ESE | GATE | PSUs

# CIVIL ENGINEERING

## REINFORCED CEMENT CONCRETE

**Text Book :** Theory with worked out Examples and Practice Questions

# 4 Reinforced Cement Concrete

**(Solutions for Text Book Practice Questions)**

## 03. Limit State Design – Singly Reinforced Beams

**01. Ans: (a)**

**Sol:** For Fe415,

$M_{u\,limit}$ = Equation (1) with $x_{u\,max}$

$= 0.138\, f_{ck} bd^2$

$= 0.138 \times 15 \times 200 \times (500)^2$

$= 103.5$ kN-m

**02. Ans: (c)**

**Sol:** Balanced (or) limiting percentage of steel (use $x_{u\,max}$)

$$C = T$$

$$0.36\, f_{ck}\, bx_{u\,max} = 0.87\, f_y\, A_{st}$$

$$0.36\, f_{ck}\, b(0.48d) = 0.87 \times 415\, A_{st}$$

$$0.36 \times 15 \times 200 \times 0.48 \times 300 = 0.87 \times 415\, A_{st}$$

$$A_{st} = 430 \text{mm}^2$$

**03. Ans: (b)**

**Sol:** $M_u = 138 \times 10^6$ N-mm

$M_u = M_{u\,limit}$

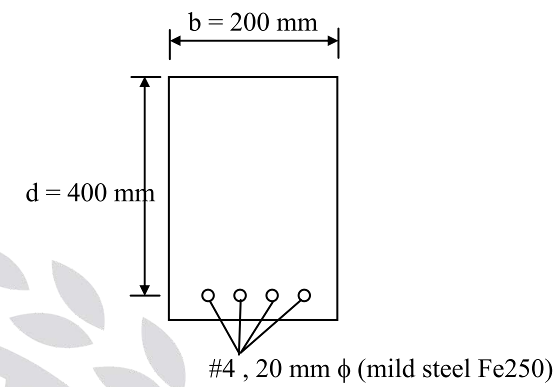
$= 0.138 \times f_{ck}\, bd^2$ – (design as BS)

$138 \times 10^6 = 0.138 \times 20 \times 200 \times d^2$

$d = 500$ mm

**04. Ans: (b)**

**Sol:**




**i)** $x_{umax} = 0.53 \times d$

$= 0.53 \times 400$

$= 212$ mm

**ii)** $x_u = ?$ C = T

$$0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$$

$$0.36 \times 15 \times 200 \times x_u = 0.87 \times 250 \times 4 \times \left(\frac{\pi}{4} \times 20^2\right)$$

$\Rightarrow 1080 x_u = 273318.5$

$x_u = 253.1$ mm

$x_u > x_{u\,max} \Rightarrow$ over reinforced section

Over reinforcement section fails suddenly

To avoid sudden fail decrease the MR to that of a balanced section

$M_{u\,limit} = 0.148 \times f_{ck}\, bd^2$

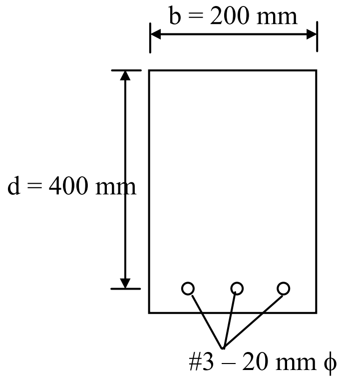
$= 0.148 \times 15 \times 200 \times 400^2$

$= 71040000$ N-mm $= 71.04$ kN-m

$\simeq 72$ kN-m

**05. Ans: (d)**

**Sol:**




**i)** $x_{u\,max} = 0.53 \times d$

$= 0.53 \times 400 = 212$ mm

**ii)** C = T

$0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$

$$0.36 \times 15 \times 200 \times x_u = 0.87 \times 250 \times \left(3\times\frac{\pi}{4}\times 20^2\right)$$

$1080\ x_u = 204988.92$

$x_u = 190$ mm

$x_u < x_{max} \Rightarrow$ Under reinforced section

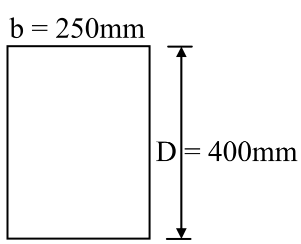
$M_u = 0.36 f_{ck}\, b x_u (d - 0.42 x_u)$

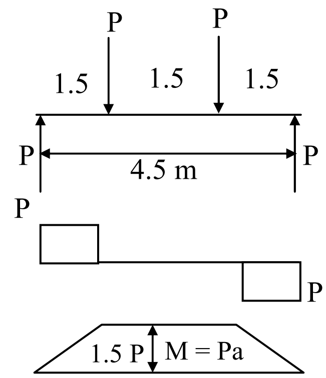
$= 0.36 \times 15 \times 200 \times 190\ (400 - 0.42 \times 190)$

$M_u = 65.7$ kN.m $\simeq 66$ kN-m

**06. Ans: 8.86 kN**

**Sol:**




Homogenous beam

$f_{cr} = 2$ MPa

Modulus of rupture/tensile stress of concrete from bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow M = f_{cr} \times z \qquad \left[\because z = \frac{bD^2}{6}\right]$$

$$= 2\left[\frac{250 \times 400^2}{6}\right] = 13.33 \times 10^6 \text{N-mm}$$

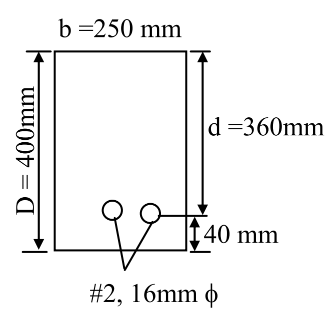
M = P.a

$13.3 = P \times 1.5$

$$P = \frac{13.3}{1.5} = 8.86\,\text{kN}$$

**07. Ans: 31.6 kN**

**Sol:**




Reinforced concrete beam

**i)** $x_{umax} = 0.48d$

$= 0.48 \times 360 = 172.8$ mm

$C = T$

$0.36 f_{ck} b x_u = 0.87\ f_y A_{st}$

$0.36 \times 20 \times 250 \times x_u = 0.87 \times 415 \times \left(2 \times \frac{\pi}{4} \times 16^2\right)$

$1800\ x_u = 145186.8$

$x_u = 80.65$ mm

$x_u < x_{max}$

$\therefore$ Under reinforced section

$M.R = 0.36 f_{ck}\ b x_u\ (d - 0.42 x_u)$

$= 0.36 \times 20 \times 250 \times 80.65\ (360 - 0.42 \times 80.65)$

$M_u = 47.5$ kN-m

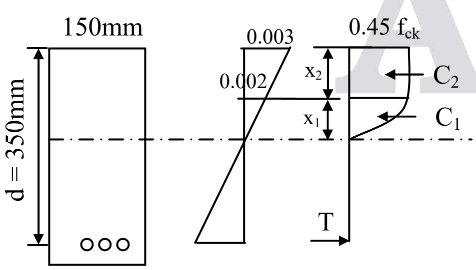
$M_u = P \times a$

$47.5 = P \times a$

$P = \frac{47.5}{1.5}$ $\qquad$ $P = 31.6$ kN

**08. Ans: 51 kN-m**

**Sol:**




$x_{u\ max} = 0.48 \times d$

$= 0.48 \times 350$

$= 168$ mm

$M_{u\ limit} = 0.36 f_{ck}\ b\ x_{u\ max}\ (d - 0.42\ x_{u\ max})$

$= 0.36 \times 20 \times 150 \times 168 (\ 350 - 0.42 \times 168)$

$= 50.70 \times 10^6$ N-m

$= 51$ kN-m

**09. Ans: 503 mm²**

**Sol:** $C = T$

$0.36\ f_{ck}\ b\ x_{u\ max} = 0.87\ f_y\ A_{st}$

$A_{st} = \frac{0.36 f_{ck} b x_{u\,max}}{0.87 \times f_y}$

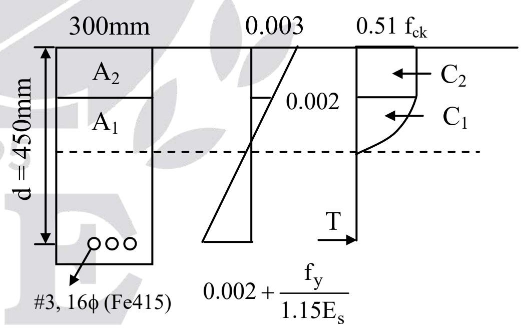
$= \frac{0.36 \times 20 \times 150 \times 168}{0.87 \times 415}$

$= 502.53\ mm^2$

$A_{st} \simeq 503\ mm^2$

**10. Ans: 196 mm**

**Sol:**




$x_{umax} = 0.003 \qquad \rightarrow (1)$

$(d - x_{umax}) = \left(0.002 + \frac{f_y}{1.1 E_s}\right)$

$450 - x_{umax} = \left(0.002 + \frac{415}{1.1 \times 2 \times 10^5}\right) \quad \rightarrow (2)$

$$\frac{450 - x_{u\max}}{x_{u\max}} = \frac{0.002 + \dfrac{415}{1.1\times 2\times 10^5}}{0.003}$$

On solving

$x_{u\,max} = 196.04$ mm

$= 196$ mm

**Conventional Practice Solutions**

**01.**

**Sol: Given:**

M20, b = 300 mm , d = 550 mm

$A_{st}$ = 4 - 20 mm $\phi$ - mild steel Fe250

MR = ?

$\sigma_{cbc} = 7\ \text{N/mm}^2$

$\sigma_{st} = 140\ \text{N/mm}^2$, m = 13

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

$$= \frac{13\times 7}{13\times 7 + 140}$$

k = 0.393

$x_c = k.d = 0.393 \times 550$

$= 216.67$ mm

$$b\frac{x_a^2}{2} = mA_{st}(d - x_a)$$

$$300\times\frac{x_a^2}{2} = 13\times 4\times\frac{\pi}{4}\times 20^2(550 - x_a)$$

$$150x_a^2 = 16.3\times 10^3(550 - x_a)$$

$$150\ x_a^2 - 8.98\times 10^6 + 16.3\times 10^3 x_a = 0$$

$x_a = 196.3$ mm

$x_a < x_c$ $\quad\therefore$ U.R.S

$$M.R = \frac{1}{2}\sigma_{cbc}bx_a\left(d - \frac{x_a}{3}\right)\ \text{(ORS)}$$

$$M.R = \sigma_{st}\,A_{st}\left(d - \frac{x_a}{3}\right)\quad\text{(URS)}$$

$$= 140\times 4\times\frac{\pi}{4}\times 20^2\left(550 - \frac{196.3}{3}\right)$$

M.R = 85.25 kN-m

**02.**

**Sol: Given:**

b = 300 mm

$l$ = 6 m

L.L = 10 kN/m

D.L = 5 kN/m

M20 = $f_{ck} = 20\ \text{N/mm}^2$

For Fe-415 $\rightarrow \sigma_{st} = 230\ \text{N/mm}^2$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times 7} = 13.33$$

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13.33\times 7}{13.33\times 7 + 230}$$

k = 0.288

$$J = 1 - \frac{k}{3} = 1 - \frac{0.288}{3} = 0.904$$

$$Q = \frac{1}{2}\sigma_{cbc}JK = \frac{1}{2}\times 7\times 0.904\times 0.288$$

Q = 0.911

$MR = Qbd^2$

If BM ≤ MR $\therefore$ S.R.B

If BM > MR $\therefore$ D.R.B

Assume d = 600 mm, b = 300 mm

$$D = 600 + 20 + \frac{20}{2} = 630\,\text{mm}$$

**Effective Span:**

i. $l + b_s = 5.7 + 0.3 = 6.0$ m

ii. $l + d = 5.7 + 0.6 = 6.3$ m

Effective span $l_{eff} = 6.0$ m

**Loads :**

D.L = 5 kN/m

L.L = 10 kN/m

Self weight of beam = $\gamma$.B.D = 25 × 0.3 × 0.63

= 4.725 kN/m

Total load = 5 + 10 + 4.725

= 19.725 kN/m

For simply supported beam

$$\text{Maximum B.M} = \frac{W L_e^2}{8}$$

$$= \frac{19.725 \times 6^2}{8}$$

= 88.76 kN-m

MR = Qbd$^2$

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{88.76 \times 10^6}{0.911 \times 300}} = 569.88 \text{ mm}$$

$d_{required} < d_{assumed}$

$\therefore$ Hence Safe

$\therefore$

**Area of Steel Reinforcement :**

$$\text{Max BM} = \sigma_{st} A_{st}\left(d - \frac{x_c}{3}\right)$$

$x_c = k.d$

= 0.288 × 600 = 172.8 mm

$$A_{st} = \frac{M}{\sigma_{st} Jd}$$

$$A_{st} = \frac{88.76 \times 10^6}{230 \times 0.904 \times 600} = 711.49 \text{ mm}^2$$

**Minimum Tension Steel:**

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

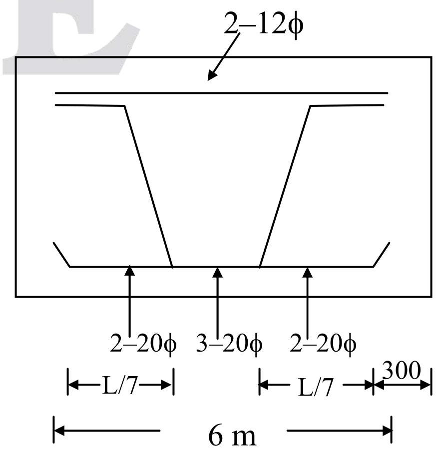
$$A_{st} = \frac{0.85 \times 300 \times 600}{415} = 368.67 \text{ mm}^2 < A_{st}$$

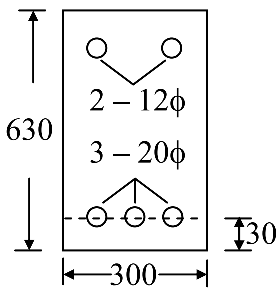
**Maximum Tension Steel :**

$$4\%bD = \frac{4}{100} \times 300 \times 630 = 7560\,\text{mm}^2$$

No. of bars required, use 20 mm $\phi$

$$n = \frac{A_{st}}{a_{st}} = \frac{711.49}{\frac{\pi}{4} \times 20^2} = 2.26 \simeq 3 \text{ Nos0}$$






**03.**

**Sol:** 300 mm × 550 mm

$A_{st} = 4 - 25\ \phi$

$= 4 \times \dfrac{\pi}{4} \times 25^2 = 1.96 \times 10^3$

Simply supported over a span

$l = 8$ m

ud$l$ = 25 kN/m

m = 15

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

$\sigma_{cbc}$ & $\sigma_{st}$ = ?

**1. Maximum Bending Moment for S.S.B**

$$M = \frac{W\ell^2}{8} = \frac{25 \times 8^2}{8} = 200 \text{ kN-m}$$

**2. Actual Depth of N.A**

$$b = \frac{x_a^2}{2} = m\ A_{st}\ (d - x_a)$$

$$300 \times \frac{x_a^2}{2} = 15 \times 1.96 \times 10^3 (550 - x_a)$$

$$150\ x_a^2 = 16.17 \times 10^6 - 29.4 \times 10^3 x_a$$

$$x_a = 244.64 \text{ mm}$$

**3. Maximum B.M required to M.R**

$$\text{M.R} = \frac{1}{2}\sigma_{cbc} b x_a \left(d - \frac{x_a}{3}\right)$$

$$= 200 \times 10^6$$

$$= \frac{1}{2} \times \sigma_{cbc} \times 300 \times 244.64 \left(550 - \frac{244.64}{3}\right)$$

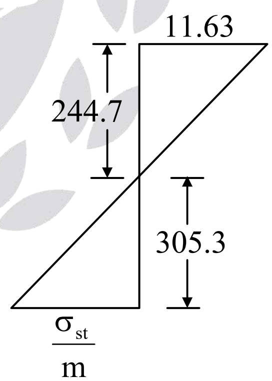
$\sigma_{cbc} = 11.63$ N/mm$^2$

From similar triangles of stress diagram

$$\frac{11.63}{244.64} = \frac{\left(\frac{\sigma_{st}}{m}\right)}{305.3}$$

$\sigma_{st} = 217.65$ N/mm$^2$ (or)

$$\text{Max. BM} = \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)$$




$$200 \times 10^6 = \sigma_{st} \times 4 \times \frac{\pi}{4} \times 25^2 \times \left(550 - \frac{244.64}{3}\right) \sigma_{st}$$

$$= 217.4 \text{ N/mm}^2$$

**04.**

**Sol:** $l = 5$ m

M20 & Fe-415

Assume effective depth, $\frac{L}{7}$ to $\frac{L}{10}$

$$d = \frac{5000}{10} = 500\,\text{mm}$$

D = 500 + 50 = 550 mm

b = 250 mm

**Loads:**

DL = 18 kN/m

LL = 12 kN/m

Self weight of the beam = $\gamma$BD

= 25 × 0.25 × 0.55

= 3.437 kN/m

Total load = 18 + 12 + 3.437

= 33.437 kN/m

$$\text{Bending moment} = \frac{33.437 \times 5^2}{8}$$

= 104.49 kN-m

Factored bending moment = 1.5 × 104.49

= 156.73 kN-m

**Checking the Effective Depth:**

$$d = \sqrt{\frac{M}{R_u.b}} = \sqrt{\frac{156.73 \times 10^6}{0.138 \times 20 \times 250}}$$

d = 476.6 mm

$d_{required} < d_{assumed}$

$\therefore$ safe

Since the depth provided is more than required hence it will be an under reinforced section.

**Area of Steel Reinforcement:**

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$$156.73 \times 10^6 = 0.87 \times 415 \times A_{st} \times 500\left[1 - \frac{A_{st} \times 415}{250 \times 500 \times 20}\right]$$

$$156.73 \times 10^6 = 180.52 \times 10^3\ A_{st} - 29.96\ A_{st}^2$$

$A_{st} = 1051.82\ \text{mm}^2$

**Minimum Tension Steel:**

$$\frac{A_{st}}{bd} = \frac{0.87}{f_y}$$

$$A_{st} = \frac{0.87 \times 250 \times 500}{415}$$

$A_{st} = 262.04 < A_{st}$ $\quad\therefore$ Ok

**Maximum tension steel:**

$$4\%\ \text{of bD} = \frac{4}{100} \times 250 \times 550 = 5500 > A_{st}$$

No. of bars required, use 16 $\phi$

$$n = \frac{A_{st}}{a_{st}} = \frac{1051.8}{\frac{\pi}{4} \times 16^2} = 5.23 \simeq 6\ \text{No's}$$

**05.**

**Sol:** Size of beam = 300 mm × 600 mm

$$A_{st} = 4 - 25\phi \quad = 4 \times \frac{\pi}{4} \times 25^2$$

$$= 1.96 \times 10^3$$

$e_c = 50$ mm

$l_e = 6$ m

w = ?

