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ESE-2020 (MAINS)

QUESTIONS WITH DETAILED SOLUTIONS

ELECTRICAL ENGINEERING

PAPER-I

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ELECTRICAL ENGINEERING

ESE_MAINS_2020_PAPER – I

Questions with Detailed Solutions

SUBJECT WISE WEIGHTAGE

S.No	NAME OF THE SUBJECT	Marks
01	Engineering Mathematic	72
02	Electrical Materials	72
03	Electric Circuits and Fields	52 + 44
04	Electrical and Electronic Measurements	84
05	Computer Fundamentals	72
06	Basic Electronics Engineering	84

SECTION - A

1(a)(i) Obtain the partial differential equation governing the equations

f(u, v) = 0, u = x + yz, v = x + y + z **[8M]**

Sol: f(u,v) = 0, where $u = x + y z$

$v = x + y + z$

Now, we can also write

$u = f(v)$

$x + yz = f(x + y + z)$

Differentiating wrt x partially

$1 + yp = f'(x + y + z)(1 + p)$ where $p = \frac{\partial z}{\partial x}$

Differentiating wrt y partially

$yq + z = f'(x + y + z)(1 + q)$, where $q = \frac{\partial z}{\partial y}$

$$\frac{1+yp}{yq+z} = \frac{f'(x+y+z)(1+p)}{f'(x+y+z)(1+q)}$$

$(1 + yp)(1 + q) = (1 + p)(yq + z)$

$1 + q + py + pqy = qy + z + pqy + pz$

$pz + z + qy - py - q - 1 = 0$

1(a)(ii) Construct a partial differential equation of all surfaces of revolution having z-axis as the axis of rotation. **[4M]**

Sol: Surface of revolution about z-axis is

$z = f(x^2 + y^2)$

$p = 2xf'(x^2 + y^2)$, where $p = \frac{\partial z}{\partial y}$

$q = 2yf'(x^2 + y^2)$ where $q = \frac{\partial z}{\partial y}$

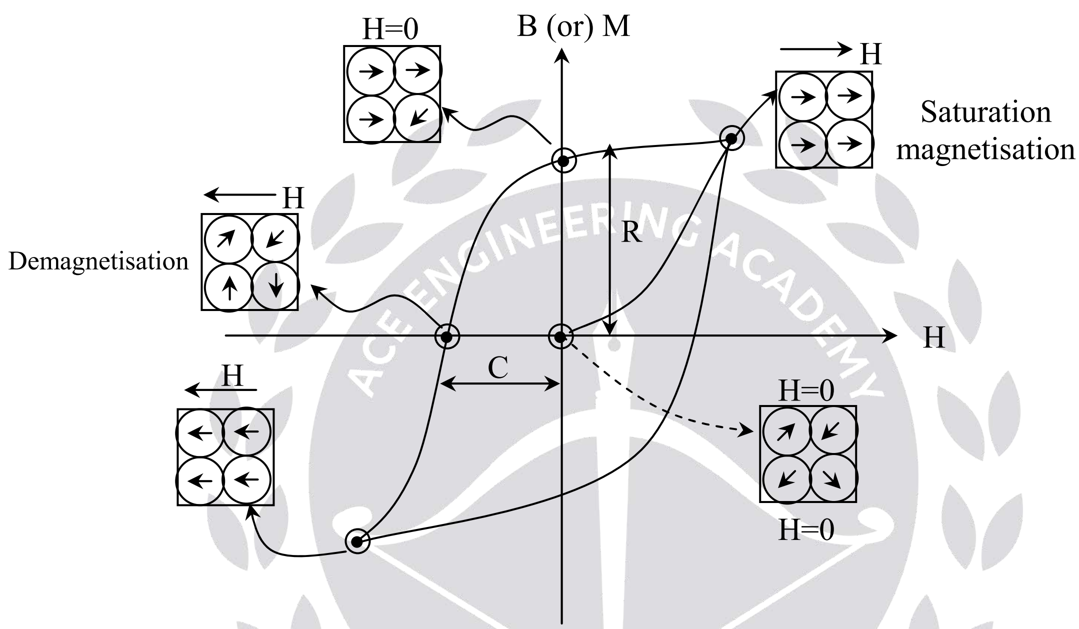
$$\frac{p}{q} = \frac{2xf'(x^2+y^2)}{2yf'(x^2+y^2)}$$

$py = qx$

$py - qx = 0$

1(b) Discuss the phenomena of Hysteresis in ferromagnetic materials. On the B-H curve show the retentivity, coercivity and saturation points. What is coercive force and the energy dissipated per unit volume of the ferromagnetic substance during the hysteresis cycle? [12M]

Sol: Hysteresis loop behaviour of ferromagnetic material:



The ferromagnetic materials generates hysteresis losses and the area of hysteresis loop represent hysteresis losses (J/m^3)

Retentivity (or) Ramnet Induction: It is the retained magnetization (or) magnetic flux density present in material without applied magnetic field (At H = 0, B = ?)

Coercivity (or) Coercive force: It is the required applied magnetic field in opposite direction to demagnetize the material

For $B = 0 \Rightarrow H = ?$

1(c) Explain and derive continuity of current equation using the principle of conservation of charge. [12M]

Sol: Continuity equation for time varying fields:

Current is nothing but charge in motion. Consider some volume. The total current flowing OUT of some volume must be equal to the rate of decrease of charge $\left(-\frac{dq}{dt}\right)$ within the volume (law of conservation of charge)

$$I_{out} = -\frac{dq}{dt}$$

q = charge within the volume

$q = \int \rho_v \, dv$

$$\therefore \ I_{out} = \frac{-d}{dt}\int \rho_v dv$$

$\oint \bar{J}.d\bar{s} = I_{out}$ From current density concept

$$\therefore \ \oint \bar{J}.d\bar{s} = -\frac{d}{dt}\int \rho_v dv$$

$$\Rightarrow \oint \bar{J}.d\bar{s} = -\int \frac{\partial \rho_v}{\partial t} dv \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

From the divergence theorem

$$\oint \bar{A}.d\bar{s} = \int \nabla.\bar{A} dv$$

$$\oint \bar{J}.d\bar{s} = \int \nabla.\bar{J} dv \qquad \ldots\ldots\ldots (2)$$

From (1) and (2)

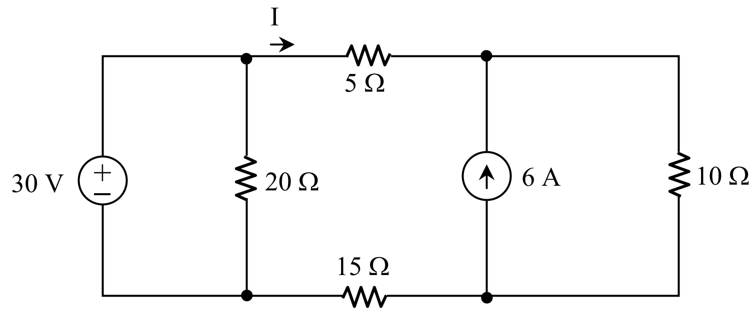
$$\therefore \ \int \nabla.\bar{J} dv = -\int \frac{\partial \rho_V}{\partial t} dv$$

$$\nabla.\bar{J} = -\frac{\partial \rho_V}{\partial t}$$

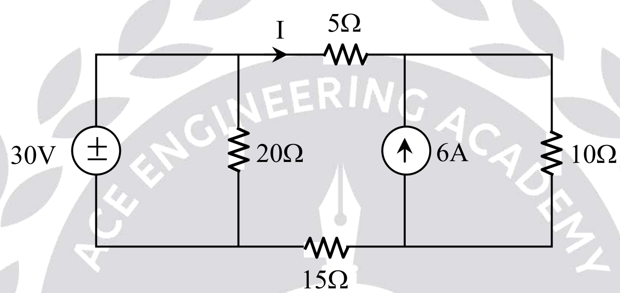
This is called continuity equation for time varying fields.

Where as $\nabla.\bar{J} = 0$ for static fields

1(d) Using the principle of superposition, determine the current I in the 5 Ω resistor in the circuit shown in the figure. [12M]

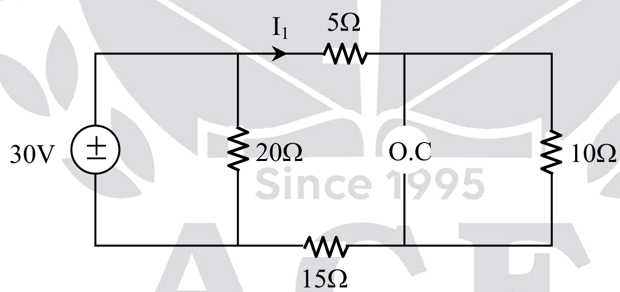


Sol:



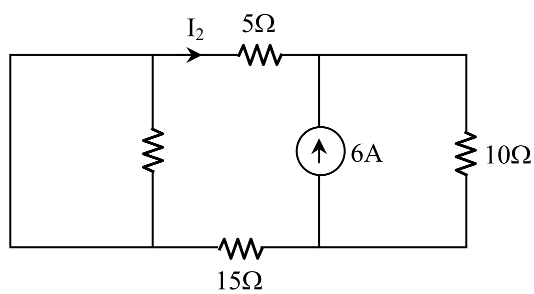
By using superposition theorem,

Take 30 V same (6A → open circuit):



$$I_1 = \frac{30}{5+10+15} = 1\text{ A}$$

Take 6 A source (30 V → short circuit):



From current division rule,

$$I_2 = \frac{-6\times 10}{30} = -2 \text{ Amps}$$

By Superposition theorem $I = I_1 + I_2$

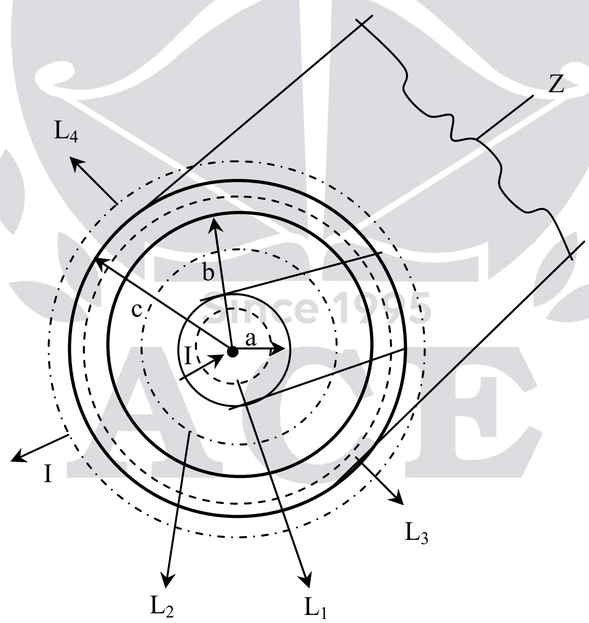
$$I = 1 - 2 = -1 \text{ Amps}$$

1(e) State Ampere's circuital law. A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find $\vec{H}$ everywhere. [12M]

Sol: Infinity long co-axial transmission line consider infinitely by co-axial transmission line with inner conductor of radius 'a' and outer conductor of radius 'b' and thickness 't' outer conductors outer radius is c.

$\therefore$ c – b = t.

Let the inner conductor carries I current and outer conductor carries –I current.



To determine $\overline{H}$ every where, four possible regions are to be taken.

