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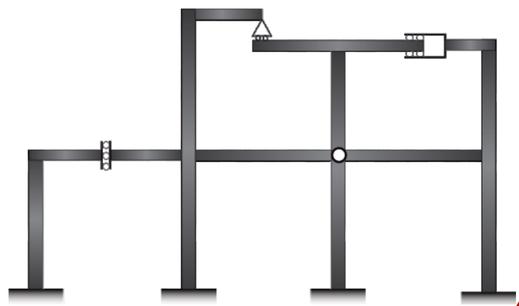


CIVIL ENGINEERING

STRUCTURAL ANALYSIS

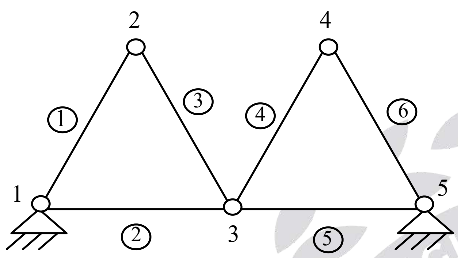
Text Book : Theory with worked out Examples and Practice Questions

2 Structural Analysis

(Solutions for Text Book Practice Questions)

01. Introduction to Structures & Static Indeterminacy

01. Ans: (d)



(i) $r = 4; j = 5; m = 6;$

$D_{se} = 4 - 3 = 1$

$D_{si} = m - (2j - 3)$

$= 6 - (2\times5 - 3)$

$= 6 - 7 = -1$

The given truss is internally unstable.

(ii) $D_{se} = r - 3$ $\quad j = 9, m = 14$

$= 6 - 3 = 3$

$D_{si} = m - (2j - 3)$

$= 14 - (18 - 3) = -1$

The given frame is internally unstable.

(iii) All supports are roller,

$\therefore$ The given truss is unstable.

(iv) $D_{se} = 4 - 3 = 1$

$D_{si} = m - (2j - 3)$

$= 15 - (20 - 3)$

$= 15 - 17 = -2$

Internally unstable.

(v) In a member, there should not be more than two internal hinges.

02. Ans: (b)

Sol: $j = 9$; $\quad m = 16;$

$D_{se} = 3 - 3 = 0$ $\quad D_{si} = m - (2j - 3)$

$= 16 - (2\times9 - 3)$

$= 16 - 15 = 1$

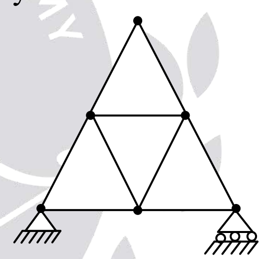
Stable but indeterminate by one

03. Ans: (c)

Sol: $D_{se} = 0;$

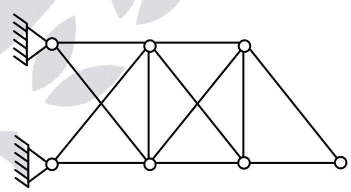
$D_{si} = m - (2j - 3) = 9 - (2\times6 - 3) = 0$

$\therefore$ The frame is internally as well as externally determinate.



04. Ans: (a)

Sol:



As the two supports are hinged total no. of reactions = 4.

The deficiency of vertical member between the supports is taken care of by the additional vertical reaction. Hence the structure is stable.

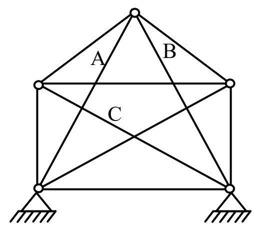
Hence D_{se} can be taken as zero.

$D_{si} = 2$ (additional members in the first two spans more than required for stability)

$D_{se} = 2$

05. Ans: (b)

Sol:



$D_{se} = 2 + 2 - 3 = 1$

$D_{si} = m - (2j - 3)$

$= 10 - (2 \times 5 - 3) = 3$

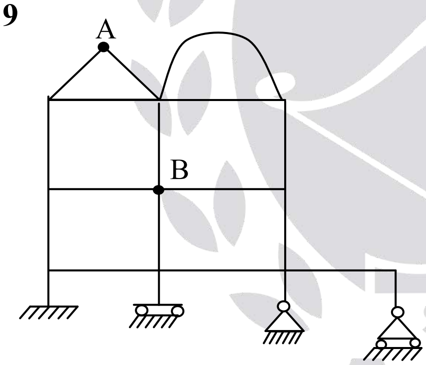
$D_s = 3+1 = 4$

Note: This is formula for internal indeterminacy of pin jointed plane trusses. We know that the basic perfect shape for pin jointed truss is triangle either by shape or by behaviour. Hence by removing three members suitably (A, B & C as shown in figure), the stability can be maintained.

$D_s = 1 + 3 = 4$

06. Ans: 19

Sol:



Number of reactions = 3 + 2 + 2 + 1 = 8

Equilibrium equations = 3

$D_{se} = 8 - 3 = 5$ $\quad D_{si} = 3c = 3 \times 6 = 18$

Force releases at A = n – 1 = 2 – 1 = 1

Force releases at B = n – 1 = 4 – 1 = 3

Where,

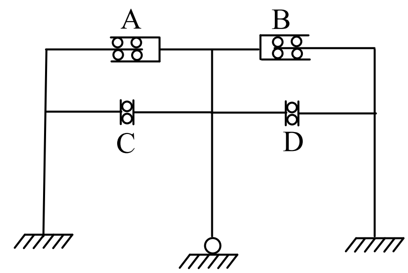
n = number of members joining at that location.

$D_s = D_{se} + D_{si}$ – no.of force releases

$= 5 + 18 - 1 - 3 = 19$

07. Ans: (d)

Sol:



No. of reactions(r) : 3 + 2 + 3 = 8

$D_{se} = r - 3$

$D_{se} = 8 - 3 = 5$

$D_{si} = 3 \times$ no.of closed boxes $= 3c = 3 \times 2 = 6$

force releases = (1 + 1 + 1 + 1) = 4

$D_s = D_{se} + D_{si}$ – no.of force releases

$= 5 + 6 - 4 = 7$

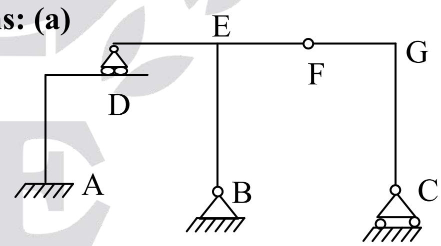
Note: A & B are horizontal shear releases. At each of them one force is released.

C & D are vertical shear releases.

At each of them one force is released.

08. Ans: (a)

Sol:



$D_{se} = (3 + 2 + 1) - 3 = 3$

$D_{si} = 0$

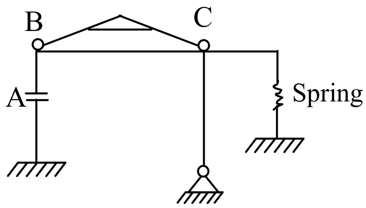
Force release at 'D' = 2

Force release at 'F' = 1

$\because D_s = 3 + 0 - 2 - 1 = 0$

09. Ans: (b)

Sol:



Reaction at fixed support = 3

Reaction at hinged support = 2

Reaction at spring support = 1

Total reactions = 6

$D_{se} = 6 - 3 = 3$ $\quad$ $D_{si} = 3 \times 2 = 6$

Horizontal force release at 'A' = 1

Moment releases at 'B' = 1

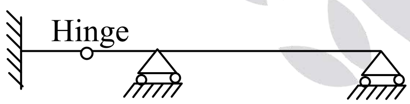
Moment releases at 'C' = 1

Note: At B and C the hinges are tangential to the horizontal beam. Hence the column and beam will have only one common rotation.

$D_s = 3 + 6 - 1 - 1 - 1 = 6$

10. Ans: (b)

Sol:



No. of reactions(r) = 3 + 1 + 1 = 5

No. of eq.eqns (E) = 3

Force releases = 1

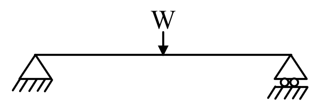
$D_{si} = 0$ $\quad$ $D_s = 5 - 3 - 1 = 1$

11. Ans: Zero

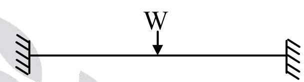
Sol: The given truss is statically determinate. Determinate structures are not subjected to stresses by lack of fit, temperature change, sinking of supports etc.

12. Ans: (c)

Sol: In statically determinate structures, stresses due to thermal changes, sinking of supports, lack of fit will not develop.



Simply Supported Beam



Fixed End Beam

Bending moment at mid span for S.S $= \dfrac{W\ell^2}{8}$

Bending moment at mid span for fixed Beam

$= \dfrac{W\ell^2}{24}$

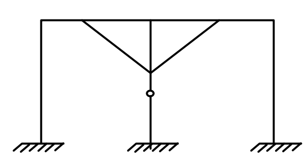
$\therefore$ As bending moment is greater in S.S Beam (determinate) than fixed beam (indeterminate), sections are more

$\therefore$ Uneconomical when compared to fixed Beam.

$\therefore$ Sections designed in determinate structures are uneconomical.

13. Ans: (c)

Sol:



Static indeterminacy $(D_s) = D_{se} + D_{si} -$ force releases

$D_{se} = 3 + 3 + 3 - 3 = 6$

$D_{si} = 3 \times C$

Where,

C = Number of closed Boxes = 2

$= 3 \times 2 = 6$

force releases = m −1

Where, m = Number of members connected to hinge

$= 2 - 1 = 1$

$\therefore\ D_s = D_{se} + D_{si}$ – force releases

$= 6 + 6 - 1 = 11$

14. Ans: (a)

Sol:

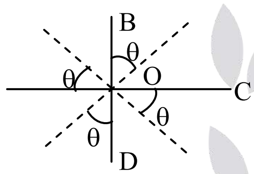


Fig: (i)

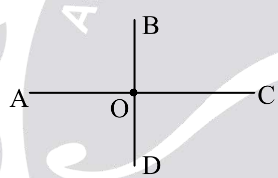


Fig: (ii)

If the joint O is considered as rigid as shown in fig (i), rotation of one member with respect to another member will be zero.

But, if hinge is considered at Joint 'O' as shown in fig (ii), OB, OC and OD will have will have rotations with respect to OA. To make three relative rotations zero, we need to apply 3- moments.

$\therefore$ Thus for a 4-members meeting at a joint, number of restraining moments required = 3

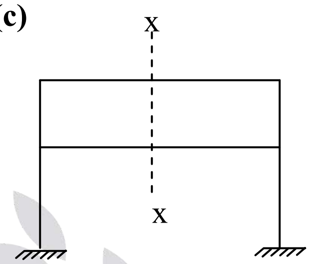
$= 4 - 1$

$\therefore$ If 'm' members meeting at a joint, number of restraining moments required = "m-1".

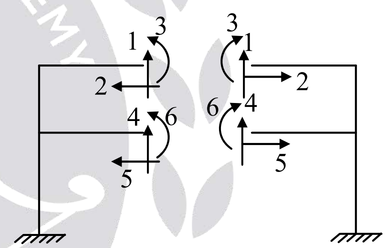
$\therefore$ While calculating static indeterminacy, due to hinge in the structure m-1 extra equilibrium equations are considered.

15. Ans: (c)

Sol:



Let us consider section x-x divides the structure in to 2-parts.



Total unknowns are 6

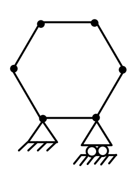
To analyze Box frame 6 equilibrium equations are required

Static indeterminacy = 6 −3 = 3

$\therefore$ Static indeterminacy of closed boxes were considered as 3

16. Ans: (a)

Sol:



External indeterminacy (D_{se}) = 2 + 1−3 = 0

Internal indeterminacy (D_{si}) $\Rightarrow n - (2j - 3)$

Where,

n = Number of members connected by hinges = 6

j = Number of pin-Joints = 6

$D_{si} = 6 - (2 \times 6 - 3)$

$= -3$

$\therefore D_{si} = -3 < 0$ (Internally unstable)

Static indeterminacy (D_s) = $D_{si} + D_{se}$

$= -3 + 0 = -3$

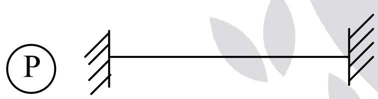
if $D_s < 0$ (It is considered as unstable)

(or) When one part of structure moves appreciably with respect to other part of structure it is classified as unstable.

02. Kinematic Indeterminacy

01. Ans: (b)

Sol:

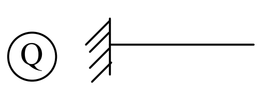


$D_k = 3j - r$ | $D_s = (3m + r) - 3j$

$j = 2, r = 6$ | $= 3 + 6 - (3\times2)$

$D_k = 6 - 6 = 0$ | $D_s = 9 - 6 = 3$

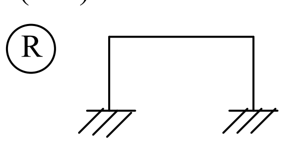
$D_k = 0$ | $D_s = 3$



$D_s = r - 3 = 3 - 3 = 0$

$D_k = 3j - r = (3\times2) - 3$

$= 3$



$D_s = 0 \quad D_k = 3$

$j = 4, m = 3, r = 6$

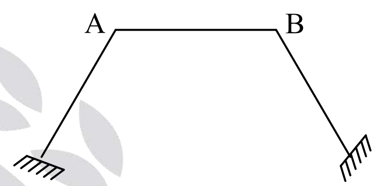
$D_s = r - 3$

$= 6 - 3 = 3$

$D_k = 3j - r = 3 \times 4 - 6 = 6$

02. Ans: (b)

Sol:



A & B are rigid joints.

The rigid joint of a plane frame will have three degrees of freedom.

Fixed supports will have zero degrees of freedom.

$\therefore$ Total number of degrees of freedom = 6 (considering axial deformations)

No.of members = 3

Neglecting axial deformations, degrees of freedom or kinematic indeterminacy

$D_k = 6 - 3 = 3$

or

Using the formula

$D_k = 3j - r$

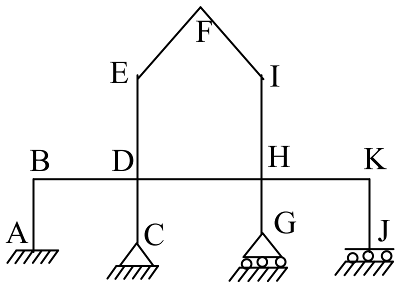
$= 3 \times 4 - 6 = 6$ (with axial deformations)

$= 6 - 3 = 3$ (Neglecting axial deformations)

Note: While using the formula supports also shall be treated as joints.

03. Ans: (b)

Sol:



D.O.F of rigid joints = 7 × 3 = 21

D.O.F of fixed support = 0

D.O.F of hinged support = 1

D.O.F of roller support = 2

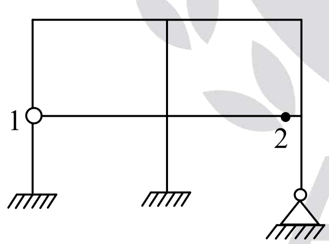
D.O.F of horizontal shear release support = 1

Total D.O.F or D_k = 21 + 0 + 1 + 2 + 1= 25
(Considering axial deformations)

Neglecting axial deformations = 25 – 11 = 14

04. Ans:22 or 12

Sol:



D.O.F of four rigid joints = 4 × 3 = 12

D.O.F of hinged joint '1' = 5
(three rotations and two translations)

D.O.F of joint 2 = 4 (two rotations and two translations. Both vertical members will have one common rotation)

D.O.F of fixed supports = 0

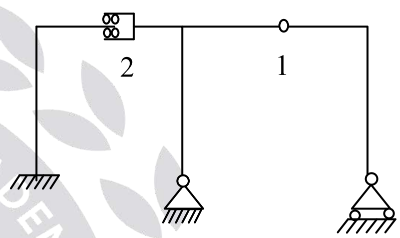
D.O.F of hinged support = 1

Total D.O.F or D_k = 12 +5+ 4 +1= 22
(considering axial deformations)

Neglecting axial deformations = 22 –10 = 12

05. Ans: 20 or 13

Sol:



D.O.F of moment release at '1' = 4

D.O.F of horizontal shear release at '2' = 4

D.O.F of 3 rigid joints = 3 × 3 = 9

D.O.F of fixed support = 0

D.O.F of hinged support = 1

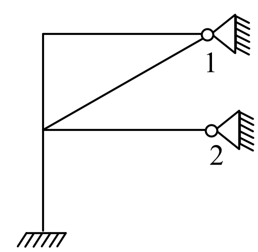
D.O.F of roller support = 2

Total D.O.F or D_k = 4 + 4 + 9 +1+2 = 20
(considering axial deformations)

Neglecting axial deformations = 20 – 7 = 13

06. Ans: 9 or 5

Sol:



D.O.F of 2 rigid joints = 2 × 3 = 6

D.O.F of fixed support = 0

D.O.F of hinged support'1' = 2

(Two members are connected to the hinged support '1'. Hence two different rotations are possible)

D.O.F of hinged support'2' = 1

Total D.O.F or D_k = 6 + 0 + 2 +1 = 9

(considering axial deformations)

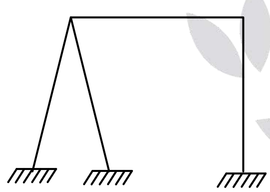
Neglecting axial deformations = 9 – 4 = 5

Note: The effect of diagonal member shall not be considered.

At hinged support '1' two rotations, at hinged support '2' one rotation, at each rigid joint one rotation. No sway. Hence five D.O.F neglecting axial deformations.

07. Ans: 6 or 2

Sol:



D.O.F of two rigid joints = 2 × 3 = 6

D.O.F of fixed support = 0

Total D.O.F or D_k = 6 + 0 = 6

(Considering axial deformations)

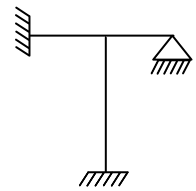
Neglecting axial deformations = 6 – 4 = 2

Note: The effect of two inclined members shall be taken as one member.

At each rigid joint one independent rotation + one sway of the frame as a whole.

08. Ans: 4 or 2

Sol:



D.O.F of 1 rigid joint = 1 × 3 = 3

D.O.F of fixed supports = 0

D.O.F of hinged support = 1

Total D.O.F or D_k = 3 + 1 = 4

(Considering axial deformations)

Neglecting axial deformations = 4 – 2 = 2

Note: As no sway the axial deformation of two beams shall be taken as one.

At rigid joint one independent rotation + one rotation at hinged support.

09. Ans: 13

Sol: For pin jointed plane frame $D_k = 2j - r$

$= 2(8) - 3$

$= 13$

10. Ans: (b)

Sol: $j = 6, r = 3,$

$D_k = 2j - r$

$= 2\times6 - 3 = 9$

$D_{se} = r - 3 = 3 - 3 = 0$

$D_{si} = m - (2j - r)$

$= 9 - (2\times6 - 3)$

$D_s = D_{se} + D_{si} = 0$

∴ Statically determinate and kinematically indeterminate by 9.

11. Ans: (b)

Sol: $j = 6, r = 3,$

$D_k = 2j - r$

$= 2\times6 - 3 = 9$

$D_{se} = r - 3 = 3 - 3 = 0$

$D_{si} = m - (2j - r)$

$= 9 - (2\times6 - 3)$

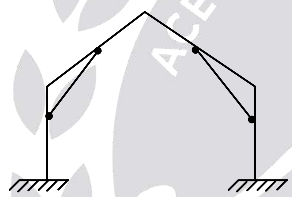
$D_s = D_{se} + D_{si}$

$= 0$

$\therefore$ Statically determinate and kinematically indeterminate by 9.

12. Ans: (a)

Sol:



At fixed supports = 0

At '3' Rigid joints = 3 × 3 = 9

At locations A, B, C & D = 4 × 4 = 16

Kinematic indeterminacy = 0 + 9 + 16 = 25

by neglecting axial deformations

$D_k = 25 - 10 = 15$

13. Ans: (b)

Sol: Degree's of freedom of pin jointed plane truss =2

Degree's of freedom of pinjointed space truss = 3

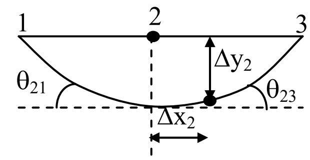
Degree's of freedom of at rigid joint for plane frame = 3

Degrees of freedom of rigid joints space frame = 6

$\therefore$ Options 3 and 4 are correct

14. Ans: (c)

Sol:



Degree's of freedom are Δy_2, Δx_2, θ_{21} and θ_{23}

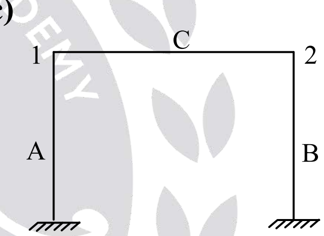
i.e at each internal hinge there are 4 degree's of freedom i.e (2 translations and 2 Rotations)

$\therefore$ Statement- (I) is true and

Statement –(II) is false

15. Ans: (c)

Sol:



at fixed supports = 0

at Two rigid joints (1) and (2) = 3 + 3 = 6

Kinematic indeterminacy = 0 + 6 = 6

D_K (neglecting axial deformations) = 6 –3

= 3

i.e. vertical axial deformations in members A and B $\Delta y_1 = 0$, $\Delta y_2 = 0$ and axial deformations in member (C) is neglected i.e

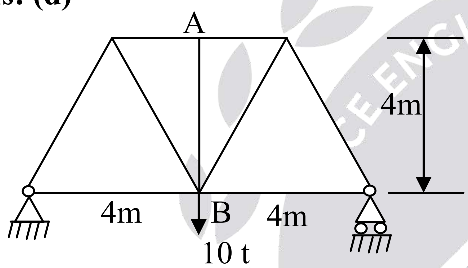
$\therefore$ $\Delta_x = 0$, Δy_1 = and, $\Delta y_2 = 0$ makes kinematic indeterminacy from 6 to 3.

03. Statically Determinate Frames

> ***Sign convention for forces***
>
> *Axial compression:* A compression member will push the joint to which it is connected.
>
> *Axial tension:* A tension member will pull the joint to which it is connected

01. Ans: (d)

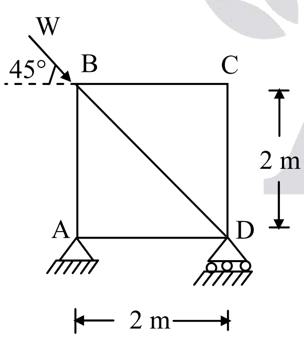
Sol:



Analyzing at 'A', two forces are in the same line, hence the 3rd force AB is zero.

02. Ans: (a)

Sol:

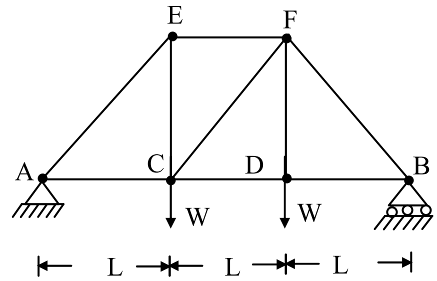


At 'C' the two forces not in the same line, hence $F_{CD} = F_{CB} = 0$

Now analyzing at 'B' $F_{BA} = 0$

03. Ans: (c)

Sol:



$F_{DC} = F_{DB}$

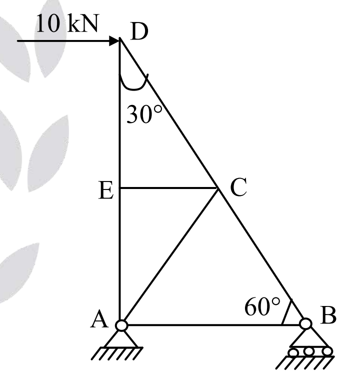
$F_{CA} = F_{CD}$

$F_{CE} = W$

$\therefore F_{CF} = 0$

04. Ans: (c)

Sol: First analyze at 'E'.



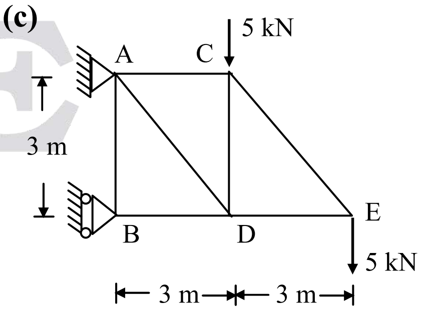
$\therefore F_{EC} = 0$

Now analyze at 'C'.

$\therefore F_{CA} = 0$

05. Ans: (c)

Sol:

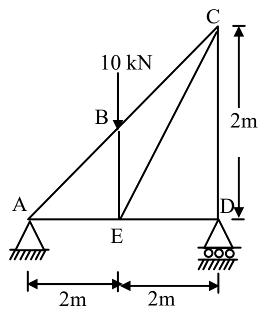


At roller support the reaction is horizontal which is in line with BD

$\therefore F_{AB} = 0$

06. Ans: (b)

Sol:



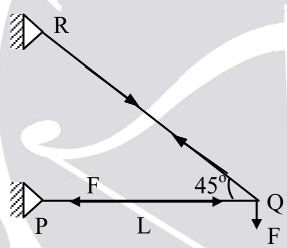
$R_D = 5$ kN ↑ $\therefore F_{CD} = 5$ kN

At 'D' as reaction is normal to the plane of rolling and DC and the vertical reaction are in the same line then $F_{DE} = 0$

$F_{BE} = 10$ kN

07. Ans: (a)

Sol:



Apply ΣV = 0 at Q.

$F_{QR} \sin 45^o = F$

$\Rightarrow F_{QR} = F\sqrt{2}$ (tension)

Now apply ΣH = 0 at Q.

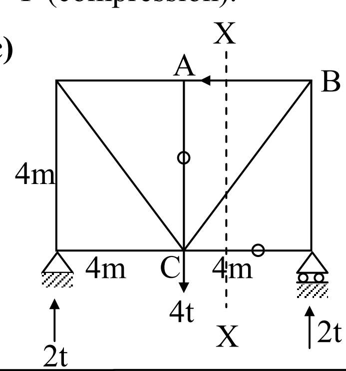
$F_{QR} .\cos 45^o = F_{QP}$

$$F\sqrt{2} \times \frac{1}{\sqrt{2}} = F_{QP}$$

$\therefore F_{QP} = F$ (compression).

08. Ans: (c)

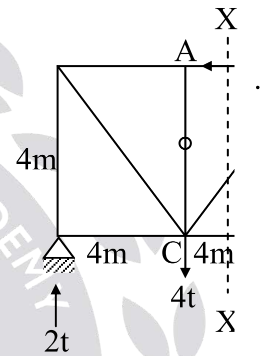
Sol:



Using method of sections. Pass a section X – X as shown through the chosen member AB and other two members so that these two other members pass through a common joint say 'C'.

Consider left side of the section.

Apply ΣM = 0 for the left side of the section.

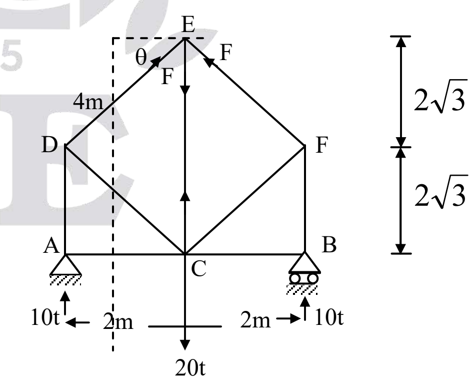


$$2t \times 4 = f_{AB} \times 4$$

$$\therefore f_{AB} = 2t \,(\text{Comp})$$

09. Ans: (a)

Sol:



$$\tan\theta = \frac{2\sqrt{3}}{2}$$

$\therefore \theta = 30^o$

Pass the section as shown in figure

Apply $\Sigma M_C = 0$ for the right part of the section.

$$\Rightarrow 10\times 2 = F\cos 30^o \times \frac{4}{\sqrt{3}}$$

$$\therefore F = 10\,t$$

Now analysis at joint E.

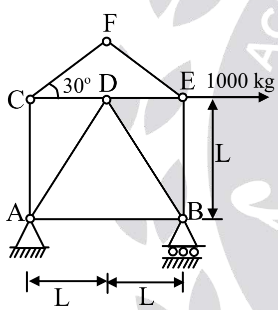
$$\Sigma F_y = 0 \;\Rightarrow\; 2F.\text{Sin}30^o = F_{CE}$$

$$2\times 10\times \frac{1}{2} = F_{CE}$$

$$F_{CE} = 10\,t\ (\text{tension})$$

10. Ans: (c)

Sol:



Consider joint F.

We know that if two members act at a joint and if they are not in the same line then each of them are zero.

Hence,

F_{CF}, F_{EF} both are zero.

Similarly Consider joint C.

$\therefore$ F_{CD}, F_{CA} both are zero

Taking $\Sigma M_B = 0$, $R_A = 500\ (\downarrow)$

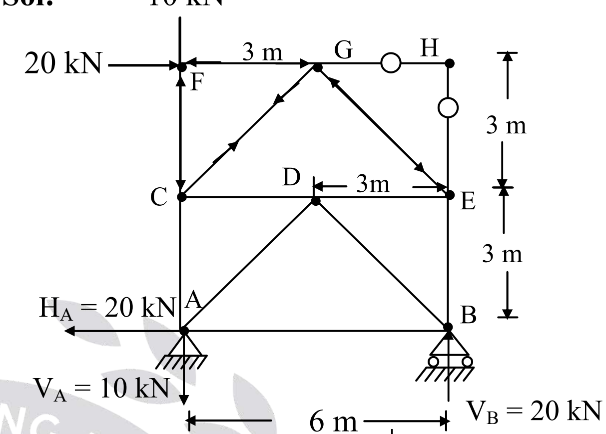
Joint (A)

$F_{AD}\sin 45 = R_A$

$\because$ $F_{AD} = 500\sqrt{2}$ (tension)

11. Ans: (c)

Sol:



Reactions are $V_A = 10$ kN $\downarrow$,

$H_A = 20$ kN $\leftarrow$

$V_B = 20$ kN $\uparrow$

$F_{HG} = F_{HE} = 0$

Apply $\Sigma V = 0$ at 'G'

$\therefore F_{AC} = F_{AE}$

Apply $\Sigma H = 0$

$F_{GE}\cos 45^o + F_{CG}\cos 45^o = 20$

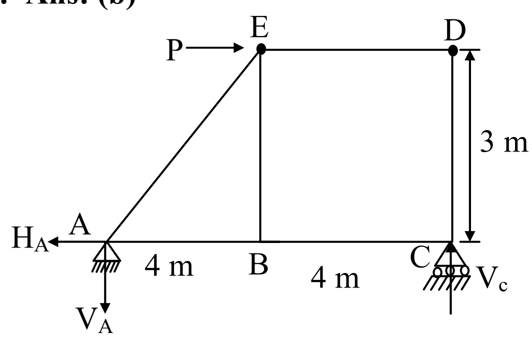
$\therefore\ 2F_{GC}\cos 45^o = 20$

$F_{GC} = 10\sqrt{2}$ (tensile)

Apply $\Sigma V = 0$ @C

$\Rightarrow F_{CA} = 0$

12. Ans: (b)



$\Sigma m_c = 0$

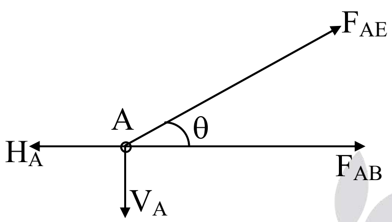
$P \times 3 - V_A \times 8 = 0$

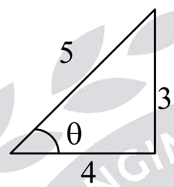
$V_A = \frac{3P}{8}$

$\Sigma F_y = 0$

$V_A = V_C = \frac{3P}{8}$

Joint A





$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$\Sigma F_y = 0$

$F_{AE} \sin\theta - V_A = 0$

$F_{AE} \times \frac{3}{5} - V_A = 0$

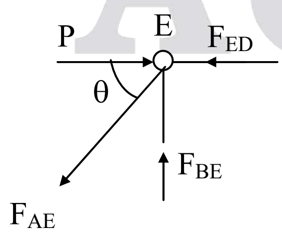
$F_{AE} = \frac{3P}{8} \times \frac{5}{3} = \frac{5P}{8}$

$\Sigma F_X = 0$

$F_{AB} + F_{AE}.\cos\theta - P = 0$

$F_{AB} = \frac{P}{2}$

Joint (E)



$\Sigma F_X = 0$

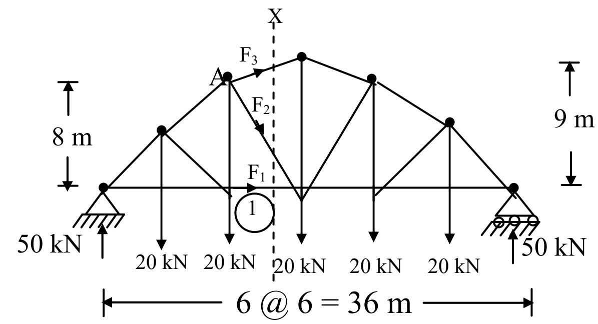
$F_{BE} - F_{AE}.\sin\theta = 0$

$F_{BE} = \frac{5P}{8} \times \frac{3}{5} = \frac{3P}{8}$

$F_{AB} : F_{BE} : F_{AE}$

$\frac{P}{2} : \frac{3P}{8} : \frac{5P}{8} = 4 : 3 : 5$

13. Ans: (b)



Taking left side

$\Sigma M_A = 0$

$50 \times 12 - 20 \times 6 - F_1 \times 8 = 0$

$F_1 = 60$ kN

14. Ans: (a)

Sol:

At any joint in planar truss, only two equilibrium conditions ($\sum F_x = 0$ and $\sum F_y = 0$) are available

Using there two equilibrium conditions, only two unknown member forces can be determined.

∴ Statement (I) and (II) are correct and statement (II) is correct explanation of statement (I).

15. Ans: (a)

By using method of sections method three unknown member forces can be determined by using three equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$ and $\sum M = 0$).

16. Ans: (a)

Ans: Generally purlins will placed at joints to eliminate bending moments in the truss members, because sections/members designed by bending is uneconomical when compare to sections / members designed by axial loads. ∴ Truss members will carry only axial loads if loads are placed at truss joints.

17. Ans: (d)

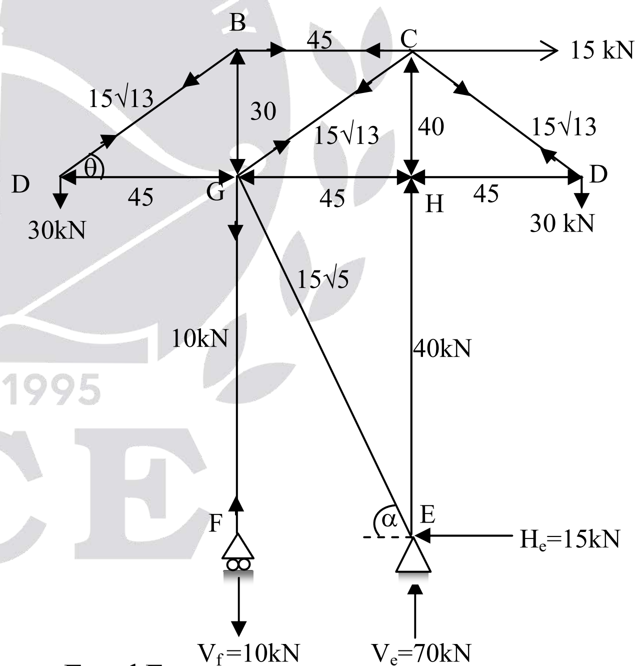
Ans: In practical, there is a possibility of reversal of stresses i.e. due to wind (or) Earthquake. During reversal of stresses/forces, members carrying zero forces (for a particular/forces, constant loading) may take forces.

∴ These zero force members cannot be removed.

Conventional Practice Solutions

01.

Sol:



$\tan\theta = \frac{2}{3}, \sin\theta = \frac{2}{\sqrt{13}}, \cos\theta = \frac{3}{\sqrt{3}},$

$\tan\alpha = 2, \sin\alpha = \frac{2}{\sqrt{5}}, \cos\alpha = \frac{1}{\sqrt{5}}$

Let V_e and V_f be the vertical reactions at the supports E and F

Let H_e be the horizontal reaction at the supports E (Note, there is no horizontal reaction at F)

Taking moment about F.

$V_e \times 3 = 30 \times 6 + 15 \times 8 - 30 \times 3$ ∴ $V_e = 70$ kN↑ ∴ $V_f = 10$kN↓

$H_e = 15$ kN ←

Joint F Resolving vertically.

$P_{fg} = 10$ kN (tensile)

Joint E Resolving horizontally.

$P_{eg} \cos\alpha = 15 \quad \therefore P_{eg} = 15\sqrt{5}$ kN (compressive)

Resolving vertically

$P_{eh} = 70 - 15\sqrt{5} \sin \alpha = 70 - 13 = 40$ kN (compressive)

Joint A Resolving vertically

$P_{ab} \sin\theta = 30 \; \therefore P_{ab} = 15\sqrt{13}$ kN (tensile)

Resolving horizontally,

$P_{ag} = 15\sqrt{13} \cos \theta = 45$ kN (compressive)

Joint D This is similar joint A

$P_{dc} = 15\sqrt{13}$ kN (tensile) and $P_{dh} = 45$ kN (compressive)

Joint B resolving vertically.

$P_{bg} = 15\sqrt{13} \sin \theta = 30$ kN (compressive)

Resolving horizontally,

$P_{bc} = 15\sqrt{13} \cos \theta = 45$ kN (tensile)

Joint H Resolving horizontally

$P_{hg} = 45$ kN (compressive)

Resolving vertically,

$P_{hc} = 45$ kN (compressive)

Joint C Resolving horizontally

$P_{cg} \cos \theta = 15 + 15\sqrt{13} \cos \theta - 45$

$P_{cg} \times \frac{3}{\sqrt{13}} = 15 + 45 - 45 \quad \therefore P_{cg} = 5\sqrt{13}$ kN (tensile)

The forces in the members for the truss are tabulated below.

Member	Force (kN)	
	Compressive	Tensile
AB		$15\sqrt{13}$
BC		45
CD		$15\sqrt{13}$
DH	45	
HE	40	
FG		10
GA	45	
BG	30	
GC		$5\sqrt{13}$
HC	40	
HG	45	
GE	$15\sqrt{5}$	

04. Basic Methods of Structural Analysis

01. Ans: (d)
Sol:
- Stiffness method deals with unknown joint displacement (degrees of freedom). It is nothing but kinematic Indeterminacy. Hence stiffness method is more suitable if kinematic Indeterminacy is less than static indeterminacy. As displacements are unknowns it is also called displacement method.
- Equilibrium equations are used at joints to analyze the structure. Hence it is also called equilibrium method.

02. Ans: (b)
Sol: In theorem of three moments, consistent deformation method unknown forces are dealt with. Hence these are force methods
Moment distribution and slope deflection method deal with displacements. Hence these are displacement methods.

03. Ans: (a)
Sol: Force methods, deal with unknown redundant forces. In pin jointed trusses, more number of degrees of freedom. Hence stiffness methods are complicated compare to force method.

04. Ans: (c)
Sol:
In Force methods, forces are kept unknowns and unknown forces are found by using geometric compatability conditions.

In displacement methods, joint displacements are kept as unknowns and joint equilibrium conditions are enforced to find unknown displacements.

05. Ans: (b)
Sol:

Description	Option
Kani's method is very much suitable for multistorey frames	∴A-4
Force method suitable if static indeterminacy is less.	∴B-3
Column analogy method suitable for box frames with varying sections and inclined members	∴ C-1
Displacement method suitable if Kinematic Indeterminacy is less	∴D-2

06. Ans: (a)
Sol: To calculate unknown forces in a structure, compatibility conditions are used to analyze.

07. Ans: (b)

Sol: Kani's Method is suitable for:

1. To analyze high storied structures.
2. Non-sway and sway analysis can be done together.
3. Number of iterations for convergence is less
4. If any mistake was done at any step, it will get adjusted automatically.

08. Ans: (a)

Sol:

(i) Moment distribution was invented by Hardy-cross (A→3)

(ii) Slope deflection method deals with displacements (B → 4)

(iii) Kani's method also known as Rotation method (C →1)

(iv) Force method also known as flexibility method (D →2)

09. Ans: (d)

Sol: For any truss, static indeterminacy (deals with forces) is less than kinematic indeterminacy (displacements).

As $D_s < D_k$

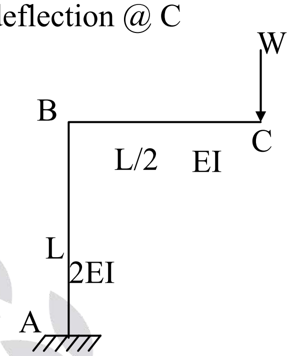
It is advisable to adopt force method

∴ In all structural Engineering packages to analyze indeterminate truss, force method is used for quick and exact solutions.

