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ESE-2020
(MAINS)

QUESTIONS WITH DETAILED SOLUTIONS

ELECTRICAL ENGINEERING

PAPER-II

All Queries related to ESE - 2020 MAINS Solutions are to be sent to the following email address hyderabad@aceenggacademy.com
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ELECTRICAL ENGINEERING

ESE _MAINS_2020_PAPER – II

Questions with Detailed Solutions

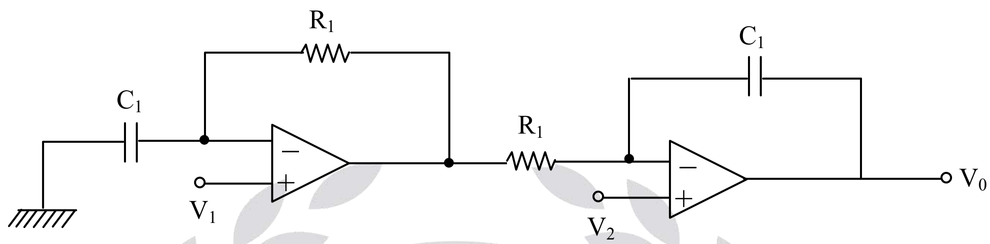
SUBJECT WISE WEIGHTAGE

S.No	NAME OF THE SUBJECT	Marks
01	Analog and Digital Electronics	52
02	Systems and signal processing	72
03	Control systems	64
04	Electrical Machines	104
05	Power Systems	104
06	Power Electronics	84

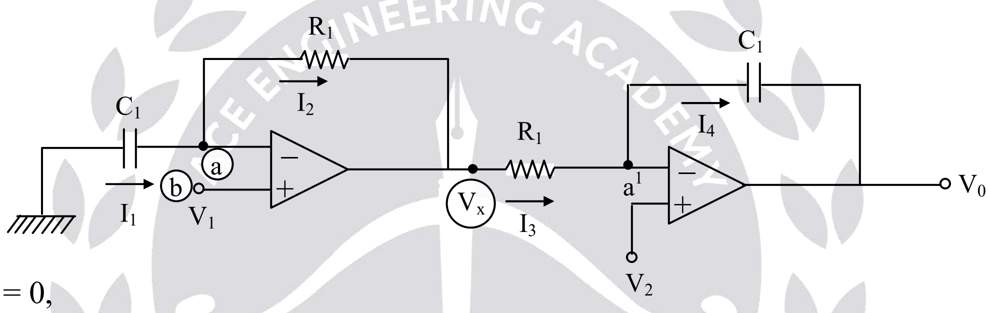
SECTION-A

Q.1

(a) For the circuit shown in the fiure below, derive the expression for output voltage and sketch the nature of the output when $V_2 = 10$ V and $V_1 = 5$ V. **(12 M)**



Sol:



Since $V_d = 0$,

$V_a = V_b = V_1$, $\quad V_a^1 = V_b^1 = V_2$

Apply KCL at ⓐ, $I_1 = I_2$

$$C_1 \frac{d}{dt}(0 - V_1) = \frac{V_1 - V_x}{R_1}$$

$$-R_1C_1 \frac{dV_1}{dt} = V_1 - V_x$$

$$V_x = V_1 + R_1C_1 \frac{dV_1}{dt} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Apply KCL at ⓐ, $I_3 = I_4$

$$\frac{V_x - V_a^1}{R_1} = C_1 . \frac{d}{dt}\left(V_a^1 - V_0\right)$$

$$\frac{V_x - V_2}{R_1C_1} = \frac{d}{dt} V_2 - \frac{dV_0}{dt}$$

Integrating on both sides

$$\int \frac{V_x}{R_1C_1}dt - \int \frac{V_2}{R_1C_1}dt = V_2 - V_0$$

$$V_0 = V_2 + \int \frac{V_2}{R_1C_1}dt - \int \frac{V_x}{R_1C_1}dt$$

$$= V_2 + \int \frac{V_2}{R_1C_1}dt - \int \frac{1}{R_1C_1}\left(V_1 + R_1C_1\frac{dV_1}{dt}\right)dt$$

$$= V_2 + \int \frac{V_2}{R_1C_1}dt - \int \frac{V_1}{R_1C_1}dt - \int \frac{dV_1}{dt}dt$$

$$V_0 = V_2 + \int \frac{V_2}{R_1C_1}dt - \int \frac{V_1}{R_1C_1}dt - V_1$$

$$V_0 = (V_2 - V_1) + \int\left(\frac{V_2 - V_1}{R_1C_1}\right)dt \quad \ldots\ldots\ldots\ldots(2)$$

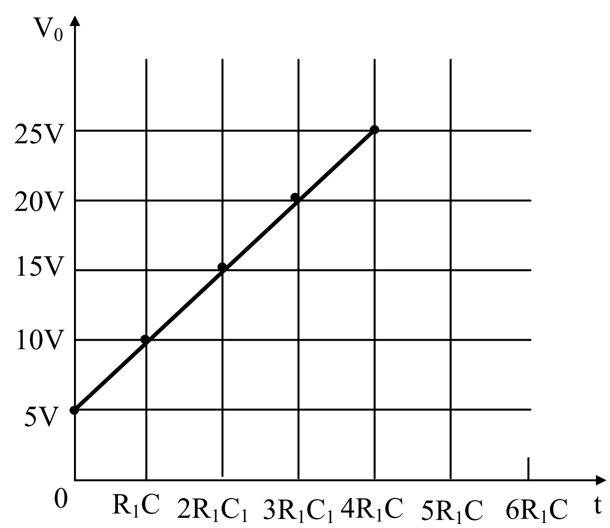
When $V_1 = 5V$, $V_2 = 10V$

$$V_0 = 5 + \int \frac{5}{R_1C_1}dt$$

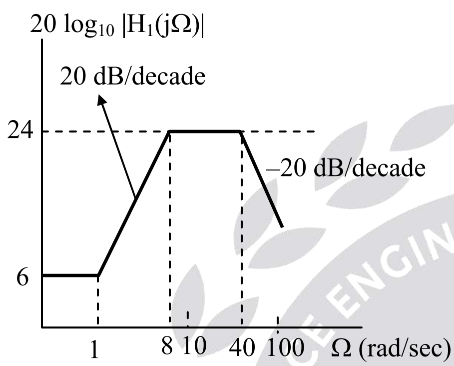
$$V_0 = 5 + 5\left(\frac{t}{R_1C_1}\right)$$

The output voltage varies linearly with time

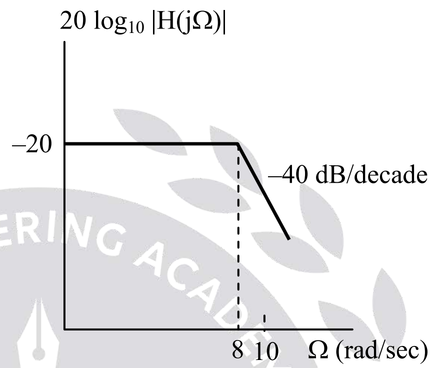
t	V_0
0	5V
R_1C_1	10V
$2R_1C_1$	15V
$3R_1C_1$	20V
$4R_1C_1$	25V



(b) A continuous LTIV system S with frequency response H(jΩ) is constructed by cascading two continuous-time LTIV system with frequency response $H_1(j\Omega)$ and $H_2(j\Omega)$, respectively. Figures a and b show the straight-line approximations of Bode magnitude plots of $H_1(j\Omega)$ and $H(j\Omega)$, respectively. Find $H_2(j\Omega)$. **(12 M)**



(a)



(b)

Sol: Given data

$H(j\Omega) = H_1(j\Omega)\, H_2s(j\Omega)$

$$\therefore\ H_2(j\Omega) = \frac{H(j\Omega)}{H_1(j\Omega)}$$

$$H_1(j\Omega) = \frac{k(1+s)}{\left(1+\frac{s}{8}\right)\left(1+\frac{s}{40}\right)}$$

20 log k = 6 dB

$\therefore$ k = 2

$$\therefore\ H_1(j\Omega) = \frac{2(1+s)}{\left(1+\frac{s}{8}\right)\left(1+\frac{s}{40}\right)}$$

$$H_1(j\Omega) = \frac{(8)(2)(40)(s+1)}{(s+8)(s+40)}$$

$$H(j\Omega) = \frac{k}{\left(1+\frac{s}{8}\right)^2}$$

20 log k = –20 dB

$\therefore$ k = 0.1

$$H(j\Omega) = \frac{0.1(8^2)}{(s+8)^2}$$

$$\therefore H_2(j\Omega) = \frac{H(j\Omega)}{H_1(j\Omega)}$$

$$= \frac{(0.1)(8^2)}{(s+8)^2} \div \frac{(8)(2)(40)(s+1)}{(s+8)(s+40)}$$

$$\therefore H_2(j\Omega) = \frac{(0.1)(8^2)}{(8)(2)(40)} \frac{(s+8)(s+40)}{(s+1)(s+8)^2}$$

$$H_2(j\Omega) = (10^{-2}) \frac{(s+40)}{(s+1)(s+8)}$$

(c) Consider a three-phase induction motor with the following parameters;

Number of poles	:	4
Supply frequency	:	50 Hz
Full load Speed	:	1470 rpm
Rotor resistance	:	0.12 Ω
Standstill reactance	:	1.12 Ω

Find the

(i) Slip for maximum torque

(ii) Ratio of maximum torque to full load torque.

Sol: Rotor Resistance R_2=0.12 ohms;

Stand still rotor Reactance X_{20} = 1.12 ohms,

Full load speed N_{rfl} = 1470 rpm,

Motor poles P = 4,

Supply frequency f = 50 Hz

Synchronous speed $N_s = \frac{120f}{4} = \frac{120\times 50}{4} = 1500$ rpm

Full load slip, $s_{fl} = \frac{N_s - N_{rfl}}{N_s} = \frac{1500-1470}{1500} = 0.02$

(i) Slip for maximum torque, $s_{Tmax} = \frac{R_2}{X_{20}} = \frac{0.12}{1.12} = 0.1071$

(ii) Ratio of maximum torque to full load torque

If the stator impedance is neglected, we have

$$T_{fl} = K\frac{s_{fl}E_{20}^2 R_2}{R_2^2 + (s_{fl}X_{20})^2}$$

$$T_{max} = K\frac{E_{20}^2}{2X_{20}}$$

$$\frac{T_{max}}{T_{fl}} = \frac{K\frac{E_{20}^2}{2X_{20}}}{K\frac{s_{fl}E_{20}^2 R_2}{R_2^2 + (s_{fl}X_{20})^2}}$$

$$\frac{T_{max}}{T_{fl}} = \frac{R_2^2 + (s_{fl}X_{20})^2}{2X_{20}R_2 s_{fl}}$$

Dividing Numerator and Denominator with X_{20}^2

$$\frac{T_{max}}{T_{fl}} = \frac{\frac{R_2^2 + (s_{fl}X_{20})^2}{X_{20}^2}}{\frac{2X_{20}R_2 s_{fl}}{X_{20}^2}}$$

$$\frac{T_{max}}{T_{fl}} = \frac{\left(\frac{R_2}{X_{20}}\right)^2 + s_{fl}^2}{\frac{2R_2 s_{fl}}{X_{20}}}$$

$$\frac{T_{max}}{T_{fl}} = \frac{\left(\frac{R_2}{X_{20}}\right)^2 + s_{fl}^2}{2\left(\frac{R_2}{X_{20}}\right)s_{fl}}$$

We know $\frac{R_2}{X_{20}} = s_{T\,max}$

$$\frac{T_{max}}{T_{fl}} = \frac{s_{T\,max}^2 + s_{fl}^2}{2s_{T\,max}s_{fl}}$$

T_{fl} is the full load torque and T_{max} is the maximum torque.

$$\frac{T_{max}}{T_{fl}} = \frac{0.10714^2 + 0.02^2}{2\times 0.10714\times 0.02} = 2.7716$$

(d) (i) What is Smart Grid? **(4 M)**

(ii) Compared to Supervisory Control and Data Acquisition (SCADA) system, what are the advantages of Phasor Measurement Unit (PMU)? **(4 M)**

(iii) Explain operation of PMU with a neat diagram **(4 M)**

Sol: **(i) Smart grid definition:**

Smart grid is an electrical power grid associated with automation, communication and IT systems which can control and regulate the power flow from generation level to consumer level.

Smart grid tries to match the generation to load in real time by optimizing the cost of power production on its own.

Smart grid can be called as intelligent grid which can take certain decisions on its own like diverting the power flow paths and curtailing the loads as well as generation.

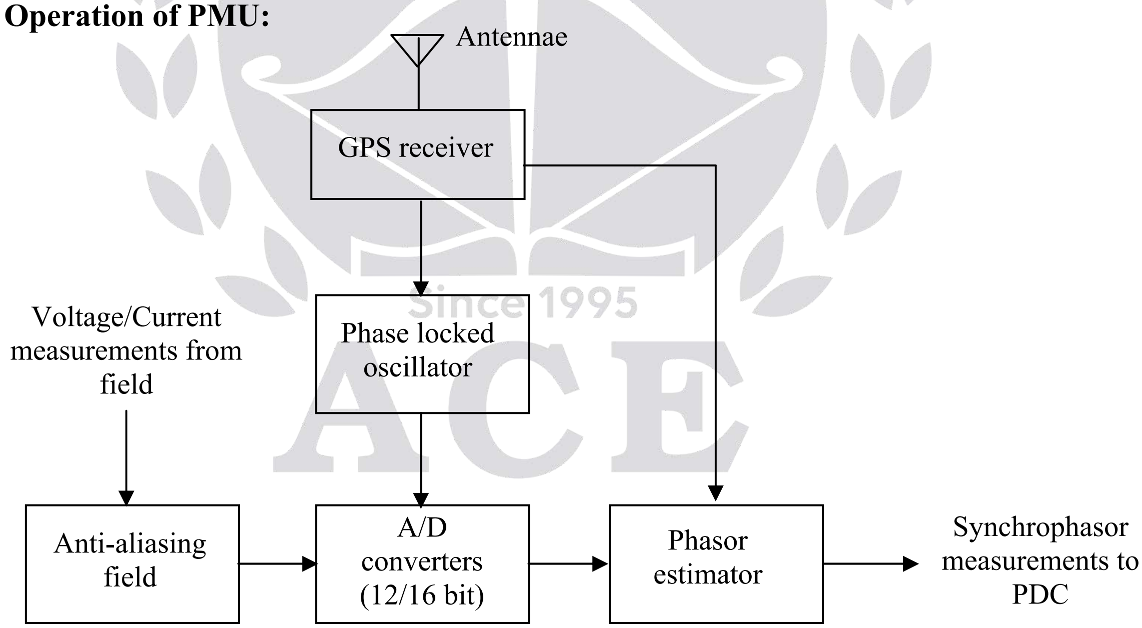
Some of the important features of smart grid are

- Real time monitoring
- Wide area management system and control
- Two way communication, two way flow of electricity
- Tracking and managing the energy usage
- Dynamic pricing of electricity

(ii) Advantages of PMU over SCADA:

- Phasor measurement unit measures the voltage angle directly with the help of synchrophasors with respect to global angle reference.
- SCADA can't measure this angle directly. It has to measure the angle with the help of various quantities like voltage, active power, reactive power, network parameters and a reference angle. So, the accuracy in voltage angle measurement is high in PMU compared to SCADA.
- Synchronized phasor measurement by PMU can provide a solution for problems in protection and automation problems. But SCADA can't do that.
- PMU is used for wide area monitoring and control but SCADA can provide only local monitoring and control
- SCADA is having a capability of observing only steady state events but PMU can observe steady state, and dynamic or transient events also

(iii) Operation of PMU:



There are two inputs to PMU

(1) The analog inputs provided by potential and current transformer secondary windings kept on the power system at the location of PMU.

(2) The GPS signal (synchronized time) given by the GPS satellite. It is taken by antenna kept at GPS receiver.

Anti aliasing filter: It collects the analog inputs from CT and PT and it filter out the high frequency signal. It is basically a low pass filter.

