



ESE | GATE | PSUs



ELECTRICAL ENGINEERING

POWER ELECTRONICS & DRIVES 

Text Book : Theory with worked out Examples
and Practice Questions

Chapter 12 Power Electronics

Solutions for Tex Book Practice Questions

1. Basics & Power Semiconductor Devices

Solutions for Objective Practice Questions

01. Ans: (i) 100μs (ii) 100.5 μs (iii) 1005μs

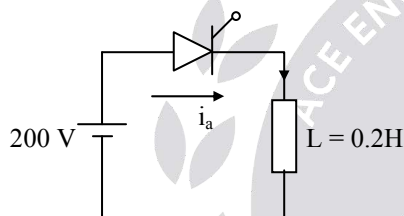
Sol: (i) $I_L = 100 \text{ mA}$

$$V = L \frac{di}{dt}$$

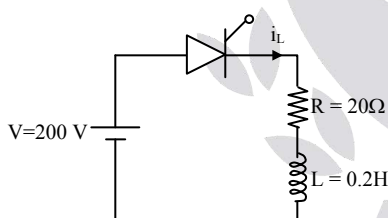
$$V t_p = L I_L$$

$$t_p = \frac{0.2 \times 100 \times 10^{-3}}{200}$$

$$= 100 \mu\text{sec}$$



(ii) $R = 20 \Omega$, $L = 0.2 \text{ H}$



$$i_L = \frac{V}{R} \left[1 - e^{-\frac{R}{L}t} \right]$$

At $t = t_p$, $i_L = 100 \text{ mA}$

$$100 \times 10^{-3} = \frac{200}{20} \left[1 - e^{-\frac{20}{0.2}t_p} \right]$$

$$10 \times 10^{-3} = [1 - e^{-100t_p}]$$

$$e^{-100t_p} = 0.99$$

$$t_p = 100.5 \mu\text{sec}$$

(iii) $R = 20 \Omega$, $L = 2 \text{ H}$

$$i_L = \frac{V}{R} \left[1 - e^{-\frac{R}{L}t} \right]$$

$$100 \times 10^{-3} = \frac{200}{20} \left[1 - e^{-\frac{20}{2}t_p} \right]$$

$$10 \times 10^{-3} = [1 - e^{-10t_p}]$$

$$e^{-10t_p} = 0.99$$

$$t_p = 1005 \mu\text{sec}$$

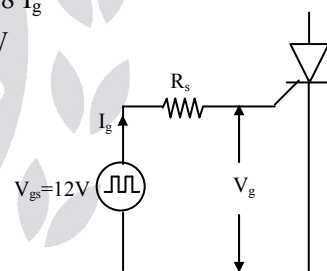
(iv) If load inductance increases, SCR requires pulse width for longer duration.

02. Ans: (i) 7Ω (ii) 1W

Sol: (i) Given

$$V_g = 1.5 + 8 I_g$$

$$V_g I_g = 5 \text{ W}$$



From KVL

$$V_{gs} = I_g R_s + V_g \dots\dots\dots (1)$$

$$V_g I_g = 5$$

$$(1.5 + 8 I_g) I_g = 5$$

$$I_g = 0.702 \text{ A}$$

$$\therefore V_g = \frac{5}{0.702} = 7.12 \text{ V}$$

From (1)

$$12 = 0.702 \times R_s + 7.12$$

$$R_s = 6.95 \Omega$$

(ii) $P_g = P_{g\text{max}} \times D$

$$= 5 \times 0.2 = 1 \text{ W}$$

03. (i) Ans: (c) (ii) (a)
Sol: (i) $I_{g \max} = 150 \text{ mA}$. Applied voltage

 $V = 10 \text{ V}$. Voltage drop of transistor, diode and gate cathode junctions are 1 V .

Write KVL to the gate circuit

$$-10 + I_{g \max} R + 1 + 1 + 1 = 0$$

$$150 \times 10^{-3} R = 7$$

$$R = 0.0467 \times 10^3 \Omega$$

$$= 46.7 \Omega$$

(ii) The time for which gate pulse should be applied along with SCR, at least anode current becomes more than latching current. Whenever SCR started conduction, the formula for anode current can be obtained as,

$$i = \frac{V}{R} \left[1 - e^{-\frac{Rt}{L}} \right]$$

$$i = \frac{200}{1} \left[1 - e^{-\frac{1 \times t}{0.15}} \right] = I_L$$

$$\Rightarrow 200 \left[1 - e^{-\frac{1t}{0.15}} \right] = 0.25$$

$$t = 187 \mu \text{sec}$$

Minimum volt second rating of transformer = voltage rating $\times$ Time for which signal should be applied

$$= 10 \times 187 = 1870 \mu \text{V-s}$$

Hence next higher rating of transformer is to be selected

$$\therefore \text{Volt-sec rating} = 2000 \mu \text{V-s}$$

04. Ans: 10 to 11
Sol: The rectifier diode can be represented as voltage drop = $IR + V$

$$\text{For } 0.8 \text{ V at } 2 \text{ A, } 0.8 = 2R + V \dots\dots\dots (1)$$

$$\text{For } 1.2 \text{ V at } 30 \text{ A, } 1.2 = 30R + V \dots\dots\dots (2)$$

$$\text{By solving (1) \& (2) } R = 14.2 \text{ m}\Omega, V = 0.7716$$

$$I_{\text{avg}} = \frac{I_m}{\pi} = \frac{30}{\pi} = 9.55 \text{ A}$$

$$I_{\text{rms}} = \frac{I_m}{2} = 15 \text{ A}$$

$$\begin{aligned} \therefore \text{Conduction loss} &= I_{\text{rms}}^2 R + V I_{\text{avg}} \\ &= (15^2 \times 14.2 \times 10^{-3}) + (9.55 \times 0.7716) \\ &= 10.564 \text{ W} \end{aligned}$$

05. Ans: (88°C)

$$\text{Sol: } T_c = 100^\circ; \quad T_j = 125^\circ \text{C}$$

$$\theta_{CA} = 0.5^\circ \text{C/W} \quad T_s = ?$$

$$T_A = 40^\circ \text{C}$$

$$\begin{aligned} P &= \frac{T_c - T_A}{\theta_{CA}} \\ &= \frac{100 - 40}{0.5} = 120 \text{ W} \end{aligned}$$

$$120 = \frac{T_s - T_A}{0.4}$$

$$T_s - T_A = 48$$

$$\Rightarrow T_s = 48 + 40 = 88^\circ \text{C}$$

06. Ans: (i) 229.17W, (ii) 8.71%

$$\text{Sol: } T_j = 125^\circ \text{C}; \quad \theta_{jc} = 0.16^\circ \text{C/W}$$

$$\theta_{cs} = 0.08^\circ \text{C/W}; \quad T_s = 70^\circ \text{C}$$

$$(i) P_{\text{av}} = \frac{T_j - T_s}{\theta_{js}}$$

$$\text{Where } \theta_{js} = \theta_{jc} + \theta_{cs}$$

$$= 0.16 + 0.08 = 0.24^\circ \text{C/W}$$

$$= \frac{125 - 70}{0.24} = 229.16 \text{ W}$$

$$(ii) \text{ Now } T_s = 60^\circ \text{C}$$

$$P_{\text{av}} = \frac{T_j - T_s}{\theta_{js}}$$

$$= \frac{125 - 60}{0.24} = 270.8 \text{ W}$$

Device rating means current rating and current rating is proportional to square root of power.

$$\begin{aligned}\% \text{ increase} &= \frac{\sqrt{270.8} - \sqrt{229.16}}{\sqrt{229.16}} \times 100 \\ &= 8.71\%\end{aligned}$$

07. Ans: (b)

Sol: $T_{ON} = 5 \mu \text{ sec}$, $I_L = 50 \text{ mA}$, $I_H = 40 \text{ mA}$

$$\begin{aligned}\text{From circuit } i &= \frac{V_s}{R} \left[1 - e^{-\frac{R}{L}t} \right] + \frac{V_s}{R} \\ &= \frac{100}{20} \left[1 - e^{-\frac{20}{0.5}t} \right] + \frac{100}{5000} \\ &= 5 \left[1 - e^{-40t} \right] + \frac{1}{50} \\ 50 \times 10^{-3} &= 5(1 - e^{-40t}) + \frac{1}{50} \\ t &= 150 \mu \text{ sec}\end{aligned}$$

08. Ans: (i) 7, 5, (ii) 22.22 k Ω , (iii) 0.094 μF

Sol: (i) $V = 11 \text{ kV}$, $I = 4 \text{ kA}$, $\eta = 90\%$

For series

$$\eta = \frac{\text{string voltage}}{n \times \text{voltage rating of SCR}}$$

$$n = \frac{11000}{0.9 \times 1800} \approx 7$$

For parallel

$$\eta = \frac{\text{string current rating}}{n \times \text{current rating of SCR}}$$

$$n = \frac{4000}{0.9 \times 1000} \approx 5$$

(ii)
$$R = \frac{n V_{bm} - V_s}{(n-1) \Delta I_b}$$

$$\therefore \Delta I_b = I_{b\max} - I_{b\min}$$

$$R = \frac{7(1800) - 11 \times 10^3}{(7-1) \times 12 \times 10^{-3}}$$

$$= 12 \text{ mA} - 0 = 12 \text{ mA}$$

$$= 22.2 \text{ k}\Omega$$

(iii)
$$C = \frac{(n-1) \Delta Q}{n V_{bm} - V_s}$$

$$= \frac{(7-1) \times 25 \times 10^{-6}}{7 \times 1800 - 11 \times 10^3} = 0.0937 \mu\text{F}$$

09. Ans: 74 to 76

Sol: Energy loss during

$$\begin{aligned}T_1 &= \int_0^{T_1} v \cdot i \, dt = 600 \times \int_0^{T_1} i \, dt \\ &= 600 \times \text{area under current curve} \\ &= 600 \times \frac{1}{2} \times 150 \times 1 \times 10^{-6} \\ &= 45 \text{ mJ}\end{aligned}$$

$$\begin{aligned}\text{Energy loss during } T_2 &= \int_0^{T_2} v \cdot i \, dt \\ &= 100 \times \int_0^{T_2} V \, dt \\ &= 100 \times \text{area under voltage curve} \\ &= 100 \times \frac{1}{2} \times 600 \times 1 \times 10^{-6} = 30 \text{ mJ}\end{aligned}$$

$$\text{Total energy loss} = 45 + 30 = 75 \text{ mJ}$$

10.

