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ESE-2020
(MAINS)

QUESTIONS WITH DETAILED SOLUTIONS

MECHANICAL ENGINEERING

PAPER-II

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MECHANICAL ENGINEERING

ESE _MAINS_2020_PAPER – II

Questions with Detailed Solutions

SUBJECT WISE WEIGHTAGE

S.No.	NAME OF THE SUBJECT	Marks
1	Engineering Mechanics	52
2	Strength of Materials	64
3	Theory of Machines	84
4	Machine Design	40
5	Production Engineering	52
6	Material Science	52
7	Mechatronics & Robotics	52
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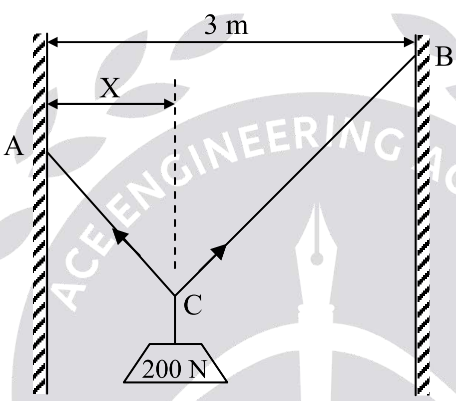
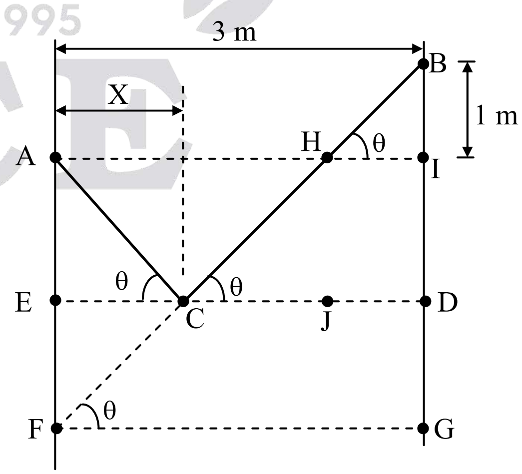
SECTION – A

01(a). A cord ACB 5 m long is attached at points A and B to two vertical walls 3 m apart as shown in the figure. A pulley C of negligible radius carries a suspended load of 200 N and is free to roll without friction along the cord. Determine the position of equilibrium as defined by the distance X, that the pulley will assume and also the tensile force in the cord.



(12 M)

Sol:



- Let $\angle BCD = \theta$
- As the tension in cord in same,

  $\angle BCD = \angle ACE = \theta$
- ΔAEC and ΔECF,

  $$\cos\theta = \frac{CE}{AC} = \frac{CE}{CF}$$

$\therefore$ AC = CF ----(i)

- AC + CB = ACB = 5 m,

  CF + CB = 5 m [$\because$ AC = CF]

- ΔBFG,

  $$\cos\theta = \frac{FG}{FB} = \frac{3}{5}$$

$\therefore$ $\sin\theta = \frac{4}{5}$

- Consider vertical distance between A and B is 1m.
- ΔAEC and ΔCHJ,

  $$\tan\theta = \frac{AE}{EC} = \frac{HJ}{CJ}$$

  $\therefore$ EC = CJ = x

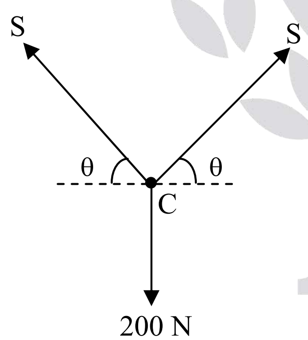
  $\therefore$ HI = 3 – 2x

- ΔBHI,

  $$\tan\theta = \frac{1}{3-2x}$$

$\Rightarrow$ x = 1.125 m

- FBD at C :



$\Sigma F_y = 0$

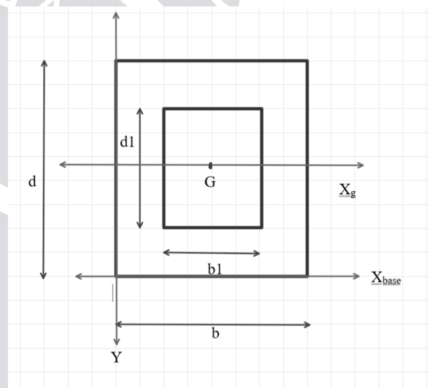
$2\,S\sin\theta = 200$

$\therefore$ $S = \frac{200}{2\sin\theta} = 125\ N$

01(b). For a beam of hollow rectangular section of outer geometry of b × d and the inner geometry of $b_1 \times d_1$, compute the area moment of inertia about its axis passing through its C.G. Also compute the area moment of inertia about a line passing through the base and also compute the same about a line passing through the vertical side. Compare the results and offer your remarks if the ratio of b : d = 1 : 2 units and in the same scaling the ratio of $b_1 : d_1 = 0.8 : 1.6$ units. (12 M)

Sol:

(i) $$I_{Xg} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{bd^3 - b\,d_1^3}{12}$$



Using parallel axis theorem,

(ii) $I_{Xbase} = I_{Xg} + A \times h^2$

$$= \frac{bd^3 - b_1 d_1^3}{12} + (b \times d - b_1 \times d_1) \times \left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3 - b_1 d_1^3}{12} + (b \times d - b_1 \times d_1) \times \left(\frac{d^2}{4}\right)$$

(iii) Using parallel axis theorem,

$I_Y = I_{Yg} + A \times h^2$

$$= \left(\frac{db^3}{12} - \frac{d_1 b_1^3}{12}\right) + (d \times b - d_1 \times b_1) \times \left(\frac{b}{2}\right)^2$$

$$= \frac{db^3 - d_1 b_1^3}{12} + (d \times b - d_1 \times b_1) \times \left(\frac{b^2}{4}\right)$$

If b:d = 1:2 and $b_1:d_1 = 0.8 : 1.6$

$\therefore$ Let , b = x, $\Rightarrow$ d = 2x

As the scaling ratio is same, $b_1 = 0.8\,x$ and $d_1 = 1.6\,x$

$$I_{Xg} = \frac{bd^3 - b_1 d_1^3}{12} = \frac{x \times (2x)^3 - (0.8x) \times (1.6x)^3}{12} = 0.3936 \times x^4 \text{ units}$$

$$I_{Xbase} = 0.3936x^4 + (x\times(2x) - 0.8x\times1.6x)\times\frac{(2x)^2}{4} = 1.1136\ x^4 \text{ units}$$

$$I_Y = \frac{db^3 - d_1b_1^3}{12} + (d\times b - d_1\times b_1)\times\left(\frac{b^2}{4}\right)$$

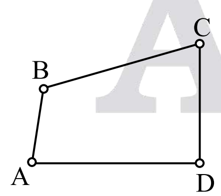
$$= \frac{(2x)\times x^3 - (1.6x)\times(0.8x)^3}{12} + (2x\times x - (1.6x)\times(0.8x))\times\left(\frac{x^2}{4}\right) = 0.2784\ x^4 \text{ units}$$

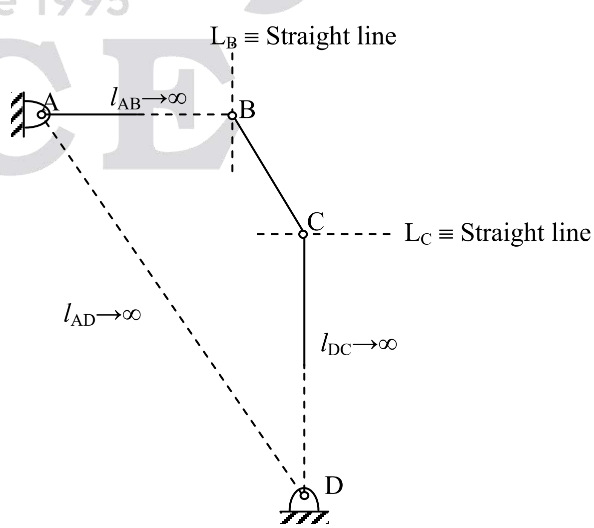
01(c).

(i). Explain about a double slider crank chain and its inversions. (8 M)

Sol: Inversion:

- It is the process of fixing different links of a chain.
- Every inversion may result in a unique mechanism and every mechanism is its own inversion.
- Number of possible inversions of a chain is equal to number of different links in chain.
- Inversion does not affect the ability to transfer the relative motion as it is fundamental property of parent kinematic chain.
- Double slider chain: In 4 bar simple planar chain if three links are infinitely long then it results in double slider chain

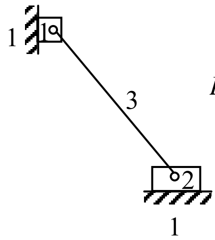




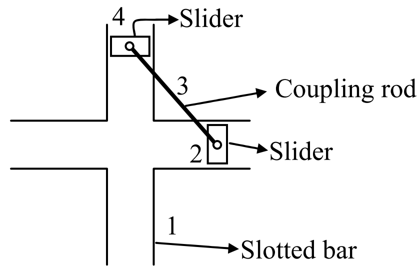
n = 4

but link 2 and 4 are identical

Therefore, number of inversions = 3



Double Slider Mechanism



Inversion (i) If link 1 i.e. slotted bar is fixed.

Rocker-Rocker mechanism

Application: Elliptical Trammel

Inversion (ii) If link 2 or link 4 i.e. slider is fixed

Crank-Rocker mechanism

Application: Scotch – Yoke Mechanism

used to convert rotation $\leftrightarrows$ SHM

Inversion (iii) If link 3 i.e. coupling rod is fixed

Crank – Crank Mechanism

Application: Oldham's coupling

01(c).

(ii). The distance between two parallel shafts is 18 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along the groove ? (4 M)

Sol: Given: distance between parallel shafts = 18 mm

$$N_{driving} = 160 \text{ rpm}$$

Maximum sliding velocity = eccentricity × angular velocity of driving shaft

$$= \frac{18}{1000} \times \frac{2\pi \times 160}{60} = 0.302 \text{ m/s}$$

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01(d). A rotor has a mass of 10 kg and is mounted midway on a 20 mm diameter horizontal shaft supported at the ends by two bearings. The bearings are 1 m apart. The shaft rotates at 2000 rpm. If the centre of the mass of the rotor is 0.11 mm away from the geometric centre of the rotor due to a certain manufacturing defect, find the amplitude of the steady-state vibration. Take E = 200 GN/m^2. Assume the shaft to be simply supported. (12 M)

Sol: Given:

Mass of rotor, (m) = 10 kg

Diameter of shaft, (d) = 20 mm

Distance between bearings = length of shaft

$l = 1$ m

Speed of shaft, N = 2000 rpm

Eccentricity of rotor, e = 0.11 mm , E = 200 GN/m^2

Calculate: *Dynamic amplitude of steady-state vibration*

Assumption: *shaft is simply supported*

Natural undamped frequency

$$\omega_n = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{48EI}{m\ell^3}}$$

$$= \sqrt{\frac{48\times 200\times 10^9}{10\times(1)^3}\times\frac{\pi}{64}\left(\frac{20}{1000}\right)^4} = 86.81 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 2000}{60} = 209.41 \text{ rad/s}$$

$$\text{Amplitude, } y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{0.11}{\left(\frac{86.81}{209.41}\right)^2 - 1}$$

$$y = -0.312 \text{ mm}$$

Negative sign indicates that the displacement is out of phase with centrifugal force.

01(e). The non-zero stress components at the critical point in the hub of a flywheel where yield is initiated are σ_{XX} = 100 MPa, σ_{YY} = – 20 MPa and τ_{XY} = 50 MPa. The flywheel material has a yield strength S_y = 300 MPa.

(i) Determine the principal stresses and maximum shear stress. Also show them in the Mohr's circle of stress.

(ii) Compare the factor of safety with Tresca's maximum shear stress theory and von Mises failure theory. (12 M)

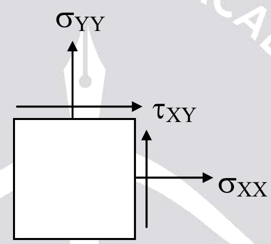
Sol:

σ_{XX} = 100 MPa,

σ_{YY} = – 20 MPa

τ_{XY} = 50 MPa.

σ_y = 300 MPa.



(i) $$\sigma_{1,2} = \left(\frac{\sigma_{XX}+\sigma_{YY}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{XX}-\sigma_{YY}}{2}\right)^2 + (\tau_{XY})^2}$$

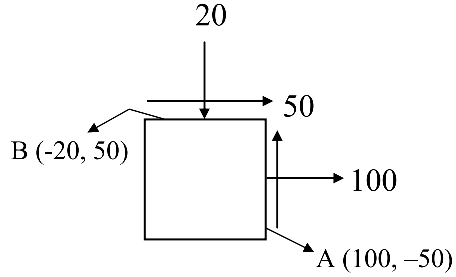
$$\sigma_{1,2} = 40 \pm \sqrt{(60)^2 + (50)^2}$$

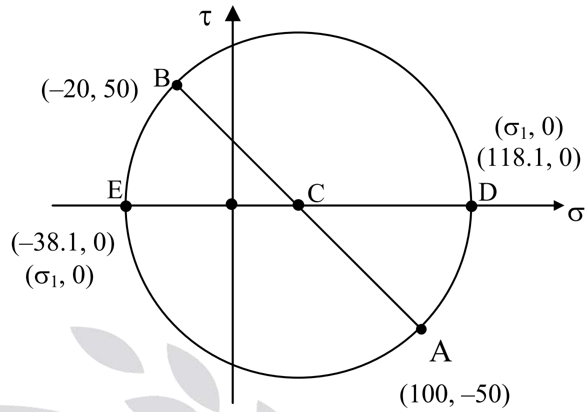
σ_1 = 118.1 MPa,

σ_2 = –38.1 MPa,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{XX}-\sigma_{YY}}{2}\right)^2 + (\tau_{XY})^2} = 78.1 \text{ MPa}$$

Mohr circle :





$$C = \text{center} = \left(\frac{\sigma_{XX} + \sigma_{YY}}{2}, 0\right)$$

$$= \left(\frac{100 - 20}{2}, 0\right) = (40, 0)$$

Radius = τ_{max} = CD

= 118.1 – 40 = 78.1 MPa

(ii) (a) Tresca's maximum shear stress theory :

$$\tau_{max} \le \tau_{all}$$

$$\text{Max. of } \left\{\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|\right\} \le \frac{\sigma_Y}{2 \times FOS}$$

$$\text{Max. of } \{78.1, 19.05, 59.05\} \le \frac{300}{2 \times FOS}$$

$$\Rightarrow 78.1 \le \frac{300}{2 \times FOS}$$

By Tresca's theory, FOS = 1.92

(b) von mises theory :

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 \le \left(\frac{\sigma_Y}{FOS}\right)^2$$

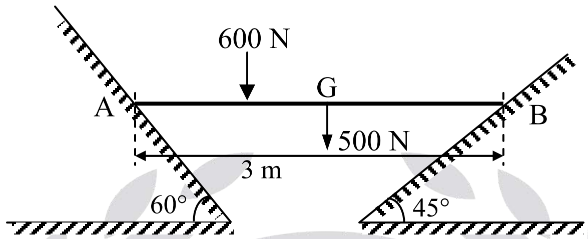
$$(118.1)^2 + (-38.1)^2 - (118.1)(-38.1) = \left(\frac{300}{FOS}\right)^2$$

$$(FOS)_{von\text{-}mises} = 2.13$$

Conclusion:

- As $(FOS)_{von\text{-}mises} > (FOS)_{Tresca}$
- That's why von-mises theory is more conservative than Tresca in this case.

