ESE | GATE | PSUs

MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

Text Book : Theory with worked out Examples
and Practice Questions

Strength of Materials

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 Simple Stresses and Strains

Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

01. Ans: (b)

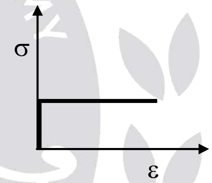
Sol:

- **Ductility:** The property of materials to allow large deformations or large extensions without failure (large plastic zone) is termed as ductility.
- **Brittleness:** A brittle material is one which exhibits a relatively small extensions or deformations prior to fracture. Failure without warning (No plastic zone) i.e. no plastic deformation.
- **Tenacity**: High tensile strength.
- **Creep:** Creep is the gradual increase of plastic strain in a material with time at constant load.
- **Plasticity:** The property by which material undergoes permanent deformation even after removal of load.
- **Endurance limit:** The stress level below which a specimen can withstand cyclic stress indefinitely without failure.
- **Fatigue:** Decreased Resistance of material to repeated reversal of stresses.

02. Ans: (a)

Sol:

- When the material is subjected to stresses, it undergoes to strains. After removal of stress, if the strain is not restored/recovered, then it is called inelastic material.
- For rigid plastic material:



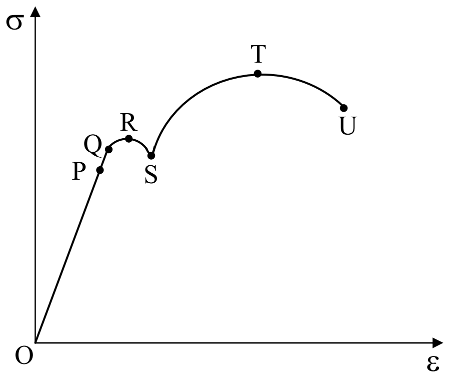
- Any material that can be subjected to large strains before it fractures is called a ductile material. Thus, it has large plastic zone.
- Materials that exhibit little or no yielding before failure are referred as brittle materials. Thus, they have no plastic zone.

03. Ans: (a)

Sol: *Refer to the solution of Q. No. (01).*

04. Ans: (b)

Sol: The stress-strain diagram for ductile material is shown below.



P – Proportionality limit
Q – Elastic limit
R – Upper yield point
S – Lower yield point
T – Ultimate tensile strength
U – Failure

From above,

OP → Stage I
PS → Stage II
ST → Stage III
TU → Stage IV

05. Ans: (b)

Sol:

- If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is **isotropic** or in other words we can say the isotropy of a material is its characteristics, which gives us the information that the properties are same in the three orthogonal directions x, y and z.
- A material is **homogeneous** if it has the same composition throughout the body. Hence, the elastic properties are the same at every point in the body in a given direction. However, the properties need not to be the same in all the directions for the material. Thus, both A and B are false.

06. Ans: (a)

Sol: **Strain hardening** increase in strength after plastic zone by rearrangement of molecules in material.

- **Visco-elastic material** exhibits a mixture of creep as well as elastic after effects at room temperature. Thus their behavior is time dependant

07. Ans: (a)

Sol: *Refer to the solution of Q. No. (01).*

08. Ans: (a)

Sol: Modulus of elasticity (Young's modulus) of some common materials are as follow:

Material	Young's Modulus (E)
Steel	200 GPa
Cast iron	100 GPa
Aluminum	60 to 70 GPa
Timber	10 GPa
Rubber	0.01 to 0.1 GPa

09. Ans: (a)

Sol: Addition of carbon will increase strength, thereby ductility will decrease.

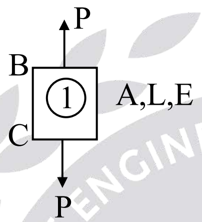
Elastic Constants and Their Relationships

01. Ans (c)

Sol: We know that,

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\therefore \quad \mu = \frac{\Delta D/8}{\dfrac{PL}{AE}/L}$$



$$\therefore \quad \mu = \frac{\Delta D}{8}\,\frac{AE}{P}$$

$$\therefore \quad 0.25 = \frac{\Delta D}{8}\,\frac{\frac{\pi}{4}(8)^2 \times 10^6}{50000}$$

$$\Rightarrow \quad \Delta D = 1.98 \times 10^{-3} \cong 0.002 \text{ cm}$$

02. Ans: (c)

Sol: We know that,

$$\text{Bulk modulus} = \frac{\delta P}{\delta V/V}$$

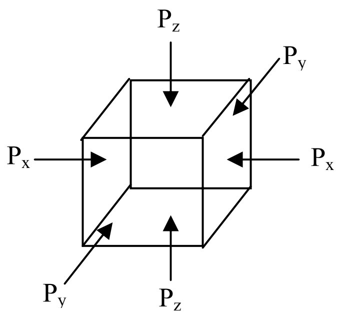
$$\Rightarrow \quad 2.5\times10^5 = \frac{200 \times 20}{\delta V}$$

$$\Rightarrow \quad \delta V = 0.016 \text{ m}^3$$

Linear and Volumetric Changes of Bodies

01. Ans: (d)

Sol:



Let $P_y = P_z = P$

$\varepsilon_y = 0$,

$\varepsilon_z = 0$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu.\frac{\sigma_x}{E}$$

$$\therefore \quad 0 = \frac{(-P)}{E} - \mu\,\frac{(-P)}{E} - \mu\frac{(P_x)}{E}$$

$$\Rightarrow \quad P = \frac{\mu.P_x}{(1-\mu)}$$

02. Ans: (a)

Sol: Given that, $\sigma_c = 4\tau$

Punching force = Shear resistance of plate

$$\therefore\ \sigma\,(\text{Cross section area}) = \tau(\text{surface Area})$$

$$\therefore\ 4\times\tau\times\frac{\pi.D^2}{4} = \tau(\pi.D.t)$$

$\Rightarrow$ D = t = 10 mm

03. Ans: (d)

Sol:

$$\sigma_S = 140\text{ MPa} = \frac{P_s}{A_s}$$

$$\Rightarrow\ P_S = \frac{140\times500}{3} \approx 23{,}300\text{ N}$$

$$\sigma_{Al} = 90\text{ MPa} = \frac{P_{A\ell}}{A_{A\ell}}$$

$$\Rightarrow\ P_{Al} = 90\times400 = 36{,}000\text{ N}$$

$$\sigma_B = 100\text{ MPa} = \frac{P_B}{A_B}$$

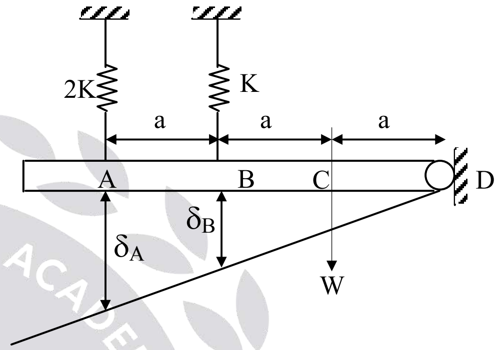
$$\Rightarrow\ P_B = \frac{100\times200}{2} = 10{,}000\text{ N}$$

Take minimum value from P_S, A_{Al} and P_B.

$\Rightarrow$ P = 10,000 N

04. Ans: (c)

Sol:



From similar triangle

$$\frac{3a}{\delta_A} = \frac{2a}{\delta_B}$$

$$3\delta_B = 2\delta_A \ \ldots\ldots (1)$$

$$\text{Stiffness } K = \frac{W}{\delta}$$

$$\therefore K_A = \frac{W_A}{\delta_A} \Rightarrow \delta_A = \frac{W_A}{2K}$$

Similarly $\delta_B = \dfrac{W_B}{K}$

From equation (1) $3\times\dfrac{W_B}{K} = 2\times\dfrac{W_A}{2K}$

$$\Rightarrow\ \frac{W_A}{W_B} = 3$$

Thermal/Temperature Stresses

01. Ans: (b)

Sol: Free expansion = Expansion prevented

$$[\ell\alpha t]_s + [\ell\alpha t]_{Al} = \left[\frac{P\ell}{AE}\right]_s + \left[\frac{P\ell}{AE}\right]_{AL}$$

$$11\times10^{-6}\times20+24\times10^{-6}\times20$$

$$=\frac{P}{100\times10^3\times200}+\frac{P}{200\times10^3\times70}$$

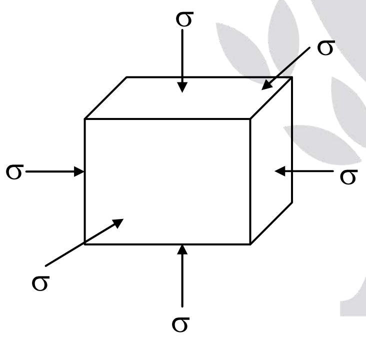
$\Rightarrow$ P = 5.76 kN

$$\sigma_s=\frac{P}{A_s}=\frac{5.76\times10^3}{100}=57.65\ \text{MPa}$$

$$\sigma_{Al}=\frac{P}{A_{al}}=\frac{5.76\times10^3}{200}=28.82\ \text{MPa}$$

02. Ans: (a)

Sol:



Strain in X-direction due to temperature,

$$\varepsilon_t=\alpha(\Delta T)$$

Strain in X-direction due to volumetric stress,

$$\varepsilon_x=\frac{\sigma_x}{E}-\mu\frac{\sigma_y}{E}-\mu\frac{\sigma_z}{E}$$

$$\therefore\quad \varepsilon_x=\frac{-\sigma}{E}(1-2\mu)$$

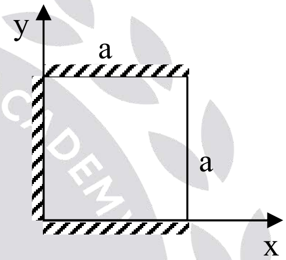
$$\therefore\quad -\sigma=\frac{(\varepsilon_x)(E)}{1-2\mu}$$

$$\therefore\quad -\sigma=\frac{\alpha(\Delta T)E}{(1-2\mu)}$$

$$\Rightarrow\quad \sigma=\frac{-\alpha(\Delta T)E}{1-2\mu}$$

03. Ans: (b)

Sol:



- Free expansion in x direction is aαt.
- Free expansion in y direction is aαt.
- Since there is restriction in y direction expansion doesn't take place. So in lateral direction, increase in expansion due to restriction is **μaαt**.

Thus, total expansion in x direction is,

$$= a\,\alpha\,t+\mu\,a\,\alpha\,t$$

$$= a\,\alpha\,t\,(1+\mu)$$

04. Ans: (b)

Sol: Stress: When force is applied on a body, it suffers a deformation. To resist this deformation, from equilibrium point of view, internal forces arise in the body giving rise to concept of stress.

$$\text{Stress} = \frac{\text{Resistance}}{\text{Area}}$$

Since the deformations arise first and are measurable, strain is a fundamental behavior and stress is derived from this

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

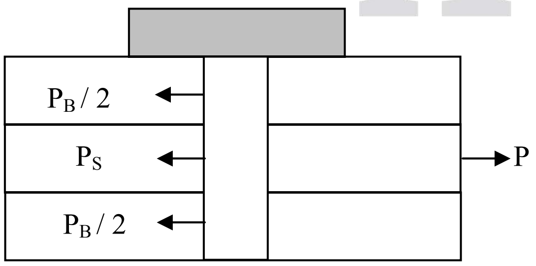
Therefore, strain has no units and SI units of stress is N/m^2 (or) Pa.

05. Ans: (a)

Sol: When a ductile material is subjected to repeating (or) cyclic loads, progressive and localized deformations occur leading to the development of residual strains in the material. When the accumulated strain energy exceeds the toughness, the material fractures and this failure called as fatigue occurs at a load much less than the ultimate load of the structure. The failure load decreases with increase in the number of loadings.

06. Ans: (b)

Sol: FBD of a single pin:



For equilibrium: $P_B + P_s = P$ --- (1)

Tension in steel bar = P_s

Tension in each brass bar = $\frac{P_B}{2}$

Elongation in steel bar = $\frac{(P_s)\times\ell}{AE_s}$;

Elongation in brass bar = $\frac{\left(\frac{P_B}{2}\right)\times\ell}{AE_s}$

But, $\delta_{steel} = \delta_{brass}$

$$\Rightarrow \quad \frac{(P_s)\times\ell}{AE_s} = \frac{\left(\frac{P_B}{2}\right)\ell}{AE_B};$$

Given data, $\frac{E_s}{E_B} = 2$

$$\frac{P_s}{2E_B} = \frac{P_B}{E_B}$$

$$\Rightarrow \quad P_S = P_B$$

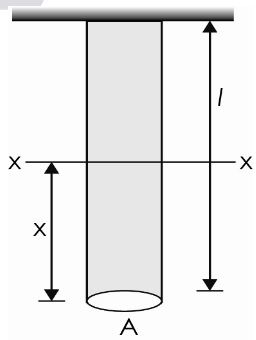
From equation (1) $P = P_B + P_B$

$$\therefore\ P_B = P_s = \frac{P}{2}$$

Shear in each pin = $\frac{P}{2}$ = 0.5 P

07. Ans: (c)

Sol: Consider a circular bar of cross-section area 'a' length 'l' unit weight 'γ'



At any section 'x-x', at distance 'x'

Stress due to self weight = $\dfrac{P_x}{A}$

$= \dfrac{\gamma \times A \times x}{A}$

$= \gamma x$

$\therefore$ Max stress (σ) = $\gamma\ell$

$\sigma_1 \propto \ell$

$\therefore$ When all the dimensions are doubled = 2ℓ

$$\frac{\sigma_2}{\sigma_1} = \frac{2\ell}{\ell} \Rightarrow \sigma_2 = 2\sigma_1$$

08. Ans: (b)

Sol: *Refer to the solution of Q.No. 04* in fundamental mechanical properties and stress-strain diagrams.

09. Ans: (b)

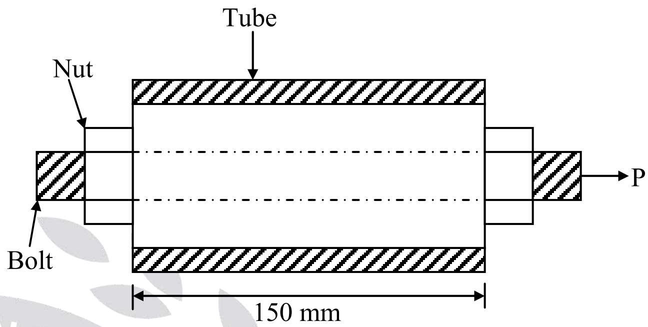
Sol: Fatigue is the progressive and localized structural damage, that occurs in a material subjected to repetitive loads. The nominal maximum stress value that cause such damage is much less than the strength of the material.

Endurance limit: It is the stress level below which a specimen can withstand cyclic stress indefinitely without exhibiting fatigue failure. Also known as fatigue limit/fatigue strength.

Conventional Practice Solutions

01.

Sol:



Pitch = 1mm/toren

Tunned by quater $\delta = \dfrac{1}{4}$ mm

(i) nut movement = comp. of tube + elongation of bolt (compatibility condition)

(ii) $P_{tube} = P_{bolt}$

From (i)

$$\frac{1}{4}\text{mm} = \left|\frac{PL}{AE}\right|_{tube} + \left|\frac{PL}{AE}\right|_{bolt}$$

$$\Rightarrow \frac{1}{4} = \frac{P(150)}{196.03\times10^6} + \frac{P(150)}{62.83\times10^6}$$

$\Rightarrow P = 79.3$ kN $= P_{tube} = P_{bolt}$

(i) $\sigma_{bolt} = \dfrac{P}{A}$

$$= \frac{79.3\times10^3}{\frac{\pi}{4}(20^2)} = 252 \text{ MPa (T)}$$

(ii) $\uparrow t = 10°C$

$P_s = P_c = P$

$$\underbrace{(\ell\alpha\Delta T)_c - \left(\frac{P\ell}{AE}\right)_c}_{\text{Tube (compression)}} = \underbrace{(\ell\alpha\Delta T)_s + \left(\frac{P\ell}{AE}\right)_s}_{\text{Bolt (tension)}}$$

$$\Rightarrow \left(18\times10^{-5}\times10\right) - \left(\frac{P}{\frac{\pi}{4}\times\left(30^2-22^2\right)\times\left(6\times10^5\right)}\right)$$

$$= \left(12\times10^{-6}\times10\right) + \left(\frac{P}{\frac{\pi}{4}\left(20^2\right)\times2\times10^5}\right)$$

$\Rightarrow P = 80$ kN

Stress in bolt only due to temperature,

$$\sigma = \frac{80\times10^3}{\frac{\pi}{4}\times20^2} = 254 \text{ MPa (T)}$$

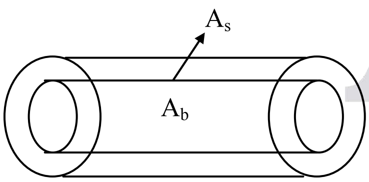
Resultant stress in bolt (tightening of nut + temperature)

= 252 + 254

= 506 MPa (T)

02.

Sol:



Let the load applied = P

Load carries by steel and brass be P_s & P_b respectively

$P_s + P_b = P$ …………(1)

Compatibility condition

Strain in steel = Strain in brass

$$\frac{P_s\ell}{A_sE_s} = \frac{P_b\ell}{A_bE_b}$$

$$P_s = \frac{P_bA_sE_s}{A_bE_b} \text{ …….(2)}$$

Put (2) in (1)

$$P_b\left[1+\frac{A_sEs}{A_bE_b}\right] = P$$

$$P_b\left[\frac{A_bE_b + A_sE_s}{A_bE_b}\right] = P$$

$$P_b = \frac{PE_bA_b}{P_bE_b + A_sE_s}$$

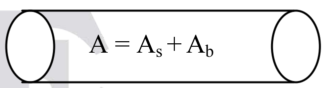
Elongation

$$= \frac{P_b\ell}{A_bE_b} = \frac{PE_bA_b\ell}{[A_bE_b + A_sE_s]}\times\frac{1}{A_bE_b}$$

$$\delta\ell = \left[\frac{P\ell}{A_bE_b + E_sA_s}\right] \text{……….(3)}$$

Let Young's modulus = E

$$\delta\ell = \frac{P\ell}{AE} = \frac{P\ell}{(A_s + A_b)E} \text{ …………(4)}$$



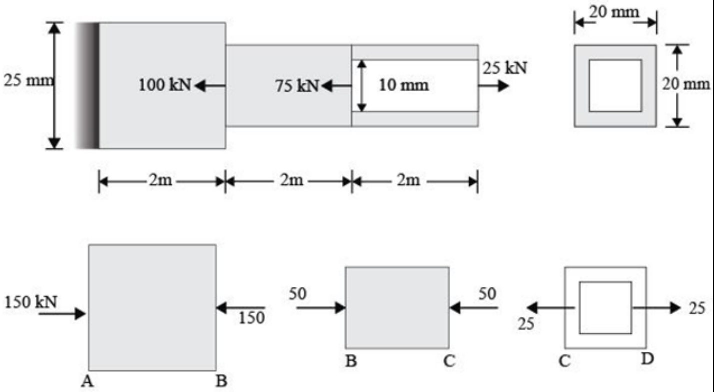
Compare (3) and (4)

$$\frac{P\ell}{A_bE_b + A_sE_b} = \frac{P\ell}{(A_s + A_b)E}$$

$$E = \frac{A_sE_s + A_bE_b}{A_s + A_b}$$

03.