Sol: Switching scheme – I

(i) Energy loss during ON condition:

$$\begin{aligned}(E_{\text{loss}})_{ON} &= \int_0^{t_{on}} V \cdot i \, dt \\ &= \int_0^{t_r} \left(400 - \frac{400}{t_r} t \right) \cdot \frac{20}{t_r} t \, dt \\ &= \int_0^{t_r} \left[400(20) \frac{t}{t_r} - \frac{400(20)}{t_r^2} \cdot t^2 \right] dt\end{aligned}$$

$$= (400)(20) \left[\frac{t_r^2}{2t_r} - \frac{t_r^3}{3t_r^2} \right]$$

$$= \frac{(400)(20)}{6} \cdot t_r$$

$$(E_{\text{loss}})_{\text{ON}} = 133.33 \mu\text{J}$$

$$(E_{\text{loss}})_{\text{OFF}} = \frac{VI}{6} t_{\text{off}}$$

$$= \frac{400 \cdot (20)}{6} \times 200 \text{ ns} = 266.66 \mu\text{J}$$

$$E_{\text{Total}} = E_{\text{ON}} + E_{\text{OFF}}$$

$$= 133.33 + 266.66 \approx 400 \mu\text{J}$$

$$(ii) E = P \times t$$

$$P = E \times f$$

$$= (400 \times 10^{-6}) \times 100 \times 10^3$$

$$P = 40 \text{ W}$$

Switching scheme-II

Energy loss during ON condition

$$(i) (E_{\text{loss}})_{\text{ON}} = \int_0^{t_r} v i \, dt$$

$$= \int_0^{t_r} \left(400 - \frac{400}{t_r} \cdot t \right) 20 \, dt$$

$$= 400(20) \int_0^{t_r} \left(1 - \frac{t}{t_r} \right) dt$$

$$= 8000 \left(t_r - \frac{t_r^2}{2t_r} \right)$$

$$= \frac{8000}{2} \cdot t_r$$

$$= \frac{8000}{2} \times 100 \times 10^{-9} = 400 \mu\text{J}$$

$$(E_{\text{loss}})_{\text{OFF}} = \frac{VI}{2} t_{\text{OFF}}$$

$$= \frac{400 \times 20}{2} \times 200 \times 10^{-6} \text{ J}$$

$$= 800 \mu\text{J}$$

$$E_{\text{Total}} = E_{\text{ON}} + E_{\text{OFF}} = 1200 \mu\text{J}$$

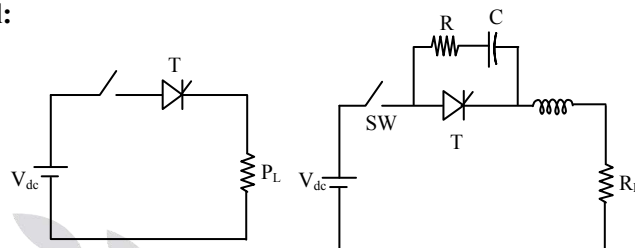
$$(ii) P = E \times f$$

$$= (1200 \times 10^{-6}) \times 100 \times 10^3$$

$$P = 120 \text{ W}$$

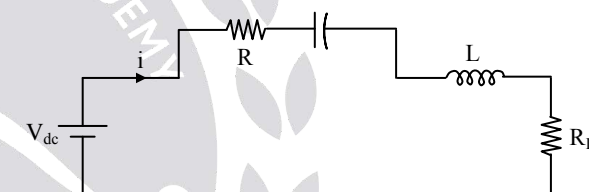
11. Ans: L = 4 μH, C = 0.16 μF and R = 5 Ω

Sol:



At the time of switch closed:

SCR is forward blocking condition



KVL $V_{\text{dc}} = (R + R_L) i + L \frac{di}{dt}$

$$i = \frac{V_{\text{dc}}}{R + R_L} [1 - e^{-t/\tau}]$$

Where $\tau = \frac{L}{R + R_L}$

$$\frac{di}{dt} = 0 - \frac{V_{\text{dc}}}{R + R_L} \left(-\frac{1}{\tau} e^{-t/\tau} \right)$$

$$\frac{di}{dt} = \frac{V_{\text{dc}}}{L} e^{-t/\tau}$$

$\frac{di}{dt}$ is maximum at $t = 0$

$$\left[\frac{di}{dt} \right]_{\text{Max}} = \frac{V_{\text{dc}}}{L}$$

$$L = \frac{V_{dc}}{(di/dt)_{\max}}$$

$$L = \frac{240\mu}{60} = 4 \mu\text{H}$$

$$L = 4 \mu\text{H}$$

Given damping ratio $\zeta = 0.5$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$0.5 = \frac{5}{2} \sqrt{\frac{C}{4}}$$

$$\Rightarrow C = 0.16 \mu\text{F}$$

As SCR is in forward blocking mode voltage

across SCR = $I_a R$

$$V_t = I_a R$$

$$\frac{dV_t}{dt} = R \frac{dI_a}{dt}$$

$$R = \frac{300}{60} = 5 \Omega$$

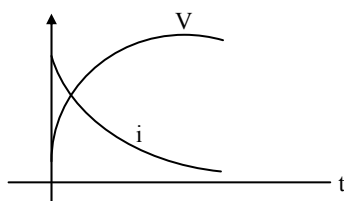
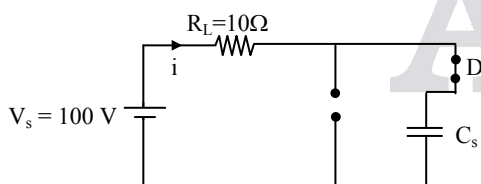
12. Ans: $R = 50 \Omega$, $C = 0.2 \mu\text{F}$

Sol: Given $\frac{dV}{dt} = 50 \frac{\text{V}}{\mu\text{s}}$,

$$I_{\text{discharge}} = 2\text{A}$$

When circuit is power up:

SCR is forward blocking condition



$$V_s = R_L i + \frac{1}{C} \int i dt$$

$$i = \frac{V_s}{R_L} e^{-t/\tau}$$

$$V_c = V_s [1 - e^{-t/\tau}]$$

$$\tau = R_L C$$

$$\frac{dV_c}{dt} = \frac{V_s}{R_L C} e^{-t/\tau}$$

$$\left(\frac{dV_c}{dt} \right)_{\max} \text{ at } t = 0$$

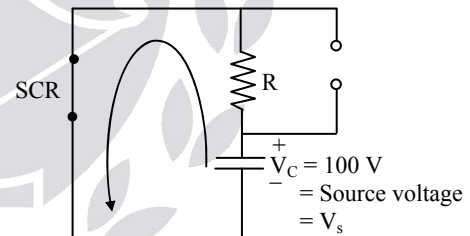
$$\frac{dV_c}{dt} = \frac{V_s}{R_L C}$$

$$\Rightarrow 50 \times 10^{-6} = \frac{100}{10 \times C}$$

$$C = 0.2 \mu\text{F}$$

When SCR is ON:

By that time capacitor is already charged source voltage, so starts discharging



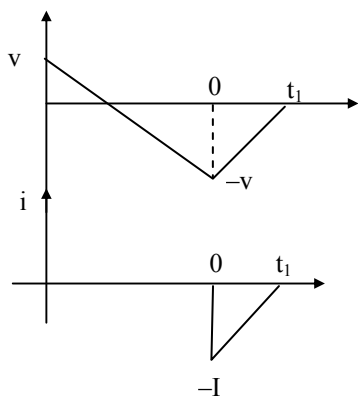
$$-100 + I_{\text{disch}} R = 0$$

$$R = \frac{100}{I_{\text{disch}}} = \frac{100}{2}$$

$$R = 50 \Omega$$

13. Ans: 1W, 900 μ C

Sol:



$$\begin{aligned}
 (E_{\text{loss}})_{\text{OFF}} &= \int_0^{t_1} v i \, dt \\
 &= \int_0^{t_1} \left(-v + \frac{v}{t_1} t \right) \left(-i + \frac{i}{t_1} t \right) dt \\
 &= \int_0^{t_1} \left(-100 + \frac{100}{t_1} t \right) \left(-300 + \frac{300}{t_1} t \right) dt \\
 &= 10^4 \int_0^{t_1} \left(-1 + \frac{t}{t_1} \right) \left(-3 + \frac{3t}{t_1} \right) dt \\
 &= 10^4 \int_0^{t_1} \left(3 - \frac{3(t)}{t_1} - \frac{3(t)}{t_1} + \frac{3t^2}{t_1^2} \right) dt \\
 &= 10^4 \left[3t_1 - \frac{6}{2t_1} t_1^2 + \frac{3}{t_1^2} \cdot \frac{t_1^3}{3} \right] \\
 &= 10^4 [3t_1 - 3t_1 + t_1] = 10^4 t_1 \\
 &= 10^4 \times 2 \times 10^{-6} \\
 &= 20 \text{ mW} \\
 P &= E \times f \\
 &= 20 \times 10^{-3} \times 50 = 1 \text{ W} \\
 Q &= \int i \, dt \\
 &= \frac{1}{2} \times 300 \times 6 \times 10^{-6} \\
 Q &= 900 \mu\text{C}
 \end{aligned}$$

14. Ans: (c)

Sol: Electronic switch described in the statement should have forward blocking state, forward

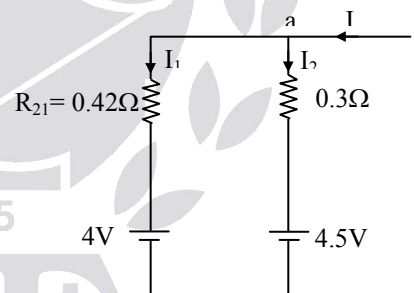
conduction state and reverse blocking state. SCR, NPN Transistor with series diode exhibits the above states.

