ESE-2020 (MAINS)

QUESTIONS WITH DETAILED SOLUTIONS

MECHANICAL ENGINEERING

PAPER-I

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MECHANICAL ENGINEERING

ESE _MAINS_2020_PAPER – I

Questions with Detailed Solutions

SUBJECT WISE WEIGHTAGE

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01.(a)

The velocity components in a two-dimensional incompressible flow are:

$u = 8x^2y - \frac{8}{3}y^3$ and $v = -8xy^2 + \frac{8}{3}x^3$.

Show that these velocity components represent a possible case of an irrotational flow.

Sol: The velocity field is given as, $u = 8x^2y - \frac{8}{3}y^3$, $v = -8xy^2 + \frac{8}{3}x^3$

The flow must satisfy continuity equation in order to be a physically realistic flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(8x^2y - \frac{8}{3}y^3\right) + \frac{\partial}{\partial y}\left(-8xy^2 + \frac{8}{3}x^3\right)$$

$$= 16\,xy - 16\,xy = 0$$

$\Rightarrow \therefore$ The given velocity field represents physically realistic 2D incompressible flow.

Now,

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{V} = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \frac{1}{2}\left[i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - \hat{j}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]$$

$$= \frac{1}{2}\hat{k}\left[\frac{\partial}{\partial x}\left(-8xy^2 + \frac{8}{3}x^3\right) - \frac{\partial}{\partial y}\left(8x^2y - \frac{8}{3}y^3\right)\right]$$

$$= \frac{1}{2}\hat{k}\left[-8y^2 + 8x^2 - 8x^2 + 8y^2\right]$$

$$= 0$$

$\Rightarrow$ The flow field is irrotational.

(b)

(i) Carnot efficiency and 2nd law efficiency of a heat engine are 70% and 90% respectively. Determine the first law efficiency.

Sol: Given:

$\eta_{carnot} = 70\%$

$\eta_{II} = 90\%$

We know

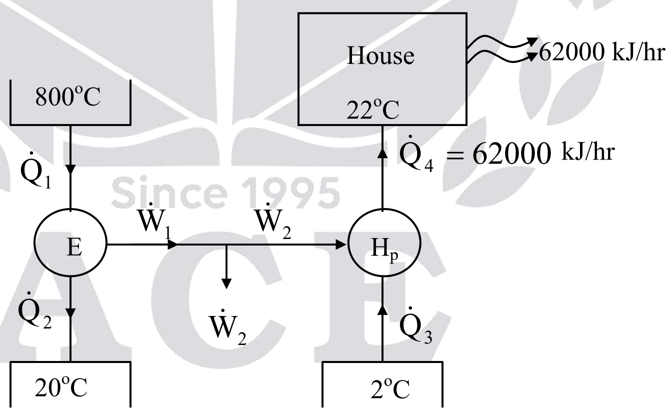
$$\eta_{II} = \frac{\eta_{actual}}{\eta_{ideal}}$$

$$\cdot 9 = \frac{\eta_{actual}}{\cdot 7}$$

$\eta_{actual} = 0.63$

(ii) A heat engine operates between two reservoirs at 800°C and 20° C. One-half of the work output of the engine is used to drive a Carnot heat pump that removes heat from the cold surroundings at 2°C and transfers it to a house maintained at 22°C. If the house is losing heat at a rate of 62,000 kJ/h, determine the minimum rate of heat supply to the heat engine required to keep the house at 22°C.

Sol: Given:



We know

$$\eta_e = 1 - \frac{273+20}{273+800} = \frac{\dot{W}_1}{Q_1}$$

$$\dot{W}_1 = 0.727\dot{Q}_1$$

$$\dot{W}_2 = \frac{\dot{W}_1}{2} = 0.363\dot{Q}_1 \quad \rightarrow (1)$$

We know

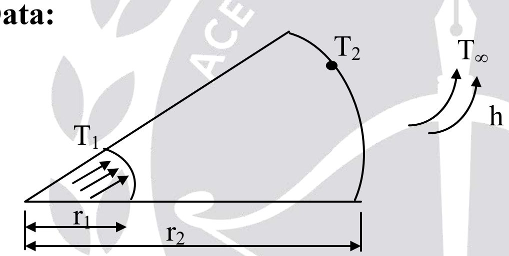
$$(COP)_{HP} = \frac{\dot{Q}_4}{\dot{W}_2} = \frac{295}{295-275}$$

$$\Rightarrow \frac{62000\text{kJ}/\text{hr}}{\cdot 363\dot{Q}_1} = 14.75$$

$$\dot{Q}_1 = 11579.5 \text{ kJ/hr}$$

(c) A hollow sphere of inside radius 3 cm and outside radius 5 cm is electrically heated at inner surface at a constant rate of heat flux of 10^5 W/m^2. The outer surface of the sphere dissipates heat to the surrounding air at 40°C. Assuming k = 15 W/mK for the sphere material and h = 400 W/m^2 K, calculate the inner and outer surface temperatures of the sphere.

Sol: Given Data:



$r_1 = 0.03$ m	$q = 10^5$ W/m^2	k = 15 W/mk
$r_2 = 0.05$ m	$T_\infty = 40°$C	h = 400 W/m^2 K

To find:

Inner surface temperature T_1 = ?

Outer surface temperature T_2 = ?

Assumptions:

i. One dimensional heat flow (radial flow)

ii. Steady state

iii. No internal heat generation within the sphere.

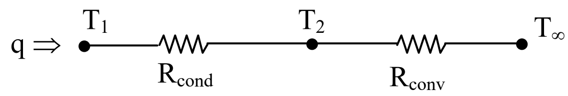
iv. Material homogenous and isotropic

v. Thermal conductivity value is constant

vi. Surfaces are isothermal

Procedure:

$$qA_1 = \frac{T_1 - T_2}{R_{cond}} = \frac{T_2 - T_\infty}{R_{conv}}$$



$$q \times 4\pi r_1^2 = \frac{\dfrac{T_1 - T_2}{r_2 - r_1}}{4\pi k r_1 r_2} = \frac{T_2 - T_\infty}{\dfrac{1}{h4\pi r_2^2}}$$

$$q \times 4\pi r_1^2 = \frac{T_2 - T_\infty}{\dfrac{1}{h4\pi r_2^2}}$$

$$q \times 4\pi r_1^2 = h4\pi r_2^2 (T_2 - T_\infty)$$

$$10^5 \times (0.03)^2 = 400 \times (0.05)^2 \times (T_2 - 40)$$

$$\mathbf{T_2 = 130^oC}$$

$$q \times 4\pi r_1^2 = \frac{T_1 - T_2}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2}}$$

$$q \times 4\pi r_1^2 = 4\pi k r_1 r_2 \left(\frac{r_1 - r_2}{r_2 - r_1}\right)$$

$$10^5 \times (0.03)^2 = 15 \times 0.03 \times 0.05 \times \left(\frac{T_1 - 130}{0.05 - 0.03}\right)$$

$$\mathbf{T_1 = 210^oC}$$

(d) A V-8 engine with 7.5 cm bores is redesigned from two valves per cylinder to four valves per cylinder. The old design had one inlet valve of 34 mm diameter and one exhaust valve of 29 mm diameter per cylinder. These are replaced with two inlet valves of 27 mm diameter and two exhaust valves of 2 mm diameter. If the maximum valve lift equals 22% of the valve diameter for all valve, calculate the increase of inlet flow area per cylinder. Also discuss the advantages and disadvantages of the new system.

Sol: V – 8 Engine

Bore = D = 7.5 cm

<u>Old</u> design Two valves per cylinder

D_i = inlet valve diameter = 34 mm

D_e = Exhaust valve diameter = 29 mm

Valve lift = 22%

Valve lift = $l = 0.22\ D_i$

$= 0.22 \times 34$

$= 7.48$ mm

Passage area of flow = A_{Pass}

$= \pi\ D_i\ l$

$= \pi \times 34 \times 7.48$

P.A. – I = 798.565 mm^2

<u>New</u> Design Four valves per cylinder

Inlet valves 2 Exhaust valves 2

D_i = Diameter inlet valve = 27 mm

D_e = Diameter exhaust valve = 2 mm

Valve lift = 22%

Valve lift = $l = 0.22 \times D_i$

$= 0.22 \times 27$

$= 5.94$ mm

Passage area of flow = $2 \times \pi\ D_i \times l$

$= \pi \times 27 \times 5.94 \times 2$

P.A – II = 1007.187 mm^2

Increase in inlet flow area

= (PA – II) – (PA – I)

= 1007.187 – 798.565

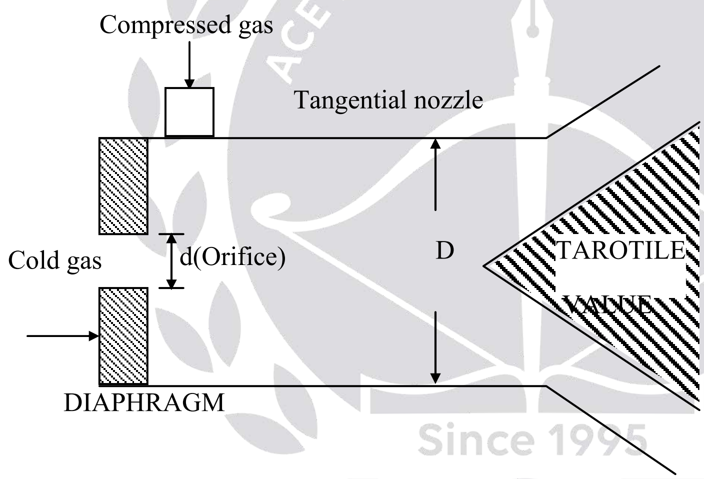
= 208.62 mm^2

On new engines with over head valves and small fast burn combustion chambers to fit the spark plug and exhaust valve and still have room for an intake valve of large enough to fit, it is becoming difficult. For this reason most engines are built with more than one intake valve per cylinder. Two or three small valves give more flow area and less flow resistance than one large valve as was used in older engines. Two or three intake valves along with usually two exhaust valves along with usually two exhaust valves can better fit into a given cylinder head size with enough clearance to maintain required structural strength.

Multiple valves required a greater complexity of design with more camshafts and mechanical linkages. Specially shaped cylinder heads and recessed piston faces just to avoid valve to valve or valve to piston contact. Only CAD can do perfect design for such systems. When two or more valves are used instead of one the valves will be smaller and lighter. This allows use of lighter springs and reduces forces in linkages. Lighter valves are opened and closed fast. Greater volumetric efficiency of multiple valves compensates the added cost of manufacture and added complexity and mechanical inefficiency.

(e) Describe briefly the working principle of a vortex tube refrigeration system mentioning its advantages and disadvantages.

Sol: It is a non conventional refrigerating system.



It consists of nozzle diaphragms, valve, hot air side, ssscold air side. The nozzles are of converging or diverging or Converging – Diverging type as per design. An efficienct nozzle is designed to have higher velocity, greater mass flow rate and minimum losses. Chamber is a portion of nozzle and facilitates the tangential entry of high velocity air stream into hot side. The chambers gradually converted to spiral form. Hot side is cylindrical in cross-section and is of different lengths as per design. Valve obstructs the flow of air through hot side and it also controls quantity of hot air through vortex tube. Diaphragm prevents left ward motion of vortex. The throttle valve opening controls the temperature and proportion of cold stream with respect to hot stream. Larger the throttle valve opening the lower the temperature of cold stream and smaller its fraction and vice versa. The throttle valve is kept sufficiently distant from nozzle and the diaphragm.

D = Vortex tube diameter

d = Orifice diameter

L = Length of vortex tube

η_h = Hot mass fraction

η_c = cold mass fraction

$d = \frac{D}{2}$ (for optimum design)

Compressed air is passed through nozzle, air expands and acquires high velocity due to particular shape of nozzle. A vortex flow is created in chamber and air travels in spiral like fashion along the periphery of hot side. The flow is restricted by valve. When pressure of air near valve is made more than outside by partly closing the valve, a reversed axial flow through the core of hot side starts from high pressure region to low pressure region. During this process, heat transfer takes place between reversed stream and forward stream. Therefore air stream through the core gets cooled below the inlet temperature of air in vortex tube, while air stream in forward direction gets heated up. The cold stream escapes through diaphragm hole into cold side, while hot stream is passed through opening by valve. By controlling valve opening the quantity of cold air and its temperature can be varied

T_i = inlet temperature of air

T_c = cold air temperature

T_h = Hot air temperature

η_c = Cold mass fraction

η_h = hot mass fraction energy balance of vortex tube

$T_i = \eta_c T_c + \eta_h T_h$

Advantages:

1. Uses air as refrigerant, so there is no leakage problem.
2. Cheap refrigerant, freely available, no toxic
3. Vortex tube is simple in design and no control systems.
4. Less costly
5. No moving parts, maintenance free
6. Light in weight and requires less space
7. Initial cost is low, working expenses are less, when compressed air is readily available.

Disadvantages

1. Low COP
2. Limited capacity
3. Very small portion of compressed air is available on cold side
4. Limitation on wide use due to availability of less cold

02.(a)

(i) Prove that equipotential lines and constant function streamlines are orthogonal to each other.

How do you distinguish between developing flow and fully developed flow?

Sol: The equation of streamline for 2D flow is given by, $\frac{dx}{u} = \frac{dy}{v}$

$$\frac{dy}{dx} = \frac{v}{u}$$

i.e the slope of stream line (m_s) is $m_s = \frac{v}{u}$ ……….. (1)

Along equipotential line the velocity potential is constant

$$\text{i.e } \phi = \text{const}$$

$$\therefore\ d\phi = 0$$

$$\text{i.e } \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy = 0$$

But as per definition of velocity potential, $u = -\frac{\partial\phi}{\partial x}$ & $v = -\frac{\partial\phi}{\partial y}$

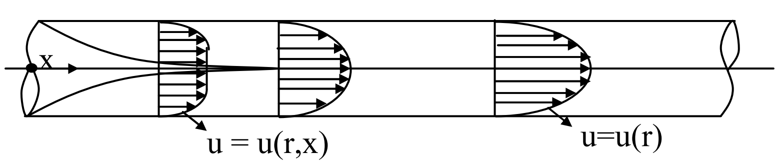
$$\therefore\ -udx + (-v)\,dy = 0$$

$$\text{i.e } \frac{dy}{dx} = \frac{-u}{v}$$

i.e slope of equipotential line (m_p) is given by, $m_p = \frac{-u}{v}$

Now, $m_s \times m_p = \left(\frac{v}{u}\right) \times \left(\frac{-u}{v}\right) = -1$

Streamlines & equipotential lines are orthogonal to each other.



