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ESE- 2020 (Prelims) - Offline Test Series Test-12

GENERAL STUDIES AND ENGINEERING APTITUDE

SUBJECT: ENGINEERING MATHEMATICS AND NUMERICAL ANALYSIS SOLUTIONS

01. Ans: (b)

Sol: We are given a system of three equations in two unknowns which will be consistent, if

$$\begin{vmatrix} 2-\lambda & 2 & 3 \\ 2 & 4-\lambda & 7 \\ 2 & 5 & 6-\lambda \end{vmatrix} = 0$$

[Operating $R_3 - R_2$]

$$\begin{vmatrix} 2-\lambda & 2 & 3 \\ 2 & 4-\lambda & 7 \\ 0 & 1+\lambda & -1-\lambda \end{vmatrix} = 0$$

[Operating $C_2 + C_3$]

$$\begin{vmatrix} 2-\lambda & 5 & 3 \\ 2 & 11-\lambda & 7 \\ 0 & 0 & -1-\lambda \end{vmatrix} = 0$$

[Expanding along R_3]

$[(-1-\lambda)(2-\lambda)(11-\lambda) - 10] = 0$

or If, $(\lambda + 1)(\lambda^2 - 13\lambda + 12) = 0$

$(\lambda + 1)(\lambda - 1)(\lambda - 12) = 0$

or if, $\lambda = -1, 1, 12$

02. Ans: (c)

Sol: The characteristic equation of A is

$|A - \lambda I| = 0$ i.e., $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$ or

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$ (On simplification)

By Cayley – Hamilton theorem, we have

$A^3 - 6A^2 + 9A - 4I = O$

Multiplying by A^{-1}

$A^2 - 6A + 9I - 4A^{-1} = 0$

$\therefore A^2 - 6A + 9I = 4A^{-1}$

03. Ans: (a)

Sol: The characteristics equation is $|A - xI| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$\therefore \lambda = 1, 1, 3$

04. Ans: (d)

Sol: Here, u is a function of x, y and z while y and z are functions of x.

$$\therefore \frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}.\frac{dy}{dx} + \frac{\partial u}{\partial z}.\frac{dz}{dx}$$

$$= e^y z.\ 1 + xe^y z\ .\ \frac{1}{2}(a^2 - x^2)^{-1/2}(-2x) + xe^y.\ 2\sin x\cos x$$

$$= e^y\left[z - \frac{x^2 z}{\sqrt{a^2 - x^2}} + x\sin 2x\right]$$

05. Ans: (b)

Sol: $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\frac{y}{x}$

$$\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial r}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial\theta}{\partial x}=\frac{1}{1+\frac{y^2}{x^2}}\left(-\frac{y}{x^2}\right)=-\frac{y}{x^2+y^2}$$

$$\frac{\partial\theta}{\partial y}=\frac{1}{1+\frac{y^2}{x^2}}\left(\frac{1}{x}\right)=\frac{x}{x^2+y^2}$$

$$\frac{\partial(r.\theta)}{\partial(x.y)}=\begin{vmatrix}\frac{\partial r}{\partial x} & \frac{\partial r}{\partial y}\\ \frac{\partial\theta}{\partial x} & \frac{\partial\theta}{\partial y}\end{vmatrix}=\begin{vmatrix}\frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}}\\ -\frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}}\end{vmatrix}$$

$$=\frac{x^2}{(x^2+y^2)^{3/2}}+\frac{y^2}{(x^2+y^2)^{3/2}}$$

$$=\frac{x^2+y^2}{(x^2+y^2)^{3/2}}=\frac{1}{\sqrt{x^2+y^2}}$$

Alternately

The given equations can be written as

$x = r\cos\theta$ and $y = r\sin\theta$

$$\frac{\partial(x.y)}{\partial(r.\theta)}=r$$

then, $\dfrac{\partial(r.\theta)}{\partial(x.y)}=\dfrac{1}{r}=\dfrac{1}{\sqrt{x^2+y^2}}$

06. Ans: (b)

Sol: Let $F(a)=\int_0^\infty \frac{e^{-x}\sin(ax)}{x}dx$

Differentiating partially with respect to a

$$F'(a)=\int_0^\infty \frac{e^{-x}}{x}x\cos(ax)dx$$

$$=\frac{a}{a^2+1}$$

$\Rightarrow F(a)=\tan^{-1}a+c$

$F(0)=0\Rightarrow c=0$

$\therefore F(a)=\tan^{-1}a$

07. Ans: (b)

Sol: The A.E is $D^3-4D^2+4D=0$

or $D(D^2-4D+4)=0$

$D(D-2)^2=0$

$D = 0, 2, 2$

Hence, the solution is

$y=c_1e^{0x}+(c_2x+x_3)e^{2x}$

or $y=c_1+(c_2x+c_3)e^{2x}$

08. Ans: (c)

Sol: $P.I=\dfrac{1}{D^3-3D^2+4}e^{2x}$

Here, the denominator vanishes when D is replaced by 2. It is a case of failure.

