PRE-GATE-2020

Mechanical Engineering

(Questions with Detailed Solutions)

GENERAL APTITUDE

Q. 01 – Q. 05 carry One Mark each.

01. Fill in the blank with an appropriate phrase

Jobs are hard to _____ these days

(A) Come by (B) Come down

(C) Come of (D) Come from

01. Ans: (A)

Sol: 'Come by' means to manage to get something.

02. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

MONKEY : TROOP:

(A) sheep : hard

(B) elephant : Parliament

(C) bacteria : Colony

(D) wolves : School

02. Ans: (C)

Sol: Troop consists of monkeys just as a colony consists of bacteria.

03. ***Choose the most appropriate word from the options given below to complete the following sentence:***

If you had gone to see him, he _______ delighted.

(A) Would have been

(B) Will have been

(C) Had been

(D) Would be

03. Ans: (A)

Ans: 'A" conditional tense type 3 grammatical code is

If +had+V3, would +have+V3

04. Which of the following options is closest in meaning to the underlined word?

European intellectuals have long debated the consequences of the **hegemony** of American popular culture around the world.

(A) regimen (B) vastness

(C) dominance (D) popularity

04. Ans: (C)

Sol: Dominance means influence or control over another country, a group of people etc.

05. How many one-rupee coins, 50 paise coins 25 paise coins in total of which the numbers are proportional to 5, 7 and 12 are together work ₹115?

(A) 50, 70, 120 (B) 60, 70, 110

(C) 70, 80, 90 (D) None of these

05. Ans: (A)

Sol: $(5 \times 1+7\times 0.5+12\times 0.25)\ x = 115$

$(5+3.5+3)x = 115$

$11.5x = 115 \Rightarrow x = 10$

$\therefore$ Number of one rupee coin = $5x = 5\times 10 = 50$

Number of 5-paise coin = $7x = 7\times 10 = 70$

Number of 25-paise coin = $12x$

$= 12\times 10 = 120$

Q. 6 – Q. 10 carry Two Marks each.

06. Critical reading is a demanding process. To read critically, you must slow down your reading and, with pencil in hand, perform specific operations on the text mark up the text with your reactions, conclusions, and questions, then you read, become an active participant.

This passage best supports the statement that

(A) Critical reading is a slow, dull but essential process.

(B) The best critical reading happens at critical times in a person's life.

(C) Readers should get in the habit of questioning the truth of what they read.

(D) Critical reading requires thoughtful and careful attention.

06. Ans: (D)

Sol: Choice (A) is incorrect because the author never says that reading is dull.

Choice (B) and (C) are not support by the paragraph.

Choice (D) is correct as it is implied in the entire passage.

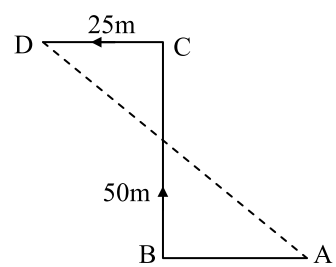
07. Anil's house faces east from the back-side of the house, he walks straight 50 metres, then turns to the right and walks 50m again finally, he turns towards left and stops after walking 25 m Now Anil is in which direction from the starting point?

(A) South-east (B) South-west

(C) North-east (D) North- west

07. Ans: (D)

Sol: The movement of Anil are shown in the adjoining figure



He starts walking from back of his house (i.e) towards west now, the final position is D, which is to the north west of his starting point A.

08. A and B enter into a partnership, A puts in ₹50 and B puts in ₹45. At the end of 4 months, A withdraws half his capital and at the end of 5 months B withdraws $\frac{1}{2}$ of his, C then enters with a capital of ₹70 at the end of 12 months, the profit of concern is ₹254, how can the profit be divided among A, B and C ?

(A) ₹76, ₹80 and ₹98

(B) ₹80, ₹76 and ₹98

(C) ₹76, ₹98 and ₹80

(D) None of these

08. Ans: (B)

Sol:

A's share	:	**B's share**	:	**C's share**
(50×4+25×8)	:	(45×5+22.5×7)	:	(70×7)
400	:	382.5	:	490
800	:	765	:	980
160	:	153	:	196

Total profit = ₹ 254

Profit of A

$$= \frac{160}{160+153+196} \times 254 = \frac{160}{509} \times 254 = ₹80$$

Profit of B = $\frac{153}{509} \times 254 = ₹76$

Profit of C = $\frac{196}{509} \times 254 = ₹98$

∵ Hence option 'B' is correct.

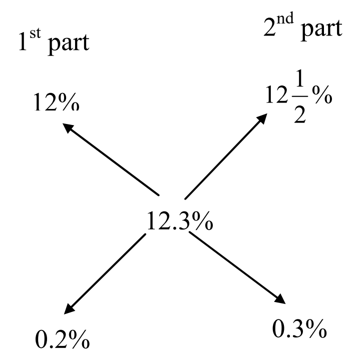
09. A sum of ₹25400 was lent out in two parts, one of 12% and the other at $12\frac{1}{2}$%. If the total annual income is ₹3124.2, the money lent at 12% is

(A) ₹15240 (B) ₹25400

(C) ₹10160 (D) ₹31242

09. Ans: (C)

Sol: Overall rate of interest $\frac{3124.2}{25400} \times 100 = 12.3\%$



∴ The sum will be divided in the ratio

0.2:0.3 (or) 2:3

$\therefore$ The sum lent at 12%

$$= 25400 \times \frac{2}{5} = ₹10160.$$

10. The following question is to be answered on the basis of the table given below.

Category of personnel	Number of staff in the year-1990	Number of staff in the year-1995
Data preparation	18	25
Data control	5	8
Operators	18	32
Programmers	21	26
Analysts	15	31
Managers	3	3
Total	80	135

What is the increase in the sector angle for operators in the year 1995 over the sector angle for operators in the year 1990?

(A) 4° (B) 3° (C) 2° (D) 1°

10. Ans: (A)

Sol: Sector angle for operators in the year 1990

$$= \frac{18}{80} \times 360° = 81°$$

Sector angle for operator in the year $1995 = \frac{32}{135} \times 360° = 85.33 \simeq 85\%$

$\therefore$ Required difference = 85° – 81° = 4°

MECHANICAL ENGINEERING

Q. 11 – Q. 35 CARRY ONE MARK EACH.

11. If $\tau = \left(\frac{du}{dy}\right)^3$ and $\vec{V} = y^2\hat{i}$, then the apparent viscosity is

(A) $2y$ (B) $2y^2$

(C) 0 (D) $4y^2$

11. Ans: (D)

Sol: For Non-Newtonian fluids,

$$\tau = A\left(\frac{du}{dy}\right)^n$$

$$\tau = \mu_{app.} \times \left(\frac{du}{dy}\right)$$

where, $\mu_{app} = A\left(\frac{du}{dy}\right)^{n-1}$

For the given of the fluid, i.e., $\tau = \left(\frac{du}{dy}\right)^3$

and $u = y^2$

$\Rightarrow$ A = 1 and n = 3

$$\mu_{app} = 1 \times \left[\frac{d}{dy}\left(y^2\right)\right]^2 = 4y^2$$

12 In thermal fully developed flows in a tube subjected to constant surface heat flux

(A) shape of the temperature profile changes along the tube

(B) shape of the temperature profile does not change along the tube
(C) shape of the temperature profile first changes then remains constant
(D) shape of the temperature profile its initially constant then changes

12. Ans: (B)

Sol: $q = h(T_s - T_m)$,

$\frac{q}{h} = T_s - T_m = \text{constant}$

$\dot{m}c_p dT_m = hp\ dx(T_s - T_m)$

[where, P = perimeter]

$\dot{m}c_p dT_m = q\,p\,dx$

$\frac{dT_m}{dx} = \frac{q\,p}{\dot{m}c_p} = \text{constant}$

13. For the function $f(x, y) = x^2 - y^2$, the point (0, 0) is
(A) a local minimum
(B) a saddle point
(C) a local maximum
(D) not a stationary point

13. Ans: (B)

Sol: Given $f(x, y) = x^2 - y^2$

$\Rightarrow f_x = 2x,\ f_y = -2y$ and

$f_{xx} = 2,\ f_{xy} = 0, f_{yy} = -2$

Consider $f_x = 0$ and $f_y = 0$

$\Rightarrow 2x = 0$ and $-2y = 0$

$\Rightarrow (0, 0)$ is a stationary point

At (0, 0) , $f_{xx}\, f_{yy} - (f_{xy})^2 = -4 < 0$

$\therefore$ f(x, y) has neither a maximum nor minimum at (0, 0).

14. The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 rpm clockwise when looking from a stern. The radius of gyration of the rotor is 0.5 m. If the ship is steering to the right in a curve of 100 m radius at a speed of 16.1 knots (1 knot = 1855 m/hr), then the gyroscopic couple is
(A) 4.15 kN-m (B) 6.52 kN-m
(C) 8.3 kN-m (D) 13.04 kN-m

14. Ans: (D)

Sol: Given:

m = 2000 kg, N = 3000 rpm

$\omega = 2\pi \times 3000/60 = 314.2$ rad/s

k = 0.5 m,

R = 100 m,

v = 16.1 knots = 16.1×1855/3600 = 8.3 m/s

Gyroscopic couple :

We know that mass moment of inertia of the rotor,

$I = m.k^2 = 2000(0.5)^2 = 500$ kg-m^2

Angular velocity of precession,

$\omega_p = v/R = 8.3/100 = 0.083$ rad/s

$\therefore$ Gyroscopic couple,

$C = I.\omega.\ \omega_p = 500 \times 314.2 \times 0.083$

$= 13040$ N-m $= 13.04$ kN-m

15. In a Rankine cycle due to increase in boiler pressure

(A) Pump work decreases

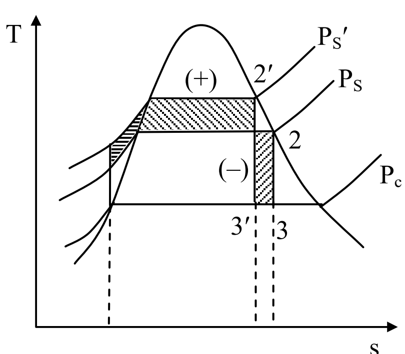
(B) Quality at exhaust increases

(C) Specific condenser load decreases

(D) Thermal efficiency remains constant.

