



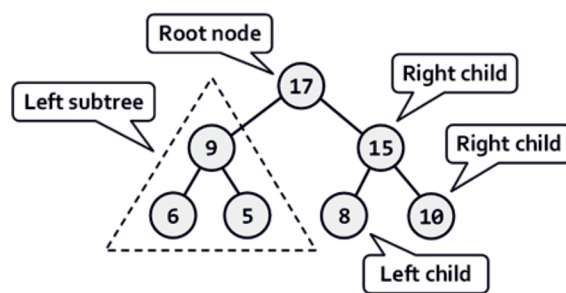
X **DEEP**
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Data Structures

(**Text Book** : Theory with worked out Examples
and Practice Questions)



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1. Arrays

01. Ans: 1010

Sol: Loc. of A (i) = $L_0 + (i-1)b * C$

$$\text{Loc of A [0]} = 1000 + (0+5) \times 2 = 1010$$

02. Ans: 1024 and 1024

Sol: (i) By RMO, the loc. of

$$A[i, j] = L_0 + [(i-b_1)(u_2-b_2+1) + (j-b_2)] * C$$

$$A[0, 5] = 1000 + [(0+2) \times 5 + (5-3)] \times 2 \\ = 1000 + 24 = 1024$$

(ii) By CMO, the loc of

$$A[i, j] = L_0 + [(j-b_2)(u_1-b_1+1) + (i-b_1)] * C$$

$$A[0, 5] = 1000 + [(5-3) \times 5 + (0+2)] \times 2 \\ = 1024$$

03. Ans: (a)

Sol: In general

$$\begin{aligned} \text{RMO} &= L_0 + (i-1)r_2 + (j-1) \\ &= 100 + (i-1)15 + (j-1) \\ &= 100 + 15i - 15 + j - 1 \\ &= 15i + j + 84 \end{aligned}$$

04. Ans: (c)

Sol: Lower triangular matrix

$$\begin{pmatrix} a & 0 & 0 & \dots & 0 \\ b & c & 0 & \dots & 0 \\ d & e & f & 0 & \dots & 0 \\ g & h & i & j & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$\begin{aligned} \text{RMO} &= L_0 + \text{the number of elements in} \\ &\quad (i-1) \text{ rows} + \text{one dimensional elements} \\ &= L_0 + (1 + 2 + \dots + i-1) + (j-1) \\ &= L_0 + i \frac{(i-1)}{2} + (j-1) \end{aligned}$$

05. Ans: (c)

Sol: CMO:

Storage:

$$\begin{array}{cccc|cccc|cccc} a_{11} & a_{21} & a_{31} & a_{41} & a_{22} & a_{32} & a_{42} & a_{33} & a_{43} & a_{44} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

Retrieval:

$$\begin{aligned} \text{loc of } A[i, j] &= L_0 + 2D + 1D \\ &= L_0 + [(j-1) \text{ cols} + (i-1)b] \end{aligned}$$

In each col., $i/b = j$.

$$\text{Loc. of } A[i, j] = L_0 + [(j-1) \text{ cols} + (i-j)]$$

In $(j-1)$ cols

The no. of elements is

$$\begin{aligned} &n + (n-1) + \dots + (n - (j-1-1)) \\ &= (j-1)n - [1 + 2 + \dots + j-2] \\ &= n(j-1) - \frac{(j-1)(j-2)}{2} \end{aligned}$$

loc. of $A[i, j]$

$$= L_0 + \left[n(j-1) - \frac{(j-1)(j-2)}{2} + (i-j) \right]$$

06. Ans: (d)

Sol: RMO:

Storage:

$$\begin{array}{cccc|cccc|cccc} a_{11} & a_{12} & a_{21} & a_{22} & a_{23} & a_{32} & a_{33} & a_{34} & a_{43} & a_{44} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

Retrieval:

$$\text{loc of } A[i, j] = L_0 + 2D + 1D$$

$$= L_0 + \text{number of elements in } (i-1) \text{ rows} \\ + (j - j/b)$$

Row	j/b
4	3
3	2
2	1

 except 1st row

$$i^{\text{th}} \quad (i-1)$$

$$\text{loc. of } A[i, j] = L_0 + [(3i-4) + j - (i-1)] \\ = L_0 + (2i + j - 3)$$

07. Ans: (a)
Sol: CMO:
Storage:

a ₁₁	a ₂₁	a ₁₂	a ₂₂	a ₃₂	a ₂₃	a ₃₃	a ₄₃	a ₃₄	a ₄₄
0	1	2	3	4	5	6	7	8	9

Retrieval:

$$\text{loc. of } A[i, j] = L_0 + 2D + 1D \\ = L_0 + (j-1)\text{cols} + (i - i/b)$$

Since i is Varying

Col	i/b
4	3
3	2
2	1
except	1 st column
j th	j-1

$$\therefore \text{loc of } A[i, j] = L_0 + [3(j-1) - 1 + i - (j-1)] \\ = L_0 + [2j + i - 3]$$

08. Ans: (b)
Sol: Storage & Retrieval:

a ₂₁	a ₃₂	a ₄₃	a ₁₁	a ₂₂	a ₃₃	a ₄₄	a ₁₂	a ₂₃	a ₃₄
0	1	2	3	4	5	6	7	8	9

 If $i - j = 1$

$$\text{loc of } A[i, j] = L_0 + 0 + (i - i/b)$$

or

$$(j - j/b)$$

i.e.,

$$\text{loc. of } A[i, j] = L_0 + 0 \\ + (i - 2)$$

or

$$(j - 1)$$

 If $i - j = 0$

$$\text{loc. of } A[i, j] = L_0 + (n-1) \\ + (i-1)$$

or

$$(j-1)$$

 If $i - j = -1$ // upper diagonal

$$\text{loc. of } A[i, j] = L_0 + 2n - 1 \\ + (i-1)$$

or

$$(j-2)$$

09. Ans: (a)
Sol: A sample 5×5 S-matrix is given below.

1	8	3	2	1
3	0	0	0	0
6	1	7	4	3
0	0	0	0	1
9	6	5	4	1

The compact representation is

 $[1, 8, 3, 2, 1, \quad 6, 1, 7, 4, 3, \quad 9, 6, 5, 4, 1, \quad 3, 1].$

10. Ans: 9

Sol: $2n - 1 = 10 - 1 = 9$

1	2	3	4
5	1	2	3
6	5	1	2
7	6	5	1

11. Ans: 190900

Sol: $n + (a - 1)(2n - a)$

$$1000 + (101 - 1)(2 \cdot 1000 - 101)$$

$$1000 + 100 \times (2000 - 101)$$

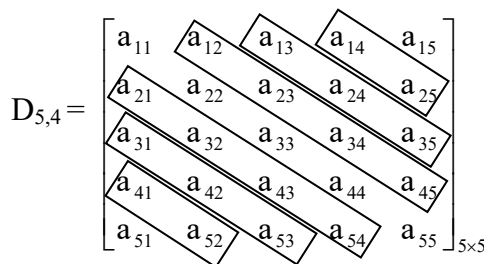
$$1000 + 100 \times 1899$$

$$1000 + 189900 = 190900$$

12. Ans: (a)

Sol: Square Band Matrix:

It is denoted by $D_{n,a}$ where n is size of matrix, a is $(a-1)$'s diagonals exists above and below the matrix diagonal.



Size:- N (diagonal elements)

$$+ 2[N-1+N-2+N-3+\dots+N-(a-1)]$$

$$= N+2[(a-1)N-(1+2+3+\dots+a-1)]$$

Total no. of elements

$$= n + 2 [n-1 + n-2 + \dots + n - (a-1)]$$

$$= n + 2 [(a-1)n - [1+2+\dots+(a-1)]]$$

$$= n + 2 \left[(a-1)n - \frac{a(a-1)}{2} \right]$$

$$= n + 2n(a-1) - a(a-1) = n + (a-1)[2n-a]$$

(b) $D_{6,4}$

In $D_{6,4} \rightarrow A(5,4)$ in 3rd diagonal

In $D_{6,5} \rightarrow A(5,4)$ in 4th diagonal

(i) $i - j$ value is remained constant through out the diagonal

(ii) As 'a' value changes, accordingly K value also changes.

$$\therefore K \text{ value} = a - (i - j)$$

$$\therefore \text{loc. of } A[i,j] = L_0 + \text{no. of elements in } (k-1)$$

dig + 1D cross

$$\text{loc. of } A[i,j] = L_0 + \text{no. of elements in } (k-1)$$

dig + $(i - i/b)$ or $(j - j/b)$

but here i/b is varying diagonal by diagonal so prefer j/b which is always 1 in each diagonal

$$= L_0 + \text{no. of elements in } (k-1) \text{ dig} + (j-1)$$

No. of elements in $(k-1)$ diagonals is

$$[n-(a-1)] + [n-(a-1)+1] + [n-(a-1)+2]$$

$$+ \dots [n-a+(k-1)]$$

$$[n-(a-1)] \rightarrow 1^{\text{st}} \text{ diagonal because it is } (a-1)^{\text{th}}$$

diagonal

$$(n-(a-1)+1) \rightarrow 2^{\text{nd}} \text{ diagonal}$$

$$[n-(a-1)+2] \rightarrow 3^{\text{rd}} \text{ diagonal}$$

$$n-a+(k-1) \rightarrow (k-1)^{\text{th}} \text{ term i.e. } (k-1)^{\text{th}}$$

diagonal

$$= (k-1)(n-a) + 1 + 2 + \dots + k-1$$

$$= (k-1)(n-a) + \frac{k(k-1)}{2}$$

\therefore loc. of $A[i, j]$

$$= L_0 + \left[(k-1)(n-a) + \frac{k(k-1)}{2} + (j-1) \right]$$

2. Stacks & Queues

01. (i). Ans: (a) (ii). Ans: (c)

Sol: Given array size m , say 9

Number of stacks n , say 3

$$0 \leq i < n \quad T[i] = B[i] = i \left\lfloor \frac{m}{n} \right\rfloor - 1$$