(i) $0 \le \rho < a$

(ii) $a \le \rho < b$

(iii) $b \le \rho < c$

(iv) $\rho \ge c$

Apply Ampere's law for the region (i) take the amperian path L_1 whose radius is less than a

$$\oint_{L_1} \overline{H}.d\overline{L} = I_{enc} = \int \overline{J}.d\overline{S}$$

Area of the cylinder = πa^2

$$\overline{J} = \frac{I}{\pi a^2}\hat{a}z$$

$$d\overline{s} = \rho\, d\rho\, d\phi\, \hat{a}z$$

$$I_{enc} = \int \overline{J}.d\overline{S} = \int_0^a \int_0^{2\pi} \frac{I}{\pi a^2}\hat{a}z \rho d\rho d\phi \hat{a}z$$

$$= \frac{I}{\pi a^2}\int_0^{\rho}\int_0^{2\pi} \rho d\rho d\phi \qquad (\because\ \rho < a)$$

$$= \frac{I}{\pi a^2}\left[\frac{\rho^2}{2}\right]_0^a [\phi]_0^{2\pi} = \frac{I}{\pi a^2}\pi a^2$$

For the region $0 \le \rho < a$

$$I_{enc} = \frac{I}{\pi a^2}\frac{\rho^2}{2}[2\pi]$$

$$I_{enc} = \frac{I\rho^2}{a^2}$$

$$\oint \overline{H} d\overline{L} = I_{enc} = \frac{I\rho^2}{a^2}$$

$$H_\phi \int dl = H_\phi \int_0^{2\pi} \rho d\phi = H_\phi \rho 2\pi = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho^2}{\rho 2\pi a^2} = \frac{I\rho}{2\pi a^2}$$

$$\overline{H} = \frac{I\rho}{2\pi a^2}\hat{a}_\phi \text{ or } 0 \le \rho < a$$

Apply Amperian path L_2 for region (ii) $a \le \rho < b$

$$\oint \overline{H}.d\overline{L} = I_{enc} = I \qquad \text{Since } L_2 \text{ contains only current I}$$

$$\int H_\phi \hat{a}_\phi . \rho d\phi \hat{a}_\phi = I$$

$$\int_0^{2\pi} H_\phi \rho d\phi = I$$

$$\rho H_\phi(2\pi) = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\overline{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \text{ for } a \le \rho \le b$$

Apply Amperian path L_3 for region (3) $b \le \rho < c$

$$\oint \overline{H}.d\overline{L} = I_{enc}$$

$$\int_0^{2\pi} H_\phi \hat{a}_\phi.\rho d\phi \hat{a}_\phi = I_{enc}$$

$$H_\phi(2\pi\rho) = I_{enc}$$

$$H_\phi(2\pi\rho) = I_{\text{of inner conductor}} + I_{\text{of outer condcutor}}$$

For $\rho < c$, the current enclosed is less than $-$ I. That is given by $\int \overline{J}.d\overline{S}$

$$H_\phi(2\pi\rho) = I + \int \overline{J}.d\overline{S}$$

$\overline{J}$ = current density of outer conductor $\overline{J}$ is in $-\hat{a}_z$ direction.

$$\overline{J} = \frac{-I}{\pi(c^2 - b^2)}\hat{a}_z$$

$$\therefore H_\phi(2\pi\rho) = I + \left[\int \frac{-I}{\pi(c^2-b^2)}\hat{a}_z.\rho d\rho d\phi \hat{a}_z\right]$$

$$= I + \left(\frac{-I}{\pi(c^2-b^2)}\right)\int_{\rho=b}^{\rho}\int_0^{2\pi} \rho d\rho d\phi$$

$$= I - \frac{I}{\pi(c^2-b^2)}\left[\frac{\rho^2}{2}\right]_b^\rho \times [\phi]_0^{2\pi}$$

$$= I - \frac{I}{\pi(c^2-b^2)}\frac{\rho^2-b^2}{2} \times 2\pi$$

$$= I - \frac{I(\rho^2-b^2)}{c^2-b^2} = I\left[1 - \frac{(\rho^2-b^2)}{c^2-b^2}\right]$$

Apply Amperian path L_4 for region 4 $\rho \geq c$

For region 4, $I_{enc} = I_{inner} + I_{outer}$

$$= I + (-I)$$

$$= 0$$

$$\therefore \oint \overline{H}.d\overline{L} = I_{enc}$$

$$\oint \overline{H}.d\overline{L} = 0$$

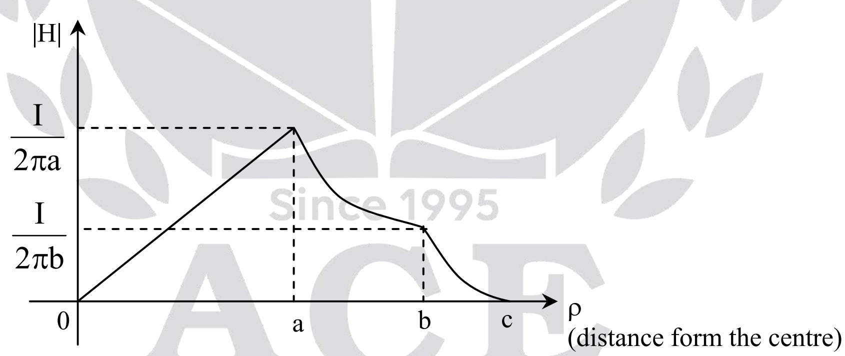
$$H_\phi = 0$$

$$\therefore \overline{H} = \frac{I\rho}{2\pi a^2}\hat{a}_\phi \text{ for } \rho < 0$$

$$= \frac{I}{2\pi\rho}\hat{a}_\phi \text{ for } a \leq \rho < b$$

$$= \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{c^2 - b^2}\right]\hat{a}_\phi \text{ for } b \leq \rho < c$$

$$= 0 \text{ for } \rho \geq c$$



2(a)(i) If a square matrix A of order n with entries in field F has n distinct eigenvalues, then prove that matrix A is similar to a diagonal matrix [10M]

Sol: Let A be an $n \times n$ matrix with n distinct eigen values

The corresponding eigen vectors of A are linearly independent

Now, $Ax_i = \lambda x_i$, $i = 1, 2, \ldots\ldots\ldots n$

Let $P = [x_1\ x_2 \ldots\ldots\ldots\ldots x_n]$

$\therefore [Ax_1\ Ax_2 \ldots\ldots\ldots\ldots\ldots Ax_n] = [\lambda x_1\ \ \lambda x_2 \ldots\ldots\ldots\ldots \lambda x_n]$

$$= [x_1 x_2 \ldots\ldots x_n]\begin{bmatrix} \lambda_1 & 0\ldots\ldots\ldots\ldots 0 \\ 0 & \lambda_2 \ldots\ldots\ldots\ldots 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ 0 & 0\ldots\ldots\ldots\ldots \lambda_n \end{bmatrix}$$

$$\therefore AP = PD \qquad \text{where } D = \begin{bmatrix} \lambda_1 & 0\ldots\ldots\ldots\ldots 0 \\ 0 & \lambda_2 \ldots\ldots\ldots\ldots 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ 0 & 0\ldots\ldots\ldots\ldots \lambda_n \end{bmatrix}$$

$P^{-1}AP = P^{-1}PD$

$P^{-1}AP = ID$

$p^{-1}AP = D$

[$\because$ columns of P are linearly independent so P is non- singular]

$\therefore$ A is similar to diagonal matrix D

$\therefore$ A is diagonalizable.

2(a)(ii) Find the matrix P which diagonalizes the matrix associated with the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$. [10M]

Sol: Given quadtratic form is $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$

Matrix of quadratic form is , $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

The characteristic eqn of A is, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

Where $S_1 = [tr(A)] = 3 + 5 + 3 = 11$

S_2 = sum of minors of principal diagonal elements

$$S_2 = \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix}$$

$= (14) + (8) + (14)$

$= 36$

$S_3 = \det(A) = 3(15 - 1) + (-3 + 1) + (1 - 5)$

$S_3 = 36$

Hence $\lambda^3 - 11\lambda^2 + 36\lambda -`36 = 0$

On solving we get, $\lambda = 2, 3, 6$

The eigen vectors are given by $(A - \lambda I)x = 0$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = 2$, $$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

That is $x_1 - x_2 + x_3 = 0$

$-x_1 + 3x_2 - x_3 = 0$

$x_1 - x_2 + x_3 = 0$

On solving, we get $X_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

When $\lambda = 3$, $$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

That is $-x_2 + x_3 = 0$

$-x_1 + 2x_2 - x_3 = 0$

$x_1 - x_2 = 0$

We get, $X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

When $\lambda = 6$, $$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

That is $-3x_1 - x_2 + x_3 = 0$

$$-x_1 - x_2 - x_3 = 0$$

$$x_1 - x_2 - 3x_3 = 0$$

We get, $X_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Hence $P = [X_1 \quad X_2 \quad X_3]$

$$P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

2(b)(i) On the basis of specific resistance ρ, discuss the difference between conductors, semi-conductors and insulators. [8M]

Sol: Conducting material may be classified into three groups: conductors, semiconductors and imperfect insulators.

Specific resistance is also known as resistivity i.e resistance perunit length of specimen.

Conductors: The specific resistance of conducting material is very low. Specific resistance of conducting materials increases with increasing temperature and hence these materials are also known as positive temperature coefficient of resistance materials.

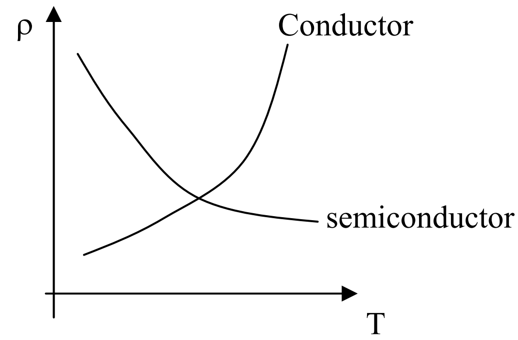
Ex; All metals

Metal	Specific resistance (ρ)
Cu	0.034×10^{-5}
Fe	32.54×10^{-5}
A*l*	0.03×10^{-5}
Ni	0.046×10^{-5}

Semiconductors: The specific resistance of semi conductor is high. Specific resistance of semiconductor decreases with increasing temperature and adding impurities and hence these materials are also known as negative temperature coefficient of resistance materials.

Ex; Si , Ge

Semiconductor	Specific resistivity
Ge	4.6×10^{-1}
Si	6.4×10^{2}

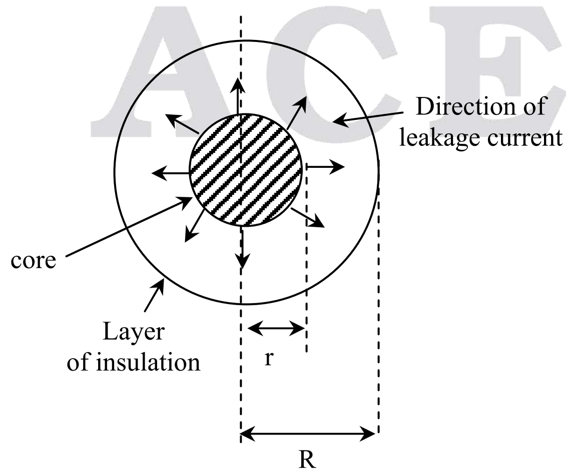


Insulators: The specific resistance of insulators is very high and these materials resist the flow of electric current due to high specific resistivity.

Insulator	Specific resistivity
Glass	10^{12}
Mica	9×10^{13}
Quartz	5×10^{16}

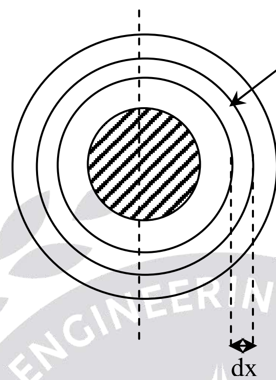
2b(ii) Prove that insulation resistance of a cable is inversely proportional to its length. Define insulation resistance. [12M]

Sol: Insulation resistance: The resistance offered by cable to path of the leakage current.



The above figure shows the section of a single core cable which is insulated with the help of layer of an insulating material.

The leakage current flows radially from ccentre towards the surface. Hence the cross section of the path of such current is not constant but changes with its length.



Consider an elementary section of a cylindrical cable of radius x and thickness dx as shown in fig.

d = distance of conductor core (2r)

D = diameter with sheath

Cross section area = $2\pi x \times l$

The resistance of this elementary cylindrical shell is $dR_i = \dfrac{\rho dx}{2\pi x \ell}$

ρ = Resistivity of the insulating material

The total insulation resistance of the cable can be obtained by integrating the resistance of an elementary ring from inner radius (r) upto outer radius (R)

$$R_i = \int_r^R dR_i = \int_r^R \rho\left(\frac{dx}{2\pi x\ell}\right) = \frac{\rho}{2\pi\ell}\int_r^R\left(\frac{dx}{x}\right) = \frac{\rho}{2\pi\ell}\ln x\Big|_r^R$$

$$R_i = \frac{\rho}{2\pi\ell}\ln\frac{R}{r}$$

The value of insulation resistance (R_i) is always high. The expression shows that the insulation resistance is inversely proportional to its length.

