# ESE | GATE | PSUs

# CIVIL ENGINEERING

## STRENGTH OF MATERIALS

**Text Book :** Theory with worked out Examples and Practice Questions

# 1 Strength of Materials

**(Solutions for Text Book Practice Questions)**

## 01. Simple Stresses and Strain

### Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

**01. Ans: (b)**

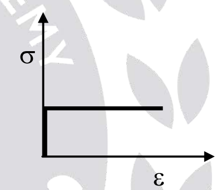
**Sol:**

- **Ductility:** The property of materials to allow large deformations or large extensions without failure (large plastic zone) is termed as ductility.
- **Brittleness:** A brittle material is one which exhibits a relatively small extensions or deformations prior to fracture. Failure without warning (No plastic zone) i.e. no plastic deformation.
- **Tenacity**: High tensile strength.
- **Creep:** Creep is the gradual increase of plastic strain in a material with time at constant load.
- **Plasticity:** The property by which material undergoes permanent deformation even after removal of load.
- **Endurance limit:** The stress level below which a specimen can withstand cyclic stress indefinitely without failure.
- **Fatigue:** Decreased Resistance of material to repeated reversal of stresses.

**02. Ans: (a)**

**Sol:**

- When the material is subjected to stresses, it undergoes to strains. After removal of stress, if the strain is not restored/recovered, then it is called inelastic material.
- For rigid plastic material:



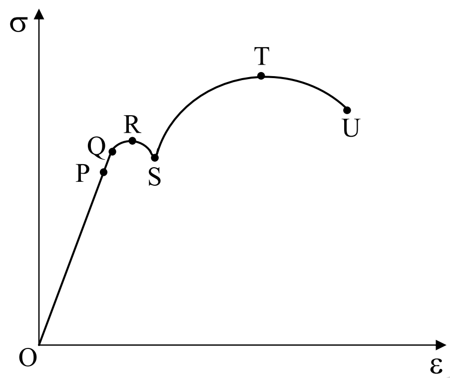

- Any material that can be subjected to large strains before it fractures is called a ductile material. Thus, it has large plastic zone.
- Materials that exhibit little or no yielding before failure are referred as brittle materials. Thus, they have no plastic zone.

**03. Ans: (a)**

**Sol:** *Refer to the solution of Q. No. (01).*

**04. Ans: (b)**

**Sol:** The stress-strain diagram for ductile material is shown below.




P – Proportionality limit
Q – Elastic limit
R – Upper yield point
S – Lower yield point
T – Ultimate tensile strength
U – Failure

From above,

OP $\rightarrow$ Stage I
PS $\rightarrow$ Stage II
ST $\rightarrow$ Stage III
TU $\rightarrow$ Stage IV

**05. Ans: (b)**

**Sol:**

- If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is **isotropic** or in other words we can say the isotropy of a material is its characteristics, which gives us the information that the properties are same in the three orthogonal directions x, y and z.
- A material is **homogeneous** if it has the same composition throughout the body. Hence, the elastic properties are the same at every point in the body in a given direction. However, the properties need not to be the same in all the directions for the material. Thus, both A and B are false.

**06. Ans: (a)**

**Sol:** **Strain hardening** increase in strength after plastic zone by rearrangement of molecules in material.

- **Visco-elastic material** exhibits a mixture of creep as well as elastic after effects at room temperature. Thus their behavior is time dependant

**07. Ans: (a)**

**Sol:** *Refer to the solution of Q. No. (01).*

**08. Ans: (a)**

**Sol:** Modulus of elasticity (Young's modulus) of some common materials are as follow:

| Material | Young's Modulus (E) |
|---|---|
| Steel | 200 GPa |
| Cast iron | 100 GPa |
| Aluminum | 60 to 70 GPa |
| Timber | 10 GPa |
| Rubber | 0.01 to 0.1 GPa |

**09. Ans: (a)**

**Sol:** Addition of carbon will increase strength, thereby ductility will decrease.

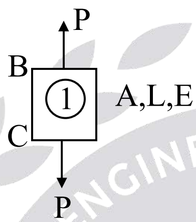
## Elastic Constants and Their Relationships

**01. Ans (c)**

**Sol:** We know that,

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\therefore \quad \mu = \frac{\Delta D/8}{\frac{PL}{AE}/L}$$




$$\therefore \quad \mu = \frac{\Delta D}{8}\frac{AE}{P}$$

$$\therefore \quad 0.25 = \frac{\Delta D}{8}\frac{\frac{\pi}{4}(8)^2 \times 10^6}{50000}$$

$$\Rightarrow \quad \Delta D = 1.98 \times 10^{-3} \cong 0.002 \text{ cm}$$

**02. Ans: (c)**

**Sol:** We know that,

$$\text{Bulk modulus} = \frac{\delta P}{\delta V/V}$$

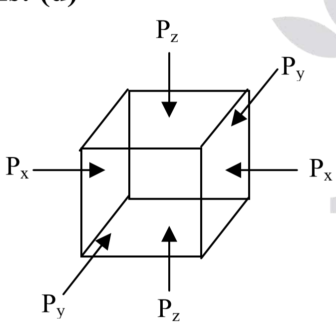
$$\Rightarrow \quad 2.5\times10^5 = \frac{200 \times 20}{\delta V}$$

$$\Rightarrow \quad \delta V = 0.016 \text{ m}^3$$

## Linear and Volumetric Changes of Bodies

**01. Ans: (d)**

**Sol:**




Let $P_y = P_z = P$

$\varepsilon_y = 0$ ,

$\varepsilon_z = 0$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu.\frac{\sigma_x}{E}$$

$$\therefore \quad 0 = \frac{(-P)}{E} - \mu\frac{(-P)}{E} - \mu\frac{(P_x)}{E}$$

$$\Rightarrow \quad P = \frac{\mu.P_x}{(1-\mu)}$$

**02. Ans: (a)**

**Sol:** Given that, $\sigma_c = 4\tau$

Punching force = Shear resistance of plate

$\therefore \ \sigma\,(\text{Cross section area}) = \tau\,(\text{surface Area})$

$$\therefore\ 4\times\tau\times\frac{\pi.D^2}{4}=\tau(\pi.D.t)$$

$$\Rightarrow D = t = 10\text{ mm}$$

**03. Ans: (d)**

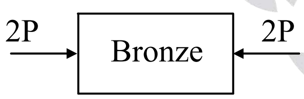
**Sol:**


$$\sigma_s = 140\text{ MPa } = \frac{P_s}{A_s}$$

$$\Rightarrow\ P_S = \frac{140\times500}{3}\ \approx 23{,}300\text{ N}$$


$$\sigma_{Al} = 90\text{ MPa } = \frac{P_{A\ell}}{A_{A\ell}}$$

$$\Rightarrow\ P_{Al} = 90\times400 = 36{,}000\text{ N}$$




$$\sigma_B = 100\text{ MPa } = \frac{P_B}{A_B}$$

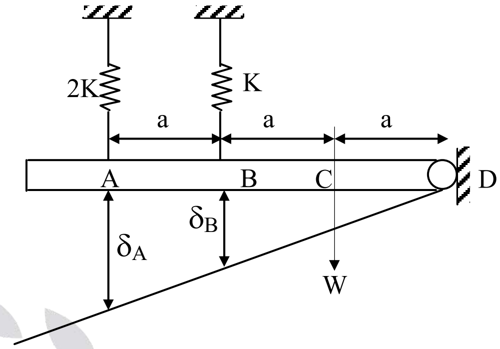
$$\Rightarrow\ P_B = \frac{100\times200}{2}\ = 10{,}000\text{ N}$$

Take minimum value from $P_S$, $A_{Al}$ and $P_B$.

$$\Rightarrow\ P = 10{,}000\text{ N}$$

**04. Ans: (c)**

**Sol:**




From similar triangle

$$\frac{3a}{\delta_A}=\frac{2a}{\delta_B}$$

$$3\delta_B = 2\delta_A\ \ldots\ldots.\ (1)$$

$$\text{Stiffness } K = \frac{W}{\delta}$$

$$\therefore K_A = \frac{W_A}{\delta_A}\Rightarrow \delta_A = \frac{W_A}{2K}$$

Similarly $\quad \delta_B = \dfrac{W_B}{K}$

From equation (1) $\quad 3\times\dfrac{W_B}{K} = 2\times\dfrac{W_A}{2K}$

$$\Rightarrow\ \frac{W_A}{W_B} = 3$$

## Thermal/Temperature Stresses

**01. Ans: (b)**

**Sol:** Free expansion = Expansion prevented

$$[\ell\,\alpha\,t]_s + [\ell\,\alpha\,t]_{Al} = \left[\frac{P\ell}{AE}\right]_s + \left[\frac{P\ell}{AE}\right]_{AL}$$

$$11\times10^{-6}\times20 + 24\times10^{-6}\times20$$

$$= \frac{P}{100\times10^3\times200} + \frac{P}{200\times10^3\times70}$$

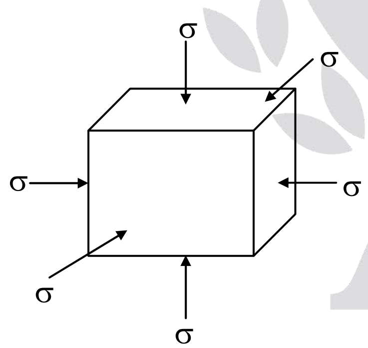
$\Rightarrow$ P = 5.76 kN

$$\sigma_s = \frac{P}{A_s} = \frac{5.76\times10^3}{100} = 57.65\ \text{MPa}$$

$$\sigma_{Al} = \frac{P}{A_{al}} = \frac{5.76\times10^3}{200} = 28.82\ \text{MPa}$$

**02. Ans: (a)**

**Sol:**




Strain in X-direction due to temperature,

$$\varepsilon_t = \alpha(\Delta T)$$

Strain in X-direction due to volumetric stress,

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\therefore\quad \varepsilon_x = \frac{-\sigma}{E}(1-2\mu)$$

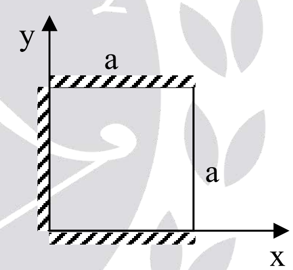
$$\therefore\quad -\sigma = \frac{(\varepsilon_x)(E)}{1-2\mu}$$

$$\therefore\quad -\sigma = \frac{\alpha(\Delta T)E}{(1-2\mu)}$$

$$\Rightarrow\quad \sigma = \frac{-\alpha(\Delta T)E}{1-2\mu}$$

**03. Ans: (b)**

**Sol:**




- Free expansion in x direction is $a\alpha t$.
- Free expansion in y direction is $a\alpha t$.
- Since there is restriction in y direction expansion doesn't take place. So in lateral direction, increase in expansion due to restriction is $\boldsymbol{\mu a \alpha t}$.

Thus, total expansion in x direction is,

$$= a\,\alpha\,t + \mu\,a\,\alpha\,t$$

$$= a\,\alpha\,t\,(1+\mu)$$

**04. Ans: (b)**

**Sol:** Stress: When force is applied on a body, it suffers a deformation. To resist this deformation, from equilibrium point of view, internal forces arise in the body giving rise to concept of stress.

$$\text{Stress} = \frac{\text{Resistance}}{\text{Area}}$$

Since the deformations arise first and are measurable, strain is a fundamental behavior and stress is derived from this

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

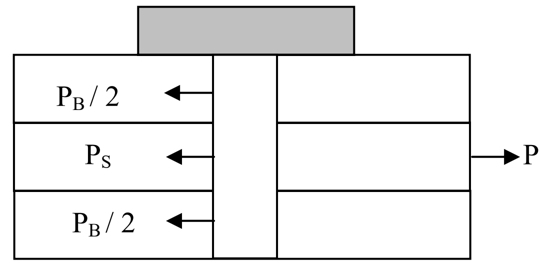
Therefore, strain has no units and SI units of stress is $N/m^2$ (or) Pa.

**05. Ans: (a)**

**Sol:** When a ductile material is subjected to repeating (or) cyclic loads, progressive and localized deformations occur leading to the development of residual strains in the material. When the accumulated strain energy exceeds the toughness, the material fractures and this failure called as fatigue occurs at a load much less than the ultimate load of the structure. The failure load decreases with increase in the number of loadings.

**06. Ans: (b)**

**Sol:** FBD of a single pin:




For equilibrium: $P_B + P_s = P$ --- (1)

Tension in steel bar = $P_s$

Tension in each brass bar = $\frac{P_B}{2}$

Elongation in steel bar = $\frac{(P_s)\times\ell}{AE_s}$;

Elongation in brass bar = $\frac{\left(\frac{P_B}{2}\right)\times\ell}{AE_s}$

But, $\delta_{steel} = \delta_{brass}$

$$\Rightarrow \quad \frac{(P_s)\times\ell}{AE_s} = \frac{\left(\frac{P_B}{2}\right)\ell}{AE_B};$$

Given data, $\frac{E_s}{E_B} = 2$

$$\frac{P_s}{2E_B} = \frac{P_B}{E_B}$$

$$\Rightarrow \quad P_S = P_B$$

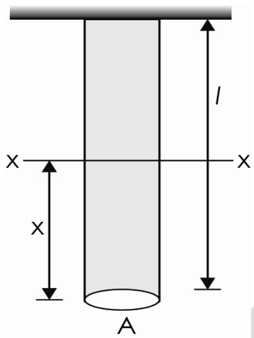
From equation (1) $P = P_B + P_B$

$$\therefore \; P_B = P_s = \frac{P}{2}$$

Shear in each pin = $\frac{P}{2}$ = 0.5 P

**07. Ans: (c)**

**Sol:** Consider a circular bar of cross-section area 'a' length '$l$' unit weight '$\gamma$'




At any section 'x-x', at distance 'x'

Stress due to self weight = $\frac{P_x}{A}$

$$= \frac{\gamma \times A \times x}{A}$$

$$= \gamma x$$

$\therefore$ Max stress ($\sigma$) = $\gamma\ell$

$$\sigma_1 \propto \ell$$

$\therefore$ When all the dimensions are doubled = $2\ell$

$$\frac{\sigma_2}{\sigma_1} = \frac{2\ell}{\ell} \Rightarrow \sigma_2 = 2\sigma_1$$

**08. Ans: (b)**

**Sol:** *Refer to the solution of Q.No. 04* in fundamental mechanical properties and stress-strain diagrams.

**09. Ans: (b)**

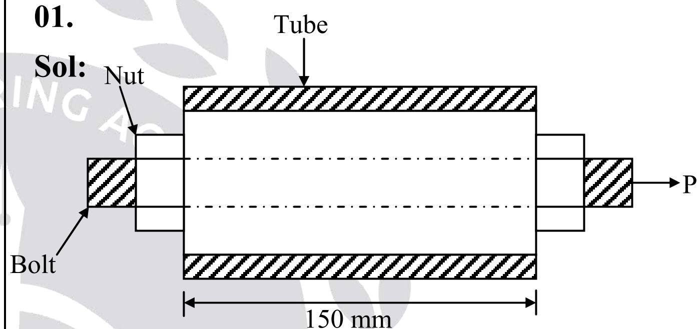
**Sol:** Fatigue is the progressive and localized structural damage, that occurs in a material subjected to repetitive loads. The nominal maximum stress value that cause such damage is much less than the strength of the material.

**Endurance limit:** It is the stress level below which a specimen can withstand cyclic stress indefinitely without exhibiting fatigue failure. Also known as fatigue limit/fatigue strength.

## Conventional Practice Solutions

**01.**

**Sol:**




Pitch = 1mm/toren

Tunned by quater $\delta = \frac{1}{4}$mm

(i) nut movement = comp. of tube + elongation of bolt (compatibility condition)

(ii) $P_{tube} = P_{bolt}$

From (i)

$$\frac{1}{4}\text{mm} = \left|\frac{PL}{AE}\right|_{tube} + \left|\frac{PL}{AE}\right|_{bolt}$$

$$\Rightarrow \frac{1}{4} = \frac{P(150)}{196.03\times10^6} + \frac{P(150)}{62.83\times10^6}$$

$$\Rightarrow P = 79.3 \text{ kN} = P_{tube} = P_{bolt}$$

(i) $\sigma_{bolt} = \frac{P}{A}$

$$= \frac{79.3\times10^3}{\frac{\pi}{4}(20^2)} = 252 \text{ MPa (T)}$$

(ii) $\uparrow t = 10°C$

$P_s = P_c = P$

$$\underbrace{(\ell\alpha\Delta T)_c - \left(\frac{P\ell}{AE}\right)_c}_{\text{Tube (compression)}} = \underbrace{(\ell\alpha\Delta T)_s + \left(\frac{P\ell}{AE}\right)_s}_{\text{Bolt (tension)}}$$

$$\Rightarrow (18\times10^{-5}\times10) - \left(\frac{P}{\frac{\pi}{4}\times(30^2-22^2)\times(6\times10^5)}\right)$$

$$= (12\times10^{-6}\times10) + \left(\frac{P}{\frac{\pi}{4}(20^2)\times2\times10^5}\right)$$

$\Rightarrow P = 80$ kN

Stress in bolt only due to temperature,

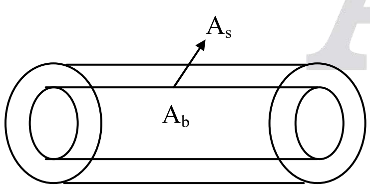
$$\sigma = \frac{80\times10^3}{\frac{\pi}{4}\times20^2} = 254 \text{ MPa (T)}$$

Resultant stress in bolt (tightening of nut + temperature)

$= 252 + 254$

$= 506$ MPa (T)

**02.**
**Sol:**




Let the load applied = P

Load carries by steel and brass be $P_s$ & $P_b$ respectively

$P_s + P_b = P$ ............(1)

Compatibility condition

Strain in steel = Strain in brass

$$\frac{P_s\ell}{A_sE_s} = \frac{P_b\ell}{A_bE_b}$$

$$P_s = \frac{P_bA_sE_s}{A_bE_b} \text{ .......(2)}$$

Put (2) in (1)

$$P_b\left[1+\frac{A_sEs}{A_bE_b}\right] = P$$

$$P_b\left[\frac{A_bE_b + A_sE_s}{A_bE_b}\right] = P$$

$$P_b = \frac{PE_bA_b}{P_bE_b + A_sE_s}$$

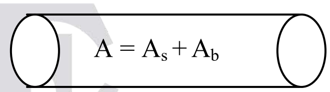
Elongation

$$= \frac{P_b\ell}{A_bE_b} = \frac{PE_bA_b\ell}{[A_bE_b + A_sE_s]}\times\frac{1}{A_bE_b}$$

$$\delta\ell = \left[\frac{P\ell}{A_bE_b + E_sA_s}\right] \text{..........(3)}$$

Let Young's modulus = E

$$\delta\ell = \frac{P\ell}{AE} = \frac{P\ell}{(A_s + A_b)E} \text{ .............(4)}$$



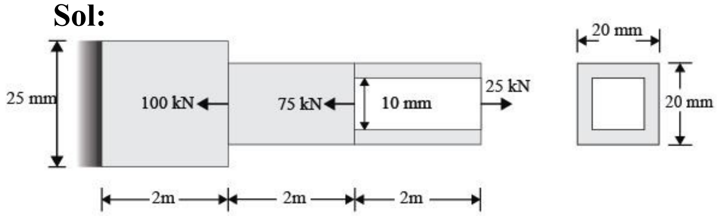

Compare (3) and (4)

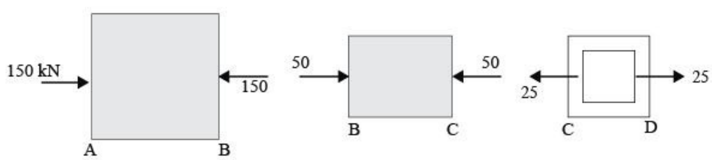
$$\frac{P\ell}{A_bE_b + A_sE_b} = \frac{P\ell}{(A_s + A_b)E}$$

$$E = \frac{A_sE_s + A_bE_b}{A_s + A_b}$$

**03.**

**Sol:**




$$Dl = Dl_{AB} + Dl_{BC} + Dl_{CD}$$

$$= -\frac{150 \times 10^3 \times 2000}{25^2 \times 2 \times 10^5} - \frac{50 \times 10^3 \times 2000}{20^2 \times 2 \times 10^5} + \frac{25 \times 10^3 \times 2000}{(20^2 - 10^2) \times 2 \times 10^5}$$

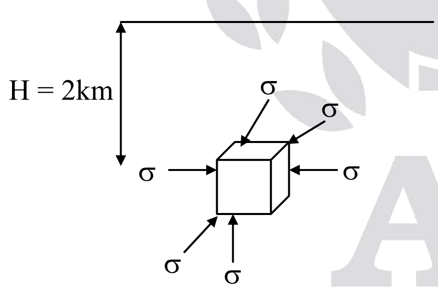
$$= \frac{10^3 \times 2000}{2 \times 10^5}\left[-\frac{150}{25^2} - \frac{50}{20^2} + \frac{25}{300}\right]$$

$$= 10[-0.24 - 0.125 + 0.0833]$$

$$= -2.816 \text{ mm (Compression)}$$

**04.**

**Sol:**




$$\sigma = \gamma_w.H$$

$$= G.\gamma_{\text{pure water}}.H$$

$$= 1.02 \times 9.81 \times 2000$$

$$= 20.01 \times 10^3 \text{ kN/m}^2$$

$$= \frac{20.01 \times 10^3 (1000\,\text{N})}{(1000\,\text{mm})^2} = 20.01 \text{ MPa}$$

$$E = 3K(1 - 2\mu)$$

$$208 \times 10^3 \text{ MPa} = 3K(1 - 2 \times 0.29)$$

$$\Rightarrow K = 165 \times 10^3 \text{ MPa}$$

$$K = \frac{\sigma}{\left(\frac{\delta V}{V}\right)}$$

$$\Rightarrow 165 \times 10^3 = \frac{20.01}{\frac{\delta V}{(100)^3}}$$

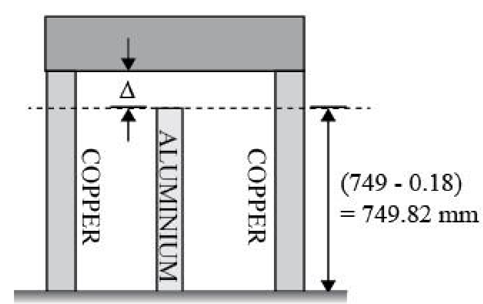
$$\delta V = 121.28 \text{ mm}^3$$

**08.**

**Sol:** $\Delta = 0.18$ mm

$\Delta T = 95°C$

$\alpha_a > \alpha_c$




$$(L\alpha\Delta T)_a - \left(\frac{PL}{AE}\right)_a = (L\alpha\Delta T)_c + \left(\frac{PL}{AE}\right)_c + \Delta$$

$$\Rightarrow (749.82) \times (23.1 \times 10^{-6}) \times (95) - \frac{P \times (749.82)}{400 \times (700 \times 10^3)}$$

$$= (750) \times (16.8 \times 10^{-6}) \times (95) + \frac{P(750)}{2(500)(120 \times 10^3)} +$$

0.18

(Two bars)

$\Rightarrow P = 8.2$ kN

Stress in each copper bar

$$\sigma_c = \frac{P}{2\times 500} = \frac{8.2\times 10^3}{2\times 500}$$

$= 8.2$ MPa (Tension)

Stress in AL bar

$$\sigma_a = \frac{P}{400} = \frac{8.2\times 10^3}{400} = 20.5 \text{ MPa (comp)}$$

**09.**

**Sol:** $\sigma_h = \left(\frac{D-d}{d}\right)E$

D = Wheel diameter (outside)

d = diameter of steel ring

$$1200 = \left(\frac{D-12}{12}\times 2\times 10^6\right)$$

$$\frac{1200\times 12}{2\times 10^6} = D-12$$

$D = 12 \times 7.2 \times 10^{-3}$

$D = 12.0072$ m

$$\Delta T = \left(\frac{D-d}{d}\right)\frac{1}{\alpha}$$

$$= \left(\frac{0.0072}{12}\right)\frac{1}{11.7\times 10^{-6}}$$

$\Delta T = 51.28^\circ$C

## 02. Complex Stresses and Strains

**01. Ans: (b)**

**Sol:** Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

Maximum shear stress $= \frac{\sigma_1 - \sigma_2}{2} = 13$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{18+(-8)}{2} = 5$$

**02. Ans: (b)**

**Sol:** Radius of Mohr's circle, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$\therefore \quad 20 = \frac{\sigma_1 - 10}{2}$$

$$\Rightarrow \quad \sigma_1 = 50 \text{ N/mm}^2$$

**03. Ans: (b)**

**Sol:** Given data,

$\sigma_x = 150$ MPa, $\sigma_y = -300$ MPa, $\mu = 0.3$

Long dam $\rightarrow$ plane strain member

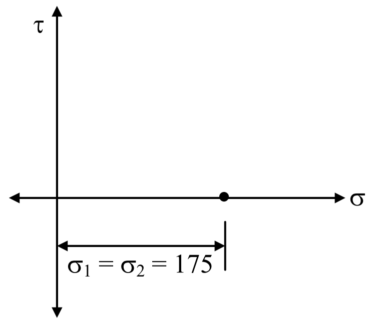
$$\varepsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\therefore 0 = \sigma_z - 0.3\times 150 + 0.3\times 300$$

$$\Rightarrow \sigma_z = 45 \text{ MPa}$$

**04. Ans: (b)**

**Sol:**




From the bove, we can say that Mohr's circle is a point located at 175 MPa on normal stress axis.