16-bit A/D Converter: Basically it converts the analog signal (coming out from anti-aliasing filter) into a digital signal

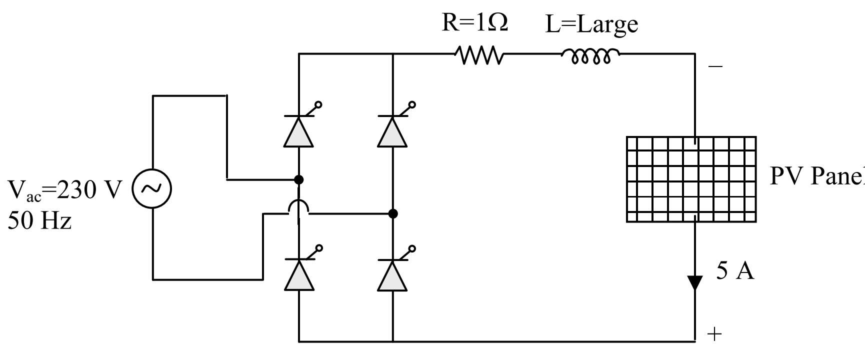
GPS receiver: The GPS receiver receives a very accurate time synchronized signal from GPS satellite kept in space. The GPS satellite collects the time synchronization signals from all other PMU's in power system. Based on the signal received by GPS receiver, it is possible to display the voltage and current waveforms in various substations on one plot (or) in one phasor diagram (which is easy for comparison).

Phase Locked Oscillators: It keeps the frequency of the reference and measured signal as equal. That is this oscillators coverts the GPS signal at one pulse per second into the required high speed timing pulses used in waveform sampling.

Phasor estimator: It consists a microprocessor which calculates the positive sequence estimates of all the voltage and current signals using DFT techniques. Finally these positive sequence components are time stamped and uploaded to phasor data concentrator (PDC). This data will be sent to GPS satellite through modems.

(e) A PV panel is connected with a single phase fully controlled converter as shown in the circuit below. The panel is supplying a current of 5 A and generated power is 1000 W. The series inductance in the circuit is large to make the current flat and continuous. Find (i) the triggering angle of the thyristor bridge, (ii) output voltage at rectifier terminal, and (iii) input power factor.

(12 M)



Sol: Given, $P_{PV} = 1000$ W and $I_0 = 5A$

$$E_V \times I_0 = 1000$$

$$E_V = \frac{1000}{I_0} = \frac{1000}{5} = 200 \text{ Volt}$$

In the given situation, the single phase fully controlled converter is operating in the inversion mode that means value of triggering angle of the thyristor bridge is greater than 90°.

$$\Rightarrow I_{0\,avg} = \frac{E_V - (-V_{0\,avg})}{R}$$

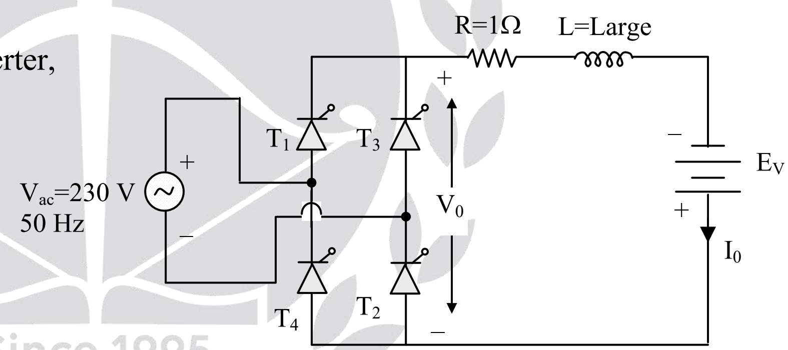
$$(I_{0\,avg} \times R) = E_V + V_{0\,avg}$$

$$(5 \times 1) = 200 + V_{0\,avg}$$

$$V_{0\,avg} = -195 \text{ Volt}$$

$\Rightarrow$ For constant 'I_0',

In single phase fully controlled converter,



$$V_{0\,avg} = \frac{2V_m}{\pi}\cos\alpha$$

$$-195 = \frac{2 \times 230\sqrt{2}}{\pi}\cos\alpha$$

$$\alpha = 160.34°$$

$\Rightarrow$ IPF = ?

$P_{i/p} = V_{s\,rms} . i_{s\,rms} . (IPF)$

$P_{o/p} = V_{0\,avg} . I_0 = V_{s\,rms} . i_{s1\,rms} . \cos\alpha$

Under Power Balance,

$P_{i/p} = P_{o/p}$

$V_{s\,rms} . i_{s\,rms} . (IPF) = V_{s\,rms} . i_{s1\,rms} . \cos\alpha$

$$IPF = \frac{i_{s1\,rms}}{i_{s\,rms}} \times \cos\alpha$$

For $i_0 = I_0$ (constant mag)

$i_{s\,rms} = I_0$

and $i_{s1\,rms} = \dfrac{2\sqrt{2}\,I_0}{\pi}$

$$IPF = \frac{\frac{2\sqrt{2}}{\pi} I_0}{I_0} \cos\alpha = \frac{2\sqrt{2}}{\pi}\cos\alpha$$

IPF = 0.848 (lag)

Ans: (i) $\alpha = 160.34°$ (ii) $V_{0\,avg} = -195$ V (iii) IPF = 0.848 (lag)

Q.2

(a) The DC – DC converter given below is operating at 30 kHz and drawing an input current of 25 A at 48 V DC.

(i) For a load current of 10 A, find

I. the duty ratio of the switch ,

II. output voltage ,

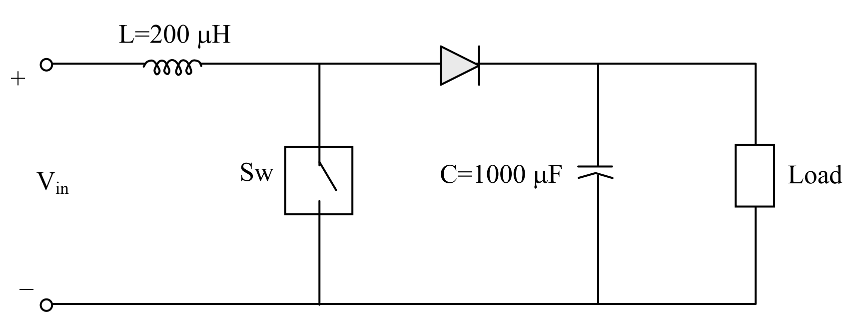
III. Peak inductor current ,

IV. output voltage ripple, and

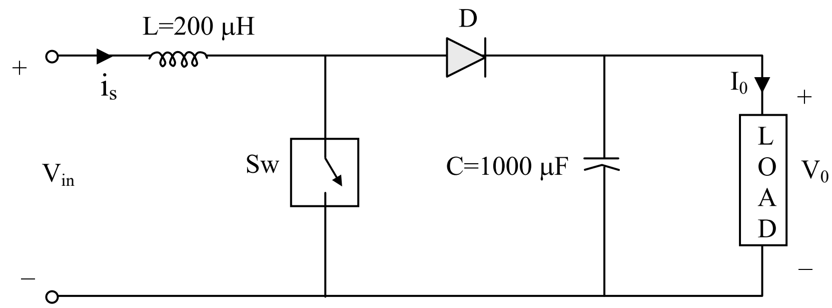
V. The load current where the inductor current just becomes discontinuous

(ii) Also find the critical value of L to keep the inductor current just continuous when the input voltage changes to 60 V with output remaining same.

(Assume lossless operation of converter components) **(20 M)**



Sol: The given DC-DC converter is a Boost converter



$\Rightarrow$ f = 30 kHz

I_s = 25 Amp

$V_s = V_{in}$ = 48V (DC)

(i) For load current of 10 Amp

I_0 = 10 Amp (cons.)

(I) D = ?, let us consider the C.C.C:

$P_{i/p} = P_{o/p}$

$V_sI_s = V_0.\ I_0$

$$V_sI_s = \frac{V_s}{1-D}.I_0$$

$$I_s = \frac{I_0}{1-D}$$

$$1 - D = \frac{I_0}{I_s}$$

$$D = 1 - \frac{I_0}{I_s}$$

$$D = 1 - \frac{10}{25}$$

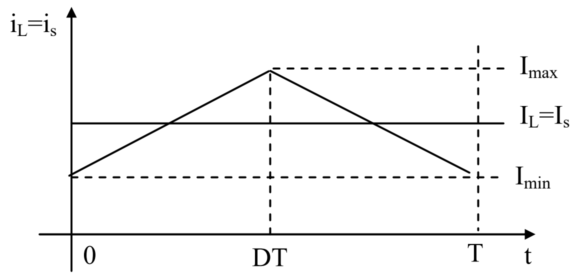
$$D = 0.6$$

(II) $V_0 = ?$

$$V_0 = \frac{V_s}{1-D} = \frac{48}{1-0.6} = 120 \text{ Volt}$$

(III) $(i_L)_{peak} = ?$

Under C.C.C



$$(i_L)_{peak} = (i_s)_{peak} = I_s + \frac{\Delta i_L}{2}$$

$\Rightarrow$ From (0 to DT)

$$V_L = V_s$$

$$L\frac{di_L}{dt} = V_s$$

$$\int_{I_{min}}^{I_{max}} di_L = \frac{V_s}{L}\int_0^{DT} dt$$

$$\Delta i_L = \frac{V_s}{L}.DT$$

$$(i_L)_{peak} = I_s + \frac{1}{2}\frac{V_s.D}{fL}$$

$$(i_L)_{peak} = 25 + \frac{1}{2}\left(\frac{48\times 0.6}{30k\times 200\mu}\right)$$

$$(i_L)_{peak} = 27.4 \text{ Amp}$$

(IV) $(\Delta V_0) = ?$

$$\Delta V_0 = \frac{\Delta Q}{C} = \frac{I_0 D.T}{C}$$

$$\Delta V_0 = \frac{I_0 .D}{f.C} = \frac{10 \times 0.6}{30\,k \times 1000\,\mu}$$

$\Delta V_0 = 0.2$ Volt

(V) The load current where the inductor current just becomes discontinuous

$(I_0)_{BCC} = ?$

$\Rightarrow P_{i/p} = P_{o/p}$

$V_s .I_s = V_0 .I_0$

$$V_s.I_s = \frac{V_s}{1-D}..I_0$$

$I_0 = I_s (1 - D)$

Where $I_s = (i_L)_{avg}$

$I_0 = (i_L)_{avg} (1 - D)$ ………(1)

and in B.C.C

$$(i_L)_{avg} = \frac{(i_L)_{peak}}{2} = \frac{1}{2} \times \frac{V_s.D}{fL}$$

From equation (1)

$$I_0 = \frac{V_s.D}{2fL}(1-D)$$

At same value of 'L' & 'D' if inductor current is just becomes discontinuous then, for it,

$$(i_0)_{avg} = \frac{48 \times 0.6 \times 0.4}{2 \times 30k \times 200\mu}$$

$(I_0) = 0.96$ Amp

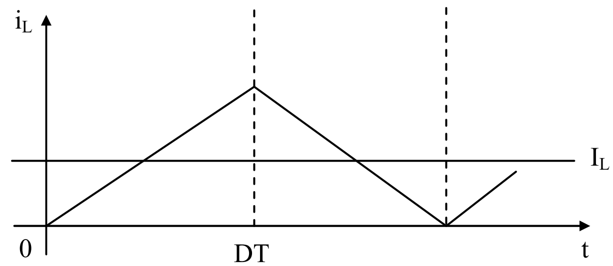
(ii) Critical value of 'L' = ? when, $V_{in} = V_s = 60$ V

Under boundary condition, $V_0 = 120$ volt

$$\Rightarrow V_0 = \frac{V_s}{1-D}$$

$$120 = \frac{60}{1-D}$$

$$D = 0.5$$



$\Rightarrow$ from (0 to DT)

$$V_L = V_s$$

$$L\frac{di_L}{dt} = V_s$$

$$di_L = \frac{V_s}{L}.dt$$

$$\int_0^{I_{max}} di_L = \int_0^{DT} \frac{V_s}{L}.dt$$

$$I_{max} = \frac{V_s}{L}.DT$$

$$\Rightarrow I_L = \frac{\Delta i_L}{2}$$

$$I_L = \frac{I_{max} - 0}{2}$$

$$\frac{I_0}{1-D} = \frac{V_s.DT}{2L_{cr}}$$

$$L_{cr} = \frac{60 \times 0.5 \times 0.5}{2 \times 30k \times 10}$$

$$L_{cr} = 25\ \mu H$$

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(b) A signal m(t) = 2 cos (20 πt) – cos (40 πt), where the unit of time is millisecond, is amplitude modulated using the carrier frequency (f_c) of 600 kHz. The AM signal is given by

$$s(t) = 5\cos 2\pi f_c\, t + m(t) \cos 2\pi\, f_c t$$

(i) Sketch the magnitude spectrum of s(t). What is its bandwidth? **(5 M)**

(ii) What is the modulation index? **(5 M)**

(iii) The AM signal is passed through a high-pass filter with cut-off frequency 595 kHz (i.e., the filter passes all frequencies above 595 kHz, and cuts off all frequencies below 595 kHz). Find an explicit time-domain expression for the quadrature component of the filter output with respect to 600 kHz frequency reference. **(10 M)**

Sol: $m(t) = 2\cos 20\pi t - \cos 40\pi t$

$f_1 = 10$ kHz $\quad f_2 = 20$ kHz

$f_c = 600$ kHz

$A_1 = 2 \quad A_2 = -1$

$$s(t) = 5\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$$

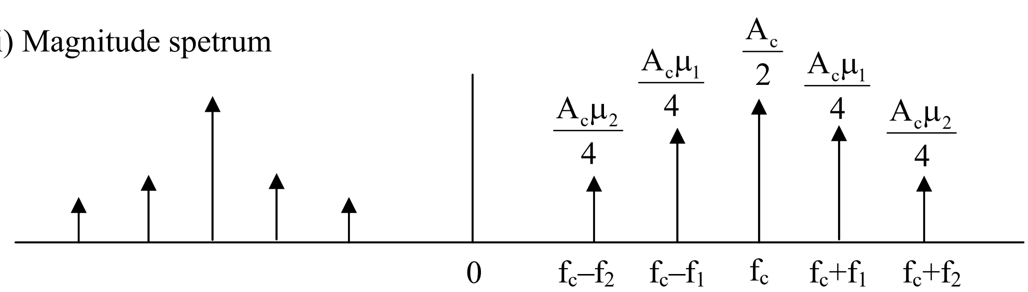
$$= 5\left[1 + \frac{1}{5} m(t)\right]\cos 2\pi f_c t$$

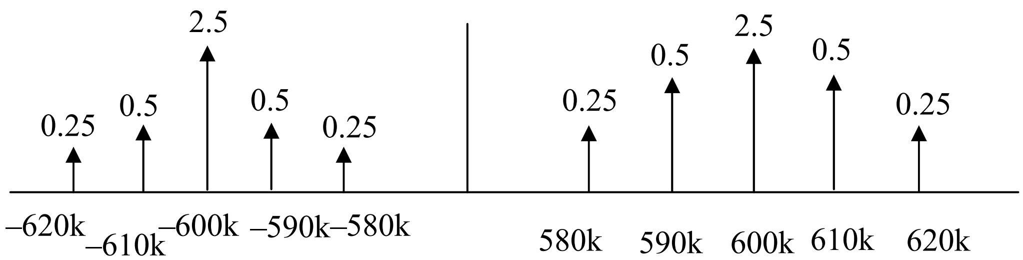
$A_c = 5 \quad k_a = \frac{1}{5}$

$$\mu_1 = k_a A_1 = \frac{1}{5}\times 2 = 0.4$$

$$\mu_2 = k_a A_2 = \frac{1}{5}\times(-1) = -0.2$$

(i) Magnitude spetrum





Band width = 620 k – 580 k

BW = 40 kHz

(ii) $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.4)^2 + (0.2)^2} = \sqrt{0.16 + 0.04} = \sqrt{0.2} = 0.447$

Modulation in dm = 0.447

(iii) If the AM signal is passed through the HPF having a cut off frequency of 545 kHz, the output of the filter consists of carrier and the two USB frequencies

Output of the HPF = $A_c \cos 2\pi f_c t + \frac{A_c\mu_1}{2} \cos 2\pi (f_c + f_1)t + \frac{A_c\mu_2}{2} \cos 2\pi (f_c + f_2)t$

$$= A_c \cos 2\pi f_c t + \frac{A_c\mu_1}{2}\cos 2\pi f_c t \cos 2\pi f_1 t - \frac{A_c\mu_1}{2}\sin 2\pi f_c t \sin 2\pi f_1 t + \frac{A_c\mu_2}{2}\cos 2\pi f_c t \cos 2\pi f_2 t - \frac{A_c\mu_2}{2}\sin 2\pi f_c t \sin 2\pi f_2 t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c\mu_1}{2}\cos 2\pi f_c t \cos 2\pi f_1 t + \frac{A_c\mu_2}{2}\cos 2\pi f_c t \cos 2\pi f_2 t - \frac{A_c\mu_1}{2}\sin 2\pi f_c t \sin 2\pi f_1 t - \frac{A_c\mu_2}{2}\sin 2\pi f_c t \sin 2\pi f_2 t$$

$$\text{Output} = \left[A_c + \frac{A_c\mu_1}{2}\cos 2\pi f_c t + \frac{A_c\mu_2}{2}\cos 2\pi f_2 t\right]\cos 2\pi f_c t + \left[-\frac{A_c\mu_1}{2}\sin 2\pi f_1 t - \frac{A_c\mu_2}{2}\sin 2\pi f_2 t\right]\sin 2\pi f_c t$$

The quadrature component at the output of HPF is

$$= -\frac{A_c\mu_1}{2}\sin 2\pi f_1 t - \frac{A_c\mu_2}{2}\sin 2\pi f_2 t$$

$A_c = 5$ $\mu_1 = 0.4$ $\mu_2 = -0.2$

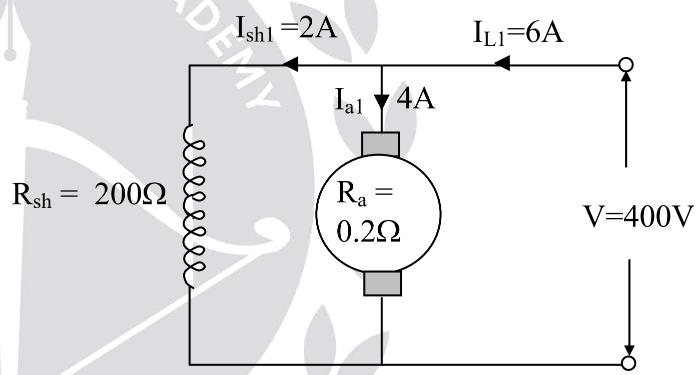
$= -\sin 2\pi f_1\, t + 0.5 \sin 2\pi\, f_2 t$

$f_1 = 10$ kHz $f_2 = 20$ kHz

(c) A 400 V DC shunt motor has armature and field resistance of 0.2 Ω and 200 Ω respectively. It draws a current of 6 A on no-load and 70 A on full-load. If its no-load and full-load speeds are the same, determine the field weakening due to load current as percentage of no-load flux.