15. Ans: (c)

$$\begin{aligned}
 \text{Sol: } I_C &= \frac{V_{CC} - V_{CE(\text{sat})}}{R_L} \\
 &= \frac{200 - 2}{10} = 19.8 \text{ A} \\
 P_{\text{on}} &= \frac{V_{CC} \times I_C}{6} \times t_{\text{on}} \times f_s \\
 &= \frac{200 \times 19.8}{6} \times 3 \mu \times 1 \text{ K} \\
 &= 1.98 \text{ W} \\
 P_{\text{off}} &= \frac{V_{CC} \times I_C}{6} \times t_{\text{off}} \times f_s \\
 &= 0.792 \text{ W}
 \end{aligned}$$

16. Ans: 9.54%

Sol: From the given data



The voltage at node a is

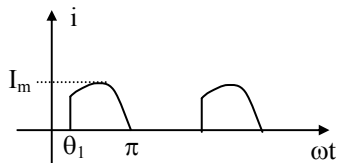
$$\begin{aligned}
 I &= \left(\frac{V_a - 4.5}{0.3} \right) + \left(\frac{V_a - 4}{0.4} \right) \\
 30 &= V_a (5.833) - (15 + 10) \\
 55 &= V_a (5.833) \\
 V_a &= 9.43 \text{ V} \\
 \Delta I &= I_2 - I_1 = -13.575 + 16.43 = 2.858 \text{ A} \\
 \frac{\Delta I}{I} \times 100 &= \frac{2.858}{30} \times 100 = 9.52\%
 \end{aligned}$$

Solutions for Conventional Practice Questions

01. For 30° 12.56 A

For 60° 18 A

Sol: For half-sine wave current as shown in below figure



$$I_{avg} = \frac{1}{2\pi} \int_{\theta_1}^{\pi} I_m \sin \theta d\theta = \frac{I_m}{2\pi} (1 + \cos \theta_1)$$

For 30° conduction angle, $\theta_1 = 150^\circ$

$$I_{avg} = 0.0213 I_m$$

$$I_{rms} = \left[\frac{I_m^2}{2\pi} \left[\frac{\pi - \theta_1}{2} + \frac{1}{4} \sin 2\theta_1 \right] \right]^{\frac{1}{2}}$$

$$I_{rms} = 0.0849035 I_m$$

$$\text{Form Factor} = \frac{0.085 I_m}{0.0213 I_m} = 3.99$$

$$\therefore I_{Tav} = \frac{50}{3.99} = 12.53 \text{ A}$$

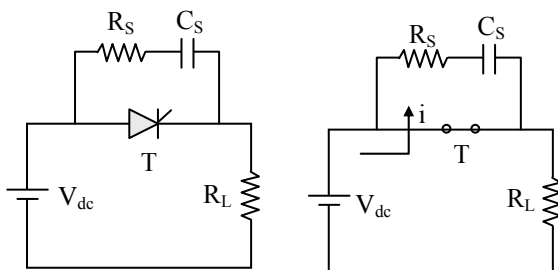
For 60° conduction angle, $\theta_1 = 120^\circ$

$$I_{av} = 0.0795 I_m$$

$$I_{Tav} = \frac{I_{Tr}}{FF} = 18 \text{ A}$$

02. $R_s = 2.5 \Omega$, $C_s = 0.15 \mu\text{F}$, 9.375 W

Sol:



$$V_{dc} = 250 \text{ V}; R_L = 7.5 \Omega$$

$$\frac{dv}{dt} = 125 \text{ V} / \mu\text{s}$$

$$I_{dis} = 100 \text{ A}$$

KVL:

$$(R_s + R_L) i + \frac{1}{C_s} \int i dt = V_{dc}$$

$$i = \frac{V_{dc}}{R_s + R_L} e^{-t/(R_s + R_L)C_s}$$

$$V_T = V_{dc} - R_L i$$

$$= V_{dc} - R_L \times \frac{V_{dc}}{R_s + R_L} e^{-t/(R_s + R_L)C_s}$$

$$\frac{dV_T}{dt} = -\frac{R_L \cdot V_{dc}}{(R_s + R_L)} e^{-t/(R_s + R_L)C_s} \left[\frac{-1}{(R_s + R_L)C_s} \right]$$

$$= \frac{R_L \cdot V_{dc}}{(R_s + R_L)^2 C_s} e^{-\frac{t}{(R_s + R_L)C_s}}$$

$$\left. \frac{dV}{dt} \right|_{t=0} = \frac{R_L}{(R_L + R_s)^2 C_s} \cdot V_{dc} \quad \dots\dots\dots(1)$$

In steady state

$$I_{dis} = \frac{V_{dc}}{R_s} = 100$$

$$\therefore R_s = \frac{250}{100} = 2.5 \Omega$$

$$\frac{7.5 \times 250}{(7.5 + 2.5)^2 \times C_s} = 125 \text{ V} / \mu\text{s}$$

$$\Rightarrow C_s = 0.15 \mu\text{F}$$

$$\begin{aligned} \text{Snubber loss} &= \left[\frac{1}{2} C_s \cdot (V_{dc})^2 \right] f_s \\ &= 9.375 \text{ W} \end{aligned}$$

2. AC-DC Converters

Solutions for Objective Practice Questions

01. Ans: $\frac{1}{4}$

Sol: In the absence of SCR $P_1 = \frac{V_{or}^2}{R} = \frac{V^2}{R}$

In the presence of SCR

$$\Rightarrow V_{or} = \frac{V_m}{\sqrt{2\pi}} \left[(\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right]^{\frac{1}{2}}$$

$$\text{For } \alpha = 90^\circ \Rightarrow V_{or} = \frac{V}{2}$$

$$\Rightarrow P_2 = \frac{V^2}{4R}$$

$$\frac{P_2}{P_1} = \frac{1}{4}$$

02. Ans: (i) 17.6V, (ii) 329V,
 (iii) 445.3V, (iv) 141.5V, 10.75A
 (v) 8.98ms

Sol: (i) Voltage across thyristor

$$\begin{aligned} V_T &= V_m \sin \alpha - E \\ &= 230 \times \sqrt{2} \sin 25^\circ - 120 \\ &= 17.46 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{(ii) Now } V_T &= V_m \sin \beta - E \\ &= 230 \times \sqrt{2} \sin 220^\circ - 120 \\ &= -329.07 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{(iii) Peak Inverse voltage} &= V_m + E \\ &= 230 \times \sqrt{2} + 120 \\ &= 445.3 \text{ V} \end{aligned}$$

(iv) Average output voltage

$$V_0 = \frac{1}{2\pi} [V_m (\cos \alpha - \cos \beta) + E(2\pi + \alpha - \beta)]$$

$$\begin{aligned} &= \frac{1}{2\pi} [230 \times \sqrt{2} (\cos 25^\circ - \cos 220^\circ) \\ &\quad + 120(2\pi + 25 \times \frac{\pi}{180} - 220 \times \frac{\pi}{180})] \end{aligned}$$

$$= 141.57 \text{ V}$$

$$\begin{aligned} \text{Current } I_0 &= \frac{V_0 - E}{R} \\ &= \frac{141.57 - 120}{2} = 10.78 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{(v) } t_c &= \frac{2\pi + \theta - \beta}{\omega} = \frac{2\pi + (21.64 - 220)}{\omega} \times \frac{\pi}{180} \\ &= \frac{2.82}{100\pi} = 8.98 \text{ ms} \end{aligned}$$

03. Ans: 18A, 1A

Sol: $R = 5 \Omega$, $L = 10 \text{ mH}$,
 $E = 80 \text{ V}$ and $V = 230 \text{ V}$

$$\begin{aligned} V_0 &= \frac{V_m}{\pi} (1 + \cos \alpha) \\ &= \frac{230 \times \sqrt{2}}{\pi} (1 + \cos 50^\circ) = 170.08 \text{ V} \\ I_0 &= \frac{V_0 - E}{R} = \frac{170.08 - 80}{5} = 18.01 \text{ A} \end{aligned}$$

If SCR damaged, the circuit will work as half wave rectifier

$$\begin{aligned} V_0 &= \frac{V_m}{2\pi} (1 + \cos \alpha) \\ &= \frac{230 \times \sqrt{2}}{2\pi} (1 + \cos 50^\circ) = 85 \text{ V} \end{aligned}$$

$$I_0 = \frac{V_0 - E}{R} = \frac{85 - 80}{5} = 1 \text{ A}$$

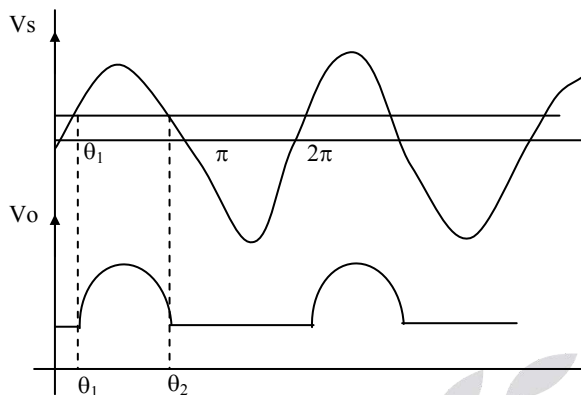
04. Ans: (c)

Sol: $R = 2 \Omega$

If SCR₂ gets open circuited then the circuit behaves like single phase half-wave rectifier.