15. Ans: (C)

Sol:



T–s diagram

Due to increase in boiler pressure

- Pump work increases.
- Quality at exhaust decreases.
- Specific condenser load decreases.
- Thermal efficiency increases as mean temperature of heat addition increases.

16. A cantilever beam of uniform strength has uniform width 'B', depth 'D' at fixed end and length 'l' is subjected to a concentrated load P at free end. If the Young's modulus of the material of beam is 'E' then the deflection at free end is (where, $I = \frac{BD^3}{12}$)

(A) $\frac{P\ell^3}{3EI}$ (B) $\frac{P\ell^3}{2EI}$

(C) $\frac{2P\ell^3}{3EI}$ (D) $\frac{P\ell^3}{8EI}$

16. Ans: (C)

Sol: As the width of beam is constant, and it is also said that the beam is of uniform strength, depth of the beam at a distance x from free end of beam is represented as

$$D_x = D\left(\frac{x}{\ell}\right)^{1/2}$$

where,

D_x = depth at a distance x from free end.

Using strain energy method, deflection at free end

$$\delta = \int_0^\ell \frac{M_x}{EI_x}\frac{\partial M_x}{\partial P}dx$$

Here, $M_x = P.x$

$$\frac{\partial M_x}{\partial P} = x$$

$$I_x = \frac{BD_x^{\ 3}}{12} = \frac{BD^3}{12}\left(\frac{x}{\ell}\right)^{3/2}$$

$$\delta = \int_0^{\ell} \frac{P\,x\,x}{E\frac{BD^3}{12}\frac{x^{3/2}}{\ell^{3/2}}} \times dx = \frac{2}{3}\frac{P\ell^3}{EI}$$

17. The solution to $x^2y^{11} + xy^1 - y = 0$ is

(A) $y = C_1x^2 + C_2\,x^{-3}$

(B) $y = C_1 + C_2x^{-2}$

(C) $y = C_1\,x + \frac{C_2}{x}$

(D) $y = C_1\,x + C_2\,x^4$

17. Ans: (C)

Sol: Put $ln\,x = t$ so that $x = e^t$ and

let $x\frac{dy}{dx} = Dy$, $\quad x^2\frac{d^2y}{dx^2} = D(D-1)\,y$

where $D = \frac{d}{dt}$

Given differential equation is

$$x^2y^{11} + xy^1 - y = 0$$

$\Rightarrow \quad D(D-1)y + Dy - y = 0$

$\Rightarrow \quad (D^2 - 1)\,y = 0$

Consider Auxiliary equation $f(D) = 0$

$\Rightarrow D^2 - 1 = 0$

$\Rightarrow D = 1, -1$ are different real roots

$\therefore$ The general solution of given equation is

$$y = c_1e^t + c_2e^{-t}$$

$$= c_1\,x + \frac{c_2}{x}$$

18. In a two-wire method if *p* denotes pitch (in mm) and *x* denotes thread angle (in degrees), then the diameter of the best size wire d (in mm) is given by

(A) $d = \left(\frac{p}{2}\right)\sec\left(\frac{x}{2}\right)$

(B) $d = \left(\frac{p}{4}\right)\sec\left(\frac{x}{2}\right)$

(C) $d = \left(\frac{p}{2}\right)\text{cosec}\left(\frac{x}{2}\right)$

(D) $d = \left(\frac{p}{2}\right)\cot\left(\frac{x}{2}\right)$

18. Ans: (A)

Sol:

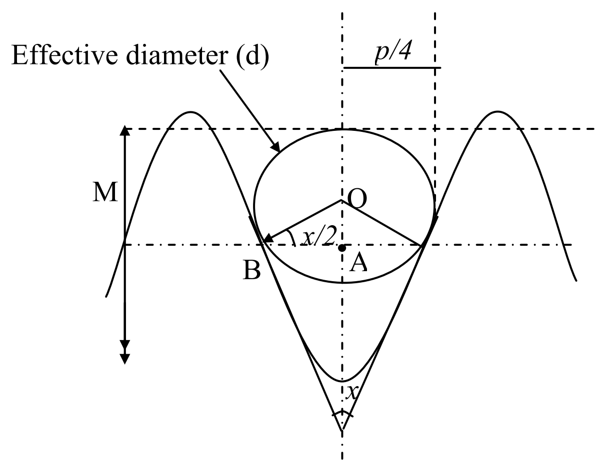


Fig: The best size wire

The above figure shows the condition achieved by the best-size wire.

In triangle OAB, sin (AOB) = AB/OB

$\sin\left(90 - \frac{x}{2}\right) = AB/OB$

$$OB = \frac{AB}{\sin(90 - x/2)} = \frac{AB}{\cos(x/2)}$$

$= AB \sec(x/2)$

Diameter of the best-size wire

$= 2(OB) = 2(AB)\sec(x/2)$.

But, AB = p / 4, where p is the pitch of the thread.

∴ Diameter of the best-size wire

(d) = (p/2) sec(x/2)

19. A 20° full depth steel spur pinion is to transmit 3 kW power at a speed of 1200 rpm. The number of teeth on the pinion is 24 and the corresponding form factor is 0.337. The suitable module and face width are 2 mm and 36 mm respectively. If the velocity factor is 1.3 and the allowable bending stress is 75 MPa, then the factor of safety is

(A) 2.38 (B) 1.41

(C) 1.83 (D) 2.72

19. Ans: (B)

Sol: Design condition :

$$F_{eff} = \frac{F_{max}}{FOS}$$

$$C_v C_s F_t = \frac{m \times b \times \sigma_b \times Y}{FOS}$$

$$1.3 \times 1 \times \frac{60P}{2\pi N} \times \frac{1}{r} = \frac{m \times b \times \sigma_b \times Y}{FOS}$$

$$1.3 \times 1 \times \frac{60 \times 3 \times 10^3}{2\pi \times 1200} \times \frac{2}{2 \times 24 \times 10^{-3}} = \frac{2 \times 36 \times 75 \times 0.337}{FOS}$$

∴ FOS = 1.41

20. For a maximization problem of LP model, the following simplex tabular is obtained.

Basic	x_1	x_2	s_1	s_2	R.H.S
z	9/2	0	5/2	0	15
x_2	3/2	1	1/2	0	3
s_2	–1/2	0	–3/2	1	0

From the above table, one can conclude that the LP model has

(A) unique optimal solution

(B) multiple optimal solutions

(C) infeasible solution

(D) degenerate solution

20. Ans: (D)

Sol: Maximization case:

All z-row elements ($z_j - c_j$) are non-negative and number of zero elements = no. of basic variables ⇒ unique optimal solution.

Unique optimal solution along with a zero basic variable ⇒ Degenerate solution.

21. A continuous random variable X has a probability density function

$f(x) = e^{-x}$, $0 < x < \infty$. Then $P(X > 2)$ is

(A) 0.1353 (B) 0.2354

(C) 0.2343 (D) 1.1353

21. Ans: (A)

Sol: $P(X > 2) = \int_2^{\infty} f(x) \cdot dx$

$$= \int_2^{\infty} e^{-x} dx$$

$$= \left. \frac{e^{-x}}{-1} \right|_2^{\infty}$$

$$= e^{-2} = 0.1353$$

22. For a 2D boundary layer flow over a flat plate, if $\eta = \frac{y}{\delta}$ and $f'(\eta) = \frac{u}{U_\infty}$, then the values of $f'(0)$ and $f'(\infty)$ are

(A) 0, 0 (B) 0, 1

(C) 0, ∞ (D) 1, 0

22. Ans: (B)

Sol: Given that $\eta = \frac{y}{\delta}$

and $f'(\eta) = \frac{u}{U_\infty}$

we know that at y = 0 i.e., η = 0

u = 0

$$\Rightarrow f'(0) = \frac{0}{U_\infty} = 0$$

Also, at y = ∞, i.e., η = ∞

$u = U_\infty$

$$\Rightarrow f'(\infty) = \frac{U_\infty}{U_\infty} = 1$$

23. In a two fluid heat exchanger the inlet and outlet temperatures of the hot fluid are 65°C and 40°C, respectively, while for the cold fluid these are 15°C and 43°C. The heat exchanger is

(A) a parallel flow heat exchanger

(B) a counter flow heat exchanger

(C) such that both parallel flow and counter flow operations are possible

(D) none of these

23. Ans: (B)

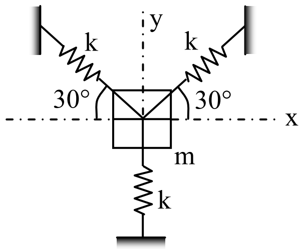
Sol: $T_{h1} = 65°C$, $T_{h2} = 40°C$

$T_{c1} = 15°C$, $T_{c2} = 43°C$

In parallel flow arrangement Th_2 cannot be less than Tc_2

Hence, arrangement is counter flow.

