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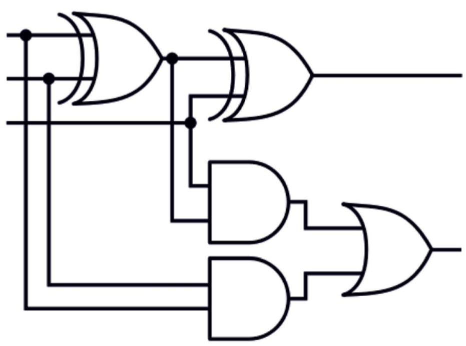
COMPUTER SCIENCE & INFORMATION TECHNOLOGY

Digital Logic

(**Text Book :** Theory with worked out Examples and Practice Questions)



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1 Digital Logic

(Solutions for Text Book Practice Questions)

1. Number Systems

01. Ans: (d)

Sol: $135_x + 144_x = 323_x$

$(1\times x^2 + 3\times x^1 + 5\times x^0)+(1\times x^2+4\times x^1+ 4\times x^0)$

$= 3x^2 + 2x^1 + 3x^0$

$\Rightarrow x^2+3x+5+x^2+4x+4 = 3x^2+ 2x + 3$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$ (Base cannot be negative)

Hence $x = 6$.

(OR)

As per the given number x must be greater than 5. Let us consider x = 6

$(135)_6 = (59)_{10}$

$(144)_6 = (64)_{10}$

$(323)_6 = (123)_{10}$

$(59)_{10} + (64)_{10} = (123)_{10}$

So that x = 6

02. Ans: (a)

Sol: 8-bit representation of

$+127_{10} = 01111111_{(2)}$

1's complement representation of

$-127 = 10000000$.

2's complement representation of

$-127 = 10000001$.

No. of 1's in 2's complement of

$-127 = m = 2$

No. of 1's in 1's complement of

$-127 = n = 1$

$\therefore m : n = 2:1$

03. Ans: (c)

Sol: In 2's complement representation the sign bit can be extended towards left any number of times without changing the value. In given number the sign bit is 'X_3', hence it can be extended left any number of times.

04. Ans: (c)

Sol: Binary representation of $+(539)_{10}$:

$$\begin{array}{r|ll}
2 & 539 & \\
2 & 269 & -1 \\
2 & 134 & -1 \\
2 & 67 & -0 \\
2 & 33 & -1 \\
2 & 16 & -1 \\
2 & 8 & -0 \\
2 & 4 & -0 \\
2 & 2 & -0 \\
 & 1 & -0
\end{array}$$

$(+539)_{10} = (1000011011)_2$

$= (00100\ 0011011)_2$

2's complement $\rightarrow 110111100101$

Hexadecimal equivalent$\rightarrow (DE5)_H$

05. Ans: 5

Sol: Symbols used in this equation are 0, 1, 2, 3. Hence base or radix can be 4 or higher

$(312)_x = (20)_x\ (13.1)_x$

$3x^2 + 1x + 2x^0 = (2x+0)(x+3x^0+x^{-1})$

$3x^2+x+2 = (2x)\left(x+3+\frac{1}{x}\right)$

$3x^2 + x + 2 = 2x^2 + 6x + 2$

$x^2 - 5x = 0 \Rightarrow x(x-5) = 0$

$x = 0$ (or) $x = 5$

x must be x > 3, So x = 5

06. Ans: 3 possible solutions

Sol: $123_5 = x8_y$

$1 \times 5^2 + 2 \times 5^1 + 3\times 5^0 = x.y^1 + 8 \times y^0$

$25 + 10 + 3 = xy+8$

$\therefore xy = 30$

Possible solutions:

i. $x = 1, y = 30$

ii. $x = 2, y = 15$

iii. $x = 3, y = 10$

3 possible solutions

07. Ans: 1

Sol: The range (or) distinct values

For 2's complement $\Rightarrow -(2^{n-1})$ to $+(2^{n-1}-1)$

For sign magnitude

$\Rightarrow -(2^{n-1}-1)$ to $+(2^{n-1}-1)$

Let n = 2 $\Rightarrow$ in 2's complement

$-(2^{2-1})$ to $+(2^{2-1}-1)$

-2 to $+1 \Rightarrow -2, -1, 0, +1 \Rightarrow x = 4$

n = 2 in sign magnitude $\Rightarrow -1$ to $+1 \Rightarrow y = 3$

$x - y = 1$

08. Ans: (c)

09. Ans: (b)

10. Ans: (b) & (d)

Sol: $[\ B \quad 9 \quad F. \quad A \quad E]_{16}$

$= [1011\ 1001\ 1111.\ 1010\ 1110]_2$

$= [5637.\ 534]_8$

2. Logic Gates & Boolean Algebra

01. Ans: (b)

Sol: Truth table of XOR

A	B	o/p
0	0	0
0	1	1
1	0	1
1	1	0

Stage 1:

Given one i/p = 1 Always.

1	X	o/p	
1	0	1	$= \overline{X}$
1	1	0	$= \overline{X}$

For First XOR gate o/p = $\overline{X}$

Stage 2:

$\overline{X}$	X	o/p
0	1	1
1	0	1

For second XOR gate o/p = 1.

Similarly for third XOR gate o/p = $\overline{X}$ & for fourth o/p = 1

For Even number of XOR gates o/p = 1

For 20 XOR gates cascaded o/p = 1.

02. Ans: 1

Sol: $f = [\overline{\overline{D}+A\overline{B}+\overline{A}C+A\overline{C}D+\overline{A}\,\overline{C}D}]$

Let $x = \overline{D}+A\overline{B}+\overline{A}C+A\overline{C}D+\overline{A}\,\overline{C}D$ [then $f=\overline{x}$]

Simplify x ⇒

$x = \overline{D}+A\overline{B}+\overline{A}C+\overline{C}D\underbrace{[A+\overline{A}]}_{1}$

$x = \overline{D}+D\overline{C}+A\overline{B}+\overline{A}C$

$x = [\overline{D}+(\overline{\overline{D}})\overline{C}]+A\overline{B}+\overline{A}C$ [∵ $p+\overline{p}q=p+q$]

$x = \overline{D}+\overline{C}+\overline{A}C+A\overline{B}$

$x = \overline{D}+[\overline{C}+(\overline{\overline{C}})\overline{A}]+A\overline{B}$

$x = \overline{D}+\overline{C}+\overline{A}+A\overline{B}$

$x = \overline{D}+\overline{C}+[\overline{A}+(\overline{\overline{A}})\overline{B}]$

$x = \overline{D}+\overline{C}+\overline{A}+\overline{B}$

$x = \overline{A}+\overline{B}+\overline{C}+\overline{D}$

$f = \overline{x} = \overline{\overline{A}+\overline{B}+\overline{C}+\overline{D}} = \overline{\overline{A}}.\overline{\overline{B}}.\overline{\overline{C}}.\overline{\overline{D}} = A.B.C.D$

Number of minterms are "one" Ans: 1.

03. Ans: (c)

Sol: $f = f_1 f_2 + f_3$

04. Ans: (c)

Sol: Let $x_1 = x_2 = x_3 = x_4$

For all cases options a, b, d not satisfy.

05. Ans: (d) **06. Ans: (b)**

07. Ans: (b) **08. Ans: (c)**

09. Ans: (a) & (d)

Sol: $(x+y)(x+\overline{y})+\overline{\overline{x}+x\overline{y}}$

using Distributive Law

F= $(x+y.\overline{y})+(\overline{\overline{x}+\overline{y}}) = x + xy = x$

10. Ans: (b) & (c)

Sol: Let B ⊙ B [Here no. of XOR=1 ⇒odd]

$\overline{B}\,\overline{B}+B.B = \overline{B}+B=1$

Let B⊙B⊙B [Here no. of XOR=2 ⇒even]

$= 1\odot B = \overline{1}.\overline{B}+1B = B$ = remain same as given variable

3. K-Maps

01. Ans: (b)

Sol:

yz \ wx	00	01	11	10
00		1	1	
01	×			1
11	×			1
10		1	1	×

$f = \overline{x}z + x\overline{z}$

02. Ans: (b)

Sol:

cd \ ab	00	01	11	10
00	1	×	×	1
01	×			1
11				
10	1			×

$\overline{b}\,\overline{c}$

$\overline{b}\,\overline{d}$

$f = \overline{b}\,\overline{d}+\overline{b}\,\overline{c}$

03.