(20 M)

Sol: **Case (i): No-load condition:**



Let speed is N_1, flux, ϕ_1

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{400}{200} = 2$ A

The armature current, $I_{a1} = I_{L1} - I_{sh1}$

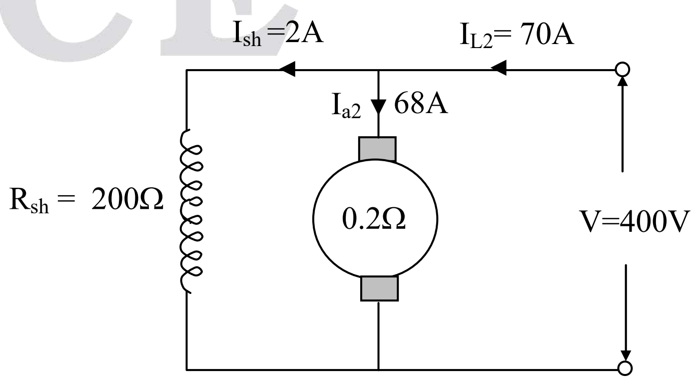
$= 6 - 2 = 4$ A

Back emf $E_{b1} = V - I_{a1}\, R_a$

$= 400 - 4 \times 0.2$

$= 399.2$ V

Case (ii): Load condition:



Let speed N_2, flux, ϕ_2

$N_2 = N_1$ (given)

$I_{sh} = \frac{V}{R_{sh}} = \frac{400}{200} = 2$ A

$I_{a2} = I_{L2} - I_{sh} = 70 - 2 = 68$ A

$E_{b2} = V - I_{a2}\, R_a$

$= 400 - 68 \times 0.2 = 386.4$ V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_1}{N_1} = \frac{386.4}{399.2} \times \frac{\phi_1}{\phi_2} = 1$$

$$\Rightarrow \frac{\phi_1}{\phi_2} = \frac{399.2}{386.4} = 1.0331 \Rightarrow \frac{\phi_2}{\phi_1} = 0.9679$$

The percentage of field weakening is $= \frac{\phi_1 - \phi_2}{\phi_1} \times 100$

$$= \left(1 - \frac{\phi_2}{\phi_1}\right) \times 100 = (1 - 0.9679) \times 100 = 3.2\%$$

Q. 3

(a) A salient pole star connected alternator is connected to infinite bus operating at 1.0 p.u voltage. The alternator has X_d = 0.75 p.u and X_q = 0.5 p.u on per phase basis. It is delivering 1.0 pu power to the infinite bus at 0.8 p.f lag. Calculate (i) the load angle and excitation voltage under this condition, (ii) the maximum power that can be delivered by the alternator with same excitation and the corresponding load angle, (iii) the armature current and p.f under maximum power condition, and (iv) the theoretical value of maximum power that the alternator can deliver when its field circuit is suddenly disconnected due to fault. **(20 M)**

Sol: Data given: V = 1.0 pu, X_d = 0.75 p.u, X_q = 0.5 pu

P = 1.0 pu, 0.8 lag PF

(i) $P = VI_a \cos\phi$

$1 = 1 \times I_a \times 0.8 \Rightarrow I_a = 1.25$ pu

$$\tan\psi = \frac{V\sin\phi \pm I_a X_q}{V\cos\phi + I_a R_a}$$ '+' lag PF, '–' lead PF

$$\tan\psi = \frac{1\times 0.6 + 1.25\times 0.5}{1\times 0.8 + 1\times 0} = 1.531$$

$\Rightarrow \psi = \tan^{-1} 1.531 = 56.85°$; $\psi \rightarrow$ internal P.F angle

$\psi = \phi + \delta$

$\therefore\ \delta = \psi - \phi = 56.85° - 36.86° = 20°$

Load angle, $\delta = 20°$

$I_d = I_a \sin\psi = 1.25 \times \sin 56.85° = 1.046$ pu

$I_q = I_a \cos\psi = 1.25 \times \cos 56.85° = 0.6835$ pu

Excitation voltage E,

$E = V\cos\delta + I_q R_a \pm I_d X_d$ '+' lag PF , '–' lead PF

$= 1 \times \cos 20° + (0.6835 \times 0) + 1.046 \times 0.75$

$= 1.724$ pu

$\therefore$ E = 1.724 pu

(ii) Power $P = \dfrac{EV}{X_d}\sin\delta + \dfrac{V^2}{2}\left(\dfrac{1}{X_q} - \dfrac{1}{X_d}\right)\sin 2\delta$

For max power output, $\dfrac{dP}{d\delta} = 0$

$$\therefore\ \frac{dP}{d\delta} = \frac{EV}{X_d}\cos\delta + V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cos 2\delta = 0$$

$$= \frac{1.724 \times 1}{0.75}\cos\delta + 1^2\left(\frac{1}{0.5} - \frac{1}{0.75}\right)\cos 2\delta = 0$$

$= 2.298\cos\delta + 0.666\cos 2\delta = 0$

$\Rightarrow 2.298\cos\delta + 0.666(2\cos^2\delta - 1) = 0$

$= 2.298\cos\delta + 1.333\cos^2\delta - 0.666 = 0$

$= \cos^2\delta + 1.724\cos\delta - 0.499 = 0$

$$\Rightarrow \cos\delta = \frac{-1.724 + \sqrt{1.724^2 - 4 \times 1 \times (-0.499)}}{2 \times 1}$$

$$= \frac{-1.724 + 2.229}{2} = 0.252$$

$\Rightarrow \delta = \cos^{-1}(0.252) = 75.35°$

The load angle at maximum power delivered

$\delta = 75.35°$

$\Rightarrow$ Maximum power, $P_{max} = \dfrac{EV}{X_d}\sin\delta + \dfrac{V^2}{2}\left(\dfrac{1}{X_q} - \dfrac{1}{X_d}\right)\sin 2\delta$

$$P_{max} = \frac{1.724\times 1}{0.75}\sin 75.35^o + \frac{1^2}{2}\left(\frac{1}{0.5} - \frac{1}{0.75}\right)\sin 2\times 75.35^o$$

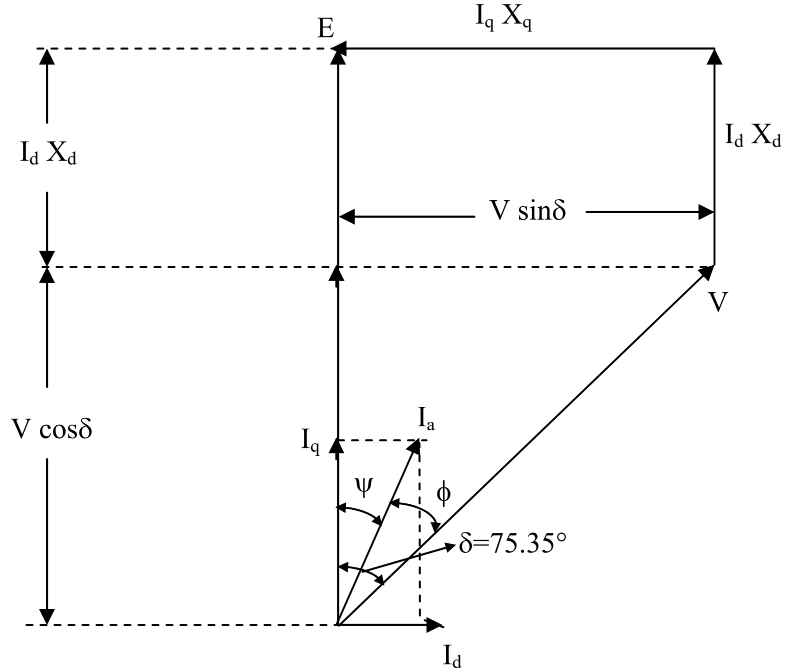
$= 2.223 + 0.163 = 2.386$ pu

$\therefore$ Maximum power delivered, $P_{max} = 2.386$ pu

(iii) under max power condition $\delta = 75.35°$,

Excitation emf E = 1.724 pu

$\Rightarrow$ At max power condition, the alternator operate at lead PF. The phasor diagram is given below



$I_d \rightarrow$ direct axis armature current

$I_q \rightarrow$ Quadrature axis armature current

$I_d = I_a \sin\psi$; $\quad I_q = I_a \cos\psi$

$E = V\cos\delta + I_d X_d$

$1.724 = 1 \times \cos 75.35° + I_d \times 0.75$

$\Rightarrow I_d = 1.961$ pu

$V \sin\delta = I_q X_q$

$1 \times \sin 75.35° = I_q \times 0.5$

$\Rightarrow I_q = 1.935$ pu

$\Rightarrow$ armature current $I_a = \sqrt{I_d^2 + I_q^2}$

$\therefore I_a = \sqrt{1.961^2 + 1.935^2} = 2.755$ pu

$\therefore I_a = 2.755$ pu

$\Rightarrow I_d = I_a \sin\psi$

$1.961 = 2.755 \sin\psi \Rightarrow \psi = 45.381°$

$\psi = \delta - \phi$

$\Rightarrow \phi = \delta - \psi = 75.35° - 45.381° = 29.97°$

$\therefore$ P.F = $\cos\phi = \cos 29.97° = 0.866$ lead

$\therefore$ P.F at max power output condition = 0.866 lead

(iv) When excitation failed, only reluctance power will be developed. The electro magnetic power is zero.

Therefore, reluctance power is maximum for $\delta = 45°$

$$\therefore P_{rel(max)} = \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)$$

$$= \frac{1^2}{2}\left(\frac{1}{0.5} - \frac{1}{0.75}\right) = 0.3335\,\text{pu}$$

$\therefore P_{rel(max)} = 0.3335$ pu

(b) A closed loop system with unity feedback and having the forward loop transfer function as

$$G(s) = \frac{14.4}{s(1+0.1s)},$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. **(20 M)**

Sol: $G(s) = \frac{14.4}{s(s+0.1s)}$, $H(s) = 1$

G(s) with cascaded compensation = $\frac{14.4k}{s(1+0.1s)}$

$$\text{CLTF} = \frac{C(s)}{R(s)} = \frac{14.4k}{0.1s^2 + s + 14.4k}$$

$$\frac{C(s)}{R(s)} = \frac{144k}{s^2 + 10s + 144k}$$

$\Rightarrow$ when $\zeta = 1$, the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot.

$\Rightarrow$ compare with $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

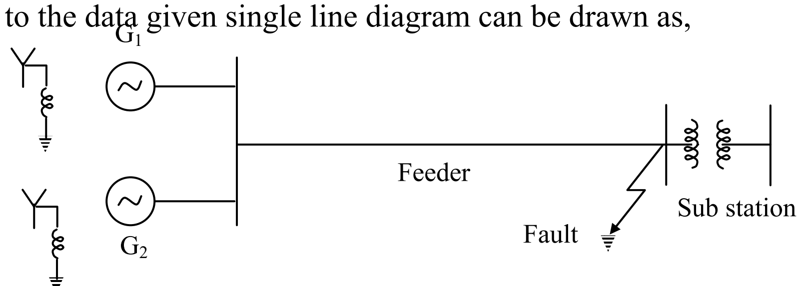
$\omega_n = \sqrt{144k} = 12\sqrt{k}$ rad/sec ....(1)

$2\zeta\omega_n = 10 \Rightarrow 2 \times 1 \times 12\sqrt{k} = 10$

$$\Rightarrow \sqrt{k} = \frac{10}{2\times 12} = 0.4167 \Rightarrow k = 0.173$$

(c) Two 11 kV, 30 MVA, three-phase synchronous generators operate in parallel supplying a sub-station through a feeder having an impedance of (0.6 + j0.8) ohms to positive and negative sequence currents and (1.0 + j2.6) ohms to zero sequence currents. Each generator has X_1 = 0.8 ohms, X_2 = 0.5 ohms and X_0 = 0.2 ohms and has its neutral grounded through a reactance of 0.2 ohms. Evaluate the fault currents in each line and the potential above earth attained by the generator neutrals, consequent to simultaneous occurrence of earth fault on the Y and B phases at the sub-station. **[20M]**

Sol: According to the data given single line diagram can be drawn as,



Data regarding apparatus,

$G_1 \equiv G_2$: $X_1 = 0.8\ \Omega$, $X_2 = 0.5\ \Omega$, $X_0 = 0.2\ \Omega$, $X_n = 0.2\ \Omega$

Feeder: $Z_{\ell_1} = Z_{\ell_2} = 0.6 + j0.8\ \Omega$

$Z_{\ell_0} = 1 + j2.6\ \Omega$

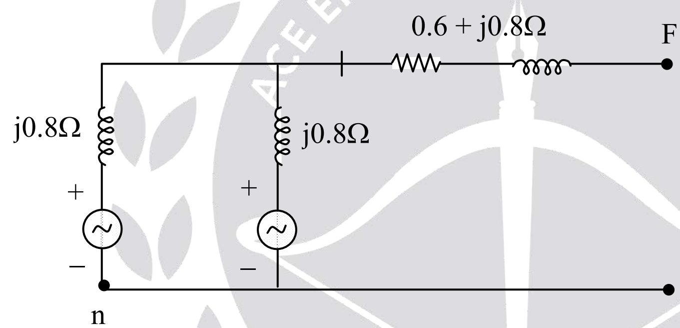
Ratings of each generator, 11 kV (LL), 30 MVA (3-ϕ)

It is said that an earth fault occurred on Y and B phases. So it can be treated as double line to ground fault on YB.

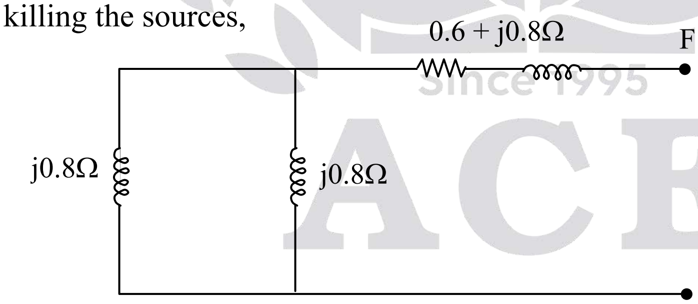
It is assumed that the substation is operated under no load condition.

For LLG fault the positive, negative and zero sequence networks will be connected in parallel.