Thus, $\sigma_1 = \sigma_2 = 175$ MPa

**05. Ans: (c)**

**Sol:** Given that, $\sigma_2 = 0$

$$\therefore \quad \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$\therefore \quad \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$\therefore \quad \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}$$

$$\therefore \quad \tau^2{}_{xy} = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

$$\therefore \quad \tau^2{}_{xy} = \sigma_x.\sigma_y$$

$$\Rightarrow \quad \tau_{xy} = \sqrt{\sigma_x.\sigma_y}$$

**06. Ans: (d)**

**Sol:** Max shear stress = $\dfrac{\sigma_1 - \sigma_2}{2}$

$\sigma_1$, $\sigma_2$ are major and minor principal stress

1. $\tau_{max} = \dfrac{\sigma - 0}{2} = \dfrac{\sigma}{2}$

2. $\tau_{max} = \dfrac{\sigma - \sigma}{2} = 0$

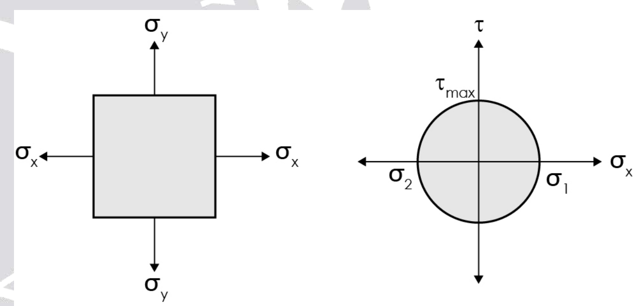
3. $\tau_{max} = \dfrac{\sigma - (-\sigma)}{2} = \sigma$

4. $\tau_{max} = \left(\sigma - \dfrac{\sigma}{2}\right)\dfrac{1}{2} = \dfrac{\sigma}{4}$

$\therefore$ Increasing order is 2, 4, 1 and 3.

**07. Ans: (b)**

**Sol:**




Since y-axis passes through centre of Mohr's circle $\Rightarrow$ centre of Mohr circle is origin

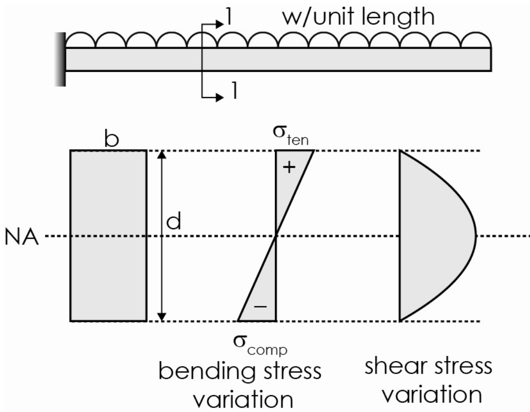
$$\Rightarrow (0, 0) = \left(\frac{\sigma_1 + \sigma_2}{2}, 0\right)$$

$$\Rightarrow \frac{\sigma_1 + \sigma_2}{2} = 0 \Rightarrow \sigma_1 = -\sigma_2$$

$\therefore$ Direct stresses are equal in magnitude but opposite in nature.

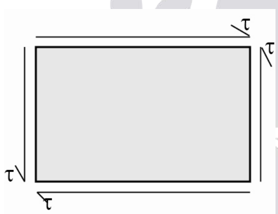
**08. Ans: (a)**

**Sol:**

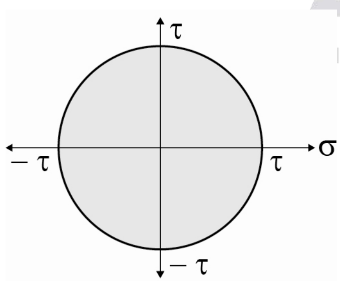



**Case - I : At Neutral axis**

Shear stress is maximum; Bending stress is zero i.e. stress condition is




and Mohr's circle is

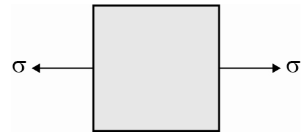



**Case - II : At extreme top fibre**

Bending stress is tensile and maximum

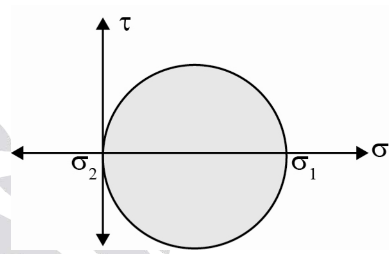
Shear stress = 0. Stress condition is

$$\therefore\ \sigma_1 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2}$$

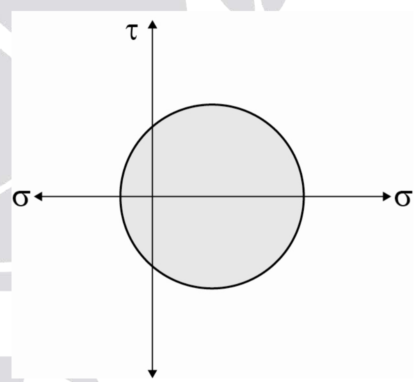



$\sigma_1 = \sigma$ ; $\sigma_2 = 0$

Corresponding Mohr's circle is




For any point above NA, and below extreme fiber Mohr’s circle will be an intermediate of the above two cases i.e.,




**Conventional Practice Solutions**

**01.**

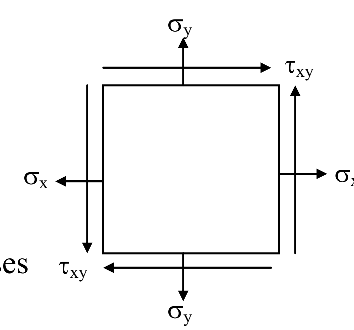
**Sol:**

$\sigma_x = 30$ MPa

$\sigma_y = 20$ MPa

$\tau_{xy} = 50$ MPa

Principal stresses



$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{30+20}{2} \pm \sqrt{\left(\frac{30-20}{2}\right)^2+50^2}$$

$\sigma_1 = 75.25$ MPa

$\sigma_2 = -25.25$ MPa

Check

$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

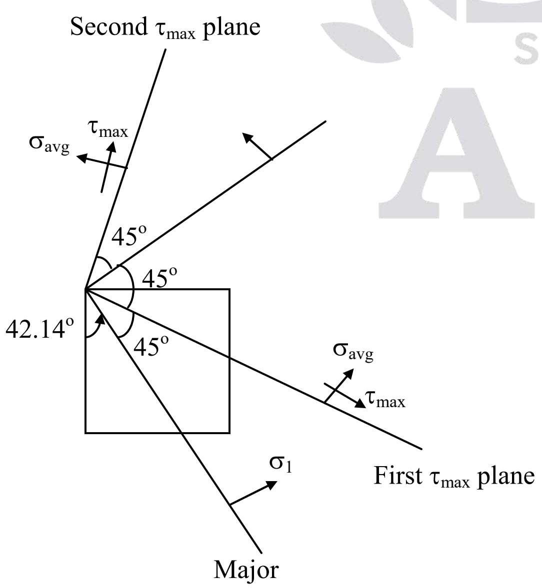
$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$ ($\theta_p$ = Angle of major principal plane)

$\theta_p = 42.14°$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \sqrt{\left(\frac{30-20}{2}\right)^2+50^2}$$

$\therefore\ \tau_{max} = 50.25$




**Or**

**Graphical**

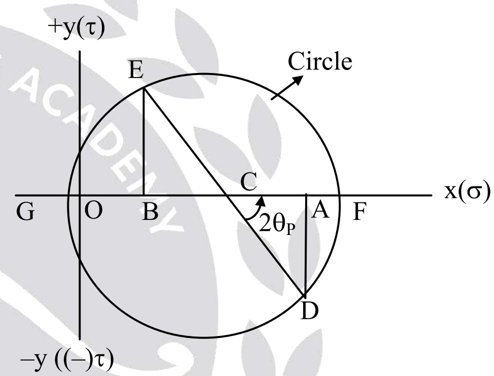
$\sigma_x = 30$ MPa = OA

$\sigma_y = 20$ MPa = OB

$\tau_{xy} = 50$ MPa = AD or BE

C : Centre, CE = CD = radius = $\tau_{max}$

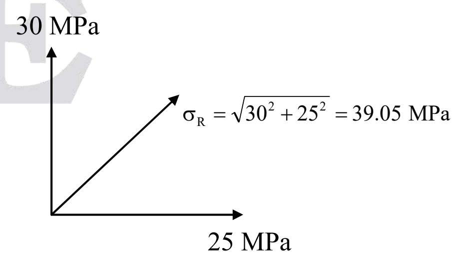
$\sigma_1$ = OF, $\sigma_2$ = OG



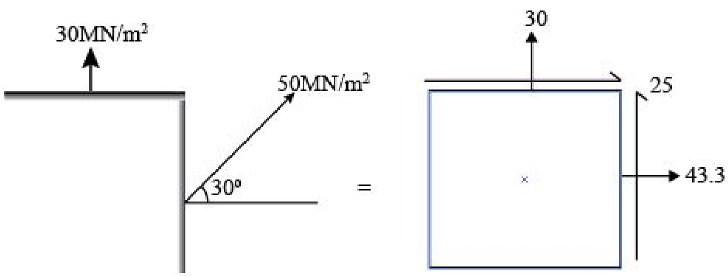

**02.**

**Sol:**

**(i) Resultant Stress on Second (Horizontal) Plane:**



**(ii)**




**Principal Plane:**

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 25}{43.3-30} = 3.76$$

$\theta_{P1} = 37.55^o$ (Major Principal plane)

$\theta_{P3} = 127.55^o$ (Minor Principal plane)

At $\theta_P = 37.55^o$ Principal stress

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_p + \tau_{xy}\sin 2\theta_P$$

$$= \left(\frac{43.3+30}{2}\right) + \left(\frac{-30+43.3}{2}\right)\cos 75.1 + 25\sin 75.1$$

$= 36.65 + (+1.71) + 24.16$

$\sigma_{P_1} = 62.52\ \text{MN/m}^2$

At $\theta_P = 127.55$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_P + \tau_{xy}\sin 2\theta_P$$

$= 36.65 - 1.71 - 24.16$

$\sigma_{P_3} = 10.78\ \text{MN/m}^2$

**(iii) Maximum Shear Stress:**

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{43.3-30}{2}\right)^2 + 25^2}$$

$\tau_{max} = 25.87$ MPa

Plane on which it occurs is

$\theta_{P_2} = 37.55^o + 45^o$

$= 82.55^o$

**04.**

**Sol:** $\sigma_x = 100$ MPa

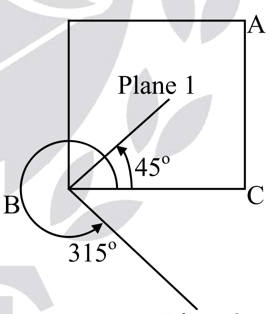
$\sigma_y = 100$ MPa

$\tau_{xy} = -20$ MPa

$$\tan 2\theta_{P_1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-40}{0} = \infty$$

$\theta_p = 45^o, 135^o$ from plane AC

$= 45^o$ & $315^o$ from plane BC




Principal strain $= \dfrac{\sigma_{P_1} - \mu\sigma_{P_3}}{E}, \dfrac{\sigma_{P_3} - \mu\sigma_{P_1}}{E}$

$$\sigma_{P1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= 100 + \frac{1}{2}\sqrt{4\times 20^2}$$

$$\frac{\sigma_{P_1}}{\sigma_{p3}} = 120, 80$$

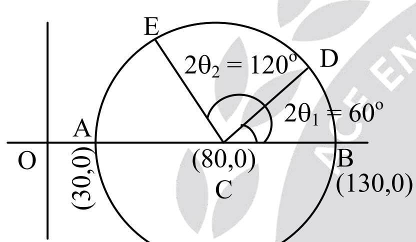
$$\sigma_{p_1} = \frac{(120 - 0.33 \times 80)}{67 \times 10^3} = 1.393 \times 10^{-3}$$

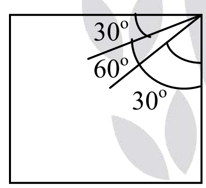
$$\sigma_{p_3} = \frac{80 - 0.333 \times 120}{67 \times 10^3} = 5.97 \times 10^{-4}$$

**05.**

**Sol:** Radius of Mohr's circle $= \dfrac{\sigma_1 - \sigma_2}{2} = 50$ MPa

Centre of Mohr's circle $= \dfrac{\sigma_1 + \sigma_2}{2} = 80$ MPa




$\theta_1 = 30^o$
$\theta_2 = 60^o$

Stress on point inclined at $30^o$ to y-axis in AC direction (Point D)

Normal stress = 80 + 50 cos$60^o$

= 105 MPa

Shear stress = 50 sin $60^o$ = 43.3 MPa

Stress at point inclined at $30^o$ to x-axis in AC direction (Point E)

Normal stress = 80 – 50 cos $60^o$ = 55 MPa

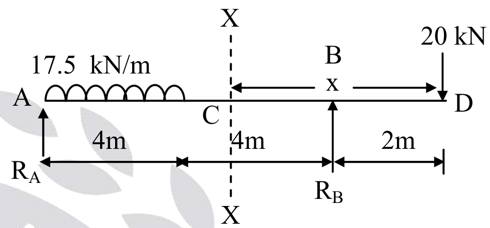
Shear stress = 50 sin $60^o$

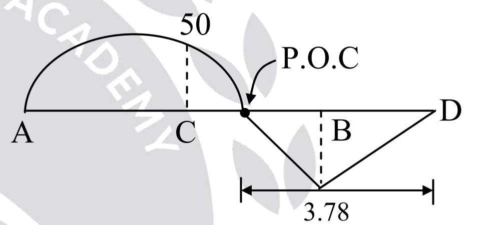
= 43.33 MPa

## 03. Shear Force and Bending Moment

**01. Ans: (b)**

**Sol:** Contra flexure is the point where BM is becoming zero.




Taking moment about A,

$\Sigma M_A = 0$

$\therefore\ 17.5 \times 4 \times \dfrac{4}{2} + 20 \times 10 - R_B \times 8 = 0$

$\therefore \quad R_B = 42.5$ kN

Now, $M_x = -20x + R_B(x - 2)$

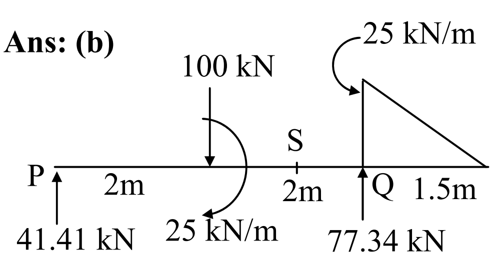
For bending moment be zero $M_x = 0$,

$-20x + 42.5(x - 2) = 0$

$\Rightarrow x = 3.78$ m from right i.e. from D.

**02. Ans: (b)**

**Sol:**



Take $\Sigma M_P = 0$

$$\frac{1}{2}\times 25\times 1.5\times\left(\frac{1.5}{3}+4\right)-\left(R_Q\times 4\right)+100\times 2+25=0$$

$\therefore \quad R_Q = 77.34\,\text{kN}$

Also, $\Sigma V = 0$

$\therefore \quad R_P + R_Q = 100+\frac{1}{2}\times 25\times 1.5 = 118.75 \text{ kN}$

$\therefore \quad R_p = 41.41\,\text{kN}$

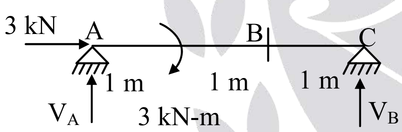
$\Rightarrow$ Shear force at P = 41.41 kN

**03. Ans: (c)**

**Sol:** $M_S = R_P(3) + 25 - (100 \times 1) = 49.2$ kN-m

**04. Ans: (c)**

**Sol:**




$-V_B \times 3 + 3 = 0$

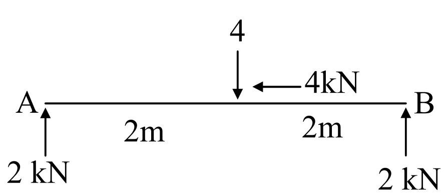
$\therefore \quad V_C = 1$ kN

$\therefore$ Bending moment at B,

$\Rightarrow M_B = V_C \times 1 = 1$ kN-m

**05. Ans: (a)**

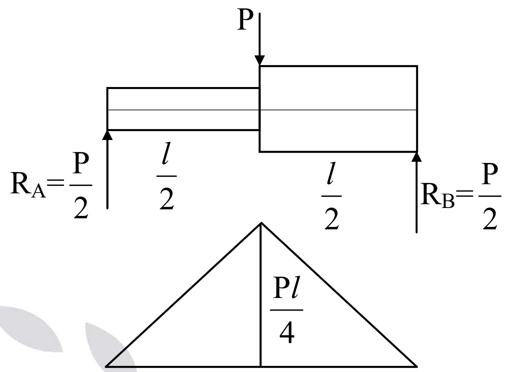
**Sol:**




Reaction at both the supports are 2 kN and in upward direction.

**06. Ans: (c)**

**Sol:**




BMD Diagram

Bending moment at $\frac{l}{2}$ from left is $\frac{Pl}{4}$.

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

**07. Ans: (a)**

**Sol:** As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

**08. Ans: (b)**

**Sol:** Shear force (V) = $\frac{dM}{dx}$

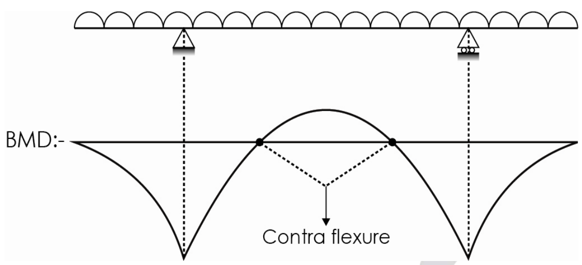
$\therefore$ For bending moment to be maximum,

$\frac{dM}{dx} = 0 \quad \Rightarrow V = 0$

When shear force changes sign it implies if it is zero at a particular section then bending moment is maximum at that section.

Point of contra flexure: Points where bending moment curve changes sign.

**Ex:**




**09. Ans: (c)**

**Sol: Point of contra flexure:** It is the point of the bending moment curve where bending moment changes its algebraic sign.

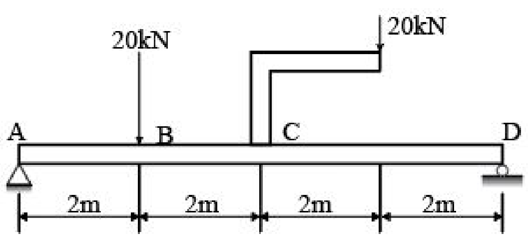
Shear force, (V) = $\dfrac{dM}{dx}$

$\therefore$ There is no relation between shear force and point of contra flexure.