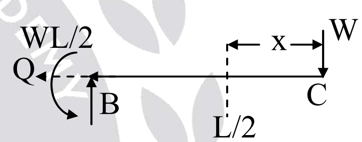
05. Energy Principles

01. Ans: (d)

Sol: Vertical deflection @ C



FBD of BC:



$M_x = +WX$

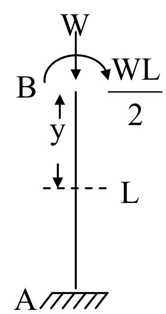
$$\frac{\partial M_X}{\partial W} = X$$

$$\delta_{VBC} = \frac{1}{EI}\int_0^{L/2} M_x \frac{\partial M_x}{\partial W}dx$$

$$= \frac{1}{EI}\int_0^{L/2}(WX)(X)dx$$

$$= \left[\frac{WX^3}{3EI}\right]_0^{L/2}$$

$$= \frac{WL^3}{24EI}$$



FBD of AB:

$$M_y = \frac{WL}{2}$$

$$\frac{\partial M_y}{\partial W} = \frac{L}{2}$$

$$\delta_{VAB} = \frac{1}{2EI}\int_0^L \left(\frac{WL}{2}\right)\left(\frac{1}{2}\right)dy$$

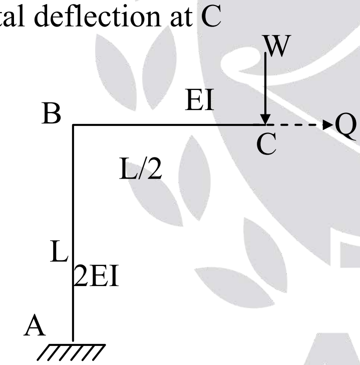
$$= \frac{1}{2EI}\frac{WL^2}{4}y\int_0^L \quad = \frac{WL^3}{8EI}$$

Total vertical deflection at

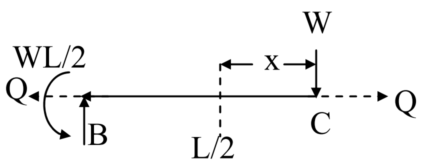
$$\delta_c = \frac{WL^3}{24EI} + \frac{WL^3}{8EI} = \frac{WL^3}{6EI}$$

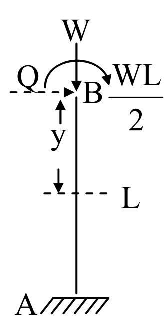
02. Ans: (b)

Sol: Horizontal deflection at C



FBD of BC:





$M_x = + WX$

$$\frac{\partial M_x}{\partial Q} = 0$$

$$\delta_{hBC} = \frac{1}{EI}\int_0^{L/2}(WX)(0)dx = 0$$

FBD of AB:

$$M_y = \frac{WL}{2} + Qy$$

$$\frac{\partial M_y}{\partial Q} = +y$$

$$\delta_{hAB} = \frac{1}{2EI}\int_0^L \left(\frac{WL}{2} + Q_y\right)(y)dy$$

$$= \frac{1}{2EI}\int_0^L \left(\frac{WL}{2}\right)(y)dy$$

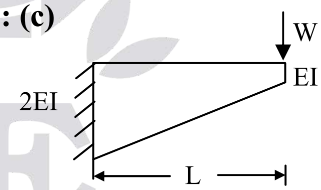
(Q= 0 as it is imaginary force)

$$= \frac{1}{2EI}\left(\frac{WL}{2}\right)\left(\frac{y^2}{2}\right)_0^L = \frac{WL^3}{8EI}$$

Total horizontal deflection = $\frac{WL^3}{8EI}$

03. Ans: (c)

Sol:



Shortcut: Strain energy is inversely proportional to I.

With uniform I, $U = \frac{w^2 l^3}{6EI}$.

With uniform 2I, $U = \frac{w^2 l^3}{12EI}$

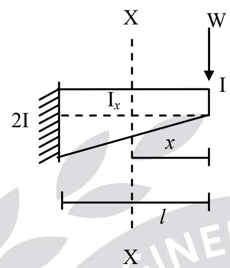
As given has I varying from I to 2I, denominator shall be in between 6 and 12.

Traditional procedure:

$M_x = wx$

$I_x = I + \frac{I.x}{l}$

$= I\left(1 + x/l\right) = \frac{I(l+x)}{l}$



$$U = \int_0^l \frac{w^2 x^2 .dx}{2E\,\frac{I(l+x)}{l}}$$

$$= \int_0^l \frac{l w^2 x^2 dx}{2EI(l+x)}$$

$$= \frac{w^2 l}{2EI}\int_0^l \frac{x^2}{l+x}dx$$

$$= \frac{w^2 l}{2EI}\int_0^l \frac{x^2 - l^2 + l^2}{l+x}dx$$

$$= \frac{w^2 l}{2EI}\left[\int_0^l \frac{(x+l)(x-l)}{(1+x)}dx + \int_0^l \frac{l^2}{(1+x)}dx\right]$$

$$= \frac{w^2 l}{2EI}\left[\left(\frac{x^2}{2} - lx\right)_0^l + \left(l^2 \log(l+x)\right)_0^l\right]$$

$$= \frac{w^2 l}{2EI}\left[\frac{l^2}{2} - l^2 + l^2 \log_e 2l - l^2 \log_e l\right]$$

$$= \frac{w^2 l}{2EI}\left[\frac{-l^2}{2} + l^2 \log_e \frac{2l}{l}\right]$$

$$= \frac{w^2 l}{2EI}\left[-0.5l^2 + l^2(0.693)\right]$$

$$U = \frac{w^2 l^3}{10.35\,EI}$$

04. Ans: (b)

Sol: $M_x = W\,R\sin\theta$

$$\frac{\partial M}{\partial W} = R\sin\theta$$

$$\delta H_B = \frac{1}{EI}\int_0^\pi WR\sin\theta \times R\sin\theta \times R\,d\theta$$

$$= \frac{WR^3}{EI}\int_0^\pi \sin^2\theta\,d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

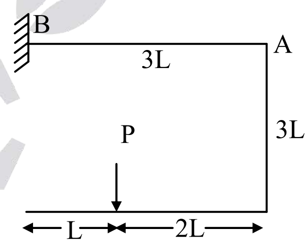
$$= \frac{WR^3}{EI}\int_0^\pi \left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

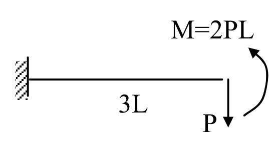
$$= \frac{WR^3}{EI}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{2}\right)_0^\pi$$

$$\delta H_B = \frac{\pi WR^3}{2EI}$$

05. Ans: (c)

Sol: Consider free body diagram Of 'AB'

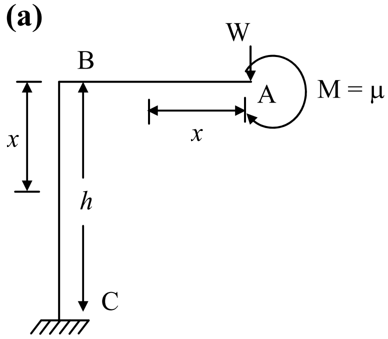




$$\delta_{VA} = \frac{P(3L)^3}{3EI} - \frac{(2PL)(3L)^2}{2EI} = 0$$

06. Ans: (a)

Sol:



For member AB

$Mx = Mz + M$

$$\frac{\partial Mx}{\partial W} = x$$

$$\delta_v = \int Mx \frac{\partial Mx}{\partial W}.\frac{dx}{EI}$$

$$\delta_v = \int_o^{\ell} (Wx + M)x.\frac{dx}{EI}$$

$\therefore W = 0$ {fictious load}

$$\delta Lv = \frac{M}{EI}\int_o^{\ell} x.ds = \frac{M\ell^2}{2EI}$$

For member BC

$M_x = W+M$

$$\frac{\partial Mx}{\partial W} = \ell$$

$$\delta_v = \int_o^h (W\ell + M)\ell \frac{dx}{EI}$$

$$\delta_v = \frac{M\ell}{EI}\int_o^h dx = \frac{M\ell h}{EI}$$

$\therefore \quad W = 0 \qquad\qquad \delta = \frac{M\ell}{EI}\left(h + \frac{\ell}{2}\right)$

$$(\delta_v)_A = \frac{\mu\ell}{EI}\left[h + \frac{\ell}{2}\right]$$

07. Ans: (d)

Sol: Strain energy (u) of Bar AB = $\frac{F^2\ell}{2AE}$

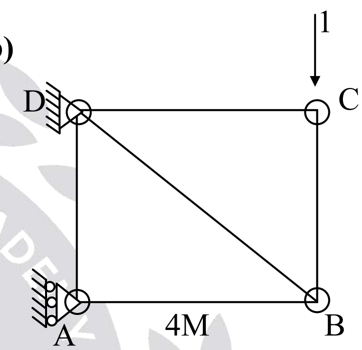
Where F = Axial force in the Bar

$F_{AB} = 0$

$\therefore u_{AB} = 0$

08. Ans: (b)

Sol:



Apply unit load in the vertical direction at 'C'. Due to this unit load $F_{CB} = 1$

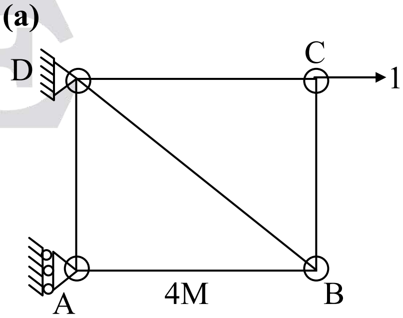
Change in length of member BC due temperature change = $\alpha\, tl$

$= 10 \times 10^{-6} \times 4000 \times 25 = 1mm$

$\therefore \delta_{VC} = \Sigma k \times \delta' = 1 \times 1 = 1mm$

09. Ans: (a)

Sol:



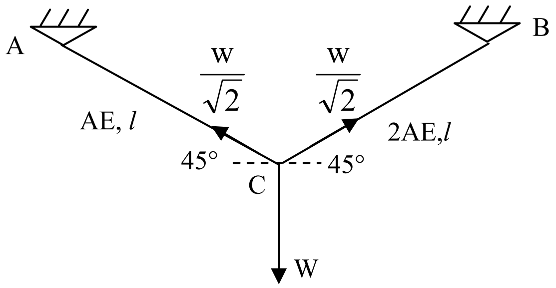
Apply unit horizontal load at 'C'.

Due to this the force in the member BC zero.

$\therefore$ Horizontal deflection @ C $= \Sigma k'\delta' = 0$

10. Ans: (d)

Sol:



Apply unit vertical load at 'C'. to get the values of k.

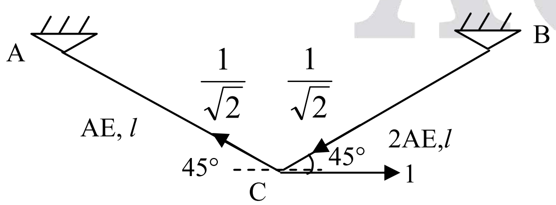
Members	Force P	k	$\frac{\ell}{AE}$	Pkl/AE
AC	$-\frac{W}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{\ell}{AE}$	$\frac{W\ell}{2AE}$
AB	$-\frac{W}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{l}{2AE}$	$\frac{W\ell}{4AE}$

$$(\delta_v)_c = \sum\frac{Pkl}{AE} = \frac{Wl}{2AE} + \frac{Wl}{4AE} = \frac{3Wl}{4AE}$$

11. Ans: (d)

Sol:

Apply unit horizontal load at 'C'. to get the values of k′



Members	P	k′	$\frac{\ell}{AE}$	$\frac{Pk'1}{AE}$
AC	$-\frac{W}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{\ell}{AE}$	$\frac{W\ell}{2AE}$
AB	$-\frac{W}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{l}{2AE}$	$\frac{-W\ell}{4AE}$

$$(\delta_H)_C = \frac{\sum Pk'l}{AE} = \frac{Wl}{2AE} - \frac{Wl}{4AE} = \frac{Wl}{4AE}$$

12. Ans: 1.5 × 10^{-3}

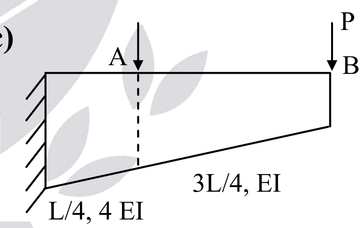
Sol: As the structure is determinate extra forces will not be generated due to lack of fit.

$\tan\theta = \left(\frac{6}{4\times10^3}\right)$ Inclination of member BC is mainly due to 6 mm extension in BD

$\theta = 1.5 \times 10^{-3}$ Radians.

13. Ans: (c)

Sol:



Maxwell's law of Reciprocal deflections:

$\delta_{ij} = \delta ji$ where

δ_{ij} = deflection @ 'i' due to unit load at 'j'

δ_{ji} = deflection @ j due to unit load at i

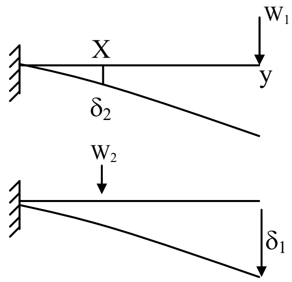
Further Maxwell's law is valid for both prismatic and non prismatic beams.

Maxwell's theorem independent of EI.

Hence option 'C'.

14. Ans: (c)

Sol:



Using Bettie's Theorem:

Virtual work done by

W_1 = virtual work done by W_2

$\therefore w_2\delta_2 = w_1\delta_1$

$$\Rightarrow \frac{\delta_1}{\delta_2} = \frac{w_2}{w_1}$$

15. Ans: (b)

Sol: According to castigliano's 1st theorem

$$\delta = \frac{\partial U}{\partial W}$$

i.e deflection at any point is equal to partial derivative of strain energy with respect to load acting at a point where deflection is desired to calculate. (A $\rightarrow$4)

If y is considered as deflection

$$\frac{dy}{dx} = \text{slope} \quad (B \rightarrow 2)$$

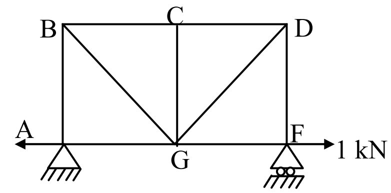
$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (C \rightarrow 3)$$

We know that,

$$V \text{ (shear force)} = \frac{dM}{dx} \quad (D \rightarrow 1)$$

16. Ans: (c)

Sol: Apply unit Horizontal load at F by Removing all External loads.



Horizontal deflection at

$$F, (\delta_H)_F = \Sigma \frac{PKL}{AE}$$

Apply $\Sigma H = 0$

$\therefore H_A = 1$ kN ($\leftarrow$)

$\therefore$ at point A, K_{AG} = 1 kN (T)

at Join F, K_{FG} = 1kN (T)

$\therefore$ Forces in rest of members are zero due to unit load at F

Due to external loading, forces in members AG and G F exists.

$\therefore$ Horizontal deflections of roller support will be the summation of deformation in members AG and GF.

17. Ans: (c)

Sol: As per Castigliano's second theorem, in any and every case of statically indetermination. Where in, an indefinite number of different values of redundant forces satisfy the conditions of static equilibrium, their actual values are those that render the strain energy stored to a minimum and this is applicable only when the redundant support do not yield.

i.e. $\frac{\partial U}{\partial R} = 0$ (or) $\frac{\partial U}{\partial M} = 0$

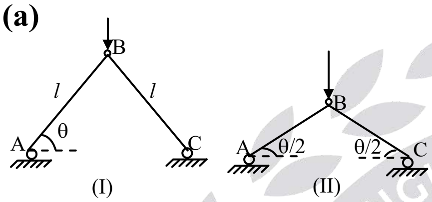
Where,

R and M are redundant forces and moments respectively.

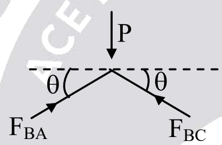
U = Strain energy stored in the members

18. Ans: (a)

Sol:



At joint B: (Fig. I)



Apply $\sum H = 0$

$F_{BA} \cos\theta = F_{BC} \cos\theta$

$F_{BA} = F_{BC}$

Apply $\sum V = 0$

$F_{BA} \sin\theta + F_{BC} \sin\theta = P$

$2F_{BA}\sin\theta = P$

$$F_{BA} = \frac{P}{2\sin\theta} (C)$$

Similarly At joint B from fig. (II)

$$F_{BA} = \frac{P}{2\sin\left(\frac{\theta}{2}\right)} \quad (C)$$

$\therefore$ For a particular

Value of θ

$(F_{BA})_I < (F_{BA})_{II}$

Deflection $(\delta) = \Sigma \frac{PKL}{AE}$

$\therefore \delta \, \alpha \, P$

$\therefore (\delta)_I < (\delta)_{II}$

$\therefore$ I will have less member force and less deflections at B compared to II

19. Ans: (a)

Sol: Generally in Rigid frames, Bending will be major criteria for design of members when compare to axial and shear forces.

$\therefore$ Strain energy due to Bending / flexure is more when compare to other.

$\therefore$ Strain energy due to flexure is considered.

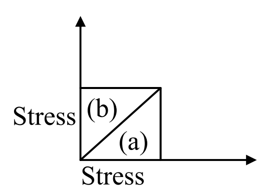
20. Ans: (b)

Sol: Deflection is calculated with the help of unit load method and this method is based on virtual work principle

Both the statements are correct, but statement (II) is not the correct explanation of statement (I).

21. Ans: (c)

Sol: Strain energy and complimentary strain energy due to gradual application of load is always equal in elastic limit.



(a) → Strain energy

(b) → Complementary strain energy

Conventional Practice Solutions

01.

Sol: Static Indeterminacy 'D_S'

$D_s = D_{se} + D_{si}$

D_{se} = external Indeterminacy = No. of unknown reactions - No. of equilibrium equations

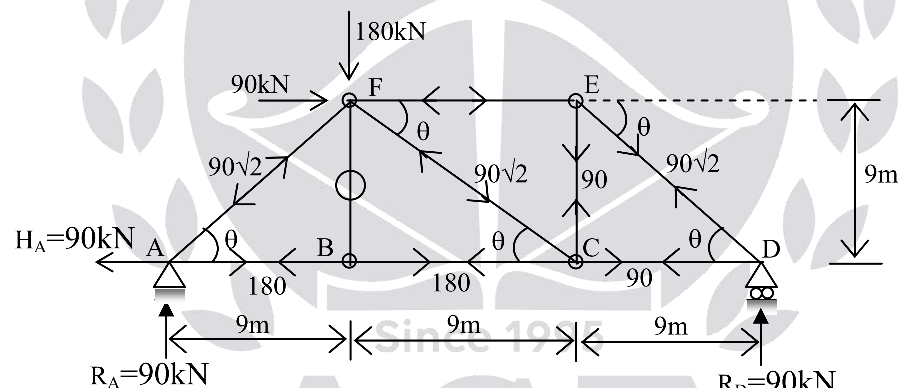
$D_{se} = 3 - 3 = 0 \rightarrow$ Externally determinate structure

D_{si} = Internally indeterminate = $m - (2j - 3)$

$D_{si} = D_{se} + D_{si}$

$D_s = 1 \rightarrow$ statically indeterminate structure

→ Let us consider that the member BE is redundant. We will now analyse the frame after removing the member BE.



p – Values

$\sum v = 0$ $\quad\quad$ $\tan\theta = 9/9$

$R_A + R_D = 180$ $\quad\quad$ $\theta = 45^0$

Taking moment about 'A'

$\sum M_A = 0$ [↶ −ve ↷ +ve]

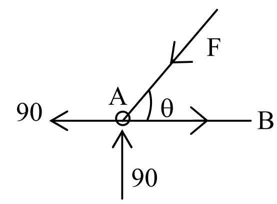
$-R_D \times 27 + 180 \times 9 + 90 \times 9 = 0$

$R_D = 90$ kN(↑)

$R_A = 180 - 90 = 90$ kN (↑)

At Joint A: -

Resolving vertically



$\sum V = 0$

$F_{AF} \sin\theta = 90$

$F_{AF} = 90\sqrt{2}$ [compression]

Resolving Horizontally

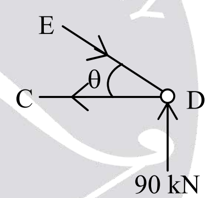
$\sum H = 0$

$F_{AB} = 90 + F_{AF} \cos\theta$

$$= 90 + 90\sqrt{2} \times \frac{1}{\sqrt{2}}$$

= 180 kN (Tension)

At Joint D: -



Resolving vertically

$\sum v = 0$

$F_{DE} \sin\theta = 90$

$F_{DE} = 90\sqrt{2}$ kN [compression]

Resolving Horizontally

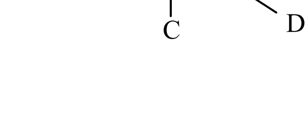
$\sum H = 0$

$F_{DC} = F_{DE} \cos 45^0$

$F_{DC} = 90\sqrt{2} \times \frac{1}{\sqrt{2}} = 90$ kN (Tension)

At Joint E: -

Resolving Horizontally



$\sum H = 0$

$F_{FE} = F_{ED} \cos\theta$

$$= 90\sqrt{2} \times \frac{1}{\sqrt{2}} = 90 \text{ kN (Compression)}$$

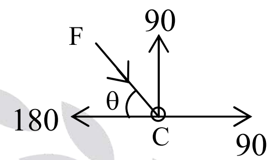
Resolving vertically

$\sum v = 0$

$F_{EC} = \; = 90\sqrt{2} \times \frac{1}{\sqrt{2}} = 90$ kN (Tension)

At Joint C: -

Resolving vertically



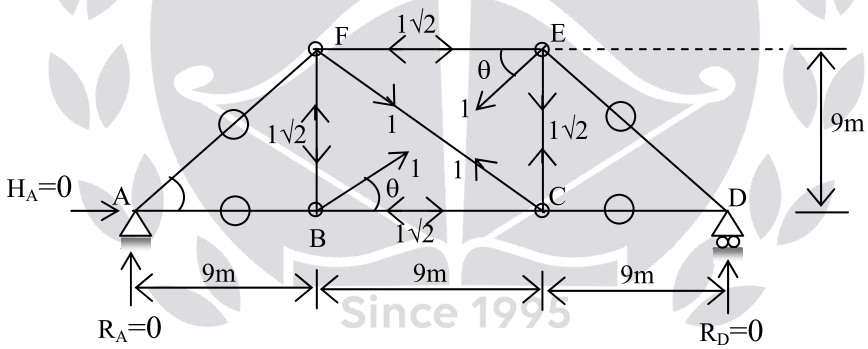
$\sum v = 0$

$F_{CF} \sin 45^0 = 90$

$F_{CF} = 90\sqrt{2}$ [compression]

Now remove the given load system and apply a pair of unit loads at B and E in place of the member BE.

K - Values:



There will be no reactions at the supports

$\frac{L}{A} = \text{constant}$

Actual force in any member 'S' = P + k X

$$X = \text{Factor} = -\frac{\sum \frac{PkL}{AE}}{\sum \frac{k^2 L}{AE}} = -\frac{\sum Pk}{\sum k^2} = -\left[\frac{-254.558}{4}\right] = 63.64$$

Compression – ve

Tension + ve

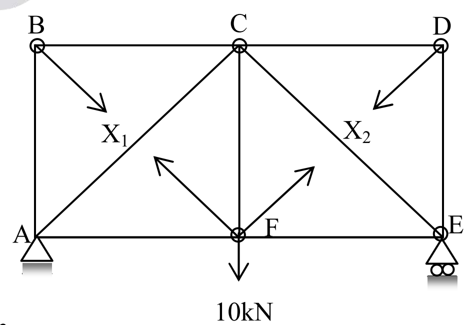
Member	P(kN)	K(kN)	Pk	K^2	S=P+kX(kN)
AB	+180	0	0	0	180 [Tension]
BC	+180	$-\frac{1}{\sqrt{2}}$	$-90\sqrt{2}$	$\frac{1}{2}$	134.99
CD	+ 90	0	0	0	90
FE	– 90	$-\frac{1}{\sqrt{2}}$	$45\sqrt{2}$	$\frac{1}{2}$	– 135
FB	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{2}$	– 45
EC	+ 90	$-\frac{1}{\sqrt{2}}$	$-45\sqrt{2}$	$\frac{1}{2}$	45
AF	$-90\sqrt{2}$	0	0	0	$-90\sqrt{2}$
FC	$-90\sqrt{2}$	1	$-90\sqrt{2}$	1	– 63.63 (Compression)
BE	–	1	0	1	63.64 (Tension)
DE	$-90\sqrt{2}$	0	0	0	$-90\sqrt{2}$
			$\sum Pk = -254.558$	$\sum k^2 = 4$	

02.

Sol: $D_{se} = r - s = 3 - 3 = 0 \rightarrow$ externally determinate

$D_{si} = m - (2j - 3) = 11 - (2 \times 6 - 3) = 2$ [Internally indeterminate]

$D_s = D_{se} + D_{si} = 2$ [Statically indeterminate]



Let us consider the members BF & DF as redundant.

Let X_1 be the tension in BF and X_2 be the tension in DF.

The truss will first be analysed after removing the members BF & DF. The forces p_1, p_2, p_3. in the members due to this condition.

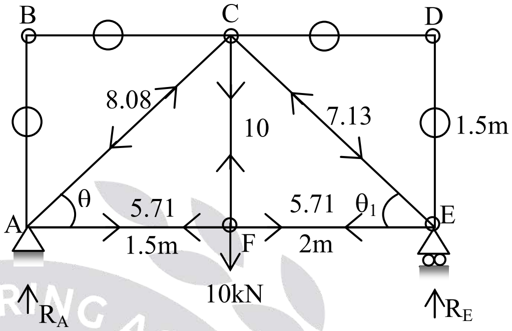
$$\cos\theta = \sin\theta = \frac{1.5}{\sqrt{1.5^2 + 1.5^2}} \qquad \cos\theta = \sin\theta = \frac{\sqrt{2}}{2} \qquad \cos\theta_1 = \frac{2}{\sqrt{1.5^2 + 2^2}}$$

P - Values:

$\cos\theta_1 = \frac{4}{5}$ $\sin\theta_1 = \frac{1.5}{\sqrt{1.5^2 + 2^2}}$

$\sin\theta_1 = \frac{3}{5}$ $\sum v = 0$

$R_A + R_E = 10$



Taking moment about 'A'

$\sum M_A = 0$ [↶ – ve ↷ +ve]

$-R_E \times 3.5 + 10 \times 1.5 = 0$

$R_E = \frac{10\times1.5}{3.5} = 4.28\,kN\ (\uparrow)$

$R_A = 10 - 4.28 = 5.72$ kN ($\uparrow$)

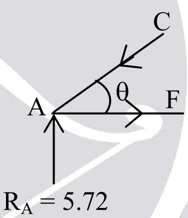
At Joint A: -

Resolving vertically

$\sum V = 0$

$F_{AC} \sin\theta = 5.72$

$F_{AC} = \frac{5.72\times2}{\sqrt{2}} = 8.08\,KN$ (Comp)



Resolving Horizontally

$\sum H = 0$

$F_{AF} = F_{AC}\cos\theta$

$F_{AF} = 8.08\times\frac{\sqrt{2}}{2} = 5.71\,kN$ (Tension)

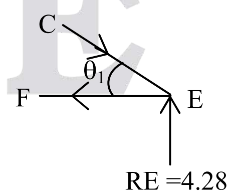
At Joint E: -

Resolving vertically

$\sum V = 0$

$F_{EC}\sin\theta_1 = 4.28$

$F_{EC} = \frac{4.28}{3}\times5 = 7.13\,kN$ (Comp)



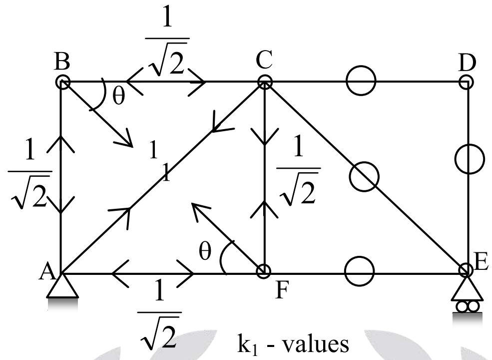
Resolving Horizontally

$\sum H = 0$

$F_{EF} = F_{EC}\cos\theta_1$

$= 7.13\times\frac{4}{5} = 5.74\,kn$ (Tension)

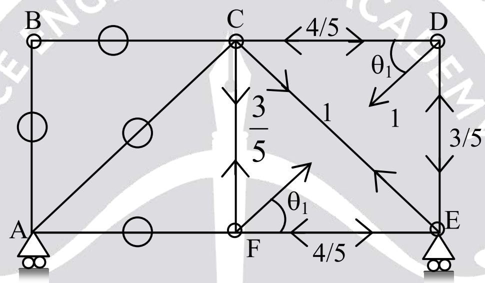
Now remove the given load system and apply a pair of unit loads at B and F in place of the member BF.



k_1 - values

There will be no reactions at the supports

Now remove the given load system and apply a pair of unit loads at D & F in place of the member DF.



k_2 - Values

k_2 - values

There will be no reactions at the supports.

The redundant quantities X_1 and X_2 are given by the condition that the strain energy stored is a minimum.

$$\sum\frac{Pk_1L}{AE}+\sum\frac{X_1k_1k_1L}{AE}+\sum\frac{X_2k_2k_1L}{AE}=0 \quad \_\_\_\_\_\_ (1)$$

$$\sum\frac{Pk_2L}{AE}+\sum\frac{X_1k_1k_2L}{AE}+\sum\frac{X_2k_2k_2L}{AE}=0 \quad \_\_\_\_\_\_ (2)$$

The unknown X_1 & X_2 can be determined by solving equations 1 & 2

Compression - ve

Tension + ve

$$\sum Pk_1L + X_1\sum k^2{}_1\, L + X_2 \sum k_1\, k_2\, L = 0$$

$$-\,33.776 + 7.24\, X_1 + 0.636\, X_2 = 0 \quad \_\_\_\_\_\_\_\_ (3)$$

$\sum Pk_2L + X_1 \sum k_1 k_2 L + X_2 \sum k^2_2 L = 0$

$-35.95 + 0.636 X_1 + 8.64 \times 2 = 0$ __________ (4)

$7.24 X_1 + 0.636 X_2 = 33.776$

$0.636 X_1 + 8.64 X_2 = 35.95$

$X_1 = 4.32$

$X_2 = 3.84$

Member	(L in m)	P	k_1	k_2	Pk_1L	Pk_2L	k_1^2L	k_2^2L	k_1k_2L	$S=P+k_1X_1+k_2X_2$
AB	1.5	0	$\frac{-1}{\sqrt{2}}$	0	0	0	0.75	0	0	–3.054 (Comp)
BC	1.5	0	$\frac{-1}{\sqrt{2}}$	0	0	0	0.75	0	0	– 3.054
CD	2	0	0	$\frac{-4}{5}$	0	0	0	1.28	0	– 3.072
DE	1.5	0	0	$\frac{-3}{5}$	0	0	0	0.54	0	– 2.304
EF	2	+5.71	0	$\frac{-4}{5}$	0	–9.13	0	1.28	0	2.638
FA	1.5	+5.71	$-\frac{1}{\sqrt{2}}$	0	–6.05	0	0.75	0	0	2.655
FC	1.5	+10	$-\frac{1}{\sqrt{2}}$	$\frac{-3}{5}$	–10.606	–9	0.75	0.54	0.636	4.641
AC	2.12	-8.08	+1	0	–17.12	0	2.12	0	0	–3.76
EC	2.5	-7.13	0	1	0	–17.82	0	2.5	0	–3.29
BF	2.12	0	+1	0	0	0	2.12	0	0	4.32
DF	2.5	0	0	1	0	0	0	2.5	0	3.84
					$\sum Pk_1^2$= -33.776	$\sum Pk_2L$= -35.95	$\sum k_1^2L$= 7.24	$\sum k_2^2L$= 8.64	$\sum k_1k_2L$= 0.636	

03.

Sol: The horizontal reaction 'H' at B is given by

$$H = -\frac{\sum \frac{PkL}{AE}}{\sum \frac{kL}{AE}} \qquad .... (1)$$

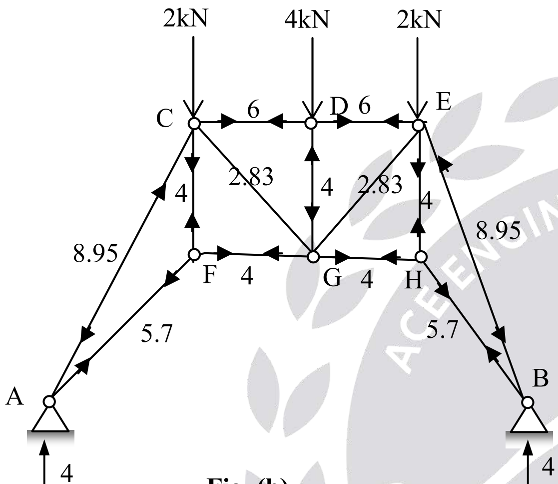


Fig. (b)

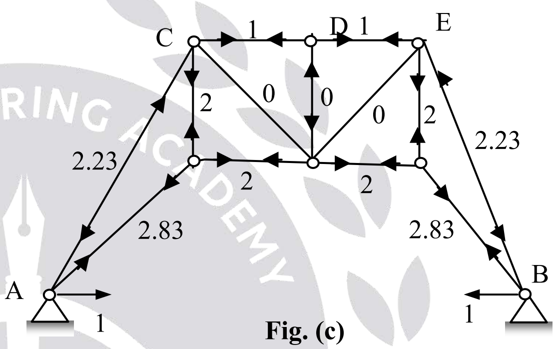


Fig. (c)

To calculate P, make the structure statically determinate by providing a roller at B, as shown in Fig. (b), where the stresses due to external loading have been marked.

To calculate k, remove the external loads and apply unit pull at the joint B as shown in Fig. (c). The stresses have been marked on the diagram.

The computations are done in the tabular form shown.

Substituting the values obtained from table in (1), we get

$$H = -\frac{-358200}{125700}$$

$$= 2.85 \text{ kN } (\leftarrow)$$

$$R_A = R_B = \sqrt{4^2 + (2.85)^2}$$

$$= 4.91 \text{ kN.}$$

(+ For Tensions; - for compression)

Member	Length L (mm)	P (kN)	K	PKL	K^2L
AC	4470	– 8.95	+ 2.23	– 89400	22210
CD	2000	– 6.0	+ 1.00	– 12000	2000
DE	2000	– 6.0	+ 1.00	– 12000	2000
EB	4470	– 8.95	+ 2.23	– 89400	22210
GH	2000	+4.0	– 2.00	– 16000	8000
FG	2000	+4.0	– 2.00	– 16000	8000
FA	2830	+5.7	– 2.83	– 45700	22640
HB	2830	+5.7	– 2.83	– 45700	22640
CF	2000	+4.0	– 2.00	– 16000	8000
CG	2830	+2.83	0	0	0
DG	2000	– 4.0	0	0	0
EG	2830	+2.83	0	0	0
EH	2000	+4.0	– 2.00	– 16000	8000
			Sum	–358200	125700

04.

Sol:

Note:

1. In case of statically determinate frames if any member is not of exact length and it is forced in position, there are no stress induced in the member of the frame.
2. In case of indeterminate frames, if the members are not of exact length, they will have to be fixed in position which will induce forces in the other members of the frame.

Force in the member having lack of fit 'x'

$$X = +\frac{\delta}{\sum \frac{K^2L}{AE}}$$

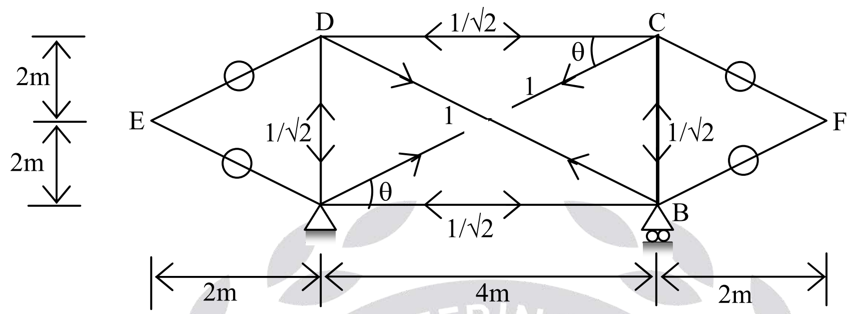
Where 'δ' is taken to be positive if the member is short in length (so as to exert pull 'X' at the joints) and negative if the member is excess in length (so as to apply push at the joints)

$D_S = D_{se} + D_{si}$

$= (3 - 3) + m - (2j - 3) = 10 - (2 \times 6 - 3) = 1 \rightarrow$ Indeterminate structure

→ To analyse the frame, it is made determinate by removing the member having lack of fit. Unit forces are applied at the joints of the members having lack of fit.

→ Member 'AC' is removed and unit loads are applied at joints 'A' and 'C'



k-values

Compression – ve, Tension +ve

A= 8 m^2, 8 × 10^2 mm^2

Member	k	L (mm)	$\frac{k^2L}{A}$	Lack of fit (kN)	Force in member due to kX
AB	$\frac{-1}{\sqrt{2}}$	4×10^3	2.5	–14.645	–7.286
DC	$\frac{-1}{\sqrt{2}}$	4×10^3	2.5	–14.645	–7.286
AD	$\frac{-1}{\sqrt{2}}$	4×10^3	2.5	–14.645	–7.286
BC	$\frac{-1}{\sqrt{2}}$	4×10^3	2.5	–14.645	–7.286
AC	1	$4\times10^3\sqrt{2}$	7.07	20.712	10.305
BD	1	$4\times10^3\sqrt{2}$	7.07	20.712	10.305
DE	0	$2\times10^3\sqrt{2}$	0	0	0
AE	0	$2\times10^3\sqrt{2}$	0	0	0
BF	0	$2\times10^3\sqrt{2}$	0	0	0
CF	0	$2\times10^3\sqrt{2}$	0	0	0
			$\sum\frac{k^2L}{A}$ =24.14		

$$X = \frac{0.25 \times 10}{\frac{24.14}{E}}$$

$$X = \frac{0.25mm \times 10}{\frac{24.14}{2 \times 10^5}} = 20712.510\,N$$

$$= 20.712 \text{ kN}$$

→ When member 'AC' is subjected to temperature of 20^0C

$$\delta = L\,\alpha\,t = 4000\sqrt{2} \times 1.1 \times 10^{-5} \times 20$$

$$= 1.244 \text{ mm}$$

Force in the member AC is given by

$$X_1 = \frac{\delta}{\sum \frac{k^2 L}{AE}}$$

$$X_1 = \frac{1.244}{\frac{24.142}{2 \times 10^5}} = 10305.691\,N$$

$$X_1 = 10.305 \text{ KN}$$

05.

Sol: Vertical deflection of the point 'C'

$$\delta_{VC} = \sum \frac{PkL}{AE} = \sum \frac{\sigma kL}{E}$$

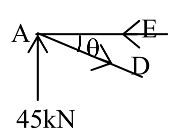
$$\cos\theta = \frac{4}{\sqrt{4^2 + 3^2}}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

@ Joint 'A': -

$$\sum V = 0$$



$$F_{AD} \sin\theta = 45 \text{ kN}$$

$$F_{AD} = \frac{45 \times 5}{3} = 75\,kN(\text{Tension})$$

$\sum H = 0$

$F_{AE} = F_{AD} \cos\theta$

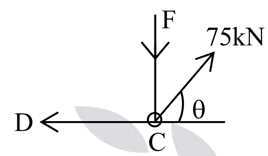
$$= \frac{75 \times 4}{5} = 60\,\text{kN(Compression)}$$

@ Joint 'C': -

$\sum H = 0$

$F_{CD} = 75 \times \cos\theta$

$$= \frac{75 \times 4}{5} = 60\ \text{kN(Tension)}$$

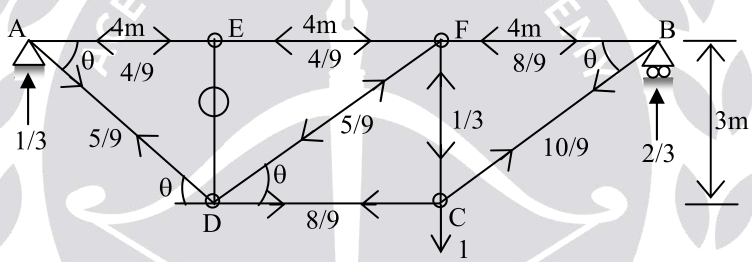


$\sum v = 0$

$F_{FC} = 75 \times \sin\theta$

$$= 75 \times \frac{3}{5} = 45\ \text{kN(compression)}$$

→ k - values:



$R_A + R_B = 1$

$-R_B \times 12 + 1 \times 8 = 0$

$$R_B = \frac{8}{12} = \frac{2}{3}\ (\uparrow)$$

$$R_A = 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}\ (\uparrow)$$

At Joint 'A': -

$\sum V = 0$

$$F_{AD} \sin\theta = \frac{1}{3}$$

$$F_{AD} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}\ \text{(Tension)}$$

$\sum H = 0$

$$F_{AE} = \frac{5}{9}\cos\theta = \frac{5}{9} \times \frac{4}{5} = \frac{4}{9}\ \text{(comp)}$$

At Joint 'D':-

$\sum V = 0$

$F_{DF} \sin\theta = \frac{5}{9} \times \sin\theta$

$F_{DF} = \frac{5}{9}$ (Comp)

$\sum H = 0$

$F_{DC} = \frac{5}{9} \times \cos\theta + \frac{5}{9} \cos\theta$

$= \frac{5}{9} \times \frac{4}{5} + \frac{5}{9} \times \frac{4}{5}$

$= \frac{8}{9}$ (Tension)

At Joint 'B': -

$\sum V = 0$

$F_{BC} \sin\theta = \frac{2}{3}$

$F_{BC} \times \frac{3}{5} = \frac{2}{3}$

$F_{BC} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$ (Tension)

$\sum H = 0$

$F_{BF} = \frac{10}{9} \times \cos\theta$

$= \frac{10}{9} \times \frac{4}{5} = \frac{8}{9}$ (Comp)

At Joint 'F': -

$F_{FC} = \frac{5}{9} \times \sin\theta$

$= \frac{5}{9} \times \frac{3}{5} = \frac{1}{3}$ (Tension)

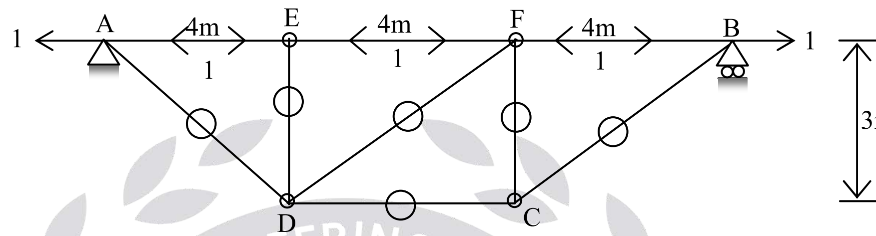
Compression –ve, Tension +ve

Member	J N/mm^2	K	L(mm)	σKL
AE EF FB DC	–80 –80 –80 +100	–4/9 –4/9 –8/9 8/9	4×10^3 4×10^3 4×10^3 4×10^3	142.2×10^3 142.2×10^3 284.4×10^3 355.55×10^3
ED FC	–80 –80	0 1/3	3×10^3 3×10^3	0 -80000
AD DF BC	100 0 100	+ 5/9 –5/9 10/9	5×10^3 5×10^3 5×10^3	277.77×10^3 0 555.55×10^3
				1677.67×10^3

$$\delta_{VC} = \frac{\sum \sigma KL}{E} = \frac{1667.67\times10^{3}}{2\times10^{5}}$$

δ_{VC} = 8.38 mm (↓)

→ Lateral displacement of the end 'B'.



$$\delta_{HB} = \sum \frac{\sigma k_1 L}{E}$$

$$= \frac{-80\times1\times4\times10^{3} - 80\times1\times4\times10^{3} - 80\times1\times4\times10^{3}}{2\times10^{5}}$$

δ_{HB} = - 4.8 mm (←)

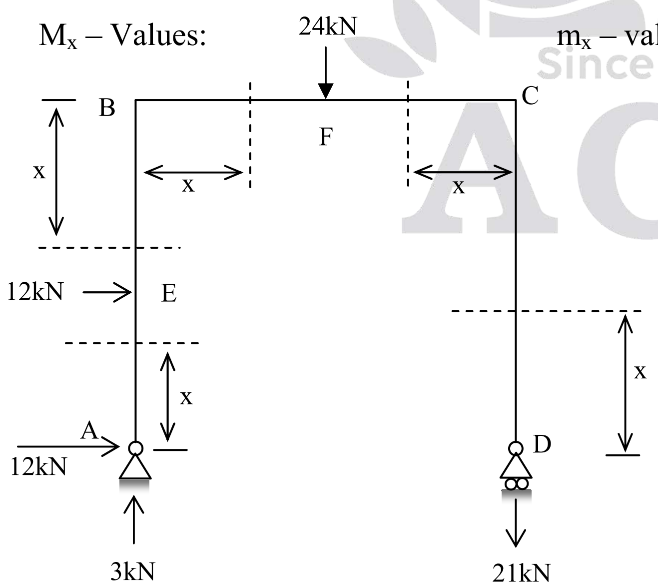
07.