Sol:



$Dl = Dl_{AB} + Dl_{BC} + Dl_{CD}$

$$= -\frac{150\times10^3\times2000}{25^2\times2\times10^5} - \frac{50\times10^3\times2000}{20^2\times2\times10^5} + \frac{25\times10^3\times2000}{(20^2-10^2)\times2\times10^5}$$

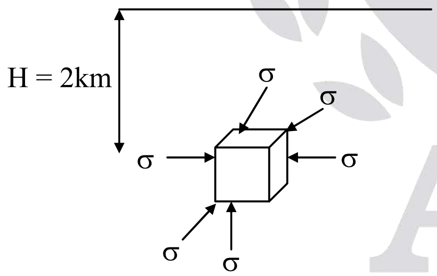
$$= \frac{10^3\times2000}{2\times10^5}\left[-\frac{150}{25^2} - \frac{50}{20^2} + \frac{25}{300}\right]$$

$= 10[-0.24 - 0.125 + 0.0833]$

$= -2.816$ mm (Compression)

04.

Sol:



$\sigma = \gamma_w.H$

$= G.\gamma_{\text{pure water}}.H$

$= 1.02\times9.81\times2000$

$= 20.01\times10^3$ kN/m^2

$$= \frac{20.01\times10^3(1000\,\text{N})}{(1000\,\text{mm})^2} = 20.01 \text{ MPa}$$

$E = 3K(1 - 2\mu)$

208×10^3 MPa $= 3K(1 - 2\times0.29)$

$\Rightarrow \quad K = 165\times10^3$ MPa

$$\Rightarrow K = \frac{\sigma}{\left(\frac{\delta V}{V}\right)}$$

$$\Rightarrow 165\times10^3 = \frac{20.01}{\frac{\delta V}{(100)^3}}$$

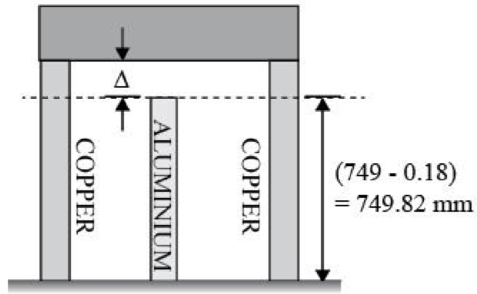
$\Rightarrow \delta V = 121.28$ mm^3

08.

Sol: $\Delta = 0.18$ mm

$\Delta T = 95°C$

$\alpha_a > \alpha_c$



$$(L\alpha\Delta T)_a - \left(\frac{PL}{AE}\right)_a = (L\alpha\Delta T)_c + \left(\frac{PL}{AE}\right)_c + \Delta$$

$$\Rightarrow (749.82)\times(23.1\times10^{-6})\times(95) - \frac{P\times(749.82)}{400\times(700\times10^3)}$$

$$= (750)\times(16.8\times10^{-6})\times(95) + \frac{P(750)}{2(500)(120\times10^3)} + 0.18$$

(Two bars)

$\Rightarrow P = 8.2$ kN

Stress in each copper bar

$$\sigma_c = \frac{P}{2\times500} = \frac{8.2\times10^3}{2\times500}$$

= 8.2 MPa (Tension)

Stress in AL bar

$$\sigma_a = \frac{P}{400} = \frac{8.2\times10^3}{400} = 20.5 \text{ MPa (comp)}$$

09.

Sol: $\sigma_h = \left(\frac{D-d}{d}\right)E$

D = Wheel diameter (outside)

d = diameter of steel ring

$$1200 = \left(\frac{D-12}{12}\times2\times10^6\right)$$

$$\frac{1200\times12}{2\times10^6} = D-12$$

$D = 12 \times 7.2 \times 10^{-3}$

D = 12.0072 m

$$\Delta T = \left(\frac{D-d}{d}\right)\frac{1}{\alpha}$$

$$= \left(\frac{0.0072}{12}\right)\frac{1}{11.7\times10^{-6}}$$

$\Delta T = 51.28^oC$

Chapter 2 Complex Stresses and Strains

01. Ans: (b)

Sol: Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

Maximum shear stress = $\frac{\sigma_1-\sigma_2}{2} = 13$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1+\sigma_2}{2} = \frac{18+(-8)}{2} = 5$$

02. Ans: (b)

Sol: Radius of Mohr's circle, $\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$

$$\therefore \quad 20 = \frac{\sigma_1-10}{2}$$

$$\Rightarrow \quad \sigma_1 = 50 \text{ N/mm}^2$$

03. Ans: (b)

Sol: Given data,

$\sigma_x = 150$ MPa, $\sigma_y = -300$ MPa, $\mu = 0.3$

Long dam $\rightarrow$ plane strain member

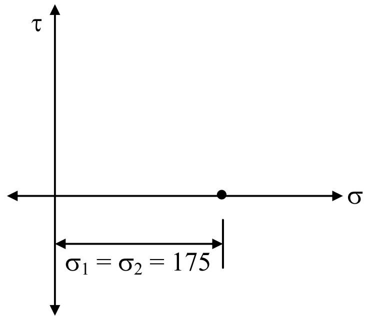
$$\varepsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\therefore 0 = \sigma_z - 0.3\times150 + 0.3\times300$$

$$\Rightarrow \sigma_z = 45 \text{ MPa}$$

04. Ans: (b)

Sol:



From the bove, we can say that Mohr's circle is a point located at 175 MPa on normal stress axis.

Thus, $\sigma_1 = \sigma_2 = 175$ MPa

05. Ans: (c)

Sol: Given that, $\sigma_2 = 0$

$$\therefore \quad \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$\therefore \quad \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$\therefore \quad \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}$$

$$\therefore \quad \tau^2{}_{xy} = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

$$\therefore \quad \tau^2{}_{xy} = \sigma_x . \sigma_y$$

$$\Rightarrow \quad \tau_{xy} = \sqrt{\sigma_x . \sigma_y}$$

06. Ans: (d)

Sol: Max shear stress = $\dfrac{\sigma_1 - \sigma_2}{2}$

σ_1, σ_2 are major and minor principal stress

1. $\tau_{max} = \dfrac{\sigma - 0}{2} = \dfrac{\sigma}{2}$

2. $\tau_{max} = \dfrac{\sigma - \sigma}{2} = 0$

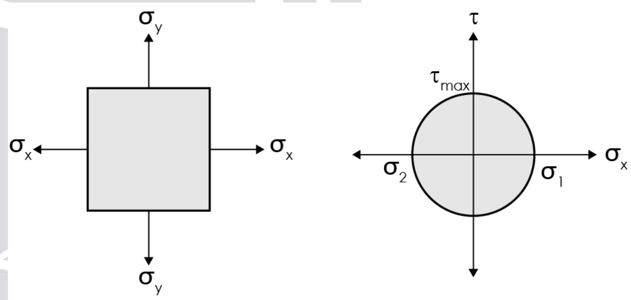
3. $\tau_{max} = \dfrac{\sigma - (-\sigma)}{2} = \sigma$

4. $\tau_{max} = \left(\sigma - \dfrac{\sigma}{2}\right)\dfrac{1}{2} = \dfrac{\sigma}{4}$

$\therefore$ Increasing order is 2, 4, 1 and 3.

07. Ans: (b)

Sol:



Since y-axis passes through centre of Mohr's circle $\Rightarrow$ centre of Mohr circle is origin

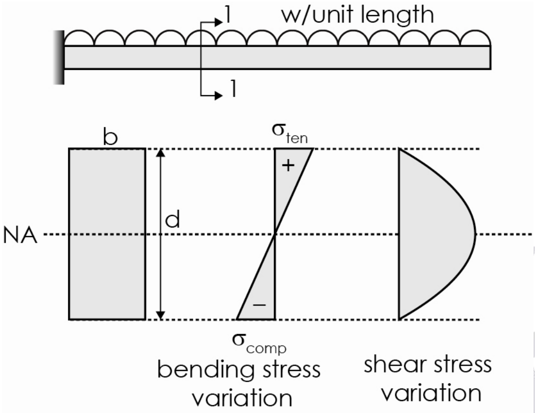
$$\Rightarrow (0, 0) = \left(\frac{\sigma_1 + \sigma_2}{2}, 0\right)$$

$$\Rightarrow \frac{\sigma_1 + \sigma_2}{2} = 0 \Rightarrow \sigma_1 = -\sigma_2$$

$\therefore$ Direct stresses are equal in magnitude but opposite in nature.

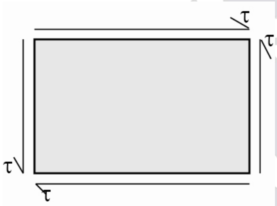
08. Ans: (a)

Sol:

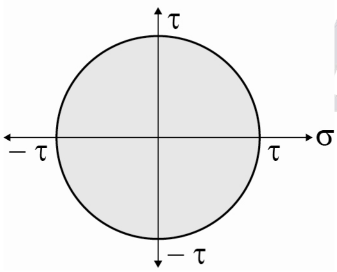


Case - I : At Neutral axis

Shear stress is maximum; Bending stress is zero i.e. stress condition is



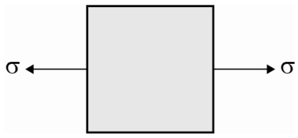
and Mohr's circle is



Case - II : At extreme top fibre

Bending stress is tensile and maximum

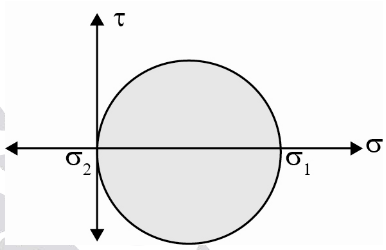
Shear stress = 0. Stress condition is



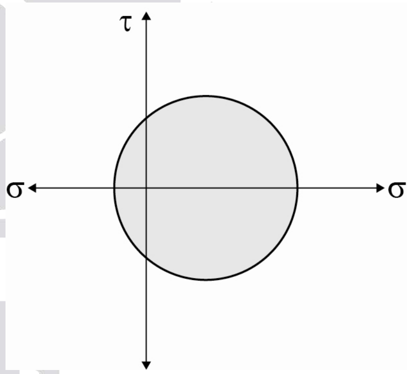
$$\therefore\ \sigma_1 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2}$$

$\sigma_1 = \sigma$; $\sigma_2 = 0$

Corresponding Mohr's circle is



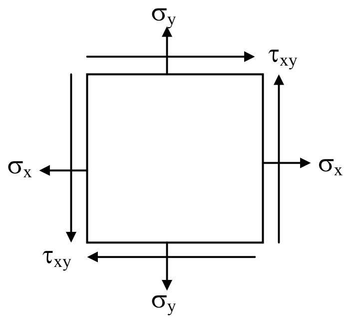
For any point above NA, and below extreme fiber Mohr's circle will be an intermediate of the above two cases i.e.,



Conventional Practice Solutions

01.

Sol:



$\sigma_x = 30$ MPa

$\sigma_y = 20$ MPa

$\tau_{xy} = 50$ MPa

Principal stresses

$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{30+20}{2} \pm \sqrt{\left(\frac{30-20}{2}\right)^2 + 50^2}$$

$\sigma_1 = 75.25$ MPa

$\sigma_2 = -25.25$ MPa

Check

$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

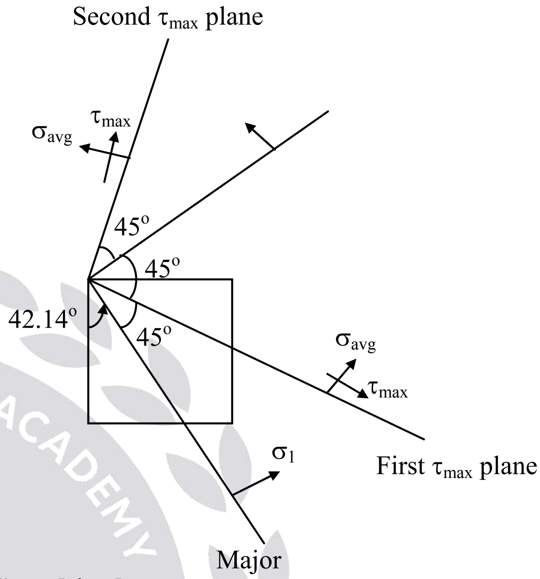
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$ (θ_p = Angle of major principal plane)

$\theta_p = 42.14°$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{30-20}{2}\right)^2 + 50^2}$$

$\therefore\ \tau_{max} = 50.25$



OR

Graphical

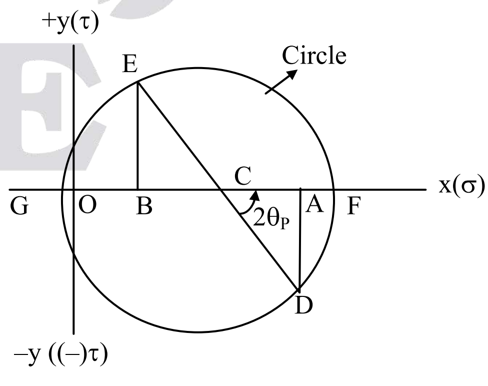
$\sigma_x = 30$ MPa = OA

$\sigma_y = 20$ MPa = OB

$\tau_{xy} = 50$ MPa = AD or BE

C : Centre, CE = CD = radius = τ_{max}

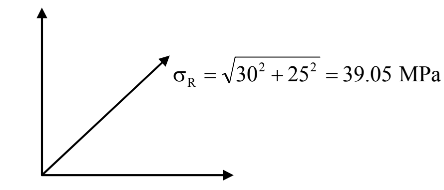
σ_1 = OF, σ_2 = OG



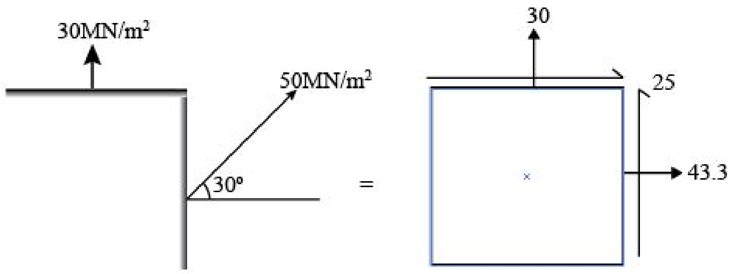
02.

Sol:

(i) Resultant Stress on Second (Horizontal) Plane:



(ii)



Principal Plane:

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 25}{43.3-30} = 3.76$$

$\theta_{P1} = 37.55^o$ (Major Principal plane)

$\theta_{P3} = 127.55^o$ (Minor Principal plane)

At $\theta_P = 37.55^o$ Principal stress

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_p + \tau_{xy}\sin 2\theta_P$$

$$= \left(\frac{43.3+30}{2}\right) + \left(\frac{-30+43.3}{2}\right)\cos 75.1 + 25\sin 75.1$$

$= 36.65 + (+1.71) + 24.16$

$\sigma_{P_1} = 62.52\ \text{MN/m}^2$

At $\theta_P = 127.55$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_P + \tau_{xy}\sin 2\theta_P$$

$= 36.65 – 1.71 – 24.16$

$\sigma_{P_3} = 10.78\ \text{MN/m}^2$

(iii) Maximum Shear Stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{43.3-30}{2}\right)^2 + 25^2}$$

$\tau_{max} = 25.87$ MPa

Plane on which it occurs is

$\theta_{P_2} = 37.55^o + 45^o$

$= 82.55^o$

04.

Sol: $\sigma_x = 100$ MPa

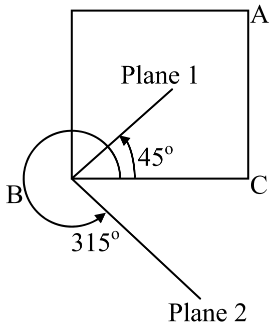
$\sigma_y = 100$ MPa

$\tau_{xy} = – 20$ MPa

$$\tan 2\theta_{P_1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-40}{0} = \infty$$

$\theta_p = 45^o, 135^o$ from plane AC

$= 45^o$ & 315^o from plane BC



Principal strain $= \dfrac{\sigma_{P_1} - \mu\sigma_{P_3}}{E}, \dfrac{\sigma_{P_3} - \mu\sigma_{P_1}}{E}$

$$\sigma_{P1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= 100 + \frac{1}{2}\sqrt{4 \times 20^2}$$

$$\frac{\sigma_{p_1}}{\sigma_{p3}} = 120, 80$$

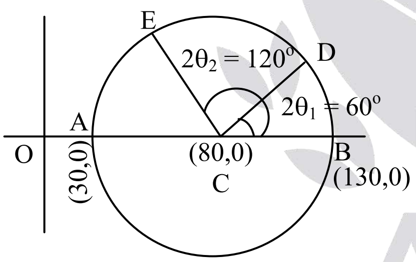
$$\sigma_{p_1} = \frac{(120 - 0.33 \times 80)}{67 \times 10^3} = 1.393 \times 10^{-3}$$

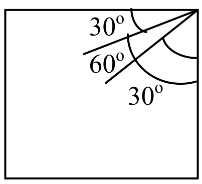
$$\sigma_{p_3} = \frac{80 - 0.333 \times 120}{67 \times 10^3} = 5.97 \times 10^{-4}$$

05.

Sol: Radius of Mohr's circle $= \dfrac{\sigma_1 - \sigma_2}{2} = 50$ MPa

Centre of Mohr's circle $= \dfrac{\sigma_1 + \sigma_2}{2} = 80$ MPa





$\theta_1 = 30^o$
$\theta_2 = 60^o$

Stress on point inclined at 30^o to y-axis in AC direction (Point D)

Normal stress = 80 + 50 cos60^o

= 105 MPa

Shear stress = 50 sin 60^o

= 43.3 MPa

Stress at point inclined at 30^o to x-axis in AC direction (Point E)

Normal stress = 80 – 50 cos 60^o

= 55 MPa

Shear stress = 50 sin 60^o

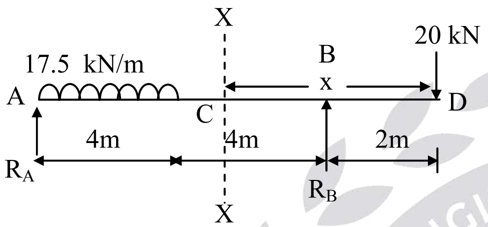
= 43.33 MPa

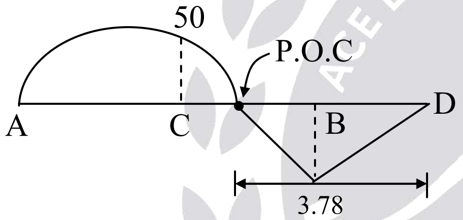
Chapter 3

Shear Force and Bending Moment

01. Ans: (b)

Sol: Contra flexure is the point where BM is becoming zero.





