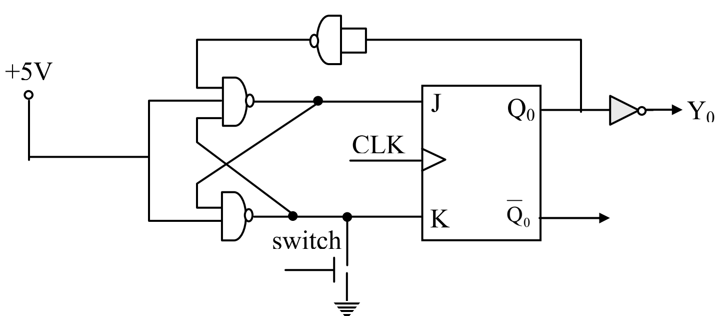
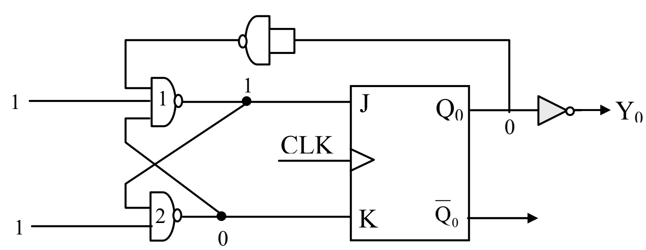
48. In the circuit shown, the input clock frequency is 9.7 MHz, the propagation delay of the logic gates and flip-flop's are 0 sec. Initially Q_0 is cleared to '0' and simultaneously the input 'K' is also cleared to '0'(with the help of push button toggle switch). The frequency of the wave form at 'Y_0' is ______ (Hz)



Ans: 0

Sol:



Observe the binary on the circuit as per given data

From the given data K = 0 (reset)

So NAND gate 1, output J = 1

Initially: J = 1, K = 0

$Q_0 = 0$, $Y_0 = 1$

After one clock edge

$Q_0 = 1$, $Y_0 = 0$

But still(from circuit) $\begin{cases} J = 1 \\ K = 0 \end{cases}$

Irrespective of number of clock pulses

$Q_0 = 1$, $Y_0 = 0$ (always)

J = 1, K = 0

$Y_0 = 0$ Hz (DC line with logic '0')

49. Let a signal x(t) be defined as $X(t) = e^{-\frac{t}{2}}u(t)$. Then the energy in the frequency band $-\pi/8 \le \omega \le \pi/8$ rad/sec is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2\pi}$

(C) $\frac{4}{\pi}$ (D) $\frac{2}{\pi}$

Ans: (D)

Sol: $x(t) = e^{-t/2}u(t)$

According Parseval's Theorem $E_t = E_\omega$

$$E_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$E_\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} |X(\omega)|^2 \, d\omega$$

$$E_\omega = \frac{1}{\frac{1}{2}+j\omega} \Rightarrow |X(\omega)|^2 = \frac{1}{\frac{1}{4}+\omega^2}$$

$$\therefore E_\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{1}{\frac{1}{4}+\omega^2} d\omega$$

$$= \frac{1}{2\pi}.\frac{1}{\left(\frac{1}{2}\right)}\tan^{-1}\left(\frac{\omega}{\frac{1}{2}}\right)_{-\frac{\pi}{8}}^{\frac{\pi}{8}}$$

$$= \frac{2}{2\pi}\tan^{-1}(2\omega)_{-\frac{\pi}{8}}^{\frac{\pi}{8}}$$

$$= \frac{1}{\pi}[1-(-1)] = \frac{2}{\pi}$$

50. A discrete sequence y(n) is defined as y(n) = $x^2(n)$, where x(n) is another discrete sequence $x(n) = \left(\frac{1}{8}\right)^n u(n)$, then the value of $y(e^{j\pi})$ is _______ (Give up to 2 decimal value).