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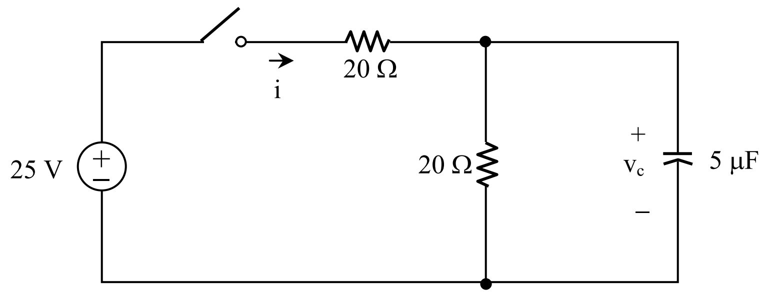
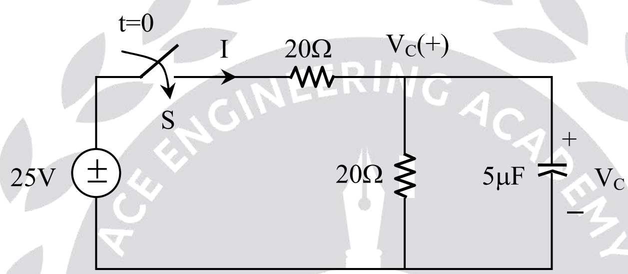
2(c) In the circuit shown in the figure given below, the switch is closed at t = 0. Obtain the current i and capacitor voltage v_c ,for t > 0. [20M]

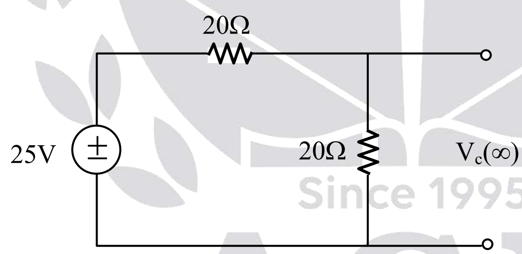


Sol:



For t > 0, S is closed,

For final value at t = ∞ (S-S), C → open circuit



By voltage division rule,

$$\frac{25\times 20}{20+20} = 12.5 \text{ volts}$$

$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]\, e^{\frac{-t}{\tau}} \quad t \geq 0$$

$$\tau = R_{eq}C = (20||20)54 = 10 \times 5 = 50 \ \mu\text{sec}$$

$$V_C(t) = 12.5 + (0-12.5)\, e^{\frac{-t}{50\mu}}$$

$$= 12.5\left(1 - e^{\frac{-t}{50\mu}}\right) \text{ volts}$$

$$i(t) = \left(\frac{25 - V_C(t)}{20}\right)$$

$$= \frac{25 - 12.5\left(1 - e^{\frac{-t}{50\mu}}\right)}{20}$$

$$= \frac{12.5 + 12.5e^{\frac{-t}{50\mu}}}{20}$$

$$= \frac{12.5}{20}\left(1 + e^{\frac{-t}{50\mu}}\right)$$

$$i(t) = 0.625\left(1 + e^{\frac{-t}{50\mu}}\right) \text{ amps}$$

3(a)(i) If the density function of a continuous random variable is given by

$$\mathbf{f(x) =} \begin{cases} 0 & x < 0 \\ ax, & 0 \le x \le 2 \\ (4 - x)a, & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

(p) Find value of a

(q) Find the cumulative distribution function (cdf)

(r) Find P(X > 2.5) **[10M]**

Sol: Given,

$$f(x) = \begin{cases} 0, & x < 0 \\ ax, & 0 \le x \le 2 \\ (4 - x)a, & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

(p) To find ‘a’, we have

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_0^2 ax\,dx + \int_2^4 (4-x)a\,dx = 1$$

$$\Rightarrow \left(\frac{ax^2}{2}\right)_0^2 + \left(4ax - \frac{ax^2}{2}\right)_2^4 = 1$$

$$\Rightarrow (2a) + \{(16a - 8a) - (8a - 2a)\} = 1$$

$$\Rightarrow (2a) + (2a) = 1$$

$$\Rightarrow a = \frac{1}{4}$$

(q) The cumulative distribution function (CDF) is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$$

For any x such that $-\infty < x \le 0$

$$F(x) = \int_{-\infty}^{0} 0dx = 0$$

For any x, where $0 < x \le 2$;

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)dx$$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{x} ax\,dx$$

$$= \left(\frac{ax^2}{2}\right)_0^x$$

$$= \frac{ax^2}{2}$$

$$= \frac{x^2}{8} \qquad (\because a = \frac{1}{4})$$

For any x, where $2 < x \le 4$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{x} f(x)dx$$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{2} axdx + \int_{2}^{x} (4-x)adx$$

$$= \int_{0}^{2} \frac{x}{4}dx + \int_{2}^{x} \frac{1}{4}(4-x)dx$$

$$= \left(\frac{x^2}{8}\right)_0^2 + \frac{1}{4}\left(4x - \frac{x^2}{2}\right)_2^x$$

$$= \frac{1}{2} + \frac{1}{4}\left\{\left(4x - \frac{x^2}{2}\right) - (8-2)\right\}$$

$$= \frac{1}{2} + \frac{1}{4}\left\{4x - \frac{x^2}{2} - 6\right\}$$

$$= \frac{1}{2} + \left(x - \frac{x^2}{8} - \frac{3}{2}\right)$$

$$= \frac{4 + 8x - x^2 - 12}{8}$$

$$= \frac{-x^2 + 8x - 8}{8}$$

For any x, where $4 < x < \infty$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{4} f(x)dx + \int_{4}^{x} f(x)dx$$

$$= \int_{0}^{2} axdx + \int_{2}^{4} (4-x)adx + \int_{4}^{x} 0dx$$

$$= \int_{0}^{2} \frac{x}{4}dx + \int_{2}^{4} \frac{1}{4}(4-x)dx$$

$$= \left(\frac{x^2}{8}\right)_0^2 + \frac{1}{4}\left(4x - \frac{x^2}{2}\right)_2^4$$

$$= \frac{1}{2} + \frac{1}{4}\{(16-8) - (8-2)\}$$

$$= \frac{1}{2} + \frac{1}{4}\{8-6\} = 1$$

Hence the distribution function is

$$F(x) = = \begin{cases} 0, & -\infty < x \le 0 \\ \dfrac{x^2}{8}, & 0 < x \le 2 \\ \dfrac{-x^2 + 8x - 8}{8} & 2 < x \le 4 \\ 1 & 4 < x < \infty \end{cases}$$

(r) $P(X>2.5) = 1 - P(X\le 2.5)$

$$= 1 - F(2.5)$$

$$= 1 - \frac{1}{8}\left\{-(2.5)^2 + 8(2.5) - 8\right\}$$

$$= 1 - \frac{1}{8}\{-6.25 + 20 - 8\}$$

$$= 1 - \frac{1}{8}\{12 - 6.25\}$$

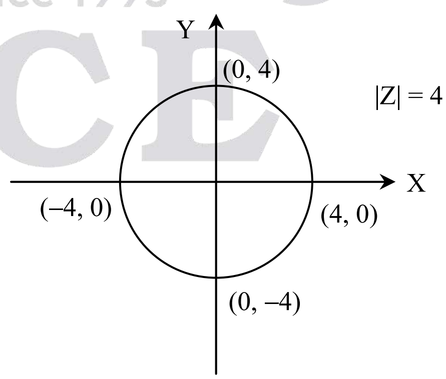
$$= 1 - \frac{5.75}{8} = \frac{2.25}{8} = 0.28125$$

3(a)(ii) Compute $\oint_c \frac{\cos z dz}{z^2 (z-\pi)^3}$, where c : |z| = 4. **[10M]**

Sol: $z = 0$ is a pole of order 2

$z = \pi$ is a pole of order 3

Residue of f(z) at z = 0 is



$$\underset{z\to 0}{\text{Lt}} \frac{d}{dz}\left\{z^2 . \frac{\cos z}{z^2 (z-\pi)^3}\right\}$$

$$= \underset{z\to 0}{\text{Lt}} \frac{d}{dz}\left\{\frac{\cos z}{(z-\pi)^3}\right\}$$

$$= \underset{z\to 0}{\text{Lt}}\left\{\frac{(z-\pi)^3(-\sin z) - (\cos z)(3)(z-\pi)^2}{(z-\pi)^6}\right\} = \frac{-3}{\pi^4}$$

Now, residue of f(z) at z = π is

$$\frac{1}{2!}\underset{z\to\pi}{Lt}\frac{d^2}{dz^2}\left\{(z-\pi)^3.\frac{\cos z}{z^2(z-\pi)^3}\right\}$$

$$=\frac{1}{2}\underset{z\to\pi}{Lt}\frac{d}{dz}\left\{\frac{z^2(-\sin z)-(\cos z)(2z)}{z^4}\right\}$$

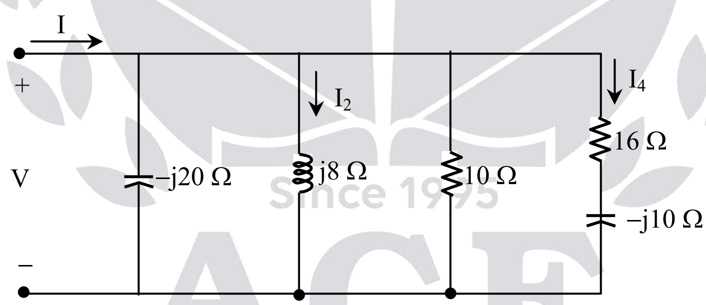
$$=\frac{1}{2}\underset{z\to\pi}{Lt}\left\{\frac{(z^4)\left[-z^2\cos z-2z\sin z+2z\sin nz-2\cos z\right]-(4z^3)\left[-z^2\sin z-2z\cos z\right]}{z^{8`}}\right\}$$

$$=\frac{1}{2}\left\{\frac{\pi^4(\pi^2+2)-4\pi^3(2\pi)}{\pi^8}\right\}$$

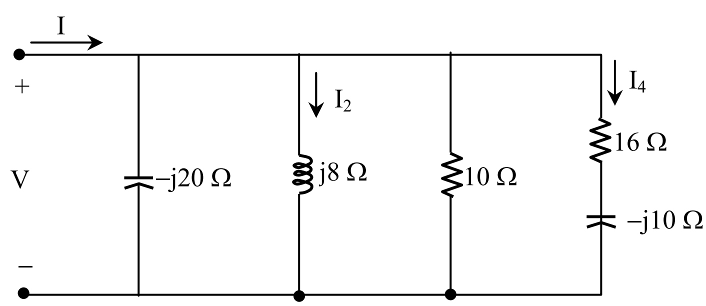
$$=\frac{1}{2}\left\{\frac{\pi^6+2\pi^4-8\pi^4}{\pi^8}\right\}$$

$$=\frac{1}{2}\left\{\frac{\pi^6-6\pi^4}{\pi^8}\right\}$$

3(b) For the circuit shown in the figure given below, the total current I entering the circuit 30.0∠–21° A. Determine the voltage V and the branch current I_2 and I_4. [20M]



Sol:



Total current I = 30∠–21° A

Total admittance

$$Y = Y_1 + Y_2 + Y_3 + Y_4$$

$$Y = \frac{1}{-j20} + \frac{1}{j8} + \frac{1}{10} + \frac{1}{16-j10}$$

$$Y = \frac{j}{20} - \frac{j}{8} + \frac{1}{10} + \frac{16+j10}{356}$$

$$Y = \left(0.1 + \frac{16}{356}\right) + j\left(\frac{1}{20} - \frac{1}{8} + \frac{10}{356}\right)$$

$$Y = (0.145 - j0.05)$$

$$V = IY = (30\angle{-21°})(0.153{-}\angle 19.0)$$

$$V = 4.59\angle{-40°} \text{ volts}$$

$$I_2 = \frac{V}{j8} = \frac{4.59\angle -40°}{8\angle 90} = 0.574\angle{-130°} \text{ A}$$

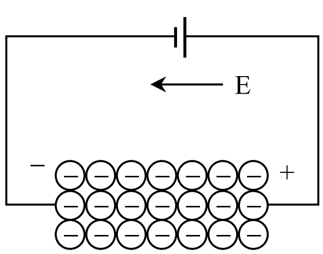
$$I_4 = \frac{V}{16-j10} = \frac{4.59\angle -40°}{18.8\angle -32}$$

$$I_4 = 0.243\angle{-8°} \text{ Amps}$$

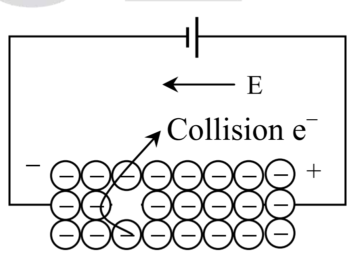
3(c)(i) Discuss the factors affecting electrical resistance of conductors. [10M]

Sol: Factor affecting resistivity of metals:

i) **Crystallography Imperfections:** With increase in defects in a material, the electrical conductivity decreases and resistivity increases because of collision of electrons at defect region



Perfect material



Defect material

ii) Temperature: With increase in temperature of material the resistivity increases and conductivity decreases because with increase in temperature atomic vibrations (or) lattice vibrations are generated due to that scattering of electrons (or) collisions of electrons takes place. At low temperature (Temp < room temp), Resistivity drastically increases with increase in temperature.