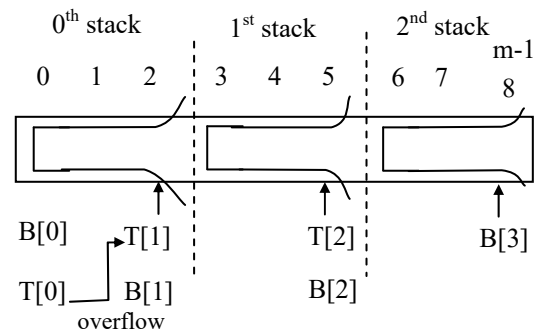
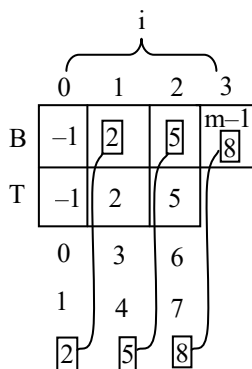
$$i = 0 \Rightarrow T[0] = B[0] = 0 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 0 - 1 = -1$$

$$i = 1 \Rightarrow T[1] = B[1] = 1 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 3 - 1 = 2$$

$$i = 2 \Rightarrow T[2] = B[2] = 2 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 6 - 1 = 5$$

$$\text{when } i = 3 \Rightarrow B[3] = m - 1 = 9 - 1 = 8$$

(i) Push = overflow = size



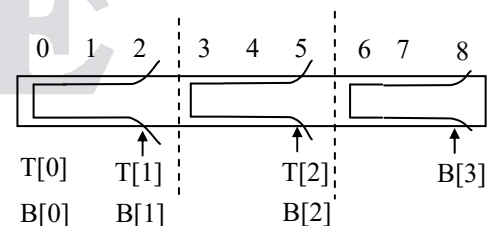
$$\left. \begin{aligned} T[0] &= B[1] \\ T[1] &= B[2] \\ T[2] &= B[3] \end{aligned} \right\} \text{ overflow cases}$$

$$\therefore T[i] = B[i+1]$$

(ii) POP = underflow = initial

	0	1	2
B	-1	2	5
T	-1	2	5

Below the table, indices 0, 3, 6 are shown with arrows pointing to the corresponding values in the B row: 0 points to -1, 3 points to 2, and 6 points to 5.



$$\left. \begin{aligned} T[0] &= B[0] \\ T[1] &= B[1] \\ T[2] &= B[2] \end{aligned} \right\} \text{ underflow cases } \therefore T[i] = B[i]$$

02. Ans: (b)

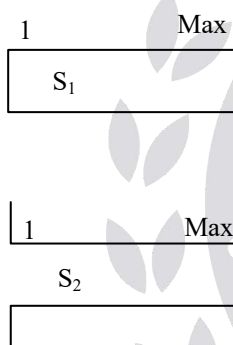
Sol:

Stack operation	Push (10)	Push (20)	Pop	Push (10)	Push (20)	Pop	Pop	Pop	Push (20)	Pop
Stack	<div>10</div>	<div>10 20</div>	<div>10</div>	<div>10 10</div>	<div>10 10 20</div>	<div>10 10</div>	<div>10</div>	<div></div>	<div>20</div>	
Pop list			20	20	20	20, 20	20, 20, 10	20, 20, 10, 10	20, 20, 10, 10	20, 20, 10, 10, 20

The sequence of popped out values \Rightarrow 20, 20, 10, 10, 20

03. Ans: (d)

Sol:



An instance of array having two stacks is shown above. Stack1 occupied from 1 to MAXSIZE and stack2 occupied from MAXSIZE to 1. Above shown array is filled completely. So condition for 'stack full' is

$$\text{Top 1} = \text{Top 2} - 1$$

04. Ans: (c)

Sol: Stack S is

2	let S ₁	S ₂	S ₃	S ₄
-1	-1			
2	2	2		
-3	-3	-3	-3	
2	2	2	2	2

$$f(0) = 0$$

$$f(S_4) = \max \{f(0), 0\} + 2 = 2$$

$$f(S_3) = \max \{f(S_4), 0\} + -3 = -1$$

$$\{ \because S_3 = \text{push}(S_4, -3) \}$$

$$f(S_2) = \max \{f(S_3), 0\} + 2 = 2$$

$$f(S_1) = \max \{f(S_2), 0\} + (-1) = 1$$

$$f(S) = \max \{f(S_1), 0\} + 2 = 3$$

$$f(S) = 3$$

05. Ans: (b)

Sol: Stack insertion order \Rightarrow 1,2,3,4,5. The only possible output sequence 3,4,5,2,1

That occurs when

Push (1)

Push (2)

Push (3)

Pop (3) \rightarrow 3

(\because There is no constraint on the order of deletion operations)

Push (4)

Pop (4) \rightarrow 3, 4

Push (5)
Pop (5) \longrightarrow 3,4,5
Pop (2) \longrightarrow 3,4,5,2
Pop (1) \longrightarrow 3,4,5,2,1

Other remaining combinations are not possible

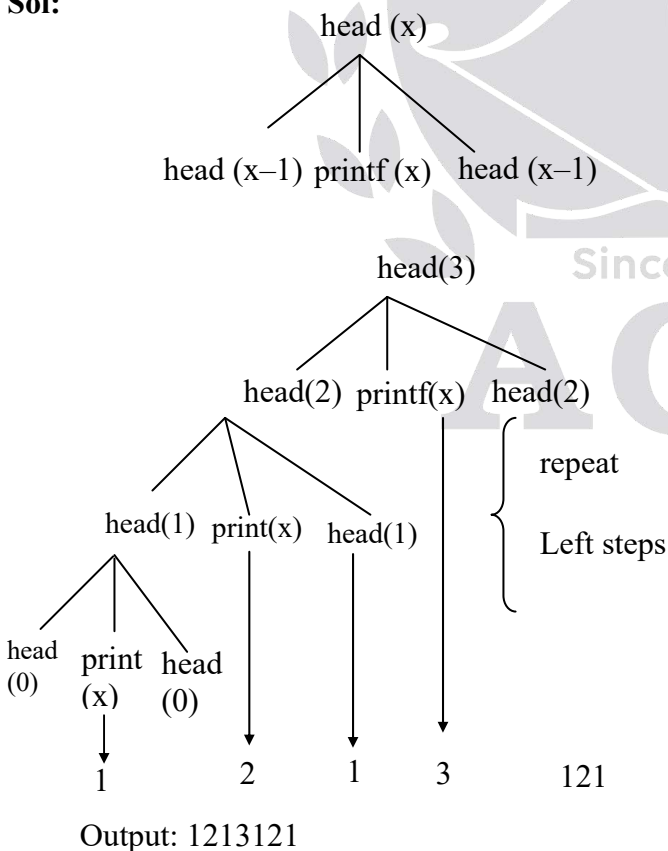
06. Ans: 321

Sol: Invocation tail (3)

$T(3) = 3$
 $T(2) = 2$
 $T(1) = 1$
 $T(0) = \text{stop}$
Output: 3, 2, 1

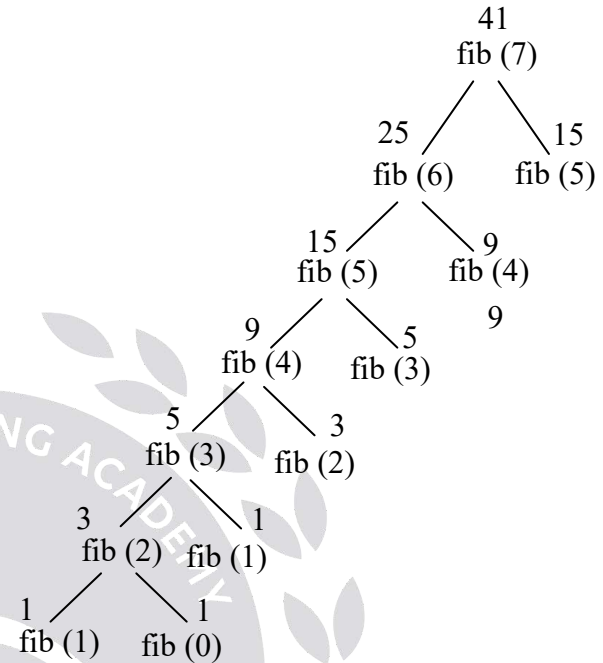
07. Ans: 1213121

Sol:



08.

Sol: (i) Ans: 41



(ii) Ans: 67

Sol: Number of calls for evaluating

$$f(n) = 2 \times f(n+1) - 1$$

The total number of calls in

$$\text{Fibonacci}(8) = 2 f(9) - 1$$

$$= 2 \times 34 - 1 = 68 - 1 = 67$$

(iii) Ans: 54

n	0	1	2	3	4	5	6	7	8	9	10
Fib(n)	0	1	1	2	3	5	8	13	21	34	55
Call:	1	1	3	5	9			41	35		
Add:				2	4						

$$\text{Additions} = f(n+1) - 1$$

$$f(9) = f(10) - 1 = 55 - 1 = 54$$

09.
Sol: Ackerman(m, n) =

$$\begin{cases} n+1 & \text{if } m=0 \\ \text{Ackerman}(m-1, 1) & \text{if } n=0 \\ \text{Ackerman}(m-1, \text{Ackerman}(m, n-1)) & \text{otherwise} \end{cases}$$