We multiply the numerator by x and differentiate the denominator w.r.t D (rule).

$$P.I=x.\frac{1}{3D^2-6D}e^{2x}$$

It is again a case of failure. We multiply the number by x and differentiate the denominator w.r.t D

$$\therefore\ P.I=x^2.\frac{1}{6D-6}e^{2x}=x^2.\frac{1}{6(2)-6}e^{2x}=\frac{x^2}{6}e^{2x}$$

09. Ans: (d)

Sol: Given equation in symbol form is $(D^2-2D+1)y=xe^x\sin x$

A.E is $D^2-2D+1=0$ or $(D-1)^2=0$ so that $D = 1, 1$

$C.F=(c_1+c_2x)e^x$

$$P.I=\frac{1}{(D-1)^2}e^x.x\sin x=e^x.\frac{1}{(D+1-1)^2}x\sin x$$

$$=e^x\frac{1}{D^2}x\sin x=e^x\frac{1}{D}\int x\sin dx$$

Integrating by parts

$$=e^x\frac{1}{D}\left[x(-\cos x)-\int 1(-\cos x)dx\right]$$

$$=e^x\frac{1}{D}(-x\cos x+\sin x)$$

$$=e^x\int(-x\cos x+\sin x)dx$$

$$=e^x\left[-\left\{x\sin x-\int 1\sin x dx\right\}-\cos x\right]$$

$= e^x [-x\sin x - \cos x - \cos x]$

$= -e^x\,(x \sin x + 2\cos x)$

Hence, the solution is

$y = (c_1 + c_2 x)e^x - e^x(x\sin x + 2\cos x)$

10. Ans: (d)

Sol: Given equation is a Cauchy's homogenous linear equation

Put $x = e^z$ i.e., $z = \log x$

$$x\frac{dy}{dx} = Dy, \quad x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

where $D = \frac{d}{dz}$

Substituting these values in the given equation, it reduces to

$[D(D-1)(D-2) + 2D(D-1) + 2]y = 0$

$(D^3 - D^2 + 2)\,y = 0$

Which is a linear equation with constant coefficients

It's A.E is

$D^3 - D^2 + 2 = 0$ or $(D+1)(D^2 - 2D + 2) = 0$

$$\therefore\; D = -1,\; \frac{2 \pm \sqrt{4-8}}{2} = -1,\; 1 \pm i$$

The solution is

$y = c_1 e^{-z} + e^z(c_2 \cos z + c_3 \sin z)$

$$= \frac{c_1}{x} + x[c_2\cos(\log x) + c_3\sin(\log x)]$$

11. Ans: (b)

Sol: Differentiating z partially with respect to x

$$p = \frac{\partial z}{\partial x} = 2f'\left(\frac{1}{x} + \log y\right)\left(-\frac{1}{x^2}\right)$$

$$-px^2 = 2f'\left(\frac{1}{x} + \log y\right) \;\ldots\ldots\ldots\ldots (1)$$

Similarly, Differentiating z partially with respect to y

$$q = \frac{\partial z}{\partial y} = 2y + 2f'\left(\frac{1}{x} + \log y\right)\left(\frac{1}{y}\right)$$

$$qy - 2y^2 = 2f'\left(\frac{1}{x} + \log y\right) \;\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2) we have

$-px^2 = qy - 2y^2$ or $x^2p + yq = 2y^2$

Which is the required partial differential equation.