For developing flow velocity profile changes in axial direction where as for fully developed flow the velocity profile remains same at any section irrespective of axial location. Hence, mathematical conditions are

$u = u\,(r, x) \rightarrow$ for developing flow

$u = u(r) \rightarrow$ for fully developed flow

It can be shown that for fully developed condition, the pressure decreases linearly in the direction of flow. Hence, practically the fully developed flow can be identified by measuring the pressure at locations equally spaced from each other. If two consecutive pressure readings have equal difference then it implies that the flow is fully developed.

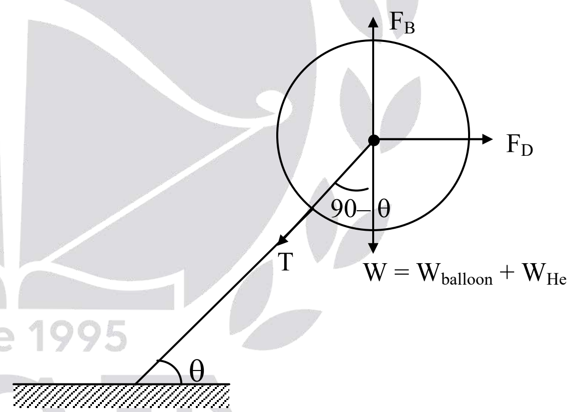
(ii) A spherical balloon having 3 m diameter weighs 130 N and contains helium having density of 0.22 kg/m^3, whereas the surrounding air has a density of 1.225 kg/m^3. The balloon is tied with the cable which is inclined to the ground. Determine the inclination of the cable to the ground when a wind of 5 m/s blows past the balloon. Take C_D = 0.2.

Sol: Given data:

$D = 3$ m ($R = 1.5$ m) , $W_{balloon} = 130$ N

$\rho_{He} = 0.22$ kg/m^3, $\rho_a = 1.225$ kg/m^3

$U_\infty = 5$ m/sec, $C_D = 0.2$



As the balloon is in equilibrium, we have:

$\Sigma F_x = 0$

$-T\sin(90-\theta) + F_D = 0$

Or $T\cos\theta = F_D$ ………… (1)

Similarly

$\Sigma F_y = 0$ gives

$-T\cos(90-\theta) - F_B - W_{balloon} - W_{He} = 0$

i.e $T\sin\theta = F_B - W_{balloon} - W_{He}$ …….. (2)

From (1) & (2)

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_B - W_{balloon} - W_{He}}{F_D}$$

$$\tan\theta = \frac{\rho_a g v - \rho_{He} g v - 130}{\frac{C_D}{2}\rho_a U_\infty^2 A}$$

$$\tan\theta = \frac{9.81\times\frac{4}{3}\times\pi\times1.5^3(1.225-0.22)-130}{\frac{0.2}{2}\times1.225\times5^2\times\pi\times1.5^2}$$

$$= \frac{9.379}{21.6475} = 0.4333$$

$$\Rightarrow \theta = 23.43^o$$

(b) A frictionless piston-cylinder device initially contains 0.01 m^3 of argon gas at 400 K and 350 kPa. Heat is now transferred to argon from a furnace at 1200 K, and the argon expands isothermally until its volume is doubled. The heat transfer takes place in such a way that there is no heat loss from argon to the atmosphere. The atmosphere is at 300 K. Determine (i) the work done by argon, (ii) the heat transferred to argon, and (iii) entropy generation and irreversibility during the process.
Take R = 0.2081 kJ/kg-K for argon.

Sol: **Given:** $V_1 = 0.01$ m^3

$V_2 = 2V_1 = 0.02$ m^3

$P_1V_1 = mRT_1$

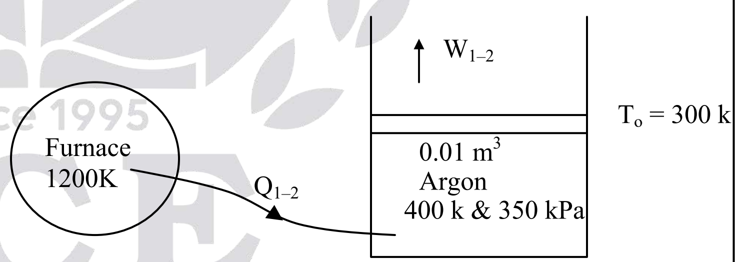
(i) $m = 0.042$ kg

Work for close system [Argon] undergoing isothermal process can given as

$$W_{12} = P_1V_1\ell n\frac{V_2}{V_1}$$

$$= 350\times0.01\ell n\frac{0.02}{0.01}$$

$W_{1\text{-}2} = 2.426$ kJ



(i) For a close system undergoing a process heat transfer is equal to work transfer

$Q_{1\text{-}2} = W_{1-2} = 2.426$ kJ

(ii) Entropy change of argon

$$(ds)_{argon} = m\left[C_V\ell n\frac{T_2}{T_1} + R\ell n\frac{V_2}{V_1}\right]$$

$$= 0.042\ [\cdot\ 208\ ln\ (2)]$$

$= 6.05 \times 10^{-3}$ kJ/k

Entropy change at furnace

$$(ds)_{furnace} = \frac{-Q_{1-2}}{1200} = \frac{-2.426}{1200} = 2.02 \times 10^{-3} \text{ kJ/K}$$

Net entropy generated

$(\delta s)_{gen} = (ds)_{argon} + (ds)_{furnace}$

$= 6.05 \times 10^{-3} - 2.02 \times 10^{-3}$

$(\delta s)_{gen} = 4.03 \times 10^{-3}$ kJ/K

Now

Irreversibility during the process

We know

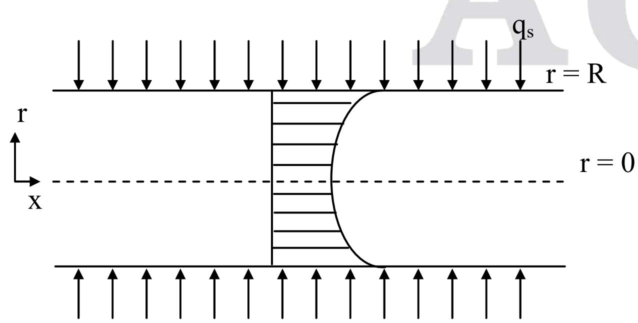
$I = T_o\,(\delta S)gen$

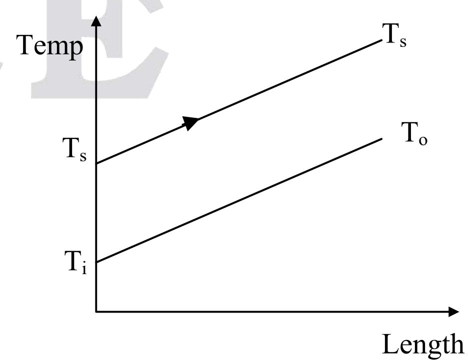
$= 300 \times 4.03 \times 10^{-3}$

$I = 1.209$ kJ

(c) Show that for fully developed laminar flow in a tube with a parabolic velocity profile $u = 2u_m\left[1-\left(\frac{r}{R}\right)^2\right]$, the Nusselt number is $\frac{48}{11}$ if the wall temperature increases linearly with x. Symbols have their usual meanings.

Sol: Shown that for fully developed laminar flow in a table with a parabolic velocity profile $u = 2\,u_m\left[1-\left(\frac{r}{R}\right)^2\right]$, the Nusselt number is $\frac{48}{11}$, if the wall temperature increases linearly with x. Symbols have their usual meanings.





T_i = Mean temperature of flow at inlet

T_o = Mean temperature of flow at outlet

T_m = Mean Temperature of flow at any location.

The governing differential equation for fully developed laminar flow through a tube is:

$$\frac{1}{\alpha} u \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}$$

If the wall temperature increases linearly with x the fluid temperature also varies linearly with x

$$\therefore \ \frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} = \text{Constant}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$\frac{1}{\alpha} u \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}$$

$$\frac{1}{\alpha} u \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

For fully developed flow $u = 2u_m\left[1-\left(\frac{r}{R}\right)^2\right]$

Put value of u in equation (1)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial T_m}{\partial x}$$

$$\therefore \ \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x} \text{ is a constant}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m r}{\alpha}\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial T_m}{\partial x}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left[r-\frac{r^3}{R^2}\right]\frac{\partial T_m}{\partial x}$$

Integrate

$$r\frac{\partial T}{\partial r} = \frac{2U_m}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right] + C_1 \quad \ldots\ldots\ldots (2)$$

At r = 0 $\frac{\partial T}{\partial r} = 0$ $C_1 = 0$

$$\frac{\partial T}{\partial r} = \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{r}{2} - \frac{r^3}{4R^2}\right]$$

Integrate

$$T = \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_2 \qquad \ldots\ldots\ldots(3)$$

$\therefore$ r = R T = T_s (Surface temperature)

$$T_S = \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{R^2}{4} - \frac{R^4}{16R^2}\right] + C_2$$

$$T_S = \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x}\frac{3R^2}{16} + C_2$$

$$C_2 = T_s - \frac{2u_m}{\alpha}\left(\frac{\partial T_m}{\partial x}\right)\left(\frac{3R^2}{16}\right)$$

Put C_2 value in equation (3)

$$T_S = \frac{2u_m}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + T_S - \frac{2u_m}{\alpha}.\frac{\partial T_m}{\partial x}.\frac{3R^2}{16}$$

$$T = T_s - \frac{2u_m R^2}{\alpha}\frac{\partial T_m}{\partial x}\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{R}\right)^4 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right] \ldots\ldots\ldots (4)$$

The bulk mean temperature is given by

$$T_m = \frac{2}{u_m R^2}\int_0^R V_{(e,x)} T_{(e,x)} r dr$$

By substitution of equation 4 and subsequent integration gives

$$T_m = T_s - \frac{11}{48}\left(\frac{u_m R^2}{\alpha}\right)\left(\frac{\partial T_m}{\partial x}\right)$$

The heat transfer coefficient

$$h = \frac{k \left.\frac{dT}{\partial}\right|_{\varepsilon=R}}{T_s - T_m}$$

$$\left.\frac{dT}{dr}\right|_{\varepsilon=R} = \frac{R}{2}\frac{u_m}{\alpha}\left(\frac{dT_m}{dx}\right) \rightarrow (5)$$

Put equation (4) and (5) in heat transfer coefficient equation

$$h = \frac{k \left.\frac{dT}{d}\right|_{\varepsilon=R}}{T_S - T_m}$$

We get $h = \frac{48}{11}.\frac{k}{D}$

$\therefore$ Nussect number $N_u = \frac{hD}{k} = \frac{48}{11}$

Hence for a laminar fully developed flow the nussect number is constant and equal to $\frac{48}{11}$.

03. (a)

(i) Prove that the total pressure which is the summation of static and dynamic pressure, also known as stagnation pressure, decreases in an irreversible adiabatic process when a gas is flowing in a steady flow device of constant cross-section without any work transfer.

Sol: Stagnation pressure

When neither the external work is done, nor the heat is transferred externally, the SFEE at the given section will be

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \Rightarrow h + \frac{V^2}{2} = \text{constant}$$

The quantity $h + \frac{V^2}{2}$ is considered as stagnation enthalpy (h_o)

$$\therefore h_o = h + \frac{V^2}{2}$$

$$C_p T_o = C_P T + \frac{V^2}{2} \text{ for an ideal gas}$$

$$T_o = T + \frac{V^2}{2C_P}$$

The quantity $\frac{V^2}{2C_P}$ is velocity temperature (or) dynamic temperature

For a reversible adiabatic process

$$\frac{P_o}{P} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

If friction is considered for the adiabatic flow

$$\frac{P_o}{P} = \left(\frac{T_o}{T}\right)^{\frac{n}{n-1}}$$

For this case $n > \gamma$ If n is increasing the stagnation pressure decreases

Ex: If $\gamma = 1.4$, $\frac{T_o}{T} = 1.2$

$$\frac{P_o}{P} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = (1.2)^{\frac{1.4}{0.4}} = 1.89$$

For the same $\frac{T_o}{T}$, if $n = 1.5$

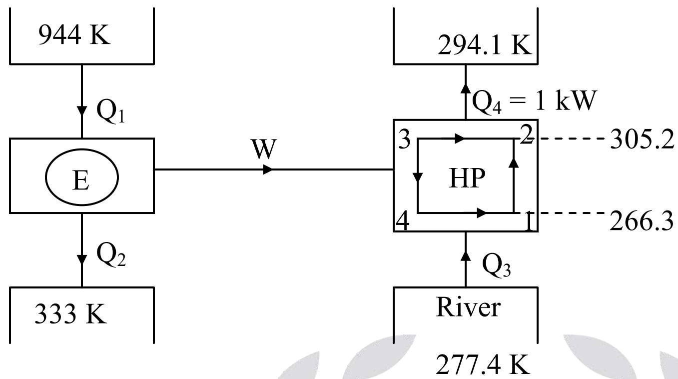
$$\frac{P_o}{P} = (1.2)^{\frac{1.5}{0.5}} = 1.728$$

From the above two examples, we can evidently decide that the stagnation pressure decreases with irreversibilities in the adiabatic process.

Even if frictional temperature rise is considered, the stagnation pressure decreases.

(ii) A heat engine operates between the maximum and minimum temperatures of 671°C and 60°C respectively, with an efficiency of 50% of its Carnot efficiency. It drives a heat pump which uses river water at 4.4°C to heat a block of flats in which the temperature is to be maintained at 21.1°C. Assume that a temperature difference of 11.1°C exists between the working fluid and the river water, on the one hand, and the required room temperature on the other. Also assume that the heat pump would be operated with a COP of 50% of the ideal COP. Find the heat input to the engine per unit heat output from the heat pump.