The SCR's are triggered by constant DC signal means

$$\alpha = \theta_1, \beta = \theta_2 = \pi - \theta_1$$



$$V_m \sin \omega t = E \quad \sin \theta_1 = \frac{E}{V_m}$$

$$\theta_1 = \sin^{-1} \left(\frac{200}{230 \times \sqrt{2}} \right) = 37.94^\circ$$

$$\begin{aligned} I_o &= \frac{1}{2\pi R} \int_{\alpha}^{\theta_2 = \pi - \alpha} (V_m \sin \omega t - E) d\omega t \\ &= \frac{1}{2\pi R} [V_m (-\cos \omega t)_{\alpha}^{\pi - \alpha} - E(\pi - 2\alpha)] \\ &= \frac{1}{2\pi R} [2 \times 230 \times \sqrt{2} \cos(37.94^\circ)] \\ &\quad - 200 \left(\pi - 2 \times 37.94^\circ \times \frac{\pi}{180} \right) = 11.90 \text{ A} \end{aligned}$$

05. Ans: 120°, 0.54A, 1.016A

$$\text{Sol: } V_o = \frac{V_m}{\pi} (1 + \cos \alpha) \text{ \& } V_{o\max} = \frac{2V_m}{\pi}$$

$$V_o = 0.25 \times V_{o\max}$$

$$\frac{V_m}{\pi} (1 + \cos \alpha) = 0.25 \times \frac{2 \times V_m}{\pi}$$

$$\alpha = 120^\circ$$

$$\therefore V_o = \frac{240 \times \sqrt{2}}{\pi} (1 + \cos 120^\circ)$$

$$= 54.01 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{54.01}{100} = 0.54 \text{ A}$$

$$\begin{aligned} V_{\text{rms}} &= \frac{V_m}{\sqrt{2\pi}} \left[(\pi - \alpha) + \frac{1}{2} (\sin 2\alpha) \right]^{1/2} \\ &= \frac{240 \times \sqrt{2}}{\sqrt{2\pi}} \left[\left(\pi - \frac{2\pi}{3} \right) + \frac{1}{2} \left(\sin \left(\frac{4\pi}{3} \right) \right) \right]^{1/2} \end{aligned}$$

$$V_{\text{rms}} = 106.02 \text{ V}$$

$$\begin{aligned} I_{\text{rms}} &= \frac{106.02}{100} \\ &= 1.061 \text{ A} \end{aligned}$$

06. Ans: 545.96 V

$$\begin{aligned} \text{Sol: } \frac{3V_{ml}}{\pi} \cos(180^\circ - \alpha) &= -E + 2I_o r_s + 2 \times V_t + \frac{3\omega L_s}{\pi} I_o \\ \frac{3 \times 415\sqrt{2}}{\pi} \cos 150^\circ &= -E + (2 \times 60 \times 0.3) \\ &\quad + (2 \times 1.5) + \frac{3 \times 100\pi \times 1.2 \times 10^{-3}}{\pi} \times 60 \\ E &= 545.96 \text{ V} \end{aligned}$$

07. Ans: 467.82 V

$$\begin{aligned} \text{Sol: } V_o &= \frac{3V_{ml}}{\pi} \cos \alpha \\ &= \frac{3 \times \sqrt{2} \times 400}{\pi} \cos 30^\circ = 467.8 \text{ V} \end{aligned}$$

$$\text{Output power} = \frac{V_{\text{rms}}^2}{R}$$

$$V_{\text{rms}} = V_{ml} \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

$$= 400\sqrt{2} \left[\frac{1}{2} + \frac{3\sqrt{3}}{8\pi} \right]^{\frac{1}{2}}$$

$$= 475.562$$

$$\text{Output power} = 22.61 \text{ kW}$$

08. Ans: (i) 67.85°, 0.36 lag (ii) 92.36V

Sol: (i) $V_0 = RI_0 + E$

$$\frac{3V_{m\ell}}{\pi} \cos \alpha = RI_0 + E$$

$$\frac{3 \times 220\sqrt{2}}{\pi} \cos \alpha = 0.2 \times 10 + 110$$

$$\alpha = 67.85^\circ$$

$$P_0 = P_{in}$$

$$\sqrt{3} V_s I_s \cos \phi = V_0 I_0$$

$$\sqrt{3} \times 220 \times 10 \sqrt{\frac{2}{3}} \times \cos \phi = (112) \cdot 10$$

$$\Rightarrow \cos \phi = 0.36 \text{ lag}$$

(ii) $V_0 = RI_0 - E$

$$\frac{3V_{m\ell}}{\pi} \cos \alpha = 0.2 \times 10 - 110 = -108$$

$$V_{m\ell} = 130.59 \text{ V}$$

$$V_s = \frac{V_{m\ell}}{\sqrt{2}} = \frac{130.59}{\sqrt{2}} = 92.34 \text{ V}$$

09. Ans: (b)

Sol: The maximum current through battery will be evaluated based on extreme condition of operation

$$I_{0(\max)} = \frac{400}{10} = 40 \text{ Amps}$$

10. Ans: (c)

Sol: kVA rating of input transformer

$$= \sqrt{3} V_i I_i$$

Where I_i = Rms value of line current on ac side

$$= I_0 \sqrt{\frac{2}{3}}$$

$$\text{kVA rating} = \sqrt{3} \times 400 \times 40 \sqrt{\frac{2}{3}}$$

$$= 22.6 \text{ kVA}$$

11. Ans: (d)

Sol: Power supplied to load = $V_0 I_0$

$$= \frac{2V_m}{\pi} I_0 = \frac{2\sqrt{2}V_s}{\pi(n)} I_0$$

VA rating of secondary winding = 2 [voltage rating of each secondary winding $\times$ current rating of each secondary winding]

$$= 2 \left[\frac{V_m}{n} \times \frac{I_0}{2} \right] = \frac{\sqrt{2}V_s I_0}{n}$$

$$\text{Primary VA rating} = V_s \cdot \frac{I_0}{n}$$

Average VA rating of transformer

$$= \frac{V_s I_0 + \sqrt{2}V_s I_0}{2n}$$

$$= 1.207 \frac{V_s I_0}{n}$$

$$\therefore \frac{\text{Average VA rating of transformer}}{\text{Power supplied to load}} = \frac{1.207}{0.9} = 1.341$$

12. Ans: (b)

Sol: Active power will be drawn by converter only due to fundamental component. Therefore,

$$\text{Active power} = V_s I_{s1} \cos \phi_1$$

$$= 100 \times 10 \cos 60$$

$$\text{Active power} = 500 \text{ watts}$$

13. Ans: (b)

Sol: Voltage applied $v = 100\sqrt{2}\sin(100\pi t)$

Current resulted

$$i = 10\sqrt{2} \sin\left(100\pi t - \frac{\pi}{3}\right) + 5\sqrt{2} \sin\left(300\pi t + \frac{\pi}{4}\right) + 2\sqrt{2} \sin\left(500\pi t - \frac{\pi}{6}\right)$$

The current flowing through converter is a combination of fundamental, 3rd harmonic and 5th harmonic components.

The current flowing through the converter is non sinusoidal component then p.f will be written as

Input power factor

$$= \frac{V_s I_{s1} \cos \phi_1}{V_s I_s} = \frac{I_{s1}}{I_s} \cos \phi_1$$

$$I_{s1} = 10 \text{ A}$$

$$I_s = \sqrt{10^2 + 5^2 + 2^2} = 11.35 \text{ A}$$

$$\text{p.f} = \frac{10}{11.35} \cos 60 = 0.44$$

14. Ans: 15.47 (Range: 14.5 to 16.5)

Sol: $P_0 = V_0 \cdot I_0 = 3000 \text{ W}$

$$\Rightarrow \left[\frac{2V_m}{\pi} \cos \alpha - \frac{2\omega L_s}{\pi} I_0 \right] I_0 = 3000$$

$$\left[\frac{2 \times 230\sqrt{2}}{\pi} \times \frac{\sqrt{3}}{2} - \frac{2 \times 100\pi \times 1.4 \times 10^{-3}}{\pi} \times I_0 \right] I_0 = 3000$$

$$\Rightarrow 179.33I_0 - 0.28I_0^2 - 3000 = 0$$

$$\Rightarrow I_0 = 17.19 \text{ A}$$

$$\cos(\alpha + \mu) = \cos \alpha - \frac{2\omega L_s}{V_m} I_0$$

$$= \frac{\sqrt{3}}{2} - \frac{2 \times 100\pi}{230\sqrt{2}} \times \frac{1.4}{1000} \times 17.19$$

$$\Rightarrow \alpha + \mu = 34.96^\circ \Rightarrow \mu = 4.96^\circ$$

$$\text{DPF} = \cos \left[\alpha + \left(\frac{\mu}{2} \right) \right] = 0.843 \text{ lag}$$

Power balance equation

$$\Rightarrow V_{s1} \times I_{s1} \cos \phi_1 = P_0$$

$$\Rightarrow 230 \times I_{s1} \times 0.843 = 3000$$

$$\Rightarrow I_{s1} = \frac{3000}{230 \times 0.843} = 15.47 \text{ A}$$

15. (i) 42.6° (ii) 11.141A

Sol: Maximum value occurs at $\alpha = 0$

$$V_0 = \frac{3V_{ml}}{2\pi} \cos \alpha = \frac{3V_{ml}}{2\pi} \cos(0)$$

$$\frac{3V_{ml}}{2\pi} \cos \alpha = 0.75 \times \frac{3V_{ml}}{2\pi}$$

$$\alpha = 41.409$$

$\alpha > 30$, so we should not use above formula

$$\text{For } \alpha > \frac{\pi}{6}, V_0 = \frac{3V_{mp}}{2\pi} \left(1 + \cos \left(\alpha + \frac{\pi}{6} \right) \right)$$

$$\frac{3V_{mp}}{2\pi} [1 + \cos(\alpha + \pi/6)] = 0.75 \times \frac{3V_{mp} \times \sqrt{3}}{2\pi}$$

$$1 + \cos(\alpha + \pi/6) = 1.2990$$

$$\alpha + 30 = 72.600$$

$$\Rightarrow \alpha = 42.600^\circ$$

$$\text{(ii) } I_0 = \frac{V_0}{R} \quad \alpha > \frac{\pi}{6}$$

$$V_0 = \frac{3V_{mp}}{2\pi} (1 + \cos(\alpha + \pi/6))$$

$$= \frac{3 \times \left(\frac{220}{\sqrt{3}} \right) \times \sqrt{2}}{2\pi} (1 + \cos(42.600 + 30))$$

$$V_0 = 111.414 \text{ and } I_0 = \frac{V_0}{R}$$

$$= 11.141 \text{ A}$$

16. Ans: 6 (Range: 5.9 to 6.1)

Sol: $\frac{2V_m}{\pi} \cos \alpha = E + RI_o$

$$\Rightarrow \frac{2 \times 200\pi}{\pi} \cos 120^\circ = -800 + 20 \times I_o \text{ A}$$

$$\Rightarrow -200 = -800 + RI_o \Rightarrow I_o = 30$$

As switches are lossless, power fed back to the source = $200 \text{ V} \times 30 \text{ A} = 6 \text{ kW}$