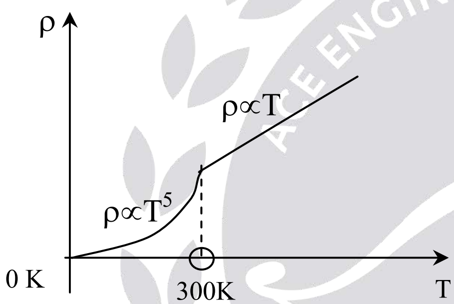
$\rho \propto T^5 \rightarrow$ At low temperature (T < Room temp)

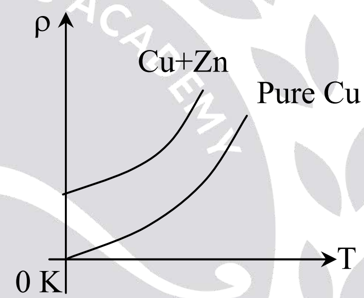
$\rho \propto T \rightarrow$ At high temperature (T > Room temperature)

At all high temperatures, the resistivity linearly increases with increase in temperature.

$R_{T2} = R_{T1}[1 + \alpha(T_2 - T_1)]$

$\rho_{T2} = \rho_{T1}[1 + \alpha(T_2 - T_1)]$





iii) Alloying (or) Solid Solution:

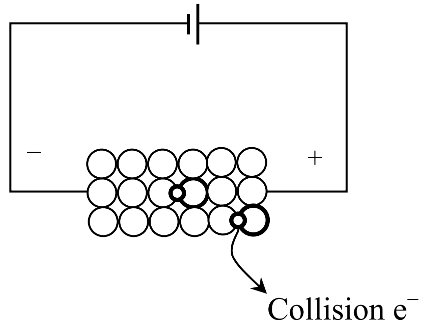
Alloy is a homogeneous mixture of solvent and Solute. Solvent is a higher concentration material in solid solution (or) alloy. Solute is a lowest concentration material in alloy.

Ex: Brass = Cu + Zn

1 kg = 90% + 10%

↓ ↓ ↓

Solid solution Solvent Solute



1) Electrical conductivity of an alloy < pure material: with increase in alloy concentration conductivity decreases because at solute atom location, localized collision of electrons takes place this is because of

(a) Difference in no. of valince electrons in solvent and solute

(b) Difference in size of host material atom and foreign atom

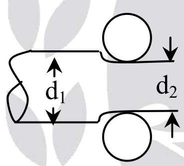
(c) Difference in Fermi energies of electrons of host material atom and impure atoms

Note:

The electrical conductivity of an alloy of 95% Cu + 5% silver is lower than pure copper even though the silver material is highest conductivity material added to copper.

iv) Cold working (or) plastic deformation (or) Hard drawn:

Cold working is a mechanical process of deforming the material by applying mechanical forces. By doing cold working operation on a material the crystallographic defects are generated and due to that electrical conductivity of material decreases.



3(c)(ii) Find the diffusion co-efficients of electrons and holes of a single silicon crystal at 27°C, if the mobilities of electrons and holes are 0.17 and 0.025 m^2/volt-sec respectively at 27°C. (Boltzmann's constant k = 1.38 × 10^{-23} jouls/degrees) **[10M]**

Sol: Given data:

Single silicon crystal

T = 27° C = 300 K

$\mu_e = 0.17\ m^2/V\text{-sec}$

$\mu_h = 0.025\ m^2/V\text{-sec}$

$K = 1.38 \times 10^{-23}$ J/°C

Formula = $\frac{D_n}{\mu_n} = \frac{D_P}{\mu_P} = \frac{kT}{e}$

(i) D_n = Diffusion coefficient of electron

$$D_n = \frac{KT\mu_n}{e} = \frac{1.33\times10^{-23}\times27\times0.17}{1.6\times10^{-19}}$$

$$= 3.95\times10^{-4}$$

(ii) D_P = Diffusion coefficient of holes

$$D_P = \frac{KT\mu_P}{e} = \frac{1.38\times10^{-23}\times27\times0.025}{1.6\times10^{-19}}$$

$$= 0.582\times10^{-4}$$

4(a)(i) State Dirichlet's conditions for existence of Fourier series of a function Determine the half range Fourier cosine series of [10M]

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x < \pi \end{cases}$$

Sol: (i) Dirchlet's conditions

→ f(x) is periodic in the given interval

→ f(x) is piece wise continuous in the given interval

→ f(x) has finite number of maxima and minima in the given interval

Now half-range cosine series of f(x) in the given interval is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx)$$

Where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$

$$= \frac{2}{\pi}\left\{\int_0^{\frac{\pi}{2}} x dx + \int_{\frac{\pi}{2}}^{\pi}(\pi - x)dx\right\}$$

$$= \frac{2}{\pi}\left\{\left(\frac{x^2}{2}\right)_0^{\pi} + \left(\pi x - x^2\right)_{\frac{\pi}{2}}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{2} + \left\{\left(\pi^2 - \pi^2\right) - \left(\frac{\pi^2}{2} - \frac{\pi^2}{4}\right)\right\}\right]$$

$$a_0 = \frac{2}{\pi}\left\{\frac{\pi^2}{2} - \frac{\pi^2}{4}\right\}$$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)dx$$

$$= \frac{2}{\pi}\left\{\int_0^{\frac{\pi}{2}} x\cos(nx)dx + \int_{\frac{\pi}{2}}^{\pi}(\pi - x)\cos(nx)dx\right\}$$

$$= \frac{2}{\pi}\left\{\left(\frac{x\sin(nx)}{n} + \frac{\cos(nx)}{n^2}\right)_0^{\frac{\pi}{2}} + \left(\frac{(\pi - x)\sin(nx)}{n} - \frac{\cos(nx)}{n^2}\right)_{\frac{\pi}{2}}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{2}{n^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2}\left(1 + (-1)^n\right)\right\}$$

$\therefore$ Half range cosine series is

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{2}{\pi}\left(\frac{2}{n^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2}\left(1 + (-1)^n\right)\right)\right\}\cos(nx)$$

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{\cos(2x)}{1^2} + \frac{\cos(6x)}{3^2} + \frac{\cos(10x)}{10^2}. + ...\right]$$

4(a)(ii) By converting into a line integral, evaluate $\iint_S \left(\nabla \times \overline{F}\right).\hat{n}dS$, where $\overline{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xy + z^2)\hat{k}$ and S is the surface of paraboloid $x^2 + y^2 + z = 4$ above xy-plane. [10M]

Sol: Given: $\vec{F} = (x^2 + y - 4)\vec{i} + 3xy\vec{j} + (2xy+z^2)\vec{k}$

Using Stoke's theorem, we have

$$\iint_S \left(\nabla \times \vec{F}\right)\hat{n}ds = \int_C \vec{F}d\vec{r}$$

$$\int_C \vec{F}d\vec{r} = \int_C (x^2 + y - 4)dx + 3xydy + (2xy + z^2)dz$$

On the xy-plane, $z = 0 \Rightarrow dz = 0$

$x^2 + y^2 = 4$

The parametric eqns of a circle are

$x = 2\cos\theta$, $y = 2\sin\theta$

$dx = -2\sin\theta d\theta$

$dy = 2\cos\theta d\theta$

$$\int_C \vec{F}d\vec{r} = \int (x^2 + y - 4)dx + 3xydy$$

$$= \int_0^{2\pi} \{(4\cos^2\theta + 2\sin\theta - 4)(-2\sin\theta d\theta) + 3(2\cos\theta)(2\sin\theta)(2\cos\theta)d\theta\}$$

$$= \int_0^{2\pi} \{-8\sin\theta\cos^2\theta - 4\sin^2\theta + 8\sin\theta + 24\sin\theta\cos^2\theta\}d\theta$$

$$= \int_0^{2\pi} 16\cos^2\theta\sin\theta d\theta - \int_0^{2\pi} 4\sin^2\theta d\theta + \int_0^{2\pi} 8\sin\theta d\theta \qquad \text{……….. (1)}$$

Put $\cos\theta = t$

$\sin\theta d\theta = -dt$

θ	0	2π
t	1	1

(1) $\Rightarrow$

$$\int_C \vec{F}.d\vec{r} = \int_{t=1}^{1} -16t^2 dt - \int_0^{2\pi} 4\left(\frac{1-\cos(2\theta)}{2}\right)d\theta + 8\{-\cos(\theta)\}_0^{2\pi}$$

$$= 2\left\{\theta - \frac{\sin(2\theta)}{2}\right\}_0^{2\pi} + 8\{(-\cos(2\pi) - (-\cos(0))\}$$

$$= 2\left\{2\pi - \frac{\sin(4\pi)}{2}\right\} + 8\{-1+1\}$$

$= 4\pi$

4(b) Discuss photoelectricity and photoemissive effect along with laws of photoemissive effect.

[20M]

Sol: A device used to convert light energy into electrical energy is called Photo Electric Cell.

Photocell is based on the phenomenon of Photoelectric effect. Photo cell are of three types.

1. Photo-Emissive Cell
2. Photo-Voltaic Cell
3. Photo-Conductive Cell

Photo-Emissive Cell: There are two types of photo-emissive cells; Vacuum type or gas filled type cells. Generally, it consists of two electrodes i.e., cathode (K) and anode (A). The cathode is in the form of semi-cylindrical plate coated with photo-sensitive material like sodium potassium or cesium i.e., alkali metals. To have large current, it is usually coated with antimony cesium alloy or combination of bismuth, silver, oxygen and cesium. The anode (A) is in the form of a straight wire made of nickel or platinum. The anode (A) faces the cathode (K). These electrodes are sealed in an evacuated glass or quartz bulb according to weather it is to be used with visible or ultra-violet light. As the current due to vacuum is small, so to increase the current, the bulb of the cells is filled with an inert gas like helium, neon, argon etc. at pressure of 1 mm of mercury.

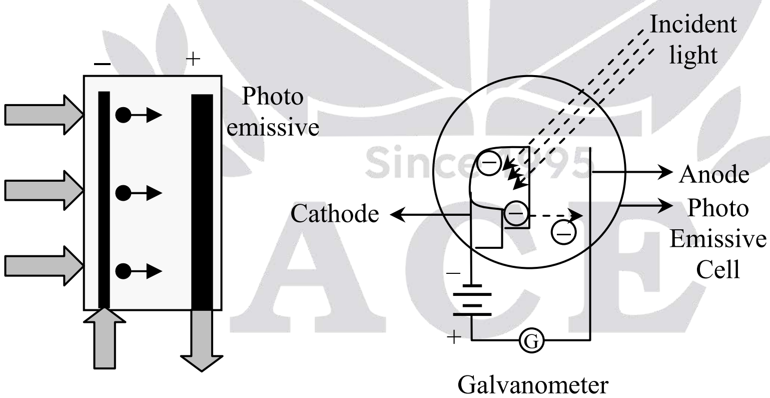


Fig. 1 Schematic and working of photo emissive cell

When photo-electrons flow from cathode to anode, they ionize the gas filled and hence the current gets modified. The main drawback of this type of cell (i.e., gas filled cell) is that the photo-electric current does not vary linearly with the intensity of the light.