Sol:



$$[\eta_e]_{actual} = 0.5[\eta_{carnot}]$$

$$= 0.5\left[1-\frac{333}{944}\right]$$

$$= 0.3236$$

$$\eta_e = \frac{W}{Q_1}$$

$$\cdot 3236 = \frac{W}{Q_1}$$

$$W = \cdot 3236 Q_1 \rightarrow (1)$$

$$(COP)_{HP} = 0.5\,[(COP)_{ideal}]$$

$$= 0.5\left[\frac{305.2}{305.2-266.3}\right] = 3.9228$$

$$(COP)_{HP} = \frac{Q_4}{W}$$

$$3.9228 = \frac{1}{\cdot 3236 Q_1}$$

$$Q_1 = 0.7877 \text{ kJ}$$

In direct heating for 1 kJ of heat supply to room we have to give 1 kJ of work input.

Using an engine heat pump combination.

$(COP)_{HP} = \dfrac{Q_4}{W}$ $\quad\quad Q_4 = 1$ kJ given

Working engine = $W = \dfrac{Q_4}{3.9228} = \dfrac{1}{3.9228} = 0.255$ kJ/kg

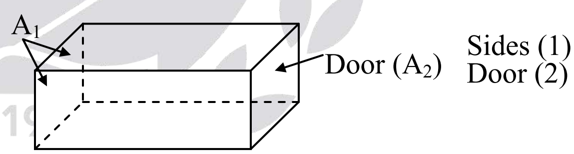
$W = 0.255$ kJ of work per kJ of heat supply to room

The second option gives lesser work input per units (1 kJ) of heat supply to room.

(b)

(i) A cubical oven has inner sides equal to 0.4 m. One of the faces of the oven forms the door. If the five other inside faces are black and maintained at 600°C, find the rate of heat loss if the oven door is kept open.
Take Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4.

Sol: A cubical oven has inner sides equal to 0.4 m. One of the faces of the oven forms the door. If the five other inside faces are black and maintained at 600°C, find the rate of heat loss if the oven door is kept open. Take Stefan Boltzman constant $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4.



$T_1 = 600°C$
$= 873$ K

A_1 = Area of the five inside faces

$A_1 = 5 \times 0.4 \times 0.4 = 0.8$ m^2

A_2 = Area of the door through which energy stream out.

$A_2 = 0.4 \times 0.4 = 0.16$ m^2

$F_{21} + F_{22} = 1, \quad F_{22} = 0$

$F_{21} = 1$

$A_1 F_{12} = A_2 F_{21}$

$F_{12} = \dfrac{A_2}{A_1} F_{21}$

$$F_{12} = \frac{A_2}{A_1} = \frac{0.16}{0.8} = 0.2$$

$F_{11} + F_{12} = 1$

$F_{11} = 1 - F_{12} = 1 - 0.2 = 0.8$

Rate of heat loss when door is open

$$Q = \sigma A_1 \varepsilon_1 T_1^4 \left[\frac{1-F_{11}}{1-(1=\varepsilon_1)F_{11}}\right]$$

$$Q = 5.67 \times 10^{-8} \times 0.8 \times 1 \times 873^4 \left[\frac{1-0.8}{1-(1-1)0.8}\right]$$

$Q = 5.273$ kW = 5273 W

(ii) Explain briefly why dropwise condensation is preferred to filmwise condensation.

Sol: In the film condensation process the surface is covered by the film which grows in thickness as it moves down the surface. The presence of a liquid film over the surface constitutes thermal resistance to heat transfer.

In dropwise condensation a significantly large part of the surface is directly exposed to the vapour. Therefore higher condensation and heat transfer rate are experienced in drop wise condensation. In dropwise condensation heat transfer coefficient is about 5 to 10 times greater than filmwise condensation. Due to this reason many surface coating and vapour additives like oleic acid have been used to promote and maintain dropwise condensation. There is no film barrier to heat transfer in dropwise condensation and a portion of the cool surface is always in contact with the vapour without insulating influence of the liquid layer. This accounts for higher heat transfer coefficient (up to 290 kW/m^2 K) associated with dropwise condensation which is certainly preferable for industrial application. Dropwise condensation offer very little thermal resistance. Dropwise condensation is difficult to maintain for longer period of time.

In dropwise condensation, condensation takes place in the form of droplet. Due to influence of gravity droplets merge together and finally leave the surface. Due to this reason large area of the surface is in contact with vapour as a result heat transfer rate is high.

(c) In an air-conditioning system, the inside conditions are dry bulb temperature 25°C and relative humidity 50%. The outside conditions are dry bulb temperature 40°C and wet bulb temperature 27°C. The room sensible heat factor is 0.8. 50% of room air is rejected to atmosphere and an equal quantity of fresh air is added before air enters the air-conditioning apparatus.

Assuming fresh air is added at a rate of 100 m^3 / minute, draw the process diagram and determine the following:

(i) Room sensible and latent heat load

(ii) Sensible and latent heat load due to fresh air

(iii) Apparatus dew point

(iv) Humidity ratio

Take density of air = 1.2 kg/m^3 at a pressure of 1.01325 bar and humid specific heat = 1.022 kJ/kg K. Bypass factor is zero.

Sol: Inside conditions:

T_i = 25°C

RH = 50%

Outside conditions :

T_o = 40°C

$(WBT))_o$ = 27°C

Mark point (i) on psychrometric chart at DBT = 25°C ϕ = 50% RH

From Chart:

h_i = 51 kJ/kg d.a

ω_i = 0.01 kgv/kg d.a

v_i = 0.86 m^3/ kg d.a

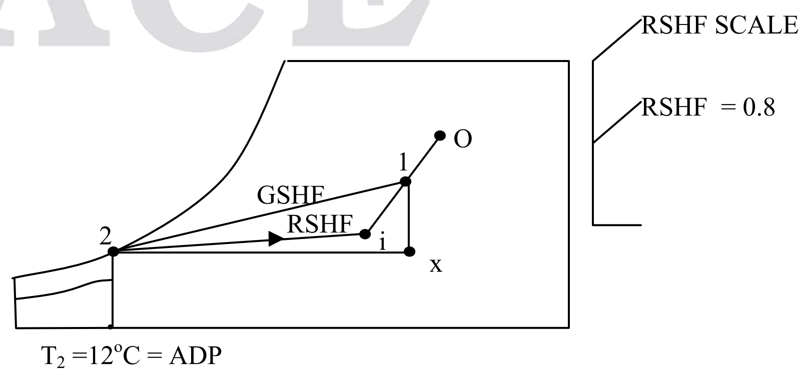
Mark point (O) on psychrometric chart at DBT = 40°C and WBT = 27°C

From Chart

h_o = 85 kJ/kg d.a

ω_o = 0.017 kg vap/kg d.a

v_o = 0.91 m^3/ kg d.a



RSHF = 0.8

Fresh air % = 50

Recirculated air % = 50

Fresh air added to room air = $\dot{V}_o$ = 100 m^3/min

Mass of fresh air added = $\dot{m}_o = \frac{\dot{V}_o}{v_o}$

$$\dot{m}_o = \frac{100}{0.91} = 109.89 \text{ kg/min}$$

Recirculated air = 109.89 kg/min

By pass factor = BPF = 0

Contact factor = CF = 100%

Mark point (1) on psychrometric chart which divides line O – i equally into two parts.

From chart

h_1 = 68 kJ/kg d.a

ω_1 = 0.0135 kg vap/kg d.a

v_1 = 0.985 m^3/kg d.a

Join RSHF 0.8 on SHF scale to point (i) and extend it to cut RH 100% line at point (2). As BPF is zero state (2) is the state of air entering the room as well as the state of air leaving the cooling coil.

From Chart

h_2 = 34 kJ/kg d.a

T_2 = 12^{o}C

ω_2 = 0.009 kg Vap/ kg d.a

(2 – i) is called RSHF line

Join 1 – 2 which is called cooling and dehumidification line

Total mass of air flowing in the system = $\dot{m}$ = 219.78 kg/min

Room total heat load = $\dot{m}(\text{kg/sec})(h_i - h_2)\text{kg/kg}$

$$\text{RTHL} = \frac{219.78}{60} \times (51 - 34)$$

= 62.271 kW

Room sensible heat load = (RSHF) (RTHL)

= (0.8) (62.271)

= 49.82 kW

Room latent heat load = RTHL – RSHL

= 62.271 – 49.82

= 12.454 kW

From point (1) drop a vertical

From point (2) draw a horizontal

Both intersect at point x

From chart: $h_x = 56.5$ kJ/kg d.a

Grand sensible heat load GSHL = $\dot{m}$ (kg/sec) $(h_x - h_2)$

$$= \frac{219.78}{60}(56.5-34) = 82.42 \text{ kW}$$

Out side air sensible heat load = GSHL – RSHL

= 82.42 – 49.82

OASHL = 32.6 kW

Grand latent heat load = GLHL

$$= \dot{m}(\text{kg/sec})(h_1 - h_x)$$

$$= \frac{219.78}{60}(68-56.5) = 42.1245 \text{ kW}$$

Outside air latent heat load = GLHL –RLHL

= 42.1245 – 12.454

OALHL = 29.67 kW

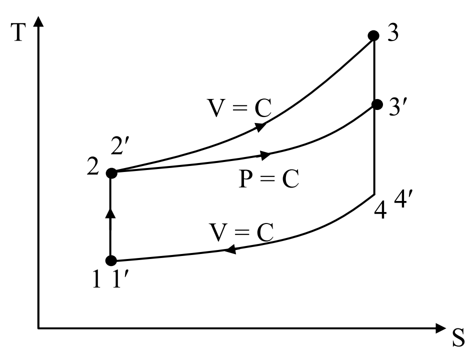
Apparatus Dew Point = $T_2 = 12°C$

Humidity ratio of air entering the room = $\omega_2 = 0.009$ kg/kg d.a.

04. (a)

(i) For the same compression ratio and heat rejection, show that the efficiency of Otto cycle is greater than Diesel cycle using T-s plot.

Sol: Same compression ratio and same reat rejection



1-2-3-4-1 ⇒ Otto cycle

1′-2′ -3′-1′ ⇒ Diesel cycle

$[Q_R]_{otto} = [Q_R]_{Diesel}$

$[Q_{in}]_{otto} > [Q_{in}]_{Diseal}$

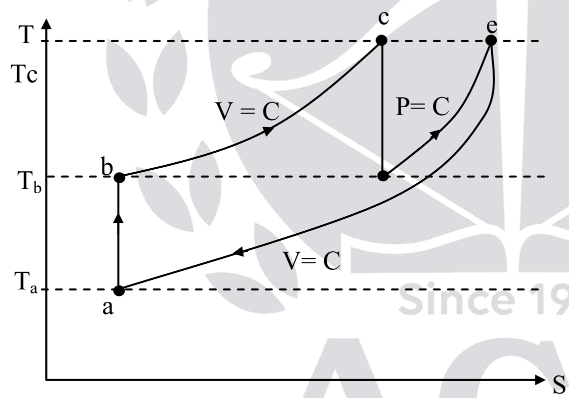
We know

$$\eta = 1 - \frac{Q_R}{Q_{in}}$$

So $\eta_{otto} > \eta_{Diseal}$

(ii) An ideal gas is compressed reversibly and adiabatically from state a to state b. It is then heated reversibly at constant volume to state c. After expanding reversibly and adiabatically to state d such that $T_b = T_d$, the gas is again reversibly heated at constant pressure to state e such that $T_e = T_c$. Heat is then rejected reversibly from the gas at constant volume till it returns to state a. Show that $T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}}$; where $\gamma = \frac{C_P}{C_V}$.

Sol:



We know

For cycle change in entropy of the system is zero

$(ds)_{cycle} = 0$

$(ds)_{a.b} + (ds)_{b-c} + (ds)_{cd} + (ds)_{de} + (ds)_{e.a} = 0 \rightarrow (A)$

$(ds)_{ab} = 0$ {Reversible adiabatic (isentropic)}

$(ds)_{cd} = 0$ {Reversible adiabatic (isentropic)}

$$(ds)_{DC} = C_v \ell n \frac{Tc}{Tb} + R \ell n \frac{V_C}{V_D}$$

$$(ds)_{d-e} = Cp\, ln\, \frac{Te}{Td} - R\ell n \frac{Pe}{Pd}$$

$$= Cp\, ln\, \frac{T_C}{T_D}$$

$$(ds)_{ea} = Cv\, ln\, \frac{Tq}{Te} + R\ell n \frac{V_a}{V_e}$$

$$= C_V \ell n \frac{Tq}{Tc}$$

Now equation (A)

$(ds)_{ab} + (ds)_{D-C} + (ds)_{cd} + (ds)_{de} + (ds)_{ea} = 0$

$$0 + C_V \ell n \frac{T_C}{T_D} + 0 + C_P \ell n \frac{T_C}{T_D} + C_V \ell n \frac{T_a}{T_C} = 0$$

$$C_V \left[\ell n \frac{Tc}{Tb} + \ell n \frac{Ta}{Tc} \right] = -Cp\ell n \frac{Tc}{Tb}$$

$$C_V \ell n \left[\frac{T_C}{T_D} \times \frac{T_a}{T_C} \right] = C_P \ell n \frac{T_b}{T_C}$$

$$C_v \ell n \left[\frac{T_a}{T_D} \right] = C_P \ell n \frac{T_b}{T_c}$$

$$\ell n \left[\frac{T_a}{T_b} = y \ell n \left[\frac{T_b}{T_c} \right] \right]$$

$$\ell n \left[\frac{T_a}{T_b} \right] = y \ell n \left[\frac{T_b}{T_c} \right]$$

$$\ell n \left[\frac{T_a}{T_b} \right] = \ell n \left[\frac{T_b}{T_c} \right]^y$$

$$\frac{T_a}{T_b} = \frac{T_b^y}{T_c^y}$$

$$T_a = \frac{[T_b]^{y+1}}{[T_c]^y}$$

(b) The following data refers to a single-stage vapour compression system:

Refrigerant used: R – 134a

Condensing temperature = 35°C

Evaporator temperature = – 10°C

Compressor : rpm = 2800

Efficiency = 0.8

Clearance volume/Swept volume = 0.03

Swept volume = 269.4 cm^3

Expansion index = 1.12

Condensate subcooling = 5°C

Determine : (i) tonnage,

(ii) power,

(iii) COP of refrigeration and

(iv) Heat rejection to condenser

Properties of R– 134a:

t, °C	P, bar	V_g, m^3/kg	h_f, kJ/kg	H_g, kJ/kg	S_f, kJ/kg K	S_g, kJ/kg K
– 10	2.014	0.0994	186.7	392.4	0.9512	1.733
35	2.870	-	249.1	417.6	1.1680	1.715

Assume: Specific heat of vapour at 8.87 bar is 1.1 kJ/kg K and that of liquid is 1.458 kJ/kg K. Suction vapour is dry saturated and compression process is isentropic. Compressor is single acting.

