



**ESE | GATE | PSUs**



# **ELECTRICAL ENGINEERING**

**POWER SYSTEMS** 

**Text Book :** Theory with worked out Examples  
and Practice Questions

# Chapter 11

# Power Systems

## Solutions for Text Book Practice Questions

### PU System, Symmetrical Components & Fault Analysis

#### Solutions for Objective Practice Questions

01. Ans: (b)

$$\text{Sol: } X_{G_1} = j0.09 \times \left(\frac{200}{100}\right) \times \left(\frac{25}{25}\right)^2$$

$$= j0.18 \text{ p.u.}$$

$$X_{T_1} = j0.12 \times \left(\frac{200}{90}\right) \times \left(\frac{25}{25}\right)^2$$

$$= j0.27 \text{ p.u.}$$

$$X_1 = X_\Omega \times \frac{MVA_{base}}{(kV_b)^2}$$

$$X_1 = j150 \times \frac{200}{(220)^2}$$

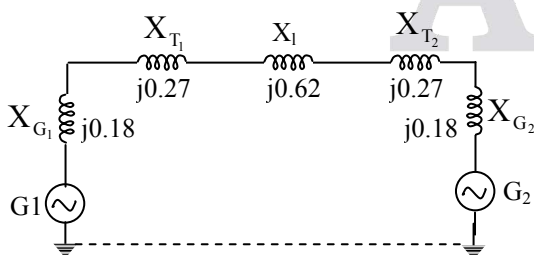
$$= j0.62 \text{ p.u.}$$

$$X_{T_2} = X_{T_1}$$

$$= j0.27 \text{ P.u.}$$

$$X_{G_2} = j0.09 \times \left(\frac{200}{100}\right) \times \left(\frac{25}{25}\right)^2$$

$$= j0.18 \text{ p.u.}$$



02. Ans: (a)

Sol: The value of the load. When referred to generator circuit in per unit is

$$Z_{P, \text{new}} = Z_{P, \text{old}} \times \frac{MVA_{\text{new}}}{MVA_{\text{old}}} \times \left(\frac{kV_{b, \text{old}}}{kV_{b, \text{new}}}\right)^2$$

$$= 0.72 \times \frac{20}{10} \times \left(\frac{69}{13.8}\right)^2 = 36 \text{ p.u.}$$

03. Ans:

Sol: Given data:

Select the base MVA as 100MVA, Base voltage as 33KV on the Generator side

Base voltage on the line side = 110 kV

$$Z_{pu \text{ new}} = Z_{pu \text{ old}} \times \frac{MVA_{\text{new}}}{MVA_{\text{old}}} \times \left(\frac{kV_{\text{old}}}{kV_{\text{new}}}\right)^2$$

**Generator:**

$$X_{pu \text{ new}} = 0.15 \times \frac{100}{100} \times \left(\frac{33}{33}\right)^2 = 0.15 \text{ pu}$$

**Transformer:**

$$X_{pu \text{ new}} = 0.09 \times \frac{100}{100} \times \left(\frac{33}{33}\right)^2 = 0.09 \text{ pu.}$$

**Transmission line:**

$$X_{pu} = 50 \times \frac{100}{(110)^2} = 0.4132 \text{ pu.}$$

**Motor 1:**

$$X_{pu, \text{ new}} = 0.18 \times \frac{100}{30} \times \left(\frac{30}{33}\right)^2 = 0.4958 \text{ pu.}$$

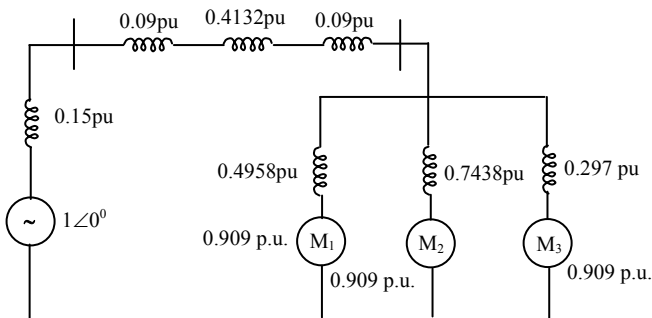
**Motor 2:**

$$X_{pu \text{ new}} = 0.18 \times \frac{100}{20} \times \left(\frac{30}{33}\right)^2 = 0.7438 \text{ pu.}$$

**Motor 3:**

$$X_{pu \text{ new}} = 0.18 \times \frac{100}{50} \times \left(\frac{30}{33}\right)^2 = 0.2975 \text{ pu.}$$

The per unit reactance diagram of the system can be given in below.



**04. Ans: (d)**

**Sol:** Given data:

$$E_a = 10\angle 0^\circ \text{V}$$

$$E_b = 10\angle -90^\circ \text{V}$$

$$E_c = 10\angle 120^\circ \text{V},$$

As both sides of the circuit are grounded we can take each branch is considered as one circuit

$$I_a = \frac{E_a}{X_a} = \frac{10\angle 0^\circ}{j2} = 5\angle -90^\circ$$

$$I_b = \frac{E_b}{X_b} = \frac{10\angle -90^\circ}{j3} = 3.33\angle -180^\circ$$

$$I_c = \frac{E_c}{X_c} = \frac{10\angle 120^\circ}{j4} = 2.5\angle 30^\circ$$

Positive sequence current,

$$I_1 = \frac{1}{3}(I_a + aI_b + a^2I_c)$$

$$\text{Where } a = 1\angle 120^\circ$$

$$I_1 = \frac{1}{3}(5\angle -90^\circ + 1\angle 120^\circ \times 3.33\angle -180^\circ + 1\angle 240^\circ \times 2.5\angle 30^\circ)$$

$$= 3.510\angle -81^\circ$$

**05. Ans:  $I_{a1} = 7.637\angle -79.1^\circ \text{ kA}$**

**Sol:** Given data:

$$I_a = 10\angle 30^\circ, I_c = 15\angle -30^\circ, I_b = ?$$

$$I_a + I_b + I_c = 0$$

$$I_b = -[I_a + I_c]$$

$$= -[10\angle 30^\circ + 15\angle -30^\circ]$$

$$= -21.79\angle 173.41^\circ$$

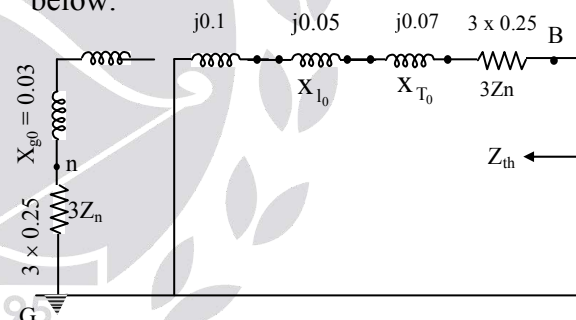
$$I_{a1} = \frac{1}{3}[I_a + K I_b + K^2 I_c]$$

$$I_{a1} = \frac{1}{3}\left[10\angle 30^\circ + 1\angle 120^\circ \times 21.79\angle 173.41^\circ + 1\angle 240^\circ \times 15\angle -30^\circ\right]$$

$$I_{a1} = 7.637\angle -79.1^\circ \text{ kA}$$

**06. Ans: (b)**

**Sol:** Per unit zero sequence reactance diagram of the given single line diagram is shown below.



Thevenin equivalent impedance,  $Z_{th}$  at 'B'

$$\text{is } Z_{th} = j0.1 + j0.05 + j0.07 + 0.75$$

$$= 0.75 + j0.22$$

**07. Ans: (b)**

**Sol:** Given data:

$$X_1 = 0.3,$$

$$X_2 = 0.4,$$

$$X_0 = 0.05$$

Fault current = Rated current

$$I_{d \text{ p.u.}} = 1.0 \text{ p.u.}$$

$$1.0 = \frac{3 E_{R1}}{X_1 + X_2 + X_0 + 3 X_n}$$

$$1.0 (X_1 + X_2 + X_0 + 3 X_n) = 3$$

$$0.3 + 0.4 + 0.05 + 3 X_n = 3$$

$$X_n = 0.75 \text{ p.u.}$$

$$X_{n(\Omega)} = 0.75 \left( \frac{k V_b^2}{\text{MVA}_b} \right)$$

$$= 0.75 \left[ \frac{13.8^2}{10 \text{ MVA}} \right] = 14.28 \Omega$$

**08. Ans: (i)**  $V_n = 1429 \text{ volts}$

**(ii)**  $V_n = 1905 \text{ volts}$

**Sol:** Given data:

$$(i) X_{1eq} = \frac{j0.1}{2} = j0.05$$

$$X_{2eq} = \frac{j0.1}{2} = j0.05$$

$$X_{0eq} = \frac{X_0 + 3X_n}{2} = j0.1$$

$$I_{R0} = I_{R1} = \frac{E_{R1}}{X_{1eq} + X_{2eq} + X_{0eq}}$$

$$= \frac{1.0}{j0.2} = 5.0 \text{ p.u.}$$

$$V_n = 3 I_{R0} X_n = 3 \times 5 \times 0.05 = 0.75 \text{ p.u.}$$

$$V_n = 0.75 \times \frac{6.6 \times 10^3}{\sqrt{3}} = 2858 \text{ volts}$$

$$V_n = \frac{2858}{2} = 1429 \text{ volts}$$

$$(ii) X_{1eq} = \frac{j0.1}{2} = j0.05$$

$$X_{2eq} = \frac{j0.1}{2} = j0.05$$

$$X_{0eq} = X_0 + 3X_n = j0.2$$

$$I_{R0} = I_{R1} = \frac{E_{R1}}{X_{1eq} + X_{2eq} + X_{0eq}} = \frac{1.0}{0.3} = 3.33$$

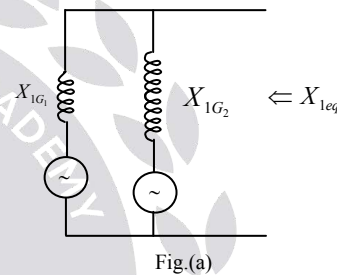
$$V_n = 3 I_{R0} X_n = 3 \times 3.33 \times 0.05 = 0.5 \text{ p.u.}$$

$$V_n = 0.5 \times \frac{6.6 \times 10^3}{\sqrt{3}} = 1905 \text{ Volts}$$

**09. Ans:  $|I_f| = 8.39 \angle -47.83 \text{ pu.}$**

**Sol:** Given data:

Two identical generators are operate in parallel and positive sequence reactance diagram is given by figure (a).

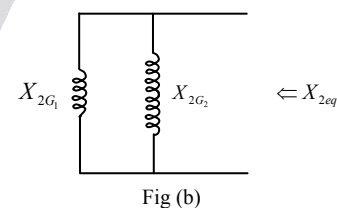


$$X_{1eq} = \frac{j0.18}{2} = 0.09 \text{ j.p.u.}$$

where  $X_{1G1}$  = positive sequence reactance in p.u. of generator (1)

$X_{1G2}$  = positive sequence reactance in p.u. of generator (2)

Negative sequence reactance diagram is given by figure (b).



$$X_{2eq} = \frac{j0.15}{2} = 0.075 \text{ j.p.u.}$$

Since the star point of the second generator is isolated. Its zero sequence reactance does not come into picture. The zero sequence reactance diagram is given by figure (c).

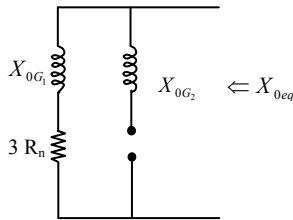


Fig.

Now all values are in p.u., then

$$R_{pu} = 0.5 \times \frac{20}{11^2} = 0.08 \text{ pu}$$

$$\therefore X_{0eq} = j0.1 + (3 \times 0.08) = 0.24 + j0.1$$

For LG Fault, Fault current

$$(I_f) = 3I_{R1} = \frac{3E_{R1}}{X_{1eq} + X_{2eq} + X_{0eq}}$$

$$I_f = \frac{3 \times 1}{j0.09 + j0.075 + j0.1 + 0.24}$$

(Assume  $E_{R1} = 1.0 \text{ p.u.}$ )

$$= \frac{3}{0.24 + j0.265}$$

$$|I_f| = 8.39 \text{ pu}$$

**10. Ans: (d)**

**Sol:** Given data:

$$Z_0 = j0.1 + j0.1 = j0.2;$$

$$Z_1 = j0.1 + j0.1 = j0.2$$

$$Z_n = 0.05$$

$$Z_1 = Z_{l_1} + Z_{g_1}$$

$$Z_2 = Z_{l_2} + Z_{g_2}$$

$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2 + 3Z_n}$$

$$= \frac{1}{j0.2 + j0.2 + 0.34j + j0.15}$$

For L-G fault

$$= -j1.12 \text{ (pu)}$$

$$I_B \text{ (Base Current)} = \frac{20 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3}$$

$$= 1750 \text{ Amp}$$

$$I_f \text{ (fault current)} = (3I_{a1}) I_B = -j 5897.6 \text{ A}$$

$$\text{Neutral voltage } V_N = I_f \cdot Z_n$$

$$\text{where } Z_n = Z_B \times 0.05 = \frac{(6.6)^2}{20} \times 0.05$$

$$= 0.1089 \Omega$$

$$V_N = 5897.6 \times 0.1089 = 642.2 \text{ volts}$$

**11. Ans: 7 kA**

**Sol:** Given data:  $X_1 = X_2 = j0.1$ ,  $X_f = j0.05$

$$I_{a1} = \frac{E}{X_1 + X_2 + X_f}$$

$$= \frac{1}{j0.1 + j0.1 + j0.05} = \frac{1}{j0.25} = 4 \text{ pu}$$

$$I_{\text{fault}} = \frac{20 \times 10^3}{\sqrt{3} \times 6.6} \times 4 = 7 \text{ kA}$$

**12. Ans:  $V_{AB} = 13.33 \text{ kV}$**

**Sol:** Given data:

$$X_{1eq} = 0.2 \text{ p.u.}, X_{2eq} = 0.3 \text{ p.u. and}$$

Alternator neutral is solidly grounded

$$(X_n = 0)$$

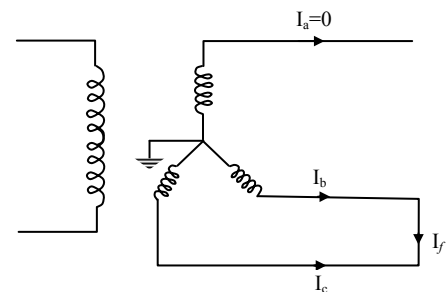


Figure (a)

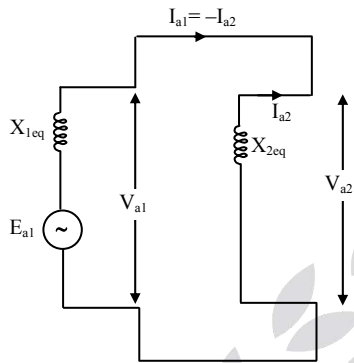
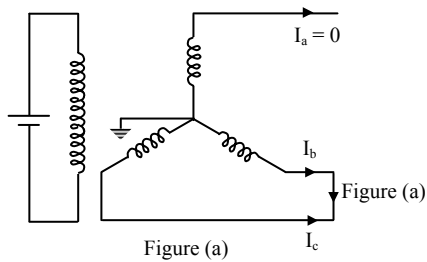


Figure (b) Sequence Network with respective Fig (a)

From figure (a),  $I_b = -I_c$

From figure (b),  $I_{a1} = -I_{a2}$

Positive sequence current

$$I_{a1} = \frac{E_{a1}}{X_{1eq} + X_{2eq}}$$

(assume pre-fault voltage  $E_{a1} = 1$  pu.)

Positive sequence current

$$I_{a1} = \frac{1 + j0}{j0.2 + j0.3} = -2j \text{ pu.}$$

Negative sequence current ( $I_{a2}$ ) =  $-I_{a1}$   
=  $2j$  pu.

A zero sequence current doesn't exist in L-L fault because this fault is not associated with the ground

$$\therefore I_{a0} = 0.$$

In this LL fault, fault current ( $I_f$ ) =  $|I_b| = |I_c|$

$$\begin{aligned} I_b &= I_{b0} + I_{b1} + I_{b2} \\ &= 0 + K^2 I_{a1} + K I_{a2} \quad (\because I_{a1} = -I_{a2}) \\ &= (K^2 - K) I_{a1} \end{aligned}$$

$$\begin{aligned} &= [(-0.5 - j0.8667) - (-0.5 + j0.8667)] I_{a1} \\ &= -j1.732 I_{a1} \end{aligned}$$

$$|I_b| = \sqrt{3} I_{a1} = \sqrt{3} \times \frac{E_{a1}}{X_{1eq} + X_{2eq}}$$

$$= \sqrt{3} \times \Rightarrow 3.464 \text{ p.u.}$$

$$\therefore \text{Fault current } (I_f) = |I_b| = |I_c| = 3.464 \text{ pu.}$$

$$\begin{aligned} \text{Base current} &= \frac{\text{Base MVA}}{\sqrt{3} \times \text{Base voltage}} \\ &= \frac{25 \times 10^6}{\sqrt{3} \times 13.2 \times 10^3} \\ &= 1093.4 \text{ A} \end{aligned}$$

$\therefore$  Fault current in amps,

$$\begin{aligned} I_{f \text{ actual}} &= I_{f \text{ pu}} \times I_{\text{base}} \\ &= 3.464 \times 1093.4 \\ &= 3787.5 \text{ A.} \end{aligned}$$

$$\begin{aligned} V_{a1} &= E_a - I_{a1} X_{1eq} \\ &= 1 + j0 - (-2j)(j0.2) \\ &= 1 - 0.4 = 0.6 \text{ p.u.} \end{aligned}$$

$$V_{a2} = -I_{a2} \times X_{2eq} = -(2j) \times (j0.3) = 0.6 \text{ pu}$$

$$\therefore |V_{a1}| = |V_{a2}| = 0.6 \text{ pu}$$

For Phase 'a',

$$\begin{aligned} V_a &= V_{a1} + V_{a2} + V_{a0} \quad (\because V_{a0} = 0) \\ &= 2V_{a1} = 2 \times 0.6 = 1.2 \text{ pu.} \end{aligned}$$

For Phase 'b',

$$\begin{aligned} V_b &= V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2} \\ &= (k^2 + k)V_{a1} \quad (\because V_{a1} = V_{a2}) \\ &= (-0.5 - j0.8667j) + (-0.5 + j0.8667j)V_{a1} \\ &= -0.6 \text{ pu.} \end{aligned}$$

But we know that  $V_b = V_c$

$$\therefore V_b = V_c = -0.6$$

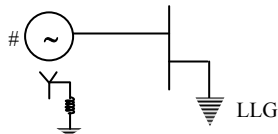
$$\begin{aligned} \text{Line voltages, } V_{ab} &= V_a - V_b \\ &= 1.2 - (-0.6) = 1.8 \text{ p.u.} \\ V_{bc} &= V_b - V_c = 0 \text{ p.u.} \end{aligned}$$

$$V_{ca} = V_c - V_a \\ = -0.6 - (1.2) = 1.8 \text{ p.u.}$$

$$V_{ab} = 1.8 \times \frac{13.2}{\sqrt{3}} = 13.33 \text{ KV,}$$

**13. Ans:  $I_f = 4.8 \text{ p.u}$**

**Sol:** Given data:



$$\text{Prefault voltage} = \frac{13.9}{13.2} = 1.05$$

Current through ground = Fault current

$$I_f = 3 I_{a0}$$

$$I_{a0} = -I_{a1} \frac{X_{2eq}}{X_{2eq} + X_{0eq}} \dots\dots (1)$$

$$I_{a1} = \frac{E_{a1}}{X_1 + \frac{X_2 X_0}{X_2 + X_0}} \\ = \frac{1.05}{0.2 + \left[ \frac{0.2 \times (3 \times 0.05 + 0.08)}{0.2 + (3 \times 0.05 + 0.08)} \right]}$$

$$= 3.42$$

Substitute  $I_{a1}$  value in equation (1)

$$\therefore I_{a0} = 3.42 \left[ \frac{0.2}{0.2 + (0.15 + 0.08)} \right] = 1.59$$

$$I_f = 3 I_{a0} = 3 \times 1.59 = 4.77 \approx 4.8 \text{ p.u}$$

$$I_{f \text{ amp}} = 4.77 \left[ \frac{15}{\sqrt{3} \times 13.2} \right] \text{ kA}$$

$$\approx 3.13 \text{ kA}$$

**14. Ans:  $I_{R1} = 6.22 \text{ kA}$**

**Sol:** Given data:

The rating each generator 20 MVA,  
6.6 kV,  $X_1 = X_2 = 0.12 \text{ pu}$ ,

$$X_0 = 0.05 \text{ pu}$$

$$X_n = 0.05$$

The sequence reactance  $X_1 = X_2 = 0.1 \text{ pu}$

$$X_0 = 0.3 \text{ pu}$$

$$X_{1eq} = \frac{j0.12}{2} + j0.1 = j0.16$$

$$X_{2eq} = X_{1eq} = j0.16$$

$$X_{0eq} = X_0 + 3X_n + X_0 \\ = j0.05 + 3(j0.05) + j0.3 = j0.5$$

$$I_{R1} = \frac{E_{R1}}{X_{1eq} + \frac{X_{2eq} X_{0eq}}{X_{2eq} + X_{0eq}}} \\ = \frac{1.0}{0.16 + \frac{0.16 \times 0.5}{0.66}} = \frac{1.0}{0.2812}$$

$$I_{R1} = 3.55 \text{ p.u} = 3.55 \times \frac{20}{\sqrt{3} \times 6.6} = 6.22 \text{ kA}$$

**15. Ans: (c)**

**Sol:** Equivalent reactance seen from the fault point

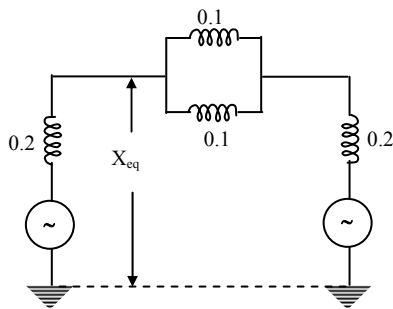
$$X_{PU} = \frac{(j0.3 + j0.08) \times (j0.1 + j0.08)}{j0.1 + j0.2 + j0.08 + j0.08 + j0.1} \\ = j0.12214$$

$$\text{Fault level current} = 1/X_{(PU)} = 1/j0.12214 \\ = -j8.1871$$

**16. Ans: (c)**

$$\text{Sol: SC MVA} = \frac{\text{Base MVA}}{X_{eq}}$$





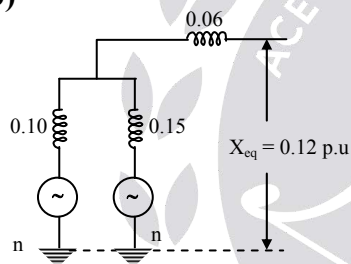
$$X_{G_2 \text{ New}} = 0.16 \left[ \frac{1000}{800} \right] = 0.2$$

$$X_{eq} = \frac{0.2 \times 0.25}{0.45} = \frac{1}{9}$$

$$\therefore \text{SC MVA} = \frac{1000}{(1/9)} = 9000 \text{ MVA}$$

**17. Ans: (b)**

**Sol:**



$X_{G_2}$  New on 15 MVA Base

$$= 0.10 \left[ \frac{15}{10} \right] [1]^2 = 0.15 \text{ p.u}$$

$$I_f = \frac{E_{R_1}}{X_{eq}} = \frac{1}{0.12} = 8.33 \text{ p.u}$$

$$I_{fG_2} = 8.33 \left[ \frac{0.1}{0.25} \right] = 3.33$$

$$\Rightarrow 3.33 \left[ \frac{15}{\sqrt{3} \times 11} \right] = 2.62 \text{ kA}$$

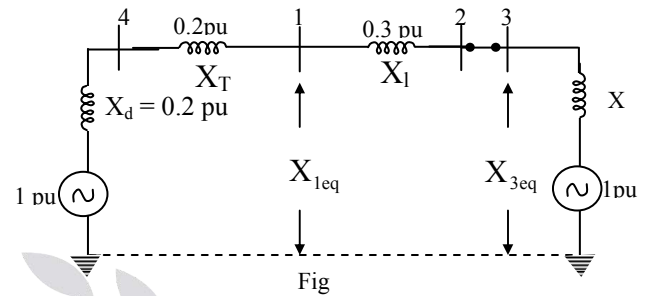
$$I_{fG_1} = 8.33 - 3.33 = 5$$

$$I_{fG1(\text{actual})} = 5 \left[ \frac{15}{\sqrt{3} \times 11} \right] = 3.93 \text{ kA}$$

**18. Ans:  $I_f = 11.43 \text{ pu}$**

**Sol:** Given data:

Per unit positive sequence reactance diagram of the given system when the breaker closed is shown in fig.



The equivalent reactance with respect to point "1" is [short circuit 1P.u sources]

$$X_{1eq} = (X_T + X_d) // (X_I + X)$$

$$= \frac{0.4 \times (0.3 + X)}{0.4 + 0.3 + X} = \frac{0.12 + 0.4X}{0.7 + X}$$

Given prefault voltage ( $V_{th}$ ) = 1pu.

$$\therefore \text{Fault current } (I_f) = \frac{V_{th}}{X_{1eq}}$$

$$= \frac{1}{\left( \frac{0.12 + 0.4X}{0.7 + X} \right)} = 5 \text{ pu}$$

$$0.7 + X = 5(0.12 + 0.4X)$$

$$\therefore X = 0.1 \text{ p.u}$$

**To find fault level at bus '3':**

The equivalent reactance w.r.t. point '3' in reactance diagram is

$$X_{3eq} = (X_d + X_T + X_I) // X$$

$$= (0.2 + 0.2 + 0.3) // 0.1$$

$$= \frac{0.7 \times 0.1}{0.8} = 0.0875 \text{ pu}$$

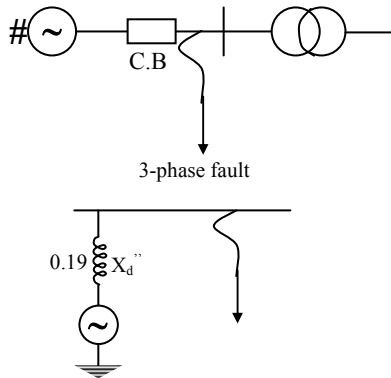
$$\therefore \text{Fault current } (I_{f3}) = \frac{V_{th}}{X_{3eq}}$$

$$= \frac{1.0}{0.0875} = 11.43 \text{ pu}$$



19. Ans: (c)

Sol:



For a 3-phase fault

$$\text{Fault current } I_f = \frac{E_{R_1}}{X_{1eq}}$$

where,  $E_{R_1} = V_{th} = 1.0 \text{ p.u.}$ ,  $X_{1eq} = X_d''$

$$\therefore I_f = \frac{1.0}{0.19} = 5.263 \text{ p.u.}$$

$$I_{base} = \frac{110 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 5773.5 \text{ A}$$

$$\therefore I_{f \text{ actual}} = I_{base} \times I_f \text{ p.u.} \\ = 5773.5 \times 5.263 = 30.39 \text{ kA}$$

20. Ans: (d)

Sol: In phasor diagram  $|V_1| > |V_2|$ , so fault may not be at location P. If fault occurs at any point, the voltage will be almost  $90^\circ$  lead with the current at that point.

In phasor diagram currents  $\bar{I}_1, \bar{I}_2$  are almost  $90^\circ$  lag with respect to  $V_{S_1}$ .

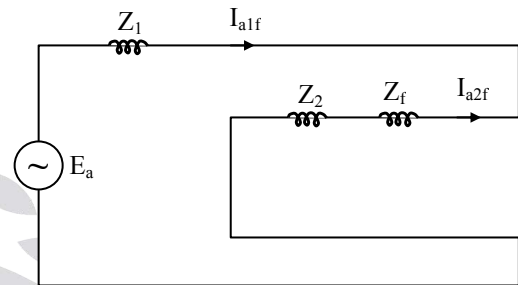
In phasor diagram current  $\bar{I}_3, \bar{I}_4$  are almost  $90^\circ$  lead with respect to  $V_{S_2}$ .

But in given diagram  $\bar{I}_3$  and  $\bar{I}_4$  are in reverse (or) out of phase.

All the conditions are satisfied if the fault occurs at a point 'S'.

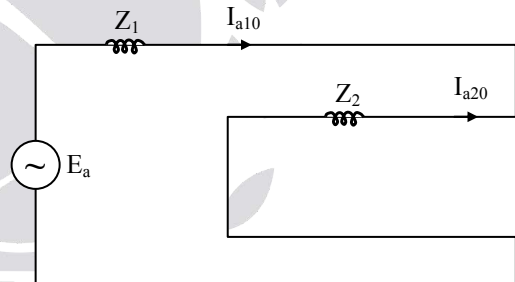
21. Ans: (a)

Sol: For a line to line fault on a generator through a fault impedance of  $Z_f$ , the sequence network is as follows.



$$\therefore I_{a1f} = \frac{E_a}{Z_1 + Z_2 + Z_f}$$

Sequence network with zero fault impedance is as follows



$$\therefore I_{a10} = \frac{E_a}{Z_1 + Z_2}$$

$$I_{f1} = k I_f$$

$$-j\sqrt{3} I_{a1f} = k (-j\sqrt{3} I_{a10})$$

$$\frac{E_a}{Z_1 + Z_2 + Z_f} = \frac{k E_a}{Z_1 + Z_2}$$

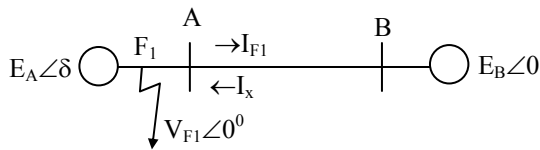
$$Z_1 + Z_2 = k Z_1 + k Z_2 + k Z_f$$

$$Z_1(1 - k) + Z_2(1 - k) = k Z_f$$

$$Z_f = \frac{(Z_1 + Z_2)(1 - k)}{k}$$

22. Ans: (c)

Sol: (i) Fault at  $F_1$



For a fault  $F_1$ :

Both Generator 1 and generator 2 are supplying the fault current the voltage at bus A is due to generator 2. The angle of generator is zero so that the voltage angle at A is negative. Hence  $V_{F1}$  lags  $I_{F1}$

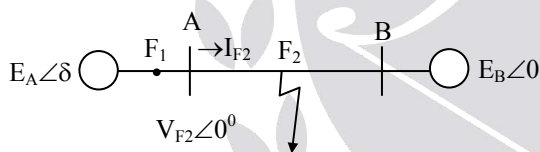
$I_x$  fault current will be  $I_x \angle -90^\circ$

$I_{F1} = -I_x = (1 \angle 180^\circ) I_x \angle -90^\circ$

$I_{F1} = I_x \angle 90^\circ$

$\rightarrow V_{F1}$  Lags  $I_{F1}$

(ii) Fault at  $F_2$



For a fault  $F_2$ :

Both Generator 1 and generator 2 are supplying the fault current the voltage at bus A due to generator 1 the angle of generator is  $\delta$  and it is positive so that the voltage angle at bus A is also positive. Hence  $V_{F2}$  Leads  $I_{F2}$

Now  $\overline{I_{F2}}$  is also  $\angle -90^\circ$

$\Rightarrow V_{F2}$  leads  $I_{F2}$

## Power Systems Dynamics & Stability

### Solutions for Objective Practice Questions

01. Ans: (i) 180 MJ

(ii) 23.54 k N-m

(iii) 184.9 Elec.deg/sec<sup>2</sup>

Sol: Given data:

$H = 9$  kW – sec/kVA

K.E = stored?

(i) Inertia constant

$$H = \frac{\text{K.E stored}}{\text{rating of the machine}}$$

K.E stored =  $H \times S$

$= 9 \times 20$  MVA

$= 180$  MW – sec  $\Rightarrow 180$  MJ

(ii) Accelerating torque  $T_a = ?$

$$P_a = T_a \omega \quad T_a = \frac{P_a}{\omega}$$

$$P_a = P_s - P_e$$

$$P_s = 26800 \times 0.735 = 1998 \text{ kW}$$

$$P_a = 19698 - 16000 = 3698 \text{ kW}$$

$$T_a = \frac{3698}{\frac{2\pi \times 1500}{60}}$$

$$= 23.54 \text{ kN – m.}$$

$$(iii) M \frac{d^2\delta}{dt^2} = P_a$$

$$M = \frac{SH}{\pi f} = \frac{180}{180 \times 50} = 0.02$$

$$0.02 \times \frac{d^2\delta}{dt^2} = 3698$$

$$\frac{d^2\delta}{dt^2} = \frac{3698}{0.02}$$

$$= 184.9 \text{ elec. deg/sec}^2$$

**02. Ans: (c)**
**Sol:** Given data:

$$N_s = 3000,$$

$$f = 60 \text{ Hz},$$

$$S = \frac{P}{\cos \phi} = \frac{60 \text{ MW}}{0.85} = 70.58 \text{ MVA}$$

$$H = \frac{\frac{1}{2} I \omega_s^2}{S} \text{ due to moment of Inertia, there}$$

is no sudden change in angular velocity

$$= \frac{\frac{1}{2} I \left( \frac{2\pi N_s}{60} \right)^2 \times 10^{-6}}{70.58}$$

$$= \frac{\frac{1}{2} (8800) \left( \frac{2\pi \times 3000}{60} \right)^2 \times 10^{-6}}{70.58}$$

$$= 6.152 \text{ MJ/MVA}$$

$$M = \frac{SH}{180f}$$

$$= \frac{70.58 \times 6.15}{180 \times 50} = 0.04825$$

**03. Ans: (d)**
**Sol:** Inertia constant,  $H \propto \frac{1}{\text{MVA rating}(S)}$ 

$$H_{A \text{ new}} = H_{A \text{ old}} \times \frac{S_{\text{old}}}{S_{\text{new}}}$$

$$= 1.6 \times \frac{250}{100} = 4.0 \text{ pu}$$

$$H_{B \text{ new}} = H_{B \text{ old}} \times \frac{S_{\text{old}}}{S_{\text{new}}}$$

$$= 1.0 \times \frac{500}{100} = 5.0 \text{ pu}$$

$$\therefore H_{eq} = H_{A \text{ new}} + H_{B \text{ new}}$$

$$= 4.0 + 5.0 = 9.0 \text{ pu}$$

**04. Ans:  $f_n = 1.53 \text{ Hz}$** 
**Sol:** Given data:

Since the system is operating initially under steady state condition, a small perturbation in power will make the rotor oscillate. The natural frequency of oscillation is given by

$$f_n = \left( \frac{\left( \frac{dp_e}{d\delta} \right)_{\delta_0}}{M} \right)^{\frac{1}{2}}$$

As load increases, load angle ( $\delta$ ) increases, there by  $\sin \delta_0$  increases.

$$\therefore \sin \delta_0 = \text{loading}$$

$$\text{At 60\% of loading } \sin \delta_0 = 0.6$$

$$\delta_0 = 36.86$$

$$\text{We know that } P_e = \frac{EV}{X} \sin \delta_0,$$

where E = no-load voltage,

V = load voltage

$$\frac{dP_e}{d\delta} = \frac{EV}{X} \cos \delta_0$$

$$\Rightarrow \frac{1.1 \times 1}{(0.3 + 0.2)} \cos 36.86 = 1.76$$

$$\text{Moment of inertia } M = \frac{SH}{\pi f},$$

where S = Rating of the machine,

f = frequency,

Inertia constant, H = 3 MW-sec/MVA

( $\therefore$  Assume rating of machine 1 pu.)

$$= \frac{1 \times 3}{\pi \times 50} \Rightarrow \frac{3}{50\pi}$$

The natural frequency of oscillation at 60% loading,

$$f_n = \left\{ \left( \frac{dPe}{d\delta} \right)_{=\delta_0} / M \right\}^{1/2}$$

$$= \left( 1.76 \times \frac{50\pi}{3} \right)^{1/2} \Rightarrow 9.6 \text{ rad/sec}$$

$$= \frac{9.6}{2\pi} \text{ Hz} = 1.53 \text{ Hz}$$

**05. Ans: 12.7**

**Sol:**  $H = \frac{1000}{250} = 4 \text{ MJ}; \delta = 10^\circ$

$$P_s = P_e = 60 \text{ MW}$$

$$\delta = \delta + \Delta\delta$$

$$\Delta\delta = \delta \frac{(\Delta t)^2}{2} = \frac{P_s - P_e}{M}$$

$$= \frac{(\Delta t)^2}{2} = \frac{60 - 0}{\frac{5H}{180f}} \times \frac{(0.1)^2}{2}$$

$$\Delta\delta = \frac{60 \times 186 \times 50}{250 \times 4} \times \frac{(0.1)^2}{2}$$

$$6 \times 180 \times 5 \times \frac{(0.1)^2}{2} = 2.7^\circ$$

$$\delta = 10 + 2.7 = 12.7^\circ$$

**06. Ans: 27 deg**

**Sol:** Given data:

$$E = 1.1 \text{ pu} \quad V = 1.0 \text{ pu}$$

Assuming inertia constant (H) = 1 pu

$$P = \frac{EV}{X} \sin \delta$$

$$X = j0.015 + j0.015 = j0.030 \text{ pu}$$

$$\sin \delta = \frac{PX}{EV}$$

$$= \frac{j0.3 \times 1}{1.1 \times 1.0} = 0.2727$$

$$\delta = 15.82^\circ$$

$$M = \frac{GH}{\pi f} = 1.11 \times 10^{-4} \text{ pu}$$

$$P_{a(+)} = \frac{1.0 - 0.0}{2} = 0.5$$

$$\alpha(0_+) = \frac{0.5}{1.11 \times 10^{-4}} = 4504 \text{ deg/sec}^2$$

$$\Delta\delta_1 = (\Delta t)^2 \alpha(0.05)^2 \times 4504 = 11.26 \text{ deg}$$

Rotor angle  $\delta_1 = \delta_0 + \Delta\delta_1$

$$= 15.82 + 11.26$$

$$= 27 \text{ deg}$$

**07. Ans:  $\delta_{cr} = 70.336^\circ$** 

**Sol:** Given data:

$$\delta = 30^\circ, P_{m2} = 0.5, P_{m3} = 1.5, P_s = 1.0$$

$$\delta_0(\text{rad}) = 0.52$$

$$\delta_{\max} = 180 - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right)$$

$$= 180 - \sin^{-1} \left( \frac{1.0}{1.5} \right)$$

$$\delta_{\max} = 180 - 41.80 = 138.18$$

$$\delta_{\max} = 138.18 \times \frac{\pi}{180} = 2.41$$

$$\delta_c = \cos^{-1} \left[ \frac{1.0(2.41 - 0.523) + 1.5 \cos 138.18 - 0.5 \cos 30^\circ}{1.5 - 0.5} \right]$$

$$= \cos^{-1} \left[ \frac{1.00 \times 1.887 + 1.5 \times -0.7452 - 0.5 \times \frac{\sqrt{3}}{2}}{1} \right]$$

$$= \cos^{-1} [1.887 + (-1.1175) - 0.433]$$

$$= \cos^{-1} [1.887 - 1.5505]$$

$$= \cos^{-1} [0.3365] = 70.336^\circ.$$

**08. Ans:  $\delta_{cr} = 55^\circ$**

**Sol:** Given data:

$$P_s = 1.0 \text{ p.u.}$$

$$P_{m1} = 1.8 \text{ p.u.}$$

$$X_{1eq} = 0.72 \text{ p.u.}$$

$$X_{2eq} = 3.0 \text{ p.u.}$$

$$X_{3eq} = 1.0 \text{ p.u.}$$

$$P_{m2} = \frac{EV}{X_2}$$

$$= \frac{EV}{X_1} \times \frac{X_1}{X_2}$$

$$P_{m2} = P_{m1} \times r_1 \text{ where } r_1 = \frac{X_1}{X_2}$$

$$P_{m3} = \frac{EV}{X_3} = \frac{EV}{X_1} \times \frac{X_1}{X_3}$$

$$P_{m3} = P_{m1} \times r_2 \text{ where } r_2 = \frac{X_1}{X_3}$$

Substitute these values to get  $P_{m2}$  &  $P_{m3}$

$$\therefore P_{m2} = 1.8 \times \frac{0.72}{3.0} = 0.416$$

$$P_{m3} = 1.245$$

$$\delta_0 = \sin^{-1} \left( \frac{P_s}{P_{m1}} \right)$$

$$\delta_0 = 35.17^\circ = 0.614 \text{ rad}$$

$$\delta_{\max} = 180 - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right)$$

$$= 126.56^\circ = 2.208 \text{ rad}$$

$$\delta_{cr} = \cos^{-1} \left[ \frac{P_s (\delta_{\max} - \delta_0) + P_{m3} \cos \delta_{\max} - P_{m2} \cos \delta_0}{P_{m3} - P_{m2}} \right]$$

$$\delta_{cr} = \cos^{-1} \left[ \frac{1.0(2.208 - 0.614) + 1.245 \cos 126.56 - 0.416 \cos 35.17}{1.245 - 0.416} \right]$$

$$\delta_{cr} = 51.82^\circ \approx 55^\circ$$

**09. Ans:  $\delta_c = 65^\circ$**

**Sol:** Given data:

$$P_s = P_{e1} = 1.0$$

$$P_{e1} = 2.2 \sin \delta$$

$$P_{m1} = 2.2$$

$$P_{e2} = 0, P_{m2} = 0$$

$$P_{m3} = 0.75 \times 2.2 = 1.65$$

$$\delta_0 = \sin^{-1} \left( \frac{P_s}{P_{m1}} \right) = \sin^{-1} \left( \frac{1}{2.2} \right)$$

$$= 27^\circ \times \frac{\pi}{180} = 0.471$$

$$\delta_m = 180 - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right)$$

$$= 180 - \sin^{-1} \left( \frac{1.0}{1.65} \right) = 142.7^\circ$$

$$\delta_m = 142.7 \times \frac{\pi}{180^\circ} = 2.48 \text{ rad}$$

$$\delta_c = \cos^{-1} \left[ \frac{P_s (\delta_m - \delta_0) + P_{m3} \cos \delta_m}{P_{m3}} \right]$$

$$\cos^{-1} \left[ \frac{1.0(2.48 - 0.471) + 1.65 \cos(142.7)}{1.65} \right]$$

$$\delta_c = \cos^{-1} \left[ \frac{(2.48 - 0.471) - 1.31}{1.65} \right]$$

$$= \cos^{-1} [0.423] = 65^\circ$$

**10. Ans:  $\delta_c = 84^\circ$**

**Sol:** Given data:

$$P_s = P_{e1} = 1.0$$

$$P_{e1} = 2.2 \sin \delta$$

$$P_{m1} = 2.2$$

$$P_{e2} = 0, P_{m2} = 0$$

$$P_{m3} = P_{m1} = 2.2$$

$$\delta_0 = 27^\circ$$

$$\delta_0(\text{rad}) = 0.471$$

$$\delta_m = 180 - \delta_0 = 153^\circ = 153 \times \frac{\pi}{180} = 2.66$$

$$\delta_c = \cos^{-1} \left[ \frac{1.0(2.66 - 0.471) + 2.2 \cos(153)}{2.2} \right]$$

$$\delta_c = \cos^{-1} \left[ \frac{2.66 - 0.471 - 1.96}{2.2} \right]$$

$$\delta_c = 84^\circ$$

**11. Ans: 0.20682 sec**

**Sol:** Given data:

$$S = 1.0, H = 5, \delta = 68.5^\circ, \delta_0 = 30^\circ, P_s = 1.0$$

$$t_c = \sqrt{\frac{2M(\delta_c - \delta_0)}{P_s}}$$

$$t_c = \sqrt{\frac{2 \times SH (\delta_c - \delta_0)}{\pi f (P_s)}}$$

$$t_c = \sqrt{\frac{2 \times 1.0 \times 5 (68.5 - 30) \times \frac{\pi}{180}}{\pi \times 50 \times 1.0}}$$

$$= 0.20682 \text{ sec}$$

**12. Ans: Permissible increase = 60.34°**

**Sol:** Given data:

$$P_s = 2.5 \text{ p.u.}$$

$$P_{\max 1} = 5.0 \text{ p.u.}$$

$$\therefore \text{Before fault } \frac{d\delta}{dt} = 0, \delta = \delta_0, P_a = 0$$

$$P_s = P_{e1}$$

$$P_s = P_{\max 1} \sin \delta_0 \Rightarrow \delta_0 = \sin^{-1} \left[ \frac{P_s}{P_{\max 1}} \right]$$

$$\delta_0 = \sin^{-1} \left[ \frac{2.5}{5} \right]$$

$$\delta_0 = 30^\circ \Rightarrow 0.523 \text{ rad}$$

$$P_{\max 2} = 2 \text{ p.u.}$$

$$P_{\max 3} = 4 \text{ p.u.}$$

$$\delta_{\max} = 180^\circ - \sin^{-1} \left[ \frac{P_s}{P_{\max 3}} \right]$$

$$= 180 - \sin^{-1} \left[ \frac{2.5}{4} \right]$$

$$= 180 - 36.68$$

$$\delta_{\max} = 141.32^\circ \Rightarrow 2.4664 \text{ rad}$$

$$\cos \delta = \frac{P_s [\delta_{\max} - \delta_0] \times \frac{\pi}{180} + P_{\max 3} \cos(\delta_{\max}) - P_{\max 2} \cos(\delta_0)}{P_{\max 3} - P_{\max 2}}$$

$$\frac{2.5[141.32 - 30] \times \frac{\pi}{180} + 4 \cos(141.32) - 2 \cos(30^\circ)}{4 - 2} =$$

$$\frac{4.84 + (-3.122) - 1.73}{2}$$

$$\cos \delta_c = -6 \times 10^{-3}$$

$$\delta_c = \cos^{-1}(-6 \times 10^{-3}) \Rightarrow 90.34^\circ$$

$$\text{Permissible increases} = \delta_c - \delta_0$$

$$= 90.34^\circ - 30^\circ$$

$$= 60.34^\circ$$

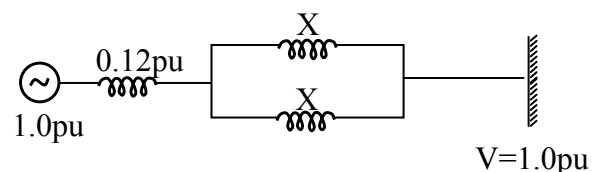
**13. Ans: (d)**

**Sol:** Given data:

$$V = 1.0 \text{ pu}$$

$$\chi_T = 0.12 \text{ pu}$$

$$|E| = 1.0 \text{ pu}$$



when one of the double circuit tripped, then

$$P_{m2} = \frac{1 \times 1}{0.12 + x} = \frac{1}{0.2} = 5 \text{ pu}$$

**14. Ans: (c)**

**Sol:** Before fault

Mechanical input to alternator

$(P_s) = \text{electrical output } (P_e) = 1.0 \text{ P.u.}$

Given  $\delta = 30^\circ$ ,  $V = 1.0 \text{ P.u.}$

During fault

$$X_{eq} = \frac{1}{0.8} \text{ pu}$$

$E = 1.1 \text{ p.u.}, V = 1.0 \text{ P.u.}$

' $\delta$ ' value cannot change instantaneously.