12. Ans: (c)

Sol: The given equation can be written as $y^2zp + x^2zq = xy^2$

Comparing with $Pp + Qq = R$, we have

$P = y^2z,\ Q = x^2z,\ R = xy^2$

$\therefore$ The auxiliary equations are

$$\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{xy^2}$$

Taking the first two members, we have $x^2\,dx = y^2\,dy$

Which an integration gives

$x^3 - y^3 = c_1$ .... (1)

Again taking the first and third members, we have $x\,dx = z\,dz$

Which on integration gives

$x^2 - z^2 = c_2$ .... (2)

From (1) and (2), the general solution is $\phi(x^3 - y^3, x^2 - z^2) = 0$

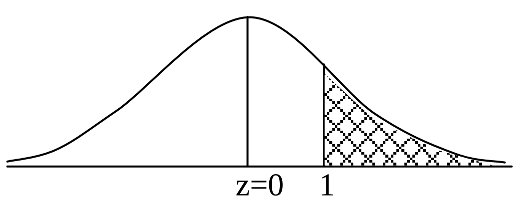
13. Ans: (a)

Sol: Here x denotes the length of life of dry battery cells.

The standard normal variable

$$z = \frac{x - \bar{x}}{\sigma} = \frac{x - 12}{3}$$

$x = 15 \Rightarrow z = 1$



$\therefore P(x > 15) = P(z > 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.4772$

$= 0.0228 = 2.28\%$

14. Ans: (a)

Sol: $f(x) = x^4 - x - 9$

$f(x) = x^4 - x - 9, \quad f'(x) = 4x^3 - 1$

The first approximation is given by

$$x_1 = x_0 - \frac{f(x_0)}{f(x_0)} = x_0 - \frac{x_0^4 - x_0 - 9}{4x_1^3 - 1}$$

$$= \frac{3x_0^4 + 9}{4x_1^3 - 1} = \frac{3(2^4)+9}{4(2^3)-1} = \frac{57}{31} = 1.8$$

15. Ans: (c)

Sol: The general solution is $y(x) = ce^{3x}$,
From this solution and the initial condition, we obtain c = 5.7
Hence the initial value problem has the solution $y(x) = 5.7e^{3x}$

16. Ans: (b)

Sol: The auxiliary is $m^2 + 1 = 0$
$\Rightarrow m = \pm i$
The solution is
$y = c_1 \cos x + c_2 \sin x$ .... (1)
$y(0) = 3 \Rightarrow c_1 = 3$
$y' = -c_1 \sin x + c_2 \cos x$
$y'(0) = -0.5$
$\Rightarrow -0.5 = c_2$
substituting the values of c_1 and c_2 in (1), we get
$y = 3\cos x - (0.5)\sin x$

17. Ans: (d)

Sol: $a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx$

$= \frac{1}{2}\int_{-1}^{1} k\,dx = k$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}dx = \frac{1}{2}\int_{-1}^{1} k\cos\frac{n\pi x}{2}dx = \frac{2k}{n\pi}\sin\frac{n\pi}{2}$$

Thus $a_n = 0$ if n in even and

$a_n = \frac{2k}{n\pi}$ if n = 1, 5, 9 …..,

$a_n = \frac{-2k}{n\pi}$ if n = 3, 7, 11, ……….

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}dx = \frac{1}{2}\int_{-1}^{1} k\sin\frac{n\pi x}{2}dx = 0$$

The fourier series is

$$f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}x - \frac{1}{3}\cos\frac{3\pi}{2}x + \frac{1}{5}\cos\frac{5\pi}{2}x - \ldots\right)$$

18. Ans: (a)

Sol: Now $u_x = 2x$ and $u_y = -2y - 1$.
Hence because of the Chauchy Riemann equation a conjugate v of u must satisfy
$v_y = u_x = 2x$; $v_x = -u_y = 2y + 1$
$f(z) = u + iv$
$f'(z) = u_x + iv_x$
$= u_x + iu_y$
$= 2x + i(2y + 1)$
$= 2z + i$ (replacing x by z and y by 0)
$f(z) = z^2 + iz + c$

19. Ans: (a)

Sol: For the unit circle, $z = e^{it}$
$\Rightarrow \bar{z} = e^{-it}$
t varies form 0 to 2π

$$\oint_C \bar{z}dz = \int_0^{2\pi} e^{-it} i e^{it} dt = 2\pi i$$

20. Ans: (b)

Sol: For any contour enclosing the point πi (counterclockwise)

$$\oint_c \frac{\cos z}{(z-\pi i)^2}dz = 2\pi i(\cos z)'\Big|_{z=\pi i} = -2\pi i\sin\pi i = 2\pi\sinh\pi$$