## Conventional Practice Solutions

**01.**

**Sol:**

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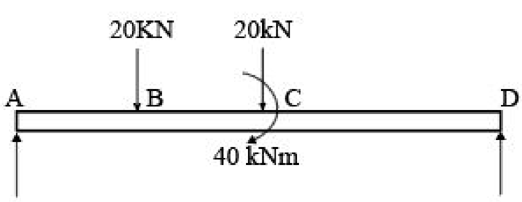


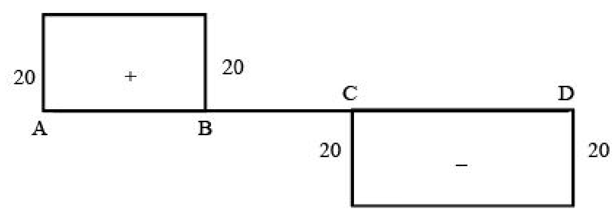

$\Sigma M_A = 0$

$20 \times 2 + 20 \times 4 + 40 = R_D \times 8$

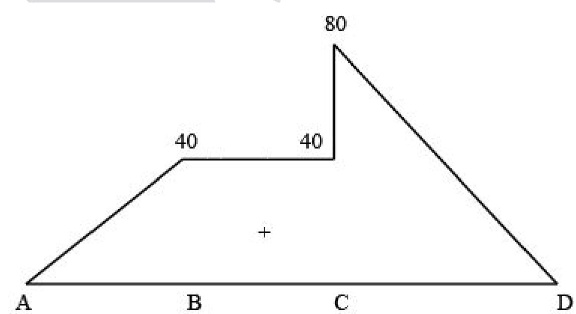
$$R_D = \frac{40+80+40}{8} = 20 \text{ kN}$$

$R_A = 20$ kN

**Shear Force Diagram (units kN)**

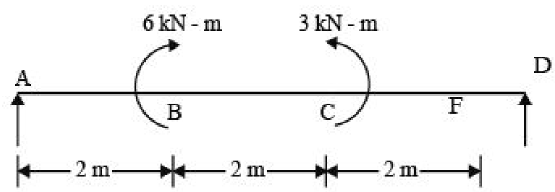



**Bending Moment diagram (units kN)**




**02.**

**Sol:**



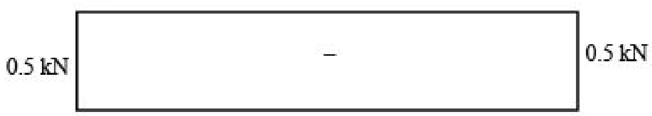

$\Sigma M_A = 0$

$R_D \times 6 + 3 - 6 = 0$

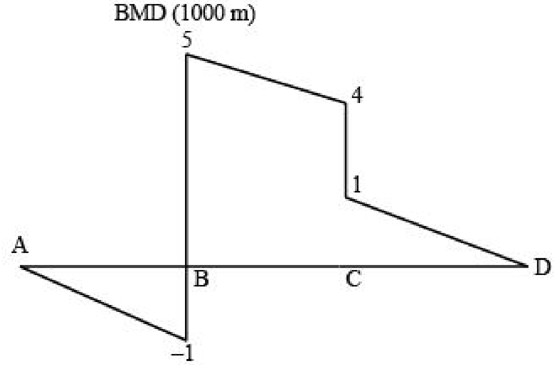
$R_D = 0.5$ kN ($\uparrow$)

$R_A = 0.5$ kN($\downarrow$)

SFD



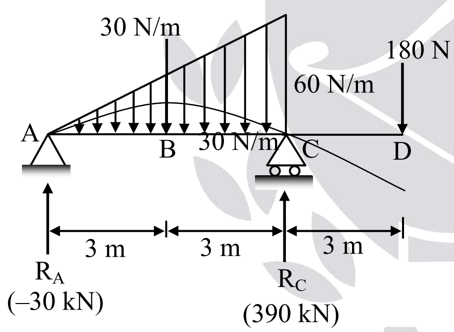

BMD (1000 m)




Point of contraflexure = 1 (point B)

**03.**

**Sol:**




**Reactions:**

$\sum F_Y = 0$

$$R_A + R_B = \frac{1}{2}\times 6\times 60 + 180$$

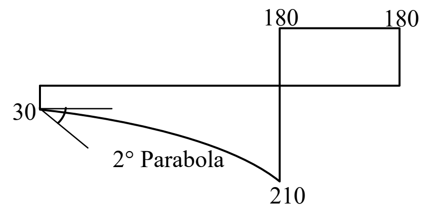
$R_A + R_B = 360$ N

$\sum M_A = 0$ (C.W )+))

$$-R_C\times 6 + \frac{1}{2}\times 6\times 60\times\left(\frac{2}{3}\times 6\right) + 180\times 9 = 0$$

$\Rightarrow R_C = 390$ N

$R_A = 360 - 390 = -30$ N




$(SF)_A$, ------- = $+R_A = +(-30) = -30$ N

$(SF)_C$ ---- = $-R_C + 180 = -390 + 180 = -210$

N or $R_A - \frac{1}{2}\times 6\times 60$

$(SF)_c$ – – – – – = +180 N

$(SF)_D$ – – – – = +180 N

BMD (SAG +ve)

$M_A = 0$

$M_C = -180\times 3 = -540$ N-m,

$M_D = 0$

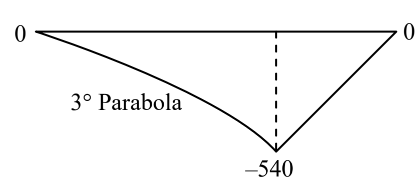
Design BM = $|540\text{ N} - \text{m}|$

Design SF = $|210\text{ N}|$

(ii) $M_B = +R_A\times 3 - \frac{1}{2}\times 3\times 30\times\left(\frac{1}{3}\times 3\right)$

$$M_B = (-30)\times 3 - \frac{1}{2}\times 90$$

$M_B = -135$ N-m




Maximum bending stress @B :

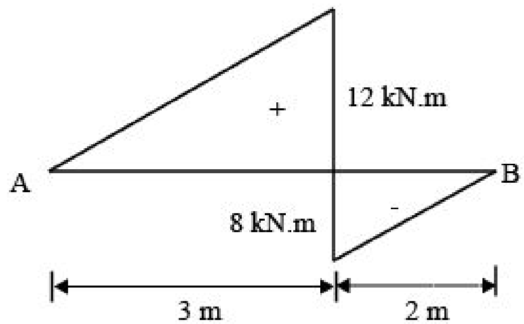
$$\frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$f_{max} = \frac{M}{I}.y_{max} = \frac{M}{Z} = \frac{135\times10^3\ N-mm}{\left(\frac{50\times100^2}{6}\right)}$$

$f_{max} = 1.62\ N/mm^2$

**04.**

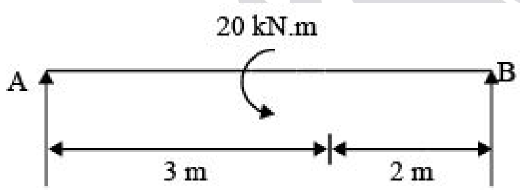
**Sol:**




At section C there is a sudden change of BM, so there is a concentrated moment in AC direction at point C of magnitude

12 – ( – 8) = 20 kNm

So loading diagram



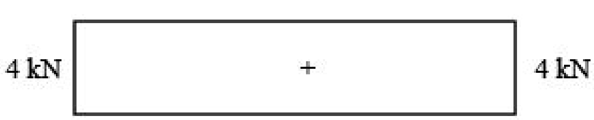

$\Sigma M_A = 0$

$R_B \times 5 = 20$

$R_B = 4\ kN\ (\downarrow)$

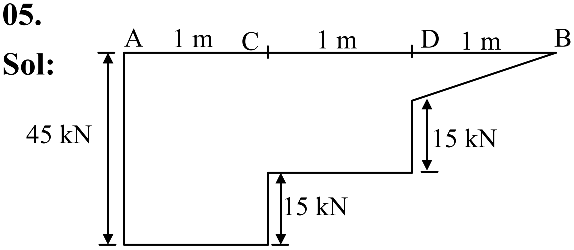
$R_A = 4\ kN\ (\uparrow)$

Shear Force Diagram




**05.**

**Sol:**

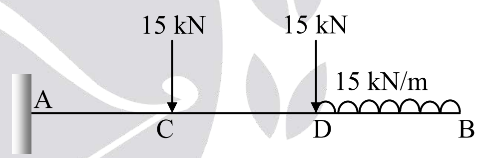



From SFD it is clear that support from at A = 45 kN

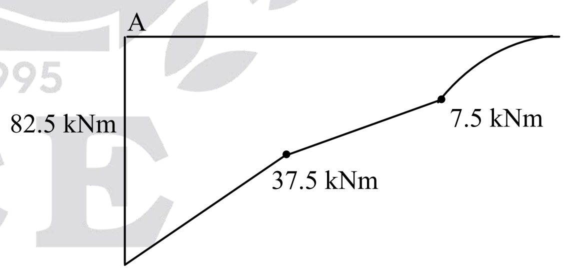
There is a concentrated point load of 15 kN at point C and D and UDL acts between points C and D of intensity

$$= \frac{15-0}{1} = 15\ kN/m$$

Loading diagram




**Bending Moment Diagram:**




Maximum Bending Stress is at point A

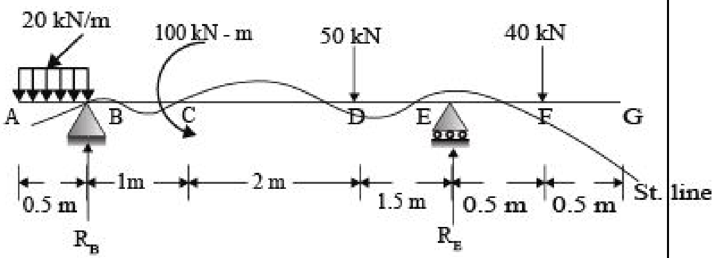
$$\delta = \frac{M}{z} = \frac{M\times6}{bd^2} = \frac{6M}{bd^2}$$

$$= \frac{6\times82.5\times10^6}{100\times250^2}$$

$= 79.2\ N/mm^2$ or 79.2 MPa

**06.**

**Sol:**




$\sum F_y = 0$

$R_B + R_E = 20\times0.5 + 50 + 40$

$= 100$ kN

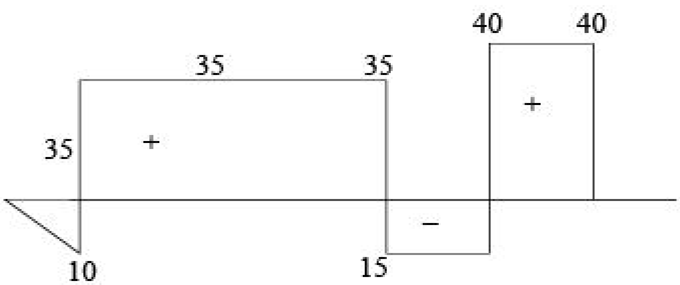
$\sum M_B = 0$ (C.W(+))

$$20\times0.5\times\frac{0.5}{2} - 100 + 50\times3$$

$-R_E\times4.5 + 40\times5 = 0$

$R_E = 55$ kN

$R_B = 100 - 55 = 45$ kN




$(SF)_A = 0$

$(SF)_B$ --- $= -20\times0.5 = -10$ kN

$(SF)_B$ -- $= -20\times0.5 + 45 = 35$ kN

$(SF)_C = -20\times0.5 + R_B = 35$ kN

$(SF)_D$ -- $= -20\times0.5 + 45 = 35$ kN

$(SF)_D$, -- $= -20\times5 + 45 - 50 = -15$ kN or $40 - R_E$

$(SF)_E$, -- $= -20\times0.5 + 45 - 50 = -15$ kN or $+40 - R_E$

$(SF)_F$ ---- $= -20\times0.3 + R_B$-------------

$(SF)_F$ ----- $= 0$

$(SF)_G = 0$

BMD (SAG + ve)

$M_A = 0$

$$M_B = -20\times0.5\times\frac{0.5}{2} = -2.5\,\text{kN}-\text{m}$$

$$M_{C,\,it} = -20\times0.5\times\left(\frac{0.5}{2}+1\right) + R_B\times1$$

$= -20\times0.5\times1.25 + 45\times1$

$= 32.5$ kN-m

$M_C$ ---S $= -20\times0.5\times1.25\times45\times1 - 100$

$= -67.5$ kN-m

$M_D = -40\times2 + R_E\times1.5$

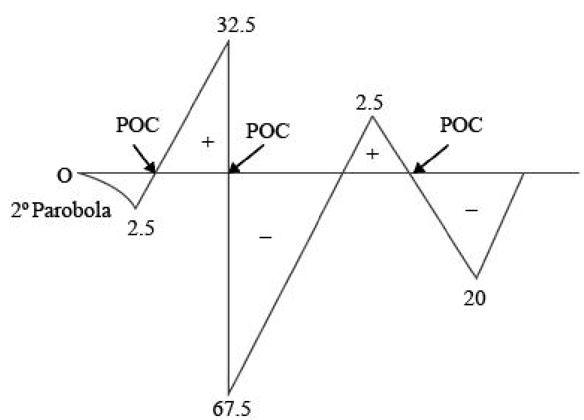
$= -40\times2 + 55\times1.5$

$= 2.5$ kN-m

$M_E = -40\times0.5 = -20$ kN-m

$M_F = 0$

$M_G = 0$



## 04. Centre of Gravity & Moment of Inertia

**01. Ans: (a)**

**Sol:** $$\bar{y} = \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2}$$

$$\Rightarrow \bar{y} = \frac{2E_2\left(h + \frac{h}{2}\right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \qquad (\because E_1 = 2E_2)$$

$$\Rightarrow \bar{y} = 1.167h \text{ from base}$$

**02. Ans: (b)**

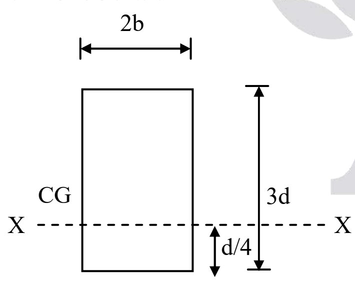
**Sol:** $$\bar{y} = \frac{A_1E_1Y_1 + A_2E_2Y_2}{A_1E_1 + A_2E_2}$$

$$= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2E_1 + 6a^2(2E_1)}$$

$$= \frac{22.5a^3E_1}{15a^2E_1} = 1.5a$$

**03. Ans: 13.875 bd³**

**Sol:**




$$y = \frac{5}{4}d$$

$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3$$

$$\text{M.I about } X - X \,|_{\text{at } d/4 \text{ distance}} = I_G + Ay^2$$

$$= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2$$

$$= \frac{111}{8}bd^3 = 13.875bd^3$$

**04. Ans: 6.885×10⁶ mm⁴**

**Sol:**

$$I_x = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12} + Ah^2\right)$$

$$= \frac{60 \times 120^3}{12} - 2\left(\frac{30 \times 30^3}{12} + (30 \times 30) \times 30^2\right)$$

$$= 6.885 \times 10^6 \text{ mm}^4$$

**05. Ans: 152146 mm⁴**

**Sol:**

$$I_x = \frac{30 \times 40^3}{12} - \frac{\pi \times 20^4}{64} = 152146 \text{ mm}^4$$

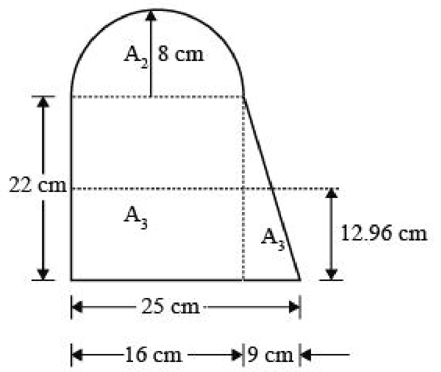
$$I_y = \frac{40 \times 30^3}{12} - \left(\frac{\pi \times 20^4}{64} + 2\left(\frac{\pi}{2} \times 10^2 \times \left(15 - \frac{4 \times 10}{3\pi}\right)^2\right)\right)$$

$$= 45801.34 \text{ mm}^4$$

## Conventional Practice Solutions

**01.**

**Sol:**




$A_1 = 22 \times 16 = 352\ cm^2$

$A_2 = \dfrac{\pi r^2}{2} = 100.48\ cm^2$

$A_3 = \dfrac{1}{2} \times 9 \times 22 = 99\ cm^2$

**Centroidal Axis location above base:**

$$= \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3}$$

$$= \frac{352 \times 11 + 100.48 \times \left(\frac{4 \times 8}{3\pi} + 22\right) + 99 \times \left(\frac{22}{3}\right)}{352 + 100.48 + 99}$$

$$= \frac{3872 + 2551.89 + 726}{551.48}$$

= 12.96 cm above base

**Second Moment of Area:**

$$I_{xx} \text{of } A_1 + A_1y_1^2 + I_{xx} \text{ of } A_2 + A_2y_2^2 + I_{xx} \text{ of } A_3 + A_3y_3^2$$

$$= \left(\frac{16 \times 22^3}{12}\right) + 16 \times 22[12.96 - 11]^2 + (0.1098 \times 8^4)$$

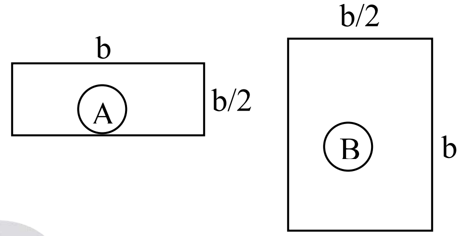
$$+ (12.43^2)(100.48) + \left(\frac{9 \times 22^3}{36}\right) + (99 \times 5.63^2)$$

$= 37323.97\ cm^4$

## 05. Theory of Simple Bending

**01. Ans: (b)**

**Sol:**




By using flexural formula, $\sigma = \dfrac{M}{Z}$

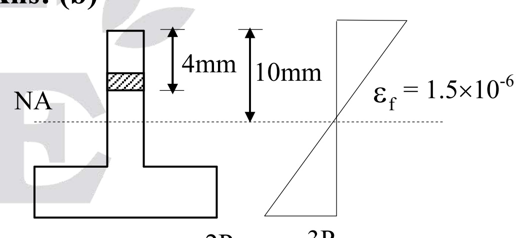
$\therefore\ \sigma \propto \dfrac{1}{Z}$ $\quad (\because M \text{ is constant})$

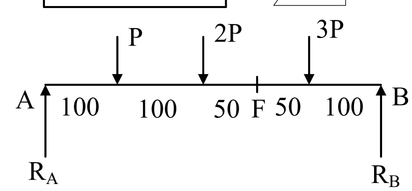
$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\dfrac{\left(\frac{b}{2} \times b^2\right)}{6}}{\dfrac{b \times \left(\frac{b}{2}\right)^2}{6}} = 2$$

$\Rightarrow \sigma_A = 2\sigma_B$

**02. Ans: (b)**

**Sol:**




$\therefore \sum M_A = 0$

$\therefore\ P \times 100 + 2P \times 200 + 3P \times 300 = R_B \times 400$

$\therefore R_B = 3.5P,\ R_A = 2.5P$

Take moments about F and moment at F

$M_F = R_B \times 150 - 3P \times 50 = 375P$

Also, $\dfrac{M_F}{I} = \dfrac{\sigma_b}{y_F}$

$$\therefore \frac{375P}{2176} = \frac{\left(1.5\times10^{-6}\times200\times10^{3}\right)}{6}$$

$\Rightarrow \quad P = 0.29$ N

**03. Ans: (b)**

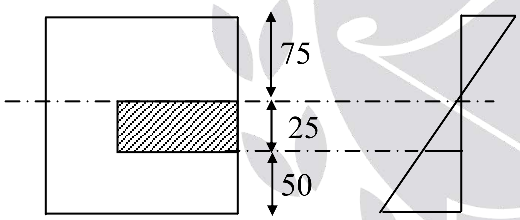
**Sol:** By using Flexural formula,

$$\frac{E}{R} = \frac{\sigma_b}{y_{max}} \Rightarrow \frac{2\times10^{5}}{250} = \frac{\sigma_b}{(0.5/2)}$$

$\Rightarrow \sigma_b = 200$ N/mm$^2$

**04. Ans: (c)**

**Sol:**




By using flexural formula,

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \frac{16\times10^{6}}{\frac{100\times150^{3}}{12}} = \frac{f}{25} \Rightarrow f = 14.22 \text{ MPa}$$

Now, Force on hatched area

= Average stress × Hatched area

$$= \left(\frac{0+14.22}{2}\right)(25\times50) = 8.9 \text{ kN}$$

**05. Ans: (b)**

**Sol:** By using flexural formula, $\dfrac{f_{Tensile}}{y_{top}} = \dfrac{M}{I}$

$$\Rightarrow f_{Tensile} = \frac{0.3\times3\times10^{6}}{3\times10^{6}}\times70$$

(maximum bending stress will be at top fibre so $y_1 = 70$ mm)

$\Rightarrow f_{Tensile} = 21$ N/mm$^2$ = 21 MN/m$^2$

**06. Ans: (c)**

**Sol:** Given data:

$P = 200$ N, $\quad M = 200$ N.m

$A = 0.1$ m$^2$, $\quad I = 1.33\times10^{-3}$ m$^4$

$y = 20$ mm

Due to direct tensile force P,

$$\sigma_d = \frac{P}{A} = \frac{200}{0.1}$$

$= 2000$ N/m$^2$ (Tensile)

Due to the moment M,

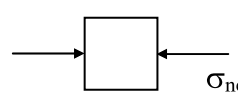
$$\sigma_b = \frac{M}{I}\times y = \frac{200}{1.33\times10^{-3}}\times20\times10^{-3}$$

$= 3007.52$ N/m$^2$ (Compressive)

$\sigma_{net} = \sigma_d - \sigma_b = 2000 - 3007.52$

$= -1007.52$ N/m$^2$

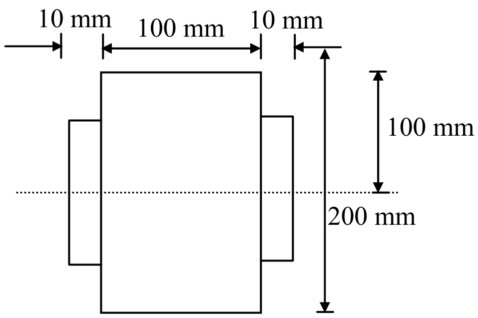
Negative sign indicates compressive stress.



$\sigma_{net} = 1007.52$ N/m$^2$

**07. Ans: 80 MPa**

**Sol:**




Maximum stress in timber = 8 MPa

Modular ratio, m = 20

Stress in timber in steel level,

$100 \rightarrow 8$

$50 \rightarrow f_w$

$\Rightarrow f_w = 4$ MPa

Maximum stress developed in steel is $= m \cdot f_w$

$= 20 \times 4 = 80$ MPa

Convert whole structure as a steel structure by using modular ratio.

**08. Ans: 2.43 mm**

**Sol:** From figure $A_1B_1 = l = 3$ m (given)

$$AB = \left(R - \frac{h}{2}\right)\alpha = l - l\alpha t_1 \text{ ------ (1)}$$

$$A_2B_2 = \left(R + \frac{h}{2}\right)\alpha = l + l\alpha t_2 \text{ ------ (2)}$$

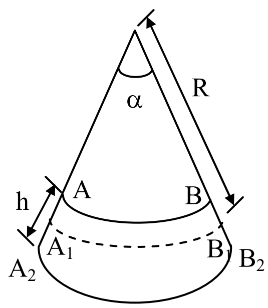
Subtracting above two equations (2) – (1)

$h(\alpha) = l\alpha\,(t_2 - t_1)$

but $A_1B_1 = l = R\alpha$

$$\Rightarrow \alpha = \frac{l}{R}$$

$$\therefore h\left(\frac{l}{R}\right) = l\alpha\,(\Delta T)$$




$$R = \frac{h}{\alpha(\Delta T)} = \frac{250}{(1.5\times10^{-5})(72-36)}$$

R = 462.9 m

From geometry of circles

$$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2} \quad \text{\{ref. figure in Q.No.02\}}$$

$$2R\cdot\delta - \delta^2 = \frac{L^2}{4} \text{ (neglect } \delta^2\text{)}$$

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8\times 462.9} = 2.43 \text{ mm}$$

***Shortcut:***

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^o - 36^o = 36^o$). Bottom temperature being more, the beam deflects down.

