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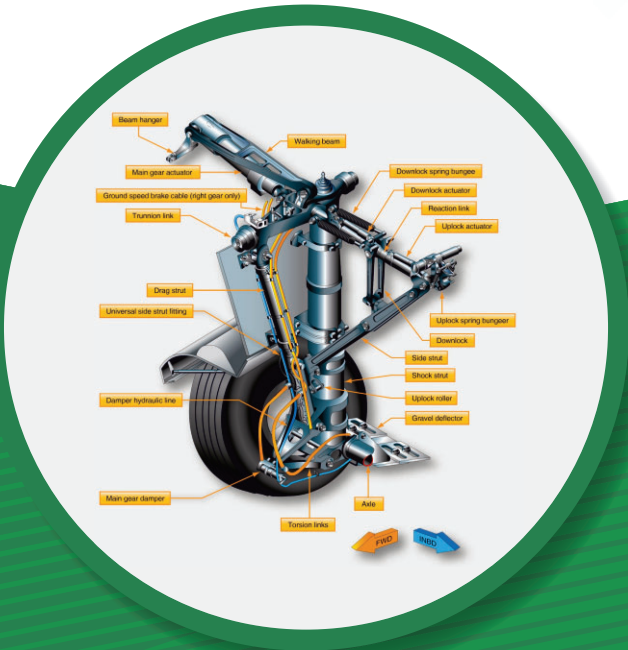


MECHANICAL ENGINEERING

THEORY OF MACHINES & VIBRATIONS

Text Book : Theory with worked out Examples and Practice Questions

Theory of Machines & Vibrations

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 Analysis of Planar Mechanisms

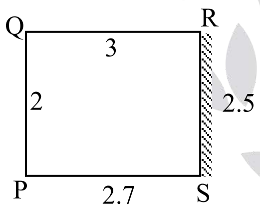
01. Ans: (a, c)

Sol:

- The pair shown has two degree of freedom one is translational (motion along axis of bar and the rotation (rotation about axis). Both motions are independent. Therefore the pair has incomplete constraint.
- Kinematic pair is a joint of two links having relative motion between them. The pair shown form a kinematic pair.

02. Ans: (c)

Sol:



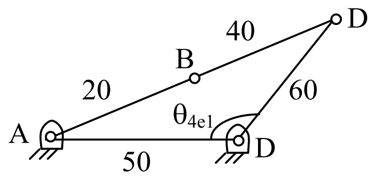
The given dimensions of the linkage satisfies Grashof's condition to get double rocker. We need to fix the link opposite to the shortest link. So by fixing link 'RS' we get double rocker.

03. Ans: (d)

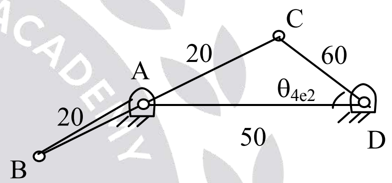
Sol: At toggle position velocity ratio is 'zero' so mechanical advantage is '∞'.

04. Ans: (d)

Sol: The two extreme positions of crank rocker mechanisms are shown below figure.



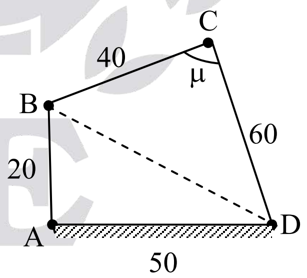
$$\theta_{4e_1} = \cos^{-1}\left(\frac{50^2+60^2-60^2}{2\times50\times60}\right) = 65.37^0$$



$$\theta_{4e_2} = \cos^{-1}\left(\frac{60^2+50^2-20^2}{2\times60\times50}\right) = 18.19^o$$

05. Ans: (a)

Sol:



Where, μ = Transmission angle

$$BD = \sqrt{20^2+50^2} = 53.85\,\text{cm}$$

By cosine rule

$$\cos\mu = \frac{BC^2+CD^2-BD^2}{2BC\times CD}$$

$$= \frac{40^2 + 60^2 - 53.85^2}{2 \times 40 \times 60} = 0.479$$

$\mu = 61.37°$

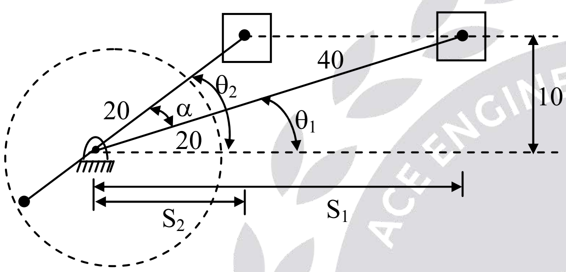
06. Ans: (c)

Sol: Two extreme positions are as shown in figure below.

Let r = radius of crank = 20 cm

l = length of connecting rod = 40 cm

h = 10 cm



Stroke = $S_1 - S_2$

$$S_1 = \sqrt{(\ell + r)^2 - h^2} = \sqrt{60^2 - 10^2} = 59.16\text{cm}$$

$$S_2 = \sqrt{(\ell - r)^2 - h^2} = \sqrt{20^2 - 10^2} = 17.32\text{cm}$$

Stroke = $S_1 - S_2$ = 59.16 – 17.32 = 41.84 cm

07. Ans: (b)

Sol: $\theta_1 = \sin^{-1}\left(\frac{h}{\ell + r}\right) = \sin^{-1}\left(\frac{10}{60}\right) = 9.55°$

$\theta_2 = \sin^{-1}\left(\frac{h}{\ell - r}\right) = \sin^{-1}\left(\frac{10}{20}\right) = 30°$

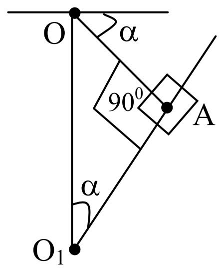
$\alpha = \theta_2 - \theta_1 = 20.41°$

Quick return ratio

$$(\text{QRR}) = \frac{180 + \alpha}{180 - \alpha} = 1.2558$$

08. Ans: (c)

Sol:



OO_1 = 40 cm , OA = 20 cm

$$\sin\alpha = \frac{OA}{OO_1} = \frac{20}{40} = \frac{1}{2}$$

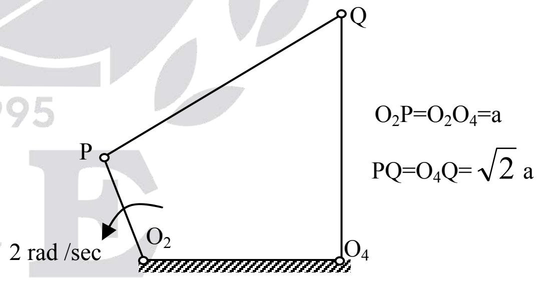
$$\Rightarrow \alpha = 30^0$$

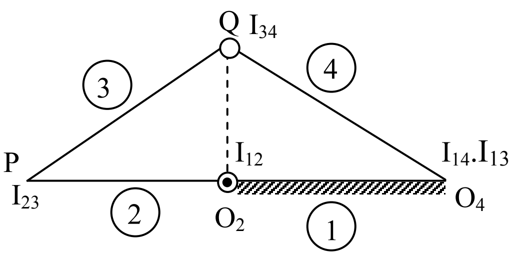
$$\text{QRR} = \frac{180 + 2\alpha}{180 - 2\alpha} = \frac{180 + 60}{180 - 60}$$

$\Rightarrow$ QRR = 2

09. Ans: (c)

Sol: $\angle O_4O_2P = 180^o$ sketch the position diagram for the given input angle and identify the Instantaneous Centers.





I_{13} is obtained by joining $I_{12}\,I_{23}$ and $I_{14}\,I_3$

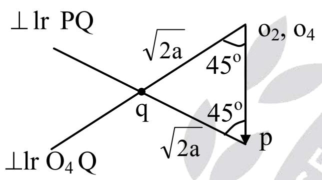
$$\frac{\omega_3}{\omega_2} = \frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \frac{a}{2a}$$

$$\frac{\omega_3}{2} = \frac{1}{2}$$

$\omega_3 = 1$ rad /sec

Alternate Method:

The position diagram is isosceles right angle triangle and the velocity triangle is similar to the position diagram.



Velocity (Diagram)

$V_{qp} = \omega_3 l_3 \Rightarrow \sqrt{2}a = \omega_3 \times \sqrt{2}a$

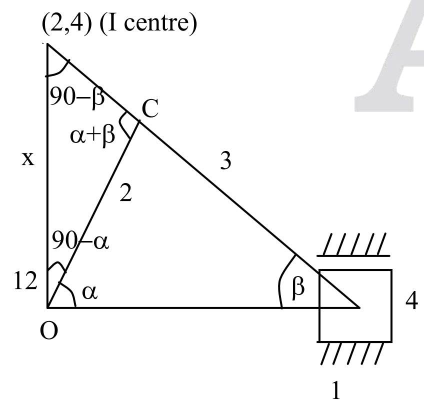
$\omega_3 = 1$

$V_q = l_4\,\omega_4 \Rightarrow \sqrt{2}a = \sqrt{2}a\,\omega_4$

$\Rightarrow \omega_4 = 1$rad/sec

10. Ans: (b)

Sol:



OC = r

Velocity of slider $V_S = (12 - 24) \times \omega_2$

$= x\,\omega_2$

$$\frac{x}{\sin(\alpha+\beta)} = \frac{r}{\sin(90-\beta)}$$

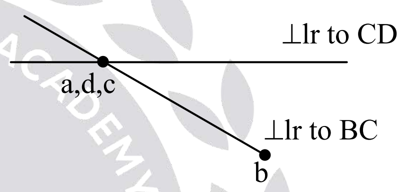
$$x = \frac{r\sin(\alpha+\beta)}{\sin(90-\beta)}$$

$V_S = r\,\omega_2 \sin(\alpha+\beta) \times \sec\beta$

$= V_C \sin(\alpha+\beta) \times \sec\beta$

11. Ans: (a)

Sol:



Velocity diagram

$V_C = 0 = dc \times \omega_{CD}$

$\therefore \omega_{CD} = 0$

Note: If input and coupler links are collinear, then output angular velocity will be zero.

12. Ans: (c)

Sol: In a four bar mechanism when input link and output links are parallel then coupler velocity(ω_3) is zero.

$\Rightarrow l_2\,\omega_2 = l_4\,\omega_4$

$l_4 = 2l_2$ (Given)

$\Rightarrow \omega_4 = \omega_2 / 2 = 2/2 = 1$ rad/s

ω_2, ω_4 = angular velocity of input and output link respectively.

Fixed links have zero velocity.

At joint 1, relative velocity between fixed link and input link = 2–0 = 2

Rubbing velocity at joint 1 = Relative velocity × radius of pin = 2×10 = 20 cm/s

At joint 2, rubbing velocity = $(\omega_2+\omega_3) \times r$

$= (2+0)\times 10 = 20$ cm/s

+ve sign means ω_2 and ω_3 are moving in opposite directions.

At joint 3, rubbing velocity = $(\omega_4+\omega_3)\times r$

$= (1+0) \times 10 = 10$ cm/s

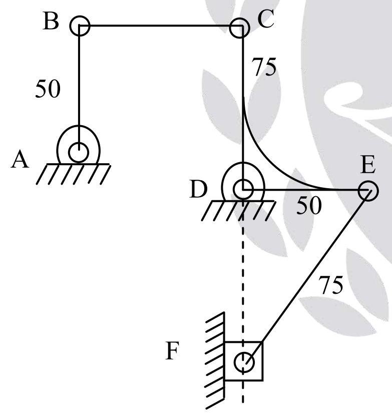
At joint 4, rubbing velocity

$= (\omega_4 - 0) \times r$

$= (1 - 0) \times 10 = 10$ cm/s

13. Ans: (a)

Sol:



Considering the four bar mechanism ABCD, $l_2 || l_4$

$$\therefore \ell_2\omega_2 = \ell_4\omega_4 \Rightarrow \omega_4 = \frac{50\times 3}{75} = 2\,\text{rad/sec}$$

CDE being a ternary link angular velocity of DE is same as that of the link DC (ω_4).

For the slider crank mechanism DEF, crank is perpendicular to the axis of the slider.

$\therefore$ Slider velocity = DE$\times\omega_4$

$= 50\times 2$

$= 100$ cm/sec (upward)

14. Ans: (a)

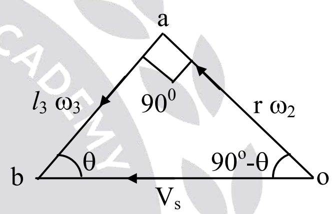
Sol: Here as angular velocity of the connecting rod is zero so crank is perpendicular to the line of stroke.

V_s = velocity of slider = $r\omega_2$

$2 = 1 \times \omega_2 \Rightarrow \omega_2 = 2$ rad/sec

15. Ans: (d)

Sol:



Here the crank is perpendicular to connecting rod

Velocity of rubbing = $(\omega_2 + \omega_3) \times r$

Where, r = radius of crank pin

From the velocity diagram V_{AB} = ab = ?

oa = $\omega_2 \times r = 10 \times 0.3 = 3$ m/sec

Δoab is right angle Δ.

$$\tan\theta = \frac{oa}{ab} = \frac{40}{30} \Rightarrow \theta = 53.13°$$

$$\tan\theta = \frac{r\omega_2}{\ell\omega_3}$$

where, $n = \frac{\ell}{r}$

$$\omega_3 = \frac{\omega_2}{n^2} = \frac{10}{\left(\frac{4}{3}\right)^2} = \frac{90}{16} = 5.625(\text{CW})$$

$V_{rb} = (\omega_2 + \omega_3) \times r$

$= (10+ 5.625) \times 2.5 = 39$ cm/s

16. Ans: (d)

Sol: As for the given dimensions the mechanism is in a right angle triangle configuration and the crank AB is perpendicular to the lever CD. The velocity of B is along CD only which is purely sliding component

$\therefore$ Velocity of the slider

$$= AB \times \omega_{AB} = 10 \times 250 = 2.5\,\text{m/sec}$$

17. Ans: (a)

Sol: $QRR = \dfrac{180+2\alpha}{180-2\alpha} = \dfrac{2}{1} \Rightarrow \alpha = 30^o$

$$\sin\alpha = \frac{OS}{OP} \Rightarrow OS = \frac{OP}{2} = 250\text{mm}$$

18. Ans: (b)

Sol: Maximum speed during forward stroke occurs when PQ is perpendicular to the line of stroke of the tool i. e. PQ, OS & OQ are in straight line

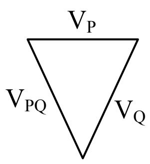
$\Rightarrow V = 250 \times 2 = 750 \times \omega_{PQ}$

$\Rightarrow \omega_{PQ} = \dfrac{2}{3}$

19. Ans: (d)

Sol:

$V_Q = V_P + V_{PQ}$

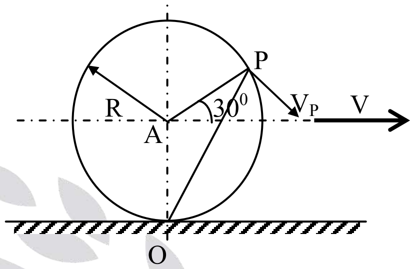


20. Ans: (a)

Sol: For rigid thin disc rolling on plane without slip. The 'I' centre lies on the point of contact.

21. Ans: (a)

Sol:



Here 'O' is the instantaneous centre

$V_P = \omega \times OP$

$V_A = R\omega$

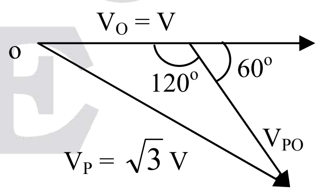
In Δ OAP , $\cos 120° = \dfrac{R^2 + R^2 - OP^2}{2R \times R}$

$$-0.5 = \frac{2R^2 - OP^2}{2R^2}$$

$OP = \sqrt{3}R$

$V_P = \sqrt{3}R \times \omega = \sqrt{3}V$

or

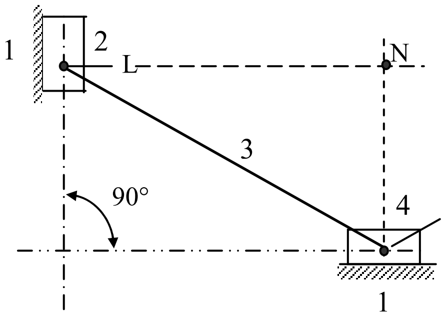


$$V_P = \vec{V}_O + \vec{V}_{PO} = \vec{V} + \overrightarrow{OP} \times \omega$$

$$= \sqrt{V^2 + V^2 + 2V^2 \cos 60} = \sqrt{3}\,V$$

22. Ans: (d)

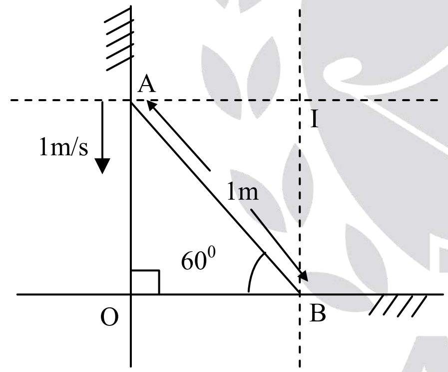
Sol:



By considering the links 1, 2 and 4 as for three centers in line theorem, I_{12}, I_{14} and I_{24} lies on a straight line I_{12} is at infinity along the horizontal direction while I_{14} is at infinity along vertical direction hence I_{24} must be at infinity

23. Ans: (a)

Sol:



$V_a = 1$ m/s

V_a = Velocity along vertical direction

V_b = Velocity along horizontal direction

So instantaneous center of link AB will be perpendicular to A and B respectively i.e at I

$$IA = OB = \cos\theta = 1 \times \cos 60^0 = \frac{1}{2}\text{m}$$

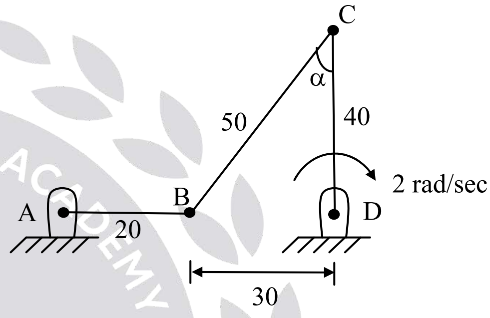
$$IB = OA = \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2}\text{m}$$

$$V_a = \omega \times IA$$

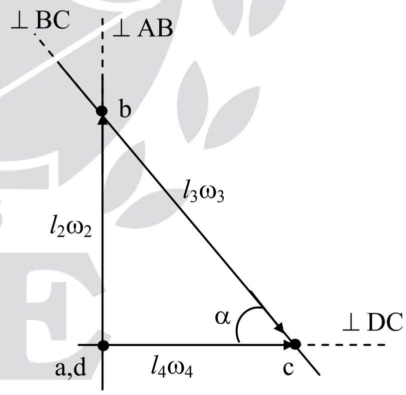
$$\Rightarrow \omega = \frac{V_a}{IA} = \frac{1}{V_2} = 2 \text{ rad/sec}$$

24. Ans: (a)

Sol:



(Position Diagram)



(Velocity Diagram)

Let the angle between BC & CD is α. Same will be the angle between their perpendiculars.

From Velocity Diagram, $\dfrac{\ell_2\omega_2}{\ell_4\omega_4} = \tan\alpha$

From Position diagram, $\tan\alpha = \dfrac{30}{40}$

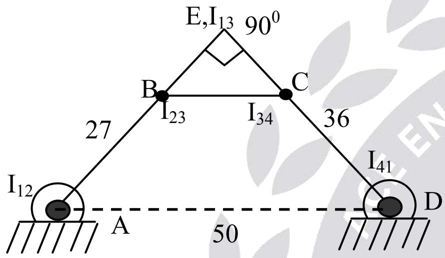
$$\therefore \omega_2 = \omega_4 \times \frac{\ell_4}{\ell_2} \times \tan\alpha = 2 \times \frac{40}{20} \times \frac{30}{40} = 3$$

$\omega_2 = 3$ rad/sec

Note: DC is the rocker (Output link) and AB is the crank (Input link).

25. Ans: (c)

Sol:



I_{13} = Instantaneous center of link 3 with respect to link 1

As AED is a right angle triangle and the sides are being integers so AE = 30 cm and DE = 40 cm

BE = 3 cm and CE = 4 cm

By 'I' center velocity method,

$V_{23} = \omega_2 \times (AB) = \omega_3 \times (BE)$

$$\omega_3 = \frac{1 \times 27}{3} = 9\,\text{rad/s}$$

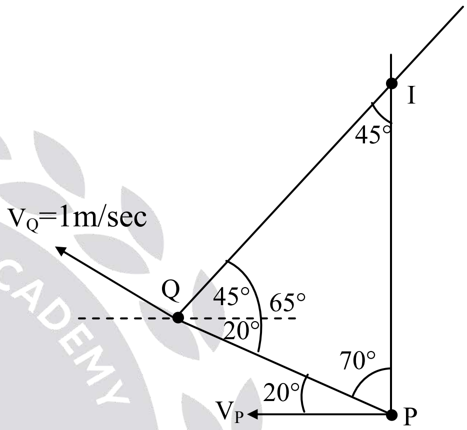
26. Ans: (a)

Sol: Similarly, $V_{34} = \omega_3 \times (EC) = \omega_4 \times (CD)$

$$\omega_4 = \frac{9 \times 4}{36} = 1\,\text{rad/s}$$

27. Ans: (d)

Sol: Refer the figure shown below, By knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P using sine rule.



'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

$$V_Q = IQ \times \omega = 1$$

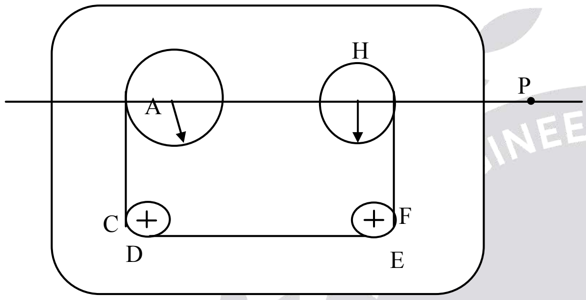
$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1$$

$$= 0.9645$$

28. Ans: (c)

Sol: Consider the three bodies the bigger spool (Radius 20), smaller spool (Radius 10) and the frame. They together have three I centers, I centre of big spool with respect to the frame is at its centre A. that of the small spool with respect to the frame is at its centre H. The I centre for the two spools P is to be located.



As for the three centers in line theorem all the three centers should lie on a straight line implies on the line joining of A and H. More over as both the spools are rotating in the same direction, P should lie on the same side of A and H. Also it should be close to the spool running at higher angular velocity. Implies close to H and it is to be on the right of H. Whether P belongs to bigger spool or smaller spool its velocity must be same. As for the radii of the spools and noting that the velocity of the tape is same on both the spools

$\omega_H = 2\omega_A$

$\therefore AP.\omega_A = HP\,\omega_H$ and

$AP = AH + HP \Rightarrow HP = AH$

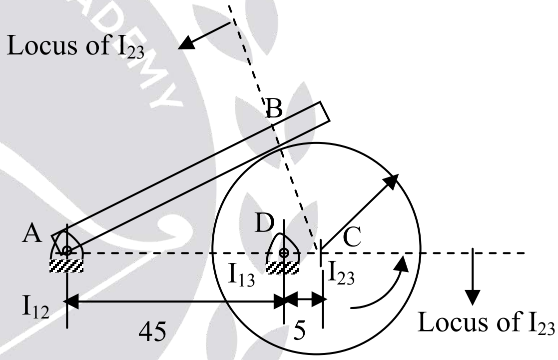
Note:

(i) If two links are rotating in same directions then their Instantaneous centre will never lie in between them. The 'I' center will always close to that link which is having high velocity.

(ii) If two links are rotating in different directions, their 'I' centre will lie in between the line joining the centres of the links.

29. Ans: (b)

Sol: I_{23} should be in the line joining I_{12} and I_{13}. Similarly the link 3 is rolling on link 2.



So the I – Center I_{23} will be on the line perpendicular to the link – 2. (I_{23} lies common normal passing through the contact point)

So the point C is the intersection of these two loci which is the center of the disc.

So $\omega_2(I_{12}, I_{23}) = \omega_3(I_{13}, I_{23})$

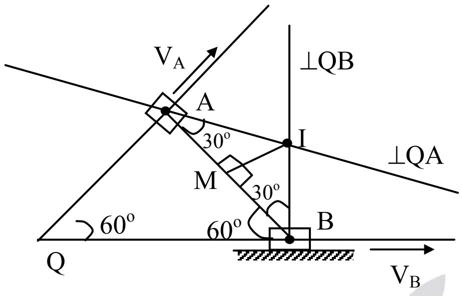
$\Rightarrow \omega_2 \times 50 = 1 \times 5$

$\Rightarrow \omega_2 = 0.1$ rad/sec

30. Ans: 1 (range 0.95 to 1.05)

Sol: Locate the I-centre for the link AB as shown in fig. M is the mid point of AB

Given, $V_A = 2$ m/sec



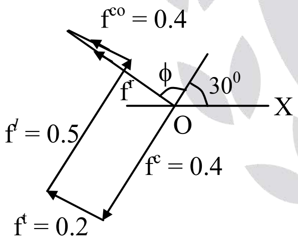
$$V_A = IA.\omega \Rightarrow \omega = \frac{V_A}{IA}$$

$$V_M = IM.\omega = IM\frac{V_A}{IA} = \frac{IM}{IA}.V_A$$

$$= \sin 30^o.V_A = \frac{1}{2}.2 = 1\text{m/sec}$$

31. Ans: (a) & 32. Ans: (b)

Sol:



Centripetal acceleration,

$f^c = r\omega^2 = 0.4$ m/s^2 acts towards the centre

Tangential acceleration, $f^t = r\alpha = 0.2$ m/s^2 acts perpendicular to the link in the direction of angular acceleration. Linear deceleration = 0.5 m/s^2 acts opposite to velocity of slider

As the link is rotating and sliding so coriolis component of acceleration acts

$f^{co} = 2V\omega = 2 \times 0.2 \times 1 = 0.4$ m/s^2

To get the direction of coriolis acceleration, rotate the velocity vector by 90^0 in the direction of ω.