$\therefore$  Initial accelerating power

$$(P_a) = P_s - P_e$$

$$P_a = 1.0 - \frac{1.1 \times 1.0}{\left(\frac{1}{0.8}\right)} \sin 30^\circ$$

$$P_a = 0.56 \text{ P.u.}$$

**Load Flow Studies**

**Solutions for Objective Practice Questions**

**01. Ans: (a)**

**Sol:** Given data:

$$Y_{23} = j10; y_{23} = -Y_{23} = -j10$$

$$z_{23} = \frac{1}{y_{23}} = j0.1$$

**02. Ans: (c)**

**Sol:**  $Y_{11} = y_{13} + y_{12}$

$$= (j0.2)^{-1} + (j0.5)^{-1} = -j7$$

$$Y_{22} = y_{21} + y_{23}$$

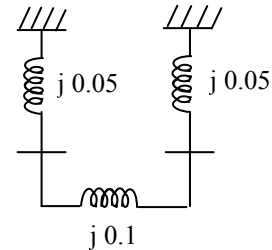
$$= (j0.5)^{-1} + (j0.25)^{-1} = -j6$$

$$Y_{33} = y_{31} + y_{32}$$

$$= (j0.2)^{-1} + (j0.25)^{-1} = -j9$$

**03. Ans: (a)**

**Sol:**



$$Y_{22} = Y_{11} = (j0.05)^{-1} + (j0.1)^{-1} = -j30$$

$$Y_{12} = Y_{21} = -(j0.1)^{-1} = j10$$

**04. Ans: (b)**

**Sol:** Given data:

We know that

$$Y_{22} = y_{21} + y_{22} + y_{23}$$

$$Y_{21} = -y_{21} \quad Y_{23} = -y_{23}$$

From the data,  $Y_{22} = -18, Y_{21} = 10,$

$$Y_{23} = 10$$

$$Y_{22} = ?$$

$$-18 = (-10) + y_{22} + (-10)$$

$$\Rightarrow y_{22} = 20 - 18$$

Shunt Susceptance,  $y_{22} = 2.$

**05. Ans:  $Y''_{13} = j0.8$**

$$\text{Sol: } Y_{\text{Bus}} = j \begin{bmatrix} -14.4 & 10 & 5 \\ 10 & -11.5 & 2.5 \\ 5 & 2.5 & -6.3 \end{bmatrix}$$

$$Y_{11} = \frac{Y'_{12}}{2} + \frac{Y'_{13}}{2} + Y_{12} + Y_{31} = -14.4$$

$$Y_{12} = -Y_{12} = j10$$

$$Y_{23} = -Y_{23} = j2.5$$



$$Y_{31} = -Y_{13} = j5$$

$$Y'_{12} + Y'_{31} = 2[-j14.4 + j10 + j5]$$

$$= j1.2 \dots\dots\dots(1)$$

Similarly

$$Y'_{12} + Y'_{23} = 2[-j11.5 + j10 + j2.5]$$

$$= j2 \dots\dots\dots(2)$$

$$Y'_{23} + Y'_{31} = 2[j(5 + 2.5 - 6.3)]$$

$$= j2.4 \dots\dots\dots(3)$$

$$Y'_{12} + Y'_{31} = j1.2 \dots\dots\dots(1)$$

Subtracting (2) and (3)

$$Y'_{12} + Y'_{23} - Y'_{23} - Y'_{31} = j2 - j2.4$$

$$\Rightarrow Y'_{12} - Y'_{31} = -j0.4 \dots\dots\dots(4)$$

Solving equation (1) & (4) we get

$$Y''_{13} = j0.8$$

**06. Ans:**  $Y_{bus} = j \begin{bmatrix} -14.76 & 10 & 5 \\ 10 & -13.72 & 4 \\ 5 & 4 & -8.64 \end{bmatrix}$

**Sol:**  $z_{12} = j0.001 \times 100 = j0.1$

$$y_{12} = -j10$$

$$z_{13} = j0.001 \times 200 = j0.2$$

$$y_{13} = -j5$$

$$y_{23} = j0.001 \times 250 = j0.25$$

$$y_{23} = -j4$$

$$y'_{12} = j0.0016 \times 100 = j0.16$$

$$y'_{13} = j0.0016 \times 200 = j0.32$$

$$y'_{23} = j0.0016 \times 250 = j0.4$$

$$Y_{11} = y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$$

$$= -j10 - j5 + j0.08 + j0.16$$

$$= -j14.76$$

$$Y_{22} = y_{12} + y_{23} + \frac{y'_{12}}{2} + \frac{y'_{23}}{2}$$

$$= -j10 - j4 + j0.08 + j0.2$$

$$= -j13.72$$

$$Y_{33} = y_{13} + y_{23} + \frac{y'_{13}}{2} + \frac{y'_{23}}{2}$$

$$= -j15 - j4 + j0.16 + j0.2$$

$$= -j18.64$$

$$Y_{12} = -y_{12} = j10, Y_{13} = -y_{13} = j5, Y_{23} = -y_{23} = j4$$

$$Y_{BUS} = j \begin{bmatrix} -14.76 & 10 & 5 \\ 10 & -13.72 & 4 \\ 5 & 4 & -8.64 \end{bmatrix}$$

**07. Ans:**  $Y_{bus} = j \begin{bmatrix} -29.76 & 20 & 10 \\ 20 & -27.72 & 8 \\ 10 & 8 & -17.64 \end{bmatrix}$

**Sol:**  $z_{12} = j0.0005 \times j0.05$

$$y_{12} = -20j$$

$$y_{13} = j0.0005 \times 200 = j0.1$$

$$y_{13} = -j10$$

$$z_{23} = j0.0005 \times 250 = j0.125$$

$$y_{23} = -j8$$

$$y'_{12} = j0.0016 \times 100 = j0.16$$

$$y'_{13} = j0.0016 \times 200 = j0.32$$

$$y'_{23} = j0.0016 \times 250 = j0.4$$

$$Y_{11} = y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$$

$$= -j20 - j10 + j0.08 + j0.16$$

$$= -j29.76$$

$$Y_{22} = y_{12} + y_{23} + \frac{y'_{12}}{2} + \frac{y'_{23}}{2}$$

$$= -j10 - j8 + j0.16 + j0.2$$

$$= -j17.64$$

$$Y_{12} = -y_{12} = j20; Y_{13} = -y_{13} = j10;$$

$$Y_{23} = -y_{23} = j8$$

$$Y_{BUS} = j \begin{bmatrix} -29.76 & 20 & 10 \\ 20 & -27.72 & 8 \\ 10 & 8 & -17.64 \end{bmatrix}$$

**08. Ans:**  $Y_{bus} = j \begin{bmatrix} -14.88 & 10 & 5 \\ 10 & -13.86 & 4 \\ 5 & 4 & -8.82 \end{bmatrix}$

**Sol:**  $z_{12} = 0.001 \times 100 = j0.1$

$$y_{12} = -j10$$

$$z_{13} = j0.001 \times 200 = j0.2$$

$$y_{13} = -j5$$

$$z_{23} = j0.001 \times 250 = j0.25$$

$$y_{23} = -j4$$

$$y'_{12} = j0.0008 \times 100 = j0.08$$

$$y'_{13} = j0.0008 \times 200 = j0.16$$

$$y'_{23} = j0.0008 \times 250 = j0.2$$

$$\begin{aligned} Y_{11} &= y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2} \\ &= -j10 - j5 + j0.04 + j0.08 \\ &= -j14.88 \end{aligned}$$

$$\begin{aligned} Y_{22} &= y_{12} + y_{23} + \frac{y'_{12}}{2} + \frac{y'_{23}}{2} \\ &= -j10 - j4 + j0.04 + j0.1 \\ &= -j13.86 \end{aligned}$$

$$\begin{aligned} Y_{33} &= y_{13} + y_{23} + \frac{y'_{13}}{2} + \frac{y'_{23}}{2} \\ &= -j5 - j4 + j0.04 + j0.1 \\ &= -j8.82 \end{aligned}$$

$$Y_{12} = -y_{12} = j10;$$

$$Y_{13} = -y_{13} = j5;$$

$$Y_{23} = -y_{23} = j4$$

$$Y_{BUS} = j \begin{bmatrix} -14.88 & 10 & 5 \\ 10 & -13.86 & 4 \\ 5 & 4 & -8.82 \end{bmatrix}$$

**09. Ans: 3500 (3500 to 3500)**

**Sol:** Given data:

$$\text{Number of Buses } (N) = 1000$$

$$\text{Number of non-zero elements} = 8000$$

$$= N + 2N_L \quad (N_L = \text{Number of transmission lines})$$

$$1000 + 2 \times N_L = 8000$$

$$N_L = 3500$$

$\therefore$  Minimum number of transmission lines and transformers = 3500

**10. Ans: 14 to 14**

**Sol:**  $G_1$  - Slack bus

$G_2$  - having reactive power

$$Q_2 \min \leq Q_2 \leq Q_2 \max$$

When it is operating at  $Q_2 \max$  means there is a reactive power divergent. Hence it is working as load bus.

$G_2 \rightarrow 2$  equations

$G_3 \rightarrow 1$  equation

$G_4 \rightarrow 1$  equation

$L_1 \rightarrow 2$  equations

$L_2 \rightarrow 2$  equations

$L_5 \rightarrow 2$  equations

$L_6 \rightarrow 2$  equations

$L_3 \rightarrow 1$  equation

$L_4 \rightarrow 1$  equation

Total No. of equations are 14

**11. Ans: (b)**

**Sol:** Total No. of buses = 100

Generator bus =  $10 - 1 = 9$

Load buses = 90

Slack bus = 1

If 2 buses are converted to PQ from PV it will add 2 unknown voltages to iteration but unknown angles remains constant.

**12. Ans: 332 to 332**

**Sol:** 183 Bus power system network,  $n = 183$

Number of PQ npq = 150

Number of PV Buses npv = 32

Remaining of PV Buses in slack bus

Number of  $|v|$ 's to be calculated = npq

Number of  $\delta$ 's to be calculated = npq + npv

Total simultaneous equations to be solved

$$= (\text{npq}) + (\text{npq} + \text{npv})$$

$$= 150 + 150 + 32 = 332$$

**13. Ans: (c)**

**Sol:** Given,

$$P = 1.4 \sin \delta + 0.15 \sin 2\delta \quad \dots\dots\dots(1)$$

$$\text{Initial guess } \delta_0 = 30^\circ = \frac{\pi}{6}$$

$$P = 0.8 \text{ pu}$$

From (1),

$$f(\delta) = P - 1.4 \sin \delta - 0.15 \sin 2\delta$$

$$f'(\delta) = -1.4 \cos \delta - 0.3 \cos 2\delta$$

$$f(\delta_0) = 0.8 - 1.4 \sin 30^\circ - 0.15 \sin (2 \times 30^\circ) \\ = -0.0299$$

$$f'(\delta_0) = -1.4 \cos 30^\circ - 0.3 \cos (2 \times 30^\circ) \\ = -1.2124 - 0.15 = -1.3624$$

According to Newton Raphson method,

$$\delta_{n+1} = \delta_n - \frac{f(\delta_n)}{f'(\delta_n)}$$

$$\delta_1 = \delta_0 - \frac{f(\delta_0)}{f'(\delta_0)}$$

$$\delta_1 = \frac{\pi}{6} - \frac{(-0.0299)}{(-1.3624)}$$

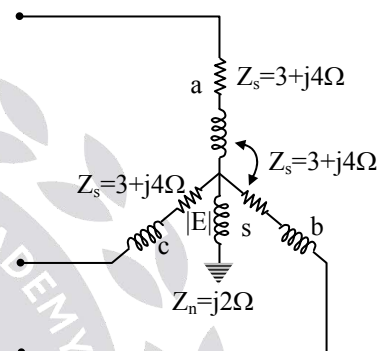
$$\delta_1 = 0.5016 \text{ rad}$$

$$\delta_1 = 28.74^\circ$$

**Solutions for Conventional Practice Questions**

01.

**Sol:**

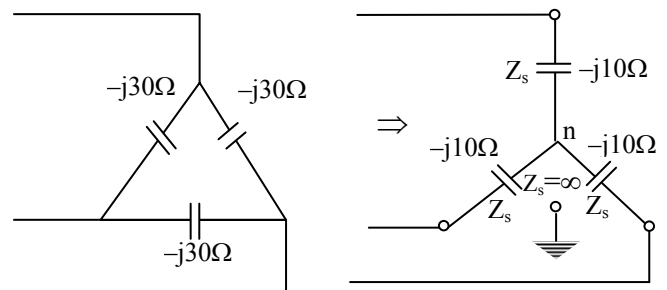


No mutual impedances exist between the phases (a, b and c) so, the sequence impedances are positive and negative sequences,

$$Z_{y1} = Z_{y2} = Z_s \\ = 3 + j4\Omega$$

$$\text{Zero sequence } Z_{y0} = Z_{y2} = Z_s \\ = 3 + j4 + (3 \times j2) \\ = 3 + j10\Omega$$

$\Delta$ -connected capacitor Bank:



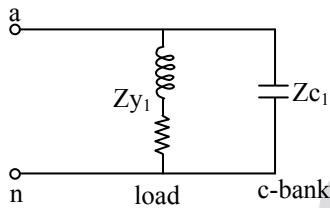
Positive sequence impedance & negative sequence impedance

$$Z_{c1} = Z_{c2} = Z_s = -j10\Omega$$

$$\text{Zero sequence } Z_{c0} = Z_s + 3 Z_n = \infty$$

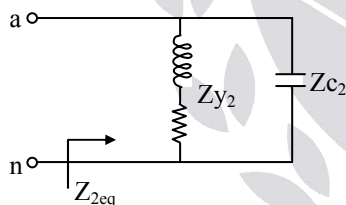
By connecting Y-load and  $\Delta$ -capacitor is in parallel

**Positive sequence equation circuit:**



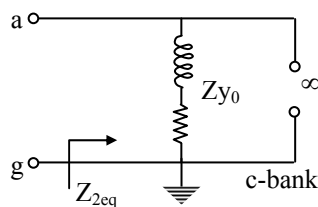
$$\begin{aligned} Z_{1eq} &= Z_{y1} // Z_{c1} \\ &= (3+j4) // (-j10) \\ &= \frac{-j10 \times (3+j4)}{3-j6} \\ &= 6.67 + j3.33\Omega \end{aligned}$$

**negative sequent equivalent circuit**



$$\begin{aligned} Z_{2eq} &= Z_{y2} // Z_{c2} \\ &= 6.67 + j3.33\Omega \end{aligned}$$

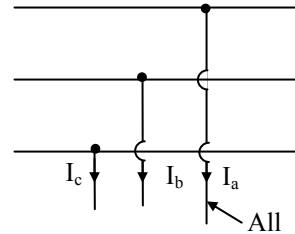
**Zero sequence circuit:**



$$Z_{0eq} = Z_{y0} = 3+j10\Omega$$

**02.**

**Sol:** With respect to 'F' point in the system. Sequence impedances (Thevenin's) are  $Z_1, Z_2, Z_0$  positive sequence voltage is 'E'



$$V_a = 0$$

$$V_b = V_c \neq 0$$

Symmetrical components for voltages:

$$\text{As } V_a = 0 \Rightarrow V_{a0} + V_{a1} + V_{a2} = 0$$

$$\text{As } V_b = V_c$$

$$\begin{aligned} V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} \\ = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2} \end{aligned}$$

Symmetrical components for currents

$$\text{As } I_b + I_c = 0$$

$$\Rightarrow I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} + I_{a2} + I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2} = 0$$

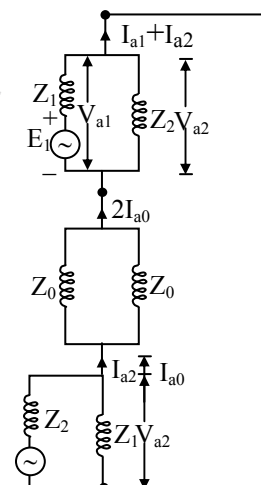
$$\text{As } \alpha + \alpha^2 = -1$$

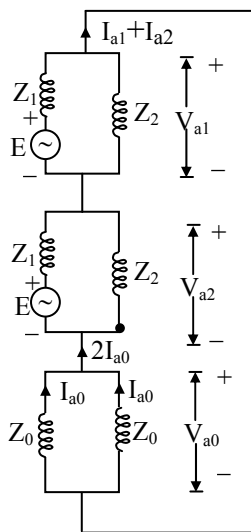
$$= 2 I_{a0} + I_{a1} (\alpha + \alpha^2) + I_{a2} (\alpha + \alpha^2) = 0$$

$$\text{as } \alpha + \alpha^2 = -1$$

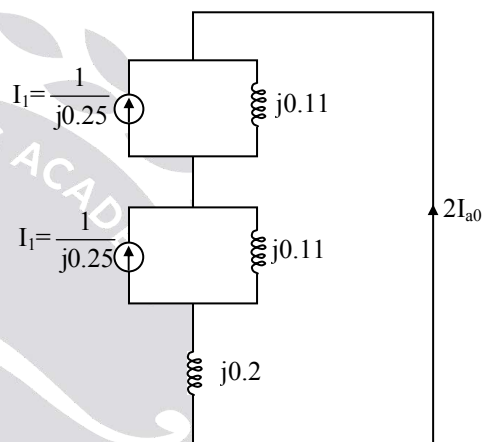
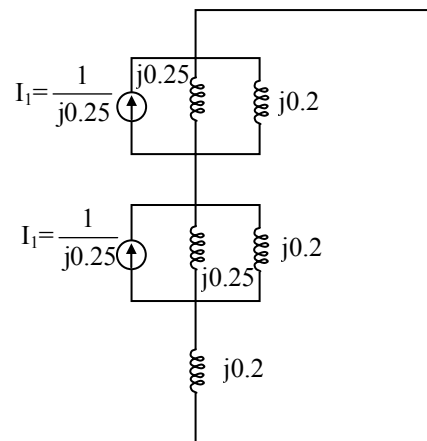
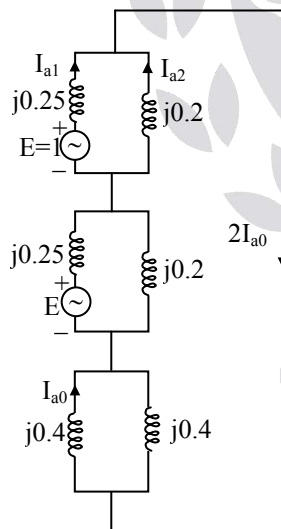
$$2 I_{a0} - I_{a1} - I_{a2} = 0$$

$$2 I_{a0} = I_{a1} + I_{a2}$$





- (a)  $Z_1 = j0.25$   
 $Z_2 = j0.2$   
 $Z_0 = j0.4$   
 $E = 1$



Now

$$2I_{a0} = \frac{0.44 + 0.44}{(j0.11 + j0.11)}$$

$$2I_{a0} = -j 2.095 \text{ p.u.}$$

$$I_{a0} = -j 1.047 \text{ p.u.}$$

$$\text{From circuit } V_{a1} = 0.44 - 2I_{a0}(j0.11)$$

$$V_{a1} = 0.44 - (-j2.095)(j0.11) = 0.2095 \text{ p.u.}$$

$$\text{As } V_{a1} = E - I_{a1}Z_1$$

$$0.2095 = 1 - I_{a1}(j0.25)$$

$$I_{a1} = -j 3.162 \text{ p.u.}$$

$$\text{As } V_{a2} = -I_{a2}Z_2$$

$$I_{a2} = -\frac{V_{a2}}{Z_2} = -\frac{V_{a1}}{Z_2}$$

$$\therefore I_{a2} = \frac{0.2095}{j0.2} = j1.047 \text{ p.u.}$$

(b) (i) Phase currents:

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$= 3I_{a0}$$

$$= 3(-j1.047)$$

$$= -j3.14 \text{ p.u.}$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$

$$I_b = (-j10.47) + (1 \angle 240^\circ)$$

$$(-j3.162) + (1 \angle 120^\circ)(j1.47)$$

$$= -3.65 \text{ p.u.}$$

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$

$$= 3.65 \text{ p.u.}$$

(ii) Ground current

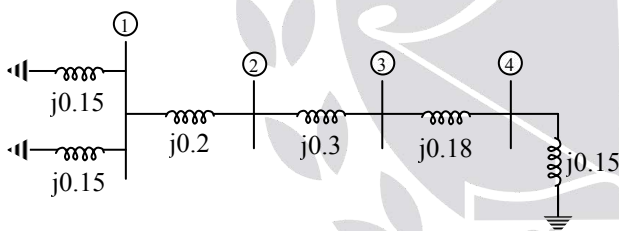
$$I_g = 3 \cdot I_{a0}$$

$$= 3(-j1.047)$$

$$= -j3.14 \text{ p.u.} \approx 3.83 \text{ pu}$$

03.

Sol: (a)(i) Positive sequence network:



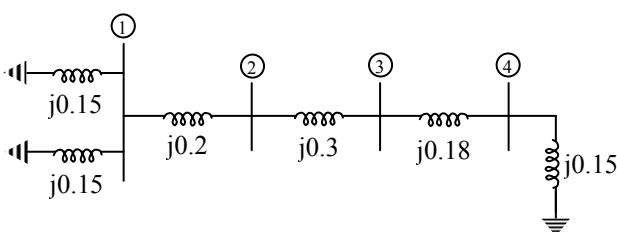
Equivalent impedance seen from fault

$$= (j0.15 \parallel j0.15) \parallel (j0.83)$$

$$= (j0.075) \parallel (j0.83)$$

$$= j0.068 \text{ pu}$$

(ii) Negative sequence network:



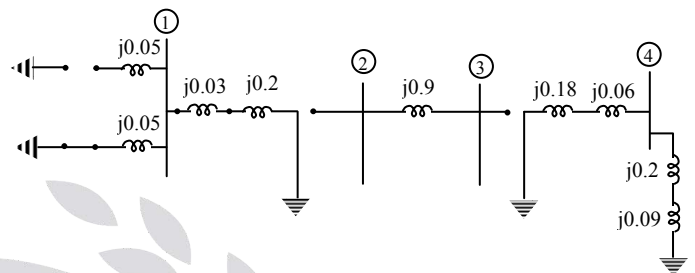
Equivalent impedance seen from fault

$$= (j0.15 \parallel j0.15) \parallel (j0.83)$$

$$= (j0.075) \parallel (j0.83)$$

$$= j0.068 \text{ pu}$$

(iii) Zero sequence network:



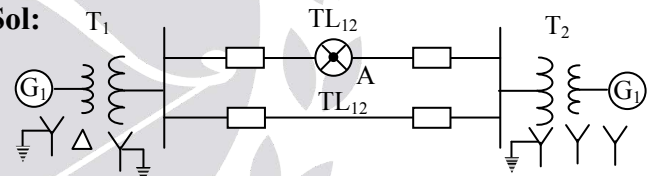
Equivalent impedance

$$= (j0.05) \parallel (j0.23)$$

$$= j0.041$$

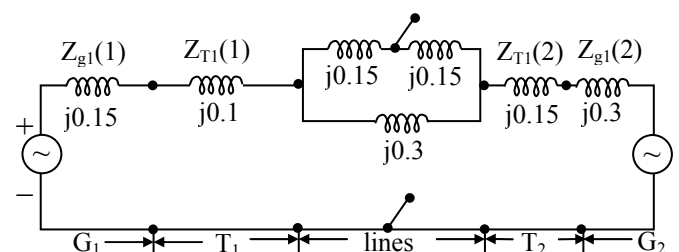
04.

Sol:

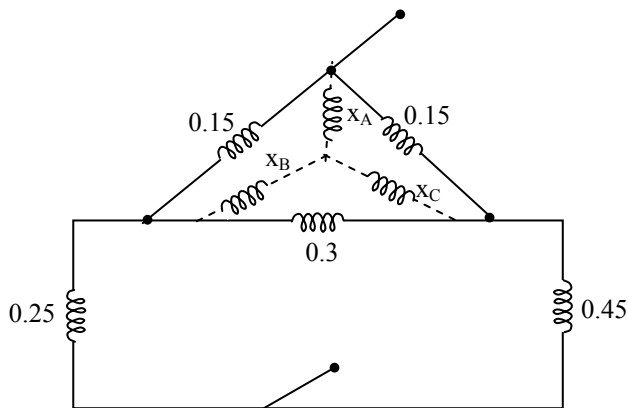


Assume that system is at No-load prior to the fault such that only positive sequence prefault voltage will exist which is assumed as 1 p.u i.e.,  $E_{a1} = 1 \text{ p.u.}$

(i) Positive sequence Network with respect to A:



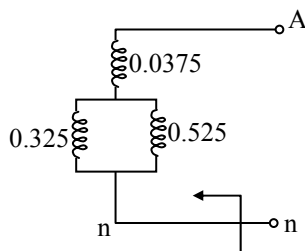
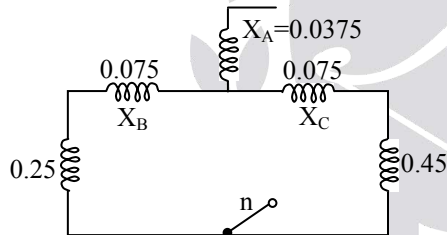
Simplified reactance diagram



$$X_A = \frac{0.15 \times 0.15}{0.15 + 0.15 + 0.3} = 0.0375$$

$$X_B = \frac{0.15 \times 0.3}{0.15 + 0.15 + 0.3} = 0.075$$

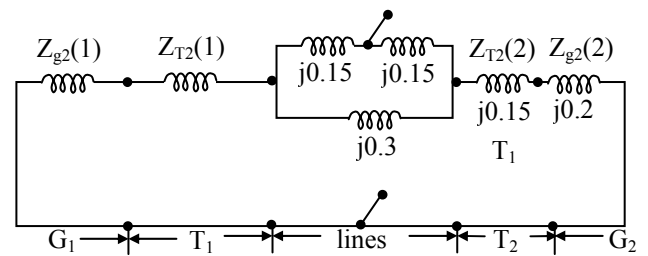
$$X_C = \frac{0.3 \times 0.15}{0.3 + 0.15 + 0.15} = 0.075$$



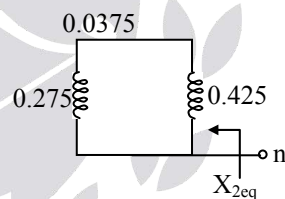
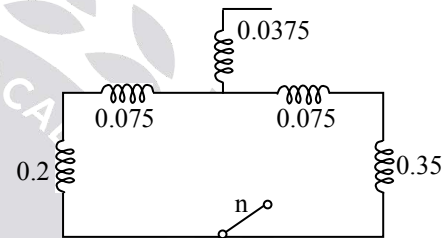
$$X_{1eq} = 0.0375 + 0.325 // 0.525$$

$$= 0.238 \text{ p.u.}$$

(ii) Negative Sequence Network:



Simplified reactance network (by following the results obtained in positive sequence network construction)

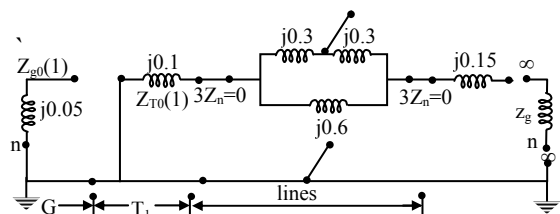


Negative sequence equivalent reactance Network:

$$X_{2eq} = 0.0375 + 0.275 // 0.425$$

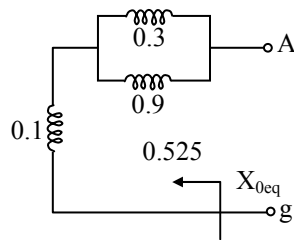
$$= 0.204 \text{ p.u.}$$

(iii) Zero sequence Network:





### Simplified reactance Diagram



$$X_{0eq} = 0.1 + 0.3 // 0.9$$

$$= 0.325 \text{ p.u.}$$

With respect to 'A' sequence thevenin's impedances are;

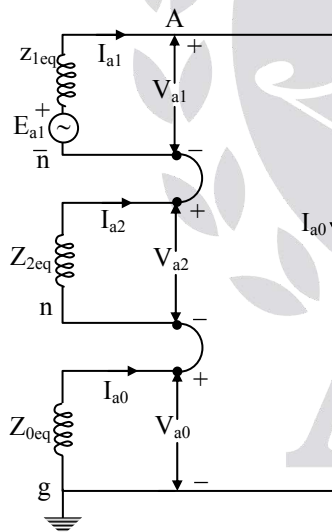
$$Z_{1eq} = jx_{1eq} = j0.238$$

$$Z_{2eq} = jx_{2eq} = j0.208$$

$$Z_{0eq} = jx_{0eq} = j0.325$$

(a) Single line to ground fault:

Three sequence circuit's are connected in series

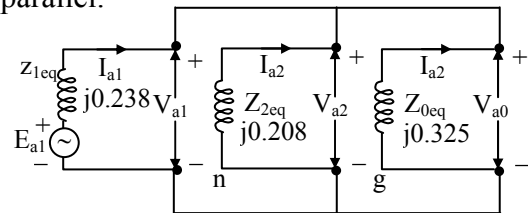


$$\text{Fault current } I_f = 3 I_{a0}$$

$$I_f = \frac{3 E_{a1}}{Z_{1eq} + Z_{2eq} + Z_{0eq}}$$

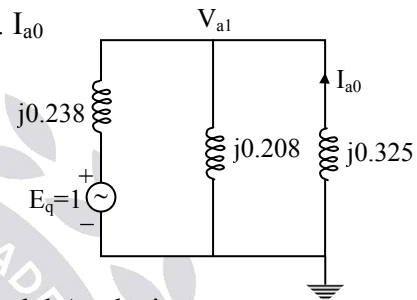
$$= \frac{3 \times 1}{j0.238 + j0.208 + j0.325} = -j3.89 \text{ pu}$$

(b) Double line to Ground fault on 'b' and 'c'  
Three sequence circuits are connected in parallel.



Ground current (or) Fault current

$$I_f = 3 I_{a0}$$



By Nodal Analysis.

$$\frac{V_{a1} - 1}{j0.238} + \frac{V_{a1}}{j0.208} + \frac{V_{a1}}{j0.325} = 0$$

$$V_{a1} \left[ \frac{1}{0.238} + \frac{1}{0.208} + \frac{1}{0.325} \right] = \frac{1}{0.238}$$

$$V_{a1} = 0.345 \text{ pu}$$

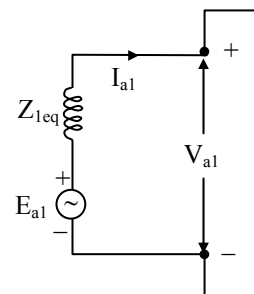
$$\text{Now, } I_{a0} = \frac{0 - V_{a1}}{j0.325} = \frac{0 - 0.345}{j0.325} = j1.05 \text{ pu}$$

$$I_f = 3 \times j1.05 = j3.15 \text{ pu}$$

(c) Three-phase fault:

$$I_f = I_a = I_{a1}$$

Sequence network will be,

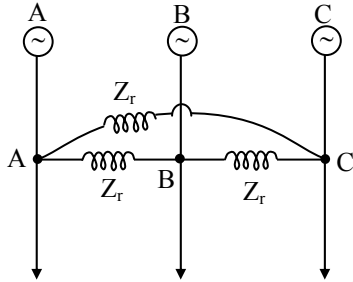


$$I_{a1} = \frac{E_{a1}}{Z_{1eq}} = \frac{1}{j0.238}$$

$$I_{a1} = -j4.2 \text{ pu}$$

05.

Sol:



400 kV feeders

Fault levels at A, B and C

$$\text{SC MVA}_A = 20 \times 10^3 \text{ MVA}$$

$$\text{SC MVA}_B = 20 \times 10^3 \text{ MVA}$$

$$\text{SC MVA}_C = 30 \times 10^3 \text{ MVA}$$

$$Z_r = j5\Omega$$

Choose the Base MVA as  $20 \times 10^3$  MVA and voltage base as 400 kV

**Station-A:**

$$X_A = \frac{\text{Base MVA}}{\text{SC MVA}_A} = \frac{20 \times 10^3}{20 \times 10^3} = 1 \text{ p.u.}$$

**Station-B:**

$$X_B = \frac{\text{Base MVA}}{\text{SC MVA}_B} = \frac{20 \times 10^3}{20 \times 10^3} = 1 \text{ p.u.}$$

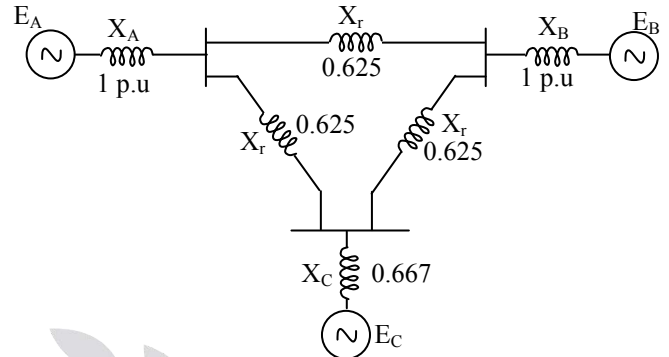
**Station-C:**

$$X_C = \frac{\text{Base MVA}}{\text{SC MVA}_C} = \frac{20 \times 10^3}{30 \times 10^3} = 0.667 \text{ p.u.}$$

$$Z_{\text{base}} = \frac{(\text{kV}_{\text{base}})^2}{\text{MVA}_{\text{base}}} = \frac{(400)^2}{20 \times 10^3} = 8 \Omega$$

$$\text{Reactor, } Z_r (\text{p.u.}) = \frac{j5}{8} = j0.625 \text{ p.u.}$$

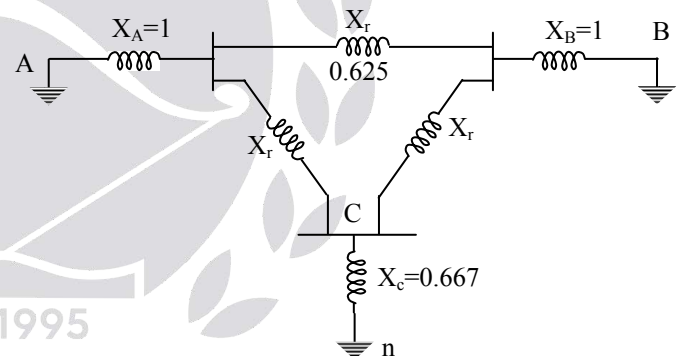
Equivalent per phase model under No-load



Short circuit capacity with respect to station-C (after connecting reactor).

$$\text{S.C capacity} = \frac{\text{Base MVA}}{X_{eqC}}$$

**To find  $X_{eqC}$ :**



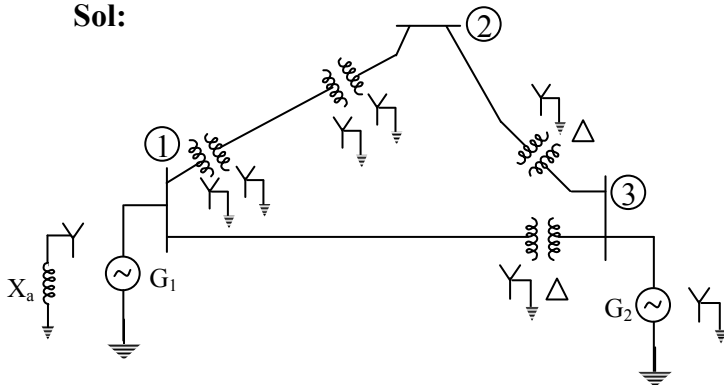
Bridge n ACB n gets balanced AB inter connection will be removed

$$X_{eqC} = 1.625 // 1.625 // 0.667 = 0.8125 // 0.667 = 0.366 \text{ p.u.}$$

$$\begin{aligned} \text{S.C capacity} &= \frac{20 \times 10^3}{0.366} \text{ MVA} \\ &= 54.64 \times 10^3 \text{ MVA} \\ &= 54.64 \text{ GVA} \end{aligned}$$

06. The given single line diagram

Sol:



$$G_1: X_1 = X_2 = 0.2, X_0 = 0.04 \text{ p.u.}, X_n = 0.02$$

$$G_2: X_1 = X_2 = 0.25, X_0 = 0.08$$

$$\text{Line (1) - (2): } Z_1 = Z_2 = j0.2$$

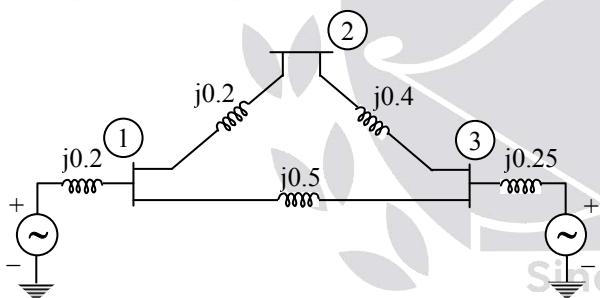
$$Z_0 = j0.4$$

$$\text{Line (2) - (3): } Z_1 = Z_2 = j0.4$$

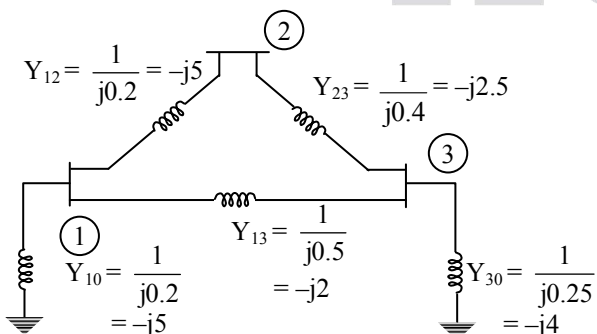
$$\text{Line (3) - (1): } Z_1 = Z_2 = j0.5$$

$$Z_0 = j1.0 \text{ pu}$$

(a) positive sequence Network:



Primitive admittance network by excluding the sources



$$Y_{BUS} \text{ size} \rightarrow 3 \times 3$$

$$[Y_{BUS}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

By direct inspection from network

$$Y_{BUS} = \begin{bmatrix} Y_{10} + Y_{12} + Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{12} + Y_{23} & -Y_{23} \\ -Y_{13} & -Y_{23} & Y_{30} + Y_{13} + Y_{23} \end{bmatrix}$$

$[Y_{BUS1}]$ , positive sequence matrix =

$$\begin{bmatrix} -j12 & j5 & j2 \\ j5 & -j7.5 & j2.5 \\ j2 & j2.5 & -j8.5 \end{bmatrix}$$

Positive sequence  $Z_{BUS}$  matrix

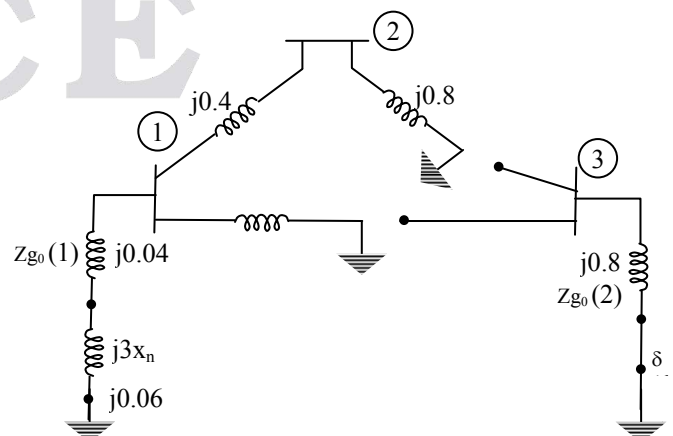
$$[Z_{BUS1}] = [Y_{BUS1}]^{-1} = \begin{bmatrix} j0.144 & j0.119 & j0.069 \\ j0.119 & j0.246 & j0.1 \\ j0.069 & j0.1 & j0.163 \end{bmatrix}$$

**Negative sequence:**

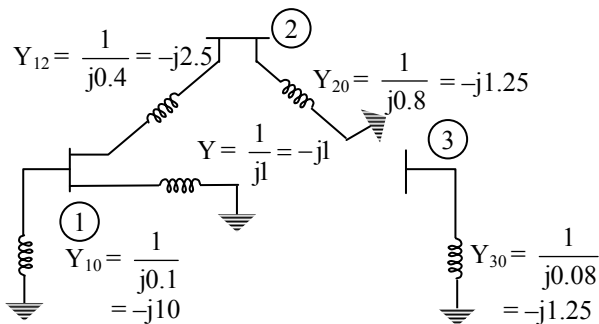
As all apparatus has equal positive and negative sequence impedance negative sequence  $Z_{BUS}$  matrix

$$[Z_{BUS2}] = [Z_{BUS1}]$$

**Zero sequence Network:**



Primitive Admittance Network,



$$[Y_{BUS}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{10} + Y + Y_{12} & -Y_{12} & 0 \\ -Y_{12} & Y_{12} + Y_{20} & 0 \\ 0 & 0 & Y_{30} \end{bmatrix}$$

$$[Y_{BUS}] = \begin{bmatrix} -j13.5 & j2.5 & 0 \\ j2.5 & -j3.75 & 0 \\ 0 & 0 & -j12.5 \end{bmatrix}$$

Zero sequence  $Z_{BUS}$  matrix

$$[Z_{BUS}] = [Y_{BUS}]^{-1}$$

$$= \begin{bmatrix} j0.084 & j0.0563 & 0 \\ j0.0563 & j0.304 & 0 \\ 0 & 0 & j0.08 \end{bmatrix}$$

$[Z_{BUS1}] = [Z_{BUS2}]$

$$= \begin{bmatrix} j0.144 & j0.119 & j0.069 \\ j0.119 & j0.246 & j0.1 \\ j0.069 & j0.1 & 0.163 \end{bmatrix}$$

$$[Z_{BUS0}] = \begin{bmatrix} j0.084 & j0.0563 & 0 \\ j0.0563 & j0.304 & 0 \\ 0 & 0 & j0.08 \end{bmatrix}$$

negative sequence current flow from (1)-(2)

$$I_{a_2(1-2)} = \frac{V_{a_2}(1) - V_{a_2}(2)}{Z_2}$$

$$= \frac{0.139 \angle -159.35 - 0.288 \angle -159.35}{j0.2}$$

$$= 0.745 \angle -69.4^\circ$$

Zero sequence current flow from (1)-(2)

$$I_{a_0(1-2)} = \frac{V_{a_0}(1) - V_{a_0}(2)}{Z_0}$$

$$= \frac{0.066 \angle -159.35^\circ - 0.335 \angle -159.35^\circ}{j0.4}$$

$$= 0.725 \angle -69.4^\circ$$

Resultant current from (1) to (2) in phase-a

$$I_{a(1-2)} = 0.745 \angle -69.4^\circ + 0.745 \angle 69.4^\circ + 0.725 \angle -69.4^\circ$$

$$= 2.12 \angle -69.4 \text{ p.u}$$

07.