Sol:

$Cp_w = 1.1$ kJ/kg.K

$Cp_l = 1.458$ kJ/kg.K

$P_2 = P_3 = 8.87$ bar

$P_1 = P_5 = 2.014$ bar

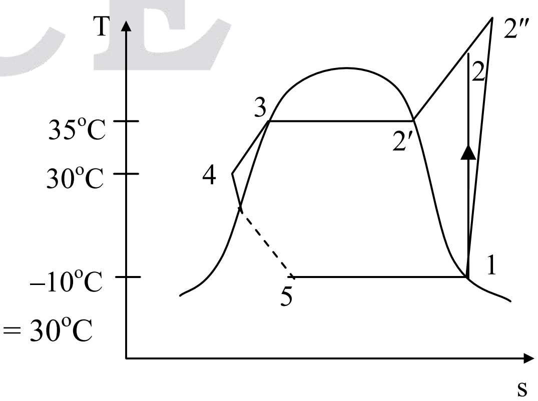
Condensing temperature = 35°C

Evaporator temperature = – 10°C

Temperature after sub cooling = $T_4 = 35 - 5 = 30$°C

Degree of sub cooling = 5°C

Speed = N = 2800 rpm



Clearance ratio = C = $\frac{V_C}{V_S}$ = 0.03

Swept volume = V_S = 269.4 cm^3

Efficiency of compressor = η_c = 0.8

Index of expansion = n = 1.12

Pressure ratio = $\frac{P_2}{P_1} = \frac{8.87}{2.014} = 4.404$

Volumetric efficiency of compressor = $\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/n}$

$\eta_{vol} = 1 + 0.03 - 0.03\,(4.404)^{1/1.12}$
$= 0.9173$

$v_1 = (v_g)_{-10^\circ C}$ = 0.0994 m^3/kg

$h_1 = (h_g)_{-10^\circ C}$ = 392.4 kJ/kg

$S_1 = (s_g)_{-10^\circ C}$ = 1.733 kJ/kg K

$S'_2 = (s_g)_{35^\circ C}$ = 1.715 kJ/kg K

$h'_2 = (h_g)_{35^\circ C}$ = 417.6 kJ/kg K

$h_3 = (h_f)_{35^\circ C}$ = 249.1 kJ/kg.K

$Cp_\ell = 1.458$ kJ/kg K $\qquad$ Cp_v = 1.1 kJ/kgK

$h_4 = h_5 = h_3 - Cp_l\,(T_3 - T_2)$
$= 249.1 - 1.458 \times 5 = 241.81$ kJ/kg K

<u>1 – 2 $\qquad$ Q = 0; s = C</u>

$s_1 = s_2 : s_2 = s'_2 + C_{pv}\ell n\frac{T_2}{T'_2}$

$\ell n\frac{T_2}{T'_2} = \frac{s_1 - s'_2}{C_{pv}} = \frac{1.733 - 1.715}{1.1}$

$= 0.0164$

$T_2 = T'_2\, e^{0.0164} = 308\, e^{0.0164}$
$= 313.09$ K

$h_2 = h'_2 + Cp_v\,(T_2 - T'_2)$
$= 417.6 + 1.1\,(313.09 - 308) = 423.2$ kJ/kg

$$\eta_c = \frac{h_2 - h_1}{h_2'' - h_1}$$

$$h_2'' = h_1 + \frac{h_2 - h_1}{\eta_c}$$

$$= 392.4 + \frac{423.2 - 392.4}{0.8} = 430.9 \text{ kJ/kg}$$

Volume flow rate (m^3/sec) = $\dot{m}_r\left(\frac{kg}{sec}\right)v\left(\frac{m^3}{kg}\right)$

$$= \left(\frac{\pi}{4}D^2L\right)\frac{N}{60} \times \eta_{vol}$$

$$= 269.4 \times 10^{-6} \times \frac{2800}{60} \times 0.9173$$

$$= 11532.3 \times 10^{-6}$$

$\dot{m}_r$ = mass flow rate of refrigerant = $\frac{11532.3 \times 10^{-6}}{v_1}$

$$\dot{m}_r = \frac{11532.3 \times 10^{-6}}{0.0994} = 0.116 \text{ kg/sec}$$

Tonnage of refrigeration = $\frac{\dot{m}_r(kg/sec)(h_1 - h_5)kJ/kg}{3.517}$

$$= \frac{0.116(392.4 - 241.81)}{3.517} = 4.967 \text{ TR}$$

Power = $(h_2'' - h_1)\frac{kJ}{kg} \times \dot{m}_r\left(\frac{kg}{sec}\right)$

= 0.116 (430.9 – 392.4) = 4.467 kW

COP = $\frac{h_1 - h_5}{h_5'' - h_1} = \frac{392.4 - 241.81}{430.9 - 392.4}$

$$= \frac{150.59}{38.5} = 3.91$$

Heat rejection to condenser = $\dot{m}_r(kg/sec)(h_2'' - h_4)$

= 0.116 (430.9 – 241.81)

= 21.934 kW

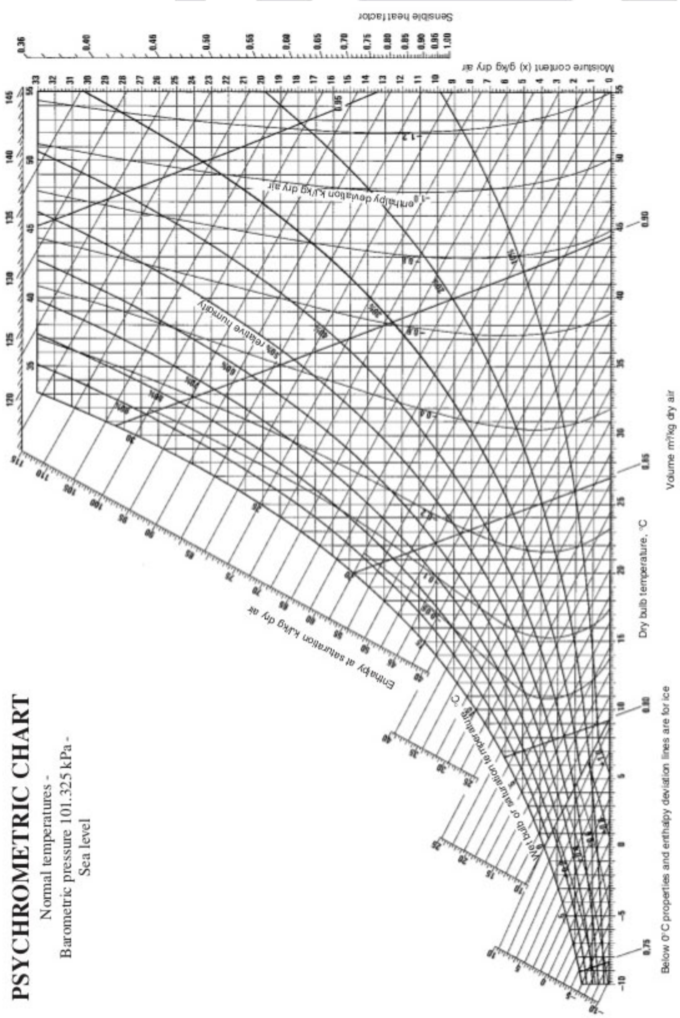
(c) **A two-stroke single cylinder SI engine of 10 cm bore having compression ratio 8.5, consume 15.75 kg/hr of fuel when running at 3500 rpm. The piston speed is 14 m/s and the indicated mean effective pressure is 5 bar. The A/F ratio is 15 : 1, the calorific value of the fuel is 44 MJ/kg. Assume 'R' for the mixture as 290 J/(kg K), the pressure and temperature of the mixture as 1.05 bar and 27°C respectively, η_{mech} = 85%.**

Calculate:

(i) The scavenging ratio   **(ii) The scavenging efficiency**

(iii) The trapping efficiency   **(iv) The ip**

(v) The bp   **(vi) The brake thermal efficiency**



Sol: SI Engine

2 Stroke

Bore = d = 10 cm = 0.1 m

Compression ratio = r_k = 8.5

Fuel consumption rate = $\dot{m}_f = 15.75\frac{kg}{hr}$

Speed = N = 3500 rpm

Piston speed = $\frac{LN}{30}$ = 14

Indicated mean effective pressure = imep = 5 bar

Air Fuel ratio = $\frac{\dot{m}_a}{m_f}$ = 15

Mechanical efficiency = η_m = 0.85

Calorific value = CV = 44000 kJ/kg

R = 290 J/kg K

P_1 = 1.05 bar $\qquad$ T_1 = 300 K

$\frac{LN}{30} = 14$

L = Stroke length = $\frac{14\times30}{N} = \frac{14\times30}{3500}$ = 0.12 m

Density of air = $\rho = \frac{P_1}{RT_1}$

$= \frac{1.05\times10^5}{290\times300}$

= 1.207 kg/m^3

Swept volume = $\frac{\pi}{4}D^2L$

$= \frac{\pi}{4}\times(0.10)^2(0.12)$

= 9.42×10^{-4} m^3

Total cylinder volume = $V = \left(\frac{r_K}{r_{K-1}}\right)V_s$

$$= \left(\frac{8.5}{8.5-1}\right) 9.42 \times 10^{-4}$$

$$= 10.676 \times 10^{-4}\ m^3$$

Ideal mass of air in total cylinder volume = ρV

$$= 1.207 \times 10.676 \times 10^{-4}$$

$$m = 12.886 \times 10^{-4}\ \text{kg per cycle}$$

Ideal mass per unit time = m (kg/cycle) N (cycle/min)

$$= 12.886 \times 10^{-4} \times 3500$$

$$= 4.51\ \text{kg/min}$$

Actual mass of air flow = $(AFR)\dot{m}_f\,(kg/min)$

$$= 15 \times \frac{15.75}{60} = 3.9375\ \text{kg/min}$$

Scavenging ratio = $R_{sc} = \dfrac{\text{Actual mass of air supplied}}{\text{ideal mass}}$

$$R_{SC} = \frac{3.9375}{4.51} = 0.873$$

Scavenging Efficiency = $1 - \exp(-R_{sc})$

$$= 1 - e^{-0.873} = 0.5823 \text{ or } 58.23\%$$

Trapping Efficiency = $\eta_{TR} = \dfrac{\eta_{SC}}{R_{SC}} = 0.5823/0.873 = 66.7\%$

Indicated power (IP) in (kW) = $\dfrac{P_{mi} LA\, Nn}{60}$

$$= \frac{500 \times 0.12 \times \frac{\pi}{4}(0.1)^2 \times 3500 \times 1}{60} = 27.475\ \text{kW}$$

Brake power (kW) = $\eta_m \times$ IP (kW)

$$= 0.85 \times 27.475$$

$$= 23.354\ \text{kW}$$

Brake thermal efficiency = $\dfrac{BP(kW) \times 3600}{\dot{m}_f\left(\frac{kg}{hr}\right) \times CV\left(\frac{kJ}{kg}\right)}$

$$= \frac{23.354 \times 3600}{15.75 \times 44000} = 0.1213 = 12.13\%$$

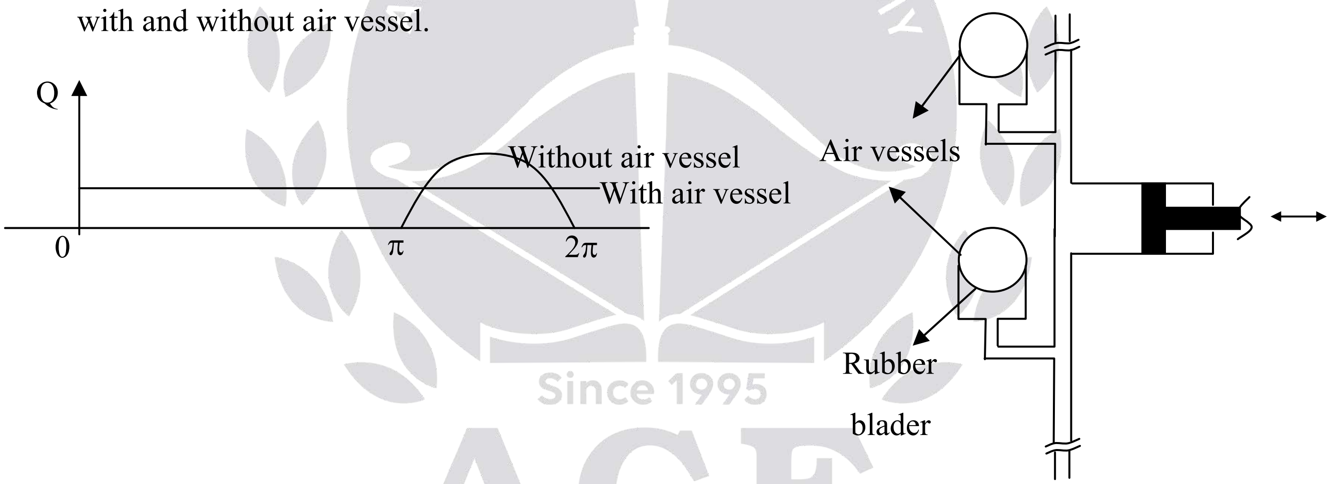
05.

(a) Reciprocating pump gives flucturating output and you want uniform or near uniform output from it. Suggest possible solutions or modifications with justifications and illustrations.