Sol: Horizontal displacement at D = $\int_{O}^{L} \frac{M_x m_x}{EI} dx$

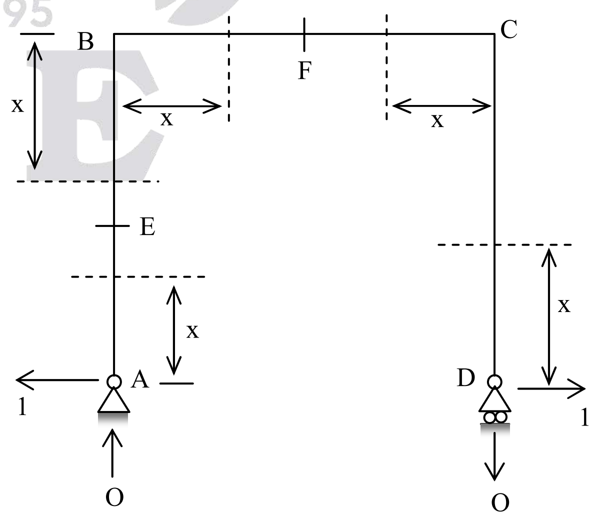
Where, M_x = BM at a section x – x due to real loads

M_x = BM at a section x – x due to vertical unit load applied where we want the deflection.

M_x – Values:



m_x – values: -

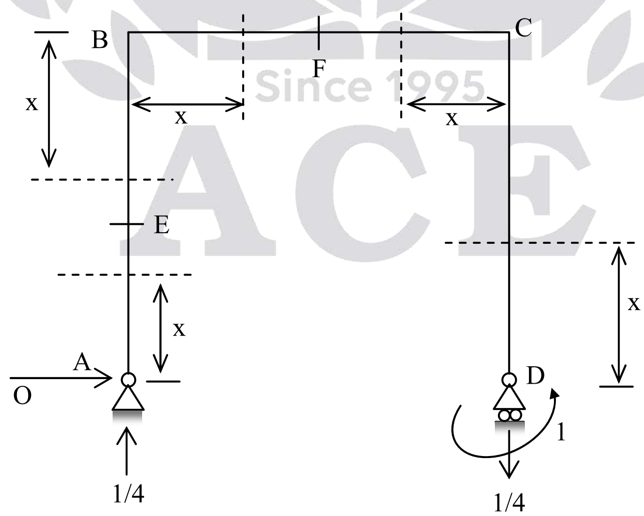


Sign conventions:
Sagging BM is +ve
Hogging BM is –ve

Member	M_x – values	m_x – values	$\int_O^L \frac{M_x m_x}{EI} dx$
DC	0	x	$\int_0^5 0$
CF	21x	5	$\int_0^2 \frac{105x}{EI} = \frac{210}{EI}$
BF	$21(4-x) - 24(2-x)$ $= 36 + 3x$	5	$\int_0^2 \frac{(36+3x)5}{EI} = \frac{390}{EI}$
BE	$12(5-x) - 12(2-x)$ $= 36$	$5 - x$	$\int_0^2 \frac{36(5-x)}{EI} = \frac{288}{EI}$
AE	12 x	x	$\int_0^3 \frac{12x^2}{EI} = \frac{108}{EI}$

$$\delta_{HD} = \frac{996}{EI}$$

Rotation at 'D' = $\int_O^L \frac{M_x m_x}{EI} dx$



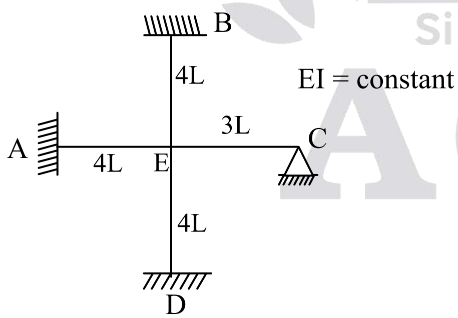
Member	M_x – Values	m_x	$\int_0^L \frac{M_x m_x}{EI} dx$
DC	0	1	$\int_0^5 0$
CF	21x	$1-\frac{x}{4}$	$\int_0^2 21x\left(1-\frac{x}{4}\right)\times\frac{1}{EI}$
BF	21(4 – x) – 24(2 – x) = 36 + 3x	$\frac{x}{4}$	$\int_0^2 (36+3x)\frac{x}{4}\times\frac{1}{EI}$
BE	12(5 – x) – 12(2 – x) = 36	0	$\int_0^2 0$
AE	12 x	0	$\int_0^3 0$

$$\theta_D = \int_0^L \frac{M_x m_x}{EI} dx = \frac{48}{EI}$$

06. Moment Distribution Method

01. Ans: (a)

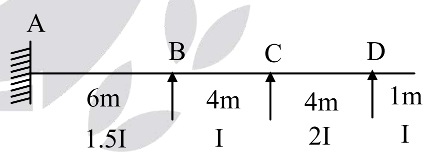
Sol:



$$(D.F)_{BE} = \frac{I/4L}{\frac{I}{4L}+\frac{I}{4L}+\frac{I}{4L}+\frac{3}{4}\times\frac{I}{3L}} = \frac{1}{4}$$

02. Ans: (c)

Sol:

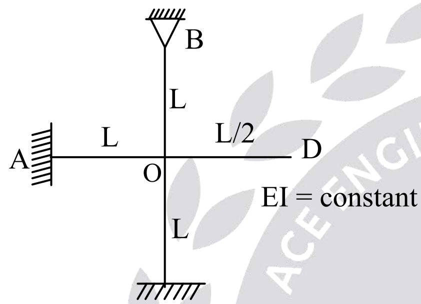


Joint	Member	Relative stiffness 'k'	Distribution factor D.F= k / Σk
B	BA	1.5I/6	0.5
	BC	I/4	0.5
C	CB	$\frac{I}{4}$	0.4
	CD	$\frac{3}{4}\left(\frac{2I}{4}\right)$	0.6

Note: Over hang present beyond 'D' does not give fixity. Hence 'D' will act like simple support. 'B' and 'C' have other supports beyond them. Hence they act like fixed supports to calculate stiffness

03. Ans: (a)

Sol:



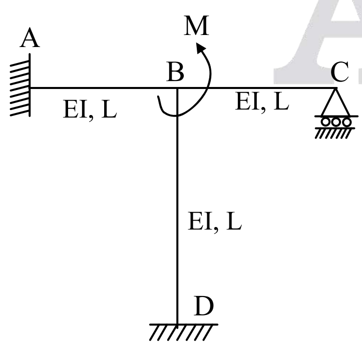
Rotational stiffness of a joint is the sum of stiffness of all members meeting at that joint

$\therefore\ K_O = K_{OA} + K_{OB} + K_{OC} + K_{OD}$

$$\Rightarrow \frac{4EI}{L} + \frac{3EI}{L} + \frac{4EI}{L} + 0 = \frac{11EI}{L}$$

04. Ans: (b)

Sol:



Rotational stiffness of joint 'B'

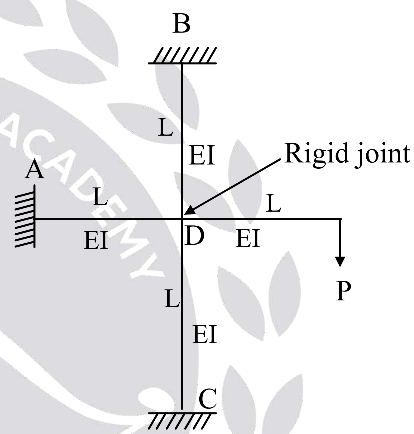
$$= \frac{11EI}{L}$$

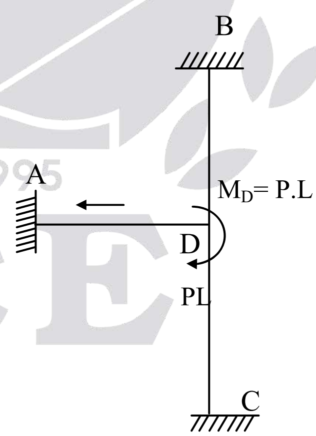
$$\Rightarrow \frac{M}{\theta} = \frac{11EI}{L},\ \theta = \frac{ML}{11EI}$$

θ = Rotation of joint 'B'.

05. Ans: (b)

Sol:





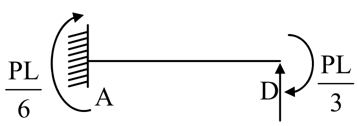
Member	D.F
DB	1/3
DA	1/3
DC	1/3

Moment at 'D' transferred from over hang,

$M_D = P.L$

Distribution factors are $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$ to DA, DB, DC respectively.

$\therefore M_{DA} = \frac{PL}{3}$

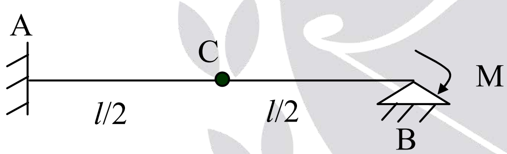


$\Rightarrow M_A = \frac{1}{2} \times \frac{PL}{3} = \frac{PL}{6}$

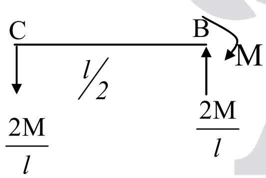
(Far end 'A' is fixed, hence the carry over moment is half of that of moment of near end 'D' of beam 'AD')

06. Ans: (d)

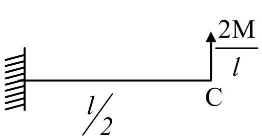
Sol:



Consider free body diagram of 'BC'



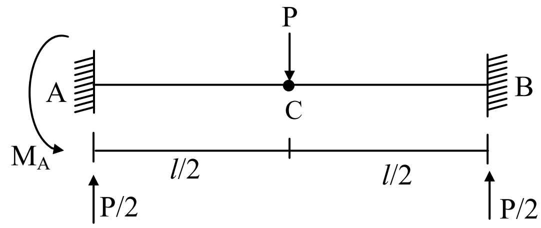
Consider free body diagram of 'AC'



Moment at 'A' = $\frac{2M}{l} \times \frac{l}{2} = M$ ↻

07. Ans: (c)

Sol:



Load is acting at center of the beam.

$\therefore R_A = R_B = \frac{p}{2}$ (↑)

As center 'C' has an internal moment hinge

$\Sigma M_C = 0$

$\therefore M_A = R_B \times \frac{L}{2}$

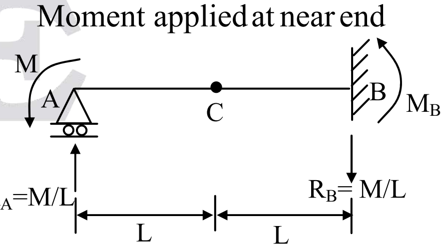
$= \frac{p}{2} \times \frac{L}{2}$

$\therefore M_A = \frac{pl}{4}$ (anticlockwise)

08. Ans: (d)

Sol: Carry over factor

$$C_{AB} = \frac{\text{Moment developed at far end}}{\text{Moment applied at near end}}$$



Let us apply moment 'M' at A

For R_A ; take moment @ C = 0

$\therefore \Sigma M_c = 0 \quad \therefore R_A \times L = M$

$R_A = M/L$ (upward)

$\& R_B = \dfrac{M}{L}$ (downward)

Again $\sum M_c = 0$ from right side

$\therefore\ M_B = R_B \times L$

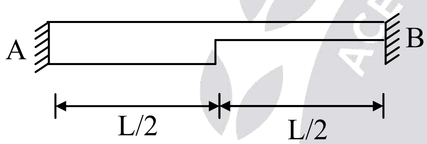
$$M_B = \frac{M}{L} \times L$$

$\therefore\ M_B = M$

$$\text{Carry over factor } = \frac{\text{Moment at B}}{\text{Moment at A}} = \frac{M}{M} = 1$$

09. Ans: (c)

Sol:



For prismatic beam with uniform EI,

The carry over factor = $\dfrac{1}{2}$

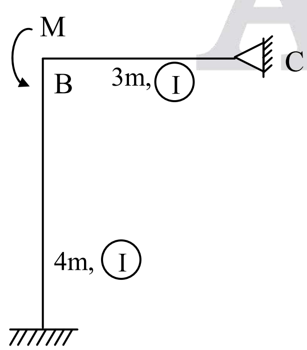
Beam towards 'A' is more stiff (has more EI)

Beam towards 'B' is less stiff (has less EI)

$\therefore$Carry over factor to 'B' is less than half

10. Ans: (a)

Sol:



	k	D.F
BA	$I/4$	$1/2$
BC	$\frac{3}{4}\frac{I}{3}$	$1/2$

$D.F_{BA} = 1/2$

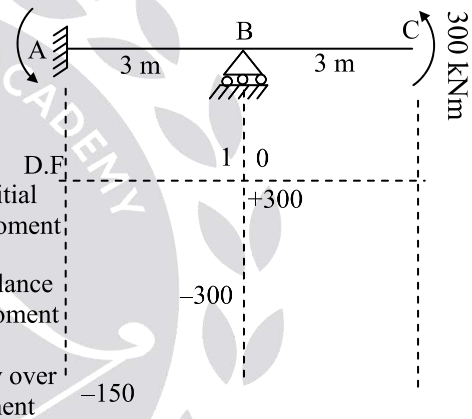
$D.F_{BC} = 1/2$

Hence applied joint moment 'M' gets equally distributed to members 'BA' and 'BC'.

$\therefore\ M_{BA} = M/2,\ M_{BC} = M/2$

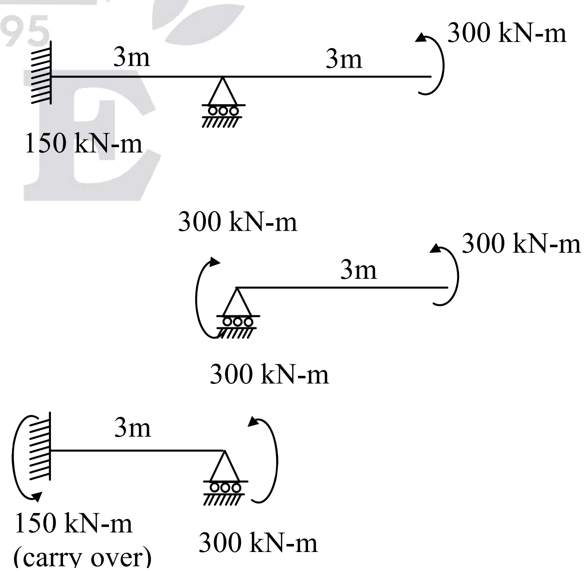
11. Ans: (a)

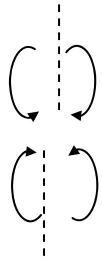
Sol:



$\therefore$ Correct answer is 150 kN.m hogging.

Shortcut:



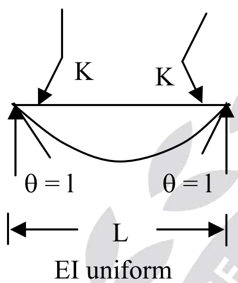


−Ve Or hogging moments

+Ve Or sagging moments

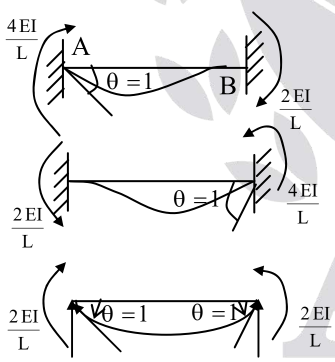
12. Ans: (b)

Sol:



We know that moment required to produce a unit rotation is called stiffness.

∵ Slope θ = 1 at both ends



Initially for $\theta = 1$ (clockwise) At A, keeping 'B' fixed.

$$M_{AB} = \frac{4EI}{L} \text{ (Clockwise)}$$

$$M_{BA} = \frac{2EI}{L} \text{ (Clockwise)}$$

Then allow $\theta = 1$ (anticlockwise) at B, keeping 'A' as fixed.

Now,

$$M_{BA} = \frac{4EI}{L} \text{ (anticlockwise)}$$

$$M_{AB} = \frac{2EI}{L} \text{ (anti clockwise)}$$

If unit rotation at both ends, as shown

$$M_{AB} = \frac{4EI}{L} - \frac{2EI}{L}$$

$$= \frac{2EI}{L} \text{ (Clockwise)}$$

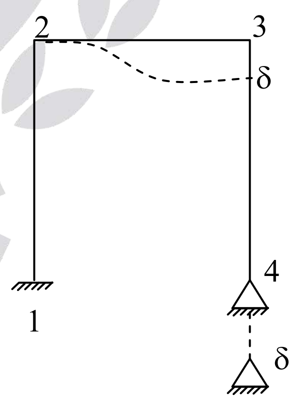
$$M_{BA} = \frac{4EI}{L} - \frac{2EI}{L}$$

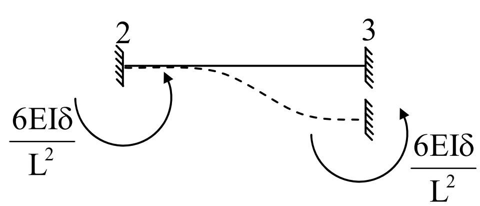
$$= \frac{2EI}{L} \text{ (Anti clockwise)}$$

Hence, $K = \frac{2EI}{L} = M$

13. Ans: (b)

Sol:





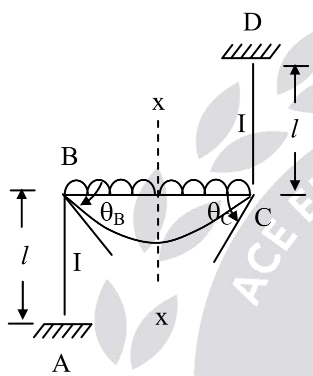
$$M_{23} = \frac{6EI\delta}{L^2}$$

$$= \frac{6EI\delta}{4^2}$$

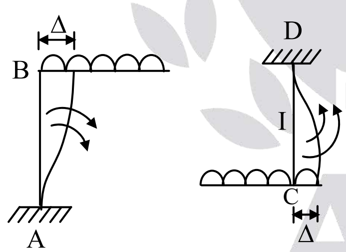
$$= \frac{6EI\delta}{16}$$

14. Ans: (b)

Sol:



Consider the section passing through the middle of the beam (x–x)



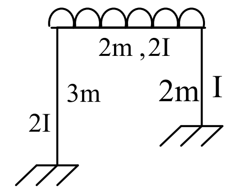
∴ Δ is present.

From the above diagram, it is seen that in the member BC rotation is clock wise at B and anticlockwise at C.

∴ $\theta_B = -\theta_C$

15. Ans : (b)

Sol:



Moment required for
sway of right column

$$= \frac{6EI\delta}{2^2} = \frac{6EI\delta}{4}$$

$$= \frac{3}{2}EI\delta = 1.5EI\delta$$

Moment required for sway of left column

$$= \frac{6(2EI)\delta}{3^2}$$

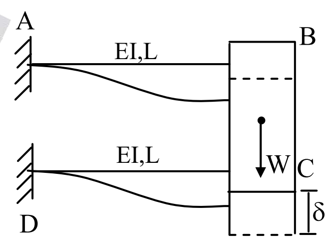
$$= \frac{4}{3}EI\delta = 1.33\,EI.\delta$$

As the left column requires less moment for sway compared to right column, the resistance of left column is less against sway.

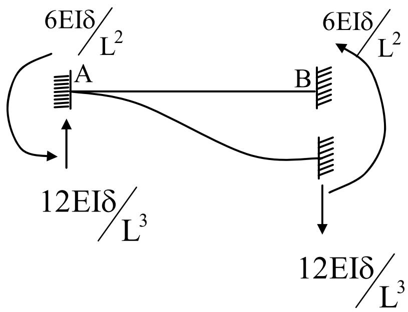
∴Frame will sway towards left

16. Ans: (b)

Sol:



Hint: As bar 'BC' is rigid it acts like sinking fixed support.



Free body diagram of 'AB'

As seen from above F.B.D. the ↓ reaction developed at B is $12\ EI\delta/L^3$.

Similarly form F.B.D of 'CD' the ↓ reaction developed at 'C' is $12EI\delta/L^3$.

$\therefore$ from vertical equilibrium condition,

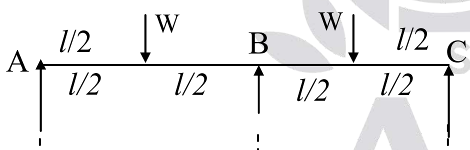
Wt. of rigid block $W = 12EI\delta/L^3 + 12EI\delta/L^3$

$= 24EI\delta/L^3$

$\Rightarrow$ down ward deflection $\delta = WL^3/24EI$

17. Ans: (a)

Sol:



	A	B	B	C
D.F		½	½	
Initial moments	$\frac{-wl}{8}$	$\frac{+wl}{8}$	$\frac{-wl}{8}$	$\frac{+wl}{8}$
Balance moments	$\frac{+wl}{8}$			$\frac{-wl}{8}$
C.O	0	$\frac{+wl}{16}$	$\frac{-wl}{16}$	0
Final Moments	0	$\frac{+3wl}{16}$	$\frac{-3wl}{16}$	0

Free moment at centre of AB = $\frac{WL}{4}$

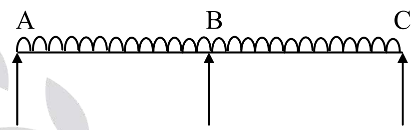
Using the Moment distribution method

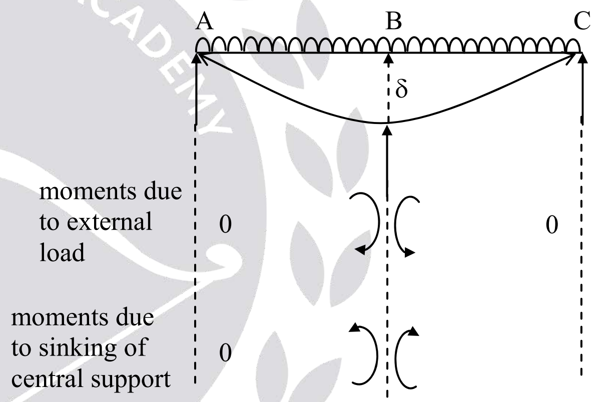
Moment at support B, $M_B = \frac{3wl}{16}$

The ratio of support moment at 'B' and free moment of AB = $\frac{3WL}{16} \times \frac{4}{WL} = 0.75$

18. Ans: (a)

Sol:





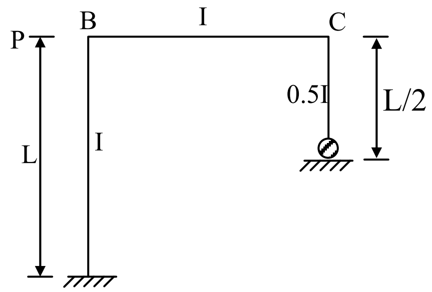
The directions of moments at central support due to external load and sinking of central support are shown.

As seen above, the net central support moment (negative moment) reduces.

From the fundamentals of redistribution of moments, if negative moment at central support decreases, the positive (sagging) moment at midspan increases.

19. Ans: 1

Sol:



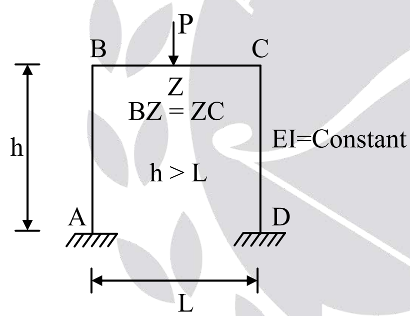
Sway moment, $M_{BA} \propto \dfrac{6EI\delta}{L^2}$

Sway moment, $M_{CD} \propto \dfrac{3E(0.5I)\delta}{(0.5L)^2} = \dfrac{6EI\delta}{L^2}$

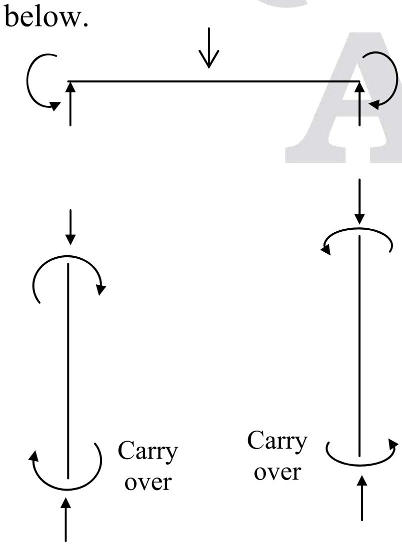
$$\therefore \frac{M_{BA}}{M_{CD}} = 1$$

20. Ans: 4

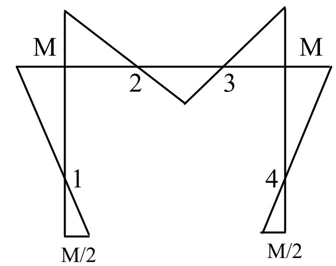
Sol:



Free body diagrams of beam and columns are shown below.



The B.M.D of the frame is shown below.



At the locations 1, 2,3and 4, the bending moment is changing sign. Hence, four points of contra flexure.

21. *Refer GATE solutions Book.(2004)*

22. *Refer GATE solutions Book.(2006)*

23. Ans: (d)

Sol: After distribution of unbalanced moment to adjacent span, one half of this amount with some sign is carried over to other end of respective sign.

$\therefore$ Option (d) is incorrect.

24. Ans: (a)

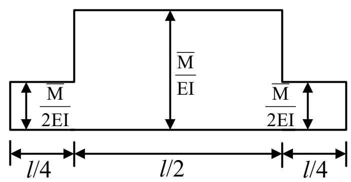
Sol: All the given statements are correct w.r.t Hardycorss (or) Moment distribution method.

25. Ans: (b)

Sol: To find out fixed end moments

Area of fixed bending moment diagram

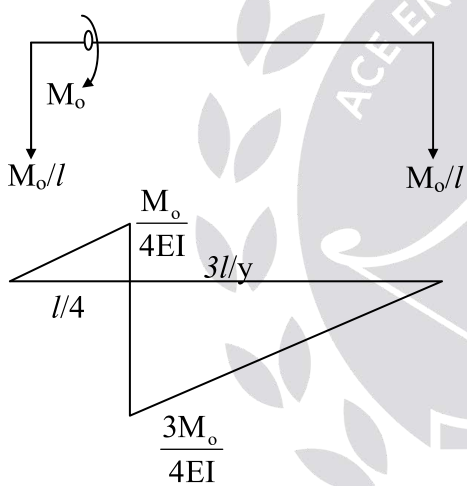
= Area of free bending moment diagram



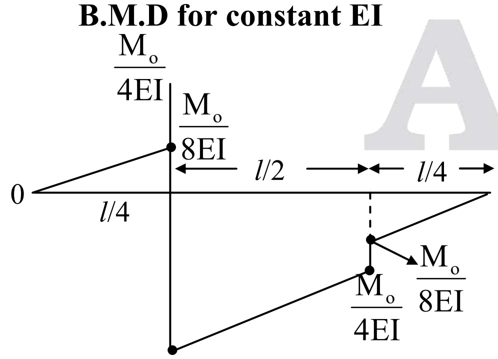
$$\text{Area of fixed BMD} = \frac{\ell}{4}\times\frac{\overline{M}}{2EI}\times 2+\frac{\overline{M}}{EI}\times\frac{\ell}{2}$$

$$=\frac{\overline{M}}{4EI}+\frac{\overline{M}}{2EI} \qquad =\frac{3}{4}\frac{\overline{M}}{EI}\ \ldots.\ (1)$$

Free BMD under load M_0 (part – 1):-



B.M.D for constant EI



B.M.D for varying EI

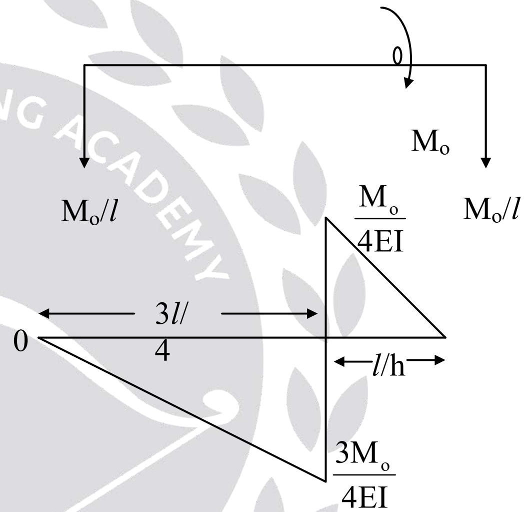
Area of the BMD

$$=-\frac{1}{2}\times\frac{\ell}{4}\times\frac{M_o}{8EI}+\frac{1}{2}\times\frac{\ell}{2}\times\left(3\frac{M_o}{4EI}+\frac{M_o}{4EI}\right)$$

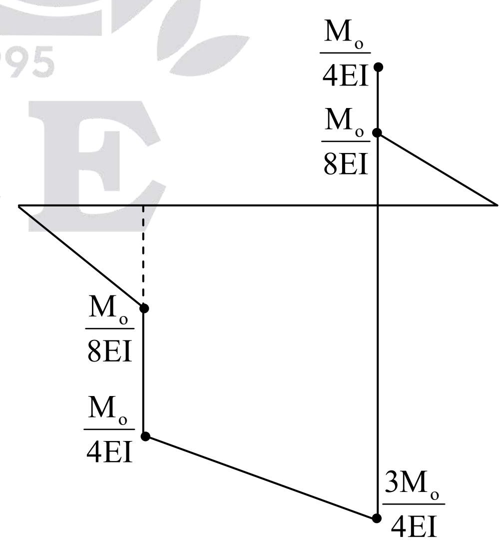
$$+\frac{1}{2}\times\left(\frac{M_o}{EI}\right)=\frac{M_o}{4EI}$$

$$=\frac{1}{2}\times\frac{\ell}{2}\left(\frac{M_o}{EI}\right)=\frac{M_o}{4EI}$$

Free BMD under load M_0 (part – 2):



B.M.D for constant EI



B.M.D for varying EI

Area of free BMD

$$= \frac{1}{2}\times\frac{\ell}{4}\times\frac{M_o}{8EI}+\frac{1}{2}\times\frac{\ell}{2}\left(\frac{M_2}{4EI}+\frac{3}{4}\ \frac{M_o}{4EI}\right)-\frac{1}{2}\times\frac{\ell}{4}\times\frac{M_o}{8EI}$$

$$= \frac{1}{2}\times\frac{\ell}{2}\left(\frac{M_o}{EI}\right)$$

$$= \frac{M_o}{4EI}$$

Total area of BMD

= Area of free BMD (Part I) + Area of free BMD (Part II)

$$= \frac{M_o}{4EI}+\frac{M_o}{4EI} \qquad = \frac{M_o}{2EI} \qquad \ldots\ldots(2)$$

Equate (1) and (2)

$$\therefore \frac{3}{4}\ \frac{\overline{M}}{EI}=\frac{M_0}{2EI}$$

$$M=\frac{2M_0}{3}$$

Conventional Practice Solutions

01.

Sol: Fixed end moments:

$$M_{FAB} = -\frac{Wab^2}{\ell^2}-\frac{6EI\delta}{\ell^2}$$

$$= -\frac{8\times2\times3^2}{5^2}-\frac{6\times2.1\times10^8\times85\times10^{-7}}{5^2}\times\frac{10}{1000} = -5.76-4.284 = -10.044 \text{ kNm}$$

$$\left[E=\frac{2.1\times10^5}{1000}\times1000^2 = 2.1\times10^8 \text{ kN/m}^2,\right.$$

$$\left. I = 85\times10^5\text{ mm}^4 = \frac{85\times10^5}{(1000)^4} = 85\times10^{-7}\text{ m}^4 \qquad \delta=\frac{10}{1000}m\right]$$

$$M_{FBA} = +\frac{Wa^2b}{\ell^2}-\frac{6EI\delta}{\ell^2} = \frac{8\times2^2\times3}{5^2}-4.284 = -0.444\text{ kNm}$$

$$M_{FBC} = -\frac{Wab^2}{\ell^2}-\frac{6EI\delta}{\ell^2} = -\frac{5\times4\times3^2}{7^2}+\frac{6\times2.1\times10^8\times85\times10^{-7}}{7^2\times1000}\times10$$

[$\therefore$ In BC, left support sinks, $\delta = -10$ mm]

$= -3.673 + 2.186 = -1.487$ kNm

$$M_{FCB} = +\frac{Wa^2b}{\ell^2} + \frac{6EI\delta}{\ell^2} = +\frac{5\times 4^2\times 3}{7^2} + \frac{6\times 2.1\times 10^8\times 85\times 10^{-7}\times 10}{7^2\times 1000}$$

$$= 4.898 + 2.186 = +7.084 \text{ kNm}$$

$$M_{FCD} = -\frac{W\ell^2}{12} = -\frac{1\times 8^2}{12} = -5.33\,\text{kNm}$$

$$M_{FDC} = +\frac{W\ell^2}{12} = +\frac{1\times 8^2}{12} = +5.33\,\text{kNm}$$

Distribution Factor:

Joint	Member	Relative Stiffness	Sum	Distribution Factor
B	BA BC	3/4 × I/ℓ =(3/4)×I/5 =3I/20 I/ℓ =I/7	(3I/20)+(I/7) =(21I+20I)/140 = 41I/140	(3I/20) × (140/41I) =0.512 (I/7)×(140/41I)=0488
C	CB CD	(I/7) (3/4)×(I/8)=(3I/32)	(I/7)+(3I/32) =(32I+21I)/224 =53I/224	(I/7)×(224/53I)=0.604 (3I/32)×(224/53I)=0.396

Moment distribution:

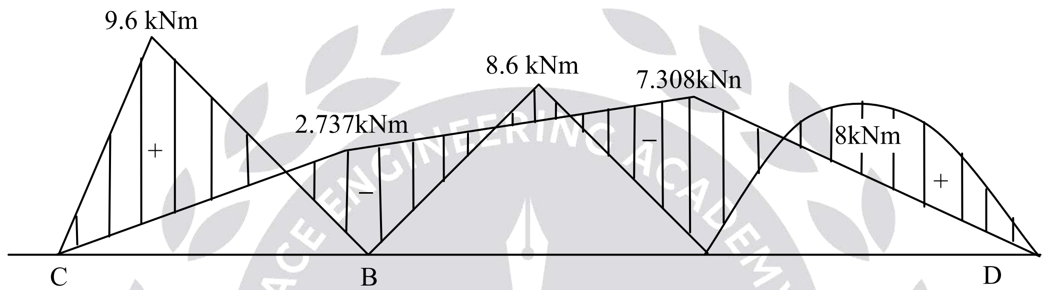
Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
Distribution Factor	–	0.512	0.488	0.604	0.396	–
FEM Release A, D and Carry over	– 10.044 +10.044	– 0.444 5.022	–1.487	7.084	–5.33 –2.665	+5.33 – 5.33
Initial moments Balance	0 –	+4.578 –1.583	–1.487 –1.508	+7.084 +0.550	–7.995 +0.361	0 –
Carry Over Balance	– –	– – 0.141	0.275 –0.134	–0.754 0.455	– 0.299	– –
Carry over Balance	– –	– –0.117	0.228 –0.111	–0.067 0.040	– 0.027	– –
Final moments	0	+2.737	–2.737	+7.308	–7.308	0

Simply supported bending moments:

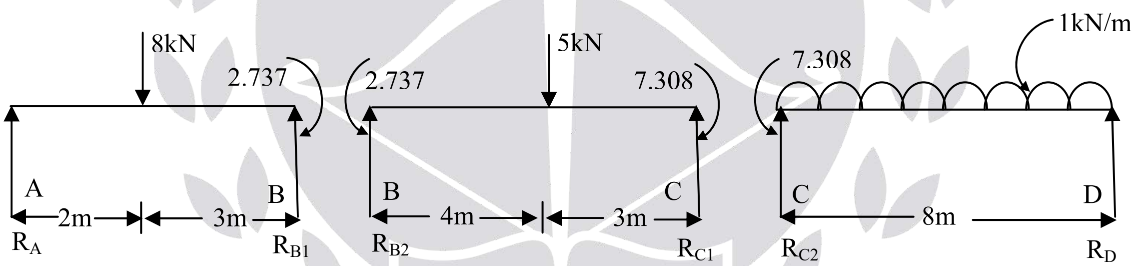
$$M_{AB} = \frac{Wab}{l} = \frac{8\times2\times3}{5} = 9.6kNm$$

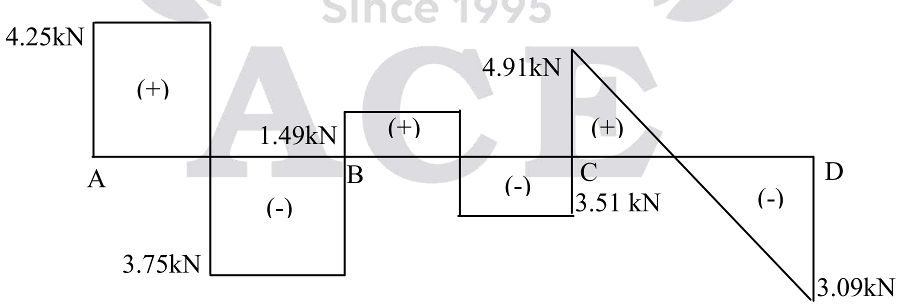
$$M_{BC} = \frac{Wab}{l} = \frac{5\times4\times3}{7} = 8.6kNm$$

$$M_{CD} = \frac{Wl^2}{8} = \frac{1\times8^2}{8} = 8\,kNm$$



Bending Moment Diagram





Shear Force Diagram

02.

