$\alpha = -3t^2$

$$\frac{d\omega}{dt} = -3t^2$$

$$\int d\omega = \int -3t^2 dt$$

$\omega = -t^3 + c_1$

From given condition at $t = 0$,

$\omega = 27$ rad/sec

$27 = -0^3 + c_1$

$\Rightarrow c_1 = 27$

$\therefore \omega = -t^3 + 27$

Wheel stops at $\omega = 0$,

$\Rightarrow 0 = -t^3 + 27$

$\Rightarrow t = 3$sec

06. Ans: (c)

Sol: angular speed, $\omega = 5$ rev/sec

$= 5 \times 2\pi$ rad/sec

$\omega = 10\pi$ rad/sec

Radius, $r = 0.1$m

If ω is constant, $d\omega = 0$

$\Rightarrow \alpha = 0 \Rightarrow a_T = 0$ (since $a_T = r\alpha$)

Since $a_T = 0$

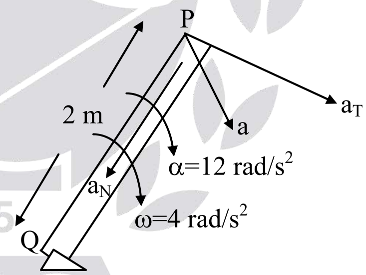
$$a = \sqrt{a_N^2 + a_T^2}$$

$$a = a_N = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$= 0.1 \times (10\pi)^2 = 10\pi^2 \text{ m/sec}^2$$

07. Ans: a = 40m/s^2

Sol:



Tangential acceleration

$a_T = r\,\alpha = 2 \times 12 = 24$m/s^2

Normal acceleration, $a_N = r\,\omega^2$

$= 2 \times 4^2 = 32$ m/s^2

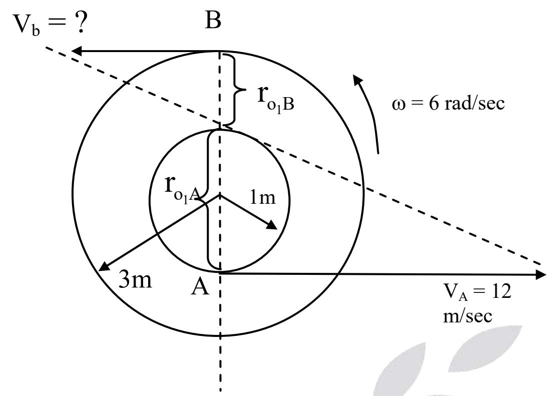
The resultant acceleration

$$a = \sqrt{a_T^2 + a_N^2}$$

$$= \sqrt{24^2 + 32^2} = 40 \text{m/s}^2$$

08. Ans: (b)

Sol:



$V_A = r_{o_1A} \times \omega$

$\Rightarrow 12 = r_{o_1A} \times 6$

$r_{o_1A} = 2m$

$4 = 2 + r_{o_1B}$

$r_{o_1B} = 2m$

$\therefore V_B = r_{o_1B} \times \omega = 2 \times 6$

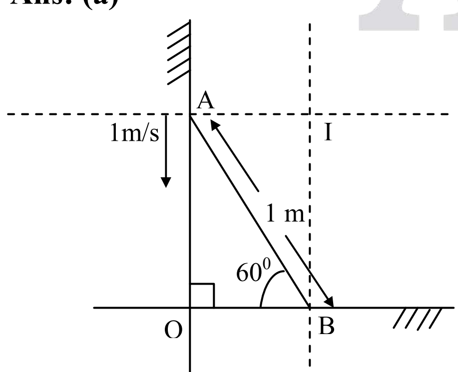
$V_B = 12$ m/sec

09. Ans: (a)

Sol: Instantaneous centre will have zero velocity because the instantaneous centre is the point of contact between the object and the floor.

10. Ans: (a)

Sol:



V_a = 1 m/s

V_a = along vertical

V_b = along horizontal

So instantaneous center of V_a and V_b will be perpendicular to A and B respectively

$$IA = OB = 1 \times \cos\theta = 1 \times \cos 60^0 = \frac{1}{2} m$$

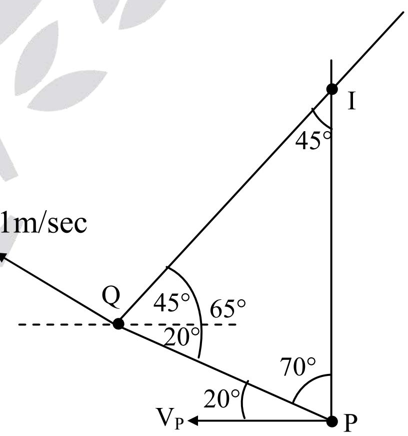
$$IB = OA = 1 \times \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2} m$$

$$V_a = \omega \times IA$$

$$\Rightarrow \omega = \frac{V_a}{IA} = 2 \text{ rad/sec}$$

11. Ans: (d)

Sol: Refer the figure shown below, by knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of ‘P using sine rule.