Resultant acceleration

$$= \sqrt{0.6^2 + 0.1^2} = 0.608\,\text{m/sec}^2$$

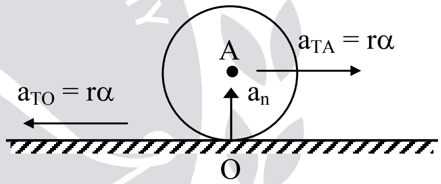
$$\phi = \tan^{-1}\left(\frac{0.6}{0.1}\right) = 80.5$$

Angle of Resultant vector with reference to

OX = 30 + φ = 30 + 80.5 = 110.53^0

33. Ans: (d)

Sol:

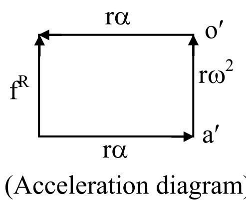


Acceleration at point 'O'

$$a_{\vec{o}} = a_{\vec{TO}} + a_{\vec{TA}} + a_{\vec{n}}$$

$a_{\vec{TO}}$ and $a_{\vec{TA}}$ are linear accelerations with same magnitude and opposite in direction.

$$\Rightarrow a_{\vec{O}} = a_{\vec{n}} = \frac{V^2}{r} = r\omega^2$$

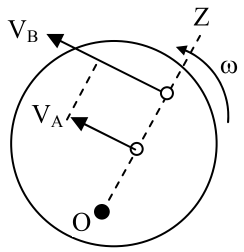


(Acceleration diagram)

Resultant acceleration, $f^R = r\,\omega^2$

34. Ans: (c)

Sol:



$V_B = OB \times \omega$

$V_A = OA \times \omega$

$V_{BA} = V_B - V_A = (OB - OA) \times \omega$

$= \omega\,(r_B - r_A)$

and direction of motion point 'B'.

35. Ans: (d)

Sol: As uniform angular velocity is given,

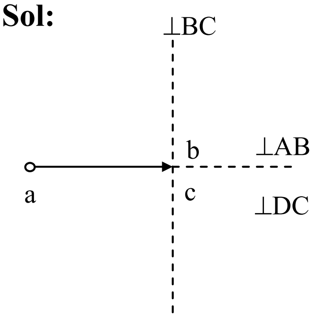
Tangential acceleration, $\alpha = 0$

Centripetal acceleration,

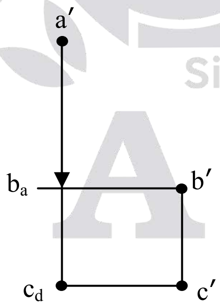
$f_{BA} = (r_B^2 - r_A^2) \times \omega$ from Z to 'O'.

36. Ans: (a)

Sol:



Velocity Diagram



Acceleration Diagram

From velocity Diagram, $V_C = V_B$

$l_4\omega_4 = l_2\omega_2$

$25 \times \omega_4 = 50 \times 0.2$

$\Rightarrow \omega_4 = 0.4$ rad/sec

From Acceleration Diagram,

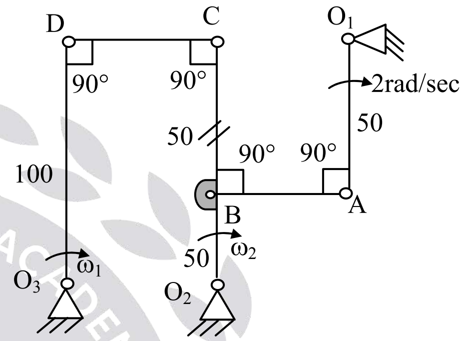
$l_4\alpha_4 = l_2\alpha_2$

$25 \times \alpha_4 = 50 \times 0.1$

$\Rightarrow \alpha_4 = 0.2$ rad/sec^2

37. Ans: (d)

Sol:



As links O_1A and O_2B are parallel then

$V_A = V_B$

$\Rightarrow 50 \times 2 = 50 \times \omega_2$

$\Rightarrow \omega_2 = 2$ rad/sec

As a O_2 C and O_3D are parallel links then

$V_C = V_D$

$\Rightarrow 100 \times 2 = 100 \times \omega_1$

$\Rightarrow \omega_1 = 2$ rad/sec

$V_D = r\omega_1$

$= 100 \times 2 = 200$ mm/sec

$\alpha = 0$ (given), so tangential acceleration a^t

$= r\alpha = 0$

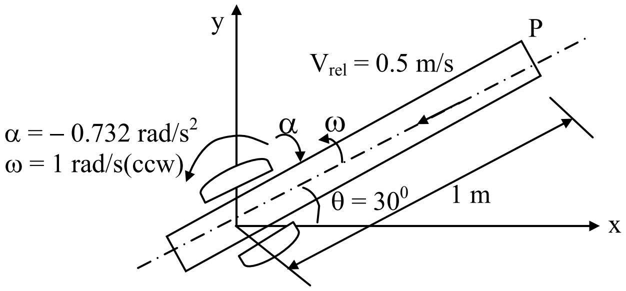
Centripetal acceleration,

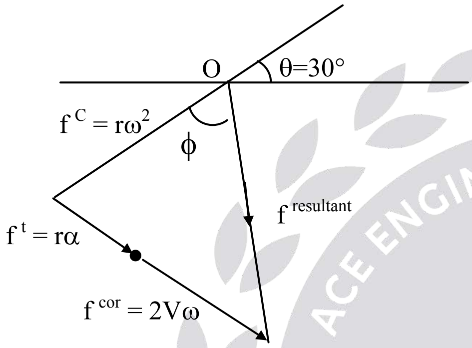
$f^c = r\omega_1^2$

$= 100 \times (2)^2 = 400$ mm/sec^2

38. Ans: 2 m/s², 270°

Sol:





Acceleration diagram

Radial relative acceleration, $f^{linear} = 0$

Centripetal acceleration, $f^c = r\omega^2$

$= 1\times1^2 = 1$ m/s² (acts towards the center)

Tangential acceleration, $f^t = r\alpha$

$= 1\times0.732 = 0.732$ m/sec²

Coriolis acceleration, $f^{cor} = 2V\omega$

$= 2 \times 0.5 \times 1 = 1$ m/sec²

Resultant acceleration,

$$f^r = \sqrt{1^2 + (1+0.732)^2} = 2 \text{ m/sec}^2$$

$$\phi = \tan^{-1}\left(\frac{1.732}{1}\right) = 60°$$

$$\theta_{reference} = 30 + 180 + 60 = 270^0$$

39. Ans: (d)

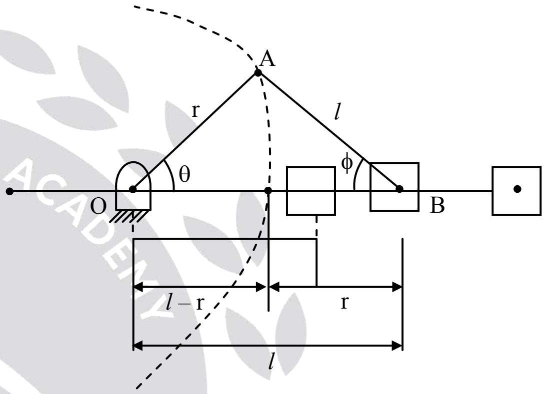
Sol: Angular acceleration of connecting rod is given by

$$a = -\omega^2 \sin\theta\left[\frac{(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}\right]$$

when n = 1, a = 0

40. Ans: (b) & 41. Ans: (a)

Sol:



F_P = 2 kN

l = 80 cm = 0.8 m

r = 20 cm = 0.2m

From the triangle OAB

$$\cos\phi = \frac{\ell^2+\ell^2-r^2}{2\ell^2}$$

$$= \frac{2\times80^2-20^2}{2\times80^2} \Rightarrow \phi = 14.36$$

$$\cos\theta = \frac{20^2+80^2-80^2}{2\times20\times80} \Rightarrow \theta = 82.82$$

Thrust connecting rod

$$F_T = \frac{F_P}{\cos\phi} = \frac{2}{\cos14.36} = 2.065\,\text{kN}$$

Turning moment,

$$T = F_T \times r = \frac{F_P}{\cos\phi}\left(\sin(\theta+\phi)\right)\times r$$

$$= \frac{2}{\cos 14.36}\times \sin(14.36+82.82)\times 0.2$$

= 0.409 kN-m

42. Ans: (b)

Sol: Calculate AB that will be equal to 260 mm

L = 260 mm, P = 160 mm

S = 60 mm, Q = 240 mm

L + S = 320

P + Q = 400

$\therefore$ L+S < P+Q

It is a Grashof's chain

Link adjacent to the shortest link is fixed

$\therefore$ Crank – Rocker Mechanism.

43. Ans: (b)

Sol: $O_2A \parallel O_4B$

Then linear velocity is same at A and B.

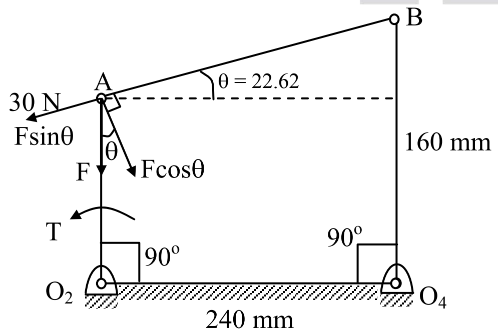
$\therefore \omega_2 \times O_2A = \omega_4 \times O_4B$

$\therefore \quad 8\times 60 = \omega_4 \times 160$

$\Rightarrow \quad \omega_4 = 3$ rad/sec

44. Ans: (c)

Sol:



$$\tan\theta = \frac{100}{240} \Rightarrow \theta = 22.62^\circ$$

As centre of mass falls at O_2

$m\bar{r}\omega^2 = 0 \quad (\because \bar{r} = 0)$

$\alpha = 0$ (Given)

Inertia torque = 0

Since torque on link O_2A is zero, the resultant force at point A must be along O_2A.

$\Rightarrow F\sin 22.62 = 30$

$$\Rightarrow F = \frac{30}{\sin 22.62} = 78\,\text{N}$$

The magnitude of the joint reaction at $O_2 = F = 78$ N

45. Ans: (d)

Sol: $I\dfrac{d^2\theta}{dt^2} = T + f(\sin\theta, \cos\theta)$

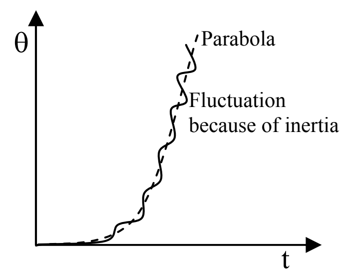
Where 'T' is applied torque, f is inertia torque which is function of sinθ & cosθ

$$\frac{d\theta}{dt} = \frac{T}{I}t + f'(\sin\theta,\cos\theta) + c_1$$

$$\theta = \frac{T}{I}t^2 + c_1 t + f''(\sin\theta,\cos\theta)$$

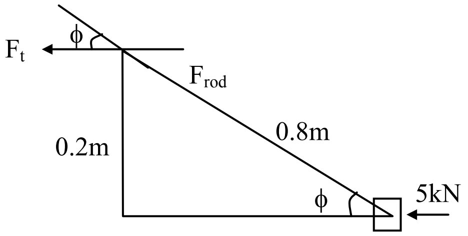
θ is fluctuating on parabola

and @ t = 0 , θ = 0 , $\dot{\theta}(\text{slope}) = 0$ (because it starts from rest)



46. Ans: 1 (range 0.9 to 1.1)

Sol:



Given $F_p = 5kN$

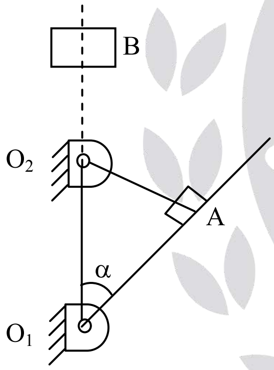
$$F_{rod} = \frac{F_p}{\cos\phi}, F_t = F_{rod}\cos\phi$$

$\therefore F_t = 5$ kN

Turning moment = $F_t.r = 5\times0.2 = 1$ kN-m

47. Ans: (a)

Sol:



N = 120 rpm ,

$$\omega_2 = \frac{2\pi N}{60} = 4\pi \text{ rad/s}$$

$\therefore l_1 = O_1 O_2 = 50$ cm

$$QRR = 1:2 = \frac{1}{2}$$

$$\frac{1}{2} = \frac{180-2\alpha}{180+2\alpha} \Rightarrow 180 + 2\alpha = 360 - 4\alpha$$

$\Rightarrow 6\alpha = 180°$

$\Rightarrow \alpha = 30°$

$$\sin 30 = \frac{O_2A}{O_1O_2}$$

$$\Rightarrow \frac{1}{2} = \frac{O_2A}{50} \Rightarrow O_2A = 25 \text{ cm}$$

$\therefore l_2 = 25$ cm

At the position given above (O_1 O_2 B) the tool post attains the maximum velocity.

At that given instant

$l_2\,\omega_2 = l_4\,\omega_4$ & velocity of slider is zero.

$l_4 = O_1B = l_1 + l_2 = 50 + 25 = 75$ cm

$\Rightarrow 25 \times 4\pi = 75 \times \omega_4$

$$\omega_4 = \frac{100\pi}{75} = \frac{4\pi}{3} = 4.19 \text{ rad/s}$$

ω_4 = angular velocity of slotted lever.

Conventional Practice Solutions

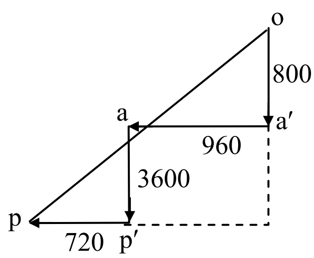
01.

Sol: $O_2A = 8$ cm, $\omega_2 = 10$ rad/sec

AP(B) = 4 cm, $\omega_3 = -30$ rad/sec^2

$\alpha_2 = 120$ rad/s^2 (CW)

$\alpha_3 = 180$ rad/s^2



(Acceleration diagram)

1. Velocity analysis

$V_{AO2} = 8 \times 10 = 80$ cm/sec ($\perp^r$ to O_2A)

$V_{PA} = \omega_3 \times AP = 30 \times 4 = 120$ cm/sec

$\therefore$ ($\perp^r$ to AP)

V_{PO2} = Absolute velocity = V_P

$\therefore V_{PO} = V_{AO2} + V_{BA} = -80+120 = 40$ cm/s

2. Acceleration analysis

$a^C_{AO} = r\omega_2^2$ (Parallel to OA & towards 'O')

$= 8 \times 100 = 800$ cm/s^2

$a^t_{AO} = OA \times \alpha_2$ (Perpendicular to OA & in direction of α_2) $= 8 \times 120 = 960$ cm/s^2

$a^c_{BA} = 4 \times 30^2 = 3600\,\text{cm/s}^2$

$a^t_{BA} = 4 \times 120 = 480\,\text{cm/s}^2$

Resultant acceleration,

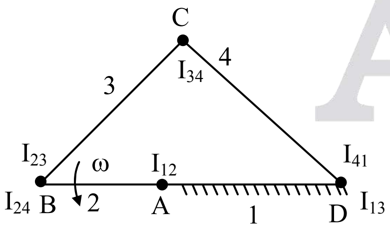
$$a_{PO} = \sqrt{(800+3600)^2 + (720+960)^2}$$

$a_{PO} = 4709.81$ cm/sec^2

02.

Sol: Given that $\omega_2 = 5$ rad/s

Need to find out ω_3 when $\angle BAD = 180°$



According to Kennedy's theorem I_{24} is at the intersection of link 1 and 3 and I_{13} at the intersection of links '2' and '4'.

$\therefore$ At A – I_{12}

B – I_{23}, I_{24}

C – I_{34}

D – I_{13}, I_{14}

From angular velocity-ratio theorem

$$\frac{\omega_4}{\omega_2} = \frac{I_{24}-I_{12}}{I_{24}-I_{14}} = \frac{\ell}{2\ell} = 0.5$$

$\omega_4 = 2.5$ rad/s (counter clockwise)

$V_2 = (I_{12}, I_{23}) \times \omega_2 = (I_{13}, I_{23}) \times \omega_3$

$\Rightarrow l \times 5 = 2l \times \omega_3$

$\Rightarrow \omega_3 = 2.5$ rad/sec

03.

Sol: N = 360 rpm, L = 0.7 m,

$F_P = F_G - m\,a_P + mg$

$$F_{CR} = \frac{F_P}{\cos\phi}$$

$$T = \frac{F_P}{\cos\phi}\sin(\theta+\phi)\times r$$

Double acting steam engine

$F_G = (P_1A_1 - P_2A_2)$

Net gas force = F_G

$$\therefore F_G = \left(0.35\times\frac{\pi}{4}(300)^2 - 0.03\times\frac{\pi}{4}(300^2-40^2)\right)$$

$F_G = 22645$ N

Acceleration of piston: $a_p = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

L = 0.7 m, s = 300 mm = 2r $\Rightarrow$ r = 0.15 m

$$n = \frac{L}{r} = \frac{0.7}{0.15} = 4.67\,\text{m}$$

$$a_p = (0.15)\left(\frac{2\pi\times360}{60}\right)^2\left(\cos(120) + \frac{\cos 2(120°)}{4.67}\right)$$

$a_p = -129.28$ m/s^2

$\therefore\ F_P = F_G - m\,a_p + mg$

$$= 22645 - \left(\frac{500}{9.81}\right)(-129.28) + 500$$

$F_P = 29734.1947$ N

$$\sin\phi = \frac{\sin\theta}{n}$$

$$\sin\phi = \frac{\sin(120°)}{4.67} \Rightarrow \phi = 10.68°$$

Turning moment,

$$T = \frac{29.734}{\cos(10.68)} \times \sin(120 + 10.68) \times 0.15$$

$T = 3.344$ kN-m

$$F_{CR} = \frac{F_P}{\cos\phi} = \frac{29.734}{\cos(10.68)} = 30.258\,\text{kN}$$

04.

Sol: Given that, $O_1O_2 = 700$ mm

B on the slider, C on the link O_2D.

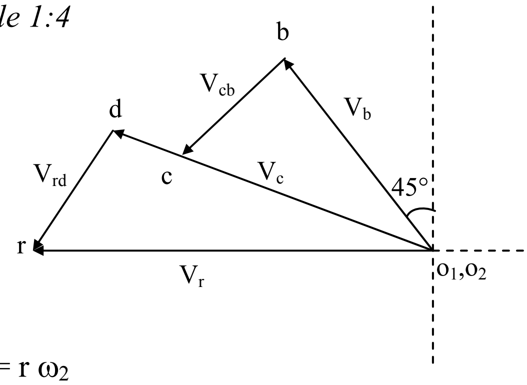
$\theta = 45°$

Procedure:

- Identify fixed points o_1, o_2 and d, Draw cutting tool velocity in horizontal direction as V_r
- Draw $V_b \perp O_1B$ which is known value
- Draw $V_d \perp O_2D$
- V_{cb} along O_2D from b.(since B,C moves along the slot)
- Identify 'd' from the relation
  $$o_2d = o_2c \times \frac{O_2D}{O_2C}$$
- Draw $V_{rd} \perp$ DR from d

Velocity Diagram

Scale 1:4



$V_b = r\,\omega_2$

$$= \frac{250}{1000} \times \frac{2\times\pi\times 40}{60} = 1.05 \text{ m/s}$$

From the figure,

$V_c = 0.875$ m/sec

Point 'C' distance on the mechanism from O_2 = 859.48 mm

$$\therefore\ \frac{V_d}{V_c} = \frac{1250}{859.48}$$

$\Rightarrow\ V_d = 1.273$ m/s

$\therefore\ V_r = 1.4235$ m/s

Velocity of the ram R

$V_r = 1.4235$ m/s (from velocity polygon)

$$V_d = O_2D \times \omega_3 \Rightarrow \omega_3 = \frac{V_d}{O_2D}$$

$$\omega_3 = \frac{1.273}{1.25} = 1.0184\,\text{rad/sec}$$

05.

Sol: Given that $r = 50$ mm, $l = 175$ mm

$$N = 400 \text{ rpm}\quad \omega = \frac{2\pi N}{60} = 41.89\,\text{rad/sec}$$

$\omega \times r = 41.89 \times 50 \times 10^{-3} = 2.09$ m/s

Velocity, $V = \omega r \left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$

Where, $n = l/r$

At

$\theta = 0$, $V = 0$

$\theta = 90°$, $V = r\omega = 2.09$ m/s

$\theta = 180°$, $V = 0$

$\theta = 270°$, $V = -2.09$ m/s

$\theta = 360°$, $V = 0$

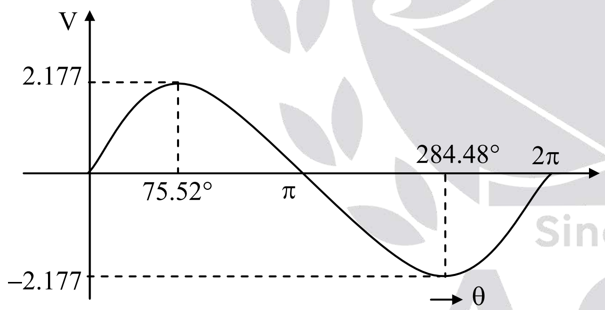
∴ Velocity at the extreme position of piston becomes zero.

For max velocity,

$$\frac{dV}{d\theta} = 0 \Rightarrow \cos\theta + \frac{\cos 2\theta}{n} = 0$$

$\Rightarrow 2\cos^2\theta - 1 + n\cos\theta = 0$, $n = 3.5$

$\cos\theta = 0.25 \Rightarrow \theta = 75.52°$, $284.478°$



Acceleration

$$a = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$r\omega^2 = 87.77$ m/s^2

$\theta = 0°$, $a = 112.8$

$\theta = 75.52°$, $a = 0$

$\theta = 90°$, $a = -25.06$ m/s^2

$\theta = 180°$, $a = -62.66$ m/s^2

$\theta = 270°$, $a = -25.06$ m/s^2

$\theta = 284.48°$, $a = 0$

$\theta = 360°$, $a = 112.8$

At maximum value of acceleration

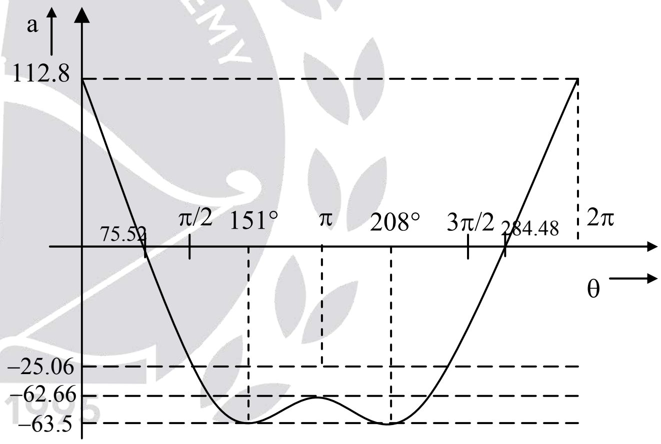
$$\frac{da}{d\theta} = 0 \quad \Rightarrow -\sin\theta - \frac{2\sin 2\theta}{n} = 0$$

$\Rightarrow (n + 4\cos\theta) = 0$

$$\cos\theta = -\frac{n}{4} = -0.875$$

$\Rightarrow \theta = 151.05°$, $208.95°$

And, $a = -63.5$ m/s^2 respectively.



Chapter 2

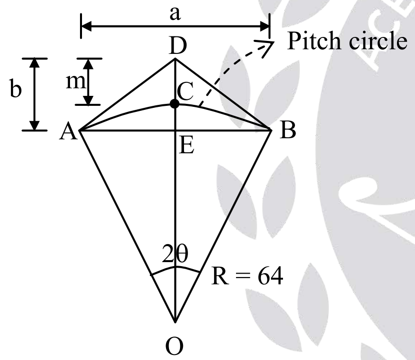
Gear and Gear Trains

01. Ans (a)

Sol: Profile between base and root circles is not involute. If tip of a tooth of a mating gear digs into this non-involute portion interference will occur.

02. Ans: (d)

Sol: Angle made by 32 teeth + 32 tooth space = 360°.



$$2\theta = \frac{360}{64} = 5.625$$

$$\theta = 2.8125$$

$$R = \frac{mT}{2} = \frac{4\times 32}{2} = 64\text{mm}$$

$$a = R\sin\theta \times 2$$

$$= 64\times\sin(2.81)\times 2 = 6.28$$

$$OE = R\cos\theta = 64\times\cos(2.8125) = 63.9 \text{ mm}$$

$$b = \text{addendum} + CE = \text{module} + (OC - OE)$$

$$= 4 + (64 - 63.9) = 4.1$$

03. Ans: (a)

Sol: When addendum of both gear and pinion are same then interference occurs between tip of the gear tooth and pinion.

04. Ans: Decreases, Increases

05. Ans: (b)

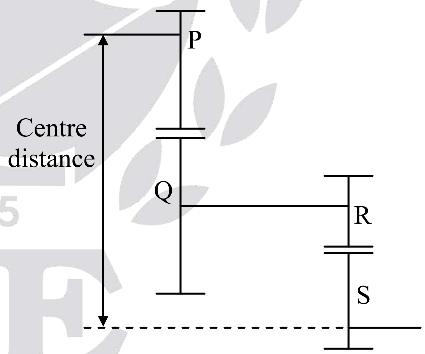
Sol: For same addendum interference is most likely to occur between tip of the gear tooth and pinion i.e., at the beginning of the contact.

06. Ans: (b)

Sol: For two gears are to be meshed, they should have same module and same pressure angle.

07. Ans: (b)

Sol:



Given $T_p = 20$, $T_Q = 40$, $T_R = 15$, $T_S = 20$

Dia of Q = 2 × Dia of R

$$m_Q.T_Q = 2m_R.T_R$$

Given, module of R = m_R = 2mm

$$\Rightarrow m_Q = 2\, m_R \frac{T_R}{T_Q} = 2\times 2\times\frac{15}{40} = 1.5\,\text{mm}$$

$m_P = m_Q = 2mm$

$m_S = m_R = 1.5$ mm

$$\text{Radius} = \text{module} \times \frac{\text{No. of teeth}}{2}$$

Centre distance between P and S is given by

$$R_P + R_Q + R_R + R_T$$

$$= m_P \frac{T_P}{2} + m_Q \frac{T_Q}{2} + m_R \frac{T_R}{2} + m_S \frac{T_S}{2}$$

$$= 1.5\left[\frac{40+20}{2}\right] + 2\left[\frac{15+20}{2}\right]$$

$= 45 + 35 = 80$ mm

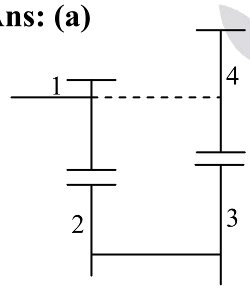
08. Ans: (c)

Sol: $$\frac{N_2}{N_6} = \frac{N_3 N_5 N_6}{N_2 N_4 N_5} = \frac{N_3 N_6}{N_2 N_4}$$

Wheel 5 is the only Idler gear as the number of teeth on wheel '5' does not appear in the velocity ratio.