DL = 4.5 kN/m$^2$

M20 , Fe-250

Effective depth = d= D – $e_c$

= 600 – 50 = 550 mm

Maximum depth of N.A

$x_{u\,max} = 0.53d$

$= 0.53 \times 550$

$= 291.5$ mm

**Actual depth of N.A :**

C = T

$0.36\, f_{ck}\, b\, x_u = 0.87 f_y\, A_{st}$

$0.36 \times 20 \times 300 \times x_u$

$= 0.87 \times 250 \times 1.96 \times 10^3$

$x_u = 197.36$ mm

$x_u < x_{u\,max}$

∴ Under reinforced section

**Moment of Resistance:**

MR = T × Z

$= 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$

$$= 0.87 \times 250 \times 4 \times \frac{\pi}{4} \times 25^2 (550 - 0.42 \times 197.36)$$

M.R = 199.4 kN-m

Maximum B.M for SSB

$$M = \frac{W_u \ell^2}{8} = \frac{W_u \times 6^2}{8}$$

Equate max B.M to MR

$$\frac{W_u 6^2}{8} = 199.4$$

$$W_u = \frac{199.4 \times 8}{6^2} = 44.3 \text{ kN/m}$$

$$W = \frac{44.3}{1.5} = 29.5 \text{ kN/m}$$

Safe load, W = DL + LL + Self weight

LL = 29.5 – 4.5 kN/m – (25 × 0.3 × 0.6)

= 20.5 kN/m

**06.**

**Sol:** Assume effective depth, d = 500 mm

D = 550 mm

b = 250 mm

Effective span, $l_e = 8$ m

**Loads :**

self weight = γBD

= 25 × 0.25 × 0.55

= 3.44 kN/m

Live load = 12 kN/m

Total load, W = 15.44 kN/m

Maximum B.M

$$M = \frac{WL^2}{8} = \frac{15.44 \times 8^2}{8} = 123.52 \text{ kN-m}$$

Checking the effective depth

$$d = \sqrt{\frac{M}{Qb}}$$

For M15 → $\sigma_{cbc}$ = 5 MPa

Fe-415 → $\sigma_{st}$ = 230 MPa

Modular ratio, $m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times 5} = 18.7$

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

$$= \frac{18.7\times 5}{18.7\times 5 + 230} = 0.289$$

$$J = 1 - \frac{k}{3} = 1 - \frac{0.289}{3} = 0.904$$

$$Q = \frac{1}{2}\sigma_{cbc} Jk$$

$$= \frac{1}{2}\times 5\times 0.904\times 0.289$$

$$= 0.65$$

$$d = \sqrt{\frac{123.52\times 10^6}{0.65\times 250}} = 871.85\,\text{mm}$$

Assume depth < Required depth

$\therefore$ Not safe (Redesign)

Provided d = 900 mm

D = 950 mm

Area of tension steel required

$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{123.52\times 10^6}{230\times 0.904\times 900}$$

$$= 660\text{ mm}^2$$

No of bars required $= \frac{A_{st}}{a_{st}} = \frac{660}{\frac{\pi}{4}\times 16^2}$

$$= 3.28 \simeq 4$$

Provided 4 – 16 mm $\phi$

**Minimum Tension Steel :**

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

$$\frac{A_s}{250\times 900} = \frac{0.85}{415}$$

$A_s = 460.8\text{ mm}^2 < A_{st}$ $\therefore$ O.K

**Maximum Tension Steel :**

$$\not> 4\%\text{ of } bD = \frac{4}{100}\times 250\times 950$$

$= 9500\text{ mm}^2 > A_{st}$ $\therefore$ ok

## 04 Limit State Design – Doubly Reinorced Beams

**01. Ans: (c)**

**Sol:** BM = 300 kN-m

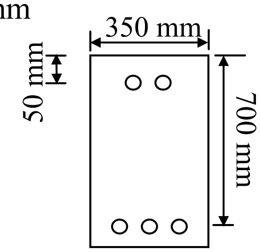
Concrete, $M_{15} = f_{ck} = 15$

Steel, $f_y = 415$

$f_{sc}$ = 353.7 MPa

Effective Cover d′ = 50mm

In LSM, we have to use Factored moment




$M_u = M \times \gamma_f$

Use $\gamma_f = 1.5$

$= 300 \times 1.5 = 450$ kN-m

To calculate $M_{u\,limit}$

$M_{u\,limit} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 15 \times 350 \times (700)^2$

$M_{u\,limit} = 355$ kN-m

$M_u = 450$ kN-m

$\therefore$ $M_u > M_{ulimit}$

So we need to use 'DRB'

$M_{ulimit} = 0.87 f_y A_{st} (d - 0.42\, x_{u\,max})$

$355 \times 10^6 = 0.87 \times 415 \times A_{st}(700 - 0.42 \times 0.48 \times 700)$

$A_{st} = 1759.31\ mm^2$

for extra moment we need to provide tensile steel & comp. steel

$M_u - M_{u\,limit} = 0.87 f_y (d - d') A_{st2}$

$(450 - 355) \times 10^6 = 0.87 \times 415 A_{st2}(700 - 50)$

$= 234682.5 A_{st2}$

$A_{st2} = 404.8 \simeq 405\ mm^2$

$A_{st} = A_{st1} + A_{st2} = 2165\ mm^2$

Now our purpose is to calculate '$A_{sc}$'

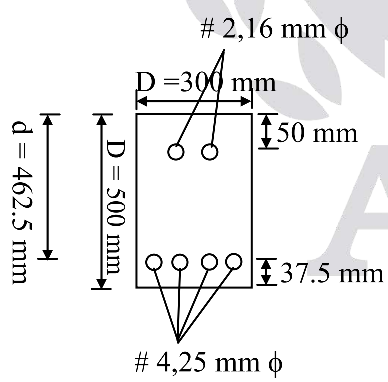
$M_u - M_{ulimit} = f_{sc} A_{sc} (d - d')$

(or) $f_{sc} A_{sc} = 0.87 f_y A_{st2}$

$A_{sc} = 413.2\ mm^2$

**02. Ans: 271 kN-m**

**Sol:**




b = 300 mm, D = 500 mm, d = 462.5 mm

$f_{ck} = 25\ N/mm^2$, $f_y = 415\ N/mm^2$,

$f_{sc} = 0.8566\, f_y$

$A_{st} = 4 \times \dfrac{\pi}{4} \times 25^2 = 1963.495\ mm^2$

$A_{sc} = 2 \times \dfrac{\pi}{4} \times 16^2 = 402.12\ mm^2$

$\Rightarrow C = T \Rightarrow C_1 + C_2 = T$

$0.36 \times f_{ck}\, b\, x_u + f_{sc} A_{sc} = 0.87\, f_y\, A_{st}$

$0.36 \times 25 \times 300 \times x_u + (0.8566 \times 415) \times 402.12$

$= 0.87 \times 415 \times 1963.495$

$x_u = 209.618$ mm

$x_{u\,max} = 0.48 \times d$

$= 0.48 \times 462.5 = 222$ mm

$x_u < x_{u\,max}$

$\therefore$ under reinforced section.

$M_u = 0.36 f_{ck}.b.x_u(d - 0.42 x_u) + f_{sc} A_{sc}(d - d^1)$

$= 0.36 \times 25 \times 300 \times 209.6$

$(462.5 - 0.42 \times 209.6) + (0.8556 \times 415)$

$\times 402.12\ (462.5 - 50) = 270.9$ kN-m

**03. Ans: 18.82 kN/m**

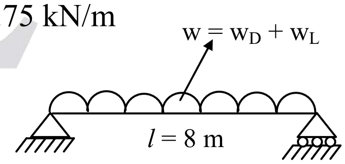
**Sol:** Working /line moment,

$M = \dfrac{270.9}{1.5} = 180.6 kN - m$

Self weight of beam, $w_D = (\gamma_c)\, b \times D$

$= \left(25 kN/m^3\right) \times (0.3 \times 0.5)$

W = 3.75 kN/m




$M = \dfrac{\left(w_D + w_L\right) \times l^2}{8}$

$180.6 = \dfrac{\left(3.75 + w_L\right) \times 8^2}{8}$

$w_L = 18.825$ kN/m

## Conventional Practice Solutions

**01.**

**Sol: Given**

b = 400 mm

d = 550 mm

Effective Cover = 50 mm

Tension steel $\rightarrow$ 4 – 20 mm $\phi$

Compression steel 4 – 16 mm $\phi$

M20 & Fe 500

$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.6 \text{ mm}^2$

$A_{sc} = 4 \times \frac{\pi}{4} \times 16^2 = 804.2 \text{ mm}^2$

$x_{u\,max} = 0.46\,d = 0.46 \times 550 = 253$ mm

**Actual depth of N.A :**

C = T

$C_1 + C_2 = T$

$0.36\, f_{ck}\, bx_u + f_{sc}\, A_{sc} = 0.87 + f_y\, A_{st} \rightarrow (1)$

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{d'}{x_4}\right]$$

$x_u = x_{u\,max}$

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{50}{253}\right] = 0.0028$$

| $f_{sc}$ | $\varepsilon_{sc}$ |
|---|---|
| 413.25 | 0.00277 |
| - | 0.0028 |
| 424.25 | 0.00312 |

$$f_{sc} = 413.25 + \frac{(424.25 - 413.25)(0.0028 - 0.00277)}{(0.00312 - 0.00277)}$$

$f_{sc} = 414.19 \text{ N/mm}^2$

$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st}$

$0.36 \times 20 \times 400 \times x_u + 414.19 \times 804.2$

$= 0.87 \times 500 \times 1256.6$

$x_u = 74.14$ mm

$x_u < x_{u\,max}$ $\therefore$ U.R.S

**Moment of resistance**

**Next trail :**

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{50}{74.14}\right] = 0.00113$$

Take b = 300 mm

$0.36 \times 20 \times 300\, x_u + 414.19 \times 4 \times 804.2$

$= 0.87 \times 500 \times 1256.6$

$x_u = 98.85 \text{ mm} \neq 253$ mm

**Next trail :**

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{50}{98.85}\right] = 0.00172$$

For $f_{sc} = 348 \rightarrow \varepsilon_{sc} = 0.00174$

Substitute in eq (1)

$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st}$

$0.36 \times 20 \times 300\, x_u + 348 \times 804.2$

$= 0.87 \times 500 \times 1256.6$

$x_u = 123.4 \text{ mm} \neq 98.85$

**Next trail:**

$$\varepsilon_{sc} = 0.0035\left(1-\frac{50}{123.4}\right) = 0.00208$$

| $f_{sc}$ | $\varepsilon_{sc}$ |
|---|---|
| 369.75 | 0.00195 |
| – | 0.00208 |
| 391.5 | 0.00226 |

$$= 369.75 + \frac{(0.00208-0.00195)(391.5-369.75)}{(0.00226-0.00195)}$$

$f_{sc}$ = 378.8 N/mm$^2$

Substitute in eq (1)

$x_u$ = 112.02 mm ≠ 123.4

$$\varepsilon_{sc} = 0.0035\left(1-\frac{50}{112.02}\right)$$

$\varepsilon_{sc}$ = 0.00193

| $f_{sc}$ | $\varepsilon_{sc}$ |
|---|---|
| 348 | 0.00174 |
| – | 0.00193 |
| 369.75 | 0.00195 |

$$= 348 + \frac{(0.00193-0.00174)(369.75-348)}{(0.00195-0.00174)}$$

$f_{sc}$ = 367.67 N/mm$^2$

Substitute in eq. 1

$0.36 \times 20 \times 300\, x_4 + 367.67 \times 804.2$

$= 0.87 \times 500 \times 1256.6$

$x_u$ = 116.17 ≠ 112

$$\varepsilon_{sc} = 0.0035\left(1-\frac{50}{116.17}\right) = 0.00199$$

| $f_{sc}$ | $\varepsilon_{sc}$ |
|---|---|
| 369.75 | 0.00195 |
| - | 0.00199 |
| 391.5 | 0.00226 |

$$369.75 + \frac{(0.00199-0.00195)(391.5-369.75)}{(0.00226-0.00195)}$$

$f_{sc}$ = 372.5 N/mm$^2$

$0.36 \times 20 \times 300x_u + 372.5 \times 804.2$

$= 0.87 \times 500 \times 1256.6$

$x_u$ = 114.3 mm ≃ 116 mm

$f_{sc}$ = 372.5 N/mm$^2$ & $x_u$ = 114 mm

$x_u < x_u$ max

Under Reinforced Section

**Moment of Resistance :**

M.R = $0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u)$

$+ f_{sc}\, A_{sc}(d - d')$

$= 0.36 \times 20 \times 300 \times 114$

$(550 - 0.42\times114) + 372.5 \times 804.2 \times (550 - 50)$

M.R = 273.42 kN-m

**02.**

**Sol:** Simply supported beam

$l$ = 5.6 m

$W_L$ = 24 kN/m

$W_D$ = 16 kN/m

M20 , Fe 415

Assume effective depth $\left(\dfrac{L}{7} \text{ to } \dfrac{L}{10}\right)$

1. Assume, d = 560 mm

$D = 560 + 40 = 600$ mm

$b = 250$ mm

2. Width of support, $b_s = 300$ mm

Effective span ($l_e$)

$l + b_s = 5.6 + 0.3 = 5.9$ m

$l + d = 5.6 + 0.56 = 6.16$ m

Effective span ($l_e$) = 5.9 m

**Loads:**

DL ($W_D$) = 16 kN/m

LL ($W_L$) = 24 kN/m

Total load,

$W = 16 + 24 = 40$ kN/m

Factored load,

$W_u = 1.5 \times 40 = 60$ kN/m

Factored bending moment,

$$M_u = \frac{60 \times 5.9^2}{8}$$

$= 261.07$ kN-m

$M_{u\,lim} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 20 \times 250 \times 560^2$

$= 216.38$ kN-m

$M_u > M_{u\,lim}$

∴ Double Reinforced Beam

$M_{u\,lim} = 0.87\, f_y\, A_{st\,1}\,(d - 0.42\, x_{u\,max})$

$216.38 \times 10^6 = 0.87 \times 415 \times A_{st\,1}$
$(560 - 0.42 \times 0.48 \times 560)$

$A_{st1} = 1340.4$ mm$^2$

Calculate $A_{st2}$ ,

$M_u - M_{u\,lim} = 0.87\, f_y\, A_{st2}\,(d - d')$

$(261.07 - 216.38) \times 10^6 = 0.87 \times 415 \times A_{st2}\,(560 - 40)$

$A_{st2} = 238$ mm$^2$

$A_{st} = A_{st1} + A_{st2}$

$= 1340.4 + 238$

$= 1578.4$ mm$^2$ $\simeq 1579$ mm$^2$

$f_{sc} \rightarrow d'/d$
$f_{sc} \rightarrow \varepsilon_{sc}$

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{40}{0.48 \times 560}\right)$$

$\varepsilon_{sc} = 0.00297$

| $f_{sc}$ | $\varepsilon_{sc}$ |
|---|---|
| 351.8 | 0.00276 |
| - | 0.00297 |
| 360.9 | 0.0038 |

$f_{sc} =$

$$351.8 + \frac{(0.00297 - 0.00276)(360.9 - 351.8)}{(0.0038 - 0.00276)}$$

$f_{sc} = 353.63$ N/mm$^2$ $\simeq 353.025$ N/mm$^2$

$M_u - M_{u\,lim} = T_2\, Z_2 = C_2\, Z_2$

$T_2 = C_2$

$0.87\, f_y\, A_{st2} = f_{sc}\, A_{sc}$

$0.87 \times 415 \times 238 = 353.025 \times A_{sc}$

$A_{sc} = 243.4$ mm$^2$

**Minimum Tension Steel :**

$$\frac{A_{st}}{bd} = \frac{0.85}{fy}$$

$$\frac{A_s}{250 \times 560} = \frac{0.85}{415}$$

$A_{st} = 286.74 < Ast \quad \therefore$ ok

**Maximum Tension Steel :**

$$4\% \text{ of bD } = \frac{4}{100} \times 250 \times 600 = 6000\,\text{mm}^2$$

Maximum compression steel

$= 4\%$ of bD

$$= \frac{4}{100} \times 250 \times 600 = 6000\,\text{mm}^2 \quad \therefore \text{ Ok}$$

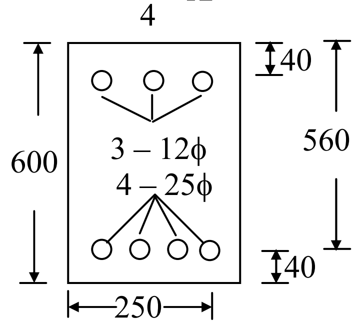
**Tension :** No of bars $= \dfrac{A_{st}}{a_{st}}$

$$= \frac{1579}{\frac{\pi}{4} \times 25^2} = 3.21 \simeq 4 \text{ No's}$$

**Compression:**

No of bars $= \dfrac{A_{sc}}{a_{st}}$

$$= \frac{243.4}{\frac{\pi}{4} \times 12^2} = 2.15 \simeq 3 \text{ No's}$$

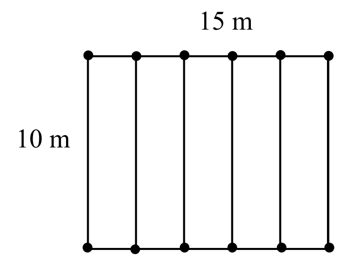



## 05. Limit State Design – Flanged Beams

**01. Ans: (c)**

**Sol:** For T-beams,

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$




Fixed to column
$l_o = 0.7\,l$

$$= \frac{0.7 \times 10}{6} + 0.25 + 6 \times 0.1$$

$= 2.01$ m $\not> c = 3$ m

$\therefore b_f = 2.01$ m

**02. Ans: (d)**

**Sol:** L – beam

$$B_f = \frac{l_0}{12} + b_w + 3D_f$$

$$= \frac{10}{12} + 0.25 + 3 \times 0.1$$

$= 1.38$ m $\not> c = 3$ m

$\therefore b_f = 1.38$ m

**03. Ans: (d)**

**Sol:** $D_f = 100$ mm, $b_w = 300$mm, $d = 500$mm,
$c = 3$m, $l = 6$m, $l_0 = 3.6$ m, $b_f = ?$

$$b_f = \frac{l_0}{6} + b_w + 6D_f \quad \not> c$$

$$= \frac{3.6}{6} + 0.3 + 6 \times 0.1$$

$= 1.5$ m $\not> c = 3$ m

$= 1.5 \times 1000$ m $= 1500$ mm

## Conventional Practice Solutions

**01.**

**Sol: Given**

$b_f = 1200$ mm

$D_f = 100$ mm

$b_w = 300$ mm

$d = 560$ mm

$MR = ?$

M20, Fe415

$A_{st} = 4 - 25\ \phi$

Maximum depth of N.A

$x_{u\,max} = 0.48\ d = 0.48 \times 560 = 268.8$ mm

**Actual depth of N.A :**

Assume $x_u \le D_f$

$$C = T$$

$$0.36\ f_{ck}\ b_f\ x_u = 0.87\ f_y\ A_{st}$$

$$0.36 \times 20 \times 1200\ x_u = 0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2$$

$x_u = 82.05$ mm $< D_f$

Assumption is true

$x_u < x_{u\,max}$ $\therefore$ U.R.S

**Moment of Resistance :**

$$M.R = T \times Z = 0.87\ f_y\ Ast\ (d - 0.42\ x_u)$$

$$= 0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2\ (560 - 0.42 \times 82.05)$$

$= 372.56$ kN-m

**02.**

**Sol: Given :**

$b_f = 1000$ mm

$D_f = 100$ mm, $b_w = 300$ mm

$A_{st} = 2600\ mm^2$

Effective Cover, $d' = 50$ mm

$d = 450$ mm

M20 & Fe-415

**Maximum depth of N.A :**

$$x_{u\,max} = 0.48\ d = 0.48 \times 450 = 216 \text{ mm}$$

**Actual depth of N.A :**

Assume $x_u \le D_f$

$$C = T$$

$$0.36\ f_{ck}\ b_f\ x_u = 0.87\ f_y\ A_{st}$$

$$0.36 \times 20 \times 1000 \times x_u = 0.87 \times 415 \times 2600$$

$x_u = 130.38$

$x_u > D_f$

Assumption is wrong

$$\frac{D_f}{d} = \frac{100}{450} = 0.22$$

$$\frac{D_f}{d} > 0.2$$

$$C = T$$

$$C_1 + C_2 = T$$

$$0.36\ f_{ck}\ b_w\ x_u + 0.446\ f_{ck}\ (b_f - b_w)\ y_f = 0.87\ f_y\ A_{st}$$

$y_f = 0.15\ x_u + 0.65\ D_f$

$y_f = 0.15\ x_u + 65$

$0.36 \times 20 \times 300\ x_u + 0.446 \times 20$

$(1000 - 300)(0.15\ x_u + 65)$

$= 0.87 \times 415 \times 2600$

$= 2160\ x_u + 936.6\ x_u + 405.86 \times 10^3$

$= 938.73 \times 10^3$

$x_u = 172$ mm

$y_f = 0.15 \times 172 + 0.65 \times 100$

$= 90.8$ mm $< D_f$

$x_u < x_{u\,max}$ $\therefore$ U.R.S

**Moment of Resistance:**

$M.R = C_1Z_1 + C_2\ Z_2$

$= 0.36\ f_{ck}\ b_w\ x_u\ (d - 0.42\ x_u) + 0.446\ f_{ck}$

$$(b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right)$$

$= 0.36 \times 20 \times 300 \times 172$

$(450 - 0.42 \times 172) + 0.446 \times 20\ (1000 - 300) \times$

$$90.8\left(450 - \frac{90.8}{2}\right)$$

$= 369.73$ kN-m

**03.**

**Sol: Given**

$l_e = 6$ m  S = 3.3 m c/c

$D_f = 130$ mm  $b_w = 300$ mm

D = 450 mm  L.L = 10 kN/m$^2$

F.F = 0.75 kN/m$^2$

Load of wall on beam = 12 kN/m$^2$

M20 , Fe415

**Loads:**

Self wt of slab = $\gamma D_f = 25 \times 0.13$

$= 3.25$ kN/m$^2$

L.L = 10 kN/m$^2$

F.F = 0.75 kN/m$^2$

Total load = 14 kN/m$^2$

Load transferred to beam = 14 × 3.3

= 46.2 kN/m

Load of wall on beam = 12 kN/m

Total load = 46.2 + 12 = 58.2 kN/m

Factored load,

$W_u = 1.5 \times 58.2 = 87.3$ kN/m

Factored bending moment,

$$M_u = \frac{W_u \ell_e^2}{8} = \frac{87.3 \times 6^2}{8} = 392.85\,\text{kN/m}$$

**Effective width of flange:**

$$b_f = \frac{\ell_o}{6} + b_w + 6D_f$$

$$= \frac{6}{6} + 0.3 + 6 \times 0.13 = 2.08\,\text{m} < \text{c/c}$$

= 2.08 < 3.3 m c/c

Assume N.A lies in the flange

$x_u = D_f$

$M_{u1} = 0.36\ f_{ck}\ b_f\ (D_f)\ [d - 0.42\ (D_f)]$

$= 0.36 \times 20 \times 2080\ (130)\ [(450 - 50)$

$0.42 \times 130]$

$= 672.45$ kN-m

$M_{u1} > M_u$

$\therefore$ Assumption is true

$$M_u = 0.87\, f_y\, A_{st}\, d\left[1 - \frac{f_y}{f_{ck}}\frac{A_{st}}{b_f d}\right]$$

$$M_u = 0.87 \times 415 \times A_{st} \times 400$$

$$\left[1 - \frac{415}{20} \times \frac{A_{st}}{2080 \times 400}\right]$$

$$392.875 \times 10^6 = 144.42 \times 10^3\, A_{st} - 3.601\, A_{st}^2$$

$A_{st} = 2935\ mm^2$

Use 30 mm $\phi$,

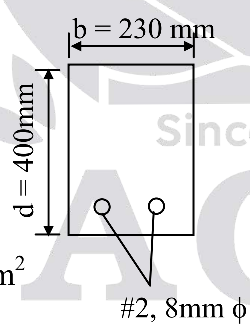
$$\text{No of bars required} = \frac{2935}{\frac{\pi}{4} \times 30^2}$$

$$= 4.15 \simeq 5 \text{ No's}$$

## 06. Limit State of Collapse - Shear

**01. Ans: (b)**

**Sol:**




$V_u = 120$ kN

$f_{ck} = 20\ N/mm^2$

Main steel, $f_y = 415\ N/mm^2$

Stirrups, $f_y = 250\ N/mm^2$

$\tau_c = 0.48\ N/mm^2$

**i)** 8mm–2 legged Stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.53\ mm^2$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{120 \times 10^3}{400 \times 230}$$

$$= 1.3\ N/mm^2$$

$\tau_v \le \tau_{c\,max}$ – safe in shear

**ii)** $\tau_v > \tau_c$ – not safe in shear reinforcement

Minimum shear reinforcement is required

$$V_{us} = \frac{(0.87 f_y) A_{sv} \times d}{S_v}$$

$$V_{us} = V_u - \tau_c b.d$$

$$= 120 \times 10^3 - 0.48 \times 400 \times 230$$

$$= 75840 \text{ N} = 75.84 \text{ kN}$$

$$75.84 \times 10^3 = \frac{0.87 \times 250 \times 100.53 \times 400}{S_v}$$

$S_v = 115$ mm c/c

**02. Ans: (c)**

**Sol:** T = 10.90 kN-m

$$V_e = V_u + \frac{1.6\, T_u}{b}$$

$$= 120 \times 10^3 + \frac{1.6 \times 10.90 \times 10^6}{230}$$

$V_e = 196$ kN

Design shear force

$$V_{us} = V_e - \tau_c.b.d$$

$$= 196 \times 10^3 - 0.48 \times 230 \times 400$$

$$V_{us} = 151.84 \times 10^3 \text{N}$$

$$= 151.84 \text{ kN}$$

**03. Ans: (d)**

**Sol:** b = 230 m, d = 450 mm

$V_u = 50$ kN

$f_{ck} = 20$ N/mm$^2$

$f_y = 250$ N/mm$^2$

$\tau_{c\,max} = 2.8$ MPa, $\tau_c = 0.75$ MPa.

$$\tau_v = \frac{V_u}{bd} = \frac{50\times10^3}{230\times450} = 0.483 \text{ MPa}$$

$\tau_v < \tau_{c,max}$ safe in shear.