‘I’ is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

$$V_Q = IQ \times \omega = 1$$

$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

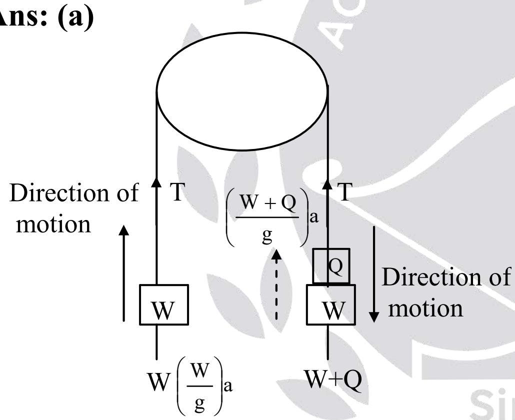
$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1$$

$$= 0.9645$$

06. Kinetics of Particle and Rigid Bodies

01. Ans: (a)

Sol:



For the left cord,

$\Sigma F_y = 0$

$$T = \left(\frac{W}{g}\right)a + W \text{ ....(1)}$$

For the right cord

$\Sigma F_y = 0$

$$T + \left(\frac{W+Q}{g}\right)a = (W+Q) \ldots(2)$$

From (1) & (2)

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W+Q}{g}\right)a$$

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W}{g}\right)a - \left(\frac{Q}{g}\right)a$$

$$Q - \frac{Qa}{g} = \frac{2Wa}{g}$$

$$Q\left(\frac{g-a}{g}\right) = \frac{2Wa}{g} \Rightarrow Q = \frac{2Wa}{g-a}$$

02. Ans: (b)

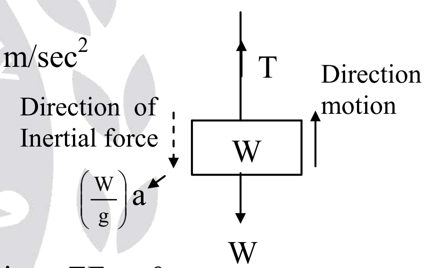
Sol: $u = 0$, $v = 1.828$ m/sec, $S = 1.825$ m,

$$v^2 - u^2 = 2as$$

$$1.828^2 - 0 = 2a \times 1.828$$

$$a = \frac{1.828}{2}$$

$$a = 0.914 \text{ m/sec}^2$$



For equilibrium, $\Sigma F_y = 0$

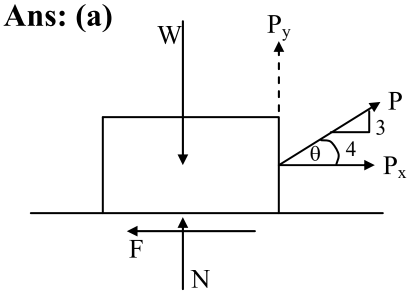
$$T = W + \left(\frac{W}{g}\right)a$$

$$= 4448 + \frac{4448}{9.81} \times 0.194$$

$$T = 4862.42 \text{ N}$$

03. Ans: (a)

Sol:



$$\tan\theta = \frac{3}{4}$$

$\theta = \tan^{-1}(3/4) = 36.86$

$(F_{net})_x = ma$

$P_x – F = \left(\frac{W}{g}\right)a$

$P\cos36.86 – F = \left(\frac{W}{g}\right)a$

$0.8P – F = \left(\frac{2224}{g}\right)(0.2g)$

$0.8P – F = 444.8$

$0.8P – F = 444.8 + F$

$P = 556+1.25F$ ……(1)

$\Sigma F_y = 0$

$N+P_y–W = 0$

$N = W – P_y$ (since $\mu = \frac{F}{N}$)

$F = \mu N$

$F = \mu\,(W – P_y)$

$= 0.2(2224 – P \sin 36.86)$

$F = 444.8 – 0.12P$ …..(2)

From (1) & (2)

$P = 556+1.25(444.8 – 0.12P)$

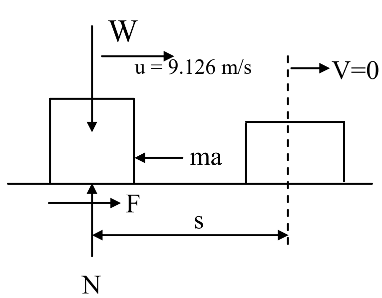
$1.15P = 1112$

$P = 966.95$

$P = 967$ N

04. Ans: (d)

Sol:



From static equilibrium condition

$\Sigma F_y = 0$

$N–W = 0$

$N = W = 44.48$N

From dynamic equilibrium condition

$\Sigma F_x = 0$

$F = ma$

$\mu N = \frac{W}{g}a$

$\mu = \frac{a}{g}$

$a = \mu g$ ….(1)

Since $v^2 – u^2 = 2as$

$0 – (9.126)^2 = 2(–a) \times 13.689$

$a = 3.042$ ….(2)

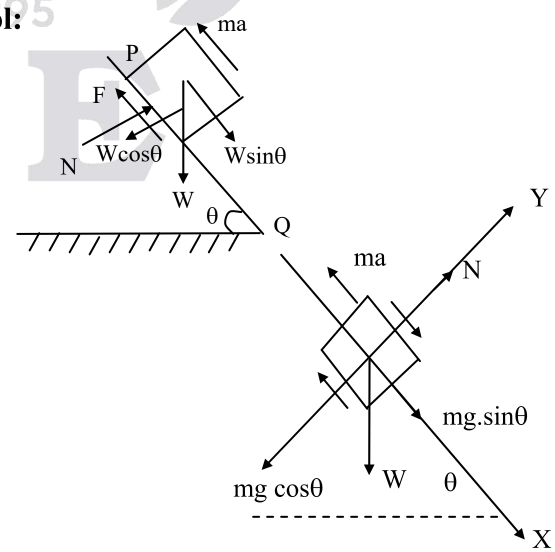
From (1) & (2)

$3.042 = \mu(9.81)$

$\Rightarrow\ \mu = 0.31$

05. Ans: (a)

Sol:



$\Sigma F_y = 0$ (static equilibrium)

$N - W\cos\theta = 0$

$N = W\cos\theta = mg\cos\theta$

Since $F = \mu N = \mu\, mg\cos\theta$….(1)

$\Sigma F_x = 0$ (Dynamic equilibrium)

$F + ma - W\sin\theta = 0$

$F = -ma + mg\sin\theta$

$F = mg\sin\theta - ma$…(2)

From (1) & (2)

$\mu mg\cos\theta = mg\sin\theta - ma$

$\Rightarrow a = g\sin\theta - \mu g\cos\theta$

$\Rightarrow a = g\cos\theta(\tan\theta - \mu)$

Given PQ = s

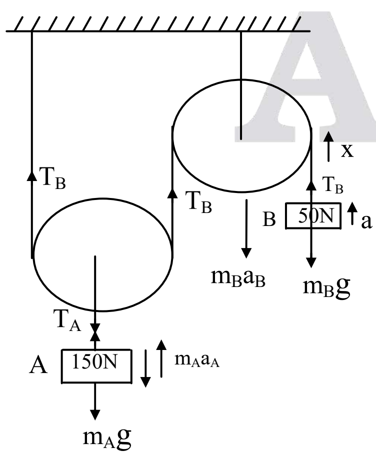
$s = ut + \frac{1}{2}at^2$

$s = 0(t) + \frac{1}{2}at^2 \quad \Rightarrow t = \sqrt{\frac{2s}{a}}$

$= \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$

06. Ans: (a)

Sol:



$T_A = 2T_B$ ….(1)

Work done by A & B equal

$T_AS_A = T_BS_B$

$2T_BS_A = T_B\, S_B$

$2S_A = S_B$

$2a_A = a_B$ ….(2)

For 'B' body

$T_B = m_Ba_B + m_Bg$ ….(3)

For 'A' body

$T_A = m_Ag - m_Aa_A$ ….(4)

(2), (3) & (4) sub in (1)

$m_A\, g - m_A\, a_A = 2(m_B(2a_A) + m_B\, g)$

$m_A\, g - m_A\, a_A = 4m_B\, a_A + 2m_B\, g$

$m_A\, a_A + 4m_B\, a_A = m_Ag - 2m_B\, g$

$$a_A = \frac{m_Ag - 2m_Bg}{m_A + 4m_B}$$

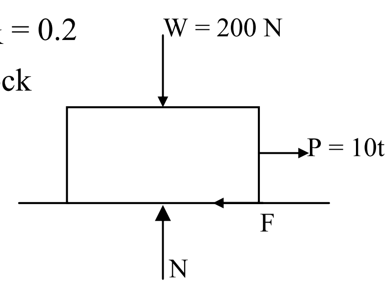
$$= \frac{150 - 2(50)}{\frac{150}{10} + 4\left(\frac{50}{10}\right)}$$

$$= \frac{50}{15+20} = \frac{50}{35} = 1.42$$

07. Ans: 4.905 m/s

Sol: $\mu_S = 0.4$; $\mu_K = 0.2$

FBD of the block



W.r.t free body diagram of the block:

$F_S = \mu_SN$;

$F_K = \mu_KN$

$\Sigma F_y = 0$

$N - W = 0$

$N = W = 200$ N

Limiting friction or static friction

$(F_S) = 0.4 \times 200 = 80$ N

Kinetic Friction

$(F_K) = 0.2 \times 200 = 40$ N

The block starts moving only when the force, P exceeds static friction, F_S

Thus, under static equilibrium

$\Rightarrow \sum F_x = 0$

$\Rightarrow P - F_S = 0 \Rightarrow 10t = 80$

$$t = \frac{80}{10} = 8 \text{ sec}$$

$\therefore$ The block starts moving only when t >8seconds

During 8 seconds to 10 seconds of time:

According to Newton's second law of motion

Force = mass × acceleration

$$(P - F_K) = m \times \frac{dv}{dt} \Rightarrow (10t - 40) = \frac{200}{9.81} \times \frac{dv}{dt}$$

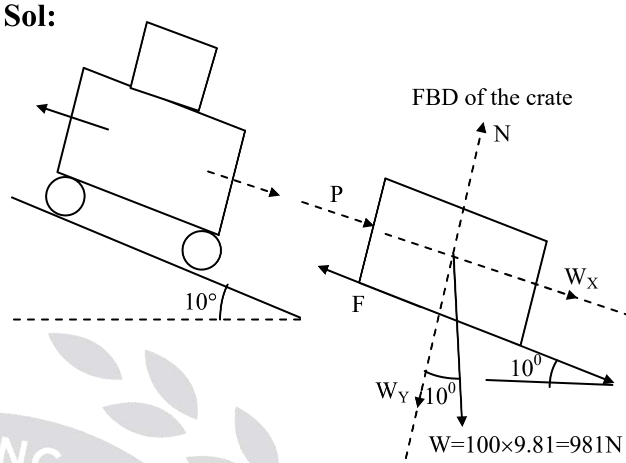
$$\int_8^{10} (10t - 40)dt = \frac{200}{9.81}\int_0^V dv$$

$$\left[5t^2 - 40t\right]_8^{10} = 20.387 \times V \Rightarrow (180 - 80) = 20.387 \times V$$

Velocity (V) = 4.905 m/s

08. Ans: 1.198 m/s^2

Sol:



W.r.t. FBD of the crate:

$W_X = W\sin 10^0 = 981 \times \sin 10^0$

$= 170.34$N

$W_Y = W\cos 10^0 = 981 \times \cos 10^0 = 966.09$ N

$\sum F_Y = 0 \Rightarrow N - W_Y = 0$

$N = W_Y = 966.09$N;

$F = \mu N = 0.3 \times 966.09 = 289.828$ N

$\sum F_X = 0 \Rightarrow P + W_X - F = 0$

$\Rightarrow P + 289.828 - 170.34 = 0$

$P = 119.488$ N

$P = ma = 119.488$ N

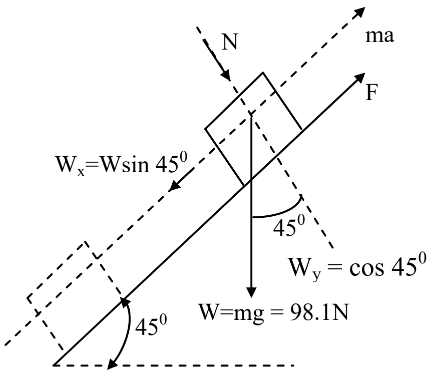
$$\Rightarrow a = \frac{119.488}{100} = 1.198 \text{ m/s}^2$$

09. Ans: 57.67 m

Sol: $W_x = W \sin 45$

$= 98.1 \times \sin 45 = 69.367$ N

$W_y = W \cos 45 = 69.367$ N



$\Sigma F_Y = 0$

$N - W_Y = 0$

$N = W_Y = 69.367$ N

$F = \mu_K N = 0.5 \times 69.367 = 34.683$N

$\Sigma F_x = 0$ (Dynamic Equilibrium

D$'$ Alembert principle)

$W_x - F - ma = 0$

$69.367 - 34.683 - 10\times a = 0$

$a = 3.468 m/s^2$

$S = ut + \frac{1}{2}at^2$

$\because$ t is unknown we can not use this equation

So use $V^2 - u^2 = 2as$

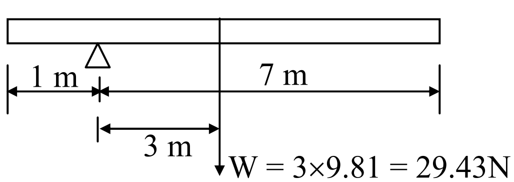
$V = 20 m/s^2$; $u = 0$; $a = 3.468 m/s^2$

$V^2 = 2as$

$$S = \frac{V^2}{2\times a} = \frac{20^2}{2\times 3.468} = 57.67 m$$

10. Ans: 2.053 rad/s²

Sol:



$M = I\alpha$

$M = 29.43 \times 3 = 88.29$N-m

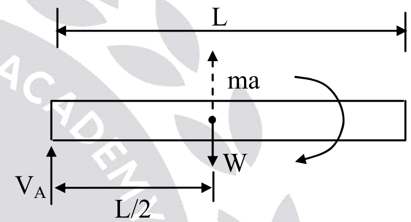
$$I = I_0 + Ad^2 = \frac{m\ell^2}{12} + md^2 = \frac{3\times 8^2}{12} + 3\times 3^2$$

$$= 16 + 27 = 43 kg - m^2$$

$$\alpha = \frac{M}{I} = \frac{88.29}{43} = 2.053 \text{ rad/s}^2$$

11. Ans: (d)

Sol:



$\Sigma F_y = 0$

$V_A + ma = W$

$V_A = m(g-a)\ldots(1)$

Where, $a = \frac{L}{2}\alpha$

Since, $M = I\alpha$

$$W\times\frac{L}{2} = \left(\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2\right)\alpha$$

$$mg\times\frac{L}{2} = \frac{4mL^2}{12}\times\frac{2a}{L}$$

$$a = \frac{3}{4}g\ldots(2)$$

from (1) & (2)

$$V_A = m\left(g - \frac{3}{4}g\right) = \frac{mg}{4}$$

$$V_A = \frac{W}{4}$$

12. Ans: (d)

Sol: $I = 5kg.m^2$

$R = 0.25m$

$F = 8N$

Mass moment of inertia, $I_x = I_y = \dfrac{mr^2}{4}$

$I_z = \dfrac{mr^2}{2}$

$M = I\alpha$

$8\times0.25 = 5\times\alpha$

$\alpha = 0.4$

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\omega^2 - 0^2 = 2(0.4)\times\pi$ (since for half revolution $\theta = \pi$)

$\omega = 1.58$ rad/sec

13. Ans: 4.6 seconds

Sol: $M = 60$ N – m

$L = 2m$, $\omega_0 = 0$,

$\omega = 200 \text{ rpm} = \dfrac{200\times2\pi}{60}$

$\omega = 20.94 \dfrac{\text{rad}}{\text{sec}}$

Moment, $M = I\alpha$

$60 = \dfrac{mL^2}{12}\times\alpha$

$\Rightarrow 60 = \dfrac{40\times2^2}{12}\times\alpha$

$\alpha = 4.5 \text{rad/sec}^2$

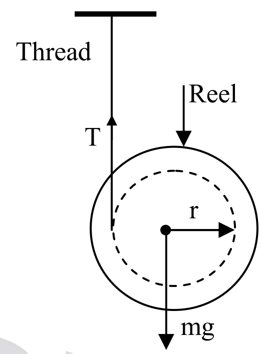
$\omega = \omega_0 + \alpha t$

$20.94 = 4.5t$

$\Rightarrow$ $t = 4.65$ sec

14. Ans: (a)

Sol:



a = linear acceleration,

k = radius of gyration

For vertical translation motion

$mg - T = ma$ ------ (1)