$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h} = \frac{1.5\times10^{-5}\times 36\times 3000^2}{8\times 250}$$

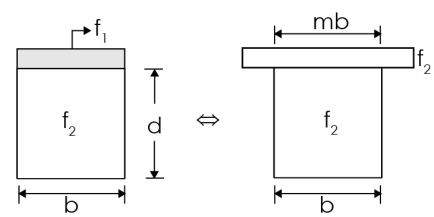
= 2.43 mm (downward)

**09. Ans: (d)**

**Sol:** For an equivalent section:

Load carried by original section = Load carried by transformed section.

$$\Rightarrow f_1A_1 = f_2A_{eq} \Rightarrow A_{eq} = \left(\frac{f_1}{f_2}\right)A_1 = mA_1$$



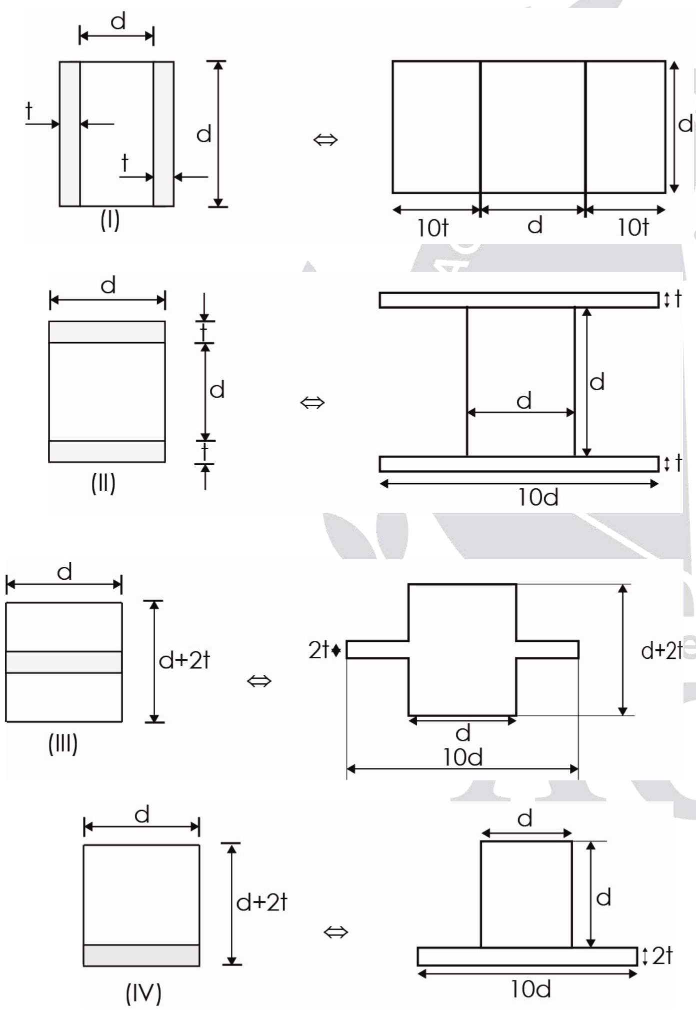

$b_{eq}d_{eq} = mb_1d_1$

Since the strain variation should be same in both the original and transformed section; depth is not changed. i.e. $d_{eq} = d_1$

$\therefore$ Width of equivalent section = (original width) × modular ratio

$\Rightarrow b_{eq} = mb_1$

**Transformed Sections:**




A beam that has larger section modulus will be stronger and support greater load.

Since in figure II, stronger material is provided at extreme fibres, moment of inertia is more. So section modulus is more.

$\therefore$ It will support greatest load.

**10. Ans: (d)**

**Sol: Assumptions in simple bending:**

- Beam is initially straight and has constant cross-section
- Material is homogeneous and isotropic i.e. same material and same elastic properties in all direction
- Beam is symmetrical about plane of bending i.e. longitudinal plane of symmetry
- Beam is composed of infinite number of fibers along longitudinal direction. Each fiber is free to expand (or) contract independently of the layer above (or) below it.
- Resultant of the applied loads, lies in the plane of symmetry i.e. cross section is symmetric about loading plane.
- Transverse sections of beam which are plane before bending remains plane after bending.
- Material obeys Hooke's law and Modulus of elasticity 'E' is same in tension and compression; Elastic limit is not exceeded.

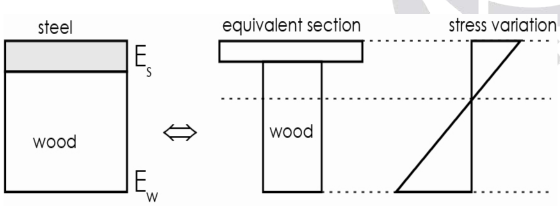
**11. Ans: (a)**

**Sol:** Flitched beam is a compound beam made of two (or) more materials, bonded together. Generally used, when large depths are required for weaker material like wood. In this case, it is bonded with a strong material like steel, thus reducing the depth required.

Since the materials are bonded rigidly, it is assumed that there is no relative movement between them. Hence all the assumptions valid in bending of homogeneous beams holds good except one assumption i.e. young's modulus 'E' is same through out the beam.

When load is applied, both the materials bend together to same radius of curvature.

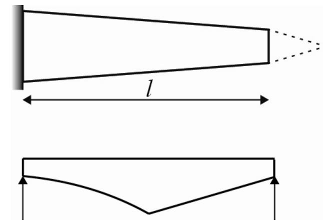
$\therefore$ Total moment of resistance = sum of moments of resistance of individual sections considering them as a single unit.




**12. Ans: (b)**

**Sol:** Generally, beams have same cross-section throughout. Since the bending moment is not maximum at all sections, cross-section dimensions can be varied along the length to resist BM at that particular section. i.e a non -prismatic beam.

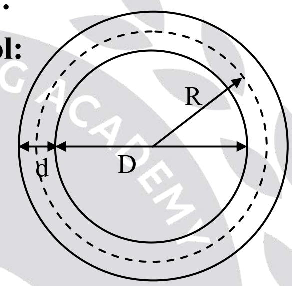
**Ex:**

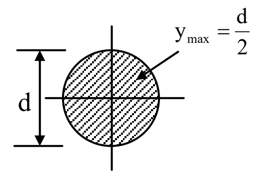



**Conventional Practice Solutions**

**01.**

**Sol:**




c/s of wire

Radius of curvature,

$$R = \frac{D}{2} + \frac{d}{2} = \frac{D}{2}$$

small therefore ignored

$$\frac{E}{R} = \frac{M}{I} = \frac{f_{max}}{y_{max}}$$

$$f_{max} = \frac{E}{R} \times y_{max} = \frac{E}{D/2} \times \frac{d}{2} = \frac{E.d}{D}$$

R = ?, if d = 1 cm, $f_{max}$ = 17000 kg/cm$^2$

E = 2×10$^6$ kg/cm$^2$

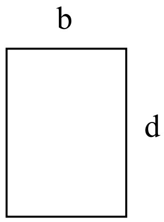
$$f_{max} = \frac{Ed}{D} = \frac{Ed}{2R}$$

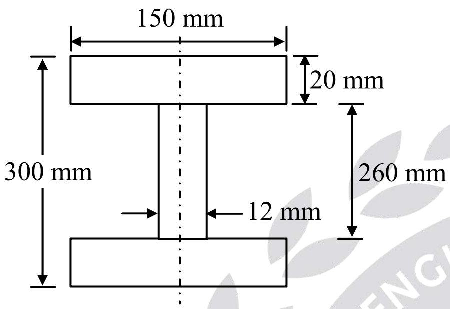
$$17000 = \frac{(2\times10^6)\times1}{2(R)} \Rightarrow R = 58.82 \text{ cm}$$

$$\frac{E}{R} = \frac{M}{I} \Rightarrow M = 1669.1 \text{ kg-cm}$$

**02.**

**Sol:**




$W = \gamma.A.l$

$W \propto A$

$\gamma$ = constant (same as material)

R = same

$A_I = A_{rect} = A$

$$\left[150\times300-(150-12)\times260\right]=(b\times2b)=\left(\frac{\pi}{4}d^2\right)$$

b = 67.53 mm

d = 107.76 mm

For I-section

$$M_I, I=\frac{150\times300^3}{12}-\frac{(150-12)(260)^3}{12}$$

$$=135.38\times10^6$$

$$y_{max}=\frac{300}{2}=150\,\text{mm}$$

$$Z=\frac{I}{y_{max}}=\frac{135.38\times10^6}{150}=902.5\times10^3\ \text{mm}^3$$

$(Z)_I : (Z)_{rect} : (Z)_{circle}$

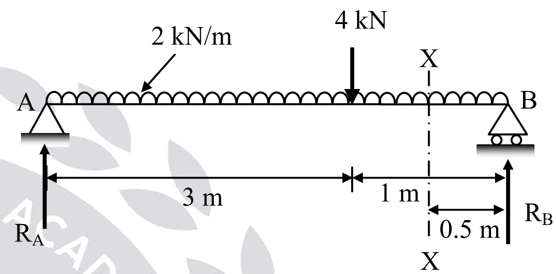
$$902.5\times10^3:\frac{(67.53)(2\times67.53)^2}{6}:\frac{\pi}{32}(107.76^3)$$

$$\frac{902.5\times10^3}{902.5\times10^3}:\frac{(67.53)(2\times67.53)^2}{6\times(902.5\times10^3)}:\frac{\pi(107.76^3)}{32\times902.5\times10^3}$$

1 : 0.227 : 0.136

**03.**

**Sol:**




$\Sigma F_y = 0 \Rightarrow R_A + R_B = (2\times3)+4$

$R_A + R_B = 10$

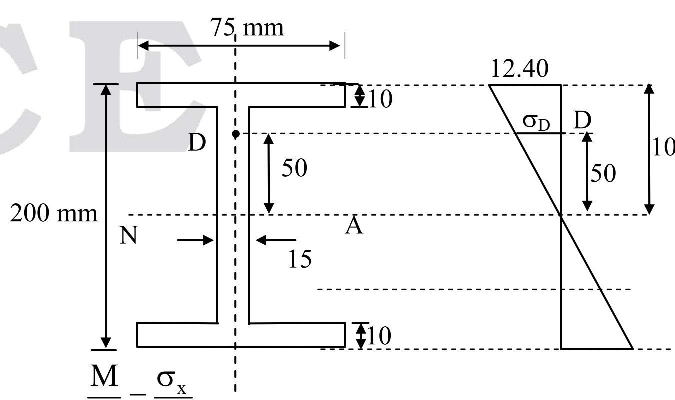
$$\Sigma M_A = 0 \Rightarrow 4\times2+2\times3\times\frac{3}{2}=R_B\times3$$

$R_B = 5.67$ kN

$R_A = 4.33$ kN

$$M_x = 5.67\times0.5-2\times0.5\times\frac{0.5}{2}$$

$M_x = 2.585$ kN-m




$$\frac{M}{I}=\frac{\sigma_x}{y}$$

$$\Rightarrow \sigma_x=\frac{M}{I}\times y$$

$$I = \frac{75 \times 200^3}{12} - \frac{60 \times 180^3}{12}$$

$I = 2084 \times 10^4\ mm^4$

$y_{max} = 100$

$M_x = 2.585 \times 10^6$ N-mm

$$(\sigma_x)_{max} = \frac{2.585 \times 10^6 \times 100}{2084 \times 10^4}$$

$(\sigma_x)_{max} = 12.40\ N/mm^2$

$100 \rightarrow \sigma_D$

$50 \rightarrow 12.40$

$$\sigma_D = \frac{12.40 \times 50}{100}$$

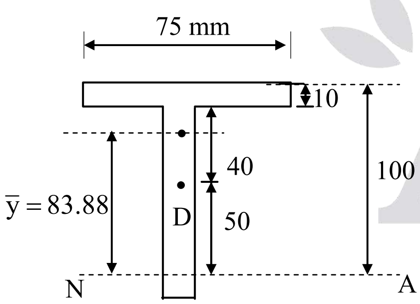
$= 6.20\ N/mm^2$ (Compression)

$\therefore\ \sigma_D = 6.20\ N/mm^2$ (Compression)

$$\tau_x = \frac{V_x A.\bar{y}}{I.b}$$

$$\tau_x = \frac{4.67 \times 10^3 \times 1350 \times 83.88}{15 \times 2084 \times 10^4}$$

$\tau_{x\ at\ D} = 1.691\ N/mm^2$




$A = 10 \times 75 + 40 \times 15$

$= 1350\ mm^2$

$I = 2084 \times 10^4\ mm^4$

$b = 15$ mm

$$\bar{y} = \frac{750 \times 5 + 600 \times 30}{750 + 600} = 16.11 \text{ mm}$$

$\bar{y} = 83.88$ mm

$V_x = 5.67 - (2 \times 0.5) = 4.67$ kN

Principal stress at point D,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2}$$

$\sigma_x = -6.2$

$\tau_x = 1.691$

$$\sigma_{1,2} = \frac{-6.2}{2} \pm \frac{1}{2}\sqrt{(6.2)^2 + (2 \times 1.691)^2}$$

$\sigma_{1,2} = -3.1 \pm 3.53$

$\sigma_1 = 0.431\ N/mm^2$ $\quad \sigma_2 = -6.63\ N/mm^2$

$\tau_{max} = 3.53\ N/mm^2$

## 06. Shear Stress Distribution in Beams

**01. Ans: (a)**

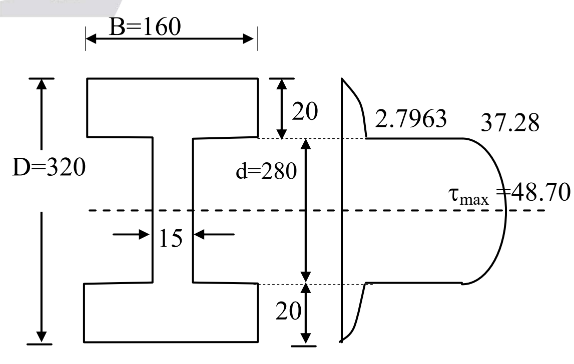
**Sol:** $\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{f}{b.d}$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$\therefore d = 250$ mm $= 25$ cm

**02. Ans: 37.3 MPa**

**Sol:**




*All dimensions are in mm*

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160\times320^3}{12} - \frac{145\times280^3}{12}$$

$$= 171.65\times10^6\ \text{mm}^4$$

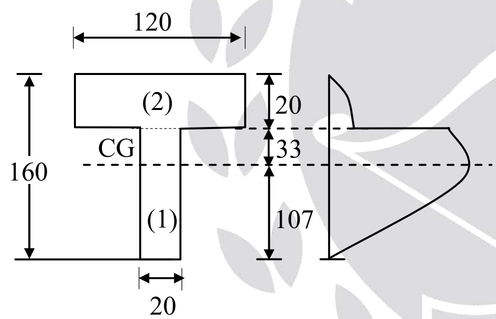
$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200\times10^3}{171.65\times10^6\times15}\times(160\times20\times150)$$

$$= 37.28\ \text{MPa}$$

**03. Ans: 61.43 MPa**

**Sol:**




*All dimensions are in mm*

$I_{NA} = 13\times10^6\ \text{mm}^4$

$y_{CG} = 107$ mm from base

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

$$A\bar{y} = (120\times20\times43) + (33\times20\times16.5)$$

$$= 114090\ \text{mm}^3$$

$$\tau_{max} = \frac{140\times10^3\times114090}{13\times10^6\times20} = 61.43\ \text{MPa}$$

**04. Ans: (a)**

**Sol:** For a shear force 'V' and cross section area 'A' average shear stress = $\tau_{avg} = \dfrac{V}{A}$

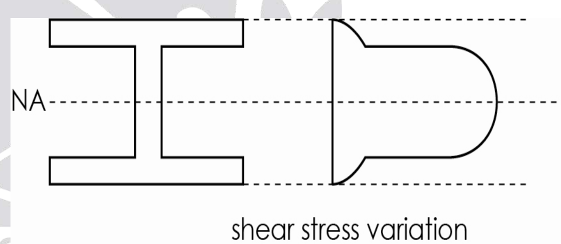
In case of rectangular cross sections, maximum shear stress = $\tau_{max} = 1.5\tau_{avg}$

In case of circular cross sections, maximum shear stress = $\tau_{max} = \dfrac{4}{3}\tau_{avg}$

For the same amount of shear force and same cross-section area, maximum shear stress is lesser in circular cross-section, so it is stronger in shear i.e. can resist more shear force compared to rectangular cross section.

**05. Ans: (b)**

**Sol:**




$$\tau = \frac{VA\bar{y}}{Ib}$$

A = Area above the section at which stress is calculated.

$\therefore\quad \tau \propto \dfrac{1}{b}$ and increases as area increases i.e. towards neutral axis.

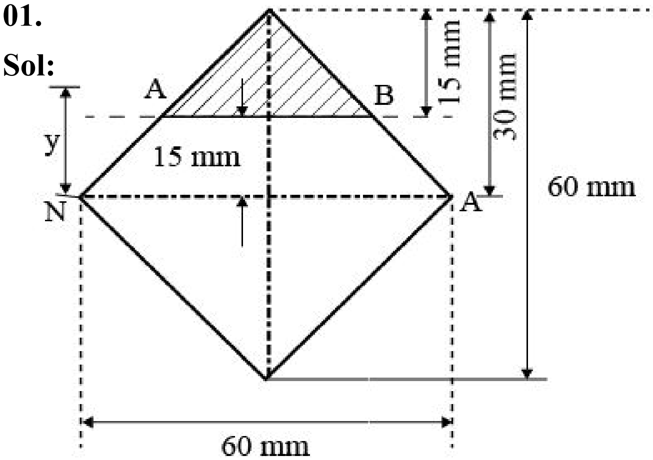
Since width of web << width of flange

$\therefore$ 'τ' increases at web and is maximum at centre of web.

## Conventional Practice Solutions

**01.**

**Sol:**




V = 5kN

$$\tau_{AB} = \frac{VA\bar{y}}{I(b_x)}$$

For diamond section:

$$I_{NA} = \frac{a.a^3}{12}$$

$$= \frac{42.43\times 42.43^3}{12}$$

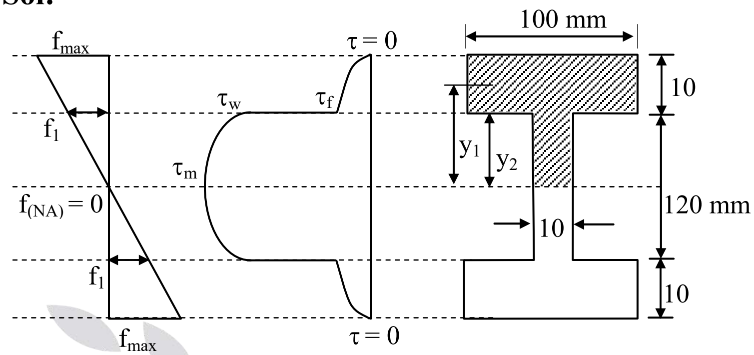
$$= 270000$$

$$\tau_{AB} = \frac{5\times10^3\times\left(\frac{1}{2}\times30\times15\right)\times\left(15+\frac{1}{3}\times15\right)}{270000\times30}$$

$$= 2.8 \text{ MPa}$$

**02.**

**Sol:**




$$I_{NA} = \frac{100\times140^3}{12} - \frac{(100-10)\times120^3}{12}$$

$$= 9.9\times10^6 \text{ mm}^4$$

$$\tau_f = \frac{VA\bar{y}}{I(b_f)} = \frac{10000\times(100\times10)\left(\frac{120}{2}+\frac{10}{2}\right)}{9.9\times10^6\times100}$$

$$= 0.66 \text{ MPa}$$

$$\tau_w = \frac{VA\bar{y}}{I(b_w)} = \frac{10000\times(100\times10)\left(\frac{120}{2}+\frac{10}{2}\right)}{9.9\times10^6\times10}$$

$$= 6.6 \text{ MPa}$$

or

$$0.66\times\frac{100}{10}$$

Maximum shear stress @ N.A

$$\tau_{NA} = \frac{V(A_1\bar{y}_1 + A_2\bar{y}_2)}{I.b_{NA}}$$

$$= \frac{10000\left(100\times10\times65+10\times60\times\frac{60}{2}\right)}{(9.9\times10^6)\times10}$$

$$= 8.38 \text{ MPa}$$

**Bending Stress:**

$$f_{max} = \frac{M}{I}.y_{max} = \frac{1000\times10^3}{9.9\times10^6}\times70$$

$$= 7.07 \text{ MPa}$$

$$f_1 = \frac{M}{I}.y_1 = \frac{1000\times10^3}{9.9\times10^6}\times60 = 6.06 \text{ MPa}$$

## 07. Torsion

**01. Ans: (a)**

**Sol:** Twisting moment = 2 × 0.5 – 1 × 0.5

= 0.5 kN-m

**02. Ans: (d)**

**Sol:** $$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{hollow}} = \frac{1}{1-K^4}$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)^4} = \frac{16}{15}$$

**03. Ans: 43.27 MPa & 37.5 MPa**

**Sol:** Given $D_o$ = 30 mm , t = 2 mm

$\therefore$ $D_i$ = 30 – 4 = 26 mm

We know that $\frac{\tau}{J} = \frac{q}{R}$

$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{max}}{\left(\frac{30}{2}\right)}$$

$q_{max}$ = 43.279 N/mm$^2$

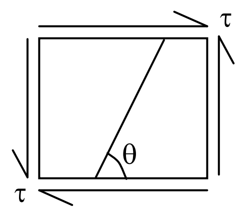
$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{min}}{\left(\frac{26}{2}\right)}$$

$q_{min}$ = 37.5 N/mm$^2$

**04. Ans: (a)**

**Sol:** Ductile material is weak in shear, so it fails in a plane where maximum shear stress occurs. Brittle material is weak in tension, so it fails in a plane where maximum tensile stress occurs.

It is a case of pure shear.

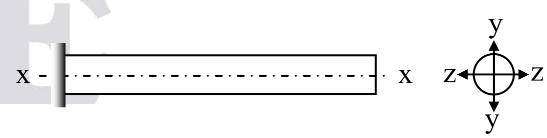



$\sigma_1 = \sigma_{max}$, at $\theta = 45°$ (maximum normal stress which causes failure of brittle material).

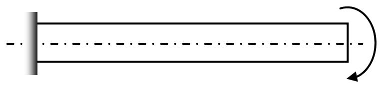
Thus, Assertion (A) and Reason (R) are correct and Reason (R) is correct explanation of Assertion (A).

**05. Ans: (b)**

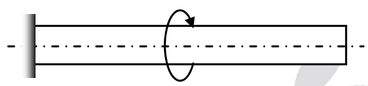
**Sol:** Let the axis of the beam is x-x as shown below.




- If the axis of the moment is perpendicular the axis of the beam (i.e. axis of the moment is either y-y or z-z), then beam will be subjected to bending. Hence, bending stress will be induced. Since there is no shear force, shear stress will not be induced.



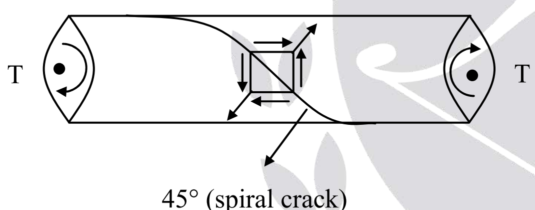
- If the axis of the moment is along the axis of the beam then beam will be subjected to torsion. Thus, only shear stress will be induced and bending stress will not be induced..