09. Ans: (a)

Sol:



$Z_1 = 16$, $Z_3 = 15$, $Z_2 = ?$, $Z_4 = ?$

First stage gear ratio, $G_1 = 4$,

Second stage gear ratio, $G_2 = 3$,

$m_{12} = 3$, $m_{34} = 4$

$Z_2 = 16 \times 4 = 64$

$Z_4 = 15 \times 3 = 45$

10. Ans: (b)

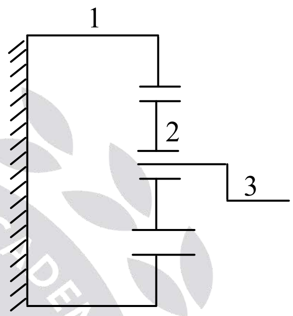
Sol: Centre distance

$$= \frac{m_{12}}{2} \times (Z_1 + Z_2) = \frac{m_{34}}{2} \times (Z_3 + Z_4)$$

$$= \frac{4}{2} \times (15 + 45) = 120\text{mm}$$

11. Ans: 5 rpm (CCW)

Sol:



$T_1 = 104$, $N_1 = 0$,

$T_2 = 96$, $N_a = 60$ rpm (CW+ve), $N_2 = ?$

$$\frac{N_2 - N_a}{N_1 - N_a} = \frac{T_1}{T_2} = \frac{104}{96}$$

$$\frac{N_2 - 60}{0 - 60} = \frac{104}{96}$$

$$N_2 = 60\left[1 - \frac{104}{96}\right] = \frac{-60 \times 8}{96} = -5 \text{ rpm CW}$$

$= 5$ rpm in CCW

12. Ans: (a)

Sol: By Analytical Approach

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{-T_2}{T_1} \times \frac{-T_4}{T_3} = \frac{45}{15} \times \frac{40}{20}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$$

13. Ans: (d)

Sol: Data given:

ω_1= 60 rpm (CW, +ve)

ω_4 = –120 rpm [2 times speed of gear -1]

We have, $\frac{\omega_1-\omega_5}{\omega_4-\omega_5}=6$

$$\Rightarrow \frac{60-\omega_5}{-120-\omega_5}=6\text{, simplifying}$$

$60-\omega_5=-720-6\omega_5$

$\omega_5 = -156$ rpm CW

$\Rightarrow$ ω_5 = 156 rpm CCW

14. Ans: (c)

Sol: ω_2 = 100 rad/sec(CW+ve),

ω_{arm} = 80 rad/s (CCW) = –80 rad/sec

$$\frac{\omega_5-\omega_a}{\omega_2-\omega_a}=\frac{-T_2}{T_3}\times\frac{T_4}{T_5}$$

$$\frac{\omega_5-(-80)}{100-(-80)}=\frac{-20}{24}\times\frac{32}{80}=-\frac{1}{3}$$

$\Rightarrow \omega_5 = -140$ CW = 140 CCW

15. Ans (c)

Sol: It also rotates one revolution but in opposite direction because of differential gear system

16. Ans: (c)

Sol: No .of Links, L = 4

No. of class 1 pairs J_1=3

No. of class 2 pairs J_2 =1 (Between gears)

No. of dof = $3(L-1)-2J_1-J_2=2$

17. Ans: (a)

Sol: r_b = base circle radius,

r_d = dedendum radius

r = pitch circle radius.

For the complete profile to be invoulte,

$r_b = r_d$

r_d = r – 1 module

$$r=\frac{mT}{2}=\frac{16\times5}{2}=40\text{ mm}$$

$\therefore r_b = r_d = 40-1\times5 = 35$ mm

$r_b = r\cos\phi \Rightarrow \phi \simeq 29^o$

18. Ans: – 3.33 N-m

Sol: $$\frac{\omega_s-\omega_a}{\omega_p-\omega_a}=\frac{-Z_p}{Z_s}$$

$$\Rightarrow\frac{0-10}{\omega_p-10}=\frac{-20}{40}$$

$\Rightarrow \omega_p$ = 30 rad/sec

By assuming no losses in power transmission

$T_p\times\omega_p+T_s\times\omega_s+T_a\times\omega_a=0$

$\Rightarrow T_p\times30+T_s\times0+5\times10=0$

$\Rightarrow T_p=\frac{-50}{30}=-1.67$ N-m, $T_p+T_s+T_a=0$

$\Rightarrow -1.67+T_s+5=0$

$\Rightarrow T_s = -3.33$ N-m

19. Ans: (a)

Sol: Train value = speed ratio

20. Ans: (d)

Sol: $T_S + 2\,T_P = T_A$ ------(1)

$$\frac{N_A - N_a}{N_P - N_a} = \frac{T_P}{T_A} \quad \text{------(2)}$$

and $$\frac{N_P - N_S}{N_S - N_G} = -\frac{T_S}{T_P} \quad \text{------(3)}$$

From (2) and (3)

$$\frac{N_A - N_a}{N_S - N_a} = -\frac{T_B}{T_A}$$

$$\Rightarrow \frac{300-180}{0-180} = -\frac{80}{T_A}$$

$\therefore$ $T_A = 120$

$80 + 2\,T_P = 120 \Rightarrow T_P = 20$

Conventional Practice Solutions

01.

Sol: Given that T = 42, t = 19,

$\phi = 20°$, m = 6 mm, addendum a_w = 6 mm

(i) $R = \dfrac{mT}{2} = \dfrac{6\times 42}{2} = 126\,\text{mm}$

$R_a = R + a_w = 126 + 6 = 132$ mm,

$r = \dfrac{mt}{2} = \dfrac{6\times 19}{2}$

r = 57 mm,

$r_a = r + a_w = 57 + 6 = 63$ mm

Path of contact,

$$= \sqrt{R_a^2 - (R\cos\phi)^2} + \sqrt{r_a^2 - (r\cos\phi)^2} - (R+r)\sin\phi$$

$$= \sqrt{132^2 - (126\times\cos 20^{o})^2} + \sqrt{63^2 - (57\cos 20^{o})^2}$$

$$- (126+57)\sin 20^{\circ}$$

= 28.93 mm

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{28.93}{\cos 20^{o}}$$

= 30.787 mm

(ii) Number of pairs of teeth in contact

$$n = \frac{\text{arc of contact}}{\text{Circular Pitch}} = \frac{30.787}{\pi m}$$

$$= \frac{30.787}{\pi\times 6} = 1.633 \approx 2$$

(iii) The angle turned by the pinion, while any one pair of teeth in contact is

$$\text{Angle of action} = \frac{\text{Arc of contact}}{2\pi r}\times 360$$

$$= \frac{30.787}{2\pi\times 57}\times 360 = 30.946^{\circ}$$

(iv) Path of approach $= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$

$$= \sqrt{132^2 - (126\times\cos 20^{o})^2} - 126\sin 20^{o}$$

= 15.259 mm

Path of recess $= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\phi$

$$= \sqrt{63^2 - (57\cos 20^{o})^2} - 57\sin 20^{o}$$

= 13.672 mm

(a) $$\frac{\text{Sliding velocity}}{\text{Rolling Velocity}} = \frac{(\omega_p + \omega_g)\times\text{Path of approach}}{\text{Pitch line velocity}(\omega_p\times r)}$$

$$= \frac{\left(\omega_p + \dfrac{19}{42}\times\omega_p\right)\times 15.259}{\omega_p\times 57} = 0.388$$

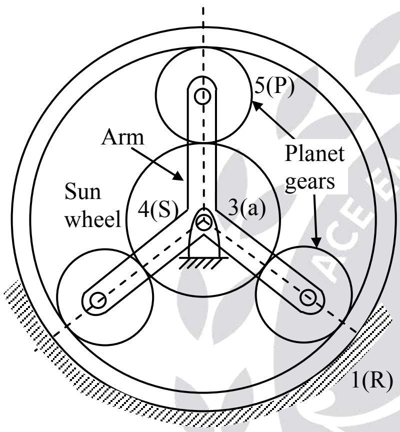
(b) $$\frac{\text{Sliding Velocity}}{\text{Rolling Velocity}} = \frac{\left(\omega_p + \frac{19}{42}\omega_p\right)\times \text{Path of recess}}{\omega_p \times r}$$

$$= \frac{\left(1+\frac{19}{42}\right)\times 13.672}{57} = 0.348$$

(c) $$\frac{\text{Sliding Velocity}}{\text{Rolling Velocity}} = \frac{(\omega_P + \omega_g)\times 0}{\text{Pitch line Velocity}} = 0$$

02.

Sol:



$N_R = 0$, $D_R = 216$ mm, $m = 4$ mm

$$Z_R = \frac{216}{4} = 54 \text{ teeth}$$

$N_s = 5\ N_a$

$$\frac{N_s - N_a}{N_R - N_a} = -\frac{Z_R}{Z_s}$$

$$\frac{5N_a - N_a}{0 - N_a} = -\frac{Z_R}{Z_s}$$

$$\therefore \frac{Z_R}{Z_s} = 4$$

$Z_R \cong 54$ (Take the nearest integer value 56)

$$Z_s = \frac{Z_R}{4} = \frac{56}{4} = 14$$

$$Z_P = \frac{3Z_R}{8} = \frac{3\times 56}{4} = 21$$

From center distance :

$$R_R = D_P + R_s$$

$$\frac{m\,Z_R}{2} = m\,Z_p + m\frac{Z_s}{2}$$

$$Z_R = 2Z_P + Z_s$$

$$Z_R = 2Z_P + \frac{Z_R}{4}$$

$$Z_P = \frac{3Z_R}{8}$$

Torque analysis : $\Sigma P_i = 0$

$$T_a\omega_a + T_s\omega_s + T_R\omega_R = 0$$

Note: idler gears can be ignored.

$\therefore T_a\omega_a + T_s\omega_s = 0$ $(\because\ \omega_R = 0)$

$$T_a = \frac{-T_s\omega_s}{\omega_a} = \frac{-T_s\times 5\omega_a}{\omega_a}$$

$\therefore\ T_a = -5 \times 19.6 = -98$ N-m

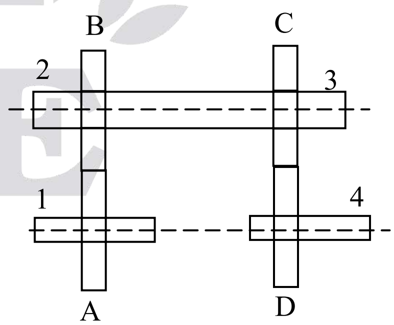
$\Sigma T_i = 0$

$\therefore\ T_s + T_a + T_R = 0$

$19.6 - 98 + T_R = 0 \Rightarrow T_R = 78.4$ N-m

03.

Sol:



$m_A = m_B = 4$ mm,

$m_C = m_D = 9$ mm

$$\frac{N_1}{N_4} = \frac{Z_B \times Z_D}{Z_A \times Z_C} = \frac{N_A}{N_D} = 12$$

$$\frac{N_D}{N_A} = \frac{1}{12},$$

$$N_D \le \frac{N_A}{12}$$

$G \ge 12 = G_1 G_2$

$\therefore \quad G_1 = G_2 = \sqrt{12}$

$$\frac{N_A}{N_D} = \frac{Z_B}{Z_A} \times \frac{Z_D}{Z_C} = 12$$

Let, $Z_{min} = 15$

$$\frac{Z_B}{Z_A} = \sqrt{12} = 3.464$$

$$\frac{Z_D}{Z_C} = 3.464$$

$$\text{Center distance} = \frac{m_1(Z_A + Z_B)}{2} = \frac{m_2(Z_C + Z_D)}{2}$$

$$\frac{4}{2}\left(Z_A + \sqrt{12}Z_A\right) = \frac{9}{2}\left(Z_C + \sqrt{12}Z_C\right)$$

$$\Rightarrow \; 2 \times 2Z_A\left(1 + \sqrt{12}\right) = 2 \times 4.5Z_C\left(1 + \sqrt{12}\right)$$

$$Z_A = \frac{9}{4}Z_C$$

Let us take $Z_C = 16$

$Z_D = \sqrt{12} \times Z_C = 55.42 = 56$

$Z_A = \frac{9}{4} \times 16 = 36$

$Z_B = \sqrt{12} \times 36 = 124.7 = 126$

$$\therefore C = \frac{m_1(Z_A + Z_B)}{2} = \frac{m_2(Z_C + Z_D)}{2}$$

$$= \frac{4}{2}(36 + 126) = \frac{9}{2}(16 + 56)$$

$= 324$ mm

04.

Sol:

Action	F	Sun A	B & D $\left(\frac{N_B}{N_A}\right)$	C $\left(\frac{N_C}{N_A}\right)$	E $\left(\frac{N_E}{N_A}\right)$
Arm F is fixed & Give +1 rev. to sun &	0	1	$\frac{50}{64}$	$-\frac{50}{80}$	$-\frac{30}{64}$
multiply with x	0	x	$\frac{50}{64}x$	$-\frac{50}{80}x$	$-\frac{30}{64}x$
add y	y	x+y	$\frac{50}{64}x + y$	$-\frac{50}{80}x + y$	$-\frac{30}{64}x + y$

Arm = F, Sun = A

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\therefore \quad N_B = N_A \times \frac{T_A}{T_B}$$

$$N_B = 1 \times \frac{50}{64}$$

$$\frac{N_C}{N_A} = \frac{-T_B \times T_A}{T_C \times T_B} = \frac{-T_A}{T_C}$$

$$\frac{N_E}{N_A} = \frac{-T_D \times T_A}{T_E \times T_B}$$

$N_{A,\,arm} = N_A - N_{arm}$

$N_A = N_{A,arm} + N_{arm}$

$N_C = 0$ (fixed)

$N_A = +\,600$ rpm (CW)

$$N_C = -\frac{50x}{80} + y = 0 \text{ -----(i)}$$

$$N_A = x + y = 600 \text{ ------(ii)}$$

x = 369.23 rpm , y = 230.76 rpm

$$N_E = -\frac{30}{64}(369.23) + 230.76 = 57.69(CW)$$

(looking from left side)

N_E = 57.69 (CCW)

(looking from right side from E)

By relative velocity method :

N_A = 600 rpm , N_C = 0 , N_E = ?

$$\frac{N_A - N_F}{N_C - N_F} = -\frac{T_C}{T_A}$$

$$\Rightarrow \frac{600 - N_F}{0 - N_F} = -\frac{80}{50}$$

3000 – 5 N_F = 8 N_F

$\Rightarrow$ N_F = 230.76 rpm

$$\frac{N_A - N_F}{N_E - N_F} = -\frac{T_B \times T_E}{T_A \times T_D}$$

$$\frac{600 - 230.76}{N_E - 230.76} = -\frac{64 \times 50}{50 \times 30}$$

$\Rightarrow$ N_E = 57.67 rpm

05.

Sol: $G = \frac{5}{3}$, A = 1 m

$$\frac{N_P}{N_G} = \frac{T_G}{T_P} = G$$

$$T_{min} = \frac{2 \times a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{1}{\frac{5}{3}}\left(\frac{1}{\frac{5}{3}} + 2\right)\sin^2 20^o} - 1}$$

T_G = 22.8 = 23

$$T_P = \frac{23}{G} = \frac{23}{\frac{5}{3}} = 13.8 \approx 15$$

$$T_G = 15 \times \frac{5}{3} = 25$$

T_G = 25,

T_P = 15

06.

Sol: Given that,

t = 15, T = 30,

ϕ = 20° , m = 5 mm

Addendum, a_m = 5 mm

N_P = 1000 rpm

But, $m = \frac{d}{t} = \frac{D}{T}$

$$r = \frac{mt}{2} = \frac{5 \times 15}{2} = 37.5\,\text{mm}$$

$$R = \frac{mT}{2} = \frac{5 \times 30}{2} = 75\,\text{mm}$$

$R_a = R + a_m$ = 75 +5 = 80 mm

$r_a = r + a_m$ = 37.5 + 5 = 42.5 mm

$$\text{Path of recess} = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$$

$$= \sqrt{(42.5)^2 - (37.5\cos 20^o)^2} - 37.5\sin 20^o$$

= 10.93 mm

$$\text{Path of approach} = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$

$$= \sqrt{80^2 - (75\cos 20^o)^2} - 75\sin 20^o$$

= 12.20 mm

Maximum sliding velocity

$$= (\omega_p + \omega_g) \times \text{maximum path}$$

$$\omega_p = \frac{2\pi \times 1000}{60} = 104.72 \text{ rad/s}$$

$$\omega_g = \omega_p \times \frac{T_p}{T_g}$$

$$= 104.72 \times \frac{15}{30} = 52.36 \text{ rad/s}$$

$$(V_s)_{max} = \frac{(104.72 + 52.36) \times 12.20}{1000} \text{ m/s}$$

$$= 1.92 \text{ m/s}$$

$$\text{Contact ratio} = \frac{\text{arc of contact}}{\text{circular pitch}}$$

$$\text{Arc of contact} = \frac{\text{path of contact}}{\cos\phi}$$

$$= \frac{10.93 + 12.2}{\cos 20°} = 24.6 \text{ mm}$$

$$\text{Circular pitch, } p = \frac{\pi d}{t}$$

$$= \frac{\pi \times (37.5 \times 2)}{15} = 15.71$$

$$\therefore \text{ Contact ratio} = \frac{24.6}{15.7} = 1.57$$

Chapter 3 Flywheels

01.

Sol: Given

$P = 80 \text{ kW} = 80\times10^3 \text{ W} = 80{,}000\text{W}$

$\Delta E = 0.9$ Per cycle

$N = 300 \text{ rpm}$

$C_S = 0.02$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 30}{60} = 31.41 \text{rad/s}$$

$\rho = 7500 \text{ kg/m}^3$

$\sigma_c = 6 \text{ MN/m}^2$

$$\sigma_c = \rho V^2 = \rho R^2 \omega^2$$

$$R = \sqrt{\frac{\sigma_C}{\rho\omega^2}} = \sqrt{\frac{6\times10^6}{7500\times31.41^2}}$$

$R = 0.9$ m

$D = 2R = 1.8$m

$N = 300\text{rpm} = 5\text{rps} \rightarrow 0.2$ Sec/rev

1 cycle = 2 revolution $(\because 4 \text{ stroke engine})$

$= 0.4 \text{ sec}$

Energy developed per cycle

$$= 0.4 \times 80 = 32 \text{ kJ}$$

$$\Delta E = E \text{ per cycle} \times 0.9$$

$$= 32\times10^3 \times 0.9 = 28800 \text{ J}$$

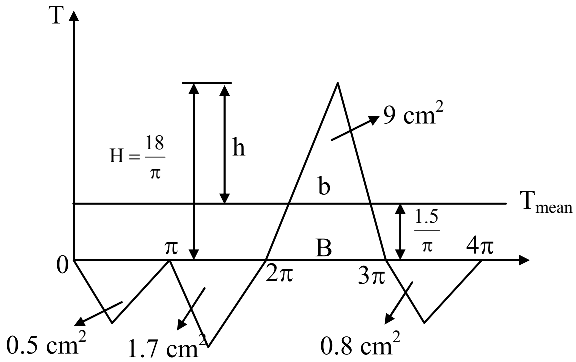
$$\Delta E = I\omega^2 C_S$$

$$I = \frac{\Delta E}{\omega^2 C_S}$$

$$I = 1459.58 \text{ kg-m}^2$$

02.

Sol:



Given: 1 cm^2 = 1400 J

Assume on x-axis 1 cm = 1 radian and on y-axis 1 cm = 1400 N-m

$a_1 = -0.5$ cm^2

$a_2 = -1.7$ cm^2

$a_3 = 9$ cm^2

$a_4 = -0.8$ cm^2

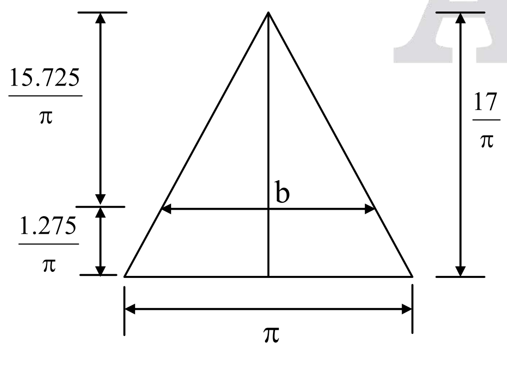
Work done per cycle $= -a_1 - a_2 + a_3 - a_4$

$= -0.5 - 1.7 + 9 - 0.8$

$= 6$ cm^2

Mean torque $T_m = \dfrac{\text{Workdone per cycle}}{4\pi}$

$= \dfrac{6}{4\pi} = \dfrac{1.5}{\pi}$ cm



Area of the triangle (expansion)

$$= \frac{1}{2}\times\pi\times H = 9$$

$H = 18/\pi$

Area above the mean torque line

$$\Delta E = \frac{1}{2}\times b\times h$$

From the similar triangles ,

$$\frac{b}{B} = \frac{h}{H} \Rightarrow b = \frac{16.5}{18}\times\pi$$

$$\Delta E = \frac{1}{2}\times b\times\frac{16.5}{\pi}$$

$$= \frac{1}{2}\times\frac{16.5}{18}\times\frac{16.5}{\pi} = 7.56 \text{ cm}^2$$

$\Delta E = 7.56\times1400 = 10587$ N-m

$N_1 = 102$ rpm, $N_2 = 98$ rpm,

$$\omega_1 = \frac{2\pi N_1}{60} = 10.68 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = 10.26 \text{ rad/s}$$

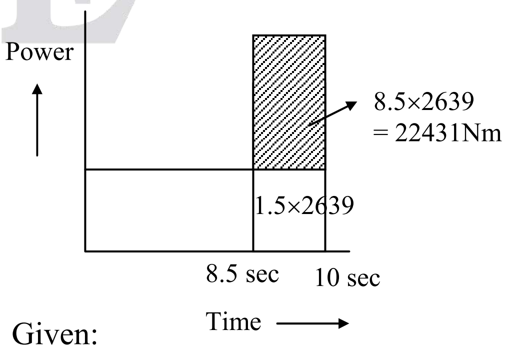
$$\Delta E = \frac{1}{2}\times I\times(\omega_1^2 - \omega_2^2)$$

$$I = \frac{2\times\Delta E}{(\omega_1^2-\omega_2^2)} = \frac{2\times10587}{10.68^2 - 10.26^2}$$

$I = 2405.6$ kg-m^2

03.

Sol:



Given:

$d = 40$ mm, $t = 30$ mm

$E_1 = 7$ N-m/mm^2, $S = 100$ mm

$V = 25$ m/s, $V_1 - V_2 = 3\%V$, $C_S = 0.03$

$A = \pi dt = \pi \times 40 \times 30$

$= 3769.9 = 3770$ mm^2

Since the energy required to punch the hole is 7 Nm/mm^2 of sheared area, therefore the Total energy required for punching one hole $= 7 \times \pi dt = 26390$ N-m

Also the time required to punch a hole is 10 sec, therefore power of the motor required $= \dfrac{26390}{10} = 2639\,\text{Watt}$

The stroke of the punch is 100 mm and it punches one hole in every 10 seconds.

Total punch travel = 200 mm

(up stroke + down stroke)

Velocity of punch = (200/10) = 20 mm/s

Actual punching time = 30/20 = 1.5 sec

Energy supplied by the motor in 1.5 sec is $E_2 = 2639 \times 1.5 = 3958.5 = 3959$ N-m

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy

$\Delta E = E_1 - E_2$

$= 26390 - 3959 = 22431$ N-m

Coefficient of fluctuation of speed

$$C_S = \frac{V_1 - V_2}{V} = 0.03$$

We know that maximum fluctuation of energy (ΔE)

$22431 = m V^2 C_S = m (25)^2 (0.03)$

$m = 1196$ kg

04. Ans: 4.27

Sol: $I = mk^2 = 200 \times 0.4^2 = 32$ kg-m^2

$$\omega_1 = \frac{2\pi \times 400}{60} = 41.86\,\text{rad/s}$$

$$\omega_2 = \frac{2\pi \times 280}{60} = 26.16\,\text{rad/s}$$

Energy released $= \dfrac{1}{2} I\left(\omega_1^2 - \omega_2^2\right) = 17086.6$ J

Total machining time $= \dfrac{60}{5} = 12\,\text{sec}$

Power of motor $= \dfrac{17086.6}{12 - 8} = 4.27\,\text{kW}$

05. Ans: (d)

Sol: Work done $= -0.5+1-2+25-0.8+0.5$

$= 23.2$ cm^2

Work done per cycle $= 23.2 \times 100 = 2320$

$\left(\because 1\text{cm}^2 = 100\text{N} - \text{m}\right)$

$$T_{mean} = \frac{\text{W.D per cycle}}{4\pi}$$

$$= \frac{2320}{4\pi} = \frac{580}{\pi}\,\text{N} - \text{m}$$

Suction = 0 to π,

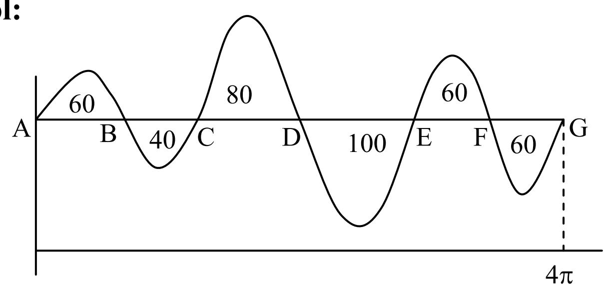
Compression = π to 2π

Expansion = 2π to 3π,

Exhaust = 3π to 4π

06. Ans: (c)

Sol:



$E_A = E$

$E_B = E + 60$

$E_C = E + 60 - 40 = E + 20$

$E_D = E + 20 + 80 = E + 100 = E_{max}$

$E_E = E + 100 - 100 = E$

$E_F = E + 60$

$E_G = E + 60 - 60 = E_{min}$

$\therefore$ R > P > Q > S

07. Ans: (b)

Sol: $I_{disk} = \dfrac{mr^2}{2}$

$I_1 = \dfrac{mr_1^2}{2}$, $C_{s1} = 0.04$

$I_2 = 4\times mr_1^2 = 4I_1$

$C_{s2} = \dfrac{I_1}{I_2}\times C_{s1} = 0.01 \Rightarrow 1\%$ reduce

08. Ans: (b)

Sol: For same ΔE and ω

$C_s \propto I$

$\dfrac{C_{S1}}{C_{S2}} = \dfrac{I_2}{I_1} = \dfrac{2I}{I}$

$C_{S2} = \dfrac{C_{S1}}{2} = \dfrac{0.04}{2} = 0.02$

09. Ans: (a)

Sol: Let the cycle time = t

Actual punching time = t/4

W = energy developed per cycle

Energy required in actual punching

= 3W/4

During 3t/4 time, energy consumed = W/4

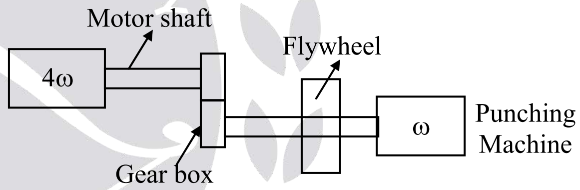
$E_{max} = \dfrac{3W}{4}$, $E_{min} = \dfrac{E}{4}$

$\Delta E = E_{max} - E_{min} = \dfrac{E}{2}$

$\dfrac{\Delta E}{E} = 0.5$

10. Ans: (c)

Sol:



$C_s = 0.032$

Gear ratio = 4

$I\omega'^2 \times C_s' = I\omega^2 C_s$

$C_s' = C_s\left(\dfrac{\omega}{\omega'}\right)^2 = \dfrac{C_s \times \omega^2}{16\,\omega^2} = \dfrac{C_s}{16}$

$= 0.0032 / 16 = 0.002$

(by taking moment of Inertia, I = constant).