24. A m kg mass is supported on three identical springs of stiffness k N/m as shown in figure below. The natural frequency of the system along x direction is



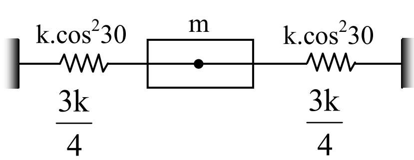
(A) $\sqrt{\frac{2k}{3m}}$ rad/s (B) $\sqrt{\frac{k}{m}}$ rad/s

(C) $\sqrt{\frac{3k}{2m}}$ rad/s (D) $\sqrt{\frac{3k}{m}}$ rad/s

24. Ans: (C)

Sol: Inclined springs are to be resolved in the given direction using,

$k\theta = k\cos^2\theta$



Along x direction :

$$k_e = \frac{3k}{4} + \frac{3k}{4} = \frac{3k}{2}$$

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{3k}{2m}}$$

25. In a CNC drilling machine, a number of holes have to be drilled on a rectangular metal plate by the execution of a single program. Part surface is parallel to X-Y plane. The holes are on the circumference of a circle with all their axes parallel to Z axis. In order to drill these holes, which of the following facilities has to be present in the CNC drilling machine ?

(A) Rotary table with rotational axis along Z axis

(B) Circular interpolation in X-Y plane

(C) Linear interpolation in X-Y plane

(D) The coordinate values of the holes to be drilled can be given in PTP mode

25. Ans: (D)

Sol: By just giving the coordinate values of the holes to be drilled can be given in PTP mode. So options (D) is Correct.

26. Patients arrive at a medical clinic with an arrival rate that is Poisson distributed with a mean as 6 per hour. Treatment time averages 6 minutes and it follows exponential distribution. The mean waiting time in the queue is

(A) 15 min (B) 6 min

(C) 25 min (D) 9 min

26. Ans: (D)

Sol: Arrival rate (λ) = 6 /hr

Service rate $(\mu) = \frac{1}{6} \times 60$ = 10 /hr

$$\rho = \frac{\lambda}{\mu} = \frac{6}{10} = 0.6$$

$$L_s = \frac{\rho}{1-\rho} = \frac{0.6}{1-0.6} = 1.5$$

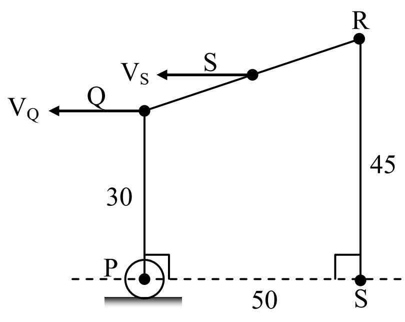
$$W_s = \frac{L_s}{\lambda} = \frac{1.5}{6} = 0.25\,\text{hr}$$

$W_q = \rho.W_s$

$= 0.6 \times 0.25$

$= 0.15$ hrs $= 9$ min

27. PQRS is a four bar mechanism. At an instant shown in figure the velocity of point 'Q' is V_Q. Velocity of mid point 'S' of QR, (i.e., V_S) at the same instant is



(A) V_Q (B) $\frac{3}{2}V_Q$

(C) $\frac{2}{3}V_Q$ (D) $2\,V_Q$

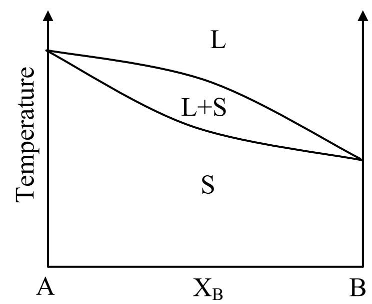
27. Ans: (A)

Sol: At the instant shown link QR will have pure sliding motion.

Any point on link with pure sliding will have same velocity.

$\therefore\ V_Q = V_S = V_R$

28. In a binary isomorphous phase diagram shown below, the degree of freedom in the two phase region is



(A) 3 (B) 2

(C) 1 (D) 0

28. Ans: (C)

Sol: Modified Gibbs phase rule gives:

$F + P = C + 1$

$P = 2(L, S)$

$C = 2(A, B)$

$F + 2 = 2 + 1$

$F = 1$ (Either temperature or chemical composition)

where, F = Degree of freedom,

P = Phase, C = Chemical constituents.

29. A hydraulic turbine operates at the following parameters at its best efficiency point: speed = 90 rpm, discharge = 200 m^3/s, net head = 55 m, density of fluid = 10^3 kg/m^3 and brake power = 100 MW. The dimensionless specific speed in radians of

this turbine is ________ . (Rounded off to two decimal places)

29. Ans: 1.15 (Range 1.05 to 1.20)

Sol: Given data:

N = 90 rpm, Q = 200 m^3/s,

H = 55 m and P = 100 MW

The dimensionless specific speed of a turbine is given as:

$$N_s^* = \frac{\omega\sqrt{P/\rho}}{(gH)^{5/4}}$$

where, ω = speed in rad/s

P = Power in Watt

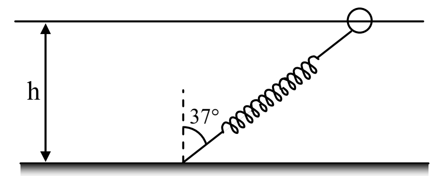
H = Net head in m

$$\text{Thus, } N_s^* = \frac{2\pi\times\frac{90}{60}\sqrt{\frac{100\times10^6}{10^3}}}{(9.81\times55)^{5/4}}$$

$$= 0.3648\,\pi = 1.15 \text{ radians}$$

30. One end of a spring of natural length 'h' (i.e., no deformation) and spring constant 'k' is fixed at the ground and the other is fitted with a smooth ring of mass 'm' which is allowed to slide on a horizontal rod fixed at a height 'h' (as shown in figure below). Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. The speed of the ring when the spring becomes vertical is $\frac{h}{n}\sqrt{\frac{k}{m}}$, the value of n is _____

(Take cos37° = 4/5 and sin 37° = 3/5°).



30. Ans: 4 (Range 3.9 to 4.1)

Sol: Elongation in spring,

$$\delta(\text{initial}) = \frac{h}{4}$$

By using conservation of mechanical energy,

$$\frac{1}{2}\times k\times\delta^2 = \frac{1}{2}mV^2$$

$$\therefore \quad V = \frac{h}{4}\sqrt{\frac{k}{m}}$$

31. If directional derivative of $\phi = 2xz - y^2$, at the point (1, 3, 2) becomes maximum in the direction of $\bar{a}$, then magnitude of $\bar{a}$ is

31. Ans: 7.48 (Range 7.4 to 7.5)

Sol: Given $\phi = 2xz - y^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{i} + \frac{\partial\phi}{\partial y}\bar{j} + \frac{\partial\phi}{\partial z}\bar{k}$$

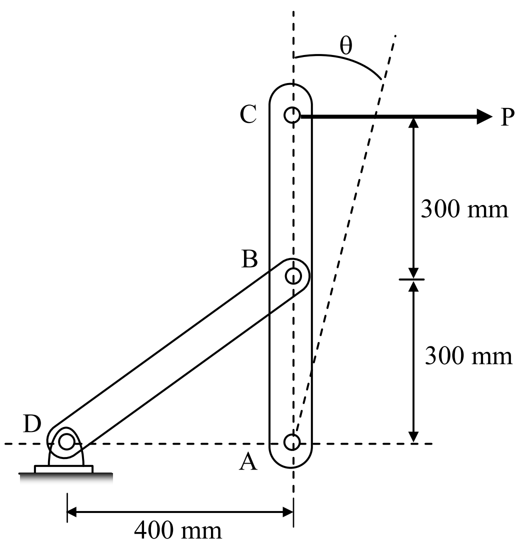
$$= 2z\,\bar{i} - 2y\bar{j} + 2x\,\bar{k}$$

$\therefore$ Required direction vector = $\bar{a}$ = $(\nabla\phi)$ at (1, 3, 2) = $(4\bar{i} - 6\bar{j} + 2\bar{k})$

Magnitude of $\bar{a} = \sqrt{16+36+4}$

$= \sqrt{56} = 7.48$

32. The rigid link ABC is initially in vertical position. A deformable link BD is attached to rigid link as shown in figure. If load P is applied at 'C', rigid link rotates by an angle θ = 0.3°, then the normal strain in the deformable link is ________ mm/m. (Rounded off to two decimal places).