21. Ans: (a)

Sol: $$\frac{1}{1+z^2} = \frac{1}{1-(-z^2)} = \sum_{n=0}^{\infty}(-z^2)^n$$

$$= \sum_{n=0}^{\infty}(-1)^n z^{2n} = 1 - z^2 + z^4 - z^6 + \ldots(|z| < 1)$$

22. Ans: (c)

Sol: We have $f'(z) = \frac{1}{(1+z^2)}$

$$\frac{1}{1+z^2} = \frac{1}{1-(-z^2)} = \sum_{n=0}^{\infty}(-z^2)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n z^{2n} = 1 - z^2 + z^4 - z^6 + \ldots \quad (|z| < 1)$$

Integrating term by term and using f (0) = 0, we get

$$\tan^{-1} z = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} z^{2n+1}$$

$$= z - \frac{z^3}{3} + \frac{z^3}{5} + \ldots (|z| < 1)$$

23. Ans: (b)

Sol: $z^3 - z^4 = z^3(1 - z)$ shows that f(z) is singular at z = 0 and z = 1.

Now z = 1 lies outside C.

So, we need the residue of f(z) at z = 0.

We find it from the Laurent series that converges for $0 < |z| < 1$.

$$\frac{1}{z^3 - z^4} = \frac{1}{z^3}(1 - z)^{-1}$$

$$= \frac{1}{z^3}(1 + z + z^2 + z^3 + \ldots..)$$

$$= \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z} + 1 + z + \ldots$$

We see from it that this residue is 1. Clockwise integration thus yields

$$\oint_c \frac{dz}{z^3 - z^4} = -2\pi i \operatorname*{Res}_{z=0} f(z) = -2\pi i$$

24. Ans: (c)

Sol: The characteristic equation is

(λ–3) (λ– 4) = 0

$\Rightarrow \lambda^2 - 7\lambda + 12 = 0$

By cayley Hamilton theorem, we have

$A^2 - 7A + 12I = O$

Multiplying both sides by A^{-1}, we get

$A - 7I + 12A^{-1} = 0$

$$\therefore A^{-1} = \frac{7I - A}{12}$$

25. Ans: (d)

Sol: $\frac{\partial M}{\partial y} = 3xy^2$ & $\frac{\partial N}{\partial x} = 2Axy^2$

Exact $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$\Rightarrow A = \frac{3}{2}$$

26. Ans: (b)

Sol: The equation can be written in Bernoullis form

$$\text{i.e, } \frac{dy}{dx} + Qy = -Py^4$$

$$y^{-4}\frac{dy}{dx} + Qy^{-3} = -P$$

If we put $y^{-3} = V$

Then the above equation will be reduced to linear function.

27. Ans: (d)

Sol: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$$P(A \cap B) = P(A).P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2}$$

28. Ans: (b)

Sol: If AX = B is an inconsistent system of equations. Then ρ(A) must be less than 3.

∴ The highest possibility is 2.

29. Ans: (b)

Sol: P (X = 5) = P (X = 6)

$$\frac{e^{-\lambda}.\lambda^5}{5!} = \frac{e^{-\lambda}.\lambda^6}{6!}$$

$$1 = \frac{\lambda}{6} \Rightarrow \lambda = 6$$

∴ mean = λ = 6

30. Ans: (c)

Sol: Given

$$f(x)=\frac{e^{Sinx}}{e^{Cosx}}=e^{Sinx-Cosx}$$

$$f'(x)=e^{Sinx-Cosx}[\cos x+Sinx]=0$$

$$\Rightarrow x=\frac{3\pi}{4},\frac{-\pi}{4} \text{ are stationary points}$$

$$f''\left(\frac{3\pi}{4}\right)<0\Rightarrow \text{maximum at } x=\frac{3\pi}{4}$$

31. Ans: (a)

Sol: The given matrix A can be obtained from the unit matrix with elementary operation $R_1 \leftrightarrow R_3$. The inverse matrix corresponding to the elementary matrix A is A itself.

32. Ans: (d)

Sol: We have $\nabla.(\bar{A}\times\bar{B})=\bar{B}.(\nabla\times\bar{A})-\bar{A}(\nabla\times\bar{B})$

Remaining three options are valid vector identities.