For rotational motion

$T \times r = I\alpha$

$Tr = mk^2\alpha = mk^2 \times \dfrac{a}{r}$

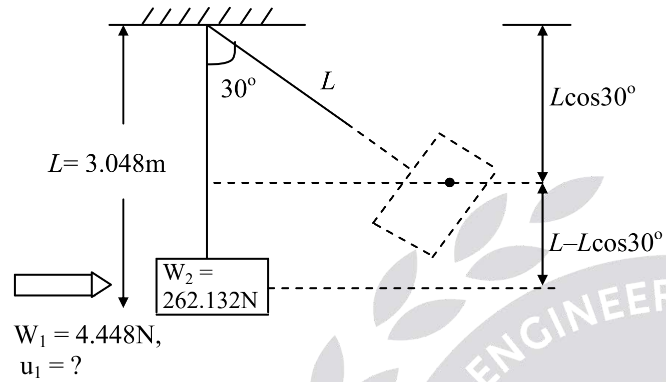
$\Rightarrow T = \dfrac{mk^2}{r^2}\times a$ ------(2)

$mg - \dfrac{mk^2}{r^2}\times a = ma \Rightarrow a = \dfrac{gr^2}{(k^2+r^2)}$

07. Work-Energy Principle and Impulse Momentum Equation

01. Ans: (a)

Sol:



The loss of KE of shell converted to do the work in lifting the sand box and shell to a height of "$L - L\cos30^{o}$"

i.e., $Wd = \frac{1}{2}mV^2$

Where $d = L - L\cos30^{o}$

$= 3.048 - 3.048\times\cos30 = 0.41$ m

$266.58\times0.41 = \frac{1}{2}\left(\frac{266.58}{9.81}\right)\times V^2$

$\Rightarrow V = 2.83$ m/sec

Where V is the velocity of block & shell

By momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Where $v_1 = v_2 = V$ & $u_1 = ?$, $u_2 = 0$

$$\frac{4.448}{9.81}\times u_1 = \frac{4.448+262.132}{9.81}\times 2.83$$

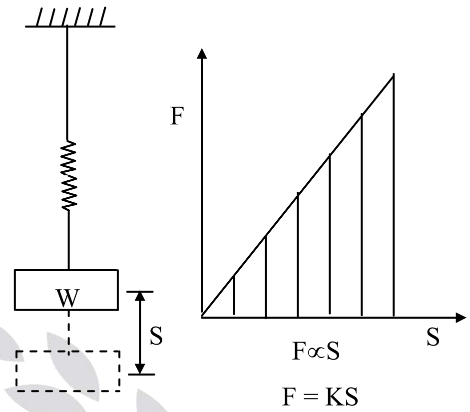
$\Rightarrow u_1 = 169.6$ m/sec

u_1 & u_2 = Initial velocity of shell and block respectively

V_1 & V_2 = Final velocity of block & shell

02. Ans: (b)

Sol:



Strain energy in spring = Area under the force displacement curve.

$$= \frac{1}{2}F\times s = \frac{1}{2}(ks)\times s = \frac{1}{2}ks^2$$

$$\frac{1}{2}ks^2 = \text{Gain of KE}$$

$$\frac{1}{2}ks^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{ks^2}{m} = \frac{ks^2}{w}g$$

$$v = \sqrt{\frac{kg}{w}}.s \qquad \left(\because m = \frac{w}{g}\right)$$

03. Ans: (a)

Sol: Given, m = 2 kg

Position at any time is given as

$$x = t + 5t^2 + 2t^3$$

At t = 0, x = 0,

At t = 3sec,

$x = 3 + 5(3^2) + 2(3^3) = 102$m

Velocity, $V = \frac{dx}{dt} = 1 + 10t + 6t^2$

Initial velocity i.e., $t = 0$, is $v_i = 1$m/s

Final velocity i.e., at $t = 3$sec,

is $v_f = 1 + 10(3) + 6(3)^2 = 85$m/s

Work done = change in KE

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}\times 2\left(85^2 - 1^2\right) = 7224 \text{ J}$$

04. Ans: (a)

Sol: Given force $F = e^{-2x}$

$$\text{Work done} = \int_{x_1}^{x_2} F dx$$

$$= \int_{0.2}^{1.5} e^{-2x} dx = \left[\frac{e^{-2x}}{-2}\right]_{0.2}^{1.5} = 0.31\text{J}$$

05. Ans: (b)

Sol: $F = 4x - 3x^2$

Potential Energy at $x = 1.7$ = work required to move object from 0 to 1.7m

$$PE = \int_0^{1.7} F dx$$

$$= \int_0^{1.7} \left(4x - 3x^2\right) dx$$

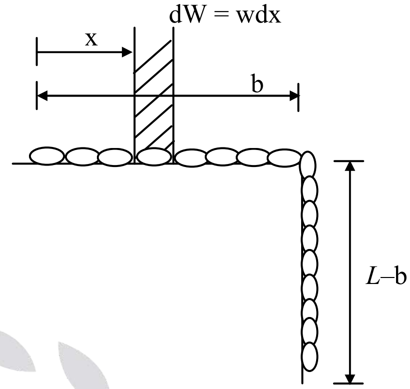
$$= \left[4\left(\frac{x^2}{2}\right) - 3\left(\frac{x^3}{3}\right)\right]_0^{1.7}$$

$$= \left[2x^2 - x^3\right]_0^{1.7}$$

$$= 2(1.7)^2 - (1.7)^3 = 0.867 \text{ J}$$

06. Ans: (c)

Sol:



Where w = weight per unit meter

dw = a small work done in moving small elemental "dx" of chain through a d/s "x"

Work done = change in KE

$$\left(\int_0^b dw \times x\right) + \left(w(L-b)\times b\right) = \frac{1}{2}\left(\frac{wL}{g}\right)v^2$$

$$\int_0^b wdx.x + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + wLb - wb^2 = \frac{1}{2}\frac{wLv^2}{g}$$

$$wLb - \frac{wb^2}{2} = \frac{1}{2}\frac{wLv^2}{g}$$

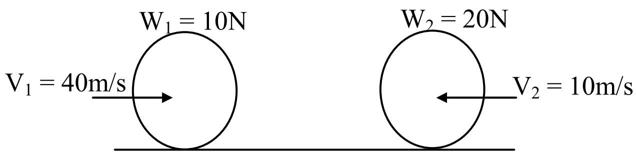
$$b\left(L - \frac{b}{2}\right) = \frac{1}{2}\frac{Lv^2}{g}$$

$$v^2 = 2gb\left(1 - \frac{b}{2L}\right)$$

$$v = \sqrt{gb\left(2 - \frac{b}{L}\right)}$$

07. Ans: (d)

Sol:



$m_1 = 1kg$, $m_2 = 2kg$,(since $g = 10m/sec^2$)

Velocities before impact

$v_1 = 40$ m/sec, $v_2 = -10m/s$

Velocities after impact

$u_1 = ?$ $u_2 = ?$

Coefficient of restitution e = 0.6

From momentum equation

$$m_1v_1+m_2v_2 = m_1u_1+m_2u_2$$

$\Rightarrow 1(40) + 2(-10) = 1(u_1) + 2(u_2)$

$\Rightarrow u_1 + 2u_2 = 20$…………………..(1)

$$e = \frac{u_2 - u_1}{v_1 - v_2} = \frac{\text{relative velocity of Seperation}}{\text{relative velocity of approach}}$$

$$0.6 = \frac{u_2 - u_1}{40-(-10)}$$

$\Rightarrow u_2 - u_1 = 30$……………………….(2)

From 1 & 2

$u_1 = -13.33$ m/sec

$u_2 = 16.66$ m/sec

08. Ans: (b)

Sol: Given, $m_1 = 3$ kg, $m_2 = 6$ kg

Velocities before impact

$u_1 = 4$ m/s, $u_2 = -1$ m/s

Velocities after impact

$v_1 = 0m/s$, $v_2 =?$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3(4) + 6(-1) = 3(0) + 6(v_2)$

$\Rightarrow 6 = 6v_2$

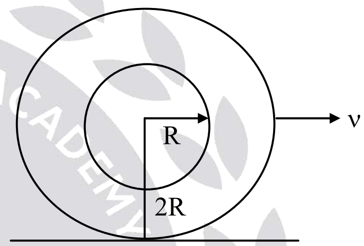
$\Rightarrow v_2 = 1m/s$

Coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$e = \frac{1-0}{4-(-1)} = \frac{1}{5}$$

09. Ans: (c)

Sol:



$$KE = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

Where, $\omega = \dfrac{V}{2R}$

$$I = \frac{1}{2}m\left((2R)^2 + R^2\right) = \frac{5}{2}mR^2$$

$$\therefore KE = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2$$

$$KE = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2$$

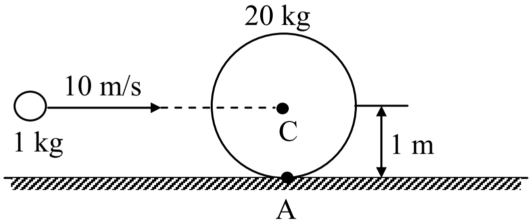
$$= \frac{1}{2}mV^2 + \frac{5}{4}mR^2 \times \frac{V^2}{4R^2}$$

$$= \frac{1}{2}mV^2 + \frac{5}{16}mV^2$$

$$KE = \frac{13mV^2}{16}$$

10. Ans: (a)

Sol:



Method I **:**

By conservation of linear momentum ,we get $1\times10 = (20+1)\times V_{cm}$ (where, V_{cm} = velocity of centre of mass)

$$\Rightarrow V_{cm} = \frac{10}{21}\,\text{m/s}$$

Applying angular momentum conservation about an axis passing through the contact point (A) and perpendicular to the plane of paper, we get

$$1\times10\times1 = I_{cm}\omega + 21\times\frac{10}{21}\times1$$

[Angular momentum about any axis passing through A can be written as, $\vec{L}_A = \vec{L}_{cm} + m\left(\vec{r}\times\vec{V}_{cm}\right)$]

$\Rightarrow \omega = 0$ rad/sec

Method II **:**

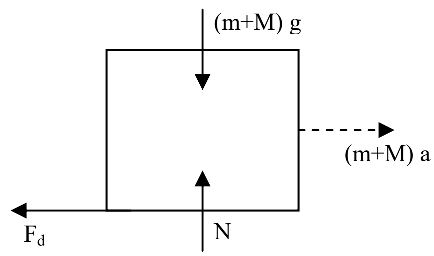
Applying angular momentum conservation about an axis passing through centre of wheel and perpendicular to the plane of paper.