Provide minimum shear reinforcement.

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y}$$

$$A_{sv} = 2\times\frac{\pi\times8^2}{4} = 100.53 \text{ mm}^2$$

$$S_v = \frac{100.53\times0.87\times250}{0.4\times230}$$

$= 237.7$ mm c/c

$S_v \not> 0.75\,d = 0.75\times450 = 337.5$ mm

$S_v \not> 300$ mm

$\therefore$ Provide spacing of 230 mm c/c

**04. Ans: (c)**

$V_u = 100$ kN

$$\tau_v = \frac{V_u}{b\times d} = \frac{100\times10^3}{230\times450} = 0.966$$

$\tau_v < \tau_{c\,max}$ – shear reinforcement safe

$\tau_v > \tau_c$ not safe in shear reinforcement

Shear reinforcement is required.

Design shear force for shear reinforcement

$V_{us} = V_u - \tau_c\,bd$

$= 100\times10^3 - 0.75\times230\times450$

$= 22.375$ kN

For vertical stirrups,

$$V_{us} = \frac{0.87f_yA_{sv}d}{S_v}$$

$$S_v = \frac{0.87\times250\times100.53\times450}{22.375\times10^3}$$

$= 439.75$ mm

**Min spacing:**

i. 439.75 mm

ii. 0.75d = 0.75 ×450 = 337.5 mm

iii. 300 mm

iv. Spacing for min shear reinforcement

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y} \Rightarrow S_v = 237.7 \text{ mm}$$

Provide min spacing of 230 mm c/c.

**05. Ans: (c)**

**Sol:** $V_u = 150$ kN

$$\tau_v = \frac{150\times10^3}{230\times450} = 1.449 \text{ MPa}$$

$\tau_v < \tau_{c,max}$ – safe in shear reinforcement

$\tau_v > \tau_c \rightarrow$ Shear reinforcement is required.

Design shear force,

$V_{us} = V_u - \tau_c\,bd$

$= 150\times10^3 - 0.75\times230\times450$

$= 72.375$ KN

Shear force taken by bent-up bars.

$V_{us1} = 0.87f_yA_{sv}\sin\alpha$

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^o$$

$$= 102.66 \text{ kN}$$

$\not> 0.5\ V_{us} = 36.18$ kN

$\therefore\ V_{us1} > 0.5\ V_{us}$

As per IS: 456 ; $V_{us1} \not> 0.5\ V_{us}$. In this case $V_{us1}$ is exceeding 0.5 $V_{us}$. Therefore limit $V_{us1}$ as 36.18 kN, the remaining S.F i.e 36.195 kN should be resisted by vertical stirrups.

**Vertical stirrups:**

For $V_{us2} = 36.195$ kN

$$36.195 \times 10^3 = \frac{0.87 f_y A_{sv}.d}{S_v}$$

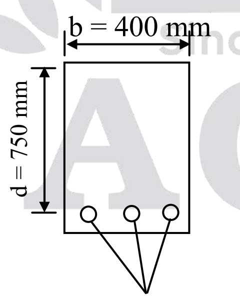
$$S_v = \frac{0.87 \times 250 \times \left(2 \times \frac{\pi}{4} \times 8^2\right) \times 450}{36.195 \times 10^3}$$

$= 271.708$ mm

Provide minimum center to center spacing of 230 mm c/c

**06. Ans: (a)**

**Sol: Beam -P**




$\tau_{c\,max} = 2.1$ MPa

$f_{ck} = 30$ N/mm$^2$

$\tau_c = 0.75$ MPa

$V_u = 400$ kN

$$\tau_v = \frac{V_u}{b \times d} = \frac{400 \times 10^3}{750 \times 400}$$

$\tau_v = 1.33$ N/mm$^2$

i) $\tau_v < \tau_{c\,max}$ –shear reinforcement safe

ii) $\tau_v > \tau_c$ Minimum shear reinforcement is required

$$V_{us} = V_u - \tau_c bd$$
$$= 400 \times 10^3 - 0.75 \times 400 \times 750$$
$$V_{us} = 175 \text{ kN}$$

**Beam –Q**

$V_u = 750$ kN

$$\tau_v = \frac{V_u}{b \times d} = \frac{750 \times 10^3}{750 \times 400} = 2.5\ \text{N/mm}^2$$

$\tau_v > \tau_{c\,max}$

The beam is not safe in shear. It should be revised.

## Conventional Practice Solutions

**01.**

**Sol: Given**

b = 250 mm    d = 450 mm

$A_{st} = 4 - 20$ mm $\phi$    $V_u = 130$ kN

M15 , Fe -250

**Design vertical stirrups:**

1. Nominal shear stress:

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{130 \times 10^3}{250 \times 450} = 1.15\ \text{N/mm}^2$$

2. Shear Resistance of concrete ($\tau_c$)

$$\% \text{ of tension steel} = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 4 \times \frac{\pi}{4} \times 20^2}{250 \times 450}$$

$= 1.11$

| $P_t$ | $\tau_c$ |
|---|---|
| 1.00 | 0.60 |
| 1.11 | – |
| 1.25 | 0.64 |

$$= 0.6 + \frac{(0.64-0.6)(1.11-1.0)}{(1.25-1.0)} = 0.617$$

$= 0.617\ N/mm^2$

$\tau_v > \tau_c$ $\therefore$ Not safe

**Design for Shear :**

Design vertical stirrups,

$V_{us} = V_u - \tau_c\, bd$

$= 130 \times 10^3 - 0.617 \times 250 \times 450$

$= 60.58 \times 10^3 = 60.58$ kN

Assumed 2 legged 8 mm $\phi$ stirrups

$$V_{us} = 0.87\, f_y\, A_{sv} \frac{d}{S_v}$$

$$= 0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 8^2 \times \frac{450}{S_v}$$

$S_v = 162.42 \simeq 160$ mm

**Minimum Shear Reinforcement :**

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

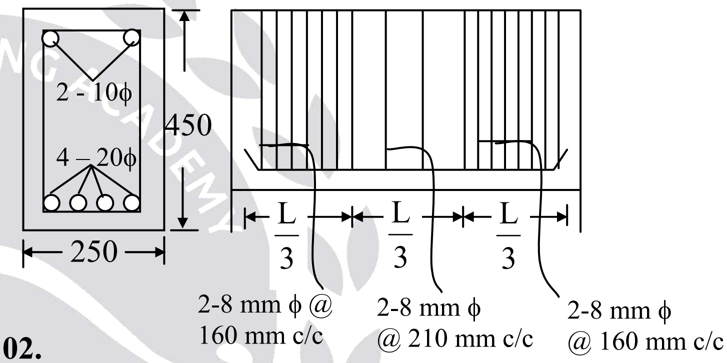
$$\frac{2 \times \frac{\pi}{4} \times 8^2}{250 \times S_v} \geq \frac{0.4}{0.87 \times 250}$$

$S_v = 218.6$ mm

**Check:**

$$\left.\begin{array}{l} \text{i.Cal } S_v = 162.42\text{ mm} \\ \text{ii.} S_v \text{ minimum } = 218.6\text{ mm} \\ \text{iii.} 0.75\,d = 0.75 \times 450 = 337.5 \\ \text{iv.} 300\text{ mm} \end{array}\right\} \text{Smaller}$$

Provide 2 legged 8 mm $\phi$ stirrups with spacing 160 mm @ supports 2-8 mm $\phi$ @ 210 mm c/c @ mid span




**02.**

**Sol: Given**

| | |
|---|---|
| c/c = 2.5 m | $l$ = 5 m |
| $D_f$ = 100 mm | $b_w$ = 300 mm |
| D = 500 mm | $f_{ck}$ = 20 MPa |
| $f_y$ = 415 MPa | $A_{st}$ = 3 – 25 mm $\phi$ |
| L.L = 6 kN/m$^2$ | |

**Design shear reinforcement:**

**Effective span:**

Assume width of support 300 mm

i. For simply supported beam

(a) $l + d = 5 + 0.3 = 5.3$ m

(b) $l + b_s = 5 + 0.45 = 5.45$ m

$d = D - e_c = 500 - 50 = 450$ mm

$l_e = 5.3$ m Provide $l_e = 5.3$ m

**Loads:**

Self weight = $\gamma D_f$ = 25 × 0.1 = 2.5 kN/m$^2$

L.L = 6 kN/m$^2$

Total load = 8.5 kN/m$^2$

Total load per meter length

= 8.5 × 2.5 (c/c distance)

= 21.25 kN/m

Self wt of beam

= 0.4 × 0.3 × 25 = 3 kN/m

Total load = 21.25 + 3 = 24.25 kN/m

Factored load, $W_u$ = 1.5 × 24.25

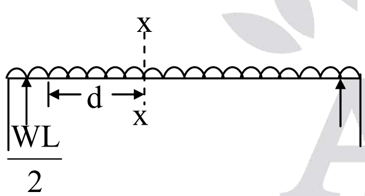
= 36.38 kN/m

Factored shear force,

$$V_u = \frac{W_u \ell_e}{2}$$

$$= \frac{36.38 \times 5.3}{2}$$

= 96.407 kN




Design Shear Force = 96.40 – W × d

$$= 96.40 - 36.38\left(\frac{0.3}{2} + 0.45\right)$$

= 74.5 kN

Nominal shear stress

$$\tau_v = \frac{74.5 \times 10^3}{300 \times 450} = 0.55\,\text{N/mm}^2$$

Shear strength of concrete ($\tau_c$)

$$P_t = \frac{100Ast}{b_w d} = \frac{100 \times 3 \times \frac{\pi}{4} \times 25^2}{300 \times 450}$$

= 1.09

| $P_t$ | $\tau_c$ |
|---|---|
| 1.00 | 0.62 |
| 1.09 | - |
| 1.25 | 0.67 |

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)(1.09 - 1.0)}{(1.25 - 1.0)}$$

$\tau_c$ = 0.638 N/mm$^2$

$\tau_v < \tau_c$

∴ Hence Safe

However provide minimum shear reinforcement

Provide 2 legged 8 mm ϕ stirrups

$$\frac{A_{sv}}{b.S_v} \geq \frac{0.4}{0.87 f_y}$$

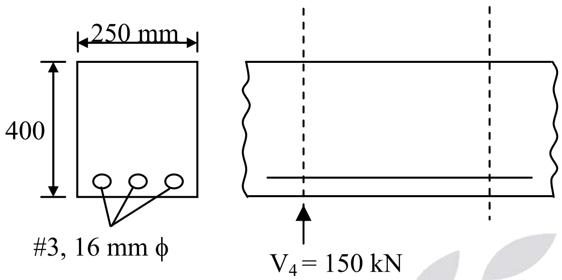
$$\frac{2 \times \frac{\pi}{4} \times 8^2}{300 \times S_v} \geq \frac{0.4}{0.87 \times 415}$$

$S_v \leq 302.47$ mm $\simeq$ 300 mm c/c

However provide 2-legged 8 mm ϕ stirrups with 300 mm c/c

## 07. Bond

**01. Ans: (c)**




**Flexural bond:**

Steel in tension ( sagging moment)

$L_d \ngtr \dfrac{M_1}{V_u} + l_0 \rightarrow$ continuous beam

$l_0 = 12\,\phi = 12 \times 16$

$$\left.\begin{aligned} &= 192\text{mm} \\ d &= 400\text{mm} \end{aligned}\right\}\text{Which is greater}$$

Take $l_0 = 400$ mm

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1} = 870 \text{ mm}$$

$x_{u,\,max} = 0.53 \times 400 = 212$ mm

$$x_u = \frac{0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 15 \times 250}$$

$= 97.18$ mm

$x_u < x_{u,max} \rightarrow$ Under reinforcement section.

$M_1 = 0.36 \times 15 \times 250 \times 97.18\,(400 - 0.42 \times 97.18)$

$= 47.12 \times 10^6$ N-mm

$$L_d \ngtr \frac{47.12 \times 10^6}{150 \times 10^3} + 400 = 714.15 \text{ mm}$$

$L_d > 714.15$

not safe in bond.

**02. Ans: (d)**

**Sol:** $\phi = 12$mm

$f_y = 415$ N/mm$^2$

$f_{ck} = 30$ N/mm$^2$, $\tau_{bd} = 2.4$ MPa

$$L_d = \frac{\phi\sigma_s}{\tau_{bd} \times 4}$$

$$= \frac{12 \times 0.87 \times 415}{(1.6 \times \tau_{bd}) \times 4} = 282.0703$$

$L_d = 282.0703$ mm

$L_d$ with 90° bend $= 282.0703 - 8\phi$

$= 282.0703 - 8 \times 12$

$= 186.1$ mm

**03. Ans: (d)**

**Sol:** Axially loaded short column

$\phi = d = 20$mm, spliced $= 16$ mm

$f_y = 415$ N/mm$^2$

$\tau_{bd} = 1.2$ MPa

$$\left.\begin{aligned} \text{lap} &\nless l_d \\ &\nless 24\phi \end{aligned}\right\}\text{max}$$

Use smaller diameter $\Rightarrow \phi = 16$ mm

$$L_d = \frac{\phi\sigma_s}{4 \times \tau_{bd}} = \frac{16 \times 0.87 \times 415}{1.25 \times 4 \times 1.2 \times 1.6}$$

$= 601.75$ mm

Lap length $\nless L_d = 601.75$ mm

$\nless 24\,\phi = 384$ mm

Use maximum, i.e., 601.75 mm

**04. Ans: (d)**

**Sol:** 1) Pull out (bond fail)

$$P_1 = \tau_{bd}[\pi D l]$$

2) Breaking of steel bar

$$P_2 = \sigma_{st}\left[\frac{\pi}{4}\times D^2\right]$$

$\Big\}$ minimum

**05. Ans: 46.8**

**Sol:** $f_{ck} = 20\ N/mm^2$,

$\tau_{bd} = 1.2$ MPa $\uparrow$60% - HYSD bars

Steel bar is in tension

$$L_d = \frac{\phi\sigma_s}{4\times\tau_{bd}} = \frac{\phi\times 360}{4\times 1.6\times 1.2} = 46.8\phi$$

**06. Ans: 290 mm**

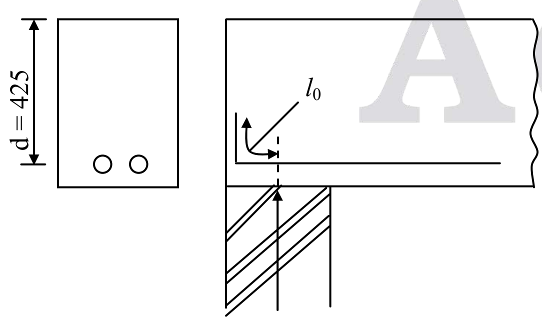
**Sol:** Given , $V_u = 220$ kN

$$A_{st} = 2\times\frac{\pi}{4}\times 16^2 = 402.12\ mm^2$$

b = 250 mm, d = 425 mm

Fe 415 ,$M_{20}$ ,$\tau_{bd} = 1.2$ MPa

$l_0$ = ? for 90° bond




$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87\times 415\times 16}{4\times 1.6\times 1.2}$$

$= 752.1875$ mm

$L_d$ (req) = 752.1875 – 8 × 16

= 624.1875 mm

$x_{u\,max} = 0.48 \times 425 = 204$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87\times 415\times 402.12}{0.36\times 20\times 250}$$

= 80.65 mm

$x_u < x_{u\,max} \rightarrow$ Under reinforced section

$M_1 = 0.87\ f_y A_{st}\ (d - 0.42\ x_u)$

$= 0.87 \times 415 \times 402.12\ (425 - 0.42 \times 80.65)$

$= 56.78 \times 10^6$ N –mm

$$L_d = \frac{1.3 M_1}{V} + l_0$$

$$624.1875 = 1.3\times\frac{56.78\times 10^6}{220\times 10^3} + l_0$$

$l_0 = 288.66$ mm

Minimum extension beyond centre of support = 290 mm

## 08. Limit State of Collapse - Torsion

**01. Ans: (d)**

**Sol:** **i)** size – 300 × 1000 mm

$V_u = 150$ kN; $M_u = 150$ kN

$T_u = 30$ kN -m

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 150\times10^3 + \frac{1.6\times30\times10^6}{300} = 310 \text{ kN}$$

$M_{e1} = M_u + M_T$

$$= M_u + \frac{T_u\left[1+\frac{D}{b}\right]}{1.7}$$

$$= 150 + \frac{30\left[1+\frac{1000}{300}\right]}{1.7} = 226.47 \text{ kN-m}$$

**02. Ans: (d)**

b = 300mm, D = 600 mm

V = 100 kN, M = 100 kN-m

T = 34 kN-m

$M_{e1} = M_u + M_T$

$$= M_u + \frac{T_u\left[1+\frac{D}{b}\right]}{1.7}$$

$$= 100 + \frac{34\left[1+\frac{600}{300}\right]}{1.7}$$

=160 kN-m

**03. Ans: (a)**

**Sol:** T = 68 kN-m

$M_{e2} = M_T - M_u$

If $M_T < M_u$ then no need of $A_{sc}$

$$M_T = \frac{T_u\left(1+\frac{D}{b}\right)}{1.7} = \frac{68\left(1+\frac{600}{300}\right)}{1.7}$$

= 120 kN-m

$M_T > M_u$ – additional compression steel is required for $M_{e2}$ i.e $M_{e2} = M_T - M_u$

= 120 – 100

= 20 kN-m

**04. Ans: (a)**

**Sol:** b = 500, D = 700 mm

d = 35 mm, V = 15 kN

M = 100kN-m, T = 10 kN-m

$\tau_c = 1.5$ MPa

If $\tau_{ve} \not> \tau_c$ ignore torsion

If $\tau_{ve} > \tau_c$ consider torsion for $A_{st}$

$V_e = V_u + V_T$

$$= V_u + 1.6\frac{T_u}{b}$$

$$= 15 + 1.6\left(\frac{10}{0.5}\right)$$

= 47 kN

$$\tau_{ve} = \frac{V_e}{b.d} = \frac{47\times10^3}{500\times(700-35)} \approx \frac{47}{0.5\times0.7}$$

= 0.14 MPa

$\tau_{ve} < \tau_c$

$\therefore$ Design BM for $A_{st}$ is $M_u$ only

$M_u = 100$ kN-m

**05. Ans: (d)**

**Sol:** $V = 20$ kN, $T = 9$ kN-m

$b = 300$ mm, $M = 200$ kN-m

gross depth = 425 mm

cover = 25 mm

$V_e = V_u + V_T$

$$= V_u + 1.6\frac{T_u}{b} = 20 + 1.6\left(\frac{9}{0.3}\right)$$

$= 68$ kN

**06. Ans: (b)**

**Sol:** As $\tau_{ve} < \tau_c$

$T_u = 0$

$M_{e1} = M_u = 200$ kN-m

$A_{st}$ based on $M_u$ only

**Conventional Practice Solutions**

01. Size of the beam = 300 × 500 mm

Effective span, $l_e = 8$ m

$M_u = 100$ kN-m

$T_u = 30$ kN-m

$V_u = 80$ kN-m

Effective cover = 40 mm

M20, Fe415

$\tau_c = 0.55$ N/mm$^2$

**Design the beam :**

Equivalent shear force,

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$V_e = 80 + \frac{1.6\times 30}{0.3} = 240\,\text{kN}$$