## Conventional Practice Solutions

**01.**

**Sol:** Cast iron: Brittle (Weak in tension)




$\sigma_1 + \tau =$ Diagonal tension

$= 80$ MPa (given)

Maximum torsional shear stress, $\tau_m = \sigma_1$

$= 80$ MPa

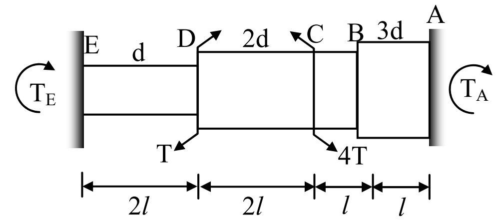
$$\tau_{max} = \frac{T}{Z_p}$$

$$\Rightarrow 80 = \frac{T}{\frac{\pi}{16}\times 30^3}$$

$$\Rightarrow T = 0.42 \text{ kN.m}$$

**02.**

**Sol:**




F.B.D:

$\sum T = 0$ (equation)

$T_A + T_E + T - 4T = 0$

Compatibility condition

$Q_{AE} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DE}$

$$Q_A - Q_E = \frac{T_A \times \ell}{G\left(\frac{\pi}{32}(3d)^4\right)} + \frac{T_A\ell}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{(T_A - 4T)(2\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{(T_A - 3T)2\ell}{G\left(\frac{\pi}{32}(d)^4\right)}$$

$T_A = 2.95$ T

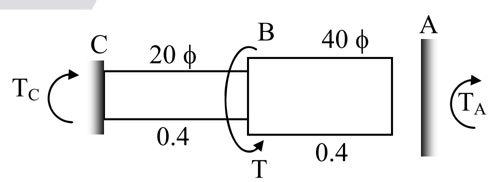
$Q_{AC} = Q_{AB} + Q_{BC}$

$$Q_A - Q_C = \frac{(T_A)(\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)} + \frac{T_A(\ell)}{G\left(\frac{\pi}{32}(2d)^4\right)}$$

$$\Rightarrow Q_c = \frac{-2.24TL}{Gd^4}$$

**03.**

**Sol:**




$\tau_{max} = 100$ MPa

Reactions:

$T_A + T_C = T$ ------------(1)

$Q_{AC} = Q_{AB} + Q_{BC}$

$$Q_A - Q_C = \frac{T_A(0.4\times1000)}{G\left(\frac{\pi}{32}\times40^4\right)} + \frac{(T_A - T)(0.4\times1000)}{G\left(\frac{\pi}{32}\times20^4\right)}$$

$T_A = 0.94$ T (kN-m)

Span AB

$$\tau_{max} = \frac{T}{Z_p}$$

$$100 = \frac{(0.94T)\times10^6}{\frac{\pi}{16}(40^3)}$$

T = 1.33 kN-m ---------(1)

**Span BC:**

$$\tau_{max} = \frac{T_{BC}}{Z_p}$$

$$100 = \frac{|0.94T - T|\times10^6}{\frac{\pi}{16}\times20^3}$$

$\Rightarrow$ T = 2.6 kN-m

Use minimum "T"

$\therefore$ T = 1.33 kN-m

**04.**

**Sol:** N= 110 rpm  P = 30 kW

$\tau_m$ = 80 MPa  d = ?

$$P = \frac{2\pi NT}{60}$$

$$30\,kN-m/s = \frac{2\pi\times110T}{60}$$

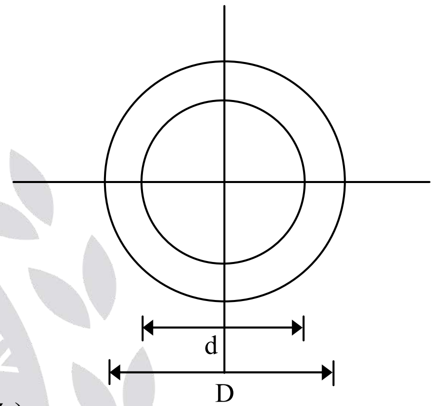
T = 2.604 kN-m

$$\tau_{max} = \frac{T}{4}$$

$$80 = \frac{2.604\times10^6}{\frac{\pi}{16}(d^3)}$$

$\Rightarrow d_s$ = 54.93 mm

Replacement: by hollow shaft $\rightarrow$ To have same strength (To transmit same power)




D = 2d

$(Z_P)_{solid} = (Z_p)_{hollow}$

$(Z_p)_{solid} = (Z_p)_{hollow}$

$$\frac{\pi}{16}(54.93^3) = \frac{\pi}{16D}(D^4 - d^4)$$

$$32543.08 = \frac{\pi}{16D}\left(D^4 - \left(\frac{D}{2}\right)^4\right)$$

$\Rightarrow$ D = 56.18 mm

d = 28.09 mm

$$\frac{W_h}{W_s} = \frac{(\gamma AR)_h}{(\gamma AL)_s} = \frac{A_h}{A_s} = \frac{\frac{\pi}{4}(D^2 - d^2)}{\frac{\pi}{4}(d_s^2)}$$

$$= \frac{56.18^2 - 28.09^2}{54.93^2} = 0.78$$

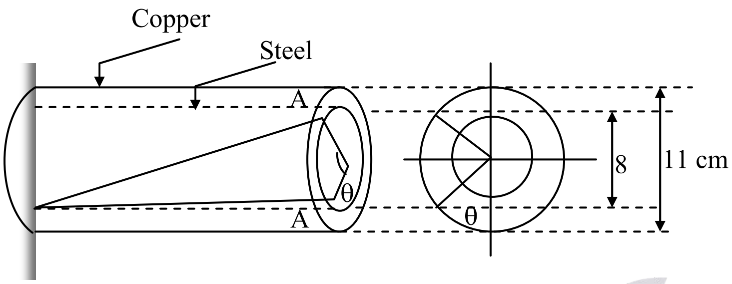
$W_h = 0.78W_s$

$W_h = (1 - 0.78)\times100$

$W_h = 22\% \downarrow W_s$

**05.**

**Sol:**




**Parallel equation :**

$T_C + T_s = T = 8$ kN-m

Compatibility condition:

$\theta_c = \theta_s$

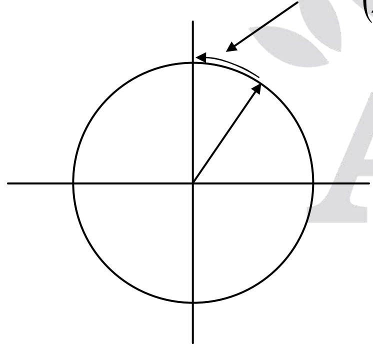
$$\frac{(T_c)(\ell)}{(40)\left[\frac{\pi}{32}\times(11^4-8^4)\right]} = \frac{(T_s)(\ell)}{(80)\left[\frac{\pi}{32}\times 8^4\right]}$$

$$\Rightarrow \frac{8-T_s}{11^4-8^4} = \frac{T_s}{8^4}$$

$\Rightarrow T_s = 3.5$ kN-m

$T_c = 8 - 3.5 = 4.5$ kN-m

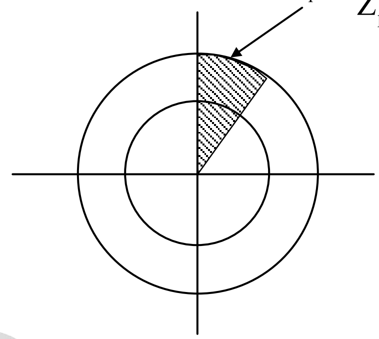
**Steel:** $\tau_{steel} = \frac{T_s}{(Z_p)_s}$



$$\tau_{steel} = \frac{T_s}{(Z_p)_s} = \frac{3.5\times10^6}{\frac{\pi}{16}(80)^3} = 34.81 \text{ kN/mm}^2$$

$= 34.81$ MPa

**Copper:** $\tau_{cop} = \frac{T_s}{Z_p}$



$$\tau_{cop} = \frac{T_c}{Z_p} = \frac{4.5\times10^6}{\frac{\pi}{16\times110}(110^4-80^4)}$$

$= 23.9$ MPa

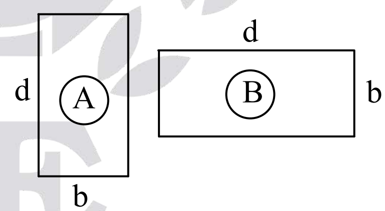
**06.**

**Sol: Refer Q. No. 4**

## 08. Slopes and Deflections

**01. Ans: (c)**

**Sol:**



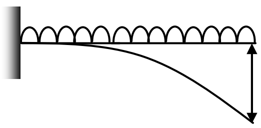

$$y_{max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

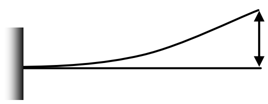
$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

**02. Ans: (b)**

**Sol:** Total load W= w$l$



$$y_{max} = \frac{W\ell^3}{8EI} \text{ (Downward)}$$



$$y_{max} = \frac{W\ell^3}{3EI} \text{ (Upward)}$$

$$y_{net} = \downarrow y_{udl} - \uparrow y_W$$

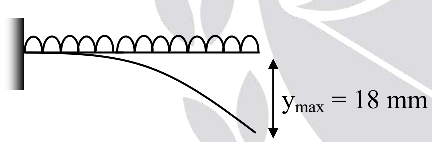
$$\text{Total Net deflection} = \frac{WL^3}{8EI} - \frac{WL^3}{3EI}$$

$$= \frac{-5WL^3}{24EI}$$

(Negative sign indicates upward deflection)

**03. Ans: (c)**

**Sol:**




$$\theta_{max} = \frac{wl^3}{6EI} = 0.02 \quad \text{--------(i)}$$

$$y_{max} = \frac{wL^4}{8EI}$$

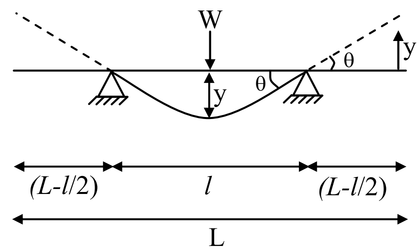
$$\therefore\ 0.018 = \left(\frac{WL^3}{6EI}\right) \times \frac{L \times 6}{8}$$

$$\therefore\ 0.018 = \frac{0.02 \times L \times 6}{8} \qquad [\because \text{ Equation (i)}]$$

$$\Rightarrow \quad L = 1.2 \text{ m}$$

**04. Ans: (a)**

**Sol:**




Conditions given

$$\downarrow y = \frac{wl^3}{48EI} \qquad \theta = \frac{wl^2}{16EI}$$

$$\tan\theta = \frac{y}{(L-\ell)/2}$$

θ is small ⇒ tan θ = θ

$$\therefore\ \theta = \frac{y}{(L-\ell)/2} \qquad \therefore y = \theta\left(\frac{L-\ell}{2}\right)$$

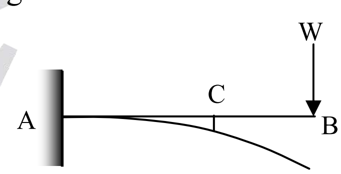
$$\uparrow y = \theta\left(\frac{L-\ell}{2}\right)$$

Thus $y\downarrow = y\uparrow$

$$\therefore\ \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-\ell}{2}\right) \Rightarrow \qquad \frac{L}{\ell} = \frac{5}{3}$$

**05. Ans: (c)**

**Sol:** By using Maxwell's law of reciprocals theorem




$$\delta_{C/B} = \delta_{B/C}$$

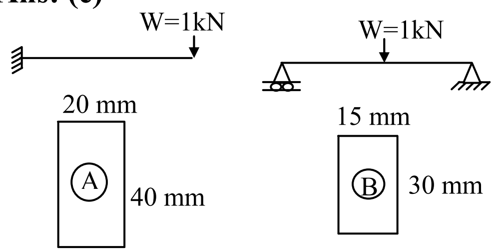
Deflection at 'C' due to unit load at 'B'

= Deflection at 'B' due to unit load at 'C'

As the load becomes half deflection becomes half.

**06. Ans: (c)**

**Sol:**

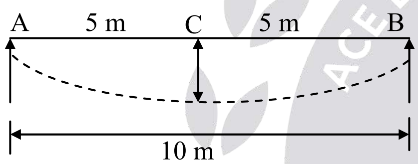



$$y_A = y_B \Rightarrow \left(\frac{wL^3}{3EI}\right)_A = \left(\frac{wL^3}{48EI}\right)_B$$

$\therefore$ $L_B$ = 400 mm

**07. Ans: 0.05**

**Sol:**




$\therefore$ Curvature, $\dfrac{d^2y}{dx^2} = 0.004$

Integrating with respect to x,

We get, $\dfrac{dy}{dx} = 0.004x$

$$y = \frac{0.004x^2}{2}$$

$$y = 0.002x^2$$

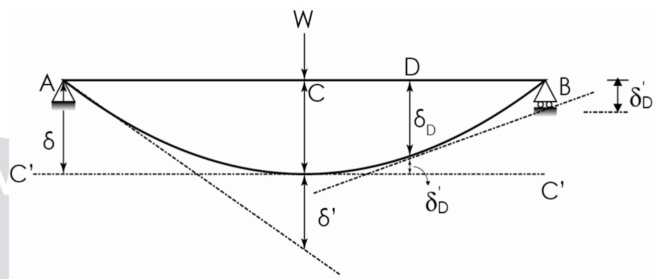
At mid span, x = 5 m

$\therefore$ $y = 0.002\,x^2$

$y = 0.05$ m

**08. Ans: (c)**

**Sol:** According to Mohr's second moment area theorem displacement of 'B' from tangent at A = moment of area of $\dfrac{M}{EI}$ diagram between A and B taken about B.




Tangent at 'C' i..e C' - C' is horizontal

Moment of area of $\dfrac{M}{EI}$ diagram between A and 'C' about 'A' gives, displacement of 'A' ($\delta_A$) w.r.t tangent at 'C' (i.e. 'δ' as shown in figure).

$\because \delta_A = 0$ and tangent at 'C' is at a distance of $\delta = \delta_c$ from the beam so we get '$\delta_c$' from the above analysis.

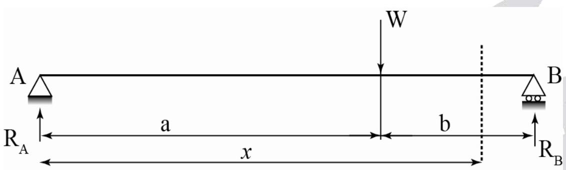
**Note:**

1) This deflection is valid only for midpoint for a simply supported beam with a load at midspan. For any other section, actual deflection is not same if calculated as above, as shown for point 'D'. $\delta_D$ is actual deflection at 'D', is the value obtained if moment of area between B and D is taken about 'B'.

2) If moment of area of $\frac{M}{EI}$ diagram between A and C about 'C' is taken it gives deflection of 'C' w.r.t tangent at 'A' i.e. $\delta'$ shown in figure (Which is not the deflection of 'C').

**09. Ans: (d)**

**Sol:**




Consider a load 'W' on a simply supported beam AB of length '$\ell$' at a distance a from A.

$$\left(a > \frac{\ell}{2}\right)$$

For equilibrium: $\Sigma F_y = 0$ ; $\Sigma M_z = 0$

$$R_A + R_B = W$$

Taking moments about A;

$$(R_B + \ell) - W(a) = 0$$

$$\Rightarrow R_B = \frac{Wa}{\ell}$$

$$R_A = \frac{W(\ell - a)}{\ell} = \frac{W(b)}{\ell}$$

**Using Macaulay's method:**

Consider a section x-x at a distance 'x' from A

$$EI\frac{d^2y}{dx^2} = -M,$$

$$= -(R_A x - W < x - a >)$$

Integrating $EI\frac{dy}{dx} = \frac{W(x-a)^2}{2} - \frac{R_A x^2}{2} + C_1$

**Integrating:**

$$EIy = \frac{W(x-a)^3}{6} - \frac{R_A}{6}x^3 + C_1 x + C_2$$

At A; $x = 0$; $y = 0$ $\quad \therefore C_2 = 0$

At B; $x = \ell$; $y = 0$

$$0 = \frac{W(\ell - a)^3}{6} - \frac{R_A \ell^3}{6} + C_1 \ell$$

$$C_1 = \frac{-1}{6}\left[\frac{W(\ell-a)^3}{\ell} - \frac{W(\ell-a)\ell^2}{\ell}\right]$$

$$= \frac{W}{6}\frac{(\ell - a)}{\ell}\left(-(\ell - a)^2 \ell^2\right)$$

$$= \frac{W}{6}\frac{(\ell - a)}{\ell}\left(-a^2 + 2a\ell\right)$$

$$\therefore EIy = \frac{W(x-a)^3}{6} - \frac{W(\ell - a)}{6\ell}x^3 + \frac{W}{6}\frac{(\ell - a)}{\ell}\left(2a\ell - a^2\right)x$$

For maximum deflection $\frac{dy}{dx} = 0$

$$EI\frac{dy}{dx} = \frac{W(x-a)^2}{2} - \frac{W(\ell - a)x^2}{2\ell} + \frac{W}{6}\frac{(\ell - a)}{\ell}\left(2a\ell - a^2\right) = 0$$

**Solving equation:**

For $x < a$; $\left(a > \frac{\ell}{2}\right)$

$$\therefore \frac{-W}{2\ell}(\ell - a)x^2 + \frac{W}{6\ell}(\ell - a)\left(2a\ell - a^2\right) = 0$$

$$\therefore\ x = \sqrt{\frac{(2a\ell - a^2)}{3}} < a$$

$$a > \frac{\ell}{2} \Rightarrow 2a > \ell$$

$$4a^2 > 2a\ell$$

$$3a^2 > 2a\ell - a^2$$

$$\frac{2a\ell - a^2}{3} < a^2$$

$$\sqrt{\frac{2a\ell - a^2}{3}} < a$$

x is real number if $2a\ell - a^2 > 0 \Rightarrow a < 2\ell$ (possible)

$$x > 0 \text{ and } a > \frac{\ell}{2}$$

$\therefore$ Max deflection occurs at $x = \sqrt{\frac{2a\ell - a^2}{3}}$

Checking if $x > \frac{\ell}{2}$

Assuming $x > \frac{\ell}{2} \Rightarrow \sqrt{\frac{2a\ell - a^2}{3}} > \frac{\ell}{2}$

$$\Rightarrow 8a\ell - 4a^2 > 3\ell^2$$

$$\Rightarrow 4a^2 - 8a\ell + 3\ell^2 < 0$$

$$\Rightarrow \left(a - \frac{\ell}{2}\right)\left(a - \frac{3\ell}{2}\right) < 0$$

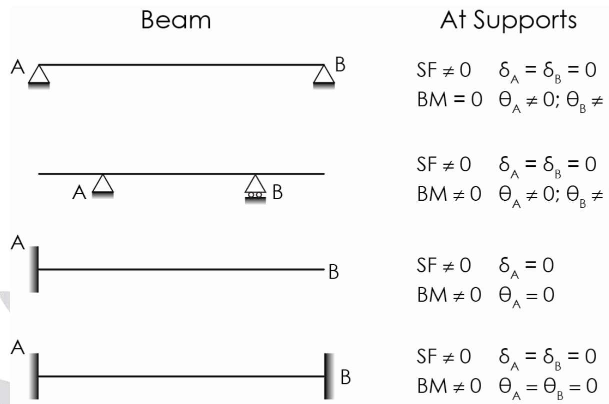
$\therefore$ Case 1: $a > \frac{\ell}{2};\ a < \frac{3\ell}{2}$ (possible)

$\therefore$ Case 2: $a < \frac{\ell}{2};\ a < \frac{3\ell}{2}$ (possible)

$\therefore$ Conclusion: $x<a;\ x > \frac{\ell}{2}$ Max deflection occurs between midspan and point of application of load.

**10. Ans: (b) or (c)**

**Sol:**




**11. Ans: (b)**

**Sol: Refer to the solution of Q.No. 08**

**12. Ans: (a)**

**Sol:** Conjugate beam is an imaginary beam for which loading = $\frac{M}{EI}$ diagram of real beam and is based on

- Slope at a section in real beam = shear force at that section in conjugate beam.
- Deflection at a section in real beam = Bending moment at that section in conjugate beam.

$\therefore$ For a simple support of real beam $\theta \neq 0$

$$\delta = 0$$

$\therefore$ Corresponding support in conjugate beam should have SF $\neq 0$

$$BM = 0$$

And the support corresponding to this condition is simple support.

**13. Ans: (c)**

**Sol:**

**1. According to Castigliano's second theorem:**

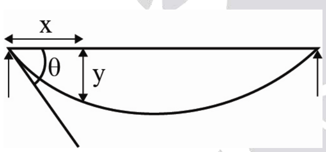
Partial derivative of strain energy w.r.t concentrated external load is the deflection of the structure at the point of application and in the direction of load.

$$\Rightarrow \frac{\partial U}{\partial P} = '\delta'$$

**2. Derivative of deflection:**

If y is the deflection; then

$$\frac{dy}{dx} = \text{slope}$$




**3. Derivative of slope:**

If $\frac{dy}{dx}$ is the slope, then Derivative of slope multiplied with EI gives Bending moment

$$\Rightarrow EI\frac{d^2y}{dx^2} = M$$

$\therefore$ There is no exact answer, but the most suitable answer is 'C'

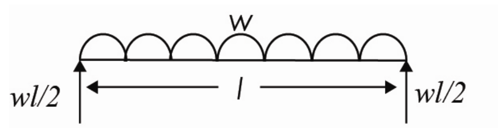
**4. Derivative of moment :**

First derivative of bending moment gives shear force.

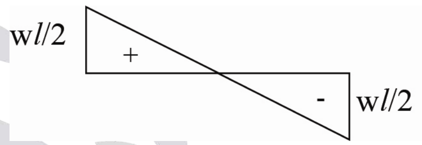
$$\frac{dM}{dx} = V$$

**14. Ans: (b)**

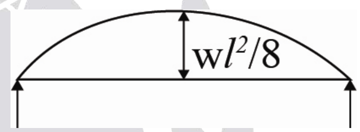
**Sol:** For a simply supported beam subjected to uniformly distributed loads.



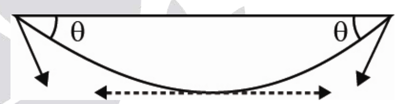

**SFD:**




**BMD:**




**Deflection profile:**




**Conclusions:**

(i) Bending moment is maximum at centre and zero at support.

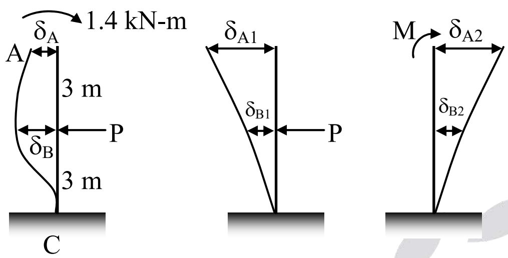
(ii) Shear force is maximum at supports and zero at centre.

(iii) Slope is maximum at supports and zero at midspan.