**Sol:** Bus impedance matrix for a 4-bus system is given

$$Z_{bus} = \begin{bmatrix} j0.15 & j0.08 & j0.04 & j0.07 \\ j0.08 & j0.15 & j0.06 & j0.09 \\ j0.04 & j0.06 & j0.13 & j0.05 \\ j0.07 & j0.09 & j0.05 & j0.12 \end{bmatrix}_{4 \times 4}$$

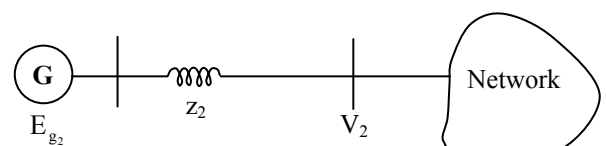
3- $\phi$  short circuit occurs at Bus-4 fault current,

$$I_{f4} = \frac{V_{pf4}}{Z_{44}}$$

$$I_{f4} = \frac{1}{j0.12} = -j8.33 \text{ p.u}$$

Generator reactance at Bus-2 is

$$Z_2 = j0.2 \text{ p.u}$$



As it is given prefault voltages are 1 p.u

$$\therefore E_{g_2} = 1 \text{ p.u}$$

If 'V<sub>2</sub>' is the post fault voltage at Bus-2 for fault at bus-4, then current supplied by generator (2)

$$I_{g_2} = \frac{E_{g_2} - V_2}{Z_2}$$

Post fault voltage,  $V_2 = V_{pf2} + Z_{24} (-I_{f4})$

$$V_2 = 1 + (j0.09)(j8.33) \\ = 0.25 \text{ p.u}$$

$$\therefore I_{g_2} = \frac{1 - 0.25}{j0.2} = -j3.75 \text{ p.u}$$

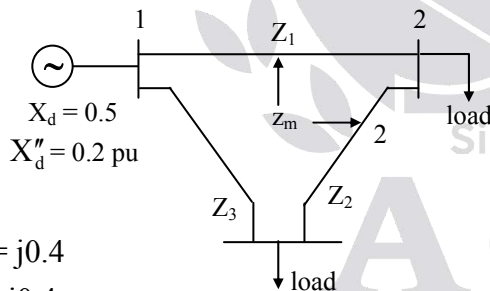
Given,  $S_{\text{base}} (3-\phi) = 150 \text{ MVA}$

$V_{\text{base(LL)}} = 230 \text{ kV}$

$$I_{\text{base}} = \frac{150}{\sqrt{3} \times 230} \text{ kA} \\ = 0.376 \text{ kA}$$

$$I_{fg2} (\text{kA}) = -j3.75 \times 0.376 \text{ kA} \\ = -j 1.41 \text{ kA}$$

**08.**  
**Sol:**



$$Z_1 = j0.4$$

$$Z_2 = j0.4$$

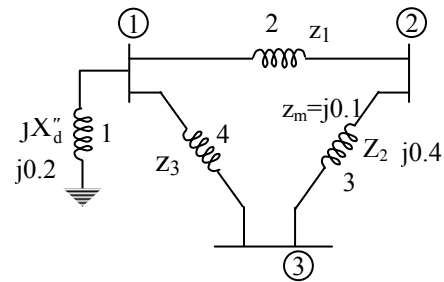
$$Z_3 = j0.3$$

$$Z_m = j0.1$$

(i) Primitive admittance matrix calculation,

$$[Y_{\text{prim}}] = [Z_{\text{prim}}]^{-1}$$

Network of primitive impedances given by excluding current injections (source & loads)



$$Z_{\text{prim}} \rightarrow 4 \times 4$$

$$Z_{\text{prim}} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & Z_{s1} & Z_{m12} & Z_{m13} & Z_{m14} \\ 2 & Z_{m12} & Z_{s2} & Z_{m23} & Z_{m24} \\ 3 & Z_{m13} & Z_{m23} & Z_{s3} & Z_{m34} \\ 4 & Z_{m14} & Z_{m24} & Z_{m34} & Z_{s4} \end{bmatrix}$$

$$[Z_{\text{prim}}] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & j0.2 & 0 & 0 & 0 \\ 2 & 0 & j0.4 & j0.1 & 0 \\ 3 & 0 & j0.1 & j0.4 & 0 \\ 4 & 0 & 0 & 0 & j0.3 \end{bmatrix}$$

Now,  $[Y_{\text{prim}}]$

$$= \begin{bmatrix} j0.2 & 0 & 0 & 0 \\ 0 & j0.4 & j0.1 & 0 \\ 0 & j0.1 & j0.4 & 0 \\ 0 & 0 & 0 & j0.3 \end{bmatrix}$$

$$[Y_{\text{prim}}] = \begin{bmatrix} -j5 & 0 & 0 & 0 \\ 0 & -j2.67 & j0.66 & 0 \\ 0 & j0.667 & -j2.69 & 0 \\ 0 & 0 & 0 & -j3.33 \end{bmatrix}$$

(ii) Bus Admittance matrix.

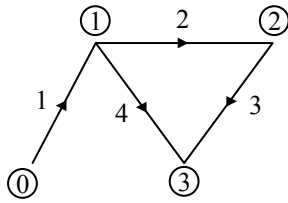
By singular Transformation method

$$[Y_{\text{BUS}}] = [A]^T \cdot [Y_{\text{prim}}] \cdot [A]$$

Reduced Bus incidence matrix,

$A \rightarrow$  number of branches  $\times$  number of Buses  
(4  $\times$  3)

Network graph will be,



$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} -1 & 0 & 0 \\ +1 & -1 & 0 \\ 0 & +1 & -1 \\ +1 & 0 & -1 \end{bmatrix} \end{matrix}$$

Now,

$$A^T, Y_{\text{prim}} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -j5 & 0 & 0 & 0 \\ 0 & -j2.67 & j0.67 & 0 \\ 0 & j0.67 & -j2.67 & 0 \\ 0 & 0 & 0 & -j3.33 \end{bmatrix}$$

$$= \begin{bmatrix} j5 & -j2.67 & j0.67 & -j3.33 \\ 0 & j3.34 & -j3.34 & 0 \\ 0 & -j0.67 & j2.67 & j3.33 \end{bmatrix}$$

Now,  $Y_{\text{BUS}} = A^T \cdot Y_{\text{prim}} \cdot A$

$$= \begin{bmatrix} j5 & -j2.69 & j0.67 & -j3.33 \\ 0 & j3.34 & -j3.34 & 0 \\ 0 & -j0.67 & j2.67 & j3.33 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -j11 & j3.34 & j2.67 \\ j3.34 & -j6.68 & j3.34 \\ j2.67 & j3.34 & -j6 \end{bmatrix}$$

(iii) Bus Impedance matrix,

$$[Z_{\text{BUS}}] = [Y_{\text{BUS}}]^{-1}$$

$$= \begin{bmatrix} -j11 & j3.34 & j2.67 \\ j3.34 & -j6.68 & j3.34 \\ j2.67 & j3.34 & -j6 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} j0.2 & j0.201 & j0.201 \\ j0.201 & j0.408 & j0.316 \\ j0.201 & j0.316 & j0.432 \end{bmatrix}$$

(iv) System is at no load condition with prefault voltage as  $1+j0$  p.u. 3- $\phi$  short circuit occurs at Bus-2

Fault current,

$$I_{f2} = \frac{V_{\text{pf}2}}{Z_{22}} = \frac{1}{j0.408}$$

$$= -j2.45 \text{ pu}$$

Post fault voltages at Buses (1) & (3)

$$\text{Bus (1)} \Rightarrow V_1 = V_{\text{pf}1} + Z_{12} (-I_{f2})$$

$$= 1 + (j0.2) (j2.45)$$

$$= 0.51 \text{ pu}$$

$$\text{Bus (3)} \Rightarrow V_3 = V_{\text{pf}3} + Z_{32} (-I_{f2})$$

$$V_3 = 1 + (j0.316) (j2.45)$$

$$= 0.225 \text{ p.u.}$$

09.

**Sol:** For a four Bus power system network

$$Y_{\text{BUS}} = \begin{matrix} & \begin{matrix} (1) & (2) & (3) & (4) \end{matrix} \\ \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix} & \begin{bmatrix} 2-j10 & -2+j4 & 0 & j5 \\ -2+j4 & 2-j3 & 0 & j2 \\ 0 & 0 & 2-j8.5 & -2+j6 \\ j5 & j2 & -2+j6 & 2-j11 \end{bmatrix} \end{matrix} \text{ p.u.}$$

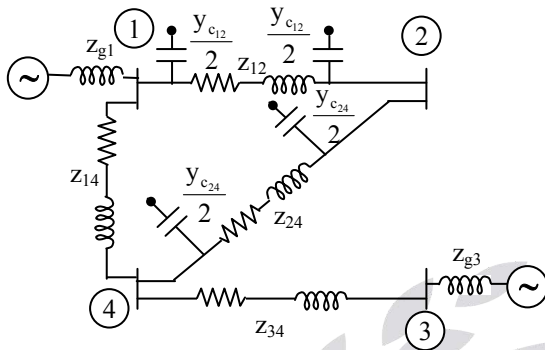
Bus (1) & (3) consists Generator's

Line 1-2 and line 2-4 are modelled in nominal- $\pi$

Line 1-4 and line 3-4 are modelled in short line modelled

No external static elements exit's

Expected network will be



Given that line charging susceptance of line 1-2 is 2p.u i.e.,  $\frac{Y_{c12}}{2} = j2$

$$\frac{Y_{c12}}{2} = j1$$

From  $Y_{BUS}$  construction by direct inspection method.

$$Y_{ik} = -y_{ik}$$

Where  $Y_{ik}$  is Y-bus element (transfer Admittance)  $Y_{ik}$  is admittance of link between Buses (i) and (k)

Now, from  $Y_{BUS}$  elements,

$$Y_{12} = -2 + j4$$

$$Y_{12} = 2 - j4$$

$$z_{12} = \frac{1}{Y_{12}} = \frac{1}{2 - j4} = 0.1 + j0.2$$

$Y_{13} = 0 \Rightarrow$  no connection between Buses (1) & (3)

$$Y_{14} = j5$$

$$Y_{14} = -j5$$

$$z_{14} = \frac{1}{Y_{14}} = j0.2$$

$$Y_{24} = j2$$

$$Y_{24} = -j2$$

$$z_{24} = \frac{1}{-j2} = j0.5$$

$$Y_{34} = -2 + j6, \quad Y_{34} = 2 - j6$$

$$z_{34} = \frac{1}{2 - j6} = 0.05 + j0.15 \text{ p.u}$$

From diagonal elements

$$Y_{11} = \frac{1}{Z_{g1}} + \frac{Y_{c12}}{2} + \frac{1}{Z_{12}} + \frac{1}{Z_{14}}$$

$$2 - j10 = \frac{1}{Z_{g1}} + j1 + \frac{1}{0.1 + j0.2} + \frac{1}{j0.2}$$

$$z_{g1} = j0.5$$

$$Y_{33} = \frac{1}{Z_{34}} + \frac{1}{Z_{g3}}$$

$$\Rightarrow 2 - j8.5 = \frac{1}{0.05 + j0.15} + \frac{1}{z_{g3}}$$

$$z_{g3} = j0.4 \text{ p.u}$$

$$\text{Poles, } S = -\xi \omega_n \pm j\omega_n \sqrt{1 - \xi^2}$$

In small distribution stability steady the poles are

$$s = -\frac{D}{2m} \pm j\sqrt{\frac{k}{m} - \left(\frac{D}{2m}\right)^2}$$

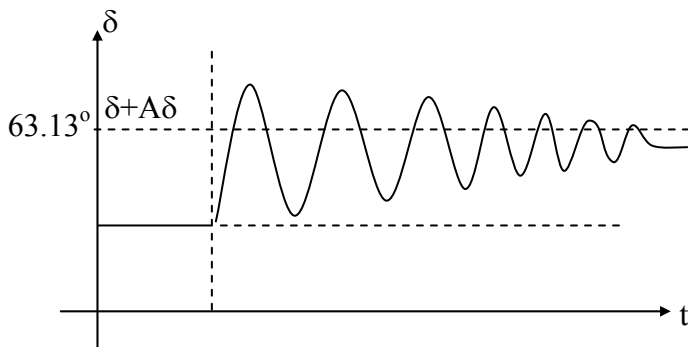
$$\therefore \xi \omega_n = \frac{D}{2m}$$

$$\xi = \frac{D}{2m \omega_n} = \frac{0.1}{2 \times \frac{1}{10\pi} \times 0.68} = 0.362$$

Expected swing curve for  $\Delta \delta = 10^\circ$   
(assumed as increment)



(c)



Expression for swing curve  $\delta(t) = \delta_0 + \Delta\delta(t)$

Second order system time response for unit step is

$$c(t) = \frac{1 - e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi)$$

For the given disturbance of  $\Delta\delta = 10^\circ$ , the response with respect to time will be

$$\Delta\delta(t) = 10^\circ \left[ 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi) \right]$$

Where  $\xi\omega_n = 0.362 \times 0.69 \times 2\pi = 1.57$

$$\sqrt{1 - \xi^2} = \sqrt{1 - (0.362)^2} = 0.87$$

$$\omega_d = 2\pi \times 0.64$$

$$= 4.02$$

$$\phi = \cos^{-1}(0.362) = 68.7^\circ$$

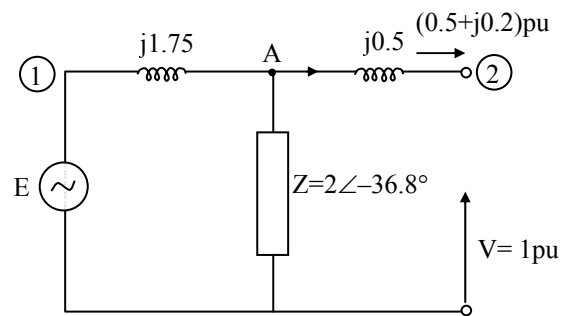
Finally  $\delta(t) = \delta_0 + 10^\circ$

$$\left[ 1 - \frac{e^{-1.57t}}{0.87} \sin(4.02t + 68.7) \right]$$

$$= \left[ 63.13^\circ - 10^\circ \times \frac{e^{-1.57t}}{0.87} \sin(4.02t + 68.7^\circ) \right]$$

10.

Sol:



Using equation and working from the infinite busbar voltage the voltage point A is given by

$$\begin{aligned} V_A &= \sqrt{\left( V + \frac{QX}{V} \right)^2 + \left( \frac{PX}{V} \right)^2} \\ &= \sqrt{\left( 1 + \frac{0.2 \times 0.5}{1} \right)^2 + \left( \frac{0.5 \times 0.5}{1} \right)^2} \\ &= 1.105 \text{ pu} \end{aligned}$$

At angle of  $5.19^\circ$  to the infinite busbar the reactive power absorbed by the line from point A to point 2 (the infinite busbar)

$$\begin{aligned} I_R^2 X &= \left( \frac{P^2 + Q^2}{V^2} \right) X \\ &= \left( \frac{0.5^2 + 0.2^2}{1^2} \right) \times 0.5 \\ &= 0.145 \text{ pu} \end{aligned}$$

The actual load taken by A (if represented by an impedance) is give by

$$\begin{aligned} \frac{V_A^2}{Z} &= \frac{1.105^2}{2 \angle -36.8^\circ} \\ &= 0.49 + j0.37 \text{ pu} \end{aligned}$$

The total load supplied by link from generator to A

$$\begin{aligned} &= (0.5 + 0.49) + j(0.2 + 0.145 + 0.37) \\ &= 0.99 + j0.715 \text{ pu} \end{aligned}$$

Internal voltage of generator E,

$$= \sqrt{\left[ \left( 1.105 + \frac{0.715 \times 1.75}{1.105} \right)^2 + \left( \frac{0.99 \times 1.75}{1.105} \right)^2 \right]}$$

$$= \sqrt{5.006 + 2.458}$$

$$= 2.73 \angle 35.02^\circ$$

Hence, the angle between E and V is

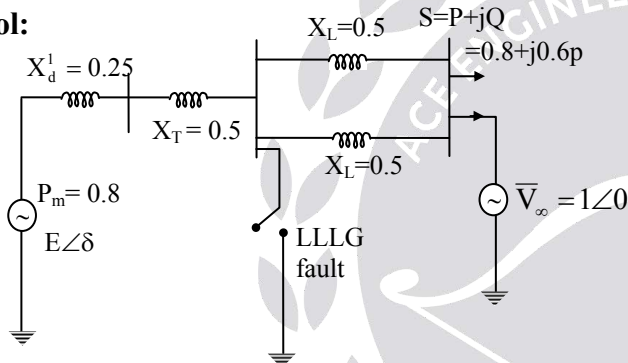
$$= 35.02^\circ + 5.19^\circ$$

$$= 40.21^\circ$$

Since this angle is much less than  $90^\circ$ , the system is stable.

11.

Sol:



(a) To find transient internal emf of machine:

$$\text{as } s = \bar{V}_\infty \cdot \bar{I}^*$$

$$\bar{I}^* = \frac{S}{\bar{V}_\infty} = 0.8 + j0.6$$

$$\bar{I} = 0.8 - j0.6 \text{ p.u.}$$

Equivalent reactance between two sources

$$X_{eq} = X_d' + X_T + \frac{X_L}{2}$$

$$X_{eq} = 0.25 + 0.5 + 0.25$$

$$= 1 \text{ p.u.}$$

$$\text{Now, } \bar{E} = \bar{V}_\infty + \bar{I} \cdot (jX_{eq})$$

$$\bar{E} = 1 \angle 0^\circ + (0.8 - j0.6) \cdot (j1)$$

$$= 1.6 + j0.8$$

$$= 1.788 \angle 26.56^\circ$$

$$|E| = 1.788 \text{ p.u.}, \delta_0 = 26.56^\circ$$

Prefault condition,

Maximum power Transfer capability,

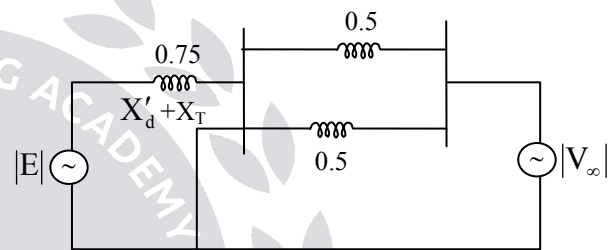
$$P_{\max 1} = \frac{|E||V_\infty|}{X_{1eq}}$$

Where  $X_{1eq} = 1 \text{ p.u.}$

$$\therefore P_{\max 1} = \frac{1.788 \times 1}{1}$$

$$= 1.788 \text{ p.u.}$$

During fault:



In this case,  $X_{2eq} = \infty$

$$P_{\max 2} = \frac{|E||V_\infty|}{X_{2eq}} = 0$$

After clearing the fault: network get restored

$$P_{\max 3} = P_{\max 1} = 1.788 \text{ p.u.}$$

$$\delta_{\max} = 180^\circ - \sin^{-1} \left( \frac{P_s}{P_{\max 3}} \right)$$

$$= 180^\circ - \sin^{-1} \left( \frac{0.8}{1.788} \right) = 153.44^\circ$$

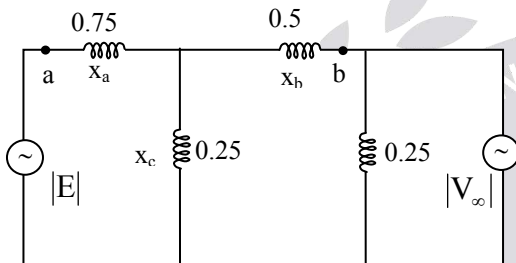
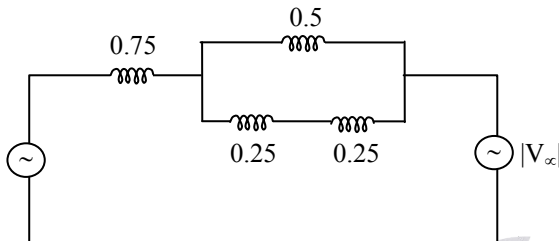
Critical clearing angle calculation

$$\cos \delta_{cr} = \left[ \frac{P_s (\delta_{\max} - \delta_0) + P_{\max 3} \cos \delta_{\max} - P_{\max 2} \cos \delta_0}{P_{\max 3} - P_{\max 2}} \right]$$

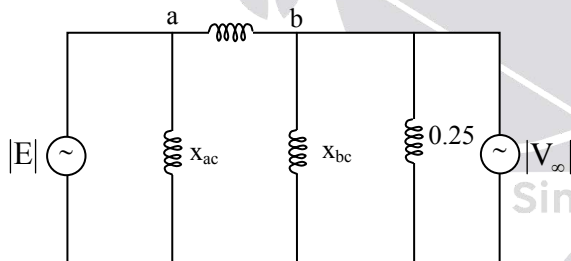
$$\cos \delta_{cr} = \left[ \frac{0.8 [153.44 - 26.56] \times \frac{\pi}{180} + 1.788 \times \cos(153.44)}{1.788 - 0} \right]$$

$$\delta_{cr} = 84.47^\circ$$

- (b) For the fault at the mid point of  
 $|E| = 1.788 \text{ p.u.}; \delta_0 = 26.56$   
 Before fault:  $P_{\max 1} = 1.788$   
 During fault: Fault at mid point on second line.



By converting star-abc to delta



Parameter 'B' for above network

$$B = j X_{ab}$$

$$= j \left[ X_a + X_b + \frac{X_a \cdot X_b}{X_c} \right]$$

$$B = j \left[ 0.75 + 0.5 + \frac{0.75 \times 0.5}{0.25} \right]$$

$$= j2.75 \text{ pu}$$

Transformer reactance during fault,

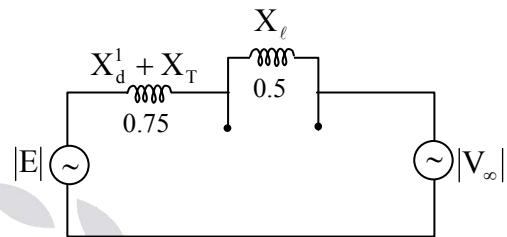
$$X_{2eq} = |B| = 2.75 \text{ p.u.}$$

$$P_{\max 2} = \frac{|E||V_{\infty}|}{X_{2eq}} = \frac{1.788 \times 1}{2.75}$$

$$= 0.65 \text{ p.u.}$$

After clearing the fault:

Faulted line was disconnected



$$\text{Transfer reactance, } X_{3eq} = 0.756 + 0.5 \\ = 1.25 \text{ p.u.}$$

$$P_{\max 3} = \frac{|E||V_{\infty}|}{X_{3eq}}$$

$$= \frac{1.788 \times 1}{1.25}$$

$$= 1.43 \text{ p.u.}$$

$$\delta_0 = 23.56$$

$$\delta_{\max} = 180 - \sin^{-1} \left( \frac{P_s}{P_{\max 3}} \right)$$

$$\delta_{\max} = 180^\circ - \sin^{-1} \left( \frac{0.8}{1.43} \right)$$

$$= 145.98^\circ$$

Critical clearing angle calculation,

$$\cos \delta_{cr} =$$

$$\left[ \frac{0.8[145.98 - 23.56] \times \frac{\pi}{180} + 1.43 \cos(145.98) - 0.65 \cos(23.5)}{1.43 - 0.65} \right]$$

$$\delta_{cr} = 97.33^\circ$$

12.

**Sol: (a)** The maximum power that can be transferred by the generator to the infinite bus and the input mechanical power to the generator are given as

$$P_{\max} = 1 \text{ pu}; P_m = 0.8 \text{ pu}$$

The initial internal angle,  $\delta_0$  can be computed as

$$\delta_0 = \sin^{-1} \left( \frac{P_m}{P_{\max}} \right)$$

$$= \sin^{-1}(0.8) = 53.13^\circ$$

The synchronous torque (or) power is given as

$$P_s = P_{\max} \cos(\delta_0) = 0.6$$

The linearized swing equation, as given can be written as

$$H = 5s, f = 50 \text{ Hz}$$

$$\frac{H}{\pi f_s} \frac{d^2 \Delta \delta}{dt^2} + P_s \Delta \delta = 0$$

$$0.0318 \frac{d^2 \Delta \delta}{dt^2} + 0.6 \Delta \delta = 0 \dots\dots\dots(1)$$

The solution of equation (1) in Laplace domain gives

$$S = \sqrt{-\frac{\pi f}{H} P_s} = \pm j4.3146$$

Since, complex pair of poles (or) roots are on the imaginary axis the system will have sustained oscillation. The natural frequency of oscillation is given as

$$\omega_n = 4.3146 \text{ rad/sec (or) } 0.691 \text{ Hz}$$

**(b)** If the damping coefficient is considered as 0.1 then the swing equation given equation (1) changes to

$$\frac{d^2 \Delta \delta}{dt^2} + 3.14 \frac{d \Delta \delta}{dt} + 18.85 \Delta \delta = 0 \dots\dots\dots(2)$$

Equation (2) can be written in Laplace domain as

$$(s^2 + 3.14s + 18.85) \Delta \delta(s) = 0 \dots\dots\dots(3)$$

Compare equation (3) with the standard second order characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

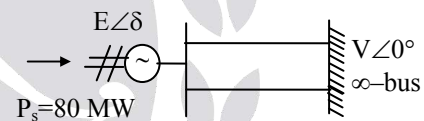
$$\omega_n = 4.3146 \text{ rad/s (or) } 0.6913 \text{ Hz};$$

$$\xi = 0.3616$$

13.

**Sol: (i)** Double circuit line with each circuit maximum power transformer as 100 MW

So, maximum power transformer capacity of system is 200 MW



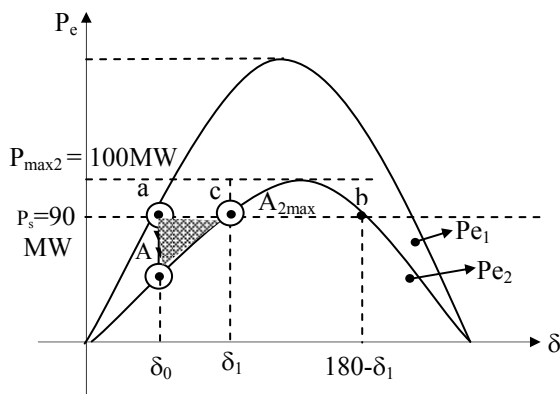
Initial power flow (or) steam input  $P_s = 80$  MW and  $P_{\max 1} = 200$  MW

$$\begin{aligned} \text{Initial rotor angle, } \delta_0 &= \sin^{-1} \left( \frac{P_s}{P_{\max 1}} \right) \\ &= \sin^{-1} \left( \frac{80}{200} \right) \\ &= 23.58^\circ \end{aligned}$$

If one of the circuits was disconnected then new  $P_{\max}$  get reduced to 100MW

Stability of this system can be estimates with the help of equal area criterion.

Power angle curves,



$$P_{e1} = P_{\max 1} \cdot \sin \delta$$

$$P_{e2} = P_{\max 2} \cdot \sin \delta$$

From graph shown,

$A_1 \rightarrow$  Acceleration area

$A_{2\max} \rightarrow$  complete deceleration area available.

$$\text{Angle, } \delta_1 = \sin^{-1} \left( \frac{P_s}{P_{\max 2}} \right)$$

$$= \sin^{-1} \left( \frac{80}{100} \right)$$

$$= 53.13^\circ$$

$$A_1 = \int_{\delta_0}^{\delta_1} P_a \cdot d\delta$$

$$= \int_{\delta_0}^{\delta_1} (P_s - P_{\max 2} \cdot \sin \delta) d\delta$$

$$A_1 = P_s(\delta_1 - \delta_0) + P_{\max 2} [\cos \delta_1 - \cos \delta_0]$$

$$= 80 (53.13 - 23.58^\circ) \frac{\pi}{180^\circ} + 100$$

$$[(\cos 53.13^\circ - \cos 23.58^\circ)]$$

$$= 9.61 \text{ MW -elec. rad}$$

Now

$$A_{2\max} = \int_{\delta_1}^{180-\delta_1} P_a \cdot d\delta = \int_{\delta_1}^{180-\delta_1} (P_s - P_{\max 2} \cdot \sin \delta) d\delta$$

$$A_{2\max} = P_s [180 - 2\delta_1] + P_{\max 2}$$

$$[\cos(180 - \delta_1) - \cos \delta_1]$$

$$= 80 [180 - 2 \times 53.13^\circ] \times \frac{\pi}{180^\circ} + 100$$

$$[\cos(180 - 53.13^\circ) - \cos(53.13^\circ)]$$

$$= -17.04 \text{ MW-elec. rad.}$$

As  $|A_{2\max}| > |A_1|$

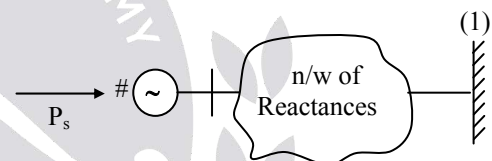
System is stable.

$\Rightarrow$  Rotor angle will not reach to the point 'x' shown in figure. Before reaching to 'x' at some point  $|P_{12}| = |A_1|$  happens and from that point onwards rotor will fall back, the system is conformal stable.

(b) Final rotor stable angle is  $\delta_1 = 53.13^\circ$

14.

Sol:



Initial power flow,  $P_{e0} = 0.5 P_{\max 1}$

(or)  $P_s = 0.5 P_{\max 1}$

During short circuit fault,  $X_{2eq} = 4 X_{1eq}$

$P_{\max}$  during fault

$$P_{\max 2} = \frac{E \cdot V}{X_{2eq}}$$

$$= \frac{E \cdot V}{4 X_{1eq}}$$

$$= \frac{1}{4} P_{\max 1}$$

$$= 0.25 P_{\max 1}$$

After clearing the fault  $P_{\max 3} = 0.75 P_{\max 1}$

inertia constant,  $H = 6.75 \text{ MJ/MVA}$

time interval gap,  $\Delta t = 0.053$

stability steady period,  $T = 0.4S$

number of interval,  $n = \frac{T}{\Delta t} = 8$

fault clearing time is 7.5 cycles

$t_c = 7.5 \times 20 \text{ ms}$

$= 0.15 \text{ s}$

Given that  $P_{e0}$  or  $P_s = 1$

$0.5 P_{\max 1} = 1$

$P_{\max 1} = 2 \text{ p.u}$

and  $P_{\max 2} = 0.25 P_{\max 1}$

$= 0.25 \times 2$

$= 0.5$

$P_{\max 3} = 0.75 P_{\max 1}$

$= 0.75 \times 2$

$= 1.5$

Initial angle,  $\delta_0 = \sin^{-1}\left(\frac{P_s}{P_{\max 1}}\right)$

$\delta_0 = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

In step by step method,

$\delta_n = \delta_{n-1} + \Delta\delta_n$

$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} \cdot P_a(n-1)$

Where angular momentum,

$M = \frac{H}{180f} \text{ s}^2 / \text{ele.rad}$

Now,  $\frac{(\Delta t)^2}{M} = \frac{(0.05)^2}{\left(\frac{6.75}{180 \times 50}\right)} = 3.33 \text{ ele.rad}$

$\therefore \Delta\delta_n = \Delta\delta_{n-1} + 3.33 P_a(n-1)$

$n=1$ :

$\delta_1 = \delta_0 + \Delta\delta_1$

Where  $\delta_0 = 30^\circ$

now,  $\Delta\delta_1 = \Delta\delta_0 + 3.33 P_a(0)$

where  $\Delta\delta_0 = 0$

as at  $t = 0$  there is a discontinuity in  $P_a$  as

$P_a(0^-) = P_s - P_{\max 1} \cdot \sin \delta_0 = 0$

$P_a(0^+) = P_s - P_{\max 2} \cdot \sin \delta_0$

$= 1 - 0.5 \sin(30^\circ)$

$= 0.75 \text{ (p.u)}$

Now,  $P_a(0)$  average  $= \frac{P_a(0^-) + P_a(0^+)}{2}$

$= \frac{0 + 0.75}{2} = 0.375$

$\therefore \Delta\delta_1 = 0 + 3.33 \times 0.375 = 1.249$

$\delta_1 = \delta_0 + \Delta\delta_1$

$= 30^\circ + 1.249$

$= 31.249^\circ \rightarrow \text{at } t = 0.053$

**$n = 2$  (or)  $t = 0.15$ :**

$\delta_2 = \delta_1 + \Delta\delta_2$

Where  $\Delta\delta_2 = \Delta\delta_1 + 3.33 P_a(0.055)$  (or)  $P_a(1)$

$P_a(1)$  (or)  $P_a(0.055) = P_s - P_{\max 2} \cdot \sin \delta_1$

$= 1 - 0.5 \sin(31.249^\circ)$

$= 0.741$

Now,  $\Delta\delta_2 = 1.249^\circ + 3.33 \times 0.741$

$= 3.716^\circ$

So,  $\delta_2 = \delta_1 + \Delta\delta_2$

$= 31.249 + 3.716^\circ = 34.96^\circ \rightarrow \text{at } t = 0.1 \text{ sec}$

**$n = 3$  (or)  $t = 0.155$**

$\delta_3 = \delta_2 + \Delta\delta_3$

Where,  $\Delta\delta_3 = \Delta\delta_2 + 3.33 P_a(2)$  (or)  $P_a(0.1s)$

$P_a(2) = P_s - P_{\max 2} \cdot \sin \delta_2$

$= 1 - 0.5 \sin(34.96^\circ) = 0.713$

$\Delta\delta_3 = 3.716 + 3.33 \times 0.713$

$= 6.09^\circ$

$\therefore \delta_3 = \delta_2 + \Delta\delta_3$

$= 34.96^\circ + 6.09^\circ = 41.05^\circ \rightarrow \text{at } t = 0.15 \text{ sec}$

**n = 4 (or) t = 0.25**

$$\delta_4 = \delta_3 + \Delta \delta_4$$

Where  $\Delta \delta_4 = \Delta \delta_3 + 3.33 P_a(3)$  (or)  $P_a(0.15) S$

$$P_a(0.15s) = P_s - P_{\max 2} \sin \delta_3$$

At  $t = 0.15s$ , fault was cleared so there is a discontinuity in the value of  $P_a$

$P_a(0.15s)$  average

$$= \frac{P_a(0.15s^-) + P_a(0.15s^+)}{2}$$

Where  $P_a(0.15s) = P_s - P_{\max 2} \sin \delta_3$

$$= 1 - 0.5 \sin(41.05^\circ)$$

$$= 0.672$$

$$P_a(0.15s^+) = P_s - P_{\max 3} \sin \delta_3$$

$$= 1 - 1.5 \sin(41.05^\circ) = 0.015$$

$$\therefore P_a(0.153) = \frac{0.672 + 0.015}{2} = 0.3435$$

$$\therefore \Delta \delta_4 = 6.09 + 3.33 \times 0.3435 = 7.234$$

$$\therefore \delta_4 = 41.05 + 7.234$$

$$= 48.284^\circ \rightarrow \text{at } t = 0.2s$$

$$\text{At } t = 0.4s \Rightarrow \delta = 66.16^\circ + 3.78^\circ = 69.94^\circ$$

$$t = 0 \rightarrow \delta = 30^\circ$$

$$t = 0.05s \rightarrow \delta = 31.25^\circ$$

$$t = 0.1s \rightarrow \delta = 35^\circ$$

$$t = 0.15s \rightarrow \delta = 41^\circ$$

$$t = 0.2s \rightarrow \delta = 48.3^\circ$$

$$t = 0.25s \rightarrow \delta = 55^\circ$$

$$t = 0.30s \rightarrow \delta = 61.14^\circ$$

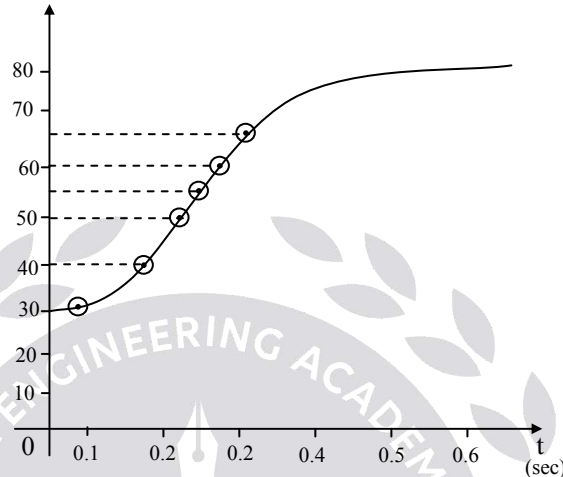
$$t = 0.35s \rightarrow \delta = 66.16^\circ$$

$$t = 0.4s \rightarrow \delta = 70^\circ$$

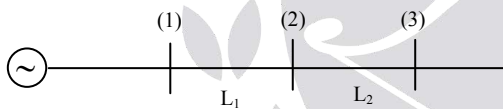
t	$P_{\max}$	$P_e = P_{\max} \sin \delta$	$P_a = 1 - P_e$	$3.33 P_a$	$\Delta \delta_n = \Delta \delta_{n-1} + 3.33 P_a$	$\delta$
$0^-$	2.0	1.0	0	-	-	$30^\circ$
$0^+$	0.5	0.25	0.75	-	-	$30^\circ$
$0_{\text{avg}}$	-	-	$\frac{0 + 0.75}{2} = 0.375$	$1.249^\circ$	$\Delta \delta = 0 + 1.249 = 1.249$	$30^\circ$
0.05s	0.5	$P_e = 0.5 \sin(31.25) = 0.287$	0.713	2.467	$\Delta \delta = 1.249 + 2.467 = 3.71^\circ$	$31.24^\circ$
0.05s	0.5	$P_e = 0.5 \sin(35) = 0.287$	$P_a = 1 - 0.287 = 0.713$	$3.33 \times 0.713 = 2.37$	$\Delta \delta = 3.71 + 2.39 = 6.08$	34.9
0.15s	0.5	$P_e = 0.328$	$P_a = 0.672$	-	-	41.04
$0.15^+$	1.5	$P_e = 0.985$	$P_a = 0.015$	-	-	41.04
$0.15_{\text{avg}}$	-	-	$P_a = \frac{0.672 + 0.015}{2} = 0.3435$	$3.33 \times 0.3435 = 1.14^\circ$	$\Delta \delta = 6.08 + 1.14 = 7.22^\circ$	41.04
0.2s	1.5	$P_e = 1.5 \sin(48.26) = 1.12$	$P_a = 1 - 1.12 = -0.12$	$3.33 \times -0.12 = -0.399$	$\Delta \delta = 7.22 - 0.399 = 6.82^\circ$	48.2



0.25s	1.5	$P_e = 1.5 \sin(55.08^\circ)$ $= 1.23$	$P_a = 1 - 1.23$ $= -0.23$	$3.33 \times -0.23$ $= -0.76$	$\Delta\delta = 6.82 - 0.76^\circ$ $= 6.06^\circ$	55.08
0.3s	1.5	$P_e = 1.5 \sin(61.14^\circ)$ $= 1.314$	$P_a = 1 - 1.314$ $= -0.314$	$3.33 \times -0.314$ $= -1.04$	$\Delta\delta = 6.06 - 1.04$ $= 5.02$	61.14
0.35s	1.5	1.372	-0.372	-1.24	$\Delta\delta = 3.78^\circ$	66.1



15.  
Sol:



Bus (1) – slack Bus,

$$|V_1| = 1.05 \text{ p.u.}, \delta_1 = 0^\circ$$

$$V_1 = 1.05 \angle 0^\circ (\text{known})$$

Bus (2) → PQ Bus,

$$P_2 = P_{g2} - P_{d2} = 0 - 2 \Rightarrow P_2 = -2$$

$$Q_2 = Q_{g2} - Q_{d2} = 0 - 1.5 \Rightarrow Q_2 = -1.5$$

Bus (3) → PV bus,  $|V_3| = 1.0 \text{ PU}$ ,

$$P_3 = P_{g3} - P_{d3} = 0 - 2 \Rightarrow P_3 = -2$$

$Y_{BUS}$  for the network,

$$[Y_{BUS}] = \begin{matrix} & \begin{matrix} (1) & (2) & (3) \end{matrix} \\ \begin{matrix} (1) \\ (2) \\ (3) \end{matrix} & \begin{bmatrix} -j5 & j5 & 0 \\ j5 & -j10 & j5 \\ 0 & j5 & -j5 \end{bmatrix} \end{matrix}$$

Flat start:

$$\text{Bus (2)} \Rightarrow |V_2| = 1.0 \text{ P.U.}$$

$$\delta_2 = 0^\circ$$

$$V_2 = 1 \angle 0^\circ \text{ P.U.}$$

$$\text{Bus (3)} \Rightarrow \delta_3 = 0^\circ$$

Complex power oriented at the 3<sup>rd</sup> Bus  $S_3$

$$= V_3 I_3^*$$

$$S_3 = V_3 I_3^*$$

$$= V_3 [Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3]^*$$

$$= 1 \angle 0^\circ [0 + j5 \times 1 \angle 0^\circ + (-j5) \times 1 \angle 0^\circ]^*$$

$$P_3 + jQ_3 = 0 \Rightarrow Q_3 = 0$$

1<sup>st</sup> iteration:

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{k+1} - \sum_{j=i+1}^n Y_{ij} V_j^k \right]$$

Bus (2)

$$\Rightarrow V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 \right]$$

$$= \frac{1}{-j10} \left[ \frac{-2 - j(-1.5)}{(1 \angle 0^\circ)^*} - (j5)(1.05 \angle 0^\circ) - (j5)(1 \angle 0^\circ) \right]$$

$$= 0.897 \angle -12.87^\circ \text{ pu}$$

With 'α' factor α = 1.5

$$V_2^1 = (V_2^1 - V_2^0)\alpha + V_2^0$$

$$= (0.897 \angle -12.87^\circ - 1 \angle 0^\circ) \times 1.5 + 1 \angle 0^\circ$$

$$= 0.865 \angle -20.26^\circ \text{ P.U}$$

Bus (3)

$$\Rightarrow V_3^1 = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 \right]$$

$$V_3^1 = \frac{1}{-j5} \left[ \frac{-2 - j0}{(1 \angle 0^\circ)^*} - 0 - (j5) \times (0.865 \angle -20.26^\circ) \right]$$

$$V_3^1 = 1.07 \angle -40.76^\circ \text{ P.U}$$

At Bus (3),  $V_3^1(\text{acc}) = (V_3^1 - V_3^0) \times \alpha + V_3^0$

$$= (1.07 \angle -40.76^\circ - 1 \angle 0^\circ) \times 1.5 + 1 \angle 0^\circ$$

$$= 1.27 \angle -55.67^\circ$$

2<sup>nd</sup> iteration:

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1}^{i-1} Y_{ij}V_j^{k+1} - \sum_{j=i+1}^n Y_{ij}V_j^k \right]$$

Bus (2)

$$\Rightarrow V_2^2 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^1)^*} - Y_{21}V_1 - Y_{23}V_3^1 \right]$$

$$= \frac{1}{-j10} \left[ \frac{-2 - j(-1.5)}{0.865 \angle 20.26^\circ} - (j5)(1.05 \angle 0^\circ) - (j5)(1.07 \angle -40.76^\circ) \right]$$

$$= 0.8537 \angle -36.362^\circ \text{ p.u}$$

16.