Positive Sequence network:



By killing the sources,

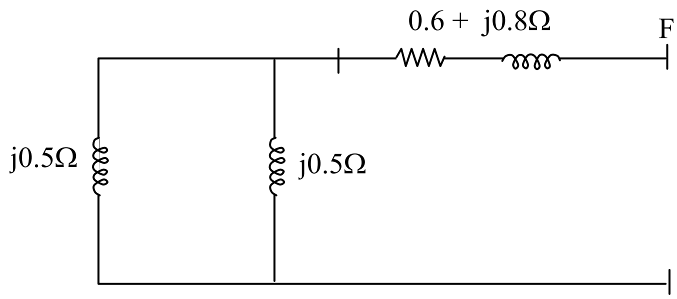


Thevenin's impedance with respect to 'F',

$$Z_{TH_1} = \frac{j0.8}{2} + 0.6 + j0.8$$

$$= 0.6 + j1.2\ \Omega$$

Negative sequence network:

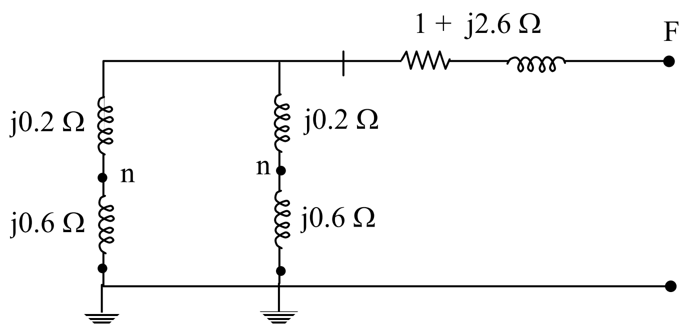


Thevenin impedance with respect to 'F'

$$Z_{TH_2} = \frac{j0.5}{2} + 0.6 + j0.8$$

$$= 0.6 + j1.05\ \Omega$$

Zero sequence network:



Thevenin impedance with respect to 'F',

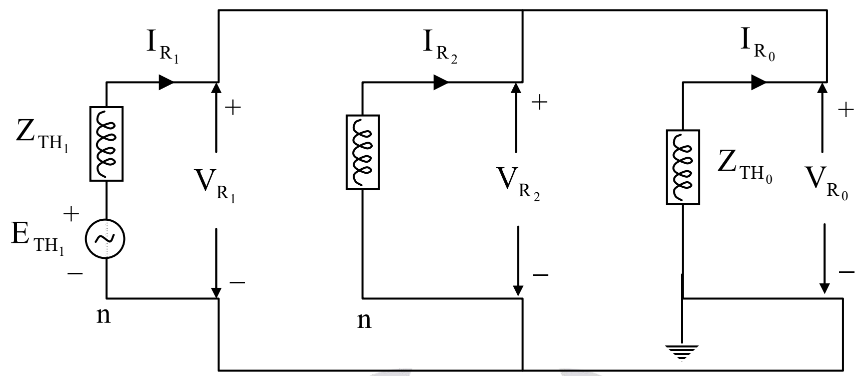
$$Z_{TH_0} = \frac{j0.8}{2} + 1 + j2.6$$

$$= 1 + j3\ \Omega$$

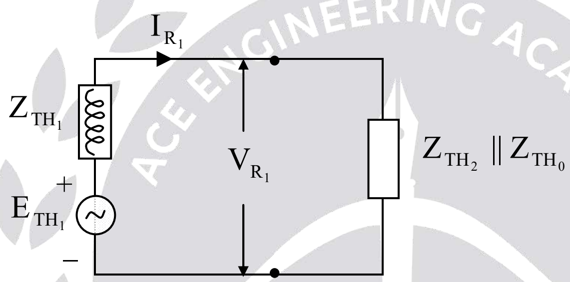
Let us assume that the prefault voltage is balanced and is rated value.

So, $E_{TH_1} = \frac{11}{\sqrt{3}}\text{kV}$

The sequence networks connection for LLG fault on YB will be,



The circuit can be redrawn as,



$$Z_{TH_2} \parallel Z_{TH_0} = \frac{(0.6 + j1.05)(1 + j3)}{0.6 + j1.05 + 1 + j3}$$

$$= 0.3935 + j0.7851\ \Omega$$

Now, positive sequence current,

$$I_{R_1} = \frac{E_{TH_1}}{Z_{TH_1} + (Z_{TH_2} \parallel Z_{TH_0})}$$

$$= \frac{11/\sqrt{3}}{(0.6 + j1.2) + (0.3935 + j0.7851)}\ \text{kA}$$

$$= 2.861\angle{-63.41^\circ}\ \text{kA}$$

By current division,

$$I_{R_2} = -I_{R_1}\frac{Z_{TH_0}}{Z_{TH_0}+Z_{TH_2}}$$

$$= -(2.861\angle{-63.41°}) \times \frac{1+j3}{1+j3+0.6+j1.05}$$

$$= 2.078\angle 119.71° \text{ kA}$$

$$I_{R_0} = -I_{R_1}\frac{Z_{TH_2}}{Z_{TH_2}+Z_{TH_0}}$$

$$= -(2.861\angle{-63.41°}) \times \frac{0.6+j1.05}{0.6+j1.05+1+j3}$$

$$= 0.795\angle 108.4° \text{ kA}$$

Fault current in Line – Y:

From symmetrical components,

$$I_Y = I_{R_0} + \alpha^2.I_{R_1} + \alpha.I_{R_2}$$

Where $\alpha = 1\angle 120°$

$$I_Y = (0.795\angle 108.4°) + (1\angle 240°)(2.861\angle{-63.41°}) + (1\angle 120°)(2.078\angle 119.71°)$$

$$= 4.245\angle{-168.18°} \text{ kA}$$

Fault current in Line – B:

From symmetrical components,

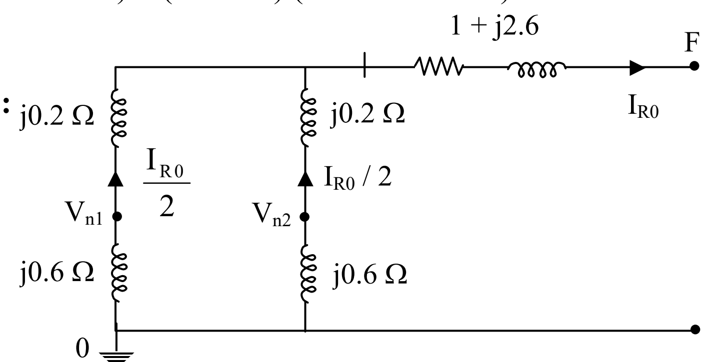
$$I_B = I_{R_0} + \alpha.I_{R_1} + \alpha^2.I_{R_2}$$

$$= (0.795\angle 108.4°) + (1\angle 120°)(2.861\angle{-63.41°}) + (1\angle 240°)(2.078\angle 119.71°)$$

$$= 4.625\angle 42.63° \text{ kA}$$

Generator neutral voltages calculation:

From zero sequence network,



By current division, current in each generator is $\frac{I_{R_0}}{2}$

G_1 neutral voltage, $V_{n_1} = -\frac{I_{R_0}}{2} \times j0.6$

$$V_{n_1} = -\left(\frac{0.795\angle 108.4^\circ}{2}\right) \times j0.6 \text{ kV} = 0.2385\angle{-71.6^\circ} \text{ kV}$$

$$|V_{n_1}| = 0.2385 \text{ kV}$$

G_2 neutral voltage, $V_{n_2} = -\frac{I_{R_0}}{2} \times j0.6$

$$= 0.2385\angle{-71.6^\circ} \text{ kV}$$

$$|V_{n_2}| = 0.2385 \text{ kV}$$

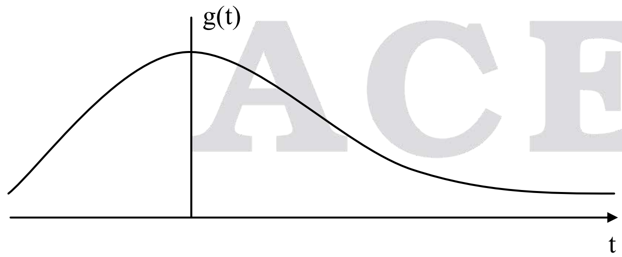
Q. 4

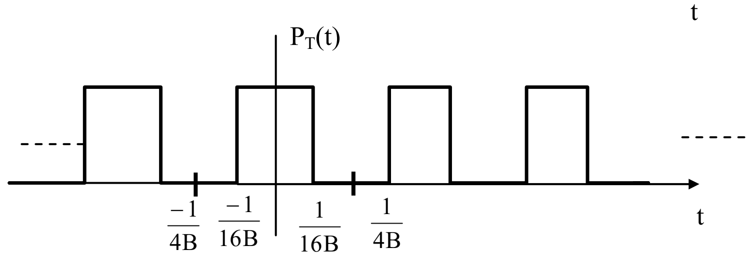
(a) A signal g(t) band limited to B Hz is sampled by a periodic pulse train $P_T(t)$ made up of a rectangular pulse of width $\frac{1}{8B}$ sec (centered at origin) repeating at Nyquist role (2B pulses per sec). Show that the sampled signal $g_s(t)$ is given by

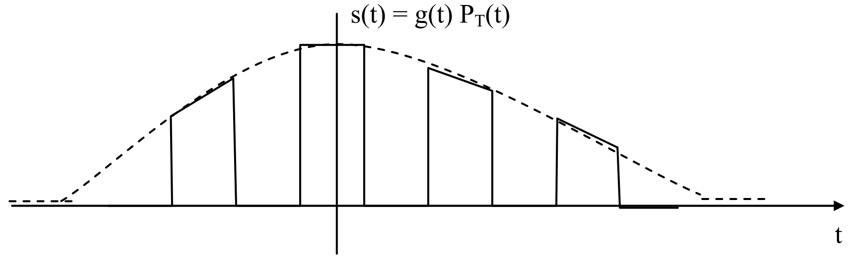
$$g_s(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos(4 n\pi Bt)$$

How will you recover g(t) from the signal $g_s(t)$? **(15 + 5M)**

Sol:







$T_s = \frac{1}{2B}$

For $P_T(t)$

$$C_0 = \frac{\frac{1}{8B}}{\frac{1}{2B}} = \frac{2B}{8B} = \frac{1}{4}$$

Here the sampling waveform s(t) consists a train of pulses having duration $\frac{1}{8B}$ and separated by $T_s = \frac{1}{2B}$. The sampled signal consists of a sequence of pulses of varying amplitude whose tops are not flat but follow the waveform of g(t).

With natural sampling, a signal sampled at the Nyquist rate may be reconstructed exactly by passing the samples through an ideal L.P.F with cut-off at "B" where "B" is highest frequency component of signal.

First we have to calculate F.S coefficient of $P_T(t)$

$$\omega_0 = \frac{2\pi}{T_S} = \frac{2\pi}{1/2B} = 4\pi B$$

Exponential F.S coefficient $C_n = \frac{1}{T_S}\int_{-1/16B}^{1/16B}(1)e^{-jn\omega_0 t}dt$

$$= \frac{1}{Ts}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-1/16B}^{1/16B} = \frac{2\sin\left(n\left(\frac{2\pi}{T_S}\right)\left(\frac{1}{16B}\right)\right)}{n\omega_0 T_S}$$

$$= \frac{2\sin\left[\frac{n2\pi(2B)}{16B}\right]}{n\frac{2\pi}{T_S}.T_S}$$

$$C_n = \frac{\sin\left(\frac{\pi n}{4}\right)}{n\pi}$$

$$\therefore\ P_T(t) = \frac{1}{4} + \sum_{n=-\infty}^{\infty} \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n} e^{jn\omega_0 t}$$

Convert to T.F.S form

$$= \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2\sin\left(\frac{\pi n}{4}\right)\cos(n\omega_0 t)}{\pi n}$$

$$= \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2\sin\left(\frac{\pi n}{4}\right)}{\pi n}\cos(4n\pi Bt)$$

∴ Naturally sampled single waveform expression is

$\therefore\ S(t) = g(t)\,P_T(t)$

$$= \frac{g(t)}{4} + \sum_{n=1}^{\infty} \frac{2\sin\left(\frac{\pi n}{4}\right)}{\pi n} g(t)\cos(4n\pi Bt)$$

A L.P.F with cut-off B will deliver an output, $S_0(t) = \frac{g(t)}{4}$

(b) A 3-phase half-controlled rectifier with free-wheeling diode is supplying a separately excited DC motor for speed control purpose. The AC input to the converter is 415 V, 3-phase, 50 Hz. The motor parameters are:

V = 220 V DC, P = 10.5 kW

Rated speed = 1100 rpm, Armature resistance $r_a = 0.4\ \Omega$.

The field current is kept constant at rated value. The motor is operated at rated speed delivering half rated torque.

(i) Find motor terminal voltage and triggering angle of thyristor bridge.

(ii) Find the speed of the motor if one of the input phases to the converter is out due to fault and the triggering angle is kept as before with same load torque.

(iii) Also find the new triggering angle if the motor speed is to be maintained at rated value with same load torque. (Neglect losses in the machine) **(20 M)**

Sol: Given: 415 V, 3-ϕ, 50 Hz

Means, $V_{ml} = 415\sqrt{2}$ Volt

Motor parameters: V = 220 V (DC)

$P = 10.5$ kW

$N_{rated} = 1100$ rpm

$R_a = 0.4\ \Omega$

Given, ϕ = constant as I_F is kept constant.

$\Rightarrow N = N_{rated} = 1100$ rpm

$$T = \frac{T_{rated}}{2}$$

Find: (i) $V_t = ?$, $\alpha = ?$

$\Rightarrow$ given, P = 10.5 kW

$$\frac{2\pi N_r T_r}{60} = 10.5 \times 10^3$$

$T_r = 91.152$ N-m

$\Rightarrow$ at rated load

$E_b.I_{ar} = 10.5 \times 10^3$

$[220 - I_{ar} \times 0.4]\, I_{ar} = 10.5 \times 10^3$

$-0.4\ I_{ar}^2 + 220\ I_{ar} - 10{,}500 = 0$

$I_{ar} = 52.795$ Amp

(i) $N_2 = N_r = 1100$ rpm

$T_2 = \frac{T_r}{2} = \frac{91.152}{2} = 45.576$ Nm

$V_0 = ?$ $\because V_0 = V_t$

And $\alpha = ?$

As we know, $T \propto \phi I_a$

$\phi = \text{cons.}\ \ T \propto I_a$

$$\frac{T_2}{T_1} = \frac{I_{a_2}}{I_{a_1}}$$

$$I_{a_2} = \frac{45.576}{91.152} \times 52.795 = 26.3975 \text{ Amp}$$

$$\Rightarrow E_{b2}\, I_{a2} = \frac{2\pi N_2 T_2}{60}$$

$$E_{b2} = \frac{2\pi \times 1100 \times 45.576}{60 \times 26.3975} = 198.88 \text{ Volt}$$

$\Rightarrow$ KVL :

$V_0 - I_{a2} R_a = E_{b2}$

$$\frac{3V_{ml}}{2\pi}[1 + \cos\alpha_2] = E_{b2} + I_{a2} R_a$$

$$\frac{3 \times 415\sqrt{2}}{2\pi}[1 + \cos\alpha_2] = 198.88 + 26.3975 \times 0.4$$

$\alpha_2 = 104.63°$

Now $V_t = V_0 = \frac{3V_{ml}}{2\pi}[1 + \cos\alpha]$

$$V_t = \frac{3 \times 415\sqrt{2}}{2\pi}[1 + \cos 104.63]$$

$V_t = 209.45$ Volt

Ans: $V_t = 209.45$ Volt

$\alpha = 104.63°$

(ii) $N_3 = ?$

$\alpha_3 = 104.63$

$T_3 = T_2 = \frac{T_r}{2} = 45.576$ N-m

If one of the input phase to the converter is out due to fault then, average output voltage of converter or motor terminal voltage will becomes equal to

$$V_{03} = \frac{V_{ml}}{\pi}[1 + \cos\alpha_3] = \frac{415\sqrt{2}}{\pi}[1+\cos 104.63]$$

$V_{03} = 139.63$ Volt

$\Rightarrow$ as $T_3 = T_2$

$I_{a3} = I_{a2} = 26.3975$

Now $\frac{2\pi N_3 T_3}{60} = E_{b3}.\ I_{a3} = [V_{03} - I_{a3}\ R_a]\ I_{a3}$

$$\frac{2\pi \times N_3 \times 45.576}{60} = [139.63 - 26.3975 \times 0.4 \times 26.3975]$$