Solution: (a) Fixed End Moments:

Span AB: $M_{FAB} = \dfrac{-Wab^2}{\ell^2} = -\dfrac{100\times 2\times (3)^2}{5^2} = -72\,\text{kNm}$

$M_{FBA} = \dfrac{Wa^2b}{\ell^2} = +\dfrac{100\times 2^2\times 3}{5^2} = +48\,\text{kNm}$

Span BC: $M_{FBC} = -\dfrac{w\ell^2}{12} = -\dfrac{20\times (3)^2}{12} = -15\,\text{kNm}$

$M_{FCB} = +\dfrac{w\ell^2}{12} = +15\,\text{kNm}$

Span BD: $M_{FBD} = M_{FDB} = 0$ (As there is no lateral load on span BD)

(b) Distribution Factors:

Joint	**Members**	**Relative Stiffness (R.S)**	**Total Stiffness or Sum (T.S)**	**Distribution Factor = R.S/T.S**
B	BA	$\dfrac{2I}{5}$	$\dfrac{2I}{5}+\dfrac{I}{3}+\dfrac{I}{4}=\dfrac{59I}{60}$	$\dfrac{2I/5}{59I/60}=\dfrac{24}{59}=0.407$
	BD	$\dfrac{I}{3}$		$\dfrac{I/3}{59I/60}=\dfrac{20}{59}=0.339$
	BC	$\dfrac{3}{4}\times\dfrac{I}{3}=\dfrac{I}{4}$		$\dfrac{I/4}{59I/60}=\dfrac{15}{59}=0.254$

(c) Moment Distribution:

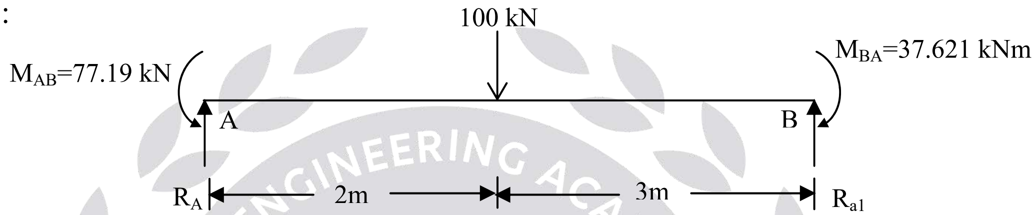
Joint	**A**	**B**			**C**
Member	**AB**	**BA**	**BD**	**BC**	**CB**
Distribution Factor	–	0.407	0.339	0.254	–
Fixed End Moments	– 72	48	0 0	– 15	15
Release C and Carry over to B				– 7.5	– 15
Initial Moments Balancing	– 72	48 – 10.379	– 8.645	– 22.5 – 6.477	0
Carry Over	– 5.19				
Final moments	– 77.19	37.621	– 8.645	– 28.977	0

We know that, A moment which rotates the near end of a prismatic beam without translation, the far end being fixed induces at the far end, a moment of one half its magnitude and in the same direction (i.e. of the same sign)

$$M_{DB} = \frac{1}{2} M_{BD} = \frac{1}{2}(-8.645) = -4.323 \text{ kNm}$$

(d) To Draw S.F.D:

Span AB:



Taking moments about B,

$R_A(5) + M_{BA} - 77.19 - 100(3) = 0$

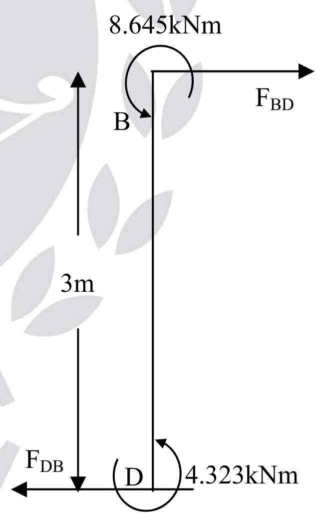
$R_A(5) + 37.621 - 77.19 - 300 = 0$

$R_A = 67.914$ kN

R_{Ba} = Total load – 67.914 = 100 – 67.914

$R_{Ba} = 32.086$ kN

Span BD:



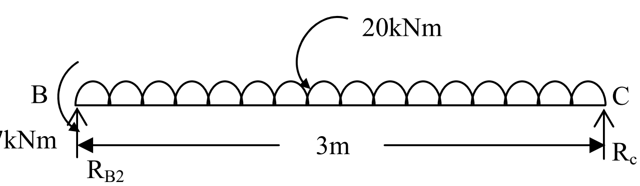
Taking moments about D, $8.645 + 4.323 - F_{BD}(3) = 0$

$F_{BD} = 4.323$ kN (→)

$F_{DB} = 4.323$ kN (←)

Span BC:

Taking moments about C,

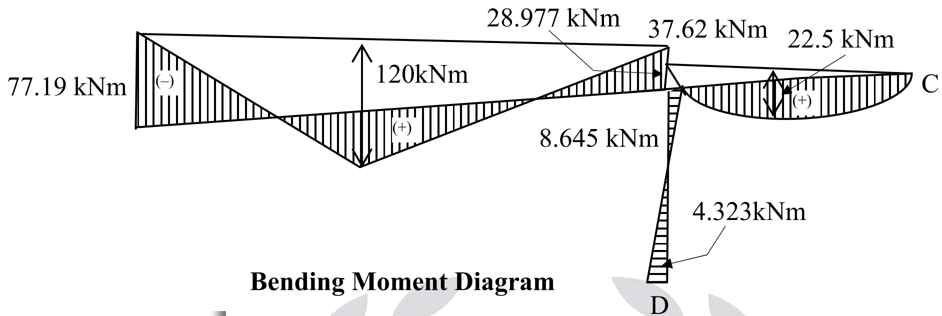


$$R_{B2}(3) - M_{BC} - \frac{w\ell^2}{2} = 0$$ M_{HC}=28.977kNm

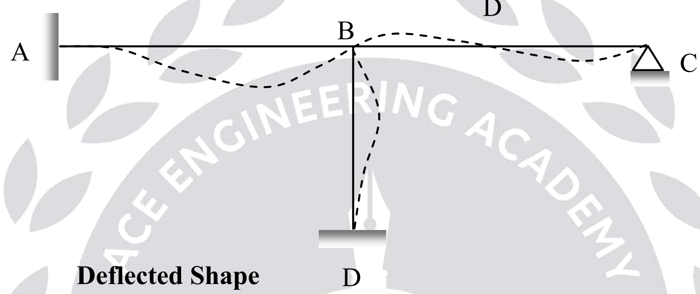
$$R_{B2}(3) - 28.977 - \frac{20 \times 3^2}{2} = 0$$

$R_{B2} = 39.659$ kN

R_C = Total load − R_{B2} = 20 × 3 − 39.659 = 20.341 kN



Bending Moment Diagram



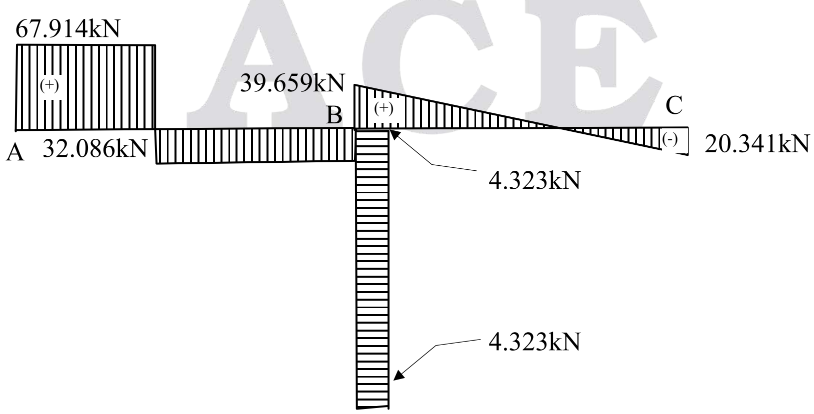
Deflected Shape

Simply supported Bending moments:

$$\text{Span AB} = \frac{Wab}{\ell} = \frac{100 \times 2 \times 3}{5} = 120\,\text{kNm}$$

Span BD = 0

$$\text{Span BC} = \frac{w\ell^2}{8} = \frac{20 \times 3^2}{8} = 22.5\,\text{kNm}$$



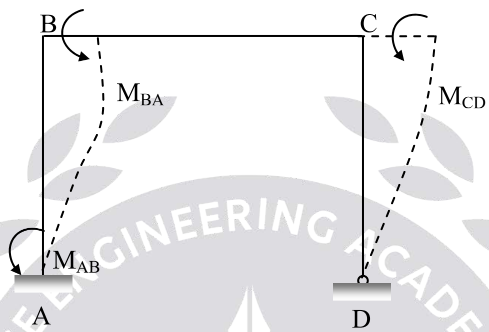
Shear force Diagram

03.

Sol: (a) Fixed End Moments:

Span AB, BC, CD: $M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$

Since the load is acting only at the joint, there will be no fixed end moments. However due to side sway, moments will be induced at joints A, B and C.



(b) Distribution Factors:

Joint	Member	Relative Stiffness	Total Stiffness	Distribution Factor
B	BA	$\frac{I}{5}$	$\frac{I}{5}+\frac{I}{5}=\frac{2I}{5}$	$\frac{I/5}{2I/5}=\frac{1}{2}$
	BC	$\frac{I}{5}$		$\frac{I/5}{2I/5}=\frac{1}{2}$
C	CB	$\frac{I}{5}$	$\frac{I}{5}+\frac{3I}{20}=\frac{7I}{20}$	$\frac{I/5}{7I/20}=\frac{4}{7}$
	CD	$\frac{3}{4}\times\frac{I}{5}=\frac{3I}{20}$		$\frac{3I/20}{7I/20}=\frac{3}{7}$

(c) Side Sway:- Under the action of the 50 kN load, there will be side sway to the right and the columns AB and CD will rotate in a clockwise direction. Thus negative moments will be induced at A, B and C in these columns. As the end 'A' is fixed and 'D' is hinged, the ratio of moments will be,

$$\frac{M_{BA}}{M_{CD}} = \frac{6EI\delta/\ell_1^2}{3EI\delta/\ell_2^2} = \frac{2}{1} = 2 \quad \Rightarrow M_{BA} = 2M_{CD} \quad (I_1 = I_2 = I;\ \ell_1 = \ell_2 = \ell)$$

Let us, first of all assume arbitrary value of these moments and find out the corresponding sway force.

Let $M_{CD} = -10$ kNm

$M_{BA} = -20$ kNm $= M_{AB}$

(D) Moment Distribution:

Joint	**A**	**B**		**C**		**D**
Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
Distribution Factor	–	1/2	1/2	4/7	3/7	–
Fixed End Moments	– 20	– 20	–	–	– 10	–
Balancing		10	10	5.71	4.29	
Carry Over	5		2.86	5		
Balancing		–1.43	– 1.43	–2.86	– 2.14	
Carry Over	–0.72		–1.43	–0.72		
Balancing		0.72	0.72	0.41	0.31	
Carry Over	0.36		0.21	0.36		
Balancing		–0.11	–0.11	–0.21	–0.15	
Carry Over	–0.06		–0.11	–0.06		
Balancing	0.06	0.05	0.06	0.03	0.03	
Final moments	–15.42	–10.76	10.77	7.66	–7.66	0

(e) Sway Forces:

Balancing moment for AB = 10.76 + 15.42 = 26.18 kNm

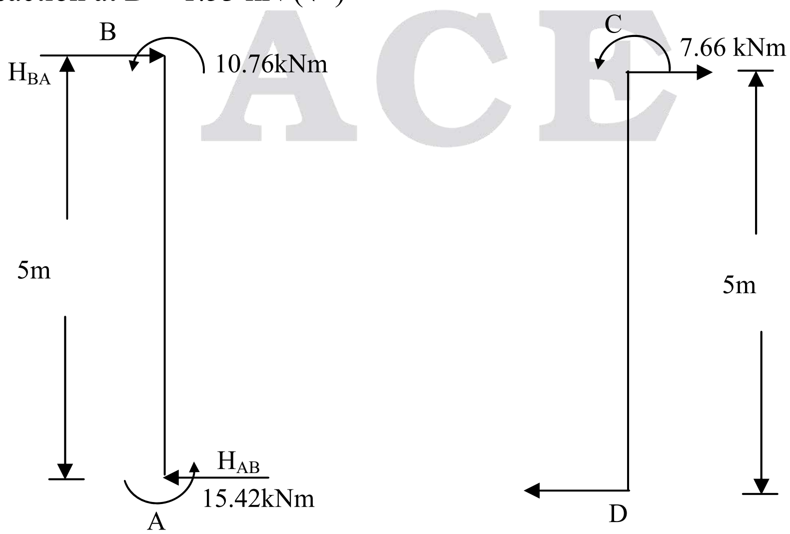
Horizontal reaction at B = (26.18)/5 = 5.24 kN (→) = H_{BA}

Horizontal reaction at A = 5.24 kN (←) = H_{AB}

Balancing moment for CD = 7.66 kNm

Horizontal reaction at C = (7.66)/5 = 1.53 kN (→)

Horizontal reaction at D = 1.53 kN (←)



The sway force causing the assumed moments = 5.24 + 1.53 = 6.77 kN (←)

But actual sway force is 50 kN; hence the moments be increased proportionately in the ratio of (50)/(6.77) = 7.38

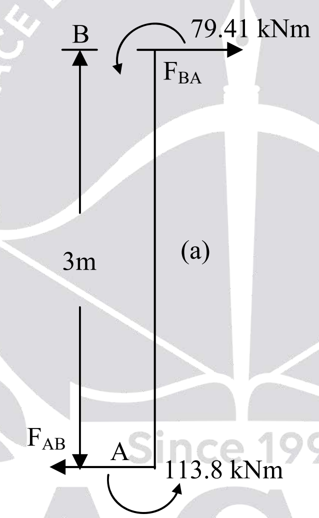
Joints	A		B		C	D
H_{Sway} = 6.77 kN	−15.42	−10.76	10.76	7.66	−7.66	0
H_{Sway} = 50 kN	−113.8	−79.41	79.41	56.53	−56.53	0

Horizontal reaction at A = 5.24 × 7.38 = 38.67 (←)

Horizontal reaction at D = 1.53 × 7.38 = 11.29 kN (←)

(*f*) To Draw S.F.D.

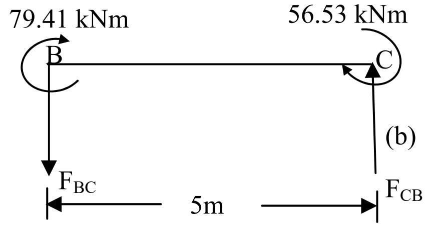
Span AB:

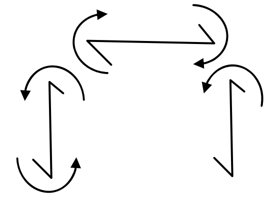


Taking moments about A, – 79.41 – 113.8 + F_{BA}(5) = 0

F_{BA} = 38.642 kN = F_{AB}

Span BC:





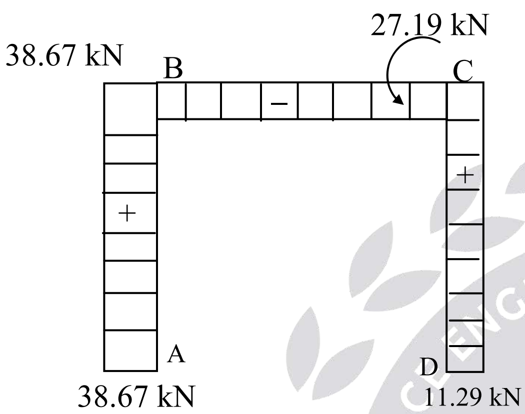
Taking moment about B, $+ 79.41 + 56.53 - F_{CB} \times 5 = 0$

$F_{CB} = 27.19 \text{ kN} = F_{BC}$

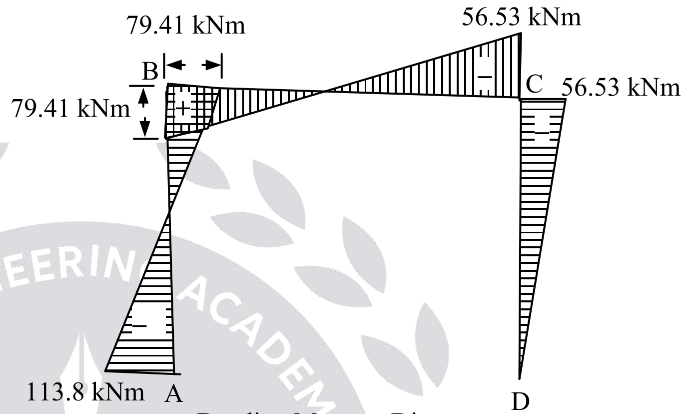
Span CD:

Reaction at D = 11.29 kN ↓

Reaction at C = 11.29 kN ↑



Shear Force Diagram



Bending Moment Diagram

04.

Sol: Distribution Factors. These are calculated in the table below.

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factor
B	BA	$\frac{3}{4}.\frac{I}{6} = \frac{3}{8}$	$\frac{3I}{8}$	$\frac{1}{3}$
	BC	$\frac{2I}{8} = \frac{I}{4}$		$\frac{2}{3}$
C	CB	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}$
	CD	$\frac{3}{4}.\frac{I}{3} = \frac{I}{4}$		$\frac{1}{2}$

(i) Non sway analysis:

Fixed end moments

$$\overline{M}_{ab} = \overline{M}_{ba} = \overline{M}_{cd} = \overline{M}_{dc} = 0$$

$$\overline{M}_{bc} = -\frac{30\times 4^2}{12} = -40\,\text{kNm},\ \ \overline{M}_{cb} = +\frac{30\times 4^2}{12} = +40\,\text{kNm}$$

The moment distribution is worked out below.

A		B		C	D
	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	
0	0	− 40.00	+ 40.00	0	0
	+ 13.33	+ 26.67	− 20.00	− 20.00	
		−10.00	+13.33		
	+ 3.33	+6.67	− 6.66	− 6.67	
		−3.33	+ 3.33		
	+ 1.11	+ 2.22	− 1.67	− 6.66	
		−0.84	+ 1.11		
	+ 0.28	+ 0.56	− 0.56	− 0.55	
		−0.28	+ 0.28		
	+ 0.09	+ 0.19	− 0.14	− 0.14	
		−0.07	+ 0.10		
	+ 0.02	+ 0.05	− 0.05	− 0.05	
0	+ 18.17	−18.17	+ 29.08	− 29.08	0

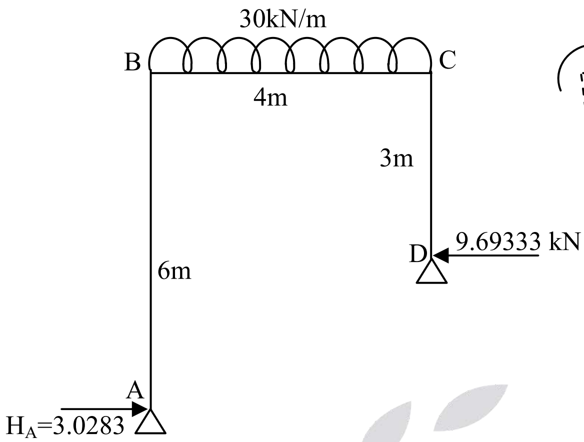
Horizontal reaction at A

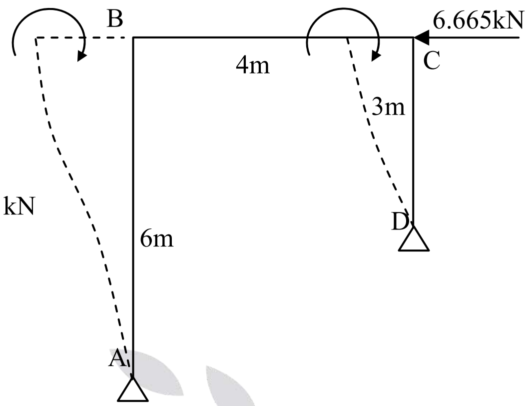
$$= \frac{0+18.17}{6} = +3.0283\,\text{kN} \rightarrow$$

Horizontal reaction at D $= \dfrac{-29.08+0}{3} = -9.6933\,\text{kN} \leftarrow$

$\therefore$ Sway force = Unbalanced horizontal force

= 9.6933 − 3.0283 = 6.665 kN ←





(ii) Sway analysis. Now the frame will be analysed or a sway force of 6.665 kN ←. Since the frame sways towards the left, the initial equivalent moments are positive. Ratio of the initial equivalent moments at the tops of the columns.

$$= \frac{I_1}{\ell_1^{\,2}} : \frac{I_2}{\ell_2^{\,2}} = \frac{I}{6^2} : \frac{I}{3^2} = \frac{1}{36} : \frac{1}{9} = 9 : 36 \,(\text{say})$$

	A	B		C		D
		$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	
	0	+ 9.00 + 3.00	0 –6.00	0 –18.00	+ 36.00 – 18.00	
		+ 3.00	–9.00 + 6.00	–3.00 + 1.50	+1.50	
		–0.25	+ 0.75 –0.50	+ 3.00 –1.50	– 1.50	
		+ 0.25	–0.75 + 0.50	–0.25 + 0.13	+ 0.12	
		–0.02	+ 0.06 + 0.04	+ 0.25 –0.12	– 0.13	
		+ 0.02	–0.06 + 0.04	– 0.02 + 0.01	+ 0.01	
Column (a)	0	+ 9.00	–9.00	–18.00	+ 18.00	0

Let the moments shown in column (a) be due to a sway force S

Horizontal reaction at A = $\frac{0+9.00}{6} = +1.50\,\text{kN} \rightarrow$

Horizontal reaction At D = $\frac{18+0}{3} = +6.00\,\text{kN} \rightarrow$

Resolving horizontally, S = 1.50 + 6.00 = 7.50 kN

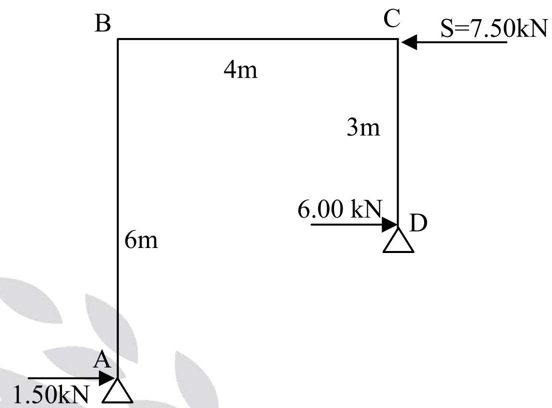
For a sway force of 7.50 kN

the sway moments are as per column (a)

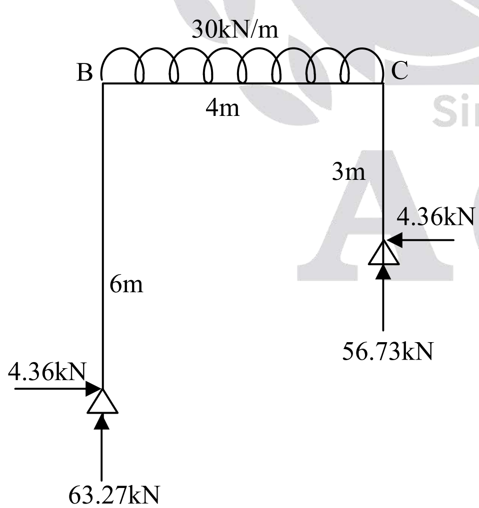
$\therefore$ For the actual sway force of 6.665 kN

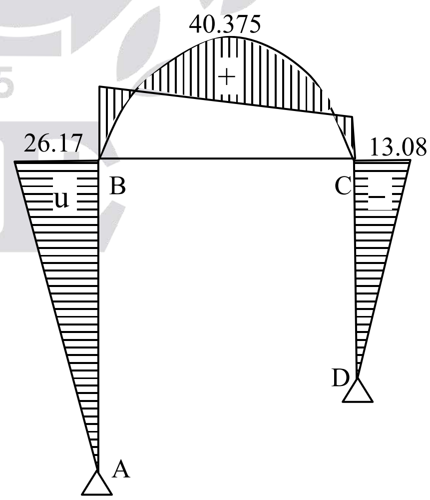
The actual sway moments will be,

$\frac{6.665}{7.50} \times \text{col(a)}$ moments



Col.(a)	0	+ 9.00	– 9.00	– 18.00	+18.00	0
Actual sway moments $= \frac{6.665}{7.50} \times \text{Col.(a)}$	0	+ 8.00	– 8.00	– 16.00	+ 16.00	0
Non sway moments	0	+18.17	– 18.17	+ 29.08	– 29.08	0
Final moments	0	+ 26.17	– 26.17	+ 13.08	– 13.08	0





05.

Sol:

Sol: Distribution Factors:

Since all the corners are alike it is sufficient to determine the distribution factors at one of the corners. Let I be the moment of inertial of the section of each member. The relevant calculations at the corner A are shown in the table below.

Joint	**Member**	**Relative Stiffness**	**Total Relative Stiffness**	**Distribution Factors**
A	AD	$\frac{I}{4}$	$\frac{3I}{4}$	$\frac{1}{3}$
	AB	$\frac{2I}{4}=\frac{I}{2}$		$\frac{1}{3}$

The distribution factors for the members at each corner are shown in figure.

Fixed end moments: Considering each member as a separate fixed member the fixed end moments are as follows.

$$\overline{M}_{ab} = -\frac{10\times 2}{8} = -2.5\,\text{kNm}$$

$$\overline{M}_{ba} = +2.5\,\text{kNm}$$

$$\overline{M}_{da} = -\frac{4\times 4}{8} = -2.00\,\text{kNm}$$

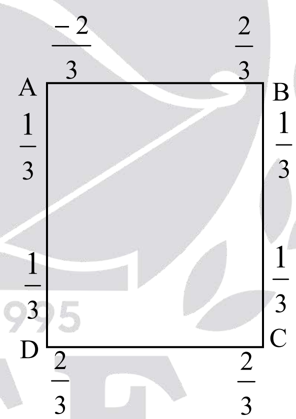
$$\overline{M}_{ad} = +2.00\,\text{kNm}$$

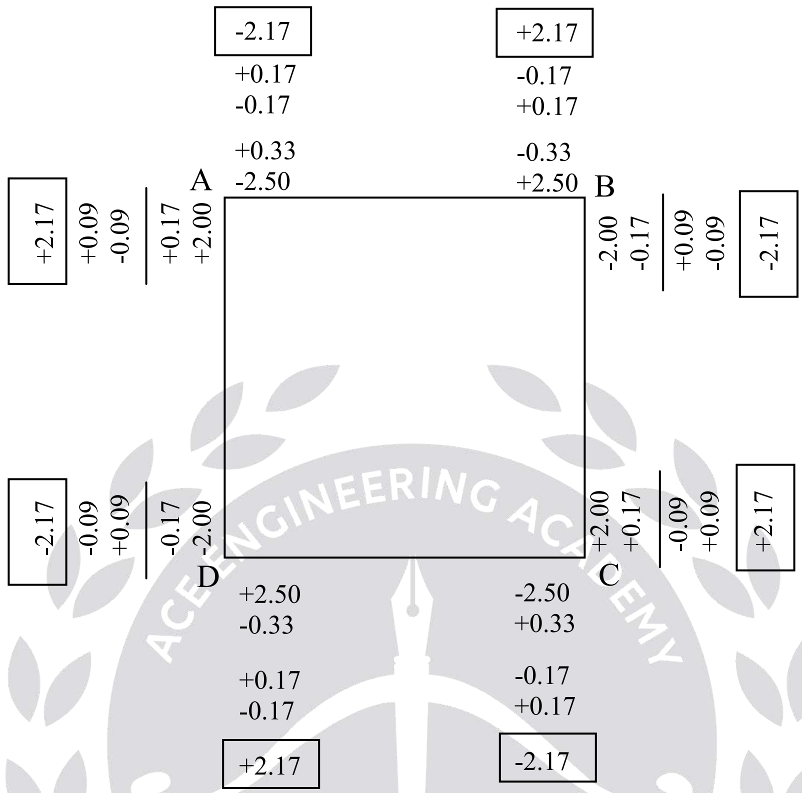
$$\overline{M}_{bc} = -2.00\,\text{kNm}$$

$$\overline{M}_{cb} = +2.00\,\text{kNm}$$

$$\overline{M}_{dc} = +2.50\,\text{kNm}$$

and $\overline{M}_{cd} = -2.50\,\text{kNm}$





The moment distribution is carried out as shown in figire.

Max. free B.M for $AB = +\dfrac{10\times 2}{4} = +5.00$ kNm.

$\therefore$ B.M at the middle of span AB

$= +5.00 - 2.17 = +2.83$ kNm

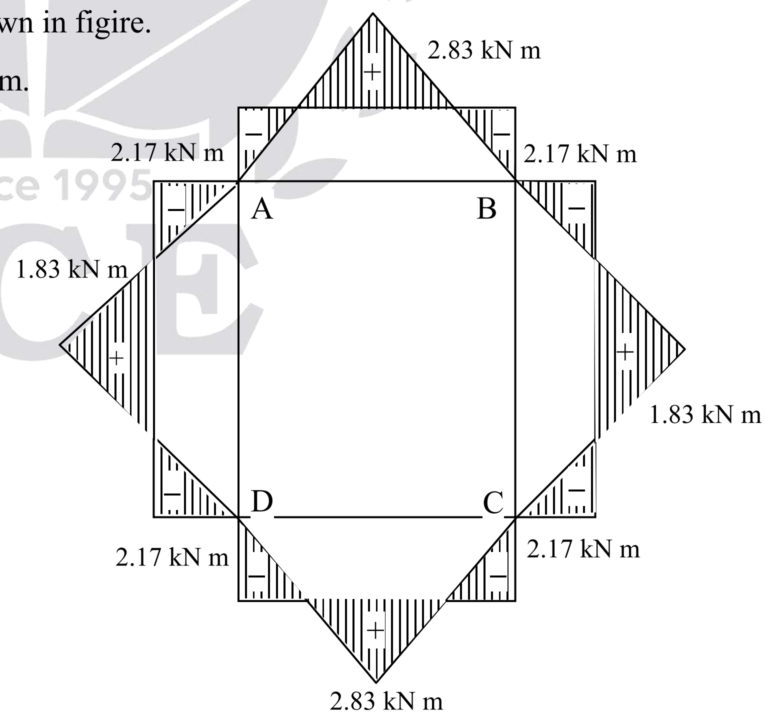
Max. free B.M. for AD

$$= +\frac{4\times 4}{4} = +4.00\,\text{kNm}$$

$\therefore$ B.M. at the middle of AD

$= +4.00 - 2.17 = +1.83$ kNm

Figure shows the B.M. diagram for the frame.



06.

Sol: Imagine a support at C.

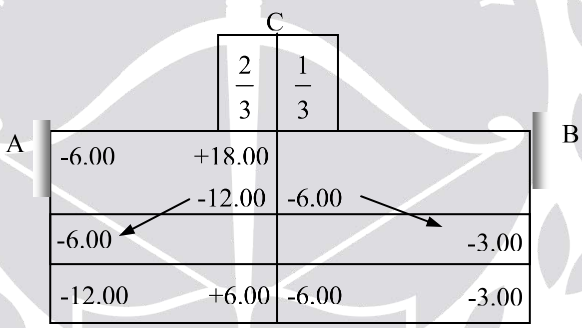
Distribution factors. These are calculate in the table below.

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factors
C	CA	$\frac{2I}{4}$	$\frac{3I}{4}$	$\frac{2}{3}$
	CB	$\frac{I}{4}$		$\frac{1}{3}$

(i) None-Sway: Assuming no vertical sway the moment distribution is now carried out.

Fixed End Moments:

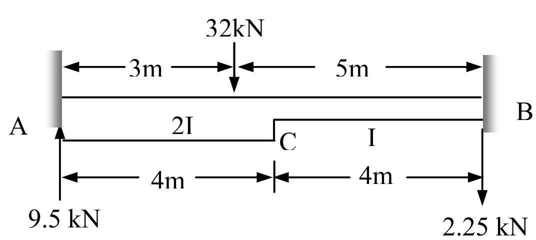
$$\overline{M}_{ac} = -\frac{32\times3\times1^2}{4^2} = -6\text{kNm};\ \overline{M}_{ca} = -\frac{32\times3^2\times1}{4^2} = +18\text{kNm};\ \overline{M}_{cb} = \overline{M}_{bc} = 0$$



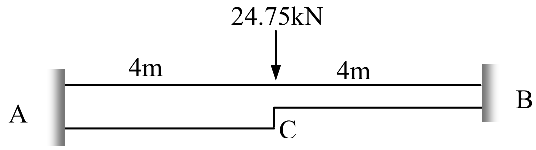
Vertical reaction at A $= -\frac{-12.00+6.00-32\times1}{4} = +9.50\text{kN}\uparrow$

Vertical reaction at B $= \frac{-6.00-3.00}{4} = -2.25\text{kN}\downarrow$

$\therefore$ Sway force = 32 + 2.25 – 9.50 = 24.75 kN ↓



Now the beam will be analysed for a vertical sway force of 24.75 kN at C



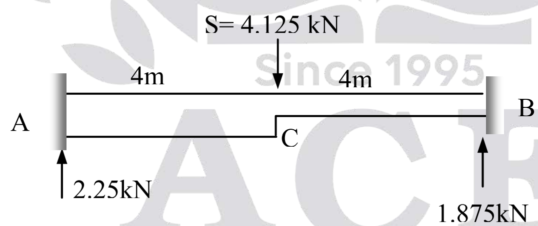
(ii) Sway Analysis:

Due to the vertical sway the fixed end moments for AC are negative, while the fixed end moments for BC are positive.

Ratio of fixed end moments for AC and BC $\frac{I_1}{\ell_1^{\,2}}:\frac{I_2}{\ell_2^{\,2}}\ \frac{2I}{4^2}:\frac{I}{4^2}=2:1$ say 6:3

Choosing the above fixed end moments, the moment distribution is carried out below:

	A	C	C	B
		$\frac{2}{3}$	$\frac{1}{3}$	
	-6.00	−6.00	+3.00	+3.00
		+2.00	+1.00	
	+1.00			+0.05
Column (a)	-5.00	−4.00	+4.00	+3.50



Let the moment shown in col. (a) be due to a sway force S.

Vertical reaction at $A = \frac{-5.00-4.00}{4} = +225\,\text{kN}$

Vertical reaction at $B = \frac{4.40+3.50}{4} = 1.875\,\text{kN}$

Resolving vertically, S = 2.25 + 1.875 = 4.125 kN

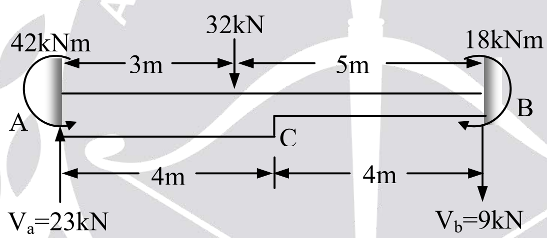
For a sway force of 4.125 kN the sway moments are as per column (a).

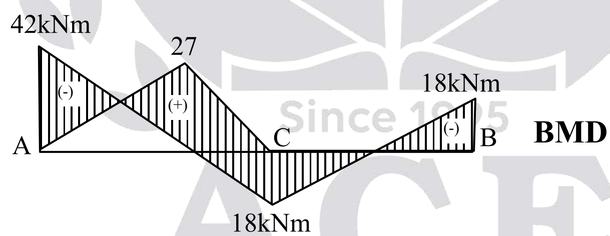
∴ For the actual sway force of 24.75 kN, the actual sway moments will be

$\frac{24.75}{4.125} \times$ column (a) moments.

	A	B		C
Column (a)	− 5.00	− 4.00	+ 4.00	+ 3.50
Actual sway moments $\frac{24.75}{4.125} \times$ Column (a)	− 30.00	− 24.00	+ 24.00	+21.00
Non-sway moments	− 12.00	+ 6.00	− 6.00	− 3.00
Final Moments	− 42.00	− 18.00	+ 18.00	+ 18.00

Actual vertical reaction at A $V_a = \frac{-42-18-32\times1}{4} = 23\,\text{kN}\uparrow$





Actual vertical reaction at B $(V_b) = \frac{18+18}{4} = 9\,\text{kN}\uparrow$

07. Slope Deflection Method

01. Ans: (a)

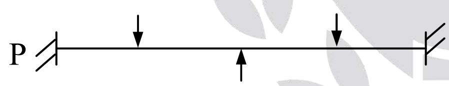
Sol: In slope deflection method deformation due to axial force and shear force are neglected. Deformations due to flexure only are considered.

02. Ans: (c)

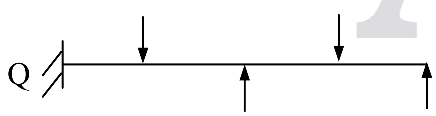
Sol: No. of unknown joint displacements is the most appropriate option. Option (b) is ambiguous as nothing is spelt about axial deformations.

03. Ans: (c)

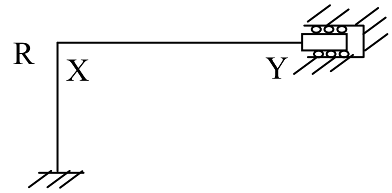
Sol: The number of equilibrium equations is
= number of unknown joint displacements.



For the above beam unknown displacement is the rotation at central support only.



For the above beam unknown displacements are the rotations at central support and right end support.



For the above frame unknown displacements are the rotation at rigid joint X and sway deflection at right support Y.

04. Ans: (a)

Sol: $M_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\delta}{L}\right]$

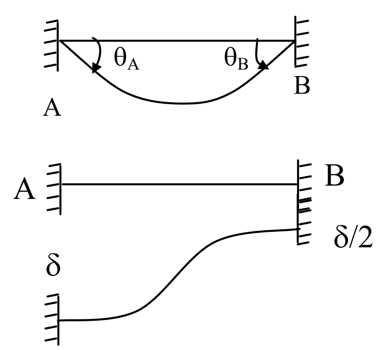
Note:

Clock wise rotations are taken as +Ve.

Anti clock wise rotations are –Ve.

δ = relative sinking of right support with respect to left support. In the standard equation right support is assumed to sink more than left support and δ is taken as +Ve.

In the given problem θ_A is clock wise hence taken as positive. θ_B is anti clock wise hence taken as negative. Further right support sinks less than that of left support.

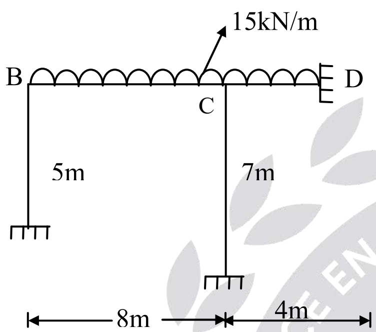


$$M_{BA} = \frac{2EI}{L}\left[-2.\left(\frac{\theta}{2}\right)+\theta-\frac{3}{L}\left(\frac{\delta}{2}-\delta\right)\right]$$

$$= \frac{3EI\delta}{L^2}$$

05. Ans: (a)

Sol:



Fixed end moment $\overline{M}_{BC} = \frac{-WL^2}{12}$

$$= -\frac{15\times 8^2}{12} = -80\text{kN.m}$$

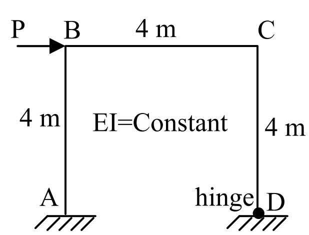
$$M_{BC} = \frac{2EI}{L}\left[2\theta_B+\theta_C-\frac{3\delta}{L}\right]+\overline{M}_{BC}$$

$$M_{BC} = \frac{2EI}{8}\left[2\theta_B+\theta_C-0\right]-80$$

$$= 0.25EI(2\theta_B+\theta_C)-80$$

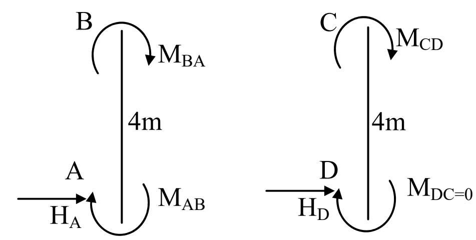
06. Ans: (c)

Sol:



Free body diagrams of columns shown below.

The joint moments are assumed clock wise



For horizontal equilibrium $\sum H = 0$

$H_A + H_D + P = 0$

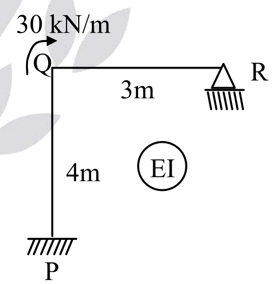
$$H_A = \frac{M_{AB}+M_{BA}}{4}$$

$$H_D = \frac{M_{CD}+M_{DC}}{4} = \frac{M_{CD}+0}{4} = \frac{M_{CD}}{4}$$

$$\frac{M_{BA}+M_{AB}}{4} + \frac{M_{CD}}{4} + P = 0$$

07. Ans: (b)

Sol:



At joint 'Q' relative stiffness

$$= \frac{M}{\theta} = \frac{30}{\theta} = \frac{3EI}{3} + \frac{4EI}{4} = 2EI$$

$$\theta_Q = \frac{15}{EI}$$

08. Ans (a)

Sol: Slope at 'R'

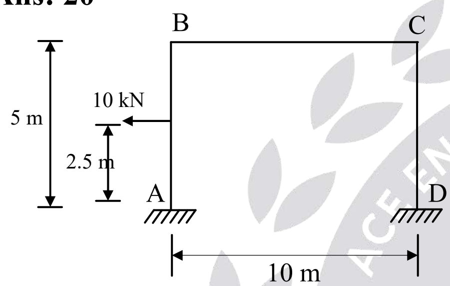
$$M_R = 0 = \frac{2EI}{3}(2\theta_R + \theta_Q)$$

$$\frac{\theta_Q}{2} = -Q_R \quad \Rightarrow \quad \theta_R = \frac{-\theta_Q}{2} = \frac{-7.5}{EI}$$

(Sign neglected)

09. Ans: 20

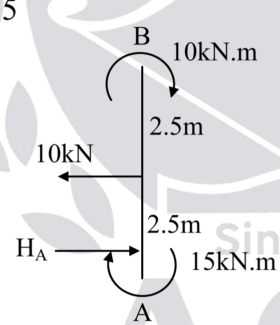
Sol:



For column AB, $\Sigma M_B = 0$

$5H_A = 15 + 10 + 10\times 2.5$

$\Rightarrow \quad H_A = 10\text{ kN} \rightarrow$



F.B.D of Column AB

Applying $\sum H = 0$

$H_A + H_D = 10$

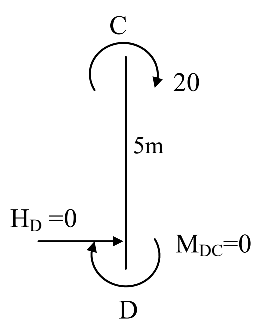
$10 + H_D = 10$

$\Rightarrow \quad H_D = 0$

$$H_D = \frac{M_{CD} + M_{DC}}{5} = 0$$

$$\frac{20 + M_{DC}}{5} = 0$$

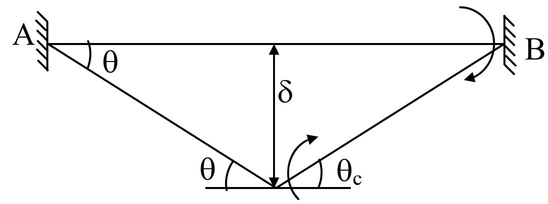
$\Rightarrow \quad M_{DC} = -20$ kN-m



F.B.D of Column CD

10. Ans: (c)

Sol:



$$M_{CB} = \overline{M}_{CB} + \frac{2EI}{L}\left[2\theta_C + \theta_B + \frac{3\delta}{\ell}\right]$$

Here $M_{CB} = 0$ (because of hinge at C) and $\overline{M}_{CB} = 0$

$$0 = \frac{2EI}{L}\left(2\theta_C + 0 + 3\left(\frac{\delta}{\ell}\right)\right)$$

Hence, $\delta = l\,\theta$ and $\theta_B = 0$ (for fixed support)

$$0 = 2\theta_c + \frac{3(\ell\theta)}{\ell}$$

$$\theta_C = \frac{-3\theta}{2}$$

$$M_{BC} = \overline{M}_{BC} + \frac{2EI}{L}\left(2\theta_B + \theta_C + \frac{3\delta}{\ell}\right)$$

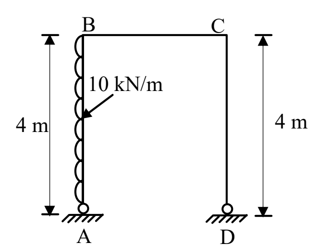
$$= 0 + \frac{2EI}{L}\left(0 + \left(\frac{-3\theta}{2}\right) + \frac{3(\ell\theta)}{\ell}\right)$$

$$= \frac{2EI}{\ell}\left[\frac{3\theta}{2}\right]$$

$$M_{BC} = \frac{3EI\theta}{L}$$

11. Ans: (b)

Sol:

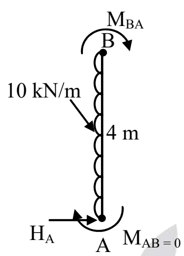


Horizontal Equilibrium equation

$H_A + H_D + 10 \times 4 = 0$

$H_A + H_D + 40 = 0$ ……(1)

Span AB:



Taking moments about point B,

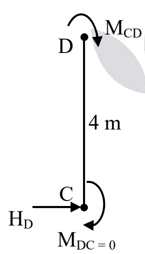
$\Sigma M_B = 0$

$-H_A \times 4 + M_{BA} + M_{AB} - 10 \times 4 \times 2 = 0$

$4H_A = M_{BA} - 80$

$$H_A = \left(\frac{M_{BA} - 80}{4}\right) kN\ (\rightarrow)$$

Span CD:



Taking moments about point D,

$\Sigma M_D = 0$

$M_{CD} + M_{DC} - H_D \times 4 = 0$

$$H_D = \frac{M_{CD}}{4} (\rightarrow)$$

Substitute H_A and H_D in eq. (1)

$$\frac{M_{BA} - 80}{4} + \frac{M_{CD}}{4} + 40 = 0$$

$M_{BA} + M_{CD} - 80 + 160 = 0$

$M_{BA} + M_{CD} = -80 \rightarrow (2)$

We know that

$M_{BA} + M_{BC} = 0 \quad \therefore M_{BA} = -M_{BC} \rightarrow (3)$

and $M_{CD} + M_{BC} = 0 \therefore M_{CD} = -M_{CB} \rightarrow (4)$

The above two conditions are equilibrium equations at joints B and C.