(i) Ans: 9
Sol: Ackerman(2, 3) = A(1, A(2, 2)) = A(1, 7)

$$A(2, 2) = A(1, A(2, 1)) = A(1, 5) = 7$$

$$A(2, 1) = A(1, A(2, 0)) = A(1, 3) = 5$$

$$A(2, 0) = A(1, 1) = 3$$

$$A(1, 1) = A(0, A(1, 0)) = A(0, 2) = 2 + 1 = 3$$

$$A(1, 0) = A(0, 1) = 2$$

$$A(0, 1) = 1 + 1 = 2$$

$$A(1, 3) = A(0, A(1, 2)) = A(0, 4) = 4 + 1 = 5$$

$$A(1, 2) = A(0, A(1, 1)) = A(0, 3) = 3 + 1 = 4$$

$$\begin{aligned} A(1, 5) &= A(0, A(1, 4)) \\ &= A(0, A(0, A(1, 3))) \\ &= A(0, A(0, 5)) \\ &= A(0, 6) = 6 + 1 = 7 \end{aligned}$$

$$\begin{aligned} A(1, 7) &= A(0, A(1, 6)) \\ &= A(0, A(0, A(1, 5))) \\ &= A(0, A(0, 7)) \\ &= A(0, 8) = 9 \end{aligned}$$

$$\text{Ackerman}(2, 3) = 9$$

(ii) Ans: 13

$$\begin{aligned} \text{Sol: Ackerman}(2, 5) &= A(1, A(2, 4)) \\ &= A(1, A(1, A(2, 3))) \\ &= A(1, A(1, 9)) \\ A(1, 9) &= A(0, A(1, 8)) \\ &= A(0, A(0, A(1, 7))) \\ &= A(0, A(0, 9)) \\ &= A(0, 10) \\ &= 11 \end{aligned}$$

$$\begin{aligned} A(1, 11) &= A(0, A(1, 10)) \\ &= A(0, A(0, A(1, 9))) \\ &= A(0, A(0, 11)) \\ &= A(0, 12) \\ &= 13 \end{aligned}$$

$$\text{Ackerman}(2, 5) = 13$$

(iii) Ans: 4
Sol: Ackerman(0, 3) = 4

(iv) Ans: 5

$$\begin{aligned} \text{Sol: Ackerman}(3, 0) &= A(2, 1) \\ A(2, 1) &= A(1, A(2, 0)) \\ A(2, 0) &= A(1, 1) \\ A(1, 1) &= A(0, A(1, 0)) \\ A(1, 0) &= A(0, 1) = 2 \\ A(1, 1) &= A(0, 1) = 3 \\ A(2, 0) &= 3 \\ A(2, 1) &= A(1, 3) \\ A(1, 3) &= 5 \quad \text{from (i)} \\ A(2, 1) &= 5 \\ \text{Ackerman}(3, 0) &= 5 \end{aligned}$$

10.
Sol: (a) After $N+1$ calls we have the first move.

So after 4 calls we have the first move.

 (b) After total calls -1 calls, we have the last move.

 (c) Total moves $2^N - 1 = 7$

 (d) Total invocations $= 2^{N+1} - 1$
 $= 2^4 - 1 = 15$
11. Ans: (b)
Sol: Postfix expression $A \ B \ C \ D \ + \ * \ F \ / + DE \ * \ +$

12. Ans: (a)

Sol: $a = -b + c * d/e + f \uparrow g \uparrow h - i * j$

Prefix:

$$a = -b + c * d/e + (\uparrow f \uparrow gh) - i * j$$

$$a = -b + *cd/e + (\uparrow f \uparrow gh) - i * j$$

$$a = -b + /*cde + \uparrow f \uparrow gh - *ij$$

$$a = +-b/*cde + \uparrow f \uparrow gh - *ij$$

$$a = ++-b/*cde \uparrow f \uparrow gh - *ij$$

$$a = -++-b/*cde \uparrow f \uparrow gh *ij$$

$$\Rightarrow a = -++-b/*cde \uparrow f \uparrow gh *ij$$

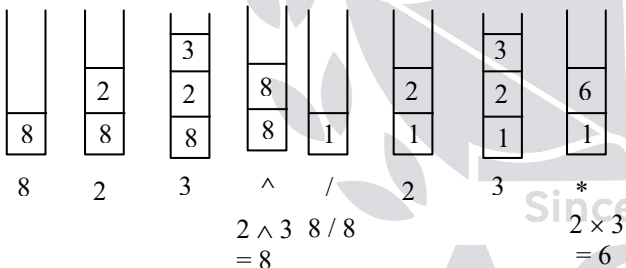
13. Ans: (a)

Sol: Infix expression: $[a+(b \times c)]-(d \wedge (e \wedge f))$

Postfix expression: $abc \times +def \wedge \wedge -$

14. Ans: (a)

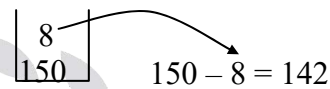
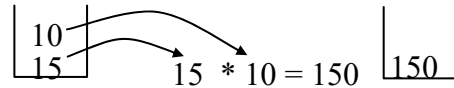
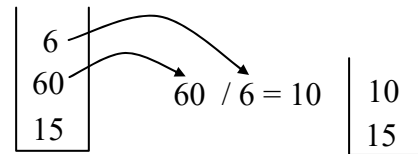
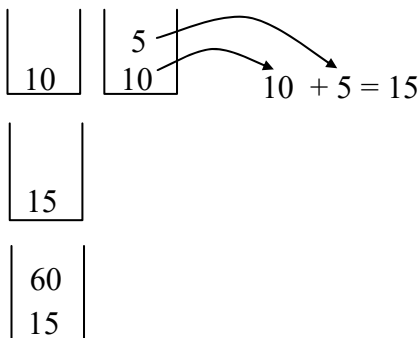
Sol:



\Rightarrow The top two elements are 6, 1

15. Ans: (c)

Sol: 10, 5, +, 60, 6, /, *, 8, -



Value of the postfix expression is 142

16. Ans: (c)

Sol: (i) ab

(ii) b

(iii) byz

(iv) yz

Output is \Rightarrow yz

17. Ans: (b)

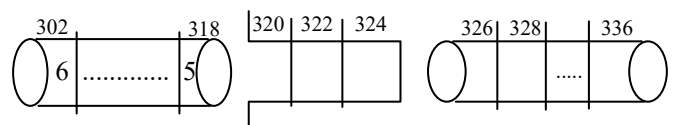
Sol:



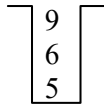
Queue rear Replace with predecessor

18. Ans: (i) 322 and (ii) 326

Sol:

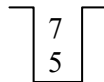


Until first '0' is encountered, stack contains

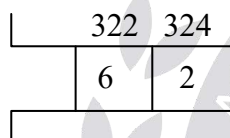


So $5+6+9 = 20$ is enqueued in Q_2 @ loc 326

Until second '0' is encountered, stack contains



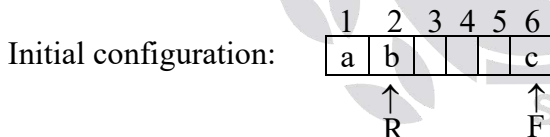
So $5+7 = 12$ is enqueued in Q_2 @ loc 328
Then simply 2 and 6 are pushed in stack



So the location of 6 and 20 are 322 and 326

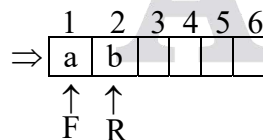
19. Ans (c)

Sol: Suppose that array contains



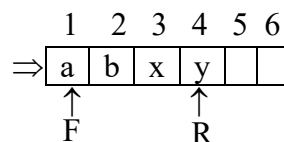
Delete element

Dequeue() → c



Now two added

enqueue (x) and
enqueue (y) :



∴ (R, F) = (4,1)

Option (c).

20. Ans: (b)

Sol: The given recursive procedure simply reverses the order of elements in the queue. Because in every invocation the deleted element is stored in 'i' and when the queue becomes empty.

Then the insert () function call will be executed from the very last invoked function call. So, the last deleted element will be inserted first and the procedure goes on.

21. Ans: (a), (b) & (c)

Sol: DCBA is possible push D, pop D, Push C, pop c, push B, pop B, push A, pop A.

BCAD is possible push D,C,B pop B Pop C, push A, pop A, pop D

A B C D is possible push D,C,A pop A,B,C,D

CABD is not possible push D, Push C, Pop C, push B, Push A, Pop A, : Last element is B so pop B will happen therefore this sequence is not possible so answer should be (a), (b), (c).

22. Ans: (a)

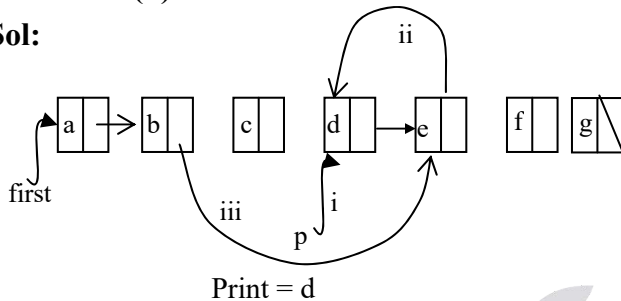
Sol: The front is the pointer in the queue from where dequeue happens. So we dequeue the element using the front. After dequeuing the front pointer moves by one, hence this is a circular array then we have to take the mod of the array length to again start from the first index after reaching the last index of the array.

Count-- // represents the number of elements is there in the queue. So after dequeuing, we also decrease the count. Return r will return the element value. Option (a) fills all these parameters that is correct.