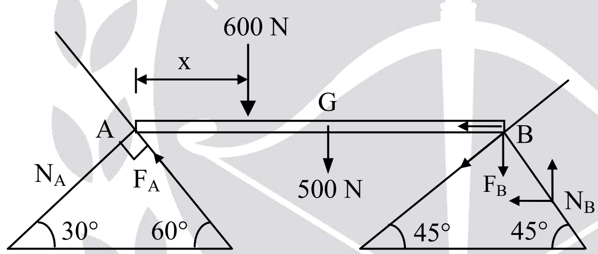
02(a). A horizontal bar AB of length 3 m and weighing 500 N is laying in a trough as shown in the figure below. Find how close to the end A and B can a load of 600 N be placed safely, if coefficient of friction between the bar and support is 0-2.



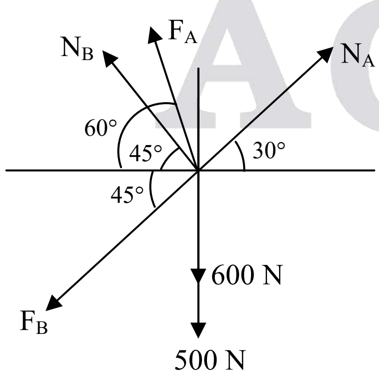
(20 M)

Sol: Given, L = 3m, W = 500 N, $\mu = 0.2$

Free body diagram is given below.



Let us assume at a distance x, the horizontal bar AB is about to slide into the trough.



$\Sigma F_x = 0$

$(N_A \cos30° - N_B \cos45° - F_A \cos60° - F_B \cos45°) = 0$ ----(1)

$\Sigma F_y = 0$

$N_A \sin30° + F_A \sin60° + N_B \sin45° – F_B \sin45° – 1100 = 0$ ----(2)

$\Sigma M_A = 0$

$(600 \times x) + (500 \times 1.5) + (–N_B \sin45° \times 3) + (– F_B \sin45° \times 3) = 0$ ----(3)

From (1),

$(0.866)N_A – 0.707\ N_B – 0.1\ N_A – 0.141\ N_B = 0$

$0.766\ N_A = 0.848\ N_B$

$\Rightarrow \quad N_A = 1.107\ N_B$

From (2),

$0.5N_A + 0.173\ N_A + 0.707\ N_B – 0.141\ N_B – 1100 = 0$

$0.5\ (1.107\ N_B) + (0.173)\ 1.107\ N_B + 0.707\ N_B – 0.141\ N_B – 1100 = 0$

$1.311\ N_B = 1100$

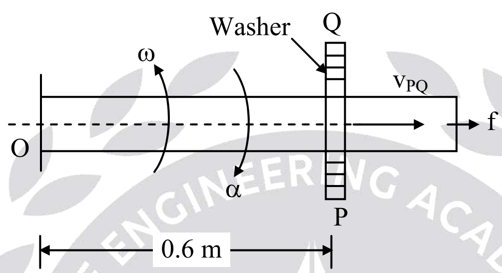
$\Rightarrow \quad N_B = 839.05\ N$

From (3),

$(600 \times x) + (750) + (–1779.89\ N_B) + (–355.97\ N_B) = 0$

$\Rightarrow \; x = 2.309\ m$

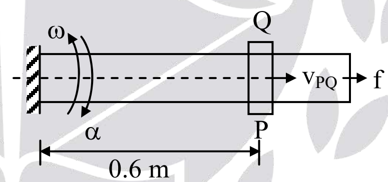
02(b). In the figure shown below, the washer is sliding outward on the rod with a velocity of 1.2 m/s when its distance from point 'O' is 0.6 m. Its velocity along the rod is increasing at the rate of 0.9 m/s². The angular velocity of the rod is 5 rad/s counter-clockwise and its angular acceleration is 10 rad/s² clockwise. Determine the absolute acceleration of a point in the washer.



(20 M)

Sol: Let point P is on washer and Q on rod and they are instantaneously coincident points.

Given



$V_{P/Q}$ = 120 m/s outward

$a_{P/Q}$ = 0.9 m/s² (acceleration)

ω_{rod} = 5 rad/s (ccw)

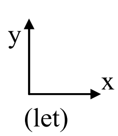
α_{rod} = 10 rad/s² (cw)

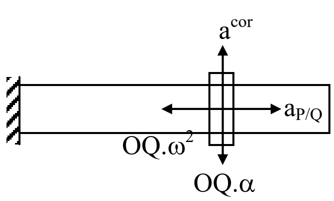
Calculate $a_{P/O} = a_P = ?$

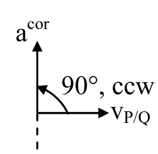
$$\vec{a}_P = \vec{a}_{Q/O} + \vec{a}_{P/Q} + \vec{a}^{\,coriolis}$$

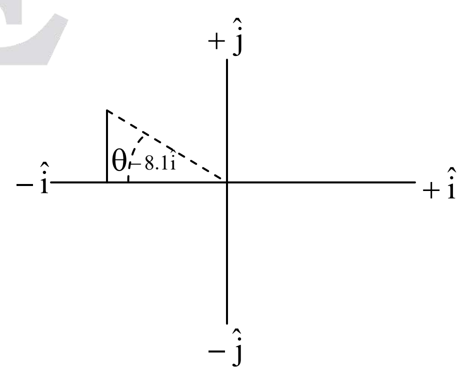
$$= \vec{a}^{\,n}_{Q/O} + \vec{a}^{\,t}_{Q/O} + \vec{a}_{P/Q} + \vec{a}^{\,coriolis}$$

$$= OQ.\omega^2_{rod}(-\hat{e}_r) + OQ.\alpha_{rod}(\hat{e}_t) + \vec{a}_{P/Q} + \vec{a}^{\,coriolis}$$









$$\overrightarrow{a_p} = OQ.\omega^2\left(-\hat{i}\right) + OQ.\alpha\left(-\hat{j}\right) + \vec{a}_{P/Q}\left(+\hat{i}\right) + a^{cor}\left(+\hat{j}\right)$$

$$= \left[0.6(5)^2\right]\left(-\hat{i}\right) + (0.6\times 10)\left(-\hat{j}\right) + 0.9\left(\hat{i}\right) + (2\times 1.20\times 5)\left(+\hat{j}\right)$$

$$= -9\hat{i} - 6\hat{j} + 0.9\hat{i} + 12\hat{j}$$

$$= \left(-8.1\hat{i} + 6\hat{j}\right)\ \text{m/s}^2$$

$$\left|\vec{a}_p\right| = \sqrt{(-8.1)^2 + 6^2} \Rightarrow \left|\vec{a}_p\right| = 10.08\ \text{m/s}^2$$

$$\tan\theta = \frac{6}{8.1}$$

$$\theta = 36.52°$$

$$\vec{a}_p = 10.08\ \angle 143.47°\ \text{m/s}^2 \quad \text{from} + x \text{ axis}$$

02(c). The shaft shown in the figure is to be designed from the standpoint of strength. Power is supplied to the pulley P by means of a flat belt and power is taken from the shaft through spur gear G. The shaft is supported by two deep groove ball bearings.

The following information has been established :

Power = 7.5 kW (steady load conditions),

speed of shaft = 1000 rev/min,

diameter of pulley = 250 mm,

pitch diameter of the gear = 250 mm,

weight of the pulley = 100 N,

weight of the gear = 100 N.

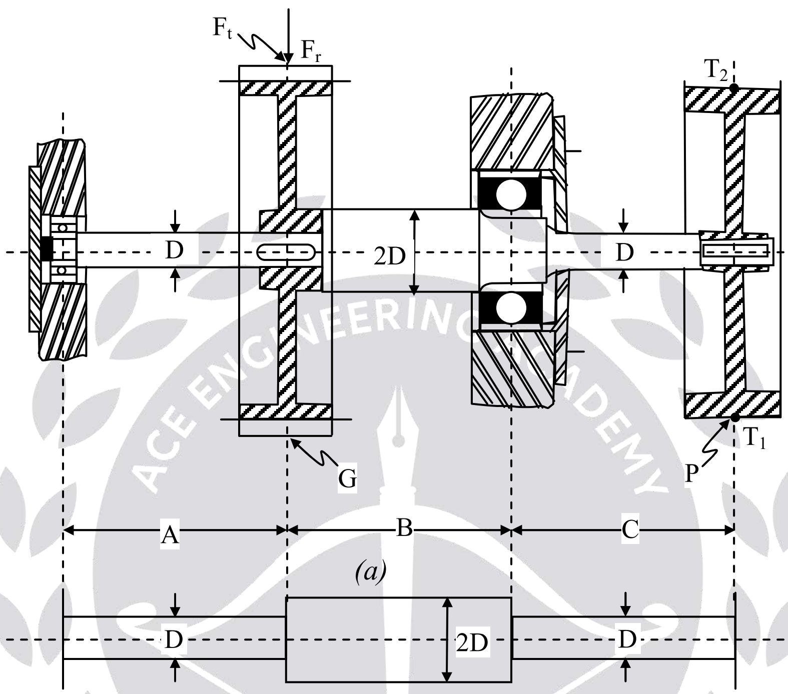
Ratio of belt tensions T_1/T_2 = 2.5,

Gear pressure angle = 20.

Dimensions A = B = C = 150 mm in the figure.

The belt forces are perpendicular to the plane of the paper, with $T_1 > T_2$ and the tangential force F_t on the gear is also perpendicular to the plane of the gear.

Shaft is to be machined from a hot rolled steel with S_{ut} = 590 MN/m^2 and S_{yt} = 380 MN/m^2. According to the ASME code, use allowable shear stress as minimum of either 0.18 S_{ut} or 0.30 S_{yt}. For steady load use k_b = 1.5 and k_t = 1.0.



(a)

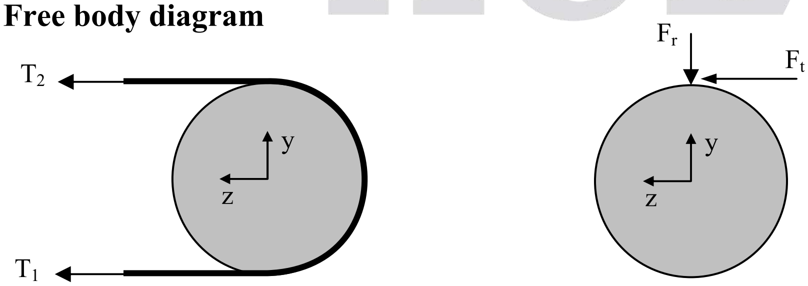
(b)

(20 M)

Sol: ***Assumptions:***

- *No power loss*
- *No fluctuations in power [∵ steady load conditions]*

Free body diagram



(1) Pulley (2) Gear

Calculation of tangential load F_t :

Power (P) = $\dfrac{2\pi NT}{60}$

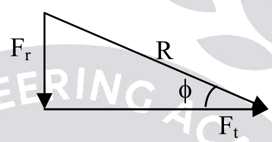
Torque transmitted (T) = $\dfrac{60P}{2\pi N} = F_t \times r$

Tangential load, $F_t = \dfrac{60P}{2\pi N} \times \dfrac{1}{r} = \dfrac{60 \times 7.5 \times 10^3}{2 \times \pi \times 1000} \times \dfrac{1}{125 \times 10^{-3}}$

Tangential load (F_t) = 573 N

Calculation of radial load :

$$\tan\phi = \frac{F_r}{F_t}$$



Radial load (F_R) = F_t. tan ϕ = 573 × tan (20°) = 208.5 N

Calculating tensions of belt :

$T_1 - T_2 = F_t$

And $\dfrac{T_1}{T_2} = 2.5$ [∵ diameter of pulley = diameter of gear]

⇒ $T_1 = 2.5\ T_2$

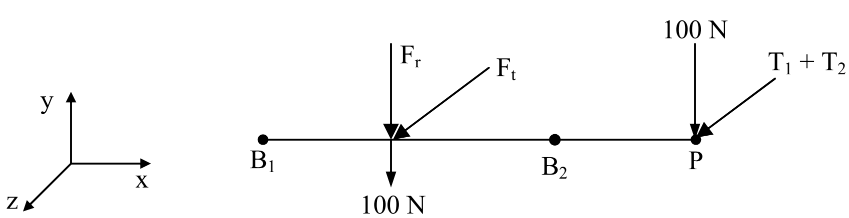
∴ $2.5\ T_2 - T_2 = F_t$

$$T_2 = \frac{F_t}{1.5} = \frac{573}{1.5}$$

T_2 = 382 N

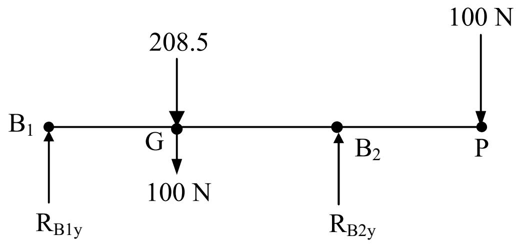
T_1 = 955 N

Loading diagram of shaft :



Where, B_1 and B_2 represent deep groove ball bearing. G represent gear, P represent pulley

Loading in x-y plane:



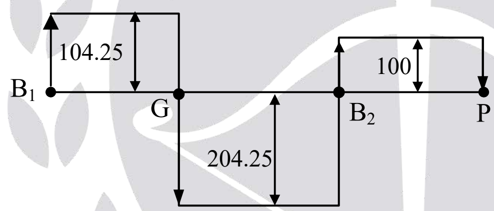
Taking moments about B_1

$R_{B2y} \times 300 = 308.5 \times 150 + 150 \times 3 \times 150$

$R_{B2y} = 304.25$ N

$R_{B1y} = 104.25$ N

SFD of shaft in xy-plane :

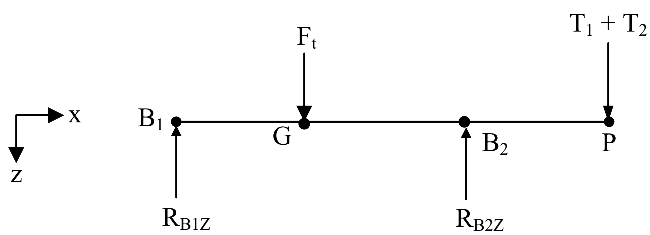


From SFD in xy plane. Bending moment is maximum at either section G (or) at section B_2 .

$(M_{xy})_G = 104.25 \times 150 \times 10^{-3} = 15.64$ N.m (Sagging)

$(M_{xy})_{B2} = 100 \times 150 \times 10^{-3} = 15$ N.m (Hogging)

Loading in xz plane :



Taking moments about B_1

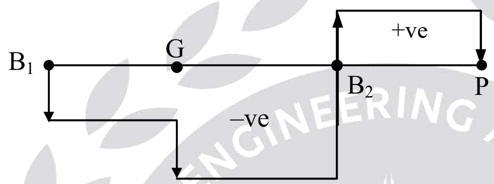
$F_t \times 150 - R_{B2Z} \times 300 + (T_1 + T_2) \times 450 = 0$

$573 \times 150 - R_{B2Z} \times 300 + (955 + 382) \times 450 = 0$

$R_{B2Z} = 2292$ N

$R_{B1Z} = -382$ N

SFD in xz plane :



As shear force is changing its sign at B_2, bending moment is maximum at section B_2. (Sagging)

$(M_{xz})_{B2} = (955 + 382) \times 150 \times 10^{-3} = 200.55$ N.m

$(M_{xz})_G = 382 \times 150 \times 10^{-3} = 57.3$ N.m

$\therefore$ Moment at G

$$M_G = \sqrt{(M_{xy})_G^{\,2} + (M_{xz})_G^{\,2}} = \sqrt{15.64^2 + 57.3^2} = 59.4 \text{ N.m}$$

$\therefore$ Moment at B_2

$$M_{B2} = \sqrt{(M_{xy})_{B2}^{\,2} + (M_{xz})_{B2}^{\,2}} = \sqrt{15^2 + 200.55^2} = 201.1 \text{ N.m}$$

As $M_{B2} > M_G$, the shaft is designed to avoid failure at section B_2.