Equivalent nominal shear stress

$$\tau_{ve} = \frac{v_e}{bd}$$

$$= \frac{240\times 10^3}{300\times 460} = 1.74\,\text{N/mm}^2$$

$\tau_{ve} > \tau_c$

$\therefore$ Not safe in torsional shear

**Design for Torsion:**

**Longitudinal Steel Design:**

$M_e = M_u + M_t$

$$M_t = T_u\frac{\left(1+\frac{D}{b}\right)}{1.7}$$

$$= 30\frac{\left(1+\frac{500}{300}\right)}{1.7}\times 10^6$$

$= 47.05$ kN-m

$M_e = M_u + M_t$

$= 100 + 47.05$

$= 147.05$ kN-m

$M_t < M_u$

No need of compression steel

$$M_e = 0.87\, f_y\, A_{st}\, d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$147.05 \times 10^6 = 0.87 \times 415 \times A_{st} \times 460$

$$\left[1-\frac{A_{st}}{300\times 460}\frac{415}{20}\right]$$

$A_{st} = 1051\ mm^2$

Use 20 mm $\phi$

$$\text{No. of bars} = \frac{1051}{\frac{\pi}{4}\times 20^2}$$

$= 3.34 \simeq 4$ No's

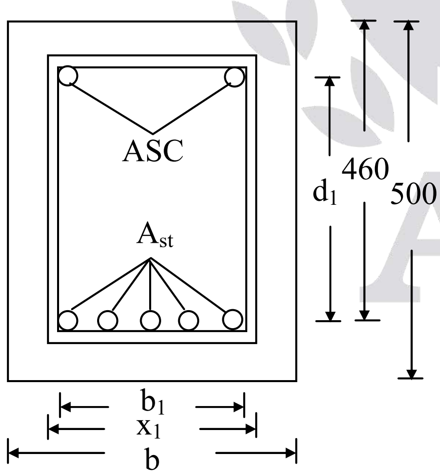
**Transverse Steel :**

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)}$$

$$b_1 = 300 - 2\times 25 = -2(8) - 2\left(\frac{20}{2}\right)$$

$b_1 = 214$ mm

$$d_1 = 460 - 25 - 8 - \frac{12}{2} = 421 \text{ mm}$$




$$x_1 = 300 - 2\times 25 - 2\times\frac{8}{2}$$

$= 242$ mm

$$y_1 = 500 - 40 + \frac{20}{2} + \frac{8}{2} - 25 - \frac{8}{2}$$

$= 500 - 40 + 10 + 4 - 25 - 4$

$y_1 = 445$ mm

$$2\times\frac{\pi}{4}\times 8^2 = \frac{30\times 10^6\times S_v}{214\times 421\times 0.87\times 415} + \frac{80\times 10^3\times S_v}{2.5\times 421\times 0.87\times 415}$$

$= 0.922\ S_v + 0.21 S_v$

$= 1.132\ S_v$

$S_v = 88.8$ mm

$$A_{sv} = \frac{(\tau_{ve} - \tau_c) b.S_v}{0.87 f_y}$$

$$2\times\frac{\pi}{4}\times 8^2 = \frac{(1.74-0.55)\times 300\times S_v}{0.87\times 415}$$

$32\pi = 980 \times 47 \times 10^{-3}\ S_v$

$S_v = 101.67$ mm

**Maximum spacing:**

i. $\not> x_1 \Rightarrow \not> 242$ mm

ii. $\not> \frac{x_1 + y_1}{4} \Rightarrow \frac{242+445}{4} = 171.75\,\text{mm}$

iii. $> 300$ mm

Provide 2 legged 8 mm $\phi$ @ 100 mm c/c

## 09. Slabs

### Conventional Practice Solutions

**01.**

**Sol: Given**

Size = 9 m × 3m

LL = 5 kN/m$^2$

M25, Fe415

Design the Slab

$\frac{L_y}{L_x} = \frac{9}{3} = 3 > 2$ i.e; One Way Slab

Assume width of slab = 1000 mm,

$b_s$ = 300 mm

Assume Effective depth;

$$\frac{\ell}{d} = 20$$

$$\frac{3000}{d} = 20$$

d = 150 mm

$$D = 150 + 20 + \frac{10}{2} = 175\,\text{mm}$$

**1. Effective span:**

(i) $L_{ex}$

$$\left.\begin{aligned}&\text{(a) } \ell_x + b_s = 3 + 0.3 = 3.3\\ &\text{(b) } \ell_x + d = 3 + 0.15 = 3.15\end{aligned}\right\}\text{minimum}$$

$L_{ex}$ = 3.15 m

(ii) $L_{ey}$

$$\left.\begin{aligned}&\text{(a) } \ell_y + b_s = 9 + 0.3 = 9.3\\ &\text{(b) } \ell_y + d = 9 + 0.15 = 9.15\end{aligned}\right\}\text{minimum}$$

$L_{ey}$ = 9.15

Aspect ratio = $\frac{L_y}{L_x} = \frac{9.15}{3.15} = 2.9 > 2$

∴ One way slab

**2. Loads:**

Self weight of slab = $\gamma$D

= 25 × 0.175 = 4.375 kN/m$^2$

L.L = 5 kN/m$^2$ F.F = 0.625 kN/m$^2$

F.F = 0.625 kN/m$^2$

Total load = 4.375 + 5 + 0.625

$W_v$ = 10 kN/m$^2$

Factored load (per m run) = 1.5 × 10

$W_u$ = 15 kN/m

Factored BM, $M_u = \frac{w_u L_{ex}^2}{8}$

$$= \frac{15 \times 3.15^2}{8}$$

$M_u$ = 18.6 kN-m

**Checking effective depth:**

$$d = \sqrt{\frac{M_u}{R_u . b}} = \sqrt{\frac{18.6 \times 10^6}{0.138 f_{ck} b}}$$

$$d = \sqrt{\frac{18.6 \times 10^6}{0.138 \times 25 \times 1000}} \quad d = 73.4\text{ mm}$$

However provided = 100 mm

$$D = 100 + 20 + \frac{10}{2} = 125\,\text{mm}$$

**3. Area of Tension Steel:**

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$$18.6\times10^6 = 0.87\times415\times A_{st}\times100$$

$$\left[1 - \frac{A_{st}}{1000\times100}\times\frac{415}{25}\right]$$

$$18.6\times10^6 = 36.105\times10^3\ A_{st} - 5.99\ A_{st}^2$$

$$A_{st} = 569\ mm^2$$

**4. Minimum Steel:**

$$0.12\%\text{ of bD} = \frac{0.12}{100}\times1000\times125$$

$$= 150\ mm^2 < A_{st}$$

**5. Maximum Steel:**

4% of bD

$$= \frac{4}{100}\times1000\times175 = 5000\,mm^2 > A_{st}\ \therefore\ \text{Ok}$$

$$\text{Spacing, } S = \frac{a_{st}}{A_{st}} = \frac{1000\times\frac{\pi}{4}\times10^2}{569}$$

$$S = 138 \simeq 130\ mm$$

**Check:**

i. 3d = 3 × 100 = 300 mm

ii. 300 mm

iii. S = 130 mm

Provide 10 mm ϕ @ 130 mm c/c

**6. Distribution Steel:**

$$0.12\%\text{ of bD} = \frac{0.12}{100}\times1000\times100$$

$$= 150\ mm^2$$

$$S = \frac{1000\times\frac{\pi}{4}\times8^2}{150}$$

$$S = 335.10 \simeq 330\ mm \simeq 300\ mm$$

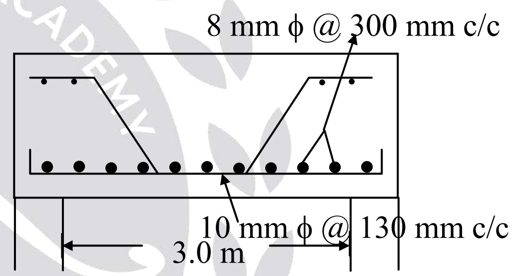
**Check:**

i. 5d = 5 × 100 = 500

ii. 450 mm

iii. S = 300 mm

Provide 8 mm ϕ @ 300 mm c/c




**02.**

**Sol: Given**

Size = 4 m × 5 m

L.L = 3 kN/m$^2$

M20, Fe415

**1.** Assume effective depth

$$\frac{\ell}{d} = 20$$

$$\frac{4000}{d} = 20$$

d = 200 mm

$$D = 200 + 20 + \frac{10}{2} = 225\,mm$$

b = 1000 mm

**2. Effective span:**

(i) $L_{ex}$

$$\left.\begin{aligned} \ell_x + d &= 4 + 0.2 = 4.2 \\ \ell_x + b_s &= 4 + 0.3 = 4.3 \end{aligned}\right\} \text{minimum}$$

$L_{ex} = 4.2$ m

(ii) $L_{ey}$

$$\left.\begin{aligned} \ell_y + d &= 5 + 0.2 = 5.2 \\ \ell_y + b_s &= 5 + 0.3 = 5.3 \end{aligned}\right\} \text{minimum}$$

$L_{ey} = 5.2$ m

Aspect ratio = $\dfrac{L_{ey}}{L_{ex}} = \dfrac{5.2}{4.2} = 1.23 < 2$

Hence design the two way slab

**Loads:**

Self weight of slab = $\gamma D$

$= 25 \times 0.225$

$= 5.625$ kN/m$^2$

L.L = 3 kN/m$^2$

Total load, W = 5.625 + 3

$= 8.625 \simeq 8.63$ kN/m

Factored load, $W_u = 1.5 \times 8.63$

$= 12.94$ kN/m$^2$

Factored load per m run = 12.94 kN/m

**Moments:**

$M_x = \alpha_x w_L L_{ex}^2$

$M_y = \alpha_y w_L L_{ex}^2$

Table 26: Page 91

| | | | | |
|---|---|---|---|---|
| 7 | | 4 | 3 | |
| 5 | | 2 | 1 | 2 |
| 7 | | 4 | 3 | 4 |

| | | | | |
|---|---|---|---|---|
| 9 | | 8 | 6 | 8 |

Four edges discontinuous -9

$\alpha_x$ =

1.2 - 0.072

1.23 - ?

1.3 - 0.079

$$0.072 + \frac{(1.23 - 1.2)(0.079 - 0.072)}{(1.3 - 1.2)}$$

$1.23 \rightarrow \alpha_x = 0.0741$

$\alpha_y = 0.056$

$M_x = \alpha_x w_L \ell_{ex}^2$

$= 0.0741 \times 12.94 \times 4.2^2$

$M_x = 16.9$ kNm

$M_y = \alpha_y w_L \ell_{ex}^2 = 0.056 \times 12.94 \times 4.2^2$

$= 12.78$ kNm

Checking effective depth:

$$d = \sqrt{\frac{M_x}{R_u b}} = \sqrt{\frac{16.9 \times 10^6}{0.138 \times 20 \times 1000}}$$

$= 78.27$ mm

However provide $d_1 = 100$ mm

$D = 100 + 20 + \dfrac{10}{2} = 125$ mm

**Area of Steel Reinforcement:**

$$M_x = 0.87 f_y A_{st} d_1 \left[1 - \frac{A_{st\,x}}{bd_1} \times \frac{f_y}{f_{ck}}\right]$$

$$16.91 \times 10^6 = 0.87 \times 415 \times A_{st\,x} \times 100$$

$$\left[1 - \frac{A_{st\,x}}{1000 \times 100} \times \frac{415}{20}\right]$$

$A_{st\,x} = 525.6\ mm^2$

$$d_2 = 100 - \frac{10}{2} - \frac{10}{2} = 90\ mm$$

$$M_y = 0.87\ f_y\ A_{st\,y}\ d_2 \left[1 - \frac{A_{st\,y}}{bd_2} \times \frac{fy}{f_{ck}}\right]$$

$$12.78 \times 10^6 = 0.87 \times 415 \times A_{st\,y} \times 90$$

$$\left[1 - \frac{A_{st\,y}}{1000 \times 90} \times \frac{415}{20}\right]$$

$A_{sty} = 437.4\ mm^2$

**Minimum Steel:**

$$0.12\%\ \text{of}\ bD = \frac{0.12}{100} \times 1000 \times 125$$

$$= 150\ mm^2 < A_{stx}$$

$$< A_{sty}$$

**Maximum Steel:**

$$4\%\ \text{of}\ bD = \frac{4}{100} \times 1000 \times 125$$

$$= 5000\ mm^2 > A_{st\,x}$$

$$> A_{st\,y} \quad \therefore\ \text{Ok}$$

**Spacing:**

Use 10 mm $\phi$ bars,

$$S_x = \frac{1000 a_{st}}{A_{st\,x}} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{525.6}$$

$a_{st} = 149.4\ mm \simeq 140\ mm$

$$S_y = \frac{1000 a_{st}}{A_{st\,y}} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{437.4}$$

$= 179.5\ mm \simeq 170mm$

**Check:**

i. $3d = 3 \times 100 = 300$ mm

ii. 300 mm

Provide 10 mm $\phi$ @ 140 mm c/c along Shorter span

Provide 10 mm $\phi$ @ 170 mm c/c along Longer span

The above steel can be provided only in the middle strips.

**Distribution Steel:**

$$0.12\%\ \text{of}\ bD\ \ \frac{0.12}{100} \times 1000 \times 125 = 150\ mm^2$$

Use 8 mm $\phi$,

$$\text{Spacing,}\ \ S = \frac{1000\, a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{150}$$

$= 335.1\ mm \simeq 300\ mm$

Provide 8 mm $\phi$ @ 300 mm as a distribution steel.

**Check:**

i. 5d = 500 mm

ii. 450 mm

The above steel can be provided only in edge strip

**Design of torsion steel:**

$$\frac{3}{4}A_{st\,x} = \frac{3}{4}\times 525.6 = 394.2\,\text{mm}^2$$

$$\text{Size of mat} = \frac{\ell_x}{5}\times\frac{\ell_x}{5} \quad \frac{4.2}{5} = 0.84\,\text{m}$$

Use 8 mm $\phi$,

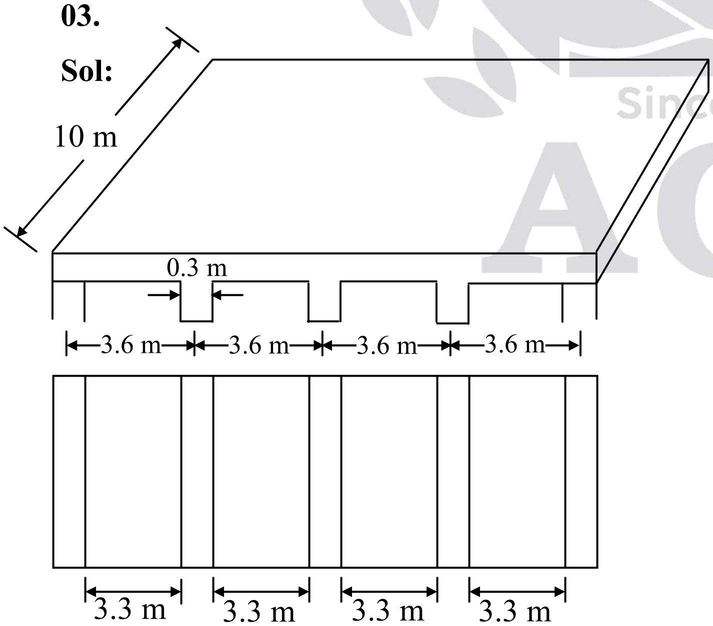
$$\text{Spacing, } S = \frac{840\times\frac{\pi}{4}\times 8^2}{394.2}$$

$= 107.11$ mm $\simeq$ 100 mm

Provide 8 mm $\phi$ @ 100 mm c/c

**03.**

**Sol:**




Live Load = 3 kN/m$^2$

Partition Loads = 1.35 kN/m$^2$

Floor Finish = 0.65 kN/m$^2$

M20 Fe415

Assume effective depth

$$d = \frac{\ell}{25} = \frac{3300}{25} = 132\,\text{mm}$$

d = 150 mm

$$D = 150 + 20 + \frac{10}{2} = 175\,\text{mm}$$

b = 1000 mm, $b_s$ = 300 mm

**Effective span:**

$\mathbf{L_{ex}}$

$$\left.\begin{array}{l} \text{i. } L_x + d = 3.3 + 0.15 = 3.45 \\ \text{ii. } L_x + b_s = 3.3 + 0.3 = 3.6 \end{array}\right\} \text{minimum}$$

$L_{ex}$ = 3.45

$\mathbf{L_{ey}}$

$$\left.\begin{array}{l} \text{i. } L_y + d = 10 + 0.15 = 10.15 \\ \text{ii. } L_y + b_s = 10 + 0.3 = 10.3 \end{array}\right\} \text{minimum}$$

$L_{ey}$ = 10.15 m

$$\frac{L_{ey}}{L_{ex}} = \frac{10.15}{3.45} = 2.94 > 2$$

**Loads:**

Self weight = $\gamma$. D= 0.175 × 25 = 4.375 kN/m$^2$

Total load = 4.375 + 1.35 + 0.65

$W_D$ = 6.375 kN/m$^2$

Factored load per m run

$W_D = 6.375$ kN/m

$W_L = 3$ kN/m$^2$

**Moment for end panel**

**Near mid span:**

$$M_1 = \frac{W_D \ell^2}{12} + \frac{W_L \ell^2}{10}$$

$$= \frac{6.375 \times 3.45^2}{12} + \frac{3 \times 3.45^2}{10}$$

$= 9.89$

$M_{u1} = 1.5 \times 9.89 = 14.835$ kN-m

**At Support:**

Next to the end support

$$M_2 = -\left(\frac{1}{10} W_D L^2 + \frac{1}{9} w_L L^2\right)$$

$$= -\left(\frac{1}{10} \times 6.375 \times 3.45^2 + \frac{1}{9} \times 3 \times 3.45^2\right)$$

$M_2 = -11.55$

$M_{u2} = -1.5 \times 11.55 = -17.33$ kN-m

**At Interior Span:**

At Middle of interior panel

$$M_3 = \frac{1}{16} W_D L^2 + \frac{1}{12} W_L L^2$$

$$= \frac{1}{16} \times 6.375 \times 3.45^2 + \frac{1}{12} \times 3 \times 3.45^2$$

$= 7.71$

$M_{u3} = 1.5 \times 7.71 = 11.57$ kN-m

**At Interior Supports:**

$$M_4 = -\left(\frac{1}{12} W_D L^2 + \frac{1}{9} W_L L^2\right)$$

$$= -\left(\frac{1}{12} \times 6.375 \times 3.45^2 + \frac{1}{9} \times 3 \times 3.45^2\right)$$

$= -10.29$

$M_{u4} = 1.5 \times 10.29 = -15.43$ kN-m

**Checking Effective Depth:**

$$d = \sqrt{\frac{\text{Maximum moment}}{R_u \times b}}$$

$$= \sqrt{\frac{17.33 \times 10^6}{0.138 \times 20 \times 1000}} = 79.24 \text{ mm}$$

However,

Provided = 125 mm, D = 150 mm

**Area of Steel Reinforcement:**

$$M_{u1} = 0.87 f_y A_{st1} d\left[1 - \frac{A_{st1}}{bd} \times \frac{f_y}{f_{ck}}\right]$$

$$= 0.87 \times 415 \times A_{st1} \times 125$$

$$\left[1 - \frac{A_{st1}}{1000 \times 125} \times \frac{415}{20}\right]$$

$14.835 \times 10^6 = 45.13 \times 10^3 \; A_{st\,1} - 7.49 \; A_{st1}^2$

$A_{st1} = 348.9$ mm$^2$

$M_{u2} = -17.33 \times 10^6$

$17.33 \times 10^6 = 45.13 \times 10^3 \; A_{st\,2} - 7.49 \; A_{st\,2}^2$

$A_{st\,2} = 412.2$ mm$^2$

$M_{u3} = 11.57 \times 10^6$

$11.57 \times 10^6 = 45.13 \times 10^3\ A_{st\,3} - 7.49\ A_{st\,3}{}^2$

$A_{st\,3} = 268.31\ mm^3$

$M_{u4} = 15.43 \times 10^6$

$15.43 \times 10^6 = 45.13 \times 10^3 A_{st\,4} - 7.49\ A_{st\,4}{}^2$

$A_{st\,4} = 363.87\ mm^2$

**Minimum steel:**

0.12% of bD

$$= \frac{0.12}{100} \times 1000 \times 150 = 180\,mm^2$$

**Maximum Steel:**

$$4\ \%\ \text{of bD} = \frac{4}{100} \times 1000 \times 150 = 6000\,mm^2$$

**Spacing :**

For $A_{st\,1}$

$$S = \frac{1000 \times \frac{\pi}{4} \times 10^2}{349} = \frac{78.53 \times 10^3}{349}$$

$= 225.04\ mm \simeq 220\ mm$

For $A_{st\,2}$

$$S = \frac{1000 \times \frac{\pi}{4} \times 10^2}{412} = 190.6\,mm \simeq 180\ mm$$

For $A_{st\,3}$

$$S = \frac{1000 \times \frac{\pi}{4} \times 10^2}{268.31} = 292.68\,mm \simeq 290\ mm$$

For $Ast_4$

$$S = \frac{1000 \times \frac{\pi}{4} \times 10^2}{363.85} = 215.83\,mm \simeq 210\ mm$$

**Check:**

Main steel

i. $3d = 3 \times 120 = 360$ mm

ii. 300 mm

Provide

10 mm $\phi$ @ 220 mm c/c

10 mm $\phi$ @ 180 mm c/c

10 mm $\phi$ @ 290 mm c/c

10 mm $\phi$ @ 210 mm c/c

**Distribution Steel:**

$$\frac{0.12}{100} \times 1000 \times 150 = 180\,mm^2$$

$$\text{Use 8 mm } \phi \text{ bar}, S = \frac{1000 \times \frac{\pi}{4} \times 8^2}{180}$$