## Conventional Practice Solutions

**01.**

**Sol:**




(i) $\delta_A = \delta_B$ (on same/left side)

$$(\delta_{B1} - \delta_{B2}) = (\delta_{A1} - \delta_{A2})$$

$$\Rightarrow \delta_{B1} = \frac{P(3)^3}{3EI}, \quad \delta_{B2} = \frac{(1.4)6^2}{8EI}$$

$$\delta_{A1} = \frac{P(3)^3}{3EI} + \frac{P(3)^2}{2EI}(3)$$

$$\delta_{A1} = \frac{1.4 \times 6^2}{2EI}$$

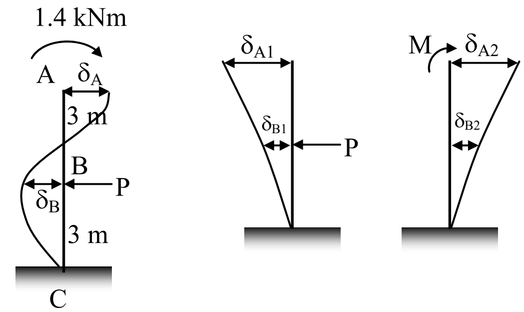
| Real | C.B |
|---|---|
| $Y_A$ | $M_A = \left(\frac{M}{EI}.\ell\right) = \frac{ML^2}{2EI}$ |
| $y_B$ | $M_B = \frac{M}{EI}\left(\frac{\ell}{2}\right)\left(\frac{\ell}{4}\right) = \frac{ML^2}{8EI}$ |

$$\frac{P(3)^3}{3EI} - \frac{1.4 \times 6^2}{8EI} = \frac{P(3)^3}{3EI} + \frac{P(3)^2 \times 3}{2EI} - \frac{1.4 \times 6^2}{2EI}$$

$$\Rightarrow \frac{216P - 151.2}{24} = \frac{54P + 81P - 151.2}{6}$$

$$\Rightarrow P = 1.4 \text{ kN}$$

(ii)




$\delta_A = \delta_B$ (on opposite side)

$(\delta B_1 - \delta B_2) = (\delta A_2 - \delta A_1)$

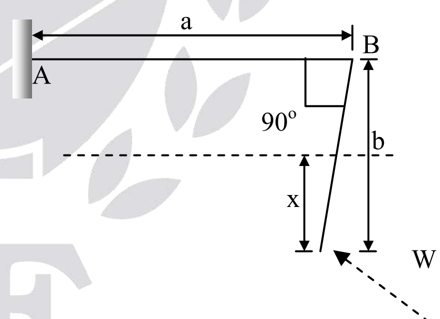
$$\Rightarrow \frac{P(3)^3}{3EI} - \frac{1.4 \times 6^2}{8EI} = \frac{1.6 \times 6^2}{2EI} - \left(\frac{P(3)^3}{3EI} + \frac{P(3)^2}{2EI}(3)\right)$$

$$\Rightarrow \frac{216P - 151.2}{24} = \frac{151.2 - 54P - 81P}{6}$$

$$\Rightarrow P = 1 \text{ kN}$$

**02.**

**Sol:**




Deflection at free end $\int_0^{\ell} \frac{Mmdn}{EI}$

M = Moment due to applied load

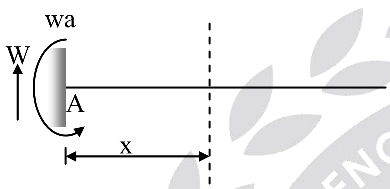
m = Moment due to unit load at which deflection is to be calculated

**In Span BC:**

$$\delta_1 = \int_0^b \frac{(W.x)(x)du}{EI}$$

$$= \left.\frac{wx^3}{3EI}\right]_o^b = \frac{wb^3}{3EI} = \frac{wb^3 64}{3E\times\pi d^4} = \frac{64wb^3}{3E\pi d^4}(\downarrow)$$

**In Span AB:**




$$\delta_2 = \int_0^a \frac{Mmdx}{EI} = \int \frac{(Wa - Wx)(a-x)dx}{EI}$$

$$= \int_0^a \frac{W(a-x)^2 dx}{EI} = \left.\frac{W(a^2x + \frac{x^3}{3} - ax^2)}{EI}\right]_0^a$$

$$= \frac{W\left(a^3 + \frac{a^3}{3} - \frac{a^2}{2}\right)}{EI} = \left(\frac{6+2-3}{6}\right)\frac{Wa^3}{EI}$$

$$= \frac{5}{6}\frac{wa^3}{EI}(\uparrow)$$

$$= \frac{5}{6}\frac{wa^3}{EI}\frac{6}{\pi d^4}(\uparrow)$$

$$\delta_2 = \frac{160wa^3}{3E\pi d^4}(\uparrow)$$

$\delta_3$ = Deflection due to for torsion in span AB

$$= \frac{TL}{GJ}\times b$$

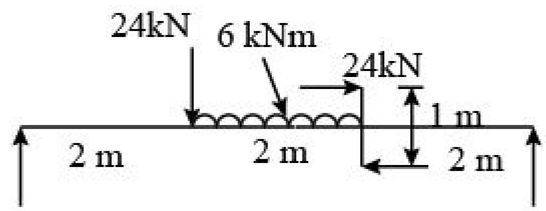
$$\delta_3 = \frac{(Wb)(a)}{G\frac{\pi}{32}d^4}b = \frac{32Wb^2a}{Gnd^4}$$

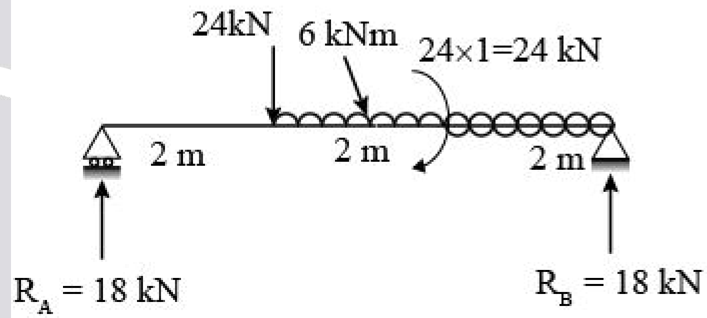
Net deflection = $\delta_1 + \delta_2 + \delta_3$

$$\delta = \frac{W}{3\pi Ed^4} = [64b^3 + 160a^3 + 96b^2a]$$

**03.**

**Sol:**




$\sum f_y = 0$

$R_A + R_B = 24 + 12 = 36$

$\sum M_A = 0$

$$-R_B\times 6 + 24 + 6\times 2\times\left(\frac{2}{2}+2\right) + 24\times 2 = 0$$

$\Rightarrow R_B = 18$ kN

$\Rightarrow R_A = 36 - 18 = 18$ kN

$$M_x = R_A(x) - 24(x-2) - 6(x-2)\left(\frac{x-2}{2}\right)$$
$$+6(x-4) - - - - - - - -$$

$$EI.\frac{d^2y}{dx^2} = 18(x) - 24(x-2) - 3(x-2)^2 + 3(x-4)^2$$
$$+24(x-4) - - - - -$$

$$EI.\frac{dy}{dx} = 18\left(\frac{x^2}{2}\right) - \frac{24}{2}(x-2)^2 0 \frac{3}{3}(x-2)^3 + \frac{3}{3}(x-4)^3$$
$$+24(x-4)' + C_1$$

@A $x = 0,\ y = 0$

$C_2 = 0$

@B $x = 6m,\ y = 0$

$$0 = \frac{18}{2}\left(\frac{6^3}{3}\right) - 4(6-2)^3 - \frac{(6-2)^4}{4} + \frac{(6-4)^4}{4} + 12(6-4)^2$$
$$+C_1 \times 6 + C_2$$

$\Rightarrow C_1 = -63.3$

Deflection @ mid point (x = 3m)

$$EI(y) = \frac{18}{2}\left(\frac{3^3}{3}\right) - \frac{24}{6}(3-2)^3 - \frac{(3-2)^4}{4} + \frac{(3-4)^4}{4} + \frac{24}{2}(3-4)^2$$
$$+(-63.3)\times 3 + 0$$

$EI(g) = -113.24$

$$y = -\frac{113.24}{EI}$$

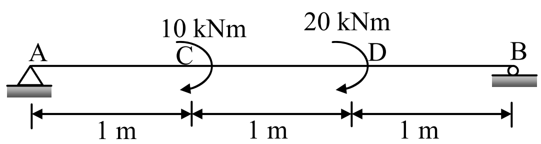
$EI = 20\ MN\text{-}m^2$

$EI = (20\times10^3)kN - m^2$

$$y = -\frac{113.24}{20\times10^3} = 5.66\times10^{-3}\ m = 5.66\ mm$$

**04.**

**Sol:**



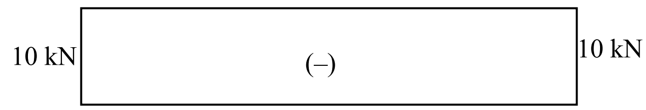

$\Sigma M_A = 0$

$10 + 20 = R_B \times 3$

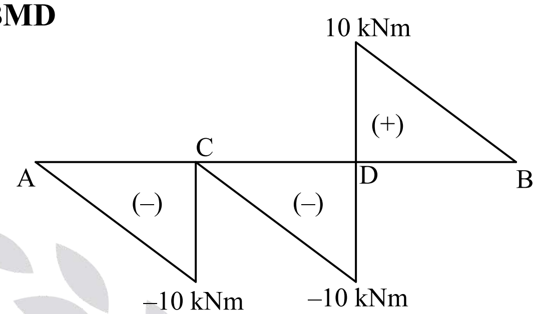
$R_B = 10$ kN (↑)

$R_A = 10$ kN (↓)

**SFD**




**BMD**

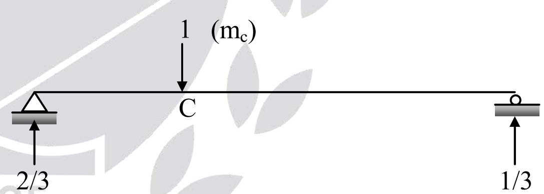


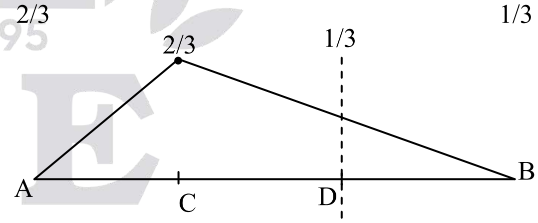

Deflection at Point C = $\int \frac{Mmdx}{EI}$

M = Actual bending moments in the beam

m = moment due to unit load at which deflection is to be calculated

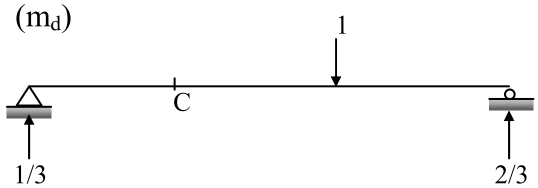
$$= \int_{AC} \frac{Mmdx}{EI} + \int_{CD} \frac{Mmdx}{EI} + \int_{DB} \frac{Mmdx}{EI}$$

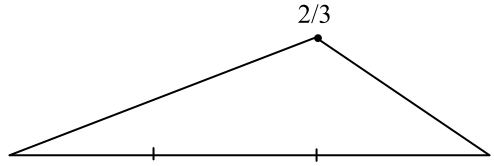



| Span | M | mC | dn | md |
|---|---|---|---|---|
| AC | –10x | 2/3x | (0–1) | $\frac{1}{3}x$ |
| CD | –10(x–1) | $\frac{2}{3}(2-x)$ | (1 –2) | $\frac{1}{3}x$ |
| DB | 10(3–x) | $\frac{1}{3}(3-x)$ | (2–3) | $\frac{2}{3}(3-x)$ |

($m_d$)




Deflection at C $\sum\int\frac{Mm_c dx}{EI}$

$$=\int_0^1\frac{(-10x)\left(\frac{2}{3}x\right)dx}{EI}+\int_1^2\frac{-10(x-1)\frac{2}{3}(2-x)dx}{EI}$$

$$+\int_2^3\frac{10(3-x)\frac{1}{3}(3-x)dx}{EI}$$

$$=\frac{-2.22}{EI}-\frac{1.11}{EI}+\frac{1.11}{EI}=\frac{-2.22}{EI}$$

**Deflection at D:**

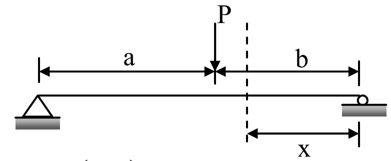
$$=\int_0^1\frac{(-10x)\left(\frac{1}{3}x\right)dx}{EI}+\int_1^2\frac{-10(x-1)\frac{1}{3}xdx}{EI}+\int_2^3\frac{10(3-x)\frac{2}{3}(3-x)dx}{EI}$$

$$=-\frac{1.11}{EI}-\frac{2.77}{EI}+\frac{2.22}{EI}=-\frac{1.657}{EI}$$

Ratio $\frac{\delta_C}{\delta_D}=\frac{-2.22}{EI}\times\frac{-EI}{1.657}=1.34$

**05.**

**Sol:**




$$M_x=\left(\frac{Pa}{L}\right)$$

$$EI\frac{d^2y}{dx^2}=\left(\frac{Pax}{L}\right)\text{ for }0\le x\le b\ \ldots\ldots\ldots..(1)$$

$$EI\frac{d^2y}{dx^2}=\left(\frac{-Pa}{L}\right)(L-x)\text{ for }b\le x\le L\ldots(2)$$

Integrate (1)

$$EI\frac{dy}{dx}=\left(\frac{Pax^2}{2L}\right)+C_1-(\ell)$$

$$EIy=\frac{Pax^3}{6L}+C_1x+C_2-(m)$$

For 2 integrate

$$EI\frac{dy}{dx}=Pax-\frac{Pax^2}{2L}+D_1-(n)$$

$$EIy=\frac{Pax^2}{L}-\frac{Pax^3}{6L}+D_1n+D_2-(0)$$

At x = u $\quad$ y = 0

At x = L $\quad$ y = 0

$\left(\frac{dy}{dx}\right)_{x=a}\ \ell=n$

$(y)_{x=0}\ \ m=0$

so solving

$$C_1=\frac{Pb}{6L}(L^2-b^2)\ C_2=0\ \ D_1=\frac{Pa}{6L}(2L^2+a^2)$$

$$D_2=\frac{Pa^2}{6EI}$$

$$EIy = -\left(\frac{Pbx}{6L}\right)(L^2 - b^2 - x^2) - (P)$$

$$EIy = \left(\frac{Pb}{6L}\right)\left[\left(\frac{L}{b}\right)(x-a)^3 + (L^2 - b^2)x - x^3\right]$$

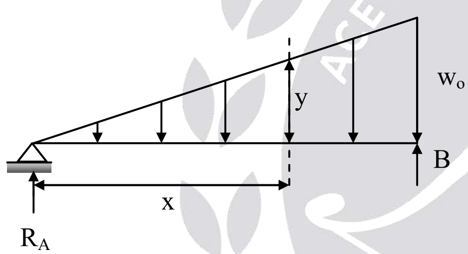
For a > b

Maximum deflection will occur in left position of beam so which equation (P) applies

$$x = \sqrt{\frac{a(a+2b)}{3}} = \sqrt{\frac{L^2 - b^2}{3}}$$

**06.**

**Sol:**




$$M = R_A x - \frac{1}{6}x^2 y \qquad \left[y = \frac{w_o x}{L}\right]$$

$$M = R_A x - \frac{w_o x^3}{6L}$$

$$EI\frac{dy}{dx} = R_A x - \frac{w_o x^3}{6L}$$

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - \frac{w_o x^4}{24L} + C_1$$

$$EIt = \frac{R_a x^3}{2} - \frac{w_o x^5}{120L} + C_1 x + C_2$$

at x = 0, y = 0, so $C_2 = 0$

At x = L, y = 0

$$\text{So, } C_1 = \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

Deflection = x

$$EIy = \frac{R_A x^3}{6} - \frac{w_o L^5}{120L} + \left(\frac{w_o L^3}{24} - \frac{R_A L^3}{2}\right)L$$

At x = L y = 0

$$R_A = \frac{w_o L}{10}$$

Slope at propped end

$$EI\left(\frac{dy}{dx}\right)_{x=0} = \frac{R_A x^2}{2} - \frac{w_o x^4}{24L} + \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

$$= \frac{R_A L^2}{2} - \frac{w_o L^3}{24} + \frac{w_o L^3}{24} - 0 - \frac{R_A L^2}{2}$$

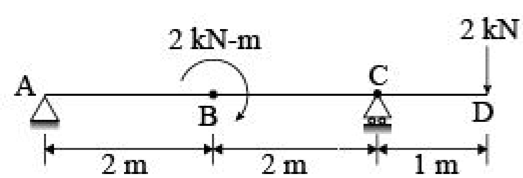
$$= \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

$$EI\left(\frac{dy}{dx}\right)_{x=0} = \frac{w_o L^3}{24} - \frac{w_o L^3}{20} = \left(\frac{-w_o L^3}{120}\right)$$

$$\left(\frac{dy}{dx}\right)_{x=0} = -\frac{w_o L^3}{120EI}$$

**07.**

**Sol:**



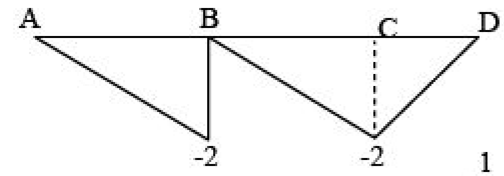

$\Sigma M_A = 0$

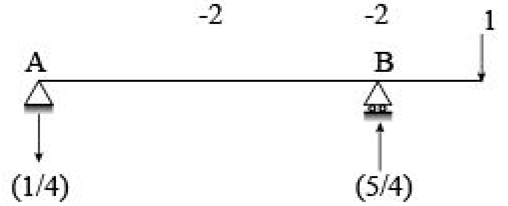
$2 + (2 \times 5) = R_c \times 4$

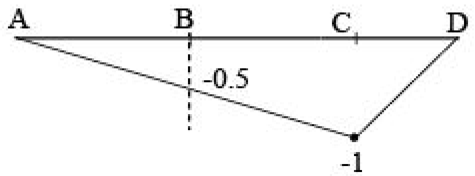
$R_c = 3$ kN (↑)

$R_A = 1$ kN (↓)

**BMD:**




| Span | M | m | dx |
|---|---|---|---|
| AB | –x | –x/4 | (0-2) |
| BC | –(x–2) | –1/4 (x) | (2-4) |
| CD | –2x | –x | (0-1) |

$$\text{Deflection } = \Sigma\int \frac{\text{Mmdx}}{\text{EI}}$$

$$\int_0^2 \frac{(-x)\left(\frac{-x}{4}\right)dx}{EI} + \int_0^4 \frac{-(x-2)\left(\frac{-1}{4}x\right)dx}{EI} + \int_0^1 \frac{)2x)(-x)dx}{EI}$$

$$\frac{0.67}{EI} + \frac{1.67}{EI} + \frac{0.67}{EI} = \delta$$

$$\frac{3}{EI} = \delta$$

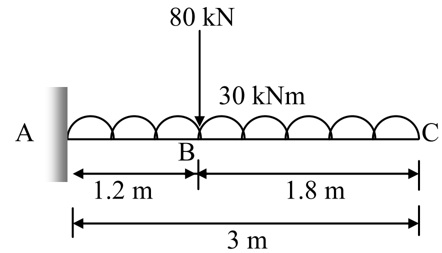
$$EI = \frac{3}{\delta}$$

$\delta = 0.001$ m

EI = 3000 kNm$^2$

**08.**

**Sol:**

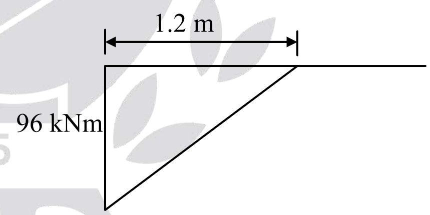


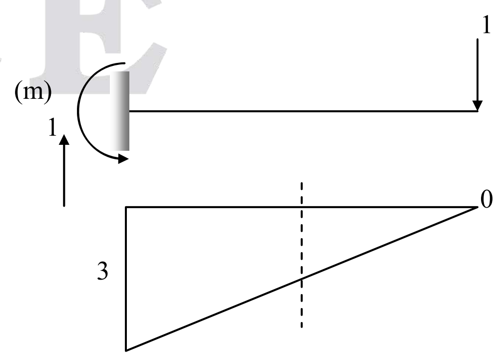

Total deflection ($\delta$) =


$$\delta_1 = \frac{w\ell^4}{8EI} = \frac{30\times 3^4}{8EI} = \frac{303.75}{EI}$$

$$\delta_2 = \frac{\text{Mmdx}}{\text{EI}}$$

**BMD**



| Span | M | m | dn |
|---|---|---|---|
| AB | (96–80x) | (3–x) | (0-1.2) |
| BC | 0 | | |

$$\delta_2 = \int \frac{Mmdx}{EI} = \int_0^{1.2} \frac{(96-90x)(3-x)dx}{EI} = \frac{149.76}{EI}$$

$$\delta = \delta_1 + \delta_2$$

$$= \frac{303.75}{EI} + \frac{149.26}{EI}$$

$$= \frac{453.51}{EI}(\downarrow)$$

## 09. Thin & Thick Pressure Vessels

**01. Ans: (b)**

**Sol:** $\tau_{max} = \sigma_1 = \frac{\sigma_h - 0}{2} = \frac{PD}{4t}$

$$\therefore \tau_{max} = \frac{1.6 \times 900}{4 \times 12} = 30 \text{ MPa}$$

**02. Ans: 2.5 MPa & 2.5 MPa**

**Sol:** Given data:

R = 0.5 m, D = 1m, t = 1mm,

H = 1 m, $\gamma = 10$ kN/m$^3$, h = 0.5 m

*At mid-depth of cylindrical wall (h = 0.5m)*:

Circumferential (hoop) stress,

$$\sigma_c = \frac{P_{at\ h=0.5m} \times D}{4t} = \frac{\gamma h \times D}{4t}$$

$$= \frac{10 \times 10^3 \times (2 \times 0.5)}{4 \times 1 \times 10^{-3}}$$

$$= 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$$

Longitudinal stress at mid-height,

$$\sigma_\ell = \frac{\text{Net weight of the water}}{\text{Cross-section area}}$$

$$= \frac{\gamma \times \text{Volume}}{\pi D \times t}$$

$$= \frac{\gamma \times \frac{\pi}{4} D^2 L}{\pi D \times t} = \frac{\gamma \times DL}{4t}$$

$$= \frac{10 \times 10^3 \times 1 \times 1}{4 \times 10^{-3}}$$

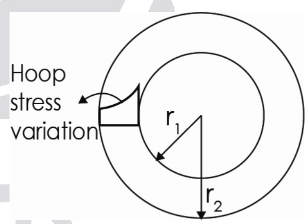
$$= 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$$

**03. Ans: (c)**

**Sol:** According to Lame's theorem

Hoop stress = $\sigma_x = \frac{b}{x^2} + a$

Stress variation is hyperbolic with maximum stress on the inner surface.