Twisting moment between gear and pulley :

$$T = F_t \times r = 573 \times 125 \times 10^{-3} = 71.625 \text{ N.m}$$

Design of shaft :

According to maximum shear stress theory

$$\tau_{max} = \frac{\tau_{yt}}{FOS}$$

$$\frac{16}{\pi D^3}\sqrt{(k_b \times M)^2 \times (k_t \times T)^2} = \frac{\tau_{yt}}{FOS}$$

$$\tau_{allowable} = \frac{\tau_{yt}}{FOS} = \min\{0.18\ \sigma_{ut}, 0.3\ \sigma_y\}$$

$$= \min\{0.18 \times 590, 0.3 \times 380\}$$

$$= \min\{106.2, 114\}$$

$$\frac{16 \times 10^{-3}}{\pi D^3}\sqrt{(1.5 \times 201.1)^2 \times (1 \times 71.625)^2} = 106.2$$

$\Rightarrow$ D = 24.6 mm

$\therefore$ The next possible diameters of transmission shaft is 25 mm.

03(a). A cantilever beam of length 'L' carries the loading as below :

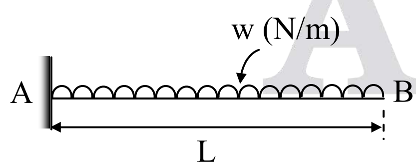
(i) Carrying a uniformly distributed load of 'w' per unit run over the whole length.

(ii) Carrying a distributed load when intensity varies from zero at the free end to w' per unit run at the fixed end.

(iii) Carrying a distributed load whose intensity varies from zero at the fixed end to 'w' per unit run at the free end.

Calculate the maximum displacement and the maximum stress for each loading case and offer your remarks on the result. (20 M)

Sol: (i)

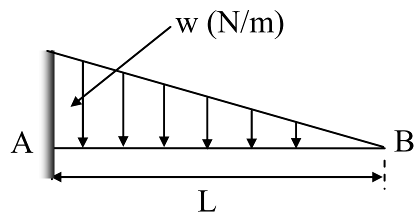


Assume section modulus = Z

$$\Delta_{max} = \Delta_B = \frac{wL^4}{8EI}$$

$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{\frac{wL^2}{2}}{Z} = \frac{wL^2}{2Z}$$

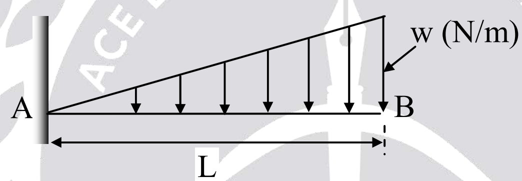
(ii)



$$\Delta_{max} = \Delta_B = \frac{wL^4}{30EI}$$

$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{\frac{wL^2}{2} \times \frac{L}{3}}{Z} = \frac{wL^2}{6Z}$$

(iii)



$$\Delta_{max} = \Delta_B = \frac{wL^4}{8EI} - \frac{wL^4}{30EI} = \frac{11\,wL^4}{120\,EI}$$

$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{\frac{wL^2}{2} \times \frac{2L}{3}}{Z} = \frac{wL^2}{3Z}$$

$\rightarrow$ As $(\Delta_{max})_{(i)} > (\Delta_{max})_{(iii)} > (\Delta_{max})_{(ii)}$

By deflection criterion (ii) is best.

$\rightarrow$ As $(\sigma_{max})_{(i)} > (\sigma_{max})_{(iii)} > (\sigma_{max})_{(ii)}$

By strength criterion (ii) is best.

$\rightarrow$ Overall in these three loading conditions case(ii) is best.

03(b).

(i). Define the terms Interference and Undercutting in the mating of a pair of teeth while transmitting the power and mention how we can avoid the same.

Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion 13 teeth. Does the interference occur and if it occurs, how can we eliminate the same ? (10 M)

Sol: **Interference:** When involute profile of gear meshes with non-involute profile of pinion it results an non-conjugate action. This non-conjugate action is known as interference. As a result of interference the involute portion of gear tooth may remove the non-involute portion of pinion tooth, this is known as undercutting.

Due to interference as well as undercutting tooth becomes weaker therefore they should be avoided.

Sometimes undercutting is also used as a method to avoid interference but since it weakens the tooth interference but since it weakens the tooth therefore as a method to avoid interference it is least preferred one.

Necessary and sufficient condition for interference:

If the addendum of gear penetrates into the base circle of pinion or the point of beginning of engagement crosses the point of tangency of pinion it will always results in interference.

Methods to avoid interference and undercutting:

1. By increasing the center distance.
2. By modifying the addendum i.e. by stubbing.
3. By properly selecting the number of teeth on pinion.

Minimum number of teeth on pinion in order to avid interface (In Gear and Pinion)

$$t_{min} = \frac{2f\lambda}{\sqrt{1+\lambda(\lambda+2)\sin^2\phi}-1}$$

addendum = fraction module

$$\lambda = \frac{w_g}{w_p} = \frac{r}{R} = \frac{t}{T}(<1)$$

4. By using cycloidal profile teeth

Given, $\phi = 20°$, involute spur gears,

$m = 10$ mm

addendum = 1 module $\quad\because\ a = f.m$

$\Rightarrow \quad f = 1$

$T = 50$

$t = 13$

$$\lambda = \frac{t}{T} = \frac{13}{50}$$

$\because$ minimum number of teeth on pinion in order to avoid interference,

$$t_{min} = \frac{2\times1\times\frac{13}{50}}{\sqrt{1+\frac{13}{50}\left(\frac{13}{50}+2\right)\sin^2 20°}-1}$$

$$= \frac{2\times0.26}{\sqrt{1+0.26(2.26)\sin^2 20°}-1}$$

$\Rightarrow t_{min} = 15.549$

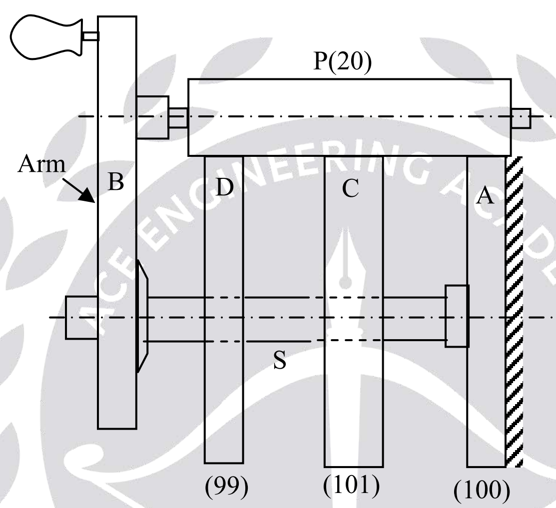
$\Rightarrow t_{min} \approx 16$

So, minimum number of teeth on pinion should be greater than or equal to 16 in order to avoid interference but in actual it is 13 so interference is occurring.

Number of teeth on pinion should increase to avoid interference.

03(b).

(ii). **The figure below shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is therefore stationary. The arm B and gear C and D are free to rotate on the shaft. Gears A, C and D have 100, 101 and 99 teeth respectively. Planet gear P has 20 teeth. Pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of the arm B.**



(10 M)

Sol: Given, $T_A = 100$, $T_C = 101$, $T_D = 99$, $T_P = 20$

Condition	Arm	T_A Gear A	T_C Gear C	T_D Gear D
Arm is fixed, gear a rotates with +x rev (cw)	0	+x	$+x.\frac{T_A}{T_C}$	$+x.\frac{T_A}{T_C}.\frac{T_C}{T_D}$ $+x.\frac{T_A}{T_D}$
Arm is rotating with +y rev	+y	x+y	$y+x\frac{T_A}{T_C}$	$y+x\frac{T_A}{T_D}$

$\because$ arm makes one rev,

$y = 1$

Gear A is fixed,

$\Rightarrow$ $N_A = 0$

$\Rightarrow$ $x + y = 0$

$\Rightarrow$ $x = -1$ rev

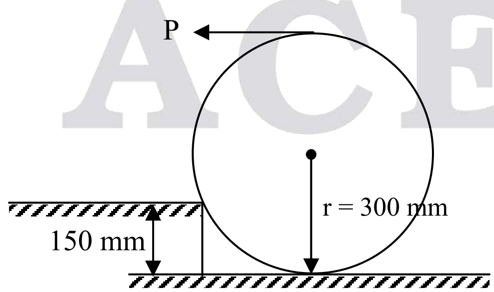
Revolutions of gear C

$$N_C = y + x.\frac{T_A}{T_C} = 1 - 1.\left(\frac{100}{101}\right) = +\frac{1}{101} \text{ rev (cw)}$$

Revolutions of gear D

$$N_D = y + x.\frac{T_A}{T_D} = 1 - 1.\frac{100}{99} = \frac{-1}{99} \text{ rev (ccw)}$$

03(c). A roller of radius r = 300 mm and weight 2000 N is to be pulled over a curb of height 150 mm by a horizontal force P applied at the end of a string wound tightly around the circumference of the roller. Find the magnitude of P required to start the roller moving over the curb. What is the least pull P through the centre of the wheel to just turn the roller over the curb ?

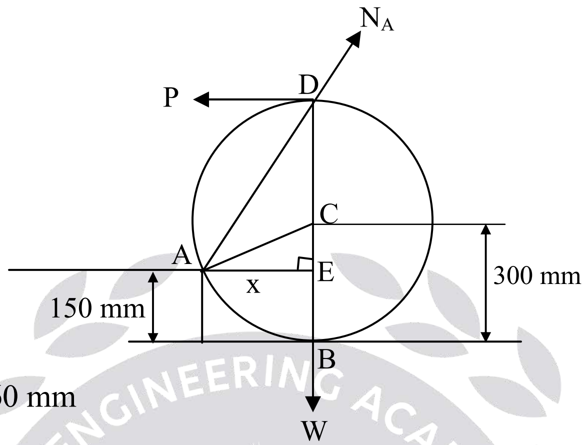


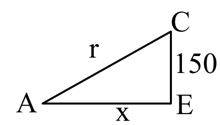
(20 M)

Sol: Given, $r = 300$ mm, $W = 20000$

$h = 150$ mm, $P = ?$

(i) Free body diagram





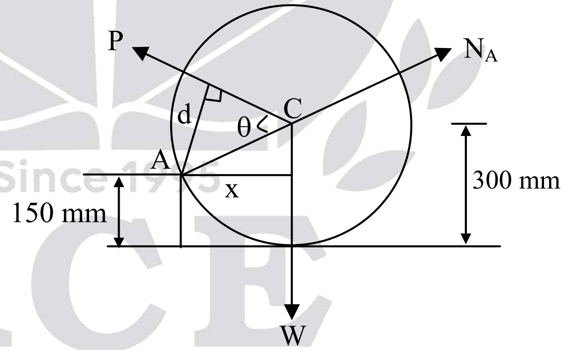
$r^2 = 150^2 + x^2 \Rightarrow x = 260$ mm

$\Sigma M_A = 0$

$(W \times x) + (-P \times 450) = 0$

$P = 1155.5 \text{ N} \approx 1.15 \text{ kN}$

(ii)



$\Sigma M_A = 0$

$(-P \times d) + (W \times x) = 0$

$$\sin\theta = \frac{d}{r}$$

$d = r \sin\theta$

$$P = \frac{W \times x}{r \sin\theta} = \frac{2000 \times 260}{300 \times \sin\theta}$$

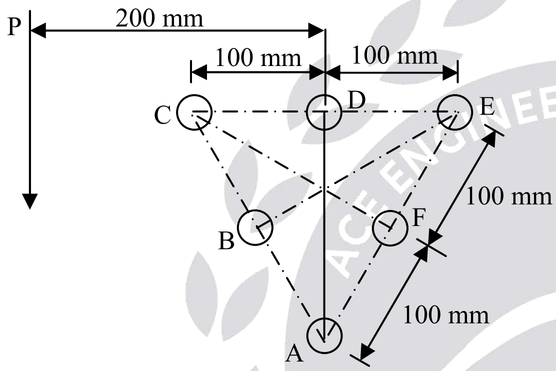
$P \rightarrow P_{min}$ (or) least

When, $\sin\theta = \text{Max} = 1 = \sin 90°$

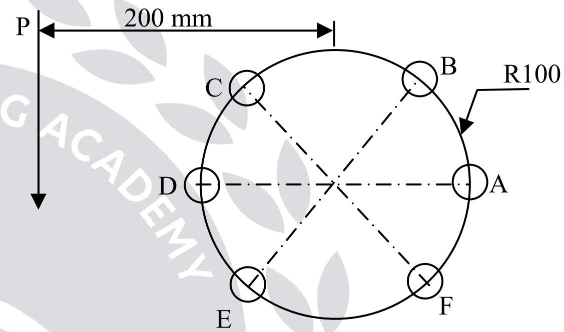
$\theta = 90°$

$$P = \frac{W \times x}{r \times (1)} = 1733.3 \text{ N} \cong 1.733 \text{ kN}$$

04(a). A bracket is to be attached to a wall with the help of six rivets. The different arrangements in which the bracket can be attached to the wall are shown in the figure. The maximum allowable stress in shear is 60 N/mm^2. Determine the way in which the rivets should be arranged so that the design is economical. The bracket is required to support a load of 60 kN with an eccentricity of 200 mm. Determine the diameter of rivets for the two arrangements.



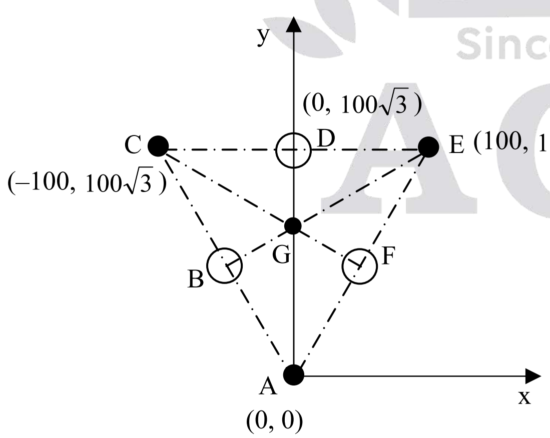
(a) Triangular Arrangement



(b) Circular Arrangement

(20 M)

Sol: Case – I :



$$G = \left(0, \frac{0 + 100\sqrt{3} + 100\sqrt{3}}{3}\right)$$

$$G = \left(0, \frac{200}{\sqrt{3}}\right)$$

$$AD = 100\sqrt{3}$$

$$AG = CG = EG = \frac{2AD}{3} = \frac{200\sqrt{3}}{3}$$

$$BG = DG = FG = \frac{AD}{3} = \frac{100\sqrt{3}}{3}$$

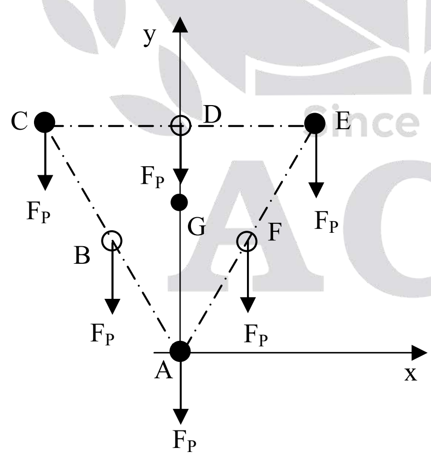
As the load P is not passing through the centre of gravity of riveting system. The loading is considered to be eccentric loading.