$\Rightarrow N_3 = 713.885$ rpm

Ans: $N_3 = 713.85$ rpm

(iii) $\alpha_4 = ?$

$N_4 = N_r = 1100$ rpm

$T_4 = T_3 = 45.576$ N-m

$T_4 = T_3$ means $I_{a4} = I_{a3} = 26.3975$

If one of the input phase to the converter is out due to fault

$$\Rightarrow V_{0\,avg} = \frac{V_{ml}}{\pi}[1 + \cos\alpha]$$

$$\frac{2\pi \times N_4 T_4}{60} = (V_{0_4} - I_{a_4} R_a).\ I_{a4}$$

$$\frac{2\pi \times 1100 \times 45.576}{60} = \left[\frac{415\sqrt{2}}{\pi}(1+\cos\alpha_4) - 26.3975 \times 0.4\right] \times 26.3975$$

$\alpha_4 = 83.04°$

⇒ under normal condition means without fault.

$$V_{0_{avg}} = \frac{3V_{ml}}{2\pi}[1 + \cos\alpha]$$

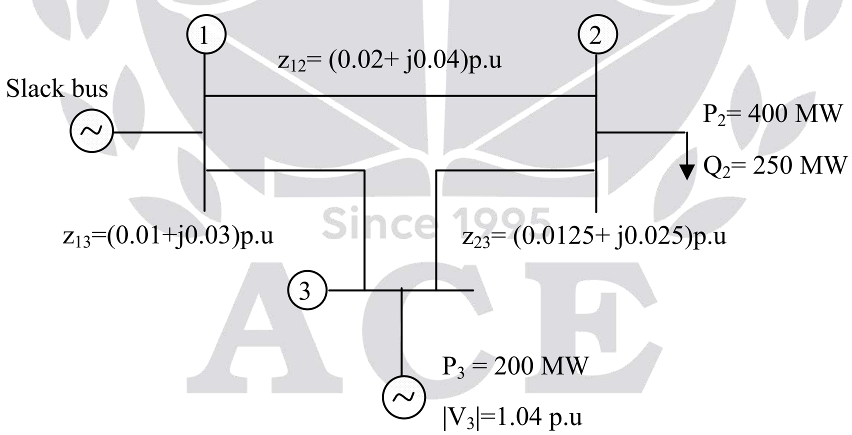
$$\frac{2\pi N_4 T_4}{60} = (V_{0_4} - I_{a_4} R_a) I_{a_4}$$

$$\frac{2\pi \times 1100 \times 45.576}{60} = \left[\frac{3 \times 415\sqrt{2}}{2\pi}(1 + \cos\alpha_4) - 26.3975 \times 0.4\right] \times 26.3975$$

$\alpha_4 = 104.63°$

Ans: under fault condition [$\alpha_4 = 83.04°$]

(c) The figure below shows single line diagram of a power system with generators at bus-1 and bus-3. The voltage at bus-1 is $1.05\angle 0°$ p.u and at bus-3, $|V| = 1.04$ p.u. Line impedances are in p.u and line charging susceptances are neglected. Obtain state vector using Fast Decoupled Load Flow (FDLF) for one iteration. **(20 M)**



Sol: In the data, there is no mention about base power (S_{base}). Voltages and impedances are given in p.u. form but powers P_2, Q_2 and P_3 are given in MW. So it is not possible to give solution for this problem.

Let us assume S_{base} = 100 MVA

P_2 (p.u.) = $\frac{400}{100}$ = 4 → Load at Bus (2)

Q_2 (p.u.) = $\frac{250}{100}$ = 2.5 → Load at Bus (2)

P_3 (p.u.) = $\frac{200}{100}$ = 2 p.u → Generation at Bus (3)

Bus Classification:

Bus (1): Slack bus, $V_1 = 1.05 \angle 0°$

$|V_1| = 1.05$ p.u., $\delta_1 = 0°$

Bus (2): Load bus (PQ bus)

$P_{L2} = 4$ p.u. , $Q_{L2} = 2.5$ p.u.

$P_{G2} = 0$, $Q_{G2} = 0$

Net real and reactive power injections by Bus (2)

$P_2 = P_{G2} - P_{L2} = -4$ p.u.

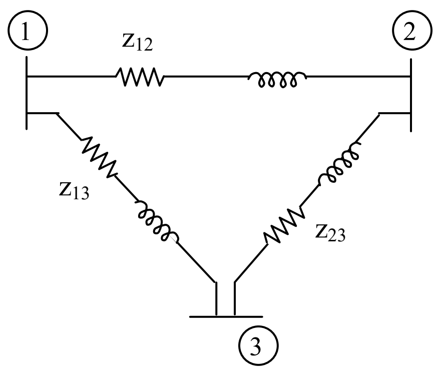
$Q_2 = Q_{G2} - Q_{L2} = -2.5$ p.u.

Bus (3): PV Bus, $|V_3| = 1.04$ p.u.

$P_{G3} = 2$ p.u., $P_{L3} = 0$

$P_3 = P_{G3} - P_{L3} = 2$ p.u.

Per phase model of the system,



Admittances of branches,

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.04} = 10 - j20 \text{ p.u}$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.0125 + j0.025} = 16 - j32 \text{ p.u}$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.01 + j0.03} = 10 - j30 \text{ p.u}$$

Bus admittance matrix by direct inspection method,

$$Y_{BUS} = \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} \overset{(1)}{y_{12} + y_{13}} & \overset{(2)}{-y_{12}} & \overset{(3)}{-y_{13}} \\ -y_{12} & y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix}$$

$$= \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} \overset{(1)}{20 - j50} & \overset{(2)}{-10 + j20} & \overset{(3)}{-10 + j30} \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Unknown states in the system are δ_2, δ_3, $|V_2|$

The corresponding equations to be solved in FDLF method,

$$\left[\frac{\Delta P}{|V|}\right] = [B^1][\Delta\delta] \text{ ……….(1)}$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B^{11}][\Delta |V|] \text{ ………..(2)}$$

Equations set (1) can be,

$$\begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} \text{ ……….(3)}$$

Equation set (2) can be,

$$\left[\frac{\Delta Q_2}{|V_2|}\right] = [-B_{22}][\Delta |V_2|] \text{ …………(4)}$$

Assuming flat start, $\delta_2^o = 0^o$, $\delta_2^o = 0^o$, $|V_2|^o = 1$ p.u.

Initial states are, $V_1 = 1.05\angle 0°$ p.u

$V_2 = 1\angle 0°$ p.u

$V_3 = 1.04\angle 0°$ p.u

Calculation of real and reactive powers (P_2, P_3, Q_2):

Complex power, $S_2^{calc} = V_2.I_2^*$

$= V_2\left[y_{21}V_1 + y_{22}V_2 + y_{23}V_3\right]^*$

$S_2^{calc.} = 1\,[(-10 + j20) \times 1.05 + (26 - j52) \times 1 + (-16 + j32) \times 1.04]^*$

$= -1.14 - j2.28$ p.u.

$P_2^{calc.} = -1.14$, $\quad Q_2^{calc.} = -2.28$

Complex power, $S_3^{calc.} = V_3.I_3^*$

$= V_3\left[y_{31}V_1 + y_{32}V_2 + y_{33}V_3\right]^*$

$S_3^{calc.} = 1.04\,[(-10+j30) \times 1.05 + (-16 + j32) \times 1 + (26 - j62) \times 1.04]^*$

$= 0.5616 + j1.0192$ p.u.

$P_3^{calc} = 0.5616$ p.u

Power mismatches calculation:

$\Delta P_2 = P_2^{SP} - P_2^{calc.}$

$= (-4) - (-1.14) = -2.86$ p.u.

$\Delta P_3 = P_3^{SP} - P_3^{calc.}$

$= 2 - 0.5616 = 1.4384$ p.u

$\Delta Q_2 = Q_2^{SP.} - Q_2^{calc.}$

$= -2.5 - (-2.28) = -0.22$ p.u.

From equations set (3),

$$\begin{bmatrix} \dfrac{-2.86}{1} \\ \dfrac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} 52 & -32 \\ -32 & 62 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = \begin{bmatrix} 52 & -32 \\ -32 & 62 \end{bmatrix}^{-1} \begin{bmatrix} -2.86 \\ 1.3831 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0605 \\ -8.908\times10^{-3} \end{bmatrix} \text{radians}$$

$$= \begin{bmatrix} -0.0605 \\ -8.908\times10^{-3} \end{bmatrix} \times \frac{180^\circ}{\pi}$$

$$= \begin{bmatrix} -3.465^\circ \\ -0.510^\circ \end{bmatrix}$$

From equations set (4),

$$\left[\frac{-0.22}{1}\right] = [5 \quad 2][\Delta|V_2|]$$

$$\Delta|V_2| = \frac{-0.22}{52} = -4.231 \times 10^{-3}$$

States of system at the end of first iteration,

$\delta_2^1 = \delta_2^0 + \Delta\delta_2$

$= -3.465^\circ$

$\delta_3^1 = \delta_3^0 + \Delta\delta_3$

$= -0.510^\circ$

$|V_2|^1 = |V_2|^o + \Delta|V_2|$

$= 1 - 4.231 \times 10^{-3}$

$= 0.996$ p.u.

States of system at the end of one iteration,

$|V_1| = 1.05, \quad \delta_1 = 0^\circ$

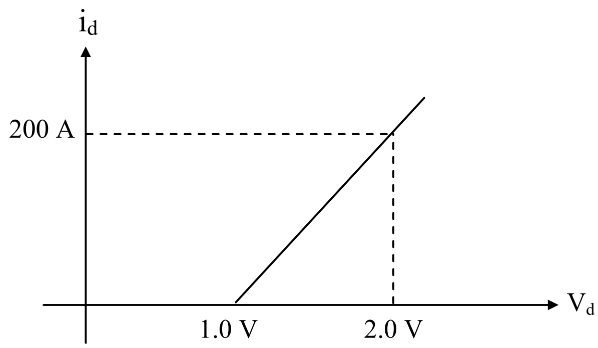
$|V_2| = 0.996, \quad \delta_2 = -3.465^\circ$

$|V_3| = 1.04, \quad \delta_3 = -0.510^\circ$

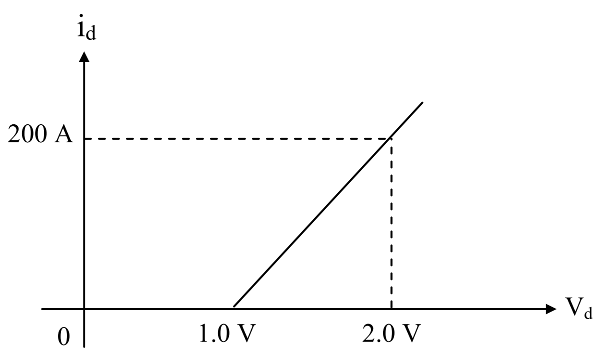
SECTION-B

Q. 5

(a) A thyristor is having the I-V characteristic as given in the figure below. It is used in a half wave rectifier circuit with resistive load operating at $\alpha = 30°$ and carrying a peak load current of 100 A. Determine the average conduction loss in the thyristor. **(12 M)**



Sol: The I-V characteristic of a thyristor is given as follows

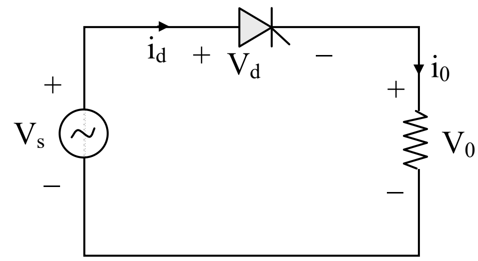


$\Rightarrow V_d = V_r + i_d . R_D$

$\because V_r = 1$ volt

$R_D = 5\ m\Omega$

$\Rightarrow$ This thyristor is used in a half wave rectifier circuit with resistive load operating $\alpha = 30°$, and given $(i_0)_{peak} = 100$ A



$\because V_s = V_m \sin \omega t$

⇒ when SCR is OFF

$i_0 = i_s = i_T = 0$

⇒ when SCR is ON [from $\omega t = 30°$ to $\omega t \approx 180°$]

$i_0 = i_s = i_T$

$\because\ i_0 \approx 100 \sin\omega t$

$$\Rightarrow (i_T)_{avg} \approx \frac{1}{2\pi}\int_{\alpha}^{\pi} 100 \sin \omega t.d\omega t$$

$$(i_T)_{avg} \approx \frac{100}{2\pi}\ [\cos\alpha - \cos\pi]$$

$$(i_T)_{avg} \approx \frac{100}{2\pi}[\cos 30 + 1] = 29.699 \text{ Amp}$$

$$\Rightarrow (i_T)_{rms} \approx \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} 100^2 \sin^2 \omega t.d\omega t}$$

$$(i_T)_{rms} \approx \sqrt{\frac{100^2}{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$(i_T)_{rms} \approx \sqrt{\frac{100^2}{4\pi}\left[\omega t\Big|_{\alpha}^{\pi} - \frac{1}{2}\sin 2\omega t\Big|_{\alpha}^{\pi}\right]}$$

$$(i_T)_{rms} \approx \sqrt{\frac{100^2}{4\pi}\left(180^{o} - 30^{o}\right)\times\frac{\pi}{180} - \frac{1}{2}\left[\sin 2\pi - \sin 2\alpha\right]}$$

$$(i_T)_{rms} \approx \sqrt{\frac{100^2}{4\pi}\left[150^{o}\times\frac{\pi}{180} + \frac{1}{2}\times\sin 60^{o}\right]}$$

$(i_T)_{rms} \approx 49.274$ Amp

$\Rightarrow$ The average conduction loss in thyristor

$(P_T)_{avg} = (i_T)^2_{rms} \times R_D + V_r \times (i_T)_{avg}$

$(P_T)_{avg} \approx ((49.274)^2 \times 5m) + (1 \times 29.699)$

$(P_T)_{avg} \approx 41.84$ W

(b) A three-phase equilateral transmission line has a total corona loss of 55 kW at 110 kV and 100 kW at 114 kV. What is the disruptive critical voltage between lines? What is the corona loss at 120 kV? **(12 M)**

Sol: It is given that,

Corona loss = 50 kW at 110 kV voltage

Corona loss = 100 kW at 114 kV voltage

Let us assume the above voltages are L-L and corona loss is in 3-ϕ manner.

From peek's formula of corona loss (P_c),

$$P_c \propto (V_s - V_d)^2$$

Where V_s is system operating voltage (L – L)

V_d is disruptive critical voltage (L – L)

P_c is 3-ϕ corona loss

$$\frac{P_{c_2}}{P_{c_1}} = \frac{(V_{s_2} - V_d)^2}{(V_{s_1} - V_d)^2}$$

$$\frac{100}{50} = \frac{(114 - V_d)^2}{(110 - V_d)^2}$$

$$2(110 - V_d)^2 = (114 - V_d)^2$$

$$2[(110)^2 - 220\,V_d + V_d^2] = (114)^2 + V_d^2 - 228\,V_d$$

$$24200 - 440\,V_d + 2\,V_d^2 = 12996 + V_d^2 - 228\,V_d$$

$$V_d^2 - 212\,V_d + 11204 = 0$$

By solving this equation,

$V_d = 111.66$ kV or $V_d = 100.34$ kV

As corona loss is non zero at 110 kV operating voltage, the disruptive voltage will be chosen as $V_d = 100.34$ kV (LL)

Now,

Corona loss $P_{c_3} = ?$ at $V_s = 120$ kV (LL)

$$\frac{P_{c_3}}{P_{c_2}} = \frac{(V_{s_3} - V_d)^2}{(V_{s_2} - V_d)^2}$$

$$P_{c_3} = 100 \times \frac{(120 - 100.34)^2}{(114 - 100.34)^2} \text{ kW}$$

$$= 207.14 \text{ kW}$$

(c) A Gaussian pulse is specified by

$$g(t) = Ae^{-\alpha^2 t^2},$$

where α is an arbitrary attenuation coefficient and A is constant. Show that the Fourier transform of g(t) is also Gaussian. **(12 M)**

Sol: $g(t) = Ae^{-kt^2}$; Assume $k = \alpha^2$

$$\downarrow$$

$$\frac{d}{dt} g(t) = -Ak(2t)e^{-kt^2}$$

$$= -2t\, kg(t)$$

$$= -2k\, tg(t) \text{ ……(I)}$$

Apply F.T

$$\frac{d}{dt} x(t) \overset{F.T}{\leftrightarrow} j\omega X(\omega)$$

Applying time & frequency differentiation properties of F.T,

$$-jt\, x(t) \leftrightarrow \frac{d}{d\omega} X(\omega)$$

(I) $\Rightarrow$

$$j\omega\, G(\omega) = \frac{2k}{j}\frac{d}{d\omega}G(\omega)$$

$$-\omega G(\omega) = 2k\,\frac{d}{d\omega}G(\omega)$$

$$\frac{d}{d\omega}G(\omega) = -\frac{1}{2k}\omega G(\omega)\ \ldots\ldots\ldots\text{(II)}$$

As equations (I) & (II) are looking like same G(ω) is same as that of g(t), except for constant multipliers.

$$\therefore\ G(\omega) = C\,e^{-k\omega^2}$$

$$G(\omega)\big|_{\omega=0} = C$$

$$G(\omega)\big|_{\omega=0} = \int_{-\infty}^{+\infty} g(t)dt = \int_{-\infty}^{+\infty} Ae^{-kt^2}\,dt$$

$$= A\sqrt{\frac{\pi}{k}} = A\sqrt{\frac{\pi}{\alpha^2}}$$

$$\therefore\ G(\omega) = A\sqrt{\frac{\pi}{k}}\,e^{-k\omega^2} = A\sqrt{\frac{\pi}{\alpha^2}}e^{-\alpha^2\omega^2}$$

$\therefore$ F.T of Gaussian is Gaussian.