3. Linked Lists

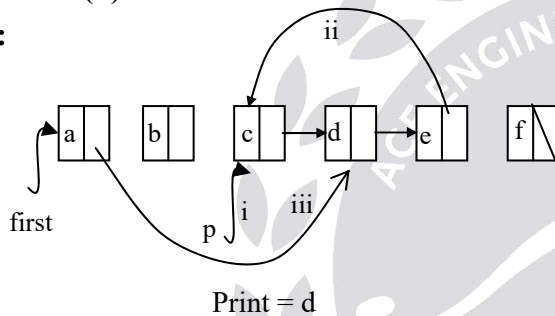
01. Ans: (d)

Sol:



02. Ans: (d)

Sol:

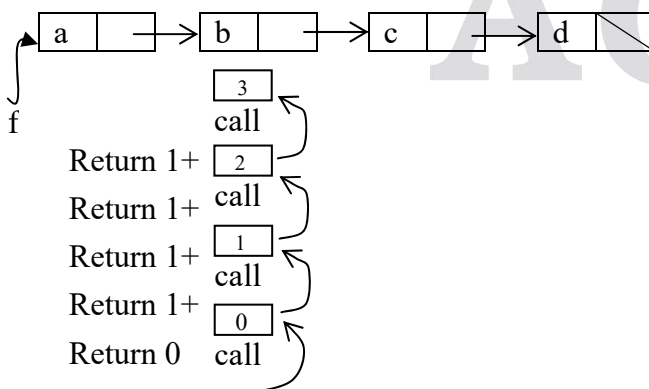


03. Ans: (a)

Sol: while (P) or while (P!= Null)
while P is pointing to somebody

04. Ans: (d)

Sol: Recursive routine for 'Count'

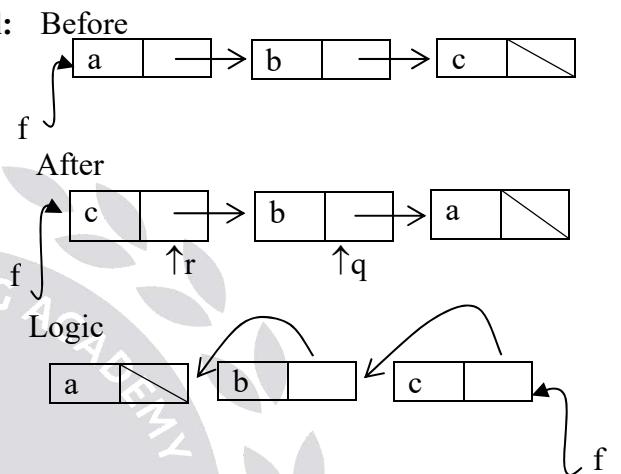


05. Ans: (b)

Sol: either causes a null pointer dereference or append list m to the end of list n.

06. Ans: (b)

Sol:

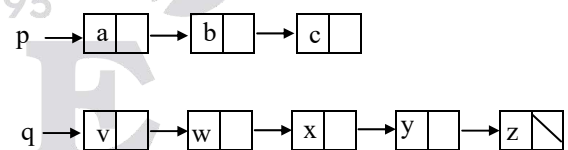


07. Ans: (b)

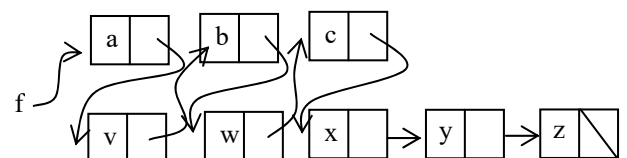
Sol: This is recursive routine for reversing a SLL.

08. Ans: (a)

Sol: Before



After



concatenation of two single linked lists by choosing alternative nodes.

09. Ans: (d)

Sol: Cur. Next = New Node (X, Cur. Next)

(i) Struct Node * n = Get Node () ;

(ii) n → data = X ;

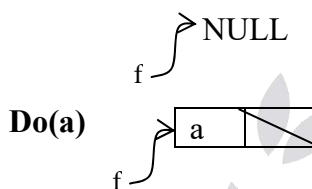
(iii) n → Next = Cur → Next ;

(iv) Cur → Next = n

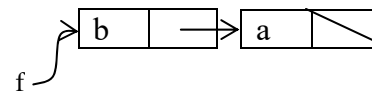
10. Ans: (a)

Sol: Linked stack push () = insert front ()

Initial Do ()



Do(b)



11.

Sol:

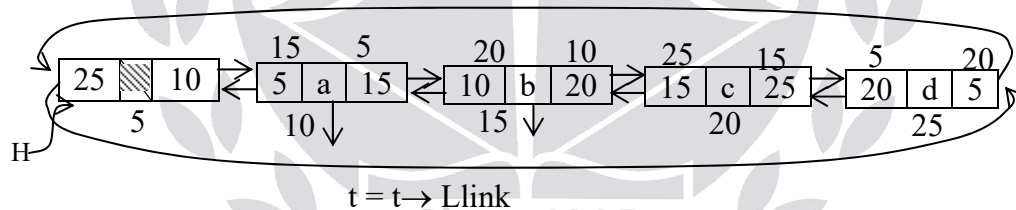
Operation	Left most	Right most	Middle
Insert	3	3	4
Delete	1	1	2

12. Ans: (b)

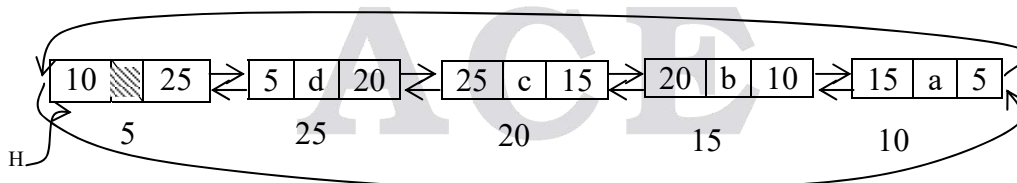
Sol: Inserts to the left of middle node in doubly linked list.

13. Ans: (a)

Sol: Before reverse:



After reverse:



14. Ans: (c)

Sol: In the doubly linked list, the maximum pointers will change if we insert them in the intermediate of the linked list. If we insert the element then we have to change the next and prev of inserting node.

Next of the previous node and prev of the next node.

15. Ans: (b)

Sol: For counting the number of nodes we have to move the temp pointer.

while(temp!=NULL)//The last node next stores the NULL

If we take the while(temp->next!=NULL) the last node will not be counted here.

4. Trees

01. Ans: (d)

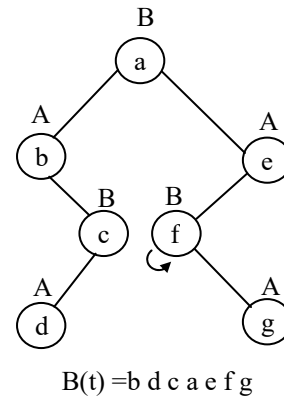
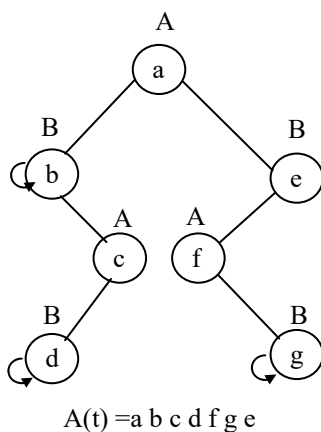
Sol: 1. Traverse the left subtree in postorder.
2. Traverse the right subtree in postorder.
3. Process the root node

02. Ans: (c)

Sol: 1. Traverse the right subtree in postorder.
2. Traverse the left subtree in postorder.
3. Process the root node

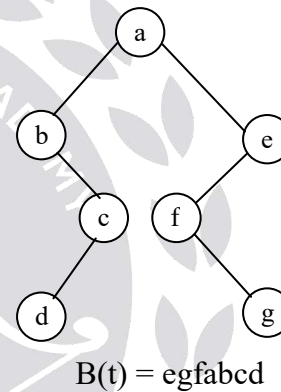
03. Ans: (c)

Sol:



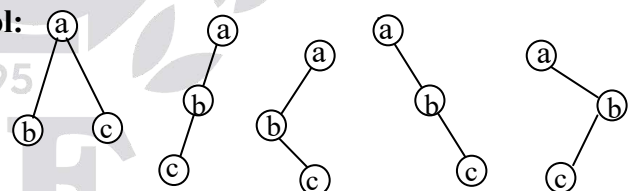
04. Ans: (b)

Sol:



05. Ans: 5

Sol:



∴ Totally 5 distinct trees possible

Note: The number of binary trees can be formulated with unlabeled nodes are

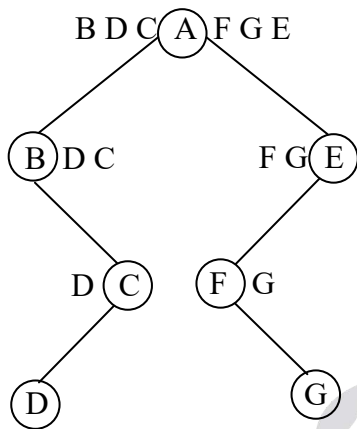
$$= \frac{2^n C_n}{n+1}$$

06. Ans: (c)

Sol: Preorder : A B C D E F G

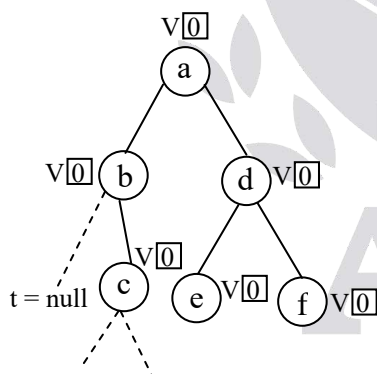
In-order : B D C A F G E

Post-order: D C B G F E A



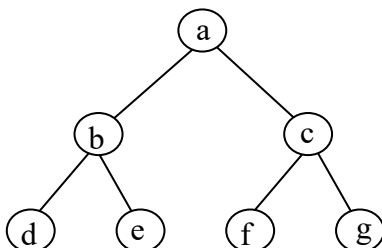
07. Ans: 3

Sol: Note: If pre-order is given, along with terminal node information & all right child information the unique pattern can be found. If post-order is given along with terminal information and all left child information the unique pattern can be identified.