Here eccentricity (e) = 200 mm

Primary load acting on each rivet,

$$F_P = \frac{\text{Total load}}{\text{No. of rivets}} = \frac{60}{6} = 10\,\text{kN}$$

The effect of primary force is shearing the rivet.



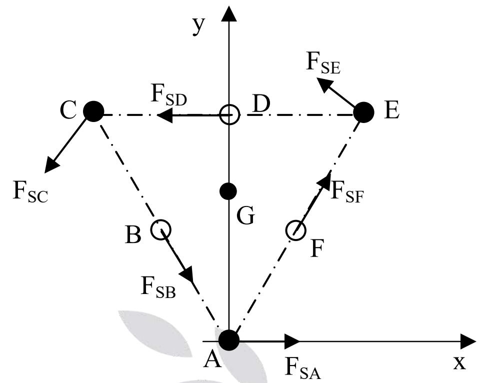
Secondary force on each rivet,

$$F_s = \frac{P \times e}{r_1^2 + r_2^2 + r_3^2} \times r$$

Where, r_1 , r_2, r_3 …. Represent the distance between centre of gravity of riveting system and centre of rivet.

$$\therefore\ r_A = r_C = r_E = \frac{200}{3}\sqrt{3}$$

$$r_B = r_D = r_F = \frac{100}{3}\sqrt{3}$$

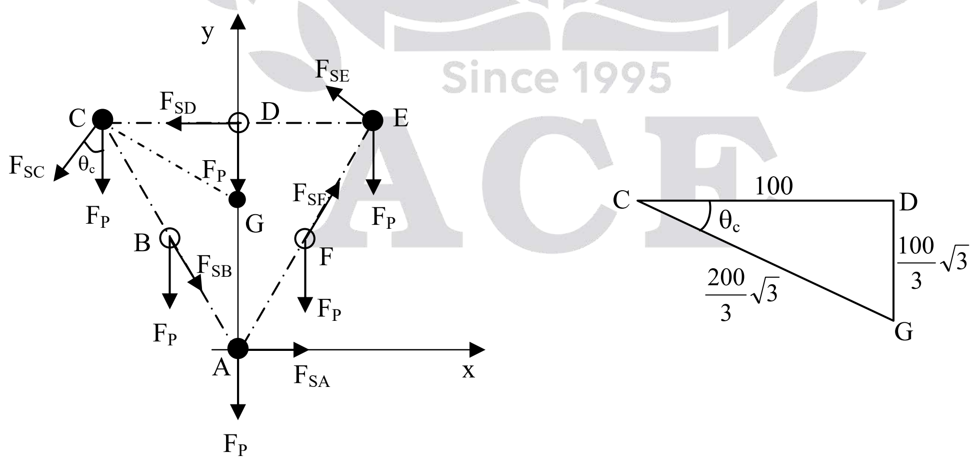


$$F_{SA} = F_{SC} = F_{SE} = \frac{60\times 200}{3\left[\left(\frac{100}{3}\sqrt{3}\right)^2 + \left(\frac{200}{3}\sqrt{3}\right)^2\right]}\times\frac{200}{3}\sqrt{3} = 16\sqrt{3}\ \text{N}$$

$$F_{SB} = F_{SD} = F_{SF} = \frac{60\times 200}{3\times\left[\left(\frac{100}{\sqrt{3}}\right)^2 + \left(\frac{200}{\sqrt{3}}\right)^2\right]}\times\frac{100}{\sqrt{3}} = 8\sqrt{3}\,\text{N}$$

Resultant load :

From resultant loading diagram, it is clear that rivet ‘C’ is critical.



$$\cos\theta_C = \frac{CD}{CG} = \frac{100}{\frac{200}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$R_{max} = \sqrt{F_P^2 + F_S^2 + 2\times F_P \times F_S \times \cos\theta}$$

$$R_{max} = \sqrt{10^2 + \left(16\sqrt{3}\right)^2 + 2\times 10\times 16\sqrt{3}\times\frac{\sqrt{3}}{2}} = 36.71 \text{ kN}$$

Design of rivet :

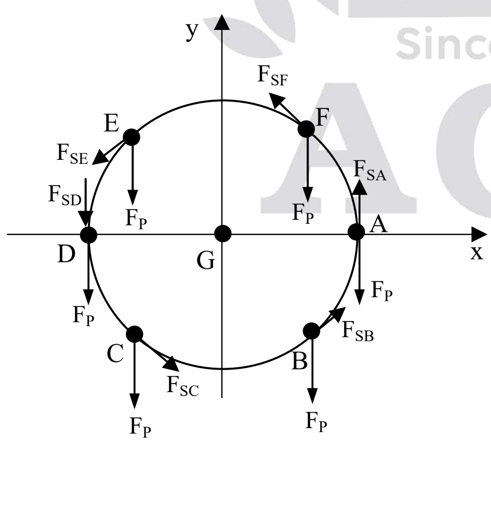
$$\tau_{max} = \frac{\tau_{yt}}{FOS}$$

$$\frac{R_{max}}{\frac{\pi}{4}d^2} = \frac{\tau_{yt}}{FOS}$$

$$\Rightarrow \quad \frac{36.71\times 10^3}{\frac{\pi}{4}\times d^2} = \frac{60}{1}$$

$$\Rightarrow \quad d = 27.91 \text{ mm}$$

Case – II :



Primary force on each rivet.

$$F_P = \frac{60}{6} = 10\,\text{kN}$$

The effect of primary force is shearing.

Secondary force on each rivet,

$$F_S = \frac{P \times e}{6 \times r^2} \times r = \frac{P \times e}{6 \times r}$$

$$F_S = \frac{60 \times 200}{6 \times 100} = 20\text{ kN}$$

From the loading diagram, it is clear that rivet – D is critical.

$\therefore$ $R_{max} = F_P + F_S = 10 + 20 = 30$ kN

Design of rivet :

$$\tau_{max} = \frac{\tau_{yt}}{FOS}$$

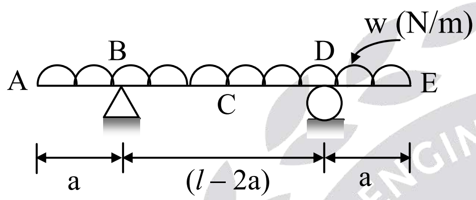
$$\Rightarrow \quad \frac{30 \times 10^3}{\frac{\pi}{4} d^2} = 60$$

$\Rightarrow$ d = 25.31 mm

Hence case – II represents economical design.

04(b). Draw the SFD and BMD of a simply supported beam with equal overhangs of length "a" carrying a uniformly distributed load of 'w' per unit run over the whole length 'l'. Show the BMD when $a = \frac{\ell}{2}$, $a < \frac{\ell}{2}$ and $a > \frac{\ell}{2}$. Offer your comments on the results. (20 M)

Sol:



$$R_B = R_D = \frac{wL}{2}$$

$$(BM)_B = (BM)_D = -wa \times \frac{a}{2} = -\frac{wa^2}{2}$$

$$(BM)_C = R_B \times BC - \frac{wL}{2} \times \frac{L}{4}$$

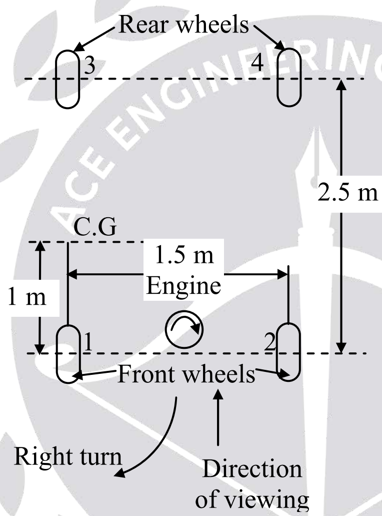
$$(BM)_C = \frac{wL}{2}\left(\frac{L-2a}{2}\right) - \frac{wL}{2} \times \frac{L}{4} = \frac{wL}{2}\left(\frac{L}{4} - a\right)$$

(i) If $a = \frac{L}{2} \rightarrow (BM)_C = (-)$

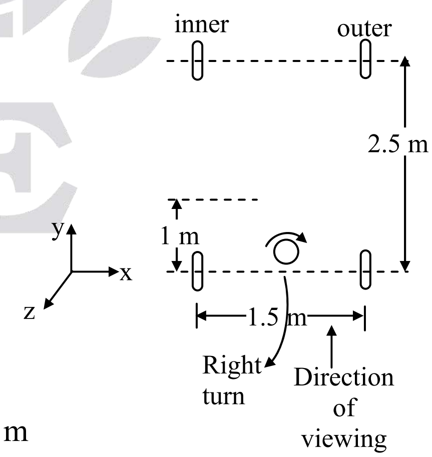
If $a < \frac{L}{2} \rightarrow$

*** *Question data is not correct, it should be like when* $a = \frac{\ell}{4}$, $a < \frac{\ell}{4}$ *and* $a > \frac{\ell}{4}$. *Then only the comparison is possible.*

04(c). A car is of total mass 200 kg. It has a wheel base equal to 2:5 m and track width equal to 1.5 m. The C.G. lies at 500 mm above ground level and 1.5 m from the rear axle. The effective diameter of each wheel is 800 mm and moment of inertia of each wheel is 1.0 kg m². The rear axle ratio (gear ratio) is 4. The equivalent mass of engine rotating parts is 140 kg with radius of gyration of 150 mm. The spin axis of the rotating engine parts is perpendicular to the spin axis of wheels. The engine parts are rotating in clockwise direction when viewed from the front. Determine the reaction at each wheel if the car takes a right turn of 100 m radius at 90 km/hour speed.



(20 M)



Sol: Total mass of car (m) = 200 kg

Wheel base = 2.5 m , Track width = 1.5 m

C.G lies 0.5 m above ground level

Diameter of each wheel, $d_w = 0.8$ m, $I_w = 1.0$ kg-m^2

gear ratio = 4

Equivalent mass of rotating parts = 100 kg

Radius of gyration of rotating parts, K = 150 mm = 0.15 m

Spin axis of rotating engine parts is perpendicular to the spin axis of wheels

Engine parts are rotating in clock wise direction when viewed from the front.
Car takes a right turn of 100 m radius at 90 kmph

(a) Reactions due to weight

Total weight = mg = 200 × 9.81

$$R_{w1,2,3,4} = \frac{mg}{4} = 50 \times 9.81$$

$\Rightarrow$ $R_{w1,2,3,4}$ = 490.5 N (upwads ↑)

(b) Reactions due to centrifugal couple

$$M_{centri} = m.\frac{v^2}{R} \times h$$

$$= 200 \times \left(\frac{90 \times 1000}{3600}\right)^2 \times \frac{0.5}{100}$$

$$= 2 \times \left(\frac{1000}{40}\right)^2 \times 0.5 = \frac{100 \times 100}{16} = 625 \text{ Nm}$$

For outer wheels = $\frac{M_{centri}}{2 \times 1.5} = 208.3\,\text{N}(\uparrow)$

inner wheels = 208.3 N (↓)

(c) Reactions due to gyroscopic couple:

(i) Due to wheels

	Axis		Plane
Spin	$+\hat{i}$	+x	yz
Precession	$-\hat{j}$	-y	xz
Reactive Gyro	$-\hat{k}$	-z	xy

Force on outer wheels : downwards

So, Reaction on outer : upwards

So, Reaction on inner wheels: downwards

$$M_G = 4I_w.\omega_w.\omega_P = 4 \times 1 \times \frac{v}{r_w}.\frac{v}{R}$$

$$= 4 \times 1 \times \frac{90 \times 1000}{0.4 \times 3600} \times \frac{90 \times 1000}{100 \times 3600} = 62.5 \text{ N-m}$$

Reaction on each wheel = $\dfrac{M_G}{2 \times 1.5}$

$R_{G,\,wheel} = 20.83$ N

(ii) Due to moving parts

$\because$ spin axis of rotating engine parts is perpendicular to the spin axis of wheels

	Axis		**Plane**
Spin	–z	$-\hat{k}$	xy
Precession	–y	$-\hat{j}$	xz
Reactive Gyro	–x	$-\hat{i}$	yz

$$M'_G = I_e.\omega_e.\omega_P$$

$$= 100(0.15)^2 \times [(\text{Gear ratio}).\omega_w] \times \frac{v}{R}$$

$$= 100(0.15)^2 \times 4 \times \frac{90 \times 1000}{0.4 \times 3600} \times \frac{90 \times 1000}{100 \times 3600} = 140.625 \text{ N-m}$$

Reaction on each wheel = $\dfrac{M'_G}{2.5 \times 2}$

$R_{G,engine} = 28.125$ N

Reaction on front wheels: downward

Reaction on rare wheels: upward

Inner wheel (Front)

▯

$\uparrow R_w$

$\downarrow R_C$

$\downarrow R_{G,wheels}$

$\downarrow R_{G,Engine}$

$R_{net,\ inner,front} = R_w - R_C - R_{G,w} - R_{G,E}$

$= 490.5 - 208.3 - 20.83 - 28.125 = 233.245$ N

Inner wheel (outer)

□

↑ R_w

↓ R_C

↓ $R_{G,wheel}$

↑ $R_{G,Engine}$

$R_{net,\ outer,front} = R_w - R_C - R_{G,wheel} - R_{G,Engine}$

$= 490.5 - 208.3 - 20.83 + 28.125 = 289.495$ N

Outer wheel (Front)

▯

↑ R_w

↑ R_C

↑ $R_{G,wheels}$

↓ $R_{G,Engine}$

$R_{net,\ outer,front} = R_w + R_C + R_{G,w} - R_{G,E}$

$= 490.5 + 208.3 + 20.83 - 28.125 = 691.505$ N

Outer wheel (Rear)

▯

↑ R_w

↑ R_C

↑ $R_{G,wheels}$

↑ $R_{G,Engine}$

$R_{net,\ outer,\ rear} = R_w + R_C + R_{G,w} + R_{G,E}$

$= 490.5 + 208.3 + 20.83 + 28.125 = 747.755$ N

SECTION – B

05(a). Prove that the atomic packing factors for BCC and FCC crystal structures are 0.68 and 0.74 respectively. (12 M)

Sol:

1. Body centered cubic structure (B.C.C):

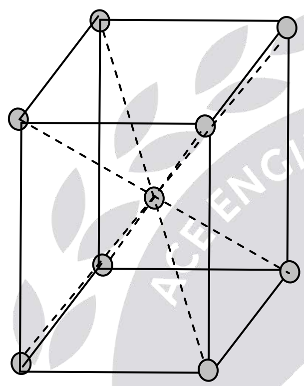


Figure shows the unit cell of B.C.C. structure. In this structure the eight corners of the cube are occupied by eight atoms and the centre of the cube is occupied by one atom. Metals that crystallize into B.C.C. structure are chromium, tungsten, iron, vanadium, molybdenum and sodium.

Number of atoms in the unit cell of B.C.C Structure:

In B.C.C. structure, the unit cell contains eight atoms at each corner of the cube and one atom in the centre of the cube. Since each corner atom is shared by eight surrounding cubes and the atom in the centre can not be shared by any other cube, the unit cell of the B.C.C. structure contains:

8 atoms at the corners $\times \dfrac{1}{8} = 1$ atom

1 centre atom = 1 atom

$\therefore$ **Total = 2 atoms**

Therefore the unit cell of B.C.C. structure contains two atoms.