**Sol:** From the given data

$$Y_{\text{bus}} = \begin{bmatrix} -j10 & j5 & j5 \\ j5 & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

$$\theta = \begin{bmatrix} -90 & 90 & 90 \\ 90 & -90 & 90 \\ 90 & 90 & -90 \end{bmatrix}$$

**Step 1:** Assume  $\delta_2 = 0^\circ$  for PV bus

$V_3 = 1 \text{ pu}$  &  $\delta_3 = 0^\circ$  for P.Q Bus

**Step 2:** Real & Reactive power flow at bus

(3)

Real Power at bus 2 can be calculated as

$$P_2 = \sum_{j=1}^3 |V_2||V_j||Y_{2j}| \cos(\delta_j - \delta_2 + \theta_{2j}) = 0$$

$$P_3 = \sum_{j=1}^3 |V_3||V_j||Y_{3j}| \cos(\delta_j - \delta_3 + \theta_{3j}) = 0$$

$$Q_3 = \sum_{j=1}^3 |V_3||V_j||Y_{3j}| \sin(\delta_j - \delta_3 + \theta_{3j})$$

$$= |V_3||V_1||Y_{31}| \sin(\delta_1 - \delta_3 + \theta_{13})$$

$$+ |V_3||V_2||Y_{32}| \sin(\delta_2 - \delta_3 + \theta_{32})$$

$$+ |V_3||V_3||Y_{33}| \sin(\theta_{33})$$

$$Q_3 = (1 \times 1.05 \times 5) \sin(90^\circ) + (1 \times 1 \times 5) \sin(90^\circ)$$

$$+ (1 \times 1 \times 10) \sin(-90^\circ)$$

$$Q_3 = 5.25 + 5 - 10 = 0.25$$

$$\Delta P_2 = P_2^{\text{spec}} - P_2^{\text{cal}} = 3 - 0 = 3$$

$$\Delta P_3 = P_3^{\text{spec}} - P_3^{\text{cal}} = -4 - 0 = -4$$

$$\Delta Q_3 = Q_3^{\text{spec}} - Q_3^{\text{cal}} = -2 - 0.25 = -2.25$$

**Step 3:**  $B^1 = \begin{bmatrix} +10 & -5 \\ -5 & +10 \end{bmatrix}$ ,  $B^{11} = [+10]$

Matrix  $[B^1]$  are the negative of imaginary component of  $Y_{\text{BUS}}$  matrix.

$$\begin{aligned}\text{Step 4: } \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} &= [B']^{-1} [\Delta P] \\ &= \frac{1}{75} \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} \\ &= \frac{1}{75} \begin{bmatrix} 30 - 20 \\ 15 - 40 \end{bmatrix} \\ &= \begin{bmatrix} +10/75 \\ -25/75 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}[\Delta V_3] &= [B'']^{-1} [\Delta Q_3] \\ &= \frac{1}{10} [-2.25] = -0.225\end{aligned}$$

$$\text{Step 5: } \delta_2^{\text{new}} = \delta_2^{\text{old}} + \Delta\delta_2 = 0.133^{\circ} = 7.62^{\circ}$$

$$\delta_3^{\text{new}} = \delta_3^{\text{old}} + \Delta\delta_3 = -0.333 = -19.07^{\circ}$$

$$V_3^{\text{new}} = V_3^{\text{old}} + \Delta V_3 = 1 - 0.225 = 0.775$$

$$P_1 = |V_1^2 Y_{11}| \cos \theta_{11} + |V_1 V_2 Y_{12}| \cos(\delta_2 - \delta_1 + \theta_{12})$$

$$\begin{aligned}&+ |V_1 V_3 Y_{13}| \cos(\delta_3 - \delta_1 + \theta_{31}) \\ &= 1.05^2 \times 10 \times \cos(-90) \\ &\quad + 1.05 \times 5 \cos(7.62 - 0 + 90) \\ &\quad + |1.05 \times 0.775 \times 5| \cos\end{aligned}$$

$$\begin{aligned}&(-19.07 - 0 + 90) \\ &= 0 + (-0.690) + (+1.329) \\ &= 0.638 \text{ pu}\end{aligned}$$

$$Q_1 = 1.05^2 \times 10 \sin(-90) + 5.25 \sin(97.62)$$

$$\begin{aligned}&+ (1.05 \times 0.775 \times 5) \sin 109.7 \\ &= -11.025 + 5.20 + 3.84 = -1.979 \text{ pu}\end{aligned}$$

$$Q_2 = |V_2 V_1 Y_{21}| \sin(\delta_1 - \delta_2 + \theta_{12}) + |V_2^2 Y_{22}| \sin \theta_{22}$$

$$+ |V_2 V_3 Y_{23}| \sin(\delta_3 - \delta_2 + \theta_{23})$$

$$\begin{aligned}Q_2 &= |1.05 \times 5| \sin(1.05 - 7.62 + 90) + 10 \sin(-90) \\ &\quad + |1 \times 0.775 \times 5| \sin(-19.07 - 7.62 + 90) \\ &= 5.215 + (-10) + 3.462 = -1.328\end{aligned}$$

	P	Q	V	$\delta$
1	0.638	-1.979	1.05	0°
2	3 P u	-1.328	1 pu	7.62°
3	-4	-2	0.775	-19.07

## 2. Transmission & Distribution

### Solutions for Objective Practice Questions

#### Basic Concepts & Transmission Line Constants:

01. Ans:  $n^2$

Sol: Given data:

For same length, same material, same power loss and same power transfer

If the voltage is increased by 'n' times, what will happen to area of cross section of conductor.

$$P_{\text{Loss } 1} = P_{\text{Loss } 2}$$

$$P_{\text{Loss } 1} = 3 I_1^2 R_1$$

$$P = \sqrt{3} V_1 I_1 \cos \phi$$

$$P_{\text{Loss } 1} = 3 \left( \frac{P_1}{\sqrt{3} V_1 \cos \phi} \right)^2 \times R_1$$

$$P_{\text{Loss } 1} = \frac{P_1^2 R_1}{V_1^2 \cos^2 \phi}$$

$$P_{\text{Loss } 1} \propto \frac{R}{V_1^2} \propto \frac{1}{a V_2^2}$$

$$\Rightarrow a V^2 \propto \frac{1}{P_{\text{Loss}}}$$

$$\Rightarrow a V^2 = \text{constant}$$

$$\therefore P_{\text{Loss}} = \text{Constant}$$

$$\frac{a_1 V_1^2}{a_2 V_2^2} = 1$$

$$\frac{V_2}{V_1} = n \rightarrow \text{given}$$

$$\Rightarrow a_2 = \frac{1}{n^2} a_1$$

In this efficiency is constant since same power loss.

**02. Ans: (b)**

**Sol:** Given data:

We know that  $P = VI \cos \phi$

$$I = \frac{P}{(V \cos \phi)} \dots \dots \dots (1)$$

Power loss  $P = I^2 R$

$$= I^2 \frac{\rho \ell}{a} \left( \because R = \frac{\rho \ell}{a} \right)$$

$$a = I^2 \frac{\rho \ell}{P} \dots \dots \dots (2)$$

Substitute eq (1) in eq. (2)

$$I = \left( \frac{P}{V \cos \phi} \right)^2 \frac{\rho \ell}{a}$$

$$a = \frac{K}{(V \cos \phi)^2}$$

$$a \propto \frac{1}{(V \cos \phi)^2}$$

$$\text{Volume} \propto \frac{1}{(V \cos \phi)^2} \quad (\because \text{volume} \propto \text{area})$$

**03. Ans: (b)**

**Sol:** Given data:

Self-inductance of a long cylindrical conductor due to its internal flux linkages is 1 kH/m.

$$L_a = \underbrace{\frac{\mu_0 \mu_r}{8\pi}}_{\psi_{\text{int}}} + \underbrace{\frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{1}{r}\right) - \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{1}{d}\right)}_{\psi_{\text{ext}}}$$

$$L_{\text{self}} = L_{\text{self}} \text{ due to } \psi_{\text{int}} + L_{\text{self}} \text{ due to } \psi_{\text{ext}}$$

$$= \frac{\mu_0 \mu_r}{8\pi} + \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{1}{r}\right)$$

$$L_{\text{mutual}} = L_{\text{mutual due to ext}} = \frac{\mu_0 \mu_r}{2\pi} \ln\left(\frac{1}{d}\right)$$

Ans: 1 K H/m ( $\because$  1<sup>st</sup> term is independent of diameter)

**04. Ans: 31.6% (Range: 30 to 32)**

**Sol:** Given data:

$L_n = 1.10$  mH/km increased 5%

$$L_n = 0.2 \ell n \left( \frac{d_1}{r_1} \right) \text{ mH/km}$$

$$1.10 \text{ mH/km} = 0.2 \ell n \left( \frac{d_1}{r_1} \right) \text{ mH/km}$$

$$1.10 = 0.2 \ell n \left( \frac{d_1}{r_1} \right)$$

$$\frac{1.10}{0.2} = \ell n \left( \frac{d_1}{r_1} \right)$$

$$5.5 = \ell n \left( \frac{d_1}{r_1} \right)$$

$$e^{5.5} = \frac{d_1}{r_1}$$

$$244.69 r_1 = d_1$$

$$(1.10) \times 1.05 = 0.2 \ell n \left( \frac{d_2}{r_2} \right)$$

$$1.155 = 0.2 \ell n \left( \frac{d_2}{r_2} \right)$$

$$e^{\frac{1.155}{0.2}} = \frac{d_2}{r_2}$$

$$322.14r_2 = d_2$$

$$\frac{d_2 - d_1}{d_1} \times 100 = \frac{322.14r_1 - 244.69r_2}{244.69r_1} \times 100$$

$$= 0.3165 \times 100$$

$$= 31.6\%$$

**05. Ans: (b)**

**Sol:** Given data:

$$d = 4;$$

(i)  $L_1$   $C_{n1}$

After Transposition

$$GMD_1 = \sqrt[3]{4 \times 4 \times 4} = 4$$

(ii)  $L_2$   $C_{n2}$

After Transposition

$$GMD_2 = \sqrt[3]{4 \times 4 \times 8} = 5.02 \text{ m}$$

$$GMD_1 < GMD_2$$

$$L_1 < L_2$$

$$C_{n1} > C_{n2}$$

$$\text{Resistances } R_1 = R_2$$

$$\uparrow Z_c = \sqrt{\frac{L \uparrow}{C \downarrow}}$$

$$\left[ Z_{c1} = \left( \frac{L_1}{C_{n1}} \right)^{1/2} \right] < \left[ Z_{c2} = \left( \frac{L_2}{C_{n2}} \right)^{1/2} \right]$$

$$\left[ SIL_1 = \left( \frac{V^2}{Z_{c1}} \right) \right] > \left[ SIL_2 = \left( \frac{V^2}{Z_{c2}} \right) \right]$$

**06. Ans: (b)**

**Sol:** Given data:

The impedance of a Transmission line

$$Z = 0.05 + 0.35j \Omega/\text{phase/km}$$

$$\text{Spacing is doubled } d_2 = 2d_1; R = 0.05$$

Radius is doubled  $r_2 = 2r_1$

$$X_L = 0.35 \Omega/\text{phase/km}$$

$$l \propto \ln \left( \frac{GMD}{GMR} \right)$$

$l$  remain constant

$$2\pi fL = 0.35$$

$$L = \frac{0.35}{2\pi f}$$

$$\text{B let } R \propto \frac{\ell}{A}; R \propto \frac{\ell}{\pi r^2}$$

$$\frac{R_2}{R_1} = \left( \frac{r_1}{r_2} \right)^2 \quad R_L = R \left( \frac{1}{2} \right)^2$$

$$= \frac{R_1}{4} = \frac{0.05}{4} = 0.0125$$

$$\therefore (Z_2)_{\text{new}} = 0.0125 + j 0.35 \Omega/\text{km}.$$

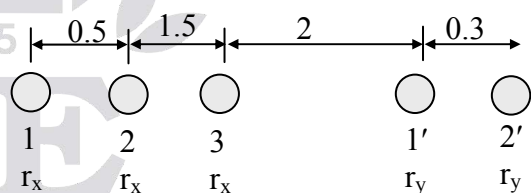
**07. Ans: (c)**

**Sol:** Given data:

$$r_x = 0.03 \text{ m}$$

$$r_y = 0.04 \text{ m}$$

$$GMD_{\text{system}} = GMD_a \cdot GMD_b$$



$$GMD_a = (d_{11'} \times d_{12'} \times d_{21'} \times d_{22'} \times d_{31'} \times d_{32'})^{1/6}$$

$$= (4 \times 4.3 \times 3.5 \times 3.8 \times 2 \times 2.3)^{1/6}$$

$$= 3.189 \text{ m}$$

$$GMD_b = GMD_a = 3.189$$

$$\therefore GMD_{\text{system}} = \sqrt{GMD_a \times GMD_b}$$

$$= 3.189 \text{ m}.$$

(Self GMD)<sub>system</sub>

$$= \sqrt{(\text{selfGMD of ststem a}) \times \text{self GMD}_b}$$

selfGMD<sub>a</sub>

$$= (r'_x \times 0.5 \times 2 \times r'_x \times 0.5 \times 1.5 \times r'_x \times 1.5 \times 2)^{1/9}$$

$$= (0.7788^3 \times (0.03)^3 \times (1.5)^2 \times (0.5)^2 \times 2^2)^{1/9}$$

$$= 0.312 \text{ m}$$

$$\text{Self GMD}_b = (r'_y \times 0.3 \times r'_y \times 0.3)^{1/4}$$

$$= \sqrt{0.7788 \times 0.04 \times 0.3}$$

$$= 0.096 \text{ m}$$

$$\therefore \text{Self GMD} = \sqrt{0.096 \times 0.312} = 0.173 \text{ m}$$

$$L = 2 \times 0.2 \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{ mH/km}$$

$$= 0.4 \ln \left( \frac{3.189}{0.162} \right) \times 10^{-6} \text{ H/m}$$

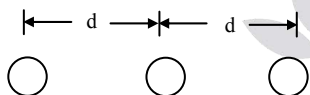
$$L = 11.63 \times 10^{-7} \text{ H/m}$$

**08. Ans: d = 2.49 m (Range: 2.2 to 2.6)**

**Sol:** Given data:

$$r = 1 \text{ cm}$$

$$L = 1.2 \text{ mH/km}$$



$$\text{GMD} = \sqrt[3]{2} \times d$$

$$0.2 \ln \left( \frac{1.2599 d}{0.7788 \times 0.01} \right) = 1.2$$

$$d = 2.49 \text{ m}$$

**09. Ans: 3.251 nF/km**

**Sol:** Given data:

$$f = 50 \text{ Hz}, d = 0.04 \text{ m}, r = 0.02 \text{ m}$$

$$V = 132 \text{ kV}$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln \left( \frac{\text{GMD}}{\text{GMR}} \right)}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times 1}{\ln \left( \frac{6}{0.02} \right)}$$

$$= 9.75 \text{ nF/km}$$

$$\text{Interline capacitance} = \frac{C}{3} = \frac{9.75}{3}$$

$$\Rightarrow 3.251 \text{ nF/km}$$

**10. Ans: 1.914 (Range: 1.85 to 1.95)**

**Sol:** Given data:

$$\text{Self GMD} = kR$$

$$\text{Self GMD} = \sqrt[3]{R^1 \times 3R \times 3R}$$

$$= \sqrt[3]{0.7788R \times 3R \times 3R}$$

$$= R \sqrt[3]{0.7788 \times 3 \times 3}$$

$$kR = 1.914 R$$

$$k = 1.914$$

### Steady State Performance analysis of Transmission lines

**01. Ans: (c)**

**Sol:** Given data:

$$A = D = 0.936 + j0.016 = 0.936 \angle 0.98^\circ,$$

$$B = 33.5 + j138 = 142.0 \angle 76.4^\circ,$$

$$C = (-5.18 + j914) \times 10^{-6},$$

$$V_r = 50 \text{ MW}, p.f = 0.9 \text{ lag},$$

$$V_s (L-L) = ?$$

$$V_{s \text{ ph}} = A V_{r \text{ ph}} + B I_{r \text{ ph}}$$

$$V_{r \text{ ph}} = \frac{220 \text{ kV}}{\sqrt{3}}$$

$$I_{rL} = \frac{P_r}{\sqrt{3} V_L \cos \phi_r}$$

$$= \frac{50 \text{ M}}{\sqrt{3} \times 220 \text{ k} \times 0.9} = 145.7 \text{ A}$$

$$I_{rph} = 145.7 \angle -\cos^{-1}(0.9) = 145.7 \angle -25.84$$

$$V_{Sph} = (0.936 \angle 0.98) \left( \frac{220 \text{ k}}{\sqrt{3}} \right) + (142 \angle 76.4)(145.7 \angle -25.84)$$

$$= 133.24 \angle 7.7^\circ \text{ kV}$$

$$V_S (L-L) = \sqrt{3} \times 133.24 = 230.6 \text{ kV}$$

$$V_R = \frac{V_s}{A}$$

$$\frac{230.6}{0.936} = 246.36 \text{ kV}$$

**02. Ans: (c)**

**Sol:** Given data:

Load delivered at nominal rating

$$V_{rl} = 220 \text{ kV}$$

$$\% V.R = \frac{\left| \frac{V_s}{A} \right| - |V_r|}{|V_r|} \times 100\%$$

$$= \frac{\frac{240}{0.94} - 220}{220} \times 100\% = 16\%$$

**03. Ans: (c)**

**Sol:** Given data:

$$A = D = 0.95 \angle 1.27^\circ ; B = 92.4 \angle 76.87^\circ$$

$$C = 0.006 \angle 90^\circ ; V_S = V_r = 138 \text{ kV}$$

R, Y are neglected

$$\therefore P_{\max} = \frac{|V_s| |V_r|}{X}$$

$$\text{In nominal-}\pi \Rightarrow B = Z$$

$$Z = 92.4 \angle 76.87^\circ = 21 + j90 \Omega$$

$$X = 90 \Omega$$

$$\therefore P_{\max} = \frac{138 \times 138}{90} = 211.6 \text{ MW}$$

**04. Ans: 81.04 kW (Range: 79 to 82)**

**Sol:** Given data:

$$A = 0.977 \angle 0.66$$

$$B = 90.18 \angle 64.12^\circ$$

$$V = 132 \text{ kV}$$

$$AD - BC = 1$$

$$C = \frac{AD - 1}{B}$$

$$V_c = \frac{132 \times 10^3}{\sqrt{3} \times 0.97} \angle -0.66$$

$$C = \frac{0.977 \angle 0.66 \times 0.977 \angle 0.66 - 1}{90.18 \angle 64.12^\circ}$$

$$= \frac{0.9545 \angle 1.32 - 1}{90.18 \angle 64.12^\circ}$$

$$= 5.62 \times 10^{-4} \angle 90.2$$

$$I_s = CV_r + BI_r$$

$$5.62 \times 10^{-4} \angle 90^\circ \times \frac{132 \times 10^3}{\sqrt{3}}$$

$$P = 3V_L I_L \cos \phi$$

$$P = 3 \times \frac{132 \times 74.184 \cos(90.2 - 0.66)}{3 \times 0.97}$$

$$P = 81.04 \text{ kW}$$

**05. Ans: (b)**

**Sol:** Given data:

Complex power delivered by load:

$$S = V I^*$$

$$= (100 \angle 60^\circ) (10 \angle 150^\circ)$$

$$= 1000 \angle 210$$

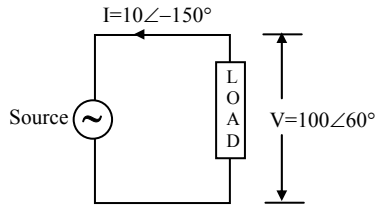


$$= -866.6 - j 500 \text{ VA}$$

Complex power absorbed by load

$$S_{\text{load}} = 866.6 + j 500 \text{ VA}$$

∴ Ans: (b) i.e., load absorbs both real and reactive power.

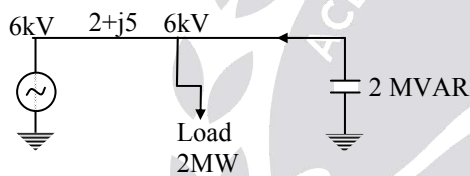


**06. Ans: 0.936 lag**

**Sol:** Given data:

Short transmission line having impedance

$$= 2 + j5 \Omega$$



$$\beta = \cos^{-1}\left(\frac{2}{\sqrt{29}}\right) = 68.2$$

$$P = \frac{V_s V_r}{B} \cos(\beta - \delta) - \frac{A V_r^2}{B} \cos(\beta - \alpha)$$

$$2 \times 10^6 = \frac{36 \times 10^6}{\sqrt{29}} [\cos(68.2 - \delta) - \cos(68.2)]$$

$$\cos(68.28 - \delta) = 0.6705$$

$$\delta = 20.309^\circ$$

$$Q = \frac{V_s V_r}{B} \sin(\beta - \delta) - \frac{A V_r^2}{B} \sin(\beta - \alpha)$$

$$= \frac{36 \times 10^6}{\sqrt{29}} [\sin(68.2 - 20.309) - \sin 68.2]$$

$$= -1.24 \text{ MW}$$

$$\therefore -1.24 + 2 = Q_c$$

$$Q_c = 0.7524 \text{ MW}$$

$$\therefore \cos \phi = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{2}{\sqrt{4 + (0.7524)^2}}$$

$$= 0.9359 \text{ lag}$$

$$\approx 0.936 \text{ lag}$$

**07. Ans: (a)**

**Sol:** Given data:

$$f = 50 \text{ Hz}$$

$$\text{Surge impedance } Z_0 = \sqrt{\frac{L}{C}} = 1$$

$$L = C$$

Velocity of wave

$$V = \frac{1}{\sqrt{LC}} = 3 \times 10^5$$

$$\frac{1}{\sqrt{LC}} = 3 \times 10^5$$

$$\frac{1}{C} = 3 \times 10^5$$

$$C = \frac{10^{-5}}{3}$$

$$X = \frac{2\pi f L}{2} \times \ell$$

$$= \pi 50 \times \frac{10^{-5}}{3} \times 400$$

$$= 0.209$$

$$y = [2\pi f c] \ell$$

$$= 2 \times \pi \times 50 \times \frac{10^{-5}}{3} \times 400$$

$$= 0.418$$

**08. Ans: (b)**

**Sol:** Given data:

$$V_s = V_r = 1,$$

$$X = 0.5,$$

$$\text{Real power } P_r = \frac{|V_s||V_r|}{|X|} \sin \delta$$

$$1 = \frac{1.0 \times 1.0}{0.5} \sin \delta$$

$$\Rightarrow \delta = \sin^{-1}(0.5) = 30^\circ$$

Reactive power

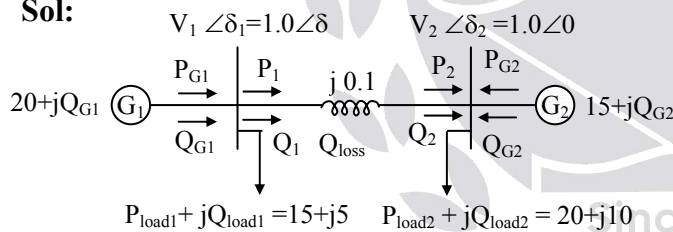
$$\begin{aligned} Q_r &= \frac{(V_s)(V_r)}{X} \cos \delta - \frac{(V)^2}{(X)} \\ &= \frac{1.0 \times 1.0}{0.5} \cos 30 - \frac{1^2}{0.5} \\ &= \left( \frac{\sqrt{3}}{2} \right) - 2 = 1.732 - 2 = -0.268 \\ &= \left( \frac{1}{2} \right) - 2 = 1.732 - 2 = -0.268 \end{aligned}$$

But  $Q_r + Q_C = 0$

$$Q_C = -Q_r = 0.268 \text{ p.u.}$$

**09. Ans: (c)**

**Sol:**



$P_1$  = Active power sent by bus (1)

$$= \frac{V_1 V_2}{X_L} \sin(\delta_1 - \delta_2)$$

$P_2$  = Active power received by bus (2)

$$= \frac{V_1 V_2}{X_L} \sin(\delta_1 - \delta_2)$$

$Q_1$  = Reactive power sent by bus (1)

$$= \frac{V_1}{X_L} (V_1 - V_2 \cos(\delta_1 - \delta_2))$$

$Q_2$  = Reactive power received by bus (2)

$$= \frac{V_2}{X_L} (V_1 \cos(\delta_1 - \delta_2) - V_2)$$

Active power balance at bus (1):

Active power balance at bus 2:

$$P_{G1} = P_1 + P_{\text{load1}}$$

$$P_2 + P_{G2} = P_{\text{load2}}$$

$$20 = P_1 + 15$$

$$P_2 + 15 = 20$$

$$P_1 = 5, P_2 = 5$$

$$\therefore P_1 = P_2 = \frac{V_1 V_2}{X_L} \sin(\delta_1 - \delta_2) = 5$$

$$\Rightarrow \frac{1 \times 1}{0.1} \sin(\delta - 0) = 5$$

$$\Rightarrow \sin \delta = 0.5$$

$$\Rightarrow \delta = 30^\circ$$

$$Q_1 = \frac{V_1}{X_L} [V_1 - V_2 \cos(\delta_1 - \delta_2)]$$

$$Q_2 = \frac{V_2}{X_L} [V_1 \cos(\delta_1 - \delta_2) - V_2]$$

$$= \frac{1}{0.1} [1 - 1 \cos 30^\circ]$$

$$= \frac{1}{0.1} [1 \cos 30^\circ - 1]$$

$$= 1.34 \text{ pu}$$

$$= -1.34 \text{ pu}$$

$$Q_{\text{line}} = Q_{\text{loss}} = Q_1 - Q_2$$

$$= 1.34 - (-1.34)$$

$$= 2.68 \text{ pu}$$

$$Q_{\text{loss}} = 2.68 \text{ pu}$$

Reactive power balance at bus (1):

Reactive power balance at bus (2):

$$Q_{G1} = Q_1 + Q_{\text{load1}}$$

$$Q_2 + Q_{G2} = Q_{\text{load2}}$$

$$Q_{G1} = 1.34 + 5$$

$$Q_{G2} = 10 - (-1.34)$$

$$Q_{G1} = 6.34 \text{ pu}$$

$$Q_{G2} = 11.34 \text{ pu}$$

$$\therefore Q_{G1}=6.34\text{pu}, Q_{G2}=11.34\text{pu}, Q_{\text{loss}}=2.68\text{pu}$$

### Transient Analysis & Wave Travelling Analysis

**01. Ans: (c)**

**Sol:** Given data:

Let "l" be the total length of line

Total reactance of line =  $0.045 \text{ p.u.} = 2\pi fL$

$$\text{Total inductance of line} = \frac{0.045}{2\pi \times 50}$$

Total susceptance of line =  $1.2 \text{ p.u.} = 2\pi fC$

$$\text{Total capacitance of line} = \frac{1}{2\pi \times 50}$$

$$\text{Inductance/km} = \frac{0.045}{2\pi \times 50 \times 1}$$

$$\text{Capacitance/km} = \frac{1.2}{2\pi \times 50 \times 1}$$

Velocity wave propagation

$$(V) = \frac{\ell}{\sqrt{\left(\frac{L}{\text{km}}\right)\left(\frac{C}{\text{km}}\right)}}$$

$$V = \frac{\ell}{\sqrt{\frac{0.045}{2\pi \times 50 \times 1} \times \frac{1.2}{2\pi \times 50 \times 1}}}$$

$$30 \times 10^5 = \frac{\ell}{7.4 \times 10^{-4}}$$

$\therefore$  Length of the line (l) = 222km

**02. Ans: (c)**

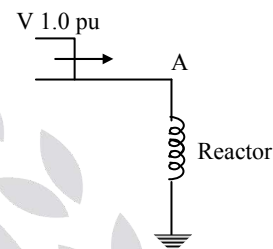
**Sol:** Since load impedance is equal to surge impedance, the voltage & current wave forms are not going to experience any reflection.

Hence reflection coefficient is zero.

$$V_{\text{reflection}} = i_{\text{reflection}} = 0.$$

**03. Ans: (c)**

**Sol:**



$$Z_s = \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{0}} = \infty$$

The Reactor is initially open circuit

$$V_2 = V + V_1 = 1.0 + 1.0 = 2.0 \text{ p.u.}$$

$V_1$  = reflected voltage

$V_2$  = Switched voltage

**04. Ans: (b)**

**Sol:** Given data:

$$V = 50 \text{ kV},$$

$$Z_L = 100 \Omega,$$

$$Z_C = 400 \Omega,$$

The transmitted (or) refracted voltage

$$V_2 = 2V \left( \frac{Z_L}{Z_L + Z_C} \right)$$

Here '2' indicates that the voltage  $V_2$  is calculating in transient condition

$$\therefore V_2 = 2 \times 50 \times 10^3 \times \left( \frac{100}{100 + 400} \right)$$

$$V_2 = 20 \text{ kV}$$

**05. Ans: (b)**

**Sol:** Given data:

$$L_{\text{cable}} = 0.185 \text{ mH/km}$$

$$C_{\text{cable}} = 0.285 \text{ } \mu\text{F/km}$$

$$L_{\text{Line}} = 1.24 \text{ mH}$$

$$C_{\text{Line}} = 0.087 \text{ } \mu\text{F/km}$$

$$\begin{aligned} Z_{C(\text{Cable})} &= \sqrt{\frac{L}{C}} \\ &= \sqrt{\frac{0.185 \times 10^{-3}}{0.285 \times 10^{-6}}} \\ &= 25.4778 \text{ } \Omega \end{aligned}$$

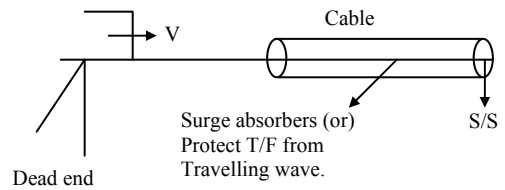
$$\begin{aligned} Z_{C(\text{Line})} &= \sqrt{\frac{L}{C}} \\ &= \sqrt{\frac{1.24 \times 10^{-3}}{0.087 \times 10^{-6}}} \\ &= 119.385 \text{ } \Omega \end{aligned}$$

$$\begin{aligned} V_2 &= 2V \left[ \frac{Z_L}{Z_L + Z_C} \right] \\ &= 2 \times 110 \text{ kV} \left[ \frac{119.385}{119.385 + 25.4778} \right] \\ &= 181.307 \text{ kV} \end{aligned}$$

**06. Ans: (d)**

**Sol:** A short length of cable is connected between dead-end tower and sub-station at the end of a transmission line. This of the following will decrease, when voltage wave is entering from overhead to cable is

- Velocity of propagation of voltage wave.
- Steepness of voltage wave.
- Magnitude of voltage wave.



Velocity of propagation

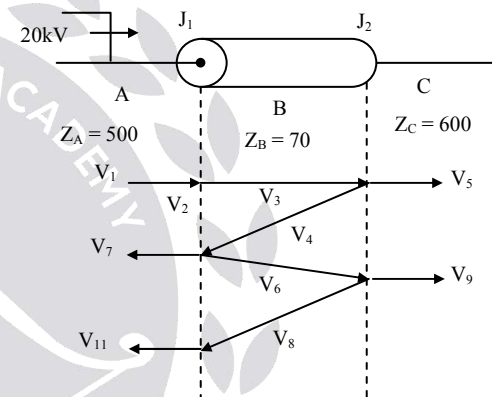
$$V_{(\text{Line})} = 3 \times 10^8$$

$$V_{(\text{Cable})} = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \text{ m/s}$$

$$V_{\text{Cable}} > V_{(\text{OH line})}$$

**07. Ans: 2.93 kV (Range: 2.8 to 3.0)**

**Sol:**



DC (or) step voltage

( $\because$  line is of infinite length)

$$\begin{aligned} V_3 &= 2V_1 \frac{Z_B}{Z_B + Z_A} \\ &= 2 \times 20 \text{ kV} \times \frac{70}{70 + 500} \\ V_3 &= 4.91 \text{ kV} \end{aligned}$$

$$\begin{aligned} V_4 (\text{Re flection of } V_3) &= V_3 \left[ \frac{Z_C - Z_B}{Z_C + Z_B} \right] \\ &= 4.91 \left[ \frac{600 - 70}{600 + 70} \right] = 3.88 \text{ kV} \end{aligned}$$

$$\begin{aligned} V_6 &= V_4 \left[ \frac{Z_A - Z_B}{Z_A + Z_B} \right] \\ &= 3.88 \text{ kV} \left[ \frac{500 - 70}{500 + 70} \right] = 2.93 \text{ kV} \end{aligned}$$

**08. Ans: (d)**

**Sol:** Given data

$$V_6 = 2.93$$

$$V_7 = 2V_4 \times \frac{500}{570}$$

$$= 6.8 \text{ kV}$$

$$V_9 = 2V_6 \times \frac{600}{670}$$

$$= 2 \times 2.93 \times \frac{600}{670} = 5.25 \text{ V}$$

### Voltage Control

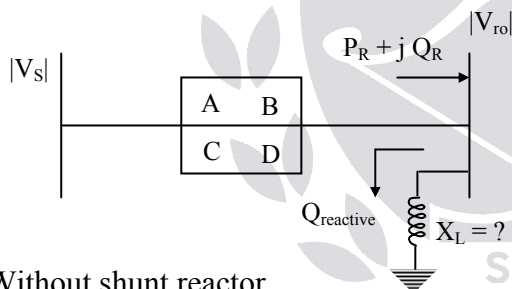
**01. Ans: (a)**

**Sol:** Given data:

$$A = D = 0.9 \angle 0^\circ$$

$$B = 200 \angle 90^\circ \Omega$$

$$C = 0.95 \times 10^{-3} \angle 90^\circ$$



Without shunt reactor

$$|V_{r0}| = \frac{|V_s|}{A}$$

By adding shunt reactor

$$|V_{r0}| = |V_s|$$

$$P_R = 0 \text{ (no load)}$$

$$Q_R = Q_{\text{reactor}}$$

$$= \frac{|V_s| |V_{r0}|}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} |V_{r0}|^2 \sin(\beta - \alpha)$$

$$Q_r = \frac{|V_r|^2}{X_L}$$

$$\text{At } |V_{r0}| = |V_s|$$

$$\frac{1}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} \sin(\beta - \alpha) = \frac{1}{X_L}$$

To get  $\delta$  at  $(|V_{r0}| = |V_s|)$

$$P_r = \frac{|V_s|^2}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_s|^2 \cos(\beta - \alpha) = 0$$

$$= \cos(\beta - \delta) - |A| \cos(\beta - \alpha)$$

$$= \cos(90 - \delta) - 0.9 \cos(90 - 0)$$

$$\cos(90 - \delta) = 0$$

$$\sin \delta = 0, \delta = 0$$

$$\frac{1}{X_L} = \frac{1}{200} \sin(90 - 0) - \frac{0.9}{200} \sin(90 - 0)$$

$$X_L = 2000 \Omega \text{ or } 2 \text{ k}\Omega$$

**02. Ans: (d)**

**Sol:** Given data:

$$P = 2000$$

$$Q = 2000 \tan(36.86^\circ)$$

$$= 2000(0.749) = 1499.46 \text{ kW}$$

$$R(S)_{\text{motor}} = 1000 - j1000$$

$$S_{\text{Total}} = S_{I_m} + S_{S_m}$$

$$= (2000 + j1499.46) + (1000 - j1000)$$

$$= 3000 + j499.46$$

$$\cos \phi = \frac{3000}{3041.29} \times 100\% = 0.986 \text{ lag}$$

**03. Ans: (a)**

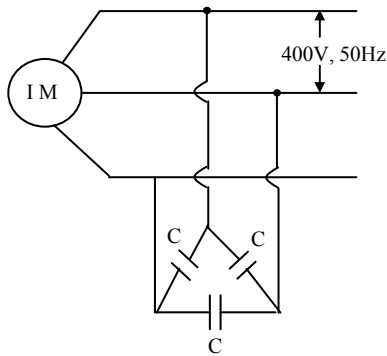
**Sol:** Given data:

$$IM = 400 \text{ V}, 50 \text{ Hz}, \text{ pf} = 0.6 \text{ lag},$$

$$\text{input} = 4.5 \text{ kVA}$$

$$\text{p.f} = 0.6 \text{ load}$$

$$\text{total supply} = ?$$



$$S = \sqrt{3} V_L I_L \quad ; \quad 4.5 \text{ kVA}$$

$$Q_{Sh (3-\phi)} = P_1 (\tan \phi_1 - \tan \phi_2)$$

$$P_1 = \text{Real power drawn by IM}$$

$$= P_{IM}$$

$$= S_{IM} \cos \phi_{IM}$$

$$= 4.5 \times 0.6 \text{ kW}$$

$$P_1 = 2.7 \text{ kW}$$

$$Q_{Sh (3-\phi)} = 2.7 [\tan(\cos^{-1} 0.6) - \tan(\cos^{-1} 0.8)]$$

$$= 1.575 \text{ kVAr}$$

$$Q_{S/ph} = \frac{1.575}{3} \text{ kVAr}$$

$$= 0.525 \text{ kVAr}$$

$$\text{Reactive power supplied} = \frac{V_s^2}{X_c} = 525$$

$$(400)^2 (2\pi \times 50) C = 525$$

$$C = 10.1 \mu\text{F}$$

**04. Ans: (c)**

**Sol:** Given data  $A = 0.85 \angle 5^\circ$

$$\alpha = 5^\circ$$

$$B = 200 \angle 75^\circ \quad \beta = 75^\circ$$

Power demand by the load = 150 MW at upf

$$P_D = P_R = 150 \text{ MW} \quad Q_D = 0$$

Power at receiving end

$$P_R = \frac{|V_s| |V_R|}{B} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_R|^2 \cos(\beta - \alpha)$$

$$\Rightarrow 150 = \frac{275 \times 275}{200} \cos(75 - \delta) - \frac{0.85}{200} (275)^2 \cos 70^\circ$$

$$\delta = 28.46^\circ$$

$$\text{So } Q_R = \frac{|V_s| |V_R|}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} |V_R|^2 \sin(\beta - \alpha)$$

$$= \frac{275 \times 275}{200} \sin(75 - 28.46) - \frac{0.85}{200} (275)^2 \sin 70^\circ$$

$$= -27.56 \text{ MVAR}$$

In order to maintain 275 kV at receiving end  $Q_R = -27.56 \text{ MVAR}$  must be drawn along with the real power.

$$\text{So } -27.56 + Q_C = 0$$

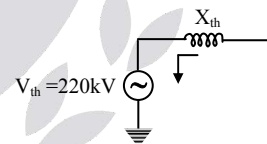
$$Q_C = 27.56 \text{ MVAR}$$

So compensation equipment must be feed in to 27.56 MVAR to the line.