Since there is no time lag between the incident light and the flow of electrons and hence current, therefore such a cell is used in television, photometry, fire alarm etc.

Photo-Voltaic Cell:

Photo-voltaic Cell is based on the principle of inner photo electric cell. This is called true cell because it generates emf without the application of any external potential difference but by only the light incident on it. It consists of a semi conductor layer formed on the surface of the metal plate by either heat treatment or cathode sputting. A film of semi-transparent metal is coated over the semi-conductor. This film maintains the electrical contact with the semi-conductor and simultaneously allows the incident light to fall on the semi-conductor.

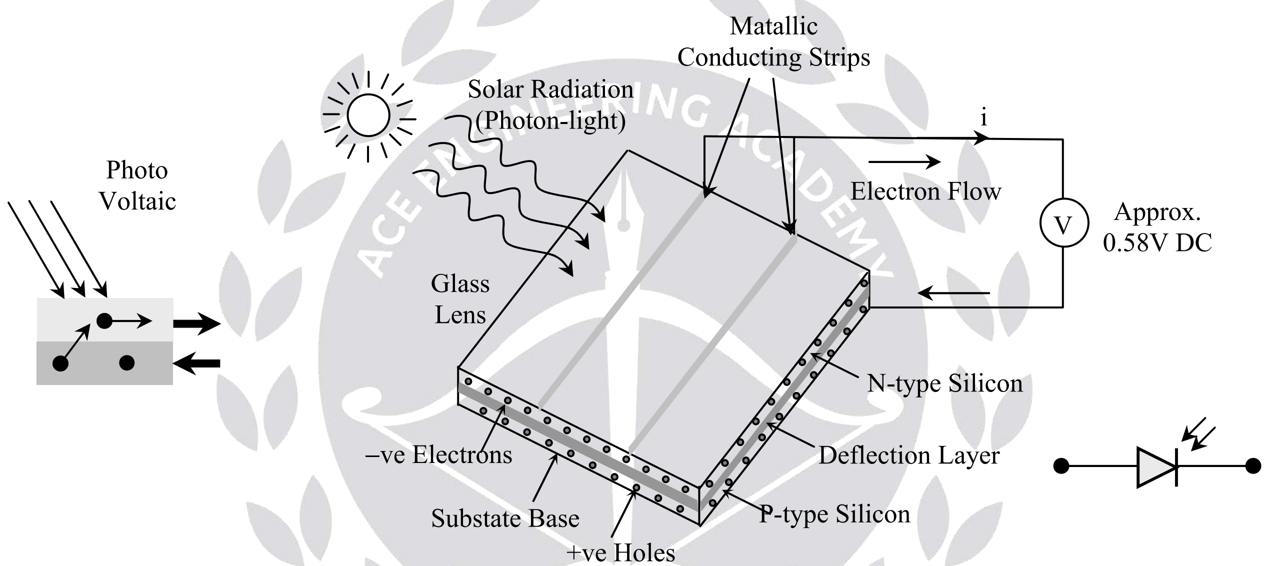


Fig. 2 Schematic and working of Photo-Voltaic Cell (Solar Cell)

When light is incident on the semi-conductor, electrons are emitted which flow in a direction opposite to the light rays. If the circuit is completed between the surface transparent film and metal base through a low resistance galvanometer (G), the current can be measured. If the resistance of the circuit is very small, the current is proportional to the intensity of incident light. The main advantage of this cell is that it requires no external voltage for its operation. This type of cell is widely used in photographic exposure meters, photometers and illumination meters etc.

Photo-conductive Cells:

Photo-conductive Cell is also based on the principle of inner photoelectric effect. It consists of a thin film of semi-conductor like Selenium or Thallium sulphide placed below a thin film of semi-transparent metal. The combination is place over the block of iron. The iron base and the transparent metal film is connected through battery and resistance. When light falls on the cell, its resistance decrease and hence the current starts flowing in the external circuit.

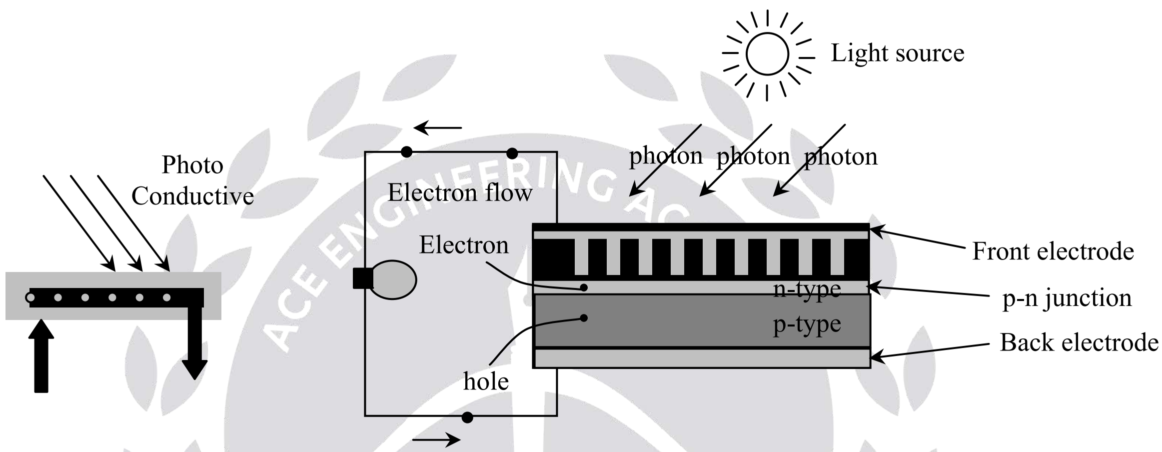


Fig. 3 Schematic and working of Photo Conductive cell

Let 'I' be the luminous intensity of an electric lamp and 'E' be the illuminance at a point distance 'd' from it. According to the inverse square law;

$$E = \frac{1}{d^2}$$

If light from the lamp be incident on the photovoltaic cell placed at a distance 'd' from it, then the photo current given out is proportional to E and if θ be the corresponding deflection shown by the microammeter then,

$$\theta \propto E$$

Or $$\theta \propto \frac{1}{d^2}$$

Or $$\theta \times d^2 = \text{constant.}$$

4(c) Show that the ratio of the amplitudes of conduction current density and displacement current density is $\frac{\sigma}{\omega\varepsilon}$ for the applied field $E = E_m \sin\omega t$. Assume $\mu = \mu_0$ [10M]

What is the amplitude ratio if the applied field is $E = E_m e^{\frac{-t}{\tau}}$, where t is real ? [10M]

Sol: Conduction current density $\vec{J}_C = \sigma\vec{E}$

Displacement current density $\vec{J}_D = \frac{\partial\vec{D}}{\partial t} = \varepsilon\frac{\partial\vec{E}}{\partial t}$

(i) In phasor form $\vec{J}_{CS} = \sigma\vec{E}_S$

And $\vec{J}_{DS} = j\omega\varepsilon\vec{E}_S$

When the given field is sinusoidal oscillatory $E = E_m \sin\omega t$

The ratio of conduction current density to displacement current density is given by

$$\frac{|\vec{J}_{CS}|}{|\vec{J}_{DS}|} = \frac{|\sigma\vec{E}_s|}{|j\omega\varepsilon\vec{E}_s|} = \frac{\sigma}{\omega\varepsilon}$$

(ii) Given electric field, $E = E\ e^{\frac{-t}{\tau}}$

$$J_C = \sigma E_m e^{\frac{-t}{\tau}}$$

$$J_D = \varepsilon E_m\left(\frac{-1}{\tau}\right)e^{\frac{-t}{\tau}}$$

$$\frac{|J_C|}{|J_D|} = \frac{\sigma E_m e^{\frac{-t}{\tau}}}{\varepsilon E_m\left(\frac{1}{\tau}\right)e^{\frac{-t}{\tau}}} = \frac{\sigma\tau}{\varepsilon}$$

Therefore the ratio of conduction current density to displacement current density is

$$\frac{|J_C|}{|J_D|} = \frac{\sigma\tau}{\varepsilon}$$

If τ (or) T_r relaxation time (time constant $T_r = \frac{\varepsilon}{\sigma}$) then, the ratio: $\frac{|J_C|}{|J_D|} = \frac{\sigma}{\varepsilon}\times\frac{\varepsilon}{J} = 1$

SECTION - B

5(a)(i) Using an iterative method, write C program segment to generate first n (n ≥ 8) Fibonacci numbers. [6M]

Sol:

```
#include <stdio.h>
int main()
{
    int n;
    printf("Enter the number of terms\n");
    scanf("%d", &n);
    if(n < 8)
     {
        printf("Sorry, Atleast 8 terms are required");
        return;
      }
      int first = 0, second = 1, next, c;
      printf("First %d terms of Fibonacci series are:\n", n);
      for (c = 0; c < n; c++)
      {
          if (c <= 1)
          next = c;
          else
           {
             next = first + second;
             first = second;
             second = next;
           }
         printf("%d\n", next);
       }
      return 0;
}
```

5(a)(ii)Find minimal disjunctive normal form (DNF) for the expressions

$\bar{x}yz + xyz + xy\bar{z}$ and $x + yz + z\bar{x}y + \bar{y}xz$. **[6M]**

Sol: $\bar{x}yz + xyz + xy\bar{z}$

x \ yz	00	01	11	10
0			1	
1			1	1

$xy + yz$

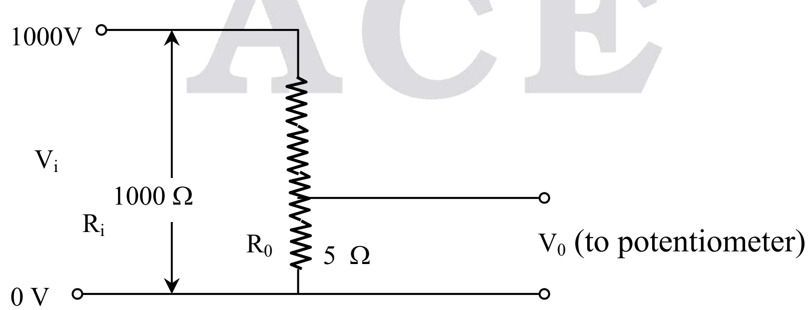
$x + yz + z\bar{x}y + \bar{y}xz$

x \ yz	00	01	11	10
0			1	
1	1	1	1	1

$x + yz$

5(b) Why and how are volt-ratio boxes utilized along with dc potentiometers? How should the value of the volt-ratio box resistance be chosen? **[12M]**

Sol: Volt-ratio box is used to measure voltages of high ranges by connecting with potentiometers



$$\frac{V_0}{V_i} = \frac{R_0}{R_i} \Rightarrow V_i = \frac{R_i}{R_0} V_0$$

Volt-ratio box has multiple input ranges. According to the requirement the range is to be selected.