Ans: 0.98 (Range 0.97 to 0.99)

Sol: $y(n) = x^2(n) = \left[\left(\frac{1}{8}\right)^n u(n)\right]^2 = \left(\frac{1}{8}\right)^{2n} u(n)$

$$y(n) = \left(\frac{1}{64}\right)^n u(n)$$

Applying Z-transform both sides

$$y(z) = \frac{z}{z-\frac{1}{64}} \qquad \text{but } z = e^{j\Omega}|_{r=1}$$

$$y(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-\frac{1}{64}}$$

$$y(e^{j\pi}) = \frac{e^{j\pi}}{e^{j\pi}-\frac{1}{64}} = \frac{-1}{-1-\frac{1}{64}}$$

$$y(e^{j\pi}) = \frac{1}{1+\frac{1}{64}} = \frac{64}{65}$$

$$y(e^{j\pi}) = \frac{64}{65} = 0.984$$

51. Given matrix $[A] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, then the system AX = O, where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ has

(a) no solution

(b) a unique solution

(c) only one independent solution

(d) two linearly independent solutions

Ans: (d)

Sol: Given $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

$R_2 \rightarrow 4R_2 - 6R_1$;

$R_3 \rightarrow 2R_3 - R_1$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$R_3 \rightarrow (10)R_3 + R_2$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

$\therefore$ Number of linearly independent solutions = Number of variables – Rank of A

$= 4 - 2 = 2$

52. The value of the double integral $\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2} \right) dx \right) dy$, using the substitution $u = \left(\frac{2x-y}{2} \right)$ and $v = \frac{y}{2}$ or otherwise is ______.

52. Ans: 4 (No range)

Sol: Given $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$

$\Rightarrow du = dx, \; dv = \frac{dy}{2}$ and $dy = 2\,dv$

If $x = \frac{y}{2}$ then $u = 0$

If $x = \frac{y}{2}+1$ then $u = 1$

If $y = 0$ then $v = 0$

If $y = 8$ then $v = 4$

$$\int_0^8 \left[\int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x-y}{2} \right) dx \right] dy = \int_{v=0}^{4} \int_{u=0}^{1} 2u \; du\,dv = 4$$

53. The surface integral $\iint_s (\bar{F}.\bar{n})\,dS$ over the surface S of the sphere $x^2 + y^2 + z^2 = 9$, where $F = (x+y)\bar{i} + (x+z)\bar{j} + (y+z)\bar{k}$ and $\bar{n}$ is the unit outward surface normal, yields ________.

Ans: 226.08 (Range 225 to 227)

Sol: $\vec{F} = (x+y)\vec{i} + (x+z)\vec{j} + (y+z)\vec{k}$

div $\vec{F} = 1 + 1 = 2$

$\iint_S \vec{F}.\vec{n}\,dS = \iiint_V \text{div}\,\vec{F}\,dx\,dy\,dz$ (By Gauss divergence theorem)

$= \iiint 2\,dx\,dy\,dz$

$= 2$

(Volume of the sphere $x^2 + y^2 + z^2 = 9$)

$= 2 \times \frac{4}{3}\pi(3)^3 = 72\,\pi$

$= 226.08$

54. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(A) 0.1587 (B) 0.3174

(C) 0.3456 (D) 0.2345

Ans: (A)

Sol: Let X = annual precipitation

We know area under normal curve in the interval $(\mu - \sigma, \mu + \sigma) = 0.6826$

Where μ is mean and σ is standard deviation

$\Rightarrow P(800 < X < 1200) = 0.6826$

Required probability = $P(X > 1200)$

$= \frac{1-0.6826}{2}$

$= 0.1587$

55. Consider the differential equation $\frac{dy}{dx}+2xy=e^{-x^2}$ with initial condition $y(0)=1$. The value of y(1) = _________.

55. Ans: 0.7357 (Range 0.73 to 0.74)

Sol: Given $\frac{dy}{dx}+2xy=e^{-x^2}$ …….. (1)

with y(0) = 1 …….. (2)

$\therefore$ I. F. = $e^{\int 2x\,dx} = e^{x^2}$

Now, the general solution of (1) is

$\Rightarrow y.\ e^{x^2} = \int e^{x^2} . e^{-x^2} dx + c$

$\Rightarrow y.\ e^{x^2} = x + c$ …….. (3)

Using (2), (3) becomes

$\Rightarrow 1 = 0 + c \Rightarrow c = 1$

$y = x\ e^{-x^2} + e^{-x^2}$

$y = (x + 1)\ e^{-x^2}$

$\therefore y(1) = 2 \times e^{-1} = 0.7357$

END OF THE QUESTION PAPER