Thus, if the flywheel is shifted from machine shaft to motor shaft when the fluctuation of energy (ΔE) is same, then coefficient of fluctuation of speed decreases by 0.2% times.

11. Ans: 0.5625

Sol: The flywheel is considered as two parts $\frac{m}{2}$ as rim type with Radius R and $\frac{m}{2}$ as disk type with Radius $\frac{R}{2}$

$$I_{Rim} = \frac{m}{2}R^2,$$

$$I_{disk} = \frac{1}{2}\times\frac{m}{2}\times\left(\frac{R}{2}\right)^2 = \frac{mR^2}{16}$$

$$I = \frac{mR^2}{2} + \frac{mR^2}{16}$$

$$= \frac{9}{16}mR^2$$

$$= 0.5625\ mR^2$$

$$\therefore\ \alpha = 0.5625$$

12. Ans: 104.71

Sol: N = 100 rpm

$$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi}(10000 + 1000\sin 2\theta - 1200\cos 2\theta)d\theta$$

$$= \frac{1}{\pi}[10000\theta - 500\cos 2\theta - 600\sin 2\theta]_0^{\pi}$$

= 10000 Nm

$$\text{Power} = \frac{2\pi NT}{60}$$

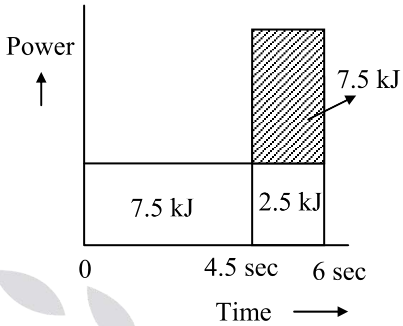
$$= \frac{2\times\pi\times100\times10000}{60} = 104719.75\ \text{W}$$

P = 104.719 kW

Conventional Practice Solutions

01.

Sol:



Given:

P = 2 kW ; K = 0.5

N = 260 rpm ; ω = 27.23 rad/s

Actual punching time = 1.5 sec

Work done per cycle = 10000 Joule per hole

Motor power = 2 kW

ΔN = 30 rpm

$\Delta\omega = 2\pi\times(30/60) = \pi$ rad/sec

600 holes/hr = 10 holes/min ⇒ 6 sec/hole

Cycle time = 6 sec

Energy withdrawn from motor

= (10000/6) = 1666.67 J

Energy stored in flywheel

$$= \frac{10000}{6}\times4.5 = 7.5\,\text{kJ}$$

Fluctuation of Energy ΔE = 7500 J

$$\Delta E = I\,\omega\,\Delta\omega = mk^2\omega\Delta\omega$$

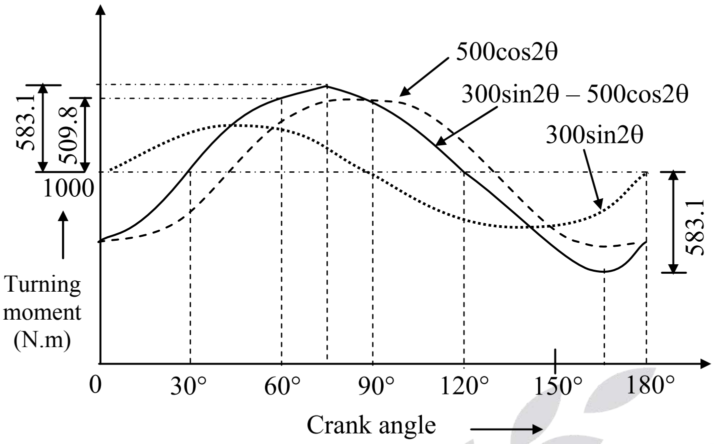
$$m = \frac{\Delta E}{k^2\omega\Delta\omega}$$

Where k = radius of gyration

$$m = \frac{7500}{0.5^2\times27.23\times\pi} = 349.5\,\text{kg}$$

02.

Sol:



(i) $T_{mean} = \frac{1}{\pi}\int_0^{\pi} Td\theta$

$= \frac{1}{\pi}\int_0^{\pi}(1000 + 300\sin 2\theta - 500\cos 2\theta)d\theta$

$= \frac{1}{\pi}\left[1000\theta - \frac{300}{2}\cos 2\theta - \frac{500}{2}\sin 2\theta\right]_0^{\pi}$

$= \frac{1}{\pi}[(1000\pi - 150 - 0) - (0 - 150 - 0)]$

= 1000 N.m

$P = T\omega = 1000 \times \frac{2\pi \times 250}{60}$

= 26180 W (or) 26.18 kW

(ii) At any instant, $\Delta T = T - T_{mean}$

$= (1000 + 300\sin2\theta - 500\cos2\theta) - 1000$

$= 300\sin2\theta - 500\cos2\theta$

ΔT is zero, when $300\sin2\theta - 500\cos2\theta = 0$

or $300\sin2\theta = 500\cos2\theta$ or $\tan 2\theta = \frac{5}{3}$

or $2\theta = 59°$ or $239°$,

$\theta = 29.5°$ or $119.5°$

$e_{max} = \int_{29.5^o}^{119.5^o} \Delta T d\theta = \int_{29.5^o}^{119.5^o} (300\sin 2\theta - 500\cos 2\theta)d\theta$

$= [-150\cos 2\theta - 250\sin 2\theta]_{29.5^o}^{119.5^o}$

= 583.1 N.m

$K = \frac{e}{mk^2\omega^2}$

$= \frac{583.1}{400 \times (0.4)^2 \times \left(\frac{2\pi \times 250}{60}\right)^2}$

= 0.01329 or 1.329%

(iii) Acceleration or deceleration is produced by excess or deficit torque than the mean value at any instant.

$\Delta T = 300\sin2\theta - 500\cos2\theta$

when $\theta = 60^o$,

$\Delta T = 259.8 - (-250) = 509.8$ N.m

or $I\alpha = mk^2\alpha = 509.8$

or $400 \times (0.4)^2 \times \alpha = 509.8$

or $\alpha = 7.966$ rad/s^2

(iv) For ΔT_{max} and ΔT_{min}

$\frac{d}{d\theta}(\Delta T) = \frac{d}{d\theta}(300\sin 2\theta - 500\cos 2\theta) = 0$

or $2\times300\cos2\theta + 2\times500\sin2\theta = 0$

$600\cos2\theta = -1000\sin2\theta$

or $\tan2\theta = -0.6$

or $2\theta = 149.04°$ and $329.04°$

or $\theta = 74.52°$ and $164.52°$

when $2\theta = 149.04°$,

$T = 1583.1$ N.m,

$\Delta T = 583.1$ N.m

when $2\theta = 329.04°$,

$T = 416.9$ N.m,

$\Delta T = -583.1$ N.m

As values of ΔT at maximum and minimum torque T are same, maximum acceleration is equal to maximum retardation.

or $\Delta T = mk^2\alpha = 583.1$

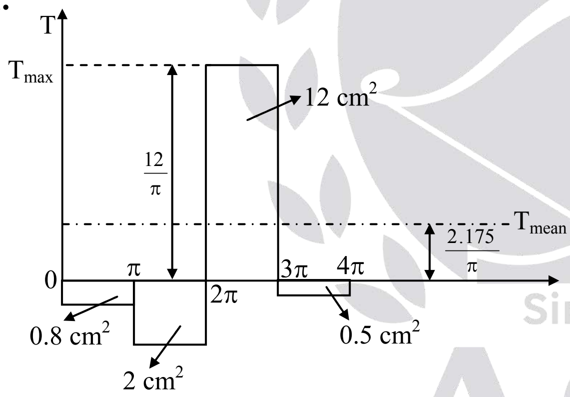
or $400\times(0.4)^2\times\alpha = 583.1$

Maximum acceleration or retardation,

$\alpha = 9.11$ rad/s^2

03.

Sol:



Given: 1 cm^2 = 1200 J

Assume on x-axis 1 cm = 1 radian and on y-axis 1 cm = 1200 N-m

$a_1 = -0.8$ cm^2, $a_2 = -2$ cm^2

$a_3 = 12$ cm^2, $a_4 = -0.5$ cm^2

Work done per cycle $= a_1 + a_2 + a_3 + a_4$

$= -0.8 - 2 + 12 - 0.5$

$= 8.7$ cm^2

Mean torque,

$$T_m = \frac{\text{Workdone per cycle}}{4\pi}$$

$$= \frac{8.7}{4\pi} = \frac{2.175}{\pi}\text{ cm}$$

Maximum torque, $T_{max} = \frac{12}{\pi}$

Area above the mean torque line

$$\Delta E = \pi \times \left(\frac{12 - 2.175}{\pi}\right) = 9.825 \text{ cm}^2$$

$\Delta E = 9.825 \times 1200$ J $= 11790$ J

$N_1 = 102$ rpm,

$N_2 = 998$ rpm,

$$\omega_1 = \frac{2\pi N_1}{60} = 105.24 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = 104.51 \text{ rad/s}$$

$$\Delta E = \frac{1}{2} \times I \times (\omega_1^2 - \omega_2^2)$$

$$I = \frac{2\times\Delta E}{(\omega_1^2 - \omega_2^2)} = \frac{2\times 11790}{105.24^2 - 104.51^2}$$

$I = 151.92$ kg-m^2

Chapter 4

Governor

01. Ans: (a)

Sol: As the governor runs at constant speed, net force on the sleeve is zero.

02. Ans: (d)

Sol: At equilibrium speed, friction at the sleeve is zero.

03. Ans: (a)

Sol: $mr\omega^2 = \frac{r}{h}\left(mg + \frac{Mg(1+k)}{2}\right)$

$k = 1$

$\omega^2 = \frac{9.8}{2\times 0.2}(10+2)$

$\omega = 17.15$ rad/sec

04. Ans: (a)

Sol: $mr\omega^2 a = \frac{1}{2} \times 200 \times \delta \times a$

$\delta = \frac{1\times 20^2 \times 0.25 \times 2}{200}$

$= 0.5 \times 2 = 1$ cm

05. Ans: (a)

Sol: $mr\omega^2 \times a = \left(\frac{F_s}{2}\right)\times a$

$F_s = 2mr\omega^2$

$= 2\times 1 \times 0.4 \times (20)^2 = 320$ N

06. Ans: (c)

Sol: A governor is used to limit the change in speed of engine between minimum to full load conditions, the sensitiveness of a governor is defined as the ratio of difference between maximum and minimum speed to mean equilibrium speed, thus,

$$\text{sensitivenss} = \frac{\text{Range of speed}}{\text{mean speed}} = \left(\frac{N_1 - N_2}{\frac{N_1 + N_2}{2}}\right)$$

Where, mean speed, $N = \frac{N_1 + N_2}{2}$

N_1 = maximum speed corresponding to no-load conditions.

N_2 = minimum speed corresponding to full load conditions.

07. Ans: (b)

08. Ans: (a)

Sol: $r_1 = 50$ cm , $F_1 = 600$ N

$F = a + rb$

$600 = a + 50\,b$

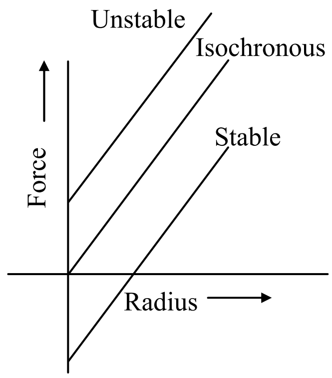
$700 = a + 60\,b$

$10\,b = 100$

$b = 10$ N/cm

$a = 100$ N

$F = 100 + 10\,r$



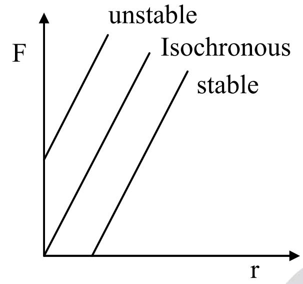
This is unstable governor. It can be isochronous if its initial compression is reduced by 100 N.

09. Ans: (d)

Sol: By increasing the dead weight in a porter governor it becomes more sensitive to speed change.

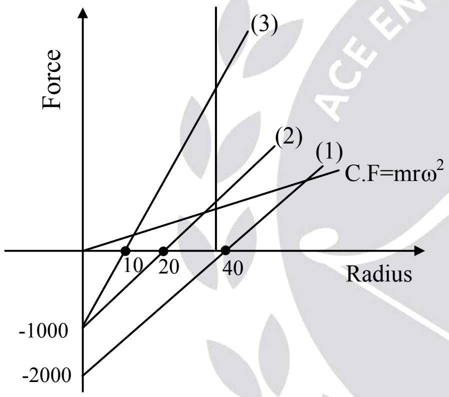
10. Ans: (d)

Sol:



11. Ans: (a)

Sol:



At radius, $r_1 = F_1 < F_2 < F_3$

$\therefore$ As Controlling force is less suitable 1 is for low speed and 2 for high speed ad 3 is for still high speed.

(1) is active after 40 cm

(2) is active after 20 cm

(3) is active after 10 cm

At given radius above 20

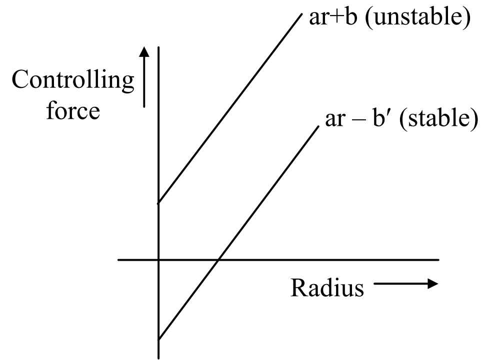
$$F_3 > F_2$$

$$mr\omega_3^2 > mr\omega_2^2$$

$$\omega_3 > \omega_2$$

12. Ans: (b)

Sol:



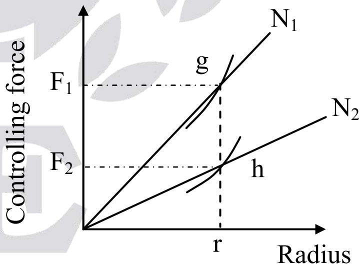
To make the governor stable spring stiffness should be decreased.

13. Ans: (c)

Sol: A governor is said to be sensitive if for a given fractional change in speed, displacement of sleeve is high.

14. Ans: (c)

Sol: If friction is taken into account, two or more controlling force are obtained as show in figure.



In all, three curves of controlling force are obtained as follows.

(a) for steady run (neglecting friction)

(b) while sleeve moves up (f positive)

(c) while sleeve moves down (f negative)

The vertical intercept *gh* signifies that between the speeds corresponding to *gh*, the radius of the ball does not change while direction of movement of sleeve does. Between speeds N_1 and N_2, the governor is insensitive.

15. Ans: (b)

Sol: A governor is stable if radius of rotation of ball is increases as the speed increases.

Centripetal force, $F = mr\omega^2$

$$\Rightarrow \quad \frac{F}{r} = m\omega^2$$

Slope of the centripetal force represents speed. Higher the slope, higher will be the speed.

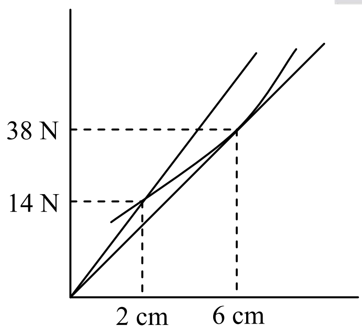
when r = 2 cm; F = 14 N

$$\therefore \quad \frac{F}{r} = \frac{14}{2} = 7$$

when r = 6 cm; F = 38 N

$$\frac{F}{r} = \frac{38}{6} = 6.33$$

As the radius increases slope of the centripetal force curve decreases and therefore speed of the governor decreases. Thus the governor is unstable.



16. Ans:

Sol: Given, m = 8 kg

$F_1 = 1500$ N at $r_1 = 0.2$ m and

$F_2 = 887.5$ N at $r_2 = 0.13$ m,

For spring controlled governor, controlling force is given by

$$F = a\,r + b$$

$1500 = a \times 0.2 + b$

$887.5 = a \times 0.13 + b$

$\therefore\ a = 8750\ ,\ b = -250$

$F = 8750\,r - 250$

At r = 0.15 m,

$F = 8750 \times 0.15 - 250 = 1062.5$ N

So, controlling force, F = 1062.5 m

$$F = mr\omega^2$$

$1062.5 = 8 \times 0.15\ \omega^2$

$\therefore \quad \omega = 29.72$ rad/s

$$N = \frac{60\,\omega}{2\pi} = 284 \text{ rpm}$$

For isochronous speed

$F = a\,r = 8750 \times 0.15 = 1312.5$ N

$$F = mr\omega^2$$

$1312.5 = 8 \times 0.5 \times \omega^2$

$\Rightarrow \quad \omega = 33.07$ rad/s

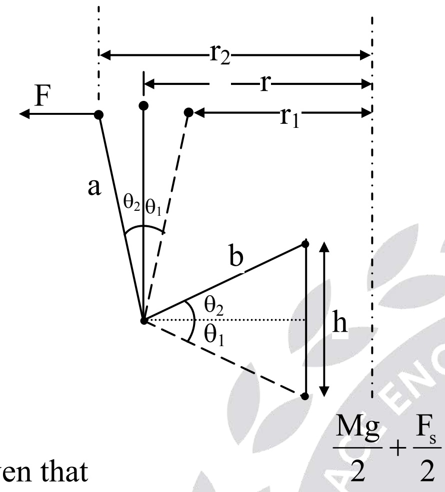
$$N = \frac{60\omega}{2\pi} = 316 \text{ rpm}$$

The increase in tension is 250 N to make the governor isochronous.

Conventional Practice Solutions

01.

Sol:



Given that

a = 10 cm,

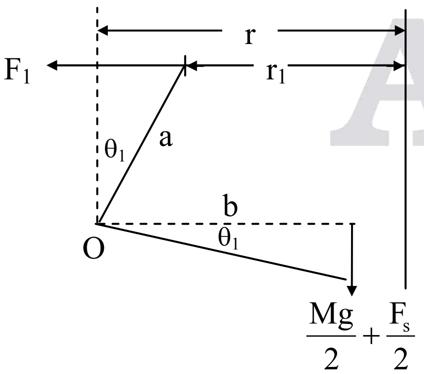
b = 9 cm,

N = 300 rpm at 'r'

$\omega = \frac{2\pi N}{60} = 10\,\pi$ rad/s at r = 12 cm

m = 2 kg, h = 2×2 = 4 cm

ω_1(minimum) = 0.96×10 π = 9.6 π rad/s.



From the above figure,

h = 40 mm, a = 100 mm, b = 90 cm

$$\frac{r-r_1}{a} = \frac{\frac{h}{2}}{b} \Rightarrow r_1 = r - \frac{ah}{2b}$$

$$\Rightarrow r_1 = 120 - \frac{100\times 20}{90} = 97.78 \text{ mm} = 9.778 \text{ cm}$$

$$F = mr\omega^2 = 2\times0.12\times(10\pi)^2 = 236.87 \text{ N}$$

$$F_1 = mr_1\omega_1^2 = 2\times(0.0978)\times(9.6\pi)^2 = 177.9 \text{ N}$$

Stiffness

$$S = 2\times\left(\frac{a}{b}\right)^2\left[\frac{F-F_1}{r-r_1}\right]$$

$$= 2\times\left(\frac{100}{90}\right)^2\left[\frac{236.87-177.9}{0.12-0.0978}\right]$$

S = 6558.78 N/m

For the maximum speed position.

$$\frac{r_2-r_1}{a} = \frac{h}{b} \Rightarrow r_2 = r+\frac{ah}{b}$$

$$r_2 = 9.778 + \frac{10\times4}{9} = 14.22\text{cm}$$

and we know that

$$S = 2\times\left(\frac{a}{b}\right)^2\left[\frac{F_2-F_1}{r_2-r_1}\right]$$

$$\Rightarrow F_2 = F_1 + \frac{(r_2-r_1)S}{2}\times\left(\frac{b}{a}\right)^2$$

$$= 177.9 + \frac{(14.22-9.778)\times10^{-2}\times6558.78}{2}\times\left(\frac{9}{10}\right)^2$$

$\Rightarrow F_2 = 295.89$ N

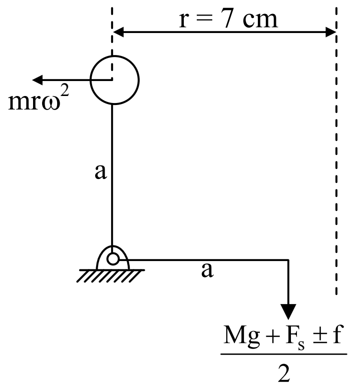
But $F_2 = mr_2\,\omega_2^2 = 295.89$

$\omega_2 = 32.25$ rad/sec

$N_2 = 307.96$ rpm

02.

Sol:



Mass of sleeve, M = 5 kg , N = 500 rpm

$\omega = \frac{2\pi \times 500}{60}$ = 52.36 rad/sec,

f = 30 N

At equilibrium friction force = 0

At r = 7 cm, f = 0

Maximum speed, with friction

ω_2 =1.05 ω = 1.05 ×52.36 = 54.98 rad/s

at minimum speed, with friction

ω_1 = 0.95ω = 0.95×52.36 = 49.74 rad/s

Also, at mid position, the change in speed of ± 1% gives the governor effort sufficient to overcome friction.

Therefore, maximum speed at mid position,

ω_2 = 1.01ω = 1.01×52.36 = 52.88 rad/s

And, minimum speed at mid position

ω_1 = 0.99ω = 0.99×52.36 = 51.84 rad/s

Also, for maximum speed at mid position,

$$(W + f) + F_S = 2F_1 \times r \times \frac{a}{b}$$

$$\Rightarrow 5\times9.81+30+F_S = 2\times m\times52.88^2\times0.07\times1$$

$$79.05+F_{S1} = 391.48 \quad \text{-------------(1)}$$

And for minimum speed at mid position

$$(W - f) + F_S = 2F_2 \times r \times \frac{a}{b}$$

$$(5\times9.81-30)+F_S = 2\times m\times(51.84)^2\times0.07\times1$$

$$19.05+F_S = 376.223 \text{----------(2)}$$

Solving equ. (1) and equ. (2)

$$m = \frac{60}{391.48 - 376.223} = 3.94 \text{ kg}$$

Thus, mass of each ball = 3.94 kg

(ii) Also, maximum speed with friction

$$(W+f)+F_{S2} = 2F_2\times\frac{a}{b}$$

$$(5\times9.81+30)+F_{S2} = 2\times3.94\times(54.98)^2\times\frac{\left(7+\frac{2}{2}\right)\times1}{100}$$

$$F_{s2} = 1826.52 \text{ N}$$

Also, minimum speed with friction

$$(W - f) + F_{S1} = 2F_1 \times \frac{a}{b}$$

$$(5\times9.81-30)+F_{S1} = 2\times3.94\times(49.74)^2\times\frac{\left(7-\frac{2}{2}\right)\times1}{100}$$

$$F_{s1} = 1150.69 \text{ N}$$

$$\text{Spring rate} = \frac{1826.52 - 1150.69}{0.02}$$

= 33.79 N/mm

(iii) Initial compression of the spring

$$\frac{F_{s1}}{k} = \frac{1150.69}{33.79} = 34\,\text{mm} = 3.4\text{cm}$$

(iv) Governor effort for 1 % change in speed

$$= c\,(W+\frac{F_{s1}+F_{s2}}{2})$$

$$= 0.01\times\left(5\times9.81+\frac{1826.52+1150.69}{2}\right)$$

= 15.38 N

(v) Power of the governor

$$= 15.38 \times 0.02 = 0.308$$

03.

Sol: Weight of each ball 'w' = 40 N

Mass of each ball $= \dfrac{40}{9.81} = 4.077\,\text{kg}$

$r_1 = 10$ cm and $r_2 = 17.5$ cm

$F_{c1} = 205$ N and $F_{c2} = 400$ N

Let $F_c = ar + b$

When $r_1 = 10$ cm = 0.1m and $F_{c1} = 205$ N

$205 = b + 0.1a$

When

$r_2 = 17.5$cm = 0.175m and $F_{c2} = 400$N

$400 = b + 0.175a$

$\therefore\ 195 = 0.075a \Rightarrow a = 2600$

$\therefore\ b = 205 - 0.1\times2600 = -55$

$\therefore\ F_c = -55 + 2600\,r$ (stable governor)

(a) For $F_c = 205$;

$$\frac{40}{g}\left(\frac{2\pi N_1}{60}\right)^2 \times 0.1 = 205\text{N}$$

$N_1 = 214.1$rpm

For $F_c = 400$; $r = 0.175$m

$$\therefore\ \frac{40}{g}\left(\frac{2\pi N_2}{60}\right)^2 \times 0.175 = 400$$

$N_2 = 226.1$rpm

At $r = 0.15$m

$F_c = -55 + 2600\times0.15 = 335$N

$335 = 4.077 \times 0.15 \times \omega^2$

$\omega = 23.404$ rad/sec

(b) Let ω_1' and ω_2' be the maximum and minimum angular velocity of the governor considering the friction.

$$F + f = m(\omega_1')^2 \times r$$

$335 + 2.5 = 4.077 \times (\omega_1')^2 \times 0.15$

$\omega_1' = 23.492$ rad/sec

$$F - f = m(\omega_2')^2 \times r$$

$335 - 2.5 = 4.077 \times (\omega_2')^2 \times 0.15$

$\omega_2' = 23.317$ rad/se

$\therefore$ Coefficient of insensitiveness

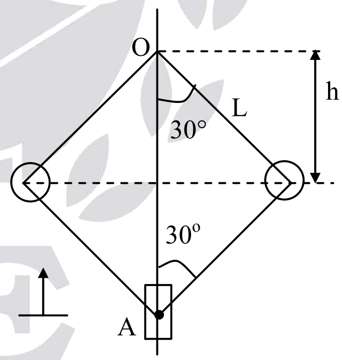
$$= \frac{(\omega_1' - \omega_2')}{\omega} = \frac{23.492 - 23.317}{23.404}$$

$$= 7.477 \times 10^{-3} = 0.747\ \%$$

04.