32. Ans: 2.51 (Range 2.4 to 2.6)

Sol: ℓ_1 = Initial length of link BD

$= \sqrt{AB^2 + AD^2}$

$= \sqrt{300^2 + 400^2} = 500$ mm

ℓ_2 = Final length of link BD

$= \sqrt{AB^2 + AD^2 - 2AB \times AD \times \cos\alpha}$

where, α = final angle between side AD and AB

= 90 + 0.3 = 90.3°

$\ell_2 = \sqrt{300^2 + 400^2 - 2\times 300 \times 400 \times \cos(90.3)}$

= 501.255 mm

Normal strain in link BD

$\varepsilon_{BD} = \dfrac{\ell_2 - \ell_1}{\ell_1}$

$= \dfrac{501.255 - 500}{500}$

= 0.00251 mm/mm

$= 0.00251 \dfrac{\text{mm}}{10^{-3}\,\text{m}}$

ε_{BD} = 2.51 mm/m

33. During plain turning of mild steel by a tool of geometry, 0°, 0°, 8°, 7°, 15°, 90°, 0 (mm) (ORS) at feed of 0.2 mm/rev, the chip thickness was found to be 0.5 mm. The shear angle (in degrees) is ______. (Rounded upto one decimal place)

33. Ans: 21.8 [Range: 21.0 to 22.6]

Sol: Given,

Tool Geometry: 0°, 0°, 8°, 7°, 15°, 90°, 0 (mm) (ORS),

Rake angle (α) = 0° = side rake angle in ORS

Feed (f) = 0.2 mm/rev

Chip thickness (t_2) = 0.5 mm

Uncut chip thickness,

$t_1 = f \sin\lambda = 0.2 \sin 90° = 0.2$ mm

$$r = \frac{t_1}{t_2} = \frac{0.2}{0.5} = 0.4$$

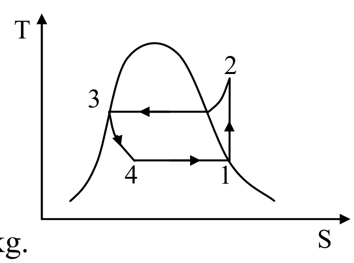
$$\phi = \tan^{-1}\left(\frac{r\cos\alpha}{1-r\sin\alpha}\right) = \tan^{-1}\left(\frac{0.4\cos 0}{1-0.4\sin 0}\right)$$

$$= 21.8°$$

34. In a vapor compression refrigeration plant, the mass flow of refrigerant is 1 kg/s. If the enthalpy at evaporator exit is 360 kJ/kg and at the condenser exit is 120 kJ/kg, then the cooling load (in kW) is ________.

34. Ans: 240 (Range: 240 to 240)

Sol:



$h_1 = 360$ kJ/kg,

$h_3 = h_4 = 120$ kJ/kg.

Cooling load = m_{ref} (kg/sec) $(h_1 - h_4)$ kJ/kg

= 1(360 – 120) = 240 kW

35. The ratio of press force required to punch a circular hole of 30 mm diameter in a 1 mm thick steel sheet to that needed to punch a circular hole of 60 mm diameter in a 2 mm thick steel sheet is_______ (Rounded to two decimal places)

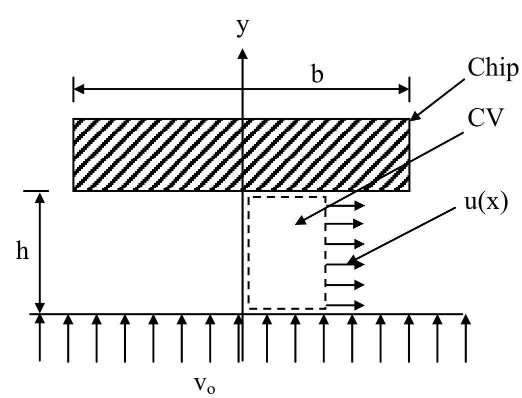
35. Ans: 0.25 (Range 0.25 to 0.25)

Sol: Ratio of force

$$= \frac{F_{1\max}}{F_{2\max}} = \frac{\pi d_1 t_1 \tau_u}{\pi d_2 t_2 \tau_u} = \frac{\pi \times 30 \times 1}{\pi \times 60 \times 2} = 0.25$$

Q. 36 – Q. 65 CARRY TWO MARKS EACH.

36. A rectangular chip floats on the top of a thin layer of air, above a bottom plate. Air is blown with an uniform velocity v_o through the holes in the bottom plate. Width of the chip perpendicular to the plane of paper is W. Assume uniform flow across any vertical section. For steady inviscid and constant density flow, the acceleration of fluid in the gap between the two plates, is



(A) $\frac{v_o^2 x}{h^2}\hat{i} - \frac{v_o^2}{h}\left(1-\frac{y}{h}\right)\hat{j}$

(B) $-\frac{v_o^2 x}{h^2}\hat{i} + \frac{v_o^2}{h}\left(1-\frac{y}{h}\right)\hat{j}$

(C) $\frac{v_o^2 x}{h^2}\hat{i} + \frac{v_o^2}{h}\left(1+\frac{y}{h}\right)\hat{j}$

(D) $\frac{v_o^2 x}{h^2}\hat{i}$

36. Ans: (A)

Sol: At any distance x and y = h, the discharge is given by

$v_o x = uh$

$$u = \frac{v_o}{h} x \text{ ------------------(1)}$$

Similarly at x and y, the discharge is given by

$vx + uy = v_o x$

$$vx + \frac{v_o}{h} xy = v_o x$$

$$v = v_o - \frac{v_o}{h} y$$

$$v = v_o\left[1 - \frac{y}{h}\right] \text{ --------------(2)}$$

Acceleration in x-direction:

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{v_o}{h} x \times \frac{v_o}{h} = \frac{v_o^2}{h^2} x$$

Acceleration in y direction:

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + v_o\left(1 - \frac{y}{h}\right) \times v_o\left[-\frac{1}{h}\right]$$

$$= -\frac{v_o^2}{h}\left(1 - \frac{y}{h}\right)$$

Thus, the acceleration of the fluid in the gap is

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{v_o^2}{h^2} x\hat{i} - \frac{v_o^2}{h}\left(1 - \frac{y}{h}\right)\hat{j}$$

37. Given matrix $[A] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, then the system AX = O, where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ has

(A) no solution

(B) a unique solution

(C) only one independent solution

(D) two linearly independent solutions

37. Ans: (D)

Sol: Given $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

$R_2 \to 4R_2 - 6R_1$;

$R_3 \to 2R_3 - R_1$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to (10)R_3 + R_2$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

$\therefore$ Number of linearly independent solutions

= Number of variables – Rank of A

= 4 – 2 = 2

38. In a sun and planet gear train, the fixed sun gear has 100 teeth. At an instant the speeds of arm and planet gears are 10 rad/s and 50 rad/s respectively. The number of teeth on planet gear is

(A) 16 (B) 20

(C) 25 (D) 100

38. Ans: (C)

Sol: A sun and planet gear train is shown in figure below.

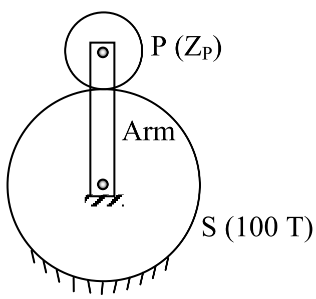
$N_S = 0$,

$Z_s = 100$ T

$N_P = 50$ rad/s,

$Z_P = ?$

$N_A = 10$ rad/s,



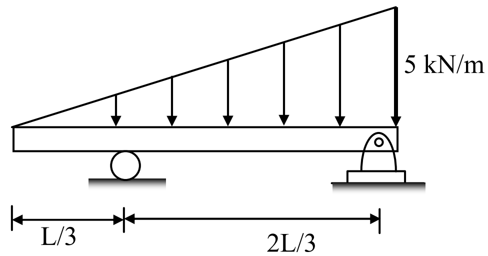
As per relative velocity method,

$$-\frac{Z_S}{Z_P} = \frac{N_P - N_A}{N_S - N_A}$$

$$-\frac{100}{Z_P} = \frac{50-10}{0-10}$$

$\therefore$ $Z_P = 25$ teeth

39. A over hanging beam of length L = 3 m is loaded as shown in figure below. The location of point of contra-flexure from left support is

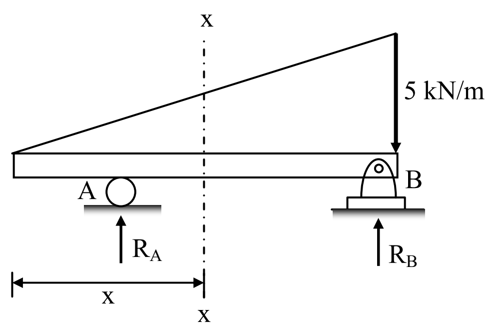


(A) 3000 mm (B) 2100 mm

(C) 1100 mm (D) 600 mm

39. Ans: (C)

Sol: The point at which bending moment changes it sign either from negative to positive or from positive to negative is said to be point of contra-flexure.



Let us consider a section - x at a distance x from left end.

Resisting bending moment about section – x

$$M_x = R_A\left(x - \frac{L}{3}\right) - \left[\frac{1}{2}\times\frac{wx}{L}\times x\right]\times\frac{x}{3}$$

$$M_x = R_A(x-1) - 1\times\frac{wx^3}{18}$$

Taking moments about support B

$$R_A \times 2 = \frac{1}{2}\times 5\times 3\times(1)$$

$$R_A = \frac{15}{4}\text{ kN}$$

$$M_x = \frac{15}{4}(x-1) - \frac{5x^3}{18}$$

At point of contraflexure,

Bending moment (M) = 0

$\therefore \quad M_x = 0$

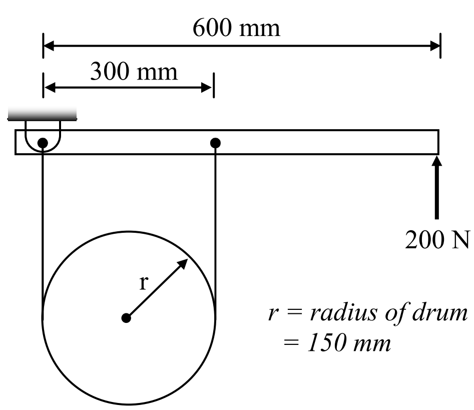
$$\frac{15}{4}(x-1) = \frac{5x^3}{18}$$

$$x^3 - 13.5\,(x-1) = 0$$

As this is cubic equation, x will have 3 roots.

It is difficult to solve the roots using on-screen calculator.