$\therefore\ 0 = I_{cm}\,\omega$

$\Rightarrow \omega = 0$ rad/sec

11. Ans: (a)

Sol:



$m_1 = m \rightarrow$ mass of bullet

$m_2 = M \rightarrow$ mass of block

$u_1 = V \rightarrow$ bullet initial velocity

$u_2 = 0 \rightarrow$ block initial velocity

$v_1 = v_2 = v \rightarrow$velocity of bullet and block after impact.

$$F_d = \mu N$$

$(M+m)a = \mu(M+m)g$

$\Rightarrow a = \mu g$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mV + m(0) = (m+M)V$

$$v = \frac{mV}{m+M}$$

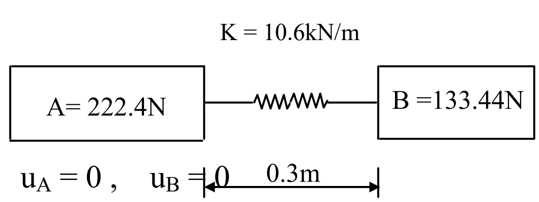
Now from $v^2 - u^2 = 2as$

$$0-\left(\frac{mV}{m+M}\right)^2 = 2\mu gs$$

$$V = \frac{m+M}{m}\sqrt{2\mu gs}$$

12. Ans: (a)

Sol:



$u_A = 0$, $u_B = 0$

From momentum equation

$m_Au_A+m_Bu_B = m_Av_A+m_Bv_B$

$0 = 222.4V_A+133.44V_B$…………..(1)

$$\frac{1}{2}ks^2 = \frac{1}{2}m_Av_A{}^2 + \frac{1}{2}m_Bv_B{}^2$$

$$10.6\times10^3\times0.15^2 = \frac{222.4}{9.81}v_A^2 + \frac{133.44}{9.81}v_B^2$$

…………(2)

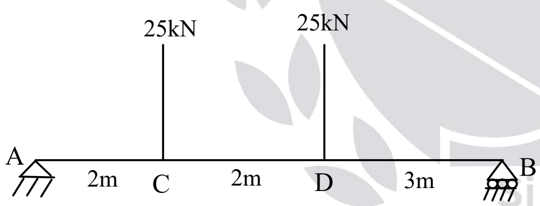
From 1 & 2

$v_A = -1.98$ m/s , $v_B = 3.3$ m/s

08. Virtual Work

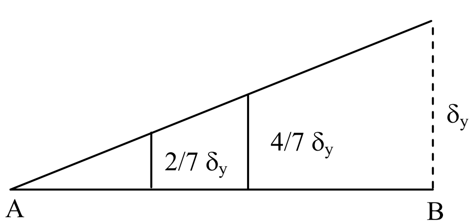
01. Ans: $\frac{200}{7}$kN, $\frac{150}{7}$kN

Sol:



Let R_A & R_B be the reactions at support A & B respectively.

Let δ_y displacement be given to the beam at B without giving displacement at 'A'



The corresponding displacement at C & D are $\frac{2}{7}\delta_y$ and $\frac{4}{7}\delta_y$

By virtual work principle,

$$R_A\times0-25\times\frac{2}{7}\delta_y - 25\times\frac{4}{7}\delta_y + R_B\times\delta_y = 0$$

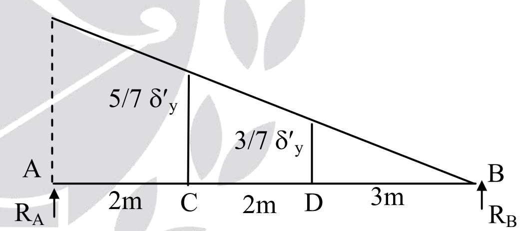
$$\Rightarrow\left(\frac{-150}{7}+R_B\right)\delta_y = 0$$

Since $\delta_y \neq 0$, $R_B - \frac{150}{7} = 0$

$$R_B = \frac{150}{7}\text{ kN}$$

Now let us give virtual displacement at A as δ_y',

Therefore corresponding displacement at C & D are $\frac{5}{7}\delta_y'$ & $\frac{3}{7}\delta_y'$



∴ By virtual work principle,

$$R_A\times\delta'_y - 25\times\frac{5}{7}\delta'_y - 25\times\frac{3}{7}\delta'_y + R_B\times0 = 0$$

$$\left(R_A - \frac{125}{7} - \frac{75}{7}\right)\delta_y' = 0$$

$$\delta'_y \neq 0,\ R_A - \frac{200}{7} = 0$$

$$R_A = \frac{200}{7}\text{ kN}$$

02. Ans: 750 N

Sol: Let the displacement of point A be 'dx' then the displacement of point B will be '3dx'.

By virtual work method the virtual work done by all the forces is zero.

$-P \times dx + 250 \times (3dx) = 0$

[$\therefore$ At A force and displacement has opposite direction]

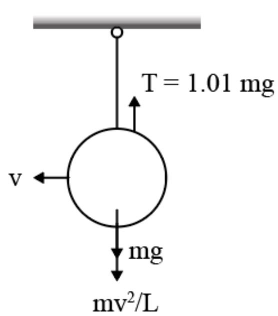
$\Rightarrow P = 750$ N

Note: At hinge there is no displacement hence work done by the reaction forces at hinge are zero.

09. Free Vibrations of Undamped SDOF System

01.