Substitute (3) and (4) in equation (2)

$-M_{BC} - M_{CB} = -80$

$\therefore M_{BC} + M_{CB} = 80$ kNm

12. Ans: (b)

Sol:

In slope deflections methods, joint displacements/Rotations are treated as unknowns. To calculate unknown joint displacements equilibrium conditions will be applied at each joint and displacement at a joint are independent of the displacements of the member at the far end of the joint.

$\therefore$ Option (b) is correct.

13. Ans: (b)

Sol: In moment distribution, slope deflection method and Kani's method unknowns are displacements/rotations.

$\therefore$ These methods are classified as displacement methods/stiffness method.

∴ Statement (I) is correct.

In moment distribution method final end moments are calculated without calculation of displacement like in slope deflection method.

∴ Statement II is correct

But, Statement II is not the correct explanation of Statement I.

14. Ans: (b)

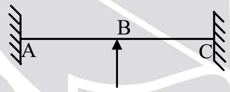
Sol:

While calculating final end moments in moment distribution method we follow,

$$\text{Total moment} = \underset{(1)}{\text{Fixed end moment}} + \underset{(2)}{\Sigma\,\text{Distributed moment at a joint}}$$

When we recall slope deflection equations, for a fixed beam shown below

$$M_{BA} = \underset{(i)}{\overline{M}_{BA}} + \underset{(ii)}{\frac{2EI}{L}(2\theta_B)}$$



$$M_{BC} = \underset{(i)}{\overline{M}_{BC}} + \underset{(ii)}{\frac{2EI}{L}(2\theta_B)}$$

1[st] term (i) is fixed end moment

2[nd] term (ii) is distributed moment.

∴ Concept of summation of fixed end moment and distributed moment in moment distribution method comes from slope deflection equations only.

In slope deflection method, displacement are calculated to analyze end moments.

∴ Statement II is correct

But Statement II is not the correct explanation of Statement I.

Conventional Practice Solutions

01.

Sol: For the data of this problem $EI = \frac{200 \times 4 \times 10^7}{10^6} = 8000\,kNm^2$

Note: $\theta_a = 0$ and $\theta_d = 0$

Span AB:

$$M_{ab} = \frac{2EI}{\ell}\left(2\theta_a + \theta_b - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{4}\left(0 + \theta_b - \frac{3 \times 3}{4000}\right) = 4000\theta_b - 9$$

$$M_{ba} = \frac{2EI}{\ell}\left(2\theta_b + \theta_a - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{4}\left(2\theta_b + 0 - \frac{3 \times 3}{4000}\right) = 8000\theta_b - 9$$

Span BC

$$M_{bc} = \frac{2EI}{\ell}\left(2\theta_b + \theta_c - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{4}\left(2\theta_b + \theta_c - \frac{3 \times 2}{4000}\right) = 8000\theta_b + 4000\theta_c - 6$$

$$M_{cb} = \frac{2EI}{\ell}\left(2\theta_c + \theta_b - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{4}\left(2\theta_c + \theta_b - \frac{3 \times 2}{4000}\right) = 4000\theta_b + 8000\theta_c - 6$$

Span CD

$$M_{cd} = \frac{2\,EI}{\ell}\left(2\theta_c + \theta_b - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{6}\left[2\theta_c + 0 - \frac{3(-5)}{6000}\right] = \frac{16000}{3}\theta_c + \frac{20}{3}$$

$$M_{dc} = \frac{2\,EI}{\ell}\left(2\theta_d + \theta_c - \frac{3\delta}{\ell}\right) = \frac{2 \times 8000}{6}\left[0 + \theta_c \frac{-3(-5)}{6000}\right] = \frac{8000}{3}\theta_c + \frac{20}{3}$$

Equilibrium condition at B, $M_{ba} + M_{bc} = 0$

$8000\,\theta_b - 9 + 8000\,\theta_b + 4000\,\theta_c - 6 = 0$

$16000\,\theta_b + 4000\,\theta_c = 15$

$\therefore\ 4\theta_b + \theta_c = \frac{3}{800}$ .... (1)

Equilibrium condition at C, $M_{cb} + M_{cd} = 0$

$$4000\theta_b + 8000\theta_c - 6 + \frac{16000}{3}\theta_c + \frac{20}{3} = 0$$

$$4000\theta_b + \frac{40000}{3}\theta_c = -\frac{2}{3}$$

$$\therefore\ \theta_b + \frac{10}{3}\theta_c = -\frac{1}{6000} \qquad .... (2)$$

Solving equations (1) and (2) we get,

$\theta_b = 0.001027$, and $\theta_c = -0.0003581$

Substituting for θ_b and θ_c we can determine

The final moments.

$M_{ab} = 4000\ (0.001027) - 9 = -4.892$ kNm

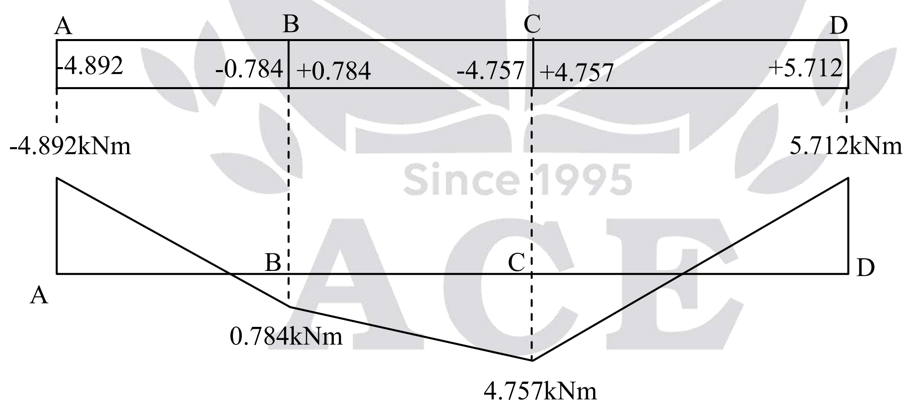
$M_{ba} = 8000\ (0.001027) - 9 = -0.784$ kNm

$M_{bc} = 8000\ (0.001027) + 4000(-0.0003581) - 6 = +0.784$ kNm

$M_{cd} = 4000\ (0.001027) + 8000(-0.0003581) - 6 = -4.757$ kNm

$$M_{cd} = \frac{16000}{3}(-0.0003581) + \frac{20}{3} = +4.757\,\text{kNm}$$

$$M_{dc} = \frac{8000}{3}(-0.0003581) + \frac{20}{3} = +5.712\,\text{kNm}$$



Slope Deflection Method:

$M_{bd} = 6.125$ kNm; $M_{db} = \frac{1}{2}(6.125) = +3.0625\,\text{kNm}$

02.

Sol: Fixed end moments:

$$\overline{M}_{ab} = \overline{M}_{ba} = \overline{M}_{cd} = \overline{M}_{dc} = \overline{M}_{ce} = \overline{M}_{ec} = 0$$

$$\overline{M}_{bc} = +\frac{100}{4} = +25\,\text{kNm}; \quad \overline{M}_{cb} = +\frac{100}{4} = +25\,\text{kNm}$$

Note $\theta_a = \theta_d = \theta_e = 0$

Span AB

$$M_{ab} = 0 + \frac{2EI}{4}(0 + \theta_b) = \frac{1}{2}EI\theta_b$$

$$M_{ba} = 0 + \frac{2EI}{4}(2\theta_b + 0) = EI\theta_b$$

Span BC

$$M_{bc} = 25 + \frac{2EI}{4}(2\theta_b + \theta_c) = 25 + EI\theta_b + \frac{1}{2}EI\theta_c;$$

$$M_{cb} = 25 + \frac{2EI}{4}(2\theta_c + \theta_b) = 25 + \frac{1}{2}EI\theta_b + EI\theta_c$$

Span CD

$$M_{cd} = 0 + \frac{2EI}{4}(2\theta_c + 0) = EI\theta_c; \; M_{dc} = 0 + \frac{2EI}{4}(0 + \theta_c) = \frac{1}{2}EI\theta_c$$

Span CE

$$M_{ce} = 0 + \frac{2EI}{4}(2\theta_c + 0) = EI\theta_c; \; M_{ec} = 0 + \frac{2EI}{4}(0 + \theta_c) = \frac{1}{2}EI\theta_c$$

Equilibrium condition at B, $M_{ba} + M_{bc} = 0$

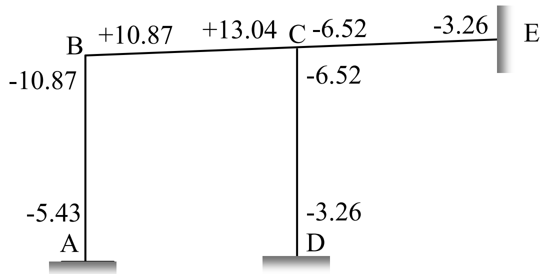
$$EI\theta_b + 25 + EI\theta_b + \frac{1}{2}EI\theta_c = 0$$

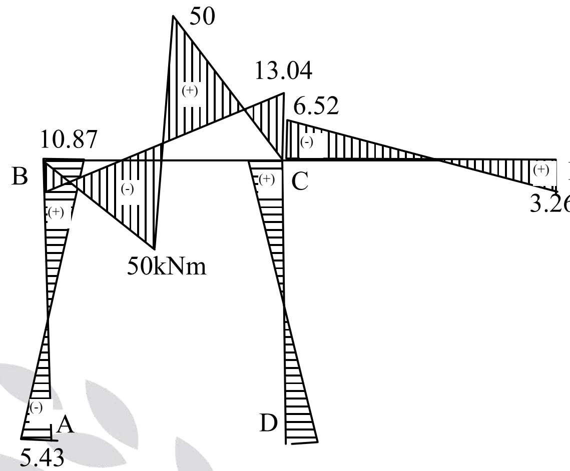
$$2EI\theta_b + \frac{1}{2}EI\theta_c = -25 \quad \therefore\ 4\,EI\,\theta_b + EI\,\theta_c = -50 \quad \text{.... (1)}$$

Equilibrium condition at C,

$$M_{cb} + M_{cd} + M_{ce} = 0; \; 25 + \frac{1}{2}EI\theta_b + EI\theta_c + EI\theta_c + EI\theta_c = 0$$

$$\frac{1}{2}EI\theta_b + 3EI\theta_c = -25 \qquad \therefore\ EI\,\theta_b + 6\,EI\,\theta_c = -50 \qquad \text{..... (2)}$$





Solving equation (1) and (2), we get

EI θ_b = - 10.8696 and EI θ_c = – 6.5217

Substituting for EI θ_b and EI θ_c, the final moments are determined

$$M_{ab} = \frac{1}{2}(-10.8696) = -5.43\,\text{kNm};\, M_{ba} = -10.87\,\text{kNm}$$

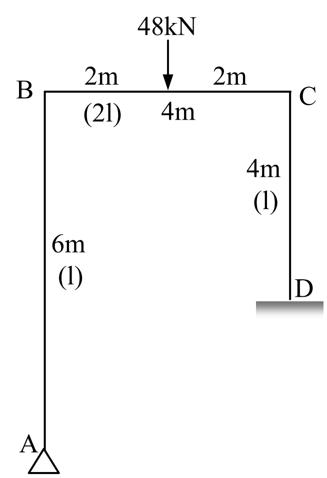
$$M_{bc} = 25 - 10.8696 + \frac{1}{2}(-6.5217) = +10.87\,\text{kNm}$$

$$M_{cb} = 25 + \frac{1}{2}(-10.8696) - 6.5217 = +13.04\,\text{kNm}$$

$$M_{cd} = -6.52\,\text{kNm}; \quad M_{dc} = \frac{1}{2}(-6.5217) = -3.26\,\text{kNm}$$

$$M_{ce} = -6.52\,\text{kNm}; \quad M_{ec} = \frac{1}{2}(-6.5217) = -3.26\,\text{kNm}$$

03.



Sol: Fixed end moments : $\overline{M}_{ab} = \overline{M}_{ba} = \overline{M}_{cd} = \overline{M}_{dc} = 0$

$$\overline{M}_{bc} = -\frac{48 \times 4}{8} = -24\,\text{kNm}; \quad \overline{M}_{cb} = +\frac{48 \times 4}{8} = +24\,\text{kNm}$$

Note A is hinged and D is fixed. $\theta_d = 0$

Member AB

$$M_{ab} = 0;\, M_{ba} = 0 - \frac{0}{2} + \frac{3E(2I)}{6}\left(\theta_b - \frac{\delta}{6}\right) = EI\theta_b - \frac{1}{6}EI\delta$$

Member BC

$$M_{bc} = -24 + \frac{2E(2I)}{4}(2\theta_b + \theta_c) = -24 + 2EI\theta_b + EI\theta_c$$

$$M_{cb} = +24 + \frac{2E(2I)}{4}(2\theta_c + \theta_b) = -24 + 2EI\theta_b + 2EI\theta_c$$

$$M_{cd} = 0 + \frac{2EI}{4}\left(2\theta_c + 0 - \frac{3\delta}{4}\right) = +EI\theta_c - \frac{3}{8}EI\delta$$

$$M_{dc} = 0 + \frac{2EI}{4}\left(0 + \theta_c - \frac{3\delta}{4}\right) = \frac{1}{2}EI\theta_c - \frac{3}{8}EI\delta$$

Equilibrium condition at B, $M_{ba} + M_{bc} = 0$

$$EI\theta_b - \frac{1}{6}EI\delta - 24 + 2EI\theta_b + EI\theta_c = 0;\ 3EI\theta_b + EI\theta_c - \frac{1}{6}EI\delta = 24$$

$\therefore$ 18 EI θ_b + 6 EI θ_c – EI δ = 144 ... (1)

Equilibrium condition at C, $M_{cb} + M_{cd} = 0$

$$+24 + EI\theta_b + 2EI\theta_c + EI\theta_c - \frac{3}{8}EI\delta = 0;\ EI\theta_b + 3EI\theta_c - \frac{3}{8}EI\delta = -24$$

$\therefore$ 8 EI θ_b + 24 EI θ_c – 3 EI δ = – 192 ... (2)

For horizontal equilibrium $H_a + H_d = 0$

$$\frac{M_{ab} + M_{ba}}{6} + \frac{M_{cd} + M_{dc}}{4} = 0$$

$$2[M_{ab} + M_{ba}] + 3[M_{cd} + M_{dc}] = 0$$

$$2\left[0 + EIi_b - \frac{1}{6}EI\delta\right] + 3\left[EIi_c - \frac{3}{8}EI\delta + \frac{1}{2}EIi_c - \frac{3}{8}EI\delta\right] = 0$$

$$2EI\theta_b + \frac{9}{2}EI\theta_c - \frac{31}{12}EI\delta = 0$$

$\therefore$ 24 EI θ_b + 54 EI θ_c – 31 EI δ = 0 .... (3)

Thus, we have the following equation.

18 EI θ_b + 6 EI θ_c – EI δ = 144 ..... (1)

8 EI θ_b + 24 EI θ_c – 3 EI δ = – 192 ...... (2)

24 EI θ_b + 54 EI θ_c – 31 EI δ = 0 ...... (3)

Solving, we get,

EI θ_b = 11.7671; EI θ_c = – 13.7852 and EI δ = – 14.9030

Substituting for EI θ_b, EI θ_c and EI δ the final moments can be determined.

$M_{ab} = 0$

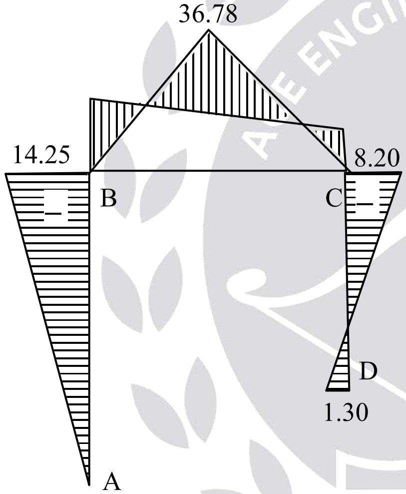
$$M_{ba} = 11.7671 - \frac{1}{6}(-14.9030) = +14.25\,\text{kNm}$$

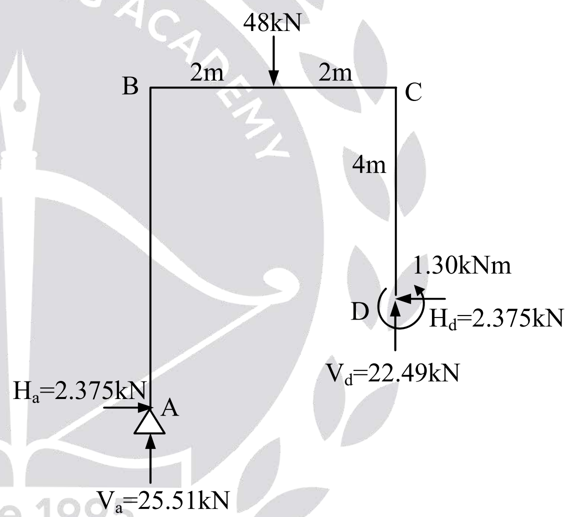
$M_{bc} = -24 + 2(11.7671) - 13.7852 = -14.25$ kNm

$M_{cb} = +24 + 11.7671 + 2(-13.7852) = +8.20$ kNm

$$M_{cd} = -13.7852 - \frac{3}{8}(-14.9030) = -8.20\,\text{kNm}$$

$$M_{dc} = \frac{1}{2}(-13.7852) - \frac{3}{8}(-14.9030) = -1.30\,\text{kNm}$$





Reaction:

$$\text{Horizontal Reaction at } A = H_a = \frac{0 + 14.25}{6} = +2.375\,\text{kN} \rightarrow$$

$$\text{Horizontal Reaction at } D = H_d = \frac{-8.20 - 1.30}{4} = -2.375\,\text{kN} \leftarrow$$

$$\text{Vertical Reaction at } D = V_d = \frac{-14.25 + 8.20 + 48 \times 2}{4} = 22.49\,\text{kN} \uparrow$$

$$\text{Vertical Reaction at } A = V_a = 48 - 22.49 = 25.51\,\text{kN} \uparrow$$

08. Plastic Theory

01. Ans: (d)

Sol: Ductile materials like mild steel are used for design using plastic theory. For ductile materials plastic deformation before Fracture is much larger than elastic deformation.

02. Ans: (c)

Sol: Shape factor is the ratio of plastic moment and yield (elastic) moment.

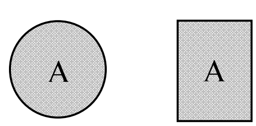
$$S = \frac{M_P}{M_e} = \frac{f_y.Z_P}{f_y.Z} = \frac{Z_P}{Z}$$

We know that section modulus represents the strength of a section both in plastic and elastic theory.

As $Z_P > Z_Y$ for all sections, shape factor indicates the increase of strengths of a section due to plastic action over elastic strength.

Hence statements 1 and 2 are correct.
Shape factor is more if area near neutral axis is more (bulk area).
For example :

i) Consider a square section and circular section of same area.



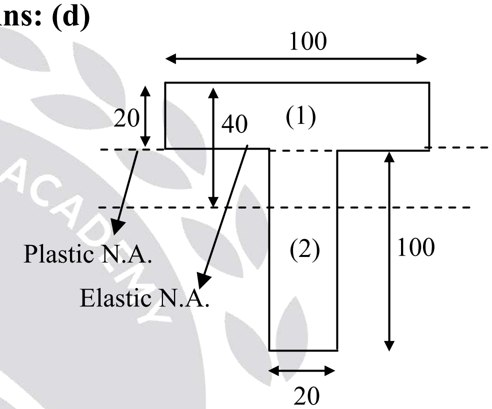
$S_{circle} = 1.7$ $\quad S_{square} = 1.5$

ii) Refer solution of Problem 3: for I section along Y axis area is more near neutral axis compared to area near X axis. Hence shape factor $S_{YY} > S_{XX}$

$\therefore$ statement 3 is wrong.

03. Ans: (d)

Sol:



Elastic N.A. distance from top of flange

$$y_e = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$$

$$y_e = \frac{100\times 20\times 10 + 100\times 20\times 70}{2000 + 2000} = 40\text{mm}$$

Plastic N.A. from top of flange;

Plastic N.A. divides the section in to two equal areas.

Total area of the section = 4000mm^2

Half of area = 2000mm^2

As the flange area is also equal to 2000mm^2, the plastic neutral axis lies at the junction of flange and web.

$\therefore$ Plastic neutral axis distances from top

$y_p = 20\text{mm}$

Distance between plastic N.A.

and Elastic N.A = 40 – 20 = 20 mm

Note: Better use calculations in cm to save time

04. Ans: (a)

05. Ans: (c)

Sol: Plastic moment $M_P = f_y \times z_p$

Given,

$M_P = 120$ kN.m

$M_P = f_y \times 5 \times 10^{-4}$

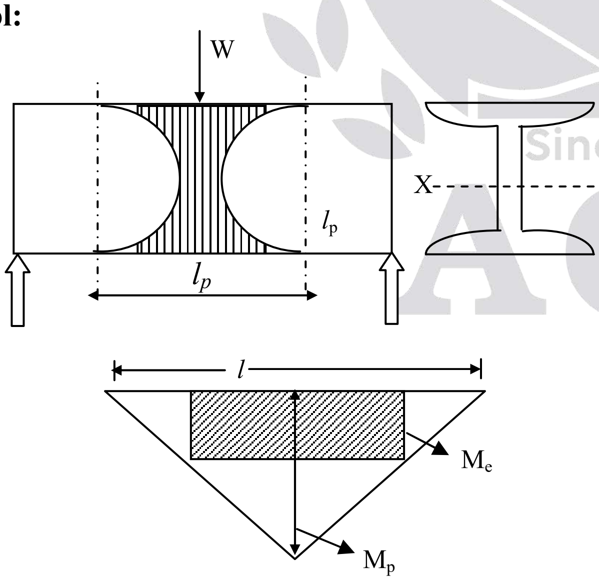
$\therefore$ Yield stress

$$f_y = \frac{120 \times 10^6}{5\times10^{-4}} = 24\times10^{10}\ \text{N/m}^2$$

$$= 240\ \text{N/mm}^2$$

06. Ans: (a)

Sol:



From similar triangles,

$$\frac{\ell_p}{(M_p - M_e)} = \frac{\ell}{M_p}$$

$$\ell_p = \frac{\ell(M_p - M_e)}{M_p}$$

$$\ell_p = \ell\left[1 - \frac{1}{S}\right]$$

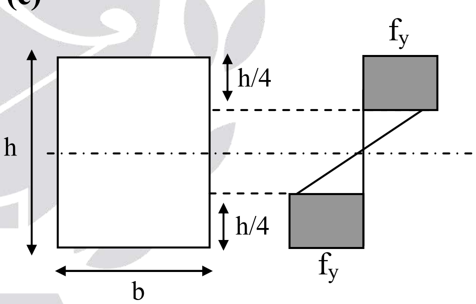
(Shape factor of I section ≈ 1.12

$$= \ell\left[1 - \frac{1}{1.12}\right]$$

$$\therefore \ell_p \approx \frac{\ell}{8}$$

07. Ans: (c)

Sol:



M_{ep} = M. R of elasto plastic section

= M.R. of elastic part + M.R.of Plastic part

= $f_y.Z + f_y.Z_p$

$$Z_{\text{elastic part}} = \frac{b}{6}\cdot\left(\frac{h}{2}\right)^2 = \frac{bh^2}{24}$$

$$Z_{\text{plastic part}} = 2\left[b\left(\frac{h}{4}\right)\left(\frac{h}{4} + \frac{.h}{8}\right)\right] = \frac{3bh^2}{16}$$

$\therefore M_{ep} = f_y.Z + f_y.Z_p$

$$= f_i\left[\frac{bh^2}{24}+\frac{3bh^2}{16}\right]=\frac{11}{48}f_y\cdot bh^2$$

Shortcut :

- M.R of fully plastic section = $f.bh^2/4$
- M.R of fully elastic section = $f.bh^2/6$
- M.R of partly plastifyed section lies between the above two values.
  $(f.bh^2/6) < M_{ep} < f.bh^2/4$
- The denominator of the above value will be between 4 and 6. Hence by elimination technique option c.

08. Ans: (d)

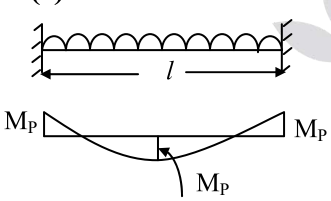
Sol: Load factor (Q)

$$= \frac{\text{Factor of safety in elastic theory} \times \text{shape factor}}{1+\text{additional \% of stress allowed for wind}}$$

$$= \frac{1.5\times1.12}{1+0.2} = 1.4$$

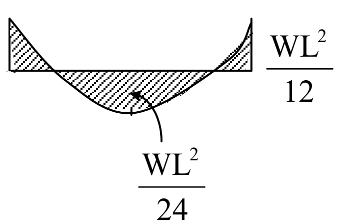
09. Ans: (c)

Sol:



$$\frac{W_cL}{8} = 2M_P \Rightarrow W_c = 16\frac{M_P}{L} \quad \text{...... (1)}$$

At the elastic limit, the centre moment is one-half of the end moment.



$$\frac{W_eL}{8} = M_e + \frac{M_e}{2}$$

$$\Rightarrow W_e = \frac{12M_e}{L} \quad \text{...... (2)}$$

From eqs. (1) & (2)

$$\frac{W_c}{W_e} = \frac{\frac{16M_p}{L}}{\frac{12M_e}{L}} = \frac{4M_P}{3M_e} = \frac{4}{3}\times \text{shape factor}$$

$$= \frac{4}{3}\times\frac{3}{2} = 2$$

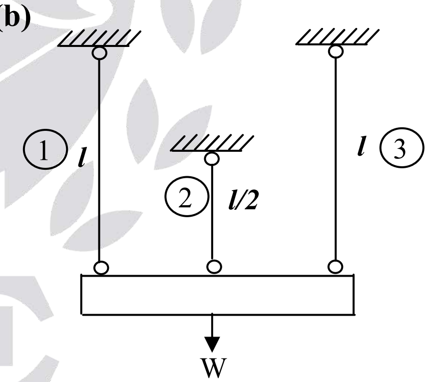
(For rectangular section S = 1.5)

Deformation is just observed means the beam is subjected to elastic failure with yield load (W_e=10kN/m)

$\therefore$ Collapse load = 2 × 10 = 20kN/m

10. Ans: (b)

Sol:



The given frame is symmetrical both in loading and configuration. The rigid block of weight W will have uniform deflection.

All the three wires will have same elongation.

Strain = change in length/original length

As central wire has half length compared to end wires, the strain of central wire is two times that of end wires. Hence the central wire will reach the yield stress 'f_y' initially.

The end wires will have half the strain of that of middle wire. Hence they reach stress of $0.5f_y$ when the middle wire yields.

The load corresponding to yielding of one of the wires

$$W_e = f_y.A + 2(0.5f_y)\,A = 2\,f_y.A$$

At plastic collapse the end wires will also reach yield stress f_y.

When the end wires are yielding, the stress in the middle wire remaines constant (f_y).

$\therefore$ collapse load = $3f_y.A$

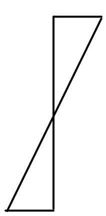
$\therefore$ ratio of collapse load and yield load = 3:2

11. Ans: (a)

Sol: In all theories, viz. elastic theory, plastic theory and limit state theory, Bernouli's assumption is valid according to which "Plane transverse sections which are plane and normal to the longitudinal axis before bending remain plane and normal after bending".

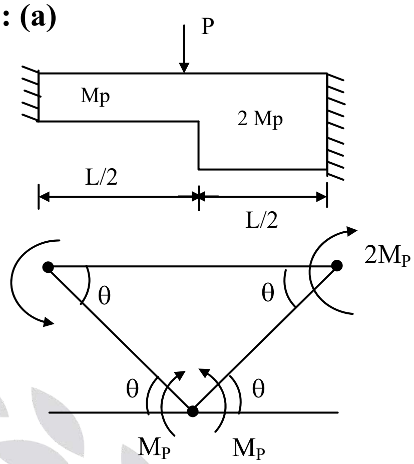
It means

Strain variation is linear as shown aside



12. Ans: (a)

Sol:



External workdone = Internal workdone

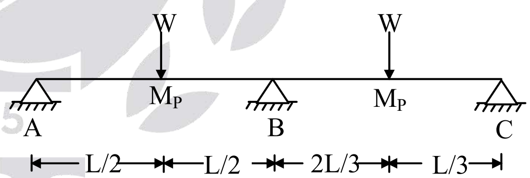
$$5\,M_p\,\theta = p \times L/2 \times \theta$$

$$\frac{10M_p}{L} = p$$

$$\text{Collapse load} = \frac{10M_P}{L}$$

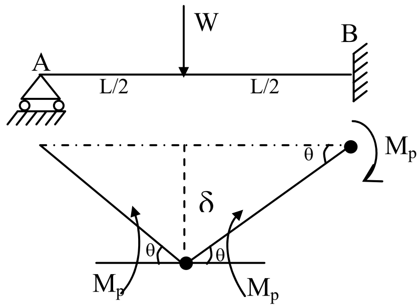
13. Ans: (d)

Sol:



The given continuous beam will have two independent mechanisms. Both will behave like propped cantilevers. Beam AB has central point load which has more B.M. compared to BC which has eccentric point load. Hence mechanism AB is sufficient to know collapse load in objective papers.

Mechanism AB:



$W_i = 3M_p\theta$

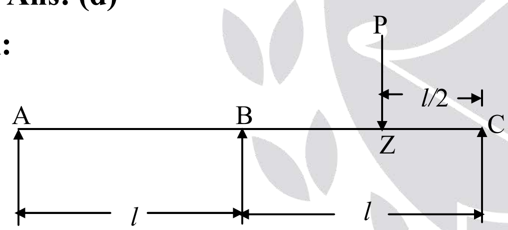
$W_e = W\delta = W\dfrac{L}{2}\cdot\theta$

$W_e = W_i$

$\Rightarrow$ Collapse load $W_C = \dfrac{6M_p}{L}$

14. Ans: (d)

Sol:

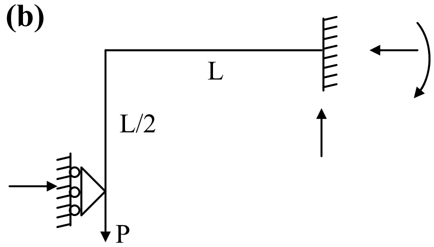


BC will act like propped cantilever with central point.

Collapse load = $P = \dfrac{6M_p}{L}$

15. Ans: (b)

Sol:

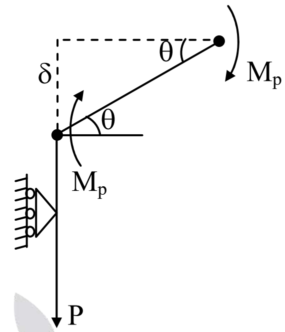


Sway mechanism only possible.

$D_S = 4-3 = 1$

Number of plastic hinges for collapse = 1+1 = 2

Plastic hinge and moment towards beam side only since no rotation towards vertical column side.



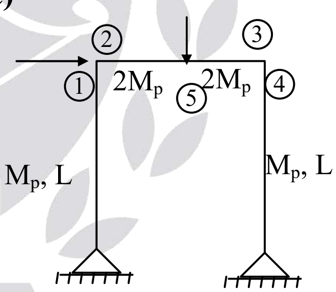
$W_i = 2M_P\theta$; $W_e = P.\delta = P.L.\theta$

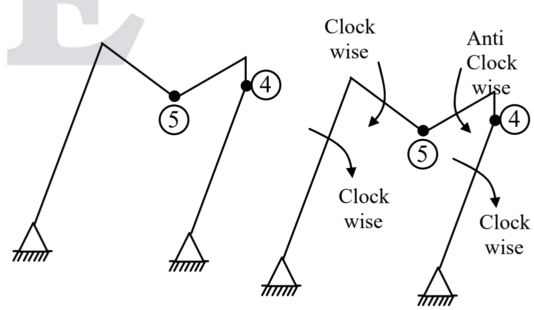
$W_e = W_i$

$\Rightarrow P = \dfrac{2M_P}{L}$

16. Ans: (c)

Sol:





$D_S = 4 - 3 = 1$

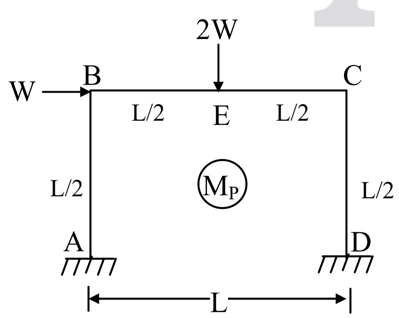
∴ Two plastic hinges will form at failure for combined mechanism. One plastic hinge will form under point load ⑤ on the beam. The second plastic hinge will form at ④ on the column side of Lee ward side node of frame as column side has M_P which is less than $2M_P$ of beam.

Reason for not having plastic hinge on windward side: As seen in the combined mechanism, the column and beam have rotations in the same direction (clock wise) and hence the initial included angle will not change.

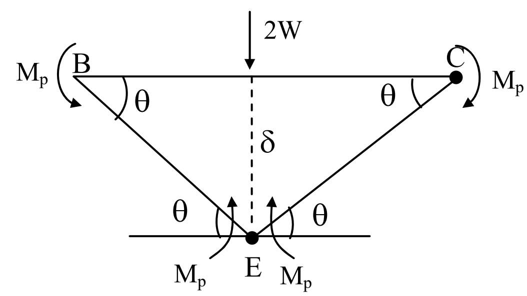
Reason for having plastic hinge on Lee ward side: As seen in the combined mechanism, the column and beam have rotations in the opposite (column clock wise and beam anti clock wise) and hence the initial included angle changes leading to plastic hinge on weaker side.

17. Ans: (b)

Sol:



(i) Beam Mechanism BC:



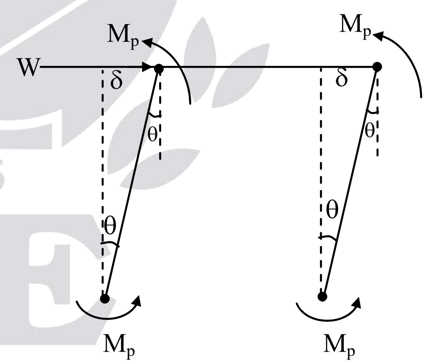
$$W_e = 2W.\delta = 2W.\left(\frac{L}{2}\right).\theta$$

$$W_i = 4M_p.\theta$$

$$W_i = W_e$$

$$\Rightarrow W = \frac{4M_P}{L} \quad \text{.... (i)}$$

(ii) Sway Mechanism:

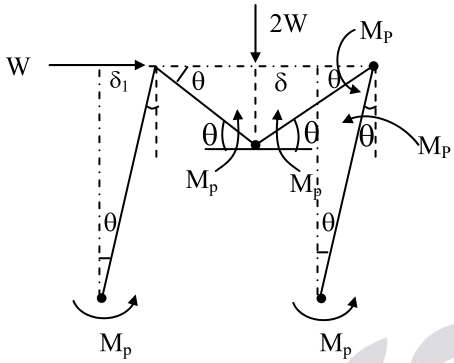


$$W_i = W_e \Rightarrow 4Mp\cdot\theta = W\cdot\delta$$

$$4M_p\theta = W\theta\times\frac{L}{2}$$

$$\Rightarrow W = \frac{8M_P}{L} \quad \text{.... (ii)}$$

(iii) Combined Mechanism:



$W_e = W{\cdot}\delta_1 + 2W{\cdot}\delta$

$= W.\left(\frac{L}{2}\right).\theta + 2W.\left(\frac{L}{2}\right).\theta$

$W_i = M_P.\theta + M_P.\theta + M_P.\theta + M_P.\theta + M_P.\theta + M_P.\theta$

$= 6M_P.\theta$

$W_e = W_i$

$\Rightarrow W = \frac{4M_p}{L}$ …. (iii)

∴Collapse load is the minimum of above three cases

$$\therefore W_C = \frac{4M_p}{L}$$

Short cut:

Compared to the columns, the beam has double the length and double the load. Hence practically the beam mechanism will govern the collapse.

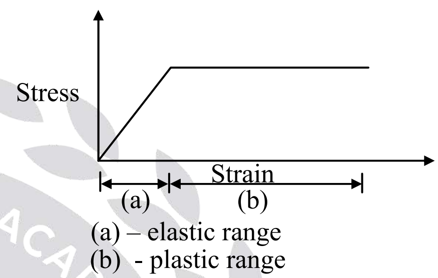
18. *Refer ESE solutions Book.(2008)*

19. *Refer ESE solutions Book.(2013)*

21. Ans: (a)

Sol: Principle of superposition valid for linear elastic structures for which stress verses strain relationship is linear.

In case of plastic theory the relationship is bi-linear (Elasto- plastic).



22. Ans: (d)

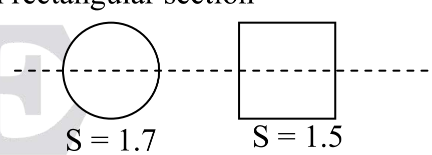
Sol: Shape factor of circular section = 1.7

and shape factor of rectangular section = 1.5

∴ statement (I) is incorrect

Compared to rectangular section, circular section has more are near to neutral axis than at the extreme fiber.

∴ Shape factor is more for circular section than rectangular section

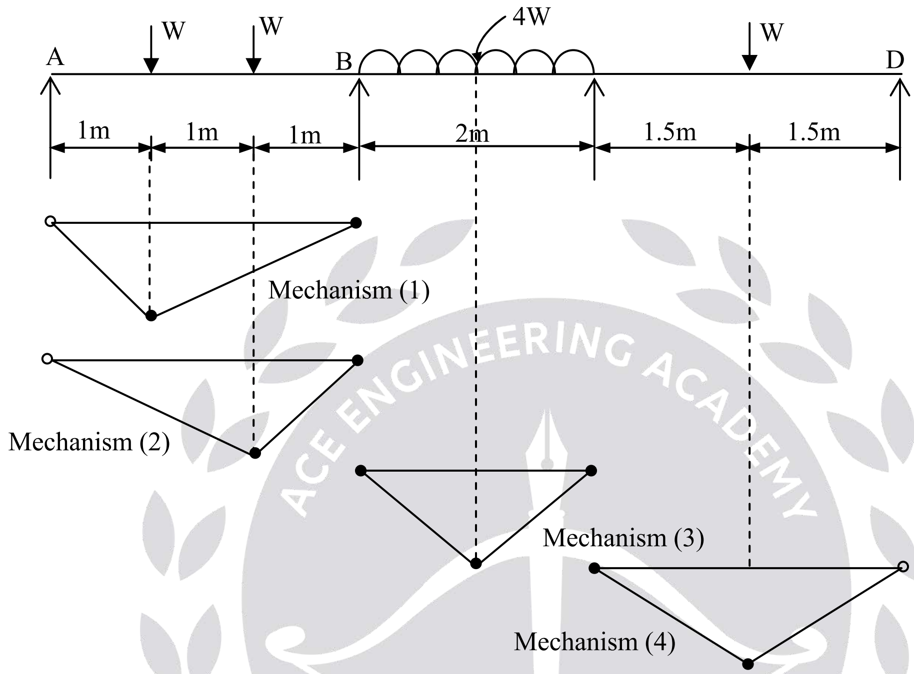


23. Ans: (b)

Conventional Practice Solutions

01.