40. A simple band brake is used for braking a rotating drum as shown in figure. The angle of wrap of band around the drum is 180° and the coefficient of friction between band and drum is 0.35. The braking torque required during clockwise rotation and counter clockwise rotation of drum are T_1 and T_2 respectively. The magnitude of $\frac{T_1}{T_2}$ is



(A) 3

(B) 1

(C) $\frac{1}{9}$

(D) $\frac{1}{3}$

40. Ans: (D)

Sol: *Clockwise rotation of drum :*

Let, P = 200 N,

ℓ = 600 mm,

a = 300 mm,

T_t = tension on tight side,

$$T_f = \frac{P \times \ell}{a}$$

$\therefore$ Braking torque,

$$T_1 = T_f\left(1 - \frac{1}{e^{\mu\theta}}\right) r$$

Counter clockwise rotation of drum :

T_s = Tension on slack side

$$T_s = \frac{P \times \ell}{a}$$

$\therefore$ Braking torque,

$$T_2 = T_s\left(e^{\mu\theta} - 1\right) r$$

$$\frac{T_1}{T_2} = \frac{T_t\left(1 - \frac{1}{e^{\mu\theta}}\right) r}{T_s\left(e^{\mu\theta} - 1\right) r}$$

$$\frac{T_1}{T_2} = \frac{1}{e^{\mu\theta}}$$

$$= \frac{1}{e^{0.35 \times \pi}}$$

$$\frac{T_1}{T_2} = 0.33$$

41. Consider the following project data :

Activity	Predecessor	Duration (days)
A	-	5
B	-	7
C	-	6
D	B, C	8
E	B, C	4
F	B	5
G	A	3
H	F, G	6
I	D, F	7
J	D, E, F	5

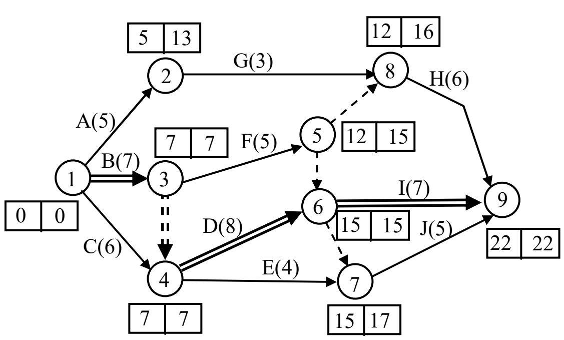
The project duration (in days) and minimum number of dummy activities required in 'AOA' diagram are respectively

(A) 22 and 0 (B) 22 and 3

(C) 22 and 4 (D) 13 and 1

41. Ans: (C)

Sol:



42. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(A) 0.1587 (B) 0.3174

(C) 0.3456 (D) 0.2345

42. Ans: (A)

Sol: Let X = annual precipitation

We know area under normal curve in the interval $(\mu - \sigma, \mu + \sigma) = 0.6826$

where μ is mean and σ is standard deviation

$\Rightarrow P(800 < X < 1200) = 0.6826$

Required probability = $P(X > 1200)$

$$= \frac{1-0.6826}{2} = 0.1587$$

43. A mild steel component is subjected to completely reserved loading. When the amplitude stress is 40 MPa the component fails exactly at 24685 cycles. When the amplitude stress is increased to 65 MPa the component fails exactly at 1035 cycles. If the amplitude stress is 25 MPa, then the life of the component is

(A) 604487 cycles (B) 10480 cycles

(C) 59061 cycles (D) 532034 cycles

43. Ans: (D)

Sol: For finite life under completely reversed loading.

$\sigma = A\,N^B$

Where,

σ = amplitude stress,

N = Number of cycles,

A, B are constants.

$40 = A\ 24685^B$ -------(1)

$65 = A\ 1035^B$ -------(2)

Dividing eq.(1) by eq.(2), we get

$$\frac{40}{65} = \left(\frac{24685}{1035}\right)^B$$

$B = -0.15307$

$A = 188.11$

When, σ = 25 MPa,

$25 = 188.11\ N^{-0.15307}$

$$\ell n\left(\frac{25}{188.11}\right) = -0.15307\ \ell n(N)$$

$\Rightarrow$ N = 532034 cycles

44. Consider the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with initial condition $y(0) = 1$. The value of y(1) = _____.

44. Ans: 0.7357 (Range 0.73 to 0.74)

Sol: Given $\frac{dy}{dx} + 2xy = e^{-x^2}$ …….. (1)

with y(0) = 1 …….. (2)

$\therefore$ I. F. $= e^{\int 2x\,dx} = e^{x^2}$

Now, the general solution of (1) is

$$\Rightarrow y.\ e^{x^2} = \int e^{x^2}.\ e^{-x^2}\,dx + c$$

$$\Rightarrow y.\ e^{x^2} = x + c \qquad …….. (3)$$

Using (2), (3) becomes

$\Rightarrow 1 = 0 + c \Rightarrow c = 1$

$$y = x\ e^{-x^2} + e^{-x^2}$$

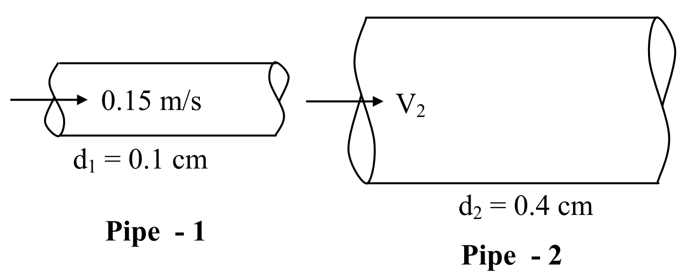
$$y = (x + 1)\ e^{-x^2}$$

$\therefore\ y(1) = 2 \times e^{-1} = 0.7357$

45. The ratio of friction factors (f_1/f_2) in two different pipes with same fluid is 0.5. The average flow velocity in pipe-1 is 0.15 m/s and the pipe diameter is 0.1 cm. The flow in the pipes can be assumed to be laminar. The radius of pipe-2 is 0.2 cm. The average velocity in pipe-2 is ______ m/s. (Rounded off to three decimal places)

45. Ans: 0.019 (Range: 0.018 to 0.020)

Sol:



Given:

$$\frac{f_1}{f_2} = 0.5$$

Flow is laminar in both pipes.

We know that in laminar flow through a pipe,

$$f = \frac{64}{Re} = \frac{64\nu}{Vd}$$

Thus,

$$\frac{f_1}{f_2} = 0.5 = \frac{64\nu}{0.15\times 0.1\times 10^{-2}} \times \frac{V_2 \times 0.4\times 10^{-2}}{64\nu}$$

On simplification

$$0.5 = \frac{V_2 \times 0.4}{0.15\times 0.1}$$

$\Rightarrow$ V_2 = 0.01875 m/s ≈ 0.019 m/s

46. The voltage arc length characteristic of a DC arc is given by V = 20 + 4L where V is the arc voltage in volts and L is the arc length in mm. The static volt ampere characteristics of the power source is approximated by a straight line with open circuit voltage as 80 V and short circuit current as 900 Amps. The maximum power for stable equilibrium conditions of welding equipment (in kW) is__________ *(Round to one decimal place)*.

46. Ans: 18.0 (Range: 17.0 to 19.0)

Sol: Given, V_o = 80 V, I_S = 900 A

V_a = 20 + 4L;

$$V_P = V_o - \frac{V_o}{I_s} \times I$$

At equilibrium condition

$$V_a = V_P$$

$$20 + 4\,L = 80 - \frac{80}{900} \times I$$

$$\frac{80}{900} I = 60 - 4L$$

$$\Rightarrow I = \frac{900}{80}(60 - 4L)$$

Power (P) = VI

$$P = (20 + 4L)\left[\frac{900}{80}(60 - 4L)\right]$$

$$= \frac{900}{80}\left[1200 - 80L + 240L - 16L^2\right]$$

$$= \frac{900}{80}\left(1200 + 160L - 16L^2\right)$$

For optimum power and optimum arc length

$$\frac{dP}{dL} = 0 \Rightarrow 160 - 32L = 0$$

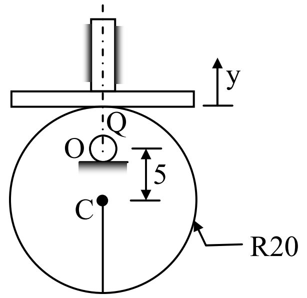
$$\Rightarrow L = \frac{160}{32} = 5\,mm$$

∴ V = 20 + 4 × 5 = 40 V

$$I = \frac{900}{80}(60 - 4\times 5) = 450\,A,$$

P = VI = 40 × 450 = 18 kW

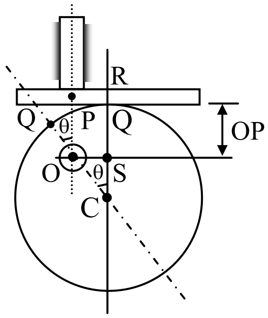
47. A circular disc cam with flat foot follower is shown in figure. Radius, eccentricity of circular cam is 20 cm and 5 cm respectively. Cam is rotating with 10 rad/s.



Maximum velocity of the follower in meters per second is ______ m/s.

47. Ans: 0.5 (Range 0.47 to 0.53)

Sol: F.B.D of cam and follower system after small rotation of cam (θ) is



Follower lift, y = OP – OQ

But from above figure,

OP = RS = RC – CS

= R – OC × cosθ

OP = (R – ρ cosθ)

∴ y = R – ρ cosθ – (R – ρ)

∴ y = ρ – ρ cosθ

y = ρ (1 – cosθ)

∴ Velocity, $V = \frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$

$$V = \rho \times \sin\theta \times \frac{d\theta}{dt}$$

$$V = \rho\, \omega \sin\theta$$

Maximum velocity is ρω,

$V_{max} = 5 \times 10$ cm/s = 50 cm/s ≈ 0.5 m/s

48. Air enters a nozzle steadily at 200 kPa and 65°C with a velocity of 35 m/s and exits at 95 kPa and 240 m/s. If the heat loss from the nozzle to the surrounding medium at 17°C is 3 kJ/kg then the irreversibility of the process (in kJ/kg) is ______ (Rounded upto one decimal place).