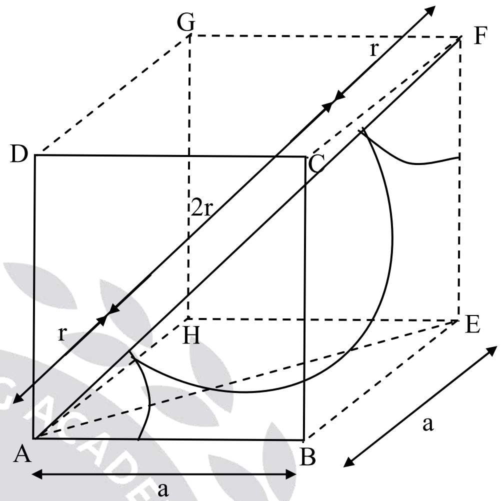
Atomic packing factor of B.C.C. structure:

From the figure

$$(AE)^2 = (AB)^2 + (BE)^2 = a^2 + a^2 = 2a^2$$

And $(AF)^2 = (AE)^2 + (EF)^2 = 2a^2 + a^2 = 3a^2$

$\Rightarrow \quad AF = a\sqrt{3}$



We have from the figure

$$AF = 4r$$

$\therefore \quad 4r = a\sqrt{3}$

$$\Rightarrow r = \frac{a\sqrt{3}}{4}$$

$\therefore$ The radius of the atom (sphere) in the B.C.C. structure is $\frac{a\sqrt{3}}{4}$. And the number of atoms in the unit cell of B.C.C. structure are two.

Volume of atoms in the unit cell $= \frac{2 \times 4\pi r^3}{3}$

$$= \frac{8\pi}{3}\left(\frac{a\sqrt{3}}{4}\right)^3 = \frac{\pi a^3\sqrt{3}}{8}$$

Volume of unit cell = a^3

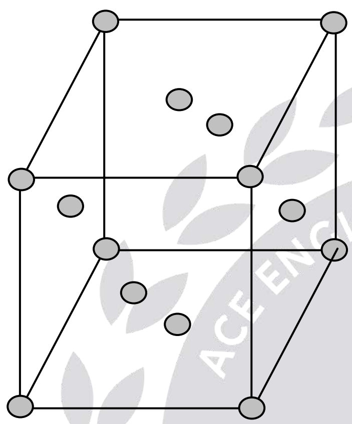
Atomic packing factor

$$= \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{\left(\frac{\pi a^3\sqrt{3}}{8}\right)}{a^3} = \frac{\pi\sqrt{3}}{8} = 0.68$$

2. Face centered cubic structure (F.C.C.) :

Figure shows the unit cell of the face centered cubic structure. In this structure the eight corners of the cube are occupied by eight atoms and six atoms occupy the centre of six faces of the cube. Metals that crystallize into F.C.C. structure are nickel, aluminum, copper, silver, gold, platinum, lead and iron.



Number of atoms in the unit cell of F.C.C. structure:

In F.C.C. structure, the unit cell contains eight atoms at each corner of the cube and six atoms at the centre of six faces of the cube. Since each corner atoms is shared by eight surrounding cubes and each of the face centered atom is shared by two surrounding cubes, the unit cell of F.C.C. structure contains.

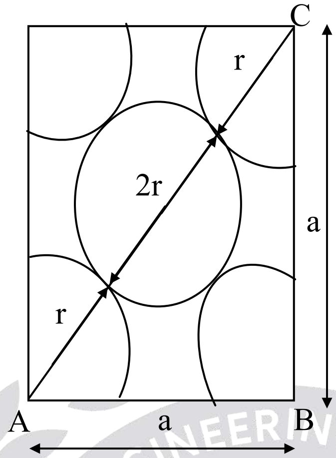
8 atoms at the corners $\times \frac{1}{8}$ = 1 atom

6 face centered atoms $\times \frac{1}{2}$ = 3 atoms

$\therefore$ **Total = 4 atoms**

Therefore the unit cell of F.C.C. structure contains four atoms.

Atomic packing factor of F.C.C. structure:



From the figure

$(AC)^2 = (AB)^2 + (BC)^2$

$= a^2 + a^2 = 2a^2 \Rightarrow AC = a\sqrt{2}$

We have from the figure

$$AC = 4r \Rightarrow 4r = a\sqrt{2} \Rightarrow r = \frac{a\sqrt{2}}{4}$$

$\therefore$ The radius of the atom (sphere) in the F.C.C. structure is $\frac{a\sqrt{2}}{4}$. And the numbers of atoms in the unit cell of F.C.C. structure are four.

$\therefore$ Volume of atoms in the unit cell $= 4 \times \frac{4\pi r^3}{3}$

$$= \frac{16\pi}{3}\left(\frac{a\sqrt{2}}{4}\right)^3 = \frac{\pi a^3\sqrt{2}}{6}$$

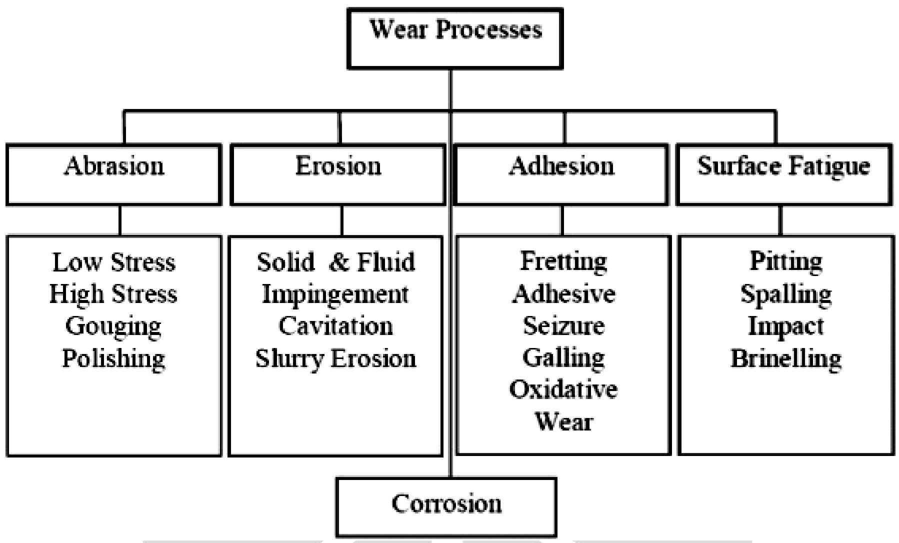
Volume of unit cell $= a^3$

$\therefore$ Atomic packing factor $= \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}}$

$$= \frac{\frac{\pi a^3\sqrt{2}}{6}}{a^3} = \frac{\pi\sqrt{2}}{6} = 0.74$$

05(b). Enlist various wear mechanisms and explain the term Wear and Debris Analysis. (12 M)

Sol:



Detection of wear particles :

Wear particles or debris generated during sliding or rolling contact between machine parts get lodged in lubricating oil.

Wear particles are of different :

(i) size ,

(ii) shape ,

(iii) composition (chemical)

Depending on the size of the wear debris in the oil, different detection methods are used.

(1) Spectroscopy (< 10 μm)

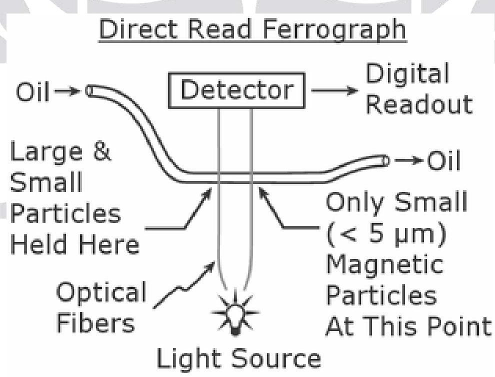
(2) Ferrography (10 μm to 100μm)

(3) Particle count (> 100 μm)

Wear debris analysis cannot be done in situ, unlike vibration signature analysis. Analysis of wear debris that are suspended in lubricating oil is done in a dedicated laboratory.
Samples of the oil and suspended particles are collected and sent to a lab.

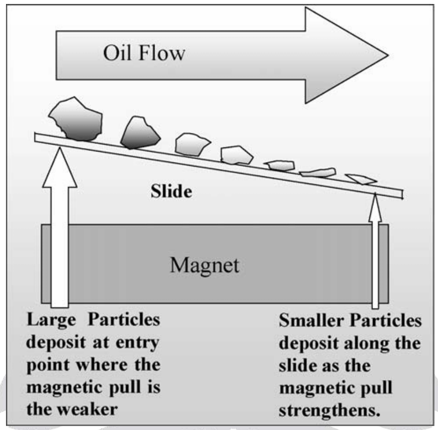
1. **Spectroscopy :** Spectroscopy analysis is done to determine the concentration of the particular chemical element in parts per million (ppm) in the sample.
   (i) Atomic absorption spectroscopy
   (ii) Atomic emission spectroscopy
   (iii) X-ray fluorescence
   It is helpful to know the chemical composition of machine components as manufactured.
   Ex: Increase in the concentration of Pb in a babbit lined journal bearing indicates a high rate of wear in the journal bearing.
   Optical microscope and SEM is also used to identify the nature of wear in the machine, by monitoring the shape of the debris particles.

2. **Ferrography method (Offline mode) :** An analysis based on magnetic field is used to separate the ferrous in an oil sample. There are two types of ferrographic analysis .
   (i) Direct Ferrography :



A contaminated oil sample is siphoned through a glass tube. Debris particles are positioned at different locations along the length of the tube, depending on the size at particles. By using optical senses, the amount and distribution of each particle size can be obtained.

(ii) Analytical Ferrography :



3. **Particle count :** When the size of the wear debris particles in the oil becomes greater than 100 microns, it is causer to count them in a given volume of contaminated oil with particle count meter. A light beam is focused on the susception of the wear debris in an oil sample, and a photo sensor is used to detect the intensity of the light that is transmitted through the suspension. The intensity of the light beam is related to the amount of wear debris present in the oil.

05(c). A work sampling study was conducted to establish the standard time for an operation. The observations of the study conducted are given below :

Total number of observations = 160

Manual (hand controlled work) = 14

Machine controlled work = 106

Machine idle time = 40

Average performance rating = 80 %

No. of parts produced = 36

Allowance for personal needs and fatigue = 10%

Study conducted for 3 days

Available working hours/day = 8 hours

Calculate the standard time per piece. (12 M)

Sol: Proportion of manual work = $\frac{14}{160} \times 100 = 8.75\ \%$

Proportion of machine work = $\frac{146}{160} \times 100 = 91.25\ \%$

Actual i.e., machine working time = $\frac{106}{160} \times 100 = 66.25\ \%$

Study time i.e., available time = 3 days × 8 hours × 60 min

= 1440 min

Manual work content = $1440 \times \frac{8.75}{100} = 126$ min

Machine work content = $1440 \times \frac{66.25}{100} = 954$ min

Normal time = Manual work content × Performance rating + Machine work content

= 126 × 0.8 + 954

= 1054.8 min

Standard time = normal time + allowances

= 1054.8 + 0.1 × 1054.8

= 1160.28 min

Standard time/piece = $\frac{1160.28}{36} = 32.23$ min

05(d). A cylindrical riser must be designed for a sand-casting mould. The casting itself is a steel rectangular plate with dimensions 3.75 cm × 6:25 cm × 1.0 cm. Previous observations have indicated that the total solidification time (T_{TS}) for this casting = 1.6 min. The cylinder for the riser will have a diameter-to-height ratio = 1.0. Determine the dimensions of the riser so that its T_{TS} = 2.0 min. Assume the value of n = 2, for mould constant.

(12 M)

Sol: Given data, $t_c = 1.6$ min, $t_r = 2.0$ min

where t_c and t_r are solidification times of casting and riser respectively.

$$t = k\left(\frac{V}{A}\right)^2$$

Where k is same because riser and casting have same metal and cast under identical conditions.

$$\frac{t_r}{t_c} = \frac{\left(\frac{V}{A}\right)_r^2}{\left(\frac{V}{A}\right)_c^2}$$

$$\Rightarrow \frac{2}{1.6} = \frac{\left(\frac{D}{6}\right)_r^2}{\left[\frac{(3.75\times 6.25\times 1.0)}{2(3.75\times 6.25+3.75\times 1+6.25\times 1)}\right]^2}$$

As D = H it is side riser

$$\therefore \left(\frac{V}{A}\right)_r = \frac{D}{6}$$

$$\Rightarrow \frac{2}{1.6} = \frac{D^2}{36}\times\left(\frac{66.875}{23.4375}\right)^2$$

$$\Rightarrow \frac{2}{1.6} = \frac{D^2}{36}\times 8.141$$

$$\Rightarrow D = \sqrt{\frac{2\times 36}{1.6\times 8.141}}$$

$\Rightarrow$ D = H = 2.35 cm

05(e). For a 4-DOF, RPPR manipulator, the joint-link transformation matrices with joint variables θ_1, d_2, d_3 and θ_4 are given as ${}^0T_1$, ${}^1T_2$, ${}^2T_3$ and ${}^3T_4$. Generate the Denavit-Hartenberg parameters table and the frames for the manipulator as per D-H rules.

${}^0T_1$	C_1	$-S_1$	0	0
	S_1	C_1	0	0
	0	0	1	0
	0	0	0	1

${}^1T_2$	1	0	0	0
	0	0	1	0
	0	-1	0	d_2
	0	0	0	1

${}^2T_3$	1	0	0	10
	0	1	0	0
	0	0	1	d_3
	0	0	0	1

${}^3T_4$	C_4	$-S_4$	0	0
	S_4	C_4	0	0
	0	0	1	1
	0	0	0	1

$C_i = \cos\theta_i$, $S_i = \sin\theta_i$,

(12 M)

Sol: For a 4-DOF, RPPR manipulator, the joint-link transformation matrices with joint variables θ_1, d_2, d_3 and θ_4 are given as ${}^0T_1$, ${}^1T_2$, ${}^2T_3$ and ${}^3T_4$.

$$^{o}T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{i-1}T_i$$

With this link 1 length (a_1) = 0 and link 1 twist (α_1) = 0

Joint 1 depth/offset is 0 & Joint 1 angle = θ_1

Above values can calculate by equations,

$${}^0T_1 = {}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i\, C\alpha_i & S\theta_i\, S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i\, C\alpha_i & -C\theta_i\, S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From this,

Link 2 length (a_2) = 0 & Twist (α_2) = $-\pi/2$

$\therefore\ S\alpha_2 = -1$, $C\alpha_2 = 0$, So $\alpha_2 = -\pi/2$

Joint 2 depth (d_2) = d_2 & Angle (θ_2) = 0°

$\therefore\ C\theta_2 = 1$, $S\theta_2 = 0$, So $\theta_2 = 0°$

$$\Rightarrow \ {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ from this matrix.}$$

Link 3 length (a_3) = 10 units, & Twist (α_3) = 0

And Joint 3 depth (d_3) = d_3 & Angle (θ_3) = 0°

$\therefore\ \cos\theta_3 = 1$ & $\sin\theta_3 = 0$, So $\theta_3 = 0°$

$$\Rightarrow \; {}^3T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ from this}$$

Link 3 length (a_4) = 0 & Twist (α_4) = 0

And Joint 4 depth (d_4) = 1 unit & Angle (θ_4) = θ_4

$\Rightarrow$ Denavit-Hartenberg parameters table.