08. Ans: 3

Sol:

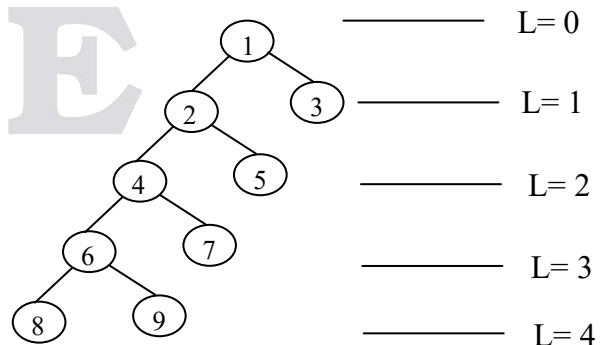
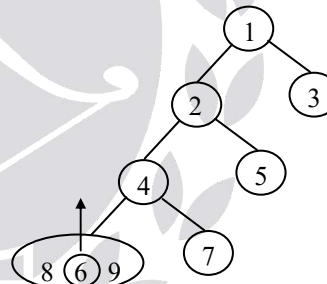
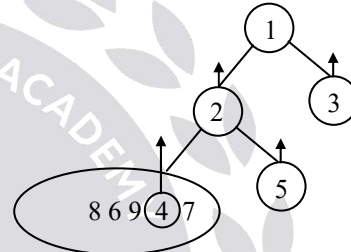
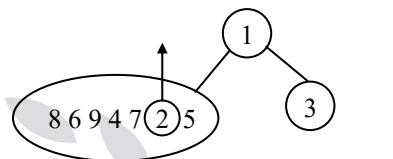


09. Ans: 4

Sol: Post 8 9 6 7 4 5 2 3 1

In 8 6 9 4 7 2 5 1 3

In 8 6 9 4 7 2 5 1 3



Height = 4

10. (a) Ans: 19

Sol: Leaf nodes (L) = Total nodes – internal nodes

$$L = In + 1 - I$$

$$L = I(n-1) + 1$$

$$L = 20$$

$$I = ?$$

$$20 = I(2-1) + 1$$

$$20 = I + 1$$

$$I = 19$$

10. (b) Ans: 199

Sol: $L = I(n-1) + 1$

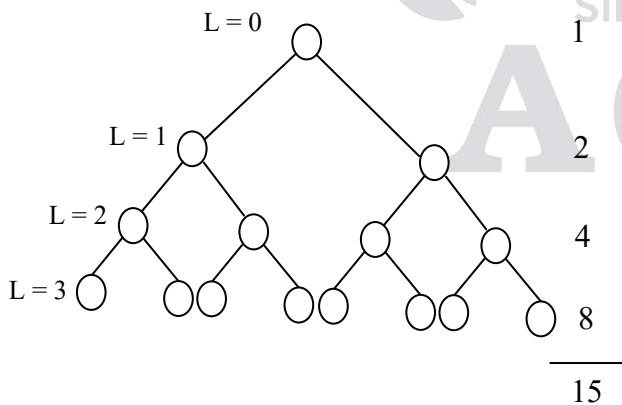
$$L = 200$$

$$200 = I + 1$$

$$I = 199$$

11. Ans: (b)

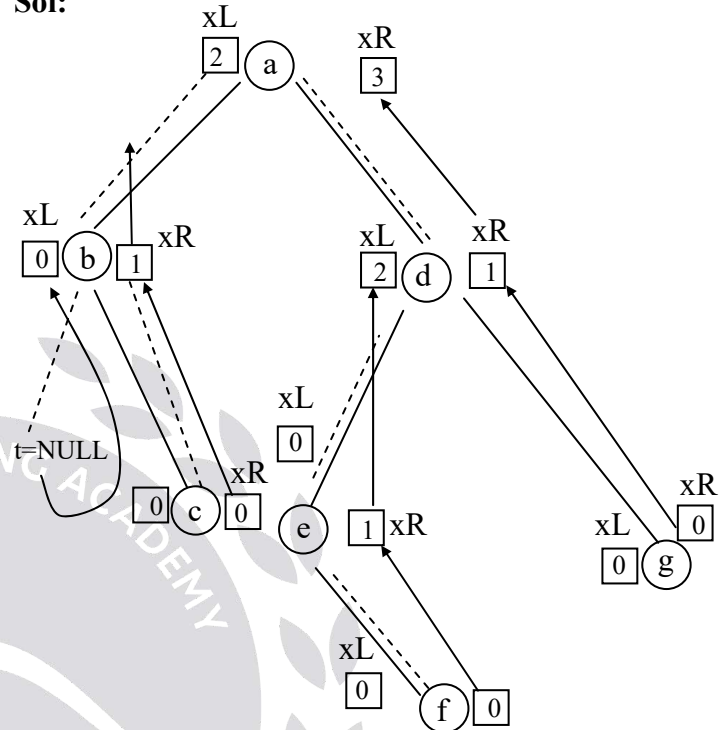
Sol:



Minimum = 3, Maximum = 14

12. Ans: 2 and 1

Sol:

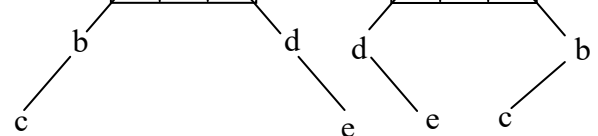
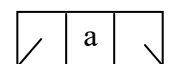
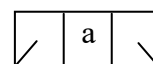
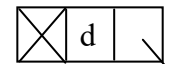
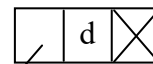
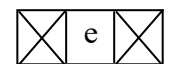
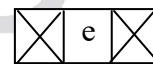
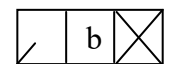
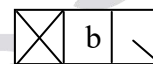


13. Ans: (a)

Sol:

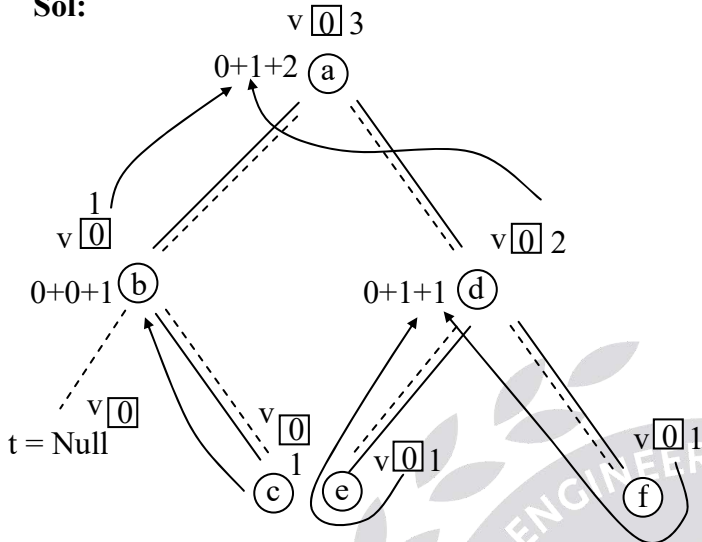
Before Swap

After swap



14. Ans: (d)

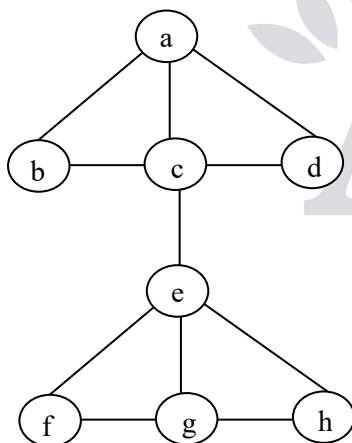
Sol:



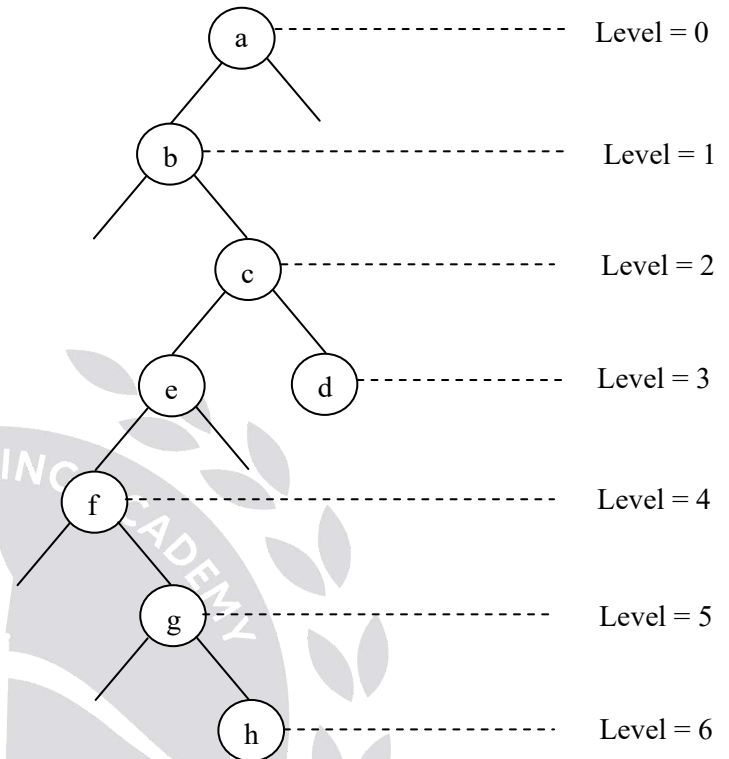
15. Ans: 6

Sol: a (b, c (e (f, g, h)), d)

Parent of f, g, h is e. i.e. internal parenthesis has children of parent which is out of parenthesis.

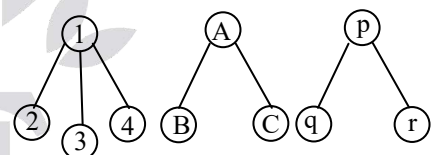


Converted Binary Tree:



16. Ans: 4

Sol: Given are 3 trees

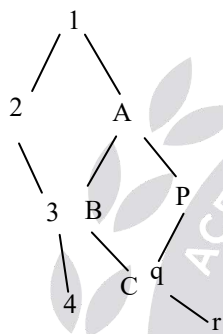
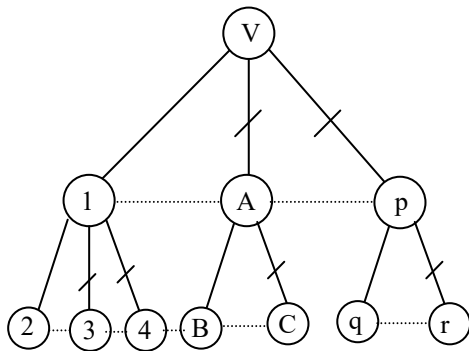


To get the converted binary tree of these given trees

∴ parent is not given we have to assume virtual parent

Among siblings

- Keep the leftmost as it is,
- Cut and connect right siblings as shown in diagram

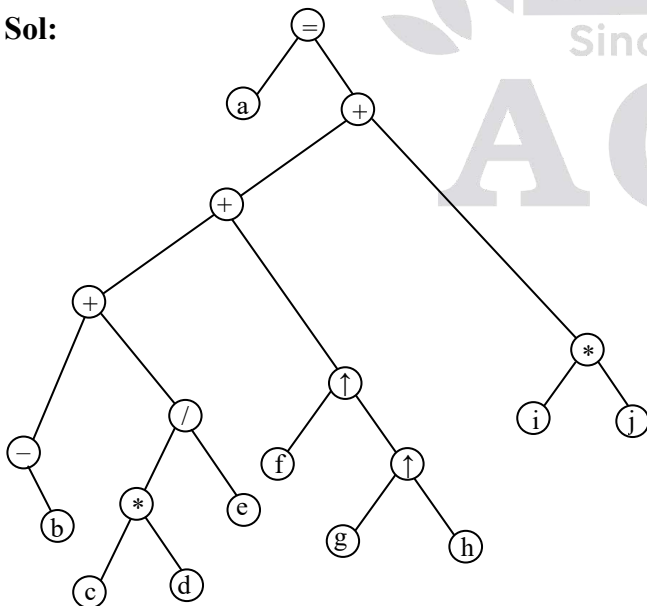


17. Ans: (d)

Sol: Count the number of trees in forest.