(Take, c_p = 1.005 kJ/kg°C ; R = 0.287 kJ/kg°C for air, γ = 1.4)

48. Ans: 36.9 (Range 36.2 to 37.6)

Sol: By using SFEE at inlet and exit of nozzle,

$$h_1 + \frac{V_1^2}{2000} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2000}$$

$$1.005 \times 65 + \frac{35^2}{2000} - 3 = 1.005 \times T_2 + \frac{240^2}{2000}$$

∴ $T_2 = 34$°C

Now, entropy generation can be calculated as,

$$s_{gen} = (s_2 - s_1) + \frac{Q}{T}$$

$$= c_p\, \ell n\left(\frac{T_2}{T_1}\right) - R\ell n\left(\frac{P_2}{P_1}\right) + \frac{3}{290}$$

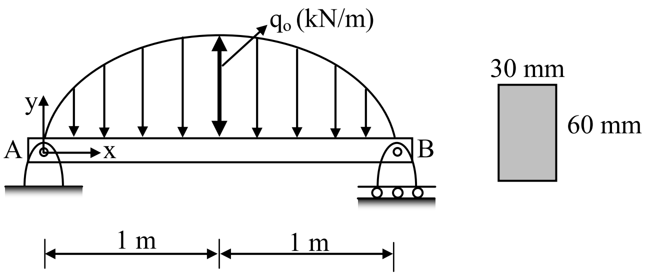
$$= 1.005\ell n\left(\frac{307}{338}\right) - 0.287\ell n\left(\frac{95}{200}\right) + \frac{3}{290}$$

= 0.127 kJ/kgK

Irreversibility = $T_o \times s_{gen}$

$= 290 \times 0.127 = 36.9$ kJ/kg

49. A simply supported beam of rectangular cross section (30 mm × 60 mm) is subjected to a parabolically distributed load of intensity, $q_x = q_o.x\ (2 - x)$, where x is in 'm', 'q_X' is in kN/m, q_o is the maximum intensity of load. If the allowable stress in bending is limited to 100 MPa then the magnitude of q_o is ______ kN/m. (Rounded upto two decimal places)



49. Ans: 4.32 (Range 4.00 to 4.50)

Sol: Given,

$q_x = q_o\, x(2 - x)$

[Here x(2 – x) is just a factor]

Shear force at a distance 'x',

$$V_x = \int q_x\, dx + C_1$$

$$= q_o \int (2 - x^2) dx + C_1$$

$$V_x = q_o\left(x^2 - \frac{x^3}{3}\right) + C_1$$

Bending moment at a distance 'x'

$$M_x = \int V_x\, dx + C_2$$

$$= q_o\left(\frac{x^3}{3} - \frac{x^4}{12}\right) + C_1\, x + C_2$$

Boundary conditions :

At x = 0 ; $M_x = 0$

$\therefore\ 0 = q_o\,(0 - 0) + C_1\,(0) + C_2$

$\therefore\ C_2 = 0$

At x = 2 ; $M_x = 0$

$$0 = q_o\left(\frac{8}{3} - \frac{16}{12}\right) + C_1 \times 2$$

$$C_1 = -q_o \times \frac{2}{3}$$

At x = 1, $M_x = M_{max}$ [$\because$ Symmetric loading]

$$M_{max} = q_o\left(\frac{1}{3} - \frac{1}{12}\right) - q_o \times \frac{2}{3} = q_o \times \frac{1}{4} - q_o \times \frac{2}{3}$$

$$= -q_o\left(\frac{5}{12}\right)\ \text{kN.m}$$

$$= -q_o\left(\frac{5}{12}\right) \times 10^6\ \text{N.mm}$$

Design condition :

$$\sigma_{allowable} = \sigma_{max}$$

$$100\ \text{N/mm}^2 = \frac{6\ M_{max}}{b \times d^2\ (\text{mm}^3)}$$

$$= \frac{6 \times q_o \times \frac{5}{12} \times 10^6}{30 \times 60^2}$$

$\Rightarrow\quad q_o = 4.32$ kN/m

50. In a slab milling operation, a cutter of 90 mm diameter with 10 numbers of teeth on cutter is used. The depth of cut and table feed are set at 3 mm and 180 mm/min respectively. If the cutting speed is 40 m/min, the average chip thickness in milling (in mm) is __________.
(Round to three decimal places)

50. Ans: 0.023 [Range: 0.020 to 0.026]

Sol: Given, slab milling operation,

D = 90 mm

Z = 10, v = 40 m/min

d = 3 mm, f = 180 mm/min

$$N = \frac{1000\times v}{\pi\times D} = \frac{1000\times 40}{\pi\times 90} = 141.47\,\text{rpm}$$

$$t_{1\max} = \frac{2f_m}{NZ}\sqrt{\frac{d}{D}}$$

$$= \frac{2\times 180}{141.47\times 10}\sqrt{\frac{3}{90}} = 0.0465\text{ mm}$$

$t_{1\,min} = 0$

$$t_{1avg} = \frac{t_{1\max}+t_{1\min}}{2}$$

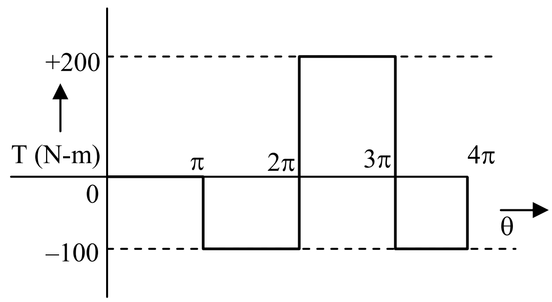
The average chip thickness in milling

$$= \frac{0.0465+0}{2} = 0.02325 \cong 0.023\text{ mm} \quad \text{(OR)}$$

$t_{1avg} = \frac{f_m}{NZ}\sqrt{\frac{d}{D}} =$ Average uncut chip thickness $= \frac{180}{141.47\times 10}\sqrt{\frac{3}{90}}$

$= 0.02325\text{ mm} \approx 0.023$

51. A simplified turning moment diagram of an engine is shown in figure. Mass moment of inertia of flywheel required to keep fluctuation within 2% of mean speed 10 rad/s is _______ (in kg-m^2). (Rounded to one decimal place).



51. Ans: 314.2 (Range 313.35 to 315.5)

Sol: E = Net energy per cycle = ΣA_i

$= 0 - 100\times\pi + 200\times\pi - 100\times\pi = 0$

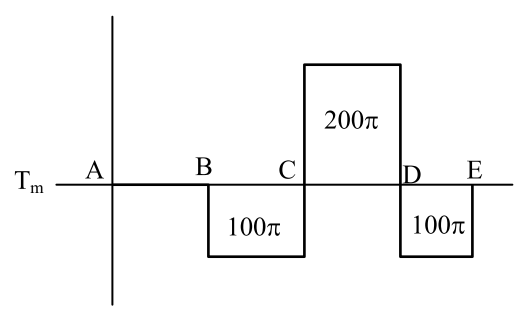
E = 0 N-m (No energy)

But E = T_m × cycle angle

$= T_m \times 4\pi = 0$

$\therefore T_m = 0$

∴ Redrawing T vs θ diagram as areas above/below mean torque line will result.



∴ ΔE = Maximum fluctuation of energy

= Area of central loop

$\Delta E = 200 \times \pi$

where, $C_s = 0.02$, $\omega = 10$ rad/s

But $\Delta E = I\,\omega^2 C_s$

$200\times\pi = I \times 10^2 \times 0.02$

$\Rightarrow \quad I = 100 \times \pi$ kg-m^2 = 314.2 kg-m^2

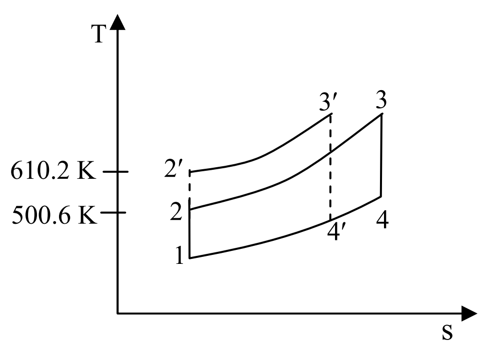
52. Consider a simple ideal Brayton cycle with air as the working fluid. The pressure ratio of the cycle is 6, and the minimum and maximum temperatures are 300 K and 1300 K, respectively. If the pressure ratio is doubled without changing the minimum and maximum temperatures in the cycle, then the change in the net work output per unit mass is _______ kJ/kg.