Joint & Link	**Link length (a)**	**Link twist (α)**	**Joint depth (d)**	**Joint angle (θ)**
1	0	0	0	θ_1
2	0	$-\pi/2$	d_2	0
3	10	0	d_3	0
4	0	0	1	θ_4

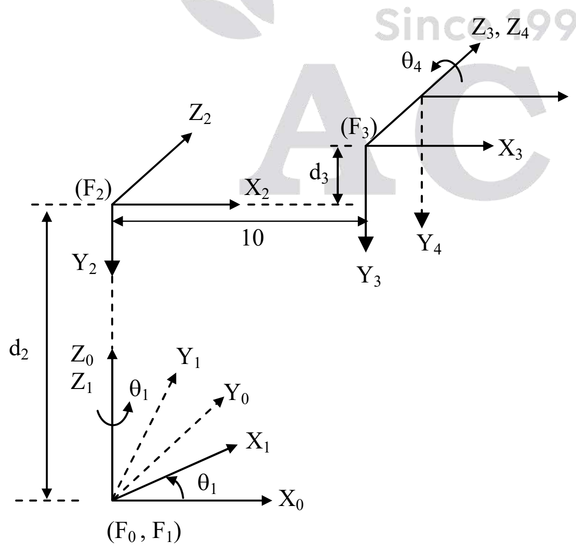
$\Rightarrow$ Co-ordinates frames…

Frame 0 $\rightarrow$ 1 $\Rightarrow$ only rotation R (z, θ_1)

Frame 1 $\rightarrow$ 2 $\Rightarrow$ Link twist, R(x, $-\pi/2$) & Joint displacement, T(z, d_2)

Frame 2 $\rightarrow$ 3 $\Rightarrow$ Link length, T(x, 10) & Joint displacement, T(z, d_3)

Frame 3 $\rightarrow$ 4 $\Rightarrow$ Joint displacement, T(z, 1) & Rotation, R (z, θ_4)



06(a). Explain electrochemical considerations of corrosion for metallic materials. Discuss the principle of cathodic protection of corrosion prevention. (20 M)

Sol: Electro-chemical Considerations :

For metallic materials, the corrosion process is normally electrochemical, that is a chemical reaction in which there is transfer of electrons from one chemical species to another. Metal atoms characteristically lose or give up electrons in what is called an oxidation reaction. The site at which oxidation takes place is called the anode; oxidation is sometimes called an anodic reaction. The electrons generated from each metal atom that is oxidized must be transferred to and become a part of another chemical species in what is termed a reduction reaction. The location at which reduction occurs is called the cathode.

Corrosion Cell : For Corrosion to take place , the information of a corrosion cell is essential. A corrosion cell essentially comprised of the following four components.

(i) Anode, (ii) Cathode , (iii) Electrolyte , (iv) Metallic path.

Anode (-ve of cell): One of the two dissimilar metal electrodes in an electrolytic cell. Electrons are released at anode, which is the more reactive metal. Electrons move through the wire in to the cathode.

Cathode (+ve of cell) : The other Electrode in the electrolytic cell .Reduction takes place at cathode and electrons are consumed .

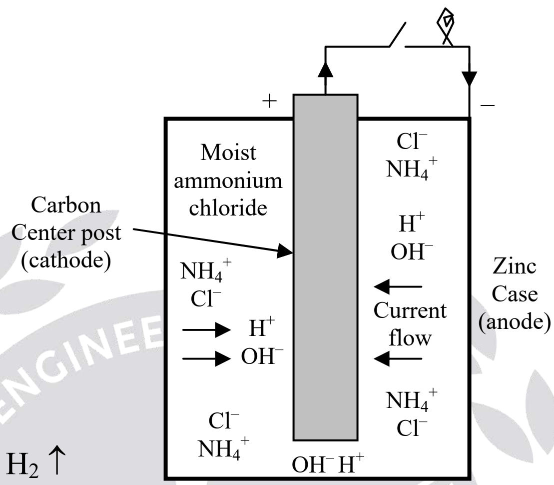
Electrolyte (e.g. salt solution) : It is the electrically conductive solution for corrosion to occur. Positive electricity passes from anode to cathode through the electrolyte as cations.

Metallic path : The two electrodes are connected externally by a metallic conductor. Current flow from (+) to (-) which really electrons flowing from (- e) to (+e).

Current flow : Conventional current flows from anode (-) to cathode (+) as Zn++ ions through the solution. The circuit is completed by passage of electrons from the anode to the cathode through the wire (outer current).

Electron Flow : The circuit is completed by negative ions (-) Which migrate from cathode (+) , through the electrolyte , towards the anode (-).

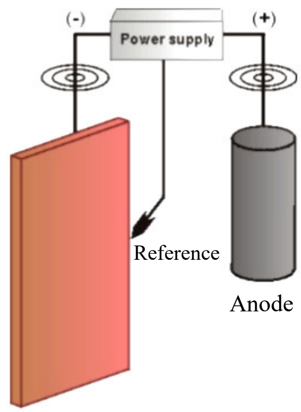
Example : In the dry Battery Zinc casing acts as anode. Carbon electrode acts as cathode. Moist ammonium chloride acts as Electrolyte . e.g. Zn ions dissolve from a zinc anode and thus carry positive current away from it, through the Aqueous electrolyte.



$$Zn + 2H^+ \longrightarrow Zn^{+2} + H_2 \uparrow$$

Cathodic Protection (CP) :

- The principle of Cathodic protection is in connecting an external anode to the metal to be protected and the passing of an electrical dc current so that all areas of the metal surface become cathodic and therefore do not corrode.
- The external anode may be a galvanic anode, where the current is a result of the potential difference between the two metals
- Or it may be an impressed current anode, where the current is impressed from an external dc power source
- CP is applied simply by maintaining a dc circuit and its effectiveness may be monitored continuously
- Commonly applied to a coated structure to provide corrosion control to areas where the coating may be damaged
- May be applied to existing structures to prolong their life
- Its main use is to protect steel structures buried in soil or immersed in water
- Cannot be used to prevent atmospheric corrosion on metals



Two basic ways:

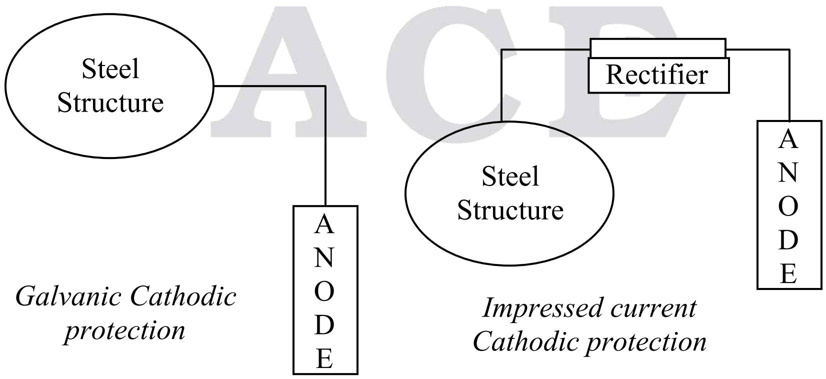
1. By Sacrificial Anode (or Galvanic Anode) Cathodic Protection
2. By Impressed Current Cathodic Protection

Sacrificial anode or Galvanic cathodic protection

- by using the galvanic series to select a more active metal, install that metal in the electrolyte and provide a metallic path.

Impressed Current Cathodic Protection

- applying a source of DC current which forces the current to flow from an installed anode (s) to the structure, causing the structure to be a cathode



- The current required for cathodic protection depends upon the metal being protected and the environment
- The potentials required to determine adequate protection (criteria) are established
- To achieve these protective potentials, current must flow from the anode to the structure being protected
- The amount of current required to protect a given structure is proportional to the area of the structure which is exposed to the electrolyte
- For coated structures, the amount of current required is much lower than for bare structures, as only those areas where the coating has been damaged or has deteriorated require or will receive current.
- Current requirements for coated structures are best determined by actual testing after the structure is installed.

Sacrificial Anode Cathodic Protection

- A galvanic cathodic protection system consists of sacrificial anodes , wiring or conductor connecting the anode to the structure and inspection station installed near the surface of the ground.
- The current required for cathodic protection is supplied by the corrosion of an active metal.
- Galvanic systems have limited life spans The current required for cathodic protection is supplied by the corrosion of an active metal
- Anode efficiency , which accounts for the anode's self corrosion rate and the corrosion rate for the amount of cathodic protection current.
- The simplest systems consist of an anode fabricated from an active metal such as zinc which is directly connected to the structure in an area where it will be exposed to the same environment as the structure being protected.

06(b). The following table gives data on normal cost and time, crash cost and time for a project. The indirect cost is ₹ 50/week.

Activity	Time (week)	Normal cost (₹)	Crash time	Cost (₹)
1 – 2	3	300	2	400
2 – 3	3	30	3	30
2 – 4	7	420	5	580
2 – 5	9	720	7	810
3 – 5	5	250	4	300
4 – 5	0	0	0	0
5 – 6	6	320	4	410
6 – 7	4	400	3	470
6 – 8	13	780	10	900
7 – 8	10	1000	9	1200

Draw the network diagram and label it. Identify critical path and find out normal project duration and corresponding cost. Crash the relevant activities systematically and determine the optimum project duration and cost. Determine the minimum project duration and corresponding cost.

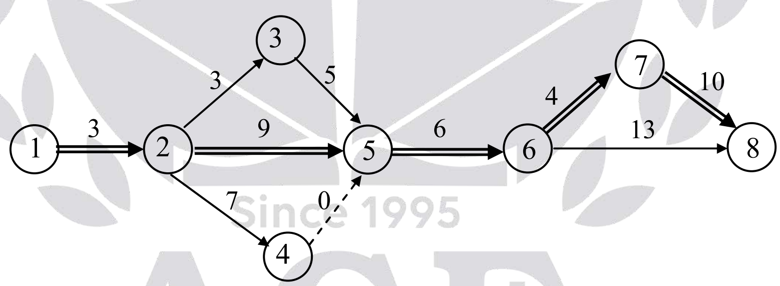
(20 M)

Sol:

Activity	Cost slope = $\frac{\Delta C}{\Delta T}$
1-2	$\frac{400-300}{3-2}=100$
2-3	-

2-4	$\frac{580-420}{7-5}=80$
2-5	$\frac{810-730}{9-7}=45$
3-5	$\frac{300-250}{5-4}=50$
4-5	0
5-6	$\frac{410-320}{6-4}=45$
6-7	$\frac{470-400}{4-3}=70$
6-8	$\frac{900-780}{13-10}=40$
7-8	$\frac{1200-1000}{10-9}=200$

Initial network :



Path	Duration
1-2-3-5-6-7-8	31
1-2-3-5-6-8	30
1-2-5-6-7-8	**32**
1-2-5-6-8	31
1-2-4-5-6-7-8	30
1-2-4-5-6-8	29

For crashing purpose, the following options available,

Option	Cost slope
1-2	100
2-5	45
5-6	45
6-7	70
7-8	200

Minimum cost slope critical activities :

(i) 2-5

(ii) 5-6

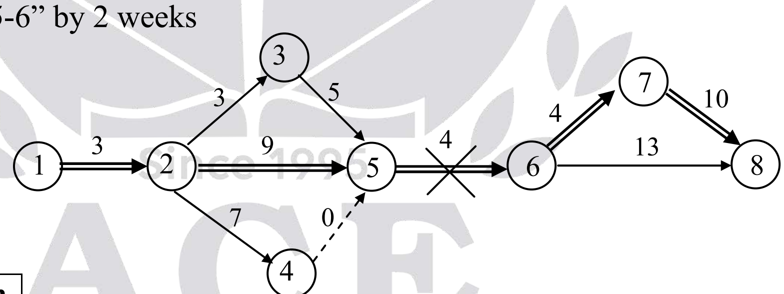
We need to choose “5-6” since, it can reduce the durations of many paths.

Crashing cost/week = 45

Indirect cost/week = 50

⇒ crashing is economical.

After crashing activity “5-6” by 2 weeks



Path	**Duration**
1-2-3-5-6-7-8	29
1-2-3-5-6-8	28
1-2-5-6-7-8	**30**
1-2-5-6-8	29
1-2-4-5-6-7-8	28
1-2-4-5-6-8	27

To reduce the project duration, the available options are

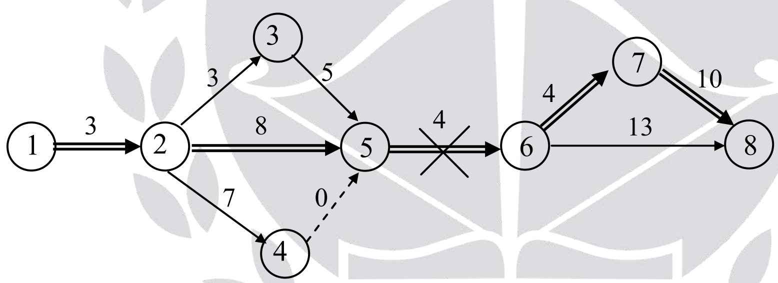
Option	Cost slope
1-2	100
2-5	45
6-7	70
7-8	200

Crashing cost/week = 45

Indirect cost/week = 50

⇒ crashing is economical.

After crashing activity “2-5” by 1 weeks



Path	Duration
1-2-3-5-6-7-8	**29**
1-2-3-5-6-8	28
1-2-5-6-7-8	**29**
1-2-5-6-8	28
1-2-4-5-6-7-8	28
1-2-4-5-6-8	27

There are 2 critical paths, hence, we need to crash both the paths simultaneously to reduce the project duration.

Option	Cost slope
1-2	100
6-7	70
7-8	200
2-5 & 3-5	45 + 50 = 95

Here,

Crashing cost/week = 70

Indirect cost/week = 50

$\Rightarrow$ crashing is not economical, hence stop crashing.

Optimum project duration = 29 weeks

Minimum project cost

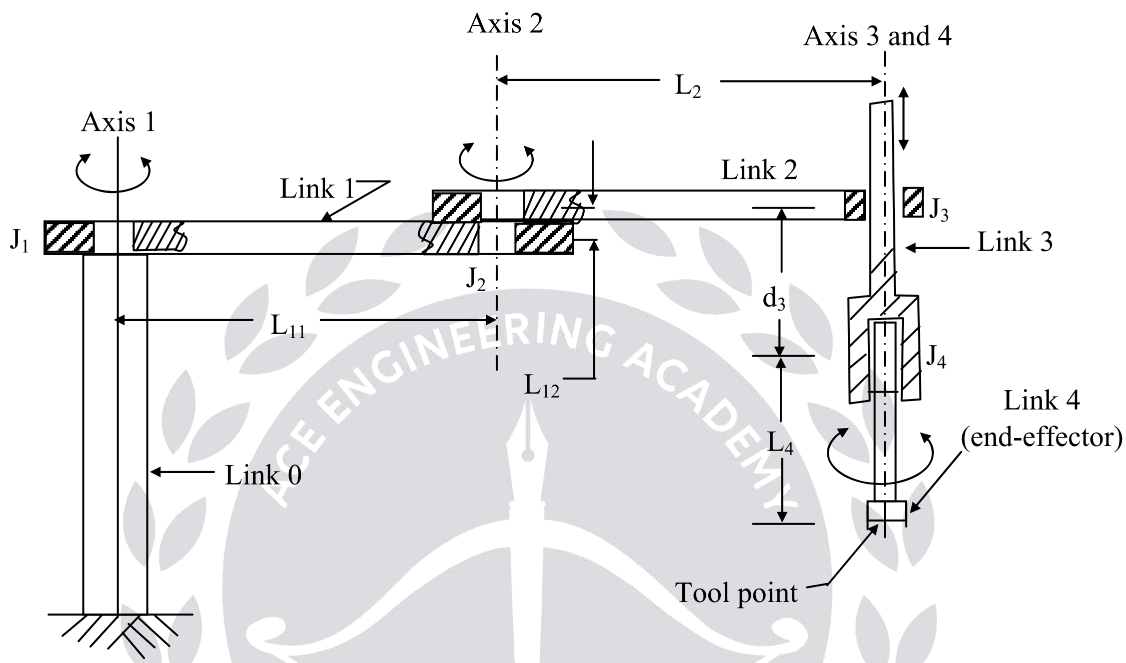
= Total normal cost + Crashing cost of 5-6 by 2 weeks + Crashing cost of 2-5 by 1 week + Total indirect cost

= [300 + 30 + 420 + 720 + 250 + 0 + 320 + 400 + 780 + 1000] + 45×2 + 45×1 + 50×29

= 4220 + 135 + 1450

= 5805 /-

06(c). Obtain the direct kinematics model of the given 4 DOF SCARA robot by developing D-H frames, D-H parameters table and individual transformation matrices.