Sol: POS

x + w

xy \ wz	00	01	11	10
00	0	×	0	0
01	0	×	1	1
11	1	1	1	1
10	0	×	0	0

y

= y(x+w)

SOP

xy \ wz	00	01	11	10
00	0	×	0	0
01	0	×	1	1
11	1	1	1	1
10	0	×	0	0

wy

xy

= xy+yw

SOP: x y + y w

POS: y(x + w)

04. Ans: (a)

Sol: For n-variable Boolean expression,

Maximum number of minterms = 2^n

Maximum number of implicants = 2^n

Maximum number of

Essential prime implicants = $\frac{2^n}{2}$

$= 2^{n-1}$

05. Ans: (c)

Sol:

C \ AB	00	01	11	10
0	1	1	0	0
1	0	1	1	0

$F(A, B, C) = \overline{A}\,\overline{C} + BC$

06. Ans: 3

Sol: $\overline{w}\,\overline{z} + \overline{w}\,x\overline{y} + \overline{x}\,y\overline{z}$

wx \ yz	00	01	11	10
00	1			1
01	1	1		1
11				
10				1

07. Ans: (c) **08. Ans: (a)** **09. Ans: (a)**

10. Ans: (b) & (d)

Sol: Given f = ΠM [1,3,4,6,9,11,12,14]+d[2,7,8,13]

AB \ CD	00	01	11	10
00		0	0	X
01	0		X	0
11	0	X		0
10	X	0	0	

$[\overline{B} + D] \Leftarrow$

$\Downarrow$

$[B + \overline{D}]$

$f_{pos} = [B + \overline{D}][\overline{B} + D]$ hence option (d) is correct

$f_{sop} = \underbrace{B.\overline{B}}_{0} + BD + \overline{B}\,\overline{D} + \underbrace{\overline{D}.D}_{0} = BD + \overline{B}\,\overline{D}$

So option (c) is wrong

$$\left.\begin{array}{l} f_{sop} = B.D + \overline{B}\,\overline{D} \\ f_{pos}[B + \overline{D}][\overline{B} + D] \end{array}\right\}$$ Both cases variables A, C are absent so Independent of "2" variables

Hence option (b) is correct

4. Combinational Circuits

01. Ans: (d)

Sol: Let the output of first MUX is "F_1"

$F_1 = AI_0 + AI_1$

Where A is selection line, I_0, I_1 = MUX Inputs

$F_1 = \overline{S}_1.W + S_1.\overline{W} = S_1 \oplus W$

Output of second MUX is

$F = \overline{A}.I_0 + A.I_1$

$F = \overline{S}_2.F_1 + S_2.\overline{F}_1$

$F = S_2 \oplus F_1$

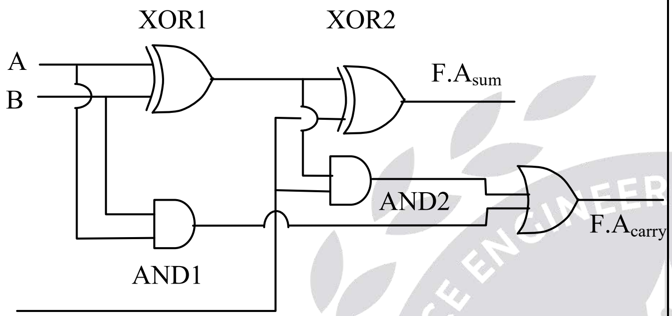
But $F_1 = S_1 \oplus W$

$F = S_2 \oplus S_1 \oplus W$

i.e., $F = W \oplus S_1 \oplus S_2$

02. Ans: 19.2

Sol: F.A using H.A



$t_{AND} = 1.2\mu s$; $t_{OR} = 1.2\mu sec$

XOR = 2×1.2= 2.4 μsec

Time for addition=XOR1 +XOR2 = 2.4+2.4
=4.8μsec

Time for carry = XOR1+AND2+OR
= 2.4+1.2+1.2=4.8μsec

for n-bit total time required

= (n–1) tc+Ma×[tc, ts]

⇒for 4 bit

(4–1) ×4.8+ Ma×[4.8, 4.8]

3×4.8+4.8=19.2μsec

03. Ans: 6

Sol: T = 0 → NOR → MUX 1 → MUX 2

2ns 1.5ns 1.5ns

Delay = 2ns + 1.5ns + 1.5ns = 5ns

T = 1 → NOT → MUX 1→NOR→ MUX 2

1ns 1.5ns 2ns 1.5ns

Delay = 1ns + 1.5ns + 2ns + 1.5ns = 6ns

Hence, the maximum delay of the circuit is 6ns

04. Ans: 195

Sol: Given t_{carry}= 12ns

t_{sum}= 15ns

n= 16 bit

Time required = (16–1) ×12+max(15, 12)

15×12+15=195nsec.