33. Ans: (a)

Sol: We have

$$\frac{xdy-ydx}{x^2} = d\left(\frac{y}{x}\right)$$

$$\frac{xdy-ydx}{xy} = d\left[\log\left(\frac{y}{x}\right)\right]$$

xdy + ydx = d(x y)

$$\frac{xdy+ydx}{xy} = d[\log(xy)]$$

34. Ans: (b)

Sol: In Euler's method, we neglect h^2, h^3 and other higher order terms.

(In the expansion of taylor series)

35. Ans: (a)

Sol: $\lambda = 2$

Let x = number of typographical errors on a page

$$P(x = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Required probability = P(x = 0) = e^{-2}

36. Ans: (b)

Sol: $r^2=x^2+y^2+z^2$

$$2r\frac{\partial r}{\partial x}=2x$$

$$\Rightarrow \frac{\partial r}{\partial x}=\frac{x}{r}$$

$$v_x=n\,r^{n-1}\frac{x}{r}$$

$$=n\,r^{n-2}x$$

$$v_{xx}=n\,r^{n-2}+n(n-2)r^{n-3}\frac{x}{r}x$$

$$=n.r^{n-2}\left\{1+(n-2)\frac{x^2}{r^2}\right\}$$

$$v_{xx}+v_{yy}+v_{zz}=n(n+1)r^{n-2}$$

Putting n = –2

$$v_{xx}+v_{yy}+v_{zz}=2r^{-4}$$

37. Ans: (c)

Sol: $z=x^n f_1\left(\frac{y}{x}\right)+y^{-n}f_2\left(\frac{x}{y}\right) = z_1 + z_2$

where, z_1 is a homogeneous function of degree 3 and z_2 is a homogeneous function of degree – 3.

By Euler's formula,

$x^2 . z_{xx} + 2xy . z_{xy} + y^2 . z_{yy}$

$= 3(3 – 1) z_1 + (–3) (– 3 – 1) z_2$

$= 6z_1 + 12z_2$

$x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy} + xz_x + y.z_y$

$$= 6x^3 f_1\left(\frac{y}{x}\right) + 12y^{-3}f_2\left(\frac{x}{y}\right)$$

38. Ans: (b)

Sol: Let $u = e^{ax+by}. f(ax – by)$

$u_x = e^{ax+by}. f^1(ax – by).a + a e^{ax+by}. f(ax – by)$

$u_y = e^{ax+by}. f^1(ax – by)(–b) + b e^{ax+by}. f(ax – by)$

$b.u_x + a.u_y = 2ab\ u$

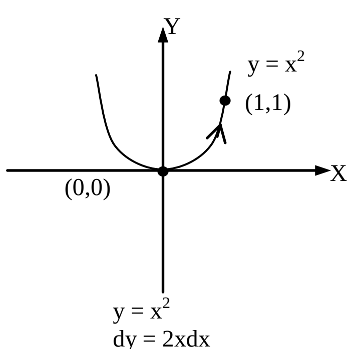
Putting a = 2 and b = 3

$3u_x + 2u_y = 12u$

39. Ans: (a)

Sol: $\vec{F} = y\vec{i} - x\vec{j}$

$$\int_c \vec{F}.d\vec{r} = \int_c ydx - xdy \qquad \ldots\ldots (1)$$



To evaluate $\int_c \vec{F}.d\vec{r}$ along $y = x^2$,

so that dy = 2x dx, from (0,0) to (1,1)

Equation (1) can be written as

$$\int_c \vec{F}.d\vec{r} = \int_c x^2dx - 2x^2\,dx$$

$$= \int_0 - x^2dx = -\left(\frac{x^3}{3}\right)_0^1$$

$$= -\frac{1}{3}$$

40. Ans: (c)

Sol: Cosec x cannot be expanded as Fourier series as it is undefined in the interval $(–\pi, \pi)$s.

41. Ans: (a)

Sol: $a_0 = \frac{2}{\ell}\int_0^\ell f(x)dx$

$$= \int_0^2 x\,dx = \left(\frac{x^2}{2}\right)_0^2 = 2$$

The constant term $= \frac{a_0}{2} = 1$

42. Ans: (c)

Sol: Given: $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$

div $\vec{F}$ = 4z –2y +y = (4z–y)

$$\iint_S \vec{F}.\vec{n}\,ds = \iiint_V \text{div}\,\vec{F}dx\,dy\,dz$$

[divergence theorem]