Taking moment about A,

$\Sigma M_A = 0$

$\therefore\ 17.5\times4\times\frac{4}{2}+20\times10-R_B\times8=0$

$\therefore\quad R_B = 42.5$ kN

Now, $M_x = -20x + R_B(x-2)$

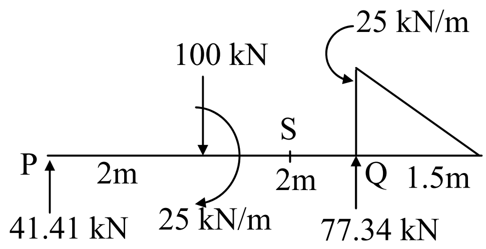
For bending moment be zero $M_x = 0$,

$-20x + 42.5(x-2) = 0$

$\Rightarrow x = 3.78$ m from right i.e. from D.

02. Ans: (b)

Sol:



Take $\Sigma M_P = 0$

$$\frac{1}{2}\times25\times1.5\times\left(\frac{1.5}{3}+4\right)-(R_Q\times4)+100\times2+25=0$$

$\therefore\quad R_Q = 77.34$ kN

Also, $\Sigma V = 0$

$\therefore\ R_P + R_Q = 100+\frac{1}{2}\times25\times1.5 = 118.75$ kN

$\therefore\ R_p = 41.41$ kN

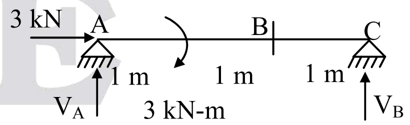
$\Rightarrow$ Shear force at P = 41.41 kN

03. Ans: (c)

Sol: $M_S = R_P(3) + 25 - (100\times1) = 49.2$ kN-m

04. Ans: (c)

Sol:



$-V_B\times3+3=0$

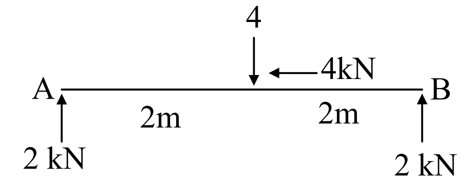
$\therefore\ V_C = 1$ kN

$\therefore$ Bending moment at B,

$\Rightarrow M_B = V_C\times1 = 1$ kN-m

05. Ans: (a)

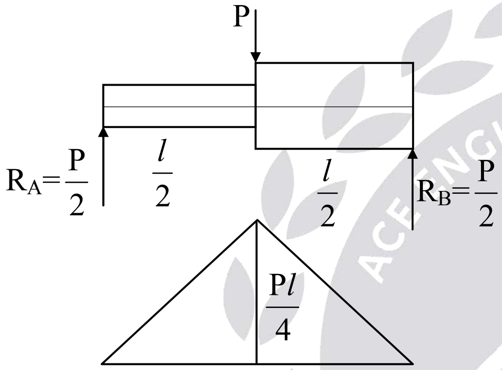
Sol:



Reaction at both the supports are 2 kN and in upward direction.

06. Ans: (c)

Sol:



BMD Diagram

Bending moment at $\frac{l}{2}$ from left is $\frac{Pl}{4}$.

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)

Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

08. Ans: (b)

Sol: Shear force (V) = $\frac{dM}{dx}$

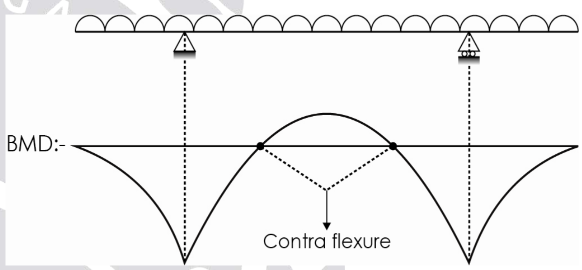
$\therefore$ For bending moment to be maximum,

$$\frac{dM}{dx} = 0 \;\; \Rightarrow V = 0$$

When shear force changes sign it implies if it is zero at a particular section then bending moment is maximum at that section.

Point of contra flexure: Points where bending moment curve changes sign.

Ex:



09. Ans: (c)

Sol: **Point of contra flexure:** It is the point of the bending moment curve where bending moment changes its algebraic sign.

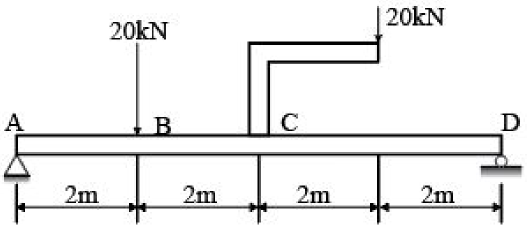
Shear force, (V) = $\frac{dM}{dx}$

$\therefore$ There is no relation between shear force and point of contra flexure.

Conventional Practice Solutions

01.

Sol:



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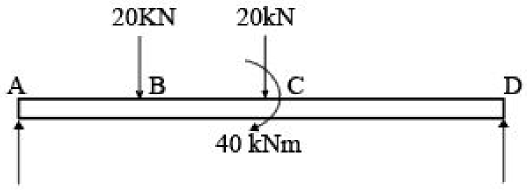


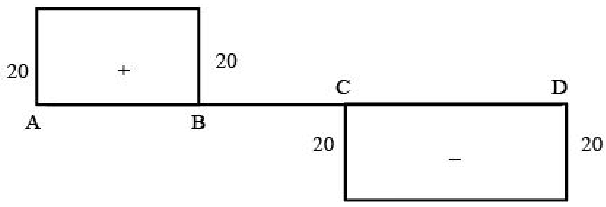
$\Sigma M_A = 0$

$20 \times 2 + 20 \times 4 + 40 = R_D \times 8$

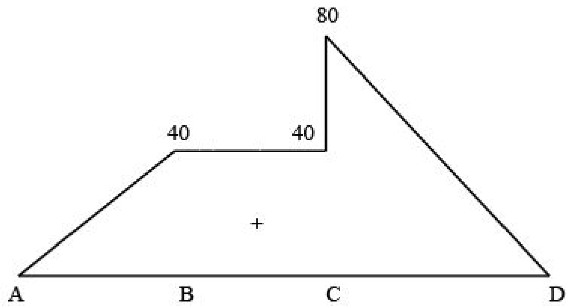
$R_D = \dfrac{40+80+40}{8} = 20\text{ kN}$

$R_A = 20\text{ kN}$

Shear Force Diagram (units kN)

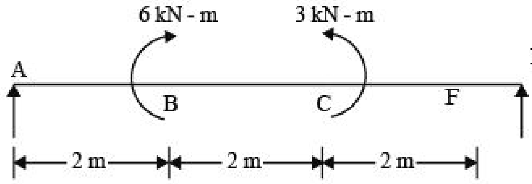


Bending Moment diagram (units kN)



02.

Sol:



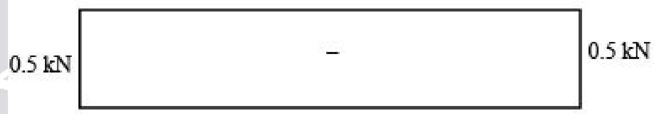
$\Sigma M_A = 0$

$R_D \times 6 + 3 - 6 = 0$

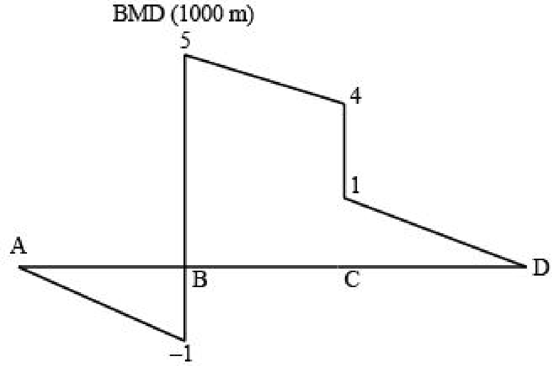
$R_D = 0.5\text{ kN} (\uparrow)$

$R_A = 0.5\text{ kN} (\downarrow)$

SFD



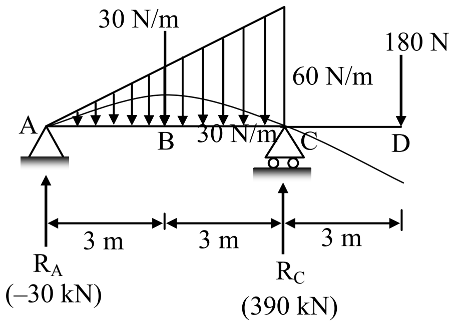
BMD (1000 m)



Point of contraflexure = 1 (point B)

03.

Sol:



Reactions:

$\sum F_Y = 0$

$$R_A + R_B = \frac{1}{2}\times 6\times 60 + 180$$

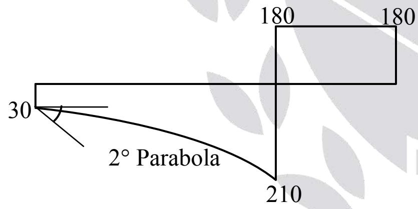
$R_A + R_B = 360$ N

$\sum M_A = 0$ (C.W)+))

$$-R_C \times 6 + \frac{1}{2}\times 6\times 60\times\left(\frac{2}{3}\times 6\right) + 180\times 9 = 0$$

$\Rightarrow R_C = 390$ N

$R_A = 360 - 390 = -30$ N



$(SF)_A$, ------- = $+R_A = +(-30) = -30$ N

$(SF)_C$ ---- = $-R_C + 180 = -390 + 180 = -210$

N or $R_A - \frac{1}{2}\times 6\times 60$

$(SF)_c ----- = +180\,N$

$(SF)_D ---- = +180\,N$

BMD (SAG +ve)

$M_A = 0$

$M_C = -180\times 3 = -540$ N-m,

$M_D = 0$

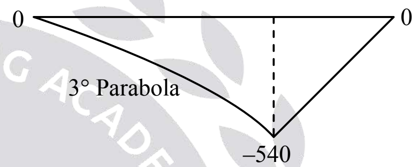
Design BM = $|540\,N - m|$

Design SF = $|210\,N|$

(ii) $M_B = +R_A \times 3 - \frac{1}{2}\times 3\times 30\times\left(\frac{1}{3}\times 3\right)$

$$M_B = (-30)\times 3 - \frac{1}{2}\times 90$$

$M_B = -135$ N-m



Maximum bending stress @B :

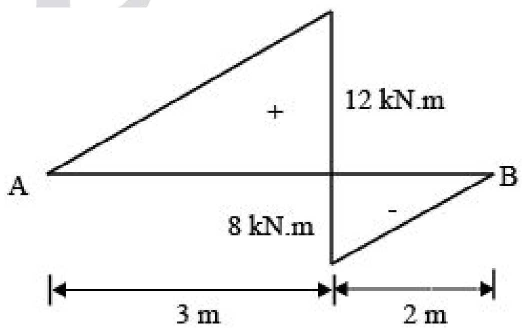
$$\frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$f_{max} = \frac{M}{I}.y_{max} = \frac{M}{Z} = \frac{135\times 10^3\ N - mm}{\left(\frac{50\times 100^2}{6}\right)}$$

$f_{max} = 1.62$ N/mm^2

04.

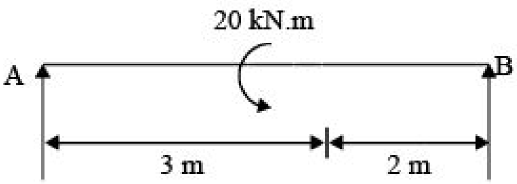
Sol:



At section C there is a sudden change of BM, so there is a concentrated moment in AC direction at point C of magnitude

12 – (– 8) = 20 kNm

So loading diagram



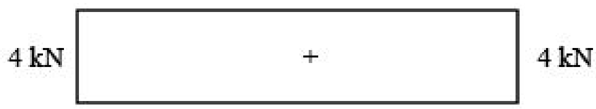
$\Sigma M_A = 0$

$R_B \times 5 = 20$

$R_B = 4$ kN (↓)

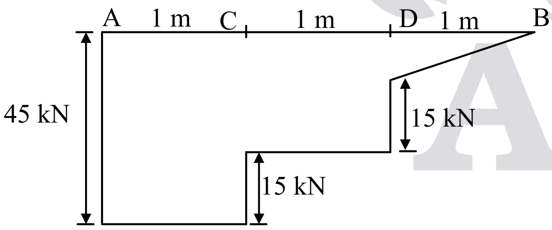
$R_A = 4$ kN (↑)

Shear Force Diagram



05.

Sol:

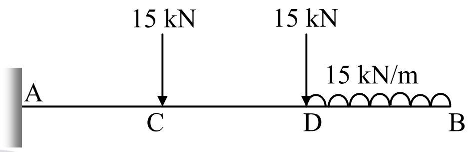


From SFD it is clear that support from at A = 45 kN

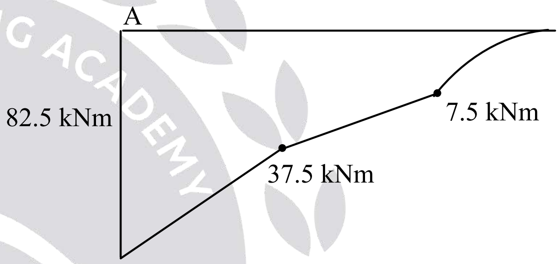
There is a concentrated point load of 15 kN at point C and D and UDL acts between points C and D of intensity

$$= \frac{15-0}{1} = 15 \text{ kN/m}$$

Loading diagram



Bending Moment Diagram:



Maximum Bending Stress is at point A

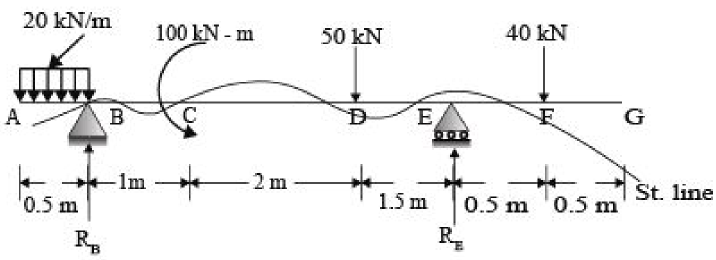
$$\delta = \frac{M}{z} = \frac{M \times 6}{bd^2} = \frac{6M}{bd^2}$$

$$= \frac{6 \times 82.5 \times 10^6}{100 \times 250^2}$$

$= 79.2$ N/mm^2 or 79.2 MPa

06.

Sol:



$\sum F_y = 0$

$R_B + R_E = 20\times0.5 + 50 + 40$

$= 100$ kN

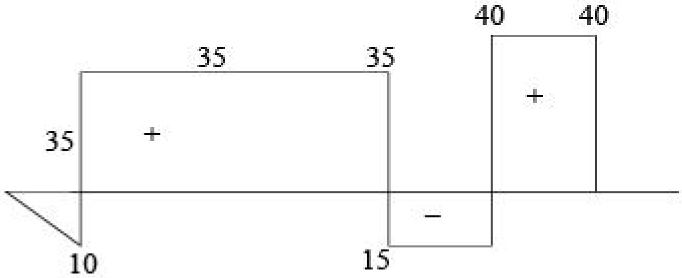
$\sum M_B = 0$ (C.W(+))

$$20\times0.5\times\frac{0.5}{2}-100+50\times3$$

$-R_E\times4.5 + 40\times5 = 0$

$R_E = 55$ kN

$R_B = 100 - 55 = 45$ kN



$(SF)_A = 0$

$(SF)_B$ --- $= -20\times0.5 = -10$ kN

$(SF)_B$ -- $= -20 \times 0.5 + 45 = 35$ kN

$(SF)_C = -20\times0.5+R_B = 35$ kN

$(SF)_D$ -- $= -20\times0.5 + 45 = 35$ kN

$(SF)_D$, -- $= -20\times5 + 45-50 = -15$ kN or 40 - R_E

$(SF)_E$, -- $= -20\times0.5 + 45 - 50 = -15$ kN or +40 - R_E

$(SF)_F$ ---- $= -20\times0.3 + R_B$-------------

$(SF)_F$ ----- $= 0$

$(SF)_G = 0$

BMD (SAG + ve)

$M_A = 0$

$$M_B = -20\times0.5\times\frac{0.5}{2} = -2.5\,\text{kN}-\text{m}$$

$$M_{C,\,it} = -20\times0.5\times\left(\frac{0.5}{2}+1\right)+R_B\times1$$

$= -20 \times 0.5 \times 1.25 + 45 \times 1$

$= 32.5$ kN-m

M_C ---S $= -20 \times 0.5 \times 1.25 \times 45\times1 - 100$

$= -67.5$ kN-m

$M_D = -40\times2 + R_E\times1.5$

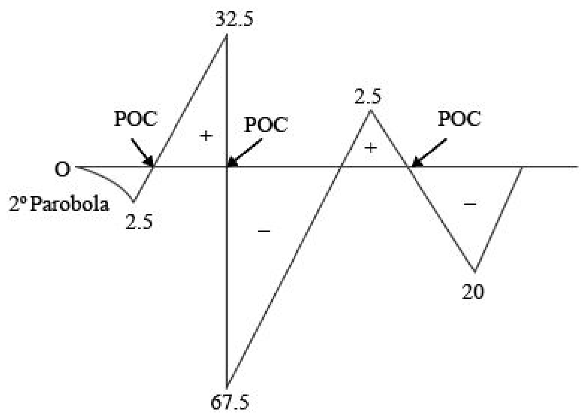
$= -40\times2 + 55\times1.5$

$= 2.5$ kN-m

$M_E = -40\times0.5 = -20$ kN-m

$M_F = 0$

$M_G = 0$



Chapter 4

Centre of Gravity & Moment of Inertia

01. Ans: (a)

Sol: $$\bar{y} = \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2}$$

$$\Rightarrow \bar{y} = \frac{2E_2\left(h + \frac{h}{2}\right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \qquad (\because E_1 = 2E_2)$$

$$\Rightarrow \bar{y} = 1.167h \text{ from base}$$

02. Ans: (b)

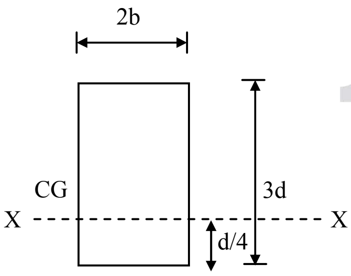
Sol: $$\bar{y} = \frac{A_1E_1Y_1 + A_2E_2Y_2}{A_1E_1 + A_2E_2}$$

$$= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2E_1 + 6a^2(2E_1)}$$

$$= \frac{22.5a^3E_1}{15a^2E_1} = 1.5a$$

03. Ans: 13.875 bd³

Sol:



$$y = \frac{5}{4}d$$

$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3$$

$$\text{M.I about } X - X\,|_{\text{at } d/4 \text{ distance}} = I_G + Ay^2$$

$$= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2$$

$$= \frac{111}{8}bd^3 = 13.875bd^3$$

04. Ans: 6.885×10⁶ mm⁴

Sol:

$$I_x = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12} + Ah^2\right)$$

$$= \frac{60 \times 120^3}{12} - 2\left(\frac{30 \times 30^3}{12} + (30 \times 30) \times 30^2\right)$$

$$= 6.885 \times 10^6 \text{ mm}^4$$

05. Ans: 152146 mm⁴

Sol:

$$I_x = \frac{30 \times 40^3}{12} - \frac{\pi \times 20^4}{64} = 152146\,\text{mm}^4$$

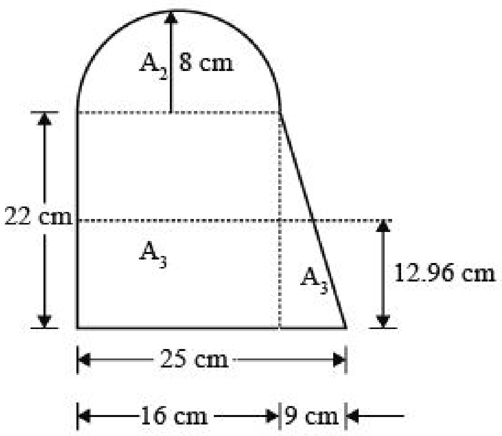
$$I_y = \frac{40 \times 30^3}{12} - \left(\frac{\pi \times 20^4}{64} + 2\left(\frac{\pi}{2} \times 10^2 \times \left(15 - \frac{4 \times 10}{3\pi}\right)^2\right)\right)$$

$$= 45801.34 \text{ mm}^4$$

Conventional Practice Solutions

01.