17. Ans: Without FD: (i) 146.42 V, (ii) 732.11 W, 732.11 VAR and

With FD: (i) 176.74 V,

(ii) 883.76 W, 366 VAR

Sol: Given data:

$$V_s = 230, f = 50 \text{ Hz}, \alpha = 45^\circ, I_o = 5 \text{ A}$$

Without Free wheeling diode

(i) It is a 1 - ϕ full converter

$$V_o = \frac{2V_m}{\pi} \cos \alpha$$

$$= \frac{2 \times 230 \times \sqrt{2}}{\pi} \cos 45^\circ$$

$$V_o = 146.4225 \text{ V}$$

(ii) $P_o = V_o I_o = 146.4225 \times 5$

$$P_o = 732.1125 \text{ W}$$

$$Q_o = V_o I_o \tan \alpha$$

$$= 146.4225 \times 5 \times \tan 45^\circ$$

$$= 732.1125 \text{ VAR}$$

If $\alpha = 60^\circ$

$$V_o = \frac{2V_m}{\pi} \cos \alpha = 103.536$$

$$= 5 \text{ A}$$

$$P_o = V_o I_o = 517.68 \text{ W}$$

$$Q_o = V_o I_o \tan \alpha$$

$$= 517.68 \times \tan 60$$

$$Q_o = 896 \text{ VAR}$$

With free wheeling diode

(i) Single phase full bridge converter with free wheeling diode will act as a single phase semi converter

$$V_o = \frac{V_m}{\pi} (1 + \cos \alpha) \frac{\sqrt{2} \times 230}{\pi} (1 + \cos 45^\circ)$$

$$V_o = 176.747 \text{ V}$$

(ii) $P_o = V_o I_o = 176.747 \times 5$
 $= 883.73 \text{ W}$

$$Q_o = V_o I_o \tan \frac{\alpha}{2}$$

$$= 883.73 \times \tan \frac{45}{2}$$

$$Q_o = 366.053 \text{ VAR}$$

(iii) If $\alpha = 60^\circ$

$$V_o = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$V_o = 155.304 \text{ V}$$

$$I_o = 5 \text{ A}$$

$$P_o = V_o I_o$$

$$P_o = 776.523$$

$$Q_o = V_o I_o \tan \frac{\alpha}{2}$$

$$= 776.523 \tan 30^\circ$$

$$Q_o = 448.32 \text{ VAR}$$

In the two cases 1 - ϕ Full converter (without F.D) and 1 - ϕ Full converter (with F.D) if ' α ' increases, active power decreases and reactive power increases.

18. Ans: (c)

Sol: $I_1 = I_{\text{load RMS}} = \sqrt{I_o^2} \times = I_o$

$$I_2 = (I_{\text{SCR}})_{\text{RMS}} = \sqrt{\frac{I_o^2 \times 60}{360}} = \frac{I_o}{\sqrt{6}}$$

$$I_3 = (I_{\text{FD}})_{\text{RMS}} = \sqrt{\frac{I_o^2 \times 3 \times 60}{360}} = \frac{I_o}{\sqrt{2}}$$

$$I_1 : I_2 : I_3 = 1 : \frac{1}{\sqrt{6}} : \frac{1}{\sqrt{2}}$$

19. Ans: 224.17

Sol: When source inductance is not taken into account, each diode will conduct for 180°

When source inductance is taken into account, each diode will conduct for $(180 + \mu)^\circ$

Where μ is overlap angle and can be determined as follows:

$$\cos \mu = 1 - \frac{2\omega L_s I_o}{V_m}$$

$$\Rightarrow \cos \mu = 1 - \frac{2 \times 100\pi \times 10 \times 10^{-3}}{220\sqrt{2}} \times 14$$

$$= 0.71727$$

$$\Rightarrow \mu = 44.17^\circ$$

$$\therefore \text{Conduction angle for } D_1 = 180 + 44.17 = 224.17^\circ$$

Solutions for Conventional Practice Questions

01.

Sol: Firing angle is not given in the data

There will be no freewheeling action if

$\alpha < 30^\circ$ and there will be freewheeling action if

$\alpha > 30^\circ$

(i) $\alpha < 30^\circ$:

Source current waveform is shown below:

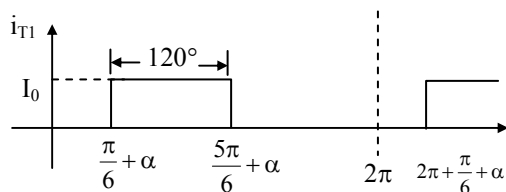


Fig. Current wave form through thyristor for $\alpha < 30^\circ$

To find Displacement factor and distortion factor, we should know Fourier coefficients for $n = 1$. Fourier series expression is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

Trigonometric expression is

$$f(t) = \frac{a_0}{2} + \sqrt{a_n^2 + b_n^2} \sum_{n=1}^{\infty} \sin(n\omega t + \theta)$$

$$\text{Where } \theta = \tan^{-1} \frac{a_n}{b_n}$$

If $n = 1$,

$$a_1 = \frac{1}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} I_o \cos \omega t \, d\omega t = \frac{-\sqrt{3}I_o}{\pi} \sin \alpha$$

$$b_1 = \frac{1}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} I_o \sin \omega t \, d\omega t = \frac{\sqrt{3}I_o}{\pi} \cos \alpha$$

Displacement angle,

$$\theta = \tan^{-1} \frac{a_1}{b_1} = \tan^{-1}(-\alpha) = -\alpha$$

Displacement factor,

$$\text{DPF} = \cos(-\alpha) = \cos \alpha$$

RMS value of fundamental source current,

$$I_{s1} = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}} = \frac{I_o}{\pi} \sqrt{\frac{3}{2}}$$

From the waveform, RMS value of source

$$\text{current, } I_s = \frac{I_o}{\sqrt{3}}$$

$$\text{Distortion factor, DF} = \frac{I_{s1}}{I_s}$$

$$= \frac{\frac{I_o}{\pi} \sqrt{\frac{3}{2}}}{\frac{I_o}{\sqrt{3}}} = \frac{3}{\pi\sqrt{2}}$$

$$\text{Power factor, DF} \times \text{DPF} = \frac{3}{\pi\sqrt{2}} \cos \alpha$$

(ii) $\alpha > 30^\circ$:

Source current waveform is shown below:

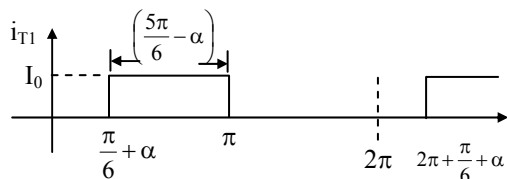


Fig. Current wave form through thyristor for $\alpha > 30^\circ$

If $n=1$,

$$a_1 = \frac{1}{\pi} \int_{\pi/6+\alpha}^{\pi} I_0 \cos \omega t \, d\omega t$$

$$= -\frac{I_0}{\pi} \sin \left(\alpha + \frac{\pi}{6} \right)$$

$$b_1 = \frac{1}{\pi} \int_{\pi/6+\alpha}^{\pi} I_0 \sin \omega t \, d\omega t$$

$$= \frac{I_0}{\pi} \left(1 + \cos \left(\alpha + \frac{\pi}{6} \right) \right)$$

$$\text{Let } \left(\alpha + \frac{\pi}{6} \right) = x$$

Then, Displacement angle,

$$\theta = \tan^{-1} \frac{a_1}{b_1} = -\frac{x}{2}$$

$$\text{Displacement factor, DPF} = \cos \frac{x}{2}$$

RMS value of fundamental source current,

$$I_{s1} = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}} = \frac{\sqrt{2}I_0}{\pi} \cos \frac{x}{2}$$

From the waveform, RMS value of source

$$\text{current, } I_s = \left[I_0^2 \times \frac{\left(\frac{5\pi}{6} - \alpha \right)}{2\pi} \right]^{\frac{1}{2}}$$

$$= I_0 \sqrt{\frac{\left(\frac{5\pi}{6} - \alpha \right)}{2\pi}}$$

$$\text{Distortion factor, DF} = \frac{I_{s1}}{I_s}$$

$$= \frac{\frac{\sqrt{2}I_0}{\pi} \cos \frac{x}{2}}{I_0 \sqrt{\frac{\left(\frac{5\pi}{6} - \alpha \right)}{2\pi}}}$$

Power factor = DF × DPF

$$\frac{\frac{\sqrt{2}I_0}{\pi} \cos^2 \left[\frac{\left(\alpha + \frac{\pi}{6} \right)}{2} \right]}{I_0 \sqrt{\frac{\left(\frac{5\pi}{6} - \alpha \right)}{2\pi}}}$$

$$= \frac{2}{\sqrt{\pi \left(\frac{5\pi}{6} - \alpha \right)}} \cos^2 \left(\frac{\alpha}{2} + \frac{\pi}{12} \right)$$

02.