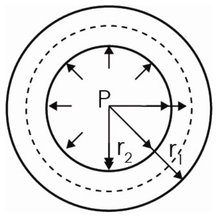



**04. Ans: (b)**

**Sol:** According to Lame's equation for thick cylinders hoop stress $\sigma_x = \frac{b}{x^2} + a$

$$b = \frac{Pr_1^2 r_2^2}{(r_1^2 - r_2^2)}$$

$$a = \frac{Pr_2^2}{(r_1^2 - r_2^2)}$$




$\therefore$ For internal pressure a, b are positive

For external pressure

$\therefore$ 'P' is negative; a, b are negative.

**05. Ans: (a)**

**Sol: Thin Cylinders:** If the thickness of the wall of the cylinders is less than of its diameters. In the design of thin cylinders, it is assumed that circumferential stress or hoop stress is uniformly distributed through the thickness of the wall.

Hoop stress, $\sigma_b = \dfrac{Pd}{2t}$

Longitudinal stress $= \dfrac{Pd}{4t}$

$= \dfrac{1}{2}\sigma_h$

**06. Ans: (d)**

**Sol:** Thin cylinder are designed based on the assumption the circumferential stress distribution is uniform over the thickness of the wall as the variation is negligible.

But in case of thick cylinders, circumferential stress is not uniform but varies from maximum at inner side to minimum at outer side.

## Conventional Practice Solutions

**01.**

**Sol:** Thin Sphere

$$\sigma_1 = \sigma_n = \frac{PD}{4t}$$

$$\sigma_2 = \dot{\sigma}_h = \frac{PD}{4t}$$

$$\sigma_3 = \sigma_R = 0$$

$$\varepsilon_V = \frac{\delta v}{V} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1-2\mu)$$

$$\frac{\delta v}{\frac{4}{3}\pi R^3} = \frac{\frac{PD}{4t} + \frac{PD}{4t} + 0}{E}(1-2\mu)$$

$$\Rightarrow \delta V = \frac{3}{2}\pi \times R^3 \times \frac{PD}{4t \times E}(1-2\mu)$$

$$\delta v = \frac{3}{8} \times \pi \times R^3 \frac{PD}{Et}(1-2\mu)$$

**02.**

**Sol:** D = 400 mm, f = 2mm, P = 1.5 N/mm$^2$

Thin sphere

$$\frac{\delta v}{V} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1-2\mu)$$

$$\sigma_1 = \sigma_2 = \frac{PD}{4t} = \sigma_h$$

$\sigma_h$ = 75 MPa

$$\frac{\delta v}{\frac{\pi}{6} \times 400^3} = \frac{2 \times 75}{1 \times 10^5}(1 - 2 \times 0.25)$$

$\Rightarrow \delta v$ = 25132.74 74 mm$^3$

Decrease in volume of water (compression of water)

$$K = \frac{\sigma}{\varepsilon_v} = \frac{P}{\left(\frac{\delta v_w}{V}\right)}$$

$$2.5\times10^3 = \frac{1.5}{\frac{\delta v_w}{\frac{\pi}{6}\times400^3}}$$

$\delta v_w = 20106.192\ mm^3$

Additional water to be added = $\delta v + \delta v_3$

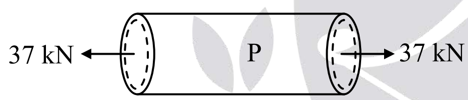
= 25132.74 + 20106.192

= $45238.932\ mm^3$

= $45.238\ cm^3$ or cc

**03.**

**Sol:**




Due to internal pressure (P)

$$\frac{\varepsilon v}{V} = \varepsilon_{v_1} = \frac{\sigma_1+\sigma_2+\sigma_3}{E}(1-2\mu) \quad \text{------(1)}$$

Due to axial force alone (tensile)

$$\Rightarrow \varepsilon_v = \frac{\sigma_1+\sigma_2+\sigma_3}{E}(1-2\mu)$$

$$\sigma = \frac{P}{A} = \frac{37\times10^3}{\frac{\pi}{4}(150)^2} = 2.093$$

$\sigma_2 = 0$

$\sigma_3 = 0$

$$\Rightarrow \varepsilon_{v_2} = \frac{2.093+0+0}{140\times10^3}(1-2\mu) \quad \text{---------(2)}$$

$\delta v_1 - \delta v_2$ = Causing pressure drop on water

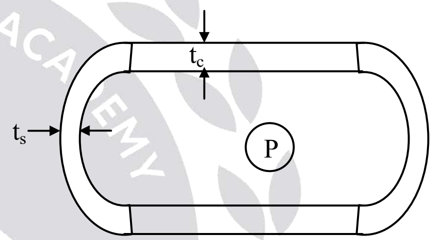
$$K = \frac{P}{\varepsilon_v} = \frac{P}{\left(\frac{\delta v_1-\delta v_2}{V}\right)}$$

$$\Rightarrow \delta v_1 - \delta v_2 = \frac{P.V}{K_{water}} = \frac{0.1\times\frac{\pi}{4}\times150^2}{2200}$$

$$= \frac{0.1V}{2200}$$

**04.**

**Sol:**




at the junction equal deformation is required

$$(\varepsilon_h)_c = (\varepsilon_h)_{sphere}$$

$$\left(\frac{\sigma_h}{E}-\mu\frac{\sigma_L}{E}-\mu\frac{\sigma_R}{E}\right) = \left(\frac{\sigma_h}{E}-\mu\frac{\sigma_h}{E}-\mu\frac{\sigma_R}{E}\right)$$

$$\frac{Pd}{2t_c}-\mu\frac{Pd}{4t_c} = \frac{Pd}{2t_s}-\mu\frac{Pd}{2t_s}$$

$$\Rightarrow \frac{Pd}{4}\left[\frac{2}{t_c}-\frac{\mu}{t_c}\right] = \frac{Pd}{2}\left[\frac{1}{t_s}-\frac{\mu}{t_s}\right]$$

$$\frac{Pd}{4}\times\frac{1}{t_c}\times(2-\mu) = \frac{Pd}{2}\times\frac{1}{t_s}\times(1-\mu)$$

$$\frac{t_c}{t_s} = \frac{2-\mu}{2(1-\mu)}$$

## 10. Columns

**01. Ans: (c)**

**Sol:** By using Euler's formula, $P_e = \dfrac{\pi^2 \times EI}{l_e^2}$

For a given system, $l_e = \dfrac{l}{2}$

$\therefore \quad P_e = \dfrac{4\pi^2 \times EI}{l^2}$

**02. Ans: (b)**

**Sol:** We know that, $P_{cr} = \dfrac{\pi^2 EI}{\ell_e^{\,2}}$

$\therefore \; P_{cr} \propto \dfrac{1}{\ell_e^{\,2}}$

$\therefore \; \dfrac{P_1}{P_2} = \dfrac{l_{2e}^2}{l_{1e}^2}$

$\therefore \; \dfrac{P_1}{P_2} = \dfrac{l^2}{(2l)^2} \Rightarrow P_1 : P_2 = 1 : 4$

**03. Ans: 4**

**Sol:** Euler's crippling load,

$$P = \frac{\pi^2}{l^2} EI$$

$\therefore \; P \propto I$

$$\Rightarrow \frac{P}{P_o} = \frac{I_{\text{bonded}}}{I_{\text{loose}}} = \frac{\left[\dfrac{b(2t)^3}{12}\right]}{2\left[\dfrac{bt^3}{12}\right]} = 4$$

**04. Ans: (c)**

**Sol:** Euler's theory is applicable for axially loaded columns.

Force in member AB, $P_{AB} = \dfrac{F}{\cos 45^\circ} = \sqrt{2}F$

$$P_{AB} = \frac{\pi^2 EI}{L_e^{\,2}}$$

$$\therefore \; \sqrt{2}\,F = \frac{\pi^2 EI}{L_e^{\,2}}$$

$$\Rightarrow \quad F = \frac{\pi^2 EI}{\sqrt{2}\,L^2}$$

**05. Ans: (a)**

**Sol:** Given data:

$L_e = L = 3$ m ,

$\alpha = 12 \times 10^{-6}$ /°C,

$d = 50$ mm $= 0.05$ m

Buckling load, $P_e = \dfrac{\pi^2 EI}{L_C^2}$

$$\therefore \quad \frac{P_e L}{AE} = L\alpha\,\Delta T$$

$$\therefore \quad \frac{\pi^2 EI \times L}{L^2 \times AE} = L\,\alpha\,\Delta T$$

$$\therefore \quad \frac{\pi^2 \times E \times \dfrac{\pi}{64} \times d^4 \times L}{L^2 \times \dfrac{\pi}{4} d^2 \times E} = L\alpha\Delta T$$

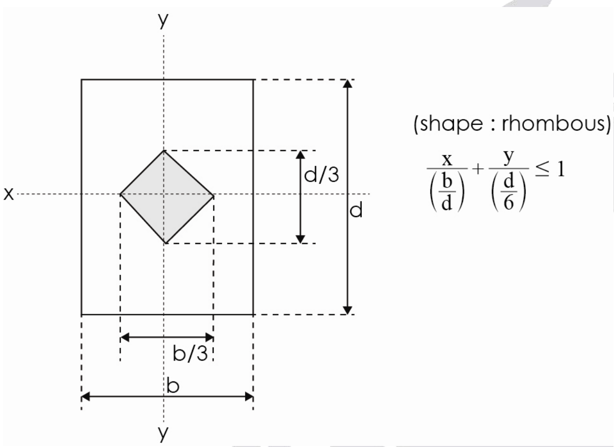
$$\therefore \quad \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}}$$

$$\Rightarrow \; \Delta T = 14.3°C$$

**06. Ans: (b)**

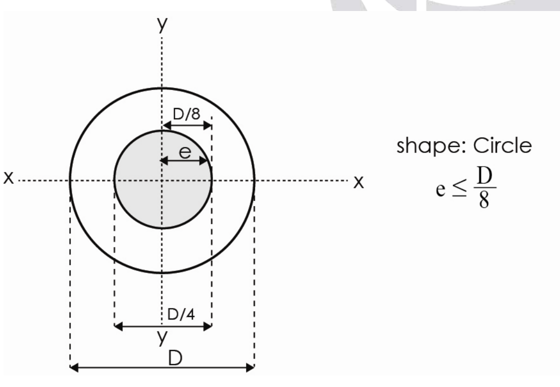
**Sol:** When the load is eccentric, it cause both direct and bending stresses in the member. For the tensile stresses, to not develop in the section; the load must lie within certain cross section of the member. This is called core (or) Kern of the section.

**For rectangle:**



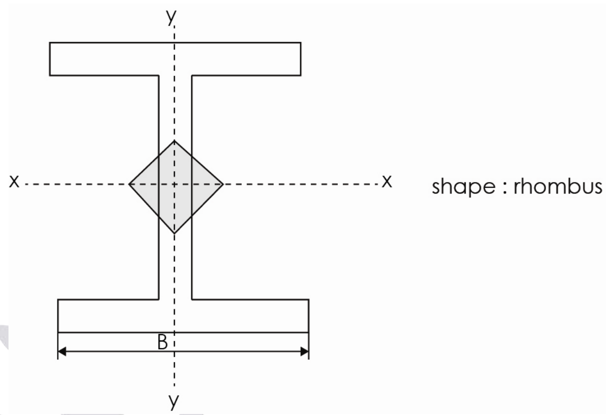

**Note:** For hollow rectangle also shape of kern is rhombus.

**For circular section:**




**Note:** For hollow circular section, also shape of kern is circle.

**For I-section:**




**07. Ans: (c)**

**Sol:** Buckling load of columns (P) = $\frac{\pi^2 EI}{\ell_o^2}$

$\ell_o$ : Effective length, depends on end conditions of the column

$P \propto \frac{1}{\ell_o^2}$ and $\ell_o = K\ell$

$\therefore P \propto \frac{1}{\ell^2}$

$\therefore$ With increasing length, of the column, buckling load decreases.

**08. Ans: (a)**

**Sol:** Modulus of elasticity of high strength alloy steel and ordinary structural steel is almost same.

So, buckling failure strength of high strength alloy steel is approximately same as that of structural steel.

Euler's buckling load = $\frac{\pi^2 EI}{\ell_e^2}$

**09. Ans: (a)**

**Sol:** Euler's buckling load = $P = \dfrac{\pi^2 EI}{\ell_0^2}$

$$\text{Stress} = \frac{P}{A} = \frac{\pi^2 EI}{\ell_e^2 \times A}$$

$$r = \text{radius of gyration} = \sqrt{\frac{I}{A}}$$

$$\Rightarrow \sigma = \frac{\pi E r^2}{\ell_e^2}$$

$$\lambda = \text{slenderness ratio} = \frac{\ell_e}{r}$$

$$\Rightarrow \sigma = \frac{\pi E}{\lambda^2}$$

$$\therefore \sigma \propto \frac{1}{\lambda^2}$$

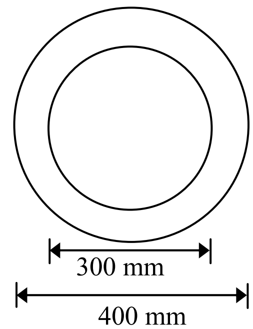
When slenderness ratio is small, stress causing failure will be high according to Euler's formula assuming ideal end conditions.

But this stress must not be greater than crushing stress. Also, in practice the end conditions will not be ideal leading to eccentricity in the loading. This results in bending moment which causes failure before the Euler's load. Hence for slenderness ratio < 120, Euler's theory is not used as it gives high value of failure stress since the crushing effect is not considered.

## Conventional Practice Solutions

**01.**

**Sol:**




D = 400 mm

d = 300 mm

L = 5m

$E = 0.75\times10^5$ N/mm$^2$

F = 5

$$\alpha = \frac{1}{1600}$$

f = 587 N/mm$^2$

**Eulers Load:**

$$P_e = \frac{\pi^2}{\ell^2} E.(I_{min})$$

$$= \frac{\pi^2}{(5000)^2} \times 0.75\times10^5 \left(\frac{\pi}{64}\left(400^4 - 300^4\right)\right)$$

$$= 25.44\times10^6 \text{ N}$$

$$\text{Safe load} = \frac{P_e}{F} = \frac{25.44\times10^6}{5}$$

$$= 5.08\times10^6 \text{ N} = 5.08 \text{ MN}$$

**Rankines Load:**

$$r = \sqrt{\frac{I_{min}}{A}} \quad = \sqrt{\frac{\frac{\pi}{64}\left(400^4 - 300^4\right)}{\frac{\pi}{4}\left(400^2 - 300^2\right)}} = 125$$

$$\lambda = \frac{\ell}{r_{min}} = \frac{5000}{125} = 40$$

$$P_R = \frac{(f)(A)}{1+(\alpha)(\lambda^2)} = \frac{587\times\frac{\pi}{4}\left(400^2-300^2\right)}{1+\left(\frac{1}{1600}\right)\left(40^2\right)}$$

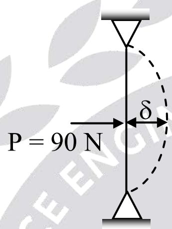
$= 16.13\times10^6$ N

safe load $= \frac{P_R}{F.O.S} = \frac{16.13\times10^6}{5} = 3.2\times10^6$

N = 3.2 MN

**02.**

**Sol:**




$$\delta = 5\,\text{mm} = \frac{P\ell^3}{48EI}$$

$$\Rightarrow 5 = \frac{(90)(1200)^3}{48EI}$$

$\Rightarrow EI = 6.4\times10^8$

**Euler's load:**

$$P_e = \frac{\pi^2 E I_{min}}{L^2} = \frac{\pi^2\times6.4\times10^8}{1200^2}$$

$= 4.4\times10^3$ N

$= 4.4$ kN

## 11. Strain Energy

**01. Ans: (d)**

**Sol:**

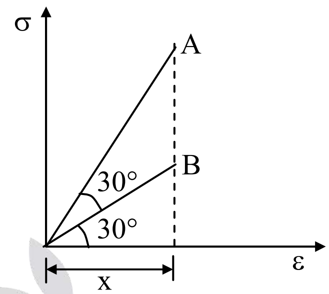
- Slope of the stress-strain curve in the elastic region is called modulus of elasticity.
  For the given curves,
  (Modulus of elasticity)$_A$ > (Modulus of elasticity)$_B$

$\therefore$ $\mathbf{E_A > E_B}$

- The material for which plastic region is more is stress-strain curve is possesed high ductility. Thus, $\mathbf{D_B > D_A}$.

**02. Ans: (b)**

**Sol:**

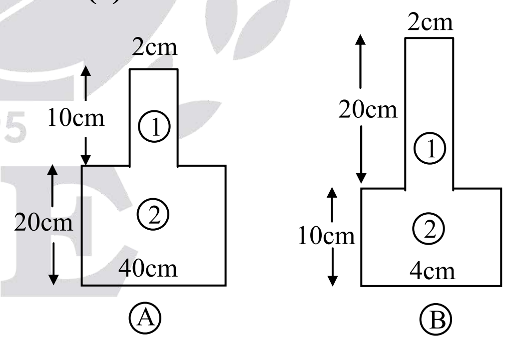



$$\frac{(SE)_A}{(SE)_B} = \frac{\text{Area under curve A}}{\text{Area under curve B}}$$

$$= \frac{\frac{1}{2}\times x\times x\tan60^\circ}{\frac{1}{2}\times x\times x\tan30^\circ} = \frac{3}{1}$$

**03. Ans: (a)**

**Sol:**




$$\frac{U_B}{U_A} = \frac{(V_1+V_2)_B}{(V_1+V_2)_A}$$

$$\therefore\ \frac{U_B}{U_A} = \frac{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_B}{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_A}$$

$$= \frac{\left[\frac{P^2}{A_1^2}\times A_1\times L_1 + \frac{P^2\times A_2\times L_2}{A_2^2}\right]}{\left[\frac{P^2\times A_1\times L_1}{A_1^2}+\frac{P^2\times A_2\times L_2}{A_2^2}\right]_A}$$

$$\Rightarrow \frac{U_B}{U_A} = \frac{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_B}{\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]_A} = \frac{7.165}{4.77} = \frac{3}{2}$$

**04. Ans: (c)**

**Sol:** $A_1$ = Modulus of resilience

$A_1 + A_2$ = Modulus of toughness

$A_1 = \frac{1}{2}\times 0.004\times 70\times 10^6 = 14\times 10^4$

$A_2 = \frac{1}{2}\times\left(0.008\times 50\times 10^6\right)+\left(0.008\times 70\times 10^6\right)$

$= 76\times 10^4$

$A_1 + A_2 = (14 + 76)\times 10^4 = 90\times 10^4$

**05. Ans: (d)**

**Sol:** Strain energy, $U = \frac{P^2}{2A^2E}.V$

$\therefore\ U \propto P^2$

Due to the application of $P_1$ and $P_2$ one after the other

$(U_1 + U_2) \propto P_1^2 + P_2^2$ …….. (1)

Due to the application of $P_1$ and $P_2$ together at the same time.

$U \propto (P_1 + P_2)^2$ ……............(2)

It is obvious that,

$(P_1^2 + P_2^2) < (P_1 + P_2)^2$

$\Rightarrow\ (U_1 + U_2) < U$

**06. Ans: 1.5**

**Sol:** Given data: $L = 100$ mm

$G = 80\times 10^3\ \ N/mm^2$

$J_1 = \frac{\pi}{32}(50)^4\ ;\ J_2 = \frac{\pi}{32}(26)^4$

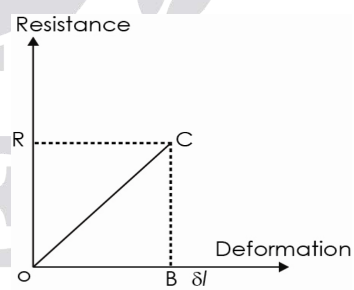
$U = U_1 + U_2 = \frac{T^2L}{2GJ_1} + \frac{T^2L}{2GJ_2}$

$\Rightarrow U = 1.5$ N-mm

**07. Ans: (c)**

**Sol:** **Strain Energy:** When a member is loaded, it deforms, behaving like a spring; resistance develops and the work is done upon it. If the elastic limit is not exceeded this work stored in the form of energy is called as strain energy.

This amount of work absorbed by the resistance (i.e. strain energy) during deformation is the area under resistance deformation curve.




$\Rightarrow U = \frac{1}{2}\times OB\times BC = \frac{1}{2}\times R\times\delta\ell$

$\Rightarrow \frac{1}{2}\times\sigma\times A\times\delta\ell = \frac{1}{2}\times\sigma\times\varepsilon\times A\times\ell$

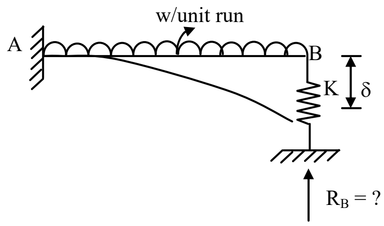
$= \frac{1}{2}\times \text{Stress}\times\text{Strain}\times\text{Volume}$

$\therefore$ Strain energy is a function of stress and strain.