Sol:



$A_1 = 22 \times 16 = 352 \text{ cm}^2$

$A_2 = \dfrac{\pi r^2}{2} = 100.48 \text{ cm}^2$

$A_3 = \dfrac{1}{2} \times 9 \times 22 = 99 \text{ cm}^2$

Centroidal Axis location above base:

$$= \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3}$$

$$= \frac{352 \times 11 + 100.48 \times \left(\dfrac{4 \times 8}{3\pi} + 22\right) + 99 \times \left(\dfrac{22}{3}\right)}{352 + 100.48 + 99}$$

$$= \frac{3872 + 2551.89 + 726}{551.48}$$

= 12.96 cm above base

Second Moment of Area:

$I_{xx} \text{ of } A_1 + A_1 y_1^2 + I_{xx} \text{ of } A_2 + A_2 y_2^2 + I_{xx} \text{ of } A_3 + A_3 y_3^2$

$$= \left(\frac{16 \times 22^3}{12}\right) + 16 \times 22[12.96 - 11]^2 + (0.1098 \times 8^4)$$

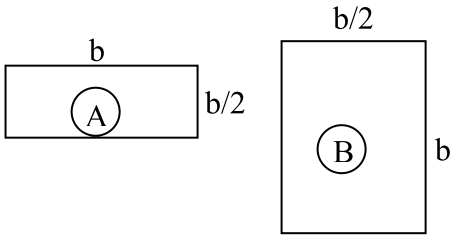
$$+ (12.43^2)(100.48) + \left(\frac{9 \times 22^3}{36}\right) + (99 \times 5.63^2)$$

$= 37323.97 \text{ cm}^4$

Chapter 5 Theory of Simple Bending

01. Ans: (b)

Sol:



By using flexural formula, $\sigma = \dfrac{M}{Z}$

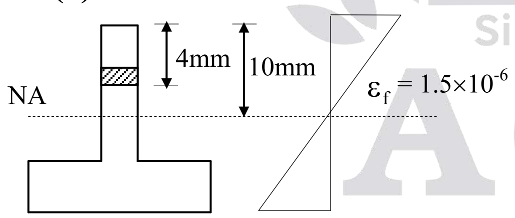
$\therefore\ \sigma \propto \dfrac{1}{Z}$ $\qquad (\because \text{M is constant})$

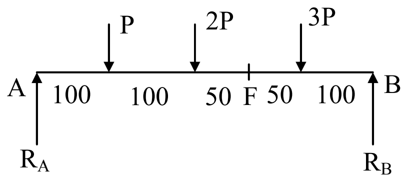
$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\dfrac{\left(\dfrac{b}{2}\times b^2\right)}{6}}{\dfrac{b\times\left(\dfrac{b}{2}\right)^2}{6}} = 2$$

$\Rightarrow \sigma_A = 2\sigma_B$

02. Ans: (b)

Sol:





$\therefore \Sigma M_A = 0$

$\therefore\ P\times100 + 2P\times200 + 3P\times300 = R_B\times400$

$\therefore R_B = 3.5P,\ R_A = 2.5P$

Take moments about F and moment at F

$M_F = R_B\times150 - 3P\times50 = 375P$

Also, $\dfrac{M_F}{I} = \dfrac{\sigma_b}{y_F}$

$$\therefore\ \frac{375P}{2176} = \frac{\left(1.5\times10^{-6}\times200\times10^{3}\right)}{6}$$

$\Rightarrow \quad P = 0.29\text{ N}$

03. Ans: (b)

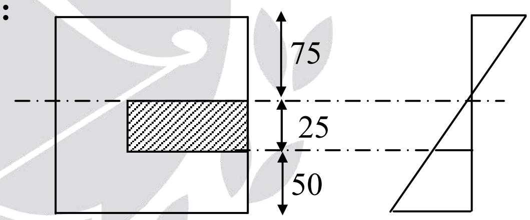
Sol: By using Flexural formula,

$$\frac{E}{R} = \frac{\sigma_b}{y_{max}} \Rightarrow \frac{2\times10^5}{250} = \frac{\sigma_b}{(0.5/2)}$$

$\Rightarrow \sigma_b = 200\text{ N/mm}^2$

04. Ans: (c)

Sol:



By using flexural formula,

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \frac{16\times10^6}{\dfrac{100\times150^3}{12}} = \frac{f}{25} \Rightarrow f = 14.22\text{ MPa}$$

Now, Force on hatched area

= Average stress × Hatched area

$$= \left(\frac{0+14.22}{2}\right)(25\times50) = 8.9\text{ kN}$$

05. Ans: (b)

Sol: By using flexural formula, $\dfrac{f_{Tensile}}{y_{top}} = \dfrac{M}{I}$

$$\Rightarrow f_{Tensile} = \frac{0.3\times3\times10^6}{3\times10^6}\times70$$

(maximum bending stress will be at top fibre so $y_1 = 70$ mm)

$\Rightarrow f_{Tensile} = 21\ N/mm^2 = 21\ MN/m^2$

06. Ans: (c)

Sol: Given data:

$P = 200$ N, $M = 200$ N.m

$A = 0.1\ m^2$, $I = 1.33\times10^{-3}\ m^4$

$y = 20$ mm

Due to direct tensile force P,

$$\sigma_d = \frac{P}{A} = \frac{200}{0.1}$$

$= 2000\ N/m^2$ (Tensile)

Due to the moment M,

$$\sigma_b = \frac{M}{I}\times y$$

$$= \frac{200}{1.33\times10^{-3}}\times20\times10^{-3}$$

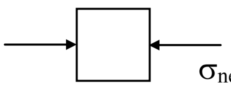
$= 3007.52\ N/m^2$ (Compressive)

$\sigma_{net} = \sigma_d - \sigma_b$

$= 2000 - 3007.52$

$= -1007.52\ N/m^2$

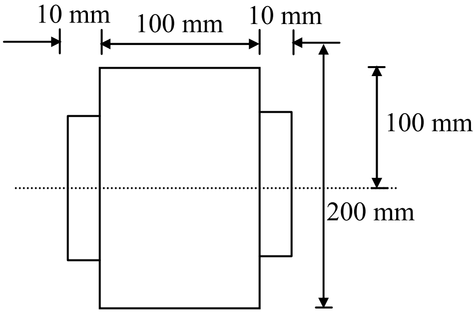
Negative sign indicates compressive stress.



$\sigma_{net} = 1007.52\ N/m^2$

07. Ans: 80 MPa

Sol:



Maximum stress in timber = 8 MPa

Modular ratio, m = 20

Stress in timber in steel level,

$100 \rightarrow 8$

$50 \rightarrow f_w$

$\Rightarrow f_w = 4$ MPa

Maximum stress developed in steel is $= m\cdot f_w$

$= 20\times4 = 80$ MPa

Convert whole structure as a steel structure by using modular ratio.

08. Ans: 2.43 mm

Sol: From figure $A_1B_1 = l = 3$ m (given)

$$AB = \left(R - \frac{h}{2}\right)\alpha = l - l\alpha t_1 \text{ ------ (1)}$$

$$A_2B_2 = \left(R + \frac{h}{2}\right)\alpha = l + l\alpha t_2 \text{ ------ (2)}$$

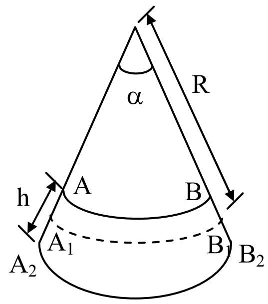
Subtracting above two equations (2) – (1)

$h(\alpha) = l\alpha\,(t_2 - t_1)$

but $A_1B_1 = l = R\alpha$

$$\Rightarrow \alpha = \frac{l}{R}$$

$$\therefore h\left(\frac{l}{R}\right) = l\alpha\,(\Delta T)$$



$$R = \frac{h}{\alpha(\Delta T)}$$

$$= \frac{250}{(1.5\times10^{-5})(72-36)}$$

$R = 462.9$ m

From geometry of circles

$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2}$ {ref. figure in Q.No.02}

$2R\cdot\delta-\delta^2 = \frac{L^2}{4}$ (neglect δ^2)

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8\times 462.9} = 2.43 \text{ mm}$$

Shortcut:

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^o - 36^o = 36^o$). Bottom temperature being more, the beam deflects down.

$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h}$$

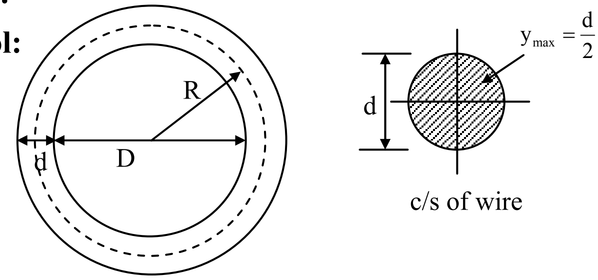
$$= \frac{1.5\times10^{-5}\times 36\times 3000^2}{8\times 250}$$

$= 2.43$ mm (downward)

Conventional Practice Solutions

01.

Sol:



c/s of wire

Radius of curvature,

$$R = \frac{D}{2} + \frac{d}{2} = \frac{D}{2}$$

small therefore ignored

$$\frac{E}{R} = \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$f_{max} = \frac{E}{R}\times y_{max} = \frac{E}{D/2}\times\frac{d}{2} = \frac{E.d}{D}$$

$R = ?$, if $d = 1$ cm, $f_{max} = 17000$ kg/cm^2

$E = 2\times10^6$ kg/cm^2

$$f_{max} = \frac{Ed}{D} = \frac{Ed}{2R}$$

$$17000 = \frac{(2\times10^6)\times 1}{2(R)}$$

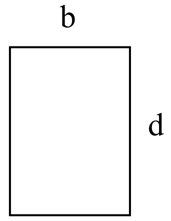
$\Rightarrow R = 58.82$ cm

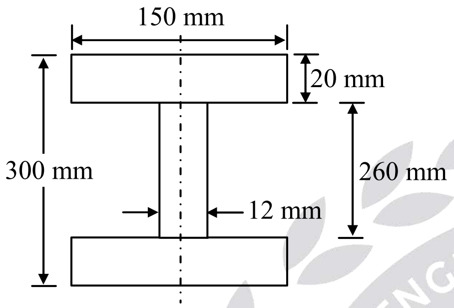
$$\frac{E}{R} = \frac{M}{I}$$

$\Rightarrow M = 1669.1$ kg-cm

02.

Sol:





$W = \gamma.A.l$

$W \propto A$

γ = constant (same as material)

R = same

$A_I = A_{rect} = A$

$$[150\times300-(150-12)\times260]=(b\times2b)=\left(\frac{\pi}{4}d^2\right)$$

b = 67.53 mm

d = 107.76 mm

For I-section

$$M_I, I=\frac{150\times300^3}{12}-\frac{(150-12)(260)^3}{12}$$

$$=135.38\times10^6$$

$$y_{max}=\frac{300}{2}=150\,\text{mm}$$

$$Z=\frac{I}{y_{max}}=\frac{135.38\times10^6}{150}=902.5\times10^3\ \text{mm}^3$$

$(Z)_I : (Z)_{rect} : (Z)_{circle}$

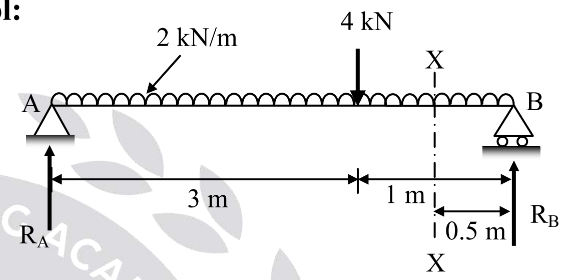
$$902.5\times10^3:\frac{(67.53)(2\times67.53)^2}{6}:\frac{\pi}{32}(107.76^3)$$

$$\frac{902.5\times10^3}{902.5\times10^3}:\frac{(67.53)(2\times67.53)^2}{6\times(902.5\times10^3)}:\frac{\pi(107.76^3)}{32\times902.5\times10^3}$$

1 : 0.227 : 0.136

03.

Sol:



$\Sigma F_y = 0 \Rightarrow R_A + R_B = (2\times3)+4$

$R_A + R_B = 10$

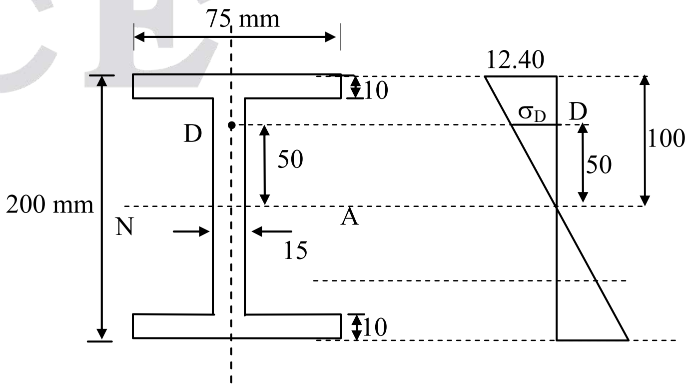
$$\Sigma M_A = 0 \Rightarrow 4\times2+2\times3\times\frac{3}{2}=R_B\times3$$

$R_B = 5.67$ kN

$R_A = 4.33$ kN

$$M_x=5.67\times0.5-2\times0.5\times\frac{0.5}{2}$$

$M_x = 2.585$ kN-m



$$\frac{M}{I} = \frac{\sigma_x}{y}$$

$$\Rightarrow \sigma_x = \frac{M}{I} \times y$$

$$I = \frac{75 \times 200^3}{12} - \frac{60 \times 180^3}{12}$$

$I = 2084 \times 10^4\ mm^4$

$y_{max} = 100$

$M_x = 2.585 \times 10^6$ N-mm

$$(\sigma_x)_{max} = \frac{2.585 \times 10^6 \times 100}{2084 \times 10^4}$$

$(\sigma_x)_{max} = 12.40\ N/mm^2$

$100 \to \sigma_D$

$50 \to 12.40$

$$\sigma_D = \frac{12.40 \times 50}{100}$$

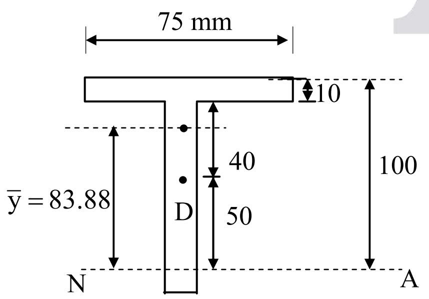
$= 6.20\ N/mm^2$ (Compression)

$\therefore\ \sigma_D = 6.20\ N/mm^2$ (Compression)

$$\tau_x = \frac{V_x A.\bar{y}}{I.b}$$

$$\tau_x = \frac{4.67 \times 10^3 \times 1350 \times 83.88}{15 \times 2084 \times 10^4}$$

$\tau_{x\ at\ D} = 1.691\ N/mm^2$



$A = 10 \times 75 + 40 \times 15$

$= 1350\ mm^2$

$I = 2084 \times 10^4\ mm^4$

$b = 15$ mm

$$\bar{y} = \frac{750 \times 5 + 600 \times 30}{750 + 600} = 16.11\ mm$$

$\bar{y} = 83.88$ mm

$V_x = 5.67 - (2 \times 0.5) = 4.67$ kN

Principal stress at point D,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2}$$

$\sigma_x = -6.2$

$\tau_x = 1.691$

$$\sigma_{1,2} = \frac{-6.2}{2} \pm \frac{1}{2}\sqrt{(6.2)^2 + (2 \times 1.691)^2}$$

$\sigma_{1,2} = -3.1 \pm 3.53$

$\sigma_1 = 0.431\ N/mm^2$

$\sigma_2 = -6.63\ N/mm^2$

$\tau_{max} = 3.53\ N/mm^2$

Chapter 6

Shear Stress Distribution in Beams

01. Ans: (a)

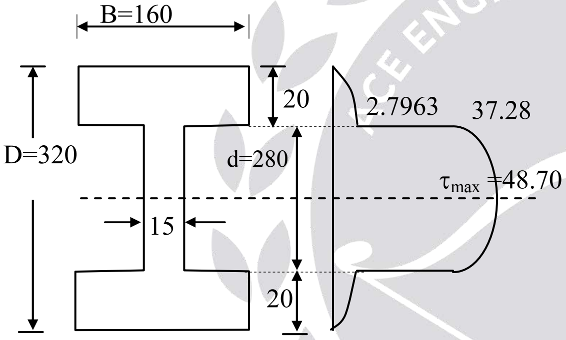
Sol: $\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{f}{b.d}$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$\therefore d = 250$ mm = 25 cm

02. Ans: 37.3

Sol:



All dimensions are in mm

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12}$$

$= 171.65 \times 10^6$ mm^4

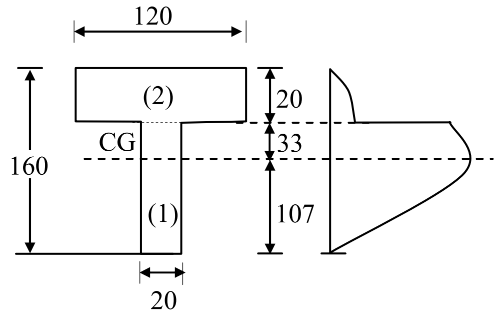
$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (160 \times 20 \times 150)$$

= 37.28 MPa

03. Ans: 61.43 MPa

Sol:



All dimensions are in mm

$I_{NA} = 13 \times 10^6$ mm^4

$y_{CG} = 107$ mm from base

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

$A\bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5)$

$= 114090$ mm^3

$$\tau_{max} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20} = 61.43 \text{ MPa}$$

04. Ans: (a)

Sol: For a shear force 'V' and cross section area 'A' average shear stress = $\tau_{avg} = \frac{V}{A}$

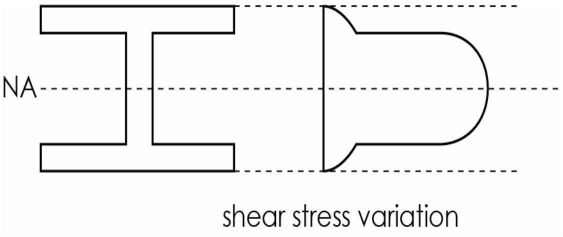
In case of rectangular cross sections, maximum shear stress = $\tau_{max} = 1.5\tau_{avg}$

In case of circular cross sections, maximum shear stress = $\tau_{max} = \frac{4}{3}\tau_{avg}$

For the same amount of shear force and same cross-section area, maximum shear stress is lesser in circular cross-section, so it is stronger in shear i.e. can resist more shear force compared to rectangular cross section.