Sol: In order to have uniform or near uniform discharge, following methods can be used:

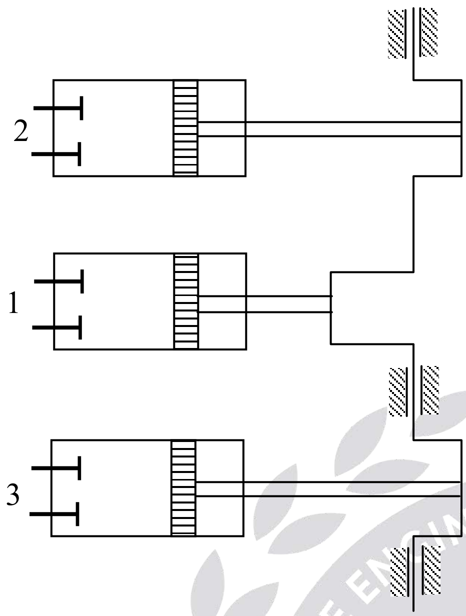
1. Use of air vessels:

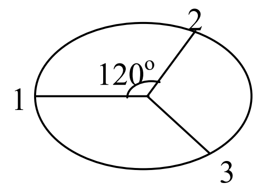
Air vessels are large closed chambers which are used to eliminate pulsation or fluctuations in discharge. Inside air vessel there is a flexible rubber bladder containing pressurized air. The bladder can compress or expand depending upon the liquid pressure inside the air vessels. The air vessels act like temporary storage reservoir for the liquid. When the discharge supplied by the pump is above average discharge then air vessel stores the extra quantity of liquid. As soon as the discharge supplied by the reciprocating pump falls below average discharge the air vessel supplies the stored liquid towards delivery pipe. In this way air vessels maintain nearly uniform discharge. Following graph indicates variation of discharge with crank angle in delivery pipe with and without air vessel.



2. Use of multiplex (multi cylinder) Pumps:

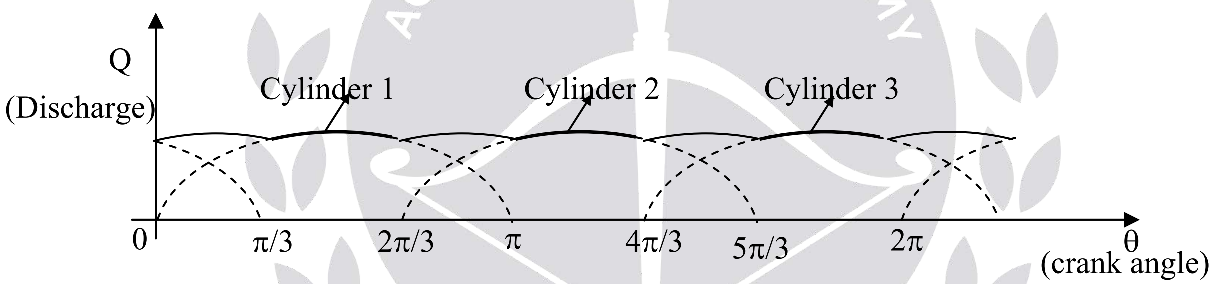
For single cylinder reciprocating pump the discharge is pulsating and intermittent. If we use two cylinders the discharge is continuous but still pulsating. However, if three or more cylinders are used then the discharge becomes continuous and nearly uniform as illustrated in the following diagram.





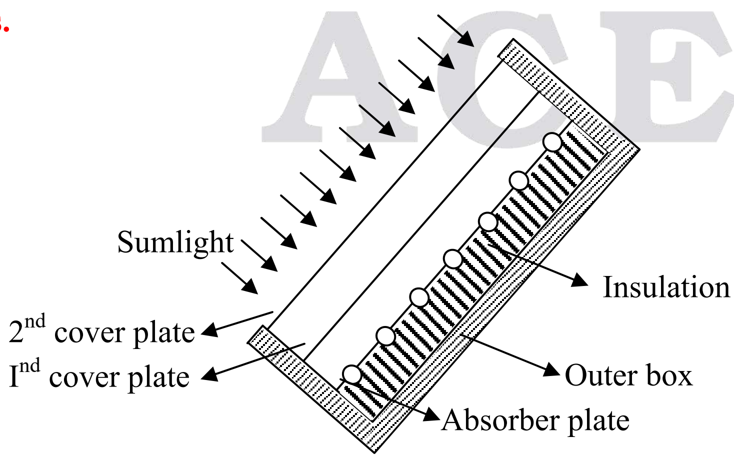
Crankshafts make

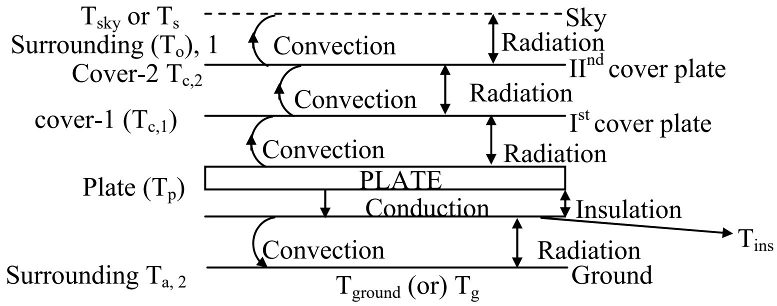
120° with each other



(b) Draw an equivalent thermal-circuit diagram of a liquid flat plate collector with two glass covers considering the thermal resistance of glass covers. Neglect thermal resistance from sides.

Sol:





Thermal Circuit Diagram

$R_{cond} \rightarrow$ Conductive Resistance of insulation below the plate

$h_{W,1} \rightarrow$ Convective HT coefficient between insulation and ground

$h_{W,2} \rightarrow$ Convective HT coefficient between second cover and ambient

$h_P - c_1 \rightarrow$ Convective HT coefficient between plate and 1st cover

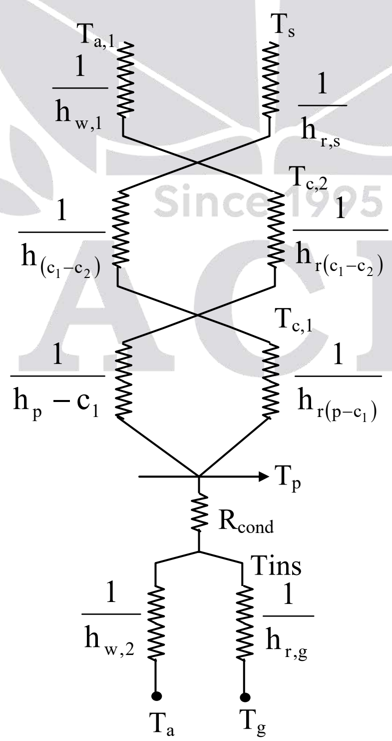
$h_p - c_2 \rightarrow$ Convective HT coefficient between 1st cover and 2nd cover

$h_{r,g} \rightarrow$ Equivalent radiation HT coefficient between insulation and ground

$h_{r,s} \rightarrow$ Equivalent radiation HT coefficient between 2nd cover and sky

$h_{r(P-C_1)} \rightarrow$ Equivalent radiation HT coefficient between plate and 1st cover

$h_{r(C_1-C_2)} \rightarrow$ Equivalent radiation HT coefficient between 1st cover and 2nd cover

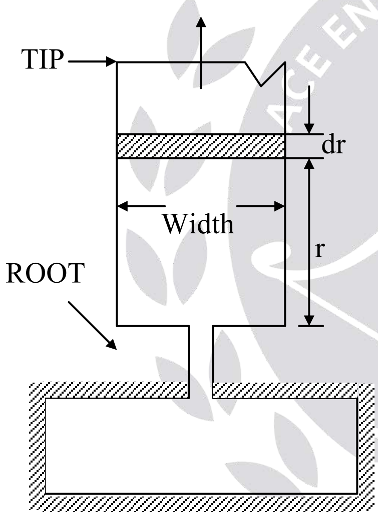


(c) What are the different types of stresses induced in stream turbine blades? How are tthese computed and designed to safely bear them?

Sol: Blades are held at one end by the rotor while other end is free. They act as cantilevers with distributed load of steam on them. They are subjected to bending stresses. As they rotate at high speeds they are subjected to centrifugal stresses also. As blade height increases both bending and centrifugal stresses also increase. Due to these stresses blade height and blade diameter get restricted. The maximum blade velocity is also limited depending on the materials of blade which is about 350-400 m/sec.

Centrifugal stresses are a function of mass of material in the blade, blade length and speed.

The component of centrifugal force acting radially outward exerts a tensile stress at root. Sufficient cross sectional area must be provided in the blade at root and material capable of with standing the stress without fatigue must also be provided.



Centrifugal force on a element dr at radius ‘r’ $dF = (\gamma\ a\ dr)\ \omega^2 r$

γ = specific weight of blade material kg/m^3

a = blade cross sectional area m^2

ω = Angular velocity rad/sec

Total centrifugal force exerted at blade root is

$$F_C = \int_{r_1}^{r_2} \gamma a dr \omega^2 r = \frac{\gamma a \omega^2}{2}\left(r_2^2 - r_1^2\right)$$

r_2 = tip radius

r_1 = root radius

$$F_C = \frac{\gamma a \omega^2}{2\pi} A = \frac{\gamma a A}{2}\left(\frac{2\pi N}{60}\right)^2$$

A= Annular area = $\pi\left(r_2^2 - r_1^2\right)$

Centrifugal stress on blade root is = $S_c = \dfrac{F_c}{a}$

$$= \gamma\, A\left(\frac{N}{23.94}\right)^2$$

If blade is tapered the mass of material is reduced, thereby reducing centrifugal stress. As stress exerted at any section of blading decreases radially reaching a minimum near the tip, a constant cross sectional area is not required for strength. Hence, where centrifugal stresses are severe blade is tapered by decreasing both its thickness and width. Impulse blades are subjected to bending from centrifugal stress and tangential force exerted by the fluid. Reaction blades have an additional bending stress due to large axial thrust because of pressure drop which occurs in blades. All turbine blades may be subjected to bending because of vibrations. The total stress at a given point on turbine blade may be found by adding the centrifugal stress at that point to the bending stress.

As maximum blade velocity is limited to 350-400 m/sec

$(V_b)_{max}$ = 350 (say) = $\dfrac{\pi D_m N}{60}$

For a 2-pole 50 hertz alternatior

$N = \dfrac{120f}{P} = \dfrac{120\times 50}{2}$ = 3000 rpm

$(D_m)_{max} = \dfrac{350\times 60}{\pi\times 3000} = 2.23\,\text{m}$

For straight blades the maximum blade height is about 20% of mean blade ring diameter.

$\dfrac{(h_b)}{D_{max}} = 0.2$

Due to flow requirements if it is necessary to exceed this the blades may be tapered or twisted thereby reducing both bending and centrifugal stresses.

In which case,

$\dfrac{h_b}{D_{max}} = 0.3$

For twisted or tapered blade

$h_b = 0.3 \times 2.23 = 0.67$ m

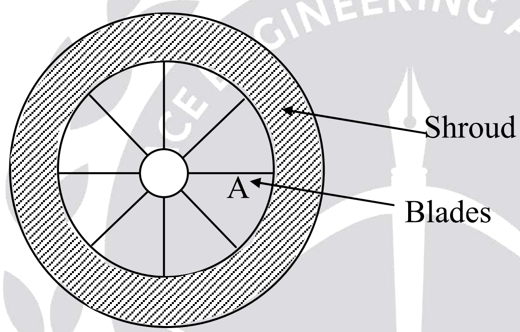
For straight blades

$h_b = 0.2 \times 2.23 = 0.446$ m

Bending is caused due to vibration which result in bending stress.

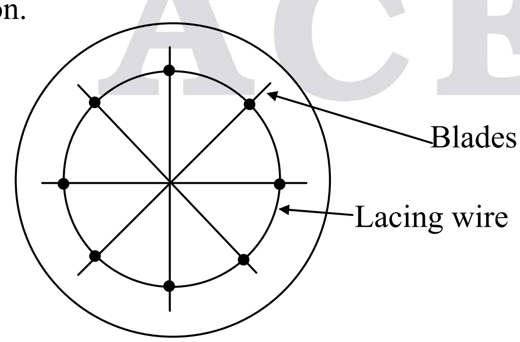
As you go from high pressure side to low pressure side the blade height goes on increasing.

The first stage height of blades is only 20 mm. Hence in high pressure stages to reduce vibrations a shroud is placed. Shroud is a band around the periphery of blade tips in order to stiffen the blades and prevent spillage of steam over the tips.



Important for reaction turbines where a pressure difference exists across the moving blades. Shrouding is beneficial in high pressure impulse blades with partial admission which are subjected to vibration. The weight of shroud adds considerably to centrifugal stresses at blade root.

In longer blades shrounding causes heavy vibrations. Hence in longer blades we use lacing wires to keep blades in alignment and to add to stiffness. Lacing wire disturbs the flow pattern and may cause some vibration.



(d) What are regenerative fuel cells? Briefly describe its working with a properly labelled diagram.

Sol: Regenrative fuel cell

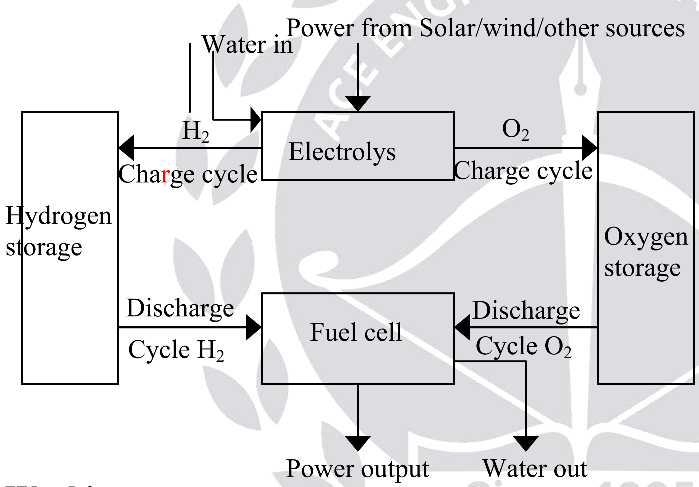
A regenerative fuel cell is one in which the fuel cell product (water) is recovered into the reactants (hydrogen and oxygen) by the several possible methods like thermal, chemical, photochemical, electrical, radio chemical etc.

There are two stages in a regenerative fuel cell.

(a) Conversion of fuel cell reactants into products while producing electrical energy.