Sol:



Degree of indeterminancy, I = V.R + E.M. – 2 = 4 + 0 – 2 = 2

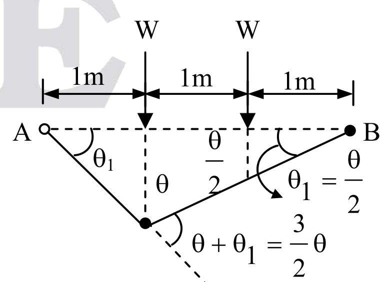
No. of possible plastic hinges = 6

No. of independent Mechanisms = 6 – 2 = 4

Mechanism (1):

$$I \times \theta = 2\theta_1, \theta_1 = \frac{\theta}{2}$$

$$\theta + \theta_1 = \theta + \frac{\theta}{2} = \frac{3\theta}{2}$$



$$\text{External Work Done} = W \times \theta + W \times \frac{\theta}{2}$$

$$= W\left(\theta + \frac{\theta}{2}\right) = \frac{3}{2} W\theta$$

$$\text{Internal Work done} = M_P \times \frac{3\theta}{2} + M_P \times \frac{\theta}{2} = M_P\left(\frac{3\theta}{2} + \frac{\theta}{2}\right) = M_P \times 2\theta$$

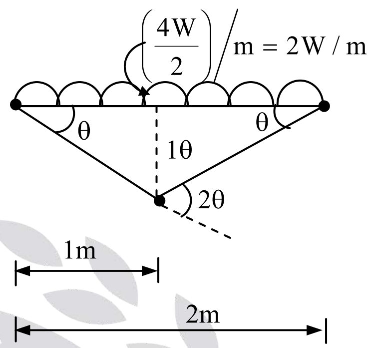
Equating external and internal works

$$\frac{3}{2}W\theta = M_P \times 2\theta,\ W = M_P \times 2\theta \times \frac{2}{3\theta}, W_C = \frac{4M_P}{3}$$

Mechanism (2):

Since This case is very similar to mechanism 1,

$$W_C = \frac{4M_P}{3}$$



Mechanism (3):

External Work Done $= 2W \times \frac{1}{2} \times 2 \times \theta = 2W\theta$

Internal work Done = $M_P \times \theta + M_P \times 2\theta + M_P \times \theta$

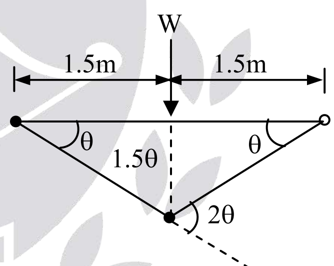
$= 4\ M_P \times \theta$

Equating external and internal works

$2W\theta = 4\ M_P\ \theta$

$W_C = 2\ M_P$ .... (1)

Mechanism (4):



External Work done $= W \times 1.5\ \theta$

Internal work done $= M_p \times \theta + M_p \times 2\theta$

$= 3M_P \times \theta$

Equating external and internal work done

$W \times 1.5\theta = 3\ M_p \times \theta$

$$W_C = \frac{2M_P}{1.5} = \frac{4}{3}M_P$$

$W_C = 2M_P$ .... (2)

Comparing (1) to (2), the least value of

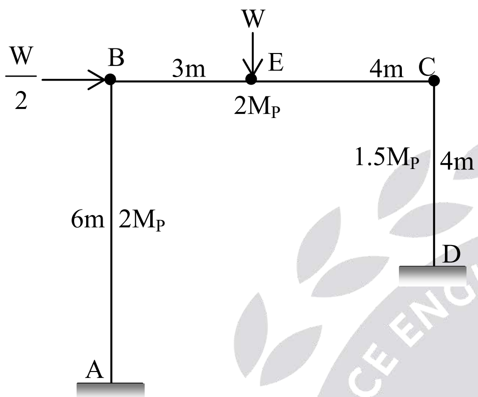
$$W_C = \frac{4}{3}M_P$$

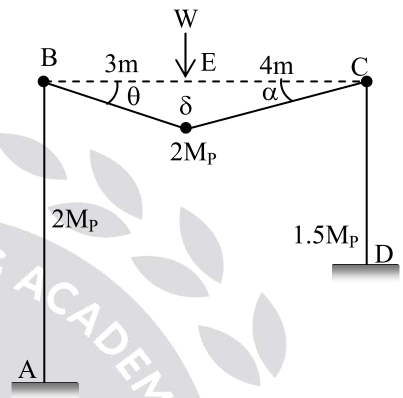
02.

Sol: We will consider various possible collapse mechanisms.

(i) Beam mechanism: In this condition plastic hinges are developed at B, C and under the load W.

Provide a small displacement as shown in figure. $\delta = 3\theta = 4\alpha$ $\quad \therefore\ \alpha = \frac{3}{4}\theta$





Equating the virtual work done to zero,

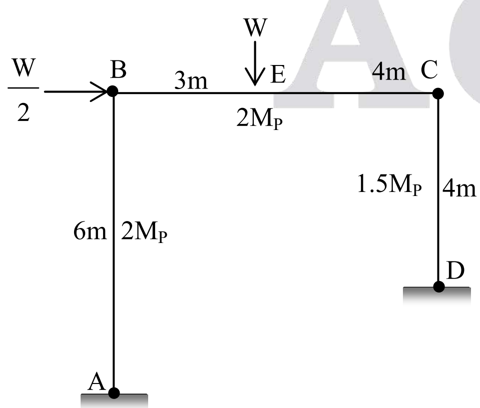
$W3\theta - 2\,M_P\theta - 2\,M_P\theta - 2\,M_P\alpha - 1.5\,M_P\alpha = 0$

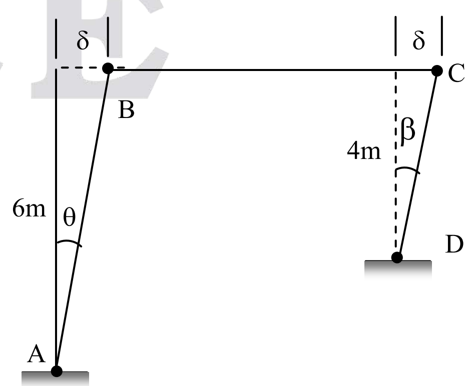
$W3\theta - 4M_P\theta - 3.5M_P \times \frac{3}{4}\theta = 0;\ 4M_P + 2.625M_P = 3W;$

$\boxed{W = 2.21M_P}$

(ii) Sway Mechanism

In this condition, plastic hinges are developed at A, B, C and D $\delta = 60 = 4\beta$; $\beta = 1.5\theta$





Provide a small displacement as shown in figure.

Equating the virtual work done to zero, $\frac{W}{2}6\theta - 2M_P\theta - 2M_P\theta - 1.5M_P\beta - 1.5M_P\beta = 0$

$3W\theta - 4M_P\theta - 3M_P \times 1.5\theta = 0;\ 3W = 8.5M_P \qquad \therefore W = 2.83M_P$

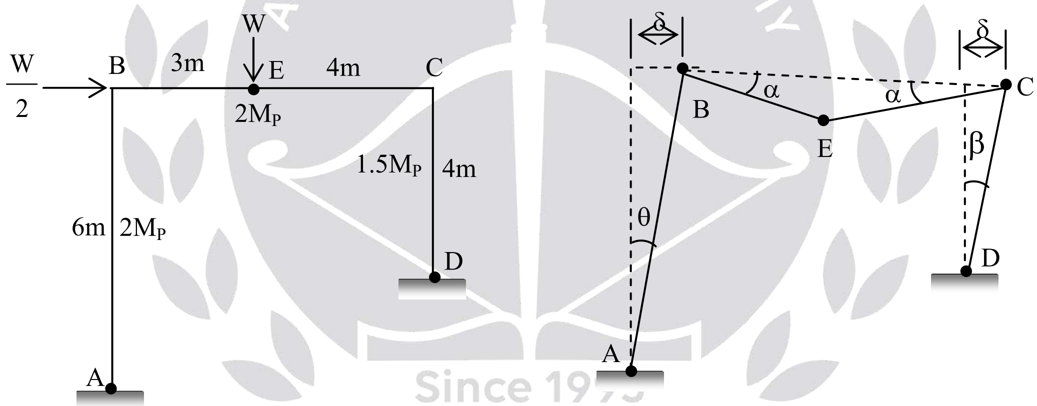
(iii) Combined Mechanism

In this condition plastic hinges are developed at A, under the load W, C and D. Provide a small displacement as shown in figure.

$$3\theta = 4\alpha \ \therefore \ \alpha = \frac{3}{4}\theta;\ 6\theta = 4\beta \ \therefore \beta = 1.5\theta$$

Equating the virtual work done to zero

$$\frac{W}{2}\times 6\theta + W\times 3\theta - 2M_P\theta - 2M_P\theta - 2M_P\alpha - 1.5M_P\alpha - 1.5M_P\beta - 1.5M_P\beta = 0$$



$6W\theta - 4M_P\theta - 6.5M_P\alpha = 0$

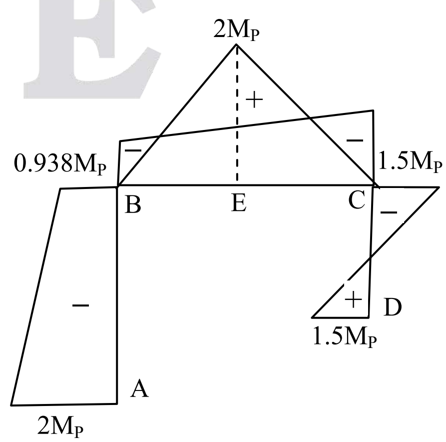
$6W\theta - 4M_P\theta - 3.5M_P\alpha - 3M_P\beta = 0$

$6W\theta - 4M_P\theta - 3.5M_P\frac{3}{4}\theta - 3M_P\times 1.5\theta = 0$

$6W = 4M_P + 2.625M_P + 4.5M_P$

$6W = 11.125M_P$

$\therefore W = 1.854M_P$



The value of W at collapse is the smallest value of W obtained above .

∴ Collapse load W = 1.854 M_P

Collapse will occur by combined mechanism

Consider the column CD

Horizontal reaction at D $= \frac{-1.5M_P - 1.5M_P}{4}$

$= -0.75\ M_P \leftarrow$

Horizontal reaction at A = $\frac{W}{2} - 0.75M_P$

$= \frac{1.854M_P}{2} - 0.75M_P$

$= +0.177\ M_P \leftarrow$

B.M at B

$= -2\ M_P + 0.177\ M_p \times 6$

$= -2\ M_P + 0.062\ M_p = -0.938\ M_p$

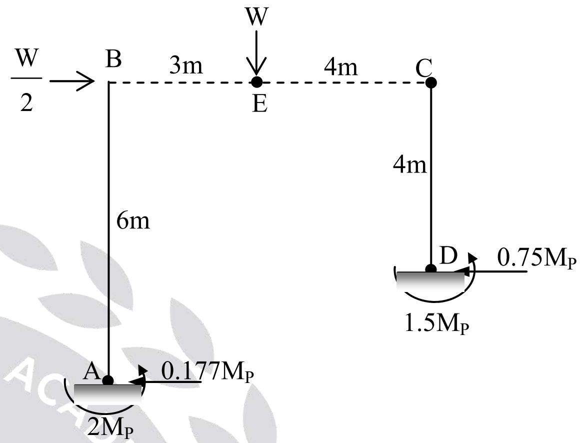


Fig. 2090 shows the collapse B.M. diagram

03.

Sol: We will consider the following possible collapse mechanisms.

1. Beam Mechanism for BC:

In this condition plastic hinges are developed at B, C and under the load W on the span BC

Provide a small displacement as shown in figure 3.

Equating the virtual work done to zero.

$$W\frac{\ell}{2}\theta - 4M_p\theta = 0$$

$$\therefore\ W = \frac{8M_P}{\ell}$$

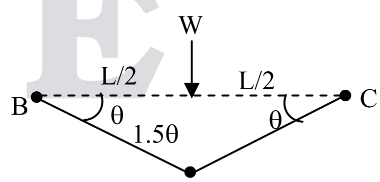


Figure 1

2. Beam mechanism for AB

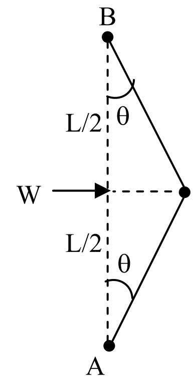


Figure 2

This condition is similar to that of beam mechanism for BC. See Fig. 4.

Hence, for this case also $W = \dfrac{8M_P}{\ell}$

3. Sway Mechanism or Panel Mechanism

In this condition plastic hinges are developed at A, B, C and D.

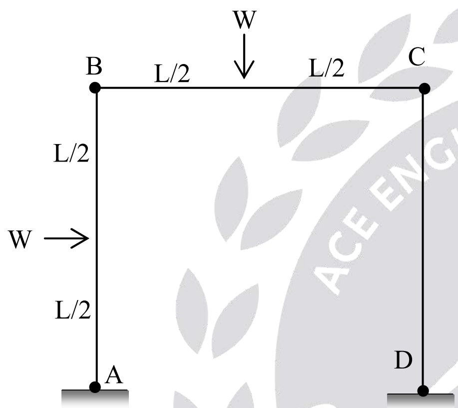


Figure 3

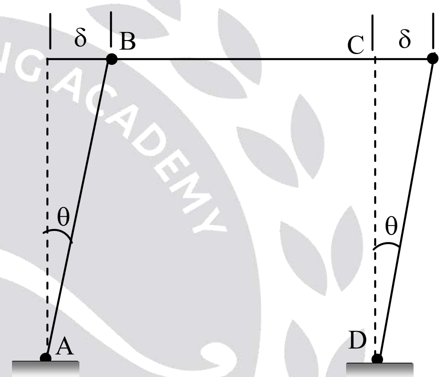


Figure 4

provide a small displacement as shown in Fig. 6.

Equating the virtual work done to zero, $W\dfrac{\ell}{2}\theta - 4M_P\theta = 0$

$\therefore\ W = \dfrac{8M_P}{\ell}$

4. Combined Mechanism

In this condition plastic hinges are developed at A, C, D and under the load W on the span BC.

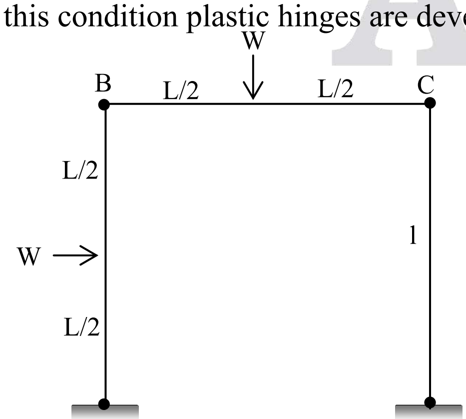


Figure 5

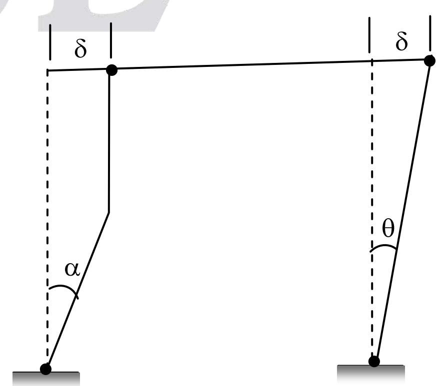


Figure 6

Provide a small displacement as shown in Fig. 6 & 7.

Equating the virtual work done to zero.

$$W\frac{\ell}{2}\theta + W\frac{\ell}{2}\theta - 6M_P\theta; \quad W = \frac{6M_P}{\ell}$$

5. Composite Mechanism

In this condition plastic hinges are developed at A, at the middle of the column AB, at C and at D provide a small displacement as shown in Fig. 6 & 7.

We know $\delta = \frac{\ell}{2}\ \alpha = \ell\theta \quad \therefore\ \alpha = 2\ \theta$

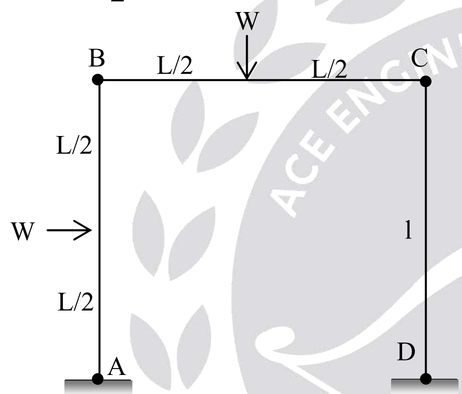


Figure 7

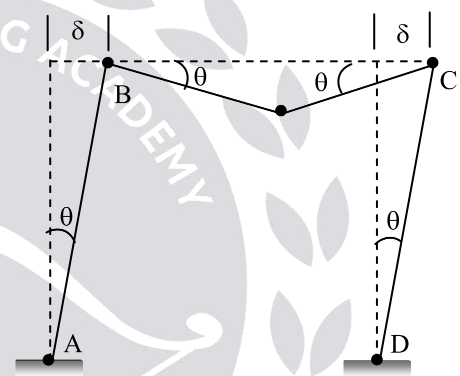


Figure 8

Equating the virtual work done to zero. $W\frac{\ell}{2}\alpha - 2M_P\alpha - 2M_P\theta = 0$

$$W\frac{\ell}{2}2\theta - 2M_P 2\theta - 2M_P\theta = 0; \quad W = \frac{6M_P}{\ell}$$

Hence the value of W at which the actual collapse will occur $W = \frac{6M_P}{\ell}$

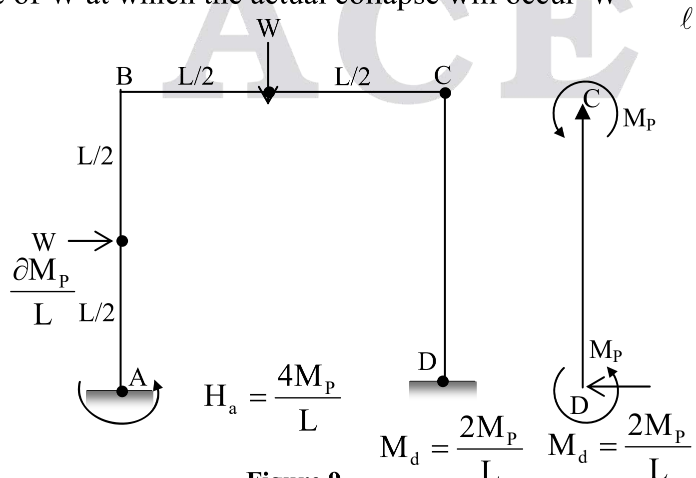


Figure 9

Fig. 9 shows the loading on the frame.

Considering CD for the equilibrium condition

$$H_d = \frac{M_P + M_P}{\ell} \quad \therefore H_d = \frac{2M_P}{\ell}$$

For the equilibrium of the frame ABCD

$$H_a = \frac{6M_P}{\ell} - \frac{2M_P}{\ell} = \frac{4M_P}{\ell}$$

B.M. at B

$$= \frac{4M_P}{\ell}\ell - M_P - \frac{6M_P}{\ell}.\frac{\ell}{2} = 4M_P - M_P - 3M_P = 0$$

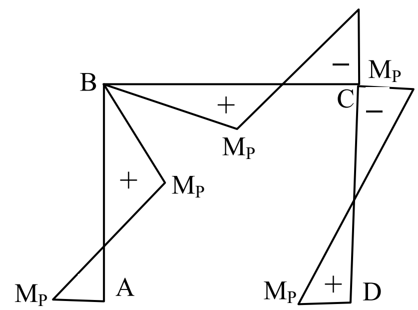


Figure 10

BMD at Collapse

04.

Sol:

$D_S = 4 - 3 = 1$

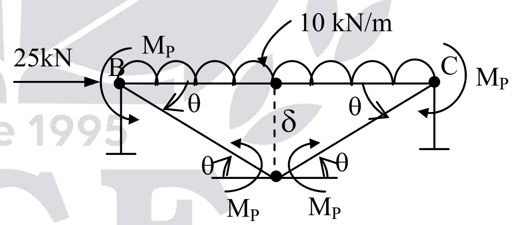
$n = D_s + 1 = 2$

$N = 3$

$I = N - D_S = 3 - 1 = 2$ [Beam mechanism, sway mechanism]

Beam Mechanism:

$$W_e = 10\left[\frac{1}{2}\times 8\times\delta\right] = 10\left[\frac{1}{2}\times 8\times 4\theta\right]$$



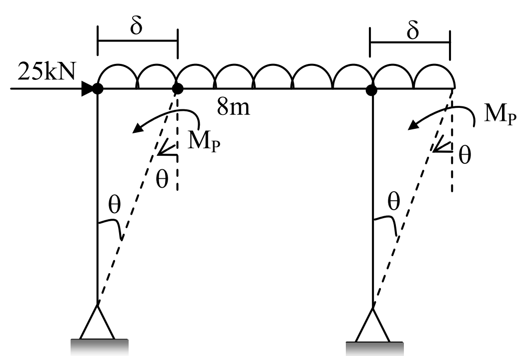
$W_i = 4M_P\theta$

$W_e = W_i$

$160\theta = 4\,M_P\theta$

$$M_P = \frac{160}{4} = 40\text{kN} - \text{m}$$

S way mechanism:-



$W_e = 25 \times \delta = 25 \times 4\theta = 100\theta$

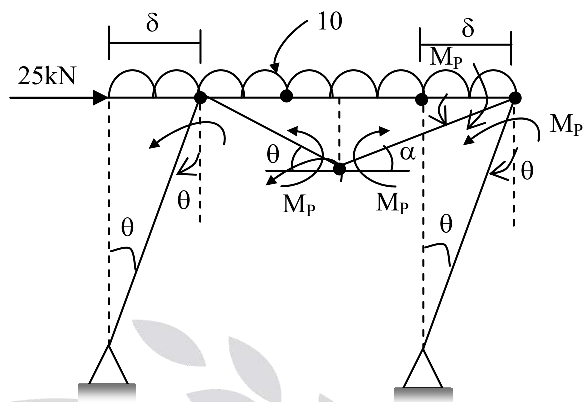
$W_i = 2\,M_P\theta$

$W_e = W_i$

$100\,\theta = 2M_P\theta$

$M_P = 50$ kN-m

Combined mechanism:-



$$W_e = 25\times\delta + 10\left[\frac{1}{2}\times 8\times\delta_1\right]$$

$$= 25\times 4\theta + 10\left[\frac{1}{2}\times 8\times x\theta\right]$$

$$= 100\theta + 40x\theta$$

$W_i = M_P\theta + M_P\alpha + M_P\,\alpha + M_P\theta$

$W_i = 2\,M_P\theta + 2M_P\alpha$

$W_e = W_i$

$100\theta + 40\,x\theta = 2M_P\theta + 2M_P\alpha$

$x\theta = (8 - x)\alpha$

$$\alpha = \frac{x\theta}{(8-x)}$$

$$100\theta + 40x\theta = 2M_P\theta + 2M_P\left[\frac{x\theta}{(8-x)}\right]$$

$$100 + 40x = 2M_P\left[1 + \frac{x}{(8-x)}\right]$$

$$M_P = \frac{100+40x}{2\left[1+\frac{x}{(8-x)}\right]}$$

$$M_P = \frac{(100+40x)(8-x)}{2[(8-x)+x]}$$

$$M_P = \frac{800+320x-100x-40x^2}{16}$$

$$M_P = \frac{800+220x-40x^2}{16}$$

$$\frac{\partial M_P}{dx} = 0$$

$$\frac{1}{16}[220-80x] = 0$$

$$x = \frac{220}{80} = 2.75\text{m}$$

$$M_P = \frac{800+220\times 2.75-40\times 2.75^2}{16}$$

$M_P = 68.906$ kN-m

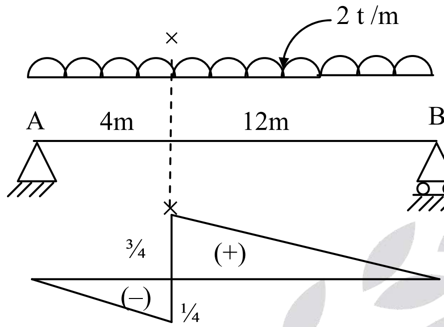
Note: - The required plastic moment will be the biggest of the three values

$M_P = 68.906$ kN-m

09. Rolling Loads & Influence Lines

01. Ans: (a)

Sol:

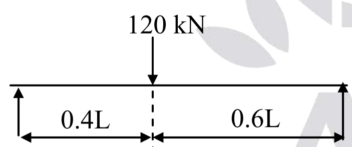


S.F @ = Intensity of u.d.*l* × area of I.L.D under u.d.*l*

$$\text{Max } V_x = 2\left[\frac{1}{2}\times\frac{3}{4}\times 12-\frac{1}{2}\times\frac{1}{4}\times 4\right]=8\text{t}$$

02. Ans: (c)

Sol:



The maximum B.M @ a section occurs if the point load is @ the section.

$$\text{Maximum B.M} = \frac{Wab}{L}=\frac{W\times 0.4L\times 0.6L}{L}$$

$$= 0.24\ WL$$

03. Ans: (b)

Sol:

$$R_A = \frac{M}{L}\downarrow \qquad R_B = \frac{M}{L}$$

Take moment at the distance 'x' from support A

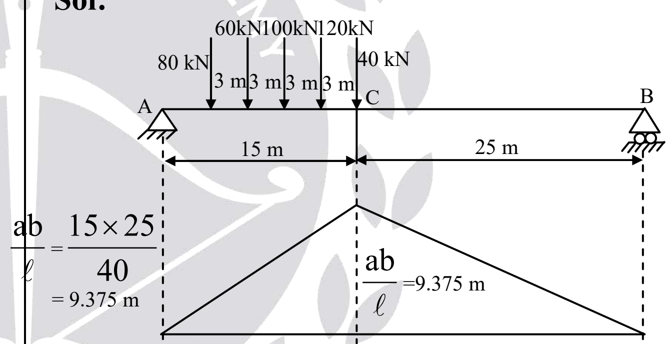
$$\because M_x = M - \frac{M}{L}.x$$

When x = L, $M_x = 0$

When x = 0, $M_{max} = M_x = M$

05. Ans: (b)

Sol:



Place 40 kN on section C

Avg load on LHS $-\dfrac{40}{25}$

$$\frac{80+60+100+120}{15}-\frac{40}{25}=22.4\text{kN}$$

∴ Allow to 40 kN to cross C and place 120 kN on section C

$$\frac{80+60+100}{15}-\frac{40+120}{25}=9.60>0$$

∴ Allow to 120 kN to cross C and place 100 kN on section C

$$\frac{80+60}{15} - \frac{40+120+100}{25} = -\,1.06<0$$

Avg load LHS   Avg load on RHS

∴ Place 100 kN on C and other load in their respective position maximum BM at C

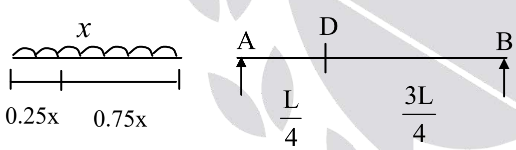
06. *Refer GATE Solutions Book*

07. *Refer GATE Solutions Book*

08. *Refer GATE Solutions Book*

09. Ans: (c)

Sol:



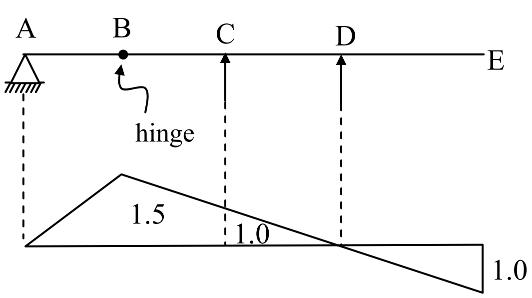
Average load on AD = Avg load on BD

The ratio of AD : DB =1:3

∴ ¾th of u.d. l has to cross the quarter section 'D'.

10. Ans: (b)

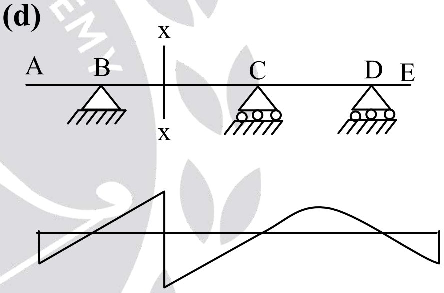
Sol:



Apply Muller Breslau's principle. To draw I.L.D for support R_C, apply unit vertical displacement at 'C'. To the left of hinge 'B', simple support 'A' exists which cannot offer resistance against rotation but offers resistance against vertical displacement only. Hence hinge 'B' rises linearly as shown. Support 'D' only can rotate. Free end 'E' can have vertical deflection also. Ordinates are proportional to distances as the I.L.D for determinate structures are linear.

11. Ans: (d)

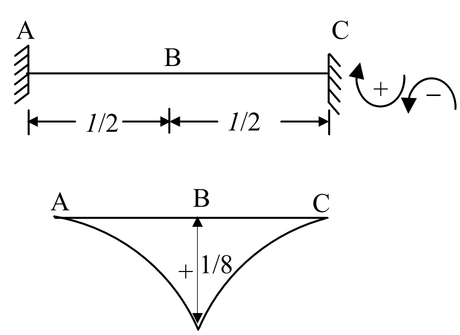
Sol:



- At x-x the I.L.D has vertical ordinate with change in sign from one side to the other side. It is the character of I.L.D for shear force.
- Using Muller Breslau's principle, release the shear constraint by assuming shear hinge at 'x'. The deflected profile is the I.L.D shown.

12. Ans: (a)

Sol:



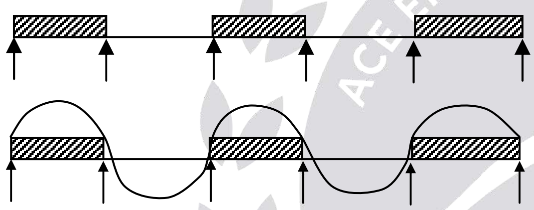
Apply unique rotation at 'B' by assuming a hinge. The deflection profile is the I.L.D for moment at 'B'.

Note: as A and B are fixed $\theta_A = \theta_B = 0$

To calculate ordinate at 'B' assume unit load is applied at 'B'. Due to this the B.M at 'B' = L / 8. Further fixed beam being statically indeterminate structure, the I.L.D will be non-linear.

13. Ans: (b)

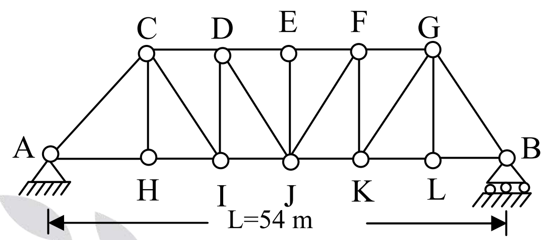
Sol:



For minimum positive moment at 'x' shown (mid point of second span), no load on second span but u.d.l on alternative spans shall be provided.

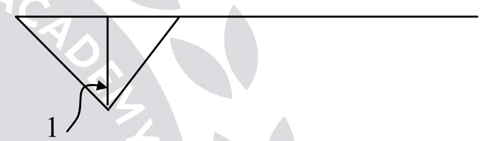
- Positive moment at 'x' means sagging in the second span. As minimum positive moment is required, don't place the load on the second span. Further to counter sagging in second span place the u.d.l on alternative spans (1, 3 and 5)
- concept can be easily understood by seeing the deflection profile shown using pattern loading.

Common Data for Questions 14 & 15



14. Ans: (c)

Sol:



I.L.D for axial force in the member 'CH'

Design force for member CH

= Intensity of u.d.l × area of I.L.D under u.d.l

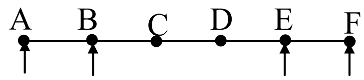
$$= (10 + 20)\left(\frac{1}{2} \times 18 \times 1\right) = 270 \text{ kN (tension)}$$

15. Ans: (d)

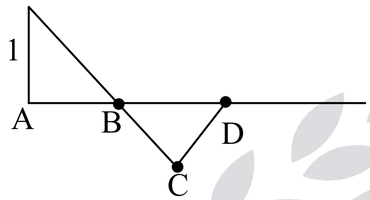
Sol: The frame shown is through type truss in which loads will be transferred to the bottom joints. Hence no load is possible at joint 'E'. Hence at 'E' three forces exists of which two are in the same line, hence the third force 'EJ' is zero.

16. Ans: (c)

Sol:



As per Muller-Breslau's principle, apply unit displacement in the direction of reaction A.



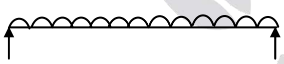
ILD for reaction A

At B, member will rotate. Due to hinge at point D, effect of unit displacement at A will not get effected.

17. Ans: (b)

Sol: When length of UDL is greater than span length

To obtain maximum B.M (or) absolute max B.M, UDL should cover entire span



∴ Statement I is correct

When length of UDL is less than span length

To obtain max B.M at section, this section should divide the load in the same ratio as it divides the span.

For absolute max B.M, center of span will coincide with C.G of load.

∴ In both cases UDL should be divided by section.

∴ Statement (II) is correct.

But statement (II) is not the explanation of statement (I)

18. Ans: (c)

Sol: ILD for indeterminate structures is curvilinear and for determinate structures is linear.

ILD can be drawn for indeterminate structure qualitatively. But, for determinate structures both qualitative and quantitative diagram can be drawn.

19. Ans: (a)

Sol: To find out the location of maximum B.M and its value, ILD is used while design of bridges.

Conventional Practice Solutions

01.

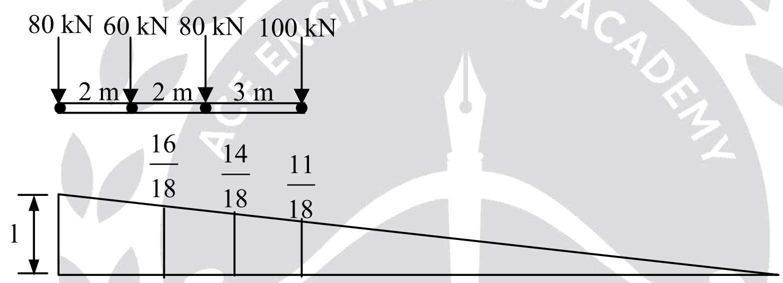
Sol: For absolute maximum negative shear, the load position is a shown in Fig. 1.

$$\text{Absolute maximum negative shear } = 80\times1+60\times\frac{16}{18}+80\times\frac{14}{18}+100\times\frac{11}{18}$$

For absolute maximum positive shear, the load position is as shown in Fig. 2.

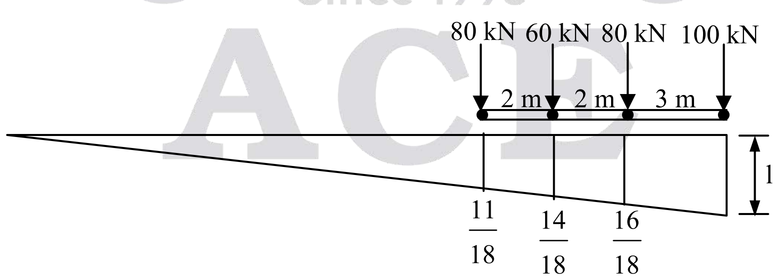
$$\text{Absolute maximum positive shear } = 80\times\frac{11}{18}+60\times\frac{13}{18}+80\times\frac{15}{18}+100$$

$$= 258.889 \text{ kN}$$



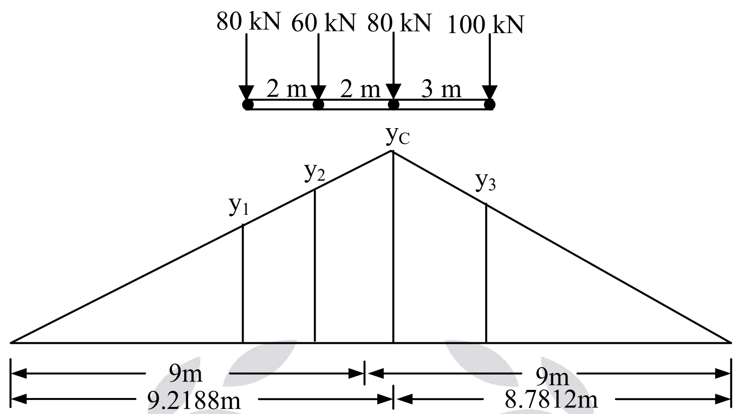
Load position for absolute maximum -ve SF

Figure 1



Load position for absolute maximum +ve SF

Figure 2



ILD for absolute maximum moment

Figure 3

Note: No additional trials are made since, end loads are not light. For finding absolute maximum moment, first C.G. of loads is to be found and the position of loads to be determined.

Taking moment about leading load 100kN

$$a = \frac{80\times 3 + 60\times 5 + 80\times 7}{100 + 80 + 60 + 80} = 3.4375\,\text{m}$$

i.e., resultant is very close to leading 80 kN load.

Hence, maximum moment is likely to occur under 80 kN leading load. The distance between this load and the resultant is d = 3.4375 − 3 = 0.4375 m.

For maximum bending moment this load should be at $= \frac{L}{2} + \frac{d}{2} = \frac{18}{2} + \frac{0.4375}{2}$

$= 9.2188$ m

ILD ordinate for a section at 9.2188 is

$$y_c = \frac{9.2188\times(18 - 9.2188)}{18}$$

$= 4.497$

ILD for this case is shown in Fig. 3.

Absolute maximum B.M.

$$= 80y_1 + 60y_2 + 80y_c + 100y_3$$

$$= \left[80\times\frac{5.2188}{5.2188} + 60\times\frac{7.2188}{9.2188} + 80 + 100\times\frac{5.7812}{8.7812}\right]4.497$$

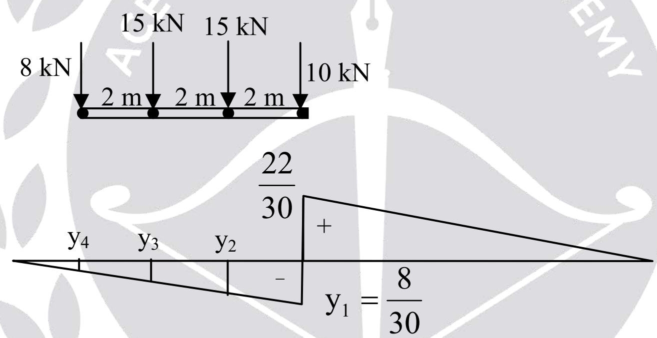
= Since, $y_c = 4.497$

= 1070.77 kNm

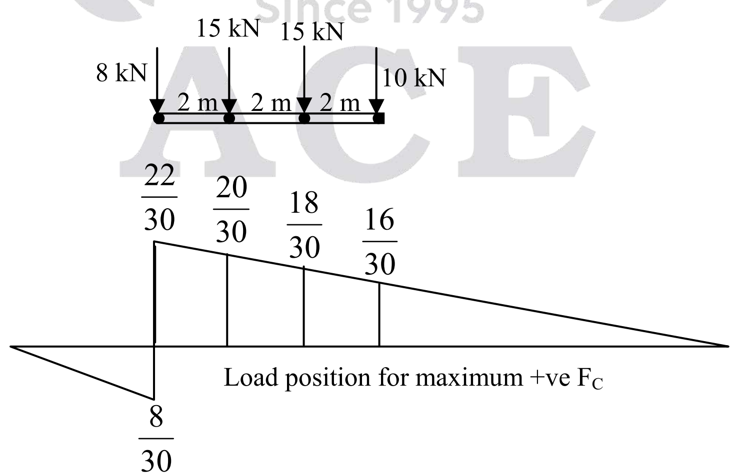
Note: Additional trial to check the moment under 60 kN load is not made since, centre of gravity is very close to the load which is heavier than 60 kN load.

02.

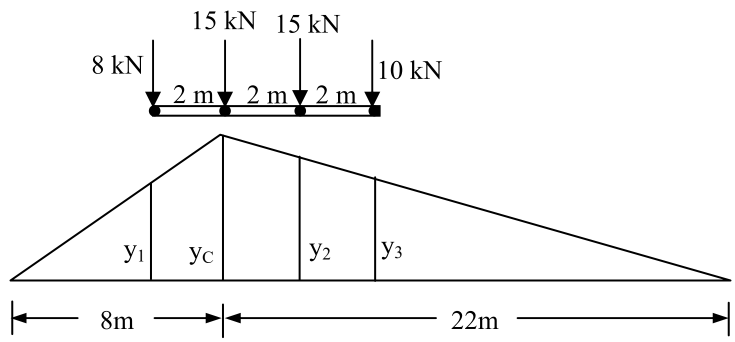
Sol: The beam is shown in Figure ILD for shear force at 8 m from left support is shown in Figure along with possible load position for maximum negative shear force. Maximum negative SF at C.



Load position for maximum –ve F_C



Load position for maximum +ve F_C



ILD for M_C

$= 10\, y_1 + 15\, y_2 + 15\, y_3 + 8\, y_4$

$= 10\times\frac{8}{30}+15\times\frac{6}{30}+15\times\frac{4}{30}+8\times\frac{2}{30}$

= 8.2 kN

For maximum positive SF at C, load position is as shown in Figure S.F. at C

$=10\times\frac{16}{30}+15\times\frac{18}{30}+15\times\frac{20}{30}+8\times\frac{22}{30}$

= 30.2 kN

Check for another position i.e., when W_S = 15 kN load is on the section

S.F at C $= 10\times\frac{18}{30}+15\times\frac{20}{30}+15\times\frac{22}{30}-8\times\frac{6}{30}$

= 25.4 kN

$\therefore$ Maximum positive shear force is = 30.2 kN

ILD for bending moment at C is as shown in Figure. The maximum ordinate

$$y_c = \frac{z(L-z)}{L} = \frac{8(30-8)}{30}$$

To find the load position for maximum moment, average load on portion AC and CB are to be found as loads crosses section C one after another.

Table: Calculations to find load position for maximum M_C

Load crossing	Average load		Remarks
	AC W_{1av}	BC W_{2av}	
10 kN	$\frac{38}{8}$	$\frac{10}{22}$	$W_{1av} > W_{2av}$
15 kN	$\frac{23}{8}$	$\frac{25}{22}$	$W_{1av} > W_{2av}$
15 kN	$\frac{8}{8}$	$\frac{40}{22}$	$W_{1av} < W_{2av}$

Hence, load position for maximum moment at C is when second 15 kN load is on C.