05. Ans: (b)

Sol:



$$\tau = \frac{VA\bar{y}}{Ib}$$

A = Area above the section at which stress is calculated.

$\therefore$ $\tau \propto \frac{1}{b}$ and increases as area increases i.e. towards neutral axis.

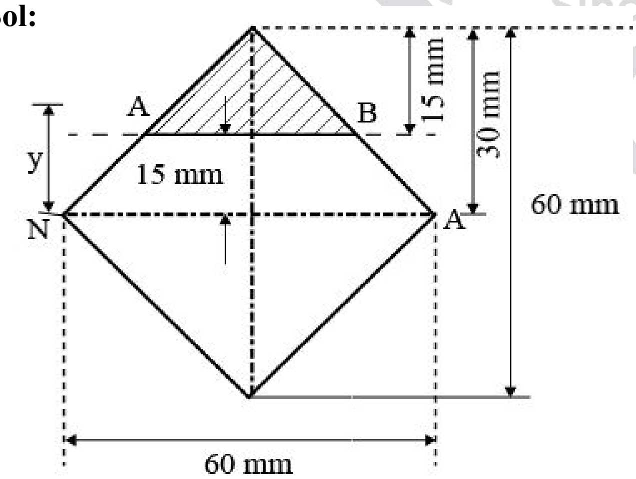
Since width of web << width of flange

$\therefore$ 'τ' increases at web and is maximum at centre of web.

Conventional Practice Solutions

01.

Sol:



V = 5kN

$$\tau_{AB} = \frac{VA\bar{y}}{I(b_x)}$$

For diamond section:

$$I_{NA} = \frac{a.a^3}{12}$$

$$= \frac{42.43 \times 42.43^3}{12}$$

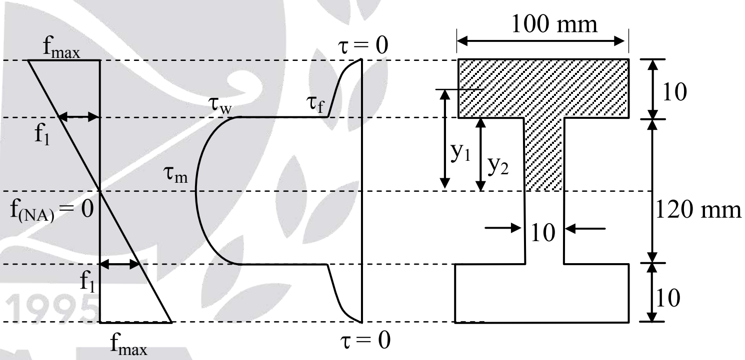
$$= 270000$$

$$\tau_{AB} = \frac{5\times10^3 \times \left(\frac{1}{2}\times30\times15\right)\times\left(15+\frac{1}{3}\times15\right)}{270000\times30}$$

$$= 2.8 \text{ MPa}$$

02.

Sol:



$$I_{NA} = \frac{100\times140^3}{12} - \frac{(100-10)\times120^3}{12}$$

$$= 9.9\times10^6 \text{ mm}^4$$

$$\tau_f = \frac{VA\bar{y}}{I(b_f)} = \frac{10000\times(100\times10)\left(\frac{120}{2}+\frac{10}{2}\right)}{9.9\times10^6\times100}$$

$$= 0.66 \text{ MPa}$$

$$\tau_w = \frac{VA\bar{y}}{I(b_w)} = \frac{10000\times(100\times10)\left(\frac{120}{2}+\frac{10}{2}\right)}{9.9\times10^6\times10}$$

$= 6.6$ MPa

or $= 0.66\times\frac{100}{10}$

Maximum shear stress @ N.A

$$\tau_{NA} = \frac{V(A_1\bar{y}_1 + A_2\bar{y}_2)}{I.b_{NA}}$$

$$= \frac{10000\left(100\times10\times65+10\times60\times\frac{60}{2}\right)}{(9.9\times10^6)\times10}$$

$= 8.38$ MPa

Bending Stress:

$$f_{max} = \frac{M}{I}.y_{max} = \frac{1000\times10^3}{9.9\times10^6}\times70$$

$= 7.07$ MPa

$$f_1 = \frac{M}{I}.y_1 = \frac{1000\times10^3}{9.9\times10^6}\times60 = 6.06 \text{ MPa}$$

Chapter 7 Torsion

01. Ans: (c)

Sol: Twisting moment $= 2\times0.5 + 1\times0.5$

$= 1.5$ kN-m

02. Ans: (d)

Sol: $$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{hollow}} = \frac{1}{1-K^4}$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)^4} = \frac{16}{15}$$

03. Ans: 43.27 MPa & 37.5 MPa

Sol: Given $D_o = 30$ mm , $t = 2$ mm

$\therefore$ $D_i = 30 - 4 = 26$ mm

We know that $\frac{\tau}{J} = \frac{q}{R}$

$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{max}}{\left(\frac{30}{2}\right)}$$

$q_{max} = 43.279$ N/mm^2

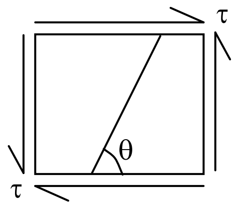
$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{min}}{\left(\frac{26}{2}\right)}$$

$q_{min} = 37.5$ N/mm^2

04. Ans: (a)

Sol: Ductile material is weak in shear, so it fails in a plane where maximum shear stress occurs. Brittle material is weak in tension, so it fails in a plane where maximum tensile stress occurs.

It is a case of pure shear.

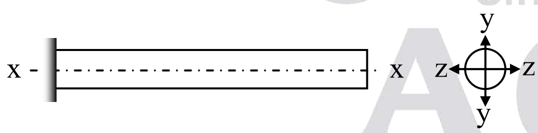


$\sigma_1 = \sigma_{max}$, at $\theta = 45°$ (maximum normal stress which causes failure of brittle material).

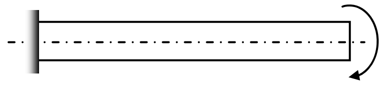
Thus, Assertion (A) and Reason (R) are correct and Reason (R) is correct explanation of Assertion (A).

05. Ans: (b)

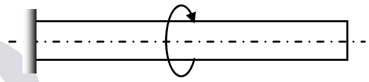
Sol: Let the axis of the beam is x-x as shown below.



- If the axis of the moment is perpendicular the axis of the beam (i.e. axis of the moment is either y-y or z-z), then beam will be subjected to bending. Hence, bending stress will be induced. Since there is no shear force, shear stress will not be induced.



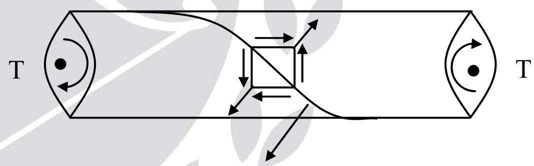
- If the axis of the moment is along the axis of the beam then beam will be subjected to torsion. Thus, only shear stress will be induced and bending stress will not be induced..



Conventional Practice Solutions

01.

Sol: Cast iron: Brittle (Weak in tension)



45° (spiral crack)

$\sigma_1 + \tau =$ Diagonal tension

$= 80$ MPa (given)

Maximum torsional shear stress, $\tau_m = \sigma_1$

$= 80$ MPa

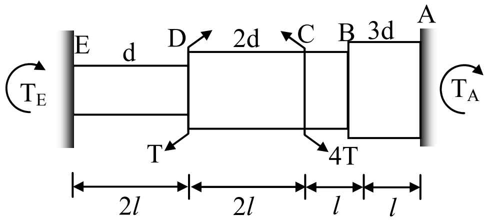
$$\tau_{max} = \frac{T}{Z_p}$$

$$\Rightarrow 80 = \frac{T}{\frac{\pi}{16} \times 30^3}$$

$\Rightarrow T = 0.42$ kN.m

02.

Sol:



F.B.D:

$\sum T = 0$ (equation)

$T_A + T_E + T - 4T = 0$

Compatibility condition

$Q_{AE} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DE}$

$$Q_A - Q_E = \frac{T_A \times \ell}{G\left(\frac{\pi}{32}(3d)^4\right)} + \frac{T_A \ell}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{(T_A - 4T)(2\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{(T_A - 3T)2\ell}{G\left(\frac{\pi}{32}(d)^4\right)}$$

$T_A = 2.95\ T$

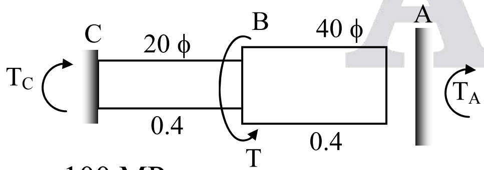
$Q_{AC} = Q_{AB} + Q_{BC}$

$$Q_A - Q_C = \frac{(T_A)(\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{T_A(\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)}$$

$$\Rightarrow Q_c = \frac{-2.24TL}{Gd^4}$$

03.

Sol:



$\tau_{max} = 100$ MPa

Reactions:

$T_A + T_C = T$ ------------(1)

$Q_{AC} = Q_{AB} + Q_{BC}$

$$Q_A - Q_C = \frac{T_A(0.4\times1000)}{G\left(\frac{\pi}{32}\times 40^4\right)} + \frac{(T_A - T)(0.4\times1000)}{G\left(\frac{\pi}{32}\times 20^4\right)}$$

$T_A = 0.94\ T$ (kN-m)

Span AB

$$\tau_{max} = \frac{T}{Z_p}$$

$$100 = \frac{(0.94T)\times10^6}{\frac{\pi}{16}(40^3)}$$

$T = 1.33$ kN-m ---------(1)

Span BC:

$$\tau_{max} = \frac{T_{BC}}{Z_p}$$

$$100 = \frac{|0.94T - T|\times10^6}{\frac{\pi}{16}\times 20^3}$$

$\Rightarrow T = 2.6$ kN-m

Use minimum "T"

$\therefore\ T = 1.33$ kN-m

04.

Sol: N= 110 rpm

P = 30 kW

$\tau_m = 80$ MPa

d = ?

$$P = \frac{2\pi NT}{60}$$

$$30\,kN-m/s = \frac{2\pi\times110T}{60}$$

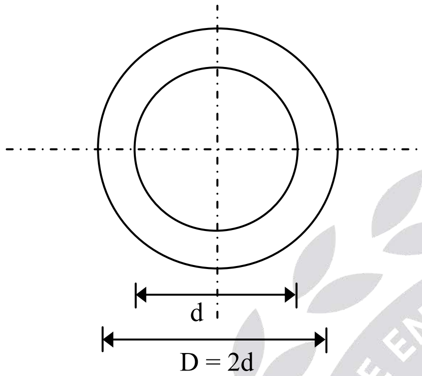
$T = 2.604$ kN-m

$$\tau_{max} = \frac{T}{4}$$

$$80 = \frac{2.604\times10^6}{\frac{\pi}{16}\left(d^3\right)}$$

$\Rightarrow d_s = 54.93$ mm

Replacement: by hollow shaft $\rightarrow$ To have same strength (To transmit same power)



$(Z_P)_{solid} = (Z_p)_{hollow}$

$(Z_p)_{solid} = (Z_p)_{hollow}$

$$\frac{\pi}{16}\left(54.93^3\right) = \frac{\pi}{16D}\left(D^4 - d^4\right)$$

$$32543.08 = \frac{\pi}{16D}\left(D^4 - \left(\frac{D}{2}\right)^4\right)$$

$\Rightarrow D = 56.18$ mm

$d = 28.09$ mm

$$\frac{W_h}{W_s} = \frac{(\gamma AR)_h}{(\gamma AL)_s} = \frac{A_h}{A_s} = \frac{\frac{\pi}{4}\left(D^2 - d^2\right)}{\frac{\pi}{4}\left(d_s^2\right)}$$

$$= \frac{56.18^2 - 28.09^2}{54.93^2} = 0.78$$

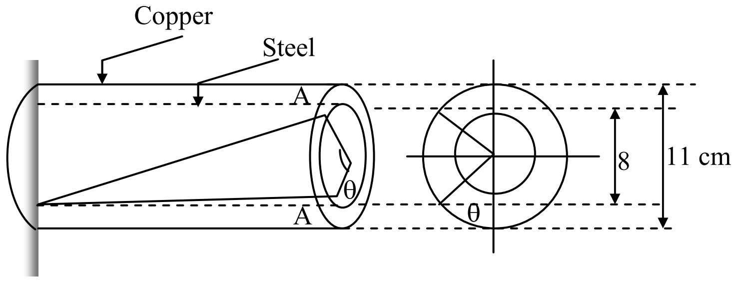
$W_h = 0.78 W_s$

$W_h = (1 - 0.78)\times100$

$W_h = 22\% \downarrow W_s$

05.

Sol:



Parallel equation :

$T_C + T_s = T = 8$ kN-m

Compatibility condition:

$\theta_c = \theta_s$

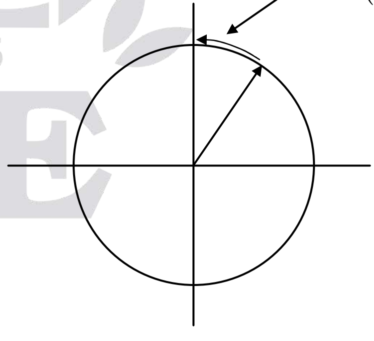
$$\frac{(T_c)(\ell)}{(40)\left[\frac{\pi}{32}\times\left(11^4 - 8^4\right)\right]} = \frac{(T_s)(\ell)}{(80)\left[\frac{\pi}{32}\times 8^4\right]}$$

$$\Rightarrow \frac{8 - T_s}{11^4 - 8^4} = \frac{T_s}{8^4}$$

$\Rightarrow T_s = 3.5$ kN-m

$T_c = 8 - 3.5 = 4.5$ kN-m

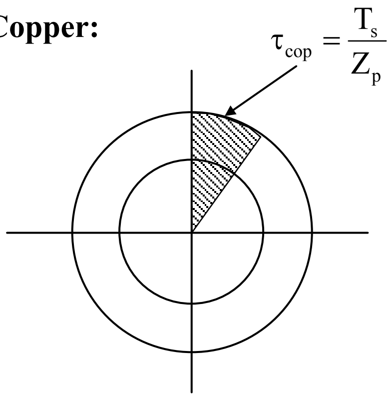
Steel: $\tau_{steel} = \frac{T_s}{(Z_p)_s}$



$$\tau_{steel} = \frac{T_s}{(Z_p)_s} = \frac{3.5\times10^6}{\frac{\pi}{16}(80)^3} = 34.81 \text{ kN/mm}^2$$

$= 34.81$ MPa

Copper:



$$\tau_{cop} = \frac{T_c}{Z_p} = \frac{4.5\times10^6}{\frac{\pi}{16\times110}\left(110^4 - 80^4\right)}$$

$= 23.9$ MPa

06.

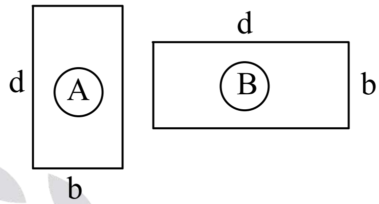
Sol: ***Refer to solution of Q. No. 4***

Chapter 8

Slopes and Deflections

01. Ans: (c)

Sol:



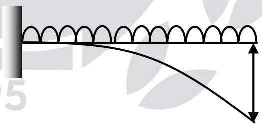
$$y_{max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

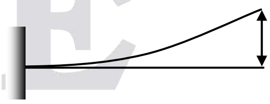
$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

02. Ans: (b)

Sol: Total load W= w*l*



$$y_{max} = \frac{W\ell^3}{8EI} \text{ (Downward)}$$



$$y_{max} = \frac{W\ell^3}{3EI} \text{(Upward)}$$

$$y_{net} = \downarrow y_{udl} - \uparrow y_w$$

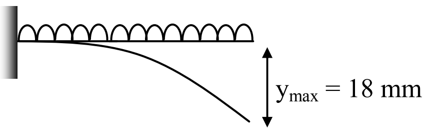
$$\text{Total Net deflection} = \frac{WL^3}{8E1} - \frac{WL^3}{3EI}$$

$$= \frac{-5WL^3}{24EI}$$

(Negative sign indicates upward deflection)

03. Ans: (c)

Sol:



$$\theta_{max} = \frac{wl^3}{6EI} = 0.02 \quad \text{-------(i)}$$

$$y_{max} = \frac{wL^4}{8EI}$$

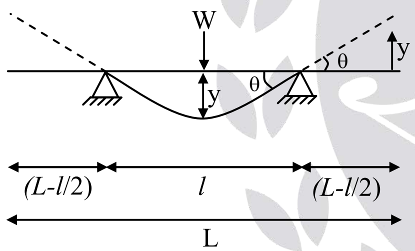
$$\therefore\ 0.018 = \left(\frac{WL^3}{6EI}\right) \times \frac{L\times 6}{8}$$

$$\therefore\ 0.018 = \frac{0.02\times L\times 6}{8} \qquad [\because \text{ Equation (i)}]$$

$$\Rightarrow \quad L = 1.2\text{ m}$$

04. Ans: (a)

Sol:



Conditions given

$$\downarrow y = \frac{wl^3}{48EI}$$

$$\theta = \frac{wl^2}{16EI}$$

$$\tan\theta = \frac{y}{(L-\ell)/2}$$

θ is small ⇒ tan θ = θ

$$\therefore\ \theta = \frac{y}{(L-\ell)/2}$$

$$\therefore y = \theta\left(\frac{L-\ell}{2}\right)$$

$$\uparrow y = \theta\left(\frac{L-\ell}{2}\right)$$

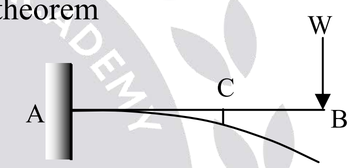
Thus $y\downarrow = y\uparrow$

$$\therefore \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-\ell}{2}\right)$$

$$\Rightarrow \quad \frac{L}{\ell} = \frac{5}{3}$$

05. Ans: (c)

Sol: By using Maxwell's law of reciprocals theorem



$$\delta_{C/B} = \delta_{B/C}$$

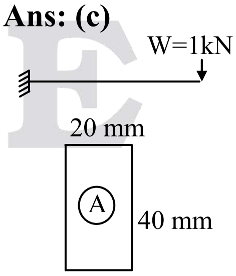
Deflection at 'C' due to unit load at 'B'

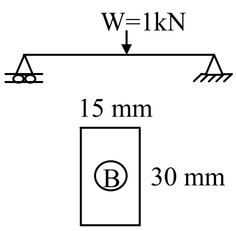
= Deflection at 'B' due to unit load at 'C'

As the load becomes half deflection becomes half.