**05. Ans: (c)**

**Sol:** Given data:

$$X_{th} = 0.25 \text{ pu} ; 250 \text{ MVA}, 220 \text{ kV}$$



To boost the voltage 4 kV shunt capacitor is used.

$$\Delta V_C = \frac{X}{|V_s|} Q_{sh \text{ Cap}}$$

$$Q_{sh \text{ Cap}} = \frac{\Delta V_C |V_s|}{X}$$

$$X_\Omega = X_{pu} \times \frac{(\text{kV}_{base})^2}{\text{MVA}_{base}}$$

$$= 0.25 \times \frac{(220^2)}{250} = 48.4$$

$$Q_{\text{sh Cap}} = \frac{4k \times 220k}{48.4} = 18.18 \text{ kVAr}$$

To reduce voltage by 2 kV, shunt reactor is used.

$$\Delta V_L = \frac{X}{|V_S|} Q_{\text{sh Ind}}$$

$$Q_{\text{sh Ind}} = \frac{2k \times 220k}{48.4} = 9.09 \text{ MVAr}$$

**06. Ans: (d)**

**Sol:** Given data:

$$V_2 = 1.1 V_1$$

$$F_2 = 0.9 f_1$$

Reactive power absorbed by reactor =

$$\frac{V^2}{X_L}$$

$$Q_1 = \frac{V_1^2}{2\pi f_1 L} = 100 \text{ MVAr}$$

Then reactive power absorbed

$$Q \propto \frac{V^2}{X} \propto \frac{V^2}{f}$$

$$\frac{Q_2}{Q_1} = \left( \frac{V_2}{V_1} \right)^2 \left( \frac{f_1}{f_2} \right)$$

$$= \left( \frac{1.1 V_1}{V_1} \right)^2 \left( \frac{f_1}{0.9 f_1} \right)$$

$$= \frac{(1.1)^2}{0.9} \times Q_1 = \frac{1.21}{0.9} \times 100 = 134.4 \text{ MVAr}$$

**07. Ans: (c)**

**Sol:** Given data:

Let characteristic impedance

$$(Z_c) = \sqrt{\frac{Z_{sc}}{Y_{oc}}} = \sqrt{\frac{1.0}{1.0}} = 1 \text{ p.u.}$$

$$= \sqrt{\frac{\text{impedance/km}}{\text{admittance/km}}}$$

Given that for a given line 30% series capacitive compensation is provided. Hence the series impedance of line is 0.7 or (70%) of original value.

$$\therefore Z_{\text{new}} = \sqrt{\frac{0.7}{1.0}} = 0.836 \text{ p.u.}$$

$$\text{Surge impedance loading (SIL)} = \frac{V^2}{Z_c}$$

$$\Rightarrow \text{SIL} \propto \frac{1}{Z_c}$$

$$\frac{(\text{SIL})_2}{(\text{SIL})_1} = \frac{Z_{c1}}{Z_{c2}}$$

$$(\text{SIL}^2) = \frac{1.0}{0.836} \times 2280 \times 10^6 = 2725 \times 10^6 = 2725 \text{ MW.}$$

**08. Ans: (b)**

**Sol:** 3 – phase, 11kV, 50Hz, 200kW load, at power factor = 0.8

kVAR demand of Load

$$(Q_1) = \frac{200 \times 10^3}{0.8} \times \sin(\cos^{-1} 0.8)$$

$$\therefore Q_1 = 150 \text{ kVAR}$$

kVAR demand of load at upf = 0

So as to operate the load at upf, we have to supply the 150 kVAR by using capacitor bank.

$\therefore$  kVAR rating of  $\Delta$ - connected

$$\text{capacitor bank} = \frac{3V_{ph}^2}{X_{C_{ph}}} = 150 \text{ kVAR}$$



$$\frac{3 \times (11000)^2}{X_{C_{ph}}} = 150 \times 10^3$$

$$X_{C_{ph}} = 2420 \, \Omega$$

$$\frac{1}{2\pi f C} = 2420 \, \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 2420}$$

$$= 1.3153 \, \mu\text{F}$$

$$\approx 1.316 \, \mu\text{F}$$

**09. Ans: (c)**

**Sol:** Given Data:

Let the initial power factor angle =  $\phi_1$

After connecting a capacitor, the power factor angle =  $\phi_2$

$$\text{Given } \phi_2 = \cos^{-1} 0.97$$

$$= 14.07^\circ$$

$P(\tan \phi_1 - \tan \phi_2) = \text{kVAR supplied by capacitor}$

$$4 \times 10^6 (\tan \phi_1 - \tan 14.07) = 2 \times 10^6$$

$$\phi_1 = 36.89^\circ$$

$$\cos \phi_1 = 0.8 \text{ lag}$$

Hence if the capacitor goes out of service the load power factor becomes 0.8 lag

**10. Ans: (d)**

**Sol:** The appearance will inject leading VARs into the system is induction generator, under excited synchronous generator, under excited synchronous motor and induction motor.

### Under ground Cables

**01. Ans: D = 3.9707 cm;**

$$E_{\text{rms}} = 90.4 \text{ kV/cm (rms)}$$

$$I_C = 20.735 \text{ A}$$

**Sol:** Given data:

$$L = 5 \text{ km}$$

$$C = 0.2 \, \mu\text{F/km}$$

$$\epsilon_r = 3.5 \quad \text{core } d = 1.5 \text{ cm,}$$

$$r = 0.75 \text{ cm}$$

$$V = 66 \text{ kV, } 50\text{Hz} = f$$

$$D = ?$$

$$E_{\text{r(rms)}} = ? \quad I_{\text{c(rms)}} = ?$$

(a) Concentric cable: core is placed exactly at the center of the cable

$$C_{\text{ph}} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(D/d)} \text{ F/M}$$

$$C = 0.2 \times 10^{-6} \times 10^3$$

$$C = 0.2 \times 10^{-3}$$

$$0.2 \times 10^{-3} = \frac{2\pi \times 8.854 \times 10^{-12} \times 3.5}{\ln\left(\frac{D}{d}\right)}$$

$$\ln\left(\frac{D}{d}\right) = \frac{2\pi \times 8.854 \times 10^{-12} \times 3.5}{(0.2 \times 10^{-3})}$$

$$= 9.731 \times 10^{13}$$

$$\ln\left(\frac{D}{d}\right) = 0.9731$$

$$\frac{D}{d} = e^{0.9731}$$

$$D = d \times e^{0.9731} = 1.5 \times e^{0.9731}$$

$$D = 3.9707 \text{ cm}$$

$$(b) \quad E_{\text{r(rms)}} = \frac{V}{r \ln\left(\frac{R}{r}\right)} \quad \frac{R}{r} = \frac{D}{d}$$

$$= \frac{66}{0.75 \ln\left(\frac{3.97}{1.5}\right)}$$

$$E_{\text{rms}} = 90.413 \text{ kV/cm}$$

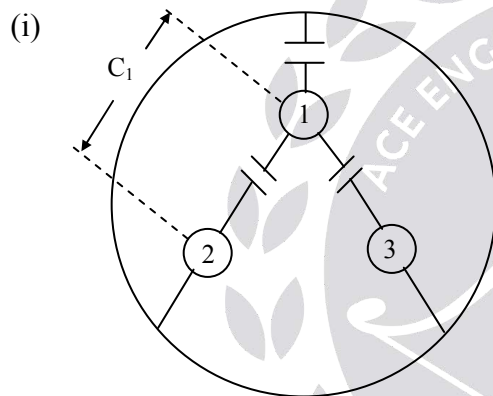
(c) At charging current  $= I_C \times l$   
 $= 4.146 \times 5$   
 $= 20.73 \text{ A}$

**02. Ans: (b)**

**Sol:** Given data

:

$$V = 11 \text{ kV}; C_1 = 0.6 \mu\text{F}; C_2 = 0.96 \mu\text{F}$$

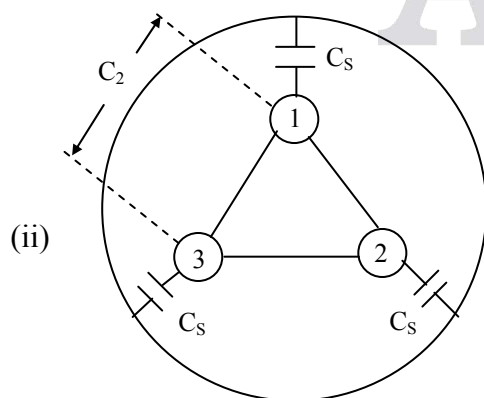


$$C_1 = 0.6 \mu\text{F} \text{ (given)}$$

From network

$$C_1 = C_S + 2 C_C$$

$$\Rightarrow C_S + 2 C_C = 0.6 \mu\text{F} \dots\dots (1)$$



$$C_2 = 0.96 \mu\text{F} \text{ (given)}$$

From network

$$C_2 = 3 C_S \Rightarrow 0.96 \mu\text{F}$$

$$C_S = 0.32 \mu\text{F}$$

From (1)

$$0.32 + 2 C_C = 0.6$$

$$C_C = 0.14 \mu\text{F}$$

Effective capacitance from core to neutral

$$C/\text{ph} = C_S + 3 C_C$$

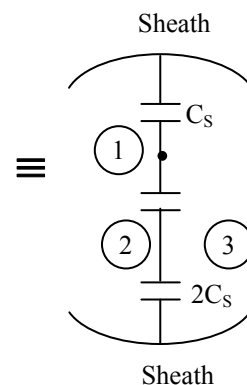
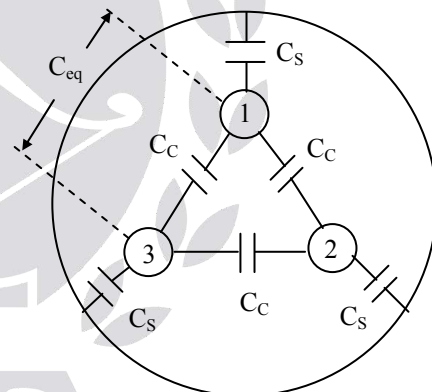
$$= 0.32 + 3 \times 0.14 = 0.74 \mu\text{F}$$

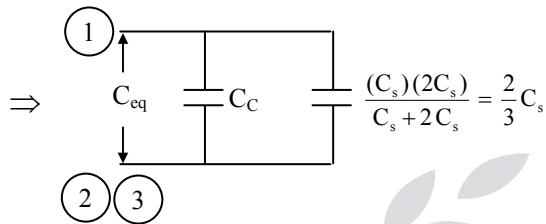
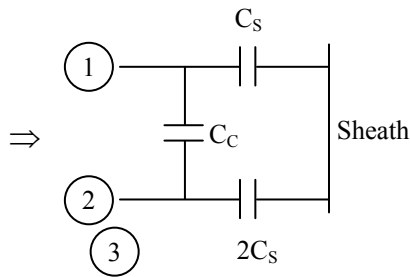
**03. Ans: (b)**

**Sol:** Given data:

$$C_c = 0.5 \mu\text{F}$$

$$C_s = 0.3 \mu\text{F}$$





$$\begin{aligned}\therefore C_{eq} &= \frac{2}{3}C_s + C_c \\ &= 2 \times 0.5 + \frac{2}{3} \times 0.3 \\ &= 1.2 \mu\text{F}\end{aligned}$$

**04. Ans: 38.32kW (Range: 37.5 to 39.5)**

**Sol:** Given data

$L = 40 \text{ km}$

3-core ground cable =  $12.77 \text{ kVAr/km}$

$f = 50 \text{ Hz}$

Dielectric material is 0.025

$\cos\phi = 0.025$

$\phi = \cos^{-1}(0.025)$

$\phi = 88.56$

$$\tan\phi = \frac{Q}{P}$$

$$\begin{aligned}P &= \frac{3 \times 12.77 \times 40}{\tan(88.56)} \\ &= 38.32 \text{ kW}\end{aligned}$$

**05. Ans: (a)**

**Sol:** Given data:

$$C_1 = 0.2 \times 10^{-6} \text{ F}, C_2 = 0.4 \times 10^{-6} \text{ F}$$

$$f = 50 \text{ Hz}$$

$$V = 11 \text{ kV}$$

$$C_{ph} = C_2 + 3C_1$$

$$= 0.4 \times 10^{-6} + 3 \times 0.2 \times 10^{-6}$$

$$= 1 \times 10^{-6} = 1 \mu\text{F}.$$

$$\therefore \text{Perphase charging current} = V_{ph} \omega C_{ph}$$

$$= \frac{11}{\sqrt{3}} \times 10^3 \times 2\pi \times 50 \times 1 \times 10^{-6} = 2 \text{ A}.$$

### Overhead line Insulators

**01. Ans: (d)**

**Sol:** Given data:

$$n = 20 ; 3-\phi;$$

$$V = 400 \text{ kV}; \eta = 80\%$$

$$\eta_{\text{string}} = \frac{V_{ph}}{n \times V_{20}}$$

$$0.8 = \frac{400 \text{ k} / \sqrt{3}}{20 \times V_{20}}$$

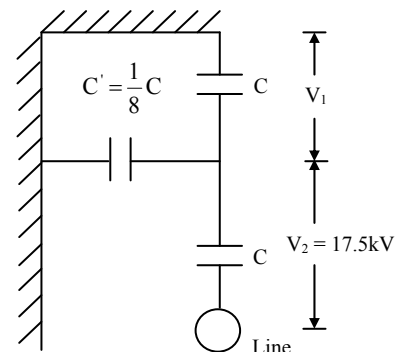
$$\therefore V_{20} = \frac{25}{\sqrt{3}} \text{ kV}$$

**02. Ans: (b)**

**Sol:** Given data:

$$V_2 = 17.5 \text{ kV}$$

$$C' = 1/8 C$$



$$V_1 + V_2 = V$$

$$V_2 = (1 + K) V_1$$

$$V_1 = \frac{V_2}{1+K} = \frac{17.5}{1+\frac{1}{8}} \text{ kV}$$

$$V_1 = 15.55 \text{ kV}$$

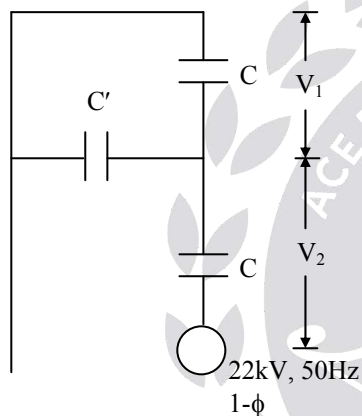
$$V = V_1 + V_2 = 33.05 \text{ kV}$$

**03. Ans: (b)**

**Sol:** Given data:

$$V = 22 \text{ kV}$$

$$f = 50 \text{ Hz}$$



$$\eta_{\text{string}} = \frac{V_1 + V_2}{2V_2} = \frac{V_1 + (1+K)V_1}{2 \times V_1(1+K)}$$

$$= \frac{2+K}{2} = \frac{2+1}{2(1+1)} = \frac{3}{4} = 75\%$$

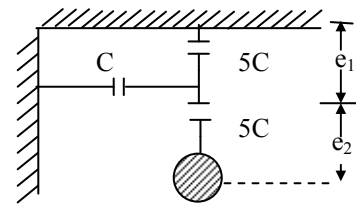
**04. Ans: (b)**

**Sol:** Given data:

$$f = 50 \text{ Hz}$$

$$V = 11 \text{ kV}$$

Capacitance of insulators is 5 times the shunt capacitance between the link and the ground.



$$e_2 = e_1 (1 + K)$$

$$e_1 + e_2 = \frac{11}{\sqrt{3}}$$

$$K = \frac{C}{5C} = \frac{1}{5} = 0.2$$

$$\therefore e_1 (1 + K) + e_1 = \frac{11}{\sqrt{3}} \times 10^3$$

$$e_1 (2 + K) = \frac{11}{\sqrt{3}} \times 10^3$$

$$e_1 = 2.8867 \approx 2.89 \text{ kV}$$

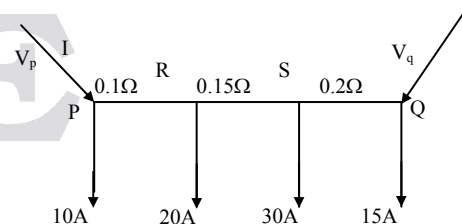
$$e_2 = e_1 (1 + K)$$

$$= 2.8867 \times 1.2 = 3.46 \text{ kV.}$$

### Distribution Systems

**01. Ans: (a)**

**Sol:** Given data:



Let " $V_D$ " be the drop of voltage in line

Applying KVL,

$$V_P - V_D - V_Q = 0$$

$$V_P - V_Q = V_D$$

$$V_D = V_P - V_Q = 3V$$

$$\text{But } V_D = (I - 10)0.1 + (I - 30)0.15 + (I - 60)0.2$$

$$3 = 0.45I - 17.5$$

$$I = \frac{20.5}{0.45} = 45.55A$$

$$\therefore V_D = 35.55 \times 0.1 + 15.55 \times 0.15 + 14.45 \times 0.2$$

Here we have to take magnitude only

$$\therefore V_D = 8.77$$

$$\therefore V_P = 220 + 8.77 = 228.7V$$

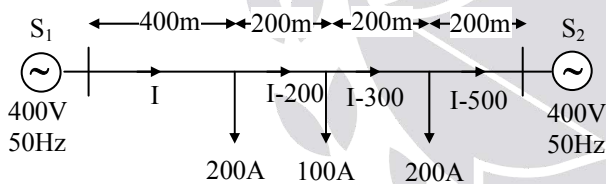
$$V_Q = V_P - 3 = 225.7V.$$

**02. Ans: (d)**

**Sol:** Given data:

All the loads are at unity factor. Let us take current in 400 m section as  $I$  such that currents in remaining sections are shown.

Assume that loop resistance feeder  $r\Omega / m$  (reactance is neglected).



KVL From  $S_1$  and  $S_2$  is given as

$$V_{S1} - V_{S2} = I(400r) + (I - 200)(200r) + (I - 300)(200r) + (I - 500)(200r)$$

$$0 = 400I + 200I - 200 \times 200 + 200I - 300 \times 200 + 200I - 500 \times 200$$

$$1000I = 200000$$

$$I = \frac{200000}{1000} \Rightarrow I = 200A \text{ as } I = 200A,$$

Contribution to load at point P from source

$S_1$  is 0A from source  $S_2$  is 100 A.

**03 Ans:  $V_s = 271.04 \angle 2.78^\circ$ ,  $pf = 0.74$  (lag)**

**Sol:** Given Data:

$$V_r = 220$$

$$I_s = 80 \angle -36.86^\circ + 50 \angle -45^\circ$$

$$= 129.9 \angle -39.98^\circ$$

$$V_s = V_r + \Delta V$$

$$\Delta V = (80 \angle -36.86^\circ)(0.15 + j0.2) +$$

$$(129.9 \angle -39.98^\circ)(0.15 + j0.2)$$

$$= 52.45 \angle -14.33^\circ$$

$$V_s = 220 \angle 0^\circ + 52.45 \angle 14.33^\circ$$

$$= 271.12 \angle 2.74^\circ$$

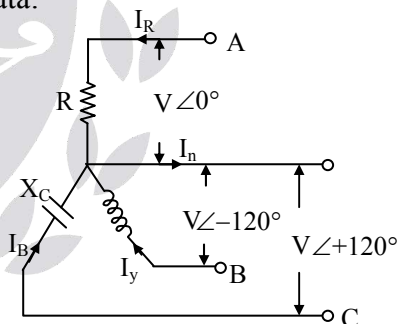
$$P.F. = \cos(\text{angle between } V_s \text{ and } I_{sc})$$

$$= \cos(42.72^\circ)$$

$$= 0.734 \text{ lag}$$

**04. Ans: (b)**

**Sol:** Given data:



$$I_R + I_y + I_B = I_n = 0$$

$$\frac{V^2}{R} = 4000, R = \frac{230^2}{4000} = 13.225$$

$$\Rightarrow I_n = 0 = \frac{V \angle 0^\circ}{R} + \frac{V \angle -120^\circ}{\omega L \angle 90^\circ} + V \omega C \angle +120^\circ \angle +90^\circ$$

$$\Rightarrow \frac{V}{R} + \frac{V}{\omega L} \angle -210^\circ + V \omega C \angle +210^\circ = 0$$

$$\Rightarrow \frac{V}{R} + \frac{V}{\omega L} \cos 210^\circ + V \omega C \cos 210^\circ = 0$$

$$\Rightarrow -\frac{V}{\omega L} \sin 210^\circ + V\omega C \sin 210^\circ = 0$$

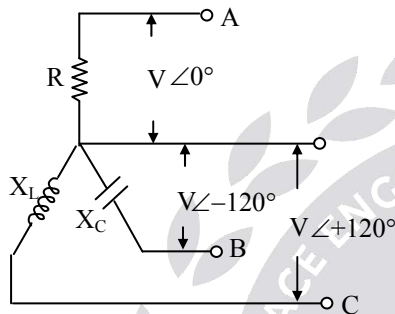
$$\omega = \frac{1}{\sqrt{LC}} \dots\dots\dots (i)$$

$$\frac{1}{R} = \left( \frac{\omega^2 LC + 1}{\omega L} \right) \times \frac{\sqrt{3}}{2}$$

$$L = 72.9 \text{ mH}$$

$$C = 139.02 \text{ } \mu\text{F}$$

If suppose ' $X_C$ ' on phase B,  $X_L$  on phase C



$$\frac{V}{R} + \frac{V}{X_C} + \frac{V}{X_L} = 0$$

$$\frac{1}{R} + \omega C \angle -30^\circ + \frac{1}{\omega L} \angle +30^\circ = 0$$

$$\frac{1}{R} + \omega C \cos 30^\circ + \frac{1}{\omega L} \cos 30^\circ \neq 0$$

$$\omega C \sin 30^\circ = \frac{1}{\omega L} \sin 30^\circ$$

1<sup>st</sup> condition never be zero, because all the positive parts never becomes zero

### HVDC Transmission

01. Ans: (d)

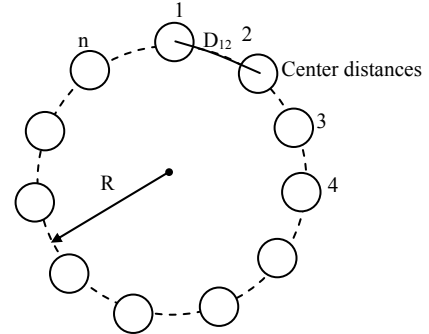
02. Ans: (c)

03. Ans: (b)

### Solutions for Conventional Practice Questions

01. **Proof:**  $\text{GMD} = R[n]^{\frac{1}{n-1}}$

**Sol:**

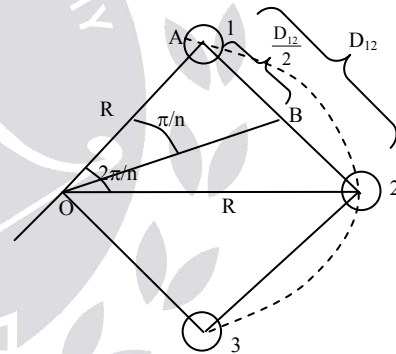


GMD among sub-conductors is given by

GMD

$$= \sqrt[n]{\text{GMD}_1 \times \text{GMD}_2 \times \text{GMD}_3 \times \dots \times \text{GMD}_n}$$

$$\text{GMD}_1 = [D_{12} \times D_{13} \times D_{14} \times \dots \times D_{1n}]^{\frac{1}{n-1}}$$



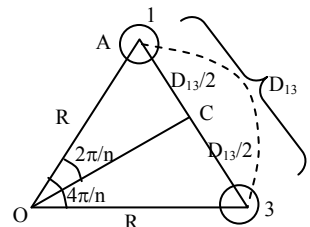
$$\text{From the } \Delta OAB \sin \frac{\pi}{n} = \frac{D_{12}/2}{R}$$

$$D_{12} = 2R \sin \frac{\pi}{n}$$

From  $\Delta OAC$

$$\sin \frac{2\pi}{n} = \frac{D_{13}/2}{R}$$

$$D_{13} = 2R \sin \frac{2\pi}{n}$$



$$\text{Similarly } D_{14} = 2R \sin \frac{3\pi}{n}$$

$$D_{15} = 2R \sin \frac{4\pi}{n} \dots\dots\dots$$

$$D_{1n} = 2R \sin \frac{(n-1)\pi}{n}$$

$$D_{12} \times D_{13} \times \dots D_{1n}$$

$$= 2R \sin \frac{\pi}{n} \times 2R \sin \frac{2\pi}{n} \times 2R \sin \frac{3\pi}{n} \times \dots \times 2R \sin \frac{(n-1)\pi}{n}$$

$$= R^{n-1} \left[ 2 \sin \frac{\pi}{n} \times 2 \sin \frac{2\pi}{n} \times 2 \sin \frac{3\pi}{n} \times \dots \times 2 \sin \frac{(n-1)\pi}{n} \right]$$

$$n = 2 \Rightarrow 2 \sin \frac{\pi}{2} = 2$$

$$n = 3 \Rightarrow 2 \sin \frac{\pi}{3} \times 2 \sin \frac{2\pi}{3} = 3$$

$$n = 4 \Rightarrow 2 \sin \frac{\pi}{4} \times 2 \sin \frac{2\pi}{4} \times 2 \sin \frac{3\pi}{4} = 4$$

$$2 \sin \frac{\pi}{n} \times 2 \sin \frac{2\pi}{n} \times \dots \times 2 \sin \frac{(n-1)\pi}{n} = n$$

$$D_{12} \times D_{13} \times \dots \times D_{1n} = R^{n-1} n$$

$$GMD_1 = [D_{12} \times D_{13} \times D_{14} \times \dots \times D_{1n}]^{\frac{1}{n-1}}$$

$$= [R^{n-1} n]^{\frac{1}{n-1}} = R [n]^{\frac{1}{n-1}}$$

$$GMD_1 = R [n]^{\frac{1}{n-1}}$$

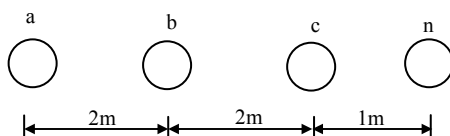
GMD of each sub conductor will be same so that overall GMD is any one of the conductor

$$GMD = GMD_1$$

$$GMD = R [n]^{\frac{1}{n-1}}$$

**02. Ans: 3.87 V/km**

**Sol:**



$$\bar{I}_a = 20 + j20A, \bar{I}_b = -15 + j25 A,$$

$$\bar{I}_c = 30 - j10A$$

Mutual magnetic flux linkage in conductors to 'n' (neutral wire)

$$\Psi_n = \bar{I}_a \times 0.2 \ln \left( \frac{1}{5} \right) + \bar{I}_b \times 0.2 \ln \left( \frac{1}{3} \right) + \bar{I}_c \times 0.2 \ln \left( \frac{1}{1} \right)$$

$$\psi_n = (20 + j20) 0.2 \ln \left( \frac{1}{5} \right) + 0.2 \ln \left( \frac{1}{3} \right) \times (-15 + j25) + 0$$

$$= 12.33 \angle -104.75 \text{ mWb-T/km (rms value)}$$

$$\text{Induced emf in neutral wire, } e_n = \frac{d\psi_n}{dt}$$

$$e_n = j\omega \psi_n$$

Magnitude of induced emf

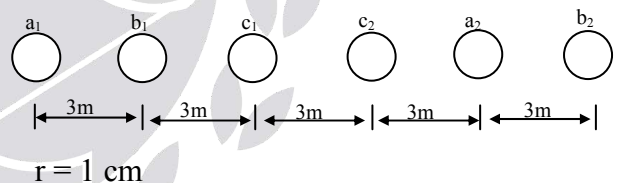
$$|e_n| = \omega |\psi_n|$$

$$= 2\pi \times 50 \times 12.33 \text{ mV/km}$$

$$= 3.87 \text{ V/km}$$

**03. Ans: 0.631 mH/km**

**Sol:**



$$L/ph = 2 \times 10^{-7} \ln \left( \frac{GMD_{\text{system}}}{\text{Self GMD}_{\text{system}}} \right)$$

$$\text{Self GMD}_{\text{system}} =$$

$$\sqrt[3]{\text{self GMD}_a \times \text{self GMD}_b \times \text{self GMD}_c}$$

$$\text{Self GMD}_a = [(r' \times d_{a1a2})(r' \times d_{a2a1})]^{\frac{1}{4}}$$

$$= [r'_1 d_{a2a1}]^{\frac{2}{4}} = \sqrt{r'_1 \times d_{a1a2}}$$

$$\text{Self GMD}_a = \sqrt{0.7788 \times 1 \times 10^{-2} \times 12}$$

$$= 0.305 \text{ m}$$

$$\text{Self GMD}_b = \sqrt{r' \times d_{b1b2}}$$



$$= \sqrt{0.7788 \times 1 \times 10^{-2} \times 12}$$

$$= 0.305 \text{ m}$$

$$\text{Self GMD}_C = \sqrt{r'_1 \times d_{c1c2}}$$

$$= \sqrt{0.7788 \times 1 \times 10^{-2} \times 3}$$

$$\text{Self GMD}_C = 0.152 \text{ m}$$

$$\text{Self GMD}_{\text{system}} = \sqrt[3]{0.305 \times 0.305 \times 0.152}$$

$$\text{Self GMD}_{\text{system}} = 0.241 \text{ m}$$

$$\text{GMD}_{\text{sy}} = \sqrt[3]{\text{GMD}_{\text{abeq}} \text{GMD}_{\text{bceq}} \text{GMD}_{\text{caeq}}}$$

$$D_{\text{abeq}} = [d_{a1b1} \times d_{a2b1} \times d_{a1b2} \times d_{a2b2}]^{\frac{1}{4}}$$

$$D_{\text{ab eq}} = (3 \times 15 \times 9 \times 3)^{\frac{1}{4}} = 5.903 \text{ m}$$

$$D_{\text{bc eq}} = [d_{b1c1} \times d_{b1c2} \times d_{c1b2} \times d_{b2c2}]^{\frac{1}{4}}$$

$$= (3 \times 6 \times 9 \times 6)^{\frac{1}{4}} = 5.58 \text{ m}$$

$$D_{\text{ca eq}} = [d_{c1a1} \times d_{c1a2} \times d_{c2a1} \times d_{c2a2}]^{\frac{1}{4}}$$

$$D_{\text{ca eq}} = 5.58 \text{ m}$$

$$\text{GMD}_{\text{system}} = \sqrt[3]{5.903 \times 5.58 \times 5.58} = 5.68 \text{ m}$$

$$L/\text{ph} = 2 \times 10^{-7} \ln \left( \frac{5.68}{0.242} \right) \text{ H/m}$$

$$L/\text{ph} = 0.631 \text{ mH/km}$$

04.

**Sol:** 1- $\phi$ , 8 kV, 50 Hz, 50 km length line.

$$r = 1.5 \text{ cm}$$

$$d = 2 \text{ m}$$

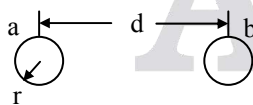
for each conductor of 50 km length  $r = 2.5$

$\Omega$

$$\text{So, } R(\text{total}) = 2 \times 2.5 = 5 \Omega$$

Inductance of each conductor / km

$$L_a = 0.2 \ell \ln \left( \frac{d}{r} \right) \text{ mH/km}$$



$$= 0.2 \ell \ln \left( \frac{2}{0.7788 \times 1.5 \times 10^{-2}} \right)$$

$$= 1.029 \text{ mH/km}$$

Similarly

$$L_b = 1.029 \text{ mH/km}$$

$$L/\text{km (Loop)} = L_a + L_b = 2.058 \text{ mH/km}$$

For  $\ell = 50 \text{ km}$

$$L = 2.058 \times 50 \text{ mH}$$

$$= 102.9 \text{ mH}$$

Inductive reactance

$$X_L = \omega L$$

$$= 2\pi \times 50 \times 102.9 \times 10^{-3}$$

$$= 32.33 \Omega$$

Capacitance of 'a' to 'n'

$$C_{an} = \frac{2\pi\epsilon_0\epsilon_r}{\ell \ln \left( \frac{d}{r} \right)}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times 1}{\ell \ln \left( \frac{2}{1.5 \times 10^{-2}} \right)}$$

$$= 11.36 \times 10^{-12} \text{ F/m}$$

$$= 11.36 \times 10^{-9} \text{ F/km}$$

Capacitance from 'a' to 'b'

$$C_{ab} = \frac{C_{an}}{2}$$

$$= \frac{11.36 \times 10^{-9}}{2} \text{ F/km}$$

$$= 5.68 \times 10^{-9} \text{ F/km}$$

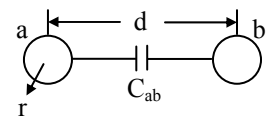
$$\text{For } \ell = 50 \text{ km, } C_{ab} = 5.68 \times 10^{-9} \times 50 \text{ F}$$

$$C_{ab} = 284 \times 10^{-9} \text{ F}$$

Shunt admittance,

$$Y = j\omega \cdot C_{ab}$$

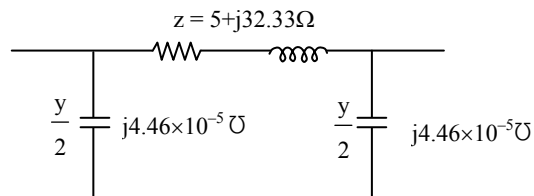
$$= j 2\pi \times 50 \times 284 \times 10^{-9}$$



$$= j8.92 \times 10^{-5} \text{ } \Omega$$

$$(i) Z = 5 + j32.33 \text{ } \Omega ; \quad Y = j8.92 \times 10^{-5} \text{ } \Omega$$

(ii) Nominal  $\pi$  network



(iii) Load data,  $V_r = 8 \text{ kV}$

$$P_r = 720 \text{ kW}$$

Voltage regulation = 25%

$$\cos \phi_r = ?$$

$$\text{voltage regulation} = \frac{\frac{|V_s|}{|A|} - |V_r|}{|V_r|}$$

for nominal- $\pi$  model

$$A = 1 + \frac{ZY}{2}$$

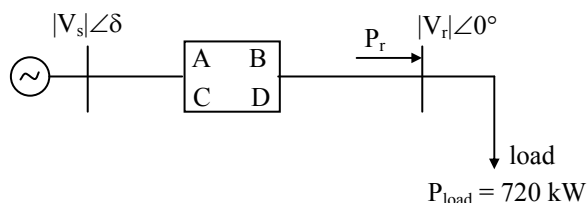
$$= 1 + \frac{(5 + j32.33)(j8.92 \times 10^{-5})}{2}$$

$$A = 0.998 \angle 0.013^\circ$$

$$\text{Now } \frac{|V_s|}{|A|} - 8 = 0.25 \times 8$$

$$\frac{|V_s|}{|A|} = 0.25 \times 8 + 8$$

$$|V_s| = 9.98 \text{ kV}$$



$$P_r = P_{\text{load}} = 720 \text{ kW}$$

Equation for ' $P_r$ ' is

$$P_r =$$

$$\frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \cos(\beta - \alpha)$$

From ABCD parameters

$$A = 0.998 \angle 0.012^\circ \rightarrow \alpha$$

$$B = Z = 5 + j32.33 = 32.68 \angle 81.2^\circ \rightarrow \beta$$

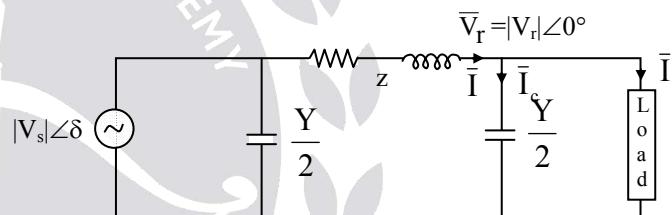
$$P_r = P_{\text{load}} = 720 \text{ kW} = 0.72 \text{ MW}$$

$$\therefore 0.72 =$$

$$\frac{9.98 \times 8}{32.68} \cos(81.2^\circ - \delta) - \frac{0.998}{32.68} (8)^2 \times \cos(81.2 - 0.013^\circ)$$

$$\delta = 15.87^\circ$$

From nominal- $\pi$  model.



$$\bar{I} = \frac{|V_s| \angle \delta - |V_r| \angle 0^\circ}{Z}$$

$$= \frac{9.98 \angle 15.87 - 8 \angle 0^\circ}{32.68 \angle 81.2^\circ} \text{ kA}$$

$$= 96.79 \angle -21.58^\circ \text{ A}$$

$$\text{Now, } \bar{I}_c = \bar{V}_r \cdot \frac{Y}{2}$$

$$= 8 \times 10^3 \times j4.46 \times 10^{-5}$$

$$= j0.356 \text{ A}$$

by KCL

$$\bar{I}_r = \bar{I} - \bar{I}_c$$

$$\bar{I}_r = (96.79 \angle -21.58^\circ) - (j0.356)$$

$$= 96.92 \angle -21.87^\circ \text{ A}$$

Power factor of load,  $\cos\phi_r = \cos(21.87)$   
 $= 0.928 \text{ lag}$

$$(iii) \eta = \frac{P_r}{P_r + P_{loss}}$$

$$P_{loss} = I^2 \times R$$

$$= (96.79)^2 \times 5$$

$$\eta = \frac{720}{720 + 46.84} \times 100 = 93.8\%$$

**05.**

**Sol: (i)** If the shunt admittance of the transmission line is ignored, the relationship between the voltages and currents on this transmission line is

$$V_S = V_R + RI + jX I$$

$$I_S = I_R = I$$

Therefore we can calculate the current in the transmission line as

$$I = \frac{V_S - V_R}{R + jX}$$

$$I = \frac{80,000 \angle 10^\circ - 76,000 \angle 0^\circ}{10.3 + j52.5 \Omega}$$

$$= 265 \angle -0.5^\circ \text{ A}$$

The real and reactive power supplied by this transmission lines is

$$P = 3V_{\phi,R} \cos\theta$$

$$= 3(76000) \times 265 \times \cos(0.5) = 60.4 \text{ MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta$$

$$= 3(76,000) \times 265 \times \sin(0.5) = 0.53 \text{ MVAR}$$

**(ii)** If the sending end voltage is changed to  $82 \angle 10^\circ \text{ kV}$ , the current is

$$I = \frac{82,000 \angle 76000 \angle 0^\circ}{10.3 + j52.52} = 280 \angle -7.7^\circ \text{ A}$$

The real and reactive power supplied by this transmission is

$$P = 3V_{\phi,R} I_\phi \cos\theta$$

$$= 3 \times 76,000 \times 280 \times \cos(7.7)$$

$$= 63.3 \text{ MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta$$

$$= 3(76,000) \times 280 \times \sin(7.7)$$

$$= 8.56 \text{ MVAR}$$

**(iii)** If the sending end voltage is changed to  $80 \angle 10^\circ \text{ kV}$ , the current

$$I = \frac{80,000 \angle 15^\circ - 76,000 \angle 0^\circ}{10.3 + j52.5 \Omega}$$

$$= 388 \angle 7.2^\circ \text{ A}$$

The real and reactive power supplied by the transmission line

$$P = 3V_{\phi,R} I_\phi \cos\theta$$

$$= 3 \times 76,000 \times 388 \times \cos(-7.2)$$

$$= 87.8 \text{ MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta$$

$$= 3 \times 76,000 \times 388 \times \sin(-7.2)$$

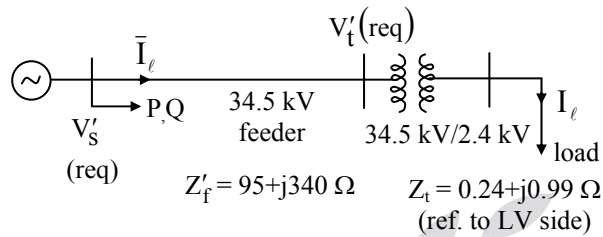
$$= -11 \text{ MVAR}$$

**(iv)** From the above results, we can see that real power flow can be adjusted by changing the phase angle between the two voltages at two ends of the transmission

line, while reactive power flow can be changed by changing of reactive magnitude of the two voltages on either side of the transmission line.

06.

Sol:



Load data given,

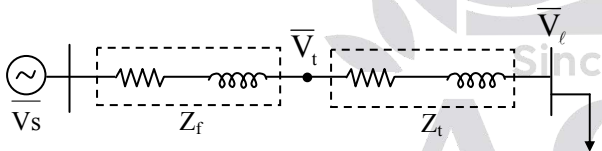
$$P_{\text{load}} = 200 \text{ kW}, \cos\phi_L = 0.85 \text{ lead}$$

$$V_\ell = 2.25 \text{ kV}$$

Impedance of feeder referred on to secondary side of transformer.

$$\begin{aligned} Z_f' &= Z_f \times K^2 \\ &= (95 + j340) \times \left(\frac{2.4}{34.5}\right)^2 \\ &= 0.46 + j1.64 \Omega \end{aligned}$$

The equivalent circuit given as



Let  $V_\ell$  as reference

$$V_\ell = 2.25 \angle 0^\circ \text{ kV}$$

$$\text{From data, } I_\ell = \frac{P_\ell}{V_\ell \cdot \cos\phi_\ell} = \frac{200}{2.25 \times 0.85}$$

$$I_\ell = 104.57 \text{ A}$$

$$\text{Load current, } \bar{I}_\ell = 104.57 \angle \cos^{-1}(0.85)$$

$$\bar{I}_\ell = 104.57 \angle 31.7^\circ$$

$$(a) \text{ by using KVL, } \bar{V}_s = \bar{V}_\ell + \bar{I}_\ell (Z_f + Z_t)$$

$$\bar{V}_s = 2250 \angle 0^\circ + (104.57 \angle 31.7^\circ) \times (0.7 + j2.63)$$

$$\bar{V}_s = 2.187 \angle 7.17^\circ \text{ kV}$$

Feeder input voltage reference to secondary side.

Now, actual input voltage of feeder

$$\begin{aligned} V_s^{-1} &= \bar{V}_s \times \frac{34.5}{2.4} \\ &= 2.187 \angle 7.17^\circ \times \frac{34.5}{2.4} \end{aligned}$$

$$V_s^1 = 31.39 \angle 7.17^\circ \text{ kV}$$

$$(b) \text{ by using KVL, } \bar{V}_t = \bar{V}_\ell + \bar{I}_\ell \times Z_t$$

$$\bar{V}_t = 2250 \angle 0^\circ + (104.57 \angle 31.7^\circ) \times (0.24 + j0.99)$$

$$\bar{V}_t = 2.22 \angle 2.6^\circ \text{ kV}$$

Input voltage of transformer referred to secondary side.

Actual Input voltage of transformer

$$\bar{V}_t' = \bar{V}_t \times \frac{34.5}{2.4} = 2.22 \angle 2.6^\circ \times \frac{34.5}{2.4}$$

$$\bar{V}_t' = 31.91 \angle 2.6^\circ \text{ kV}$$

(c) complex power supplied by secondary side of feeder

$$S = \bar{V}_s \cdot \bar{I}_\ell \text{ (ref. to secondary side)}$$

$$= 2.187 \angle 7.17^\circ \times 104.57 \angle -31.7^\circ \text{ kVA}$$

$$= 207.9 - j95.2 \text{ kVA} \rightarrow \text{same value ref to primary side.}$$

07.