(d) What are the advantages and limitations of Lead and Lag networks in a practical control system? **(12 M)**

Sol: **Lead compensator:**

Advantages:

1. Lead compensator is a high pass filter. Hence bandwidth of the system is increases
2. As bandwidth increases, the transient behavior of the system improves
3. The rise time and settling time are reduced.
4. Gain margin and phase margin are improved. Hence relative stabilities improved
5. The steady state error of the system is not effected.

Limitations:

1. Due to large bandwidth, noise enter into the system. Hence signal to noise ratio is reduce.
2. It require high gain amplifier, which could be costly.
3. The maximum phase lead available from a single-stage phase – lead controller is less than 90°. Thus, if a phase lead of more than 90° is required, a multistage controller should be used

Lag Compensator:

Advantages:

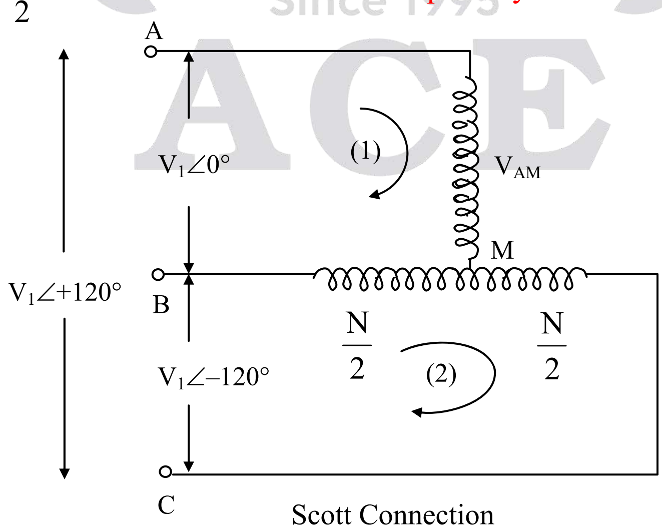
1. Lag compensator is a low pass filter. Hence bandwidth of the system is reduced.
2. As bandwidth reduced, noise is eliminated, signals to noise ratio is improved.
3. Steady state performance is improved.
4. Gain margin and phase margins are improve. Hence relative stability improved.

Limitations:

1. As bandwidth reduced, the system is slow.
2. The rise time and settling time of system are usually large.
3. The system is more sensitive to parameter variations.

(e) For a Scott connected transformer, prove that the number of turns on primary of the teaser transformer is $\frac{\sqrt{3}}{2}$ times the number of turns in primary of main transformer. **(12 M)**

Sol:



Scott Connection

Applying KVL in loop - (1)

$V_1\angle 0° = V_{AM} - V_{BM}$ …….(i)

Applying KVL in loop- (2)

$V_1\angle -120° = 2V_{BM}$

$$V_{BM} = \frac{V_1}{2}\angle -120°$$

Substituting in equation (i)

$$V_{AM} = V_1\angle 0° + \frac{V_1}{2}\angle -120°$$

$$= \frac{\sqrt{3}}{2}V_1\angle -30°$$

$\therefore$ The number of turn required in the primary of Teaser transformer = $\frac{\sqrt{3}}{2}N$

$= \frac{\sqrt{3}}{2}\times$ number of turns in primary of main transformer .

Q. 6

(a) A 15 kW, 400 V, 3-phase, star connected synchronous motor has synchronous impedance of 0.4 + j4 Ω. Find the motor excitation voltage for full load output at 0.866 leading power factor. Take the armature efficiency of 95%. **(20 M)**

Sol: Power output, P_{out} = 15 kW;

$$\text{Power input, } P_{in} = \frac{P_{out}}{\eta} = \frac{15\times 10^3}{0.95} = 15.789 \text{ kW}$$

$Z_s = 0.4 + j4 = 4.02\ \angle 84.29° = \theta$

Full load current at 0.866 leading is power factor

$$I_a = \frac{P_{in}}{\sqrt{3}V_L\cos\phi} = \frac{15.789\times 10^3}{\sqrt{3}\times 400\times 0.866} = 26.315 \text{ A}$$

$V_L = 400$ V, $V_{ph} = \dfrac{400}{\sqrt{3}} = 230.94$ V; $\phi = 30°$

$E = V\angle 0 - I_a\angle \pm\phi \; Z_s \angle\theta$

$= 230.94\angle 0 - 26.315\angle 30° \times 4.02 \angle 84.29°$

$= 230.94 - 105.78\angle 114.29°$

$= 230.94 - (105.78 \cos 114.29 + j105.78 \sin 114.29)$

$= 230.94 - (-43.51 + j96.41)$

$= 274.45 - j96.41$

$= 290.89\angle -19.35°$

$\therefore E_{ph} = 290.89$ V

$E_L = \sqrt{3} \times 290.89 = 503.83$ V

(OR)

$$E = \sqrt{(V\cos\phi - I_aR_a)^2 + (V\sin\phi \mp I_aX_s)^2}$$

'–' lag pF

'+' lead pF

$$= \sqrt{(230.94 \times \cos 30 - 26..315 \times 0.4)^2 + (230.94 \sin 30 + 26.315 \times 4)^2}$$

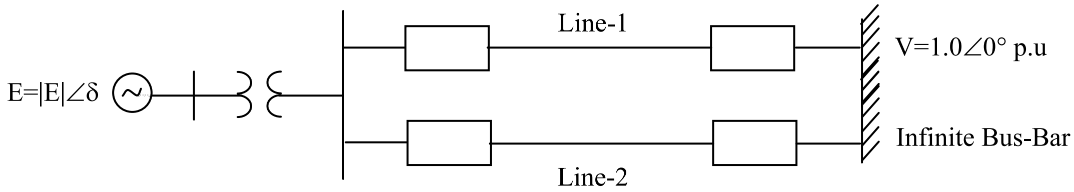
$$= \sqrt{189.473^2 + 220.73^2}$$

$= 290.89$ V

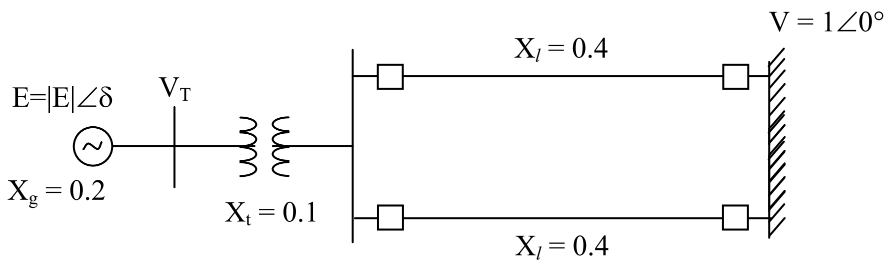
$\therefore E_{ph} = 290.89$ V

$E_L = \sqrt{3} \times 290.89 = 503.83$ V

(b) A synchronous machine is connected to an infinite bus through a transformer and a double circuit line shown in the figure. The infinite bus voltage is V = 1.0 ∠0° p.u. The direct axis transient reactance of the machine is 0.20 p.u., the transformer reactance is 0.10 p.u. and the reactance of each of the transmission lines is 0.4 p.u., all to a base of the rating of the synchronous machine. Initially the machine is delivering 0.8 p.u power with a terminal voltage of 1.05 p.u. The inertia constant H = 5 MJ/MVA. All resistances are neglected. Determine the equation of motion of the machine rotor. **(20 M)**



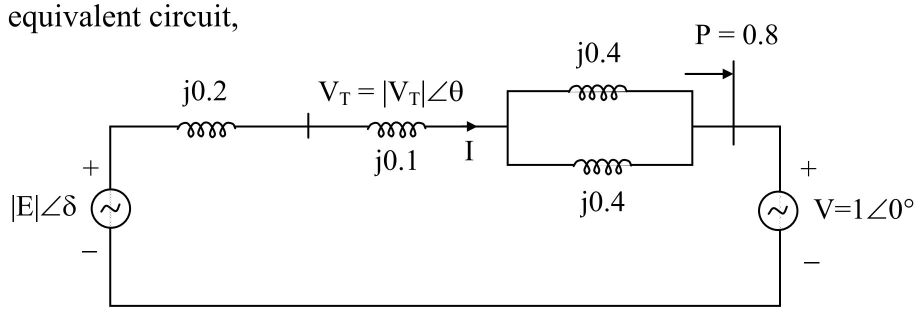
Sol: The single line diagram along with data given,



Terminal voltage of alternator $|V_T| = 1.05$ p.u

Let $V_T = |V_T|\angle\theta = 1.05\angle\theta$

Per phase equivalent circuit,



Real power flow in transmission line, P = 0.8 p.u

By real power equation,

$$\frac{|V_T||V|}{0.1+\frac{0.4}{2}}\sin\theta = 0.8$$

$$\frac{1.05}{0.3}\sin\theta = 0.8$$

$\sin\theta = 0.22857$

$\theta = 13.213°$

Current in the network,

$$I = \frac{V_T - V}{j0.3}$$

$$= \frac{1.05\angle 13.213^\circ - 1\angle 0^\circ}{j0.3}$$

$$= 0.8034 \angle -5.286^\circ \text{ p.u.}$$

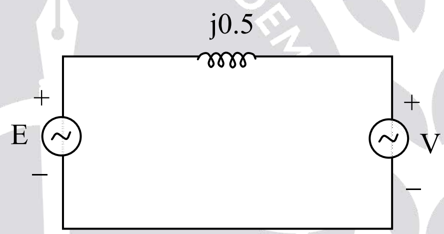
Internal emf of generator,

$E = V_T + (j0.2)\, I$

$= 1.05\angle 13.213^\circ + (j0.2)\,(0.8034\angle -5.286^\circ)$

$= 1.111\angle 21.093^\circ$ p.u.

The equivalent network will be



Maximum power transfer, $P_{max} = \frac{|E|.|V|}{0.5}$

$$= \frac{1.111 \times 1}{0.5}$$

$$= 2.222 \text{ p.u.}$$

Power equation, $P_e = 2.222 \sin\delta$

Equation of motion of machine rotor,

$$M.\frac{d^2\delta}{dt^2} = P_s - P_e$$

Where, $M = \frac{H}{180\,f} = \frac{5}{180 \times 50} = \frac{1}{1800}$ s^2/elec.deg

$P_s = 0.8$ p.u.

Now, $\frac{1}{1800}.\frac{d^2\delta}{dt^2} = 0.8 - 2.22 \sin\delta$

(c) The open loop transfer function of unity feedback control system is given by

$$G(s) = \frac{K}{s(s+a)(s+b)} \quad 0 < a \le b$$

(i) Find the range of the gain constant K (> 0) for stability using Routh-Hurwitz criterion.

(ii) What type of control do you use if the system is required to have zero steady-state error for ramp input? Let 'A' be the parameter that can be varied in the introduced control. Find the range of 'K' for stability in terms of parameters a, b and A using Routh-Hurwitz criterion.

(20 M)

Sol: (i) $G(s) = \frac{k}{s(s+a)(s+b)}$ $[0 < a \le b]$, $H(s) = 1$

CE: $1 + G(s) = 0$

CE $s(s+a)(s+b) + k = 0$

$s(s^2+(a+b)s+ab) + k = 0$

CE $s^3 + (a+b)s^2 + abs + k = 0$

Routh – Hurwitz Criterion:

$$\begin{array}{c|cc} s^3 & 1 & ab \\ s^2 & (a+b) & k \\ s^1 & \left[\frac{ab(a+b)-k}{a+b}\right] > 0 \text{ for stability} & \\ s^0 & k > 0 \text{ for stability} & \end{array}$$

$[ab\,(a+b) - k] > 0 \Rightarrow k < [ab\,(a+b)]$

For CL stability $0 < k < [ab\,(a+b)]$

(ii)

Require PI controller to get zero steady state error for ramp input.

Transfer function of PI controller $= \left(1 + \frac{1}{As}\right) = \left(\frac{As+1}{As}\right)$

G(s) with controller $= \dfrac{k(As+1)}{s(s+a)(s+b)As}$, H(s) = 1

$= \dfrac{k(As+1)}{As^2(s+a)(s+b)}$, H(s) = 1

CE $\Rightarrow 1 + G(s) = 0$

$\Rightarrow As^2 (s + a)(s + b) + k(As + 1) = 0$

$\Rightarrow As^2 [s^2 +(a+b)s + ab] +kAs + k = 0$

$\Rightarrow As^4 + A(a+b)s^3 + Aabs^2 + kAs + k = 0$

$\Rightarrow s^4 + (a+b)s^3 + abs^2 + ks + \dfrac{k}{A} = 0$

Routh – Hurwitz Criterion:

$$
\begin{array}{c|ccc}
s^4 & 1 & ab & \dfrac{k}{A} \\
s^3 & (a+b) & k & \\
s^2 & \dfrac{ab(a+b)-k}{(a+b)} & \dfrac{k}{A} & \\
s^1 & \dfrac{\dfrac{[ab(a+b)-k]k}{a+b} - \dfrac{k(a+b)}{A}}{\left[\dfrac{ab(a+b)-k}{a+b}\right]} & & \\
s^0 & \dfrac{k}{A} & &
\end{array}
$$

For stability:

$\Rightarrow \dfrac{k}{A} > 0 \Rightarrow k > 0$

$\Rightarrow \left[\left[\dfrac{ab(a+b)-k}{(a+b)}\right]k - \dfrac{k(a+b)}{A}\right] > 0$

$$\Rightarrow\left[\frac{ab(a+b)-k}{a+b}-\frac{(a+b)}{A}\right]>0$$

$$\Rightarrow\left[\frac{ab(a+b)-k}{a+b}\right]>\frac{(a+b)}{A}$$

$$\Rightarrow k<\frac{(a+b)^2}{A}-ab(a+b)$$

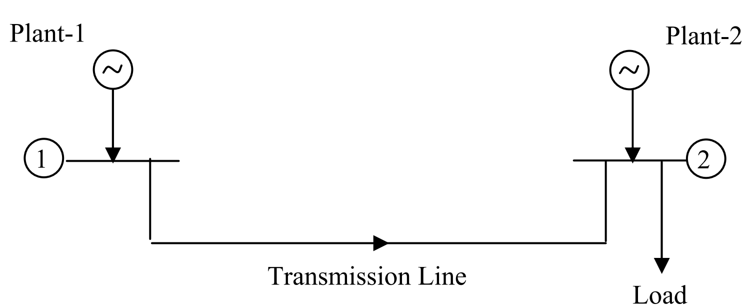
For stability $\Rightarrow 0<k<\left(\frac{(a+b)^2}{A}-ab(a+b)\right)$

Q. 7

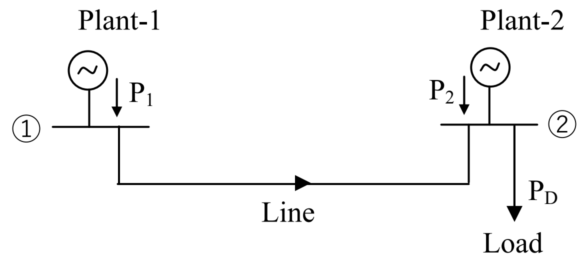
(a) A system consists of two plants connected by a transmission line and a load is at power plant-2 as shown in the figure. Data for the loss equation consists of the information that 200 MW transmitted from plant-1 to the results in transmission loss of 20 MW. Find the optimum generation schedule considering transmission losses to supply a load of 204.41 MW. Also evaluate the amount of financial loss that may be incurred if at the time of scheduling transmission losses are not coordinated. The incremental fuel cost characteristics of plant-1 and plant-2 are given by **(20 M)**

$$\frac{df_1}{dP_1}=0.025\,P_1+14 \quad ₹\text{/Mw-hr}$$

$$\frac{df_2}{dP_2}=0.05\,P_2+16 \quad ₹\text{/MW-hr}$$



Sol: Given two plant system



It is said that there is a loss of 20 MW for 200 MW power transfer from plant -1.