06. Ans: (c)

Sol:



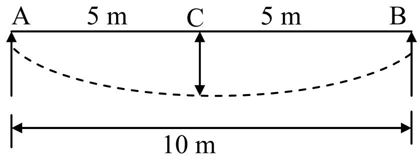


$$y_A = y_B \Rightarrow \left(\frac{wL^3}{3EI}\right)_A = \left(\frac{wL^3}{48EI}\right)_B$$

$$\therefore\ L_B = 400\text{ mm}$$

07. Ans: 0.05

Sol:



$\therefore$ Curvature, $\dfrac{d^2y}{dx^2} = 0.004$

Integrating with respect to x,

We get, $\dfrac{dy}{dx} = 0.004x$

$y = \dfrac{0.004x^2}{2}$

$y = 0.002x^2$

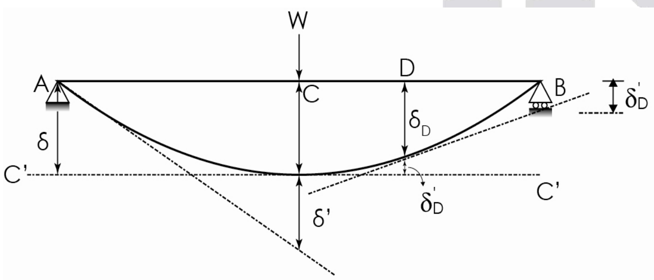
At mid span, x = 5 m

$\therefore\ y = 0.002\ x^2$

$y = 0.05$ m

08. Ans: (c)

Sol: According to Mohr's second moment area theorem displacement of 'B' from tangent at A = moment of area of $\dfrac{M}{EI}$ diagram between A and B taken about B.



Tangent at 'C' i..e C' - C' is horizontal

Moment of area of $\dfrac{M}{EI}$ diagram between A and 'C' about 'A' gives, displacement of 'A' (δ_A) w.r.t tangent at 'C' (i.e. 'δ' as shown in figure).

$\because \delta_A = 0$ and tangent at 'C' is at a distance of $\delta = \delta_c$ from the beam so we get 'δ_c' from the above analysis.

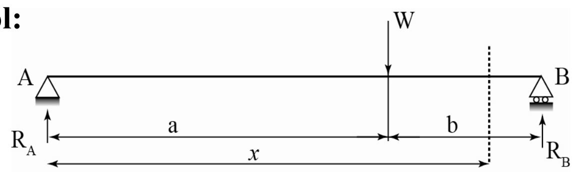
Note:

1) This deflection is valid only for midpoint for a simply supported beam with a load at midspan. For any other section, actual deflection is not same if calculated as above, as shown for point 'D'. δ_D is actual deflection at 'D', is the value obtained if moment of area between B and D is taken about 'B'.

2) If moment of area of $\dfrac{M}{EI}$ diagram between A and C about 'C' is taken it gives deflection of 'C' w.r.t tangent at 'A' i.e. δ′ shown in figure (Which is not the deflection of 'C').

09. Ans: (d)

Sol:



Consider a load 'W' on a simply supported beam AB of length 'ℓ' at a distance a from A.

$$\left(a > \frac{\ell}{2}\right)$$

For equilibrium: $\Sigma F_y = 0$; $\Sigma M_z = 0$

$$R_A + R_B = W$$

Taking moments about A;

$$(R_B + \ell) - W(a) = 0$$

$$\Rightarrow R_B = \frac{Wa}{\ell}$$

$$R_A = \frac{W(\ell - a)}{\ell} = \frac{W(b)}{\ell}$$

Using Macaulay's method:

Consider a section x-x at a distance 'x' from A

$$EI\frac{d^2y}{dx^2} = -M,$$

$$= -(R_A x - W < x - a >)$$

Integrating $EI\frac{dy}{dx} = \frac{W(x-a)^2}{2} - \frac{R_A x^2}{2} + C_1$

Integrating:

$$Ely = \frac{W(x-a)^3}{6} - \frac{R_A}{6}x^3 + C_1 x + C_2$$

At A; x = 0; y = 0 $\quad \therefore C_2 = 0$

At B; x = ℓ ; y = 0

$$0 = \frac{W(\ell - a)^3}{6} - \frac{R_A \ell^3}{6} + C_1 \ell$$

$$C_1 = \frac{-1}{6}\left[\frac{W(\ell - a)^3}{\ell} - \frac{W(\ell - a)\ell^2}{\ell}\right]$$

$$= \frac{W}{6}\frac{(\ell - a)}{\ell}\left(-(\ell - a)^2 \ell^2\right)$$

$$= \frac{W}{6}\frac{(\ell - a)}{\ell}\left(-a^2 + 2a\ell\right)$$

$$\therefore Ely = \frac{W(x-a)^3}{6} - \frac{W(\ell - a)}{6\ell}x^3 + \frac{W}{6}\frac{(\ell - a)}{\ell}\left(2a\ell - a^2\right)x$$

For maximum deflection $\frac{dy}{dx} = 0$

$$EI\frac{dy}{dx} = \frac{W(x-a)^2}{2} - \frac{W(\ell - a)x^2}{2\ell} + \frac{W}{6}\frac{(\ell - a)}{\ell}\left(2a\ell - a^2\right) = 0$$

Solving equation:

For $x < a; \left(a > \frac{\ell}{2}\right)$

$$\therefore \frac{-W}{2\ell}(\ell - a)x^2 + \frac{W}{6\ell}(\ell - a)\left(2a\ell - a^2\right) = 0$$

$$\therefore x = \sqrt{\frac{\left(2a\ell - a^2\right)}{3}} < a$$

$$a > \frac{\ell}{2} \Rightarrow 2a > \ell$$

$$4a^2 > 2a\ell$$

$$3a^2 > 2a\ell - a^2$$

$$\frac{2a\ell - a^2}{3} < a^2$$

$$\sqrt{\frac{2a\ell - a^2}{3}} < a$$

x is real number if $2a\ell - a^2 > 0 \Rightarrow a < 2\ell$ (possible)

$$x > 0 \text{ and } a > \frac{\ell}{2}$$

$\therefore$ Max deflection occurs at $x = \sqrt{\frac{2a\ell - a^2}{3}}$

Checking if $x > \frac{\ell}{2}$

Assuming $x > \frac{\ell}{2}$

$$\Rightarrow \sqrt{\frac{2a\ell - a^2}{3}} > \frac{\ell}{2}$$

$$\Rightarrow 8a\ell - 4a^2 > 3\ell^2$$

$$\Rightarrow 4a^2 - 8a\ell + 3\ell^2 < 0$$

$$\Rightarrow \left(a - \frac{\ell}{2}\right)\left(a - \frac{3\ell}{2}\right) < 0$$

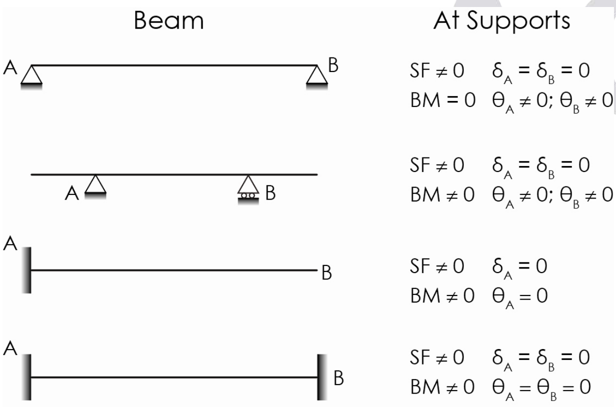
$\therefore$ Case 1: $a > \frac{\ell}{2}$; $a < \frac{3\ell}{2}$ (possible)

$\therefore$ Case 2: $a < \frac{\ell}{2}$; $a < \frac{3\ell}{2}$ (possible)

$\therefore$ Conclusion: x<a; $x > \frac{\ell}{2}$ Max deflection occurs between midspan and point of application of load.

10. Ans: (b) or (c)

Sol:



11. Ans: (b)

Sol: Refer to the solution of Q.No. 08

12. Ans: (a)

Sol: Conjugate beam is an imaginary beam for which loading = $\frac{M}{EI}$ diagram of real beam and is based on

- Slope at a section in real beam = shear force at that section in conjugate beam.
- Deflection at a section in real beam = Bending moment at that section in conjugate beam.

$\therefore$ For a simple support of real beam $\theta \neq 0$

$$\delta = 0$$

$\therefore$ Corresponding support in conjugate beam should have SF $\neq 0$

$$BM = 0$$

And the support corresponding to this condition is simple support.

13. Ans: (c)

Sol:

1. According to Castigliano's second theorem:

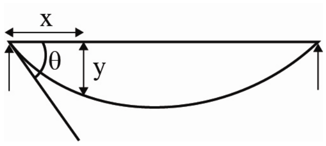
Partial derivative of strain energy w.r.t concentrated external load is the deflection of the structure at the point of application and in the direction of load.

$$\Rightarrow \frac{\partial U}{\partial P} = '\delta'$$

2. Derivative of deflection:

If y is the deflection; then

$$\frac{dy}{dx} = \text{slope}$$



3. Derivative of slope:

If $\frac{dy}{dx}$ is the slope, then Derivative of slope multiplied with EI gives Bending moment

$$\Rightarrow EI\frac{d^2y}{dx^2} = M$$

$\therefore$ There is no exact answer, but the most suitable answer is 'C'

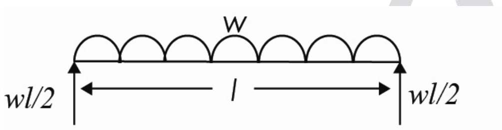
4. Derivative of moment :

First derivative of bending moment gives shear force.

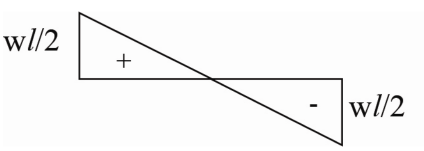
$$\frac{dM}{dx} = V$$

14. Ans: (b)

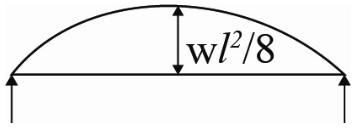
Sol: For a simply supported beam subjected to uniformly distributed loads.



SFD:



BMD:



Deflection profile:

Conclusions:

(i) Bending moment is maximum at centre and zero at support.

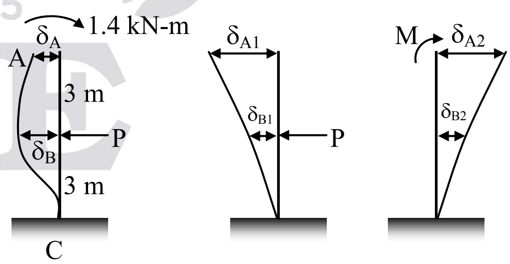
(ii) Shear force is maximum at supports and zero at centre.

(iii) Slope is maximum at supports and zero at midspan.

Conventional Practice Solutions

01.

Sol:



(i) $\delta_A = \delta_B$ (on same/left side)

$$(\delta_{B1} - \delta_{B2}) = (\delta_{A1} - \delta_{A2})$$

$$\Rightarrow \delta_{B1} = \frac{P(3)^3}{3EI}, \quad \delta_{B2} = \frac{(1.4)6^2}{8EI}$$

$$\delta_{A1} = \frac{P(3)^3}{3EI} + \frac{P(3)^2}{2EI}(3)$$

$$\delta_{A1} = \frac{1.4 \times 6^2}{2EI}$$

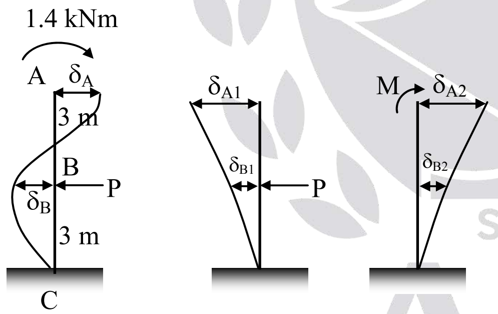
Real	C.B
Y_A	$M_A = \left(\frac{M}{EI}.\ell\right) = \frac{ML^2}{2EI}$
y_B	$M_B = \frac{M}{EI}\left(\frac{\ell}{2}\right)\left(\frac{\ell}{4}\right) = \frac{ML^2}{8EI}$

$$\frac{P(3)^3}{3EI} - \frac{1.4 \times 6^2}{8EI} = \frac{P(3)^3}{3EI} + \frac{P(3)^2 \times 3}{2EI} - \frac{1.4 \times 6^2}{2EI}$$

$$\Rightarrow \frac{216P - 151.2}{24} = \frac{54P + 81P - 151.2}{6}$$

$\Rightarrow$ P = 1.4 kN

(ii)



$\delta_A = \delta_B$ (on opposite side)

$(\delta B_1 - \delta B_2) = (\delta A_2 - \delta A_1)$

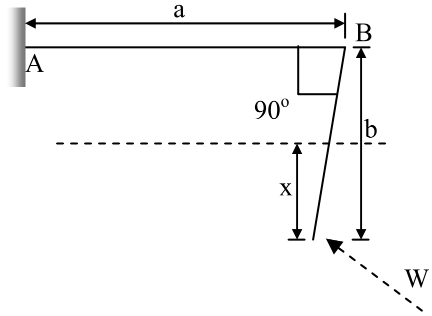
$$\Rightarrow \frac{P(3)^3}{3EI} - \frac{1.4 \times 6^2}{8EI} = \frac{1.6 \times 6^2}{2EI} - \left(\frac{P(3)^3}{3EI} + \frac{P(3)^2}{2EI}(3)\right)$$

$$\Rightarrow \frac{216P - 151.2}{24} = \frac{151.2 - 54P - 81P}{6}$$

$\Rightarrow$ P = 1 kN

02.

Sol:



Deflection at free end $\int_0^{\ell} \frac{Mmdn}{EI}$

M = Moment due to applied load

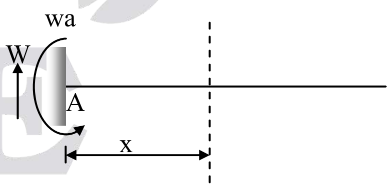
m = Moment due to unit load at which deflection is to be calculated

In Span BC:

$$\delta_1 = \int_0^b \frac{(W.x)(x)du}{EI}$$

$$= \left.\frac{wx^3}{3EI}\right]_o^b = \frac{wb^3}{3EI} = \frac{wb^3\,64}{3E \times \pi d^4} = \frac{64wb^3}{3E\pi d^4}(\downarrow)$$

In Span AB:



$$\delta_2 = \int_0^a \frac{Mmdx}{EI} = \int \frac{(Wa - Wx)(a-x)dx}{EI}$$

$$= \int_0^a \frac{W(a-x)^2 dx}{EI} = \left.\frac{W(a^2x + \frac{x^3}{3} - ax^2)}{EI}\right]_0^a$$

$$=\frac{W\left(a^3+\frac{a^3}{3}-\frac{a^2}{2}\right)}{EI}=\left(\frac{6+2-3}{6}\right)\frac{Wa^3}{EI}$$

$$=\frac{5}{6}\frac{wa^3}{EI}(\uparrow)$$

$$=\frac{5}{6}\frac{wa^3}{EI}\frac{6}{\pi d^4}(\uparrow)$$

$$\delta_2=\frac{160wa^3}{3E\pi d^4}(\uparrow)$$

δ_3 = Deflection due to for torsion in span AB

$$=\frac{TL}{GJ}\times b$$

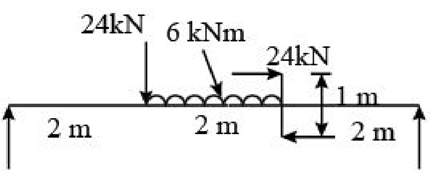
$$\delta_3=\frac{(Wb)(a)}{G\frac{\pi}{32}d^4}b=\frac{32Wb^2a}{Gnd^4}$$

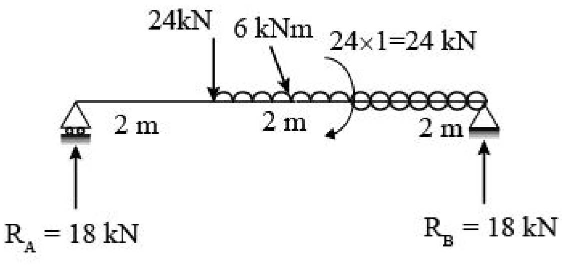
Net deflection = $\delta_1+\delta_2+\delta_3$

$$\delta=\frac{W}{3\pi Ed^4}=[64b^3+160a^3+96b^2a]$$

03.

Sol:





$\sum f_y=0$

$R_A+R_B=24+12=36$

$\sum M_A=0$

$$-R_B\times 6+24+6\times 2\times\left(\frac{2}{2}+2\right)+24\times 2=0$$

$\Rightarrow R_B=18$ kN

$\Rightarrow R_A=36-18=18$ kN

$$M_x=R_A(x)-24(x-2)-6(x-2)\left(\frac{x-2}{2}\right)$$
$$+6(x-4)--------$$

$$EI.\frac{d^2y}{dx^2}=18(x)-24(x-2)-3(x-2)^2+3(x-4)^2$$
$$+24(x-4)-----$$

$$EI.\frac{dy}{dx}=18\left(\frac{x^2}{2}\right)-\frac{24}{2}(x-2)^2 0\frac{3}{3}(x-2)^3+\frac{3}{3}(x-4)^3$$
$$+24(x-4)'+C_1$$

@A x = 0, y = 0

$C_2=0$

@B x = 6m, y = 0

$$0=\frac{18}{2}\left(\frac{6^3}{3}\right)-4(6-2)^3-\frac{(6-2)^4}{4}+\frac{(6-4)^4}{4}+12(6-4)^2$$
$$+C_1\times 6+C_2$$

$\Rightarrow C_1=-63.3$

Deflection @ mid point (x = 3m)

$$EI(y)=\frac{18}{2}\left(\frac{3^3}{3}\right)-\frac{24}{6}(3-2)^3-\frac{(3-2)^4}{4}+\frac{(3-4)^4}{4}+\frac{24}{2}(3-4)^2$$
$$+(-63.3)\times 3+0$$

$EI(g)=-113.24$

$$y=-\frac{113.24}{EI}$$

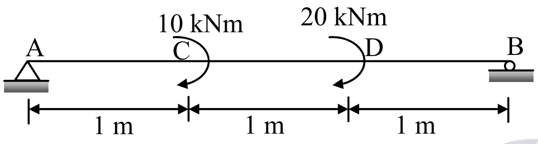
$EI = 20\ \text{MN-m}^2$

$$EI = \left(20\times10^3\right)\text{kN}-\text{m}^2$$

$$y = -\frac{113.24}{20\times10^3} = 5.66\times10^{-3}\ \text{m} = 5.66\ \text{mm}$$

04.