18. Ans: (b)

Sol:



19. Ans: 6

Sol: Expanded as

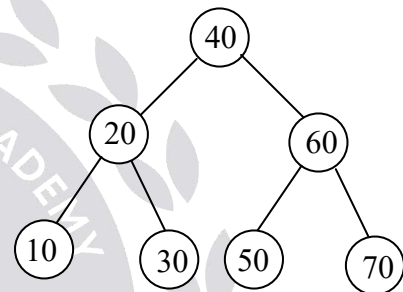
$$((1+1) - (0 - 1)) + ((1 - 0) + (1+1)) \\ = 3 + 3 = 6$$

20. Ans: -2

$$(0 + 0) - (1 - 0) + (0 - 1) + (0 + 0) \\ = -1 + (-1) = -2$$

21. Ans: 4

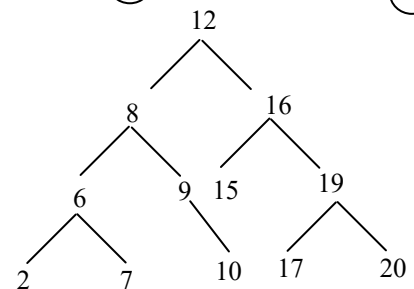
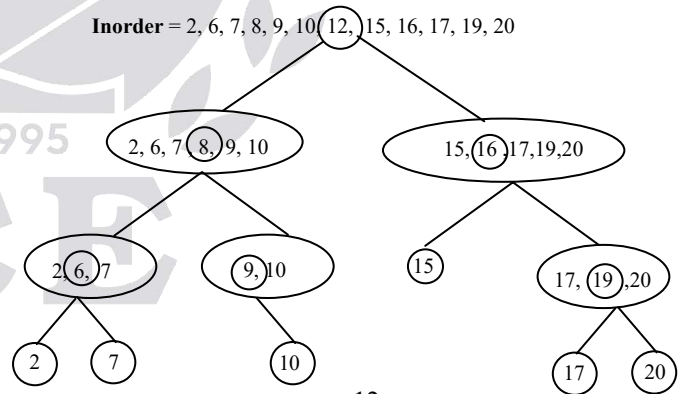
Sol:



22. Ans: (b)

Sol: Preorder = 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20

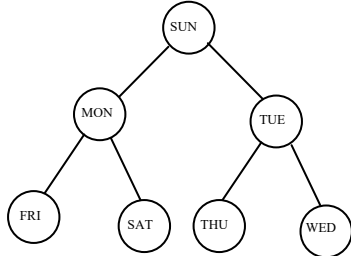
Inorder = 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20



$$= 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12$$

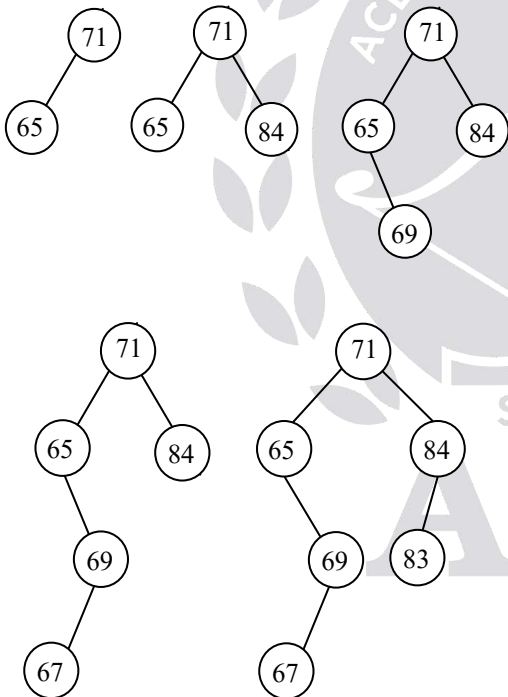
23. Ans: 4

Sol:



24. Ans: 67

Sol: 71, 65, 84, 69, 67, 83 insert into empty binary search tree

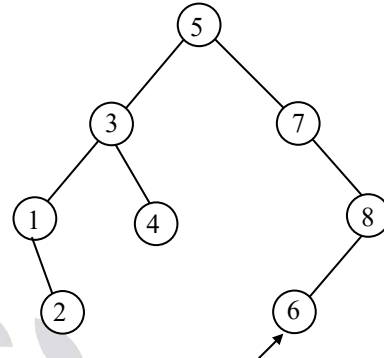


∴ Element in the lowest level is 67

25. Ans: 30

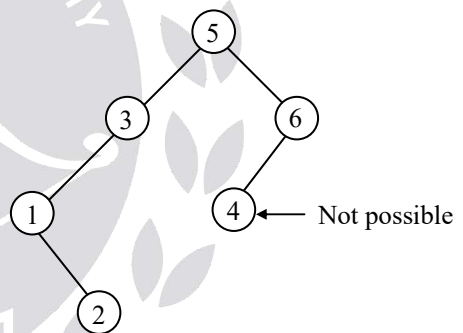
26. Ans: (d)

Sol: (a) 5 3 1 2 4 7 8 6



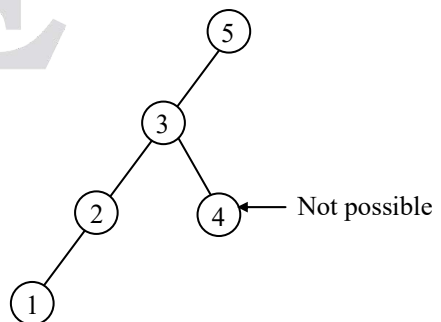
Not possible
IN: not sorted order

(b) 5 3 1 2 6 4 8 7



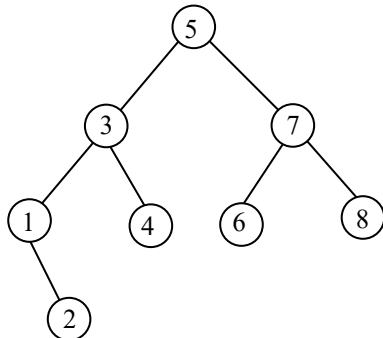
Not possible

(c) 5 3 2 4 1 6 7 8



Not possible

(d) 5 3 1 2 4 7 6 8



27. Ans: 15

Sol: 1. Jump right
2. Go on descend left

28. Ans: 88

$$\text{Sol: } N(H) = \begin{cases} 1 & L=H=0 \\ 2 & L=H=1 \\ 1+N(H-1)+N(H-2) & (L=H)>1 \end{cases}$$