Sol: Loop resistance of dc link = 40 Ω

Transformer secondary voltage = 120 kV at each end

Bridge connected converters on both sides has

Rectifier: $\alpha = 15^\circ$, $X = 15 \Omega$

Inverter: $\beta = 25^\circ$, $X = 15 \Omega$

Equivalent circuit of rectifier,

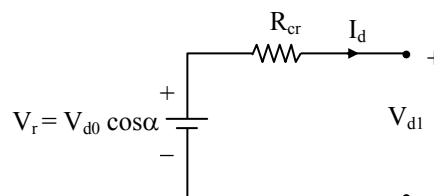


Fig. 1: Equivalent circuit

$$\begin{aligned}\text{Where } V_{d0} &= \frac{3\sqrt{3}E_{\text{mph}}}{\pi} \\ &= \frac{3E_{\text{mL}}}{\pi} \\ &= \frac{3 \times 120 \times \sqrt{2} \text{ kV}}{\pi} \\ &= 162.05 \text{ kV}\end{aligned}$$

Equivalent commutation resistance,

$$\begin{aligned}R_{\text{cr}} &= \frac{3}{\pi} X_{\text{cr}} \\ &= \frac{3}{\pi} \times 15 = 14.324 \Omega\end{aligned}$$

Equivalent circuit for Inverter,

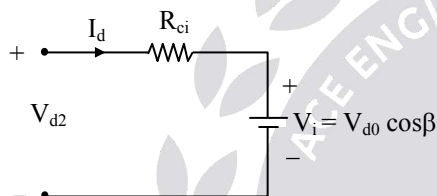


Fig. 2: Equivalent circuit

Where $V_{d0} = 162.05 \text{ kV}$

(Because same secondary voltage of transformer)

$$\begin{aligned}\text{Equivalent resistance, } R_{\text{ci}} &= \frac{3}{\pi} \times i = \frac{3}{\pi} \times 15 \\ &= 14.324 \Omega\end{aligned}$$

The equivalent DC circuit will be,

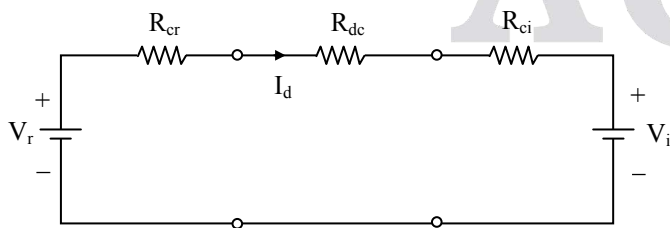


Fig. 3: Equivalent circuit

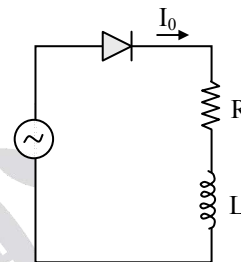
$$\begin{aligned}V_r &= 162.05 \cos(15^\circ) \\ &= 156.53 \text{ kV}\end{aligned}$$

$$\begin{aligned}V_i &= 162.05 \cos(25^\circ) \\ &= 146.87 \text{ kV}\end{aligned}$$

$$\begin{aligned}\text{Now, } I_d &= \frac{V_r - V_i}{R_{\text{cr}} + R_{\text{dc}} + R_{\text{ci}}} \\ &= \frac{(156.53 - 146.87) \times 10^3}{14.324 + 40 + 14.324} \\ &= 140.7 \text{ A}\end{aligned}$$

03. $12.023 \sin(31468.3^\circ) - 2.3614e^{-125t}$

Sol: When SCR is ON:



$$Ri_0 + L \frac{di_0}{dt} = V_m \sin \omega t$$

$$i_0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \phi)$$

$$- \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\alpha - \phi) \cdot e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$

$$\text{Where } \phi = \tan^{-1} \frac{\omega L}{R}$$

$$\phi = 1.68343$$

$$i_0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} = 12.025$$

$$\sin(\alpha - \phi) = 1.68343$$

$$i_0 = 12.025 \sin(314t - 68.3^\circ) - 2.362 e^{-125t}$$

- 04. (i) 179.33V (ii) 20A**
(iii) 18A (iv) 0.866
(v) 0.70 (vi) 48.43%
(vii) 230V (viii) 80.3%

(ix) 3586.6W (x) 2070.7244 Var

Sol: $I_0 = 20A$; $V_S = 230V$, $\alpha = 30^\circ$

$$(i) V_0 = \frac{2V_m}{\pi} \cos \alpha = \frac{2 \times 230 \times \sqrt{2}}{\pi} \cos 30^\circ = 179.33 V$$

$$(ii) I_S = I_0 = 20A$$

$$(iii) I_{S1} = \frac{2\sqrt{2}}{\pi} I_0 = 18A$$

$$(iv) DPF = \cos \alpha = 0.866 \text{ lag}$$

$$(v) IPF = \frac{2\sqrt{2}}{\pi} \cos \alpha = 0.781 \text{ a g}$$

$$(vi) THD \text{ in is} = 48.43\%$$

$$(vii) V_{or}^2 = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t d(\omega t)$$

$$\Rightarrow V_{or} = \frac{V_m}{\sqrt{2}} = V_s = 230V$$

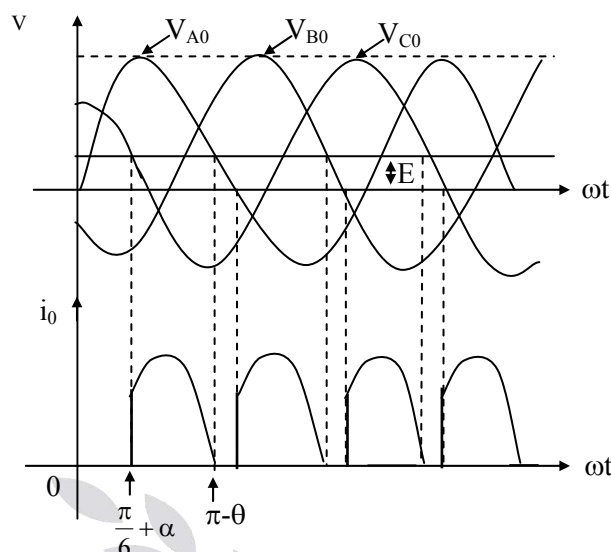
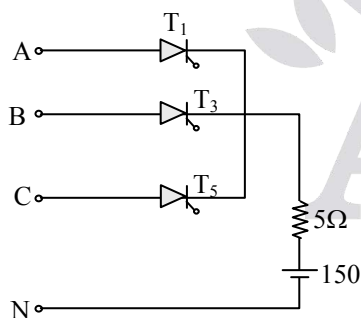
$$(viii) RF = \frac{\sqrt{V_{or}^2 - V_0^2}}{V_0} \times 100 = 80.3\%$$

$$(ix) P_{in} = V_{s1} I_{s1} \cos \phi_1 = 3585.3 W$$

$$(x) Q_{in} = V_{s1} I_{s1} \sin \phi_1 = 2070 \text{ VAR}$$

05. 19.95 A

Sol:



$$\theta = \sin^{-1} \left[\frac{150}{230\sqrt{2}} \right] = 27.46^\circ$$

$$\alpha = 30^\circ$$

$$I_0 = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi-\theta} \left[\frac{V_{mp} \sin \omega t - E}{R} \right] d\omega t$$

$$= \frac{3}{2\pi R} \left[V_{mp} (-\cos \omega t) \Big|_{\frac{\pi}{6}+\alpha}^{\pi-\theta} - E(\omega t) \Big|_{\frac{\pi}{6}+\alpha}^{\pi-\theta} \right]$$

$$= \frac{3}{2\pi R} \left[V_{mp} \left[\cos \left(\frac{\pi}{6} + \alpha \right) - \cos(\pi - \theta) \right] - E \left(\pi - \theta - \frac{\pi}{6} - \alpha \right) \right]$$

$$I_0 = 19.95A$$

3. DC-DC Converters

Solutions for Objective Practice Questions

01. Ans: 5A, 5.104A, 4.896A

Sol: $f = 2 \text{ kHz}$, $V_{dc} = 100V$, $\frac{L}{R} = 6 \text{ msec}$

$$R_L = 10\Omega$$

$$I_0 = \frac{V_0}{R} = \frac{50}{10} \Rightarrow 5A$$

$$\Delta I_L = \frac{V_{dc}}{L} D(1-D)T$$

$$= \frac{100}{60 \times 10^{-3}} \times 0.5(0.5) \times \frac{1}{2 \times 10^3}$$

$$= \frac{50 \times 0.5 \times 0.5}{60} = 0.208 \text{ A}$$

$$I_{L \max} = I_L + \frac{\Delta I_L}{2} = 5 + \frac{0.208}{2} = 5.104 \text{ A}$$

$$I_{L \min} = I_L - \frac{\Delta I_L}{2} = 4.896 \text{ A}$$

02. (i) Ans: (c) (ii) 2.5 A, 37.5 μH

Sol: (i) $D = 0.5$, $f = 100 \text{ kHz}$,

$$\Delta i_c = 1.6 \text{ A}, I_0 = 5 \text{ A}$$

$$\Delta I_C = \Delta I_L$$

$$I_{L \max} = I_L + \frac{\Delta I_L}{2}$$

$$= 5 + \frac{1.6}{2} = 5.8 \text{ A}$$

(ii) Average switch current

$$= \frac{1}{T_s} \left[4.2 \times \frac{T_s}{2} + \frac{1}{2} \times (5.8 - 4.2) \times \frac{T_s}{2} \right]$$

$$= 2.5 \text{ A}$$

$$\Delta I_L = \frac{V_{dc}}{L} D(1-D)T$$

$$= \frac{24}{L} \times 0.5(1-0.5) \times \frac{1}{100}$$

$$= 1.6$$

$$L = 37.5 \mu\text{H}$$

03. (i) $\frac{1}{3}$

(ii) 333.333 μH

(iii) 312.5 μH

(iv) 8.33mH, 12.5 nF

Sol: $\Delta V_0 = 10 \text{ mV}$

$$\Delta I_L = 0.5 \text{ A}, T = 50 \mu\text{s}$$

(i) Duty cycle ratio

$$V_o = DV_{dc}$$

$$D = \frac{V_o}{V_{dc}} = \frac{5V}{15} = \frac{1}{3}$$

(ii) filter Inductance

$$\Delta I_2 = \frac{V_{dc}}{L} D(1-D)T$$

$$\Rightarrow 0.5 = \frac{5 \times \frac{1}{3} \times \frac{2}{3} \times 50 \mu}{L}$$

$$L = 333.33 \mu\text{H}$$

$$\text{(iii) } \Delta V_0 = \frac{V_{dc}}{8LCf^2} D(1-D)$$

$$10 \times 10^{-3} = \frac{15}{8 \times 333.33 \times 10^{-6} \times C \times 20 \times 10^3} \cdot \frac{1}{3} \left(1 - \frac{1}{3} \right)$$

$$C = 312.5 \mu\text{F}$$

$$\text{(iv) } L_{cr} = \frac{(1-D)R}{2f}$$

$$= \frac{(1-0.33)500}{2 \times 20 \times 10^3} = 8.33 \text{ mH}$$

$$C = \frac{1}{16L} (1-D)T^2$$

$$C = \frac{1}{16 \times 8.33 \times 10^{-3}} \times \frac{2}{3} \times \left(\frac{1}{400 \times 10^6} \right) = 12.5 \text{ nF}$$