Sol:



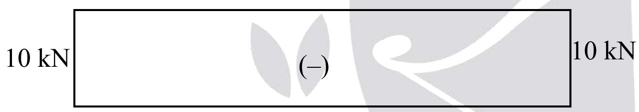
$\Sigma M_A = 0$

$10 + 20 = R_B \times 3$

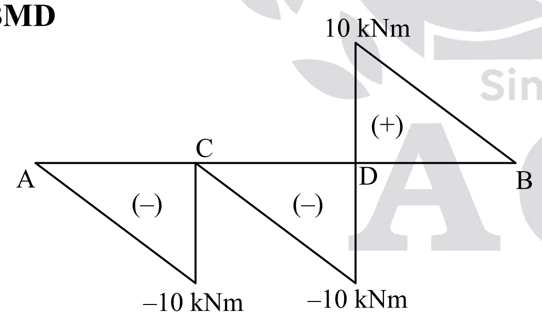
$R_B = 10\ \text{kN}\ (\uparrow)$

$R_A = 10\ \text{kN}\ (\downarrow)$

SFD



BMD

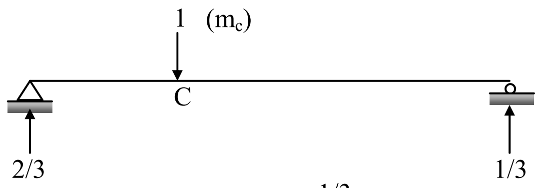


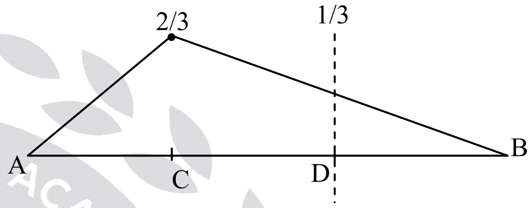
Deflection at Point C = $\int \frac{Mmdx}{EI}$

M = Actual bending moments in the beam

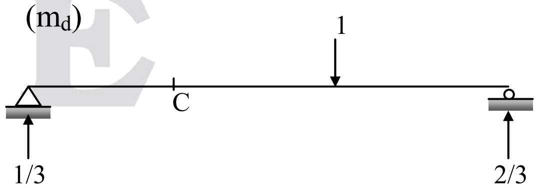
m = moment due to unit load at which deflection is to be calculated

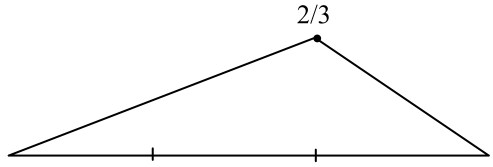
$$= \int_{AC} \frac{Mmdx}{EI} + \int_{CD} \frac{Mmdx}{EI} + \int_{DB} \frac{Mmdx}{EI}$$





Span	M	mC	dn	md
AC	–10x	2/3x	(0–1)	$\frac{1}{3}x$
CD	–10(x–1)	$\frac{2}{3}(2-x)$	(1 – 2)	$\frac{1}{3}x$
DB	10(3–x)	$\frac{1}{3}(3-x)$	(2–3)	$\frac{2}{3}(3-x)$





Deflection at C, $\sum\int \frac{Mm_c\,dx}{EI}$

$$= \int_0^1 \frac{(-10x)\left(\frac{2}{3}x\right)dx}{EI} + \int_1^2 \frac{-10(x-1)\frac{2}{3}(2-x)dx}{EI}$$

$$+ \int_2^3 \frac{10(3-x)\frac{1}{3}(3-x)dx}{EI}$$

$$= \frac{-2.22}{EI} - \frac{1.11}{EI} + \frac{1.11}{EI} = \frac{-2.22}{EI}$$

Deflection at D:

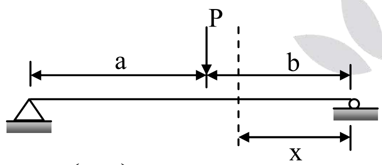
$$= \int_0^1 \frac{(-10x)\left(\frac{1}{3}x\right)dx}{EI} + \int_1^2 \frac{-10(x-1)\frac{1}{3}x\,dx}{EI} + \int_2^3 \frac{10(3-x)\frac{2}{3}(3-x)dx}{EI}$$

$$= -\frac{1.11}{EI} - \frac{2.77}{EI} + \frac{2.22}{EI} = -\frac{1.657}{EI}$$

Ratio $\frac{\delta_C}{\delta_D} = \frac{-2.22}{EI} \times \frac{-EI}{1.657} = 1.34$

05.

Sol:



$$M_x = \left(\frac{Pa}{L}\right)$$

$$EI\frac{d^2y}{dx^2} = \left(\frac{Pax}{L}\right) \text{ for } 0 \le x \le b \text{ ………..(1)}$$

$$EI\frac{d^2y}{dx^2} = \left(\frac{-Pa}{L}\right)(L-x) \text{ for } b \le x \le L \text{…(2)}$$

Integrate (1)

$$EI\frac{dy}{dx} = \left(\frac{Pax^2}{2L}\right) + C_1 - (\ell)$$

$$EIy = \frac{Pax^3}{6L} + C_1x + C_2 - (m)$$

For 2 integrate

$$EI\frac{dy}{dx} = Pax - \frac{Pax^2}{2L} + D_1 - (n)$$

$$EIy = \frac{Pax^2}{L} - \frac{Pax^3}{6L} + D_1n + D_2 - (0)$$

At x = u y = 0

At x = L y = 0

$\left(\frac{dy}{dx}\right)_{x=a}$ $\ell = n$

$(y)_{x=0}$ $m = 0$

so solving

$$C_1 = \frac{Pb}{6L}(L^2 - b^2)\ C_2 = 0\ \ D_1 = \frac{Pa}{6L}(2L^2 + a^2)$$

$$D_2 = \frac{Pa^2}{6EI}$$

$$EIy = -\left(\frac{Pbx}{6L}\right)(L^2 - b^2 - x^2) - (P)$$

$$EIy = \left(\frac{Pb}{6L}\right)\left[\left(\frac{L}{b}\right)(x-a)^3 + (L^2 - b^2)x - x^3\right]$$

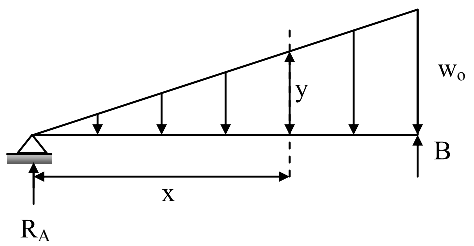
For a > b

Maximum deflection will occur in left position of beam so which equation (P) applies

$$x = \sqrt{\frac{a(a+2b)}{3}} = \sqrt{\frac{L^2 - b^2}{3}}$$

06.

Sol:



$$M = R_A x - \frac{1}{6}x^2 y \qquad \left[y = \frac{w_o x}{L}\right]$$

$$M = R_A x - \frac{w_o x^3}{6L}$$

$$EI\frac{dy}{dx} = R_A x - \frac{w_o x^3}{6L}$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - \frac{w_o x^4}{24L} + C_1$$

$$EIt = \frac{R_a x^3}{2} - \frac{w_o x^5}{120L} + C_1 x + C_2$$

at x = 0, y = 0, so $C_2 = 0$

At x = L, y = 0

$$\text{So, } C_1 = \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

Deflection = x

$$EIy = \frac{R_A x^3}{6} - \frac{w_o L^5}{120L} + \left(\frac{w_o L^3}{24} - \frac{R_A L^3}{2}\right)L$$

At x = L y = 0

$$R_A = \frac{w_o L}{10}$$

Slope at propped end

$$EI\left(\frac{dy}{dx}\right)_{x=0} = \frac{R_A x^2}{2} - \frac{w_o x^4}{24L} + \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

$$= \frac{R_A L^2}{2} - \frac{w_o L^3}{24} + \frac{w_o L^3}{24} - 0 - \frac{R_A L^2}{2}$$

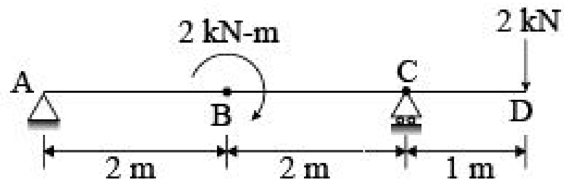
$$= \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

$$EI\left(\frac{dy}{dx}\right)_{x=0} = \frac{w_o L^3}{24} - \frac{w_o L^3}{20} = \left(\frac{-w_o L^3}{120}\right)$$

$$\left(\frac{dy}{dx}\right)_{x=0} = -\frac{w_o L^3}{120EI}$$

07.

Sol:



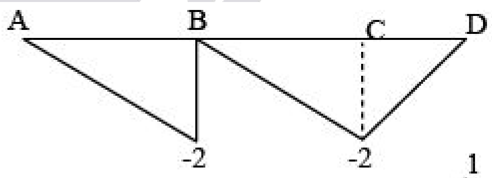
$\Sigma M_A = 0$

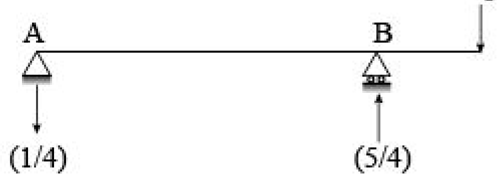
$2 + (2 \times 5) = R_c \times 4$

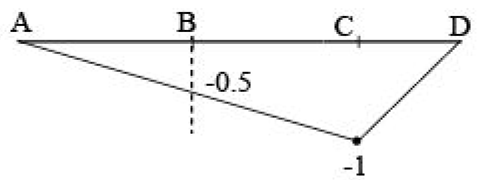
$R_c = 3$ kN (↑)

$R_A = 1$ kN (↓)

BMD:







Span	M	m	dx
AB	–x	–x/4	(0-2)
BC	–(x–2)	–1/4 (x)	(2-4)
CD	–2x	–x	(0-1)

$$\text{Deflection } = \Sigma\int\frac{\text{Mmdx}}{\text{EI}}$$

$$\int_0^2\frac{(-x)\left(\frac{-x}{4}\right)dx}{EI}+\int_0^4\frac{-(x-2)\left(\frac{-1}{4}x\right)dx}{EI}+\int_0^1\frac{)2x)(-x)dx}{EI}$$

$$\frac{0.67}{EI}+\frac{1.67}{EI}+\frac{0.67}{EI}=\delta$$

$$\frac{3}{EI}=\delta$$

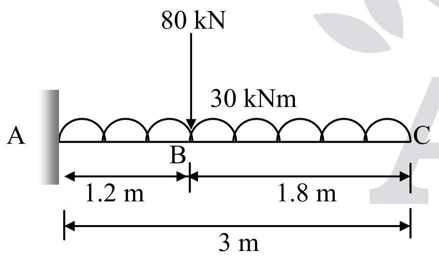
$$EI=\frac{3}{\delta}$$

$\delta = 0.001$ m

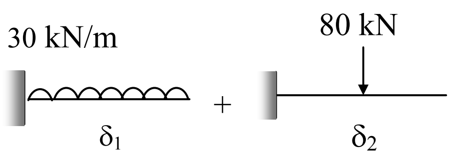
EI = 3000 kNm^2

08.

Sol:



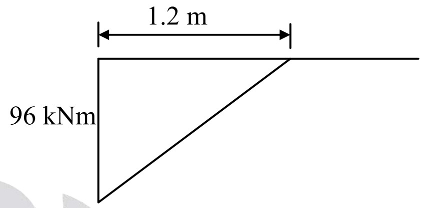
Total deflection (δ) =

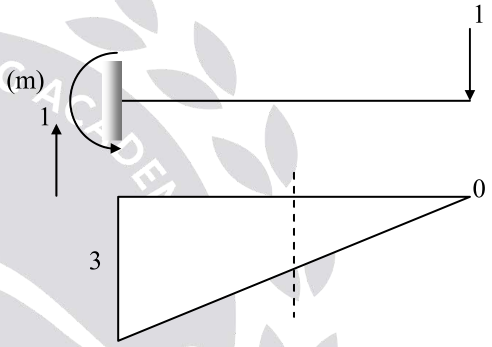


$$\delta_1=\frac{w\ell^4}{8EI}=\frac{30\times3^4}{8EI}=\frac{303.75}{EI}$$

$$\delta_2=\frac{\text{Mmdx}}{EI}$$

BMD





Span	M	m	dn
AB	(96–80x)	(3–x)	(0-1.2)
BC	0		

$$\delta_2=\int\frac{\text{Mmdx}}{EI}=\int_0^{1.2}\frac{(96-90x)(3-x)dx}{EI}=\frac{149.76}{EI}$$

$\delta = \delta_1 + \delta_2$

$$=\frac{303.75}{EI}+\frac{149.26}{EI}$$

$$=\frac{453.51}{EI}(\downarrow)$$

Chapter 9 Thin Pressure Vessels

01. Ans: (b)

Sol: $\tau_{max} = \sigma_l = \dfrac{\sigma_h - 0}{2} = \dfrac{PD}{4t}$

$\therefore \tau_{max} = \dfrac{1.6 \times 900}{4 \times 12} = 30 \text{ MPa}$

02. Ans: 2.5 MPa & 2.5 MPa

Sol: Given data:

R = 0.5 m, D = 1m, t = 1mm,

H = 1 m, $\gamma = 10 \text{ kN/m}^3$, h = 0.5 m

At mid-depth of cylindrical wall (h = 0.5m):

Circumferential (hoop) stress,

$$\sigma_c = \frac{P_{at\ h=0.5m} \times D}{4t} = \frac{\gamma h \times D}{4t}$$

$$= \frac{10 \times 10^3 \times (2 \times 0.5)}{4 \times 1 \times 10^{-3}}$$

$= 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$

Longitudinal stress at mid-height,

$$\sigma_\ell = \frac{\text{Net weight of the water}}{\text{Cross-section area}}$$

$$= \frac{\gamma \times \text{Volume}}{\pi D \times t}$$

$$= \frac{\gamma \times \frac{\pi}{4} D^2 L}{\pi D \times t} = \frac{\gamma \times DL}{4t}$$

$$= \frac{10 \times 10^3 \times 1 \times 1}{4 \times 10^{-3}}$$

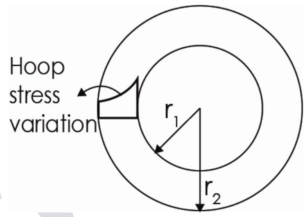
$= 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$

03. Ans: (c)

Sol: According to Lame's theorem

Hoop stress = $\sigma_x = \dfrac{b}{x^2} + a$

Stress variation is hyperbolic with maximum stress on the inner surface.

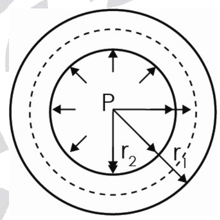


04. Ans: (b)

Sol: According to Lame's equation for thick cylinders hoop stress $\sigma_x = \dfrac{b}{x^2} + a$

$$b = \frac{P r_1^2 r_2^2}{(r_1^2 - r_2^2)}$$

$$a = \frac{P r_2^2}{(r_1^2 - r_2^2)}$$



$\therefore$ For internal pressure a, b are positive

For external pressure

$\therefore$ 'P' is negative; a, b are negative.

05. Ans: (a)

Sol: **Thin Cylinders:** If the thickness of the wall of the cylinders is less than of its diameters. In the design of thin cylinders, it is assumed that circumferential stress or hoop stress is uniformly distributed through the thickness of the wall.

Hoop stress, $\sigma_b = \frac{Pd}{2t}$

Longitudinal stress = $\frac{Pd}{4t}$

$= \frac{1}{2}\sigma_h$

06. Ans: (d)

Sol: Thin cylinder are designed based on the assumption the circumferential stress distribution is uniform over the thickness of the wall as the variation is negligible.

But in case of thick cylinders, circumferential stress is not uniform but varies from maximum at inner side to minimum at outer side.

Conventional Practice Solutions

01.

Sol: Thin Sphere

$$\sigma_1 = \sigma_n = \frac{PD}{4t}$$

$$\sigma_2 = \dot{\sigma}_h = \frac{PD}{4t}$$

$$\sigma_3 = \sigma_R = 0$$

$$\varepsilon_V = \frac{\delta v}{V} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1-2\mu)$$

$$\frac{\delta v}{\frac{4}{3}\pi R^3} = \frac{\frac{PD}{4t} + \frac{PD}{4t} + 0}{E}(1-2\mu)$$

$$\Rightarrow \delta V = \frac{3}{2}\pi \times R^3 \times \frac{PD}{4t \times E}(1-2\mu)$$

$$\Rightarrow \delta v = \frac{3}{8} \times \pi \times R^3 \frac{PD}{Et}(1-2\mu)$$

02.

Sol: D = 400 mm, f = 2mm, P = 1.5 N/mm^2

Thin sphere

$$\frac{\delta v}{V} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1-2\mu)$$

$$\sigma_1 = \sigma_2 = \frac{PD}{4t} = \sigma_h$$

σ_h = 75 MPa

$$\frac{\delta v}{\frac{\pi}{6} \times 400^3} = \frac{2 \times 75}{1 \times 10^5}(1 - 2 \times 0.25)$$

$\Rightarrow \delta v$ = 25132.74 74 mm^3

Decrease in volume of water (compression of water)

$$K = \frac{\sigma}{\varepsilon_v} = \frac{P}{\left(\frac{\delta v_w}{V}\right)}$$

$$2.5 \times 10^3 = \frac{1.5}{\frac{\delta v_w}{\frac{\pi}{6} \times 400^3}}$$

δv_W = 20106.192 mm^3

Additional water to be added = $\delta v + \delta v_3$

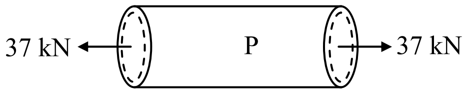
$= 25132.74 + 20106.192$

$= 45238.932 \text{ mm}^3$

$= 45.238 \text{ cm}^3$ or cc

03.

Sol:



Due to internal pressure (P)

$$\frac{\varepsilon v}{V} = \varepsilon_{v_1} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1 - 2\mu) \text{ -------(1)}$$

Due to axial force alone (tensile)

$$\Rightarrow \varepsilon_v = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1 - 2\mu)$$

$$\sigma = \frac{P}{A} = \frac{37 \times 10^3}{\frac{\pi}{4}(150)^2} = 2.093$$

$\sigma_2 = 0$

$\sigma_3 = 0$

$$\Rightarrow \varepsilon_{v_2} = \frac{2.093 + 0 + 0}{140 \times 10^3}(1 - 2\mu) \text{ ---------(2)}$$

$\delta v_1 - \delta v_2$ = Causing pressure drop on water

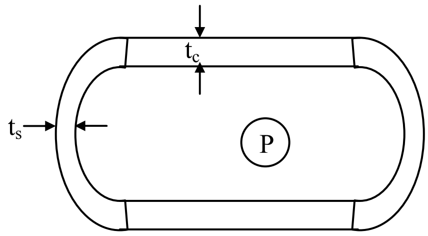
$$K = \frac{P}{\varepsilon_v} = \frac{P}{\left(\frac{\delta v_1 - \delta v_2}{V}\right)}$$

$$\Rightarrow \delta v_1 - \delta v_2 = \frac{P.V}{K_{water}} = \frac{0.1 \times \frac{\pi}{4} \times 150^2}{2200}$$

$$= \frac{0.1V}{2200}$$

04.