$$N(H) = 1 + N(H-1) + N(H-2)$$

$$N(2) = 1 + N(1) + N(0)$$

$$= 1 + 2 + 1$$

$$= 4$$

$$N(3) = 1 + N(2) + N(1)$$

$$= 1 + 4 + 2$$

$$= 7$$

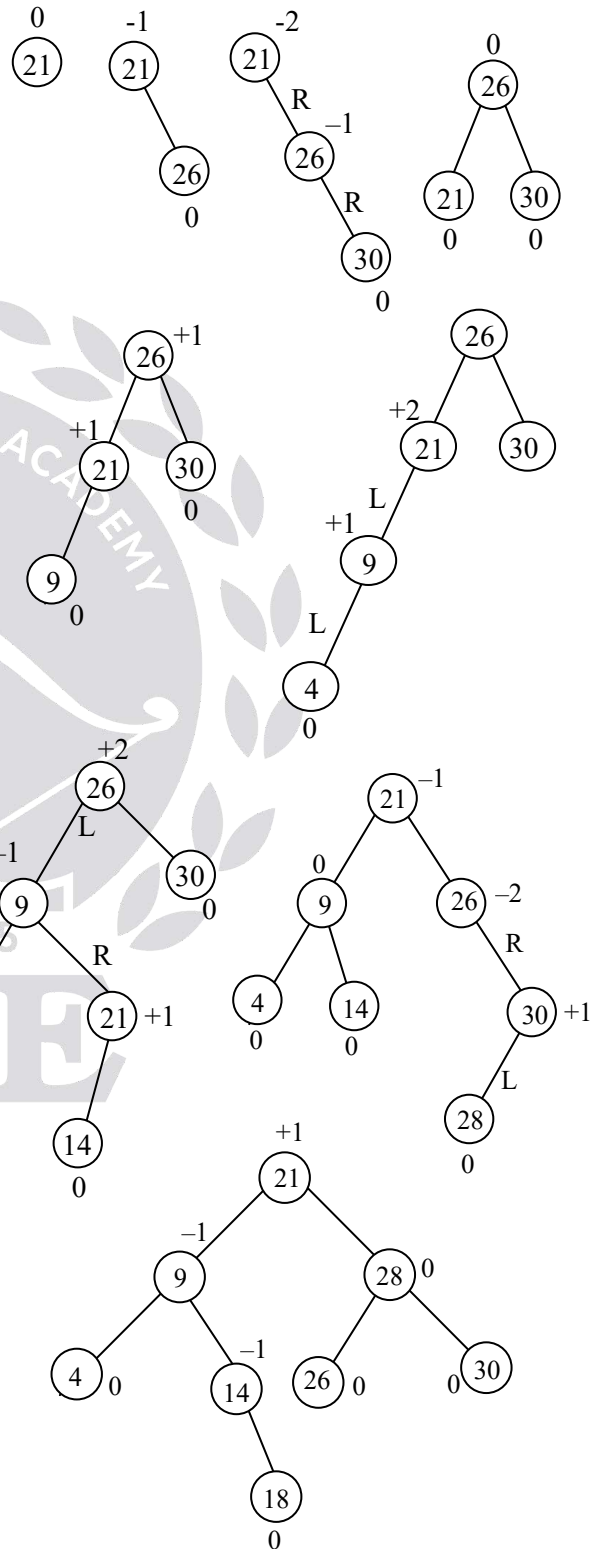
$$N(8) = 1 + N(7) + N(6)$$

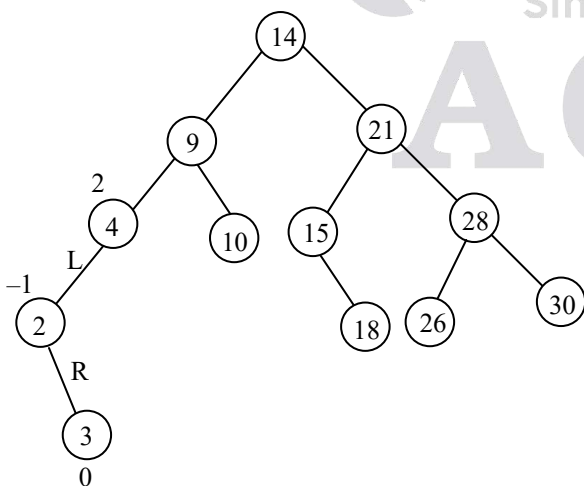
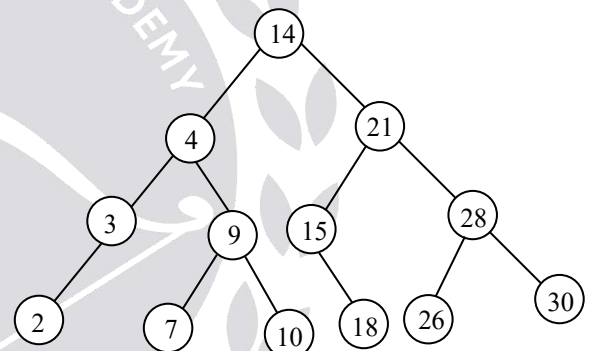
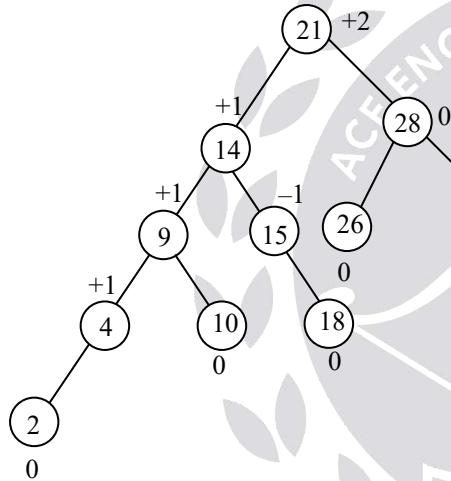
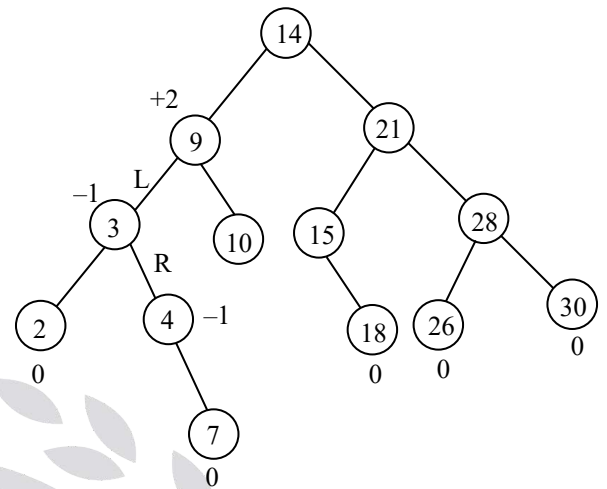
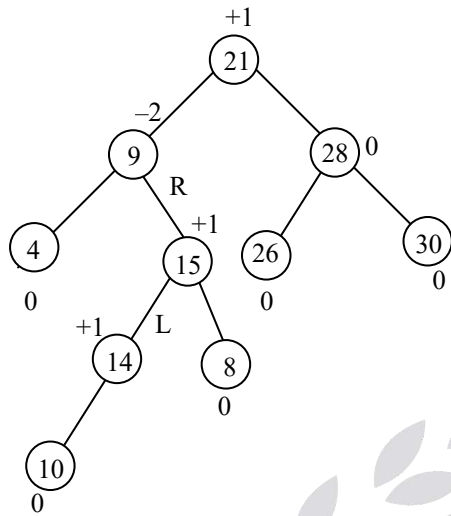
$$= 1 + 54 + 33 = 88$$

H	0	1	2	3	4	5	6	7	8
N(H)	1	2	4	7	12	20	33	54	88

29. Ans: 14

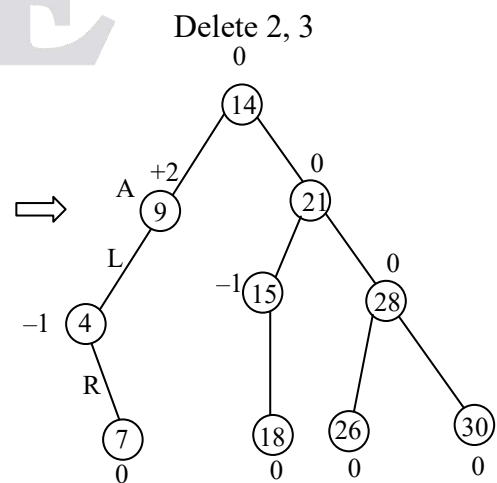
Sol: 21, 26, 30, 9, 4, 14, 28, 18, 15, 10, 2, 3, 7



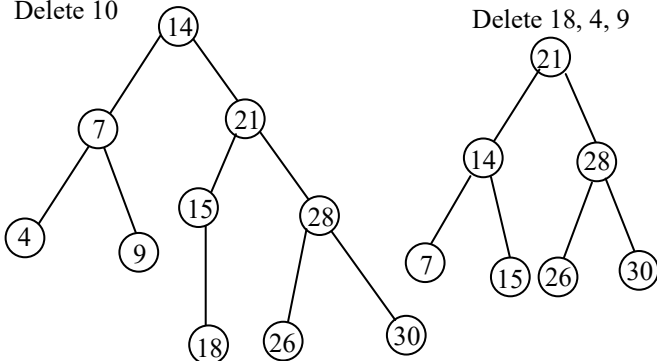


30. Ans: 28

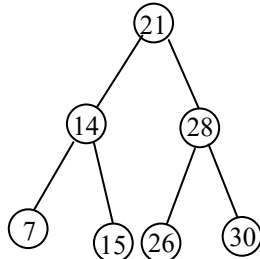
Sol:



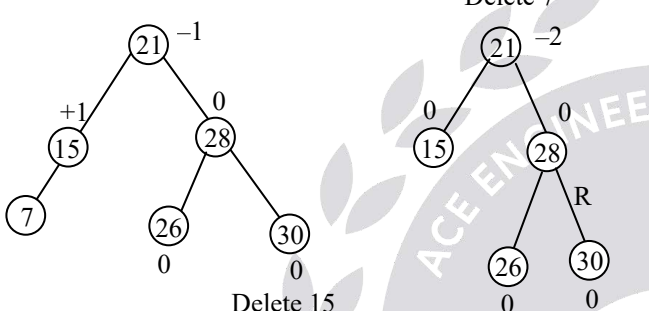
Delete 10



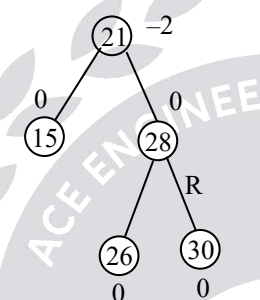
Delete 18, 4, 9



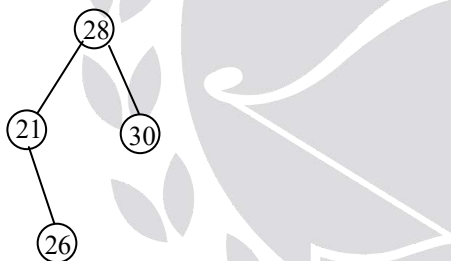
Delete 14



Delete 7



Delete 15



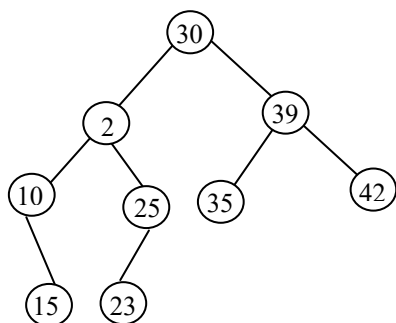
31. Ans: (a), (b) & (c)

Sol: The inorder of the binary search tree is always the sorted order.

Inorder: 10 15 20 23 25 30 35 39 42

To make a unique tree the inorder and the pre-order is sufficient.

A unique tree is:



Post-order traversal is:

15 10 23 25 20 35 42 39 30

So answer should be (a), (b), (c)

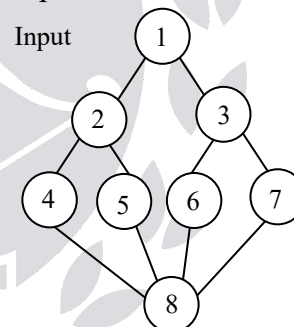
32. Ans: (a), (b) & (c)

Sol: Every option follows the same tree apart from the option d. In option d : 6 come before the 7 which makes store the 6 in place of 7. Hence option d will not be the same considering this given tree.