$= 279.25\,mm \simeq 270\ mm$

**Check:**

i. $5d = 5 \times 120 = 600$ mm

ii. 450 mm

Provide 8 mm $\phi$ @ 270 mm c/c

## 10. Limit State of Collapse - Compression

**01. Ans: (c)**

**Sol:** b = 300 mm

d = 600 mm

$f_y = 415$ MPa

$f_{ck} = 20$ MPa

$P_u = 0.40 f_{ck} A_c + 0.67\, f_y A_{sc}$

$A_{sc} = 0.8\%\ A_g$

$= \frac{0.8}{100}(300\times600) = 1440\ \text{mm}^2$

$A_c = A_g - A_{sc}$

$= 300 \times 600 - 1440$

$= 178560\ \text{mm}^2$

$P_u = 0.4 \times 20 \times 178560 + 0.67 \times 415 \times 1440$

$P_u = 1829$ kN

**02. Ans: (d)**

**Sol:** d = 300 mm; $f_{ck} = 20\ \text{N/mm}^2$

$f_y = 415\ \text{N/mm}^2$;

$P_u = 1.05[0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$

$A_{sc} = \left(\frac{\pi}{4}\times300^2\right)\times\frac{1}{100} = 706.85\text{mm}^2$

$A_c = A_g - A_{sc}$

$= \left(\frac{\pi}{4}\times300^2\right) - 706.85$

$= 69978.98\ \text{mm}^2$

$P_u = 1.05(0.4 \times 20 \times 69978.98 + 0.67 \times 415 \times 706.85)$

$= 794.19$ kN

**03. Ans: (d)**

**Sol:** $A_g = 300 \times 300$ mm

$f_{ck} = 20\ \text{N/mm}^2$,

$A_c = A_g$ (neglecting $A_{sc}$)

$f_y = 415\ \text{N/mm}^2$

$A_{sc} = 4\times\frac{\pi}{4}\times20^2 = 1256.63$

$P_u = 0.4\times20\times300\times300 + 0.67 \times 415 \times 1256.63$

$= 1069$ kN

**04. Ans: (d)**

**Sol:** $m = \frac{E_{strong}}{E_{weak}} = \frac{E_{steel}}{E_{conc}}$

compatability condition for composite (RCC) members

$\delta_s = \delta_c$

$\frac{P_s l}{A_s E_s} = \frac{P_c l}{A_c E_c}$

$\frac{P_s}{P_c} = \frac{A_s}{A_c}\left(\frac{E_s}{E_c}\right) \quad = \frac{1\%\,A_c}{A_c}\times10 = 10\%$

**Conventional Practice Solutions**

**01.**

**Sol: Given**

$P_u = 1900$ kN

Square column

M20 , Fe250

$e_{min} < 0.05$ D

Assume longitudinal steel $A_{sc} = 1\%$ of $A_g$

Size of column & $A_{sc}$

$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}$

$$1900 \times 10^3 = 0.4 \times 20 \times \left( A_g - \frac{1}{100} \times A_g \right) + 0.67 \times 250 \times \frac{1}{100}\, Ag$$

$A_g = 198.03 \times 10^3\ mm^2$

Side of square column $= \sqrt{198.03 \times 10^3}$

$= 445$ mm

Provide 450 mm × 450 mm

Longitudinal steel

$A_{sc} = 1\%$ of Ag

$$= \frac{450 \times 450}{100} = 2025\, mm^2$$

No of bars required, use 4 – 20 mm $\phi$ & 4 – 16 mm $\phi$

**Transverse steel (Laterals Ties):**

**Diameter:**

i. $\frac{1}{4}\phi_{LLD} + \frac{1}{4} \times 20 = 5\, mm$

ii. 8 mm

**Pitch:**

i. LLD = 450 mm

ii. $16\, \phi_{SL} = 16 \times 16 = 256$ mm

iii. 300 mm

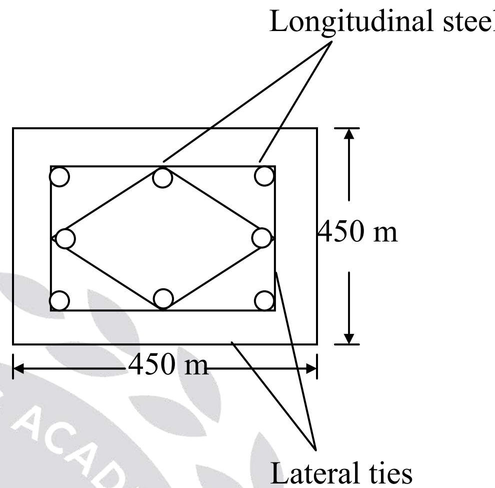
Provided 8 mm $\phi$ @ 250 mm

**Longitudinal Steel:**

4 – 20 mm $\phi$ & 4 – 16 mm $\phi$

**Tension Steel:**

8 mm $\phi$ @ 250 mm c/c




**02.**

**Sol: Given:**

P = 1000 kN

M20 , Fe 415

It is assuming that the column design with helical reinforcement.

1. Assume longitudinal steel:

   $A_{sc} = 0.8\%$ of $A_g$

2. $P_u = 1.5 \times 1000 = 1500$ kN

   Size of column & longitudinal steel

   $P_u = 1.05\, (0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc})$

   $1500 \times 10^3 = 1.05\, (0.4 \times 20 \times (A_g - A_{sc}) + 0.67\, f_y\, A_{sc})$

   $1500 \times 10^3 = 1.05$

$$\left(0.4 \times 20 \times \left(A_g - \frac{0.8}{100} A_g\right)\right) + 0.67 \times 415 \times \frac{0.8}{100} A_g$$

$$\frac{\pi}{4} D^2 = 140.6 \times 10^3$$

D = 423.1 mm $\simeq$ 430 mm

**Longitudinal steel:**

$$A_{sc} = \frac{0.8}{100} \times \frac{\pi}{4} \times 430^2 = 1161.76 \text{ mm}^2$$

No of bars required, use 16 mm $\phi$

$$n = \frac{A_{sc}}{a_{sc}} = \frac{1161.76}{\frac{\pi}{4} \times 16^2} = 5.77 \simeq 6 \text{ No's}$$

**Transverse Steel (Helical Reinforcement):**

**Diameter:**

i. $\frac{1}{4}\phi_{LLD} = \frac{1}{4} \times 16 = 4 \text{ mm}$

ii. 8 mm

Provide 8 mm $\phi$

**Calculation of Pitch :**

$$\frac{V_h}{V_k} \geq 0.36\left[\frac{A_g}{A_k} - 1\right]\frac{f_{ck}}{f_y}$$

Core dia = $\phi_k$ = D – 2c = 430 – 2 × 40

= 350 mm

Core distance $d_h = D - 2(c) - 2\left(\frac{\phi_h}{2}\right)$

$$= 430 - 2 \times 40 - 2 \times \frac{8}{2}$$

= 342 mm

**Area of Gross Cross Section :**

$$A_g = \frac{\pi}{4} \times 430^2 = 145.22 \times 10^3 \text{ mm}^3$$

Area of core ,

$$A_k = \frac{\pi}{4} \times 350^2 = 96.21 \times 10^3 \text{ mm}^2$$

Volume of helical reinforcement,

$$V_h = \pi d_h \left(\frac{\pi}{4}\phi_h^2\right)$$

$$= \pi \times 342 \times \frac{\pi}{4} \times 8^2 = 54 \times 10^3$$

$$V_k = \frac{\pi}{4} \times \phi_k^2 \times P$$

$$= \frac{\pi}{4} \times 350^2 \times P$$

$$\frac{54 \times 10^3}{\frac{\pi}{4} \times 350^2 \times P} \geq 0.36\left[\frac{145.22 \times 10^3}{96.21 \times 10^3} - 1\right]\frac{20}{415}$$

P ≤ 63.5 mm

**Pitch**

i. $\not>$ 75 mm

ii. $\not> \frac{1}{6}\phi_k = \frac{1}{6} \times 350 = 58.33 \text{ mm}$

iii. $\not<$ 25 mm

iv. $\not<$ 3 $\phi_h$ = 3 × 8 = 24 mm

Provide 55 mm pitch.

## 11. Footings

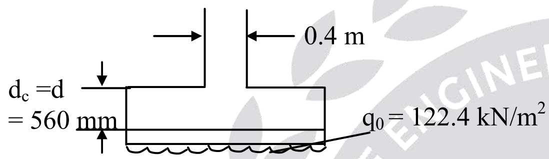
**01. Ans: (b)**

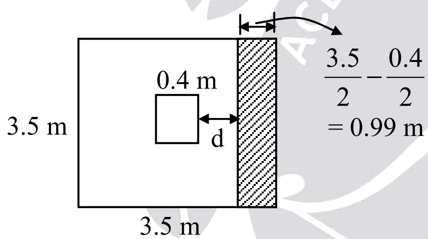
**Sol:** B = 3.5m

column size = 400 mm

d = 560 mm

$q_0 = 122.4\ \text{kN/m}^2$




For one way shear

$V_u = q_0$[hatched area]

$= 122.4\ [0.99 \times 3.5]$

$= 425$ kN

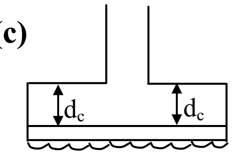
$$\tau_v = \frac{V_u}{b.d_c} = \frac{425 \times 10^3}{3500 \times 560}$$

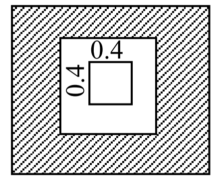
$= 0.22\ \text{N/mm}^2$

$= 0.22$ MPa

**02. Ans: (c)**

**Sol:**




$$B = 0.4 + \frac{0.56}{2} + \frac{0.56}{2}$$

$= 0.96$

$V_u = q_0$[hatched area]

$= 122.4 \times [3.5^2 - 0.96^2]$

$= 1386$ kN

$$\tau_v = \frac{V_u}{pd} = \frac{1386 \times 10^3}{(4 \times 960)(560)}$$

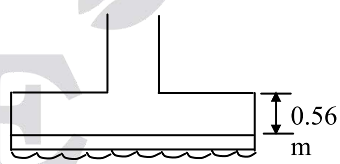
$= 0.64$ MPa

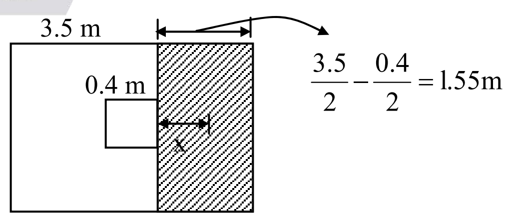
$V_u$ is more for 2-way

2-way shear is critical

**03. Ans: (a)**

**Sol:**

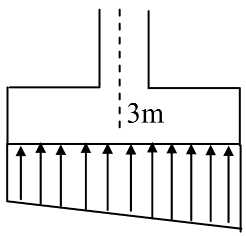


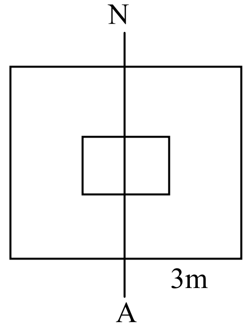




$M_u = q_0$[hatched area $\times\ \bar{x}$]

$$= 122.4\left[3.5 \times 1.55 \times \frac{1.55}{2}\right] = 515\ \text{kN}$$

**04. Ans: (a)**




$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{450}{3\times 2} \pm \frac{60}{\left(\dfrac{2\times 3^2}{6}\right)}$$

$\sigma_{max}$ = 95 kN/m$^2$ compression

$\sigma_{min}$ = 55 kN/m$^2$ compression

As per IS 456 –2000 the assumed pressure distribution below the footing is uniform

**05. Ans: (a)**

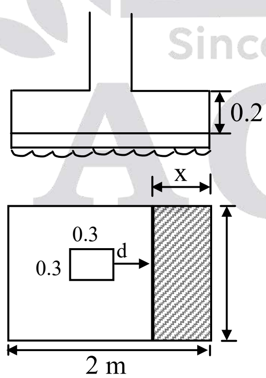
**Sol:** $l$ = 2m ; d = 200 mm

column size = 300 ×300 mm

$q_0$ = 320 kN

$\tau_v$ = ?

$$q_0 = \frac{320}{2\times 2} = 80\,\text{kN/m}^2$$




$$x = \frac{2}{2} - \frac{0.3}{2} - 0.2$$

= 1 – 0.15 – 0.2

= 0.65

One way shear $V_u = q_0$ [hatched area]

= 80[0.65 ×2] = 104 kN

$$\tau_v = \frac{V_u}{bd_c} = \frac{104\times 10^3}{2000\times 200} = 0.26$$

## Conventional Practice Solutions

01. A square footing **3.5 m × 3.5 m** is used for a square column 300 mm × 300 mm carrying a total ultimate load of 1500 kN. Safe bearing capacity of soil is 100 kN/m$^2$ under service loads, using grades M20 and Fe-415. Shear strength of concrete is 0.4 MPa. Design the pad type footing.

**Sol: Given :**

Size of column : 300 mm × 300 mm

Column load (Factored) : 1500 kN

S.B.C of soil $q_o$ : 100 kN/m$^2$

M20, Fe-415

**Size of Footing :**

Net upward soil pressure

$$= P_o = \frac{W}{A} = \frac{1000}{3.5\times 3.5}$$

= 81.6 kN/m$^2$ < 100 kN/m$^2$ ∴ O.K.

**Depth of Footing:**

**i.** Based on B.M

$$M = \frac{P_o B}{8}[B-b]^2$$

$$= \frac{81.6\times 3.5}{8}[3.5-0.3]^2 = 365.56\text{ kN-m}$$

$M_u$ = 1.5 × 365.56 = 548.35 kN-m

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} B}} = \sqrt{\frac{548.35\times 10^6}{0.138\times 20\times 3500}} = 238.25\,\text{mm}$$

**ii.** Based on one way shear

Shear Force, $V = P_o B \left[\frac{B}{2} - \frac{b}{2} - d\right]$

$= 81.6 \times 3.5 \left[\frac{3.5}{2} - \frac{0.3}{2} - d\right]$

$V_u = 1.5V$

$= 1.5 \times 81.6 \times 3.5 \left[\frac{3.5}{2} - \frac{0.3}{2} - d\right]$

$= 428.4\,[1.6 - d]$

Nominal shear stress, $\tau_v = \frac{V_u}{Bd}$

$= \frac{428.4[1.6 - d]}{3.5 \times d}$

$\tau_c = 0.4\text{ N/mm}^2 \rightarrow 400\text{ kN/m}^2$

For safety $\tau_v \le k\tau_c$

Assume $D \ge 300$ mm, $k = 1$

$\frac{428.4[1.6 - d]}{3.5 \times d} \le 400$

$122.4[1.6 - d] \le 400\,d$

$[1.6 - d] \le 3.26\,d$

$0.37\text{ m} \le d$

$d \ge 0.37$ m

$d \ge 370$ mm

Take maximum of above two cases

Adopted D =500 mm, d = 500 – 60

= 440 mm

**Check for punching Shear:**

$V = P_o\,[B^2 - (b + d)^2]$

$= 81.6\,[3.5^2 - (0.3 + 0.44)^2]$

= 954.9 kN

$V_u = 1.5\,V = 1.5 \times 954.9$

= 1432 kN

**Punching Shear Stress:**

$\tau_v = \frac{V_u}{b_o d}$

$= \frac{1432 \times 10^3}{4(300 + 440)440} = 1.09\text{ N/mm}^2$

Permissible shear stress = $K_s\,\tau_c$

$K_s = 0.5 + \beta_c \not> 1$

$= 0.5 + \frac{0.3}{0.3} = 1.5 > 1$ $\quad\therefore K_s = 1$

$\tau_c = 0.25\sqrt{f_{ck}}$

$= 0.25\sqrt{20} = 1.118\text{ N/mm}^2$

$\tau_v \le K_s\,\tau_c$ $\quad\therefore$ Safe

**Area of Tension Steel Required:**

$M_u = 0.87\,f_y\,A_{st}d\left[1 - \frac{A_{st}}{Bd} \times \frac{f_y}{f_{ck}}\right]$

$= 0.87 \times 415 \times A_{st} \times 440 \left[1 - \frac{A_{st}}{3500 \times 440} \times \frac{415}{20}\right]$

$548.35 \times 10^6 = 158.86 \times 10^3\,A_{st} - 2.14\,A_{st}^2$

$A_{st} = 3629\text{ mm}^2$

Spacing, $S = \frac{B \times a_{st}}{A_{st}}$ (use 16 mm $\phi$ bars)

$$= \frac{3500 \times \frac{\pi}{4} \times 16^2}{3629} = 193.8\,\text{mm}$$

Provide 16 mm $\phi$ @ 190 mm C/C

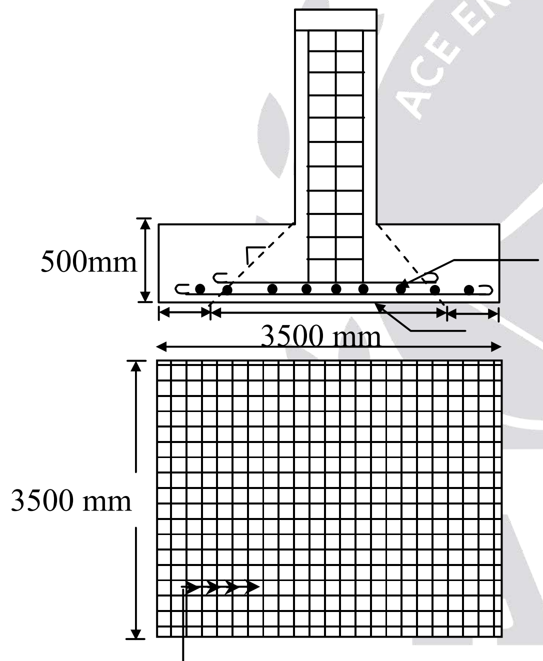
**Check for Development Length:**

$L_d = 47\ \phi = 47 \times 16 = 752$ mm

Available length of footing $= \frac{1}{2}(B - b) - 60$

$$= \frac{1}{2}(3500 - 300) - 60$$

$= 1540$ mm $> L_d$ $\therefore$ OK




**02.**

**Sol: Given**

Size of column = 450 mm × 600 mm

DL = 880 kN, LL = 1420 kN

SBC of soil ,$q_o$ = 140 kN/m$^2$

M20 & Fe415

**Design Rectangular Footing:**

**Size of Footing:**

Area of footing $= \frac{W + W'}{q_o}$

$$= \frac{1420 + 880 + \frac{10}{100}(1420 + 880)}{140}$$

$= 18$ m$^2$

**Dimensions of Footing:**

$$\frac{B}{L} = \frac{b}{a}$$

$B \times L = 18$

$$\frac{b}{a} \times L \times L = 18$$

$$\frac{0.45}{0.6} \times L^2 = 18$$

$L = 4.89 \simeq 5$ m

$B = 3.67$ m

Provide 5.0 m × 3.75 m

Net upward soil pressure

$$P_o = \frac{W}{A_P} = \frac{1420 + 880}{5 \times 3.75}$$

$= 122.67$ kN/m$^2$

$P_o < q_o$ $\therefore$ Safe

**Depth of footing:**

**i. From maximum bending moment:**

$$M_1 = \frac{P_o B}{8}(L-a)^2$$

$$= \frac{122.67 \times 3.75}{8}(5-0.6)^2$$

= 1113.2 kNm

$M_{u1}$ = 1.5 × 1113.2 = 1669.84 kN-m

$$M_2 = \frac{P_o L}{8}(B-b)^2$$

$$= \frac{122.67 \times 5}{8}(3.75-0.45)^2$$

= 834.9 kN-m

$M_{u2}$ = 1.5 × 834.9 = 1252.38 kN-m

**Effective depth:**

$$d = \sqrt{\frac{M_{u1}}{R_u . B}}$$

$$= \sqrt{\frac{1669.84 \times 10^6}{0.138 \times 20 \times 3.75 \times 10^3}}$$

d = 401.66 mm

**ii. From one way shear :**

$$V = P_o B\left[\frac{L}{2} - \frac{a}{2} - d\right]$$

$$= 122.67 \times 3.75\left[\frac{5}{2} - \frac{0.6}{2} - d\right]$$

V = 460 [2.2 – d]

$V_u$ = 1.5 × V= 690 [2.2 – d]

**Nominal shear stress :**

$$\tau_v = \frac{V_u}{Bd}$$

$$= \frac{690[2.2-d]}{3.75 \times d}$$

$$\tau_v = \frac{184[2.2-d]}{d}$$

For safety $\tau_v \le k\, \tau_c$

Assume $\tau_c$ = 0.3 N/mm$^2$ for an under reinforced section

$\tau_v = \tau_c$

$$\frac{184[2.2-d]}{d} = 300 \text{ kN/m}^2$$

404.8 – 184 d = 300 d

d = 836 × 10$^{-3}$ m

d = 836 mm

**iii. From two way shear :**

V = $P_o$[ L× B – (a+d) (b+d)]

= 122.67 [5 × 3.75 – [(0.6 + 0.836) (0.45 + 0.836)]