Sol:



At the junction equal deformation is required

$$(\varepsilon_h)_c = (\varepsilon_h)_{sphere}$$

$$\left(\frac{\sigma_h}{E} - \mu\frac{\sigma_L}{E} - \mu\frac{\sigma_R}{E}\right) = \left(\frac{\sigma_h}{E} - \mu\frac{\sigma_h}{E} - \mu\frac{\sigma_R}{E}\right)$$

$$\frac{Pd}{2t_c} - \mu\frac{Pd}{4t_c} = \frac{Pd}{2t_s} - \mu\frac{Pd}{2t_s}$$

$$\Rightarrow \frac{Pd}{4}\left[\frac{2}{t_c} - \frac{\mu}{t_c}\right] = \frac{Pd}{2}\left[\frac{1}{t_s} - \frac{\mu}{t_s}\right]$$

$$\frac{Pd}{4} \times \frac{1}{t_c} \times (2 - \mu) = \frac{Pd}{2} \times \frac{1}{t_s} \times (1 - \mu)$$

$$\frac{t_c}{t_s} = \frac{2 - \mu}{2(1 - \mu)}$$

Chapter 10

Columns

01. Ans: (c)

Sol: By using Euler's formula, $P_e = \frac{\pi^2 \times EI}{l_e^2}$

For a given system, $l_e = \frac{l}{2}$

$\therefore \quad P_e = \frac{4\pi^2 \times EI}{l^2}$

02. Ans: (b)

Sol: We know that, $P_{cr} = \frac{\pi^2 EI}{\ell_e^2}$

$\therefore P_{cr} \propto \frac{1}{\ell_e^2}$

$\therefore \frac{P_1}{P_2} = \frac{l_{2e}^2}{l_{1e}^2}$

$\therefore \frac{P_1}{P_2} = \frac{l^2}{(2l)^2} \Rightarrow P_1 : P_2 = 1 : 4$

03. Ans: 4

Sol: Euler's crippling load,

$$P = \frac{\pi^2}{l^2} EI$$

$\therefore \quad P \propto I$

$$\Rightarrow \frac{P}{P_o} = \frac{I_{bonded}}{I_{loose}} = \frac{\left[\frac{b(2t)^3}{12}\right]}{2\left[\frac{bt^3}{12}\right]} = 4$$

04. Ans: (c)

Sol: Euler's theory is applicable for axially loaded columns.

Force in member AB, $P_{AB} = \frac{F}{\cos 45^o} = \sqrt{2}F$

$$P_{AB} = \frac{\pi^2 EI}{L_e^2}$$

$$\therefore \sqrt{2}\ F = \frac{\pi^2 EI}{L_e^2}$$

$$\Rightarrow F = \frac{\pi^2 EI}{\sqrt{2} L^2}$$

05. Ans: (a)

Sol: Given data:

$L_e = L = 3$ m ,

$\alpha = 12 \times 10^{-6}$ /°C,

d = 50 mm = 0.05 m

Buckling load, $P_e = \frac{\pi^2 EI}{L_C^2}$

$$\therefore \quad \frac{P_e L}{AE} = L\alpha \Delta T$$

$$\therefore \quad \frac{\pi^2 EI \times L}{L^2 \times AE} = L\alpha\Delta T$$

$$\therefore \quad \frac{\pi^2 \times E \times \frac{\pi}{64} \times d^4 \times L}{L^2 \times \frac{\pi}{4} d^2 \times E} = L\alpha\Delta T$$

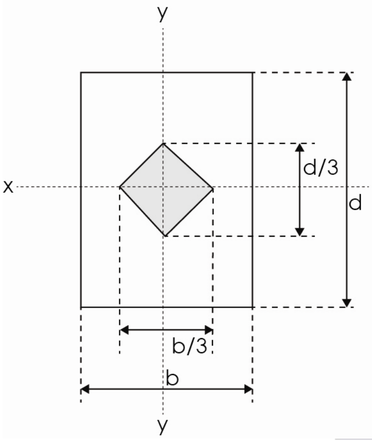
$$\therefore \quad \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}}$$

$$\Rightarrow \Delta T = 14.3°C$$

06. Ans: (b)

Sol: When the load is eccentric, it cause both direct and bending stresses in the member. For the tensile stresses, to not develop in the section; the load must lie within certain cross section of the member. This is called core (or) Kern of the section.

For rectangle:

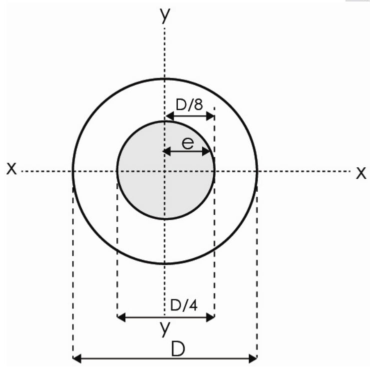


(shape : rhombous)

$$\frac{x}{\left(\frac{b}{d}\right)} + \frac{y}{\left(\frac{d}{6}\right)} \le 1$$

Note: For hollow rectangle also shape of kern is rhombus.

For circular section:

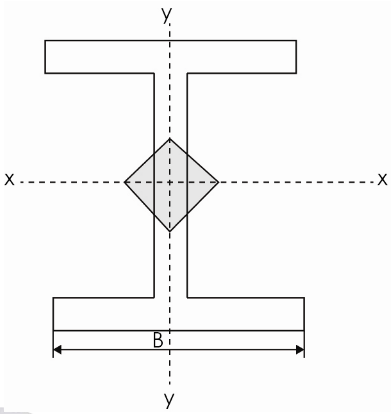


shape: Circle

$$e \le \frac{D}{8}$$

Note: For hollow circular section, also shape of kern is circle.

For I-section:



shape : rhombus

07. Ans: (c)

Sol: Buckling load of columns (P) = $\dfrac{\pi^2 EI}{\ell_o^2}$

ℓ_o: Effective length, depends on end conditions of the column

$P \propto \dfrac{1}{\ell_o^2}$ and $\ell_o = K\ell$

$\therefore P \propto \dfrac{1}{\ell^2}$

$\therefore$ With increasing length, of the column, buckling load decreases.

08. Ans: (a)

Sol: Modulus of elasticity of high strength alloy steel and ordinary structural steel is almost same.

So, buckling failure strength of high strength alloy steel is approximately same as that of structural steel.

Euler's buckling load = $\dfrac{\pi^2 EI}{\ell_e^2}$

09. Ans: (a)

Sol: Euler's buckling load = $P = \dfrac{\pi^2 EI}{\ell_0^2}$

Stress = $\dfrac{P}{A} = \dfrac{\pi^2 EI}{\ell_e^2 \times A}$

r = radius of gyration = $\sqrt{\dfrac{I}{A}}$

$\Rightarrow \sigma = \dfrac{\pi E r^2}{\ell_e^2}$

λ = slenderness ratio = $\dfrac{\ell_e}{r}$

$\Rightarrow \sigma = \dfrac{\pi E}{\lambda^2}$

$\therefore \sigma \propto \dfrac{1}{\lambda^2}$

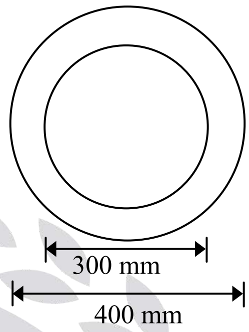
When slenderness ratio is small, stress causing failure will be high according to Euler's formula assuming ideal end conditions.

But this stress must not be greater than crushing stress. Also, in practice the end conditions will not be ideal leading to eccentricity in the loading. This results in bending moment which causes failure before the Euler's load. Hence for slenderness ratio < 120, Euler's theory is not used as it gives high value of failure stress since the crushing effect is not considered.

Conventional Practice Solutions

01.

Sol:



D = 400 mm

d = 300 mm

L = 5m

E = 0.75×10^5 N/mm^2

F = 5

$\alpha = \dfrac{1}{1600}$

f = 587 N/mm^2

Eulers Load:

$$P_e = \frac{\pi^2}{\ell^2} E.(I_{min})$$

$$= \frac{\pi^2}{(5000)^2} \times 0.75 \times 10^5 \left(\frac{\pi}{64}\left(400^4 - 300^4\right)\right)$$

$= 25.44\times10^6$ N

Safe load = $\dfrac{P_e}{F} = \dfrac{25.44\times10^6}{5}$

$= 5.08\times10^6$ N = 5.08 MN

Rankines Load:

$$r = \sqrt{\frac{I_{min}}{A}}$$

$$= \sqrt{\frac{\frac{\pi}{64}\left(400^4 - 300^4\right)}{\frac{\pi}{4}\left(400^2 - 300^2\right)}} = 125$$

$$\lambda = \frac{\ell}{r_{min}} = \frac{5000}{125} = 40$$

$$P_R = \frac{(f)(A)}{1+(\alpha)(\lambda^2)} = \frac{587 \times \frac{\pi}{4}\left(400^2 - 300^2\right)}{1+\left(\frac{1}{1600}\right)\left(40^2\right)}$$

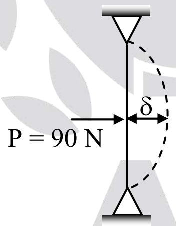
$$= 16.13\times10^6 \text{ N}$$

$$\text{safe load} = \frac{P_R}{F.O.S} = \frac{16.13\times10^6}{5} = 3.2\times10^6$$

N = 3.2 MN

02.

Sol: $\delta = 5\,\text{mm} = \dfrac{P\ell^3}{48EI}$



$$\Rightarrow 5 = \frac{(90)(1200)^3}{48EI}$$

$$\Rightarrow EI = 6.4\times10^8$$

Euler's load:

$$P_e = \frac{\pi^2 E I_{min}}{L^2}$$

$$= \frac{\pi^2 \times 6.4\times10^8}{1200^2}$$

$$= 4.4 \times 10^3 \text{ N}$$

$$= 4.4 \text{ kN}$$

Chapter 11 Strain Energy

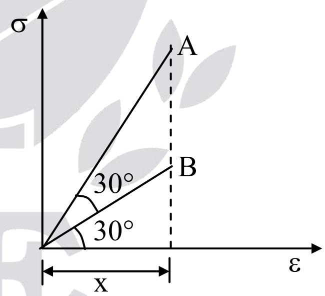
01. Ans: (*)

Sol:

- Slope of the stress-strain curve in the elastic region is called modulus of elasticity.
  For the given curves,
  (Modulus of elasticity)$_A$ > (Modulus of elasticity)$_B$
  $\therefore$ $\mathbf{E_A > E_B}$
- The material for which plastic region is more is stress-strain curve is possesed high ductility. Thus, $\mathbf{D_B > D_A}$.

02. Ans: (b)

Sol:

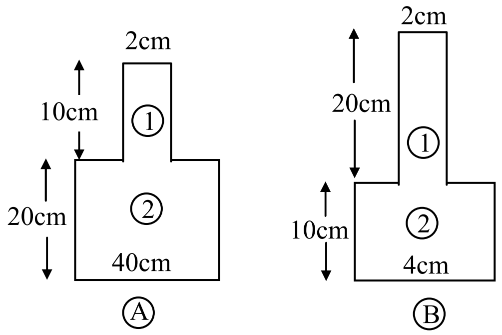


$$\frac{(SE)_A}{(SE)_B} = \frac{\text{Area under curve A}}{\text{Area under curve B}}$$

$$= \frac{\frac{1}{2}\times x \times x \tan 60^o}{\frac{1}{2}\times x \times x \tan 30^o} = \frac{3}{1}$$

03. Ans: (a)

Sol:



$$\frac{U_B}{U_A}=\frac{(V_1+V_2)_B}{(V_1+V_2)_A}$$

$$\therefore\ \frac{U_B}{U_A}=\frac{\left[\frac{\sigma_1^2}{2E}\times V_1+\frac{\sigma_2^2}{2E}\times V_2\right]_B}{\left[\frac{\sigma_1^2}{2E}\times V_1+\frac{\sigma_2^2}{2E}\times V_2\right]_A}$$

$$=\frac{\left[\frac{P^2}{A_1^2}\times A_1\times L_1+\frac{P^2\times A_2\times L_2}{A_2^2}\right]}{\left[\frac{P^2\times A_1\times L_1}{A_1^2}+\frac{P^2\times A_2\times L_2}{A_2^2}\right]_A}$$

$$\Rightarrow \frac{U_B}{U_A}=\frac{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_B}{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_A}=\frac{7.165}{4.77}=\frac{3}{2}$$

04. Ans: (c)

Sol: A_1 = Modulus of resilience

$A_1 + A_2$ = Modulus of toughness

$A_1 = \frac{1}{2}\times 0.004\times 70\times 10^6 = 14\times 10^4$

$A_2 = \frac{1}{2}\times\left(0.008\times 50\times 10^6\right)+\left(0.008\times 70\times 10^6\right)$

$= 76\times 10^4$

$A_1 + A_2 = (14 + 76)\times 10^4 = 90\times 10^4$

05. Ans: (d)

Sol: Strain energy, $U = \frac{P^2}{2A^2E}.V$

$\therefore\ U \propto P^2$

Due to the application of P_1 and P_2 one after the other

$(U_1 + U_2) \propto P_1^2 + P_2^2$ …….. (1)

Due to the application of P_1 and P_2 together at the same time.

$U \propto (P_1 + P_2)^2$ ……..........(2)

It is obvious that,

$(P_1^2 + P_2^2) < (P_1 + P_2)^2$

$\Rightarrow (U_1 + U_2) < U$

06. Ans: 1.5

Sol: Given data:

$L = 100$ mm

$G = 80\times 10^3\ \text{N/mm}^2$

$J_1 = \frac{\pi}{32}(50)^4; J_2 = \frac{\pi}{32}(26)^4$

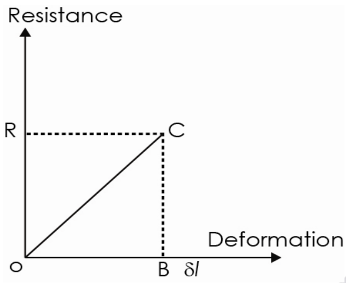
$U = U_1 + U_2 = \frac{T^2L}{2GJ_1}+\frac{T^2L}{2GJ_2}$

$\Rightarrow U = 1.5$ N-mm

07. Ans: (c)

Sol: **Strain Energy:** When a member is loaded, it deforms, behaving like a spring; resistance develops and the work is done upon it. If the elastic limit is not exceeded this work stored in the form of energy is called as strain energy.

This amount of work absorbed by the resistance (i.e. strain energy) during deformation is the area under resistance deformation curve.



$$\Rightarrow U = \frac{1}{2} \times OB \times BC = \frac{1}{2} \times R \times \delta\ell$$

$$\Rightarrow \frac{1}{2} \times \sigma \times A \times \delta\ell = \frac{1}{2} \times \sigma \times \varepsilon \times A \times \ell$$

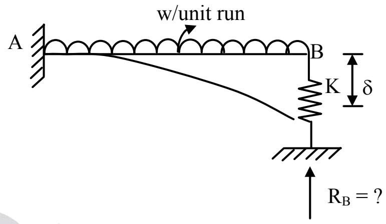
$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

∴ Strain energy is a function of stress and strain.

Chapter 12 Propped and Fixed Beams

01. Ans: (d)

Sol:



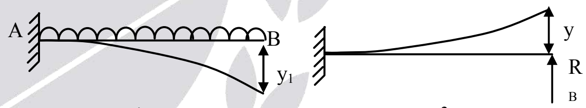
$$K = \text{Stiffness} = \frac{\text{Load}}{\text{deflection}}$$

$$\therefore K = \frac{R_B}{\delta}$$

∴ Compatibility condition

Deflection @ B = δ

$$\because K = \frac{R_B}{\delta} \Rightarrow \delta = \frac{R_B}{K}$$



$$y_1 = \frac{w\ell^4}{8EI} \qquad y_2 = \frac{R_B\ell^3}{3EI}$$

↓(+)

↑(–)

$$y_1 - y_2 = \delta$$

$$\therefore \frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \delta$$

$$\frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \frac{R_B}{K}$$

$$\frac{w\ell^4}{8EI} = \frac{R_B}{K} + \frac{R_B\ell^3}{3EI}$$

$$\frac{w\ell^4}{8EI} = R_B\ell^3\left[\frac{1}{K\ell^3} + \frac{1}{3EI}\right]$$

$$\frac{w\ell^4}{8EI} = R_B\left[\frac{3EI + K\ell^3}{3EI \times K\ell^3}\right] \times \ell^3$$

$$\frac{w\ell}{8EI} = \frac{R_B}{3EI}\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

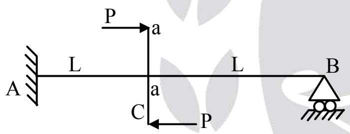
$$\frac{3w\ell}{8} = R_B\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

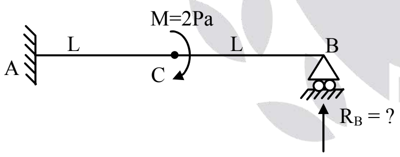
$$\frac{3w\ell}{8} = R_B\left[1 + \frac{3EI}{K\ell^3}\right]$$

$$R_B = \frac{3w\ell/8}{1 + \dfrac{3EI}{K\ell^3}}$$

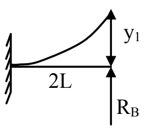
02. Ans: $\frac{9pa}{8L}$

Sol:

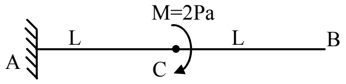




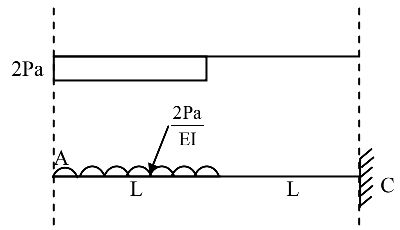
Applying, superposition principle



$$y_1 = \frac{R_B(2L)^3}{3EI} = \frac{8R_BL^3}{3EI}$$



By conjugate beam method



$\therefore y_c$ = deflection @C

= B.M.D. @ C by conjugate beam

$$y_c = \frac{2Pa}{EI} \times L \times \left[L + \frac{L}{2}\right]$$

$$= \frac{2Pa}{EI} \times L \times \frac{3L}{2}$$

$$= \frac{3PaL^2}{EI}$$

Compatibility Condition (y_B) = 0

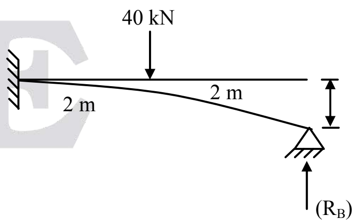
$\therefore \quad y_1 = y_c$

$$\frac{8R_BL^3}{3EI} = \frac{3PaL^2}{EI}$$

$$R_B = \frac{9Pa}{8L}(\uparrow)$$

03. Ans: 12.51 kN

Sol:



E = 200 GPa

I = 2 × 10^{+6} mm^4

As per compatablity

$$\frac{(R_B)(4000)^3}{3EI} = \frac{(40\times10^3)(2000)^3}{3\times EI} + \frac{40\times10^3\times(2000)^2}{2EI}\times2000 + 1\text{mm}$$

$$\frac{R_B(2\ell)^3}{3EI} = \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(b) + 1\text{mm}$$

$$\left[\text{use } a = b = \frac{L}{2} = 2000\text{ mm}\right]$$

where EI = 4×10^{11} N/mm^2

$$\therefore \frac{R_B(4000)^3}{3\times4\times10^{11}} = \frac{40\times10^3\times(2000)^3}{3\times4\times10^{11}} + \frac{40\times10^3\times(2000)^3}{2\times4\times10^{11}} + 1$$

R_B = 12.51 kN