04. (i) Ans: 937.5 W (ii) 1041.7 W

Sol: $f = 20 \text{ kHz}$, $T = 50 \mu\text{s}$

$D = 0.5$, Given circuit is boost converter

(i) When switch is ON [$0 < t < DT$]

$$L \frac{di_L}{dt} = 100 \Rightarrow \frac{di_L}{dt} = \frac{100}{L}$$

When switch is OFF

$$L \frac{di_L}{dt} = 100 - 300 = -200$$

$$\frac{di_L}{dt} = -\frac{200}{L}$$

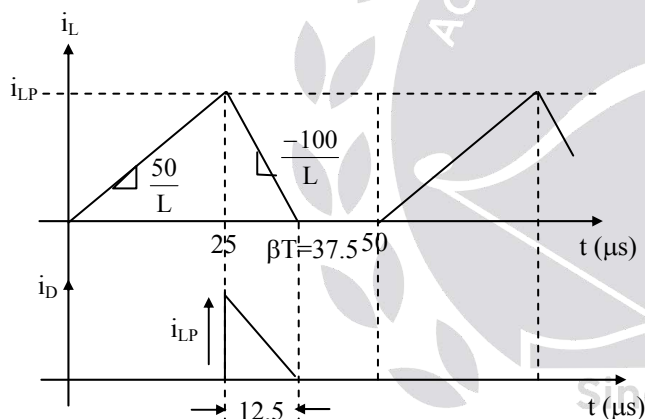
From the given slopes, given condition is discontinuous conduction mode.

$$\therefore \frac{V_0}{V_{dc}} = \frac{\beta}{\beta - D}$$

$$\Rightarrow \frac{300}{100} = \frac{\beta}{\beta - D} \Rightarrow \beta = 0.75$$

$$\text{From } i_L \text{ waveform, } i_{Lp} = \frac{100}{L} \times DT$$

$$= \frac{100}{100 \times 10^{-6}} \times 25 \times 10^{-6} \\ = 25 \text{ A}$$



$$\therefore I_{Dav} = \frac{\frac{1}{2} \times i_{Lp} \times (\beta - D)T}{T} \\ = \frac{\frac{1}{2} \times 25 \times 12.5 \times 10^{-6}}{50 \times 10^{-6}} = 3.125 \text{ A}$$

$$P_{out} = V_2 \times I_{Dav} = 300 \times 3.125 \\ = 937.5 \text{ W}$$

$$\therefore \text{Power transferred from } B_1 \text{ to } B_2 = 937.5$$

W

(ii) When switch is ON $[0 < t < DT]$

$$L \frac{di_L}{dt} = 100 \Rightarrow \frac{di_L}{dt} = \frac{100}{L}$$

When switch is OFF

$$L \frac{di_L}{dt} = 100 - 250 = -150$$

$$\frac{di_L}{dt} = -\frac{150}{L}$$

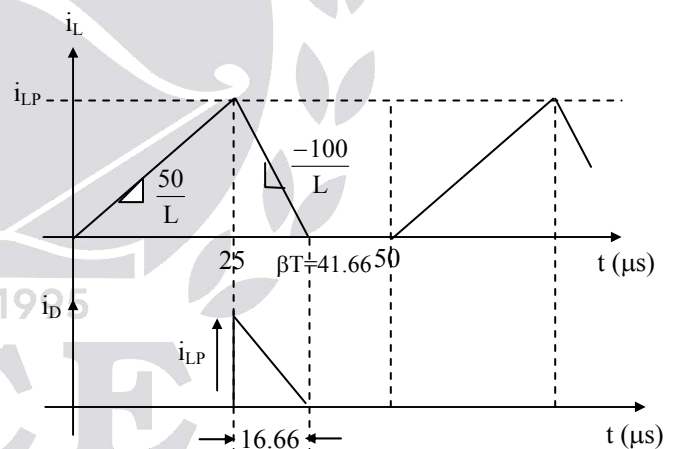
From the given slopes, given condition is discontinuous conduction mode.

$$\therefore \frac{V_0}{V_{dc}} = \frac{\beta}{\beta - D}$$

$$\Rightarrow \frac{250}{100} = \frac{\beta}{\beta - D} \Rightarrow \beta = \frac{5}{6}$$

$$\text{From } i_L \text{ waveform, } i_{Lp} = \frac{100}{L} \times DT$$

$$= \frac{100}{100 \times 10^{-6}} \times 25 \times 10^{-6} = 25 \text{ A}$$



$$\therefore I_{Dav} = \frac{\frac{1}{2} \times i_{Lp} \times (\beta - D)T}{T} \\ = \frac{\frac{1}{2} \times 25 \times 16.66 \times 10^{-6}}{50 \times 10^{-6}} \\ = 4.166 \text{ A}$$

$$P_{out} = V_2 \times I_{Dav} = 250 \times 4.166$$

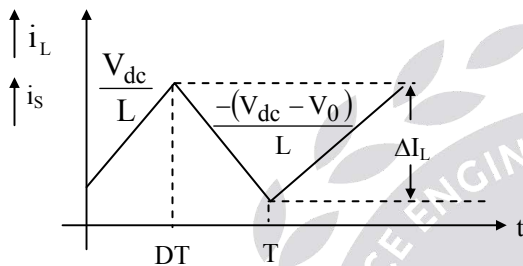
$$= 1041.667 \text{ W}$$

$$\therefore \text{Power transferred from } B_1 \text{ to } B_2 = 1041.667 \text{ W}$$

05.

Sol: (i) Ans: (b) (ii) Ans: (c)

Given circuit is Boost converter circuit. In this circuit source current and inductor current are same.



Average output voltage

$$V_0 = \frac{V_{dc}}{1-D} = \frac{12}{1-0.4} = 20\text{V}$$

Average output current

$$I_0 = \frac{V_0}{R} = \frac{20}{20} = 1\text{A}$$

Source current

$$I_s = \frac{I_0}{1-D} = \frac{1}{1-0.4} = \frac{1}{0.6} = \frac{10}{6} = \frac{5}{3} \text{ A}$$

Peak to peak source current ripple.

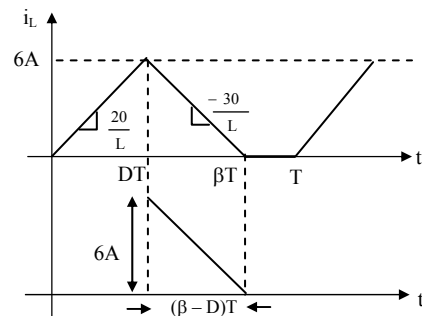
$$\Delta I_L = \Delta I_s = \frac{V_{dc}}{L} DT$$

$$= \frac{12}{100 \times 10^{-6}} \times 0.4 \times \frac{1}{250 \times 10^3}$$

$$= 0.192\text{A}$$

06. Ans: 2 (Range 2 to 2)

Sol:



In continuous conduction mode,

$$V_0 = \frac{V_{dc}}{1-D} = \frac{20}{1-0.5} = 40\text{V}$$

But given $V_0 > 40 \text{ V}$, so it is discontinuous mode of operation.

Power balance equation $P_0 = P_{in}$

$$\Rightarrow V_{dc} \cdot I_s = 50 \times \left(\frac{50}{50} \right)$$

$$I_s = \frac{50}{20} = 2.5 \text{ A}$$

$$\frac{V_0}{V_{dc}} = \frac{\beta}{\beta-D} = \frac{50}{20} = 2.5$$

$$\Rightarrow \beta = 2.5 \times \beta - (2.5 \times 0.5)$$

$$\beta = 0.833$$

$$I_L = \frac{1}{2} \times I_{L \text{ Max}} \times \frac{\beta T}{T} = 2.5$$

$$I_{L \text{ Max}} = 6 \text{ A}$$

$$I_{D, \text{rms}} = \left[\frac{6^2}{3} \times \left(\frac{5}{6} - \frac{1}{2} \right) \times \frac{T}{T} \right]^{\frac{1}{2}}$$

$$= \left[\frac{6^2}{3} \times \frac{2}{6} \right]^{\frac{1}{2}}$$

$$I_{D, \text{rms}} = 2 \text{ A}$$

07. Ans: (i) 0.6

(ii) 0.72 A

(iii) 1.61 A

(iv) 34.09 mA

(v) 72μH, 500nF

Sol: Given

$$I_0 = 0.5 \text{ A}, V_0 = 15 \text{ V}, V_{DC} = 6 \text{ V}$$

$$(i) V_0 = \frac{V_{dc}}{1-D}$$

$$\Rightarrow \text{Duty cycle } D = 0.6$$

$$(ii) \Delta I_L = \frac{V_{dc}}{L} DT$$

$$= \frac{6}{250 \times 10^{-6}} \times 0.6 \times \frac{1}{20K}$$

$$= 0.72 \text{ A}$$

$$(iii) I_{L_{max}} = I_L + \frac{\Delta I_L}{2} = \frac{20}{1-D} + \frac{\Delta I_L}{2}$$

$$= 0.5 + \frac{0.72}{2} = 1.61 \text{ A}$$

$$(iv) \Delta V_c = \frac{I_D \cdot DT}{C} = \frac{0.5 \times 0.6 \times \frac{1}{20 \times K}}{440 \times 10^{-6}}$$

$$= 34.1 \text{ mV}$$

$$(v) L_{cr} = \frac{D(1-D)^2 RT}{2}$$

$$= \frac{0.6(1-0.6)^2}{2} \times 30 \times \frac{1}{20k}$$

$$R = \frac{V_0}{I_0} = \frac{15}{0.5} = 30$$

$$L_{cr} = 72 \mu\text{H}$$

$$C_{cr} = \frac{(1-D)I_0 DT}{2V_{DC}}$$

$$= \frac{(1-0.6)0.5 \times 0.6 \times \frac{1}{20k}}{2 \times 6} = 0.5 \mu\text{F}$$

08. Ans: 3.51 A (Range: 3.0 to 4.0)

Sol: For continuous inductor current,

$$V_o = \frac{V_{dc}}{1-D} \Rightarrow 1-D = \frac{360}{400} \Rightarrow D = 0.1$$