5. Graphs

01. Ans: (b)

Sol: V_8 is pushed in for two times



Output: 1, 2, 4, ⑧, 5 step back
⑧, 6, 3, 7.

V_8 : Two times

02. Ans: 2

Sol: Sequence of exploration

$V_5 \rightarrow V_2 \rightarrow V_1 \rightarrow V_3 \rightarrow$

$V_6 \rightarrow V_8 \rightarrow V_7 \rightarrow V_4$

Sequence of stack contents

Not pushed vertices are $\rightarrow V_4, V_7$

Vertices are pushed in more than once

$\rightarrow V_1, V_2, V_3, V_6, V_5$

V_8
V_8
V_6
V_3
V_1
V_2
V_5

03. Ans: 3

Sol: Sequence of exploration

$V_8 \rightarrow V_4 \rightarrow V_2 \rightarrow V_1 \rightarrow V_3 \rightarrow V_6 \rightarrow V_7 \rightarrow V_5$

Sequence of stack contents

V_2
V_3
V_3
V_1
V_2
V_4
V_8

Not pushed vertices are $\rightarrow V_6, V_7, V_5$

Vertices are not pushed in more than once

$\rightarrow V_1, V_4, V_8$

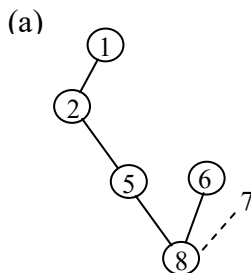
04. Ans: (a)

Sol: (a) invalid

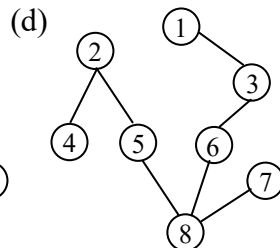
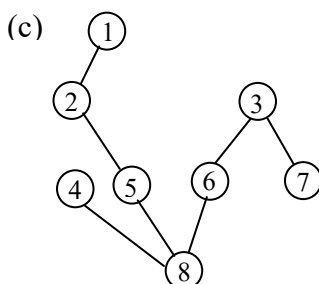
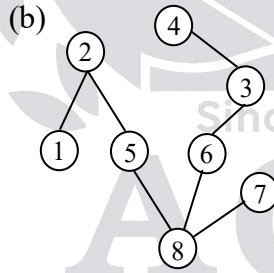
(b) valid

(c) valid

(d) valid



Step back only when already explored vertices are there



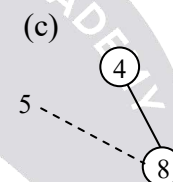
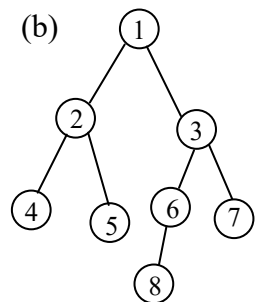
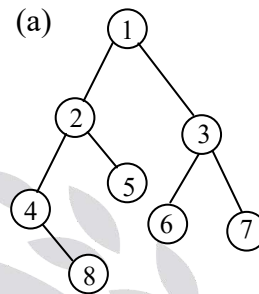
05. Ans: (c)

Sol: (a) valid

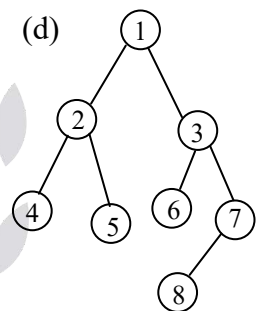
(b) valid

(c) invalid

(d) valid

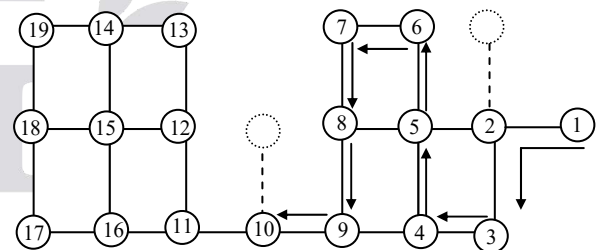


Step back only when already explored vertices are there



06. Ans: 19

Sol:

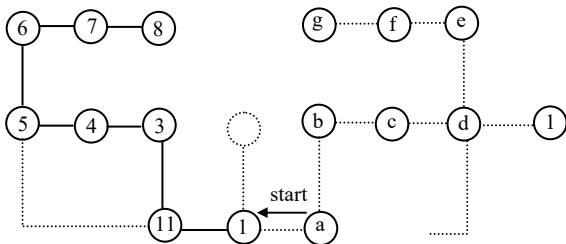


Maximum possible recursion depth = 19

(The dashed link 'nodes' are explored while stepping backward.)

07. Ans: 8

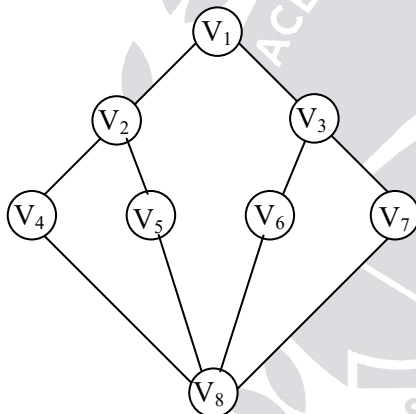
Sol:



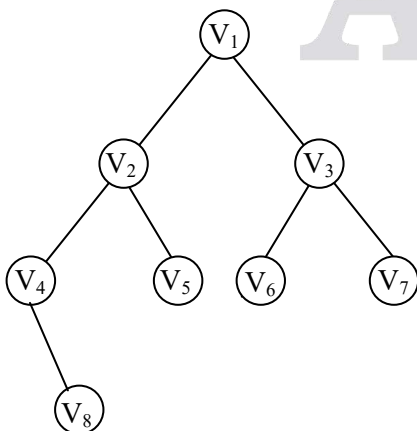
Minimum possible recursion depth = 8
(The dashed link 'nodes' are explored while stepping backward.)

08. Ans: (d)

Sol:



Traversal: BFS



queue

V ₂	V ₃	
----------------	----------------	--

Dequeue

queue

V ₃	V ₄	V ₅	
----------------	----------------	----------------	--

Dequeue

queue

V ₄	V ₅	V ₆	V ₇
----------------	----------------	----------------	----------------

Dequeue

queue

V ₅	V ₆	V ₇	V ₈
----------------	----------------	----------------	----------------

09. Ans: (d)

10. Ans: (d)

11. Ans: (a), (b) & (c)

Sol: S1: True. If DFS finds no back edges, then the graph is acyclic. Removing any back edges found doesn't change the traversal order of DFS, so running DFS again on the modified graph would produce no back edges.

S2: True. Each needs to keep track of the vertices that have already been visited.

6. Hashing

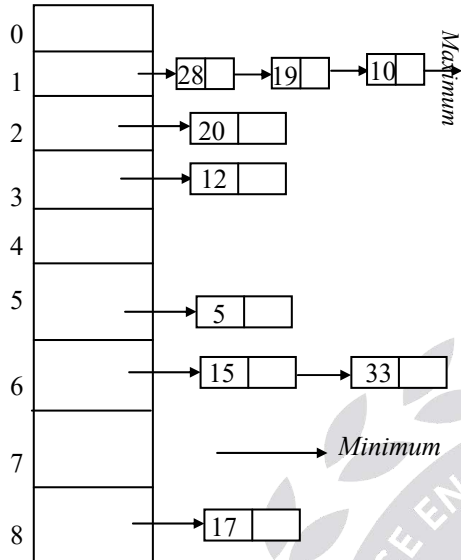
01. Ans: (d)

Sol:

1	to	1000
---	----	------

02. Ans: (a)

Sol:



$$\frac{3+1+1+1+2+1}{9} = 1 \text{ (average)}$$

03. Ans: 80

Sol: Slots = 25

Elements = 2000

$$\begin{aligned} \text{Load factor} &= \frac{\text{elements}}{\text{slots}} \\ &= \frac{2000}{25} = 80 \end{aligned}$$

04. Ans: (b)

Sol: Hash function

$$h(x) = (3x + 4) \% 7$$

$$h(1) = (3+4) \% 7 = 0$$

$$h(3) = (9+4) \% 7 = 6$$

$$h(8) = (24+4) \% 7 = 0$$

$$h(10) = (30+4) \% 7 = 6$$

Assume Linear probing for collision resolution

The table will be like

1	8	10				3
0	1	2	3	4	5	6

05. Ans: (d)

Sol: After inserting all keys, the hash table is

Key	43	36	92	87	11	4	71	13	14
Loc	10	3	4	10	0	4	5	2	3

0	1	2	3	4	5	6	7	8	9	10
87	11	13	36	92	4	71	14			43

Last element is stored at the position 7

06. Ans: (c)

Sol: Resultant hash table.

In linear probing, we search hash table sequentially starting from the original location. If a location is occupied, we check the next location. We wrap around from the last table location to the first table location if necessary.

07. Ans: (c)

Sol:

	×	×	✓	×
	A	B	C	D
0				
1				
2	42	42	42	42
3	52	23	23	23
4	34	34	34	23
5	23	52	52	34
6	46	33	46	46
7	33	46	33	52
8				
9				

08. Ans: (c)

Sol: Case (I): To store 52

Variable part			Fixed part		
42	23	34	52	46	33
42	34	23	52	46	33
23	42	34	52	46	33
23	34	42	52	46	33
34	42	23	52	46	33
34	23	42	52	46	33

$$3! = 6$$

Case (II): To store 33

Variable part			Fixed part	
42	23	34	52	33
42	23	34	46	

$$! = 24$$

Since 46 is not getting collided with any other key, it can be moved to the variable part.

Case (I) & Case (II) are mutually exclusive

$$\text{Case (I)} + \text{Case (II)} = 24 + 6 = 30$$

Total 30 different insertion sequences.

09. Ans: (a), (b) & (c)

Sol: Hash table: There is no restriction of deletion any item can be deleted

Priority Queue: It works on the priority and if the priority of 1 is higher then it may be deleted first.

Search tree: There is no restriction on deletion, any item can be deleted from the search tree. Queue: Queue follows FIFO, in the queue the first item should be deleted first which is 6.