V = 2073.52 kN

$V_u$ = 1.5 × 2073.52 = 3110.28 kN

Punching shear stress ,

$$\tau_v = \frac{V_u}{b_o . d} = \frac{V_u}{2[(a+d)(b+d)]d}$$

$$= \frac{3110.28 \times 10^3}{2[(600+836)+(450+836)]836}$$

$\tau_v$ = 0.68 N/mm$^2$

Shear resistance of concrete = $K_s\, \tau_c$

$K_s = 0.5 + \beta_c$

$= 0.5 + \frac{0.45}{0.6}$

$= 1.25 > 1$

$K_s = 1.0$

$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20}$

$= 1.118\ N/mm^2$

$\tau_v < K_s\,\tau_c$

$\therefore$ Safe

Provide $d_1 = 900$ mm

$D = 950$ mm

**Area of Tension Steel:**

$$M_{u1} = 0.87\, f_y\, A_{st1}\, d_1\left[1 - \frac{A_{st1}}{Bd} \times \frac{f_y}{f_{ck}}\right]$$

$$1669.84 \times 10^6 = 0.87 \times 415\, A_{st1} \times 900 \left[1 - \frac{A_{st1} \times 415}{3750 \times 900 \times 20}\right]$$

$$1669.84 \times 10^6 = 324.9 \times 10^3\, A_{st1} - 1.99\, A_{st1}^2$$

$A_{st\,1} = 5312\ mm^2$

Use 16 mm $\phi$ bars,

$$\text{Spacing, } S = \frac{Ba_{st}}{A_{st}} = \frac{3750 \times \frac{\pi}{4} \times 16^2}{5312}$$

$= 142\ mm \simeq 140\ mm$

Provide 16 mm $\phi$ @ 140 mm c/c

$d_2 = 900 - 16 = 884$ mm

$$M_{u2} = 0.87\, f_y\, A_{st2}\, d_2\left[1 - \frac{A_{st2}}{L.d_2} \times \frac{f_y}{f_{ck}}\right]$$

$$1252.38 \times 10^6 = 0.87 \times 415 \times A_{st2} \times 884 \left[1 - \frac{A_{st2} \times 415}{5000 \times 884 \times 20}\right]$$

$$1252.38 \times 10^6 = 319.168 \times 10^3\, A_{st2} - 1.498\, A_{st2}^2$$

$A_{st\,2} = 4000\ mm^2$

(i) Area of steel provided in middle band

$$A_{st2B} = \frac{2A_{st2}}{\beta + 1}$$

$$= \frac{2 \times 4000}{\frac{5}{3.75} + 1} = 3428.57\ mm^2$$

Use 16 mm $\phi$ bars,

$$\text{Spacing, } S = \frac{Ba_{st}}{A_{st}}$$

$$= \frac{3750 \times \frac{\pi}{4} \times 16^2}{3428.57} = 219.9\ mm$$

Provide 16 mm $\phi$ @ 210 mm c/c

(ii) Area of steel provided in end band:

$$A_{st2A} = \frac{A_{st2} - A_{st2B}}{2}$$

$$= \frac{4000 - 3428.57}{2} = 285.715\ mm^2$$

$$\text{No. of bars required} = \frac{A_{st}}{a_{st}} = \frac{285.715}{\frac{\pi}{4} \times 16^2}$$

$= 1.42$

However provide '3' No's

**Check for development length:**

$L_d \ngtr \frac{1}{2}(B-b) - \text{Cover}$

$47\,\phi = 47 \times 16 = 752$ mm

Available length

$$= \frac{1}{2}[3750 - 450] - 50$$

$$= 1600 \text{ mm} > L_d$$

$\therefore$ Safe

**03.**

**Sol:** Size of column = 450 mm × 450 mm

$A_{sc} = 8 - 18$ mm$\phi$

$W = 1480$ kN

$q_o = 120$ kN/m$^2$

**Design a Sloped Footing :**

**Size of Footing:**

$$A = \frac{W + W'}{q_o} = \frac{1480 + \frac{10}{100} \times 1480}{120}$$

$$= 13.57 \text{ m}^2$$

Side of square footing $= \sqrt{13.57} = 3.68$ m

Provide $(A_p) = 3.75$ m × 3.75 m

Net upward soil pressure $P_o = \frac{W}{A_p}$

$$P_o = \frac{1480}{3.75 \times 3.75}$$

$$P_o = 105.24 \text{ kN/m}^2$$

**Depth of footing:**

Bending moment:

$$M = \frac{P_o B}{8}(B-b)^2$$

$$= \frac{105.24 \times 3.75}{8}(3.75 - 0.45)^2$$

$$M = 537.2 \text{ kNm}$$

$M_u = 1.5\ M = 1.5 \times 537.2$

$= 805.8$ kN-m

Effective depth required,

$$d = \sqrt{\frac{M_u}{R_u b}} = \sqrt{\frac{805.8 \times 10^6}{0.138 \times 20 \times 450}}$$

$$= 805.5 \text{ mm}$$

Provide at edges 200 mm thickness and at centre 850 mm

**Check for one way shear:**

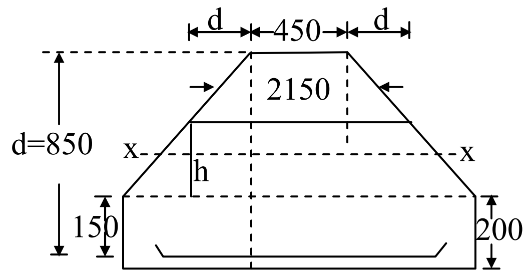
$$V = P_o B\left[\frac{B}{2} - \frac{b}{2} - d\right]$$

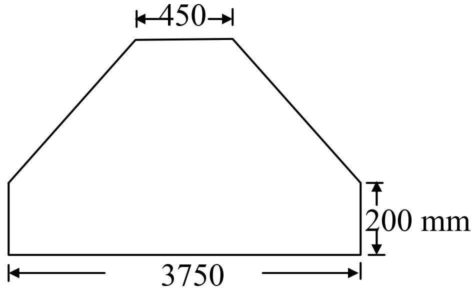
$$= 105.24 \times 3.75\left[\frac{3.75}{2} - \frac{0.45}{2} - 0.85\right]$$

$V = 315.72$ kN

$V_u = 1.5 \times 315.72$

$= 473.58$ kN






$$\frac{h}{700} = \frac{\left(\frac{3750-450}{2}\right)-850}{\left(\frac{3750-450}{2}\right)}$$

$h = 340$ mm

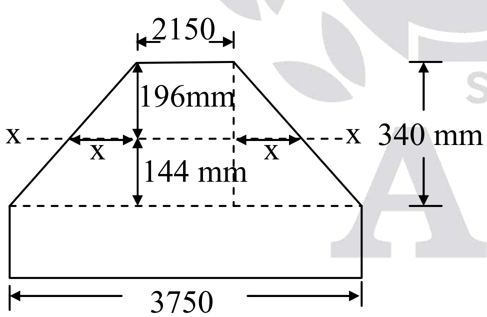
Effective depth at critical section

$d' = 340 + 150 = 490$ mm

For balanced section $\Rightarrow x_{u\,max} = 0.48\ d$

For under reinforced section

$x_u = 0.4d = 0.4 \times 490 = 196$ mm




$b_n = 2150 + 2x$

$= 2150 + 2 \times 461$

$= 3072$ mm

$$\frac{x}{\frac{3750-2150}{2}} = \frac{196}{196+144}$$

$x = 461$ mm

Nominal shear stress

$$\tau_v = \frac{V_u}{b_n d'}$$

$$\tau_v = \frac{473.58\times10^3}{3072\times490} = 0.314\ \text{N/mm}^2 < \tau_c$$

$\therefore$ Safe

**Check for two way shear:**

$$V = P_o\left[B^2-(b+d)^2\right]$$

$$= 105.24\left[3.75^2-(0.45+0.85)^2\right]$$

$= 1302$ kN

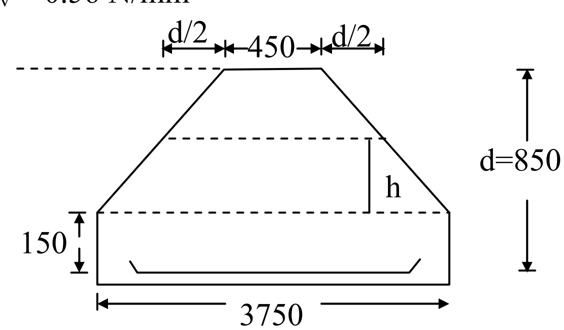
$V_u = 1.5\ V = 1953$ kN

Punching shear stress,

$$\tau_v = \frac{V_u}{b_o d_2}$$

$$= \frac{1953\times10^3}{4(450+850)\times670}$$

$\tau_v = 0.56$ N/mm$^2$



$$\frac{\left(\frac{3750-450}{2}\right)-\frac{850}{2}}{\left(\frac{3750-450}{2}\right)}=\frac{h}{700}$$

$h = 519.69 \simeq 520$ mm

Shear resistance = $K_s\ \tau_c$ = 1.11 N/mm$^2$

$\tau_v < k_s\ \tau_c$

$\therefore$ Safe

**Area of Tension Steel:**

$$M_u = 0.87 f_y A_{st}\ d\left[1-\frac{A_{st}}{b.d}\times\frac{f_y}{f_{ck}}\right]$$

$805.8\times10^6 = 0.87\times415\times A_{st}\times850$

$$\left[1-\frac{A_{st}}{3750\times850}\times\frac{415}{20}\right]$$

$805.8\times10^6 = 306.89\times10^3\ A_{st}\ [1-6.50\times10^{-6}\ A_{st}]$

$805.8\times10^6 = 306.89\times10^3\ A_{st} - 1.997\ A_{st}^2$

$1.997\ A_{st}^2 = 306.89\times10^3\ A_{st} + 805.8\times10^6 = 0$

$A_{st} = 2672$ mm$^2$

Use 16 mm $\phi$ bars,

**Spacing,** $S=\frac{B.a_{st}}{A_{st}}=\frac{3750\times\frac{\pi}{4}\times16^2}{2672}$

$= 282$ mm

Provide 16 mm $\phi$ @ 285 mm c/c

**Check for development length**

$L_d \not> \frac{1}{2}[B-b]-50$

$$L_d=\frac{\phi\sigma_{st}}{4\tau_{bd}}=\frac{16\times0.87\times415}{4\times1.6\times1.2}=752\,\text{mm}$$

Available length = $\frac{1}{2}[3750-450]-50$

$L_d$ = 1600 mm

$L_d$ = 752.18 < 1600 mm

$\therefore$ Hence it is safe

## 13. Retaining Walls

**01. Ans: 1.4**

**Sol:** As per IS 456, a factor of 0.9 is multiplied for stabilising force (or moments) in FOS calculation if dead load is a stabilising force (or moment).

Factor of safety against sliding is given by

$$\left[\frac{0.9(\text{Stabilising force})}{(\text{Overturning force})}\right]\ge1.4$$

$$\left[\frac{0.9\mu W}{p_a\cos\theta}\right]\ge1.4$$

**Note:** If stabilising force is not multiplied by 0.9 then FOS $\ge$ 1.55

**02. Ans: (c)**

**Sol:** FOS = $\left[\frac{0.9(\text{Stabilising force})}{(\text{Overturning force})}\right]\ge1.4$

**Note:** If stabilising moment is not multiplied by 0.9 then FOS $\ge$ 1.55

**03. Ans: (b)**

**Sol:** Economical spacing of counterforts is H/3 to H/2, where H is height of the retaining wall.

Generally, the spacing is between 3 to 3.5m.

**04. Ans: 0.12%**

**Sol:** Minimum reinforcement required in any direction in retaining wall is 0.12%

**05. Ans: (b)**

In a cantilever retaining wall (with no surcharge) , the variation of earth pressure is linear with zero at top and ($k_a\gamma H$) at bottom. The total earth pressure P is given by

$P = \frac{1}{2}k_a\gamma H^2$ acting at height of $\frac{H}{3}$ from base.

**07. Ans: (a)**

**Sol:** According to Rankine's formula, the minimum depth of foundation is given by

$$\frac{p}{w}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

Note: It does not consider the loads acting on the foundation.

**08. Ans: (c)**

**Sol:** The width of the stem at the top should not be less than 200mm for proper placement of reinforcement.

**09. Ans: (b)**

**Sol:** In retaining walls, the loads are due to self weight and earth pressure which can be classified in dead loads category. For a continuous beam in interior span, the negative moment coefficient for dead loads is 1/12. In case of counterfort retaining wall, the counterforts gives full fixity to held slab at its ends. Hence the moment coefficient for end span can be taken same as interior span = 1/12

Bending moment in heel slab at counterfort = $-wl^2/12$

**10. Ans: (b)**

**Sol:** For a cantilever retaining wall, with horizontal backfill total earth pressure is

$$p = \frac{1}{2}k_a wH^2 \text{ where } k_a = \frac{1-\sin\phi}{1+\sin\phi}$$

**11. Ans: (d)**

**Sol:** The midspan moment can be considered as the location of midspan of interior support in a continuous beam. The positive moment coefficient for dead loads at midspan of interior span is 1/16.

Bending moment in heel slab between counterfort = $\frac{w\ell^2}{16}$

## Conventional Practice Solutions

**01.**

**Sol:** $W_1$ = Weight of stem

$W_2$ = Weight of base slab

$W_3$ = Weight of soil above the heel slab

$H_1$ = Height of stem

$H_1 = 6 - 0.4 = 5.6$ m

$W_1 = 25 \times 0.35 \times 1 \times 5.6 = 49$ kN

$W_2 = 0.4 \times 3 \times 1 \times 25 = 30$ kN

$W_3 = 1.65 \times 5.6 \times 1 \times 16 = 147.84$ kN

$\sum W = 226.84$ kN

**Pressure Distribution,**

**At Toe:**

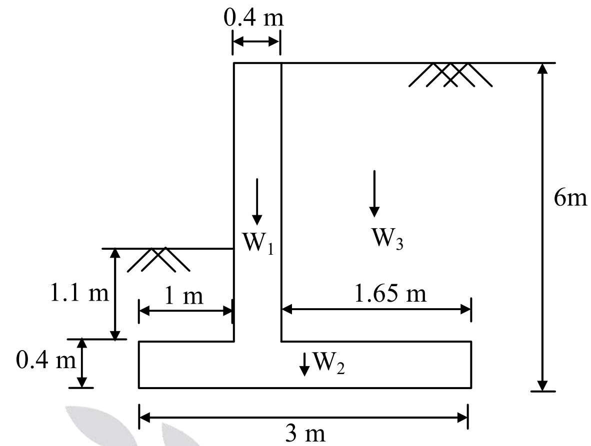
$$P_1 = \frac{\Sigma W}{b}\left(1 + \frac{6e}{b}\right)$$

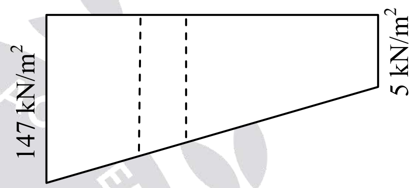
$$P_1 = \frac{226.84}{3}\left(1 + \frac{6 \times 0.47}{3}\right) = 147 \text{ kN/m}^2$$

**At Heel:**

$$P_2 = \frac{\Sigma W}{b}\left(1 - \frac{6e}{b}\right)$$

$$P_2 = \frac{226.84}{3}\left(1 - \frac{6 \times 0.47}{3}\right) = 5 \text{ kN/m}$$




At the junction between stem and toe slab.

$$P_3 = 5 + \frac{2}{3} \times 142 = 100 \text{ kN/m}^2$$

At the junction between stem and heel slab

$$P_4 = 5 + \frac{1.65}{3} \times 142 = 85 \text{ kN/m}^2$$

**Design of Heel Slab:**

The following forces are consider weight of heel slab and its C.G = 2.5 × 1.65 × 1 × 0.4

$= 1.65 \text{ kN} \downarrow @ \frac{1.65}{2}$

Weight of soil above heel slab and its C.G =

$16 \times 1.6 \times 1 \times 5.6 = 147.84 \text{ kN} \downarrow @ \frac{1.65}{2}$

Total soil pressure below heel slab and its CG

$= \frac{1}{2}(83 + 5)1.65 = 72.6 \text{ kN} \uparrow$

$$C.G = \left[\frac{a+2b}{a+b}\right]\frac{h}{3} = \left[\frac{83+2)5)}{83+5}\right]\frac{165}{3} = 0.58 \text{ m}$$

Net moment about junction between stem and heel slab

$$M = 16.5\times\frac{1.65}{2} + 147.84\times\frac{1.65}{2} = 72.6 \times 0.58$$

M = 93.4 kNm

$M_u = 1.5M = 1.5 \times 93.4 = 140.2$ kNm

Effective depth required, $d = \sqrt{\frac{M_u}{R_u B}}$

$$d = \sqrt{\frac{140.2\times10^6}{0.138\times20\times1000}} = 225 \text{ mm}$$

Available effective depth = 400 – 60

= 340 mm > 285 mm

$\therefore$ U.R.S

**Area of Main Steel Required:**

$$A_{st} = \frac{0.5f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}Bd^2}}\right]Bd$$

$$A_{st} = \frac{0.5\times20}{415}\left[1-\sqrt{1-\frac{4.6\times140.2\times10^6}{20\times1000\times340^2}}\right]1000\times340$$

$A_{st} = 1235$ mm$^2$

**Minimum Steel:**

$$\frac{0.12}{100}bD = \frac{0.12}{100}\times1000\times400$$

$= 480$ mm$^2 < A_{st}$ $\quad\therefore$ OK

$$\text{Spacing, } S = \frac{1000a_{st}}{A_{st}} = \frac{1000\times\frac{\pi}{4}\times16^2}{12.35}$$

= 162 mm

Provide 16 mm $\phi$ @ 160 mm c/c @ Fe415

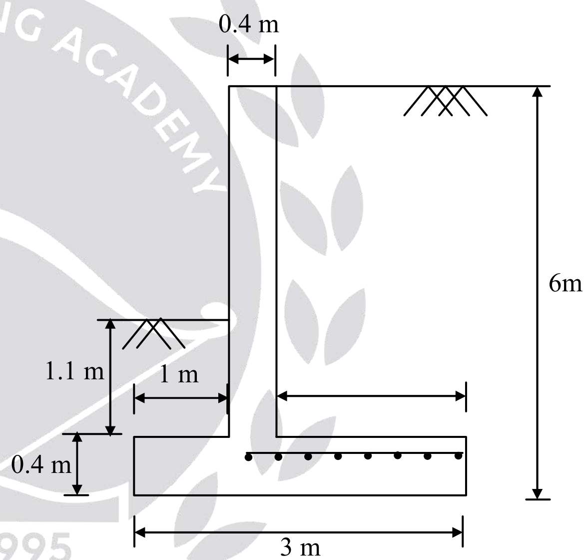
**Distribution Steel:**

Provide minimum steel

$$\text{Spacing, } S = 1000\frac{a_{st}}{A_{st}} = 1000\times\frac{\frac{\pi}{4}\times10^2}{480}$$

= 163 mm

Provide 10 mm $\phi$ @ 160 mm c/c@ Fe415




**Design S.F at Critical Section:**

Total soil pressure below the base slab

$$= \frac{1}{2}[147+5]\times3 = 228 \text{ kN}$$

$V = W_1 + W_2 + W_3 - W_4$

V = 49 + 30 + 147.8 – 228 = – 1.2 kN

V = – 1.2 kN ('–' Indicates resistance is high)

$V_u = 1.5V = 1.5 \times 1.2 = 1.8$ kN

## 14. Water tanks

**01. Ans: (b)**

**Sol:** When shrinkage stresses are allowed, permissible tensile stress (direct and bending) are increased by 33.33 %.

**02. Ans: (a)**

**Sol:** In water tanks, Reinforcement is designed such that entire hoop tension is resisted by steel only.

**03. Ans: (b)**

**Sol:** Minimum grade of concrete for reinforced concrete structures is M20.

**05. Ans: (c)**

**Sol:** In case of intze tanks, bottom dome is conical and generally the rise is $1/8^{th}$ of the span.

**07. Ans: (a)**

**Sol:** Top ring beam of a intze tank is subjected to hoop tension, weight of the dome and self weight.

**09. Ans: (a)**

**Sol:** Vertical wall of a intze tank is subjected to hoop tension due to water pressure ,vertical loads and self weight.

**10. Ans: (a)**

**Sol:** Permissible direct tensile stress for M20 grade in RCC water tanks is 1.2 MPa and bending tensile stress is 1.7MPa.

**13. Ans: 0.2%**

**Sol:** Minimum percentage of reinforcement for sections upto 450mm thick is 0.2%.
Minimum percentage of reinforcement for sections upto 100mm thick is 0.3%.

**14. Ans: (d)**

**Sol:** Permissible direct tensile stress for M30 grade in RCC water tanks is 1.5 MPa and bending tensile stress is 2.0 MPa.

**15. Ans: (b)**

**Sol:** Minimum percentage of reinforcement for sections upto 100mm thick is 0.3%.

## Conventional Practice Solutions

**01.**

**Sol: Given**

Size : 6 m × 8 m × 4 m

B × L × D

Ignore cantilever effect

M20, Fe-415

Design the walls above 1 m

It is given that ignore cantilever effect. For cantilever effect, vertical steel shall be provided. In this problem no need to calculate vertical steel.

**1. Constants**

For M20, Fe-415

$\sigma_{cbc}$ = 7 MPa, $\sigma_{st}$ = 150 MPa

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times7} = 13.33$$

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}} = \frac{13.33\times7}{13.33\times7+150} = 0.384$$

$$J = 1-\frac{k}{3} = 1-\frac{0.384}{3} = 0.872$$

$$Q = \frac{1}{2}\sigma_{cbc}Jk$$

$$= \frac{1}{2}\times7\times0.872\times0.384$$

$$= 1.17$$

$$\frac{L}{B} = \frac{8}{6} = 1.33 < 2$$

**Horizontal Frame Action:**

The critical section is at a height, $h = \frac{H}{4}$

(or)1 m which ever is maximum.