Sol: $m = 0.5$ kg , $M = 2$ kg

At lowest position:

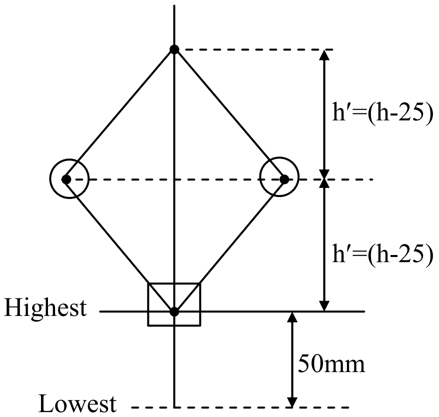


$\alpha = \beta$

$k = \dfrac{\tan\beta}{\tan\alpha} = 1$;

$$h = \frac{(mg + Mg + F_f)}{m\omega^2}$$

$$250\cos30^\circ = \frac{mg + Mg + F_f}{m\omega^2}$$



$$250\cos 30^\circ - 25 = \frac{mg + Mg - F_f}{m\omega^2}$$

$$\frac{250\cos 30^\circ}{250\cos 30^\circ - 25} = \frac{mg + Mg + F_f}{mg + Mg + -F_f}$$

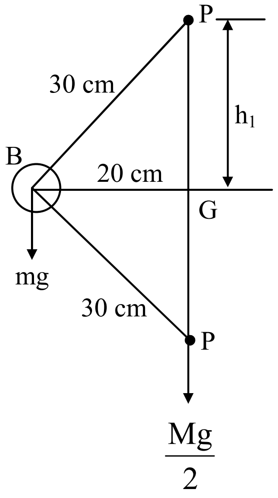
$F_f = 1.5N$

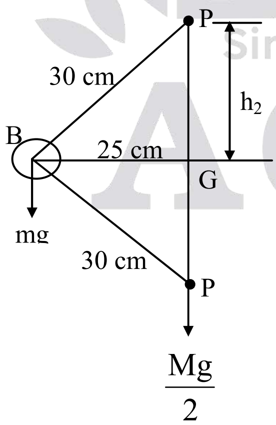
05.

Sol: Given, BP = BD = 30 cm,

$m = 7$ kg, $M = 54$ kg,

$r_1 = 20$ cm, $r_2 = 25$ cm





Let,

N_1 = minimum speed when r_1 = BG = 20 cm,

N_2 = minimum speed when r_2 = BG = 25 cm,

Height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$$

$$= \sqrt{30^2 - 20^2} = 22.36 \text{ cm}$$

we know that,

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{7+54}{7} \times \frac{895}{\left(\frac{22.36}{100}\right)}$$

$N_1 = 186.8$ rpm

Height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2}$$

$$= \sqrt{30^2 - 25^2} = 16.58 \text{ cm}$$

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{7+54}{7} \times \frac{895}{\left(\frac{16.58}{100}\right)}$$

$N_2 = 216.89$ rpm

Range of speed of governor,

$= N_2 - N_1 = 216.89 - 186.8 = 30.1$ rpm

06.

Sol: Given data,

a = b, d = 140 mm,

N = 500 rpm, h = 30 mm,

M = 5 kg, f = 20 N

$$\omega = \frac{2\pi \times 500}{60} = 52.36 \text{ rad/s}$$

Assuming power of governor is sufficient to overcome friction by 1% change in speed on each side of mid position.

(i) Considering the friction at the mid position,

$$m\,r\,\omega_1^2\,a = \frac{1}{2}\left(Mg + F_s + f\right)b$$

$$m\times\left(\frac{0.140}{2}\right)\times\left(52.36\times1.01\right)^2 = \frac{1}{2}\left(\left(5\times9.81\right)+F_s+20\right)$$

(a = b) -----(1)

$$mr\omega_2^2\,a = \frac{1}{2}\left(Mg + F_s - f\right)b$$

$$m\times\left(\frac{0.140}{2}\right)\times\left(52.36\times0.99\right)^2 = \frac{1}{2}\left(5\times9.81+F_s-20\right)$$

-------------(2)

Subtracting (1) from (2)

$$m\times0.07\times\left(52.36\right)^2\left[\left(1.01\right)^2-\left(0.99\right)^2\right] = \frac{1}{2}\times\left(20+20\right)$$

$\Rightarrow$ m = 2.605 kg

(ii) In the extreme positions,

$$mr_2\omega_2^2a = \frac{1}{2}\left(Mg + F_{s2} + f\right)b$$

$$2.605\times\left(0.07+\frac{0.03}{2}\right)\times\left(52.36\times1.05\right)^2 = \frac{1}{2}\left(5\times9.81+F_{s2}+20\right)$$

(a = b)

F_{S2} = 1269.5 N

$$mr_1\omega_1^2a = \frac{1}{2}\left(Mg + F_{s1} - f\right)b$$

$$2.605\times\left(0.07+\frac{0.03}{2}\right)\times\left(52.36\times0.95\right)^2 = \frac{1}{2}\left(5\times9.81+F_{s2}-20\right)$$

F_{s1} = 639.95 N

$h_1s = F_{s2} - F_{s1}$

0.03 × s = 1269.5 – 639.95

s = 20985 N/m (or) 20.98 N/mm

(iii) Initial compression $= \dfrac{F_{s1}}{s} = \dfrac{639.95}{20.98}$

= 30.50 mm

Chapter 5 Balancing

01. Ans: (c)

Sol: unbalanced force (F_{un}) $\propto$ $mr\omega^2$

Unbalance force is directly proportional to square of speed. At high speed this force is very high. Hence, dynamic balancing becomes necessary at high speeds.

02. Ans: (a)

Sol: Dynamic force $= \dfrac{W}{g}e\,\omega^2$

Couple $= \dfrac{W}{g}e\,\omega^2\,a$

Reaction on each bearing $= \pm\dfrac{W}{g}e\,\omega^2\,\dfrac{a}{l}$

Total reaction on bearing

$$= \left(\frac{W}{g}e\,\omega^2\,\frac{a}{l}\right) - \left(\frac{W}{g}e\,\omega^2\,\frac{a}{l}\right) = 0$$

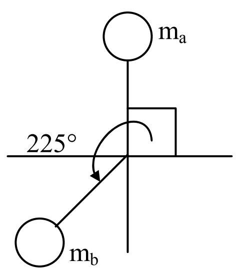
03. Ans: (b)

Sol: Since total dynamic reaction is zero the system is in static balance.

04. Ans: (a)

05. Ans: (b)

Sol:



m_a = 5 kg, r_a = 20 cm

m_b = 6 kg, r_b = 20 cm

m_c = ? , r_c = 20 cm

m_d = ?, θ_c = ? , θ_d = ?

Take reference plane as ‘C’

For complete balancing

$\sum mr = 0$ & $\sum mrl = 0$

$2m_d\cos\theta_d - 9\sqrt{2} = 0$

$\Rightarrow m_d\cos\theta_d = 9\sqrt{2}$

$2m_d\sin\theta_d - 5 - 9\sqrt{2} = 0$

$m_d\sin\theta_d = = \frac{1}{2}\left(5+9\sqrt{2}\right)$

$$m_d = \sqrt{\left(\frac{9}{\sqrt{2}}\right)^2 + \left[\frac{1}{2}\left(5+9\sqrt{2}\right)\right]^2} = 10.91\text{kg}$$

$$\theta_d = \tan^{-1}\left[\frac{\frac{1}{2}\left(5+9\sqrt{2}\right)}{\frac{9}{\sqrt{2}}}\right] = 54.31^0$$

$= 90 - 54.31 = 35.68$ w.r.t ‘A’

$m_c\cos\theta_c + m_d\cos\theta_d - 3\sqrt{2} = 0$

$\Rightarrow m_c\cos\theta_c + 10.91\cos 54.31 - 3\sqrt{2} = 0$

$m_c\cos\theta_c = -2.122$

$m_c\sin\theta_c + m_d\sin\theta_d - 3\sqrt{2} + 5 = 0$

$m_c\sin\theta_c + 10.91\sin 54.31 - 3\sqrt{2} + 5 = 0$

$m_c\sin\theta_c = -9.618$

$$m_c = \sqrt{\left(-2.122\right)^2 + \left(-9.618\right)^2} = 9.85\text{kg}$$

$$\tan\theta_c = \frac{-9.618}{-2.122}$$

$\theta_c = 257.56$ or $257.56 - 90$ w.r.t ‘A’

$= 167.56$

S.No	m	(r×20)cm	(l×20)cm	θ	mrcosθ	mrsinθ	mrlcosθ	mrlsinθ
A	5	1	–1	90	0	5	0	–5
B	6	1	3	225	$-3\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-9\sqrt{2}$
C	m_c	1	0	θ_c	$m_c\cos\theta_c$	$m_c\sin\theta_c$	0	0
D	m_d	1	2	θ_d	$m_d\cos\theta_d$	$m_d\sin\theta_d$	$2m_d\cos\theta_d$	$2m_d\sin\theta_d$

Common data Q. 06 & 07

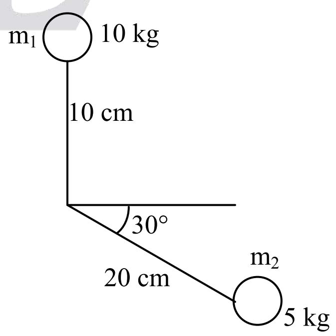
06. Ans: (a)

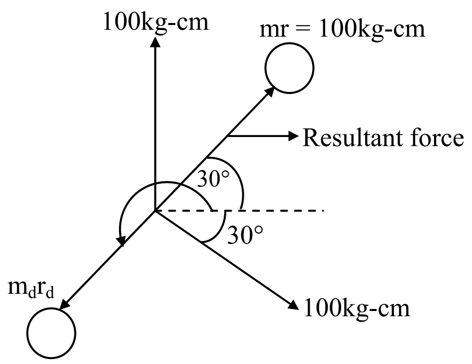
Sol: m_1 = kg , m_2 = 5kg , r_1 = 10cm

r_2 = 20cm, m_d = ?, r_d = 10cm

m_1r_1 = 100 kg cm

m_2r_2 = 100kg cm





Keep the balancing mass m_d at exactly opposite to the resultant force

$\therefore m_dr_d = 100$kg-cm

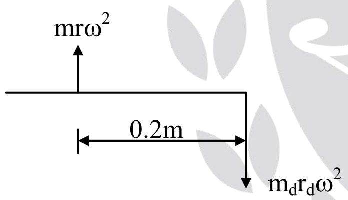
$\Rightarrow m_d \times 10 = 100$ kg-cm

$m_d = 10$kg cm

$\theta_d = 180 + 30 = 210$

07. Ans: (d)

Sol:



mr = 100kg-cm = 1kgm

N = 600 rpm $\Rightarrow \omega = \dfrac{2\pi N}{60} = 20\pi \text{rad/s}$

Couple 'C' = $mr\omega^2 \times 0.2 = 1\times(20\pi)^2\times0.2$

$= 789.56$ Nm

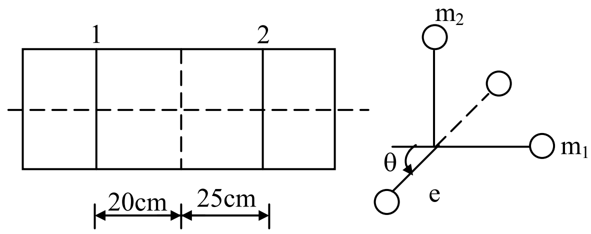
Reaction on the bearing

$$= \frac{\text{couple}}{\text{distance between bearing}}$$

$$= \frac{789.56}{0.4} = 1973.92\text{N}$$

08. Ans: (a)

Sol:



$r_1 = 10$ cm, $r_2 = 10$ cm, $m_1 = 52$ kg

$m_2 = 75$ kg, $\theta_1 = 0$ (Reference)

$\theta_2 = 90°$, m = 2000kg , e = ?, θ = ?

$me\cos\theta = m_1r_1 = 520$

$me\sin\theta = m_2r_2 = 750$

$$me = \sqrt{(m_1r_1)^2 + (m_2r_2)^2} = \sqrt{520^2 + 750^2}$$

$$= 913 \text{ kg} - \text{cm}$$

$$e = \left(\frac{913}{2000}\right) = 0.456\text{cm}$$

$$\theta = \tan^{-1}\left(\frac{m_2r_2}{m_1r_1}\right) = \tan^{-1}\left(\frac{75}{52}\right) = 55.26^0$$

$= 180 + 55.26 = 235.26°$

w.r.t mass '1'.

09. Ans: (a)

Sol:

Plane	m (kg)	r (m)	L (m) (reference Plane A)	θ	F_x (mrcosθ)	F_y (mrsinθ)	C_x (mr*l*cosθ)	C_y (mr*l*sinθ)
D	2 kg.m		0.3	0	2	0	0.6	0
A	$-m_a$	0.5m	0	θ_a	$-0.5m_a\cos\theta_a$	$-0.5m_a\sin\theta_a$	0	0
B	$-m_b$	0.5m	0.5	θ_b	$-0.5m_b\cos\theta_b$	$-0.5m_b\sin\theta_b$	$-\dfrac{m_b}{4}\cos\theta_b$	$-\dfrac{m_b}{4}\sin\theta_b$

$$C_x = 0 \Rightarrow \frac{m_b\cos\theta_b}{4} = 0.6$$

$$C_y = 0 \Rightarrow \frac{m_b\sin\theta_b}{4} = 0$$

$\Rightarrow m_b = 2.4\text{kg}$, $\theta_b = 0$

$\Sigma F_x = 0$

$\Rightarrow 2 - 0.5\, m_a\cos\theta_a - 0.5\, m_b\cos\theta_b = 0$

$$\Rightarrow \frac{m_a}{2}\cos\theta_a = 0.8$$

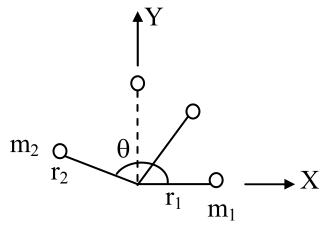
$$\Sigma F_y = 0 \Rightarrow \frac{m_a}{2}\sin\theta_a = 0$$

$\therefore \theta_a = 0°$, $m_a = 1.6$ kg

(Note: mass is to be removed so that is taken as –ve).

10. Ans: (a)

Sol:



$$\frac{F_x}{\omega^2} = m_1r_1 + m_2r_2\cos\theta$$

$= 20 \times 15 + 25 \times 20 \cos 135$

$= -53.55$ gm-cm

$$\frac{F_y}{\omega^2} = m_2r_2\sin\theta_2 = 25\times 20\sin 135$$

$= 353.553$ gm-cm

$$m_br_b = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$\Rightarrow m_b = \frac{\sqrt{F_x^{\,2} + F_y^{\,2}}}{r_b}$$

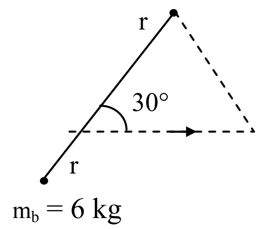
$$= \frac{\sqrt{(-53.55)^2 + (353.553)^2}}{20} = 17.88$$

gm

$$\theta_b = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{353.553}{-53.55}\right) = 98.7°$$

11. Ans: 30 N

Sol:



Crank radius

= stroke/2 = 0.1 m,

ω = 10 rad/sec

Unbalanced force along perpendicular to the line of stroke = $m_b r\omega^2 \sin 30°$

$= 6 \times (0.1) \times (10)^2 \sin 30°$

$= 30$ N

12. Ans: (b)

Sol:

- Primary unbalanced force = $mr\omega^2\cos\theta$

  At θ = 0° and 180°, Primary force attains maximum.

  Secondary force = $\frac{mr\omega^2}{n}\cos 2\theta$ where n is obliquity ratio. As n > 1, primary force is greater than secondary force.

- Unbalanced force due to reciprocating mass varies in magnitude. It is always along the line of stroke.

13. Ans: (b)

Sol: In balancing of single-cylinder engine, the rotating balance is completely made zero and the reciprocating unbalance is partially reduced.

14. Ans: (b)

Sol: m = 10 kg, r = 0.15 m ,

c = 0.6 , θ = 60° , ω = 4 rad/sec

Residual unbalance along the line of stroke

$= (1 - c)\, m r\omega^2 \cos\theta$

$= (1 - 0.6)\times 10 \times 0.15 \times 4^2 \cos 60$

= 4.8 N

15. Ans: 2

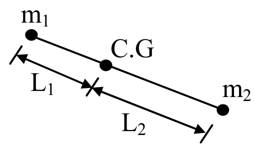
Sol: By symmetric two system is in dynamic balance when

$mea = m_1e_1a_1$

$$m_1 = m\frac{e}{e_1}.\frac{a}{a_1} = 1\times\frac{50}{20}\frac{2}{2.5} = 2\text{kg}$$

16. Ans: (a)

Sol:



$$m_1 = \frac{mL_2}{L_1 + L_2} = \frac{100\times 60}{100} = 60\text{kg}$$

$$m_2 = \frac{mL_1}{L_1 + L_2} = \frac{100\times 40}{100} = 40\text{kg}$$

$I = m_1L_1^2 + m_2L_2^2$

$= 60 \times 40^2 + 40\times 60^2$

= 240000 kg cm^2

= 24 kg m^2

17. Ans: (d)

Sol: For primary forces balances

$\sum r \cos\theta_i = 0$

Sl.No.	θ	$\cos\theta$
1	α	$\cos\alpha$
2	$180 + \beta$	$\cos(180 + \beta)$
3	$180 - \beta$	$\cos(180 - \beta)$
4	$360 - \alpha$	$\cos(360 - \alpha)$

$\therefore \sum m\cos\theta_i = R_1\cos\alpha + R_2\cos(180+\beta) + R_2\cos(180-\beta) + R_1\cos(360-\alpha) = 0$

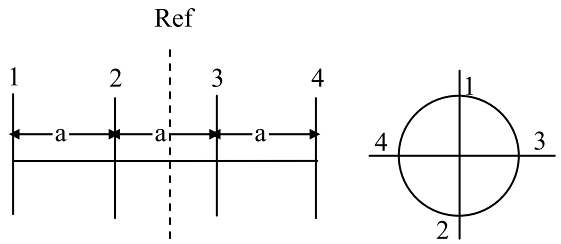
$= R_1\cos\alpha - R_2\cos\beta - R_2\cos\beta + R_1\cos\alpha = 0$

$\therefore 2R_1\cos\alpha = 2R_2\cos\beta$

$\therefore R_1\cos\alpha = R_2\cos\beta$

18. Ans: (a)

Sol:



	θ	2θ	L
1	0	0	$\frac{3a}{2}$
2	180	360	$\frac{a}{2}$
3	270°	540° = 360+180	$-\frac{a}{2}$
4	90°	180°	$-\frac{3a}{2}$

Resultant primary unbalanced force is given by

$$\sum\vec{F}_P = mr\omega^2\cos 0° + mr\omega^2\cos 180 + mr\omega^2\cos 270 + mr\omega^2\cos 90 = 0$$

Resultant secondary unbalanced force is given by

$$\sum\vec{F}_S = \frac{mr\omega^2\cos 0}{n} + \frac{mr\omega^2\cos 360}{n} + \frac{mr\omega^2\cos(360+180)}{n} + \frac{mr\omega^2\cos(180)}{n}$$

$= 0$

$\therefore$ All primary and secondary forces are balanced.

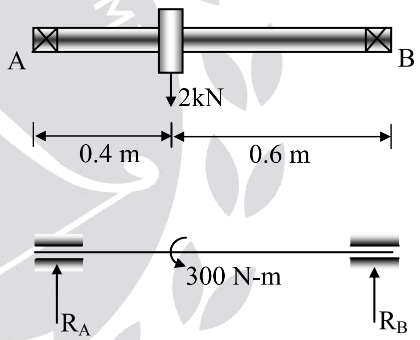
19. Ans: (d)

Sol:

- For primary direct crank total unbalanced mass is $\frac{3W}{2g}$. Therefore primary direct force is equal to $\frac{3W}{2g}r\omega^2$.
- As primary reverse crank is balanced, primary reverse force is equal to zero.
- Primary direct crank speed is ω.
- Primary reverse crank speed is equal and opposite to the primary direct crank speed.

20. Ans: (d)

Sol:



If the shaft is statically balanced then Reactions due to unbalanced couple

$\therefore R_A \times 1\text{ m} = 300\text{ N-m}$

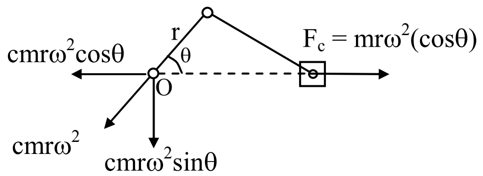
$\therefore R_A = 300\text{ N}$

$R_B = -300\text{ N}$

21. Ans: (a)

22. Ans: (a)

Sol:



Let c be the fraction of reciprocating mass

Primary force balance by the mass

$= cmr\omega^2\cos\theta$

Unbalance force along Line of action

$= (1 - c)mr\omega^2\cos\theta$

Unbalance force perpendicular to line of action $= cmr\omega^2\sin\theta$

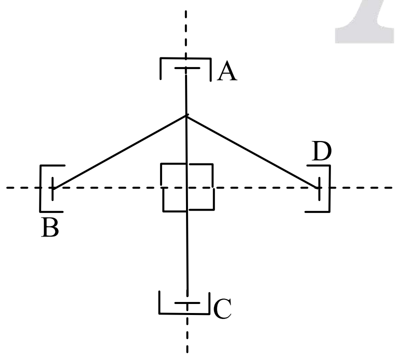
$\therefore$ Resultant unbalanced force

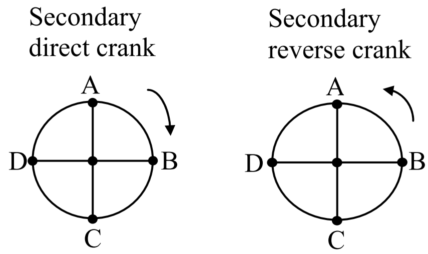
$$= \sqrt{(1-c)^2 mr\omega^2\cos^2\theta + c^2\sin^2\theta(mr\omega^2)}$$

Resultant unbalanced force is minimum when $c = \frac{1}{2}$. But common practice is to use $\frac{2}{3}$ of the reciprocating mass to minimize the effect of unbalanced force along line of stroke.

23. Ans: (a)

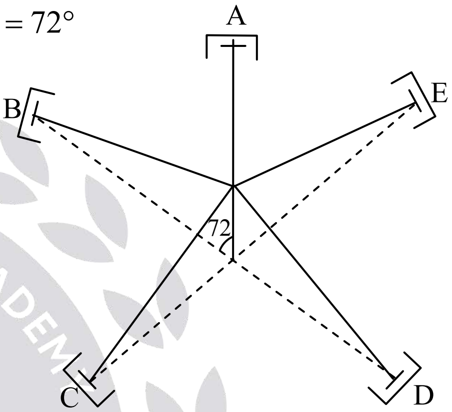
Sol: For Four cylinders :

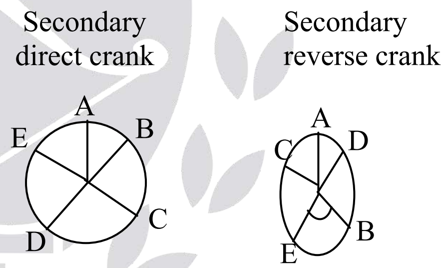




For five cylinder

$\theta = \frac{2\pi}{5} = 72°$





So both secondary direct and reverse cranks are balanced completely.

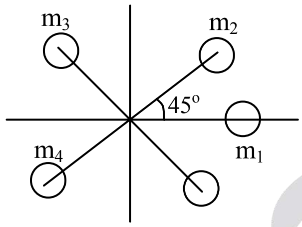
Conventional Practice Solutions

01.

Sol: **Note:** All masses are in same plane

$\therefore \sum mr = 0$

$\therefore$ Static balance is required



S.no	m	r	θ	mrcosθ	mrsinθ
1.	160	0.2	0	32	0
2	225	0.2	45°	31.82	31.82
3	220	0.3	75+45 =120	–30	51.96
4	312	0.25	135+45+75 =225	–20.19	–75.34
5	B	0.25	θ	0.25Bcosθ	0.25Bsinθ

$\sum F_y = 0$

$\therefore$ 0.25B sinθ + 8.44 = 0

0.25Bsinθ = – 8.44 …..(1)

$\sum F_x = 0$

$\therefore$ 0.25Bcosθ + 13.63 = 0

0.25Bcosθ = – 13.63 …..(2)

3. $$\frac{eq(1)}{eq(2)} = \frac{0.25B\sin\theta}{0.25B\cos\theta} = \frac{-8.44}{-13.63}$$

$$\therefore \tan\theta = \frac{-8.44}{-13.63}$$ (θ lies in 3rd quadrant)

$\therefore$ θ = 180 + 31.76

θ = 211.76°

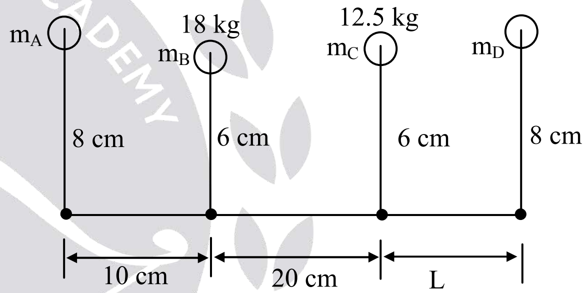
4. Substituting θ in eq (1)

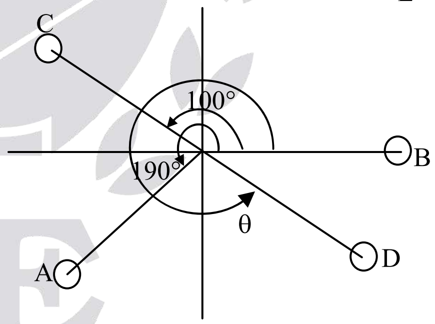
0.25B sin(211.76) = – 84

Balancing mass, B = 64.13 kg

02.