(b) Reconversion of fuel cell products into reactants

Labelled Diagram:



Working

(i) In electrolyser, electrical energy plus some heat drawn from the environment is used to split water into H_2 and O_2.

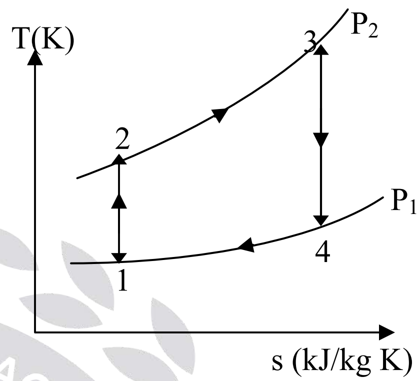
$$H_2O + \text{electricity} + \text{heat} \rightarrow H_2 + \frac{1}{2} O_2$$

(ii) In the fuel cell the reverse reaction takes place as hydrogen is recombined with oxygen with the production of electricity, water and heat.

$$H_2 + 1/2\, O_2 \rightarrow H_2\, O + \text{Electricity} + \text{heat}$$

(e) A gas turbine power plant operating on an ideal Brayton cycle takes in air at the initial conditions of 5°C and 1.03 bar. The pressure ratio is 7 and the maximum temperature is 816°C. Determine the work ratio nad the mass flow rate of air for a net output of 3750 kW. For air C_p = 1.005 kJ/kg K and γ = 1.4.

Sol: Given data:



Gas turbine power plant-Brayton cycle

Initial temperature (T_1) = 5 + 273 = 278 K

Initial pressure (P_1) = 1.03 bar

= 103 kPa

Pressure ratio (r_p) = 7

Maximum temperature (T_3) = 816 + 273

= 1089 k

Work ratio = ?

Mass flow rate of air $(\dot{m}_a)$ = ?

Net power output, $\dot{W}_{net} = 3750\,\text{kW}$

C_P = 1.005 kJ/kgK and γ = 1.4

Assumptions

(a) Air is working material and is treated as ideal gas

(b) Changes in KE and PE are neglected

(c) All the processes are occurring in a steady flow devices.

(d) Frictional pressure drop is negligible

(e) All the processes are reversible

Process: 1 – 2 Isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_P)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \times (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 278(7)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 484.73 \text{ K}$$

Process: 3 – 4 Isentropic expansion

$$\frac{T_3}{T_4} = (r_P)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1089}{(7)^{\frac{0.4}{1.4}}} = 624.55 \text{ k}$$

Compressor work per kg of air, $W_C = C_p (T_2 - T_1)$

$= 1.005 (484.73 - 278) = 207.76$ kJ/kg

Turbine work per kg of air, $W_T = C_p (T_3 - T_4)$

$= 1.005 (1089 - 624.55) = 466.766$ kJ/kg

Net Work (W_{net}) = $W_T - W_C = 466.766 - 207.76 = 259$ kJ/kg

$$\text{Work ratio (WR)} = \frac{W_{net}}{W_T} = \frac{259}{466.766} = 0.5548$$

Net power = $\dot{m}_a \times$ Network

$3750 = \dot{m}_a \times 259$

$$\therefore \text{Mass flow rate of air } (\dot{m}_a) = \frac{3750}{259} = 14.478 \text{ kg/sec}$$

06. (a)

An axial compressor stage has the following data:

Temperature and pressure at entry = 300 K, 1.0 bar

Degree of reaction = 5%

Mean blade ring diameter = 36 cm

Rotational speed = 18000 rpm

Blade height at entry = 6 cm

Air angles at rotor and stator exit = 25°

Axial velocity = 180 m/s

Work done factor = 0.88

Stage efficiency = 85%

Mechanical efficiency = 96.7%

Determine:

(i) Air angles at rotor and stator entry

(ii) Mass flow rate of air

(iii) Power required to drive the compressor

(iv) Loading coefficient

(v) Pressure ratio developed by the stage.

Sol: **Given Data:**

Type of compressor……..Axial Flow

Initial Temperature (T_1) = 300^{o}C

Initial pressure (P_1) = 1 bar = 100 kPa

Degree of reaction = 50%

Mean blade ring diameter (D) = 36 cm = 0.36 m

Rotational speed (N) = 18000 rpm

Blade height at entry (h_1) = 6 cm = 0.06 m

Air angles at rotor and stator exit = 25^o, $\alpha = \phi = 25^o$

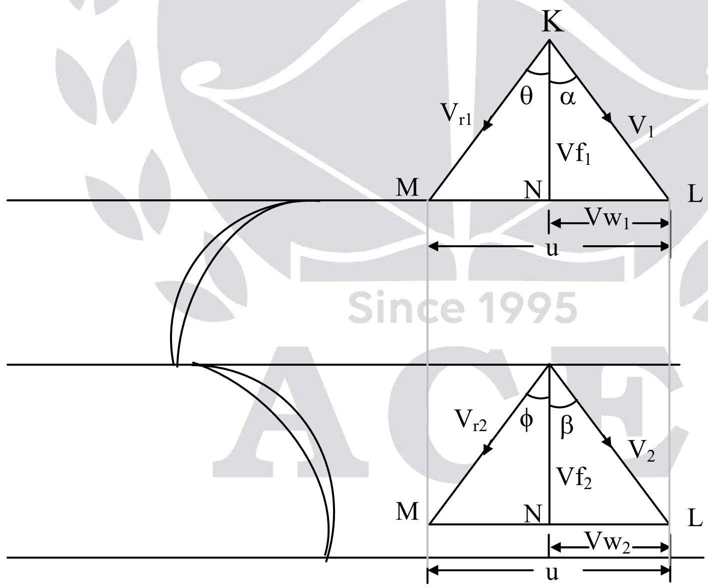
Axial velocity (V_f) = 180 m/s

Work done factor (ϕ_w) = 0.88

Stage efficiency (η_s) = 0.85

Mena blade speed (u) = $\frac{\pi DN}{60} = \frac{\pi(0.36)(18000)}{60} = 339.292$ m/s

$Vf_1 = Vf_2 = Vf = 180$ m/sec



(i) Air angles at rotor and stator entry i.e θ = ? & β = ?

From triangle kNL

$\tan\alpha = \frac{Vw_1}{Vf_1} \Rightarrow Vw_1 = Vf_1 \tan\alpha = 180 \tan(25^o) = 83.935$ m/sec

$MN = u - Vw_1 = 339.292 - 83.935 = 255.357$ m/sec

$$\tan\theta = \frac{MN}{Vf_1} \Rightarrow \theta = \tan^{-1}\left(\frac{255.357}{180}\right) = 54.82^o$$

Since degree of reaction is 50%

$$\theta = \beta = 54.82^o$$

(b) Mass flow rate of air

$$\rho_1 = \frac{P_1}{RT_1} = \frac{1.0\times 10^5}{287\times 300} = 1.161 \text{ kg/m}^3$$

Mass flow rate $(\dot{m}) = \rho\, AV_f = \rho\, \pi dh\, (V_f)$

$$= 1.161 \times \pi\,(0.36 \times 0.06) \times 180$$

$$= 14.18 \text{ kg/sec}$$

(c) Specific work

$w = \phi_w\, u\, V_f\, (\tan\theta - \tan\phi)$

$= 0.88 \times 339.292 \times 180\, (\tan 54.82^o - \tan 25^o)$

$= 51182.08$ J/kg

$$\text{Power} = \frac{W}{\eta_{mech}} = \frac{\dot{m}_a \times w}{\eta_{mech}} = \frac{14.18\times 51182.08}{0.967}$$

$$= 750.529 \text{ kW}$$

(d) Loading coefficient $(\psi) = \dfrac{w}{u^2} = \dfrac{51182.08}{(339.292)^2} = 0.444$

(e) Pressure ratio per stage

$w = C_p\,(T_2 - T_1)/\eta_{satge}$

$$\frac{51182.08}{1000} = \frac{1.005(T_2 - 300)}{0.85}$$

$T_2 = 343.28$ K

$$\frac{T_2}{T_1} = (r_P)^{\frac{\gamma-1}{\gamma}}$$

$$r_P = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{343.28}{300}\right)^{\frac{1.4}{0.4}} r$$

$r_p = 1.6$

(b) A centrifugal pump with backward curved vanes is running at 1200 rpm against a head of 35 m. The discharge through the pump is 0.28 m^3/s. If the blade angle at outlet is 30^o, flow velocity at outlet is 4 m/s and hydraulic or manometric efficiency is 0.85, determine diameter and width of impeller at outlet. Draw velocity triangles.

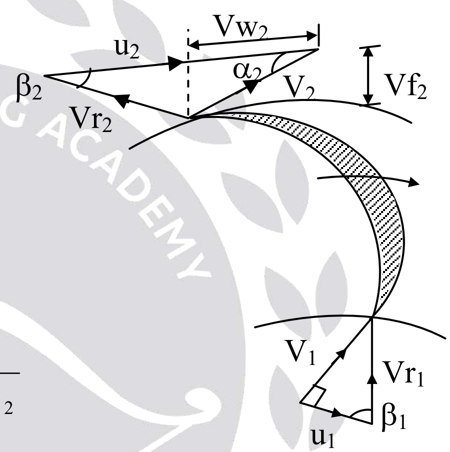
Sol: **Given data:**

$N = 1200$ rpm $\qquad H_m = 35$ m

$\beta_2 = 30^o$ $\qquad Vf_2 = 4$ m/s

$\eta_{mano} = 0.85$ $\qquad Q = 0.28$ m^3/s

The manometric efficiency is given by,



$\alpha_1 = 90^o$
$Vw_1 = 0$
$Vf_1 = V_1$

$$\eta_{mano} = \frac{gH_m}{u_2 Vw_2}$$

$$0.85 = \frac{9.81 \times 35}{u_2 . Vw_2}$$

$$\therefore\ u_2 = \frac{403.9}{Vw_2} \ \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From the exit velocity triangle, $\tan\beta_2 = \dfrac{Vf_2}{u_2 - Vw_2}$

i.e $\tan(30^o)\ \dfrac{4}{\left(u_2 - \dfrac{403.9}{u_2}\right)}$

$\Rightarrow u_2 = 23.86$ m/s

$$\therefore\ Vw_2 = \frac{403.9}{23.86} = 16.93 \text{ m/s}$$

But $u_2 = \dfrac{\pi D_2 N}{60}$

$$\therefore\ 23.86 = \frac{\pi \times D_2 \times 1200}{60}$$

$\therefore D_2 = 0.38$ m

The discharge through the impeller is given by,

$Q = \pi D_2 B_2 Vf_2$

i.e $0.28 = \pi \times 0.38 \times B_2 \times 4$

$\therefore B_2 = 0.059$ m

(c) (i) How is biogas production related to sustainable waste management?
(ii) What is meant by biogas enrichment?
(iii) Explain the working of a power generation set-up using municipal organic waste.

Sol: (i) Biogas is produced by Anaerobic digestion. It is the process of converting the organic wastes into usable products including biogas, Renewabe Natural Gas (RNG) as well as valuable organic fertilizer and compost.

These biogas system turn a waste management issue into a revenue opportunity for India's farm, dairies, food processing and wastewater treatment industries.

(ii) Biogas-Enrichment

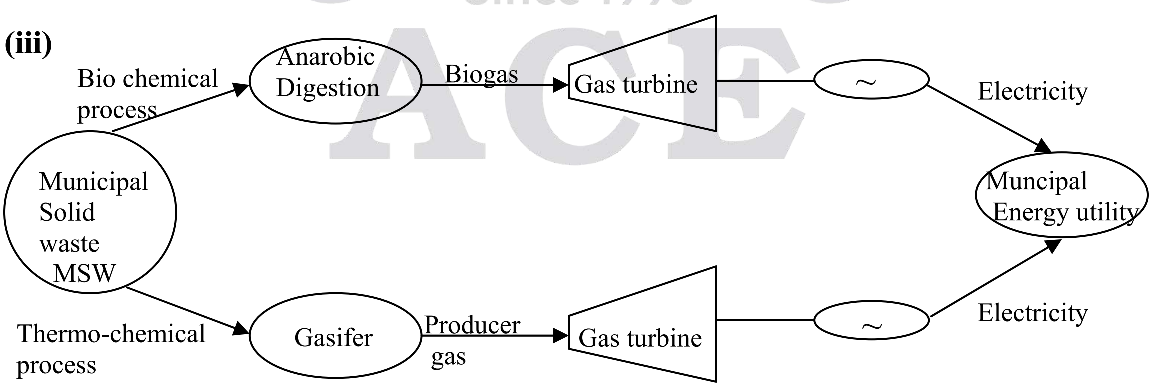
The composition of biogas is

CH_4 (55 to 65%); CO_2 (30 to 40%); H_2, H_2S, N_2 (< 10%)

Enrichment method of biogas is the process of removing unwanted gases (CO_2, H_2S) from biogas to increase the calorific value so that it is more economical to compress and transport to longer distribution or move to other areas.

- The removal of CO_2 and H_2S can increase the percentage of biomethane in biogas.
- **Hydrogen sulphide (H_2S):** It is a corrosive compound that attacks on the tank in which biogas is stored.
  It's concentration must be below 400 – 500 ppm
- **CO_2** → This product is not a real impurity but it is necessary to separate it if we need to obtain more concentrated mehane gas

(iii)



- MSW can be converted into biogas or producer gas by biochemical and thermo chemical methods.

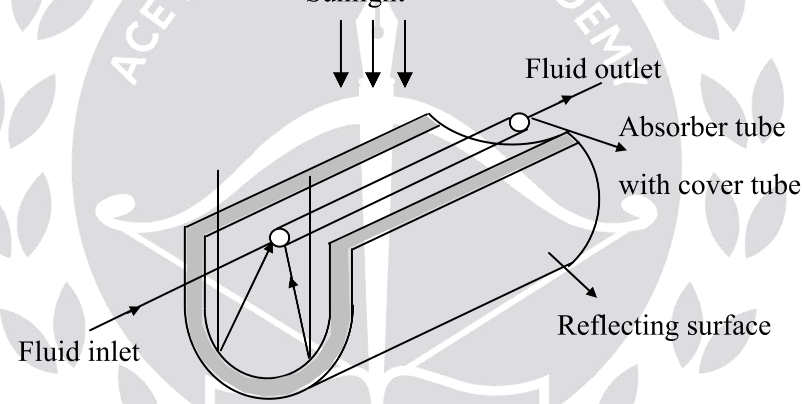
- After cleaning those gases, they are ready to use as a fuel and can replace coal in thermal power plant or combustible gases in gas turbines.
- In the field of thermo chemical conversion incineration is a widely used technology.