**Sol:** 20 km long, 50 Hz, 1- $\phi$ , 2-wire line resistance of each conductor,  $r = 0.02 \Omega/\text{km}$  total resistance for  $l = 20 \text{ km}$  is  $R = 2 \times 0.02 \times 20 \Omega = 0.8 \Omega$

Inductance of each conductor,

$$L = 0.4 \text{ mH/km}$$

$$\text{Loop inductance per km, } L_{\text{Loop}} = 2 \times 0.4 \\ = 0.8 \text{ mH/km}$$

$$\text{For } l = 20 \text{ km, } L = 0.8 \text{ mH/km} \times 20 \text{ km} \\ 20 \times 0.8 = 16 \text{ mH}$$

$$\text{Inductive reactance } X = 2\pi fL \\ = 2\pi \times 50 \times 16 \times 10^{-3} \Omega \\ = 5.02 \Omega$$

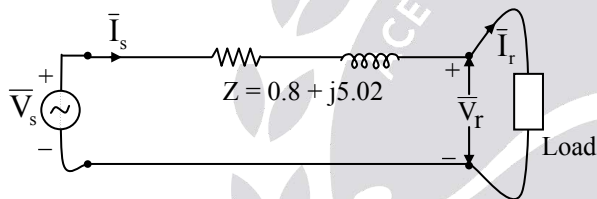
Load kept at receiving end

$$P_r = 5 \text{ MW}$$

$$\cos\phi_r = 0.8 \text{ lag}$$

$$V_r = 10 \text{ kV (fixed)}$$

(i) equivalent circuit is given as



Let ' $V_r$ ' as the reference

$$\text{So, } \bar{V}_r = 10,000 \angle 0^\circ \text{ V}$$

$$\text{as } P_r = V_r \cdot I_r \cdot \cos\phi_r$$

$$I_r = \frac{5 \times 10^6}{10 \times 10^3 \times 0.8} \\ = 625 \text{ A}$$

$$\text{Now, } \bar{I}_r = 625 \angle -\cos^{-1}(0.8) \\ = 625 \angle -36.86^\circ \text{ A}$$

By KVL for loop

$$\bar{V}_s = \bar{V}_r + \bar{I}_r \cdot Z \\ = 10000 \angle 0^\circ + (625 \angle -36.86^\circ)(0.8 + j5.02) \\ = 12562.02 \angle 10.5^\circ \text{ kV}$$

$$\text{Voltage regulation} = \frac{\left| \frac{V_s}{A} \right| - |V_r|}{|V_r|}$$

As  $|A| = 1$ , for short line,

$$\text{Voltage regulation} = \frac{|V_s| - |V_r|}{|V_r|} \times 100\% \\ = \frac{12.48 - 10}{10} \times 100\% \\ = 24.8\%$$

(ii) The required voltage regulation of line

$$\text{will be voltage regulation} = \frac{24.8}{2} \% =$$

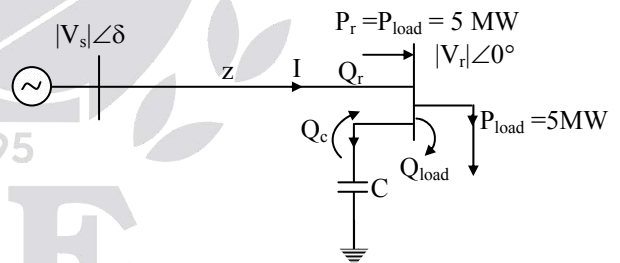
$$12.4\%$$

$$\text{i.e., } \frac{|V_s| - |V_r|}{|V_r|} = 0.124$$

where  $|V_r|$  fixed to 10 kV

$$\therefore \frac{|V_s| - 10}{10} = 0.124$$

$$|V_s| = 11.24 \text{ kV}$$



Equation for ' $P_r$ ' is

$$P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \cos(\beta - \alpha)$$

Where  $|V_s| = 11.24 \text{ kV}$

$$|V_r| = 10 \text{ kV}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 \angle 0^\circ & 5.08 \angle 80.96^\circ \\ 0 & 1 \angle 0^\circ \end{bmatrix}$$

$$|A| = 1, \alpha = 0^\circ, |B| = 5.08 \Omega$$

$$\beta = 80.96$$

From equation (1),

$$5 = \frac{11.24 \times 10}{5.08} \cos(80.96 - \delta) - \frac{1}{5.08} (10)^2 \cos(80.96^\circ)$$

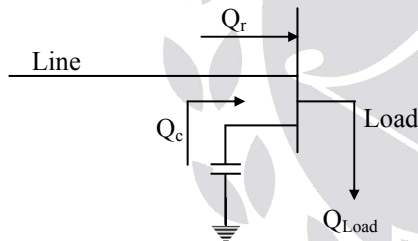
Net reactive power injected into receiving end bus after keeping 'C' bank.

$$Q_r = \frac{|V_s||V_r|}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \sin(\beta - \alpha)$$

$$= \frac{11.24 \times 10}{5.08} \sin(80.96 - 12.4) - \frac{1}{5.08} (10)^2 \sin(80.96 - 0)$$

$$= 1.15 \text{ MVar}$$

at receiving end bus,



$$Q_{\text{Load}} = \frac{P_{\text{load}}}{\cos \phi_{\text{load}}} \times \sin \phi_{\text{load}}$$

$$= \frac{5}{0.8} \times 0.6 \text{ MVar}$$

$$= 3.75 \text{ MVar}$$

$$\text{as } Q_r + Q_c = Q_{\text{load}}$$

$$Q_c = Q_{\text{load}} - Q_r = 3.75 - 1.15$$

$$= 2.6 \text{ MVar}$$

$$\text{as } Q_c = \frac{V_r^2}{X_c}$$

$$X_c = \frac{V_r^2}{Q_c} = \frac{100}{2.6}$$

$$\frac{1}{\omega C} = 38.46 \Omega$$

$$C = \frac{1}{38.6 \times 2\pi \times 50} = 82.8 \mu\text{F}$$

(iii) Case (i):  $P_r = 5 \text{ MW}$

$$P_s = P_r + P_{\text{loss1}}$$

$$P_{\text{loss1}} = I_1^2 R$$

$$\text{Where, } I_1 = 625 \text{ A}$$

$$P_{\text{loss1}} = (625)^2 \times 0.8$$

$$= 0.312 \text{ MW}$$

$$\eta = \frac{P_r}{P_s} = \frac{P_r}{P_r + P_{\text{loss}}} = \frac{5}{5 + 0.312}$$

$$= 94.1\%$$

Case(ii):  $P_r = 5 \text{ MW}$

$$P_s = P_r + P_{\text{loss2}}$$

$$I_2 = \frac{|V_s| \angle \delta - |V_r| \angle 0^\circ}{Z}$$

$$= \frac{11.24 \angle 12.4^\circ - 10 \angle 0^\circ}{5.08 \angle 80.96^\circ} \text{ kA}$$

$$I_2 = 0.512 \angle -12.9^\circ \text{ kA}$$

$$\text{Now, } P_{\text{loss2}} = (0.512)^2 \times 0.8 \text{ MW}$$

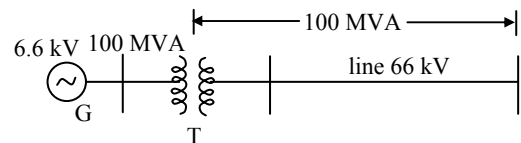
$$= 0.21 \text{ MW}$$

$$\text{Efficiency } \eta = \frac{P_r}{P_s} = \frac{5}{5 + 0.21}$$

$$= 95.9\%$$

08.

Sol:



$$G: 6.6 \text{ kV}, 100 \text{ MVA}, X_g = 0.8 \text{ pu}$$

$$T: 6.6 \text{ kV}/66 \text{ kV}, 100 \text{ MVA},$$

$$Z_T = 0.1 + j0.4 \text{ pu}$$

Line:  $Z_l$  (series) =  $10 + j30\Omega$

$Y_l$  (shunt) =  $j3.2 \times 10^{-4} \text{ } \bar{U}$

Choose base values as 6.6 kV, 100 MVA  
at 'G' location on the Transmission line

$$Z_{\text{base}} = \frac{[\text{kV}_{\text{base(LL)}}]^2}{\text{MVA}_{\text{base(3-\phi)}}}$$

$$= \frac{66^2}{100}$$

$$= 43.56 \Omega$$

For Transmission line

$$Z_{\ell} = \frac{Z_{\ell}(\Omega)}{Z_{\text{base}}}$$

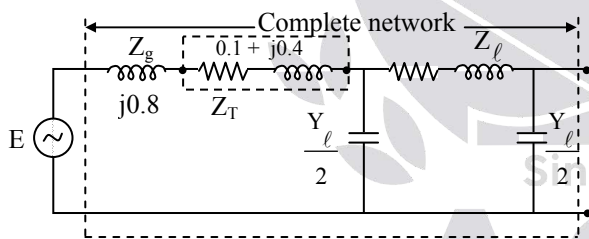
$$= \frac{10 + j30}{43.56} = 0.23 + j0.68 \text{ pu}$$

$$Y_{\ell}(\text{pu}) = Y_{\ell}(\bar{U}) \times Z_{\text{base}}(\Omega)$$

$$= j3.2 \times 10^{-4} \times 43.56$$

$$= j0.014 \text{ pu}$$

Per phase equivalent circuit in pu form



Given that  $E_{\text{(LL)}} = 7 \text{ kV}$

$$E_{\text{pu}} = \frac{E(\text{kV})}{\text{kV}_{\text{base}}} = \frac{7}{6.6} = 1.06 \text{ pu}$$

ABCD of Transmission line

$$\begin{bmatrix} A_{\ell} & B_{\ell} \\ C_{\ell} & D_{\ell} \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_{\ell} Y_{\ell}}{2} & Z_{\ell} \\ Y_{\ell} \left( 1 + \frac{Z_{\ell} Y_{\ell}}{4} \right) & 1 + \frac{Z_{\ell} Y_{\ell}}{2} \end{bmatrix}$$

$$A_{\ell} = D_{\ell} = 1 + \frac{(0.23 + j0.68) \times j0.014}{2}$$

$$= 0.995 \angle 0.096$$

$$B_{\ell} = Z_{\ell}$$

$$= 0.23 + j0.68$$

$$= 0.718 \angle 71.31 \text{ pu}$$

$$C_{\ell} = j0.014 \left[ 1 + \frac{(0.718 \angle 71.3) j0.014}{4} \right]$$

$$= 0.014 \angle 90.05 \text{ pu}$$

ABCD of complete network

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} 1 & 0.1 + j1.2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_{\ell} & B_{\ell} \\ C_{\ell} & D_{\ell} \end{bmatrix}$$

$$\text{Resultant, } A_0 = A_{\ell} + C_{\ell} (0.1 + j1.2)$$

$$= 0.995 \angle 0.096 + (0.014 \angle 90.05)(0.1 + j1.2)$$

$$A_0 = 0.978 \angle 0.176$$

Resultant no load voltage

$$\frac{|V_r|}{A_0} = \frac{|E|}{A_0} \Rightarrow \frac{1.06}{0.978} = 1.08 \text{ pu}$$

$$V_{r0}(\text{kV}) = V_{r0}(\text{pu}) \times \text{kV}_{\text{base(LL)}}$$

$$= 1.08 \times 66 \text{ kV}$$

$$= 71.2 \text{ kV}$$

To calculate power loss under no-load condition: (input real power at no-load)

$$\text{Let } \bar{E} = 1.06 \angle 0^\circ$$

$$\bar{V}_{r0} = \frac{1.06 \angle 0^\circ}{0.978 \angle 0.176^\circ}$$

$$= 1.08 \angle -0.176^\circ$$

from ABCD standard equations.

$$\bar{I}_s = C_{\ell} \bar{V}_r + D_{\ell} \bar{I}_r$$

at no-load,

$$\bar{I}_s = C_0 \bar{V}_{r0}$$

$$= C_{\ell} \bar{V}_{r0} \quad (\because C_0 = C_r)$$



$$= (0.014 \angle 90.05^\circ) \times 1.08 \angle -0.176^\circ$$

$$= 0.015 \angle 89.874^\circ$$

$$P_{\text{loss(at NL)}} = \text{real part } (\bar{E} \cdot \bar{I}_s^*)$$

$$= \text{real part } [1.06 \angle 0^\circ \times 0.015 \angle -89.874^\circ]$$

$$= 3.4965 \times 10^{-5} \text{ pu}$$

$$P_{\text{loss(actual)}} = P_{\text{loss(pu)}} \times S_{\text{base(3-}\phi\text{)}}$$

$$= 3.4965 \times 10^{-5} \times 100 \times 10^6 \text{ W}$$

$$= 3496.5 \text{ W}$$

$$= 3.4965 \text{ kW}$$

09.

**Sol:**  $A = D = 0.91 \angle 2.13^\circ$

$$B = 173.3 \angle 69.9^\circ \Omega$$

$$C = 1.067 \times 10^{-3} \angle 90.7^\circ \text{ S}$$

**Case (i):** Equivalent T – network of a long transmission line

$$C = Y = 1.067 \times 10^{-3} \angle 90.7^\circ$$

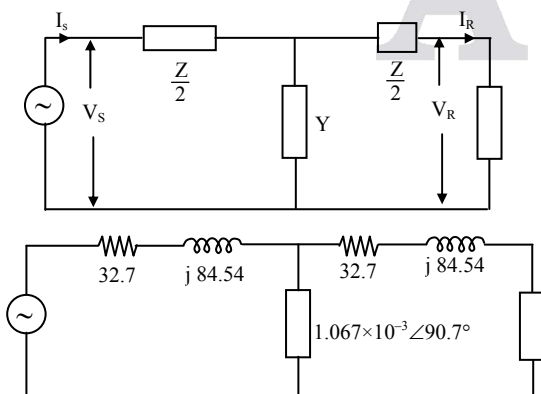
$$A = D = 1 + \frac{ZY}{2} \Rightarrow Z = \frac{2(A-1)}{C}$$

$$Z = \frac{2(0.91 \angle 2.13^\circ - 1)}{1.067 \times 10^{-3} \angle 90.7^\circ}$$

$$= \frac{0.193 \angle 159.53^\circ}{1.067 \times 10^{-3} \angle 90.7^\circ}$$

$$= 181.321 \angle 68.83^\circ$$

$$= 65.5 + j 169.1$$



**Case (ii):**

Equivalent –  $\pi$  model

$$A = D = 1 + \frac{YZ}{2}$$

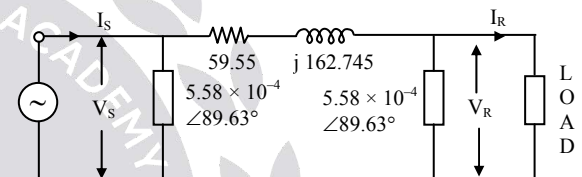
$$B = Z$$

$$C = Y + \frac{1}{4} Y^2 Z$$

$$Y = \frac{2(A-1)}{B} = \frac{2(0.91 \angle 2.13^\circ - 1)}{173.3 \angle 69.9^\circ}$$

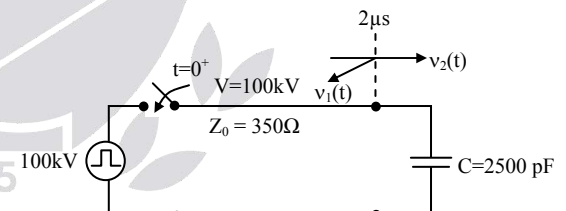
$$= \frac{.193 \angle 159.53^\circ}{173.3 \angle 69.9^\circ} = 1.16377 \times 10^{-3} \angle 89.63^\circ$$

$$\frac{Y}{2} = 5.58 \times 10^{-4} \angle 89.63^\circ$$



10.

**Sol: Rectangular surge:**



$$\text{Refracted voltage } v_2(t) = 2V \left[ 1 - e^{-\frac{t}{Z_0 C}} \right]$$

This will be valid upto  $0 < t < 2 \mu\text{s}$

$$\tau = Z_0 C = 350 \times 2500 \times 10^{-12}$$

$$= 8.75 \times 10^{-7}$$

$$\tau = 0.875 \mu\text{sec}$$

$$v_2(t) = 2 \times 100 \left( 1 - e^{-\frac{t}{0.875 \mu}} \right) \text{ kV} (0 < t < 2 \mu\text{s})$$

$$v_2(t) = 200 \left( 1 - e^{-t \times 1.14 \times 10^6} \right) \text{ kV} \quad (0 < t < 2 \mu\text{s})$$

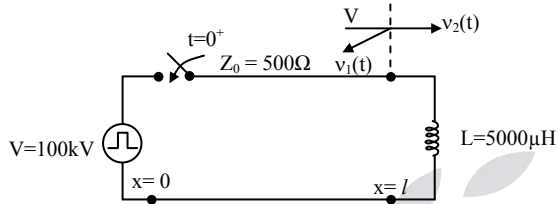
Reflected voltage  $v_1(t) = v_2(t) - V$

$$v_1(t) = 200 \left[ \left( 1 - e^{-t \times 1.14 \times 10^6} \right) \right] - 100 \text{ kV}$$

$$(0 < t < 2 \mu\text{s})$$

**11. Ans: 163.74 kV**

**Sol:**



Refracted surge into 'L':

$$v_2(t) = 2V e^{-t \frac{Z}{L}}$$

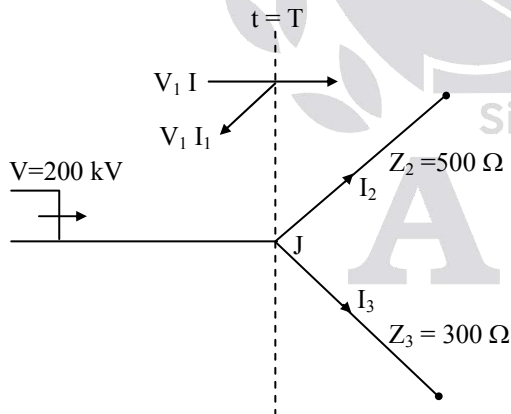
Maximum surge voltage transmitted into inductor

$$V_2(s) = \frac{2VLS}{S(Z+SL)} = \frac{2VL}{L \left( \frac{Z}{L} + S \right)}$$

$$\Rightarrow v_2(t) = 2V e^{-t \frac{Z}{L}} \Rightarrow v_2 = 163.74 \text{ V}$$

**12.**

**Sol:**



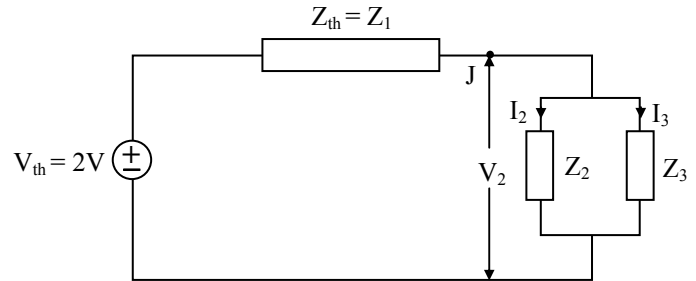
$Z_1, Z_2, Z_3 \rightarrow$  surge impedances

In diagram shown  $V, I$  respect incident

$V_1, I_1$  respect reflections

$V_2, I_2, I_3$  respect refracted quantities

The Thevenin's equivalent circuit constructed in the view of refracted quantities at J at  $t = T$  will be



$$Z_{eq} = Z_2 \parallel Z_3$$

$$= \frac{500 \times 300}{500 + 300} = 187.5 \Omega$$

(i) Refracted voltage,  $V_2 = 2V \frac{Z_{eq}}{Z_{eq} + Z_1}$

$$V_2 = 2 \times 200 \times \frac{187.5}{187.5 + 400}$$

$$V_2 = 127.66 \text{ kV}$$

Refracted currents

$$I_2 = \frac{V_2}{Z_2} = \frac{127.66}{500} \text{ kA}$$

$$= 0.25 \text{ kA}$$

$$I_3 = \frac{V_2}{Z_3} = \frac{127.66}{300}$$

(ii) Reflected voltage  $V_1 = V_2 - V$

$$V_1 = 127.66 - 200$$

$$= -72.34 \text{ kV}$$

Incident current  $I = \frac{V}{Z_1} = \frac{200}{400} = 0.5 \text{ kA}$

Reflected current  $I_1 = (I_2 + I_3) - I$

$$I_1 = (0.25 + 0.42) - 0.5 = 0.18 \text{ kA}$$

$$\begin{aligned} \text{(iii) Co-efficient voltage refraction} &= \frac{V_2}{V} \\ &= \frac{127.66}{200} = 0.638 \end{aligned}$$

$$\begin{aligned} \text{Co-efficient of voltage reflection} &= \frac{V_1}{V} \\ &= \frac{-72.34}{200} = 0.362 \end{aligned}$$

$$\begin{aligned} \text{Co-efficient of current refraction} &= \frac{(I_2 + I_3)}{I} \\ &= \frac{0.25 + 0.42}{0.5} = 1.362 \end{aligned}$$

$$\begin{aligned} \text{Co-efficient of current reflection} &= \frac{I'}{I} \\ &= \frac{0.18}{0.5} = 0.362 \end{aligned}$$

13.

**Sol:**  $Z = 400 \Omega$ ,  $V = 3 \times 10^8$

Loss less Transmission line with,

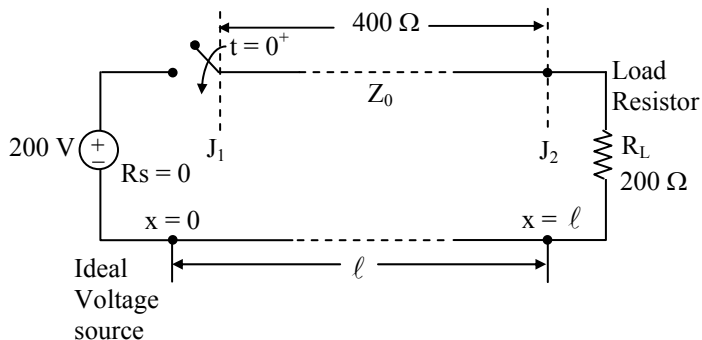
$$Z_0 = 400 \Omega$$

$$\ell = 30 \text{ km}$$

$$v = 3 \times 10^8 \text{ m/s}$$

$$\text{Load resistor, } R_L = 200 \Omega,$$

$$\text{Voltage surge, } V = 200 \text{ V},$$



$$\begin{aligned} \text{Propagation delay } T &= \frac{\ell}{v} \\ &= \frac{30}{3 \times 10^5} \frac{\text{km}}{\text{km/s}} = 0.1 \text{ ms} \end{aligned}$$

To find load voltage at 0.4 ms time.

Transient analysis is done by Bewly's lattice diagram method.

Coefficient at  $J_2$ :

$$V_{\text{refraction}} = \frac{2 \times 200}{200 + 400} = 0.667$$

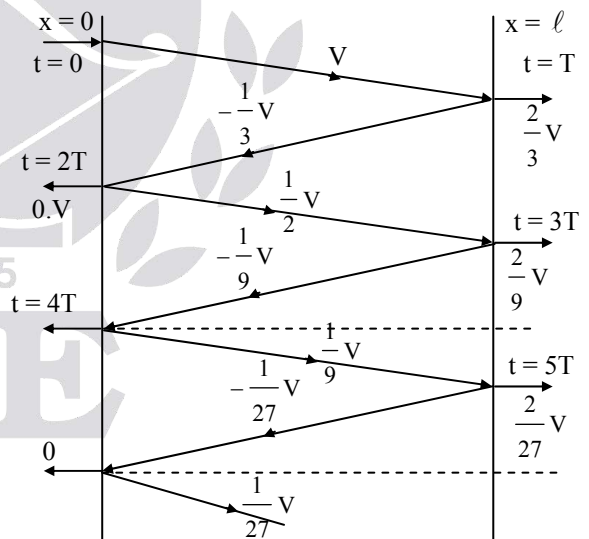
$$V_{\text{reflecion}} = \frac{200 - 400}{200 + 400} = -0.333$$

Coefficient at  $J_L$ :

$$V_{\text{refraction}} = 0$$

$$V_{\text{refraction}} = -1$$

( $\therefore$  Source internal resistance = 0)



Where  $V = 200$  volts in the diagram

$$T = 0.1 \text{ ms}$$

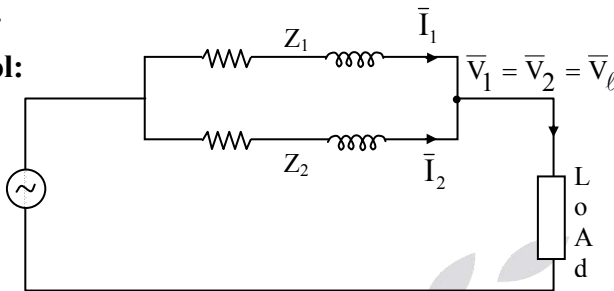
Load voltage at 0.4 ms will be represented as

$$v(\ell, 4T) = \frac{2}{3V} + \frac{2}{9}V$$

$$= \left(\frac{2}{3} + \frac{2}{9}\right) \times 200V = 177.7V$$

14.

Sol:



Feeder-1 impedance  $z_1 = 1.6 + j2.5 \Omega$

load data,  $V_l(LL) = 33 \text{ kV}$

$P_{\text{load}}(3-\phi) = 10 \text{ MW}$

Power factor = 0.8 lag

Power carried by feeder-1

$P_1 = 4.46 \text{ MW}(3-\phi)$

$\cos\phi_1 = 0.72 \text{ lag}$

$V_1 = 33 \text{ kV}(LL)$

Let us take  $V_l$  (phase) as the reference

$$\bar{V}_\ell = \frac{33}{\sqrt{3}} \angle 0^\circ \text{ kV}$$

from load

$$P_{\text{load}}(3-\phi) = \sqrt{3} \cdot V_{\ell} \cdot I_{\ell} \cdot \cos\phi_{\ell}$$

$$I_{\ell} = \frac{10M}{\sqrt{3} \times 33k \times 0.8} = 218.7 \text{ A}$$

Now,  $\bar{I}_{\ell} = 218.7 \angle -\cos^{-1}0.8$

$$\bar{I}_{\ell} = 218.7 \angle -36.86^\circ \text{ A}$$

$$\text{Feeder-1 current } I_1 = \frac{P_1(3-\phi)}{\sqrt{3} \cdot V_1(LL) \cos\phi_1}$$

$$I_1 = \frac{4.46M}{\sqrt{3} \times 33k \times 0.72}$$

$$= 108.37 \text{ A}$$

Now  $\bar{I}_1 = 108.37 \angle -\cos^{-1}0.72$

$$= 108.37 \angle -43.94^\circ$$

From KCL,  $\bar{I}_2 = \bar{I}_{\ell} - \bar{I}_1$

$$= 218.7 \angle -36.86^\circ - 108.37 \angle -43.94^\circ$$

$$= 111.9 \angle -30^\circ \text{ A}$$

as feeders are in parallel

$$\bar{I}_1 Z_1 = \bar{I}_2 Z_2$$

$$Z_2 = \frac{\bar{I}_1 Z_1}{\bar{I}_2}$$

$$= \frac{(108 \angle -43.94^\circ)(1.6 + j2.5)}{111.9 \angle -30^\circ}$$

$$Z_2 = 2 + j2\Omega$$

15.

Sol: 3- $\phi$  IM output power

Metric = 736 W = 1HP

British = 746 W = 1HP

$$P_{\text{out}} = 500\text{HP}$$

$$= 500 \times 746 \text{ W} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{According to british}$$

$$= 373.3 \text{ kW}$$

Efficiency  $\eta = 90\%$

Power drawn by IM from supply

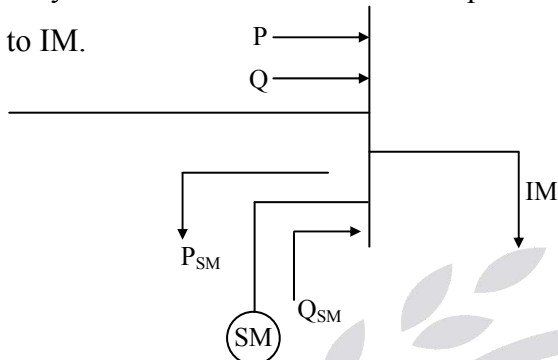
$$P_{\text{IM}}(\text{input}) = \frac{P_{\text{out}}}{\eta}$$

$$= \frac{373.3}{0.9} = 414.44 \text{ kW}$$

Apparent power drawn by induction motor

$$S_{IM} = 600 \text{ kVA}$$

A synchronous motor connected in parallel to IM.



$$P_{IM} = 414.44 \text{ kW}$$

$$Q_{IM} = \sqrt{600^2 - 414.44^2}$$

$$= 433.87 \text{ kVAr}$$

Power balance equations

$$P = P_{SM} + 414.44 \dots\dots\dots(1)$$

$$Q = Q_{IM} - Q_{SM}$$

$$Q = 433.87 - Q_{SM} \dots\dots\dots(2)$$

Overall Power factor  $\cos\phi = 0.9$  lag

$$\phi = 25.84^\circ$$

Synchronous motor power factor,  $\cos\phi_{SM}$

= 0.3 lead

$$\phi_{SM} = 72.54^\circ$$

Taking  $\frac{Q}{P}$  from equation (2) & (1)

$$\frac{Q}{P} = \frac{433.87}{P_{SM} + 414.44}$$

$$\tan\phi = \frac{433.87 - P_{SM} \cdot \tan\phi_{SM}}{P_{SM} + 414.44}$$

$$\tan(25.84) = \frac{433.87 - P_{SM} \tan(72.54)}{P_{SM} + 414.44}$$

$$P_{SM} = 63.66 \text{ kW}$$

(ii) Total real power drawn by the combination of IM & SM

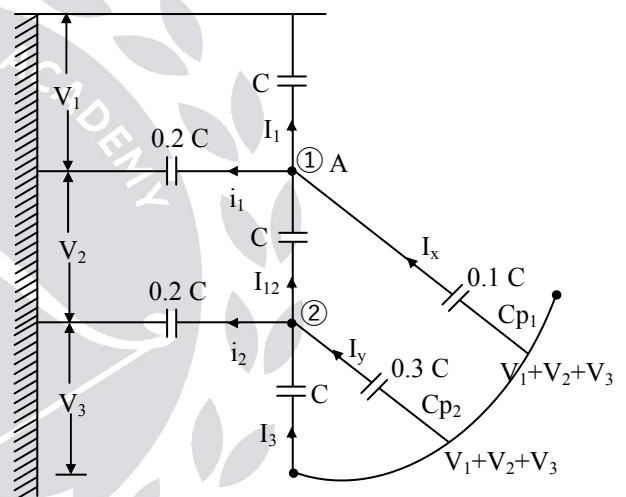
$$P = P_{SM} + P_{IM}$$

$$= 63.66 + 414.44$$

$$= 478.1 \text{ kW}$$

16.

Sol:



Applying KCL at (1) gives

$$I_x + I_2 = I_1 + i_1$$

$$(V_2 + V_3) \times j\omega(0.1C) + V_2 \times j\omega C = V_1 \times j\omega C + V_1 \times j\omega(0.2C)$$

$$0.1(V_2 + V_3) + V_2 = V_1 + V_1(0.2)$$

$$1.1 V_2 + 0.1 V_3 = 1.2 V_1 \dots\dots\dots(1)$$

KCL at (2) gives

$$I_3 + I_y = I_2 + i_2$$

$$V_3 \times j\omega C + (V_1 + V_2) \times j\omega(0.3C) = V_2 \times j\omega C + (V_1 + V_2) \times j\omega(0.2C)$$

$$1.3 V_3 = V_2 + (V_1 + V_2) (0.2)$$

$$1.3 V_3 = 1.2 V_2 + 0.2 V_1 \dots\dots\dots(2)$$

From (1)

$$V_1 = \frac{1.1V_2 + 0.1V_3}{1.2} \dots\dots\dots(3)$$

Substitute equation (3) in equation (2)

$$1.3 V_3 = 1.2 V_2 + \frac{0.2}{1.2} (1.1V_2 + 0.1V_3)$$

$$1.3 V_3 - \frac{0.02}{1.2} V_3 = 1.2 V_2 + \frac{0.2 \times 1.1}{1.2} V_2$$

$$V_3(1.28) = V_2(1.38)$$

$$\frac{V_2}{V_3} = \frac{1.28}{1.38} = 0.927$$

From equation (3)  $V_1$

$$= \frac{1.1V_2 + 0.4 \left( \frac{1.38}{1.28} V_2 \right)}{1.2}$$

$$V_1 = 1.006 V_2$$

Disc-1 voltage  $\Rightarrow V_1$

Disc-2 voltage  $\Rightarrow V_2 = 0.994 \text{ V}$

Disc-3 voltage  $\Rightarrow V_3 = \frac{1.38}{1.28} V_2$

$$V_3 = \frac{1.38}{1.28} \times 0.994 \text{ V}$$

$$V_3 = 1.07 V_1$$

$$\eta_{\text{string}} = \frac{V_1 + V_2 + V_3}{3 \times V_3}$$

$$= \frac{(1 + 0.994 + 1.07)V_1}{3 \times 1.07 V_1} = 95.38\%$$

$$C_p = \frac{P \times kC}{n - P}$$

$$\text{If } C_{p_2} \text{ is maintained as } C_{p_2} = \frac{2 \times 0.2C}{3 - 2}$$

$$= 0.4 C$$

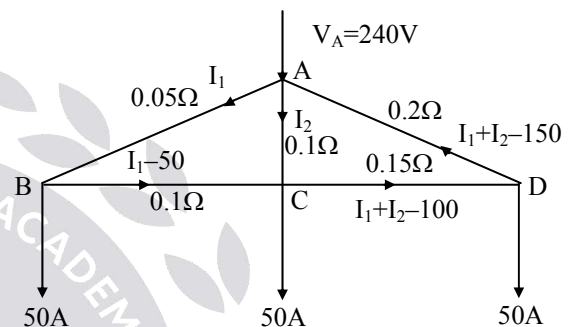
$$\text{and } C_{p_1} \text{ is maintained as } C_{p_1} = \frac{1 \times 0.2C}{3 - 1}$$

$$= 0.1C$$

The string efficiency turns to 100%

17.

Sol:



Assume that current's in AB and AC sections as  $I_1$  and  $I_2$  respectively

The currents in each section are obtained as shown in diagram by following KCL.

KVL for loop ABCA

$$I_1(0.05) + (I_1 - 50)(0.1) - I_2(0.1) = 0$$

$$I_1(0.15) - I_2(0.1) = 5 \dots\dots\dots(1)$$

KVL for loop ACDA,

$$I_2(0.1) + (I_1 + I_2 - 100)(0.15) + (I_1 + I_2 - 150)(0.2) = 0$$

$$I_1(0.35) + I_2(0.45) = 45 \dots\dots\dots(2)$$

By solving (1) & (2)

$$I_1 = 65.85, I_2 = 48.78 \text{ A}$$

Load-B voltage,

$$V_B = V_A - I_1(0.05)$$

$$= 240 - (65.85)(0.05) = 236.7 \text{ V}$$

Load-C voltage

$$V_C = V_A - I_2(0.1)$$

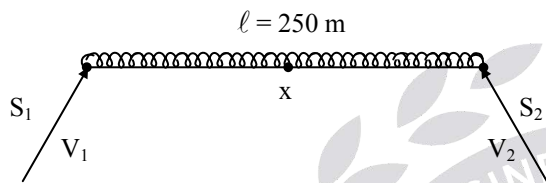
$$V_C = 240 - (48.78)(0.1) = 235.12 \text{ V}$$

Load-D voltage

$$\begin{aligned} V_D &= V_A + (I_1 + I_2 - 150)(0.2) \\ &= 240 + (65.85 + 48.78 - 150)(0.2) \\ &= 232.92 \text{ V} \end{aligned}$$

18.

Sol:



Uniformly loaded,  $i = 2 \text{ A/m}$

loop resistance,  $r = 0.2 \Omega/\text{km}$

$$= 0.2 \times 10^{-3} \Omega/\text{m}$$

Source voltages  $V_1 = 225 \text{ V}$ ,  $V_2 = 220 \text{ V}$

$V_{\min}$  occurs at a distance of

$$x = \frac{\ell}{2} + \frac{V_1 - V_2}{ir\ell}$$

$$x = \frac{250}{2} + \frac{225 - 220}{2 \times 0.2 \times 10^{-3} \times 250}$$

$x = 175 \text{ m}$  from  $s_1$

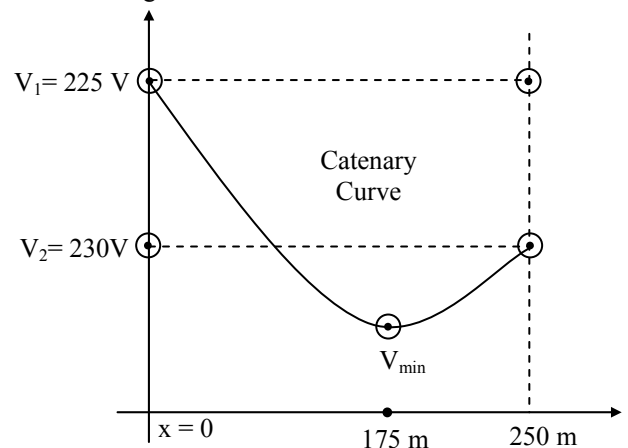
(or)  $x = 75 \text{ m}$  from  $s_2$

Minimum voltage,

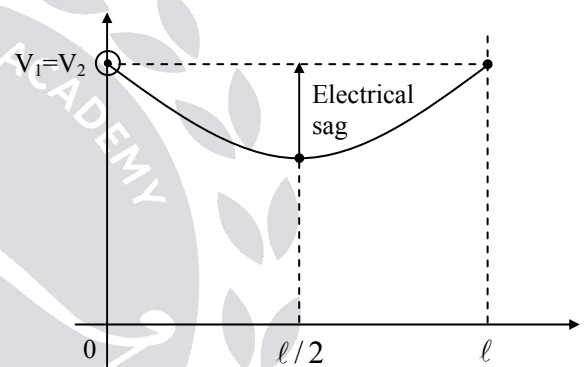
$$V_{\min} = V_1 - \frac{i^2 r x^2}{2}$$

$$\begin{aligned} V_{\min} &= 225 - \frac{2 \times 0.2 \times 10^{-3} \times 175^2}{2} \\ &= 218.87 \text{ V} \end{aligned}$$

Voltage Profile



If  $V_1 = V_2$



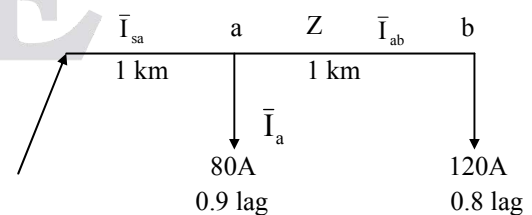
19.

Sol: Loop impedance,

$$Z / \text{km} = 0.05 + j 0.1 \Omega$$

$$V_b = 230 \text{ V}$$

Assume  $V_b$  as Reference



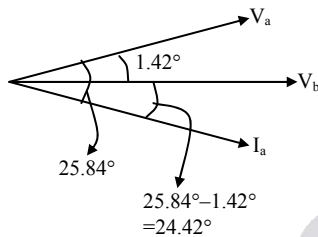
$$\bar{V}_n = 230 \angle 0$$

$$\bar{I}_b = 120 \angle -\cos^{-1}(0.8) = 120 \angle -36.86^\circ \text{ A}$$

$$\bar{I}_{ab} = \bar{I}_b = 120 \angle -36.86^\circ$$



$$\begin{aligned}\bar{V}_a &= \bar{V}_b + \bar{I}_{ab} Z \\ &= 230 \angle 0^\circ + (120 \angle -36.86^\circ)(0.05 + j0.1) \Omega \\ &= 242.07 \angle +1.42^\circ \text{ V} \\ \bar{I}_a &= 80 \angle -(\cos^{-1}(0.9) + 1.42^\circ) \\ &= 80 \angle (-25.84^\circ + 1.42^\circ) \\ &= 80 \angle -24.42^\circ\end{aligned}$$



The Pf is with  $\bar{V}$  only  
 $= 80 \angle -24.42^\circ$  with Reference

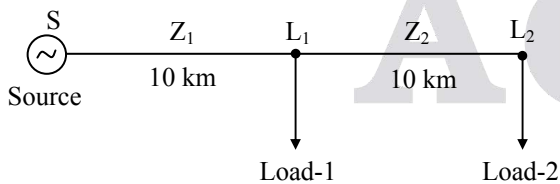
$$\begin{aligned}\bar{I}_{sa} &= \bar{I}_a + \bar{I}_{ab} \\ &= 80 \angle -24.42^\circ + 120 \angle -36.86^\circ \\ &= 198.86 \angle -31.8^\circ\end{aligned}$$

$$\begin{aligned}\text{Source voltage } \bar{V}_s &= \bar{V}_a + \bar{I}_{sa} Z \\ &= 246.1 \angle 1.42^\circ + (198.86 \angle -31.8^\circ)(0.05 + j0.1) \\ &= 265.5 \angle 3.86^\circ\end{aligned}$$

(ii) Phase angle between source & far end voltage.  
 $\delta = 3.86^\circ$

20.