Sol:





S.No	m	r (cm)	θ	L	mrcosθ	mrsinθ	mrLcosθ	mrLsinθ
1	m_A	8	190	30+L	–7.87 m_A	–1.38 m_A	–7.87 m_A (L+30)	–1.39 m_A (L+30)
2	18	6	0	20+L	108	0	108(L+20)	0
3	12.5	6	100	L	–13.02	73.86	–13.02L	73.86L
4	m_D	8	θ	0	8 m_D cosθ	8 m_D sinθ	0	0

Note: Assuming the plane (D) having maximum unknowns as the reference plane

$\sum M_y = 0$

$-1.39\, m_A\,(L + 30) + 73.86L = 0$

$-1.39\, m_A\,(L + 30) = -73.86L$ ….(1)

$\sum M_x = 0$

$-7.87\, m_A\,(L + 30) + 108(L + 20) - 13.02L = 0$

$-7.87\, m_A\,(L+30) = 13.02L - 108(L + 20)$…(2)

$$\frac{eq(1)}{eq(2)} = \frac{-1.39}{-7.87} = \frac{-73.86L}{13.02L - 108(L+20)}$$

$2.299L - 19.072L - 381.456 = -73.86L$

$\therefore$ $L = 6.683$ cm

Substituting in equation (1)

$-1.39\, m_A\,(6.683 + 30) = -73.86 \times 6.683$

$\therefore\ m_A = 9.68$ kg

$-7.87 \times 9.68 + 108 - 13.02 + 8\, m_D \cos\theta = 0$

$-76.89 + 108 - 13.02 + 8\, m_D \cos\theta = 0$

$8\, m_D \cos\theta = -18.8$ …..(3)

$\Sigma F_y = 0$

$-1.39 \times 9.68 + 73.86 + 8\, m_D \sin\theta = 0$

$8\, m_D \sin\theta = -60.4$ …..(4)

$$\frac{eq(4)}{eq(3)} = \frac{8m_D \sin\theta}{8m_D \cos\theta} = \frac{-60.4}{-18.8}$$

$\theta = 72.71$

$\theta = 180 + 72.71 = 252.7$

(θ lies in 3rd quadrant)

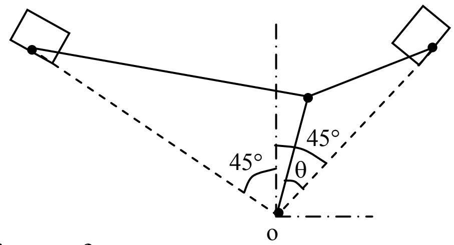
Substitute in (3)

$8\, m_D \cos(252.7) = -18.8$

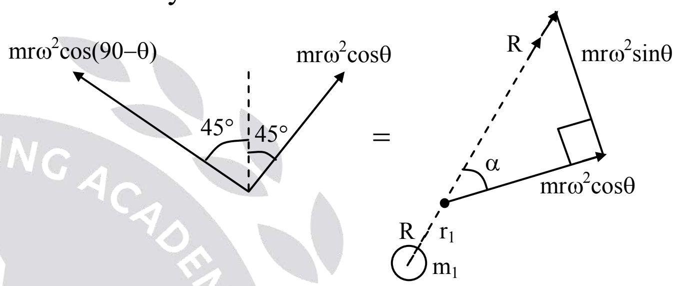
$\Rightarrow$ $m_D = 7.9$ kg

03.

Sol: $\omega = \dfrac{2\pi N}{60} = 251.2\,\text{rad/s}$



Primary forces:



Resultant primary forces,

$$R = \sqrt{(mr\omega^2 \sin\theta)^2} + \sqrt{(mr\omega^2 \cos\theta)^2} = mr\omega^2$$

Given,

$m = 1.5$kg, $2r = 0.14$m, $l = 0.28$m

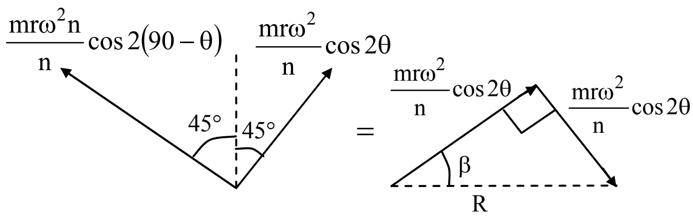
$\therefore$ $R = 1.5 \times 0.07 \times (251.2)^2 = 6625.65$ N

$$\tan\alpha = \frac{mr\omega^2 \sin\theta}{mr\omega^2 \cos\theta} = \tan\theta$$

$\alpha = \theta$

$\therefore$ primary forces can be balanced by keeping a mass m_1 at a distance r_1 such that $m_1r_1 = m_2r_2$

Secondary forces :



Secondary force,

$$R_S = \sqrt{\left(\frac{mr\omega^2}{n}\cos 2\theta\right)^2 + \left(\frac{mr\omega^2}{n}\cos 2\theta\right)^2}$$

$$= \sqrt{2}.\frac{mr\omega^2}{n}\cos 2\theta$$

$\tan\beta = 1 \Rightarrow \beta = 45°$

As R_S magnitude is varying and direction is not varying so it can't be balanced.

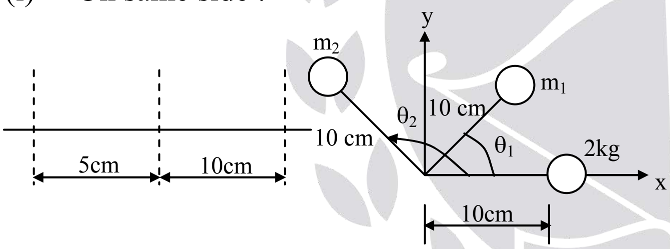
$$n = \frac{\ell}{r} = \frac{28}{7} = 4$$

$$\therefore\ (R_S)_{max} = \sqrt{2}\,\frac{mr\omega^2}{n} = 2342.5\,N$$

04.

Sol:

(i) On same side :



$\sum mr\cos\theta = 0$

$2\times10 + 10\times m_1\cos\theta_1 + 10\times m_2\cos\theta_2 = 0$

$\sum mr\sin\theta = 0$

$m_1\cos\theta_1 + m_2\cos\theta_2 = -2$ ------- (1)

$m_1\sin\theta_1 + m_2\sin\theta_2 = 0$ -------- (2)

$\sum mrl\cos\theta = 0$

$0 + m_1\times10\times5\cos\theta_1 + m_2\times10\times15\cos\theta_2 = 0$

$5m_1\cos\theta_1 + 15m_2\cos\theta_2 = 0$

$m_1\cos\theta_1 + 3m_2\cos\theta_2 = 0$ -----(3)

$\sum mrl\sin\theta = 0$

$m_1\sin\theta_1 + 3m_2\sin\theta_2 = 0$ -----(4)

From (3) & (4)

$\tan\theta_1 = \tan\theta_2$

$\theta_1 = \theta_2$ or $(\pi + \theta_2)$

Take $\theta_1 = \theta_2$

$m_1 = -3m_2$ is not valid

$\therefore\ \theta_1 = \pi + \theta_2$

$m_1 = 3m_2$

From (2)

$(m_1 - m_2)\sin\theta_2 = 0$

$\theta_2 = 0, \pi$

From (1)

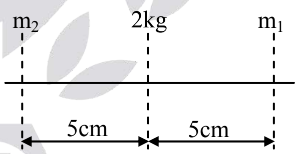
$(m_1 - m_2)\cos\theta_2 = -2$

$2m_2\cos\theta_2 = -2$

$\therefore\ \theta_2 = 180°,\ m_2 = 1$ kg, $m_1 = 3$kg

$\therefore\ \theta_2 = 180°,\ \theta_1 = 360°$ or $0°$

(ii) On opposite sides of crank :



$\sum mrl\cos\theta = 0$ about crank plane

$m_1\cos\theta_1\,5 - m_2\cos\theta_2\,5 = 0$ -----(5)

$m_1\sin\theta_1\,5 - m_2\sin\theta_2\,5 = 0$ ------(6)

From (5) & (6)

$\tan\theta_1 = \tan\theta_2$

$\theta_1 = \theta_2$ or $\pi + \theta_2$

Take $\theta_1 = \theta_2$

$(m_1 - m_2)\cos\theta_1 = 0$ ------(7)

$(m_1 + m_2)\cos\theta_1 = -2$ ---------(8)

From (7) & (8)

$\therefore\ m_1 - m_2 = 0$

$\Rightarrow m_1 = m_2$

$m_1\cos\theta_1 = -1$

$2m_1\sin\theta_1 = 0$ (from (2))

$\therefore\ \theta_1 = \pi$,

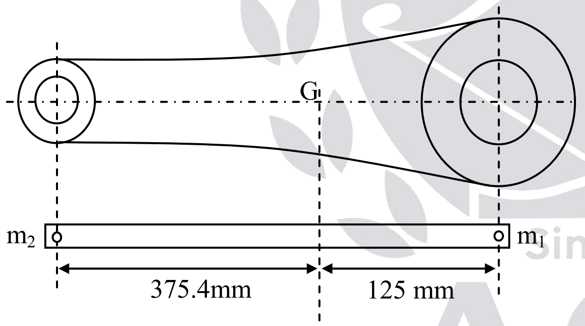
$\theta_2 = \pi$,

$m_1 = 1$ kg,

$m_2 = 1$ kg

05.

Sol:



L = 500 mm,

m = 18 kg,

r = 100 mm

$L_1 = 375$ mm ,

$L_2 = ?$

$L_1 L_2 = k^2$

Radius of gyration = k

$$f = \frac{1}{2\pi}\sqrt{\frac{gL}{(k^2+L^2)}} \text{ cycle/sec}$$

$$f = 43 \text{ cycle/min} = \frac{43}{60}\text{cycle/sec}$$

$$\frac{43}{60} = \frac{1}{2\pi}\sqrt{\frac{9.81\times 0.375}{k^2+(0.375)^2}}$$

k = 0.202 m = 202 mm

$L_1 L_2 = k^2$

$$L_2 = \frac{(202)^2}{L_1} = \frac{(202)^2}{375} = 108.8\text{mm}$$

$$m_1 = \frac{mL_2}{L_1+L_2} = \frac{18\times 108.8}{108.8+375} = 4.05\text{kg}$$

$$m_2 = \frac{mL_1}{L_1+L_2} = \frac{18\times 375}{108.8+375} = 13.95\text{kg}$$

Chapter 6

Cams

01. Ans: (d)

Sol: Pressure angle is given by

$$\tan\phi = \frac{\frac{dy(\theta)}{d\theta} - e}{y(\theta) + \sqrt{(r_p)^2 - (e)^2}}$$

where, ϕ is pressure angle ,
θ is angle of rotation of cam
e is eccentricity
r_p is pitch circle radius
y is follower displacement

02. Ans: (d)

Sol: Cycloidal motion

$$y = \frac{h}{2\pi}\left(\frac{2\pi\theta}{\phi} - \sin\left(\frac{2\pi}{\phi}\theta\right)\right)$$

$$\dot{y}_{max} = \frac{2h\omega}{\phi} \qquad \text{------(1)}$$

Simple harmonic motion :

$$\dot{y}_{max} = \left(\frac{\pi}{2}\frac{h\omega}{\phi}\right) \qquad \text{------(2)}$$

Uniform velocity :

$$\dot{y} = \frac{h\omega}{\phi} \qquad \text{------(3)}$$

From (1), (2) and (3) we observe that

$V_{cyclodial} > V_{SHM} > V_{UV}$

03. Ans: (b)

04. Ans: (b)

Sol: L = 4 cm , $\phi = 90^o = \pi/2$ radian ,

$\omega = 2$ rad/sec , $\theta = \frac{2}{3}\times 90 = 60°$

$$\frac{\theta}{\phi} = \frac{2}{3}$$

$$s(t) = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\phi}\right)$$

$$= 2(1 - \cos 120) = 3\text{cm}$$

$$V(t) = \frac{L}{2}\times\frac{\pi}{\phi}\times\omega\times\sin\left(\frac{\pi\theta}{\phi}\right)$$

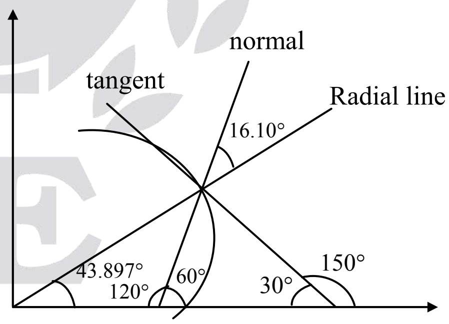
$$= \frac{4}{2}\times 2\times 2\sin(120) = 7\text{cm/s}$$

$$a(t) = \frac{L}{2}\left(\frac{\pi}{\phi}\right)^2\times\omega^2\times\cos\left(\frac{\pi\theta}{\phi}\right)$$

$$= \frac{4}{2}\times 2^2\times 2^2\times\cos(120) = -16\text{cm/sec}^2$$

05. Ans: (b)

Sol:



x = 15cosθ ,

y = 10 + 5sinθ

$$\tan\phi = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\left(\frac{dx}{d\theta}\right)} = \frac{5\cos\theta}{-15\sin\theta}$$

at $\theta = 30°$,

$$\tan\phi = \frac{5\times\frac{\sqrt{3}}{2}}{-15\times\frac{1}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow \phi = 150°$$

$$\tan\theta = \frac{y}{x} = \frac{10+5\sin\theta}{15\cos\theta} = \frac{10+5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

Pressure angle is angle between normal and radial line = 16.10° .

or $x = 15\cos\theta$,

$y = 10 + 5\sin\theta$ at $\theta = 30^o$

$$\left(\frac{x}{15}\right)^2 + \left(\frac{y-10}{5}\right)^2 = 1$$

$$x = \frac{15\sqrt{3}}{2}\ , \quad y = 125$$

$$\frac{2x}{15^2} + \frac{2(y-10)}{5^2}.\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{(y-10)9} = \frac{-15\sqrt{3}}{2\left(\frac{3}{2}\right)\times 9} = \frac{-1}{\sqrt{3}}$$

$$\tan\theta = \frac{-1}{\sqrt{3}}$$

Then normal makes with x-axis

$\tan^{-1}(\sqrt{3}) = 60^o$

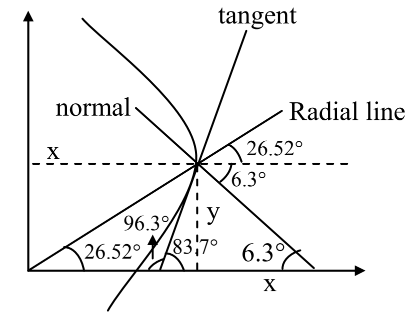
$$\tan\theta = \frac{y}{x} = \frac{10+5\sin\theta}{15\cos\theta} = \frac{10+5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

With follower axis angle made by normal (pressure angle) = 60°–43.897° = 16.10^o

06. Ans: (a)

Sol:



Let α be the angle made by the normal to the curve

$$\left(\frac{dy}{dx}\right)_{(4,2)} = 9$$

$$\tan\alpha = \frac{dy}{dx} = 4x - 7$$

At $x = 4$ & $y = 2$,

$\alpha = \tan^{-1}(9) = 83.7°$

The normal makes an angle

$$= \tan^{-1}\left(\frac{-1}{9}\right) = 6.3° \text{ with x axis}$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.52°$$

Pressure angle is angle between normal and radial line = 26.52 + 6.3 = 32.82°

07. Ans: (b)

Sol: For the highest position the distance between the cam center and follower

= (r + 5) mm

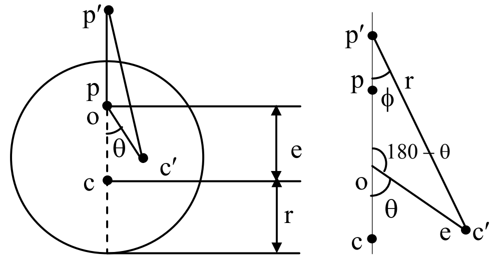
For the lowest position it is (r – 5) mm

So the distance between the two positions

= (r + 5) – (r – 5) = 10 mm

08. Ans: (a)

Sol:



When ‘c’ move about ‘o’ through ‘θ’, point ‘p’ moves to p′. ‘ϕ’ is angle between normal drawn at point of contact which always passes through centre of circle and follower axis. So this is pressure angle.

From Δle $p'oc'$

$$\frac{r}{\sin(\pi-\theta)} = \frac{e}{\sin\phi}$$

$$\sin\phi = \frac{e}{r}\sin\theta$$

ϕ is maximum $\theta = 90°$

$$\sin\phi = \frac{e}{r}$$

Pressure angle s maximum at pitch point

$$\phi = \sin^{-1}\left(\frac{e}{r}\right) = 30°$$

09. Ans: (c)

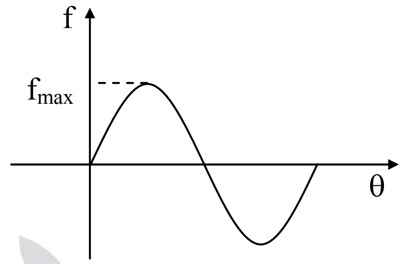
Sol: For a cycloidal motion displacement is given by

$$s = \frac{h}{\pi}\left[\frac{\pi\theta}{\phi} - \frac{1}{2}\sin\frac{2\pi\theta}{\phi}\right]$$

Velocity, $v = \dfrac{ds}{dt} = \dfrac{h\omega}{\phi}\left(1-\cos\dfrac{2\pi\theta}{\phi}\right)$

Acceleration, $f = \dfrac{dv}{dt} = \dfrac{2h\pi\omega^2}{\phi}\sin\dfrac{2\pi\theta}{\phi}$

$\therefore$ Shape of acceleration curve is a sine curve as shown below:



10. Ans: (b)

Sol: By providing offset in a radial cam translating follower pressure angle is decreased during ascent of the follower.

let, ϕ is pressure angle ,

θ is angle of rotation of cam

e is eccentricity

r_p is pitch circle radius

y is follower displacement

11. Ans: (d)

Sol: The cam in contact with a follower is case of successful constraint.

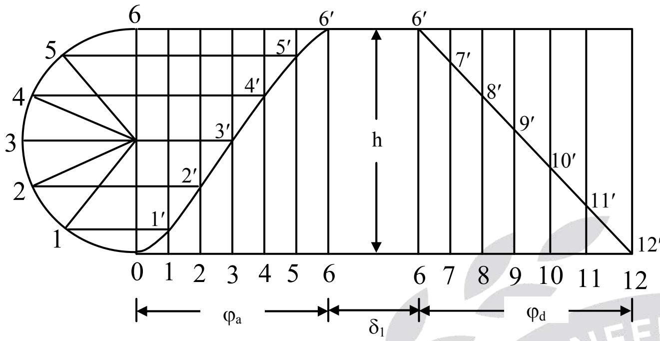
12. Ans: (a)

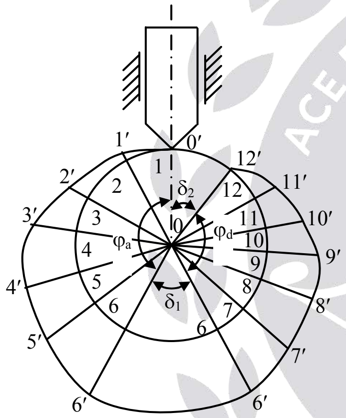
Sol: By providing offset to the follower, pressure angle decreases. As a result side thrust reduces and prevents jamming of follower in its guide. Wear between follower and cam surface also decreases.

Conventional Practice Solutions

01.

Sol:





h = 30 mm $\varphi_a = 150°$

N = 120 rpm $\delta_1 = 60°$

$r_c = 20$ mm $\varphi_d = 100°$

$\delta_2 = (360° - 150° - 100° - 60°) = 50°$

Procedure:

Draw the displacement diagram of the follower as discussed earlier taking a convenient scale. Construct the cam profile as follows refer fig.

(i) Draw a circle with radius r_c

(ii) If the cam rotates clockwise and the follower remains in vertical direction, the cam profile can be drawn by assuming that the cam is stationary and the follower rotates about the cam in the counter-clockwise direction.

From the vertical position, mark angles φ_a, δ_1, φ_d, and δ_2 in the counter-clockwise direction, representing angles of ascent, rest or dwell, descent and rest respectively.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, each has been divided into 6 equal parts.

(iv) Draw radial lines O-1, O-2, O-3 etc, O-1 represents that after an interval of $\varphi_d/6$ of the cam rotation in the clockwise direction it will take the vertical position of O-O′.

(v) On the radial lines produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e., 1–1′, 2–2′, 3–3′, etc.

(vi) Draw a smooth curve passing through O′, 1′, 2′,……10′, 11′ and 12′. Draw an arc of radius O–6′ for the dwell period δ_1.

During ascent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57\ \text{rad/s}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a}$$

(or)

$$v_{max} = \frac{30}{2} \times \frac{\pi \times 12.57}{150 \times \frac{\pi}{180}} = 226.3\,\text{mm/s}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{(or)}$$

$$f_{max} = \frac{30}{2} \times \left(\frac{\pi \times 12.57}{150 \times \frac{\pi}{180}}\right)$$

$$= 3413\ \text{mm/s}^2 \text{ or } 3.413\ \text{m/s}^2$$

During descent

$$v_{max} = h\frac{\omega}{\varphi_d}$$

$$v_{max} = 30 \times \frac{12.57}{100 \times \frac{\pi}{180}} = 216\,\text{mm/s}$$

$f_{max} = f = 0$

Note that to draw the cam profile, it is not necessary that the interval δ_1 is taken in the displacement diagram. Also, the scales of φ_a and φ_d can be taken different and of any magnitudes.

02.

Sol: e = 30 mm,

m = 3 kg,

s = 5 N/mm = 5000 N/m,

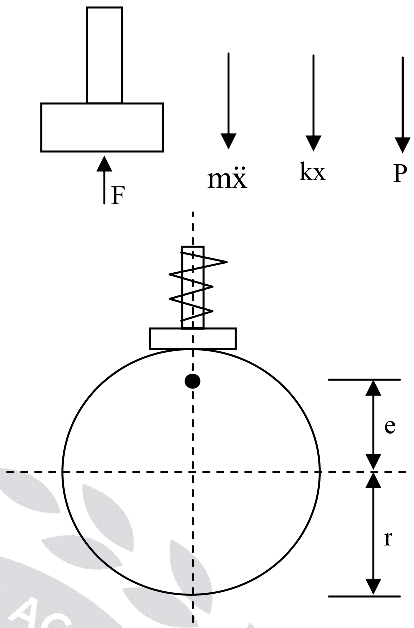
δ = 12 mm

Spring force = 5×12 = 60 N

P = 60 N + mg

= (60 + 3 × 9.81) N = 89.43 N

Considering free body diagram of follower



Force excreted by cam

$$F = m\ddot{x} + kx + P$$

To make, F = 0

$$m\ddot{x} + kx + P = 0$$

On taking , $x = e - e\cos\omega t$

We have equation

$$(se + P) + (m\omega^2 - s)\, e\cos\omega t = 0$$

When ωt = 180° jump may occur at a particular speed

$$(se + P) = e(m\omega^2 - s)$$

$$\omega = \sqrt{\frac{2se + P}{me}} = 63.89\ \text{rad/sec}$$

$$= \sqrt{\frac{2 \times 5 \times 30 + 89.43}{3 \times \frac{30}{100}}} = 65.78\ \text{rad/s}$$

$$\therefore\ N = \frac{60\omega}{2\pi} = 628\,\text{rpm}$$

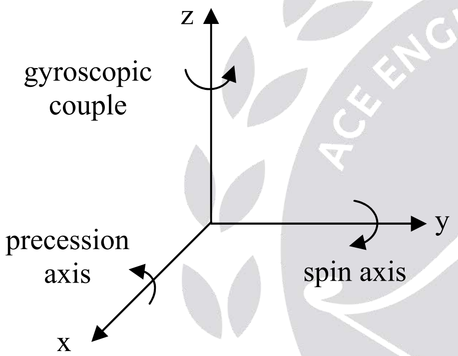
Chapter 7 Gyroscope

01. Ans: (c)

Sol: Due to Gyroscopic couple effect and centrifugal force effect the inner wheels tend to leave the ground.

02. Ans: (d)

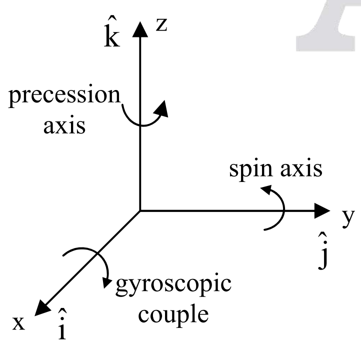
Sol: Pitching is angular motion of ship about transverse axis.



Due to pitching gyroscopic couple acts about vertical axis.

03.

Sol: m = 1000 kg, r_k = 200 mm



$I = 1000 \times (0.2)^2 = 40$ kg-m^2

N = 5000 rpm (CCW) looking from stern

$$\omega = \frac{2\pi \times 5000}{60} = 523.33 \text{ rpm}$$

$$\vec{\omega} = -523.33\ \hat{j}$$

Precession velocity

$$\omega_p = \frac{V}{r} = \frac{25 \times 0.514}{400} = 0.032125 \text{ rad/s}$$

$$\vec{\omega}_p = 0.0312\ \hat{k}$$

$$\text{Gyroscopic couple} = I(\vec{\omega} \times \vec{\omega}_p)$$

$$G = 40(-523.33\,\hat{j} \times 0.032125\,\hat{k})$$

$$= -672\ \hat{i} \text{ N-m}$$

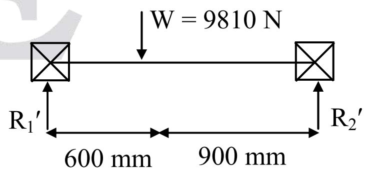
Now,

$$R_1 = -R_2 = \frac{M}{L} = -\frac{672}{1.5} = 448\,\text{N}$$

R_1 = 448 N (Acting downwards)

R_2 = 448 N (Acting upwards)

Now reaction due to weight



$$R_1' = \frac{9810 \times 900}{1500} = 5886\,\text{N (upwards)}$$

$$R_2' = \frac{9810 \times 600}{1500} = 3924\,\text{N (upwards)}$$

Total bearing reaction at A

$= R_A + R_A'$

$= 5886 - 448 = 5438$ N

Total bearing reaction at B

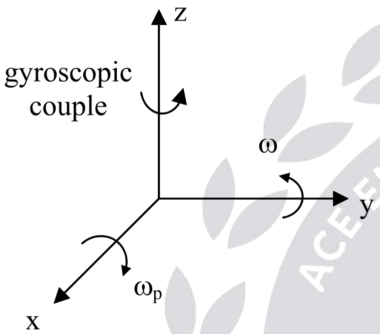
$= R_B + R_B'$

$= 3924 + 448 = 4372$ N

Bow falls and stern rises.