$h = 1$ m

$P_h = \gamma\,(H-h)$

$= 9.8 \times (4-1)$

$= 29.4$ kN/m$^2$

Fixed end moments are

$$\frac{P_hL^2}{12} = \frac{29.4\times8^2}{12} = 156.8\text{ kN-m}\rightarrow\text{in long wall}$$

$$\frac{P_hB^2}{12} = \frac{29.4\times6^2}{12} = 88.2\text{ kN-m}\rightarrow\text{in short wall}$$

Since thickness of short and long walls are maintained same, distribution factors at joints are as shown in below.

| S.No | Member | Stiffness | Total stiffness | D.F |
|---|---|---|---|---|
| 1 | Short wall | $\frac{4EI}{6}$ | $\frac{7}{6}$EI | $\frac{4}{7}$ |
| 2 | Long wall | $\frac{4EI}{8}$ | | $\frac{3}{7}$ |

| | S.W | L.W |
|---|---|---|
| D.F | $\frac{4}{7}$ | $\frac{3}{7}$ |
| FEM | – 88.2 | +156.8 |
| | –39.2 | – 29.4 |
| | –127.4 | + 127.4 |

Corner moment, $M_c$ = 127.4 kN-m

Effective thickness required

$$= \sqrt{\frac{M_c}{Qb}} = \sqrt{\frac{127.4\times10^6}{1.17\times1000}}$$

d = 329.98 mm

Provided D = 380 mm

d = 380 – 30 = 350 mm

Eccentricity, $x = \frac{D}{2} - \text{Cover}$

$$= \frac{380}{2} - 30 = 160\,\text{mm}$$

Direct pull on longwall and short wall

$$T_L = P_h \times \frac{B}{2} = 29.4\times\frac{6}{2} = 88.2\,\text{kN}$$

$$T_B = P_h \times \frac{L}{2} = 29.4\times\frac{8}{2} = 117.6\,\text{kN}$$

**For Longwall (Horizontal reinforcement)**

**i. At corner**

Design moment = $M_c - T_L.x$

= 127.4 – 88.2 × 0.16

= 113.28 kN-m

$$A_{st_1} = \frac{\text{Design moment}}{\sigma_{st} Jd}$$

$$= \frac{113.28\times10^6}{150\times0.872\times350} = 2474.44\,\text{mm}^2$$

$$A_{st_2} = \frac{T_L}{\sigma_{st}} = \frac{88.2\times10^3}{150} = 588\,\text{mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 3062.44\ \text{mm}^2$$

Use 20 mm ϕ bars

$$\text{Spacing },S = \frac{1000\times\frac{\pi}{4}\times20^2}{3062.44} = 102.58\,\text{mm}$$

Provide 20 mm ϕ @ 100 mm c/c

**ii. At mid span :**

$$\text{Design moment} = \frac{P_h L^2}{8} - M_c$$

$$= \frac{29.4\times8^2}{8} - 127.4$$

= 107.8 kN-m

$$A_{st_1} = \frac{\text{Design moment}}{\sigma_{st} Jd}$$

$$= \frac{107.8\times10^6}{150\times0.872\times350}$$

= 2354.74 mm$^2$

$$A_{st} = \frac{T_L}{\sigma_{st}} = \frac{88.2\times10^3}{150} = 588\,\text{mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 2942.74\,\text{mm}^2$$

$$\text{Spacing, } S = \frac{1000\times\frac{\pi}{4}\times20^2}{2942.74} = 106.75\,\text{mm}$$

Provide 20 mm ϕ @ 100 mm c/c

**For Short Wall:**

**i. At corner**

Design moment = $M_c - T_B .x$

= 127.4 – 117.6 × 0.16

= 108.58 kN-m

$$A_{st_1} = \frac{\text{Design moment}}{\sigma_{st} Jd}$$

$$= \frac{108.58\times10^6}{150\times0.872\times350} = 2371.87\,\text{mm}^2$$

$$A_{st_1} = \frac{T_B}{\sigma_{st}} = \frac{117.6\times10^3}{150} = 784\ \text{mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 3155.87\ \text{mm}^2$$

$$\text{Spacing, } S = \frac{1000\times\frac{\pi}{4}\times20^2}{3155.87} = 99.54\ \text{mm}$$

Provide 90 mm c/c –20 mm $\phi$

**ii. At mid span:**

$$\text{Design moment} = \frac{P_h B^2}{8} - M_c$$

$$= \frac{29.4\times6^2}{8} - 127.4$$

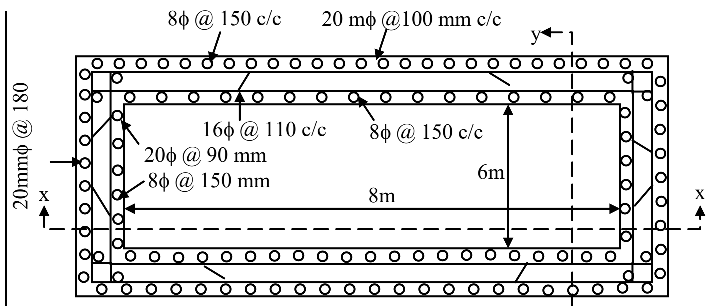
$$= 4.9\ \text{kN-m}$$

$$= \frac{4.9\times10^6}{150\times0.872\times350} = 107.03\,\text{mm}^2$$

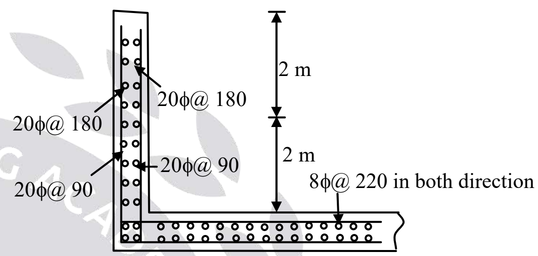
$$A_{st_2} = \frac{T_B}{\sigma_{st}} = \frac{117.6\times10^3}{150} = 784\,\text{mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 891.03\ \text{mm}^2$$

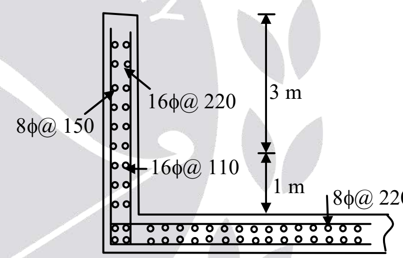
Since the area of steel is very less hence provide 20 mm $\phi$ @ 180 mm c/c at centre



Sectional plan at base



Section through short (Section y-y) wall



Section through long wall (Section y-y)

## 15. Staircases

**01. Ans: (a)**

**Sol:** The depth (thickness) of the waist slab is the minimum thickness perpendicular to the soffit of the staircase.

**02. Ans: (a)**

**Sol:** For staircases spanning on edge of landings, parallel to the risers:

The effective span = Going + half width of the slab at each end or 1m whichever is smaller.

= 2.25 + min (1.5/2, 1) + min(2.2,1)

= 2.25 +0.75+1 = 4.0 m

**03. Ans: (b)**

**Sol:** Staircase: IS 456

Water tanks: IS 3370

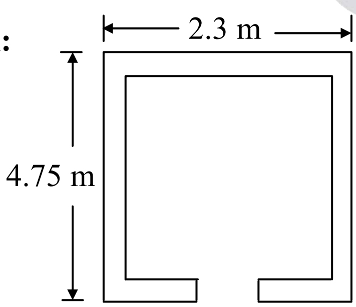
Earth quake resistant design: IS 4326 and IS 1893

Prestressed concrete: IS 1343

**Conventional Practice Solutions**

**01.**

**Sol:**




M20 , Fe 415

H = 3.6 m

L.L = 5 kN/m$^2$

$R_u$ = 0.138 $f_{ck}$ = 0.138 × 20 = 2.76

Assume rise = 150 mm

Tread = 250 mm

Providing two flights

Height of each fight = $\frac{3.6}{2} = 1.8\text{m}$

= 1800 mm

No. of risers = $\frac{1800}{150} = 12$

No. of Treads = 12 – 1 = 11

Total space covered by steps

= 11 × 250 = 2750 mm = 2.75 m

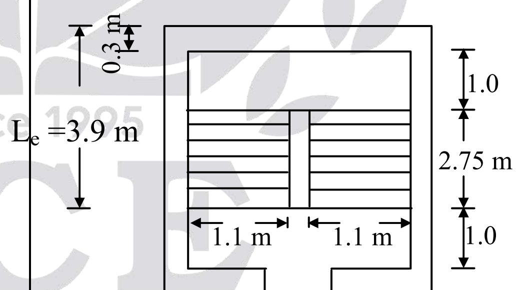
Assume landing width = 1.0 m

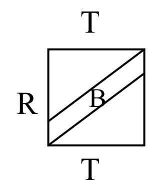
Free passage = 4.75 – 2.75 – 1.0 = 1.0 m

Assuming wall thickness = 300 mm

= 0.3 m

Effective span = $2.75 + 1 + \frac{0.3}{2} = 3.9\,\text{m}$




**Loads:**

$$B = \sqrt{R^2 + T^2}$$

$$= \sqrt{150^2 + 250^2} = 291.55\text{ m}$$

Assume waist slab thickness = 200 mm

$$\text{D.L} = \left(\frac{RT}{2} + WB\right)\frac{25}{T}$$

$$= \left[\frac{0.15\times 0.25}{2} + 0.2\times 0.2915\right]\frac{25}{0.25}$$

$= 7.705$ kN/m$^2$

Floor finish (F.F) = 0.75 kN/m$^2$

Live load (L.L) = 5 kN/m$^2$

Total load = 7.705 + 0.75 + 5

= 13.45 kN/m$^2$

Maximum bending moment $= \dfrac{w\ell e^2}{8}$

$$= \frac{13.45\times 3.9^2}{8} = 25.57 \text{ kN-m}$$

$M_u = 1.5 \times 25.57 = 38.36$ kN-m

Effective depth $d = \sqrt{\dfrac{M_u}{R_u.b}}$

$$= \sqrt{\frac{38.36\times 10^6}{2.76\times 1000}}$$

$d = 117.9 \simeq 120$ mm

D = 200 mm

**Area of Steel Reinforcement :**

$$M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$38.36 \times 10^6 = 0.87 \times 415 \times A_{st} \times 120$

$$\left[1 - \frac{A_{st}}{1000\times 120}\times\frac{415}{20}\right] 38.36\times 10^6$$

$= 43.326 \times 10^3 A_{st} - 7.49 A_{st}^2$

$A_{st} = 1091.24$ mm$^2$

No of bars required $= \dfrac{A_{st}}{a_{st}} = \dfrac{1091.24}{\frac{\pi}{4}\times 12^2}$

$= 9.64 \simeq 10$ No's

Assuming dia of bar 10 no's 12 mm

**Distribution Steel :**

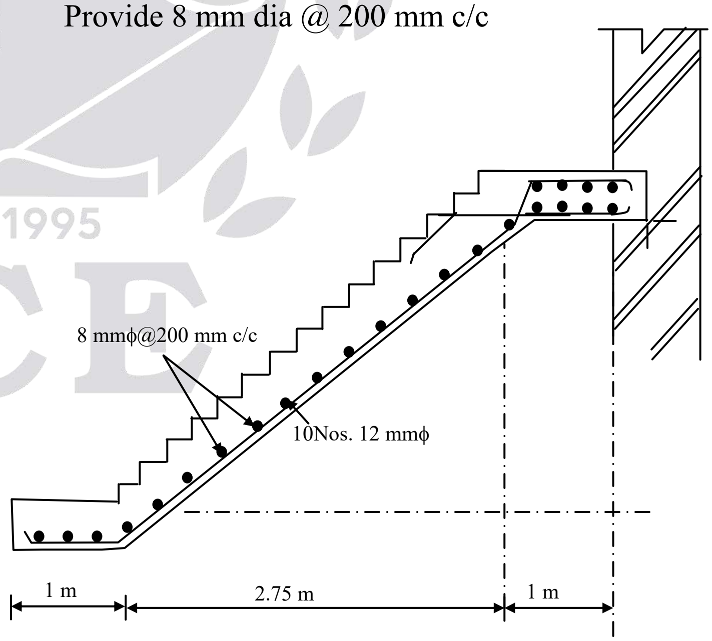
$$0.12\%\ bD = \frac{0.12}{100}\times 1000\times 200 = 240\,\text{mm}^2$$

Spacing, $S = \dfrac{a_{st}}{A_{st}}\times 100$

$$= \frac{\frac{\pi}{4}8^2}{240}\times 1000$$

$= 209.4 \simeq 200$ mm

Provide 8 mm dia @ 200 mm c/c



## 17. Fundamentals of Prestressed Concrete

**Refer theory in Volume I Material**

## 18. Analysis of Prestressed Concrete Members

**01. Ans: (b)**

**Sol:** Prestressing force, P = 2500 kN

Effective span, $l$ = 10 m

udl on the beam, w = 40 kN/m

For load balancing

$$P.e = \frac{w\ell^2}{8}$$

$$(2500)(e) = \frac{(40)(10)^2}{8}$$

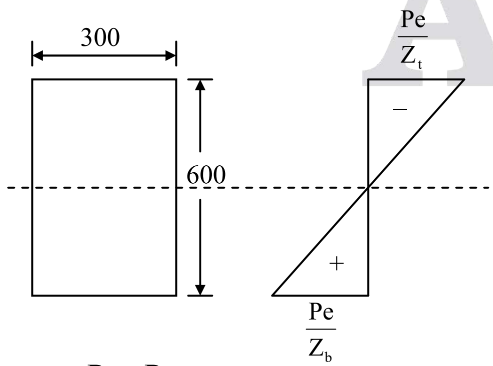
e = 0.2 m = 200 mm

**02. Ans: (b)**

**Sol:** $\gamma_c = 24\ kN/m^3$

$\sigma_t = 2$ MPa

$\sigma_b = 20$ MPa




$$\sigma_b = \frac{P}{A} + \frac{Pe}{z} \text{ ------------------ (1)}$$

$$\sigma_t = \frac{P}{A} - \frac{Pe}{z} \text{ ------------------ (2)}$$

Adding (1) & (2)

$$20 = \frac{P}{A} + \frac{Pe}{z}$$

$$-2 = \frac{P}{A} - \frac{Pe}{z}$$

$$18 = \frac{2P}{A}$$

P = 1620 kN

$$\sigma_b = \frac{P}{A} + \frac{Pe}{z}$$

$$20 = \frac{1620\times10^3}{300\times600} + \frac{1620\times10^3\times6\times e}{300\times600^2}$$

e = 122 mm

e ≃ 135 mm

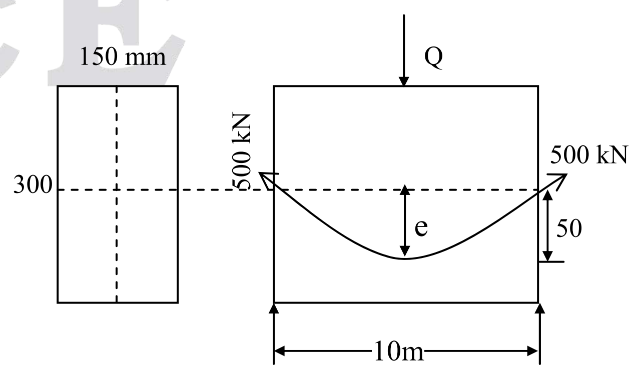
**03. Ans: (a)**

**Sol:** 150 × 300 mm

$l$ = 10 m, e at support = 0mm

e = 50 mm (center), P = 500 kN

Q =? (at center of span)




$$Pe = \frac{Q\times l}{4}$$

$$500\times\frac{50}{1000}=\frac{Q\times 10}{4}$$

$100 = Q \times 10$

$Q = 10$ kN

**04. Ans: (b)**

**Sol:** Self weight

$w_D = \gamma_c \times b \times D$

$= (24\ \text{kN/m}^3)\times 0.15 \times 0.3$

$= 1.08$ kN/m

P – line at upper kern point ($\sigma_b = 0$)

$$M_D = \frac{w_D l^2}{8} = \frac{1.08\times 10^2}{8} = 13.5$$

$$\sigma_b = 0 = \frac{P}{A}+\frac{Pe}{z}-\frac{M_D}{z}-\frac{M_L}{z}$$

$$= \frac{500\times 10^3}{300\times 150}+\frac{500\times 10^3\times 50}{\left(\frac{150\times 300^2}{6}\right)}-\frac{13.5\times 10^6}{\left(\frac{150\times 300^2}{6}\right)}-\frac{M_L}{\left(\frac{150\times 300^2}{6}\right)}$$

$$0 = 11.11+11.11-6-\frac{M_L}{225\times 10^4}$$

$M_L = 16.22 \times 225 \times 10^4$

$M_L = 36.5$ kN-m,

$$M_L = \frac{Ql}{4}$$

$$36.5 = \frac{Q\times 10}{4}$$

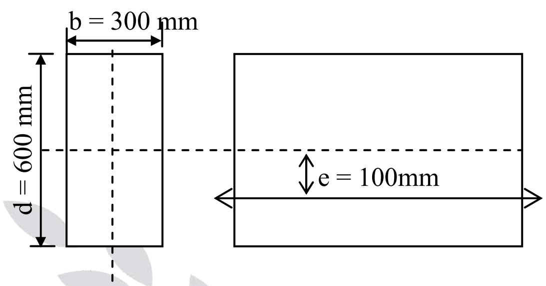
$146 = Q \times 10$

$Q = 14.6$ kN

**05. Ans: (c)**

**Sol:** $l = 6$ m, b = 300 mm, d = 600 mm

e = 100 mm, P = 1000 kN,




Neglecting self weight of the beam

$$\sigma_b = \frac{P}{A}+\frac{Pe}{z}$$

$$= \frac{1000\times 10^3}{300\times 600}+\frac{1000\times 10^3\times 100}{\left(\frac{300\times (600)^2}{6}\right)}$$

$= 5.55+5.55 = 11.11$ MPa

**06. Ans: (b)**

**Sol:** b = 200 mm, D = 250 mm

A = 500 mm$^2$, P = 1000 MPa

m = 10

$\varepsilon_s = \varepsilon_e$

$$\frac{\sigma_s}{E_s}=\frac{\sigma_c}{E_c}$$

$$\sigma_c = \sigma_s\left(\frac{\varepsilon_c}{\varepsilon_s}\right)=\frac{\sigma_s}{m}=\frac{1000}{10}$$

$\sigma_e = 100$ MPa

Prestressing force on steel = $\sigma_s.A_s$

$= 1000 \times 500 = 500 \times 10^3$N

Compression force in concrete = 500 kN

$= \sigma_c . A_c$

Compression stress in concrete $\sigma_c = \dfrac{P_c}{A_c}$

$$= \frac{500\times10^3}{200\times250} = 10 \text{ MPa}$$

**Conventional Practice Solutions**

**01.**

**Sol: Given**

$l = 8$ m

e = 150 m

P = 100 kN

$W_u = 2$ kN/m

(a) prestress + self weight ($\gamma = 24$ kN/m$^3$)

(b) prestress + self weight + LL

Area of the section

$A = 2(200 \times 60) + 280 \times 80$

$A = 46400$ mm$^2$

$$\text{M.I} = I = \frac{200\times400^3}{12} - \frac{120\times280^3}{12}$$

$= 847.14 \times 10^6$ mm$^4$

$$Z = \frac{I}{y} = \frac{847.14\times10^6}{\frac{400}{2}}$$

$Z = 4235.7 \times 10^3$ mm$^3$

Self weight = $W_D = \gamma_A$

$W_D = 24 \times 46400 \times 10^{-6} = 1.11$ kN/m

$$M_D = \frac{W_D L^2}{8} = \frac{1.11\times8^2}{8} = 8.88 \text{kN} - \text{m}$$

**(a) At mid span due to prestress + self weight**

**Prestress + selfweight**

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_D}{Z}$$

$$= \frac{100\times10^3}{46,400} - \frac{100\times10^3\times150}{4235.7\times10^3} + \frac{8.88\times10^6}{4235.7\times10^3}$$

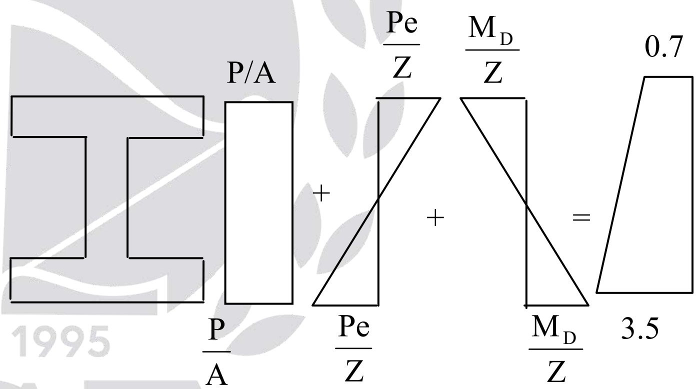
$= 2.155 - 3.541 + 2.096$

$= 0.7104$ N/mm$^2$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_D}{Z}$$

$= 2.155 + 3.541 - 2.096$

$= 3.5$ N/mm$^2$




**(b) At corner:**

**Prestress + self weight + L.L**

$$\text{LL Moment } M_L = \frac{w_u L^2}{8} = \frac{2\times8^2}{8}$$

$= 16$ kN-m

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$= \frac{100\times10^3}{46,400} - \frac{100\times10^3\times150}{4235.7\times10^3} + \frac{8.88\times10^6}{4235.7\times10^3} + \frac{16\times10^6}{4235.7\times10^3}$$

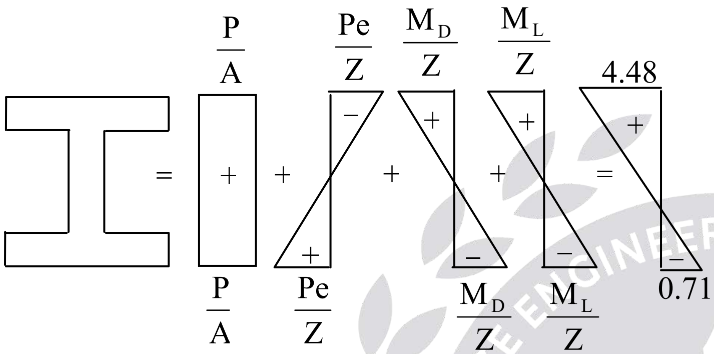
$= 2.155 - 3.541 + 2.096 + 3.77$

$= 4.48 \text{ N/mm}^2$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z}$$

$= 2.155 + 3.541 - 2.096 - 3.77$

$= -0.17 \text{ N/mm}^2$




**02.**

**Sol:** Area = 200 × 300 = 60000 mm$^2$

10 – 5 mm $\phi$ bars – bottom-65 mm

3 – 5 mm $\phi$ bars – top – 25 mm

Effective stress in steel wire = 840 N/mm$^2$

$\sigma_o = 840 \text{ N/mm}^2$

**i.** Prestress + self weight

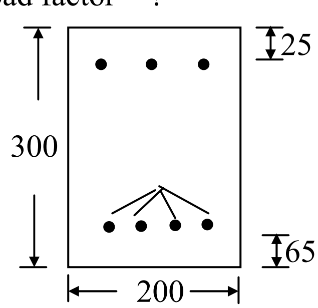
$l = 6$ m

**ii.** If $W_L = 6$ kN/m

$f_{cmax} = ?$

**iii.** $f_{cr} = 5 \text{ N/mm}^2$

Load factor = ?