Sol:



$$\begin{aligned}\text{Source: } & 3\text{-}\phi, 400 \text{ V, } 50 \text{ Hz} \\ V_{\text{sph}} &= \frac{400}{\sqrt{3}} = 230.9 \text{ V}\end{aligned}$$

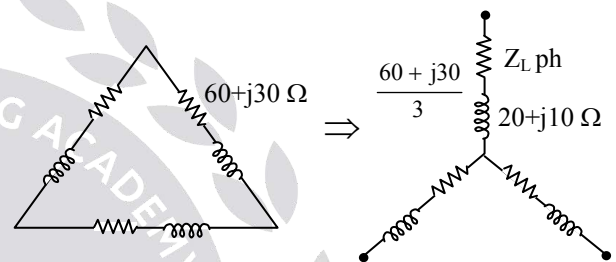
$$\text{Feeder: } z_f = 3 + j6 \Omega \text{ for } 20 \text{ km}$$

$$z_1 = z_2 = \frac{3 + j6}{2} = 1.5 + j3 \Omega$$

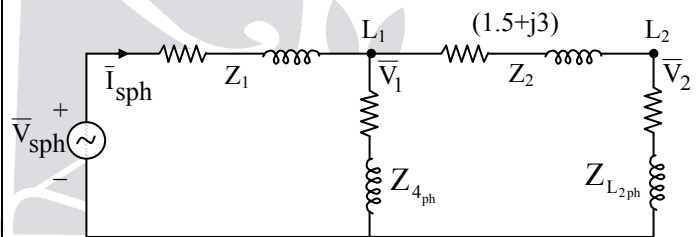
Load1:  $\Delta$ -load of each phase impedance  $60 + j30 \Omega$

Load2: Y-load of each phase impedance  $40 - j40 \Omega$

To draw per phase equivalent circuit delta connected load 1 should be converted into star such that,



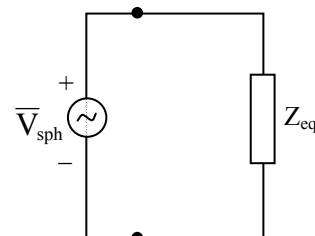
Per phase equivalent circuit



$$Z_{L2ph} = 40 - j40 \Omega$$

let  $V_{\text{sph}}$  as reference  $\bar{V}_{\text{sph}} = 230.9 \angle 0^\circ \text{ V}$

(i) The simplified equivalent circuit



$$\begin{aligned}
 Z_{eq} &= Z_1 + Z_{L1ph} // (Z_2 + Z_{L2ph}) \\
 &= Z_1 + (20 + j10) // (41.5 - j37) \\
 &= Z_1 + \frac{(20 + j10)(41.5 - j37)}{20 + j10 + 41.5 - j37}
 \end{aligned}$$

$$\begin{aligned}
 Z_{eq} &= Z_1 + 18.33 + j2.7 \\
 &= (1.5 + j3) + 18.33 + j2.7 \\
 &= 19.83 + j5.7 \Omega
 \end{aligned}$$

Now source current,

$$\begin{aligned}
 \bar{I}_{sph} &= \frac{\bar{V}_{sph}}{Z_{eq}} = \frac{230.9 \angle 0^\circ}{19.83 + j5.7} \\
 &= 11.19 \angle -16.19^\circ \text{ A}
 \end{aligned}$$

Complex power supplied by source (or) fed into the feeder.

$$\begin{aligned}
 s &= 3 \cdot \bar{V}_{sph} \cdot \bar{I}_{sph} \\
 &= 3(230.9 \angle 0^\circ)(11.19 \angle -16.19^\circ) \\
 &= 7443.9 + j2161.24 \text{ VA}
 \end{aligned}$$

$$\begin{aligned}
 \text{Load-1 voltage, } \bar{V}_{1ph} &= \bar{V}_{sph} - \bar{I}_{sph} Z_1 \\
 \bar{V}_{1ph} &= 230.9 \angle 0^\circ - (11.19 \angle -16.19^\circ)(1.5 + j3) \\
 &= 207.26 \angle -7.64^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_{1(LL)} &= \sqrt{3} \cdot V_{1ph} \\
 &= \sqrt{3} \times 207.26 \text{ V} \\
 &= 358.98 \text{ V}
 \end{aligned}$$

$$\bar{V}_1(LL) = 358.98 \angle -7.64^\circ + 30^\circ$$

$$\begin{aligned}
 \text{Load-2 voltage, } \bar{V}_{2ph} &= \bar{V}_{1ph} \frac{Z_{L2ph}}{Z_{L2ph} + Z_2} \\
 &= (207.26 \angle -7.64^\circ) \frac{(40 - j40)}{40 - j40 + 1.5 + j3} \\
 &= 210.87 \angle -10.9^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \bar{V}_{2(LL)} &= \sqrt{3} \times 210.87 \angle -10.9^\circ + 30^\circ \\
 &= 365.81 \angle 19.1^\circ \text{ V}
 \end{aligned}$$

(iii) Source Power factor

$$\begin{aligned}
 \cos \phi_s &= \cos(\text{angle between } \bar{V}_{sph} \text{ \& } \bar{I}_{sph}) \\
 &= \cos(0 + 16.19^\circ) = 0.96 \text{ lag}
 \end{aligned}$$

## Circuit Breakers

### Solutions for Objective Practice Questions

**01. Ans: (a)**

**Sol:** Given data:

$$L = 15 \times 10^{-3} \text{ H}$$

$$C = 0.002 \times 10^{-6} \text{ F}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{15 \times 10^{-3} \times 0.002 \times 10^{-6}}} = 29 \text{ kHz}$$

**02. Ans: (b)**

**Sol:** Given data:

$$I = 10 \text{ A, } C = 0.01 \times 10^{-6} \text{ F,}$$

$$L = 1 \text{ H}$$

$$\frac{1}{2} Li^2 = \frac{1}{2} CV^2 \Rightarrow Li^2 = CV^2$$

$$V = i \sqrt{\frac{L}{C}} = 10 \left[ \sqrt{\frac{1}{0.01 \times 10^{-6}}} \right] = 100 \text{ kV}$$

**03. Ans: (a)**

**Sol:** Given data:

Maximum voltage across circuit breakers contacts at current zero point = Maximum value of Restriking voltage ( $V_{max}$ )

$$V_{r\max} = 2 \text{ ARV}$$

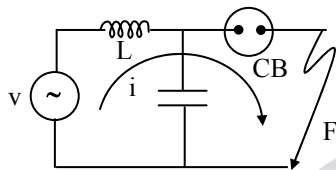
$$\text{ARV} = K_1 K_2 K_3 V_{\max} \sin \phi$$

$$K_1 = 1 \rightarrow \text{No Armature reaction}$$

$$K_2 = 1 \rightarrow \text{Assuming fault as grounded fault}$$

$$K_3 = 1 \rightarrow \text{ARV/phase}$$

$$V_{\max} = \frac{17.32}{\sqrt{3}} \times \sqrt{2}$$



$$V_{r\max} = 2 \left[ 1 \times 1 \times 1 \times \frac{17.32}{\sqrt{3}} \times \sqrt{2} \times 1 \right]$$

$$= 28.28 \text{ kV}$$

**04. Ans: (d)**

$$\text{Sol: Making current} = 2.55 \times I_B$$

$$= 2.55 \left[ \frac{2000}{\sqrt{2} \times 25} \right] = 144.25 \text{ kA}$$

**05. Ans: (a)**

$$\text{Sol: For 1-}\phi, \text{ breaking current} = \left[ \frac{2000 \text{ MVA}}{25 \text{ kV}} \right]$$

$$= 80 \text{ kA}$$

$$\text{Making current} = 2.55 [80 \text{ kA}]$$

$$= 204 \text{ kA}$$

**06. Ans: (c)**

$$\text{Sol: } R = 0.5 \sqrt{\frac{L}{C}}$$

$$= 0.5 \sqrt{\frac{25 \text{ mH}}{0.025 \mu\text{H}}} = 500 \Omega$$

**07. Ans: (c)**

$$\text{Sol: A.R.V} = K_1 K_2 V_m \sin \phi$$

$K_1$  – first pole clearing factor

$$K_1 = 1.5 \text{ (LLL fault)}$$

$K_2$  – Due to armature reaction

$$K_2 = 1 \text{ (Armature reaction not given)}$$

$\phi$  - p.f angle of the fault

$$\cos \phi = 0.8 \Rightarrow \phi = 36.86^\circ$$

$V_m$  = maximum value of phase voltage of the system

$$V_m = \frac{132 \text{ kV}}{\sqrt{3}} \times \sqrt{2}$$

$$\text{A.R.V} = 1.5 \times \frac{132}{\sqrt{3}} \times \sqrt{2} \times \sin 36.86$$

$$= 96.7 \text{ kV}$$

#### Solutions for Conventional Practice Questions

**01.**

**Sol:** In a short circuit test, 132 kV, 3 $\phi$ , CB

Pf of fault,  $\cos \phi = 0.3$  lag

Recovery voltage = 0.95 of full line Rated voltage.

The natural frequency  $f_n = 16000 \text{ Hz}$

Avg RRRV =

$$\frac{2 \text{ ARV}}{\pi \sqrt{LC}} = 4 \frac{\text{ARV}}{2\pi \sqrt{LC}} = 4 f_n (\text{ARV})$$

$$= 4 \times 16000 \times \text{ARV}$$

(i). for LLL fault,

$$\text{ARV} = K_1 K_2 K_3 V_{\max} \sin \phi$$

$$= 0.95 \times 1.5 \times 1 \times$$

$$\frac{132}{\sqrt{3}} \sqrt{2} \times 0.953 = 146.5 \text{ kV}$$

$$V_{\max} = \frac{132}{\sqrt{3}} \times \sqrt{2} \text{ kV},$$

$$\sin\phi = \sqrt{1 - \cos^2\phi}$$

$$= \sqrt{1 - (0.3)^2} = 0.953$$

$$\Rightarrow \text{ARV} = 146.5 \text{ kV}$$

$$\text{Avg RRRV} = 4 \times 16000 \times 146.5$$

$$\text{kV/sec}$$

$$= 9.36 \times 10^6 \text{ kV/sec}$$

$$\text{Avg RRRV} = 9.36 \text{ kV/}\mu\text{sec}$$

(ii) For LLLG fault,

$$\text{ARV} = k_1 k_2 k_3 V_{\max} \sin\phi$$

$$= 0.95 \times 1 \times 1 \times \frac{132}{\sqrt{3}} \times \sqrt{2} \times 0.954$$

$$= 97.63 \text{ kV}$$

$$\text{Avg RRRV} = 4 \times 16000 \times 97.67 \text{ kV/sec}$$

$$= 6.25 \text{ kV/}\mu\text{sec}.$$

**Note:**

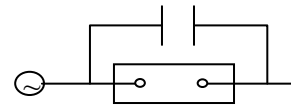
- For kilometric faults (fault occurs at few no. of kilometer from (B))  $\Rightarrow$  L&C are very small
- Rate of Rise Restricting voltage is very

$$\text{large} \therefore \text{RRRV} = \frac{2\text{ARV}}{\pi\sqrt{LC}}$$

**02.**

**Sol:** 50 Hz, 11 kV (L-L), 3- $\phi$  alternator with earthed neutral,  $X = 50 \Omega/\text{ph}$

$$C = 0.02 \mu\text{F/ph}$$



$$C = 0.08 \mu\text{F/ph}$$

Max voltage across Contacts of CB at the instant of current zero Are interruption

$$= 2(\text{ARV})$$

$$(i) \text{ARV} = k_1 k_2 k_3 V_{\max} \sin\phi$$

Assure grounded fault  $\rightarrow k_2 = 1$

$$k_1 = 1, k_2 = 1, k_3 = 1, V_{\max} = \frac{11\text{k}}{\sqrt{3}} \sqrt{2}$$

$$\sin\phi = \sqrt{1 - \cos^2\phi}$$

No Resistance given for alternator ,

Hence  $90^\circ$  lag current will flow

$\Rightarrow$  fault power factor = 0

$$\text{ARV} = \frac{11\text{k}}{\sqrt{3}} \times \sqrt{2} \times 1 \times 1 \times 1$$

$$= 8.98 \text{ kV}$$

$$V_{r\max} = 2(\text{ARV}) = 2 \times 8.98 \text{ k}$$

$$= 17.96 \text{ kV}$$

(ii) Frequency of oscillations

$$(f_n) = \frac{1}{2\pi \times \sqrt{LC}}$$

$$\text{as, } x = 2\pi fL \Rightarrow L$$

$$= \frac{x}{2\pi f} = \frac{50}{2\pi \times 50} = 0.159 \text{ H}$$

$$\Rightarrow f_n = \frac{1}{2\pi \sqrt{0.159 \times 0.02\mu}} = 2.82 \text{ kHz}$$

(iii) RRRV upto first peak

$$= \text{Avg RRRV upto first peak}$$

$$= \frac{2(\text{ARV})}{\pi\sqrt{LC}} = 4f_n(\text{ARV})$$

$$= 4 \times 2.82 \text{ k} \times 8.98 \text{ kV/sec}$$

$$= 101.29 \times 10^3 \text{ kV/sec}$$

$$= 0.101 \text{ kV}/\mu\text{sec}$$

**03.**
**Sol:** 132 kV system  $c = 0.01 \mu\text{F}$ ,  $L = 6\text{H}$ ,  $i = 10\text{A}$ 

$$\text{Prospective voltage } V = i \sqrt{\frac{L}{C}} = 10 \sqrt{\frac{6}{0.01 \mu}} = 244.9 \text{ kV}$$

The value of resistance to be used across the contacts to eliminate the restriking voltage

$$R = \frac{1}{2} \sqrt{\frac{L}{C}} = \frac{1}{2} \sqrt{\frac{6}{0.01 \times 10^{-6}}} = 12.247 \Omega$$

**04.**
**Sol:**  $f = 50 \text{ Hz}$   $V_{ph} = 8.5 \text{ kV}$ ;  $X_L = 4.2 \Omega$ 

$$\Rightarrow 2\pi fL = 4.2 \Rightarrow L = \frac{4.2}{2\pi \times 50} = 13.37 \times 10^{-3} \text{ H}$$

$$C = 0.015 \mu\text{F}$$

$$(a) (V_{TRV})_{\max} = 2V_m$$

$$= 2(\sqrt{2}V_{ph})$$

$$= 2\sqrt{2} \times 8.5$$

$$= 24.04 \text{ kV}$$

time at  $(V_{TRV})_{\max}$  occurs

$$t_m = \pi \sqrt{LC}$$

$$= \pi \sqrt{13.37 \times 10^{-3} \times 0.015 \times 10^{-6}}$$

$$= 44.49 \times 10^{-6} \text{ sec}$$

$$= 44.49 \mu\text{sec}$$

$$(b) \text{ Avg Restriking voltage} = \frac{(V_{TRV})_{\max}}{t_m}$$

$$\Rightarrow \frac{24.04 \times 10^3}{44.49 \times 10^{-6}}$$

$$\Rightarrow 540.346 \times 10^6 \text{ V} = 0.540 \text{ kV}/\mu\text{sec}$$

$$(c) R = \frac{1}{2} \sqrt{\frac{L}{C}}$$

$$= \frac{1}{2} \sqrt{\frac{13.37 \times 10^{-3}}{0.015 \times 10^{-6}}}$$

$$= 0.472 \text{ k}\Omega$$

## Protective Relays

### Solutions for Objective Practice Questions

**01. Ans: (d)**

$$\text{Sol: Relay current setting} = 50\% \times 5$$

$$\Rightarrow 0.5 \times 5 = 2.5$$

$$\text{PSM} = \frac{\text{primary current (fault current)}}{\text{relay current setting} \times \text{CT ratio}}$$

$$= \frac{2000}{\frac{400}{5} \times 0.5 \times 5} = 10$$

**02. Ans: (c)**

**Sol:** The minimum value of current required for relay operation is the plug setting value of current.

$\therefore$  Minimum value of negative sequence

Current required for relay operation

$$= 0.2 \times \frac{5}{1} = 1\text{A}$$

But for a line to line fault,  $I_{R_2} = -I_{R_1}$

$$\text{And fault current } (I_f) = \sqrt{3} I_{R_2}$$

$$= \sqrt{3} \times 1 = 1.732\text{A}$$

∴ Minimum fault current required  
= 1.732 A.

**03. Ans: (a)**

**Sol:** From figure, it is clear that zone 2 of relay1 and relay 2 are overlapped. If there is a fault in overlapped section (line2), the fault should be clear by relay 2. Hence zone 2 operating time of relay2 must be less than zone1 operating time. ( $TZ2_{R1} > TZ2_{R2}$ )

**04. Ans: (b)**

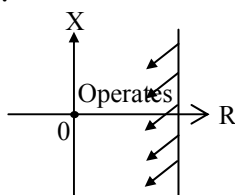
**Sol:**  $\frac{I_2}{i_2}$ ;  $I_2 = 400 \times \frac{11}{66} = \frac{400}{6} = 66.66$

$$i_2 = \frac{5}{\sqrt{3}} = 2.88$$

$$\frac{I_2}{i_2} = 23:1$$

**05. Ans: (b)**

**Sol:** The active power restrained over current relay will have characteristics in R-X plane.



**06. Ans: (b)**

**Sol:** CT ratio =  $400/5 = 80$

$$\begin{aligned} \text{Relay current setting} &= 50\% \text{ of } 5A \\ &= 0.5 \times 5A \\ &= 2.5A \end{aligned}$$

$$\begin{aligned} \text{PSM} &= \frac{\text{Primary current (fault current)}}{\text{Relay current setting} \times \text{CT ratio}} \\ &= \frac{1000}{2.5 \times 80} = 5 \end{aligned}$$

The operating time from given table at PSM 5 is 1.4 the operating time for TMS of 0.5 will be  
 $0.5 \times 1.4 = 0.7 \text{ sec}$

**07. Ans: (b)**

**Sol:**  $T_{\max} \propto \cos(\theta - \tau)$

When  $\cos(\theta - \tau) = 1$ ,  $T_{\max} = 10$

$\tau = 90^\circ$

Impedance of relay  $0.1 + j0.1$   
 $= 0.1414 \angle 45^\circ$

$\theta = 45^\circ$

Operating torque  $\frac{T_1}{T_{\max}} = \frac{\cos(45 - 90)}{1}$

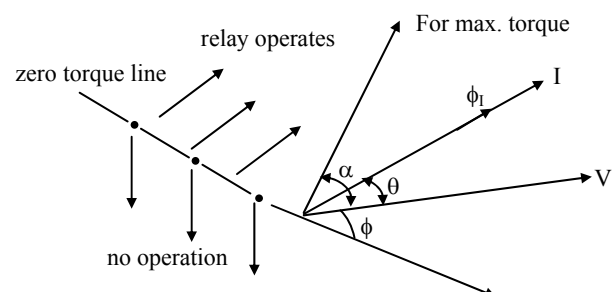
$$\frac{T_1}{10} = \frac{\cos(-45)}{1}$$

$T_1 = 7.07 \text{ N-m.}$

### Solutions for Conventional Practice Questions

**01.**

**Sol:** Operating torque  $T_{\text{op}} = \phi_v \phi_I \sin(\phi + \theta)$



$$\phi = \tan^{-1} \left[ \frac{x}{R} \text{ of voltage coil} \right]$$

$$Z_v = 6 + j 8$$

$$\phi = \tan^{-1} \left( \frac{8}{6} \right) = 53.13^\circ$$

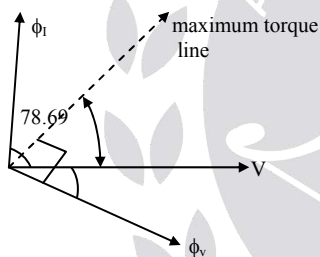
$$\begin{aligned} \text{line impedance angle } \theta &= \tan^{-1} \left( \frac{x}{R} \right) \\ &= \tan^{-1} \left( \frac{10}{2} \right) = 78.69^\circ \end{aligned}$$

$$T_{op} = kVI \sin (\phi + \theta)$$

$$= kVI \sin [78.69^\circ + 53.13^\circ]$$

$$= kVI \times 0.7452$$

$$T_{op} = 0.7452 \text{ kV}$$



02.

**Sol:** CT Ratio =  $\frac{1000}{5} = 200$

$$\begin{aligned} \text{Relay current setting} &= 166.66 \% \text{ of } 5A \\ &= 1.66 \times 5 = 8.3 A \end{aligned}$$

$$\text{Plug setting} = 8.3 A$$

$$\text{PSM} = \frac{\text{secondary current}}{\text{Relay current setting}}$$

$$= \frac{\text{primary current (fault current)}}{\text{relay current Setting} \times \text{CT ratio}}$$

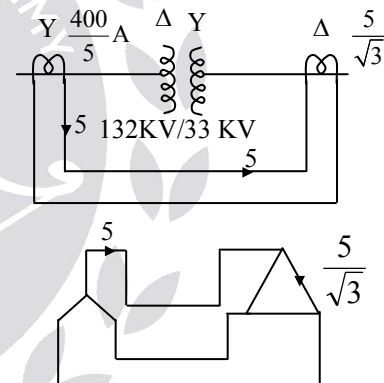
$$= \frac{10 \times 10^3}{8.3 \times 200} \cong 6$$

- The operating time from the given diagram at PSM of 6 is 0.6 sec. This time is for TMS = 1
- The operating time for TMS of 0.8 will be equal to  $0.6 \times 0.8 = 0.48 \text{ sec}$

03.

**Sol:** The current transformer is connected opposite connection

i.e.,  $\Delta$ -side winding Y connection current transformer is taken to avoid phase angle.



The primary rating of current transformer is obtained by

$$400 \times 132 = 33 \times x$$

$$\Rightarrow x = 1600 A$$

The phase current of secondary side of HV

$$CT = 5A$$

$\therefore$  The pilot current = 5A = Line current



∴ The phase current of  $\Delta$  connected current transformer =  $\frac{5}{\sqrt{3}}$  A.

∴ The current transformer ratio on LT side

$$= \frac{1600}{5/\sqrt{3}}$$

$$= \frac{1600\sqrt{3}}{5} = 320\sqrt{3}$$

Shortcut: The current in pilot wire and always taken as line current and current transformer rating is taken as phase currents.

04.

**Sol:** The impedance of the transmission line

$$= 50\Omega$$

$$\text{The C.T ratio} = \frac{500}{5 \text{ A}}$$

$$= 100 \text{ A}$$

A fault is occurred at the middle of the transmission lines & impedance seen by the relay is  $75\Omega$

P.T?

$$Z_{\text{seen}} = \frac{\text{C.T ratio}}{\text{P.T ratio}} \times Z_{\text{actual}}$$

$$\text{P.T ratio} = \frac{\text{C.T ratio}}{Z_{\text{seen}}} \times Z_{\text{actual}}$$

$$\text{P.T} = \frac{100}{75} \times 50$$

$$= 66.6667$$

## Fundamentals of Power Economics

### Solutions for Objective Practice Questions

01. Ans: (i)  $P_1 = 20 \text{ MW}$ ,  $P_2 = 20 \text{ MW}$ ,

$$\lambda = 18.4 \text{ Rs/MWhr}$$

(ii)  $P_1 = 125 \text{ MW}$ ,  $P_2 = 125 \text{ MW}$ ,

$$\lambda = 32.5 \text{ Rs/MWhr}$$

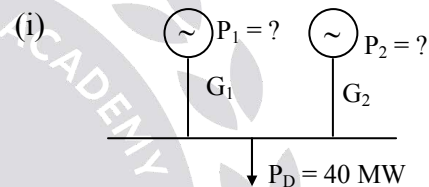
(iii)  $P_1 = 63.636 \text{ MW}$ ,  $P_2 = 86.364 \text{ MW}$ ,

$$\lambda = 26.36 \text{ Rs/MWhr}$$

(iv)  $40 \text{ MW}$ ; (v)  $70 \text{ MW}$ ; (vi)  $250 \text{ MW}$

(vii)  $235 \text{ MW}$

**Sol:**



As per equality constraints  $P_1 + P_2 = 40$

...(1)

From the coordination equation  $I_{c1} = I_{c2}$

$$0.1P_1 + 20 = 0.12P_2 + 16$$

$$0.1P_1 - 0.12P_2 = -4 \dots\dots\dots (2)$$

Solving (1) & (2)

$$P_1 = 3.6363 \text{ MW}$$

$$P_2 = 36.3636 \text{ MW}$$

$G_1$  violated its min power limit

$$P_1 = P_{1 \text{ min}} = 20 \text{ MW}$$

$$P_2 = 40 - 20 = 20 \text{ MW}$$

$$I_{C1} = 0.1 \times 20 + 20 = 22 \text{ Rs/MWhr}$$

$$I_{C2} = 0.12 \times 20 + 16 = 18.4 \text{ Rs/MWhr}$$

$\lambda$  always decided by the unit (or) group of units which are participated in the economic dispatch.

$$\therefore \lambda = 18.4 \text{ Rs/MWhr}$$

(ii)  $P_D = 250$  MW

$$\therefore P_1 + P_2 = 250 \text{ MW} \dots\dots\dots (3)$$

$$0.1 P_1 + 20 = 0.12 P_2 + 16$$

$$\Rightarrow 0.1 P_1 - 0.12 P_2 = -4 \dots\dots\dots (4)$$

Solving (3) and (4)

$$P_1 = 118.1 \text{ MW}, P_2 = 131.81 \text{ MW}$$

$G_2$  is violating its maximum power limit.

$$\therefore P_2 = 125 \text{ MW}$$

$$\Rightarrow P_1 = 250 - 125 = 125 \text{ MW}$$

$$IC_1 = 0.1 \times 125 + 20 = 32.5 \text{ Rs/MWhr}$$

$$IC_2 = 0.12 \times 125 + 16 = 31 \text{ Rs/MWhr}$$

' $\lambda$ ' is always decided by the unit or group of units which are participated in the economic dispatch.

$$\therefore \lambda = 32.5 \text{ Rs/MWhr}$$

(iii)  $P_D = 150$  MW

$$P_1 + P_2 = 150 \dots\dots\dots (5)$$

$$0.1 P_1 + 20 = 0.12 P_2 + 16$$

$$\Rightarrow 0.1 P_1 - 0.12 P_2 = -4 \dots\dots\dots (6)$$

Solving (5) and (6)

$$\Rightarrow P_1 = 63.63 \text{ MW}$$

$$P_2 = 86.36 \text{ MW}$$

$$IC_1 = 0.1 P_1 + 20 = 26.363 \text{ Rs/MWhr}$$

$$IC_2 = 0.12 P_2 + 16 = 26.363 \text{ Rs/MWhr}$$

$$\therefore \lambda = 26.363 \text{ Rs/MWhr}$$

(iv)  $P_{D \min} = P_{1 \min} + P_{2 \min} \dots\dots P_{N \min}$

$$P_{D \min} = 40 \text{ MW}$$

(v)  $P_{D \min}$  economic dispatch

$P_{D \min}$  ED:

It is the minimum demand on two generators, such that both generators operate at the economic dispatch.

To solve for  $P_{D \min}$  ED

Solve for  $P_D = P_{D \min}$

Calculate  $P_1$ ,  $P_2$  and  $IC_1$ ,  $IC_2$

$$I_{C \max} = \max \{IC_1, IC_2\}$$

$$IC_1 = I_{C \max} \Rightarrow P_1 = ?$$

$$IC_2 = I_{C \max} \Rightarrow P_2 = ?$$

$P_{D \min}$  ED:

$$P_D = 40, P_1 = 20; P_2 = 20$$

$$IC_1 = 22, IC_2 = 18.4$$

$$I_{C \max} = 22 \text{ Rs/MWhr}$$

$$IC_1 = I_{C \max}$$

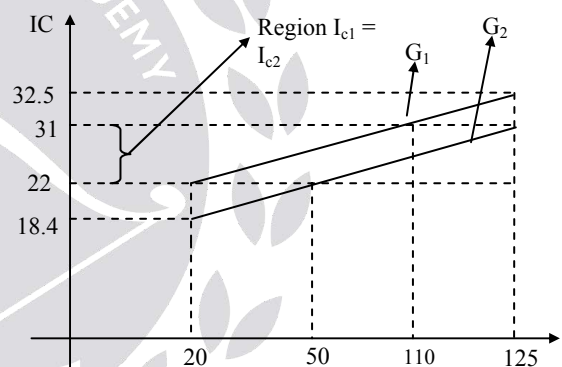
$$0.1 P_1 + 20 = 22$$

$$P_1 = 20 \text{ MW}$$

$$0.12 P_2 + 16 = 22$$

$$P_2 = 50 \text{ MW}$$

$$P_{D \min} \text{ ED} = 20 + 50 = 70 \text{ MW}$$



$$\text{vi) } P_{D \max} = P_{1 \max} + P_{2 \max} \\ = 125 + 125 = 250 \text{ MW}$$

vii) To solve for  $P_{D \max}$  ED

Solve for  $P_D = P_{D \max}$

Calculate  $P_1$ ,  $P_2$  and  $IC_1$ ,  $IC_2$

$$I_{C \min} = \min \{IC_1, IC_2\}$$

$$IC_1 = I_{C \min} \Rightarrow P_1 = ?$$

$$IC_2 = I_{C \min} \Rightarrow P_2 = ?$$

$$P_{D \max} \text{ ED} = P_1 + P_2$$

$$P_{D \max} \text{ ED}$$

$$P_D = P_{D \max} = 250 \text{ MW}$$

$$P_1 = 125 \text{ MW} ; P_2 = 125 \text{ MW}$$

$$I_{C1} = 32.5 \text{ \& } I_{C2} = 31$$

$$I_{C \text{ min}} = 31$$

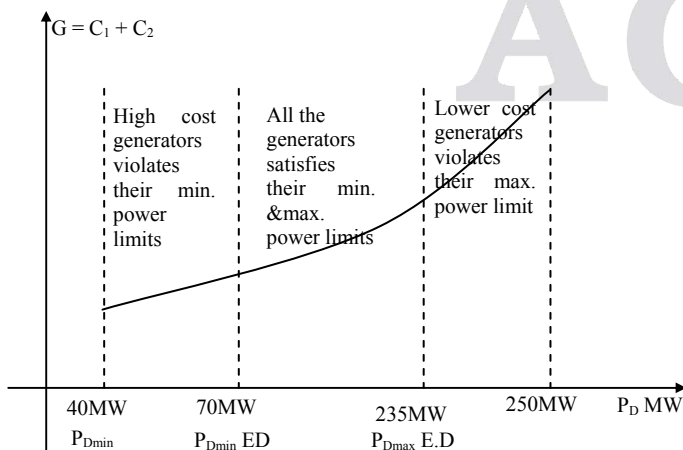
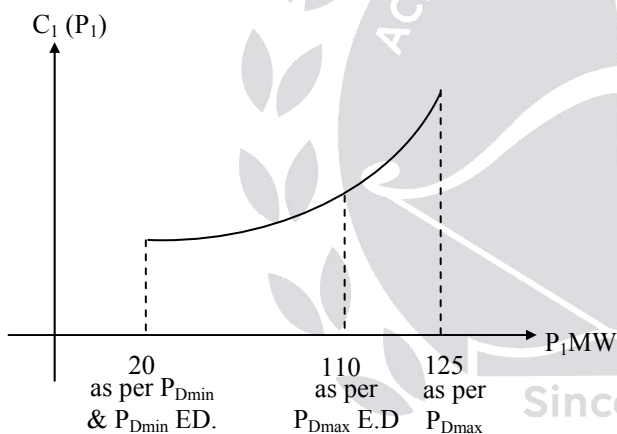
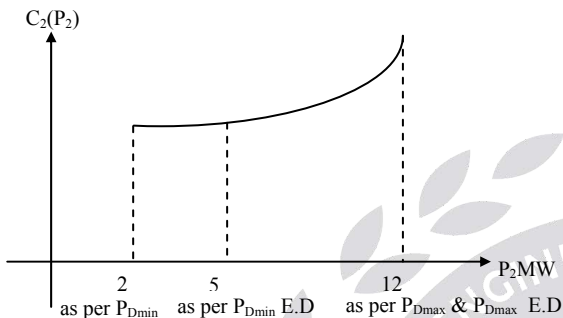
$$I_{C1} = 31 \Rightarrow 0.1P_1 + 20 = 31$$

$$P_1 = 110 \text{ MW}$$

$$I_{C2} = 31 \Rightarrow 0.12P_2 + 16.31$$

$$P_2 = 125 \text{ MW}$$

$$P_{D(\text{max})} \text{ ED} = 110 + 125 = 235 \text{ MW}$$



**02. Ans:  $P_{G1} = 212.44 \text{ MW}$**

**$P_{G2} = 56.51 \text{ MW}$**

**$P_{G3} = 230.98 \text{ MW}$**

**Sol:**  $\frac{dF_1}{dP_1} = -2.5I_{C1}^2 + 60I_{C1} - 120$

$$\frac{dF_2}{dP_2} = -2I_{C2}^2 + 40I_{C2} - 140$$

$$\frac{dF_3}{dP_3} = -1.5I_{C3}^2 + 50I_{C3} - 90$$

$$P_1 + P_2 + P_3 = 500 \text{ MW}$$

Optimal generation schedule

$$I_{C1} = I_{C2} = I_{C3} = \lambda$$

$$-2.5\lambda^2 + 60\lambda - 120 - 2\lambda^2 + 40\lambda - 140$$

$$-1.5\lambda^2 + 50\lambda - 90 = 500$$

$$-6\lambda^2 + 150\lambda - 350 = 500$$

$$6\lambda^2 - 150\lambda + 850 = 0$$

$$\lambda = 16.31; \lambda = 8.68$$

$$P_{G1} = -2.5(8.68)^2 + 60(8.68) - 120 = 212.44 \text{ MW}$$

$$P_{G2} = -2(8.68)^2 + 40(8.68) - 140 = 56.51 \text{ MW}$$

$$P_{G3} = -1.5(8.68)^2 + 50(8.68) - 90 = 230.98 \text{ MW}$$

**03. Ans:  $P_1 = 326.6 \text{ MW}$**

**$P_2 = 273.33 \text{ MW}$**

**Sol:**  $C_1(PG_1) = 0.006PG_1^2 + 8PG_1 + 350 \rightarrow$

$$100\text{MW} \leq PG_1 \leq 650 \text{ MW}$$

$$C_2(PG_2) = 0.009PG_2^2 + 7PG_2 + 400 \rightarrow$$

$$50\text{MW} \leq PG_2 \leq 500 \text{ MW}$$

$$P_D = 600 \text{ MW}$$

$$\frac{dC_1}{dP_1} = 0.012PG_1 + 8$$

$$\frac{dC_2}{dP_2} = 0.018 P_{G_2} + 7$$

$$\lambda \left[ \frac{1}{0.012} + \frac{1}{0.018} \right] = 600 + \left[ \frac{8}{0.012} + \frac{7}{0.018} \right]$$

$$\lambda [138.88] = 1655.55$$

$$\lambda = 11.92$$

$$P_1 = \frac{\lambda - \alpha_1}{\beta_1} = \frac{11.92 - 8}{0.012} = 326.6 \text{ MW}$$

$$P_2 = \frac{\lambda - \alpha_2}{\beta_2} = \frac{11.92 - 7}{0.018} = 273.33 \text{ MW}$$

**04. Ans: (c)**

**05. Ans:  $P_1 = 80 \text{ MW}$**

**$P_2 = 50 \text{ MW}$**

**Sol:**  $F_1 = 0.2P_1^2 + 30P_1 + 100 \text{ Rs/hr} \rightarrow 20 \leq P_1 \leq 80$

$F_2 = 0.25P_2^2 + 40P_2 + 150 \text{ Rs/hr} \rightarrow 40 \leq P_2 \leq 100$

$$\frac{dF_1}{dP_1} = 0.4 P_1 + 30 \text{ Rs/MWhr}$$

$$\frac{dF_2}{dP_2} = 0.5 P_2 + 40 \text{ Rs/MWhr}$$

$$\lambda \left[ \frac{1}{0.4} + \frac{1}{0.5} \right] = 130 + \left[ \frac{30}{0.4} + \frac{40}{0.5} \right]$$

$$4.5\lambda = 285 \Rightarrow \lambda = 63.33$$

$$P_1 = \frac{\lambda - \alpha_1}{\beta_1} = \frac{63.33 - 30}{0.4} = 83.32 \text{ MW}$$

$$P_2 = \frac{\lambda - \alpha_2}{\beta_2} = \frac{63.33 - 40}{0.5} = 46.66 \text{ MW}$$

**Note:** Here Generator 1 is violating upper limit which cannot be allowed instead it is fix to generate 80 MW and remaining rest of the load is shared by unit 2

$$\therefore P_D = P_1 + P_2$$

$$130 = 80 + P_2$$

$$P_2 = 50 \text{ MW}$$

$$\therefore P_1 = 80 \text{ MW}, P_2 = 50 \text{ MW}$$

**06. Ans: 79716 Rs/annum**

**Sol:** Given:

Alternators capacity = 200 MW

Load = 300 MW

$$\frac{dF_1}{dP_1} = 0.1 P_1 + 20 \text{ Rs/MWhr}$$

$$\frac{dF_2}{dP_2} = 0.12 P_2 + 15 \text{ Rs/MWhr}$$

When the load is economically divided between two generators

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$\Rightarrow 0.1 P_1 - 0.12 P_2 = -5 \dots\dots\dots (1)$$

$$P_1 + P_2 = 300 \dots\dots\dots (2)$$

Solving (1) and (2)

$$\Rightarrow P_1 = 140.91 \text{ MW and } P_2 = 159.09 \text{ MW}$$

$$F_1 = 0.1 \frac{P_1^2}{2} + 20P_1 + x$$

$$= 0.05P_1^2 + 20P_1 + x \text{ Rs/hr}$$

$$F_2 = 0.12 \frac{P_2^2}{2} + 15P_2 + y$$

$$= 0.06P_2^2 + 15P_2 + y \text{ Rs/hr}$$

Substitute  $P_1$  and  $P_2$  values in the above equation

$$F_1 = 0.05(140.91)^2 + 20 \times 140.91 + x \text{ Rs/hr}$$

$$F_2 = 0.06(1589.09)^2 + 15 \times 159.09 + y \text{ Rs/hr}$$

$$\therefore (F_1 + F_2)_{\text{Economic}} = (3810.98 + x + 3904.92 + y)$$

$$= (7715.9 + x + y) \text{ Rs/hr}$$

When the load is equally shared

$$\Rightarrow P_1 = 150 \text{ MW}, P_2 = 150 \text{ MW}$$

Substitute  $P_1$  and  $P_2$  values  $F_1$  and  $F_2$

$$\therefore F_1 = 0.05(150)^2 + 20 \times 150 + x \text{ Rs/hr}$$

$$F_2 = 0.06(150)^2 + 15 \times 150 + y \text{ Rs/hr}$$

$$\therefore (F_1 + F_2)_{\text{equals}} = (7725 + x + y) \text{ Rs/hr}$$

$$\therefore \text{saving} = (F_1 + F_2)_{\text{equal}} - (F_1 + F_2)_{\text{economic}} \\ = 9.1 \text{ Rs/hr}$$

$$\therefore \text{savings in fuel cost per annum}$$

$$= 9.1 \times 365 \times 24 \text{ Rs/annum}$$

$$= 79716 \text{ Rs/annum.}$$

**07. Ans: (c)**

**Sol:** Incremental fuel cost of generator 'A' for maximum power generation

$$= 600 \text{ Rs/ MWhr}$$

Incremental fuel cost of generator 'B' for minimum power generation

$$= 650 \text{ Rs / MWhr}$$

As the incremental fuel cost for maximum generation of generator 'A' is less than the incremental fuel cost for minimum generation of generator 'B' is Hence we can operate the generator 'A' at its maximum output of 450 MW and the remaining will be generated by generator 'B'.

**08. Ans: (c)**

**Sol:**  $\frac{dF_1}{dP_1} = b + 2CP_1 \text{ RS/MWhr}$

$$\frac{dF_2}{dP_2} = b + 4CP_2 \text{ RS/MWhr}$$

For most economic generation

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$b + 2CP_1 = b + 4CP_2$$

$$P_1 = 2P_2$$

$$\text{Given } P_1 + P_2 = 300$$

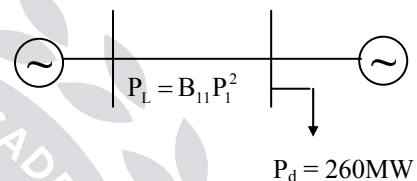
$$\therefore 2P_2 + P_2 = 300$$

$$P_1 = 200 \text{ MW}, P_2 = 100 \text{ MW.}$$

**09. Ans:  $P_1 = 300 \text{ MW}, P_2 = 50 \text{ MW}$**

$$P_L = 90 \text{ MW}$$

**Sol:**



$$B_{11} = \frac{10}{100^2} = 0.001$$

$$P_1 + P_2 - P_L = P_d = 260$$

$$P_1 + P_2 - B_{11} P_1^2 = 260$$

$$P_1 + P_2 - 0.001 P_1^2 = 260 \dots\dots\dots (1)$$

Assuming lossless problem

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = 0.02P_1 + 16 = 0.04P_2 + 20$$

$$-4 + 0.02P_1 = 0.04P_2$$

$$P_2 = \frac{0.02P_1 - 4}{0.04}$$

$$P_2 = 0.5P_1 - 100 \dots\dots\dots (2)$$

Substitute (2) in (1)

$$P_1 + 0.5P_1 - 100 - 0.001 P_1^2 = 260$$

$$1.5P_1 - 0.001 P_1^2 - 360 = 0$$

$$0.001 P_1^2 - 1.5P_1 + 360 = 0$$

$$\Rightarrow P_1 = 300 \text{ MW}$$

$$P_2 = 0.5 \times 300 - 100$$

$$= 50 \text{ MW}$$

$$P_L = 0.001 \times 300^2$$

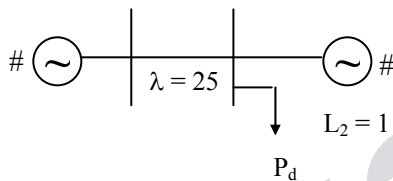
$$= 90 \text{ MW}$$

**10. Ans:  $P_1 = 133.33 \text{ MW}$**

$$P_2 = 100 \text{ MW}$$

$$P_d = 215.53 \text{ MW}$$

**Sol:**



$$\frac{dF_1}{dP_1} L_1 = \frac{dF_2}{dP_2} L_2 = \lambda$$

$$0.06P_2 + 19 = 25$$

$$P_2 = \frac{25-19}{0.06} = 100 \text{ MW}$$

$$\frac{dF_1}{dP_1} L_1 = \lambda$$

$$(0.01P_1 + 17) = \lambda \left[ 1 - \frac{dP_L}{dP_1} \right]$$

$$(0.01P_1 + 17) = 25(1 - 2B_{11}P_1)$$

$$P_L = B_{11} P_1^2$$

$$10 = B_{11} (100)^2$$

$$B_{11} = 10^{-3}$$

$$(0.01P_1 + 17) = 25(1 - 2 \times 0.001P_1)$$

$$0.01P_1 + (0.025P_1)2 = 25 - 17$$

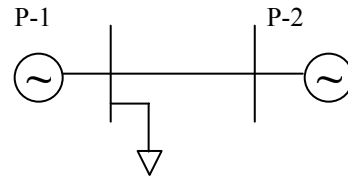
$$0.06P_1 = 25 - 17$$

$$P_1 = 133.33 \text{ MW}$$

$$\begin{aligned} \text{Power received} &= P_d = P_1 + P_2 - P_L \\ &= 133.33 + 100 - (0.001)(133.3)^2 \\ &= 215.53 \text{ MW} \end{aligned}$$

**11. Ans: (b)**

**Sol:**



$$B_{11} = B_{12} = B_{21} = 0, B_{22} \neq 0$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0} = 1$$

$$\begin{aligned} L_2 &= \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1 - 2B_{22}P_2} \\ &= \frac{1}{1 - (2 \times 10^{-3} \times 100)} = 1.25 \end{aligned}$$

**12. Ans: (c)**

**Sol:**



$$\Delta P_D = 5 \text{ MW}$$

$$\Delta P_1 = 8 \text{ MW}$$

$$\Delta P_L = 8 - 5 = 3 \text{ MW}$$

$$\frac{P_L}{P_1} = \frac{\Delta P_L}{\Delta P_1} = 3/8 = 0.375$$

$$\text{Penalty factor } L_1 = \frac{1}{1 - \frac{\Delta P_L}{\Delta P_1}} = 1.6$$

**13. Ans:  $L_1 = 1.5625, L_2 = 1.25$**

$$\text{Sol: } \frac{dc_1}{dp_1} = 0.15P_1 + 150 \text{ Rs/MWhr}$$

$$\frac{dc_2}{dp_2} = 0.25P_2 + 175 \text{ Rs/MWhr}$$

$$P_1 = P_2 = 200 \text{ MW}$$

$$\frac{dP_L}{dP_2} = 0.2$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1 - 0.2} = 1.25$$

$$L_1 I_{c1} = L_2 I_{c2}$$

$$L_1 = 1.25 \left( \frac{0.25 \times 200 + 175}{0.15 \times 200 + 150} \right) = 1.5625$$

14. Ans: (b)

15. Ans: (c)

16. Ans: (a)

17. Ans: (a)

**Sol:** Unit commitment is optimally out of the available generating sources to meet the expected load and provide a specified margin of operating reserve over a specified period of time.