Power loss, $P_{loss} = B_{11}.\ P_1^2$

$20 = B_{11}\ .\ (200)^2$

$B_{11} = 5 \times 10^{-4}\ MW^{-1}$

$P_{loss} = (5 \times 10^{-4}).\ P_1^2$

Load demand, $P_D = 204.41$ MW

Power balance equation, $P_1 + P_2 = P_D + P_{loss}$

$P_1 + P_2 = 204.41 + (5 \times 10^{-4}).\ P_1^2$ ………..(1)

Coordination equation for optimal dispatch,

$$L_1.\frac{dF_1}{dP_1} = L_2.\frac{dF_2}{dP_2}$$

Penalty factors,

$$L_1 = \frac{1}{1 - \frac{dP_{loss}}{dP_1}} = \frac{1}{1 - 10^{-3}.P_1}$$

$$L_2 = \frac{1}{1 - \frac{dP_{loss}}{dP_2}} = \frac{1}{1-0} = 1$$

Now, $\left(\frac{1}{1 - 0.001P_1}\right)(0.025\ P_1 + 14) = 1 \times (0.05\ P_2 + 16)$

$$0.05\ P_2 = \left(\frac{0.025P_1 + 14}{1 - 0.001P_1}\right) - 16$$

$$= \frac{0.025P_1 + 14 - 16 + 0.016P_1}{1 - 0.001P_1}$$

$$0.05\ P_2 = \frac{0.041P_1 - 2}{1 - 0.001P_1}$$

$$P_2 = \frac{0.82P_1 - 40}{1 - 0.001P_1} \quad \ldots\ldots\ldots.(2)$$

Substitute equation (2) in (1)

$$P_1 + \left(\frac{0.82P_1 - 40}{1 - 0.001P_1}\right) = 204.41 + (5 \times 10^{-4})\ P_1^2$$

$$P_1(1 - 0.001\ P_1) + (0.82\ P_1 - 40) = (204.41 + 5 \times 10^{-4} \times P_1^2) \times (1 - 0.001\ P_1)$$

$$P_1 - 0.001\,P_1^2 + 0.82\ P_1 - 40 = 204.41 - 0.20441\ P_1 + 5 \times 10^{-4}P_1^2 - 5 \times 10^{-7}\ P_1^3$$

$$(5 \times 10^{-7}).\ P_1^3 - (1.5 \times 10^{-3}).P_1^2 + 2.02441\ P_1 - 244.41 = 0$$

By solving above equation,

$P_1 = 133.315$ MW

By using equation (2),

$$P_2 = \frac{0.82 \times 133.315 - 40}{1 - 0.001 \times 133.315}$$

$= 79.981$ MW

By considering and coordinating losses into optimum dispatch problem, the generation schedule,

$P_1 = 133.315$ MW, $P_2 = 79.981$ MW

$P_{loss} = (5 \times 10^{-4}) \times (133.315)^2$

$= 8.886$ MW

Fuel costs of plants,

$$F_1 = \int \frac{dF_1}{dP_1}.dP_1$$

$= \int (0.025\,P_1 + 14).dP_1$

$= 0.025\ \frac{P_1^2}{2} + 14\ P_1 + C_1$

$= 0.0125\ P_1^2 + 14\ P_1 + C_1$ Rs/hr

$F_2 = \int \frac{dF_2}{dP_2}.dP_2$

$= \int (0.05\,P_2 + 16).dP_2$

$= 0.05 \times \frac{P_2^2}{2} + 16\,P_2 + C_2$

$= 0.025\ P_2^2 + 16\ P_2 + C_2$ R_2/hr

Total cost, $F_T = F_1 + F_2$

At optimum schedule,

$F_{T_1} = 0.0125 \times (133.315)^2 + 14 \times 133.315 + C_1 + 0.025 \times (79.981)^2 + 16 \times 79.981 + C_2$

$= 3528.19 + C_1 + C_2$ Rs/hr

Optimum schedule without coordinating losses:

Coordination equation, $\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$

Power balance equation, $P_1 + P_2 = P_D + P_{loss}$

From coordination equation,

$0.025\ P_1 + 14 = 0.05\ P_2 + 16$

$0.05\ P_2 = 0.025\ P_1 - 2$

$P_2 = 0.5\ P_1 - 40$ ....(1)

From power balance equation,

$P_1 + 0.5\ P_1 - 40 = 204.41 + (5 \times 10^{-4})\,P_1^2$

$(5 \times 10^{-4}).\ P_1^2 - 1.5\ P_1 + 244.41 = 0$

By solving above equation,

$P_1 = 172.905$ MW

From (1), $P_2 = 0.5 \times 172.905 - 40$

$= 46.4525$ MW

Optimum schedule if losses are not coordinated,

$P_1 = 172.905$ MW, $P_2 = 46.4525$ MW

$P_{loss} = 14.948$ MW

Total cost of generation when losses are coordinated,

$F_{T_2} = F_1 + F_2$

$= 0.0125 \times (172.905)^2 + 14 \times 172.905 + C_1 + 0.025 \times (46.4525)^2 + 16 \times 46.4525 + C_2$

$= 3591.56 + C_1 + C_2$ Rs/hr

Financial loss incurred when the losses are not coordinated is $F_{T_2} - F_{T_1}$

$= (3591.56 + C_1 + C_2) - (3528.19 + C_1 + C_2)$

$= 63.38$ Rs/hr

(b) A continuous-time integrator has a system function $H_a(s) = \dfrac{1}{s}$

(i) Design a discrete-time integrator using bilinear transformation and find the difference equation relating the input x[n] to the output y[n] of the discrete-time system. **(10 M)**

(ii) Find the frequency response of the discrete-time integrator found in part (i) and determine whether or not this system is a good approximation of the continuous time system **(10 M)**

(For $\theta << 1$, $\sin\theta = \theta$ and $\cos\theta = 1 - \dfrac{\theta^2}{2}$)

Sol: Applying Bilinear transformation $s = \dfrac{2}{T}\left[\dfrac{1-z^{-1}}{1+z^{-1}}\right]$

Digital integrator $H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-Z^{-1}}{1+Z^{-1}}\right]} = \dfrac{1}{\dfrac{2}{T}\left[\dfrac{1-z^{-1}}{1+z^{-1}}\right]}$

$$H(z) = \frac{T}{2}\left[\frac{1+z^{-1}}{1-z^{-1}}\right];\ |z| > 1$$

$$\downarrow \text{I.Z.T}$$

Impulse response $h(n) = \frac{T}{2}u(n) + \frac{T}{2}u(n-1)$

$$\frac{Y(z)}{X(z)} = \frac{T}{2}\left[\frac{1+z^{-1}}{1-z^{-1}}\right]$$

$$Y(z) - z^{-1}\,Y(z) = \frac{T}{2}X(z) + \frac{T}{2}z^{-1}X(z)$$

$$\downarrow \text{I.Z.T}$$

$$y(n) = y(n-1) = \frac{T}{2}x(n) + \frac{T}{2}x(n-1)$$

$$y(n) = \frac{T}{2}[x(n) + x(n-1)] + y(n-1)$$

This system is not implementable since it has a pole on the circle and is not stable.

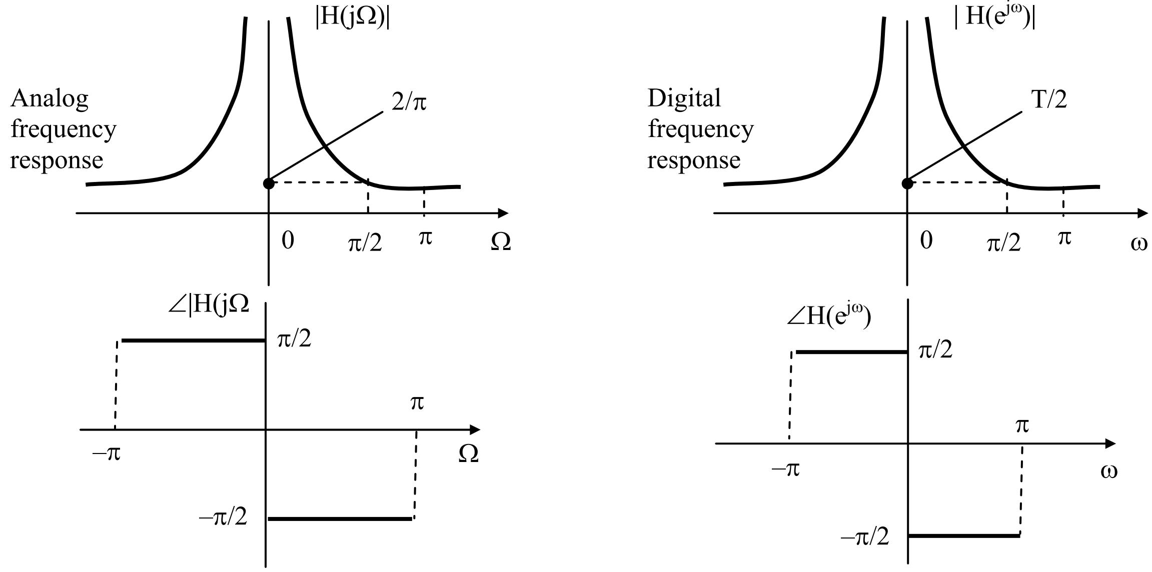
Since the system is not stable, it doesn't strictly has frequency response if we ignore

$$H(e^{j\omega}) = \frac{T}{2}\left[\frac{1+e^{-j\omega}}{1-e^{-j\omega}}\right] = \frac{T}{2}\frac{e^{-j\omega/2}\left[e^{j\omega/2} + e^{-j\omega/2}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

$$= \frac{T}{2j}\cos\left(\frac{\omega}{2}\right)$$

$$= \frac{T}{2}\frac{2\cos(\omega/2)}{2j\sin(\omega/2)}$$

$$H(s)\Big|_{s=j\Omega} = \frac{1}{j\Omega}$$



In general, we will not be able to approximate high frequencies, but we can approximate the lower frequencies if we choose $T = 4/\pi$

(c) For a 3-phase, 50 Hz, 415 V, 4-pole induction motor, the standstill resistance and reactance are 3.0 Ω and 5.0 Ω at 50 Hz respectively. The machine has magnetizing inductance of 350 mH and stator resistance of 1.2 Ω. The machine is supplied from a 3-phase voltage source inverter with quasi square wave output voltage waveform per phase as shown in the figure below. The DC bus voltage is 500 V.

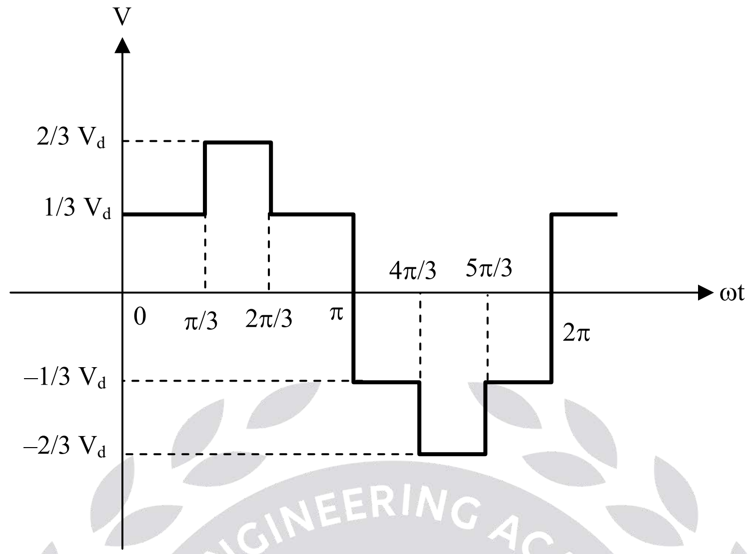
If the machine is operating at 4% slip, find

(i) the fundamental input current,

(ii) harmonic copper losses in the machine up to 13 harmonics, and

(iii) input power factor.

Assume negligible core losses, equal distribution of stator and rotor leakage reactances and linear magnetic circuit. **(20 M)**



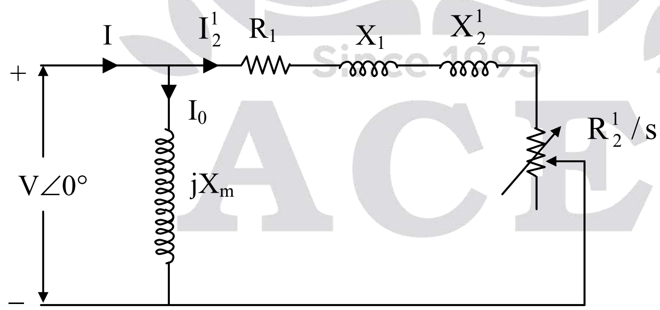
Sol: **Name plate details of motor:**

f = 50 Hz, 415, 4-pole, $R_2^1 = 3\Omega$, $X_2^1 = 5\Omega$

L_m = 350 mH, R_1 = 1.2 Ω, X_1 = 5Ω(∵ given equal distribution of stator & rotor leakage reactance) s = 4% and

V_d = 500V

The equivalent circuit of Induction motor for fundamental is shown below.



The r.m.s output voltage of inverter is $\dfrac{2V_d}{n\pi\sqrt{2}} = \dfrac{\sqrt{2}\,V_d}{n\pi}$

Where n = 1, 3, 5, 7, 9, 11, 13 …………

The third harmonic and triplent harmonics doesn't produce rotating magnetic field in machine.

$V_1\text{(r.m.s)} = \dfrac{\sqrt{2}\,V_d}{\pi} = \dfrac{\sqrt{2}\times 500}{\pi} = 225.07$

$V_5\text{ (r.m.s)} = \dfrac{\sqrt{2}\times 500}{5\pi} = 45.01\text{ V}$

$V_7\text{(r.m.s)} = 32.15\text{ V}$

$V_{11}\text{(r.m.s)} = 20.46\text{ V}$

$V_{13}\text{ (r.m.s)} = 17.31\text{ V}$

Resultant voltage $V_{R.M.S} = \sqrt{V_1^2 + V_5^2 + V_7^2 + V_{11}^2 + V_{13}^2} = 233.311\text{ V}$

$S_1 = \dfrac{N_{S_1} - N_r}{N_{S_1}} = \dfrac{1500 - 1440}{1500} = 0.04$

$S_5 = \dfrac{-5\times 1500 - 1440}{-5\times 1500} = 1.192$

$S_7 = \dfrac{7\times 1500 - 1440}{7\times 1500} = 0.862$

$S_{11} = \dfrac{-11\times 1500 - 1440}{-11\times 1500} = 1.08$

$S_{13} = \dfrac{13\times 1500 - 1440}{13\times 1500} = 0.92$

From circuit,

Fundamental component, $I_{21}^1 = \dfrac{V_1(\text{r.m.s})}{\left(R_1 + \dfrac{R_2^1}{S_1}\right) + j\left(X_1 + X_2^1\right)}$

$$= \frac{225.07\angle 0^\circ}{\left(1.2 + \dfrac{3}{0.04}\right) + j(5+5)}$$

$$= 2.92\angle -7.47^\circ$$

Similarly, $I_{25}^1 = \dfrac{V_5(\text{r.m.s})}{\left(R_1 + \dfrac{R_2^1}{S_5}\right) + j\left(X_1 + X_2^1\right)\times 5} = \dfrac{45.01\angle 0^\circ}{\left(1.2 + \dfrac{3}{1.192}\right) + j(5+5)5}$

$I^1_{25} = 0.8977\angle{-85.74°}$

Similarly $I^1_{27} = 0.458\angle{-86.17°}$

$I^1_{211} = 0.185\angle{-87.92}$

$I^1_{213} = 0.133\angle{-88.03}$

Magnetizing current, $I_{01} = \frac{V_1(\text{r.m.s})}{jX_{ml}} = \frac{225.07}{j(2\pi f_1 L_m)} = \frac{225.07}{j(109.9)} = 2.04\angle{-90°}$

Fundamental current, $I_1 = I_{01} + I^1_{21} = 2.04\angle{-90°} + 2.92\angle{-7.47°}$

$= 3.77\angle{-39.88}$

Fundamental input current = 3.77 A

Fundamental power factor = cos (39.88) = 0.767 lag

Per phase Harmonic Copper losses:

$= \left[(I^1_{25})^2 + (I^1_{27})^2 + (I^1_{211})^2 + (I^1_{213})^2\right](R_1 + R^1_2)$

$= 1.0675 \times [1.2+3]$

$= 4.48$ W

8(a) A 50 Hz, 3-phase induction motor has a slip of 0.2 for maximum torque, when operated on rated frequency and rated voltage. If the motor is run on 60 Hz supply with application of rated voltage, find the ratio of

(i) Starting Currents **(7 M)**

(ii) Starting torques **(7 M)**

(iii) Maximum torques **(6 M)**

With respecivet values at 50 Hz

Neglect the stator impedance.