$^{i-1}T_i =$	$C\theta_i$	$-S\theta_i\,C\alpha_i$	$S\theta_i\,S\alpha_i$	$a_i\,C\theta_i$
	$S\theta_i$	$C\theta_i\,C\alpha_i$	$-C\theta_i\,S\alpha_i$	$a_i\,S\theta_i$
	0	$S\alpha_i$	$C\alpha_i$	d_i
	0	0	0	1

(20 M)

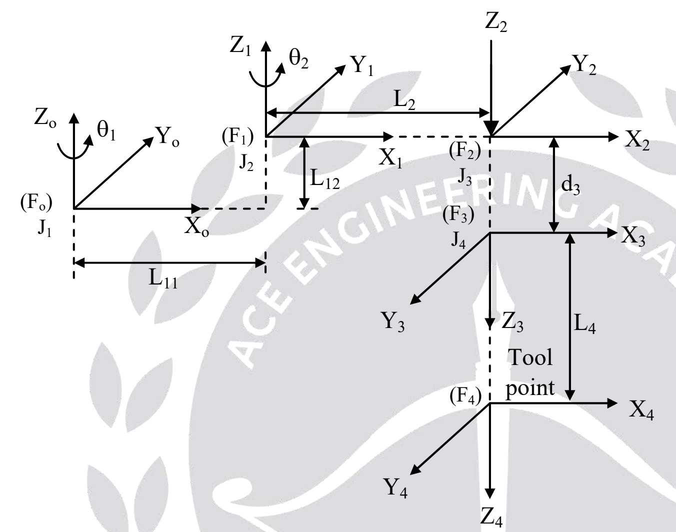
06(c).

Sol: Direct kinematics model of the given 4 DOF SCARA robot

→ D-H frames,

→ D-H parameters table

→ Individual transformation matrices.



⇒ D-H frames (F_o, F_1, F_2, F_3, F_4 at joints J_1, J_2, J_3, J_4 tool points respectively)

⇒ F_2 at J_3 (Z_2 can be vertical may be upwards or down wards)

⇒ As per up/down wards link twists α has to substitute in 3rd row (or) 2nd row as 180° (π).

D-H parameter table:

Link	Link length (a)	Link twist (α)	Joint depth (d)	Joint angle (θ)
1	L_{11}	0	L_{12}	θ_1
2	L_2	0	0	θ_2
3	0	180°	$-d_3$	0
4	0	0	L_4	θ_4

$\Rightarrow$ Individual transformation matrices

$$^{o}T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & L_{11}C_1 \\ S_1 & C_1 & 0 & L_{11}S_1 \\ 0 & 0 & 1 & L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \; ^{3}T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note:

(i) F_2 (x_2 , y_2, z_2) , if z_2 axes taken downward. Slight changes in D-H table parameters.

(ii) But final tool position & orientation w.r.t base frame analysis and values are same.

07(a). In an orthogonal turning operation of a mild steel bar of 60 mm diameter, cutting speed was 30 m/minute, rake angle of tool 30°, feed rate 0.10 mm/revolution, tangential force 3000 N, feed force 1300 N, length of continuous chip in one revolution 100 mm. Calculate coefficient of friction, shear plane angle, velocity of chip along tool face and chip thickness.

(20 M)

Sol: Given data,

$D_o = 60$ mm, $V_c = 30$ m/min,

$\alpha_o = 30°$, $f = 0.10$ mm/rev

$F_c = 3000$ N,

$F_f = F_a = 1300$ N [$C_s = 0$, $\lambda = 90°$]

$l_c = l_f = 100$ mm,

As, $F_a = F_T = F_f = 1300$ N

$\therefore$ $C_s = 0$, $\lambda = 90°$

$\therefore$ $f = t_o$

$\therefore$ $t_o = f = 0.10$ mm

$$\therefore \quad r = \frac{V_f}{V_c} = \frac{L_f}{L_o} = \frac{L_c}{L_o} = \frac{100}{\pi \times 60} = \frac{100}{188.49} = 0.53$$

$$r = \frac{t_o}{t_c} \Rightarrow t_c = \frac{0.10}{0.53} = 0.18\,\text{mm}$$

$\Rightarrow$ $t_c = 0.18$ mm $\rightarrow$ thickness of chip

$$r = \frac{V_f}{V_c}$$

$\therefore$ $V_f = r\ V_c$

$V_f = 0.53 \times 30 = 15.9$ m/min $\rightarrow$ velocity of chip along rake face

$$\tan\phi = \frac{r\cos\alpha_o}{1 - r\sin\alpha_o}$$

$$\tan\phi = \frac{0.53 \times \cos 30^o}{1 - 0.53 \times \sin 30^o}$$

$$\tan\phi = \frac{0.458}{1 - 0.265}$$

$\Rightarrow$ $\phi = 31.928°$ $\rightarrow$ shear plane angle

$$\tan(\beta - \alpha_o) = \frac{F_T}{F_C}$$

$$\tan(\beta - 30^\circ) = \frac{1300}{3000}$$

$$\beta = 53.42^\circ$$

$$\tan\beta = \mu = 1.347$$

As μ is > 1, we can use,

Use kronenberg's equation,

$$\mu = \frac{\ell n\left(\frac{1}{r}\right)}{\frac{\pi}{2} - \alpha_o}$$

$$\mu = \frac{\ell n\left(\frac{1}{0.53}\right)}{\frac{\pi}{2} - 30\times\frac{2\pi}{360}}$$

$$\mu = \frac{\ell n\left(\frac{1}{0.53}\right)}{1.047}$$

$\Rightarrow$ $\mu = 0.606 \rightarrow$ coefficient of friction.

07(b). Determine the most economical order quantity when annual usage is 8000 parts. Unit commodity cost is ₹ 60 and the cost of placing an order is ₹ 150 and the annual inventory carrying cost is 30% of the average inventory. Also find out the most economical order quantity for the variable price schedule given below :

Lot Size	Unit Price
1 – 200	₹ 62
201 – 500	₹ 60
501 and above	₹ 56

(20 M)

Sol: Annual usage = 8000 units,

Ordering cost = Rs. 150

Carrying cost = 30% of average inventory.

Lot size	Unit price
1-200	Rs 62
201 – 500	Rs 60
501 & above	Rs 56

Let, $C_u = 56$

$$EOQ\big|_{C_u=56} = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2\times 8000\times 150}{0.3\times 56}} = 377.96 \text{ units}$$

It does not satisfy the given quantity range, hence, it is discarded.

$$TC\big|_{\substack{Q=501,\\ C_u=45}} = \frac{Q}{2}\times C_c + \frac{D}{Q}\times C_o + D\times C_u$$

$$= \frac{501}{2}\times 0.3\times 56 + \frac{8000}{501}\times 150 + 8000\times 56$$

$$= 4208.4 + 2395.2 + 448000 = 4{,}54{,}603\ /-$$

Let $C_u = 60$

$$EOQ\big|_{C_u=60} = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2\times 8000\times 150}{0.3\times 60}} = 365 \text{ units}$$

$$TC\big|_{\substack{EOQ=365,\\ C_u=60}} = \sqrt{2DC_oC_c} + D\times C_u$$

$$= \sqrt{2\times 8000\times 150\times 0.3\times 60} + 8000\times 60$$

$$= 6572.6 + 448000 = 4{,}86{,}572\ /-$$

Order quantity	Total cost
Q = 501 ; C_u = 56	4,54,603 /-
EOQ = 365 ; C_u = 60	4,86,572 /-

Best order quantity = 501 units.

07(c). What are nanomaterials ? Cite examples of the special properties of nanomaterials. Discuss the salient features of nanomaterials characterization tools. (20 M)

Sol: Nanomaterial means a material that meets at least one of the following criteria:

- Consists of particles with one or more external dimensions in the size range of 1–100 nm for more than 1% of their number.
- Internal/surface structures in one or more dimensions in the size range of 1–100 nm.
- Specific surface-to-volume ratio > 60 m^2/cm^3, excluding materials consisting of particles with a size less than 1 nm.

Nanomaterials can be nanoscale in one dimension (e.g., surface films) two dimensions (e.g., strands or fibers) or three dimensions (e.g., precipitates, colloids). They can exist in single, fused, aggregated, or agglomerated forms with spherical, tubular, and irregular shapes. Nanostructures are the ordered system of one, two, or three dimensions of nanomaterials, assembled with nanometer scale in certain pattern that includes nanosphere, nanotubes, nanorod, nanowire, and nanobelt. Nanostructured materials are classifie as zero, one, two, and three-dimensional nanostructures, showing typical examples with varied dimensionality in nanomaterials as in Figure.

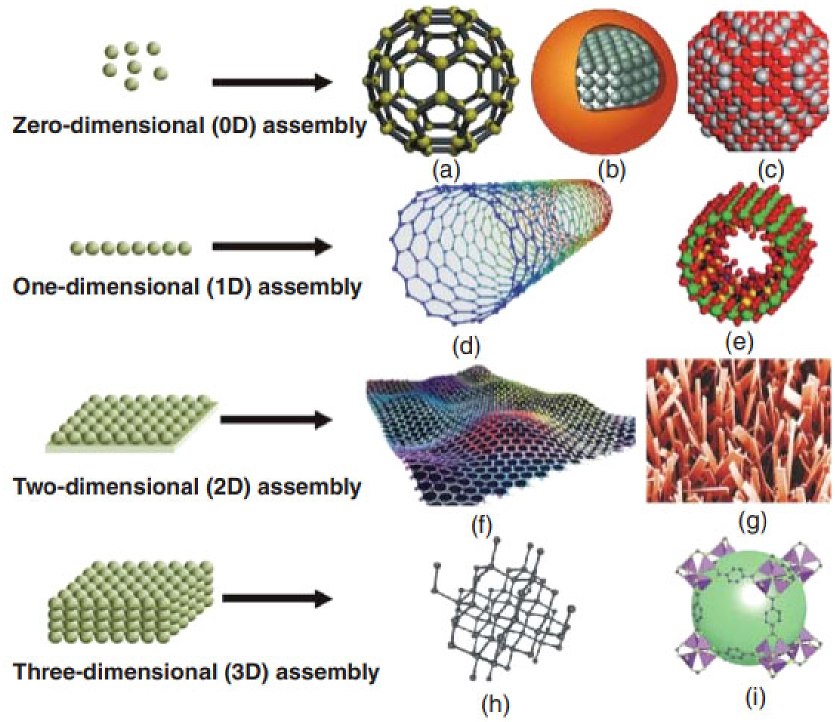


Figure: Typical examples showing varied dimensionality in nanomaterials: (a) fullerene; (b) quantum dot; (c) metal cluster; (d) carbon nanotube; (e) metal oxide nanotube; (f) graphene; (g) metal oxide nanobelts; (h) nanodiamond; (i) metal organic frameworks (MOFs).

PROPERTIES OF NANOMATERIALS

The interest in nanostructured materials arises from the fact that because of the small size of the building blocks and the high density of interfaces (surfaces, grain, and phase boundaries) and other defects such as pores, new physical and chemical effects are expected or known properties can be improved substantially. The physical and chemical properties of nanostructured materials (such as optical absorption and flu orescence, melting point, catalytic activity, magnetism, electric and thermal conductivity, etc.) differ significantl from the corresponding coarser bulk material. Roughly two types of nanostructure-induced effects can be distinguished:

- The size effect, in particular the quantum size effects, where the normal bulk electronic structure is replaced by a series of discrete electronic levels.

- The surface- or interface-induced effect, which is important because of the enormously increased specifi surface in particle systems.

Although the size effect is mainly considered to describe physical properties, the surface- or interface-induced effect plays an eminent role for chemical processing, in particular in connection with heterogeneous catalysis. Experimental evidence of the quantum size effect in small particles has been provided by different methods, while the surface-induced effect could be evidenced by the measurement of thermodynamic properties such as vapor pressure, specifi heat, thermal conductivity, and melting point of small metallic particles. Both types of size effects have also been clearly separated in the optical properties of metal cluster composites. Very small semiconductor (< 10 nm), or metal particles in glass composites, and semiconductor/polymer composites show interesting quantum effects and nonlinear electrical and optical properties.

These special properties of nanomaterials are mainly due to quantum size confinement in nanoclusters and an extremely large surface-to-volume ratio relative to bulk materials, thus leading to the presence of a high percentage of atoms/molecules lying at reactive boundary surfaces. For example, in a particle with 10 nm diameter only around 20% of all atoms are forming the surface, whereas in a particle of 1 nm diameter this figure can reach more than 90%. The increase in the surface-to-volume ratio results in an increase in the surface energy of the particle, which leads to, for example, a decrease in melting point or an increase in sintering activity. Furthermore, large surface area of particles may significantly raise the level of otherwise kinetically and thermodynamically unfavorable reactions. For instance, even gold, which is a very stable material, becomes reactive when the particle size is small enough. Fundamentally, there are seven key characteristics that contribute to the uniqueness of nanomaterials and these are summarized in Table.

TABLE : Characteristics of Nanomaterial and Their Importance

Characteristic	Importance
Size	Key definin criteria for a nanomaterial
Shape	Carbon nanosheets with a flat geodesic (hexagonal) structure show improved performance in epoxy composites versus carbon fiber
Surface charge	Surface charge is as important as size or shape. Can impact adhesion to surfaces and agglomeration characteristics. Nanoparticles are often coated or "capped" with agents such as polymers (PEG) or surfactants to manage the surface charge issues
Surface area	This is a critical parameter as the surface-to-weight ratio for nanomaterials is huge. For example, 1 g of an 8-nm-diameter nanoparticle has a surface area of 32 m^2 . Nanoparticles may have occlusions and cavities on the surface
Surface porosity	Many nanomaterials are characterized with zeolite-type porous surfaces. These engineered surfaces are designed for maximum absorption of a specifi coating or to accommodate other molecules with a specifi size
Composition	The chemical composition of nanomaterials is critical to ensure the correct stoichiometry being achieved. The purity of nanomaterials, impact of different catalysts used in the synthesis, and presence of possible contaminants need to be assessed along with possible coatings that may have been applied
Structure	Knowledge of the structure at the nanolevel is important. Many nanomaterials are heterogeneous, and information concerning crystal structure and grain boundaries is required

TABLE : Common Investigation Techniques for the Characterization of Nanoparticles

Techniques	Characterization Parameters
Dynamic light scattering (DLS)/particle size analyzer	Size and size distribution of nanoparticles suspended in a liquid phase
Zeta potential analyzer	Surface charge of nanoparticles in aqueous solutions or suspensions
Scanning electron microscopy (SEM)	Shape and surface structure
Transmission electron microscopy (TEM)	Size, shape, and morphology (including internal structure), especially useful for biological specimen
Atomic force microscopy (AFM)	Shape and surface morphology of nanoparticles with high lateral and vertical resolutions
Scanning tunneling microscopy (STM)	Surface images with atomic-scale lateral resolution; modificatio of material at atomic/ molecular / nanometer scale with high precision
Laser scanning confocal microscopy (LSCM)	Noninvasive technique provides information about migration of nanoparticles into bio-barrier; 3D morphology of nanoparticles
Brunauer–Emmett–Teller (BET) technique	Surface area analysis, porosity, and adsorption capability
X-ray diffraction (XRD) technique	Crystal structure, phase, and average particle size
X-ray photoelectron spectroscopy (XPS, ESCA)	Chemical composition (both elemental & chemical states) information on nanoparticle's surface
Fourier transform infrared spectroscopy (FTIR)	Assisted analytical tool for chemical composition
Differential scanning calorimetry (DSC)	Thermal analysis and phase transition studies
High-performance liquid chromatography (HPLC)	Detection, separation, and quantificatio of nanoparticles / nanomaterials with different particle size

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08(a). Prepare the forward kinematic model for a 3-link planar arm with link lengths l_1, l_2 and l_3 respectively by developing

(i) Respective coordinate frames as per D-H rules,

(ii) D-H parameters table,

(iii) Individual transformation matrices, ${}^{i-1}T_i$ and

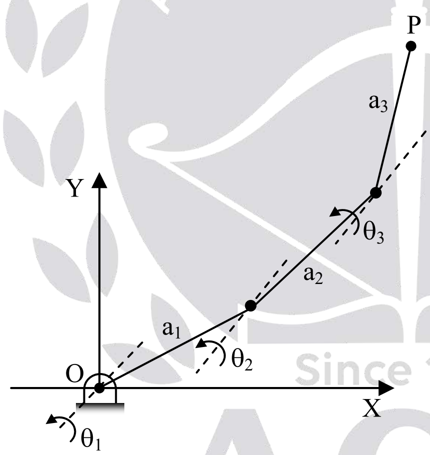
(iv) The transformation matrix of the last frame with reference to the base frame.