(Take, $c_p = 1.005$ kJ/kgK ;

$R = 0.287$ kJ/kgK for air, $\gamma = 1.4$)

52. Ans: 30.4 (Range 29.4 to 31.4)

Sol:



For $r_p = 6$:

$$T_2 = T_1 \times (6)^{\frac{\gamma-1}{\gamma}} = 500.6\,K$$

$$T_4 = T_3 \times \left(\frac{1}{6}\right)^{\frac{\gamma-1}{\gamma}} = 779.1\ K$$

$W_{net} = 1.005\ [(T_3 - T_4) - (T_2 - T_1)]$

$= 1.005\ [(1300 - 779.1) - (500.6 - 300)]$

$= 321.9$ kJ/kg

For $r_p = 12$:

$$T_2' = 300 \times (12)^{\frac{\gamma-1}{\gamma}} = 610.2\,K$$

$$T_4' = 1300 \times \left(\frac{1}{12}\right)^{\frac{\gamma-1}{\gamma}} = 639.2\ K$$

$W_{net} = 1.005\ [(T_3' - T_4') - (T_2' - T_1)]$

$= 1.005\ [(1300 - 639.2) - (610.2 - 300)]$

$= 352.3$ kJ/kg

$\Delta W_{net} = 352.3 - 321.9 = 30.4$ kJ/kg

53. The value of the double integral $\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2}\right) dx \right) dy$, using the substitution $u = \left(\frac{2x-y}{2}\right)$ and $v = \frac{y}{2}$ or otherwise is ______.

53. Ans: 4 (Range 3.9 to 4.1)

Sol: Given $u = \dfrac{2x-y}{2}$ and $v = \dfrac{y}{2}$

$\Rightarrow du = dx,\ dv = \dfrac{dy}{2}$ and $dy = 2\,dv$

If $x = \dfrac{y}{2}$ then $u = 0$

If $x = \frac{y}{2} + 1$ then $u = 1$

If $y = 0$ then $v = 0$

If $y = 8$ then $v = 4$

$$\int_0^8 \left[\int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x - y}{2} \right) dx \right] dy = \int_{v=0}^{4} \int_{u=0}^{1} 2u \; du \, dv = 4$$

54. The dimension of an annular cylinder of outside diameter 8 cm, inside diameter 3 cm and height 5 cm steel casting are to be made. Assume the cylindrical side riser with height is equal to diameter. For tabulated shape factor values given below, the diameter of the riser (in cm) is________ *(Round to one decimal place)*

Shape factor	3	5	7	9	11
Riser volume / Casting volume	0.85	0.625	0.525	0.45	0.375

54. Ans: 4.9 Range: (4.5 to 5.5)

Sol: In casting shape factor (SR) = $\frac{L + W}{t}$

Weight (W) = 5 cm

Length, $L = \pi \times D_{mean} = \pi\left(\frac{8+3}{2}\right)$

$= 17.27$ cm

$t = \frac{8-3}{2} = 2.5$ cm

Shape factor $= \frac{17.27 + 5}{2.5} = 8.91 \approx 9$

From the given table for shape factor, the ratio

$$\frac{V_r}{V_c} = 0.45$$

$V_r = 0.45 \times V_c$

$$\frac{\pi}{4} D^3 = 0.45 \times \frac{\pi}{4}\left(8^2 - 3^2\right) \times 5$$

$\Rightarrow$ D = 4.98 cm

55. A bucket full of water is placed in a 50 m^3 room at 15°C with initial relative humidity 40%, 361 grams of water will evaporate to maintain equilibrium. If the room temperature is increased by 5°C, then the how much water (in grams) will evaporate __________

At, 15°C , $P_{sat} = 1.6$ kPa

At, 20°C , $P_{sat} = 2.4$ kPa

55. Ans: 286 (Range: 282 to 290)

Sol: Initial water vapour in room $= \frac{P_v \times V}{R_{H_2O} \times T_1}$

$$= \frac{0.4 \times 1.6 \times 50}{0.461 \times 288}$$

(T_1 = Initial room temperature = 15°C)

= 241 gm

As, water vapour will evaporate to maintain equilibrium till saturation pressure is reached. Water vapour in the room at equilibrium at 15°C.

$$= \frac{1.6 \times 50}{0.461 \times 288}$$

= 602.5 gm

Final water vapour in room in equilibrium at 20°C

$$= \frac{P_{sat}(at\ 20^\circ C) \times V}{R_{H_2O} \times T_2}$$

$$= \frac{2.4 \times 50}{0.461 \times 293}$$

(T_2 = Final room temperature = 20°C)

= 888.5 gm

Amount of water evaporated

= 888.5 – 602.5 = 286 gm

56. In a manufacturing company the daily requirement is 100 units and the same is produced @ 120 units per day. The company works for 250 days in a year. The setup cost per setup is Rs. 500 and inventory holding cost is estimated as Rs. 2.167 per unit per month. The maximum inventory level is ______.

56. Ans: 400 (Range: 399 to 401)

Sol: Rate of consumption (d) = 100 units/day

Rate of production (k) = 120 units/day

Annual demand (D) = d × 250

= 100 × 250 = 25000 units

Setup cost (C_o) = Rs. 500/setup

Carrying cost (C_c) = 2.167×12/unit/year

= Rs. 26/unit/year

$$EBS = \sqrt{\frac{2DC_o}{C_c}}\sqrt{\frac{k}{k-d}}$$

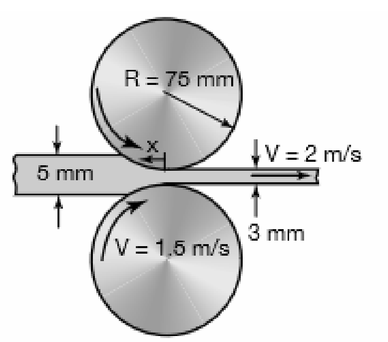
Maximum inventory level,

$$(Q_{max}) = EBS.\left(\frac{k-d}{k}\right)$$

$$= \sqrt{\frac{2 \times 25000 \times 500}{26}}\sqrt{\frac{120}{120-100}}\left(\frac{120-100}{120}\right)$$

= 2400×0.167 = 400.28 units

57. An aluminium metallic strip having a thickness of 5 mm is to be rolled using hardened steel rolls, each of 150 mm diameter under the condition shown in the figure below. It is noted that there are front and back tensions that have not been specified. The surface roughness of the rolls is 0.02 μm and rolling temperature is 210°C. The position x_n (in mm) of the neutral point is _____. (Rounded to two decimal places)



57. Ans: 8.64 [Range: 8.00 to 9.00]

Sol: Given,

Inlet conditions: $H_0 = 5$ mm, $V_0 = ?$

Outlet conditions: $V_1 = 2$ m/sec, $H_1 = 3$ mm

At neutral point: H, V

D = 150 mm ⇒ r = 75 mm

Volume before = Volume after

$V_0\, b_0\, H_0 = V_1\, b_1\, H_1$

$V_0 \times 5 = 2 \times 3$

$V_0 = 1.2$ m/s

At the neutral point the velocity is the roll velocity

$VH = V_1 H_1$

$1.5 \times H = 2 \times 3$

H = 4 mm

Assuming the material is incompressible,

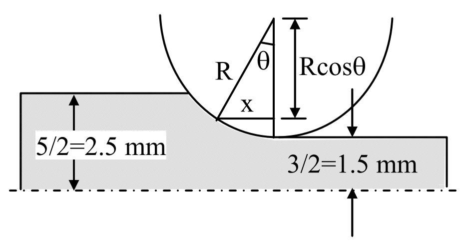


Fig: The rolls bite geometry.

θ can be calculated from above diagram:

$$75 - 75\cos\theta = \frac{4-3}{2}$$

θ = 6.62°.

∴ The position x_n (in mm) of the neutral point

= Rsinθ = 75sin6.62° = 8.64 mm

58. The surface integral $\iint_s (\bar{F}.\bar{n})\, dS$ over the surface S of the sphere $x^2 + y^2 + z^2 = 9$, where $F = (x+y)\bar{i} + (x+z)\bar{j} + (y+z)\bar{k}$ and $\bar{n}$ is the unit outward surface normal, yields ______.

58. Ans: 226.08 (Range 226 to 227)

Sol: $\vec{F} = (x+y)\vec{i} + (x+z)\vec{j} + (y+z)\vec{k}$

div $\vec{F} = 1 + 1 = 2$

$\iint_S \vec{F}.\vec{n}\, dS = \iiint_V \text{div}\,\vec{F}\, dx\, dy\, dz$ (By Gauss divergence theorem)

$= \iiint 2\, dx\, dy\, dz$

= 2 (volume of the sphere $x^2 + y^2 + z^2 = 9$)

$$= 2 \times \frac{4}{3}\pi(3)^3 = 72\,\pi = 226.08$$

59. Air at 80 kPa, 27°C and 220 m/s enters a diffuser at a rate of 2.5 kg/s and leaves at 42°C. The air is estimated to lose 18 kW of heat during this process. If the exit area of diffuser is 400 cm^2 then the exit pressure (in kPa) of the air is _____. (Round off one decimal place)

(Take, $c_p = 1.005$ kJ/kg°C ;

R = 0.287 kJ/kg°C for air, γ = 1.4)

59. Ans: 91.1 (Range 90.7 to 91.5)

Sol: By using S.F.E.E at inlet and exit of nozzle,

$$\dot{m}\left(h_1+\frac{V_1^2}{2000}\right)+\frac{dQ}{dt}=\dot{m}\left(h_2+\frac{V_2^2}{2000}\right)$$

$$2.5\left(1.005\times300+\frac{220^2}{2000}\right)-18=2.5\left(1.005\times315+\frac{V_2^2}{2000}\right)$$

$\therefore\ V_2 = 62$ m/s

$$\dot{m}=\rho_2\ A_2\ V_2$$

$$2.5=\frac{P_2}{RT_2}\times400\times10^{-4}\times62$$

$\therefore\ P_2 = 91.1$ kPa

60. In an orthogonal cutting with a tool of 0º rake angle, the cutting force is 200 N and the cutting velocity is 90 m/min. Assuming that 90% of the total work has been utilized for plastic deformation and shearing and gets converted to heat, out of which 10% heat goes into the workpiece, the heat flow (in W) in the workpiece will be______.