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AIR 8 HEMANT KUMAR ME
AIR 8 YOGESH KUMAR CE
AIR 9 DEEPITA ROY EE
AIR 9 E&T
AIR 9 ME
AIR 9 ANKIT KUMAR CE
AIR 10 ANKITA SHARMA EE
AIR 10 E&T
AIR 10 SUMIT BHAMBOO ME
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$$= \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1}(4z-y)\,dx\,dy\,dz$$

$$= \int_{x=0}^{1}\int_{y=0}^{1}\left(2z^2-yz\right)_0^1 dx\,dy$$

$$= \int_{x=0}^{1}\int_{y=0}^{1}(2-y)\,dx\,dy$$

$$= \int_0^1\left(2y-\frac{y^2}{2}\right)_0^1 dx$$

$$= \int_0^1\left(2-\frac{1}{2}\right)dx = \frac{3}{2}$$

43. Ans: (a)

Sol: $\iint_S \left(\nabla\times\vec{A}\right).\hat{n}\,ds = \iiint_V \text{div}\left(\nabla\times\vec{A}\right)dx\,dy\,dz$

$$= \iiint_V \text{div}\left(\text{curl}\vec{A}\right)dx\,dy\,dz$$

$$= 0 \qquad \left[\because \text{div}\left(\text{curl}\vec{F}\right)=0\right]$$

44. Ans: (c)

Sol: $f(x, y) = 4 – 2xy$

$x_0 = 0$, $y_0 = 0.2$, $f_1 = 0.1$

By Euler's formula

$y_1 = y_0 + h\, f(x_0, y_0) = 0.2 + 0.1(4 – 0)$

$= 0.6$

45. Ans: 1.1961

Sol: $f(x, y) = x + \sin y$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$

$k_1 = h(f_0, y_0)$

$= 0.2(0+ \sin 1)$

$= 0.2(0.8414) = 0.1682$

$k_2 = hf(x_0 + h, y_0 + k_1)$

$= 0.2(0.2 + \sin (1.1682))$

$= 0.2(0.2 + 0.9200) = 0.2(1.1200)= 0.2240$

$y_1 = 1+ \frac{1}{2}(0.1682 + 0.2240) = 1.1961$

46. Ans: (a)

Sol:

X	1	2	3
P(X)	$\frac{2+5P}{5}$	$\frac{1+3P}{5}$	$\frac{1.5+2P}{5}$

We have $\Sigma P_i = 1$

$$\Rightarrow \frac{2+5P}{5}+\frac{1+3P}{5}+\frac{1.5+2P}{5}=1$$

$\therefore$ P = 0.05

47. Ans: (a)

Sol: For continuous random variable

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^2 k\left(5x-2x^2\right)dx = 1$$

$$\Rightarrow k\left[5\left(\frac{x^2}{2}\right)_0^2 - 2\left(\frac{x^3}{3}\right)_0^2\right] = 1$$

$$\Rightarrow k\left(10-\frac{16}{3}\right) = 1$$

$$\therefore k = \frac{3}{14}$$

48. Ans: (a)

Sol: $f(z)=\frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}$ is the probability density function of Normal Distribution

$$\therefore \int_{-\infty}^{\infty} f(z)\,dz = 1$$

$\therefore$ The value of given integral = $\sqrt{2\pi}$

49. Ans: (c)

Sol: $$\frac{1-e^{2z}}{z^4}=\frac{1-\left\{1+(2z)+\frac{(2z)^2}{2!}+\frac{(2z)^3}{3!}+......\right\}}{z^4}$$

$$=\frac{\left(-2z-2z^2-\frac{4z^3}{3}-.....\right)}{z^4}$$

$$=-\frac{2}{z^3}-\frac{2}{z^2}-\frac{4}{3z}-.....$$

$$\underset{z=0}{Res.}f(z)=-\frac{4}{3}$$

50. Ans: (b)

Sol: Let f(z) = $\frac{z^2+z}{(z-1)^{10}}$.

Then the singular point of f(z) is z = 1 and the singular z = 1 lies inside the circle |z| = 2.

Now, f(z) = $\frac{\phi(z)}{[z-z_0]^{n+1}}$ = $\frac{z^2+z}{[z-1]^{9+1}}$

$\therefore$ By Cauchy's Integral formula, we have

$$\oint_C f(z)\,dz = \frac{2\pi i}{9!}\left[\frac{d^9}{dz^9}(z^2+z)\right]_{z=1}$$

$$= \frac{2\pi i}{9}(0) = 0$$