Sol: At lowest pint,

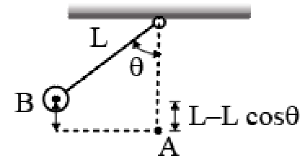


$$T = mg + \frac{mv^2}{L}$$

$$0.01mg = mg + \frac{mv^2}{L}$$

$$0.01 \times g = \frac{v^2}{L} \text{ ....(1)}$$

At extreme position



Total energy at A = Total energy at B

$$\frac{1}{2}m.v^2 = mg(1-\cos\theta)$$

$$\therefore \frac{v^2}{2} = g.L \times (1-\cos\theta)$$

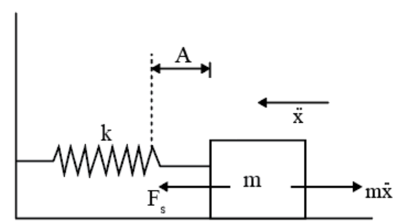
From (1)

$$\frac{0.01}{2} = (1-\cos\theta)$$

$$\therefore \theta \simeq 0.1 \text{ rad}$$

02. Ans: (b)

Sol: At extreme position



At extreme position,

Spirng force, $F_s = m \times \ddot{x}$

$$= m \times A\,\omega^2$$

$$= m \times A \times \left[\frac{2\pi}{T}\right]^2$$

$$= 4 \times 2 \times \left[\frac{2\pi}{2}\right]^2$$

$$\simeq 78.96 \text{ N}$$

03. Ans : (b)

Sol : The time period of the pendulums is

$$T_1 = 2\pi\sqrt{\left(\frac{0.6}{g}\right)}$$

$$T_2 = 2\pi\sqrt{\left(\frac{0.63}{g}\right)}$$

Let the time be t in which the longer length pendulum makes n oscillations

While the shorter one makes (n + 1) oscillations.

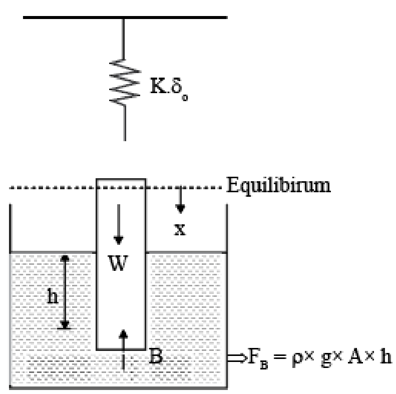
Then

$T = (n + 1)\ T_1 = nT_2$

Using (1) and (2) in (3), we find n = 40.5 and t = 64.49 s

04.

Sol:



F_B = Buoyance force

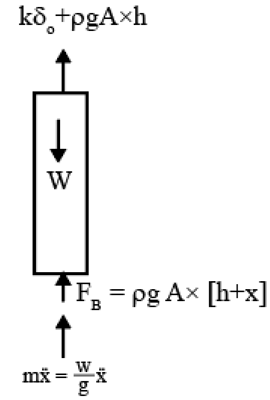
$= \rho.\ g\ \theta$

At equilibrium

$W = k\delta_o + \rho gA \times h$

$$W = k\delta_o + k.z + \rho gAh + \rho gAx + \frac{W}{g}.\ddot{x}$$

$$\therefore \frac{W}{g}.\ddot{x} + x[k + \rho gA] = 0$$



$$\therefore \ddot{x} + \frac{[k + \rho gA]}{W} \times x = 0$$

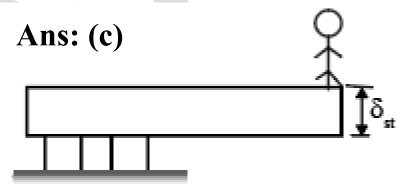
$$\therefore \omega = \left[\frac{g[k + \rho gA]}{W}\right]^{1/2} = \frac{2\pi}{T}$$

T = 0.78 sec

05. Ans: (b)

06. Ans: (c)

Sol:



$$\omega = \sqrt{\frac{g}{\delta_{st}}} = \frac{2\pi}{T}$$

$$= \sqrt{\frac{9.81}{\delta_{st}}} = \frac{2\pi}{0.6}$$

$\delta_{st} = 0.08938$ m

= 89.4 mm

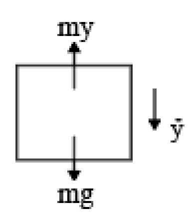
07. Ans: 0.156

Sol:

1. Frequency, $f = 10\text{Hz} = \dfrac{\omega}{2\pi}$

$\omega = 2\pi \times 10 = 20\pi$ rad/sec

ii) At the highest point,



$\therefore mg = m\ddot{y}$

$\ddot{y} = g - \ddot{y}_{max}$

$\ddot{y} = g = A \times \omega^2$

$= (A \times \omega) \times \omega$

$= v_{max} \times \omega$

$$\therefore v_{max} = \frac{g}{\omega} = \frac{9.81}{20\pi}$$

$= 0.156$ m/s

08. Ans: (b, c)

Sol:

1. Let, K = spring stiffness of original spring

   Stiffness of cut spring = 2 × k

   $$K \alpha \frac{1}{N}$$

2. For series arrangement,

   $$\frac{1}{K_e} = \frac{1}{2K} + \frac{1}{2K} \qquad K_e = K$$

   $$\omega = \sqrt{\frac{K}{m}} = \frac{2\pi}{T} \ \ldots\ldots\ldots\ldots(1)$$

3. For parallel arrangement,

   $K_e = 2K + 2K = 4\ K$

   $$\therefore \omega = \sqrt{\frac{4K}{m}} = \frac{2\pi}{T} \ \ldots\ldots\ldots\ldots\ldots(2)$$

   From (1) and (2)

   $\omega_{series} < \omega_{parallel}$

   $T_{series} > T_{parallel}$