04.

Sol:



k = 220 mm, m = 210 kg

$I = 210 \times (0.22)^2 = 10.164$ kg-m^2

$$\omega = \frac{2\pi \times 1800}{60} = 1884.95 \text{ rad/s}$$

$$\omega_p = \frac{1200 \times \frac{5}{18}}{3800} = 0.0877 \text{ rad/s}$$

$M = I\,\omega\,\omega_p$

$= 10.164 \times 0.0877 \times 1884.95$

$= 1681$ N-m

05. Ans: 200

Sol: R = 100 m, v = 20 m/sec,

$$\omega_p = \frac{V}{R} = 0.2\frac{\text{rad}}{\text{sec}} \qquad \omega_s = 100 \text{ rad/sec}$$

I = 10 kg-m^2

Gyroscopic moment

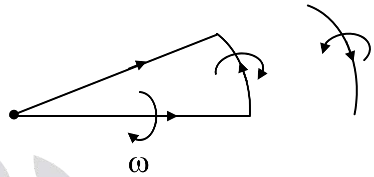
$$= I\omega_s\omega_p = 10 \times 0.2 \times 100 \text{N-m}$$

$$= 200 \text{N-m}$$

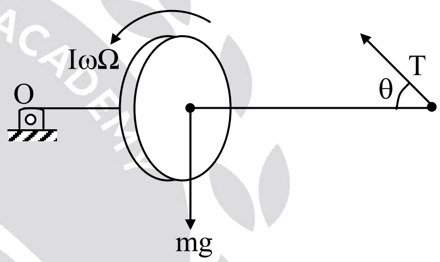
06. Ans:

Sol:

(i)



The gyroscopic couple is = IωΩ



$\sum M_o = 0$

$2a.T\sin\theta + I\omega\Omega = mg \times a$

$$\frac{2a.T.b}{\sqrt{4a^2+b^2}} + \frac{mr^2}{2}\omega\Omega = mg \times a$$

$$T = \frac{\sqrt{4a^2+b^2}}{2ab}\left(mga - \frac{mr^2}{2}\omega\Omega\right)$$

For clockwise rotation of precession

(ii) $\sum M_o = 0$

$2a.T\sin\theta - I\omega\Omega = mg \times a$

$$T = \frac{\left(mga + \frac{1}{2}mr^2\omega\Omega\right)\left(b^2+4a^2\right)^{\frac{1}{2}}}{2ab}$$

Conventional Practice Solutions

01.

Sol: Let V = limiting velocity of the vehicle.

Angular velocity, $\omega = \frac{V}{r} = \frac{V}{0.3}\text{rad/s}$

Precession velocity = $\omega_p = \frac{V}{R} = \frac{V}{120}\text{rad/s}$

(i) Reaction due to gyroscopic couple.

(a) Gyroscopic couple due to four wheels:

$C_w = 4I_\omega\omega\omega_p$

$= 4\times2.2\times\frac{V}{0.3}\times\frac{V}{120} = 0.25V^2\text{N.m}$

(b) Gyroscopic couple due to engine parts:

$C_e = I_eG\omega\omega_p$

$= 1.25\times3.2\times\frac{V}{0.3}\times\frac{V}{120} = 0.11\ V^2\text{N.m}$

Total gyroscopic couple:

$C_g = C_w+C_e = 0.25\ V^2 + 0.11$

$= 0.36\ V^2\ \text{N.m}$

Reaction due to total gyroscopic couple on each outer wheel:

$$R_g = \frac{C_g}{2b} = \frac{0.36V^2}{2\times1.6} = 0.1125V^2\text{N}\ (\uparrow)$$

Reaction due to total gyroscopic couple on each inner wheel = $0.1125V^2\text{N}\ (\downarrow)$

(ii) Reaction due to centrifugal couple:

Centrifugal force,

$$F_c = \frac{mV^2}{R} = \frac{2050\times V^2}{120} = 17.083V^2\text{N}$$

Overturning couple due to centrifugal force

$C_c = F_c\times h$

$= 17.083V^2\times0.52 = 8.883\ V^2\text{N.m}$

Vertical downward reactions on each inner wheel is,

$$R_c = \frac{C_c}{2b} = \frac{8.883V^2}{2\times1.6} = 2.776\ V^2\text{N}\ (\downarrow)$$

(iii) Reaction due to weight of the vehicle

$$R_w = \frac{mg}{4} = \frac{2050\times9.81}{4} = 5027.625\ \text{N}\ (\uparrow)$$

The limiting condition to avoid lifting of the inner wheels from the road surface is

$R_i = R_w - R_c - R_g > 0$

$R_w > R_c + R_g$

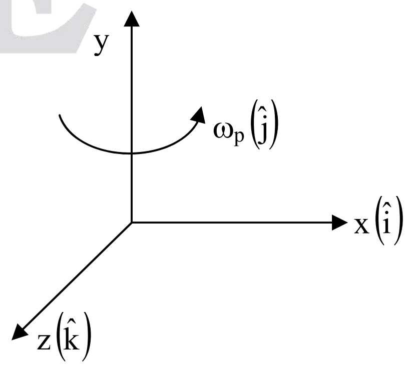
$5027.625 \geq 2.776V^2+0.1125V^2$

$V = 41.72$ m/s

$= 41.72\times\frac{18}{5} = 150.19$ km/hr

02.

Sol:



$I = mk^2$,

$m = 6000$ kg ,

$k = 0.45$ m ,

$\omega = 2400$ rpm

$= 80\pi$ rad/sec = 251.2 rad/sec

$$\omega_p = \frac{18\times1860}{60\times3600} = 0.155\,\text{rad/sec}$$

$$\omega_p = \frac{V}{R}\hat{j}$$

(i) Gyroscope couple

$$C_g = \vec{H}\times\vec{\omega}_p$$

$$= I\omega\times\omega_p\left(\hat{i}\times\hat{j}\right) = mk^2\omega\times\omega_p\times\hat{k}$$

$$= 6000\times0.45^2\times251.2\times0.155$$

$$= 47.3\ \hat{k}\ \text{kN-m}$$

Bow portion is raised.

(ii) Pitching amplitude, $A = 7.5°$

$\alpha = A \sin\omega t$

$\tau = 18$ sec ,

$$f = \frac{1}{18}\text{Hz}$$

$$\omega = \frac{2\pi}{18}\text{rad/sec}$$

Maximum angular velocity of precession,

$\omega_p = A\ \omega$

$$= 7.5\times\frac{\pi}{180}\times\frac{2\pi}{18} = 0.0457\left(-\hat{k}\right)\ \text{rad/sec}$$

$$\vec{H} = I\omega\,\hat{i} = 6000\times0.45^2\times80\pi\hat{i}$$

$$= 30536.28\ \hat{i}$$

$$C_g = \vec{H}\times\vec{\omega}_p$$

$$= I\omega\times\omega_p\left(\hat{i}\times-\hat{k}\right)$$

$$= 6000\times0.45^2\times80\pi\,\hat{i}\times0.0457\left(-\hat{k}\right)$$

$$= 13.955\ \left(\hat{j}\right)\ \text{kN-m}$$

(as the bow portion is lowered, the ship turns towards left or port side)

Maximum acceleration = $A\ \omega^2$

$$= 7.5\times\frac{\pi}{180}\times\left(\frac{2\pi}{18}\right)^2\ \text{rad/sec}^2$$

$$= 0.016\ \text{rad/sec}^2$$

(iii) $\omega_{rolling} = 0.035$ rad/sec

$\omega_p = 0$ during rolling

$C_g = \vec{H}\times\vec{\omega}_p = 0$ (No gyroscope effect)

Chapter 8

Mechanical Vibrations

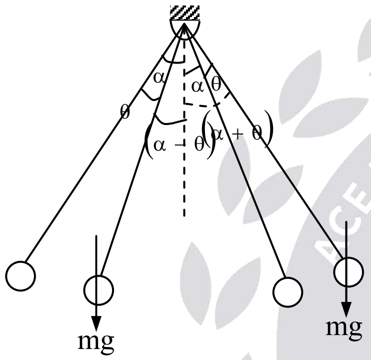
01. Ans: (b)

Sol: $T = 2\pi\sqrt{\dfrac{L}{g}} \Rightarrow 0.5 = 2\pi\times\sqrt{\dfrac{L}{9.81}}$

$\Rightarrow \quad L = 62.12$ mm

02. Ans: (d)

Sol:



Let the system is displaced by θ from the equilibrium position. It's position will be as shown in figure.

By considering moment equilibrium about the axis of rotation (Hinge)

$$I\ddot{\theta} + mg\ell\sin(\alpha+\theta) - mg\ell\sin(\alpha-\theta) = 0$$

$$I = m\ell^2 + m\ell^2 = 2m\ell^2$$

After simplification

$$2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha\ \sin\theta = 0$$

For small oscillations (θ is small) $\sin\theta = \theta$

$$\therefore 2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha.\theta = 0$$

$$\omega_n = \sqrt{\frac{2mg\ell\cos\alpha}{2m\ell^2}} = \sqrt{\frac{g\cos\alpha}{\ell}}$$

03. Ans: (c)

Sol: Let, V_o is the initial velocity,

'm' is the mass

Equating Impulse = momentum

$$mV_o = 5kN\times10^{-4}\text{ sec}$$

$$= 5\times10^3\times10^{-4} = 0.5\text{ sec}$$

$$\therefore V_0 = \frac{0.5}{m} = 0.5\text{ m/sec}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10000}{1}} = 100\text{ rad/sec}$$

When the free vibrations are initiate with initial velocity,

The amplitude

$$X = \frac{V_0}{\omega_n}\text{ (Initial displacement)}$$

$$\therefore X = \frac{V_0}{\omega_n} = \frac{0.5\times10^3}{100} = 5\text{ mm}$$

04. Ans: (a)

Sol: **Note:** ω_n depends on mass of the system not on gravity

$$\therefore \omega_n \propto \frac{1}{\sqrt{m}}$$

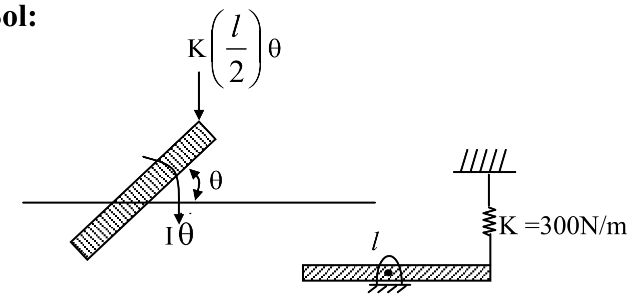
$$\text{If } \omega_n = \sqrt{\frac{g}{\delta}}, \quad \delta = \frac{mg}{K}$$

$$\therefore \omega_n = \sqrt{\frac{g}{\left(\frac{mg}{K}\right)}} = \sqrt{\frac{K}{m}}$$

$\therefore\ \omega_n$ is constant every where.

05. Ans: (c)

Sol:



By energy method

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}Kx^2 = \text{constant}$$

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}K\times\left(\frac{\ell}{2}\theta\right)^2 = \text{constant}$$

Differentiating w.r.t 't'

$$\frac{dE}{dt} = I\ddot{\theta} + \frac{K}{2}\times\frac{\ell^2}{4}\times 2\theta = 0$$

$$I = \frac{m\ell^2}{12}$$

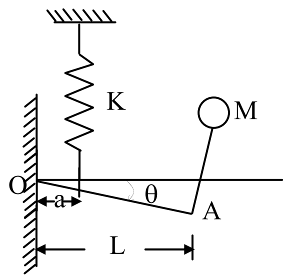
$$\frac{m\ell^2}{12}\ddot{\theta} + \frac{K\ell^2}{4}\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{3K}{m}\theta = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{3K}{m}} = 30\,\text{rad/sec}$$

06. Ans: (a)

Sol:



Assume that in equilibrium position mass M is vertically above 'A'. Consider the displaced position of the system at any instant as shown above figure.

If Δ_{st} is the static extension of the spring in equilibrium position, its total extension in the displaced position is $(\Delta_{st} + a\theta)$.

From the Newton's second law, we have

$$I_0\ddot{\theta} = Mg(L + b\theta) - k(\Delta_{st} + a\theta)a \quad ...(1)$$

But in the equilibrium position

$MgL = k\Delta_{st}\,a$

Substituting the value in equation (1), we have $I_0\ddot{\theta} = (Mgb - ka^2)\theta$

$$\Rightarrow I_0\ddot{\theta} + (ka^2 - Mgb)\theta = 0$$

$$\omega_n = \sqrt{\frac{ka^2 - Mgb}{I_0}}$$

$$\tau = 2\pi\sqrt{\frac{I_0}{ka^2 - Mgb}}$$

The time period becomes an imaginary quantity if $ka^2 < Mgb$. This makes the system unstable. Thus the system to vibrate the limitation is

$$ka^2 > Mgb$$

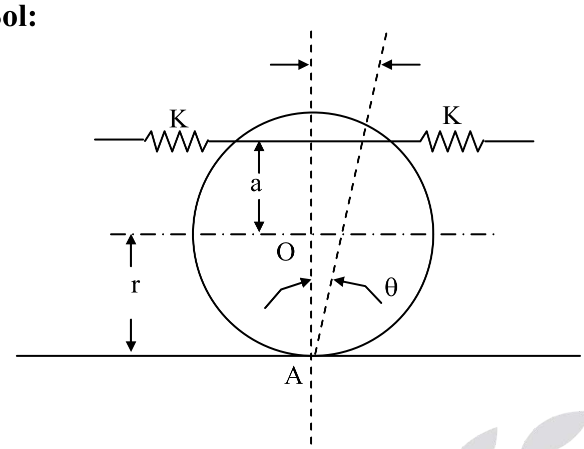
$$b < \frac{ka^2}{Mg}$$

Where W = Mg

07. Ans: (a)

08.

Sol:



$$KE = \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2$$

$$= \frac{3}{4}mr^2\dot{\theta}^2$$

$$PE = \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2 = Kx^2$$

$$x = (r + a)\theta$$

$$\Rightarrow PE = K\{(r + a)\theta\}^2$$

$$\frac{d}{dt}KE + \frac{d}{dt}PE = 0$$

Substituting in the above equation

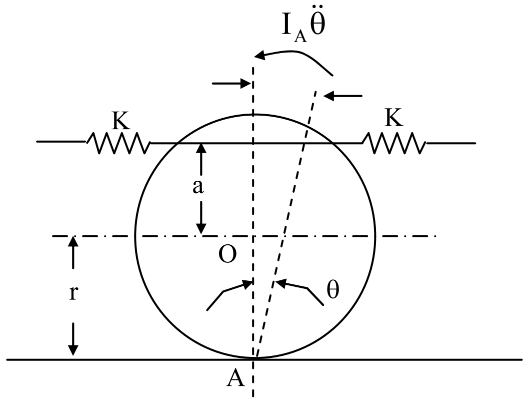
$$\frac{3}{2}mr^2\ddot{\theta} + 2K(r + a)^2\theta = 0$$

Natural frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{4K(r+a)^2}{3mr^2}}$$

So $f_n = 47.74$ Hz.

OR



Taking the moment about the instantaneous centre 'A'.

$$I_A\ddot{\theta} + 2K(r+a)\,\theta\,(r+a) = 0$$

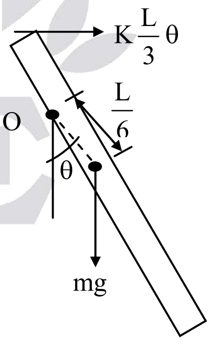
$$I_A = \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

$$\frac{3}{2}mr^2\ddot{\theta} + 2k(r + a)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{2k(r+a)^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$$

09. Ans: (b)

Sol:



By considering the equilibrium about the pivot 'O'

$$I_O\ddot{\theta} + mg\times\frac{L}{6}\sin\theta + K\frac{L}{3}\theta\times\frac{L}{3} = 0$$

$$\frac{mL^2}{9}\ddot{\theta}+\left(mg\times\frac{L}{6}+\frac{KL^2}{9}\right)\theta=0 \quad (\because \sin\theta \approx \theta)$$

$$\omega_n=\sqrt{\frac{mg\times\frac{L}{6}+\frac{KL^2}{9}}{\frac{mL^2}{9}}} \Rightarrow \omega_n=\sqrt{\frac{3g}{2L}+\frac{K}{m}}$$

10. Ans: (d)

Sol: $X_0 = 10$ cm, $\omega_n = 5$ rad/sec

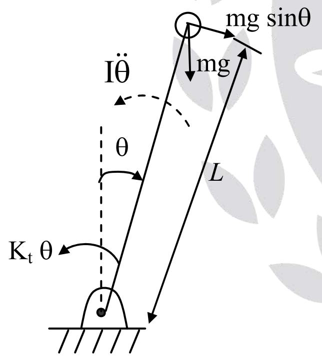
$$X=\sqrt{x_0^2+\left(\frac{v_0}{\omega_n}\right)^2}$$

If $v_0 = 0$ then $X = x_0$

$\therefore X = x_0 = 10$cm

11. Ans: (c) & 12. Ans: (c)

Sol:



$I = mL^2$

The equation of motion is

$$mL^2\ddot{\theta}+(k_t-mgL)\theta=0$$

Inertia torque = mL^2

Restoring torque = $k_t - mgL\sin\theta$

$= (k_t - mgL)\theta$

13. Ans: 0.0658 N.m^2

Sol: For a Cantilever beam stiffness, $K=\frac{3EI}{\ell^3}$

Natural frequency, $\omega_n=\sqrt{\frac{K}{m}}=\sqrt{\frac{3EI}{m\ell^3}}$

Given $f_n = 100$ Hz

$\Rightarrow \omega_n = 2\pi f_n = 200\pi$

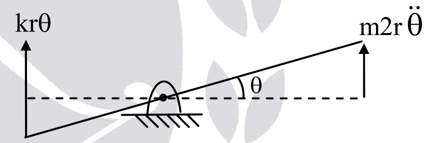
$$200\pi=\sqrt{\frac{3EI}{m\ell^3}}$$

Flexural Rigidity

$$EI=\frac{(200.\pi)^2.m\ell^3}{3}=0.0658 \text{ N.m}^2$$

14. Ans: (d)

Sol: Free body diagram



Moment equilibrium about hinge

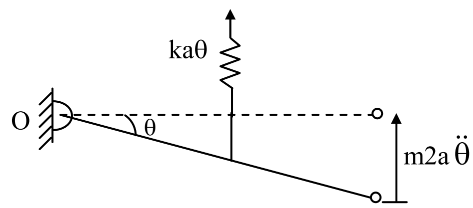
$$m2r\ddot{\theta}.2r+k\theta.r=0$$

$$4mr^2\ddot{\theta}+kr^2\theta=0$$

$$\omega_n=\sqrt{\frac{kr^2}{4mr^2}}=\sqrt{\frac{k}{4m}}=\sqrt{\frac{400}{4}}$$

15. Ans: (a)

Sol:



By taking the moment about 'O', $\Sigma m_o = 0$

$(m2a\ddot{\theta} \times 2a) + (ka\theta \times a) = 0$

$\Rightarrow 4a^2 m\ddot{\theta} + ka^2\theta = 0$

Where, $m_{eq} = 4a^2m$, $k_{eq} = ka^2$

Natural frequency, $\omega_n = \sqrt{\dfrac{k_{eq}}{m_{eq}}}$

$$= \sqrt{\frac{ka^2}{4a^2m}} = \sqrt{\frac{k}{4m}}\frac{rad}{sec}$$

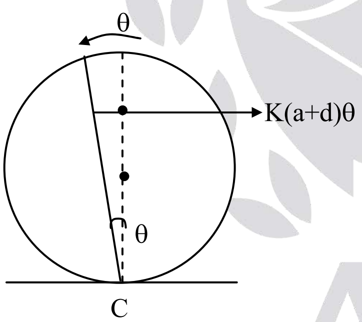
$[\because \omega_n = 2\pi f]$

$$\Rightarrow f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \times \sqrt{\frac{k}{4m}}\,Hz$$

16. Ans: (a)

Sol: Moment equilibrium above instantaneous centre (contact point)

$$-k(a+d)\theta.(a+d) = I_c\ddot{\theta}$$



$I_c = \dfrac{3}{2}Ma^2$,

$$\omega_a = \sqrt{\frac{k(a+d)^2}{\frac{3}{2}Ma^2}}$$

$$\omega_n = \sqrt{\frac{2k(a+d)^2}{3Ma^2}}$$

17. Ans: 10 (range 9.9 to 10.1)

Sol: $KE = \dfrac{1}{2}m\dot{x}^2 + \dfrac{1}{2}I\dot{\theta}^2$

$m = 5$ kg, $\theta = \dfrac{x}{r}$

$I = \dfrac{20 \times r^2}{2} = 10r^2$

$KE = \dfrac{1}{2}5\dot{x}^2 + \dfrac{1}{2}10r^2.\dfrac{\dot{x}^2}{r^2} = \dfrac{1}{2}(15)\dot{x}^2$

$\therefore m_{eq} = 15$

$PE = \dfrac{1}{2}kx^2$

$\therefore k_{eq} = k = 1500N/m$

Natural frequency

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{1500}{15}} = 10rad/sec$$

18. Ans: (b)

Sol: In damped free vibrations the oscillatory motion becomes non-oscillatory at critical damping.

Hence critical damping is the smallest damping at which no oscillation occurs in free vibration

19. Ans: (a)

Sol: $\omega_n = 50$ rad/sec $= \sqrt{\dfrac{5}{m}}$

If mass increases by 4 times

$$\omega_{n_1} = \sqrt{\frac{k}{4m}} = \frac{1}{2} \times \sqrt{\frac{k}{m}} = \frac{50}{2} = 25\,rad/sec$$

Damped frequency natural frequency,

$$\omega_d = \sqrt{1-\xi^2} \times \omega_n$$

$$\Rightarrow\ 20 = \sqrt{1-\xi^2} \times 25 = 0.6 = 60\%$$

20. Ans: (a)

Sol: $K_1, K_2 = 16$ MN/m

$K_3, K_4 = 32$ MN/m

$K_{eq} = K_1 + K_2 + K_3 + K_4$

m = 240 kg

$$\omega_n = \sqrt{\frac{K_e}{m}}$$

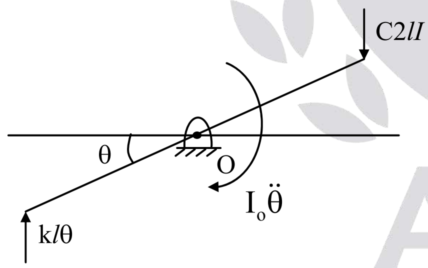
$$K_{eq} = \left((16\times 2)+(32\times 2)\right)\times 10^6 = 96\times 10^6\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{96\times 10^6}{240}} = 632.455\ \text{rad/sec}$$

$$N = \frac{\omega_n \times 60}{2\pi} = 6040\ \text{rpm}$$

21. Ans: (a)

Sol:



For slender rod, $I_o = \left[\rho \frac{x^3}{3}\right]_{-\ell}^{2\ell}$

$$= \frac{\rho}{3}\times\left(8\ell^3 + \ell^3\right) = \frac{9\rho\ell^3}{3} = 3\rho\ell^3 = m\ell^2$$

Where, $\rho = m/3l$

Considering the equilibrium at hinge 'O'.

$$I_o\ \ddot{\theta}\ + c2l\dot{\theta}\ \times 2l + kl\theta \times l = 0$$

$$\Rightarrow\ ml^2\ddot{\theta}\ + 4l^2c\ \dot{\theta} + kl^2\theta\ = 0$$

$I_{equivalent} = ml^2$, $C_{eq} = 4l^2c$, $k_{eq} = kl^2$

22. Ans: (b)

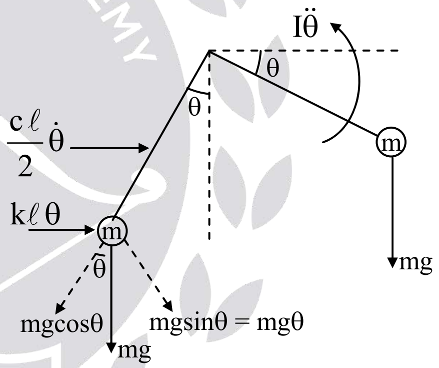
Sol: Damping ratio, $\xi = \frac{c}{c_c} = \frac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}}$

$$= \frac{4\ell^2 c}{2\times\sqrt{k\ell^2 \times m\ell^2}}$$

$$= \frac{4\ell^2 c}{2\times\sqrt{mk\ell^4}} = \frac{2c}{\sqrt{km}}$$

23. Ans: (a)

Sol:



$I = m(2l)^2 + ml^2 = 5ml^2$

The equation motion is

$$\left(m\times(2\ell)^2 + m\ell^2\right)\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$= 5m\ell^2\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{k\ell^2 + mg\ell}{5m\ell^2}}$$

$$= \sqrt{\frac{400}{5\times 10}} = 3.162\ \text{rad/s}$$

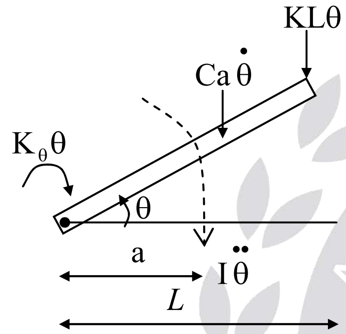
24. Ans: (a)

Sol: $$\xi = \frac{c_{eq}}{2\sqrt{k_{eq} m_{eq}}} = \frac{\left(\frac{c\ell^2}{4}\right)}{2\sqrt{(k\ell^2 + mg\ell)\times 5m\ell^2}}$$

$$= \frac{\frac{400\times 1^2}{4}}{2\sqrt{(400\times 1^2 + 10\times 9.81\times 1)\times 5\times 10\times 1^2}} = 0.316$$

25. Ans: (a)

Sol:



By moment equilibrium

$$I\ddot{\theta} + Ca^2\dot{\theta} + KL^2\theta + K_\theta\theta = 0$$

$$\frac{mL^2}{3}\ddot{\theta} + Ca^2\dot{\theta} + \left(KL^2 + K_\theta\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{KL^2 + K_\theta}{mL^2/3}}$$

$$\omega_n = \sqrt{\frac{1500}{0.833}} = 42.26 \text{ rad/sec}$$

26. Ans: (c)

Sol: Refer to the above equilibrium equation

$$C_{eq} = Ca^2$$

$$= 500\times 0.4^2 = 80\frac{\text{N}-\text{m}-\text{sec}}{\text{rad}}$$

$\Rightarrow$ C = 80 Nms/rad

Note: For angular co-ordinate

Unit of Equivalent inertia = $\frac{\text{N}-\text{m}}{\text{rad/s}^2} = \text{kg}-\text{m}^2$

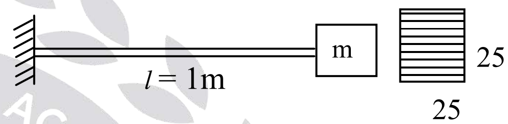
Unit of equivalent damping coefficient = $\frac{\text{N}-\text{m}}{\text{rad/s}}$

Unit of equivalent stiffness = N-m/rad

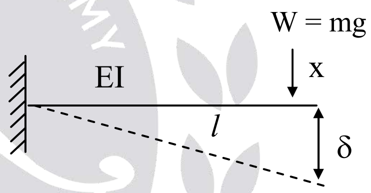
27. Ans: (a)

Sol: Given length of cantilever beam,

l = 1000 mm = 1m, m = 20 kg



Cross section of beam = square



Moment of inertia of the shaft,

$$I = \frac{1}{12}bd^3 = \frac{25\times(25)^3}{12} = 3.25\times 10^{-8}\,\text{m}^4$$

$E_{steel} = 200\times 10^9$ Pa

Mass, M = 20kg

Stiffness, $K = \frac{3EI}{\ell^3}$

Critical damping coefficient,

$$C_C = 2\sqrt{Km} = 1250\text{ Ns/m}$$

28. Ans: (c)

29. Ans: (d)

Sol: x = 10 cm at $\frac{\omega}{\omega_n} = 1$;

$\xi = 0.1$

At resonance $x = \frac{x_0}{2\xi} = 10$ cm

$\Rightarrow x_0 = 2 \times 0.1 \times 10 = 2$ cm

x_0 = static deflection

At $\frac{\omega}{\omega_n} = 0.5$,

$$x = \frac{x_0}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$x = \frac{2}{\sqrt{\left[1-(0.5)^2\right]^2 + (2\times 0.1\times 0.5)^2}} = 2.64\,\text{cm}$$

30. Ans: (a)

Sol: $m\ddot{x} + Kx = F\cos\omega t$

m = ?