05. Ans: (b), (c) & (d)

Sol: For "n" bit number addition we require

= "n" F.A (or) 1 H.A, (n–1) F.A (or)

(2n–1) H.A, (n–1) OR Gates

06. Ans: (a) & (c)

Sol: Output =Y= $\sum m(0, 1, 2, 6)+\overline{C}$

A \ BC	00	01	11	10
0	1	1		1
1				1

$$Y = \overline{A}\,\overline{B} + B\,\overline{C} + \overline{C}$$

$$= \overline{A}\,\overline{B} + \overline{C}[B+1] = \overline{C} + \overline{A}\,\overline{B} \rightarrow \text{Option (c)}$$

$$\Rightarrow Y = \overline{C} + \overline{A}\,\overline{B} = (\overline{C}+\overline{A})(\overline{C}+\overline{B}) = (\overline{A}+\overline{C})(\overline{B}+\overline{C})$$

→Option (d)

5. Sequential Circuits

01. Ans: 4

Sol: In the given first loop of states, zero has repeated 3 times. So, minimum 4 number of Flip-flops are needed.

02. Ans: (b)

Sol:

CLK	Serial in= $B \oplus C \oplus D$	A B C D
0		1 0 1 0
1	1 →	1 1 0 1
2	0 →	0 1 1 0
3	0 →	0 0 1 1
4	0 →	0 0 0 1
5	1 →	1 0 0 0
6	0 →	0 1 0 0
7	1 →	1 0 1 0

03. Ans: (b)

Sol:

J	K	Q_n	$\overline{Q}_n$	$T = (J+Q_n)(K+\overline{Q}_n)$	Q_{n+1}
0	0	0	1	0.1 = 0	0 $\}$ Q_n
0	0	1	0	1.0 = 0	1
0	1	0	1	0.1 = 0	0 $\}$ 0
0	1	1	0	1.1 = 1	0
1	0	0	1	1.1 = 1	1 $\}$ 1
1	0	1	0	1.0 = 0	1
1	1	0	1	1.1 = 1	1 $\}$ $\overline{Q}_n$
1	1	1	0	1.1 = 1	0

J \ KQ_n	00	01	11	10
0			1	
1	1		1	1

$T = J\,\overline{Q_n} + KQ_n = (J+Q_n)\,(K+\overline{Q_n})$

04. Ans: (c) **05. Ans: (d)** **06. Ans: 3**

07. Ans: (c)

Sol: From circuit we can write

$D = X \oplus Q$

but for D FF $\Rightarrow Q_{n+1} = D$

$Q_{n+1} = D = X \oplus Q$

$Q_{n+1} = \overline{X}\,Q + X\,\overline{Q}$

[This is same as T FF output]

T FF output $Q_{n+1} = \overline{T}\,Q + TQ$

Therefore given circuit act as T Flip-Flop

08. Ans: (d)

Sol: $D_A = Q_B \odot Q_C$; $D_B = Q_A$; $D_C = Q_B$

Given initially $Q_A = Q_B = Q_C = 0$

CLOCK	D_A	D_B	D_C	Q_A	Q_B	Q_C
0	-	-	-	0	0	0
1	1	0	0	1	0	0
2	1	1	0	1	1	0
3	0	1	1	0	1	1
4	1	0	1	1	0	1
5	0	1	0	0	1	0
6	0	0	1	0	0	1

The output observed at Q_A is 0110100-----

09. Ans: (b) & (c)

Sol: Total states in 'n' bit Johnson counter = 2^n

number of used states = $2 \times n$

Here n = 4 bits

So, total number of states = $2^4 = 16$

number of used states = $2 \times 4 = 8$

unused states = Total states – used states

Unused states = 16 – 8

= 8