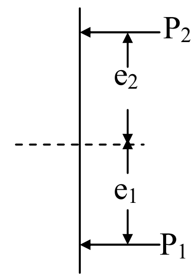



Self weight = $\gamma A$

$= 24 \times 0.2 \times 0.3$

$W_D = 1.44$ kN/m

Dead load moment,




$$M_D = \frac{W_D \ell^2}{8}$$

$$M_D = \frac{1.44 \times 6^2}{8} = 6.48 \text{ kN/m}$$

$$P_1 = 840 \times \left(10 \times \frac{\pi}{4} \times 5^2\right)$$

$= 164.93$ kN

$A = 60000 \text{ mm}^2$

$$Z = \frac{200 \times 300^2}{6} = 3 \times 10^6 \text{ mm}^4$$

$e_1 = 150 - 65 = 85$ mm

$e_2 = 150 - 25 = 125$ mm

$$P_2 = 840 \times \left(3 \times \frac{\pi}{4} \times 5^2\right)$$

$P_2 = 49.48$ kN

**Stresses:**

$$\sigma_t = \frac{P_1}{A} - \frac{P_1 e_1}{Z} + \frac{P_2}{A} + \frac{P_2 e_2}{Z} + \frac{M_D}{Z}$$

$$= \frac{164.93 \times 10^3}{6 \times 10^4} - \frac{164.93 \times 10^3 \times 85}{3 \times 10^6}$$

$$+ \frac{49.48 \times 10^3}{6 \times 10^4} + \frac{49.48 \times 10^3 \times 125}{3 \times 10^6} + \frac{6.48 \times 10^6}{3 \times 10^6}$$

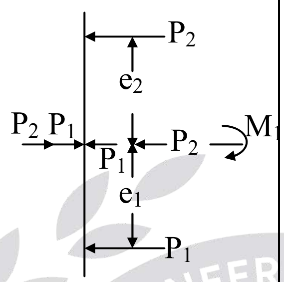
$= 2.74 - 4.67 + 0.82 + 2.06 + 2.16$

$= 3.11 \text{ kN/mm}^2$

$$\sigma_b = \frac{P_1}{A} + \frac{P_1 e_1}{Z_1} + \frac{P_2}{A} - \frac{P_2 e_2}{Z} - \frac{M_D}{Z}$$

$\sigma_b = 2.74 + 4.67 + 0.82 - 2.06 - 2.16$

$= 4.01 \text{ N/mm}^2$




**ii. Live Load Moment:**

$$M_L = \frac{W_L \ell^2}{8}$$

$$= \frac{6 \times 6^2}{8} = 27 \text{ kN-m}$$

$$\sigma_t = \frac{P_1}{A} - \frac{P_1 e_1}{Z} + \frac{P_2}{A} + \frac{P_2 e_2}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$= 2.74 - 4.67 + 0.82 + 2.06 + 2.16 + \frac{27 \times 10^6}{3 \times 10^6}$$

$= 12.1 \text{ N/mm}^2$

$$\sigma_b = \frac{P_1}{A} + \frac{P_1 e_1}{A} + \frac{P_2}{A} - \frac{P_2 e_2}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z}$$

$= 2.74 + 4.67 + 0.82 - 2.06 - 2.16 - 9$

$= -4.99 \text{ N/mm}^2$

$$\text{Load Factor} = \frac{\text{Cracking load}}{\text{Live load}} = \frac{M_{cr}}{M_L}$$

$$\text{Load Factor} = \frac{W_{cr}}{W_L} = \frac{M_{cr}}{M_L}$$

$$-5 = \frac{P_1}{A} + \frac{P_1 e_1}{Z} + \frac{P_2}{A} - \frac{P_2 e_2}{Z} - \frac{M_D}{Z} - \frac{M_{cr}}{Z}$$

$-5 = 2.74 + 4.67 + 0.82$

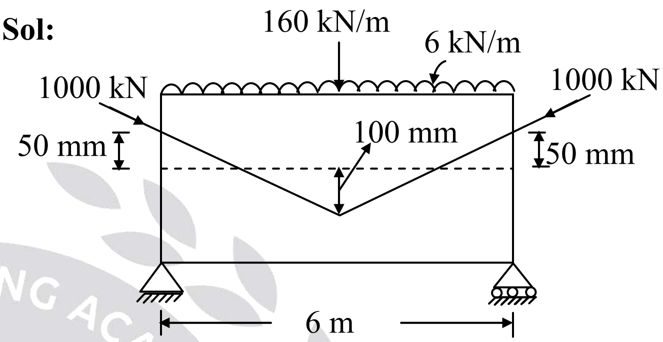
$$-2.06 - 2.16 - \frac{M_{cr}}{3 \times 10^6}$$

$M_{cr} = 27.03$ kN-m

$$\text{Load factor} = \frac{M_{cr}}{M_L} = \frac{27.03}{27} = 1$$

**03.**

**Sol:**




**At End:**

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$\sigma_t = \frac{1000 \times 10^3}{400 \times 600} - \frac{1000 \times 10^3 (-50)}{\frac{400 \times 600^2}{6}}$$

$\sigma_t = 4.167 - (-2.08) = 6.247 \text{ N/mm}^2$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z}$$

$$= \frac{1000 \times 10^3}{400 \times 600} + \frac{1000 \times 10^3 (-50)}{\frac{400 \times 600^2}{6}}$$

$\sigma_b = 4.167 + (-2.08) = 2.087 \text{ N/mm}^2$

**At Mid span:**

$W_D = 6$ kN/m

$$M_D = \frac{W_D L^2}{8} = \frac{6 \times 6^2}{8} = 27 \text{ kN-m}$$

$$M_L = \frac{WL}{4} + \frac{W_D L^2}{8}$$

$$= \frac{160\times 6}{4} + \frac{6\times 6^2}{8} = 267 \text{ kN-m}$$

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$\sigma_t = \frac{1000\times 10^3}{400\times 600} - \frac{1000\times 10^3 \times 100}{\frac{400\times 600^2}{6}} + \frac{27\times 10^6}{\frac{400\times 600^2}{6}} + \frac{267\times 10^6}{\frac{400\times 600^2}{6}}$$

$= 4.167 - 4.167 + 1.125 + 11.125$

$\sigma_t = 12.25 \text{ N/mm}^2$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z}$$

$= 4.167 + 4.167 - 1.125 - 11.125$

$= -3.916 \text{ N/mm}^2$

## 19. Losses in Prestressed Concrete

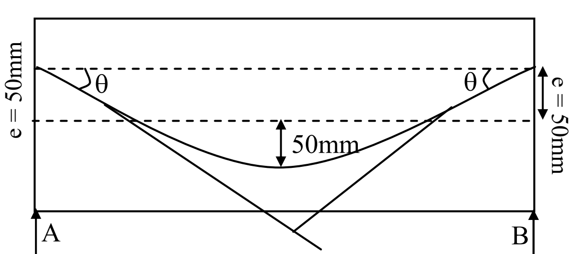
**01. Ans: (b)**

**Sol:** $l = 10$ m, $b = 100$ mm,

$D = 300$ mm $A = 200$ sq-mm,

$e = 50$ mm, $\mu = 0.35$;

$k = 0.0015$ per m




$\alpha = 2\theta$

$h = 2e = 100$mm

$l = 10$m

Initial stress in wires = 1200 MPa

Loss of stress in wires = $\sigma(\mu\alpha + kx)$

$= 1200[0.35 \times \alpha + 0.0015\times 10]$

From equation of parabola

$$\theta = \frac{4\times 0.1}{10} = 0.04 \text{ radians}$$

$\alpha = 2 \times \theta = 0.08$

Loss = $1200[0.35 \times 0.08 + 0.0015 \times 10]$

Loss of stress = 51.6 MPa

$$\% \text{ loss of stress} = \frac{51.6}{1200}\times 100$$

$= 4.28 \simeq 4.3\%$

**02. Ans: (b)**

**Sol:**

Tensioning from both the ends % loss of stress

$$= \frac{\%\text{loss of stress}}{2} = \frac{4.28}{2} = 2.15$$

**03. Ans: (b)**

**Sol:** Straight tendon tensioned from one end

Loss of stress in wires = $\sigma[\mu\alpha + kx]$

($\because \alpha = 0$)

$1200(0.35 \times (0) + 0.0015 \times 10) = 18$ MPa

$$\% \text{ of loss} = \frac{18}{1200}\times 100$$

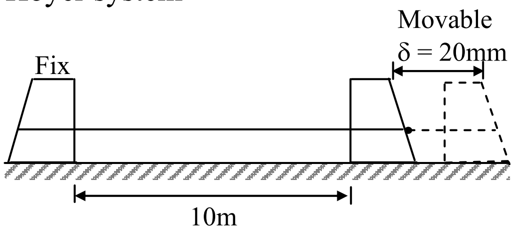
$= 1.5\%$

If tensioned from two ends

$$\frac{\%\text{ of loss}}{2} = \frac{1.5}{2} = 0.75\%$$

**04. Ans: (c)**

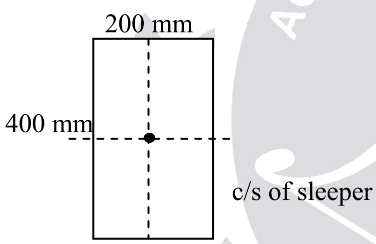
**Sol:** Hoyer system




$$\delta = \frac{PL}{AE} \qquad \left(\text{as } \sigma = \frac{P}{A}\right)$$

Prestress induced in steel wire, $\sigma = \dfrac{\delta E}{L}$

$$\sigma = \frac{20 \times 2 \times 10^5}{10{,}000} = 400 \text{ MPa}$$




Eccentricity of Prestress, e = 0

Prestressing force in steel wire = $P = \sigma_s . A_s$

$= 400 \times 500 \text{ mm}^2$

$= 200$ kN

$$f_c = \frac{P}{A} + \frac{Pe}{I}(e) = \frac{200 \times 10^3}{200 \times 400} = 2.5 \text{ MPa}$$

Loss due to elastic shortening

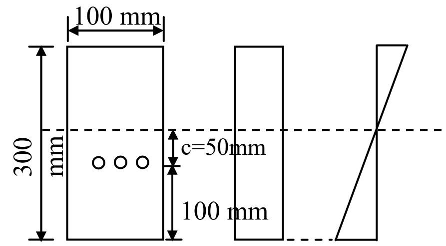
$$= m \times f_c = \left(\frac{E_s}{E_c}\right) f_c$$

$$\sigma = \left(\frac{200{,}000}{20{,}000}\right) \times 2.5 = 25 \text{ MPa}$$

$$\% \text{ loss of Prestress} = \frac{25}{400} \times 100 = 6.25\%$$

**05. Ans: (d)**

**Sol:**




$$f_c = \frac{P}{A} + \frac{P}{I}(e)^2$$

Initial stress in steel wire =1200 MPa

Prestressing force in each steel wire

$P = \sigma_s . A_s$

$P = 1200 \times 50 = 60$ kN

$$f_c = \frac{60 \times 10^3}{100 \times 300} + \frac{60 \times 10^3}{\left(\dfrac{100 \times 300^3}{12}\right)} \times (50)^2$$

$f_c = 2.66$ MPa

Simultaneous tensioning = loss of prestress is zero

**06. Ans: (a)**

**Sol:** Successive tensioning of the 3 cables

$$= \frac{n(n-1)}{2}(m.f_c)$$

$$= \frac{3(3-1)}{2}(6 \times 2.66)$$

$$= 48.0 \text{ MPa}$$

$$\% \text{ of loss} = \frac{48.0}{1200} \times 100 = 4\%$$

(or) For pretensioning system

Loss = $n(m \times f_c)$

$= 3(6 \times 2.66) = 48.0$ MPa

**07. Ans: (c)**

**Sol:** Anchorage slip = 3 mm

$l = 30$ m, $\sigma = 1200$ MPa

$E = 2.1 \times 10^5$ MPa

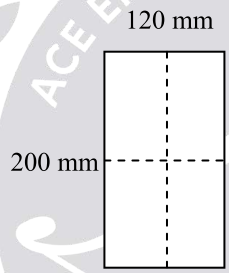
$$E = \frac{\delta E}{l} = \frac{3 \times 2.1 \times 10^5}{30 \times 10^3}$$

$\sigma = 21$ MPa

$$\% \text{ of loss} = \frac{21}{1200} \times 100 = 1.73\%$$

**08. Ans: (b)**

**Sol:**




$P = 150$ kN, $e = 20$ mm

$A = 187.5 \text{ mm}^2$

$E_s = 2.1 \times 10^5$ MPa

$E_c = 3.0 \times 10^4$ MPa

$$f_c = \frac{P}{A} + \frac{Pe}{I}.e$$

$$= \frac{150 \times 10^3}{187.5} + \frac{150 \times 10^3 \times 20^2}{\left(\frac{120 \times 200^3}{12}\right)}$$

$= 800 + 0.75$ MPa

$f_c = 800.75$ MPa

loss due to elastic shortening = $m.f_c$

$$= \left(\frac{E_s}{E_c}\right).f_c = \frac{2.1 \times 10^5}{3.0 \times 10^4} \times 7 = 4.9$$

Percentage loss in the prestressing steel due to elastic deformation

$$= \frac{4.9}{800.75} \times 100 = 6.12\%$$

**09. Ans: (c)**

**Sol:** $\varepsilon = \varepsilon_{shrink} + \varepsilon_{creep}$

$= 0.0008$

Loss of prestress on steel = $\varepsilon \times E_s$

$= 0.0008 \times 200 \times 10^3$

$= 160$ MPa

Stress remaining after loss = Initial stress–Loss

$= 200 - 160$

$= 40$ MPa

## Conventional Practice Solutions

**01.**

**Sol: Given**

$n = 50$

dia = 3 mm

$P = 400$ kN

$E_s = 210 \text{ kN/mm}^2$

$E_c = 32 \text{ kN/mm}^2$

$\varepsilon_{cu} = 35 \times 10^{-6} \text{ mm/N/mm}^2$ of stress

Relaxation of steel stress = 6% of initial stress

$\varepsilon_{sh}$ = Total shrinkage

$= 250 \times 10^{-6}$ per unit length

**Elastic Deformation:**

$$m = \frac{E_s}{E_c} = \frac{210}{32} = 6.56$$

$$f_c = \frac{P}{A} + \frac{P.e}{Z} = \frac{400 \times 10^3}{300 \times 300} = 4.44 \text{ N/mm}^2$$

$\Delta\sigma = mfc$

$= 6.56 \times 4.44 = 29.12 \text{ N/mm}^2$

**Loss of stress due to shrinkage of concrete:**

$\Delta\sigma = 250 \times 10^{-6} \times 210 \times 10^3$

$= 52.5 \text{ N/mm}^2$

**Loss of stress due to relaxation of steel :**

$\Delta\sigma$ = 6% initial prestress

$$\text{Prestress} = f = \frac{400\times10^3}{\frac{\pi}{4}\times3^2\times50} = 1131.76$$

$$\Delta\sigma = \frac{6}{100}\times1131.76 = 67.9 \text{ N/mm}^2$$

**Loss of stress due to creep of concrete :**

$\Delta\sigma = \varepsilon_{cu}\, f_c\, E_s$

$= 35 \times 10^{-6} \times 4.44 \times 210 \times 10^3$

$= 32.63 \text{ N/mm}^2$

After all losses stresses in tendon

$= 1131.76 - 29.12 - 52.5 - 67.9 - 32.63$

$= 949.61 \text{ N/mm}^2$

$$\text{Prestressing force } P = 949.61\times\frac{\pi}{4}\times3^2\times50$$

$P = 335.61$ kN

Final stress in concrete,

$$= \frac{P}{A} = \frac{335.61\times10^3}{300\times300} = 3.729 \text{ N/mm}^2$$

$$\%\text{ loss} = \frac{\Delta\sigma}{\sigma_o}\times100$$

$$= \frac{29.12+52.5+67.9+32.63}{1131.76}\times100$$

$= 16.09\%$

**02.**

**Sol: Given:**

$b = 250$ mm $\quad D = 360$ mm

$l = 12$ m $\quad A_s = 350 \text{ mm}^2$

$e = 60$ mm $\quad \sigma_o = 1250 \text{ N/mm}^2$

Determine the percentage loss of stress

i. Pretensioning

ii. Post tensioning

$E_s = 210 \text{ kN/mm}^2$, $E_c = 35 \text{ kN/mm}^2$

$\varepsilon_{cu} = 45 \times 10^{-6} \rightarrow$ Pre tensioning

$= 22 \times 10^{-6} \rightarrow$ Post tensioning

$\varepsilon_{sh} = 300 \times 10^{-6} \rightarrow$ Pre tensioning

$= 215 \times 10^{-6} \rightarrow$ Post tensioning

Relaxation of steel stress = 5% of initial stress

$\Delta = 1.25$ mm

$K = 0.00015$/m

$$\text{Modular ratio, } m = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

Prestress force $P = \sigma_o A_s = 1250 \times 350$

$= 437.5$ kN

Stress in concrete at the level of tendon

$$f_c = \frac{P}{A} + \frac{Pe}{I}.e$$

$$= \frac{437.5\times10^3}{250\times360} + \frac{437.5\times10^3\times60}{\frac{250\times360^3}{12}}\times60$$

$= 4.86 + 1.62 = 6.48 \text{ N/mm}^2$

**i. In Pretensioning**

- Loss of prestress due to elastic deformation

$\Delta\sigma = mfc = 6 \times 6.48 = 38.88$ N/mm$^2$

- Loss of prestress due to relaxation of steel

$\Delta\sigma = 5\%$ of $\sigma_o$

$= \frac{5}{100} \times 1250$

$= 62.5$ N/mm$^2$

- Loss of prestress due to shrinkage of concrete

$\Delta\sigma = \varepsilon_{sh} E_s = 300 \times 10^{-6} \times 210 \times 10^3$

$= 63$ N/mm$^2$

- Loss of prestress due to creep of concrete

$\Delta\sigma = \varepsilon_{cu} f_c E_s$

$= 45 \times 10^{-6} \times 6.48 \times 210 \times 10^3$

$= 61.24$ N/mm$^2$

Total loss of prestress

$\Delta\sigma = 38.88 + 62.5 + 63 + 61.24$

$= 225.62$ N/mm$^2$

% loss of stress $= \frac{\Delta\sigma}{\sigma_o} \times 100$

$= \frac{225.62}{1250} \times 100 = 18.05\%$

**ii. In post Tensioning:**

- Loss of prestress due to elastic deformation

$\Delta\sigma = 0$

- Loss of prestress due to relaxation of steel

$\Delta\sigma = 5\%$ of $\sigma_o = 62.5$ N/mm$^2$

- Loss of prestress due to shrinkage of concrete

$\Delta\sigma = \varepsilon_{sh} E_s$

$= 215 \times 10^{-6} \times 210 \times 10^3$

$= 45.15$ N/mm$^2$

- Loss of prestress due to creep of concrete

$\Delta\sigma = \varepsilon_{cu} f_c E_s$

$= 22 \times 10^{-6} \times 6.48 \times 210 \times 10^3$

$= 29.94$ N/mm$^2$

- Loss of prestress due to anchorage slip

$\Delta\sigma = \frac{\Delta}{L} E_s = \frac{1.25}{12000} \times 210 \times 10^3$

$= 21.875$ N/mm$^2$

- Loss of prestress due to friction

$\Delta\sigma = \sigma_o [\cancelto{0}{\mu\alpha} + Kx]$

$= 1250 [0 + 0.00015 \times 12]$

$= 2.25$ N/mm$^2$

$\alpha = 0 \rightarrow$ if $\theta = 0 \rightarrow$ for straight cables

Total loss of prestress

$\Delta\sigma = 0 + 62.5 + 45.15 + 29.94 + 21.875 + 2.25$

$= 161.751$ N/mm$^2$

% Loss of stress $= \frac{\Delta\sigma}{\sigma_o} \times 100$

$= \frac{161.715}{1250} \times 100 = 12.94$ %