18. Ans: (d)

### Solutions for Conventional Practice Questions

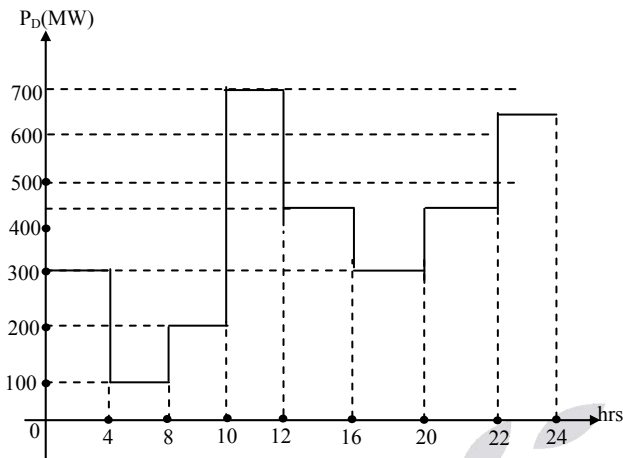
01.

**Sol:**

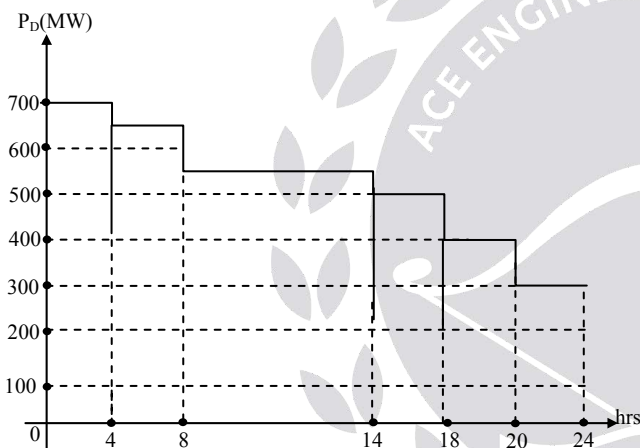
Max.Demand	0-4	4-8	8-10	10-12	12-16	16-20	20-22	22-24
A=100MW	✓	✓	×	✓	×	✓	×	×
B=150MW	×	×	×	×	✓	✓	✓	✓
C=200MW	✓	×	×	✓	✓	×	×	✓
D=100MW	×	×	✓	✓	✓	×	×	×
E=300MW	×	×	✓	✓	×	×	✓	✓
	300MW	100MW	400MW	700MW	450MW	250MW	450MW	650MW



**(A) Load curve:**



**Load duration curve:**



(B) Maximum demand = 700 MW

(C) Diversity factor

$$= \frac{\text{sum of individual maximum demands}}{\text{simultaneous maximum demand}}$$

$$= \frac{100 + 150 + 200 + 100 + 300}{700}$$

$$= 1.21$$

(D) Load factor =  $\frac{\text{Average demand}}{\text{maximum demand}}$

Average demand

$$= \frac{300 \times 4 + 100 \times 4 + 400 \times 2 + 700 \times 2 + 450 \times 4 + 250 \times 4 + 450 \times 2 + 650 \times 2}{24}$$

$$= 366.6 \text{ MW}$$

$$\therefore \text{load factor} = \frac{366.66}{700} = 0.52$$

(E) Plant capacity = Reserve capacity +  
Maximum demand = 100 + 700 = 800 MW

**02.**

**Sol:**  $I_{C1} = 1.0P_1 + 85 \text{ Rs/MWhr}$

$$I_{C2} = 1.2P_2 + 72 \text{ Rs/MWhr}$$

$$B_{11} = 0.015 \text{ MW}^{-1}$$

$$B_{22} = 0.02 \text{ MW}^{-1}$$

$$B_{12} = -0.001 \text{ MW}^{-1}$$

$$\lambda = 150 \text{ Rs/MWhr}$$

$$P_D = 30 \text{ MW}$$

$$\Delta\lambda = 15 \text{ Rs/MW}$$

$$L_1 I_{C1} = L_2 I_{C2} = \lambda \dots \dots \dots (1)$$

$$P_G = P_D + P_L \dots \dots \dots (2)$$

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1 + 2B_{12}P_2$$

$$= 0.03P_1 - 0.002P_2$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$$

$$= 0.04P = \frac{1}{1 - 0.03P_1 + 0.002P_2}$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{22}P_2 + 2B_{12}P_1 - 0.002P_1$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}}$$

$$= \frac{1}{1 - 0.04P_2 + 0.002P_1}$$

$$L_1 I_{C1} = \lambda$$

$$\frac{1}{1 - 0.03P_1 + 0.002P_2} (P_1 + 85) = 150$$

$$P_1 + 85 = 150 - 4.5P_1 + 0.3P_2$$

$$5.5P_1 - 0.3P_2 = 65 \dots\dots\dots (3)$$

$$L_2 I_{C2} = \lambda$$

$$\left[ \frac{1}{1 - 0.04P_2 + 0.002P_1} \right] (1.2P_2 + 72) = 150$$

$$1.2P_2 + 72 = 150 - 6P_2 + 0.3P_1$$

$$-0.3P_1 + 7.2P_2 = 78 \dots\dots\dots (4)$$

Solving (3) & (4)

$$P_1 = 12.43 \text{ MW}$$

$$P_2 = 11.35 \text{ MW}$$

$$P_L = 0.015 (12.43)^2 + 0.02 (11.35)^2 - 2 \times 0.001 \times 12.43 \times 11.35$$

$$= 4.61 \text{ MW}$$

$$P_G = P_D + P_L$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = P_1 + P_2 - P_L = 19.17 \text{ MW}$$

$\therefore$  Demand calculated is less than Demand expected so generation needs to be increased

$$\text{So } \lambda = \lambda^\circ + \Delta\lambda$$

$$= 165 \text{ Rs/MWhr}$$

Now again

$$L_1 I_{C1} = \lambda$$

$$\left( \frac{1}{1 - 0.03P_1 + 0.002P_2} \right) (P_1 + 85) = 165$$

$$5.95P_1 - 0.33P_2 = 80$$

$$L_2 I_{C2} = \lambda$$

$$\left[ \frac{1}{1 - 0.04P_2 + 0.002P_1} \right] [1.2P_2 + 72] = 165$$

$$-0.33P_1 + 7.81P_2 = 93$$

**03.**

$$\text{Sol: } \frac{dF_1}{dP_1} = 0.010 P_1 + 8.5$$

$$\frac{dF_2}{dP_2} = 0.015 P_2 + 9.5$$



Load is located at plant "2"

$\therefore$  The losses in the line will not be affected by generator of plant 2

$$P_L = B_{11} P_1^2$$

$$16 \text{ MW} = B_{11} (200 \text{ MW})^2$$

$$B_{11} = \frac{16 \text{ MW}}{(200 \text{ MW})^2}$$

$$= 4 \times 10^{-4}$$

$$B_{11} = 0.0004$$

Coordination equation

$$\frac{dF_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda \quad \dots\dots\dots (1)$$

$$P_L = 0.0004 P_1^2$$

$$\frac{dP_L}{dP_1} = 0.0008 P_1$$

Substitute in equation (1)

$$0.010P_1 + 8.5 + \lambda (0.0008P_1) = 12.5$$

$$0.010P_1 + 0.01P_1 = 4$$

$$P_1 (0.02) = 4$$

$$P_1 = \frac{4}{0.02}$$

$$= 200 \text{ MW}$$

$$\frac{dF_2}{dP_2} = 0.015P_2 + 9.5$$

$$0.015P_2 + 9.5 = 12.5$$

$$P_2 = \frac{3}{0.015}$$

$$= 200 \text{ MW}$$

$$\text{The transmission loss } P_L = 0.0004(200)^2$$

$$= 16 \text{ MW}$$

Power received by the load

$$P_D = 200 + 200 - 16 = 384 \text{ MW}$$

∴ Power supplied by the generator 1 is

$$= 200$$

And power supplied by the generator 2 is

$$= 200 \text{ MW}$$

**04.**

$$\text{Sol: } \lambda = (\text{Penalty factor}) \times \frac{dc}{dp}$$

Let us assume

$$\frac{dc_1}{dp_1} = \text{incremental fuel cost for the plant(1)}$$

$$\frac{dc_1}{dp_1} = 275/- \text{ per MWh}$$

$$\frac{dc_2}{dp_2} = \text{incremental fuel cost for the plant(2)}$$

$$\frac{dc_2}{dp_2} = 300/- \text{ per MWh}$$

Coordination equation with losses is

$$\lambda = L_1 \frac{dc_1}{dp_1} = L_2 \frac{dc_2}{dp_2}$$

Since the system  $\lambda$  should satisfy the above equation

$$\lambda = (275) L_1 = (300) L_2$$

Where  $L_1$  = penalty factor for the plant(1)

$L_2$  = penalty factor for the plant(2)

Must be  $L_1 > L_2$

Hence the penalty factor of the plant 1 is high.

Given that the cost per hour of increasing the load on the system  $\lambda = 341/-$  per MWh.

From coordination equation,

$$341 = 275 L_1 \Rightarrow L_1 = 1.24$$

$$300 L_2 = 341 \Rightarrow L_2 = 1.13$$

### Load Frequency Control

#### Solutions for Objective Practice Questions

**01. Ans: (c)**

**Sol:** Given data:

Nominal frequency is 60 Hz,

Regulation is 0.1.

When load of 1500 MW,

$$\begin{aligned}\text{The regulation} &= \frac{0.1 \times 60}{1500} \\ &= \frac{6}{1500} \text{ Hz / MW}\end{aligned}$$

**02. Ans: (a)**

**Sol:** Given data:

$D = 2$ ,  $R = 0.025$ ,

We know that Change in load

$$\Delta P_D = -\left(D + \frac{1}{R}\right) \Delta f,$$

where  $\Delta f$  = change in frequency

$$= D + \frac{1}{R} \Rightarrow 2 + \frac{1}{0.025} = 42 \text{ MW / Hz}$$

$\therefore \text{AFRC} = 42 \text{ MW / Hz}$

**03. Ans: (b)**

**Sol:** Given data:

$f = 50 \text{ Hz}$ , generator rating = 120 MVA

Generator frequency decreases 0.01

$$\frac{\Delta f}{f} = \frac{0.06X}{120}$$

$$\Rightarrow X = \frac{0.01}{50} \times \frac{120}{0.06} = 0.4 \text{ MW}$$

**04. Ans: (c)**

**Sol:** Given data:

The energy stored at no load =  $5 \times 100$   
 $= 500 \text{ MJ}$

Before the steam valves open the energy lost by the rotor =  $25 \times 0.6 = 15 \text{ MJ}$

As a result of this there is reduction in speed of the rotor and,

$\therefore$  reduction in frequency

$$\begin{aligned}f_{\text{new}} &= \sqrt{\frac{500 - 15}{500}} \times 50 \\ &= 49.24 \text{ Hz}\end{aligned}$$

**05. Ans: (c)**

$$\begin{aligned}\text{Sol: \% regulation} &= \frac{\frac{\Delta f}{f}}{\frac{\Delta p}{p}} = \frac{\frac{50 - 48}{50}}{\frac{100}{100}} \times 100 \\ &= \frac{2}{50} \times 100 = 4\%\end{aligned}$$

### Generating Stations

#### Solutions for Conventional Practice Questions

### Thermal Plants

**01.**

**Sol: Water treatment plant:** Boilers require clean and soft water for longer life and better efficiency. However, the source of boiler feed water is generally a river or lake which may contain suspended and dissolved impurities, dissolved gases etc. Therefore, it is very important that water is first purified

and softened by chemical treatment and then delivered to the boiler.

The water from the source of supply is stored in storage tanks. The suspended impurities are removed through sedimentation, coagulation and filtration. Dissolved gases are removed by aeration and degasification. The water is then 'softened' by removing temporary and permanent hardness through different chemical processes. The pure and soft and soft water thus available is fed to the boiler for steam generation.

**02.**

**Sol:** Super Thermal Power Stations (STPS) or Super Power Station is a series of ambitious power projects planned by the Government of India. With India being a country of chronic power deficits, the Government of India has planned to provide 'power for all' by the end of the eleventh plan. The capacity of thermal power is 1000 MW and above. This would entail the creation of an additional capacity of at least 100,000 Megawatts by 2012. The Ultra Mega Power Projects, each with a capacity of 4000 megawatts or above, are being developed with the aim of bridging this gap.

**03.**

**Sol:** The term 'super-critical' is used for power plants with operating pressures above critical pressure. Thermodynamic cycles which operate at parameters above critical point (at 225.56 kg/cm<sup>2</sup> and 374.15 °C ) are called 'supercritical cycles'. At critical point, density of water and steam are same.

**04.**

**Sol: Electro static precipitator (ESP):** The use of electrostatic precipitator is to remove fine, dust particles from flue gas. It is connected to high D.C. voltage about 30 kV. It is placed between combustion chamber and chimney.

**05.**

**Sol: (i) Boilers:** Boilers or steam generators convert water into steam and form one of the major equipments in a steam power plant. Boilers used in steam power plants are of two types namely fire tube boilers and water tube boilers. In fire tube boilers the tubes containing hot gases of combustion inside are surrounded with water while in water tube boilers the water is inside the tubes and hot gases outside the tubes. Fire tube boilers are compact in size, have low initial cost and have the ability to raise rapidly large

quantities of steam per unit area of fire grate but have the following drawbacks.

As water and steam, both are in the same shell, higher pressure of steam are not possible, the maximum pressure which can be had is about  $17.5 \text{ kg/cm}^2$  and with a capacity of 15,000 kg of steam per hour. For higher pressure or higher rates of evaporation, the shell and fire tube boilers become extremely heavy and unwieldy. In the event of a sudden and major tube failure, steam explosions may be caused in the furnace due to rush of high pressure water into the hot combustion chamber which may generate large quantities of steam in the furnace.

**06.**

**Sol:** Let  $x \text{ cal/kg}$  be the calorific value of fuel  
 heat produce by 1.0 kg of coal =  $1.0 \times x \text{ cal}$

$$1 \text{ kWh} = 800 \text{ k cal}$$

$$\eta_{\text{overall}} = \frac{\text{electric output in heat units}}{\text{Heat of combustion}}$$

$$0.15 = \frac{860}{1.0x}$$

$$x = \frac{860}{1.0 \times 0.15}$$

$$x = 1720 \text{ k cal/kg}$$

**07.**

**Sol:** Flue gas is the gas exiting to the atmosphere via a flue, which is a pipe or channel for conveying exhaust gases from a fireplace, oven, furnace, boiler or steam generator. Quite often, the flue gas refers to the combustion exhaust gas produced at power plants. Its composition depends on what is being burned, but it will usually consist of mostly nitrogen (typically more than two-thirds) derived from the combustion of air, carbon dioxide ( $\text{CO}_2$ ), and water vapor as well as excess oxygen (also derived from the combustion air). It further contains a small percentage of a number of pollutants, such as particulate matter (like soot), carbon monoxide, nitrogen oxides, and sulfur oxides.

**08.**

**Sol:** Given data

$$\begin{aligned} \text{Thermal efficiency of power station} &= 30\% \\ &= 0.3 \end{aligned}$$

$$\begin{aligned} \text{Electrical efficiency of power station} &= 80\% \\ &= 0.8 \end{aligned}$$

$$\text{Calorific value of coal} = 6400 \text{ kcal/kg}$$

$$\text{capacity of steam plant} = 75 \text{ MW}$$

$$\text{Overall efficiency} = \text{Thermal } \eta \times \text{electrical efficiency}$$

$$= 0.3 \times 0.8$$

$$= 0.24$$

Units generated per day at full load

$$= 75 \times 10^3 \times 24 \text{ kWh}$$

$$= 1800 \times 10^3 \text{ kWh}$$

Overall efficiency

$$= \frac{\text{units generated per day in kcal}}{\text{heat produced per day in kcal}}$$

(Heat equivalent of 1 kWh = 860 kcal)

So heat produced per day

$$= \frac{1800 \times 10^3 \times 860}{0.24}$$

$$= 6,450 \times 10^6 \text{ kcal}$$

Coal consumption per day

$$= \frac{\text{Heat produced per day}}{\text{calorific value of coal}}$$

$$= \frac{6,450 \times 10^6 \text{ kcal}}{6400 \text{ kcal/kg}}$$

$$= 1007812.5 \text{ kg}$$

$$= 1007.812 \text{ tonnes.}$$

### Hydel Plants

01.

**Sol: Water hammer effect:**

When a pipe is suddenly closed at the outlet (downstream), the mass of water before the

closure is still moving, thereby building up high pressure and a resulting shock wave. In domestic plumbing this is experienced as a loud banging resembling a hammering noise. Water hammer can cause pipelines to break if the pressure is high enough. Air traps or stand pipes (open at the top) are sometimes added as dampers to water systems to absorb the potentially damaging forces caused by the moving water.

In hydroelectric generating stations, the water traveling along the tunnel or pipeline may be prevented from entering a turbine by closing a valve. For example, if there is 14 km (8.7 mi) of tunnel of 7.7 m (25 ft) diameter full of water travelling at 3.75 m/s (8.4 mph), that represents approximately 8,000 megajoules (2,200 kWh) of kinetic energy that must be arrested. This arresting is frequently achieved by a surge shaft open at the top, into which the water flows. As the water rises up the shaft its kinetic energy is converted into potential energy, which causes the water in the tunnel to decelerate. At some hydroelectric power (HEP) stations, such as the Saxon Falls Hydro Power Plant in Michigan, what looks like a water tower is actually one of these devices, known in these cases as a surge drum.



**Water hammer effect can be prevented by:**
**1. Remove the cause of the hammer:**

Some causes can be resolved by arranging for the elimination or control of the problem item. Apart from the items previously discussed, this might include vibrating pressure relief valves, fast emergency shutdown valve closures, and some manual valve closures eg butterfly valves. Soft starters can assist with some water hammer problems induced by pumps.

**2. Reduce the pumping velocity.**

This can be done using a larger pipe diameter or lower flowrate.

**3. Make the pipe stronger.**

This can be expensive but might be a solution if the pipe specification is only slightly exceeded.

**4. Slow down valves, or use ones with better discharge characteristics in the pipe system.**
**5. Use surge tanks.** These allow liquid to leave or enter the pipe when water hammer occurs, and are normally only seen on water systems.

**6. Use surge alleviators.** These are similar to pulsation dampers commonly fitted to positive displacement pumps, only much larger.

7. Use pump flywheels. These can be used when water hammer is a consequence of a pump slowing too quickly following a trip.

8. Use pressure relief valves. These are not suitable with toxic materials unless a catch system is provided.

9. Use air inlet valves. These are not suitable if ingress of air or other possible external materials is not permissible.

10. A novel solution would be the injection of nitrogen or air into the fluid. The author has not seen this used in practice and its use would require care, but it is theoretically possible.

**02.**

**Sol:** Cavitation is a phenomenon in which rapid changes of pressure in a liquid lead to the formation of small vapor-filled cavities, in places where the pressure is relatively low.

When subjected to higher pressure, these cavities, called "bubbles" or "voids", collapse and can generate an intense shock wave.

Cavitation is a significant cause of wear in some engineering contexts. Collapsing voids that implode near to a metal surface cause cyclic stress through repeated implosion. This results in surface fatigue of the metal causing a type of wear also called "cavitation". The most common examples of

this kind of wear are to pump impellers, and bends where a sudden change in the direction of liquid occurs.

### Cavitation Prevention

1. Check filters and strainers – clogs on the suction, or discharge side can cause an imbalance of pressure inside the pump
2. Reference the pump's curve – Use a pressure gauge and/or a flowmeter to understand where your pump is operating on the curve. Make sure it is running at its best efficiency point
3. Re-evaluate pipe design – Ensure the path the liquid takes to get to and from your pump is ideal for the pump's operating conditions

03.

**Sol: Specific speed** is defined as “the speed of an ideal pump geometrically similar to the actual pump, which when running at this speed will raise a unit of volume, in a unit of time through a unit of head”.

The performance of a centrifugal pump is expressed in terms of pump speed, total head, and required flow. This information is available from the pump manufacturer's published curves. Specific speed is calculated from the following formula, using data from these curves at the pump's best efficiency point (BEP):

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

N = The speed of the pump in revolutions per minute (rpm.)

Q = The flow rate in liters per second ( for either single or double suction impellers)

H = The total dynamic head in meters

### Runaway speed

The runaway speed of a water turbine is its speed at full flow, and no shaft load. The turbine will be designed to survive the mechanical forces of this speed. The manufacturer will supply the runaway speed rating. It is 1.5 to 3 times of the normal speed

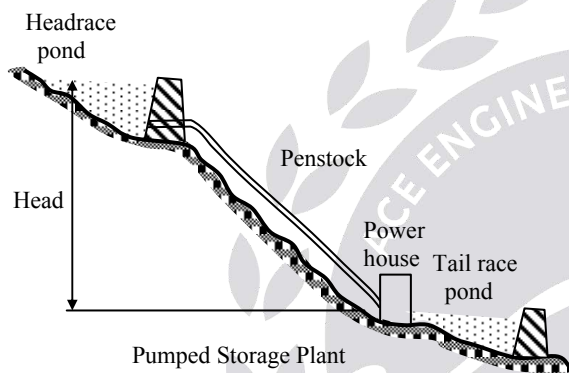
04.

### Sol: Pumped storage plant.

Pumped storage plants are a special type of power plants which work as ordinary hydro power plants for part of the time and when such plants are not producing power, they can be used as pumping stations which pump water from tail race to the head race. During this time, these plants utilize power available from the grid to run the pumping set. Thus, pumped storage plants can operate only if these plants are interconnected in a large grid.

The pumped storage plant thus consists of two ponds, one at a high level and the other

at a low level with power house near the low level pond. The two ponds are connected through a penstock as shown in below figure. It is an ingenious way of conserving the limited water resources on the hand and balancing the load on the distribution system, on the other hand. The plant operates as a source of electric energy during system peak hours and as a sink during off-peak hours.



The modern trend is to use a reversible pump turbine unit. While generating, the turbine drives the electric generator and in the reverse operation, the generator runs as a motor driving the turbine, which, now acts as a pump. The following are the advantages of a pumped storage plant:

**Advantages:**

- (a) Free from environmental pollution.
- (b) Readily adaptable to automatic and remote controls.
- (c) Greater flexibility in the operational schedules of the system
- (d) Economical as a peaking power station.

- (e) Improves load factor of the overall plant as it works as a load during off-peak periods of the system.

**05.**

**Sol:**

1. Initial cost of thermal power plant is lower than Hydro electric power plant.
2. Thermal power plants are located near the load centers where as Hydro electric power plants are located away from load centers. Therefore, transmission and distribution costs are quite low in thermal power plants.
3. Maintenance cost is quite high as skilled operating staff is required in thermal power plant than Hydro electric power plant.
4. Cost of fuel transportation is maximum because huge amount of coal is transported to the plant site where as in hydro electric power plant, cost of fuel transportation is nil.
5. Running cost higher in thermal power plant than hydro electric power plant.
6. The simplicity and cleanness of hydro electric plant is simple and clean, whereas in thermal causes air pollution disposal of ash is another problem.
7. Field applications

Hydro electric: can be used to supply peak load or base load.

Thermal: Generally used to supply base load.

8. Reliability of hydro electric is simple, robust and most reliable, thermal is less reliable compare to hydro.

9. Hydro electric plant need a large space for civil engineering construction work such as dams etc. The buildings has to be much larger than that required for other type of plants.

Much more space than Diesel electric stations but much less when compare to with hydro stations is required in thermal power plant. A huge space is required for storage of fuel(i.e., coal).

(Note: Problem was misprinted)

06.

Sol: Given data:

Annual load factor = 0.2

Annual plant capacity factor = 0.15

Annual load factor

$$= \frac{\text{energy generated during 1 year}}{\text{maximum load} \times 8760}$$

$$0.2 = \frac{438 \times 10^4}{\text{maximum load} \times 8760}$$

$$\begin{aligned} \text{maximum load} &= \frac{438 \times 10^4}{0.2 \times 8760} \\ &= 2500 \text{ kW} \end{aligned}$$

$$= 2.5 \text{ MW}$$

Capacity factor

$$= \frac{\text{maximum load}}{\text{plant capacity}} \times \text{load factor}$$

$$0.15 = \frac{\text{maximum load}}{\text{plant capacity}} \times 0.2$$

$$\frac{\text{maximum load}}{\text{plant capacity}} = \frac{0.15}{0.2}$$

$$= 0.75$$

$$\text{Plant capacity} = \frac{2.5}{0.75} = 3.333 \text{ MW}$$

$$\text{Reverse capacity} = 3.333 - 2.5 = 0.833 \text{ MW}$$

## Nuclear Plants

01.

Sol: Merits of Nuclear power plant:

- (i) The amount of the fuel required is quite small. Therefore, there is a considerable saving in the cost of fuel transportation.
- (ii) A nuclear power plant requires less space as compared to any other type of the same size.
- (iii) It has low running charges as a small amount of fuel is used for producing bulk electrical energy and it is very economical for producing bulk electrical power.
- (iv) It can be located near the load centers because it does not require large quantities of water and need not be near coal mines. Therefore, the cost of primary distribution is reduced.

- (v) There are large deposits of nuclear fuels available all over the world. Therefore, such plants can ensure continued supply of electrical energy for thousands of years.
- (vi) It ensures reliability of operation.

**Demerits of Nuclear power plant:**

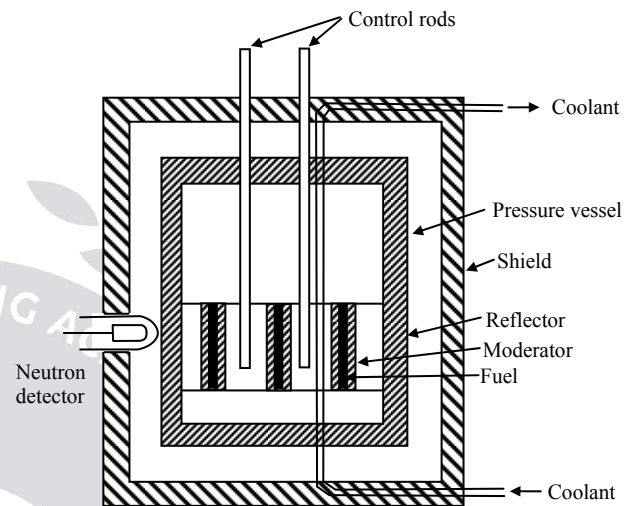
- (i) Nuclear power plants are not suitable for variable load since the reactor cannot be easily controlled to respond quickly to load changes. They are used at a load factor not less than 80%.
- (ii) The capital cost on a nuclear power plant is very high as compared to other types of plants.
- (iii) The fuel used is expensive and is difficult to recover.
- (iv) The skilled persons are required to handle the plant therefore, maintenance cost is more.
- (v) The fission-by-products are generally radio active and may cause a dangerous amount of radioactive pollution.

**02.**

**Sol: Nuclear Reactor:** A nuclear reactor is a device in which energy is made available through controlled nuclear reaction. The main parts of a reactor are

- (i) a core in which the nuclear reaction takes place and energy is release.
- (ii) a control system used for controlled the rate of energy release.

- (iii) a method of extracting the energy such as a cooling system which could remove heat from the core.
- (iv) a biological shield to protect the personnel against radiations emitted from the reactor.



**Basic Components of a Nuclear Reactor**

**Reactor Core:**

The core consists of a number of fuel rods made of fissile material. The material is used for cladding the nuclear fuel should be resistant to abrasion, should have a low neutron cross section, bond well to uranium and should be cheaply available. The materials used are aluminium, stainless steel and zirconium. There should be good metallurgical bond between the fuel and the cladding material, otherwise heat transfer across the interface will be poor and will result in hot spots in the fuel, leading to malfunctioning of the reactor. It has been



observed that the materials, which have been found suitable for cladding fuel elements, are useful for other structural (conduits for cooling, structure for control rods etc.) purposes as well.

It is desirable to use reactor core as cubical or cylindrical in shape rather than spherical, as it facilitates the refueling operation and simplifies the process of circulation of coolant through the core. With this configuration, the core has a series of parallel fuel elements in the form of thin plates or small rods, with coolant flowing axially and additional moderator or reflector material surrounding the assembly. If the reactor is to be used for converting the fertile material into fissionable material, the material to be converted should be placed around the core so that the neutron, which otherwise would escape the core, would be utilized for conversion. This arrangement also simplifies the process of separation of the converted material during fuel reprocessing.

**Moderator:** The moderator is used to slow down the neutrons, by absorbing some of the kinetic energy of the neutrons by direct collision, thereby increasing the chances of fission. From the requirement of moderator, it is clear that the material should have a

light weight nucleus, so that it does not absorb the neutron as it collides. The material used are: graphite, ordinary water and heavy water.

Graphite is simple to fabricate and handle and does not pose any containment problem. However, if continued neutron bombing is maintained, this may create some stress problems. Light water, after dissolved impurities are removed, is the cheapest of all the moderating materials. This can be used as a coolant at moderate temperature and pressure. Heavy water is costlier per unit weight, as compared to graphite or ordinary water; as a result containment is a serious problem for heavy water than for ordinary water. For the same power output the size of the reactor, using heavy water is more compact as compared to one using ordinary water.

**Reflector:** A neutron reflector is placed around the core and used to avoid the leakage of neutron from the core. If a neutron tries to escape the core it is reflected back, by the reflector, and used for the conversion of non-fissionable material to fissionable material, thereby improving the efficiency of the reactor. The material normally used is a high purity or reactor-grade graphite. A reflector also helps in

bringing a more uniform distribution of heat production in the core, which simplifies the arrangements to be made for removing the heat. With this, it is possible to use uniform coolant flow at different locations throughout the core.

**Reactor Control:** The most common method of control involves insertion of a material, having high absorption cross section for thermal neutrons, into the core. Cadmium and boron are the two most commonly used materials. Boron is frequently alloyed with steel or aluminum and is used in the form of control rods or plates which may be inserted or removed from the system, depending upon the requirement.

A reactor usually has three different types of control rods – (a) safety rod, (b) shim rod and (c) regulating rod.

The safety rods, as long as, are inserted into the core, the reactor stops generation and when they are removed completely from the core, it starts generating. In case, there is an earthquake of high severity or excessive power generation or failure of control systems or any similar event leading danger to health hazards and safety, the safety rods are inserted manually or automatically.

The fission products in nuclear power plant are analogous to ash in coal fired plant, and these slow down the output of the

reactor. Shim rods are withdrawn from the core through small displacement at intervals, so as to compensate for fission product built-up in the fuel. These rods are usually partially withdrawn during start-up and are left in one position for a long period at constant level operation.

The load on the system keeps on changing from time to time. These changes are taken care of by the regulating rod. Also, if we keep all the control rods in one position, the radioactive material (e.g. Uranium) continues to decay and hence the power output goes on decreasing. Regulating rods are used to take care of the effect also. In order to maintain constant power output, continuous, adjustment of the regulating rod about a mean position is required.

**Coolant System:** For large nuclear power plants closed loop coolant system is used, which means the coolant passing through the reactor is re-circulated and is not passed through the turbines and discharged. With this, the discharge of the radioactive material into the atmosphere or rivers is avoided, thereby providing safety to the people residing in nearby areas. Also, by designing a suitable heat exchanger, it is possible to obtain suitable combinations of



temperature and pressure for higher efficiency, in a secondary fluid than in the primary fluid.

Boiling water as coolant is being used in United States of America. Liquid metal like sodium or sodium-potassium alloy is being used as a coolant, as it has better heat transfer properties. Sodium prevent problems of containment, reactivity with water and if it is released to atmosphere accidentally, this will lead to health hazards. However, these problems are being tackled and liquid metal may find a good future as a coolant.

British nuclear power plants are using carbon dioxide at high pressure as the coolant, as it has good heat transfer properties and poses no health hazard problem in case of leakage to the atmosphere or to secondary steam.

**Shielding:** Shielding is provided around a reactor to minimise the possible dosage of radiation acquired by personnel living nearby the reactor. Of the four types of radiations  $\alpha$ -and  $\beta$ -radiations do not cause much concern as the shielding provided against  $\alpha$ -and neutron radiation will be sufficient to stop  $\alpha$ -and  $\beta$ -radiations. The shielding material should be cheaply available and it should not pose any problem

in giving suitable shape to the shielding structure. Concrete is found to be the most commonly used shielding material.

In order to understand the control of nuclear reactor, we define here, what is known as multiplication factor( $K$ ). It is defined as the ratio of the total number of neutrons produced during a small time, to the total number of neutrons absorbed or lost during the same time. In order to keep the power output constant,  $K$  must be kept equal to unity i.e., one neutron and only one neutron from each fission must split another nucleus. When  $K$  is less than unity, the power developed decreases and when  $K$  is more than unity, the power developed increases.

When a reactor is started the value of  $K$  is taken slightly greater than unity (say 1.005, please note it should not be high), thereby the power level increases.  $K$  is reduced to unity immediately after required level of power is reached. Similarly if the power level is to be lowered, value of  $K$  is made slightly less than unity and is again made equal to unity after the desired lower level is reached. The reactor can be shut down when  $K$  is made less than unity for a long time.

03.

**Sol:** The basic working principle of running a nuclear power plant with a pressurized water reactor can be simplified in these 4 steps:

1. Obtaining thermal energy by nuclear fission of the nucleus of atoms of nuclear fuel.
2. Generate steam in the heat generator by means of the thermal energy obtained previously.
3. Operate a set of turbines using the steam obtained.
4. Take advantage of the mechanical energy of the turbines to drive an electric generator. This electric generator will generate electricity.

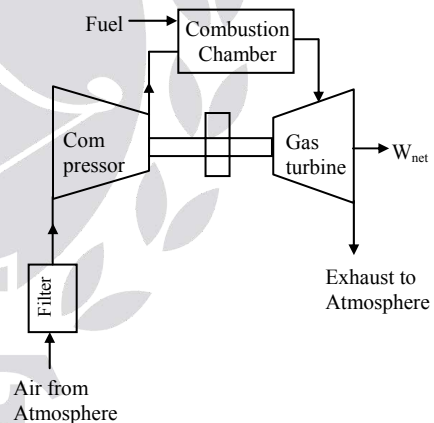
From a physical point of view several changes of energy are observed: initially we have nuclear energy (that keeps the nuclei of the atoms cohesion), later, when it is broken, it becomes thermal energy. Part of the thermal energy is converted into internal energy of water by becoming steam according to the principles of thermodynamics. The internal energy and the heat energy of the water are transformed into kinetic energy when the turbine is actuated. Finally, the generator converts the kinetic energy into electrical energy.

## Gas Plants

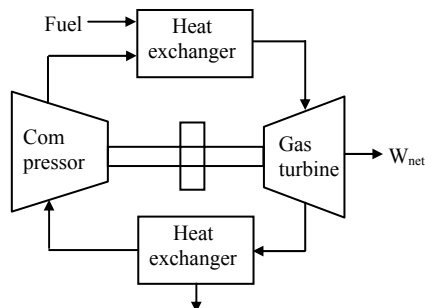
01.

**Sol: Gas Turbine Power Plant with Regeneration Reheating and intercooling:**

A simple Cycle Gas turbine follows the Brayton cycle. In many aircrafts gas turbine engines are used in their simple forms as the aircraft is needed to be light. While gas turbines which are used in land or marine application can be equipped with additional parts to increase the efficiency. Modifications that are usually seen in Gas Turbine cycle plants are regeneration, reheating and intercooling. Open cycle and close cycle gas turbine

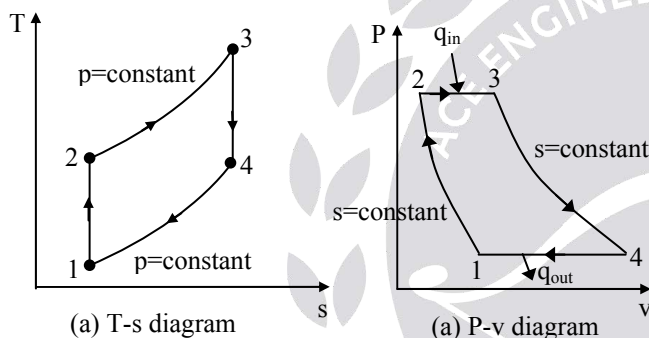


It is clearly visible that in open cycle GT the exhaust gas is not been used in any other purpose. It is discharged to atmosphere. But in closed cycle gas turbine the exhaust gas is used in an heat exchanger as a result the working fluid is reusable. It is also a clean cycle as the working fluid is recirculated.



### Gas Turbine or Brayton Cycle With Reheat , Regeneration and Intercooling:

Lets have a look at the T-s and P-V diagram of an ideal Brayton Cycle or Gas Turbine cycle.

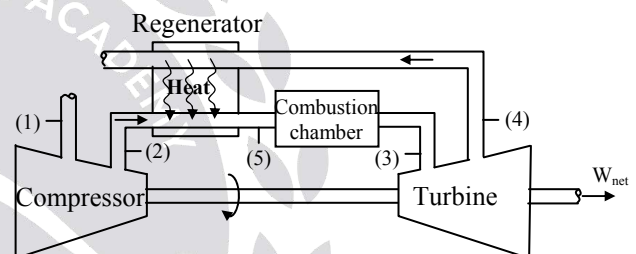


The simple gas turbine cycle consists of two isentropic and two isobaric processes. But there are some deviations in the actual cycle of gas turbine in comparing to the ideal Brayton cycle. The following diagram will focus on the deviations. The deviations are mainly due to the irreversibilities.

### Regeneration of Gas Turbine Plant

Regeneration process involves the installation of a heat exchanger in the gas turbine cycle. The heat-exchanger is also known as the recuperator. This heat

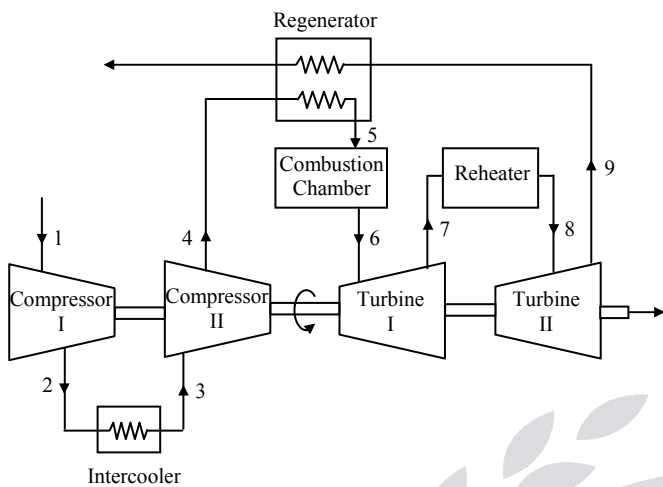
exchanger is used to extract the heat from the exhaust gas . This exhaust gas is used to heat the compressed air. This compressed and pre-heated air then enters the combustors. When the heat exchanger is well designed , the effectiveness is high and pressure drops are minimal. And when these heat exchangers are used an improvement in the efficiency is noticed. Regenerated Gas turbines can improve the efficiency more than 5 % . Regenerated Gas Turbine work even more effectively in the improved part load applications.



### Gas Turbine Power Plant with Intercooling:

In Intercooling a heat exchanger is used to cool the compressor gases at the time of compression process. When the compressor involves the high and low pressure unit in it, the intercooler could be installed between them to cool down the flow. This cooling process will decrease the work needed for the compression in the high pressure unit. The cooling fluid can be water, air. In marine gas turbines the sea water is used to cool the fluid. It is observed that a successful implementation of the

intercooler can improve the gas turbine output.



Gas turbine with regeneration, reheating and

02.

**Sol: Open loop gas Turbine:**  
**Advantages:**

1. **Warm-up time:** Once the turbine is brought up to the rated speed by the starting motor and the fuel is ignited, the gas turbine will be accelerated from cold start to full load without warm-up time.
2. **Low weight and size:** The weight in kg per kW developed is less.
3. **Fuels:** Almost any hydrocarbon fuel from high-octane gasoline to heavy diesel oils can be used in the combustion chamber.
4. Open cycle plants occupies less space compared to close cycle plants.
5. The stipulation of a quick start and take-up of load frequently are the points in

favor of open cycle plant when the plant is used as peak load plant.

6. Component or auxiliary refinements can usually be varied in open cycle gas turbine plant to improve the thermal efficiency and can give the most economical overall cost for the plant load factors and other operating conditions envisaged.
7. Open cycle gas turbine power plant, except those having an intercooler, does not need cooling water. Therefore, the plant is independent of cooling medium and becomes self-contained.

**Disadvantages:**

1. The part load efficiency of the open cycle gas turbine plant decreases rapidly as the considerable percentage of power developed by the turbine is used for driving the compressor.
2. The system is sensitive to the component efficiency; particularly that of compressor. The open cycle gas turbine plant is sensitive to changes in the atmospheric air temperature, pressure and humidity.
3. The open cycle plant has high air rate compared to the closed cycle plants, therefore, it results in increased loss of heat in the exhaust gases and large diameter duct work is needed.

4. It is essential that the dust should be prevented from entering into the compressor to decrease erosion and depositions on the blades and passages of the compressor and turbine. So damages their profile. The deposition of the carbon and ash content on the turbine blades is not at all desirable as it reduces the overall efficiency of the open cycle gas turbine plant.

**Closed loop power plant**

Advantages and Disadvantages:

**Advantages:**

1. It requires less space for installation.
2. The installation and running cost of gas turbines are less compare to others.
3. It has very high power to weight ratio.
4. It generates less vibration compare to reciprocating engine.
5. It starts easily and quickly.
6. It can work in changing load condition easily.
7. Its efficiency is higher than IC engines.
8. It can develop uniform torque, which is not possible in IC engines.

**Disadvantages:**

1. Starting problem. It cannot start easily because compressor is driven by the turning itself. So an external unit is

required to rotate the compressor to start the turbine.

2. Most of power is used to drive the compressor so it gives less output.
3. Overall efficiency of turbine is low because exhaust gases contain most of heat.

**03.****Sol: Advantage of Gas Turbine Plant**

- Smaller in size and weight as compared to and equivalent steam power plant
- Natural gas is a very suitable fuel
- The gas turbine plants are subjected to less vibration
- The initial cost lower than an equivalent steam Plant
- The installation and maintenance cost are less than thermal power plants.
- There are no standby losses in gas turbine plants
- It requires less water as compared to a steam plant.
- Any quantity of fuels can be used in gas turbine plants.
- It can be started quickly
- The Exhaust of gas turbine is free from smoke
- Gas turbines can be built relatively Quicker and requires less space.

**Disadvantages:**

- The part load efficiency is poor
- The unit is operated at high temperature and pressure so special metals are required to maintain the unit
- Major part of the work about 66% is developed in the turbine is used to travel in the drive the compressor
- The devices that are operated at high temperature are complicated.