(v) Also draw the orientation and position of the last frame with reference to the base frame, if link lengths are

$$a_1 = a_2 = a_3 = 10 \text{ units}, \quad \theta_1 = 0°, \theta_2 = 45° \text{ and } \theta_3 = 45°.$$

Given that ${}^{i-1}T_i$ matrix form is as following.

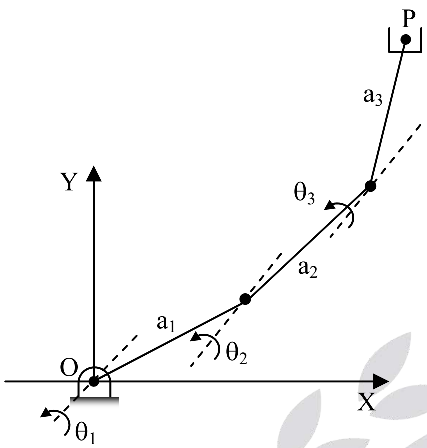
Note : angles are measured in counter-clockwise direction.

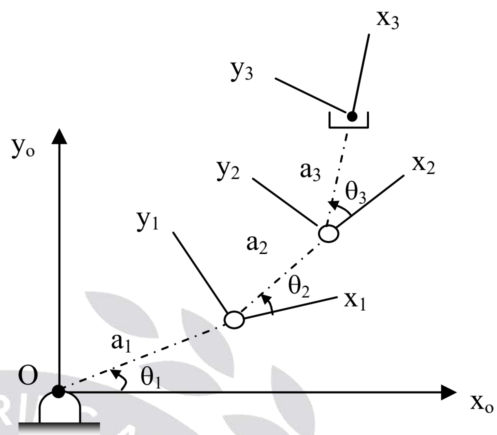


$${}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(20 M)

Sol:





(i) Co-ordinate frames as per D-H rules,

The joint axes Z_o, Z_1, Z_2 are normal for the page.

Base (link 0) has co-ordinate frame $\Rightarrow x_o\ y_o\ z_o$ (Joint J_1)

Base (link 1) has co-ordinate frame $\Rightarrow x_1\ y_1\ z_1$ (Joint J_2)

Base (link 2) has co-ordinate frame $\Rightarrow x_2\ y_2\ z_2$ (Joint J_3)

Base (link 3) has co-ordinate frame $\Rightarrow x_3\ y_3\ z_3$ (Point P)

(ii) D-H parameters table :

Joint, Link	Link length (a)	Link twist (α)	Joint depth (d)	Joint angle (θ)
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

(iii) Individual transformation matrices, ${}^{i-1}T_i$ (for j = 1,2, 3….)

$$^{0}T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iv) Last frame (3) matrix with reference to base frame (0)

$$^{0}T_3 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3$$

The resultant matrix after multiplication of above 3 matrix in sequence.

$$^{0}T_3 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3$$

$$= \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1c_1 + a_2c_{12} + a_3c_{123} \\ S_{123} & C_{123} & 0 & a_1s_1 + a_2s_{12} + a_3s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$ and $S_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$

(v) If link lengths are

$$a_1 = a_2 = a_3 = 10 \text{ units } \& \ \theta_1 = 0°,\ \theta_2 = 45°,\ \theta_3 = 45°$$

Position of last frame w.r.t base frame.

$\Rightarrow P_x = a_1 C_1 + a_2 C_{12} + a_3 C_{123} = 10.C.0° + 10.C.45° + 10.C.90°$

$\Rightarrow P_y = a_1 S_1 + a_2 S_{12} + a_3 S_{123} = 10.S.0° + 10.S.45° + 10.S.90°$

$\Rightarrow P_z = 0$

$\Rightarrow P_x = 10 + \dfrac{10}{\sqrt{2}} = 10 + 5\sqrt{2}$

$\Rightarrow P_y = 10 + \dfrac{10}{\sqrt{2}} = 10 + 5\sqrt{2}$

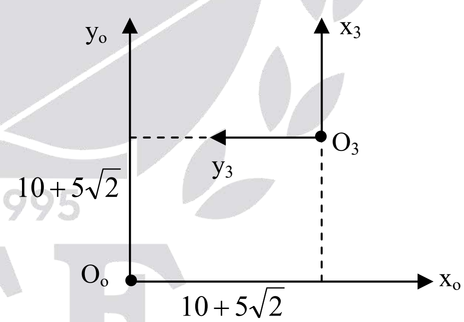
$\Rightarrow P_z = 0$

Orientation of last frame w.r.t base frame

$$\Rightarrow \begin{bmatrix} C_{123} & -S_{123} & 0 \\ S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C90^o & -S90^o & 0 \\ S90^o & C90^o & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{ So angles between frame axes.}$$

$$\begin{array}{c|ccc} \backslash & x_3 & y_3 & z_3 \\ \hline x_o & & & \\ y_o & & \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \\ z_o & & & \end{array} = \begin{bmatrix} C90^o & -S(90+90) & C90^o \\ C0^o & C\,90^o & C90^o \\ C90^o & C90^o & C0^o \end{bmatrix}$$



$\Rightarrow$ Axes Z_0, Z_3 are normal the page and both are parallel.

$\Rightarrow$ Both frames (F_0 , F_3) are displaced in xy plane by $(10+5\sqrt{2}\,,10+5\sqrt{2}\,) \rightarrow$ Position displacement.

$\Rightarrow$ Frame (F_3) rotated around Z axis by 90° in anti-clock wise w.r.t frame (F_o) $\rightarrow$ Orientation of last frame w.r.t. base frame.

08(b). The following data is available for a machine in a manufacturing unit.

Number of hours worked per day = 8

Working days per month = 25

Number of operators = 1

Standard time per unit of production

Machine time = 22 min

Operator time = 08 min

Total time/unit = 30 min

(i) If plant is operated at 75% efficiency, and the operator is working at 100% efficiency, what is the output per month ?

(ii) It the machine productivity is increased by 10% over the existing level, what will be the output per month ?

(iii) If the operator efficiency is reduced by 20% over the existing level, what will be the output per month ? (20 M)

Sol:

(i) Total time/unit = 22 + 8 = 30 min

Available time = $8 \times 25 \times 60 = 12000$ min

Output per month = $\frac{12000}{30} \times 0.75 = 300$ units

(ii) If the machine productivity increased @ 10% then machining time = $\frac{1}{1.1} \times 22 = 20$ min

Total time/unit = 20 + 8 = 28 min

Output per month = $\frac{12000}{28} \times 0.75 = 321.42$ units

(iii) If the operator efficiency is reduced @ 20% then operator time = $\frac{1}{0.8} \times 8 = 10$ min

Total time/unit = 22 + 10 = 32 min

Output per month = $\frac{12000}{32} \times 0.75 = 281.25$ units

08(c). Explain the methods of numerical evaluation of surface texture. Compare their merits and demerits. Describe the construction and working of a Talysurf surface roughness tester.
(20 M)

Sol: SURFACE TEXTURE:

" The characteristics quality of an actual surface due to small departures from its general geometrical form which, occurring at regular or irregular intervals, tend to form a pattern or texture on the surface"

NUMERICAL EVALUATION OF SURFACE TEXTURE

- Practice adopted for obtaining numerical evaluation of surface texture is to use a parameter based on the cross-sectional profile of the surface under examination.
- There are two system for Numerical evaluation of surface finish, i.e:
  a. Centre line average (CLA)
  b. Root mean square (RMS) value of height undulations of the surface

The CLA system is more popular in UK, while the RMS is more popular in USA.

Analysis of Surface Traces :

It is required to assign a numerical value to surface roughness in order to measure its degree. This will enable the analyst to assess whether the surface quality meets the functional requirements of a component. Various methodologies are employed to arrive at a representative parameter of surface roughness. Some of these are 10-point height average (Rz), root mean square (RMS) value, and the centre line average height (Ra), which are explained in the following paragraphs

1. Ten-point Height Average Value

It is also referred to as the peak-to-valley height. In this case, we basically consider the average height encompassing a number of successive peaks and valleys of the asperities. As can be seen in Fig., a line AA parallel to the general lay of the trace is drawn. The heights of five consecutive peaks and valleys from the line AA are noted down.

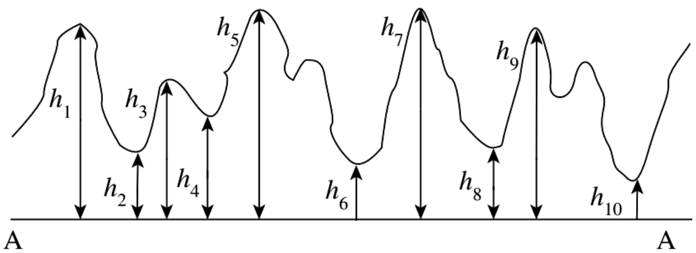


Fig. Measurement to calculate the 10-point height average

The average peak-to-valley height Rz is given by the following expression:

$$Rz = \frac{(h_1 + h_3 + h_5 + h_7 + h_9) - (h_2 + h_4 + h_6 + h_8 + h_{10})}{5} \times \frac{1000}{\text{vertical magnification}} \mu m$$

2. Root Mean Square Value

Until recently, RMS value was a popular choice for quantifying surface roughness; however, this has been superseded by the centre line average value. The RMS value is defined as the square root of the mean of squares of the ordinates of the surface measured from a mean line. Figure illustrates the graphical procedure for arriving at an RMS value.

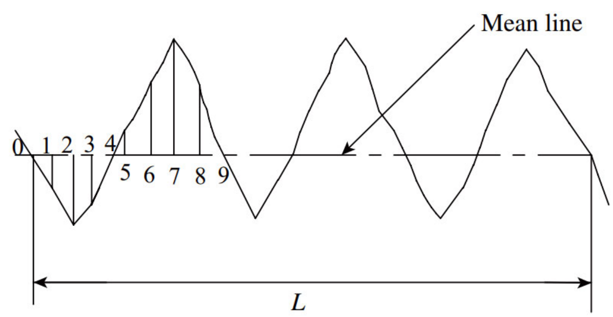


Fig. Representation of an RMS value

With reference to this figure, if h_1, h_2, ..., h_n are equally spaced ordinates at points 1, 2, ... , n, then

$$h_{RMS} = \frac{\sqrt{(h_1^2 + h_2^2 + \ldots\ldots + h_n^2)}}{n}$$

3. Centre Line Average Value (CLA)

The Ra value is the prevalent standard for measuring surface roughness. It is defined as the average height from a mean line of all ordinates of the surface, regardless of sign.

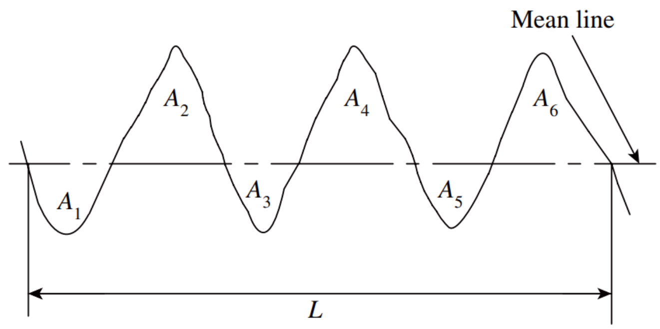


Fig. Representation of Ra value

With reference to Fig., it can be shown that

$$Ra = \frac{A_1 + A_2 + ... + A_N}{L} = \frac{\Sigma A}{L}$$

It should be mentioned here that the Ra value is an index for surface texture comparison and not a dimension. This value is always much less than the peak-to-valley height. It is generally a popular choice as it is easily understood and applied for the purpose of measurement.

Taylor–Hobson Talysurf surface roughness tester :

- This instrument is a stylus and skid type of instrument working on carrier modulating principle.
- The variation in the surface profile is sensed by the probe , which is attached to the armature.
- The gap between the armature and E-shaped arm varies according to the surface profile and due to this amplitude of the AC current flowing in the coil is modulated.

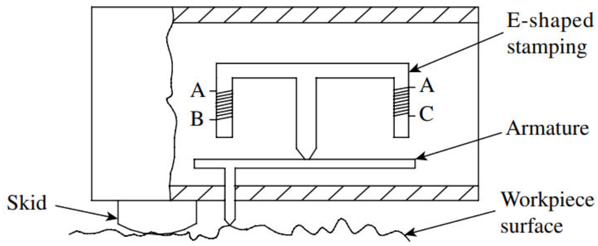


Fig. Taylor–Hobson talysurf

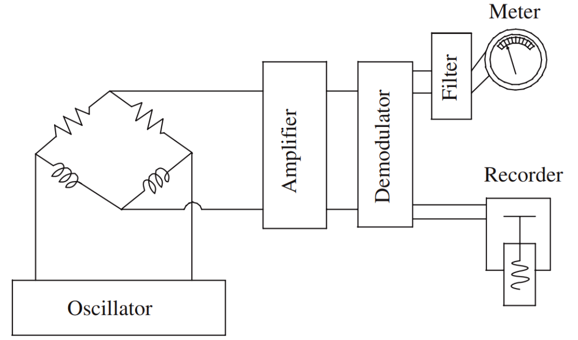


Fig. Bridge circuit and electronics

Construction :

- As shown in figure, the stylus is mounted on the armature , which is pivoted at the central limb of an E-shaped soft iron head.
- The outer limbs of the head are provided with two induction coils and a small air gap is left between the armature and outer limbs of the head.

Working :

- A downward movement of the stylus results in decreasing the air gap of the primary coil and in an equal increase of the air gap at the secondary coil.
- Thus the amplitude of the original A.C. current flowing in the coils is modulated.
- The output of the bridge thus consists of modulation only as shown in figure.
- This is further demodulated so that the current now is directly proportional to the vertical displacement of the stylus only.
- The demodulated output is caused to operate a pen recorder to produce a permanent record and the meter to give a numerical assessment directly.
- The response of this instrument is more rapid and accurate as compared to Tomlinson surface tester.