5(c) The self-capacitance or distributed capacitance of a coil is measured using Q meter. The first measurement is carried out at 2.5 MHz, when the tuning capacitor is set at 425 pF. The second measurement is carried out by increasing the frequency to 6 MHz, when the tuning capacitor is set at 60 pF. Determine the distributed capacitance of the coil. [12M]

Sol: $f_1 = 2.5$ MHz; $C_1 = 425$ pF

$f_2 = 6$ MHz; $C_2 = 60$ pF

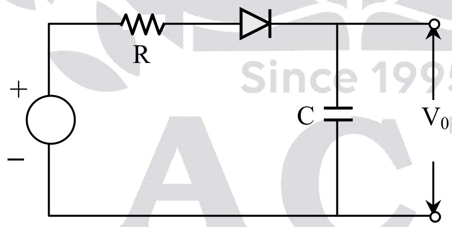
$C_d = ?$

$$\Rightarrow n = \frac{f_2}{f_1} = \frac{6}{2.5} = 2.4$$

$$C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$$

$$= \frac{425 - (2.4)^2(60)}{(2.4)^2 - 1} = \frac{79.4}{4.76} = 16.6 \text{ pF}$$

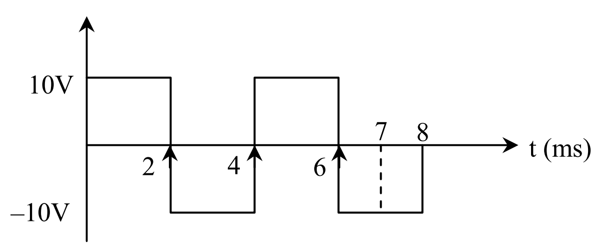
5(d) Calculate the output voltage V_0 at 7 ms in the figure shown below if a ±10V square wave of 250 Hz source is applied to R = 10 Ω, C = 20 μF. The diode is ideal and capacitor is initially uncharged. [12M]



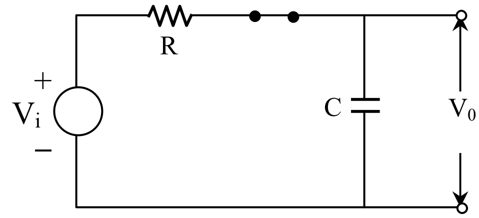
Sol: f = 250 Hz, T = 4 msec

Given that the capacitor is initially uncharged

$$V_C = V_0 = 0 \text{ V}$$



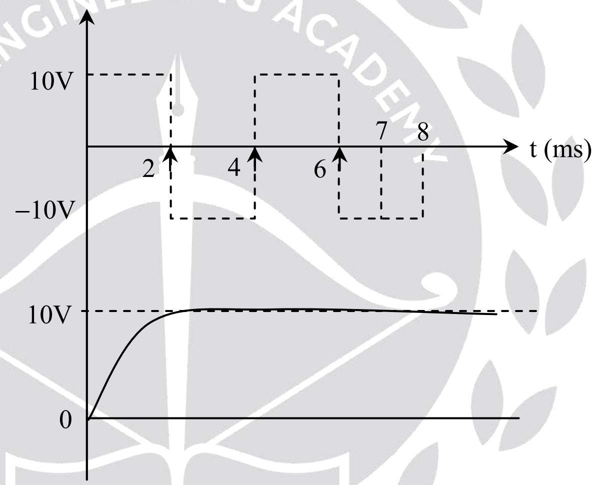
During +ve cycle: D becomes ON then the capacitor starts charging towards +10 V with $\tau = RC =$ 0.2 msec. Then $V_0 = V_C = V\left(1 - e^{-\frac{T_1}{RC}}\right)$



$$= 10\left(1 - e^{\frac{-2}{0.2}}\right)$$

$$= 9.999546 \text{ V} \Rightarrow V_0 \approx +10 \text{ V}$$

During –ve cycle: D-OFF, then there is no discharging path to capacitor then the capacitor holds its previous value until diode becomes ON. To become Diode ON the required value of input is greater than 10 V. So for remaining cycles D-OFF



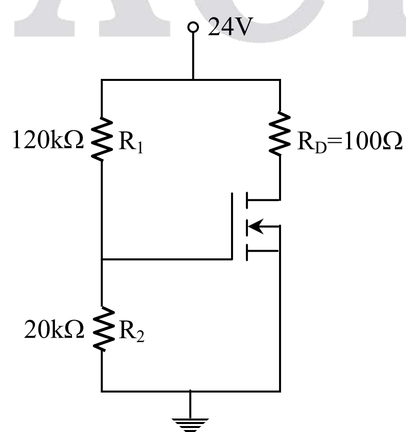
Then $V_0 = 10$ V

At t = 7 msec

$V_0 = 10$ V

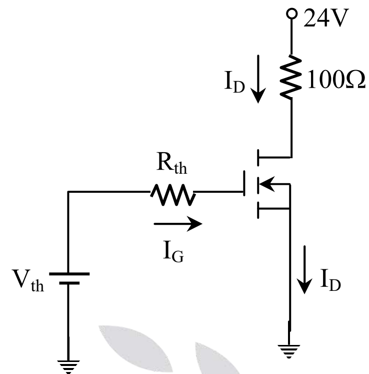
5(e) Determine V_{GS} and V_{DS} for the E-MOSFET circuit shown in the figure below. The minimum values of $I_{D(ON)} = 200$ mA at $V_{GS} = 4$ V and Gate to source threshold voltage $V_{GS(Th)} = 2$ V.

[12M]



Sol: The Thevenin's equivalent circuit is

$I_{Th} = 24\left(\frac{20}{140}\right)$



$V_{th} = 3.428$ V

$R_{th} = 20k||120k$

$= 17.143$ kΩ

As $I_G = 0$, $V_G = V_{th} \approx 3.428$ V

$V_S = 0$

$\Rightarrow V_{GS} = 3.428$ V

$V_{GS} > V_{GS(th)}$

Then conducting parameter is

$$K = \frac{I_{D(ON)}}{\left[V_{GS(ON)} - V_{GS(th)}\right]^2} = \frac{200\times10^{-3}}{(4-2)^2}$$

$K = 50$ mA/V^2

Then the drain current is, $I_D = K[V_{GS} - V_{th}]^2$

$= 50 \times 10^{-3}[3.428 - 2]^2$

$= 101.96$ mA

Then $V_{DS} = 24 - I_D R_D$

$= 24 - 101.96 \times 10^{-3} \times 100$

$= 24 - 10.196$

$= 13.804$ V

$\therefore V_{GS} = 3.428$ V

$V_{DS} = 13.804$ V

6(a)(i) Explain in brief the following and differentiate between them: **[12M]**

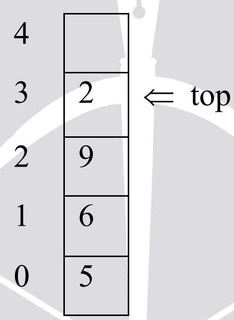
I. Stack and Queue

II. Sort and Search

Sol: I. Stack

A stack is a linear data structure in which elements can be inserted and deleted only from one side of the list, called the top. A stack follows the LIFO (Last In First Out) principle, i.e., the element inserted at the last is the first element to come out. The insertion of an element into stack is called push operation, and deletion of an element from the stack is called pop operation. In stack we always keep track of the last element present in the list with a pointer called top.

The diagrammatic representation of stack is given below:

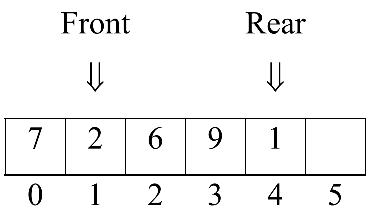


Stack

Queue:

A queue is a linear data structure in which elements can be inserted only from one side of the list called rear, and the elements can be deleted only from the other side called the front. The queue data structure follows the FIFO (First In First Out) principle, i.e. the element inserted at first in the list, is the first element to be removed from the list. The insertion of an element in a queue is called an enqueue operation and the deletion of an element is called a dequeue operation. In queue we always maintain two pointers, one pointing to the element which was inserted at the first and still present in the list with the front pointer and the second pointer pointing to the element inserted at the last with the rear pointer.

The diagrammatic representation of queue is given below:



Queue

Difference between Stack and Queue Data Structures

STACKS	QUEUES
Stacks are based on the LIFO principle, i.e., the element inserted at the last, is the first element to come out of the list.	Queues are based on the FIFO principle, i.e., the element inserted at the first, is the first element to come out of the list.
Insertion and deletion in stacks takes place only from one end of the list called the top.	Insertion and deletion in queues takes place from the opposite ends of the list. The insertion takes place at the rear of the list and the deletion takes place from the front of the list.
Insert operation is called push operation.	Insert operation is called enqueue operation.
Delete operation is called pop operation.	Delete operation is called dequeue operation.
In stacks we maintain only one pointer to access the list, called the top, which always points to the last element present in the list.	In queues we maintain two pointers to access the list. The front pointer always points to the first element inserted in the list and is still present, and the rear pointer always points to the last inserted element.
Stack is used in solving problems works on recursion.	Queue is used in solving problems having sequential processing.

II. Searching:

Searching is the process of finding a particular item in a collection of items. A search typically answers whether the item is present in the collection or not. Searching requires a key field such as name, ID, code which is related to the target item. When the key field of a target item is found, a pointer to the target item is returned. The pointer may be an address, an index into a vector or array, or some other indication of where to find the target. If a matching key field isn't found, the user is informed.

The most common searching algorithms are:

- Linear search
- Binary search
- Interpolation search
- Hash table

Sorting:

Sorting is the process of placing elements from a collection in some kind of order. For example, a list of words could be sorted alphabetically or by length. Efficient sorting is important to optimize the use of other algorithms that require sorted lists to work correctly. The importance of sorting is To represent data in more readable format and Optimize data searching to high level.

The most common sorting algorithms are:

- Bubble Sort
- Insertion Sort
- Selection Sort
- Quick Sort
- Merge Sort

6(a)(ii) Write a pseudo code or in any standard programming language for interchanging the values of two variables: [8M]

I. Using a third variable

II. Not using any extra variable.

Sol: I. Using a third variable:

```
#include<stdio.h>
int main()
{
  double first, second, temp;
   printf("Enter first number: ");
   scanf("%lf", &first);
```

```
    printf("Enter second number: ");
    scanf("%lf", &second);
    temp = first;
    first = second;
    second = temp;
    printf("\nAfter swapping, firstNumber = %.2lf\n", first);
    printf("After swapping, secondNumber = %.2lf", second);
    return 0;
  }
```

II. Not using any extra variable:

```
#include <stdio.h>
int main()
{
     double a, b;
     printf("Enter a: ");
     scanf("%lf", &a);
     printf("Enter b: ");
     scanf("%lf", &b);
       // Swapping
       // a = (initial_a - initial_b)
          a = a – b;
       // b = (initial_a - initial_b) + initial_b = initial_a
         b = a + b;
       // a = initial_a - (initial_a - initial_b) = initial_b
          a = b - a;
      printf("After swapping, a = %.2lf\n", a);
       printf("After swapping, b = %.2lf", b);
       return 0;
}
```

6(b) A 230 V, single phase, watt hour meter has a constant load of 5A passing through it for 8 hours at unity power factor. If the meter constant is 460 revolutions per kWh, how many revolutions does the meter disc make during this period? If the same meter makes 1638 revolutions when operating at 230 V and a constant load of 6 A passing through it for certain duration at a power factor of 0.86, determine the duration of operation of the meter in hours. [20M]

6(b)

Sol: Given data: $V_L = 230$ V, $I_L = 5$ A, t = 8 hrs

$\cos\phi = 1$, M.C = 460 rev/kWh, No. of rev = ?

Energy consumption = kWh × t

$$= \frac{V_L I_L \cos\phi}{1000} \times t$$

$$= \frac{230 \times 5 \times 1}{1000} \times 8 = 9.2 \text{ kWh}$$

$$\text{Meter constant} = \frac{\text{rev}}{\text{kWh}}$$

$\Rightarrow$ Rev = M.C × kWh = 460 × 9.2 = 4232 rev

If meter makes 1635 rev

$I_L = 6$ A

$\cos\phi = 0.86$

t = ?