07.(a)

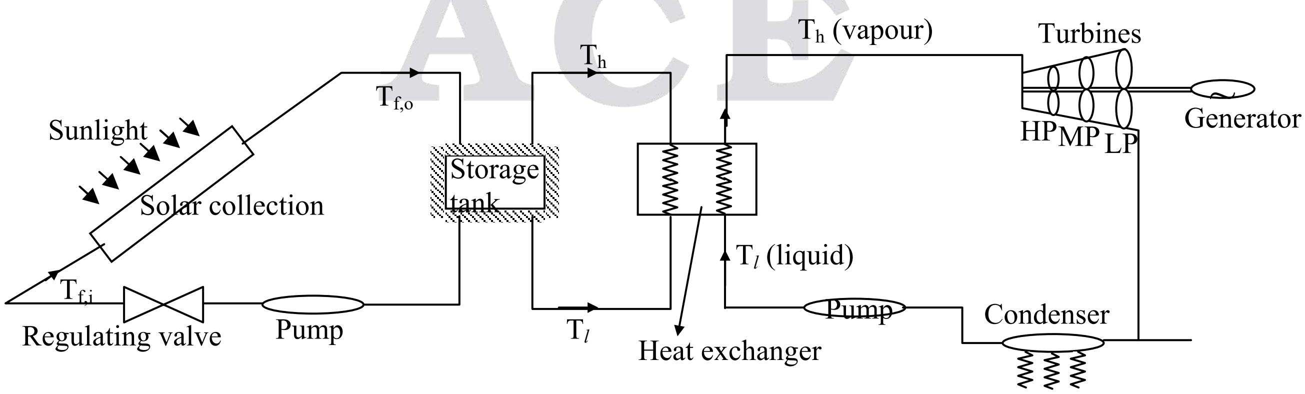
Parabolic trough collector based solar thermal power plants with thermal storage are becoming popular as they can generate power even during off-sunshine hours. Explain the working of such a plant with neat sketch. Also explain the basic thermodynamic cycle, on which such plant operate, using T-s plot.

07. (a)

Sol: Parabolic trough collector / compound parabolic collector



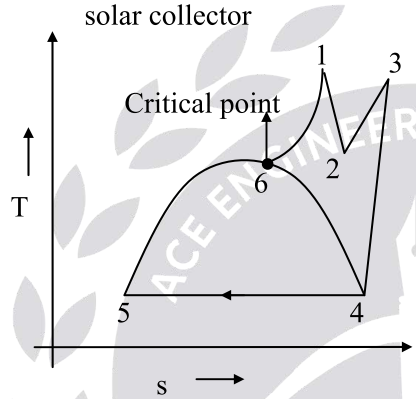
By proper tracking sun's rays are focussed on the absorber tube by which a fluid can be heated.



- It's working is like a ordinary power plant except heat is not supplied by burning fossil fuel but it is supplied from solar collector.
- It can work over subcritical Rankine cycle or the Regenerative open air Brayton cycle or on a combination of both
- In the layout it was working over Rankine cycle (subcritical)

Water → Steam → Rotate the turbine → Rotate the generator → Electricity

↑ Heat input from solar collector (to Steam)



1-2 → High pressure turbine expansion

2-3 → Reheat

3-4 → Low pressure turbine expansion

4-5 → Condenser

5-6 → Feed water heating-pumping

6-1 → Boiler

(b) A Kaplan turbine generates 12 MW of shaft power under 20 m head. If inlet guide vane angle is 30° and diameters of runner and hub are taken to be 6 m and 4 m respectively, determine (a) runner vane angles (inlet and outlet) (b) guide vane angle at outlet, and (c) speed of runner. The absolute velocity at the outlet should be kept minimum. Assume hydraulic efficiency as 80% and overall efficiency as 75%. Draw velocity triangles.

Sol: Given Data:

S.P = 12 MW, $\eta_h = 80\%$

H = 20 m $\eta_o = 75\%$

$\alpha_1 = 30^o$

$D_o = 6$ m

$D_i = 4$ m

$S.P = \eta_o \rho g Q H$

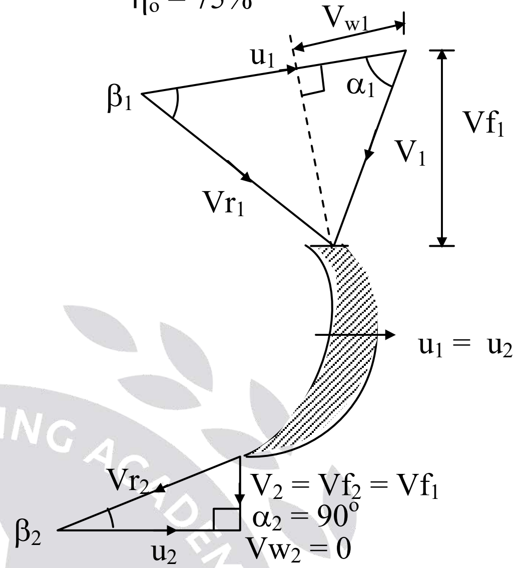
$12 \times 10^6 = 0.75 \times 9810 \times Q \times 20$

$\Rightarrow Q = 81.55$ m³/sec

For Kaplan turbine, the velocity of flow at inlet and exit remains same and it is given by,

$$Vf_1 = Vf_2 = \frac{4Q}{\pi\left(D_0^2 - D_i^2\right)} = \frac{4\times 81.55}{\pi\left(6^2-4^2\right)} = 5.19 \text{ m/s}$$

$\eta_h = 80\%$
$\eta_o = 75\%$



From inlet velocity triangle

$$\tan\alpha_1 = \frac{Vf_1}{Vw_1}$$

$$\therefore\ Vw_1 = \frac{Vf_1}{\tan\alpha_1} = \frac{5.19}{\tan 30} = 8.99 \text{ m/sec}$$

The hydraulic efficiency is given by

$$\eta_h = \frac{u_1 Vw_1}{gH}$$

$$\therefore\ 0.80 = \frac{u_1\times 8.99}{9.81\times 20}$$

$\therefore$ $u_1 = 26.2$ m/s

For Kaplan turbine the velocity of whirl at exit (Vw_2) is zero for minimum absolute velocity at exit condition. Hence, the angle α_2 (Guide vane angle at exit) is 90^o.

$\therefore$ $\alpha_2 = 90 \rightarrow$ (Ans)

For Kaplan turbine, velocity triangles are different at each radial section. In the question it is not clearly mentioned the location where velocity triangles are to be considered. Hence we assume that the velocity triangles are drawn at mean blade radius.

$\therefore$ The mean diameter (D_m) is given by,

$$D_m = \frac{D_i + D_o}{2} = \frac{6+4}{2} = 5\,\text{m}$$

$$\text{Now, } u_1 = \frac{\pi D_m N}{60}$$

$$\therefore\ 26.2 = \frac{\pi\times 5\times N}{60}$$

$\Rightarrow$ N = 100.1 rpm

From inlet velocity triangle

$$\tan(\beta_1) = \frac{Vf_1}{u_1 - Vw_1} = \frac{5.19}{26.2-8.99}$$

$\Rightarrow$ $\beta_1 = 16.8^o$

Similarly from exit velocity triangle

$$\tan\beta_2 = \frac{Vf_2}{u_2}$$

But for Kaplan turbine $Vf_1 = Vf_2 = \dfrac{4Q}{\pi\left(D_o^2 - D_i^2\right)}$

$$\& \ u_1 = u_2 = \frac{\pi D_m N}{60}$$

$$\therefore \tan\beta_2 = \frac{Vf_1}{u_1} \qquad \{\because u_1 = u_2 \ \& \ Vf_1 = Vf_2\}$$

$$= \frac{5.19}{26.2} \Rightarrow \beta_2 = 11.2^o$$

(c)

(i) If the circulation ratio is 12.5, find the dryness fraction at the top of a riser tube of a boiler.

Sol: Circulation ratio $= \dfrac{\text{Mass of water circulation}}{\text{Mass of steam produced}}$

$$12.5 = \frac{1}{x}$$

$$x = \frac{1}{12.5} = 0.08$$

(ii) Feed water enters the economizer at 170°C and leaves at 336.75°C whereas flue gas enters the economizer at 815°C and leaves at 450°C, If overall heat transfer coefficient is 70 W/m^2 K and het transfer through economizer is 511134 kW, determine the outside surface area.

Sol: **Given data:**

Feed water entering temperature (t_{w1}) = 170°C

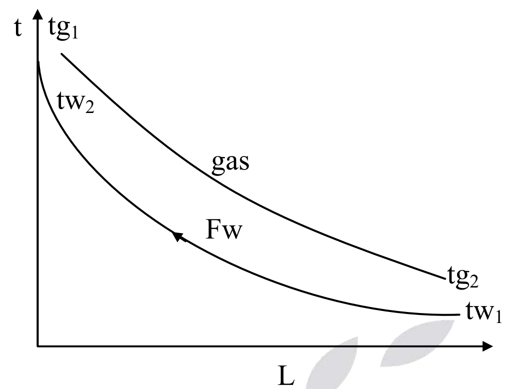
Feed water leaving temperature (t_{w2}) = 336.75°C

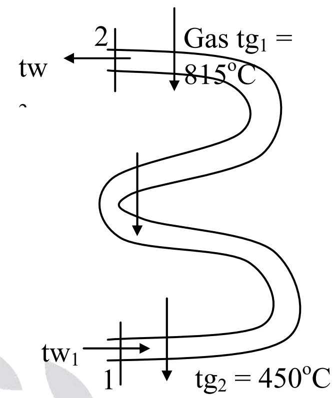
Gas entering temperature (tg_1) = 815°C

Gas leaving temperature (tg_2) = 450°C

Overall heat transfer coefficient (U) = 70 W/m^2 K

Heat transfer in economiser $(\dot{Q})$ = 511134 kW





$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{478.25 - 280}{\ell n\left(\frac{478.25}{280}\right)} = 370.32^{\circ}\text{C}$$

Rate of heat transfer in economiser

$$\dot{Q} = \dot{m}_g C_{pg}(tg_1 - tg_2) = UA_o(\text{LMTD})$$

$$511134 \times 10^3 = 70\,(A_o)\,(370.32)$$

$$\therefore \text{ Outside surface area } (A_o) = \frac{511134 \times 10^3}{70 \times 370.32}$$

$$A_o = 19717.85\ \text{m}^2$$

(iii) A surface condenser receives 250 t/h of steam at 40°C with 12% moisture. The cooling water enters at 32°C and leaves at 38°C. The pressure inside the condenser is found to be 0.078 bar. The velocity of circulating water is 1.8 m/s. The condenser tubes are of 25.4 mm outer diameter and 1.25 mm thickness. Taking overall heat transfer coefficient as 2600 W/m^2 K, determine the rate of flow of cooling water, the rate of air leakage into the condenser shell, the length of tubes and number of tubes. At 40°C, h_{fg} = 2407 kJ/kg, p_{sat} = 0.07375 bar, v_f = 0.001008 m^3/kg and v_{fg} = 19.544 m^3 / kg.

Sol: Mass of steam entering to condenser

$$(\dot{m}_s) = 250\,\text{t/hr} = \frac{250 \times 1000}{3600}\ \text{kg/s}$$

Steam entering temperature (t_{sat}) = 40°C

Dryness fraction of steam (x) = 1 – 0.12 = 0.88

Cooling water inlet temperature (t_{c1}) = 32°C

Cooling water outlet temperature (t_{c2}) = 38°C

Pressure inside condenser (P_c) = 0.078 bar

Velocity of circulating water (V) = 1.8 m/sec

Outside diameter of condenser (d_o) = 25.4 mm

Thickness of condenser tube (t) = 1.25 mm

Overall heat transfer coefficient (U_o) = 2600 W/m^{2}K

Rate of cooling water required $(\dot{m}_w)$ = ?

Rate of air leaving into condenser $(\dot{m}_a)$ = ?

Length of the condenser tubes (l) = ?

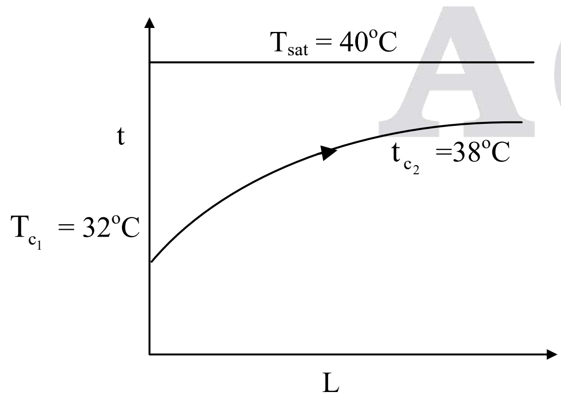
No. of condenser tubes (n) = ?