Maximum $M_c = 8y_1 + 15y_c + 15y_2 + 10y_3$

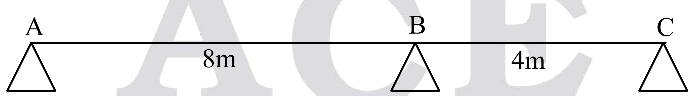
$$= 8\left(\frac{6}{8}\right)y_c + 15y_c + 15\left(\frac{20}{22}\right)y_c + 10\left(\frac{18}{22}\right)y_c$$

$= 251.21$ kNm, Since $y_c = 5.867$

04.

Sol: When the unit load is on AB, distant x from A.

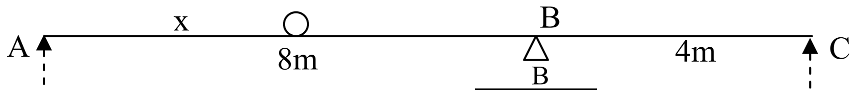
Fixed end moments



$$\overline{M}_{ab} = -\frac{x(8-x)^2}{64};\overline{M}_{ba} = +\frac{x^2(8-x)}{64}$$

The distribution factors at B for BA and BC are respectively,

$$D_{ba} = \frac{4}{8+4} = \frac{1}{3} \text{ and } D_{bc} = \frac{8}{8+4} = \frac{2}{3}$$



A	B		C
	$\frac{1}{3}$	$\frac{2}{3}$	
$-\frac{x(8-x)^2}{64}$	$+\frac{x^2(8-x)}{64}$	0	0
$+\frac{x(8-x)^2}{64}$	$+\frac{x(8-x)^2}{128}$		
0	$+\frac{x(64-x^2)}{128}$	0	0
	$-\frac{x(64-x^2)}{384}$	$-\frac{2x(64-x^2)}{384}$	
0	$+\frac{x(64-x^2)}{192}$	$-\frac{x(64-x^2)}{192}$	0

The moment distribution is carried out and the final moments are calculated. See moment table. The reactions at the supports are given by,

$$V_a = -\left[\frac{\frac{x(64-x^2)}{192} - 1(8-x)}{8}\right] = 1 - \frac{x}{8} - \frac{x(64-x^2)}{1536}$$

$$V_c = -\frac{x(64-x^2)}{192}\frac{1}{4} = -\frac{x(64-x^2)}{768}$$

$$V_b = 1 - V_a - V_c = 1 - 1 + \frac{x}{8} + \frac{x(64-x^2)}{1536} + \frac{x(64-x^2)}{768} = \frac{x(128-x^2)}{512}$$

The ordinates of the influence lines for V_a, V_b and V_c at 1 metre intervals as the unit load moves from A to B are tabulated below.

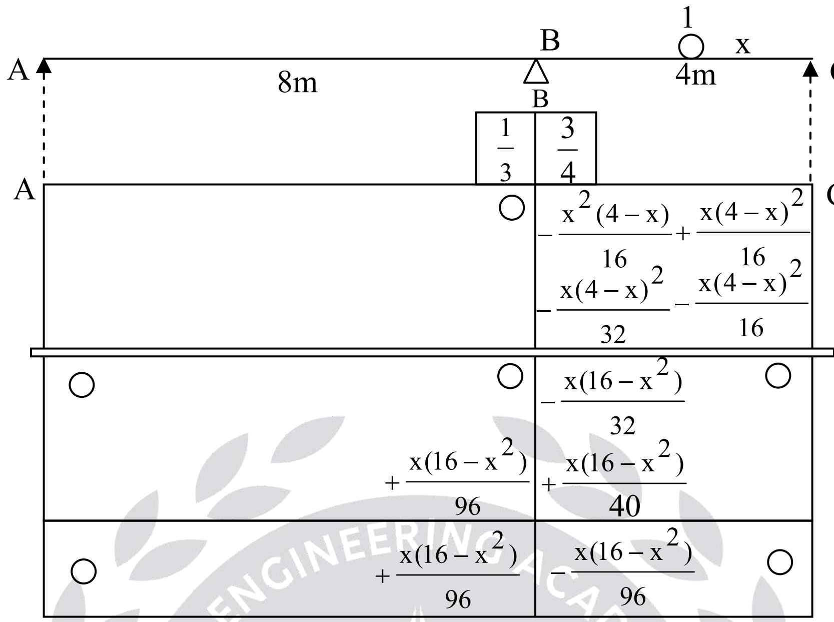
Distance of the unit load from A (m)	$V_a = 1 - \frac{x}{8} - \frac{x(64-x^2)}{1536}$	$V_b = \frac{x(128-x^2)}{512}$	$V_c = \frac{-x(64-x^2)}{768}$
0	1	0	0
1	0.8341	0.2480	-0.0821
2	0.6719	0.4844	-0.1563
3	0.5176	0.6972	-0.2148
4	0.3750	0.8750	-0.2500
5	0.2481	1.0058	-0.2539
6	0.1407	1.0781	-0.2188
7	0.0566	1.0801	-1.1367
8	0	1	0

When the unit load is on BC at a distance x from C

Fixed end moments:

$$\overline{M}_{bc} = -\frac{(4-x)x^2}{16} \text{ and } \overline{M}_{cb} = +\frac{(4-x)^2 x}{16}$$

The moment distribution is carried out and the final moments are calculated. See moment table.



The reactions at the supports are now given by,

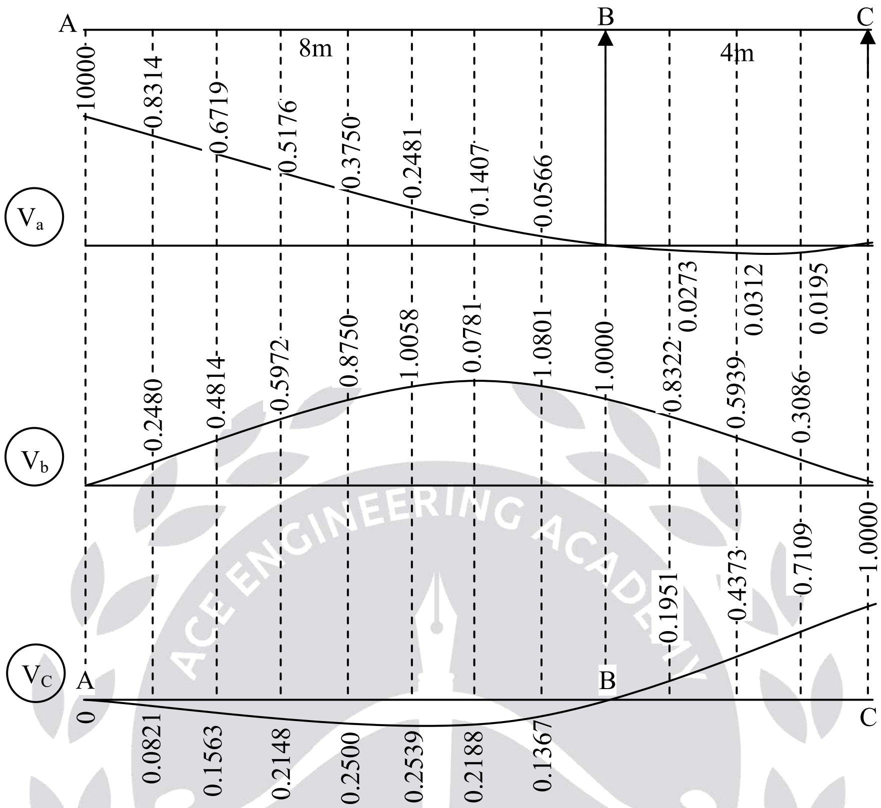
$$V_a = -\frac{x(16-x^2)}{96}\cdot\frac{1}{8} = -\frac{x(16-x^2)}{768}$$

$$V_c = \left[\frac{\dfrac{-x(16-x^2)}{96}+1(4-x)}{4}\right] = 1-\frac{x}{4}-\frac{x(16-x^2)}{384}$$

$$V_b = 1 - V_a - V_c = 1+\frac{x(16-x^2)}{768} - 1 + \frac{x}{4} + \frac{x(16-x^2)}{384} = \frac{x(80-x^2)}{256}$$

The ordinates of the influence lines for V_a, V_b and V_c at 1 metres intervals, as the unit load moves from C to B are tabulated below.

Distance of the unit load from C(m)	$V_a = -\frac{x(16-x^2)}{768}$	$V_b = -\frac{x(80-x^2)}{256}$	$V_C = 1-\frac{x}{4}-\frac{x(16-x^2)}{384}$
0	0	0	1
1	– 0.0195	0.3086	0.7109
2	– 0.0312	0.5938	0.4374
3	– 0.0273	0.8322	0.1951
4	0	1	0



Above figure shows the influence line for V_a, V_b and V_c.

05.

Sol: (a) I.L for R_B

$$R_B = \frac{y_{XB}}{y_{BB}} \quad \text{.................. (1)}$$

To computer y_{XB}, apply a unit vertical load at B, as shown in Fig. b.

At any section X distant x from B, we have

$$EI\frac{d^2y}{dx^2} = -M_x = 1.x$$

Integrating, $EI\frac{dy}{dx} = \frac{x^2}{2} + C_1$

At $x = L$, $\frac{dy}{dx} = 0$

$\therefore\ C_1 = -\frac{L^2}{2}$

Hence $EI\frac{dy}{dx} = \frac{x^2}{2} - \frac{L^2}{2}$

Integrating further,

$$EI\,y = \frac{x^3}{6} - \frac{L^2}{2}x + C_2$$

At $x = L, y = 0$

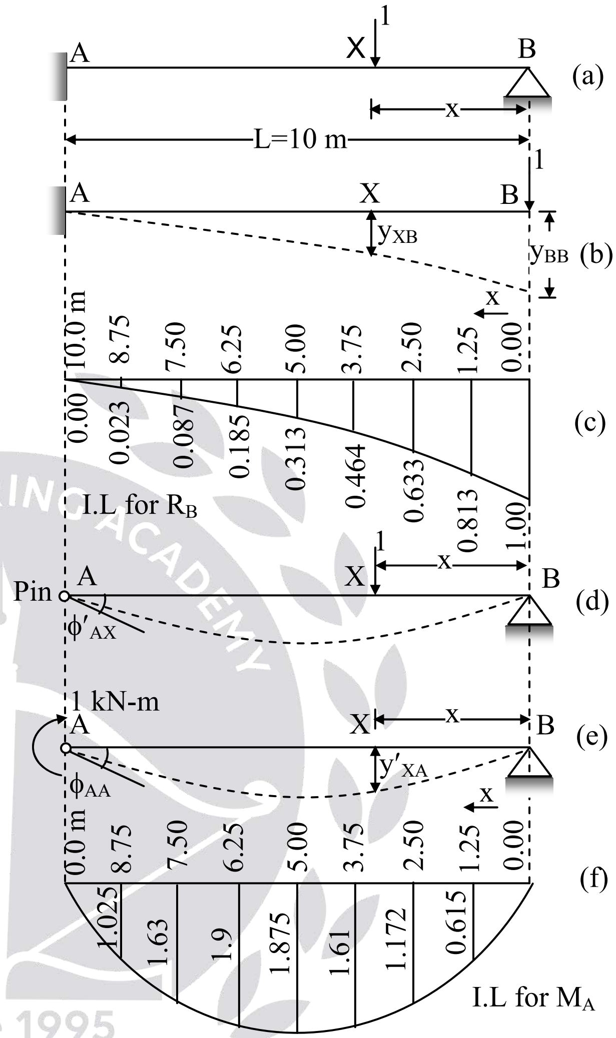
$$\therefore\ C_2 = \frac{L^3}{2} - \frac{L^3}{6} = \frac{L^3}{3}$$

Hence $Ely = \frac{x^3}{6} - \frac{L^2}{2}x + \frac{L^3}{3}$ ....(2)

At $x = 0,\ y = y_{BB} = \frac{L3}{3EI}$

At $x = x$

$$y = y_{XB} = \frac{1}{EI}\left(\frac{x^3}{6} - \frac{L^2}{2}x + \frac{L^3}{3}\right)$$



Substituting these in (1), we get

$$R_B = \left(\frac{x^3}{6} - \frac{L^2}{2}x + \frac{L^3}{3}\right)\frac{3}{L^3}$$

or $\quad R_B = \frac{1}{2}\left(\frac{x^3}{L^3} - \frac{3x}{L} + 2\right)$

$$R_B = \frac{1}{2}(n^3 - 3n + 2), \text{where} \frac{x}{L} = n$$

The ordinates of I.L for R_B are computed in Table below.

Table

x(m)	0	1.25	2.50	3.75	5	6.25	7.5	8.75	10
n = x/L	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
R_B	1	0.813	0.633	0.464	0.313	0.185	0.087	0.023	0

The I.L for R_B is shown in Fig. c.

(b) I.L for M_A:

In order to draw the I.L for M_A, replace the fixed support at A by a pin, as shown in Fig. d. Remove the external unit load and apply a unit couple at A, as shown in Fig. e.

$$M_A = \frac{y'_{XA}}{\phi_{AA}} S$$

Where y'_{XA} = vertical deflection at X due to unit couple at A

ϕ_{AA} = slope at A due to unit couple at A,

Let R'_B = Reaction at B, when unit moment is acting at $A = \frac{1}{L}\uparrow$

$$EI\frac{d^2y}{dx^3} = -M_x = -R'_{B.}x = -\frac{x}{L}$$

$$EI\frac{dy}{dx} = -\frac{x^2}{2L} + C_1$$

and $EI\,y = -\frac{x^3}{6L} + C_1 x + C_2$

At x = 0, y = 0. $\therefore$ $C_2 = 0$; At x = L, y = 0, $\therefore$ $C_1 = L/6$

Hence $EI\frac{dy}{dx} = -\frac{x^2}{2L} + \frac{L}{6}$ and $EI\,y = -\frac{x^3}{6L} + \frac{Lx}{6}$

$$\text{At } x = L,\ \frac{dy}{dx} = \phi_{AA} = \frac{1}{EI}\left(-\frac{L^2}{2L} + \frac{L}{6}\right) = -\frac{L}{3}$$

$$\text{At } x = x,\ y = y'_{XA} = \frac{1}{EI}\left(-\frac{x^3}{6L} + \frac{Lx}{6}\right)$$

Substituting these values in (3), we get

$$M_A = \left(\frac{x^3}{6L} - \frac{Lx}{6}\right) \times \frac{3}{L} = \frac{1}{2}\left(\frac{x^3}{L^2} - x\right)$$

This is thus the equation of the influence line for M_A. The ordinates are calculated in the tabular form in table below.

Table

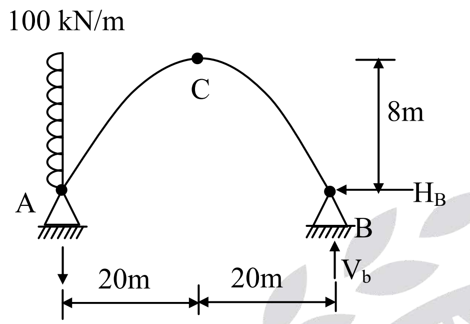
x(m)	0	1.25	2.5	3.75	5	6.25	7.5	8.75	10
x^2/L^2	0	0.0195	0.156	0.53	1.25	2.45	4.24	6.7	10.0
M_A	0	-0.615	-1.172	-1.61	-1.875	-1.9	-1.63	-1.025	0

The minus sign shows that the direction of M_A is in reverse direction to that of the unit moment applied at A, i.e., M_A acts in anti-clockwise direction. The I.L for M_A is shown in Fig. f.

10. Arches & Cables

01. Ans: (a)

Sol:



Take moment about A $\Sigma M_A = 0$

$40 \times V_b = 100 \times 8 \times 8/2$

$V_b = 80$ kN

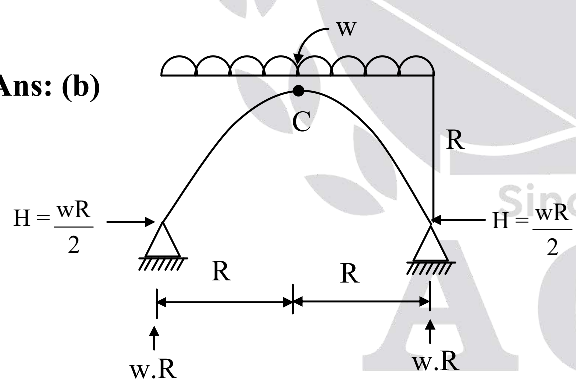
$\Sigma M_C = 0$, from RHS

$80 \times 20 = H_B \times 8$

$H_B = 200$ kN

02. Ans: (b)

Sol:



$\Sigma M_c = 0$

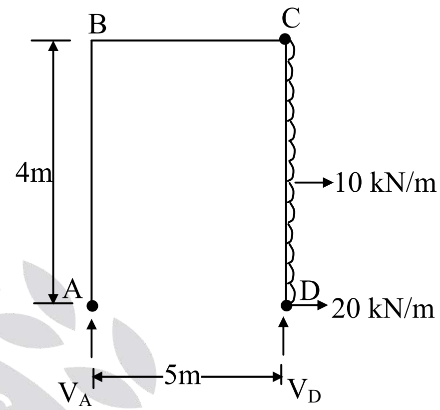
$H.R = wR.R - wR.\ R/2$

$H \times R = wR^2 - \frac{wR^2}{2} = H \times R$

$$H = \frac{wR}{2}$$

03. Ans: (c)

Sol:



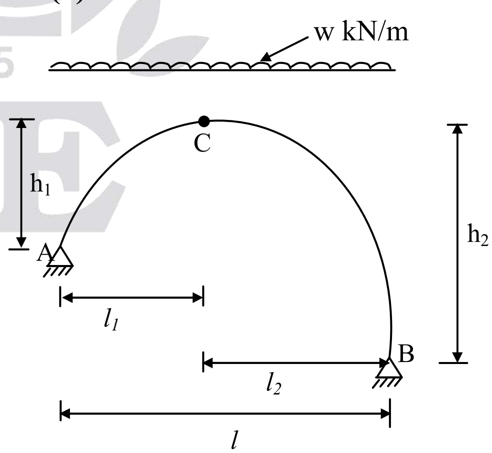
As the support are at same level, the vertical reactions can be worked to similar to that of S.S beam

$\Sigma M_D = 0$ from left

$5V_A = 10 \times 4 \times 2 = 80\ kN \Rightarrow V_A = 16$ kN

04. Ans: (a)

Sol:



Equation for parabola can be taken as

$$\frac{x^2}{y} = \text{constant}$$

$$\therefore \frac{x}{\sqrt{y}} = \text{constant}$$

$$\therefore \frac{\ell_1}{\sqrt{h_1}} = \frac{\ell_2}{\sqrt{h_2}} = \frac{\ell_1+\ell_2}{\sqrt{h_1}+\sqrt{h_2}} = \frac{\ell}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\therefore \ell_1 = \frac{\ell\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \quad \text{and} \quad \ell_2 = \frac{\ell\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

Taking moments on left portion about C

$$\therefore V_A \times \ell_1 - H \times h_1 - w(\ell_1^2)/2 = 0$$

$$\therefore V_A = \frac{w\ell_1}{2} + \frac{Hh_1}{\ell_1} \text{ …….. (1)}$$

Similarly taking moments on right portion about C,

$$-V_B \times \ell_2 + H \times h_2 + w(\ell_2^2)/2 = 0$$

$$\therefore V_B = H.\left(\frac{h_2}{\ell_2}\right) + \frac{w\ell_2}{2} \text{ …….. (2)}$$

Apply $\Sigma V = 0$,

$V_A + V_B = w(l_1 + l_2) = wl$

Substitute V_A and V_B in above equation

$$\frac{w\ell_1}{2} + H\left(\frac{h_1}{\ell_1}\right) + H\left(\frac{h_2}{\ell_2}\right) + \frac{w\ell_2}{2} = w\ell$$

$$H\left(\frac{h_1}{\ell_1} + \frac{h_2}{\ell_2}\right) + w\left(\frac{\ell_1+\ell_2}{2}\right) = w\ell$$

$$H\left(\frac{h_1}{\ell_1} + \frac{h_2}{\ell_2}\right) = w\ell - w\left(\frac{\ell}{2}\right) = \frac{w\ell}{2}$$

Substitute l_1 *and* l_2 in above equation

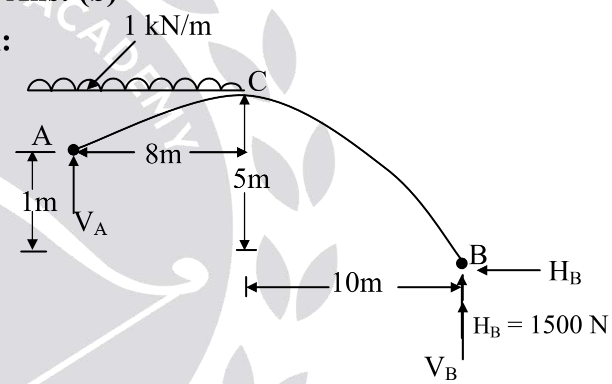
$$\therefore H\left(\frac{h_1}{\left(\frac{\ell\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right)} + \frac{h_2}{\left(\frac{\ell\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}\right)}\right) = \frac{w\ell}{2}$$

$$H\left[(\sqrt{h_1}+\sqrt{h_2})\sqrt{h_1} + \sqrt{h_2}(\sqrt{h_1}+\sqrt{h_2})\right] = \frac{w\ell^2}{2}$$

$$\therefore H = \frac{w\ell^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

05. Ans: (b)

Sol:



Supports are at different levels $\Sigma M_c = 0$ from right

$V_b \times 10 = 5\ H_b \qquad \therefore V_b = 0.5\ H_b$ …. (1)

$\Sigma M_c = 0$, from left.

$4H_A + 1 \times 8 \times 4 = V_A \times 8$

$\therefore V_A = 0.5H_A + 4$ …. (2)

$V_a + V_b = 8 \times 1 = 8$

$H_A = H_b = H$

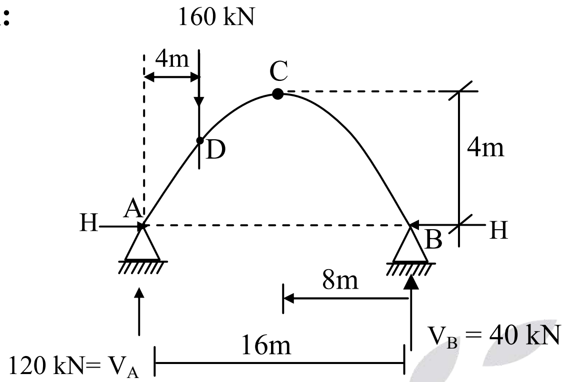
$V_a + V_b = 0.5\ H_b + 0.5H_a + 4$

$V_a + V_b = H + 4$

$8 = H + 4 \qquad \therefore H = 4$ kN

06. Ans: (c)

Sol:



$$\therefore V_A = \frac{160\times12}{16} \qquad V_B = \frac{160\times4}{16}$$

$= 120 \qquad\qquad = 40$ kN

Take $\Sigma M_c = 0 \quad H \times 4 = 40 \times 8$

$\Rightarrow H = 80$ kN.m

Calculation of vertical ordinate at section 'D' where the point load is acting

$$y = \frac{4h}{l^2}(x)(l - x)$$

$$= \frac{4\times4}{16^2}\times(4)\times(16-4)$$

$$= \frac{1}{16}\times4\times12 = 3\text{m}$$

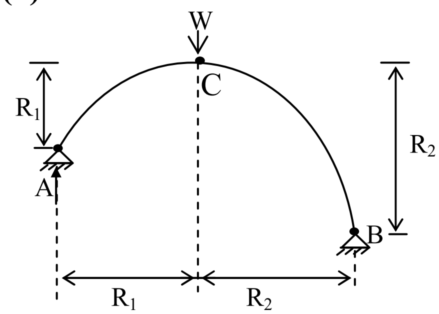
Note: The height at quarter of a parabola is $= 3h/4$

$M_D = 120 \times 4 - 80 \times 3 = 480 - 240$

$= 240$ kN-m

07. Ans: (b)

Sol:



$\Sigma M_C = 0$ from left

$V_A.R_1 = H.R_1 \Rightarrow V_A = H$

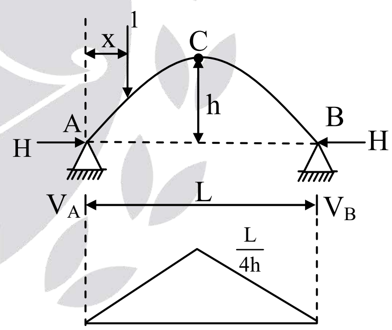
Similarly $V_B = H$

$V_A + V_B = 2H = W$

$$\Rightarrow H = \frac{W}{2}$$

08. Ans: (d)

Sol:



ILD for H

Assume a unit load rolls on the span from left to right. The horizontal and vertical reactions will change at the supports as the load moves on the span.

Assume the unit load be at a distance x from A.

Then

$$V_A = \frac{L-x}{L} \text{ and } V_B = \frac{x}{L}$$

Assume H=The horizontal thrust at supports.

Apply $\Sigma M_C = 0$ from right

$$H.h = \frac{x}{L}.\frac{L}{2}$$

$$\therefore H = \frac{x}{2h}$$

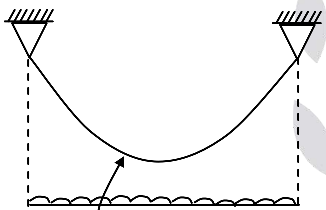
For horizontal thrust to be maximum

$$x = \frac{L}{2} \text{ i.e., at the crown.}$$

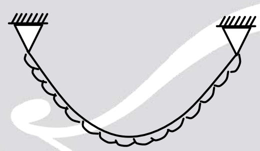
$\Rightarrow$ Maximum horizontal reaction of $\frac{L}{4h}$ is possible if the load is at the crown.

09. Ans: (d)

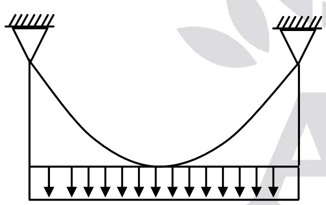
Sol: When resolved it can be axial force



Parabolic shape
If load is along
horizontal span



Catenaries shape
If load is along
the length



Load transferred from cross girders to the longitudinal girder in cable bridges.

10. Ans: (b)

Sol: Figure shows an arch (either two-hinged or three-hinged arch) subjected to an external load system. Consider any section X. Consider the equilibrium of the part AX of the arch. This part is in equilibrium under the action of the following

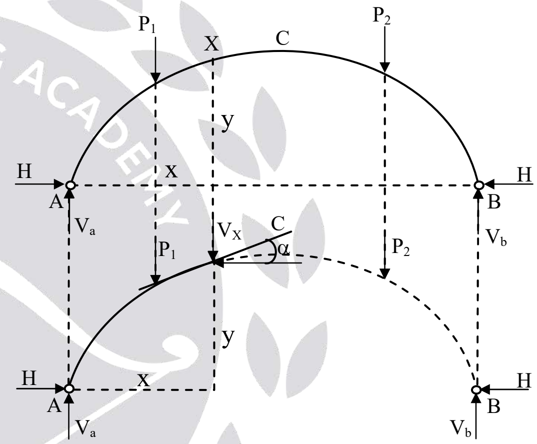
i) Reaction V_a and H at A

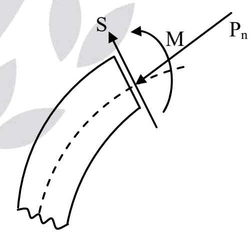
ii) External loads between A and X

iii) Reacting forces V_X and H_X provided by the part XB on the part XA at X

iv) Reacting moment (bending moment) at X.

Resolving the forces on the part AX vertically and horizontally, we can determine the vertical and the horizontal reacting forces V_X and H_X at D.





Arch section subjected to normal thrust P_n radial shear S, bending moment M.

Let the tangent to the centre line of the arch at X be inclined at α to the horizontal.

The component of the reacting forces at X perpendicular to the tangent at X is called the Shear Force (or) Radical Shear at X.

$\therefore$ Shear at X= S

$= H_X \sin\alpha - V_X \sin\alpha$ (or) $V_X \cos\alpha - H_X \sin\alpha$

The component of reacting forces at X along the tangent X is called the Normal thrust at X.

$\therefore$ Normal thrust at X = $P_n = H_X\cos\alpha + V_X\sin\alpha$

(H_X = H) from F.B.D

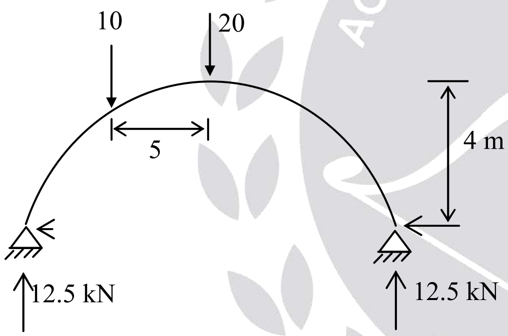
(Neglecting sign)

11. Ans: (c)

Sol: $\therefore H_{max}.h = \frac{w}{2}.\frac{l}{2} \Rightarrow H_{max} = \frac{wl}{4h}$ s

(due to rolling point load)

$\therefore$ In the problem, here. Place 20 kN at centre.

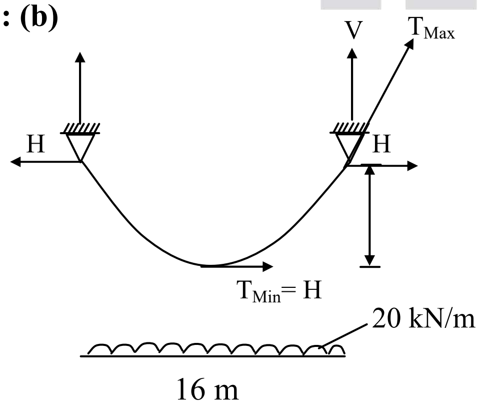


$$\Sigma M_c = 0$$

$$12.5 \times 10 = H \times 4$$

$$H = \frac{12.5\times10}{4} = 31.25 \text{ kN}$$

12. Ans: (b)



Sol: $V = \frac{wl}{2} = \frac{20\times16}{2} = 160\,\text{kN}$

$$H = \frac{wl^2}{8h} = \frac{20\times16}{8\times4}$$

$$= 160 \text{ kN}$$

$$T_{max} = \sqrt{V^2+H^2} = 160\sqrt{2}\,\text{kN}$$

$$T_{min} = H = 160 \text{ kN}$$

13. Ans: (c)

Sol: When unit load is in b/w A and C

Considering RHS of C.

$$H \times h = V_B \times \frac{L}{2}$$

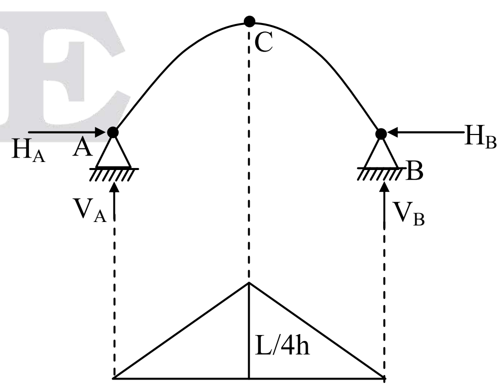
$$H = \frac{x}{L}\times\frac{L}{2}\times\frac{1}{h} = \frac{x}{2h}$$

When unit load is in b/w C and B.

Considering LHS

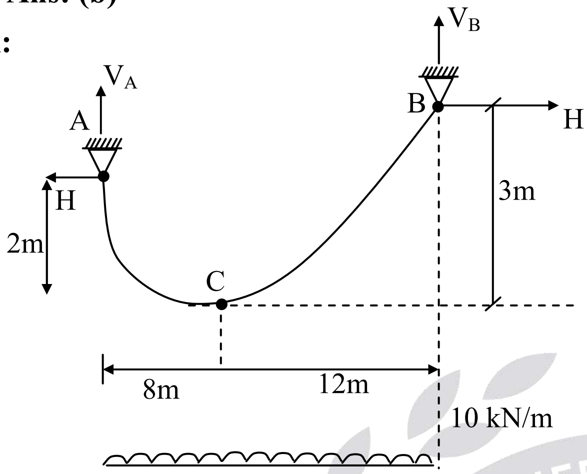
$$V_A \times \frac{L}{2} = H\times h$$

$$H = \frac{(L-x)}{L}\times\frac{L}{2h} = \frac{L-x}{2h}$$



14. Ans: (b)

Sol:



$\Sigma M_c = 0$, from left

$V_A \times 8 = H \times 2 + 10 \times 8 \times 4$

$V_A = 0.25\,H + 40$ …. (1)

$\Sigma M_c = 0$ from right

$12V_b = 3H + 10 \times 12 \times 6$

$V_b = 0.25H + 60$ …. (2)

$V_a + V_b = 200$ kN

$\therefore 400 = 0.25H + 40 + 0.25\,H + 60$

$400 = 0.5\,H + 100$

$\Rightarrow H = 200$ Kn

15. Ans: (c)

Sol: H = 200 kN

$V_b = 0.25 \times 200 + 60 = 110$ kN

Maximum tension occurs at highest support (B)

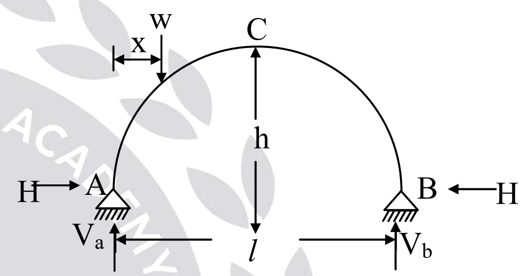
$\therefore T_{max.} = \sqrt{H^2 + Vb^2} = \sqrt{110^2 + 200^2}$

16. Ans: (a)

Sol: Stresses will generate due to change in temperature, rib shortening, and lack of fit in two hinged arches only, but not in three hinged arches because two hinged arches are indeterminate structures.

17. Ans: (b)

Sol:



Let 'W' be single rolling load placed at a distance x from left end support A,

$$V_a = W\left(\frac{\ell - x}{\ell}\right) \text{ and } V_B = \frac{Wx}{\ell}$$

Taking moments about point C on right hand section, $\Sigma M_c = 0$

$$-V_b \times \frac{\ell}{2} + H \times h = 0$$

$$H = \frac{V_b \ell}{2h}$$

The maximum bending moment occurs under the load "W"

$(B.M)_{x-x} = (V_a)\,(x) - H \times y$

$$= \frac{W(\ell - x)}{\ell}x - \left(\frac{Wx}{2h}\right)\frac{4hx(\ell - x)}{\ell^2}$$

$$= \frac{Wx(\ell - x)}{\ell} - \frac{2Wx^2(\ell - x)}{\ell^2}$$

For maximum $\frac{d}{dx}(B.M)_{x-x} = 0$

$$\frac{w}{\ell}(\ell-2x)-\frac{2w}{\ell^2}\left[2\ell x-3x^2\right]=0$$

$(l - 2x)\,(l) - 2(2\,lx - 3x^2) = 0$

$l^2 - 2xl - 4lx + 6x^2 = 0$

$6x^2 - 6lx + l^2 = 0$

$$x=\frac{+6\ell\pm\sqrt{(6\ell)^2-4(6)(\ell^2)}}{2(6)}$$

$$x=\frac{6\ell\pm\sqrt{12\ell^2}}{12} \Rightarrow \quad x=\frac{6\ell\pm2\sqrt{3}\ell}{12}$$

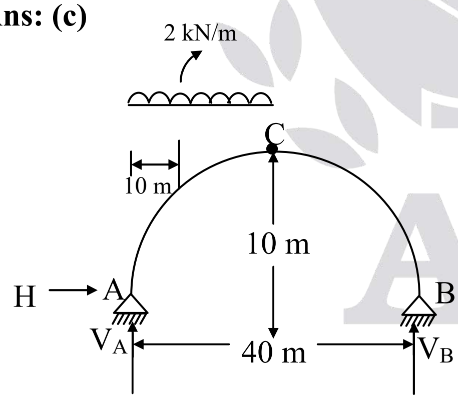
$$\therefore x=\frac{\ell(6+2\sqrt{3})}{12} \text{ and } x=\frac{\ell(6-2\sqrt{3})}{12}$$

$= 0.788l$ and $0.211\,l$

$\therefore$ Absolute maximum B.M occurs from $0.211\,l$ from both supports

18. Ans: (c)

Sol:



We know that equation of arch is

$$y=\frac{4hx(\ell-x)}{\ell^2}$$

$$\frac{dy}{dx}=\frac{4h}{\ell^2}(\ell-2x)$$

$$\tan\alpha=\left(\frac{dy}{dx}\right)_{at\,x=10}$$

$$=\frac{4\times10}{40^2}(40-2\times10)$$

$$=\frac{120}{40}=\frac{1}{2}$$

$\therefore\ \alpha = \tan^{-1}(1/2)$

Statement 1 is correct

Taking moments about point A, $\Sigma M_A = 0$

$-V_b \times 40 + 2 \times 20 \times 10 = 10$

$V_b = 10$ kN

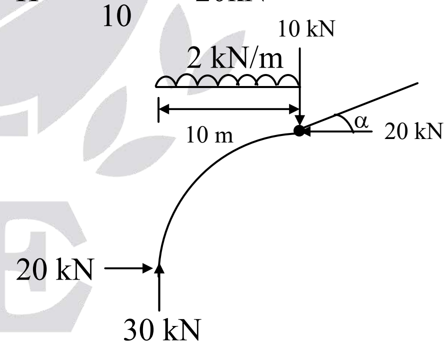
$\therefore V_A = 2 \times 20 - 10$

$V_A = 30$ kN

Taking moments about right hand portion of hinge C, $\Sigma M_c = 0$

$-V_b \times 20 + H \times 10 = 0$

$$H=\frac{10\times20}{10}=20\text{kN}$$



Net horizontal load at D = 20 kN (←)

Net vertical load at D = – 2 × 10 + 30

= 10 kN (↓)

$\therefore$ Normal thrust at D = 20 cos α + 10 sin α

$$=20\times\frac{2}{\sqrt{5}}+10\times\frac{1}{\sqrt{5}}$$

$$= \frac{40}{\sqrt{5}} + \frac{10}{\sqrt{5}}$$

$$= \frac{50}{\sqrt{5}}$$

$$= 10\sqrt{5}\text{kN}$$

$\therefore$ Statement 2 is incorrect

Shear force (S) = (20) sinα – 10cosα

$$= 20 \times \frac{1}{\sqrt{5}} - 10 \times \frac{2}{\sqrt{5}} = 0$$

$\therefore$ Statement 3 is correct.

Bending moment (B.M)

$$= 30 \times 10 - 20 \times y_{\text{at } x = 10} - 2 \times 10 \times 5$$

$$y_{\text{at } x=10} = \frac{4 \times 10 \times 10(40-10)}{40^2} = 7.5 \text{ m}$$

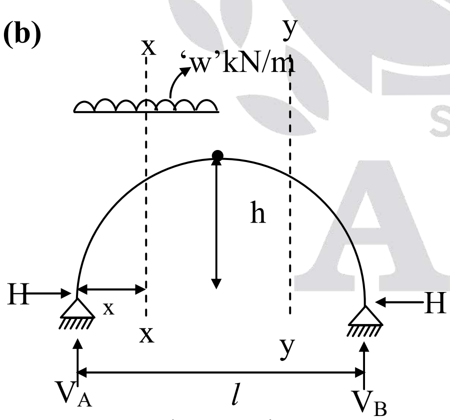
$$(\text{B.M})_{x=10} = 30 \times 10 - 20 \times 7.5 - 2 \times 10 \times 5$$

$$= 50 \text{ kNm}$$

$\therefore$ Statement 4 is incorrect

19. Ans: (b)

Sol:



Taking moments about point A, $\Sigma M_A = 0$

$$-V_b \times \ell + w \times \frac{\ell}{2} \times \frac{\ell}{4} = 0$$

$$V_b = \frac{w\ell}{8} \text{kN}$$

Apply $\Sigma V = 0$

$$V_A + V_B = \frac{w\ell}{2}$$

$$\therefore V_A = \frac{w\ell}{2} - \frac{w\ell}{8}$$

$$V_A = \frac{3}{8} w\ell$$

$$H = \frac{1}{2}\left(\frac{w\ell^2}{8h}\right) = \frac{w\ell^2}{16h}$$

$$(\text{B.M})_{X-X} = (V_A)(x) - \frac{wx^2}{2} - H_y$$

$$= \frac{3w\ell x}{8} - \frac{wx^2}{2} - \frac{w\ell^2}{16h} \times \frac{4hx(\ell - x)}{\ell^2}$$

$$= \frac{3w\ell x}{8} - \frac{wx^2}{2} - \frac{wx(\ell - x)}{4}$$

$$= \frac{3w\ell x}{8} - \frac{wx^2}{2} - \frac{w\ell x}{4} + \frac{wx^2}{4}$$

$$= \frac{w\ell x}{8} - \frac{wx^2}{4}$$

At x = 0 ; $(\text{B.M})_{x-x} = 0$

At x = l/4 ; $(\text{B.M})_{x-x} = \frac{w\ell^2}{64}$

$\therefore$ $(\text{B.M})_{x-x}$ is sagging moment between A and C

$$(\text{B.M})_{y-y} = (V_b)(x) - \frac{w\ell^2}{16h} \times y$$

$$= \frac{w\ell x}{8} - \frac{w\ell^2}{16h} \times \frac{4hx(\ell - x)}{\ell^2}$$

$$= \frac{w\ell x}{8} - \frac{wx(\ell - x)}{4}$$

$$= \frac{w\ell}{8} - \frac{w\ell x}{4} + \frac{wx^2}{4}$$

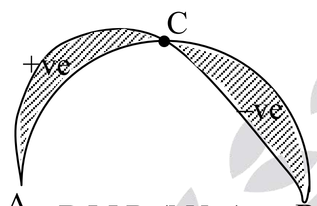
$$= \frac{-w\ell x}{8} + \frac{wx^2}{4}$$

at x = 0 $(B.M)_{y-y} = 0$

at x = $l/4$ $(B.M)_{y-y} = \frac{-w\ell^2}{64}$

at x = $l/2$ $(B.M)_{y-y} = 0$

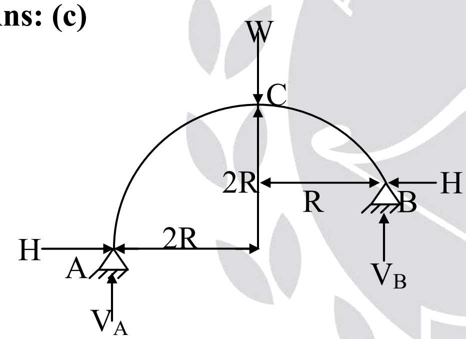
$\therefore$ $(B.M)_{y-y}$ is hogging moment between C and B



B.M.D (kNm)

20. Ans: (c)

Sol:



Taking moments about point C on right hand portion $\Sigma M_c = 0$

$$-V_B \times R + H \times R = 0$$

$$V_B = H$$

Taking moments about point C on left hand portion, $\Sigma M_c = 0$

$$(V_A)(2R) - H \times 2R = 0$$

$$V_A = H$$

But we know that

$$V_A + V_B = W$$

$$H + H = W$$

$\therefore$ H = W/2

$\therefore$ $H_A = H_B = H/2$

and $V_A = V_B = H/2$

Resultant load at A = $\sqrt{V_A^2 + H_A^2}$

$$= \sqrt{\left(\frac{H}{2}\right)^2 + \left(\frac{H}{2}\right)^2} = \left(\frac{H}{2}\right)\sqrt{2} = \frac{H}{\sqrt{2}}$$

Inclination angle with horizontal

$$\tan\theta = \frac{V_A}{H_A} = \frac{H/2}{H/2} = 1$$

$\therefore$ $\theta = 45^o$

21. Ans: (d)

Sol: In three hinged arches

$$\left(\frac{\Delta H}{H}\right) = -\left(\frac{\Delta h}{h}\right)$$

Where, H = Horizontal thrust

Δ H = Change in Horizontal thrust

h = Rise of an arch

Δh = change in rise of an arch.