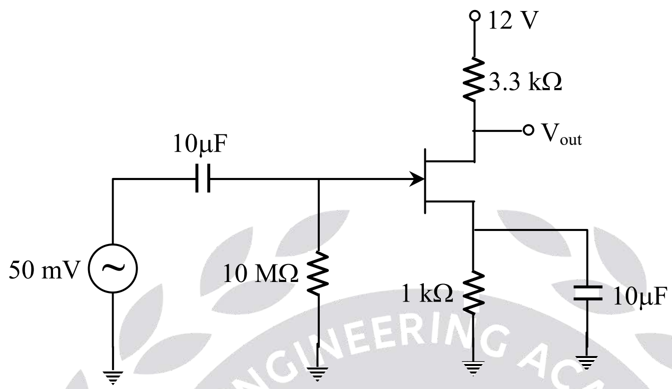
$$\text{MC} = \frac{\text{rev}}{\text{kWh}}$$

$$460 = \frac{1638}{\frac{230 \times 6 \times 0.86}{1000} \times \text{h}}$$

$$460 = \frac{1638}{1.1868 \times h}$$

Time = 3 hrs

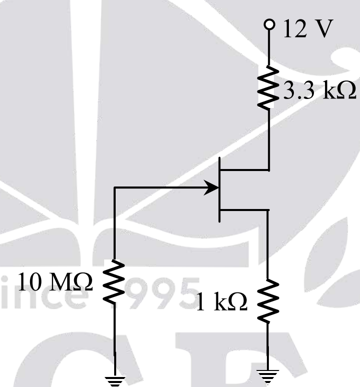
6(c)(i) Find the voltage gain of JFET amplifier shown in the figure below for the drain to source current with gate shorted, I_{DSS} = 10 mA, cut-off voltage $V_{GS(OFF)} = -4$ V and I_D = 2 mA. If a load resistance of 4.7 kΩ is a.c coupled to the output of this amplifier, calculate the percentage change in voltage gain. [10M]



Sol: From DC Analysis

As $I_G \approx 0$, $V_G = 0$, $V_S = I_DR_S = 2V$

$V_{GS} = -2$ V



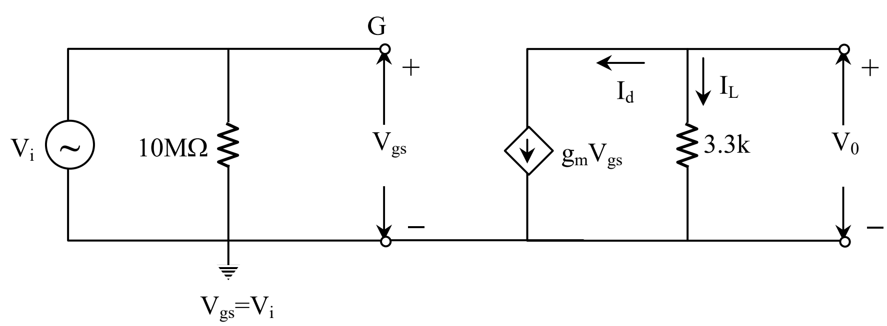
$$g_m = \frac{-2I_{DSS}}{V_{GS(off)}}\left[1-\frac{V_{GS}}{V_{GS(off)}}\right]$$

$$= \frac{-2\times10\times10^{-3}}{-4}\left[1-\frac{-2}{-4}\right]$$

$$= 5\times10^{-3}(0.5)$$

$\Rightarrow g_m = 2.5$ m ℧

From AC Analysis;



$V_{gs} = V_i$

$V_0 = I_L(3300)$

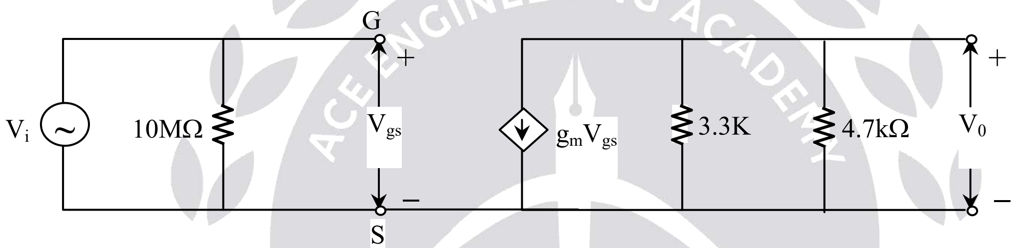
$= -g_m V_{gs}(3300)$

$$\frac{V_0}{V_{gs}} = A_V = \frac{V_0}{V_i} = -g_m \times 3300$$

$$= -2.5 \times 10^{-3}(3300)$$

$$= -8.25$$

For a load resistance of 4.7 kΩ,



$R'_L = 3.3k \,||\, 4.7k$

$= 1.93375$ kΩ

$A_V = -g_m(3.3k||4.7k)$

$= -2.5(1.93875)$

$= -4.8468$

The percentage change in voltage gain

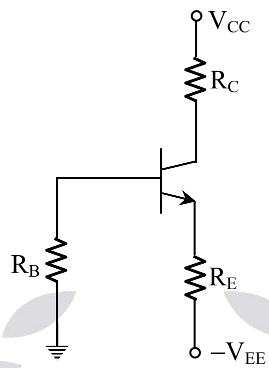
$A_{V1} = -8.25$, $|A_{V1}| = 8.25$

$A_{V2} = -4.8468$, $|A_{V2}| = 4.8468$

$$\%A = \frac{|A_{V1}| - |A_{V2}|}{|A_{V1}|} \times 100 = \frac{8.25 - 4.8468}{8.25} = 41.25\%$$

The gain reduced by 41.25%

6(c)(ii) Find R_B in the figure shown below for silicon transistor with β = 100 and negligible leakage current, if V_{CC} = 20 V, V_{EE} = 5 V, R_E = 100 Ω, R_C = 2 kΩ and I_C = 6 mA. [10M]



Sol: Given $\beta = 100$, $I_{CO} = 0$, $I_C = 6$ mA

Let transistor in active region

$$I_B = \frac{I_C}{\beta} = 60\ \mu A,\ I_E = (1+\beta)I_B$$

$$= 6.06\ mA$$

Then $V_{CE} = 20 - I_CR_C - I_ER_E - (-5)$

$$= 25 - 6(2) - 6.06(0.1)$$

$$= 12.394\ V$$

As $V_{CE} > 0.2$ V → Si transistor in active region

Then apply KVL to BE loop

$$0 - I_BR_B - 0.7 - I_ER_E - (-5) = 0$$

$$R_B = \frac{4.3 - 6.06(0.1)}{I_B}$$

$$= \frac{3.694}{60 \times 10^{-6}}$$

$$\Rightarrow R_B = 61.566\ k\Omega$$

7(a)(i) Execution of a sequence of instructions of a program involves 200 instruction fetch operations, 100 memory operand read operations, and 80 memory operand write operations. Find the average memory access time in executing this sequence of instructions if the memory access time is 2 ns for a read operation with a hit in cache, 5 ns for a read operation with a miss in cache, 3 ns for write operation with a hit in cache and 10 ns for a write operation with a miss in cache. The cache hit ratio is 0.9. Consider the time taken for fetch operation to be equal to that of read operation. [10M]

Sol: Total fetch instruction – 200, read – 100 and write – 80 → Total = 380

As, fetch and read is same, we consider 300 for read for calculation

Read access time = 0.9 * 2 + 0.1 * 7 = 2.5 nsec

Write access time = 0.9 * 3 + 0.1 * 13 = 4 nsec

So, total average = (300 * 2.5 + 80 * 4) / 380

= 750 + 320 / 380

= 1070 / 380

= 2.815 nsec

7(a)(ii) Derive the expression for the number of comparisons required in the worst case for sorting an array of 'n' elements using Bubble sort. Calculate it for an array of 100 elements. When will such maximum number comparisons occur in Bubble sort? [10M]

Sol: In bubble sort, we normally try to compare every consecutive elements and if first element is larger than second one, then we swap the first and second element, else we keep the first and second element as it is, and continue comparison for second and third element.

In this way, we assure that in first pass, the highest element of the list is bring down to the bottom of the list. Similarly, we then continue the same process from the elements left out and in such way, we assure that the highest element always get down to the bottom of the list. The algorithm would terminate when there is no change of position of any element in any pass as compared to its previous pass.

The worst number of comparisons would be required whenever the entire given array of elements are sorted in descending order, so we need to scan every element with every other remaining element in each pass.

Consider the following example:

Given Array (Completely sorted in descending order)

5, 4, 3, 2, 1

Here, we need to compare in following order for pass 1 for entire list

1. 5 and 4 – 5 is higher so it becomes 4, 5, 3, 2, 1 after swapping
2. 5 and 3 – 5 is higher so it becomes 4, 3, 5, 2, 1 after swapping
3. 5 and 2 – 5 is higher so it becomes 4, 3, 2, 5, 1 after swapping
4. 5 and 1 – 5 is higher so it becomes 4, 3, 2, 1, 5 after swapping

So, total 4 comparisons would be required.

Result of Pass 1 → 4, 3, 2, 1, 5

Similarly, we need to scan for the first 4 elements of the list in pass 2, where the comparison would happen from 1 to 4, as 5 now is already sorted as follows:

1. 4 and 3 – 4 is higher so it becomes 3, 4, 2, 1, 5 after swapping
2. 4 and 2 – 4 is higher so it becomes 3, 2, 4, 1, 5 after swapping
3. 4 and 1 – 4 is higher so it becomes 3, 2, 1, 4, 5 after swapping

So, total 3 comparisons would be required.

Result of pass 2 → 3, 2, 1, 4, 5

Similarly, for pass 3 → 2 comparison and pass 4 → 1 comparison would be required.

So, in total 4 + 3 + 2 + 1 = 10 comparisons would be required in worst case.

If we take this scenario for any larger value n, it would give as follows:

Total comparisons for n = (n – 1) + (n – 2) + (n – 3) + ….. + 1 = (n * (n – 1)) / 2 by generic mathematical formula.

If we take n = 100, then total comparisons = 100 (99) / 2 = 99 * 50 = 4950 comparisons.

Ideally this is the worst case scenario, and would happen when the given array is completely sorted in descending order or reversely sorted.

7(b) A spring controlled, electrodynamic voltmeter has a range of 100 V, has a square law scale response, and it takes 0.08 A on dc for full scale deflection of 120°. The control constant is 1×10^{-6} N-m/degree. The true potential difference across the instrument is 100.42 V, when it reads 100 V at 50 Hz. Determine the initial mutual inductance of the instrument. [20M]

Sol: Given, V = 100 V,

$\theta \propto I^2 \propto V^2$

$\theta \propto V^2$

$I = 0.08$ A DC, $\theta_{max} = 120°$, $K_C = 1 \times 10^{-6}$ N-m/deg

$(V_m)_t = 100.42$ V, when reads 100 V

f = 50 Hz, M = ?

$$\theta = \frac{V^2}{K_C}\frac{dM}{d\theta} \quad \text{-------- (1)}$$

$$\frac{\theta_2}{\theta_1} = \left(\frac{V_2}{V_1}\right)^2$$

$$\frac{\theta_2}{120} = \left(\frac{100.42}{100}\right)^2$$

$$\theta_2 = 121.01°$$

From (1)

$$120 \times \frac{\pi}{180} = \frac{(100)^2}{1\times10^{-6}}\frac{dM}{d\theta}$$

$$\frac{dM}{d\theta} = 0.209 \text{ nH/deg}$$

$dM = 0.209 \times 10^{-9} \times (121.01 - 120°)$

$= 0.211$ nH

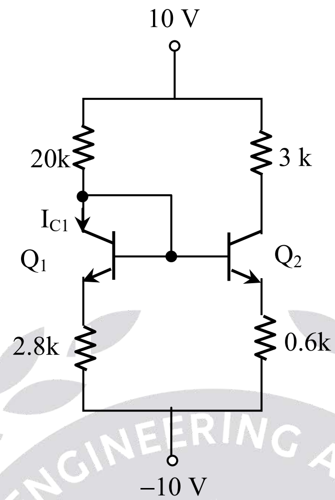
Mutual inductance at 121.01° is $M = 0.209\times10^{-9} \times 121.01 = 25.29$ nH.