60. Ans: 27 [Range: 26 to 28]

Sol: Given,

Orthogonal cutting, $\alpha = 0°$,

Cutting force, $F_C = 200$ N

Cutting velocity, $V_C = 90$ m/min

The deformation power along the shear plane is = 90% of total work

$= 0.9 \times F_C \times V_C$

$= 0.9 \times 200 \times 90/60 = 270$ W

Total heat generated in the shearing zone

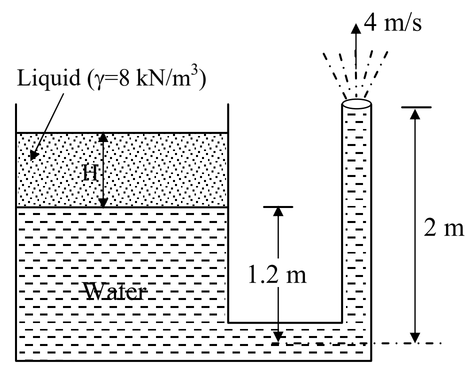
= 270 W

Heat flow in the workpiece

$= 270 \times 0.1 = 27$ W

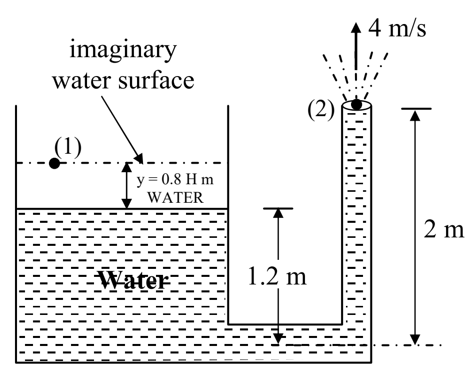
61. A large tank contains water and a liquid (Specific weight = 8 kN/m^3) as shown in the figure. Water is leaving steadily at 4 m/s through the small pipe. The depth, H of liquid is _____ m.

(Assume g = 10 m/s^2)



61. Ans: 2

Sol:



Equivalent height (y) of H m of liquid in terms of m of water column is

$$\gamma \times H = \gamma_w \times y$$

Or, $$y = \frac{\gamma}{\gamma_w} H = \frac{8}{10} H = 0.8H$$

Applying Bernoulli's equation for points (1) (lying on the imaginary water surface) and (2) (exit of the pipe), we get

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2$$

where, $P_1 = P_2 = P_{atm} = 0$;

$V_1 = 0$; $V_2 = 4$ m/s;

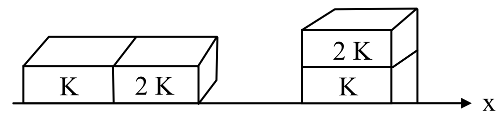
$Z_1 = 1.2 + 0.8$ H and $Z_2 = 2$ m

$$0 + 0 + 1.2 + 0.8H = 0 + \frac{4^2}{2 \times 10} + 2$$

$$0.8H = 0.8 + (2 - 1.2)$$

$$\Rightarrow H = \frac{1.6}{0.8} = 2\,m$$

62. Two rectangular blocks, having identical dimensions can be arranged either in configuration I or in configuration II as shown in the figure. The blocks have thermal conductivities K and 2K, as shown in the figure below. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 sec to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat from the hot end to cold end in the configuration II is ____ sec.



(Configuration I) **(Configuration II)**

62. Ans: 2

Sol: Heat transfer rate through the bocks for given configurations is

$$\frac{Q}{t} = \frac{\Delta T}{R_{th}}$$

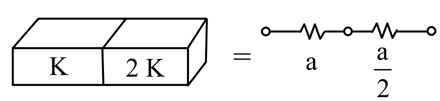
where Q is amount of heat flow, R_{th} is the corresponding thermal resistance and t is the time taken to transport the heat.

$\therefore$ For same amount of heat flow and temperature difference :

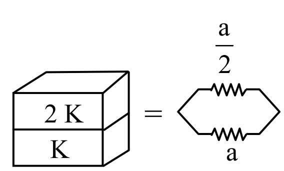
$\Rightarrow t \propto R_{th}$

or $$\frac{t_1}{t_2} = \frac{R_{th1}}{R_{th2}} \quad \text{------(i)}$$

Let L be the length of the blocks parallel to x-axis and $\frac{L}{KA} = a$



(Configuration I)



(Configuration II)

For configuration I,

$$R_{th1} = \frac{L}{KA} + \frac{L}{2KA} = a + \frac{a}{2} = \frac{3a}{2} \quad \text{-----(ii)}$$

For configuration II,

$$R_{th2} = \frac{\left(a.\frac{a}{2}\right)}{a + \frac{a}{2}} = \frac{\frac{a^2}{2}}{\frac{3a}{2}} = \frac{a^2}{2} \times \frac{2}{3a} = \frac{a}{3} \quad \text{-----(iii)}$$

From eq.(i), (ii) and (iii), with $t_1 = 9$ sec (given)

then $$\frac{9}{t_2} = \frac{\frac{3a}{2}}{\frac{a}{3}} \Rightarrow t_2 = 2 \text{ sec}$$

The required time to transport heat in configuration II is 2 seconds.

63. The stress matrix for a particle is given by

$$\sigma_{ij} = \begin{bmatrix} 40 & 0 & 27 \\ 0 & 20 & 0 \\ 27 & 0 & -32 \end{bmatrix} \text{MPa}$$

where, *i* represents the direction of area, *j* represents the direction of load.

For the given state of stress, the largest possible diameter of Mohr's circle (in MPa) is _________

63. Ans: 90

Sol: Given data,

$\sigma_{xx} = 40$, $\tau_{xy} = 0$

$\sigma_{yy} = 20$, $\tau_{yz} = 0$

$\sigma_{zz} = -32$, $\tau_{xz} = 27$

There is no shear stress in y-direction.

Hence, σ_{yy} is considered to be the principal stress.

$\sigma_{yy} = \sigma_2 = 20$ MPa

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \frac{40 - 32}{2} + \sqrt{\left(\frac{40 + 32}{2}\right)^2 + (27)^2}$$

$= 4 + 45 = 49$ MPa

$$\sigma_3 = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$= 4 - 45 = -41$ MPa

τ_{max} = largest possible radius of Mohr's circle

$$= \max\left\{\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|\right\}$$

Largest possible diameter of Mohr's circle

$= \max\{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\}$

$= \max\{|49 - 20|, |20 + 44|, |-41 - 49|\}$

$= 90$ MPa

64. A flow of 0.1 kg/s of exhaust gas at 700 K from a gas turbine is used to preheat the incoming air, which is at the ambient temperature of 300 K . It is desired to cool the exhaust gas to 400 K. It is estimated that with an overall heat transfer coefficient of 30 W/m^2 K, it can be achieved in a balanced counter flow heat exchanger. Assume that the specific heat of exhaust gases the same

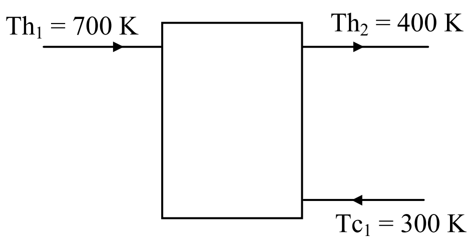
as for air, which is 1000 J/kg K. The area required for the counter flow heat exchanger is ______m^2.

64. Ans: 10

Sol: Given data:

$U = 30\ W/m^2K \qquad \dot{m} = 0.1\ kg/s$

$C = 1000\ J/kg\text{-}K \qquad R = 1$



$$C_{min} = \dot{m}C = 0.1 \times 1000 = 100\ W/K$$

$$NTU = \frac{UA}{C_{min}} = \frac{30 \times A}{100}$$

$$\varepsilon = \frac{Q_{act}}{Q_{max}} = \frac{\dot{m}_h C_h (Th_1 - Th_2)}{C_{min}(Th_1 - Tc_1)} = \frac{700-400}{700-300} = 0.75$$

$$\varepsilon = \frac{NTU}{1+NTU}$$

$$NTU = \frac{\varepsilon}{1-\varepsilon} = 3$$

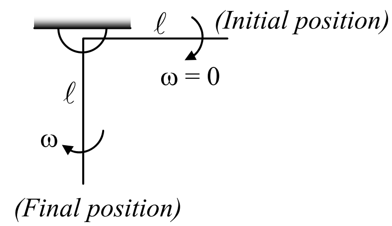
$$NTU = \frac{UA}{C_{min}} = \frac{30\,A}{100} = 3$$

$$\Rightarrow A = 10\ m^2$$

65. A uniform rod of mass 'm' hinged at one of its end and is released from a horizontal position. When it reaches the vertical position, the force that it exert on the hinge is cmg (where c is positive constant and g is acceleration due to gravity), then the value of c is _________

65. Ans: 2.5 (Range 2.5 to 2.5)

Sol:



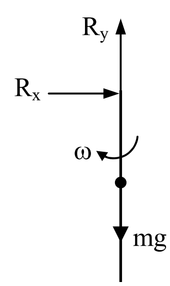
Let, ℓ = length of rod

By conservation of mechanical energy between initial and final position,

$$mg\frac{\ell}{2} = \frac{1}{2} \times \frac{m\ell^2}{3} \times \omega^2$$

$$\therefore \omega = \sqrt{\frac{3g}{\ell}}$$

F.B.D of rod at final position :



By writing equation of motion along vertical direction,

$$R_y - mg = m \times (a_r)_{cm} = m \times \omega^2 \times \frac{\ell}{2}$$

$$R_y - mg = \frac{3mg}{2}$$

$$\therefore R_y = 2.5\,mg$$