## 12. Propped and Fixed Beams

**01. Ans: (d)**

**Sol:**



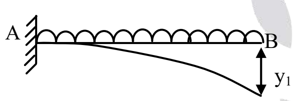

$$K = \text{Stiffness} = \frac{\text{Load}}{\text{deflection}}$$

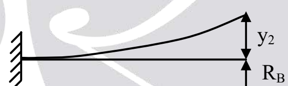
$$\therefore K = \frac{R_B}{\delta}$$

$\therefore$ Compatibility condition

Deflection @ B = δ

$$\because K = \frac{R_B}{\delta} \Rightarrow \delta = \frac{R_B}{K}$$




$$y_1 = \frac{w\ell^4}{8EI} \qquad y_2 = \frac{R_B\ell^3}{3EI}$$

↓(+)

↑(–)

$$y_1 - y_2 = \delta$$

$$\therefore \frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \delta$$

$$\frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \frac{R_B}{K}$$

$$\frac{w\ell^4}{8EI} = \frac{R_B}{K} + \frac{R_B\ell^3}{3EI}$$

$$\frac{w\ell^4}{8EI} = R_B\ell^3\left[\frac{1}{K\ell^3} + \frac{1}{3EI}\right]$$

$$\frac{w\ell^4}{8EI} = R_B\left[\frac{3EI + K\ell^3}{3EI \times K\ell^3}\right] \times \ell^3$$

$$\frac{w\ell}{8EI} = \frac{R_B}{3EI}\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

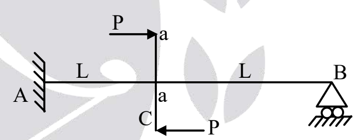
$$\frac{3w\ell}{8} = R_B\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

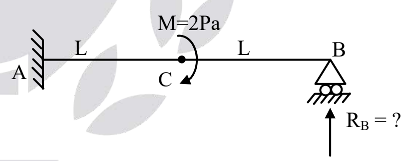
$$\frac{3w\ell}{8} = R_B\left[1 + \frac{3EI}{K\ell^3}\right]$$

$$R_B = \frac{3w\ell/8}{1 + \dfrac{3EI}{K\ell^3}}$$

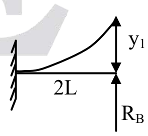
**02. Ans: $\dfrac{9pa}{8L}$**

**Sol:**

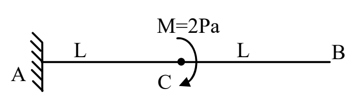


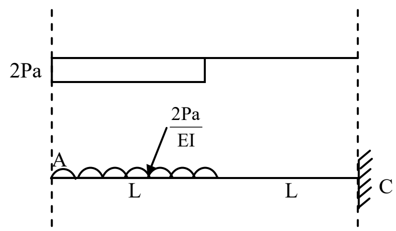

Applying, superposition principle




$$y_1 = \frac{R_B(2L)^3}{3EI} = \frac{8R_BL^3}{3EI}$$



By conjugate beam method




$\therefore y_c$ = deflection @C

= B.M.D. @ C by conjugate beam

$$y_c = \frac{2Pa}{EI} \times L \times \left[L + \frac{L}{2}\right]$$

$$= \frac{2Pa}{EI} \times L \times \frac{3L}{2}$$

$$= \frac{3PaL^2}{EI}$$

Compatibility Condition $(y_B) = 0$

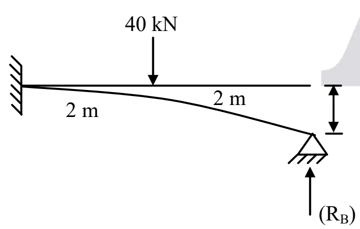
$\therefore \quad y_1 = y_c$

$$\frac{8R_B L^3}{3EI} = \frac{3PaL^2}{EI}$$

$$R_B = \frac{9Pa}{8L} (\uparrow)$$

**03. Ans: 12.51 kN**

**Sol:**




E = 200 GPa

$I = 2 \times 10^{+6}$ mm$^4$

As per compatablity

$$\frac{(R_B)(4000)^3}{3EI} = \frac{(40\times10^3)(2000)^3}{3\times EI} + \frac{40\times10^3\times(2000)^2}{2EI}\times 2000 + 1\text{mm}$$

$$\frac{R_B(2\ell)^3}{3EI} = \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(b) + 1\text{mm}$$

$$\left[\text{use } a = b = \frac{L}{2} = 2000\text{ mm}\right]$$

where EI = $4\times10^{11}$ N/mm$^2$

$$\therefore \frac{R_B)(4000)^3}{3\times4\times10^{11}} = \frac{40\times10^3\times(2000)^3}{3\times4\times10^{11}} + \frac{40\times10^3\times(2000)^3}{2\times4\times10^{11}} + 1$$

$R_B$ = 12.51 kN

## 13. Springs

**01. Ans: (b)**

**Sol:** In the analysis of closed coiled helical spring, bending and axial force in spring are negligible.

**Correct answer is (b).**

**02. Ans: (b)**

**Sol:** Stiffness of Spring (S)

$$S = \frac{Gd^4}{64nR^3} = \frac{G(2r)^4}{64nR^3}$$

$$S = \frac{Gr^4}{4nR^3}$$

**03. Ans: (d)**

**Sol:** Spring stiffness, $k = \dfrac{G d^4}{64 R^3 n}$

$\therefore \quad k \propto d^4$

Let $d_1 = d$.

If d is doubles i.e $d_2 = 2d$ .

$$\therefore \frac{k_1}{k_2} = \frac{d_1^4}{d_2^4}$$

$$\Rightarrow \frac{k_1}{k_2} = \frac{d^4}{(2d)^4}$$

$\Rightarrow \; k_2 = 16\, k_1$

**04. Ans: (a)**

**Sol:** Spring deflection,

$$\delta = \frac{64 W R^3 n}{G d^4}$$

$\therefore \; \delta \propto R^3$

$$\Rightarrow \frac{\delta_1}{\delta_2} = \frac{R_1^3}{R_2^3} = \frac{R_1^3}{\left(\dfrac{R_1}{2}\right)^3} = 8$$

**05. Ans: (a)**

**Sol:** For springs in series: effective stiffness is

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

Therefore, $K_e = \dfrac{K_1 K_2}{K_1 + K_2}$

**06. Ans: (d)**

**Sol:** Deflection of closely coiled spring

$$\delta = \frac{64 R^3 W n}{G d^4}$$

$\therefore \; \delta \propto n$

**07. Ans: (d)**

**Sol:** For springs connected in series

$$\frac{1}{K_e} = \frac{1}{S} + \frac{1}{2S} \Rightarrow K_e = \frac{2S}{3}$$

For springs connected in parallel

$(K_e) = K_1 + K_2 = S + 2S = 3S$

$$\therefore \frac{(K_e)_{series}}{(K_e)_{parallel}} = \frac{2S/3}{3S} = \frac{2}{9}$$

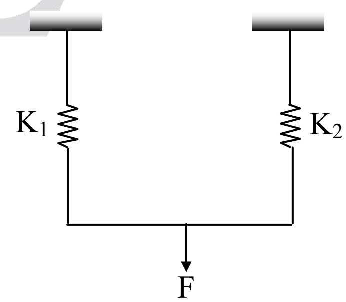
**08. Ans: (d)**

**Sol:** When one spring placed in other then those two springs will be in parallel. Hence combined stiffness is given by

$K_e = K_A + K_B$

**09. Ans: (a)**

**Sol:** Equivalent Load Diagram:




$K_{eq} = K_1 + K_2 = 300 + 100$

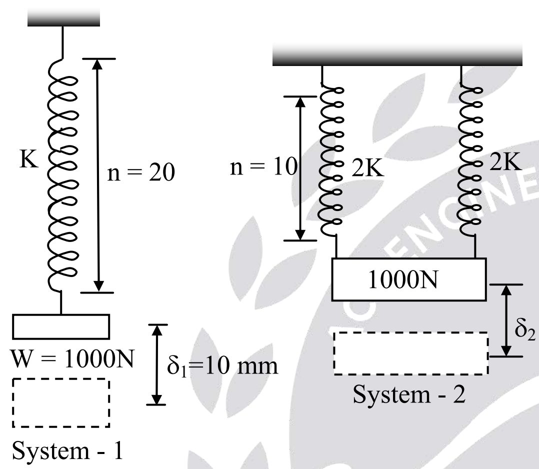
$K_{eq} = 400$ MN/m

$$\delta = \frac{F}{K_{eq}} = \frac{400kN}{400\times10^3\ kN/m}$$

$$= \frac{1}{1000}m = 1mm$$

**10. Ans: (d)**

**Sol:**




From system (1)

$$\delta_1 = \frac{1000}{K}$$

$$K = \frac{1000}{10} = 100\ N/mm$$

From system (2)

$K_{eq} = 2K + 2K = 4K$

$K_{eq} = 4\times 100 = 400$ N/mm

$$\delta_2 = \frac{W}{K_{eq}} = \frac{1000}{400} = 2.5\ mm$$

**11. Ans: (a)**

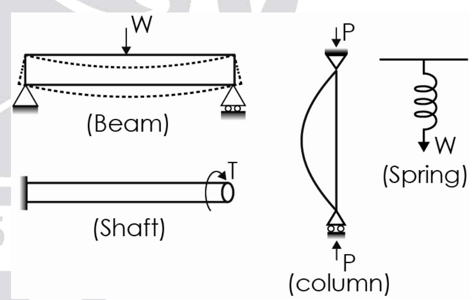
**Sol:** **Beam:** It is structural member subjected to transverse loading on its axis thus causing flexural bending.

**Column:** It is a structural member that is subjected to axial loading which may cause buckling in the member.

**Circular section Shaft: I**t is a member subjected to twisting. For pure torsion, cross-section should be circular and prismatic. Can be solid (or) hollow.

**Close Coiled Helical Springs:** These are the elastic members, which deform due to load and regain original shape after the removal of the load. A spring is used to absorb energy in the form of strain energy which may be restored when required.

For a closed coil helical spring pitch is very small.




**Note:** When torque is applied to non circular sections, shear stress distribution is non-uniform and also warping occurs i.e. plane sections do not remain plane after twisting.

## Conventional Practice Solutions

**01.**

**Sol:** Given data:

D = 2R = mean coil dia,

$\tau_{max} = 350$ MPa

$\frac{D}{d} = \frac{2R}{d} = 6 \Rightarrow R = 3d$

W = 500 N; $\delta = 30$ mm

$G = 80 \times 10^3$ MPa

$$\delta = \frac{64WR^3n}{Gd^4} = \frac{64 \times 500 \times (3d)^3 \times n}{80 \times 10^3 \times d^4} = 30$$

$$\tau_{max} = \frac{16WR}{\pi d^3}$$

$$350 = \frac{16 \times 500 \times 3d}{\pi d^3}$$

$\Rightarrow d = 4.67$ mm

$$\Rightarrow 30 = \frac{64 \times 500 \times (3d)^3 \times n}{80 \times 10^3 \times d^4}$$

$\Rightarrow n = 12.97 = 13$

$\therefore$ n = 13

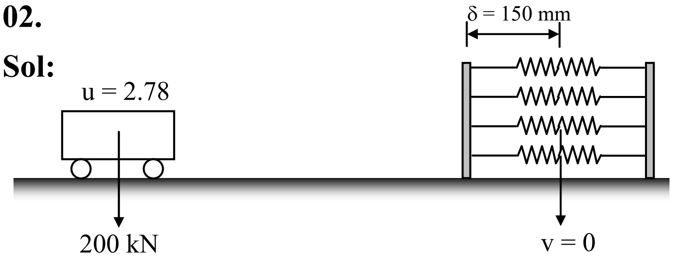
Length of spring = $(2\pi R)n = 2\pi \times 3d \times n$

$= 2\pi \times 3 \times 4.67 \times 13$

L = 1144.356 mm

**02.**

**Sol:**




K.E of wagon = Energy observed by spring

$$\frac{1}{2}(m)(v^2 - u^2) = -\frac{1}{2}(K_e)\delta^2 \text{ (spring compression)}$$

$$\Rightarrow \frac{1}{2}\left(\frac{200 \times 10^3}{g}\right)(0 - 2.78^2) = -\frac{1}{2} \times K_e \times \left(\frac{IS}{1000}\right)$$

$\Rightarrow K_e = 7 \times 10^3$ N / mm

(equation stiffness of found ------ springs)

$4K = K_e$

stiffness of each spring, $K = \frac{K_e}{4} = \frac{7 \times 10^3}{4}$

$= 1.75 \times 10^3 \frac{N}{mm}$

$$K = \frac{W}{\delta} = \frac{Gd^4}{64R^3n}$$

$$\Rightarrow 1.75 \times 10^3 = \frac{(80 \times 10^3)d^4}{64\left(\frac{200}{2}\right)^3(8)} \Rightarrow d = 57.85$$

**03.**

**Sol:** $K = \frac{Gd^4}{64R^3n}$ $\qquad d^4 = \frac{K \times 64R^3n}{G}$

$$= \frac{5 \times 64 \times \left(\frac{60}{2\pi n}\right)^3 \times n}{G}$$

n is not given in Q.

## 14. Shear Centre

**01. Ans: (a)**

**Sol:**

- Shear centre is related to torsion
- On principal plane shear stress is zero
- At fixed end slope is zero.
- Middle third rule is to avoid tension in columns.

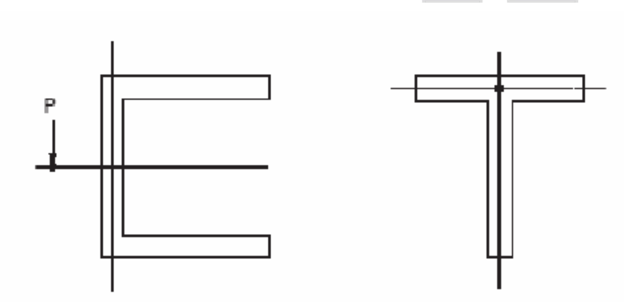
**02. Ans: (b)**

**Sol:** If the resultant force is acting through shear centre torsion developed in the c/s is zero.

**03. Ans: (d)**

**Sol:**

**Shear centre:** When the loads are perpendicular to the axis of the beam and beam is symmetrical, bending takes place without twisting. When the beams do not have longitudinal axis of symmetry, applied loads may induce torsion. In such case, the point (inside or outside) where the load can be applied so that no torsion is induced, is called as shear centre.




## 15. Theories of Failure

**01. Ans: (d)**

**Sol:** $\sigma = \sigma_y = 2500$ kg/cm$^2$

$\sigma_1 = 2000$ kg/cm$^2$

$\sigma_3 = ?$

Maximum shear stress theory

$$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2} \not> \frac{\sigma_y}{2}$$

$$= \frac{2000 - \sigma_3}{2} = \frac{2500}{2}$$

$\sigma_3 = -500$ (comp)

**02. Ans: (b)**

**Sol:** D = 100 cm

P = 10 kg/cm$^2$

$\sigma = \sigma_y = 2000$ kg/cm$^2$

FOS = 4 t = ?

Maximum Principal stress theory

$$\sigma_1 = \sigma_h = \frac{PD}{2t} \not> \sigma_y$$

$$\frac{10 \times 100}{2 \times t} = 2000$$

t = 2.5 mm

Safe thickness of plate = 2.5 × F.O.S

= 2.5 ×4

= 10 mm

**03. Ans: (b)**

**Sol:** $\sigma_1 = 1.5$ (T) $\qquad \sigma_2 = \sigma$ (T)

$\sigma_3 = -\sigma/2$ (C) $\qquad \sigma_y = 2000$ kg/cm$^2$

$\mu = 0.3$

In which theory of failure $\sigma = 1000$ kg/cm$^2$

Check

(a) Maximum principal stress theory

$$\sigma_1 = \sigma_y$$

$$1.5\sigma_1 = 2000$$

$$\sigma_1 = 1333 \text{ kg/cm}^2$$

(b) Maximum shear stress theory

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \frac{\sigma_y}{2}$$

$$\left(\frac{1.5\sigma + \frac{\sigma}{2}}{2}\right) = \frac{2000}{2}$$

$$\frac{4}{2}\sigma = 2000$$

$$\sigma = 1000 \text{ kg/cm}^2$$

**04. Ans: (c)**

**Sol:** $\sigma_1 = 800$ kg/cm$^2$

$\sigma_2 = 400$ kg/cm$^2$

$\mu = 0.25$

$$\varepsilon_1 \le \frac{\sigma_y}{E}$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \frac{\mu\sigma_3}{E} = \frac{\sigma_y}{E}$$

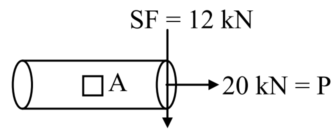
$$\frac{800}{E} - 0.25\frac{(400)}{E} = \frac{\sigma_y}{E}$$

$$\sigma_y = 800 - 100 = 700 \text{ kg/cm}^2$$

## Conventional Practice Solutions

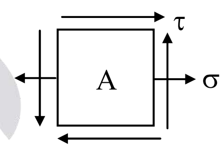
**01.**

**Sol:**




$$\sigma = \frac{P}{A} = \frac{20\times10^3}{A}$$

$$\tau = \frac{SF}{A} = \frac{12\times10^3}{A}$$




Principal stresses

$$\sigma_1 \text{ and } \sigma_3 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{10^3}{A}\left[\frac{20+0}{2} \pm \sqrt{\left(\frac{20-0}{2}\right)^2 + 12^2}\right]$$

$$\sigma_1 = \frac{25.62\times10^3}{A}$$

$$\sigma_3 = \frac{-5.62\times10^3}{A}$$

$$\sigma_2 = 0$$

$$f_y = \frac{25\times10^3}{(10\,\text{mm})^2} = 250\,\text{N/mm}^2$$

$$f = \frac{f_y}{F.O.S} = \frac{250}{2} = 125\,\text{N/mm}^2$$

(i) Maximum principal stress theory

$\sigma_1 \ngtr f$

$$\Rightarrow \frac{25.62\times10^3}{A} = 125$$

$\Rightarrow A = 204.96\ \text{mm}^2$

(ii) Maximum shear stress theory

$$\frac{\sigma_1-\sigma_3}{2} \ngtr \frac{f}{2}$$

$$\Rightarrow \left(\frac{25.62\times10^3}{A}\right)-\left(\frac{-562\times10^3}{A}\right)=125$$

$\Rightarrow A = 249.92\ \text{mm}^2$

(iii) Maximum distortion energy theory

$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 \ngtr 2f^2$

$$\left(\frac{25.62\times10^3}{A}-0\right)^2+\left(0-\left(\frac{-5.62\times10^3}{A}\right)\right)^2+\left(\frac{-5.62\times10^3}{A}-\frac{25.62\times10^3}{A}\right)$$

$= 2\times125^2$

$\Rightarrow A = 230.74\ \text{mm}^2$

Use maximum area for design

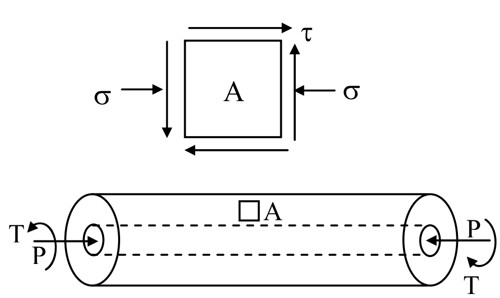
$\therefore A = 249.92$

$$\frac{\pi}{4}d^2 = 249.92$$

$$d=\sqrt{\frac{249.92\times4}{\pi}} = 17.83\ \text{mm}$$

**02.**

**Sol:**




State of stress at any point on surface of shaft.

$$\tau_{xy}=\tau_T=\frac{T}{z_p}=\frac{800\times10^3}{\frac{\pi}{16\times50}\left(50^4-30^4\right)}=37.4\,\text{MPa}$$

$$\sigma_x=\sigma=\frac{P}{A}=\frac{40\times10^3}{\frac{\pi}{4}\left(50^2-30^2\right)}$$

= +31.8 MPa (comp)

(use comp +ve)

$\sigma_y = 0$

Principal stresses:

$$\sigma_1 \text{ and } \sigma_3=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$=\frac{31.8+0}{2}\pm\sqrt{\left(\frac{31.8-0}{2}\right)^2+37.4^2}$$

$\sigma_1 = +56.5$ comp

$\sigma_2 = 0$

$\sigma_3 = -24.7$ tension

(i) Maximum principal stress theory

$\sigma_1 \ngtr f$

$$\Rightarrow 56.5=\frac{f_y}{FOS} \quad \Rightarrow \text{F.O.S}=\frac{280}{56.5}=4.96$$

F = 4.96

(ii) Maximum shear stress theory

$$\frac{\sigma_1-\sigma_3}{2}\ngtr\frac{f_y}{2}$$

$$\Rightarrow \frac{56.5-(-24.7)}{2}=\frac{280}{2\times \text{F.O.S}}$$

$\Rightarrow F = 3.45$

(iii) Distontion energy theory

$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \ngtr 2f^2$

$\Rightarrow (56.5 - 0)^2 + (0 - (-24.7))^2 + (-24.7 - 56.5)^2 = 2\times\left(\frac{f_y}{FOS}\right)^2$

$\Rightarrow F = 3.88$

$\therefore$ Use maximum F.O.S

F = 4.96

**03.**

**Sol:**

**(1) Maximum Shear Stress Theory:**

$$(\sigma_1 - \sigma_3) \le \frac{\sigma_y}{FOS}$$

$$40 - (-70) \le \frac{200}{FOS}$$

$$FOS = \frac{200}{110} = \frac{2}{1.1} = 1.82$$

**(2) Maximum Strain Energy Theory:**

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \ngtr \left(\frac{\sigma_y}{FOS}\right)^2$$

$$40^2 + 70^2 + 2\times 0.3\times 40\times 70 \le \frac{200^2}{FOS}$$

$$FOS = \frac{200}{\sqrt{1600 + 4900 + 1680}}$$

$$= \frac{200}{90.44} = 2.211$$

**(3) Maximum Shear Strain Energy Theory**

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \left(\frac{\sigma_y}{FOS}\right)^2$$

$$40^2 + 70^2 - 40\times(-70) \le \frac{200^2}{FOS^2}$$

$$FOS = \frac{200}{\sqrt{9300}} = \frac{200}{96.44} = 2.07$$

**04.**

**Sol:** $\frac{T}{J} = \frac{\tau}{R}$

$$\tau = \frac{TR}{J} = \frac{T\times(d/2)}{\frac{\pi}{32}\left[d^4 - \left(\frac{d}{2}\right)^4\right]} = \frac{16Td}{\pi d^4\left[1 - \frac{1}{16}\right]}$$

$$= \frac{16^2 T}{\pi d^3 \times 15}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$y = \frac{M_y}{I} = \frac{M\times(d/2)}{\frac{\pi}{64}\left[d^4 - \left(\frac{d}{2}\right)^4\right]} = \frac{(32\times 16)M}{\pi d^3 \times 15}$$

$$\frac{\sigma_{p1}}{\sigma_{p3}} = \left(\frac{f}{2}\right)I = \frac{1}{2}\sqrt{f^2 + 4\tau^2}$$

$$= \frac{1}{2}\left[f \pm \sqrt{f^2 + 4\tau^2}\right]$$

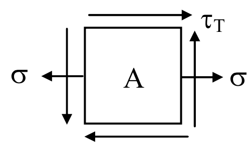
$$= \frac{1}{2}\left[32M \pm \sqrt{32^2 M^2 + 32^2 \times \tau^2}\right]\frac{16}{\pi d^3 \times 15}$$

$$= \frac{16\times 16}{\pi d^3 \times 15}[M \pm \sqrt{M^2 + T^2}] = \frac{5.43}{d^3}[40 \pm 64.03]$$

$$= \frac{564.9}{d^3}, \frac{-130.48}{d^3}$$

**As per maximum shear stress theory**

$$\sigma_{p_1} - \sigma_{p_3} \le \frac{\sigma_y}{FOS}$$




$$10^3 \times \frac{695.38}{d^3} \le \frac{280}{2}$$

$d^3 = 4967$

$d = 17.06$ mm

**05.**

**Sol:** $$\frac{TR}{J} = \frac{T \times 10^3 \times 25}{\frac{\pi}{32}(50^4 - 20^4)} = \frac{32 \times 10^3 T \times 25}{\pi(50^4 - 20^4)}$$

$$= \frac{0.131T}{\pi}$$

$$y = \frac{My}{I} = \frac{0.5T \times 10^3 \times 25}{\frac{\pi}{64}(50^4 - 20^4)} = \frac{32T \times 10^3 \times 25}{\pi(50^4 - 20^4)}$$

$$= \frac{0.131T}{\pi}$$

$$\frac{\sigma_{p_1}}{\sigma_{p_3}} = \frac{1}{\pi}\left(\frac{0.131T}{2}\right) \pm \frac{1}{2}\sqrt{(0.131T)^2 + (2 \times 0.131T)^2}$$

$0.0655 \pm 0.146T$

$$\frac{\sigma_{p_1}}{\sigma_{p_3}} = \frac{0.2106T}{\pi}, -\frac{0.0805T}{\pi}$$

As per Tresca Theory

$$\sigma_{p_1} - \sigma_3 \le \frac{\sigma_y}{FOS}$$

$$\frac{0.2911T}{\pi} \le \frac{250}{2}$$

$T \le 1348.33$ N/mm

$T \le 1.348$ Nm