So that the initial mutual inductance is

$(25.291 - 0.211) = 25.08$ nH

$= 25.08$ nH

7(c)(i) Find I_{C1} in the circuit shown in the figure below. Assume that the two transistors are matched and V_{BE} = 0.7 V, β = 100 [12M]

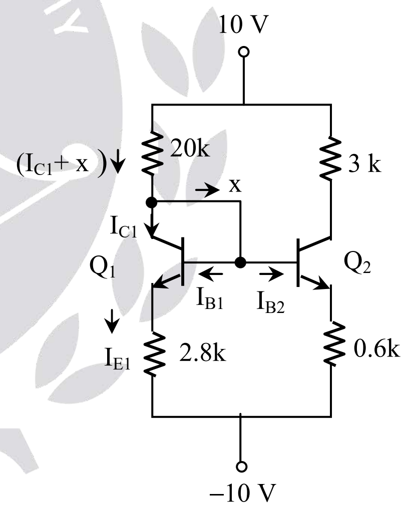


Sol: For matched transistors

$I_{B1} = I_{B2}$, $x = I_{B1} + I_{B2}$

$x = 2I_{B1} = 2I_{B2}$

Apply KVL for 10 V to –10V through Q_1



$10 - 20 \times 10^3(I_{C1} + x) - 0.7 - 2.8 \times 10^3(I_{E1}) - (-10) = 0$

$20 \times 10^3 (\beta I_{B1} + 2I_{B1}) + 2.8 \times 10^3(1 + \beta)I_{B1} = 19.3$

$I_{B1} \times 10^3[20\beta + 40 + 2.8(1 + \beta)] = 19.3$

$I_{B1} \times 10^3[2322.8] = 19.3$

$$\Rightarrow I_{B1} = \frac{19.3}{2322.8 \times 10^3}$$

$= 8.3089$ μA

$I_{C1} = \beta I_{B1}$

$= 0.83089$ mA

$I_{C1} \approx 0.8309$ mA

7(c)(ii) Find the feedback factor β of the negative feedback network required for an amplifier with open loop gain A_0 = 2000 ± 200 to reduce the variation to less than ±0.2%. Find the overall gain of the system with feedback. [8M]

Sol: Given: $A_0 = 2000 \pm 200$,

$$\frac{dA_f}{A_f} = 0.2\%$$

$A_0 = 2000$, $dA_0 = 200$

$$\frac{dA_0}{A_0} = \frac{200}{2000}\times 100 = 10\%$$

Then $\dfrac{dA_f}{A_f} = \dfrac{1}{1+\beta A_0}\left(\dfrac{dA_0}{A_0}\right)$

$$0.2 = \frac{1}{1+\beta A_0}(10) \Rightarrow 1 + \beta A_0 = \frac{10}{0.2} = 50$$

$$\beta = \frac{49}{A_0} = \frac{49}{2000}$$

$\beta = \dfrac{49}{2000}$ or 0.0245

The overall gain of the system $A_f = \dfrac{A_0}{1+\beta A_0}$

$$\Rightarrow A_f = \frac{2000}{50} = 40$$

8(a) Name the layers of Open Systems Interconnection (OSI) model created by the International Organisation for Standardisation for different computer systems to communicate with each other using standard protocols. Mention the important functions of each of these layers in brief. [20M]

Sol: The Open Systems Interconnection (OSI) model describes seven layers that computer systems use to communicate over a network. It was the first standard model for network communications, adopted by all major computer and telecommunication companies in the early 1980s

The modern Internet is not based on OSI, but on the simpler TCP/IP model. However, the OSI 7-layer model is still widely used, as it helps visualize and communicate how networks operate, and helps isolate and troubleshoot networking problems.

OSI was introduced in 1983 by representatives of the major computer and telecom companies, and was adopted by ISO as an international standard in 1984.

The OSI model has 7 layers are as follows

Transmit Data — Sender's side

Application Layer	Layer - 7
Presentation Layer	Layer - 6
Session Layer	Layer - 5
Transport Layer	Layer - 4
Network Layer	Layer - 3
Data-Link Layer	Layer - 2
Physical Layer	Layer - 1

Receive Data — Receiver's side

1. Physical Layer

The physical layer is responsible for the physical cable or wireless connection between network nodes. It defines the connector, the electrical cable or wireless technology connecting the devices, and is responsible for transmission of the raw data, which is simply a series of 0s and 1s, while taking care of bit rate control.

2. Data Link Layer

The data link layer establishes and terminates a connection between two physically-connected nodes on a network. It breaks up packets into frames and sends them from source to destination. This layer is composed of two parts—Logical Link Control (LLC), which identifies network protocols, performs error checking and synchronizes frames, and Media Access Control (MAC) which uses MAC addresses to connect devices and define permissions to transmit and receive data.

3. Network Layer:

The network layer has two main functions. One is breaking up segments into network packets, and reassembling the packets on the receiving end. The other is routing packets by discovering the best path across a physical network. The network layer uses network addresses (typically Internet Protocol addresses) to route packets to a destination node.

4. Transport Layer:

The transport layer takes data transferred in the session layer and breaks it into "segments" on the transmitting end. It is responsible for reassembling the segments on the receiving end, turning it back into data that can be used by the session layer. The transport layer carries out flow control, sending data at a rate that matches the connection speed of the receiving device, and error control, checking if data was received incorrectly and if not, requesting it again.

5. Session Layer:

The session layer creates communication channels, called sessions, between devices. It is responsible for opening sessions, ensuring they remain open and functional while data is being transferred, and closing them when communication ends. The session layer can also set checkpoints during a data transfer—if the session is interrupted, devices can resume data transfer from the last checkpoint.

6. Presentation Layer:

The presentation layer prepares data for the application layer. It defines how two devices should encode, encrypt, and compress data so it is received correctly on the other end. The presentation layer takes any data transmitted by the application layer and prepares it for transmission over the session layer.

7. Application Layer:

The application layer is used by end-user software such as web browsers and email clients. It provides protocols that allow software to send and receive information and present meaningful data to users. A few examples of application layer protocols are the Hypertext Transfer Protocol (HTTP), File Transfer Protocol (FTP), Post Office Protocol (POP), Simple Mail Transfer Protocol (SMTP), and Domain Name System (DNS).

8(b) In an oscilloscope, the deflection factor of CRT is 80 V/cm and the accelerating voltage is 2500 V. What is the minimum distance required from center of deflection plates to screen that allows full deflection of 4 cm on the oscilloscope screen? [20M]

Sol: G = 80 V/Cm

Accelerating voltage, $V_a = 2500$ V

Deflection of the electron beam on the screen, D = 4 cm

l = Length of the deflection plates.

L = Distance between the screen and the mid of the deflection plates

d = Distance between the deflection plates

$D = \frac{\ell L V_d}{2dV_a}$ and $G = \frac{1}{S} = \frac{V_d}{D}$

$\therefore\ G = \frac{V_d}{\left(\frac{\ell L V_d}{2dV_a}\right)} = \frac{2dV_a}{\ell L}$ ………… (1)

For the minimum distance L, from the geometry, $\frac{L}{D} = \frac{\ell}{D}$ i.e., $L = \frac{D\ell}{d}$ … (2)

From eq. (1), $d = \frac{\ell L G}{2V_a}$ substitute in eq. (2)

$L^2 = \frac{2DV_a}{G} = \frac{2\times 4\times 10^{-2}\times 2500}{80\times 10^2}$

$L^2 = 0.025$

L = 0.158 m

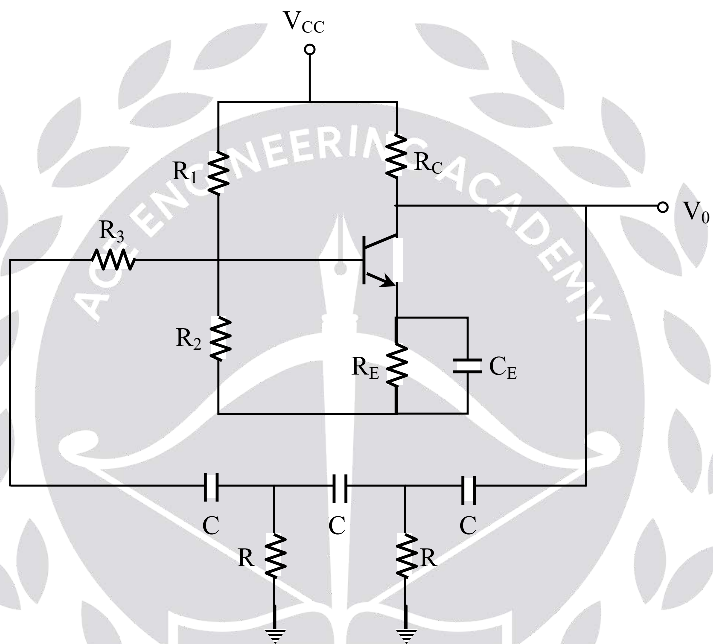
8(c) Derive the expression for the frequency and the condition for starting of sustained oscillation in a transistorized R-C phase shift oscillator. Neglect h_{oe} and h_{re}. Assume R >> h_{ie} and a load resistance R_L is a.c. coupled to the oscillator. [20M]

Sol: **<u>BJT-RC-PSO:</u>**

$R_B = R_1//R_2$ and in generally $R_B > R_i$

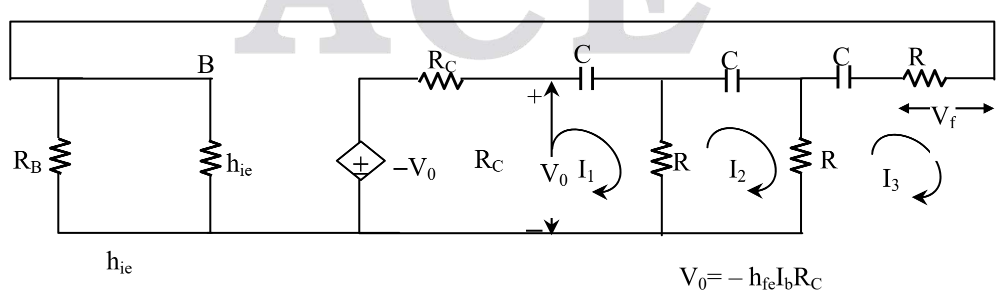
$R_i = h_{ie}$

$R_B//h_{ie} \approx h_{ie}$



$R_3 + h_{ie} \approx R$

The equivalent circuit is



Apply KVL to loops

$(R + R_C - jX_C)I_1 - I_2R = V_0$ ………… (1)

$-I_1R + (2R - jX_C)I_2 - I_3R = 0$ ……….. (2)

$-I_2R + (2R - jX_C)I_3 = 0$ ………. (3)

Then

$$\begin{bmatrix} (R + R_C - jX_C) & -R & 0 \\ -R & 2R - jX_C & -R \\ 0 & -R & 2R - jX_C \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix}$$

$\Delta = (R + R_C - jX_C)[(2R - jX_C)^2 - R^2] + R[-R(2R - jX_C)]$

$= (R + R_C - jX_C)[(2R - jX_C)^2 - R^2] - R^2(2R - jX_C)$

$$= R^3\left[\left\{\left(1 + \frac{R_C}{R} - j\frac{X_C}{R}\right)\right\}\left\{\left(2 - j\frac{X_C}{R}\right)^2 - 1\right\} - \left(2 - j\frac{X_C}{R}\right)\right]$$

Assume, $\frac{RC}{R} = K$, $\frac{X_C}{R} = \alpha$

$= R^3\{(1 + K - j\alpha)(3 - \alpha^2 - j4\alpha) - (2 - j\alpha)\}$

$= R^3\{3 - \alpha^2 j4\alpha + 3K - \alpha^2K - j4\alpha K - j3\alpha + j\alpha^3 - 4\alpha^2 - 2 + j\alpha\}$

$\Delta = R^3[1 + 3K - (5 + K)\alpha^2 - j\{(6 + 4K)\alpha - \alpha^3\}]$

$$\Delta_3 = \begin{bmatrix} R + R_C - jX_C & -R & V_0 \\ -R & 2R - jX_C & 0 \\ 0 & -R & 0 \end{bmatrix} = V_0R^2$$

$= -h_{fe}I_b.R_CR^2$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-h_{fe}I_bR_CR^2}{R^3\left[(1 + 3K - (5 + K)\alpha^2) - j\{(6 + 4K)\alpha - \alpha^3\}\right]}$$

But $I_3 = I_b$ then

$$\frac{-h_{fe}R_C}{R\left\{1 + 3K - (5 + K)\alpha^2 - j\{(6 + 4K)\alpha - \alpha^3\}\right\}} = 1$$

$V_0 = -h_{fe}I_bR_C$, $V_f = I_3R$

$$\frac{V_f}{V_0} = \frac{1}{\{(1 + 3K) - \alpha^2(5 + K)\} - j\{(6 + 4K)\alpha - \alpha^3\}}$$

To get undamped oscillations imaginary part must be zero

$(6+4K)\alpha = \alpha^3$

$\beta \rightarrow$ feedback factor $= \dfrac{1}{(1+3K)-\alpha^2(5+K)}$

$\alpha^2 = 6+4K$

$\beta = \dfrac{1}{(1+3K)-(6+4K)(5+K)}$

$\alpha = \sqrt{6+4K}$

$= \dfrac{1}{1+3K-30-26K-4K^2}$

$\dfrac{X_C}{R} = \sqrt{6+4K}$

$= \dfrac{-1}{4K^2+23K+29}$

$\dfrac{1}{\omega RC} = \sqrt{6+4K}$

$\omega = \dfrac{1}{RC\sqrt{6+4K}}$

Where $K = \left(\dfrac{RC}{R}\right)$