K = 3000 N/m,

X = 50 mm = 0.05 m

F = 100 N,

$\omega = 100\,\text{rad/sec}$

$$X = \frac{F}{K - m\omega^2}$$

$$\Rightarrow m = \frac{K}{\omega^2} - \frac{F}{X\omega^2} = 0.1\,\text{kg}$$

31. Ans: (a)

Sol: $\delta = ln\left(\frac{x_1}{x_2}\right) = ln\,2 = 0.693$

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$= \frac{0.693}{\sqrt{4\pi^2 + 0.693^2}} = 0.109$$

$c = 2\xi\sqrt{k\,m} = 2\times 0.109 \times \sqrt{100\times 1}$

= 2.19 N-sec/m

32. Ans: (b)

Sol: x_{static} = 3mm, ω = 20 rad/sec

As $\omega > \omega_n$

So, the phase is 180°.

$$-x = \frac{x_{static}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$x = \frac{3}{\sqrt{\left(1-\left(\frac{20}{10}\right)^2\right) + \left(2\times 0.109 \times \frac{20}{10}\right)^2}}$$

= 1 mm opposite to F.

33. Ans: (c)

Sol: At resonance, magnification factor = $\frac{1}{2\xi}$

$$\Rightarrow 20 = \frac{1}{2\xi}$$

$$\Rightarrow \xi = \frac{1}{40} = 0.025$$

34. Ans: (c)

Sol: M = 100 kg, m = 20 kg, e = 0.5 mm

K = 85 kN/m, C = 0 or $\xi = 0$

$\omega = 20\pi$ rad/sec

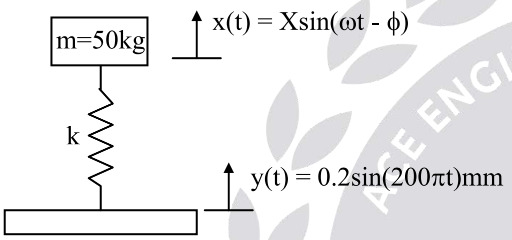
Dynamic amplitude

$$X = \frac{me\omega^2}{\pm\left(k - M\omega^2\right)} = \frac{20\times 5\times 10^{-4}\times(20\pi)^2}{\pm\left(8500 - 100\times(20\pi)^2\right)}$$

$$= 1.27\times 10^{-4}\ \text{m}$$

35. Ans:

Sol:



$\omega = 200\pi$ rad/sec, $-X = 0.01$ mm

Y = 0.2 mm

$$\frac{X}{Y} = \frac{k}{k - m\omega^2}$$

$$\Rightarrow \frac{-0.01}{0.2} = \frac{k}{k - 50\times(200\pi)^2}$$

$\Rightarrow$ k = 939.96 kN/m

36. Ans: (b)

Sol: m = 5 kg, c = 20 ,

k = 80, F= 8, $\omega = 4$

$$x = \frac{F}{\left(k - m\omega^2\right) + (c\omega)^2}$$

$$= \frac{8}{\sqrt{\left(80 - 5\times 4^2\right) + (20\times 4)^2}} = 0.1$$

$$\text{Magnification factor} = \frac{x}{x_{static}}$$

$$x_{static} = \frac{F}{k} = \frac{8}{80} = 0.1$$

$$\text{Magnification factor} = \frac{0.1}{0.1} = 1$$

37. Ans: (c)

Sol: Given, m = 250 kg

K = 100, 000 N/m

N = 3600 rpm

$\xi = 0.15$

$$\omega_n = \sqrt{\frac{K}{m}} = 20\ \text{rad /sec}$$

$$\omega = \frac{2\pi\times N}{60} = 377\ \text{rad/sec}$$

$$TR = \frac{\sqrt{1 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} = 0.0162$$

38. Ans: 10 N.sec/m

Sol: Given systems represented by

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

For which, $X = \dfrac{F}{\sqrt{\left(K - m\omega^2\right)^2 + (C\omega)^2}}$

Given, K = 6250 N/m, m = 10 kg, F = 10 N

$\omega = 25$ rad/sec, $X = 40\times 10^{-3}$

$$\omega_n = \sqrt{\frac{K}{m}} = 25\ \text{rad/sec}$$

$\omega t = 25t \Rightarrow \omega = 25$ rad/sec

$\omega = \omega_n$ or $K = m\omega_n^2$

$$\therefore X = \frac{F}{C\omega} \Rightarrow C = \frac{F}{X\omega}$$

$$= \frac{10}{40\times10^{-3}\times25} = 10\frac{N-sec}{m}$$

39. Ans: (b)

Sol: Transmissibility (T) reduces with increase in damping up to the frequency ratio of $\sqrt{2}$. Beyond $\sqrt{2}$, T increases with increase in damping

40. Ans: (c).

Sol: Because f = 144 Hz execution frequency.

f_{R_n} (Natural frequency) is 128.

$$\frac{\omega}{\omega_{R_n}} = \frac{f}{f_{R_n}} = \frac{144}{128} = 1.125$$

It is close to 1, which ever sample for which $\frac{\omega}{\omega_n}$ close to 1 will have more response, so sample R will show most perceptible to vibration

41. Ans: (b)

Sol: Given Problem of the type

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

for which, $X = \dfrac{F}{(k-m\omega^2)^2 + (c\omega)^2}$

or $$X = \frac{F/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Given F = 10, $\omega_n = 10\omega$

$k = 150\ N/m$ or $\dfrac{\omega}{\omega_n} = \dfrac{1}{10} = 0.1$

$\xi = 0.2$

$$X = \frac{10/150}{\sqrt{(1-0.1)^2 + (2\times0.2\times0.1)^2}}$$

$$= 0.0669 \simeq 0.07$$

42. Ans: 6767.7 N/m

Sol: Given f = 60 Hz, m = 1 kg

$\omega = 2\pi f = 120\pi rad/sec$

Transmissibility ratio, TR = 0.05

Damping is negligible, C = 0 , K =?

We know $TR = \dfrac{K}{K-m\omega^2}$ when C = 0

As TR is less than 1 $\Rightarrow \omega/\omega_n >> \sqrt{2}$

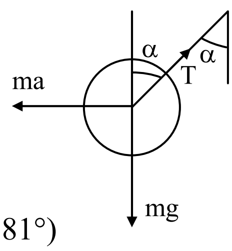
TR is negative

$$\therefore \quad -0.05 = \frac{K}{K-m\omega^2}$$

Solving we get K = 6767.7 N/m

43. Ans: (c)

Sol:



Where, a = acceleration of train

T cos α = mg

T sinα = ma

$$\tan\alpha = \frac{ma}{mg}$$

a = g tanα = 9.81 tan(9.81°)

= 1.69 m/s^2

44. Ans: (a)

45. Ans: (b)

Sol: $e = 2\text{mm} = 2\times10^{-3}\text{m}$,

$\omega_n = 10$ rad/s,

$N = 300$ rpm

$$\omega = \frac{2\pi N}{60} = 10\pi\,\text{rad/sec}$$

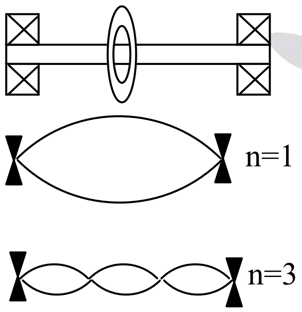
$$X = \frac{me\omega^2}{k - m\omega^2} = \frac{e\omega^2}{\left(\frac{k}{m}\right)^2 - \omega^2} = \frac{e\omega^2}{\omega_n^2 - \omega^2}$$

$$X = \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{2\times10^{-3}\times\left(\frac{10\pi}{10}\right)^2}{\pm\left(1-\left(\frac{10\pi}{10}\right)^2\right)}$$

$= 2.25\times10^{-3}$ m $= 2.25$ mm

46. Ans: (a)

Sol: Number of nodes observed at a frequency of 1800 rpm is 2



n-mode number

The whirling frequency of shaft,

$$f = \frac{\pi}{2}\times n^2\sqrt{\frac{gEI}{WL^4}}$$

For 1st mode frequency, $f_1 = \frac{\pi}{2}\times\sqrt{\frac{gEI}{WL^4}}$

$$f_n = n^2 f_1$$

As there are two nodes present in 3rd mode,

$f_3 = 3^2 f_1 = 1800$ rpm

$\therefore f_1 = \frac{1800}{9} = 200$ rpm

$\therefore$ The first critical speed of the shaft = 200 rpm

47. Ans: (b)

Sol: Critical or whirling speed

$$\omega_c = \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta}}\,\text{rad/sec}$$

If N_C is the critical or whirling speed in rpm

then $\frac{2\pi N_C}{60} = \sqrt{\frac{g}{\delta}}$

$\Rightarrow \frac{2\pi N_C}{60} = \sqrt{\frac{9.81\text{m/s}^2}{1.8\times10^{-3}\text{m}}}$

$\Rightarrow N_C = 705.32$ rpm ≈ 705 rpm

Conventional Practice Solutions

01.

Sol: Total energy (T) of the system = kinetic energy (K.E) + Potential energy (P.E)

$$K.E = \frac{1}{2}I\theta^2 + \frac{1}{2}m_2(b\dot{\theta})^2 + \frac{1}{2}m_1(c\theta)^2$$

$$P.E = \frac{1}{2}k_2(b\theta)^2 + \frac{1}{2}k_3(a\theta)^2 + \frac{1}{2}k_1(c\theta)^2$$

$$T = K.E + P.E$$

$$\frac{d(T)}{dt} = 0$$ as the sum of energy is constant

It gives ,

$$\left(I + m_2b^2 + m_1c^2\right)\ddot{\theta} + (k_2b^2 + k_1c^2 + k_3a^2)\theta = 0$$

It is of the form

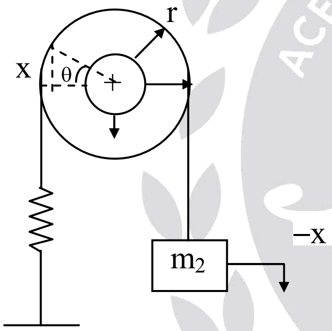
$$I\ddot{\theta} + k_t\theta = 0$$

So natural frequency of the system can be expressed as

$$\omega_n = \sqrt{\frac{\left(k_2b^2 + k_1c^2 + k_3a^2\right)}{\left(I + m_2b^2 + m_1c^2\right)}}\text{rad/sec}$$

02.

Sol:



The equation of motion is

$$I\ddot{\theta} + kxr + m_2\ddot{x}\,r = 0$$

$$I\ddot{\theta} + k\theta r^2 + m_2\ddot{\theta}r^2 = 0 \qquad \left(\because\ \theta = \frac{x}{r}\right)$$

$$\left(I + m_2r^2\right)\ddot{\theta} + kr^2\theta = 0 \qquad (\because x = \theta r)$$

$$\left(I + m_2r^2\right)\ddot{\theta} + kr^2\theta = 0 \qquad (\because\ \ddot{x} = \ddot{\theta}r\ )$$

$$M_{eq} = I + m_2\,r^2$$

$$\therefore\ \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{Kr^2}{I + m_2\,r^2}}$$

03.

Sol: m = 250 kg,

K = 5 N/mm

Let $X_3 = X$ and $X_4 = 0.8X$

$$\Rightarrow \frac{X_4}{X_3} = 0.8$$

but $\frac{X_1}{X_2} = \frac{X_2}{X_3} = \frac{X_3}{X_4}$

(i) $$\ell n\frac{X_3}{X_4} = \delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow$$

$$\frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \ell n\left(1/0.8\right) = 0.223$$

$$\Rightarrow 1 - \zeta^2 = 793.87\zeta^2$$

$$\zeta = 0.0355$$

Damping coefficient C = $2m\omega_n\zeta$

$$= 2 \times 250 \times \sqrt{\frac{5\times10^3}{250}} \times 0.0355 \times 10^{-3}$$

= 79.38 N-s/m

(ii) Critical damping coefficient C_c

$C_c = 2\sqrt{km} = 2263.0679\,\text{N/m/S}$

(iii) damping factor, $\zeta = 0.0355$

(iv) $$\frac{\text{Damped frequency}}{\text{Undamped Frequency}} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{1-\zeta^2}\,\omega_n}{\omega_n}$$

$$= \sqrt{1-\zeta^2} = \sqrt{1-(0.0355)^2}$$

$$= 0.999$$

04.

Sol: Given that m = 100 kg,

$\delta_{static} = 8$ mm

$$\omega_n = \sqrt{\frac{g}{\delta_{static}}} = \sqrt{\frac{9.81}{8\times10^{-3}}} = 35.017\,\text{rad/s}$$

Let F be the vertical harmonic force at 80% of resonance frequency

$\therefore \omega = 0.8\ \omega_n = 0.8 \times 35.017$

$= 28.0136$ rad/sec

$$\Rightarrow \frac{\omega}{\omega_n} = 0.8$$

Let A_1 is amplitude without damping

$$A_1 = \frac{F/K}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]} = \frac{F/K}{(1-0.8^2)} \text{-----------(1)}$$

Let A_2 is the amplitude with damping at resonance, i.e., $\frac{\omega}{\omega_n} = 1$

$$\Rightarrow A_2 = \frac{F/K}{\sqrt{\left(2\zeta\frac{\omega}{\omega_n}\right)^2 + \left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}}$$

$$= \frac{F/K}{2\zeta} \text{----------- (2)}$$

$$\frac{\text{equ.(1)}}{\text{equ.(2)}} \Rightarrow \frac{A_1}{A_2} = \frac{2\zeta}{1-0.8^2}$$

$$\zeta = \frac{A_1}{2A_2}\times 1-0.8^2 = \frac{5}{2\times2}(1-0.8^2)$$

$\zeta = 0.45$

(i) Damping coefficient

$c = 2\,m\omega_n\zeta = 2\times100\times35.97\times0.45$

$= 3151.53$ N/m/s

(ii) Critical damping coefficient

$$c_c = \frac{c}{\zeta} = \frac{3151.53}{0.45} = 7003.4 \text{ N/m/s}$$

(iii) Damping ratio, $\zeta = 0.45$

(iv) Logarithmic decrement

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi\times0.45}{\sqrt{1-0.45^2}} = 3.166$$

(v) Damping force $= c\dot{x} = cA_2\,\omega$

$$= 3151.53\times\frac{2}{1000}\times35.017$$

$= 220.714$ N

05.

Sol: Given that total motor mass m = 100 kg

Eccentric mass, m = 30 kg,

r = 0.5 mm

Let total stiffness of the springs = 5k

Damping is neglected, i.e $\xi = 0$

$$\text{Transmissibility, TR} = \frac{F_T}{F} = \frac{1}{11}$$

$\Rightarrow$ where F_T = transmitted force

F = applied force

$$\text{But TR} = \frac{1}{11} = \pm\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2} \quad (\because \xi = 0)$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times1400}{60} = 146.6\,\text{rad/s}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 - 1 = 11 \Rightarrow \omega_n = \frac{\omega}{\sqrt{12}} = 42.32\,\text{rad/s}$$

(i) $\omega_n = \sqrt{\frac{5k}{M}} = 42.32 \Rightarrow k = 35.82$ kN/m

(ii) The unbalanced force that is exciting vibrations = $mr\omega^2$

$= 30\times0.5\times10^{-3}\times(146.6)^2$

$F = 322.37$ N

Dynamic force transmitted to the floor

$$F_T = F\times TR = \frac{322.37}{11} = 29.31\,\text{N}$$

06.

Sol: Given that M = 68 kg, k = 100 N/mm

$\zeta = 0.2$, m = 2 kg, $\omega = 314$ rad/s,

s = 75 mm = 2r

$$r = \frac{s}{2} = \frac{75}{2} = 37.5\,\text{mm}$$

N = 3000 rpm

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{100\times10^3}{68}} = 38.34\,\text{rad/s}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times3000}{60} = 314.159\,\text{rad/s}$$

$$\frac{\omega}{\omega_n} = \frac{314.159}{38.34} = 8.188$$

$$\text{Amplitude } X = \frac{\frac{mr}{M}\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{\frac{2\times37.5}{68}(8.188)^2}{\sqrt{\left(1-8.188^2\right)^2 + (2\times0.2\times8.188)^2}}$$

X = 1.118 mm

$$\text{Transmissibility, } \varepsilon = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$\varepsilon = \frac{\sqrt{1+(2\times0.2\times8.188)^2}}{\sqrt{\left(1-8.188^2\right)^2 + (2\times0.2\times8.188)^2}}$$

$\varepsilon = 0.0517$

But the force produced due to reciprocating part

$F = mr\omega^2 = 2\times37.5\times10^{-3}\times(314.159)^2$

$= 7402.19$ N

But $\varepsilon = \frac{F_T}{F} \Rightarrow F_T = \varepsilon F$

$\therefore$ Transmitted force,

$F_T = 7402.19 \times 0.0517$

$= 382.69$ N

Phase angle,

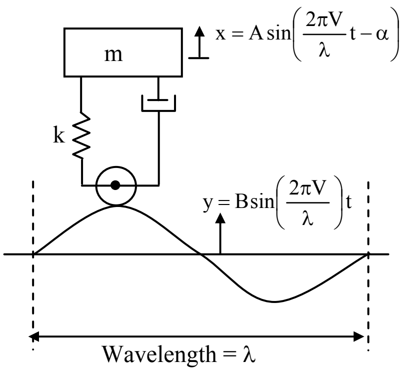
$$\tan\phi = \frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{2\times0.2\times8.188}{1-8.188^2}$$

$\phi = -2.83°$

$\phi = 180 - 2.83 = 177.17°$

07.

Sol:



$\delta_s = 10$ cm, $\quad A = 8$ cm,

$\lambda = 15$ m, $\quad \xi = 0.05$

Road induced vibrations

$y = B\sin\omega t$

$\lambda = Vt$

$T = \text{time period} = \dfrac{\lambda}{v}$

$$T = \frac{2\pi}{\omega} = \frac{\lambda}{v}$$

$$\therefore \omega = \frac{2\pi v}{\lambda}$$

$$V = 75\text{ km/hr} = 75\times\frac{5}{18} = 20.833\text{ m/s}$$

$$\omega = \frac{2\times\pi\times 20.833}{15} = 8.726\text{ rad/s}$$

Support motion:

$$\therefore \frac{A}{B} = \frac{\sqrt{1+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}}$$

Resonance occurs if $\omega = \omega_n$

$$\therefore\ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_s}} = \sqrt{\frac{9.81}{0.1}} = 9.964\text{ rad/s}$$

$$\omega_n = \frac{2\times\pi\times V_{critical}}{\lambda}$$

$$V_{critical} = \frac{9.964\times 15}{2\pi}$$

$$= 23.64\text{ m/s} = 85.123\text{ km/hr}$$

$$q = \frac{\omega}{\omega_n} = \frac{8.722}{9.964} = 0.88$$

$$\frac{A}{8} = \frac{\sqrt{1+(2\times0.05\times0.88)^2}}{\sqrt{(1-0.88^2)^2+(2\times0.05\times0.88)^2}}$$

$A = 33.17$ cm

08.

Sol: Given that vertical, shaft in the bearings.

Shaft diameter $d = 15$ mm

Distance between the bearings

$l = 100$ cm $= 1$ m

weight of disc, $W = 147.15$ N

Eccentricity, $e = 0.3$ mm

$E = 19.6\times10^6$ N/cm^2 = 196 GPa,

$\sigma_{allow} = 6867$ N/cm^2 = 68.67 MPa

Because of the eccentricity, the amount of dynamic deflection (r) is depends on the allowable stress.

(i) Natural frequency $\omega_n = \sqrt{\dfrac{g}{\delta_{st}}}$

Where, δ_{st} = static deflection

Since the shaft is held in long bearings, therefore it is assumed to be fixed at both the ends, we know that the static direction at the centre of the shaft.

$$\delta_{st} = \frac{mg\ell^3}{192EI}$$

$$I = \frac{\pi}{4}\times\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 = 2.5\times10^{-9}\,m^4$$

$$\delta_{st} = \frac{47.15\times(1)^3}{192\times196\times10^9\times\left(\frac{\pi}{64}\times15^4\right)\times10^{-12}}$$

$\delta_{st} = 1.57\times10^{-3}$ m

Critical speed

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = 78.93 \text{ rad/sec}$$

Critical speed of the shaft is equal to the speed at the natural frequency

$$N_c = \frac{60\omega_n}{2\pi} = 753.74\text{rpm}$$

(ii) Maximum allowable stress in the shaft due to bending moment produced by the whirling

$$\frac{M}{I} = \frac{\sigma_{allow}}{d/2}$$

Where bending moment, $M = \frac{m_d g\ell}{8}$

Here m_d is the dynamic mass due to the whirling

$$\frac{m_d g\ell}{8} = \frac{\sigma I}{d/2}$$

$$m_d = \frac{8\times2\times68.67\times10^6\times2.5\times10^{-9}}{0.015\times9.81}$$

$= 18.67$ kg

Static deflection $\delta_{st} \propto$ mass of the system for same stiffness

$\Rightarrow \delta_{st} \propto m$ and $r \propto m_d$

$$\Rightarrow r = \frac{W_d}{W}\times\delta_{st} = \frac{183.15}{147.15}\times1.57\times10^{-3}$$

$= 1.958\times10^{-3}$m

$$r = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

$$\left(\frac{\omega_n}{\omega}\right)^2 - 1 = \left(\frac{e}{r}\right) = \frac{0.3\times10^{-3}}{1.9\times10^{-3}} = \pm0.1532$$

<u>Case 1:</u>

$$\left(\frac{\omega_n}{\omega}\right)^2 = 1.1532 \Rightarrow \omega^2 = \frac{\omega_n^2}{1.1532}$$

$$\omega^2 = \frac{(78.93)^2}{1.1532}$$

$\omega = 73.5$ rad/s

$$\Rightarrow N = \frac{60\times\omega}{2\pi} = \frac{60\times73.5}{2\pi} = 702 \text{ rpm}$$

<u>Case 2</u>

$$\left(\frac{\omega_n}{\omega}\right)^2 = 1-0.1532 = 0.8468$$

$\omega = 85.77$ rad/sec

$$N = \frac{60\times85.77}{2\pi} = 819 \text{ rpm}$$

$\therefore$ Range of speed is 702 to 819 rpm.

09.

Sol: Given that m = 10 kg, d = 20 mm

Bearing span l = 1 m

e = 0.1 mm,

$$\omega = \frac{2\pi \times 3000}{60} = 100\pi$$

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}}$$

Where 'δ_{st}' is static deflection of simply supported beam with midpoint load.

$$\delta_{st} = \frac{mg\ell^3}{48EI}$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times \left(2 \times 10^{-2}\right)^4$$

$$= 7.854 \times 10^{-9} \text{ m}^4$$

$$\delta_{st} = \frac{10 \times 9.81 \times 1^3}{48 \times 2 \times 10^{11} \times 7.854 \times 10^{-9}}$$

$$= 1.301 \times 10^{-3} \text{ m}$$

$$\omega_n = \sqrt{\frac{9.81}{1.301 \times 10^{-3}}} = 86.83 \text{ rad/s}$$

$$y = \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = \frac{0.1 \times 10^{-3}\left(\frac{314.16}{86.83}\right)^2}{1 - \left(\frac{314.16}{86.83}\right)^2}$$

$$y = -1.0827 \times 10^{-3} \text{ m}$$

The negative sign indicates that the displacement is out of phase with the centrifugal force.

Dynamic force on the bearings = $m\omega_n^2 y$

$$= 10 \times (86.83)^2 \times 1.0827 \times 10^{-3}$$

$$= 86.63 \text{ N}$$