Sol: At 50 Hz, $s_{Tmax} = \frac{R_2}{X_{20}} = 0.2$, where s_{Tmax} is the slip at maximum torque.

$R_2 = 0.2\ X_{20}$

If stator impedance and no-load current are neglected, the rotor current can be expressed as

$$I_2 = \frac{sE_{20}}{\sqrt{R_2^2 + (sX_{20})^2}}$$

E_{20} = EMF induced in the rotor winding at starting

At starting, s = 1, then rotor current at staring current $I_{2st} = \frac{E_{20}}{\sqrt{R_2^2 + (X_{20})^2}}$

Stator current at starting current $I_1 = I_{2st} = \frac{E_{20}}{\sqrt{R_2^2 + (X_{20})^2}}$

$E_{20}(1)$ is the induced voltage in the rotor at standstill with Rated voltage (V_1) and at rated frequency = 50 Hz.

$X_{20}(1)$ is the standstill rotor leakage reactance at 50 Hz = X_{20}

(i) Ratio of starting currents:

At rated voltage (V_1) and at rated frequency = 50 Hz

Stator current at starting current $I_1(1) = I_{2st}(1) = \frac{E_{20}(1)}{\sqrt{R_2^2 + (X_{20}(1))^2}}$

Stator current at starting current $I_1(1) = I_{2st}(1) = \frac{E_{20}(1)}{\sqrt{R_2^2 + (X_{20})^2}}$

$E_{20}(2)$ is the induced voltage in the rotor at standstill with Rated voltage (V_2) and at rated frequency = 60 Hz.

$X_{20}(2)$ is the standstill rotor leakage reactance at 60 Hz

$X_{20}(2) = \frac{6}{5}X_{20}(1) = \frac{6}{5}X_{20}$

At voltage (V_2) and at rated frequency = 60 Hz.

Stator current at starting current $I_1(2) = I_{2st}(2) = \frac{E_{20}(2)}{\sqrt{R_2^2 + (X_{20}(2))^2}}$

Stator current at starting current $I_1(2) = I_{2st}(2) = \frac{E_{20}(2)}{\sqrt{R_2^2 + \left(\frac{6}{5}X_{20}\right)^2}}$

Voltage $V_2 = V_1$; $E_{20}(1) = E_{20}(2)$

$$\frac{I_1(2)}{I_1(1)} = \frac{\dfrac{E_{20}(2)}{\sqrt{R_2^2 + \left(\frac{6}{5}X_{20}\right)^2}}}{\dfrac{E_{20}(1)}{\sqrt{R_2^2 + (X_{20})^2}}} = \frac{\sqrt{R_2^2 + (X_{20})^2}}{\sqrt{R_2^2 + \left(\frac{6}{5}X_{20}\right)^2}} = \frac{\sqrt{\left(\frac{R_2}{X_{20}}\right)^2 + 1}}{\sqrt{\left(\frac{R_2}{X_{20}}\right)^2 + \left(\frac{6}{5}\right)^2}} = \frac{\sqrt{(0.2)^2 + 1}}{\sqrt{(0.2)^2 + (1.2)^2}} = 0.838$$

$I_1(2) = 0.838\, I_1(1)$

(ii) If the stator impedance is neglected, the starting torque $T_{st} = \frac{180}{2\pi N_s}\frac{E_{20}^2 R_2}{R_2^2 + X_{20}^2}$

At rated voltage (V_1) and at rated frequency = 50 Hz

$$T_{st}(1) = \frac{180}{2\pi N_s(1)}\frac{E_{20}^2(1) R_2}{R_2^2 + (X_{20}(1))^2}$$

$N_s(1) = N_s$; $X_{20}(1) = X_{20}$

$$T_{st}(1) = \frac{180}{2\pi N_s}\frac{E_{20}^2(1) R_2}{R_2^2 + (X_{20})^2}$$

At rated voltage (V_2) and at frequency = 60 Hz

$$T_{st}(2) = \frac{180}{2\pi N_s(2)}\frac{E_{20}^2(2) R_2}{R_2^2 + (X_{20}(2))^2}$$

$N_s(2) = \frac{6}{5}N_s(1) = \frac{6}{5}N_s$; $X_{20}(2) = \frac{6}{5}X_{20}(1) = \frac{6}{5}X_{20}$

$$T_{st}(2) = \frac{180}{2\pi \frac{6}{5} N_s}\frac{E_{20}^2(2) R_2}{R_2^2 + (\frac{6}{5}X_{20})^2}$$

Voltage $V_2 = V_1$; $E_{20}(1) = E_{20}(2)$

Ratio of starting Torque,

$$\frac{T_{st}(2)}{T_{st}(1)} = \frac{\dfrac{180}{2\pi\frac{6}{5}N_s}\dfrac{E_{20}^2(2)R_2}{R_2^2+(\frac{6}{5}X_{20})^2}}{\dfrac{180}{2\pi N_s}\dfrac{E_{20}^2(1)R_2}{R_2^2+(X_{20})^2}} = \frac{5}{6}\frac{R_2^2+(X_{20})^2}{R_2^2+(\frac{6}{5}X_{20})^2}$$

$$= \frac{5}{6}\frac{\left(\dfrac{R_2}{X_{20}}\right)^2+1}{\left(\dfrac{R_2}{X_{20}}\right)^2+\left(\dfrac{6}{5}\right)^2} = \frac{5}{6}\frac{(0.2)^2+1}{(0.2)^2+(1.2)^2} = 0.585$$

$T_{st}(2) = 0.585\ T_{st}(1)$

(iii) If the stator impedance is neglected, the maximum torque, $T_{max} = \dfrac{180}{2\pi N_s}\dfrac{E_{20}^2}{2X_{20}}$

At rated voltage (V_1) and at rated frequency = 50 Hz

$$T_{max}(1) = \frac{180}{2\pi N_s(1)}\frac{E_{20}^2(1)}{2X_{20}(1)}$$

$N_s(1) = N_s$; $X_{20}(1) = X_{20}$

$$T_{max}(1) = \frac{180}{2\pi N_s}\frac{E_{20}^2(1)}{2X_{20}}$$

At rated voltage (V_2) and at frequency = 60 Hz

$$T_{max}(2) = \frac{180}{2\pi N_s(2)}\frac{E_{20}^2(2)}{2X_{20}(2)}$$

$N_s(2) = \dfrac{6}{5}N_s(1) = \dfrac{6}{5}N_s$; $X_{20}(2) = \dfrac{6}{5}X_{20}(1) = \dfrac{6}{5}X_{20}$

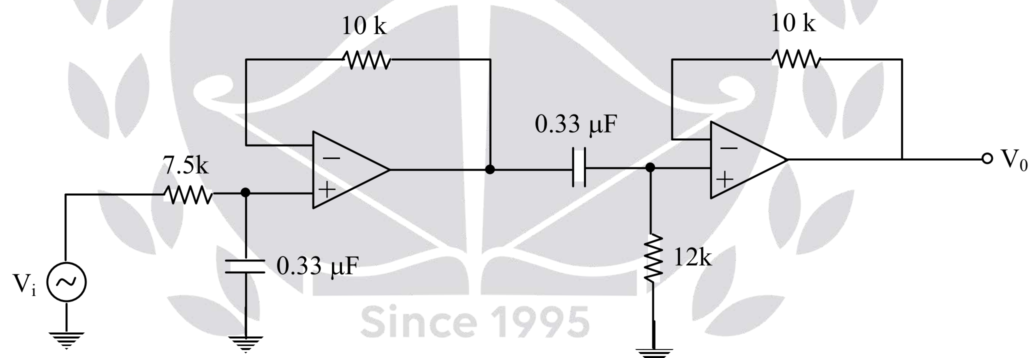
$$T_{max}(2) = \frac{180}{2\pi\frac{6}{5}N_s}\frac{E_{20}^2(2)}{2\times\frac{6}{5}X_{20}}$$

Ratio of maximum Torque,

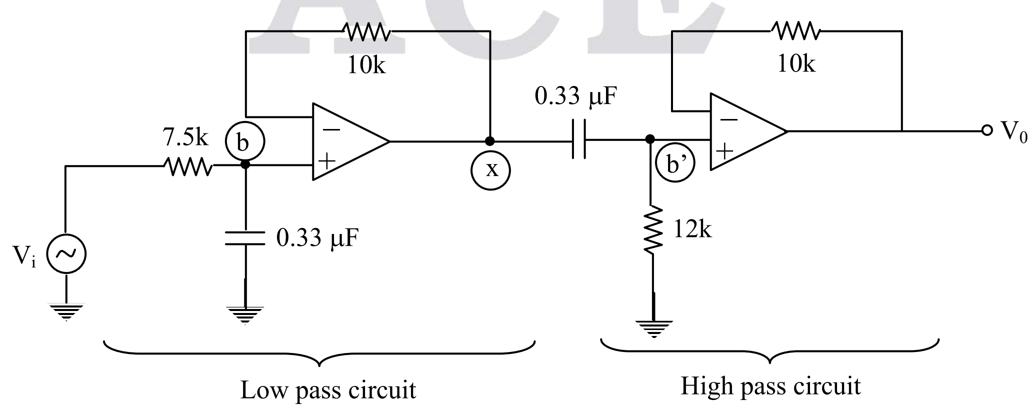
$$\frac{T_{max}(2)}{T_{max}(1)} = \frac{\dfrac{180}{2\pi\frac{6}{5}N_s}\,\dfrac{E_{20}^2(2)}{2\times\frac{6}{5}X_{20}}}{\dfrac{180}{2\pi N_s}\,\dfrac{E_{20}^2(1)}{2X_{20}}} = \left(\frac{5}{6}\right)^2 = 0.694$$

$T_{max}(2) = 0.694\ T_{max}(1)$

(b) The current of an induction motor is sensed through a suitable arrangement and converted to equivalent voltage. The current contains fundamental and higher order 5th and 7th harmonics. In order to separate the fundamental, the equivalent voltage waveform is passed through the following circuit as given in figure. Find the (i) Cut-off frequencies of each section (ii) Overall gain attenuation in dB for fundamental, 5th & 7th harmonics and (iii) Overall phase shift of the measured fundamental current. **(20 M)**



Sol:



Since $V_d = 0$, $V_b = V_x$, $V_0 = V_b'$

$$V_b = V_i\left(\frac{\frac{1}{SC_1}}{\frac{1}{SC_1}+R_1}\right) = V_i\left(\frac{1}{1+SC_1R_1}\right)$$

$$V_b = V_i\left(\frac{\frac{1}{R_1C_1}}{S+\frac{1}{R_1C_1}}\right) \rightarrow \text{upper cut off frequency of LPF is } \frac{1}{R_1C_1} = 404.04 \text{ rad/sec}$$

$$V_b' = V_x\left(\frac{R_2}{R_2+\frac{1}{SC_2}}\right) = V_x\left(\frac{SC_2R_2}{1+SC_2R_2}\right) = V_x\left(\frac{S}{S+\frac{1}{R_2C_2}}\right)$$

Lower cutoff frequency of HP circuit is

$$(f)_L = \frac{1}{R_2C_2} = \frac{1}{12000\times 0.33\times 10^{-6}} = 252.52 \text{ rad/sec}$$

(ii) The cutoff frequency of 1st section (LPCkt) 404 rad/sec

2nd section (HPCkt) 252.52 rad/sec

(ii) Then the transfer function is

$$H(s) = H_1(s).H_2(s) = \frac{SC_2R_2}{(1+SC_1R_1)(1+SC_2R_2)}$$

Put $S = j\omega$ $\quad H(s) = \dfrac{j\omega C_2R_2}{(1+j\omega C_1R_1)(1+j\omega C_2R_2)}$

The magnitude of transfer function is $\qquad R_1C_1 = 2.475$ ms

$$|H(\omega)| = \frac{\omega C_2R_2}{\sqrt{1+(\omega C_1R_1)^2}\sqrt{1+(\omega C_2R_2)^2}}$$

$R_2C_2 = 3.96$

Approximate value is ω (for fundamental frequency f = 50 Hz)

$\omega_1 = 314.159$ rad/sec

$\omega_5 = 1570.795$ rad/sec

$\omega_7 = 2199.113$ rad/sec

$\omega_1 R_1 C_1 = 314.159 \times 2.475 \times 10^{-3} = 0.77754$

$\omega_1 R_2 C_1 = 1.244$

$$H_1(\omega) = \frac{1.244}{\sqrt{(1.6045)(2.5475)}} = \frac{1.244}{2.021} = 0.6153$$

$(H_1(\omega)) = 20 \log(0.6153)$

$= -4.218$ dB

For 5th Harmonic: $\omega_5 = 5\omega_1 = 1570.796$

$\omega_5 R_1 C_1 = 3.8877$

$\omega_5 R_2 C_2 = 6.2203$

$$|H_5(\omega)| = \frac{6.2203}{\sqrt{(16.114)(39.692)}} = \frac{6.2203}{25.29} = 0.245956$$

$H_5(\omega) = 20 \log (0.245956)$

$= -12.18$ dB

For 7th harmonic:

$\omega_7 = 7\omega_1 = 350 \times 2\pi = 2199.113$ rad/sec

$\omega_7 R_1 C_1 = 5.4428$

$\omega_7 R_2 C_2 = 8.7085$

$$|H_7(\omega)| = \frac{8.7085}{\sqrt{(30.624)(76.8379)}} = \frac{8.7085}{48.5086} = 0.17952$$

$|H_7(\omega)| = 20 \log (0.17952)$

$= -14.9175$ dB

(iii) The Transfer function is

$$H(\omega) = \frac{j\omega C_2 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)}$$

The overall phase shift measured is $\phi = \dfrac{90^{o}}{\tan^{-1}(\omega C_1 R_1) + \tan^{-1}(\omega C_2 R_2)}$

$\phi = 90 - \tan^{-1}(\omega C_1 R_1) - \tan^{-1}(\omega C_2 R_2)$

For fundamental freuquency

$$\phi = 90 - \tan^{-1}(0.77754) - \tan^{-1}(1.244)$$
$$= 90 - 37.866 - 51.20$$
$$= 0.9283°$$

(c) Given the following facts about a real signal x(t) with Laplace transform X(s):

A: X(s) has exactly two poles

B: X(s) has no zeros in the infinite s-plane

C: X(s) has a pole at $s = -1 + j$

D: $e^{2t}\,x(t)$ is not absolutely integrable

E: X(0) = 8

Determine X(s) and Specify its region of convergence **(10M +10M)**

Sol: Since x(t) is real, poles of X(s) must occur in conjugate pair.

$$X(s) = \frac{k}{(s+1-j)(s+1+j)} \qquad \text{From “C”} = \frac{k}{s^2+2s+2}$$

As X(0) = 8 from (E)

$$\downarrow$$

$$\frac{k}{2} = 8 \Rightarrow k = 16$$

$$\therefore X(s) = \frac{16}{s^2+2s+2} = \frac{16}{(s+1)^2+(1)^2}$$

There are 2 possible ROCs, $Re\{s\} > -1$ (or) $Re\{s\} < -1$

From fact (D), $y(t) = e^{2t}\,x(t)$ $\qquad x(t)\,e^{s_0 t} \leftrightarrow X(s - s_0)$

$Y(s) = X(s-2)$ $\qquad ROC = R + Re\{s_0\}$

ROC of X(s) as

If we choose, $Re\{s\} > -1$, then Roc of $x(t)\,e^{2t}$ is $\sigma > -1 + 2$

$\sigma > 1 \rightarrow$ satisfying fact(D) which is not including jω-axis

$$\therefore X(s) = \frac{16}{s^2+2s+2};\ \sigma > -1$$