$\therefore$ In three hinged arches as temperature increases, horizontal thrust decreases.

$\therefore$ Statement (I) is incorrect

As temperature increases, change in length occurs as there is a free moment at hinge C.

$\therefore$ Statement (II) is correct.

22. Ans: (a)

Sol: Due to temperature rise in two hinged arches, horizontal thrust will be generated at each supports.

Horizontal thrust, $H = \frac{EI(\alpha T \ell)}{\int y^2 ds}$

∴ Bending moment on arch is due to horizontal thrust H .

B.M = (–H) (y)

∴ Shape of bending moment diagram will correspond to shape of arch

∴ Statement (I) and (II) are correct and Statement (II) is the correct explanation of statement I.

23. Ans: (a)

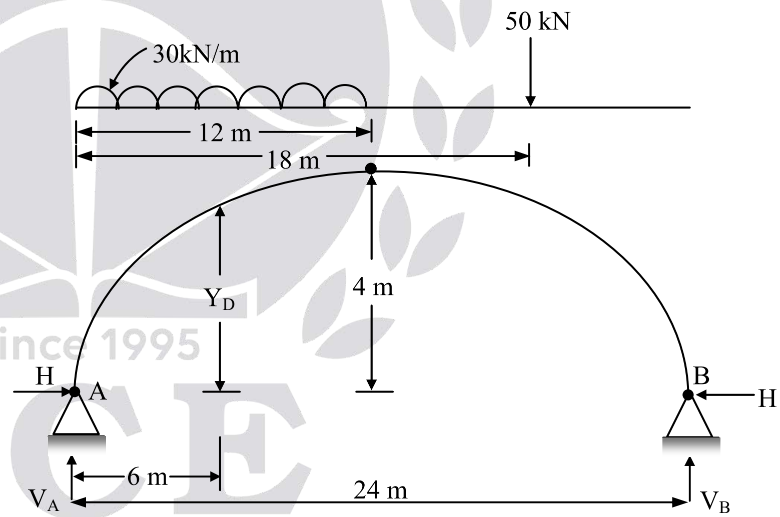
Sol: $(B.M)_{x-x}$ in arch = Beam moment – 'H' moment.

'H' moment (or) moment due to "H" will reduces the bending moment in arch. This is called arching action. Due to this cross-section of arch reduce, which is practically used in the construction of dams in irrigation field.

Conventional Practice Solutions

01.

Sol: The arch is shown in figure below. Taking moment about B, we get



$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$

$V_A = 282.50$ kN

$V_B = 30 \times 12 + 50 - 282.50 = 127.5$ kN

Taking moment about crown C,

$V_B \times 12 - H \times 4 - 50 \times 6 = 0$

$127.5 \times 12 - H \times 4 + 50 \times 6 = 0$

$H = 307.5$ kN

At 6 m from the left support,

$$M = V_A \times 6 - Hy_D - 30 \times \frac{6^2}{2}$$

In the parabolic arch, $y = \frac{4hx(L-x)}{L^2}$

Therefore, at x = 6 m $\quad y_D = \frac{4 \times 4 \times 6(24-6)}{24^2} = 3$ m

$$M = 282.5 \times 6 - 307.5 \times 3 - 30 \times \frac{6^2}{2} = 232.5 \text{ kNm}$$

Vertical shear at D,

$$V = V_A - 30 \times 6$$

$$= 282.5 - 30 \times 6 = 102.5 \text{ kN}$$

Equation of crown is given by

$$y = \frac{4hx(L-x)}{L^2}$$

$$\therefore \frac{dy}{dx} = \tan\theta = \frac{4h(L-2x)}{L^2}$$

Therefore, at x = 6 m,

$$\tan\theta = \frac{4\times 4(24-2\times 6)}{24\times 24}$$

$$\therefore \theta = 18.435^0$$

$$\therefore N = V \sin\theta + H\cos\theta$$

$$= 102.5 \sin 18.435^0 + 307.5 \cos 18.435^0$$

$$= 324.133 \text{ kN}$$

and radial shear

$$Q = V\cos\theta - H\sin\theta$$

$$= 102.5 \cos 18.435^0 - 307.5 \sin 18.435^0$$

$$= 0$$

02.

Sol: Referring to figure below,

L = 20 m, h = 3m , z = 5m

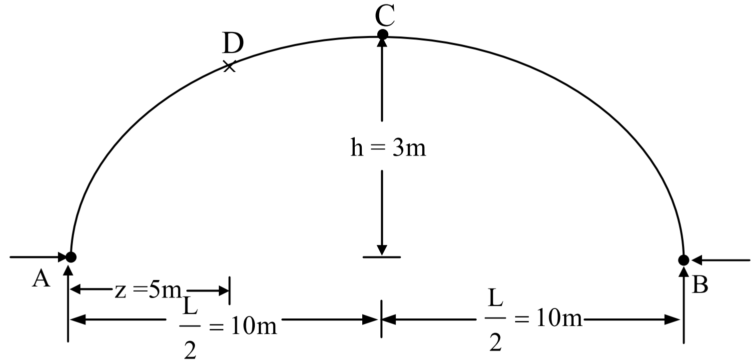
ILD ordinate are $= \frac{z(L-z)}{L} = \frac{5(20-5)}{20} = 3.75$

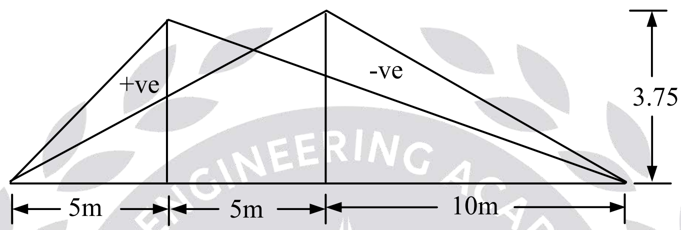
Maximum bending moment occurs when the load is on the section and its value is given by

= Load × ILD ordinate

$$= 300\left[3.75 - \frac{5}{10}\times 3.75\right]$$

= 562.5 kNm





ILD for M_D

Maximum negative moment occurs when the load is on the central hinge and its value

$= W \times$ ordinate at centre

$$= 300\left[3.75 - \frac{10}{15} \times 3.75\right]$$

= 375 kNm

Absolute maximum positive moment occurs at section

$= 0.2113 \times L$

$= 0.2113 \times 20$

= 4.226 m, from either support

Therefore, absolute maximum positive moment = 0.096225 WL

$= 0.96225 \times 300 \times 20$

= 577.35 kNm

Absolute maximum negative moment occurs when the load is at the quarter span and its value

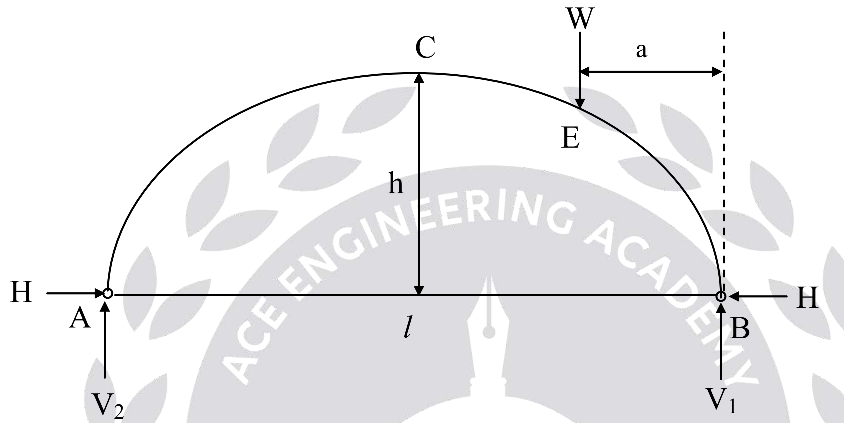
$$= \frac{WL}{16} = \frac{300 \times 20}{16}$$

= 375 kNm

03.

Sol: Figure shows the parabolic arch carrying the load W at D at a distance ‘a’ from the end A. Let the vertical reactions at A and B be V_1 and V_2 respectively. Let H be the horizontal thrust, at each support.

Above figure shows the same arch, but carrying the load W at E, at a distance of a from the right end B.



The vertical reaction at A and B will now be V_2 and V_1 respectively. But the horizontal thrust would still be H, at each support.

Below figure shows the same arch carrying two concentrated loads each of magnitude W one at D, the other at E.

For this load system the horizontal thrust would be 2H, at each support.

Let us consider this case

Each vertical reaction = W.

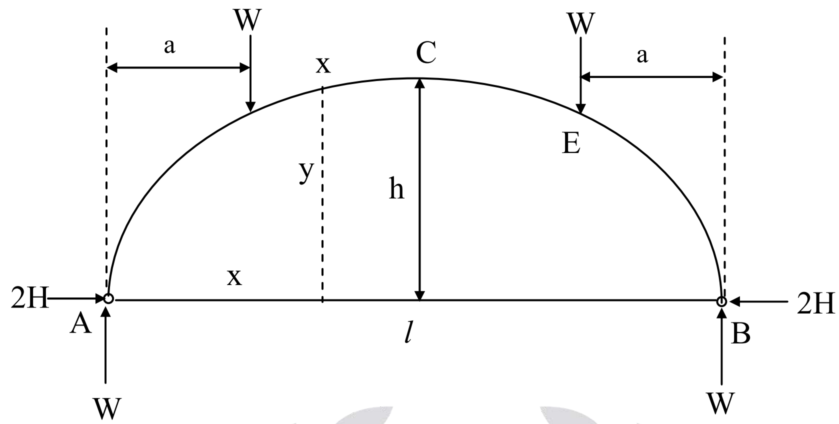
At any section distant x from A (x < a) the beam moment at the section = M = W_x

But any section X distant x from A (x > a and < l/2) the beam moment

$$= M = Wx - Wx\,(x - a) = Wa = \text{constant}$$

$$y = \frac{4h}{\ell^2}x(\ell - x)$$

$$\therefore \text{ Horizontal thrust} \quad 2H = \frac{\int M y\,dx}{\int y^2 dx}$$



$$2H = \frac{2\left[\int_0^a Wx.\frac{4h}{\ell^2}x(\ell - x)dx + \int_0^{\ell/2} Wa.\frac{4h}{\ell^2}x(\ell - x)dx\right]}{\int_0^{\ell/2}\frac{16h^2}{\ell^4}x^2(\ell - x^2)dx}$$

$$2H = \frac{\frac{4Wh}{\ell^2}\int_0^a x^2(\ell - x)dx + \frac{4Wha}{\ell^2}\int_a^{\ell/2} x(\ell - x)dx}{\frac{16h^2}{\ell^4}\int_0^{\ell/2} x^2(\ell - x)^2 dx} \quad \text{---------- (1)}$$

Numerator of equations (1)

$$= N = \frac{4Wh}{\ell^2}\int_0^a x^2(\ell - x)dx + \frac{4Wha}{\ell^2}\int_a^{\ell/2} x(\ell - x)dx$$

$$= \frac{4Wh}{\ell^2}\left[\frac{\ell a^3}{3} - \frac{a^4}{4}\right] + \frac{4Wha}{\ell^2}\left[\frac{\ell}{2}\left(\frac{\ell^2}{4} - a^2\right) - \frac{1}{3}(\ )\right]$$

$$= \frac{4Wha}{\ell^2}\left[\frac{\ell a^2}{3} - \frac{a^3}{4} + \frac{\ell^3}{8} - \frac{\ell a^2}{2} - \frac{\ell^3}{24} + \frac{a^2}{3}\right]$$

$$= \frac{4Wha}{\ell^2}.\frac{1}{24}\left[8\ell a^2 - 6a^3 + 3\ell^3 - 12\ell a^2 - \ell^3 + 8a^3\right]$$

$$= \frac{Wha}{6\ell^2}\left[2\ell^3 - 4\ell a^2 + 2a^3\right] = \frac{Wha}{6\ell^2}.2\left[\ell^3 - 2\ell a^2 + a^3\right]$$

$$= \frac{Wha}{3\ell^2}\left[\ell^3 - 2\ell a^2 + a^3\right] = \frac{Wha}{3\ell^2}\left[\ell^3 - \ell a^2 - \ell a^2 + a^3\right]$$

$$= \frac{Wha}{3\ell^2}\left[\ell\left(\ell^2 - a^2\right) - a^2\left(\ell - a\right)\right]$$

$\therefore$ $$N = \frac{Wha}{3\ell^2}(\ell - a)\left(\ell^2 + \ell a - a^2\right)$$

Denominator of equation (1)

$$D = \frac{16h^2}{\ell^4}\int_0^{\ell/2} x^2\left(\ell - x^2\right)dx = \frac{16h^2}{\ell^4}\int_0^{\ell/2}\left(\ell^2 x^2 - 2\ell x^3 + x^4\right)dx$$

$$= \frac{16h^2}{\ell^4}\left[\ell^2\frac{1}{3}.\frac{\ell^3}{8} - 2\ell.\frac{1}{4}.\frac{\ell^4}{16} + \frac{1}{5}.\frac{\ell^5}{32}\right] = \frac{4}{15}h^2\ell$$

$\therefore$ Horizontal thrust

$$= 2H = \frac{N}{D} = \frac{Wha}{3\ell^2}(\ell - a)\left(\ell^2 + \ell a - a^2\right)\times\frac{15}{4h^2\ell}$$

$$= \frac{5}{4}.\frac{W}{h\ell^3}.a(\ell - a)\left(\ell^2 + \ell a - a^2\right)$$

$\therefore$ horizontal thrust when one of the two point loads is present.

$$= H = \frac{5}{8}.\frac{W}{h\ell^3}a(\ell - a)\left(\ell^2 + \ell a - a^2\right)$$

Horizontal thrust $\ell = 20$ m

a = 3 m, h = 6 m, W = 10 kN

$$H = \frac{5}{8}\times\frac{10}{6\times 20^3}\times 3(20 - 3)\left(20^2 + 20\times 3 - 3^2\right)$$

= 2.99 kN

$H \cong 3$ kN

11. Matrix Methods

01. Ans: (b)

Sol: $d \propto \frac{1}{EI}$

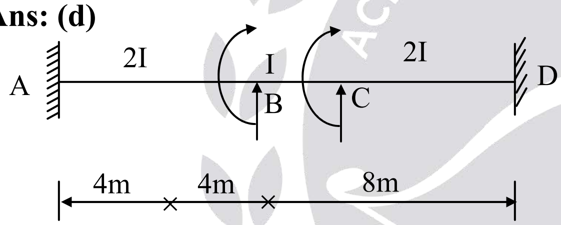
$$\frac{d_1}{d_2} = \frac{(EI)_2}{(EI)_1}$$

$$\frac{d_1}{d_2} = \frac{2EI}{EI}$$

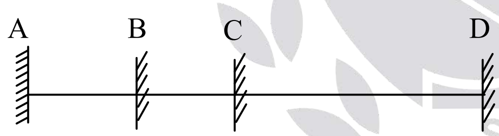
$\mathbf{d_2} = d_1/2$

02. Ans: (d)

Sol:

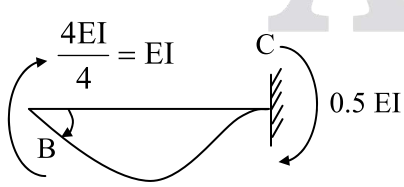


Assume restrained structure at 1 & 2



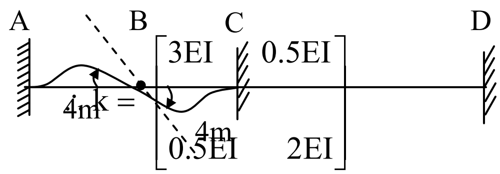
Restrain at 'C'. Apply unit rotation at B

Consider BC



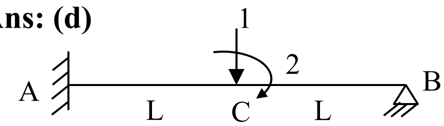
k_{21} = force developed at 1 due to unit rotation at

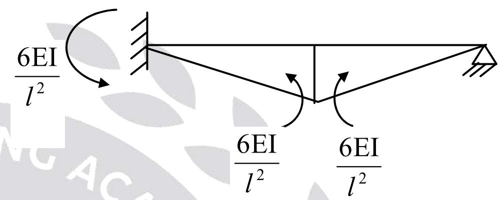
$k_{21} = k_{12} = 0.5$ EI



03. Ans: (d)

Sol:





$$\frac{6EI}{l^2} - \frac{6EI}{l^2} = 0$$

$$\frac{6EI}{l^2} - \frac{3EI}{l^2} = \frac{3EI}{l^2}$$

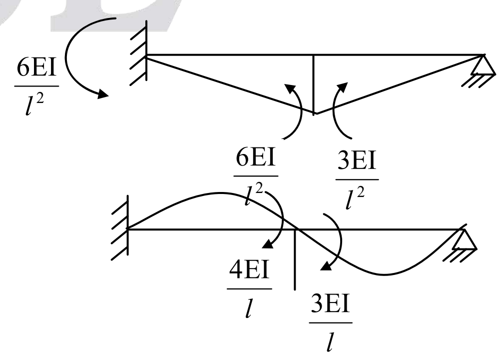
For portion AC (↑) R_B

$$= \frac{\frac{6EI}{l^2} + \frac{6EI}{l^2}}{l} = \frac{12EI}{l^3}$$

For portion BC(↑) $R_B = \frac{3EI}{l^3} = \frac{3EI}{l^3}$

$$K_{11} = \frac{12EI}{l^3} + \frac{3EI}{l^3} = \frac{15EI}{l^3}$$

$$K_{21} = \frac{-3EI}{l^2}$$



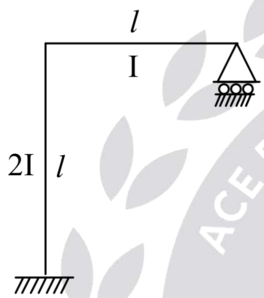
$$K_{22} = \frac{4EI}{l} + \frac{3EI}{l} = \frac{7EI}{l}$$

$$K_{12} = \frac{-6EI}{l^2} + \frac{3EI}{l^2} = \frac{-3EI}{l^2}$$

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} \frac{15EI}{l^3} & \frac{-3EI}{l^2} \\ \frac{-3EI}{l^2} & \frac{7EI}{l} \end{bmatrix}$$

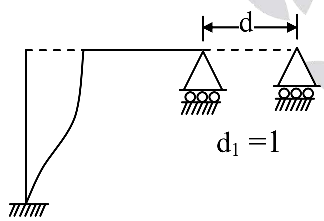
04. Ans: (a)

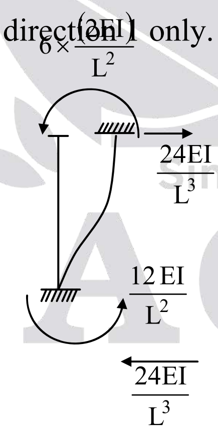
Sol:



Initially restrain the structure @ co-ordinates 1 & 2.

Allow unit defection in the direction 1 only.





$$\therefore\ k_{11} = \frac{24EI}{L^3}$$

With this value of k_{11} only option (a).

05. Ans: (d)

Sol:

$$\text{Stiffness} \propto \frac{1}{\text{flexibility}}$$

$\therefore [K] \rightarrow$ Stiffness matrix

$[\delta] \rightarrow$ flexibility matrix

$\therefore\ [k]\ [\delta] = I$

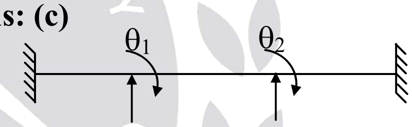
$\therefore$ Flexibility matrix $[\delta] = [k]^{-1}$

$$\text{Given } [k] = \frac{2EI}{L}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix}$$

$$\therefore\ \delta = [k]^{-1} = \frac{L}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

06. Ans: (c)

Sol:



Order of stiffness matrix = degree of kinematic indeterminacy = degrees of freedom

$\therefore$ Order of stiffness matrix = [2 × 2]

07. Ans: (c)

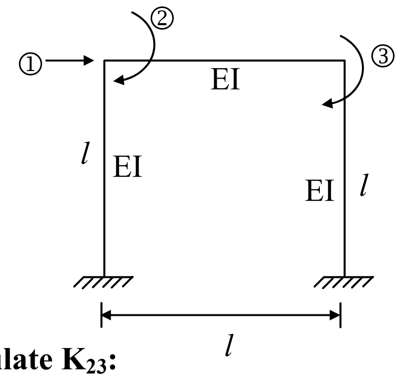
Sol: Flexibility is defined as displacement obtained due to unit applied load .

$\therefore$ Flexibility matrix contains displacements elements.

- Flexibility matrix is a square and symmetric matrix.
- In flexibility matrix all the leading diagonal elements are always positive and other diagonal element may be positive (or) negative.

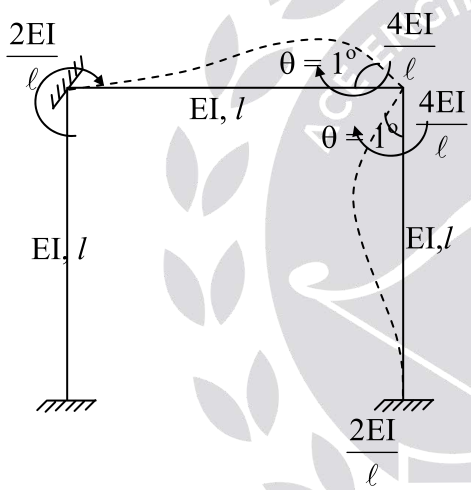
08. Ans: (d)

Sol:



To calculate K_{23}:

Apply unit Rotation in the direction of degree of freedom '3' and lock other degrees of freedom i.e. (1 and 2)



K_{23} = Moment developed in the direction of degree of freedom '2' due to unit rotation in the direction of degree of freedom 3.

$$= \frac{2EI}{\ell}$$

09. Ans: (d)

Sol: Stiffness is defined as force required due obtain to unit displacement.

∴ To obtain 1st column of stiffness matrix, Release unit displacement in the direction of degree of freedom '1' by look other degree's of freedom.

10. Ans: (a)

Sol: Elements in the leading diagonal in flexibility matrix are always positive because, displacement will occur in the direction of force applied.

But Rest of elements other than leading diagonal elements may be Negative.

∴ Statement (I) and (II) are correct and statement (II) is correct explanation of statement (I).

11. Ans: (c)

Sol: As per maxwell's reciprocal theorem, deflection at point A due to unit load at B is equal to deflection at point B due to unit load at A.

i.e $f_{21} = f_{12}$ (or) $k_{12} = k_{21}$

Diagonal elements (i.e. k_{11}, k_{22}, k_{33} etc) of the matrix are not same.

∴ Statement (II) is flame.

12. Ans: (d)

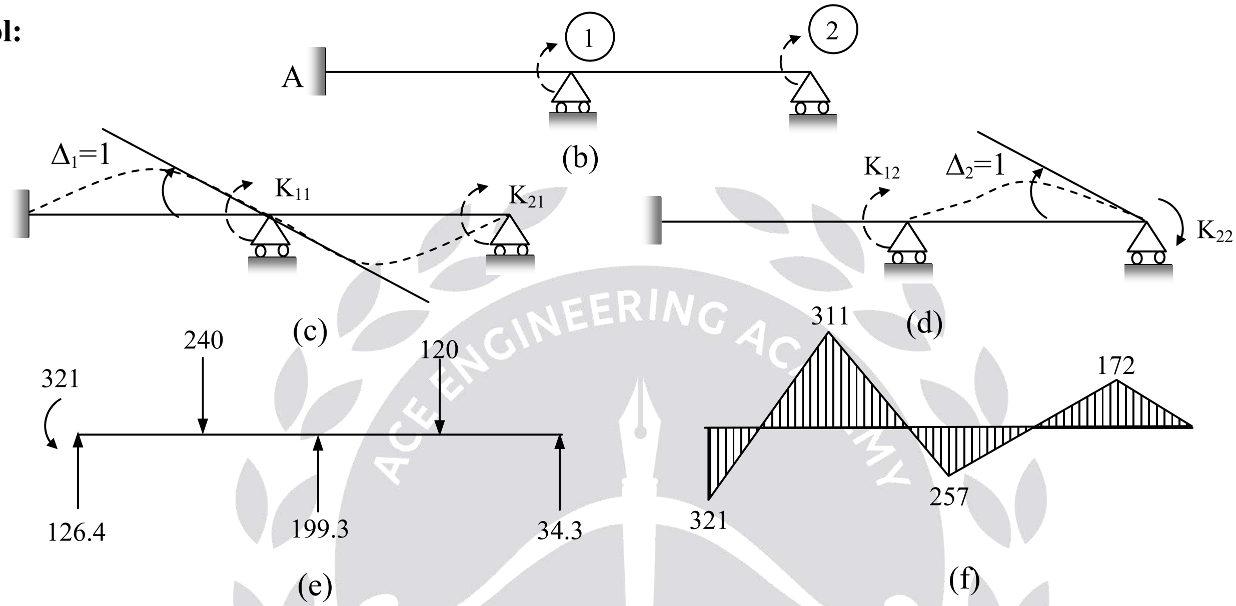
Sol: Size of stiffness matrix and flexibility matrix are different because stiffness matrix deals with unknown displalcements and flexibility matrix deals with unknown forces.

∴ Size of stiffness matrix is equal to number of unknown displacements/Rotations, and size of flexibility matrix is equal to number of unknown forces/Moment.

Conventional Practice Solutions

01.

Sol:



In this problem the only two independent displacement components are the rotation at δ and C. hence the degree of freedom is two. Coordinates 1 and 2 many be assigned to the rotations at B and C as shown in Fig. b. Locking joints B and C, the fixed-end moments due to the applied loads are

$$M'_{AB} = -\frac{240\times5\times5^2}{10^2} = -300\text{kN.m}$$

$$M'_{BA} = \frac{240\times5\times5^2}{10^2} = 300\text{kN.m}$$

$$M'_{BC} = -\frac{120\times5\times5^2}{10^2} = -150\text{kN.m}$$

$$M'_{CB} = \frac{120\times5\times5^2}{10^2} = 150\text{kN.m}$$

As the supports are unyielding, there are no additional fixed-end moments due to the settlement of supports. Hence,

$$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = 0$$

Therefore forces P'_1 and P'_2 at coordinates 1 and 2 for the fixed-end conditions are

$P'_1 = 300 - 150 = 150$ kN.m

$P'_2 = 150$ kN.m

Next, the stiffness matrix with reference to coordinates 1 and 2 may be developed. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 as shown in Fig. c.

$$k_{11} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$$

$$k_{21} = \frac{2EI}{10} = 0.2\,EI$$

Similarly, to generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 as shown in Fig. d.

$$k_{12} = \frac{2EI}{10} = 0.2\,EI$$

$$k_{22} = \frac{4EI}{10} = 0.4\,EI$$

As there are no external loads at coordinates 1 and 2,

$P_1 = P_2 = 0$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = -\begin{bmatrix} 0.8EI & 0.2EI \\ 0.2EI & 0.4EI \end{bmatrix}^{-1} \begin{bmatrix} 150 \\ 150 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -107.14 \\ -321.43 \end{bmatrix}$$

Knowing the displacements, the end moments may be calculated by using the slope-deflection.

$$M_{AB} = -300 + \frac{2EI}{10}\left[-\frac{107.14}{EI}\right] = -321\,\text{kN.m}$$

$$M_{BA} = 300 + \frac{2EI}{10}\left[2\left(-\frac{107.14}{EI}\right)\right] = 257\,\text{kN.m}$$

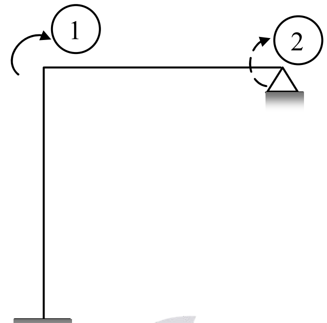
$$M_{BC} = -150 + \frac{2EI}{10}\left[2\left(-\frac{107.14}{EI}\right) + \left(-\frac{321.43}{EI}\right)\right] = -257\,\text{kN.m}$$

$$M_{CB} = 150 + \frac{2EI}{10}\left[2\left(-\frac{321.43}{EI}\right) + \left(-\frac{107.14}{EI}\right)\right] = 0$$

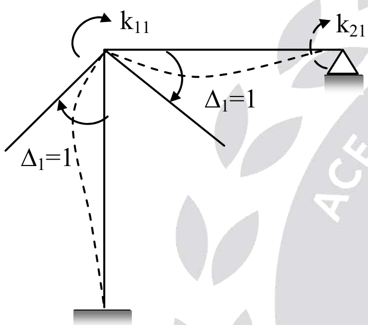
The free-body diagram and the bending - moment diagram drawn on the compression side are shown in Fig. (e) and (f) respectively.

02.

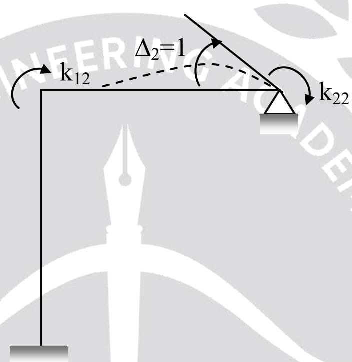
Sol:



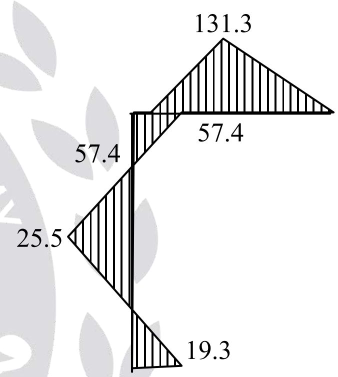
(b)



(c)



(d)



(e)

As the frame cannot sway, rotations at B and C are the two independent displacement components. Hence coordinates 1 and 2 may be chosen as shown in Fig. (b). Forces P_1' and P_2' at coordinates 1 and 2 respectively, due to the external loads other than those acting at the coordinates when no displacement is permitted at the coordinates, may be computed first. Considering member AB as fixed ended, the end moments are

$$M'_{AB} = -\frac{50\times 2\times 3^2}{5^2} = -36\,\text{kN.m}$$

$$M'_{BA} = -\frac{50\times 3\times 2^2}{5^2} = 24\,\text{kN.m}$$

Similarly, considering member BC as fixed ended, the end moments are

$$M'_{BC} = -\frac{160\times 2\times 2^2}{4^2} = -80\,\text{kN.m}$$

$$M'_{CB} = -\frac{160\times 2\times 2^2}{4^2} = 80\,\text{kN.m}$$

Hence,

$$P'_1 = M'_{BA} + M'_{BC} = 24 - 80 = -56\,kN.m$$

$$P'_2 = M'_{CB} = 80\,kN.m$$

As there are no external forces at coordinates 1 and 2,

$$P_1 = P_2 = 0$$

The stiffness matrix may now be developed. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 without any displacement at coordinate 2 as shown in Fig. (c) and compute the forces at coordinates 1 and 2.

$$k_{11} = \frac{4EI}{5} + \frac{4E(2I)}{4} = 2.8EI$$

$$k_{21} = \frac{2E(2I)}{4} = EI$$

Similarly, to generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 without any displacement at coordinate 1 as shown in Fig. (d) and compute the forces at coordinates 1 and 2.

$$k_{12} = \frac{2E(2I)}{4} = EI$$

$$k_{22} = \frac{4E(2I)}{4} = 2EI$$

Hence, the stiffness matrix [k] is given by the equation.

$$[k] = \begin{bmatrix} 2.8EI & EI \\ EI & 2EI \end{bmatrix}$$

Substituting into Eq. (6.11),

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 2.8EI & EI \\ EI & 2EI \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -56 \\ 80 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} \dfrac{41.74}{EI} \\ -\dfrac{60.87}{EI} \end{bmatrix}$$

Knowing the displacements, the end moments are obtained by using the slope – deflection equation.

$$M_{AB} = -36 + \frac{2EI}{5}\left(0 + \frac{41.74}{EI}\right) = -19.3kN.m$$

$$M_{BA} = 24 + \frac{2EI}{5}\left(\frac{2\times 41.74}{EI} + 0\right) = 57.4\,kN.m$$

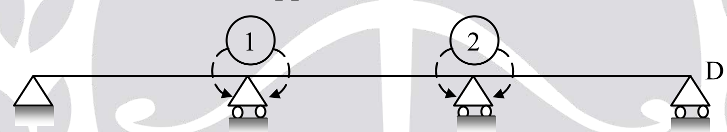
$$M_{BC} = -80 + \frac{2E(2I)}{4}\left(\frac{2\times 41.74}{EI} - \frac{60.87}{EI}\right) = -57.4kN.m$$

$$M_{CB} = 80 + \frac{2E(2I)}{4}\left(-\frac{2\times 60.87}{EI} + \frac{41.74}{EI}\right) = 0$$

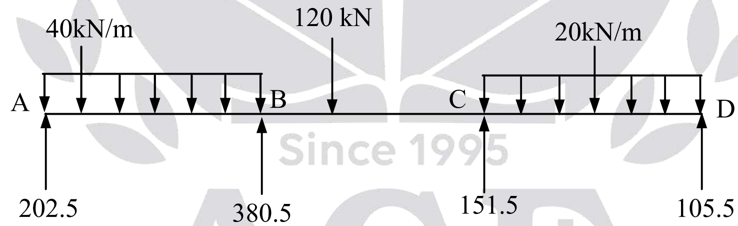
The bending-moment diagram drawn on the compression side is shown in Fig. (e).

03.

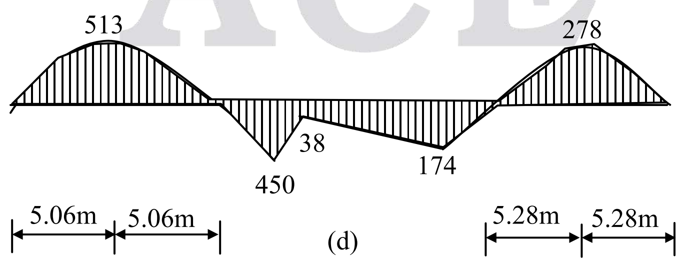
Sol: The beam is statically indeterminate to the second degree. The released structure may be obtained by inserting hinges at B and C as shown in Fig. (b). So that the released structure comprises a series of three simply supported beams. The chosen coordinates 1 and 2 correspond to the released bending moments at B and C respectively. The displacements in the released structure at coordinates 1 and 2 due to the applied loads.



(b)



(c)



(d)

The rotation at B is span $AB = \frac{40\times 12^3}{24EI} = \frac{2880}{EI}$ (Counter – clock wise)

The rotation at B in span $BC = \frac{120\times 4\times 8\times 20}{6\times 12EI}$

$$= \frac{3200}{3EI} \text{(Colckwise)}$$

Hence, the displacement at coordinate I due to the applied loads,

$$\Delta_{1L} = \frac{2880}{EI} + \frac{3200}{3EI} = \frac{11840}{3EI}$$

The rotation at C in span $BC = \frac{120\times 4}{6\times 12EI}(12^2 - 4^2) = \frac{2560}{3EI}$ (Counter-clockwise)

The rotation at C in span $CD = \frac{20\times 12^3}{24EI} = \frac{1440}{EI}$ (Clockwise)

Hence, the displacement at coordinate 2 due to the applied loads,

$$\Delta_{2L} = \frac{2560}{3EI} + \frac{1440}{EI} = \frac{6880}{3EI}$$

The flexibility matrix may be developed by applying a unit force successively at coordinates 1 and 2 and using Table 2.16.

$$\delta_{11} = \frac{12}{3EI} + \frac{12}{3EI} = \frac{8}{EI}$$

$$\delta_{12} = \delta_{21} = \frac{12}{6EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{12}{3EI} + \frac{12}{3EI} = \frac{8}{EI}$$

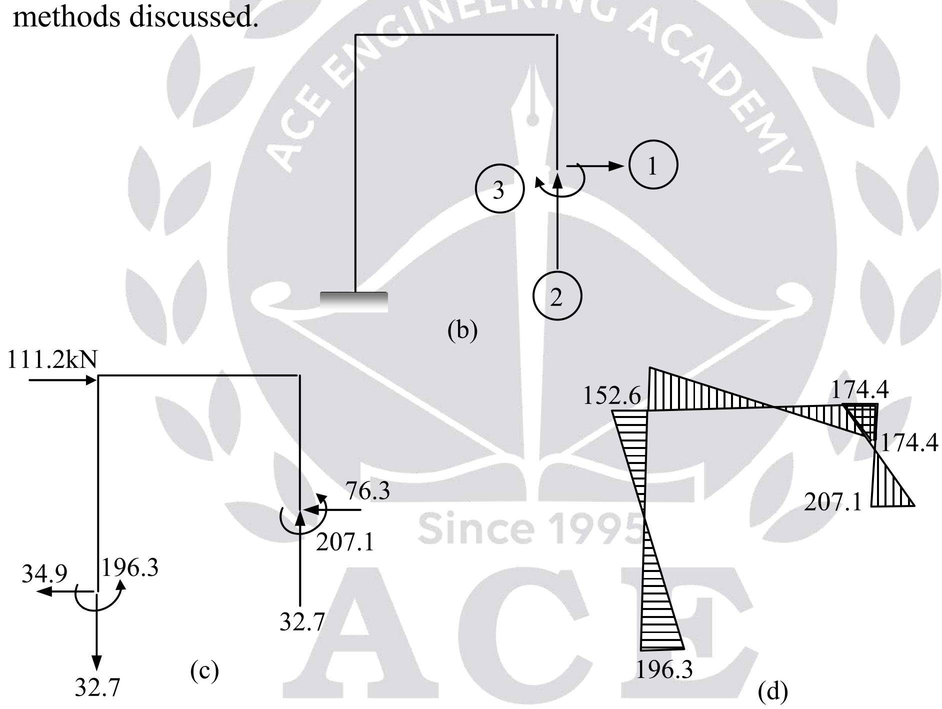
Substituting into Eq. (5.3)

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} \frac{8}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix}^{-1} \begin{bmatrix} \frac{11840}{3EI} \\ \frac{6880}{3EI} \end{bmatrix} = \begin{bmatrix} -450 \\ -174 \end{bmatrix}$$

Hence, P_1 = - 450 kNm and P_2 = - 174 kN m. All the reaction components may now be computed by using the equations of static equilibrium. Hence, the free - body diagram shown in Fig. (c) can be drawn. The bending-moment diagram for the continuous beam drawn on the compression side is shown in Fig. (d).

04.

Sol: The frame is statically indeterminate to the third degree. The released structure may be obtained by removing the support at D and thereby releasing three reaction components. Coordinates 1, 2 and 3 may be assigned to these reaction components as shown in Fig. (b). The displacements at the chosen coordinates in the released structure due to the applied loads may be calculated by using any one of the methods discussed.



$$\Delta_{1L} = \frac{6950}{3EI}$$

$$\Delta_{2L} = -\frac{13900}{EI}$$

$$\Delta_{3L} = \frac{1390}{EI}$$

Next, the flexibility matrix for the frame with reference to the chosen coordinates may be developed.

$$[\delta] = \frac{1}{6EI}\begin{bmatrix} 750 & 375 & -150 \\ 375 & 2000 & -225 \\ -150 & -225 & 60 \end{bmatrix}$$

As the supports are unyielding, ,

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = -6EI\begin{bmatrix} 750 & 375 & -150 \\ 375 & 2000 & -225 \\ -150 & -225 & 60 \end{bmatrix}^{-1}\begin{bmatrix} \frac{6950}{3EI} \\ -\frac{13900}{EI} \\ \frac{1390}{EI} \end{bmatrix} = \begin{bmatrix} -76.3 \\ 32.7 \\ -207.1 \end{bmatrix}$$

Knowing the reactive forces at D, the reactive forces at A can be calculated by statics. Hence the free- body diagram of the entire frame as shown in Fig. (c) may be drawn. Fig. (d) shows the bending- moment diagram for the frame drawn on the compression side.