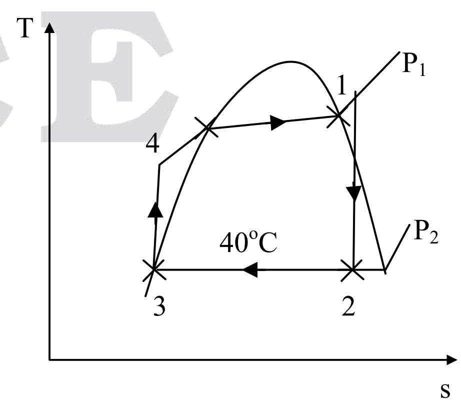
At 40°C, h_{fg} = 2407 kJ/kg

P_{sat} = 0.07375 bar

v_f = 0.001008 m^3/kg

v_{fg} = 19.544 m^3/kg





$h_2 = h_{f2} + x_2 h_{fg2}$ $\qquad\qquad$ $h_3 = h_{f2}$

$h_2 - h_3 = h_{f2} + x_2 h_{fg2} - h_{f2}$

$= x_2 h_{fg2} = 0.88 \times 2407 = 2118.16$ kJ/kg

By energy balance

$$\dot{m}_s(h_2 - h_3) = \dot{m}_c C_{PC}(t_{c_2} - t_{c_1})$$

$$\frac{250\times1000}{3600}\times 2118.16 = \dot{m}_c(4.187)(38-32)$$

$\dot{m}_c = 5855.2$ kg/sec

Total pressure in condenser (P) = $P_{sat} + P_{air}$

$0.078 = 0.07375 + P_{air}$

$P_{air} = 0.078 - 0.07375$

$P_{air} = 0.078 - 0.07375$

$= 0.00425$ bar

$= 0.425$ kPa

$v_2 = v_{f2} + x_2 v_{fg2} = 0.001008 + 0.88\ (19.544)$

$= 17.2\ m^3/kg$

Now,

$$P_{air}\dot{m}_s v_2 = \dot{m}_a R_{air} T_{sat}$$

$$0.425\left(\frac{250\times1000}{3600}\right)\times 17.2 = \dot{m}_{air}(0.287\times313)$$

$\dot{m}_{air} = 5.651$ kg/s

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{8-2}{\ell n\left(\frac{8}{2}\right)} = 4.33^{o}C$$

$Q = U_oA_o\,(LMTD) = \dot{m}_s\left(h_2 - h_3\right)$

$2.6\,(A_o)\,(4.33) = \dfrac{250\times1000}{3600}\times 2118.16$

$A_o = 13066\ m^2$

$\dot{m}_c = n\left(\dfrac{\pi}{4}d_i^2\right)\rho_w\ Vel = 5855.2\ kg/sec$

$n\dfrac{\pi}{4}(25.4-2.5)^2\times10^{-6}\times1000\times1.8 = 5855.2$

No of tubes (n) = $\dfrac{5855.2\times4\times10^6}{524.41\times\pi\times1.8} = 7898$

Again $(A_o) = n\,\pi\,d_o\,l = 13066\ m^2$

$\therefore$ length of tubes (l) = $\dfrac{13066}{\pi\left(25.4\times10^{-3}\times7898\right)} = 20.73\ m$

08.

(a) **Wind is blowing at a speed of 12 m/s. It enters a turbine wheel at standard atmospheric pressure and 15°C. The turbine wheel has a cross-sectional area of 90 m². Determine the power of the incoming wind, theoretical maximum possible power available accoriding to Bet'z criterion and a reasonably attainable turbine power in kW assuming 40% efficiency of the turbine. Find out the torque if the turbine wheel rotates at 30 RPM. Also determine the axial thrust if the turbine were operating at maximum efficiency.**

Sol: $u_o = 12\ m/s$ $\qquad$ $P = 101325\ Pa$

$A_1 = 90\ m^2$ $\qquad$ $T = 288\ k$

$P = \rho RT$

$\rho = \dfrac{P}{RT} = \dfrac{101325}{287\times288} = 1.23\ kg/m^3$

Power of incoming wind (P_o)

$P_o = \dfrac{1}{2}\rho.A_1.u_0^3$

$= \dfrac{1}{2}\times1.23\times90\times(12)^3 = 95.6\ kW$

Theoritical Maximum power available for extraction by Bet'z (P_{max})

$P_{max} = C_{p,\,max} \,.\, P_o$

$C_{p,\,max} = 0.593$

$P_{max} = 0.593 \times 95.6 = 56.72$ kW

Reasonably attainable power ($\eta = 40\%$) (P_T)

$P_T = \eta .\, P_o$

$= 0.4 \times 95.6 = 38.24$ kW

Torque:

$$N = 30 \text{ RPM}, \ \sigma = \frac{2\pi N}{60}$$

Torque at maximum efficiency means Bet'z limit.

$T_{sh} = C_{T,\,max} \,.\, T_m$

$$= \frac{C_{p.max}}{\lambda} \times \frac{P_o \lambda}{\omega}$$

$$= \frac{0.593 \times 95.6}{\left(\frac{2\pi \times 30}{60}\right)}$$

$T_{sh} = 18.04$ kN-m

Axial thrust

Axial thrust at maximum efficiency $F_A = \frac{8}{9} F_{A,Max}$

$$= \frac{8}{9} . \frac{1}{2} \rho A_1 u_o^2$$

$$= \frac{8P_o}{9u_o}$$

$$F_A = \frac{8}{9} . \frac{95.6}{12} = 7.08 \text{ kN}$$

(b) The following data refers to a two-row velocity compounded impulse wheel which forms the first stage of a combination turbine:

Steam velocity at nozzle outlet : 630 m/s

Mean blade velocity : 125 m/s

Nozzle angle : 16^o

Outlet angle, first row of moving blades : 18°
Outlet angle, fixed guide blades : 22°
Outlet angle, second row of moving blades : 36°
Steam flow rate : 2-6 kg/s
The ratio of the relative velocity at outlet to that at inlet is 0.84 for all the blades, Calculate:

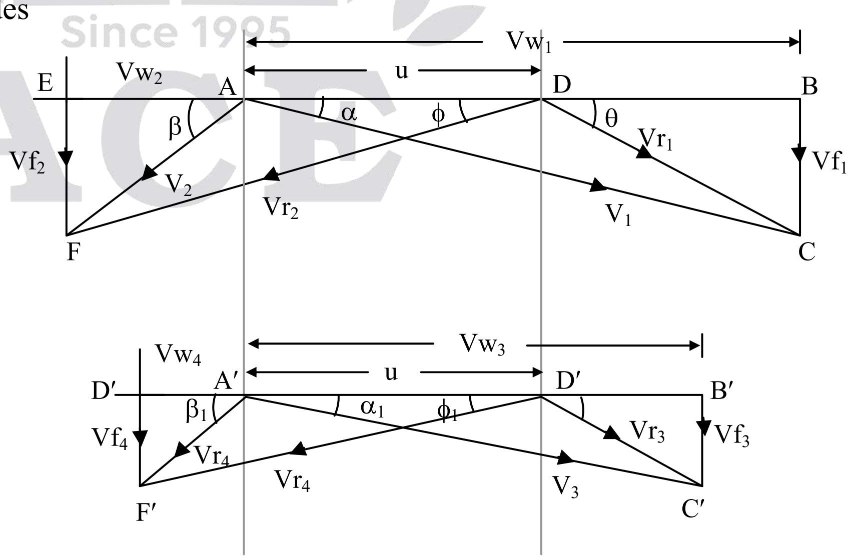
(i) The velocity of whirl
(ii) The tangential thrust on the blades
(iii) The axial thrust on the blades
(iv) The power developed
(v) The blade efficiency

Sol: Given data:

Stream velocity at inlet of turbine (V_1) = 630 m/sec
Blade speed (u) = 125 m/sec
Nozzle angle (α) = 16°
First row moving blade Outlet angles (ϕ) = 18°
Outlet angles of fixed blades (α_1) = 22°
Outlet angles of second row moving blades (ϕ_1) = 36°
Mass flow rate of steam $(\dot{m}_s)$ = 2.6 kg/sec
Friction factor (k) = 0.84

To find

i. Whirl velocity
ii. Tangential thrust on the blades
iii. Axial thrust on the blades
iv. Power developed
v. Blade efficiency



From Triangle

$\text{Sin}\ \alpha = \dfrac{Vf_1}{V_1} \Rightarrow Vf_1 = V_1 \sin\alpha = 630 \sin(16^o)$

$Vf_1 = 173.65$ m/sec

$\text{Cos}\ \alpha = \dfrac{Vw_1}{V_1} \Rightarrow Vw_1 = V_1 \cos\alpha = 630 \cos(16^o) = 605.6$ m/sec

$DB = Vw_1 - u = 605.6 - 125 = 480.6$ m/sec

$$\tan\theta = \frac{V_{f1}}{BD} \Rightarrow \theta = \tan^{-1}\left(\frac{173.65}{480.6}\right) = 19.86^o$$

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \Rightarrow Vr_1 = \frac{Vf_1}{\sin\theta} = \frac{173.65}{\sin(19.86)} = 511 \text{ m/sec}$$

$Vr_2 = kV_{r1} = 0.84\ (511) = 429.36$ m/sec

$$\sin\phi = \frac{V_{f_1}}{V_{r_2}} \Rightarrow Vf_2 = Vr_2 \sin(\phi)$$

$V_{f2} = 429.36 \sin(18^o)$

$= 132.68$ m/sec

$$\cos(\phi) = \frac{u + Vw_2}{Vr_2} \Rightarrow Vw_2 = Vr_2 \cos\phi - u$$

$Vw_2 = 429.36 \cos(18^o) - 125$

$= 283.345$ m/sec

$$\tan\beta = \frac{Vf_2}{Vw_2} \Rightarrow \beta = \tan^{-1}\left(\frac{Vf_2}{Vw_2}\right)$$

$$\beta = \tan^{-1}\left(\frac{132.68}{283.345}\right)$$

$= 25.09^o$

$V_3 = kV_2$

$= 0.84\ (312.87)$

$= 262.81$ m/sec

$V^2{}_2 = Vf^2{}_2 + Vw^2{}_2$

$V_2^2 = 132.68^2 + 283.345^2$

$V_2 = 312.87$ m/sec

$\sin\alpha_1 = \dfrac{Vf_3}{V_3} \Rightarrow Vf_3 = V_3 \sin\alpha_1 = 262.81 \sin(22^o)$

$= 98.45$ m/sec

$\cos\alpha_1 = \dfrac{Vw_3}{V_3} \Rightarrow Vw_3 = V_3 \cos\alpha_1 = 262.81 \cos(22^o)$

$Vw_3 = 243.67$ m/sec

$D'B' = Vw_3 - u = 243.67 - 125 = 118.67$ m/sec

$Vr_3^2 = Vf_3^2 + (D'B')^2 = (98.45)^2 + (118.67)^2$

$Vr_3 = 154.19$ m/sec

$\tan\theta_1 = \dfrac{Vf_3}{D'B'} \Rightarrow \theta_1 = \tan^{-1}\left(\dfrac{98.45}{118.67}\right) = 39.67^o$

$Vr_4 = kVr_3 = 0.84\,(154.19) = 129.51$ m/sec

$\cos\phi_1 = \dfrac{u + Vw_4}{Vr_4} \Rightarrow Vw_4 = Vr_4 \cos\phi_1 - u$

$Vw_4 = 129.51 \cos(36^o) - 125$

$= -20.22$ m/sec

$\sin\phi_1 = \dfrac{V_{f_4}}{V_{r_4}}$

$Vf_4 = 129.51\,(\sin 36^o) = 76.12$ m/sec

(i) Whirl velocity $(V_w) = Vw_1 + Vw_2 + Vw_3 + Vw_4$

$= 605.6 + 283.345 + 243.67 - 20.22$

$= 1112.39$ m/sec

(ii) Tangential thrust $(F) = \dot{m}_s (V_w)$

$= 2.6\,(1112.39)$

$= 2892.21$ N

(iii) Axial thrust $(Fa) = \dot{m}_s (Vf_1 - Vf_2 + Vf_3 - Vf_4)$

$= 2.6\,(173.65 - 132.68 + 98.45 - 76.12) = 164.58$ N

(iv) Power developed $(P) = F \times u$

$= 2892.2 \times 125$

$= 361525$ Watts

$= 361.252$ kW

(v) Blade efficiency $(\eta) = \dfrac{\text{Power}}{\frac{\text{KE}}{\text{s}}} = \dfrac{361.525}{515.97} \times 100 = 70.06\%$

where,

$$\frac{\text{KE}}{\text{s}} = \frac{1}{2}\dot{m}_s V_1^2$$

$$= \frac{1}{2}(8.6)(630)^2$$

$$= 515.97 \text{ kW}$$

(c) A Francis turbine is running at 500 rpm under a head of 190 nm. The blade angle at inlet is 50^o and guide vane angle at inlet is 20^o. If the peripheral speed of runner at inlet is 35 m/s and discharge is 9 m^3/s, determine.

(i) power developed by the runner

(ii) diameter and width of the runner at inlet, and

(iii) hydraulic efficiency of the turbine.

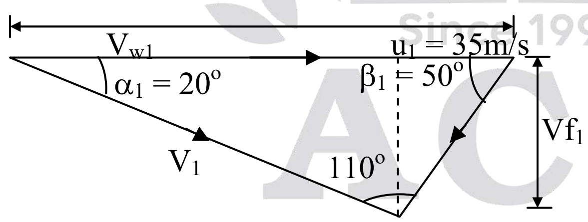
Draw velocity triangle at inlet.

Sol: Given data:

$N = 500$ rpm, $H = 190$ m, $\beta_1 = 50^o$, $\alpha_1 = 20^o$

$u_1 = 35$ m/sec, $Q = 9\ m^3$/sec

With respect to given data the velocity triangle at inlet can be drawn as



As $\beta_1 < 90$ the velocity triangle corresponds to fast runner Francis turbine.

From sine rule

$$\frac{u_1}{\sin(110)} = \frac{v_1}{\sin(50)} = \frac{v_{r_1}}{\sin(20)}$$

$$\therefore\ V_1 = 35 \times \frac{\sin(50^\circ)}{\sin(110^\circ)} = 28.53\,\text{m/sec}$$

$$V_{w_1} = V_1 \cos\alpha_1 = 28.53\cos(20) = 26.81\ \text{m/sec}$$

$Vf_1 = V_1 \sin\alpha_1 = 9.759$ m/s

The power developed by the runner is given by,

$R.P = \rho Q V w_1\, u_1$

$= 1000 \times 9 \times 26.81 \times 35$

$= 8.445$ MW

$$u_1 = \frac{\pi D_1 N}{60}$$

$$\therefore\ 35 = \frac{\pi \times D_1 \times 500}{60}$$

$\therefore\ D_1 = 1.337$ m

The discharge through the Francis turbine runner is given by,

$Q = \pi\, D_1\, B_1\, Vf_1$

$\therefore\ 8 = \pi \times 1.337 \times B_1 \times 9.759$

$\Rightarrow B_1 = 0.195$ m

The hydraulic efficiency is given by

$$\eta_h = \frac{u_1 V w_1}{gH} = \frac{35 \times 26.81}{9.81 \times 190}$$

$= 0.503$