As output power is 4 kW at 400 V, $I_o = 10 \text{ A}$

From power balance, $P_{in} = P_{out}$ or $V_{dc} \times I_i = V_o \times$

$$I_o, \text{ it will give } I_i = \frac{4000}{360} = 11.11 \text{ A}$$

As the switching frequency is not given in the question, we cannot proceed further

But by assuming one condition i.e. the current through supply line is constant of 11.11 A then r.m.s value of switch current

$$= 11.11 \times \sqrt{\frac{DT}{T}} = 11.11 \sqrt{D}$$

$$= 11.11 \sqrt{0.1} = 3.51 \text{ A}$$

09. Ans: (i) 83.3μs (ii) 83.3A

Sol: (i) Buck-boost converter, $f = 10 \text{ kHz}$

$$I_{L_{max}} = \frac{100}{L} DT \quad \dots\dots\dots (1)$$

$$= \frac{500}{L} (T - DT) \quad \dots\dots\dots (2)$$

From (1) & (2)

$$\frac{100}{L} DT = \frac{500}{L} (T - DT)$$

$$DT = \frac{500}{600} T$$

$$= \frac{5}{6} \times \frac{1}{10 \times 10^3} = 83.3 \mu\text{sec}$$

(ii) Peak current through switch

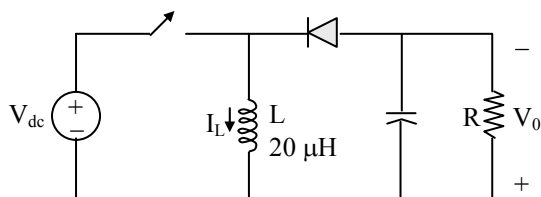
$$I_{L_{max}} = \frac{V_{dc}}{L} DT$$

$$= \frac{100}{100 \times 10^{-6}} \times 83.3 \times 10^{-6}$$

$$= 83.3 \text{ A}$$

10. Ans: (c)

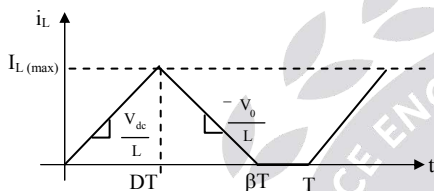
Sol:



In CCM Buck Boost,

$$V_o = V_{dc} \left[\frac{D}{1-D} \right] = 20 \times \frac{0.6}{0.4} = 30 \text{ V}$$

But given $V_o > 30 \text{ V}$, so it is discontinuous mode of operation.



From i_L waveform

$$\frac{20}{L} DT = \frac{60}{L} (\beta T - DT)$$

$$\Rightarrow 4DT = 3\beta T \Rightarrow \beta = \frac{4}{3} \times D = 0.8$$

$$\therefore I_{L \max} = \frac{V_{dc}}{L} \times DT$$

$$= \frac{20}{20\mu} \times 0.6 \times 20\mu = 12 \text{ A}$$

$$I_{L \text{ (avg)}} = \frac{1}{2} \times 12 \times 0.8 \times 20\mu = 4.8 \text{ A}$$

11. Ans: (i) 18V

(ii) 0.163V

(iii) 1.152A

(iv) 4.326A

(v) 38.4 μH and 1μF

Sol: (i) $V_o = \frac{D}{1-D} V_{dc}$

$$= \frac{0.6}{1-0.6} \times 12$$

$$= 18 \text{ V}$$

$$(ii) \Delta V_c = \frac{I_o}{L} DT$$

$$= \frac{1.5}{250 \times 10^{-6}} \times 0.6 \times \frac{1}{25 \text{ K}} = 0.144 \text{ V}$$

$$(iii) \Delta I_L = \frac{V_{dc}}{L} DT$$

$$= \frac{12}{250 \times 10^{-6}} \times 0.6 \times \frac{1}{25 \text{ K}}$$

$$= 1.152 \text{ A}$$

$$(iv) I_{L \max} = I_L + \frac{\Delta I_L}{2} = \frac{1.5}{1-0.6} + \frac{1.152}{2}$$

$$= 4.326$$

$$(v) L_{cr} = \frac{(1-D)^2 RT}{2} = 38.4 \mu\text{H}$$

$$C_{cr} = \frac{I_o T (1-D)}{2 V_{dc}}$$

$$= \frac{1.5 \times \frac{1}{25 \times 10^3} (1-0.6)}{2 \times 12}$$

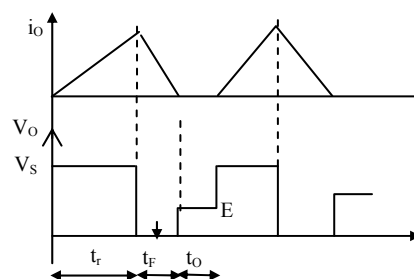
$$= 1 \mu\text{F}$$

12. Ans: (c)

Sol: For step down chopper for R_L

$$\text{Load } V(t) = V_o = D V_s = \frac{T_{\text{ON}}}{T} V_s$$

For RLE load, the output waveform is



From above diagram $t = t_r + t_f + t_o$

The terminal voltage exists only for the periods t_r and t_o , remaining time zero

$$\therefore \text{The average voltage} = \frac{V_s t_r + E_b t_o}{t}$$

13. Ans: (a)

Sol: In Buck boost converter, $V_o = \frac{D}{1-D} V_{dc}$

When $V_{dc} = 32 \text{ V}$,

$$\frac{D}{1-D} = \frac{48}{32} \Rightarrow D = \frac{3}{5} = 0.6$$

When $V_{dc} = 72 \text{ V}$, $\frac{D}{1-D} = \frac{48}{72} \Rightarrow D = \frac{2}{5} = 0.4$

$\therefore$ The range of D will be $\frac{2}{5} < D < \frac{3}{5}$

or $0.4 < D < 0.6$

14. Ans: 40

Sol: Given circuit is buck boost converter.

Source current is same switch current.

Peak value of switch current means,

$$i_{sw, peak} = I_{L, max} = I_L + \frac{\Delta I_L}{2}$$

$$\Delta I_L = \frac{V_{dc}}{L} DT$$

$$= \frac{50}{0.6 \times 10^{-3}} \times 0.6 \times 0.1 \times 10^{-3} = 5 \text{ A}$$

$$I_L = \frac{I_o}{1-D} = \frac{\left(\frac{75}{5}\right)}{1-0.6} = 37.5 \text{ A}$$

$$\therefore I_{L, max} = 37.5 + \frac{5}{2} = 40 \text{ A}$$

15. Ans: (d)

$$\begin{aligned} \text{Sol: } i &= V_o \sqrt{\frac{C}{L}} \sin \omega_0 t \\ &= V_o \sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}} t \\ &= 100 \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} \sin \frac{1}{\sqrt{10 \times 10^{-6} \times 10^{-3}}} t \\ i &= 10 \sin(10^4 t) \text{ A} \end{aligned}$$

16. Ans: (a)

Sol: Given circuit is a current commutation circuit.

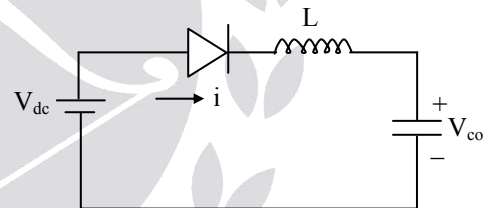
When ever both the currents are equal and opposite then the thyristor will be turned off.

$$10 \sin 10^4 t = 5 \text{ (Refer previous solution)}$$

$$t = 52 \mu\text{sec}$$

17. Ans: 35.1 μs , 189.4 V

Sol:



$$i_L(0) = I_0$$

$$\frac{L di}{dt} + \frac{1}{C} \int i dt = V_{dc}$$

$$L[SI(s) - I_0] + \frac{1}{C} \int \frac{I(s)}{s} + \frac{CV_{co}}{s} = \frac{V_{dc}}{s}$$

$$I(s) \left[SL + \frac{1}{Cs} \right] - LI_0 + \frac{V_{co}}{s} = \frac{V_{dc}}{s}$$

$$I(s) = \frac{V_{dc} - V_{co}}{s} \times \frac{1}{SL + \frac{1}{Cs}} + LI_0 \times \frac{1}{SL + \frac{1}{Cs}}$$

$$= \frac{V_{dc} - V_{co}}{s} \times \frac{CS}{S^2 + LC + 1} + LI_0 \times \frac{CS}{S^2 + LC + 1}$$

$$= \frac{V_{dc} - V_{co}}{LC} \times \frac{C}{S^2 + \omega^2} + \frac{LI_0}{LC} \times \frac{CS}{S^2 + \omega^2}$$

$$i(t) = \frac{V_{dc} - V_{co}}{\omega L} \sin \omega t + I_0 \cos \omega t$$

$$V_c(t) = V_{dc} - \frac{L di}{dt}$$

$$= V_{dc} - L \left[\frac{V_{dc} - V_{co}}{L} \right] \cos \omega t - I_0 \sin \omega t (\omega)$$

$$v_c(t) = V_{dc} - (V_{dc} - V_{co}) \cos \omega t + I_0 \sqrt{\frac{L}{C}} \sin \omega t$$

$$\text{If } V_{co} = V_{dc} \Rightarrow i(t) = I_0 \cos \omega t$$

$$t_{on} = \frac{\pi}{2\omega} = \frac{\pi}{2} \sqrt{LC}$$

$$t_{on} = \frac{\pi}{2} \sqrt{10 \times 10^{-6} \times 50 \times 10^{-6}}$$

$$= 35.1 \mu s$$

$$\text{If } V_{co} = V_{dc} \Rightarrow v_c(t) = V_{dc} + I_0 \sqrt{\frac{L}{C}} \sin \omega t$$

$$\text{At turn OFF } \omega t = \frac{\pi}{2}$$

$$\therefore V_c = V_{dc} + I_0 \sqrt{\frac{L}{C}}$$

$$= 100 + 200 \sqrt{\frac{10\mu}{50\mu}} = 189.4V$$

18. Ans: (i) 1075μs, (ii) 427.85A,
(iii) 137.5μs, (iv) 20V

Sol: Given data

$$V_s = 220V, R = 0.5\Omega, L = 2mH, E = 40V$$

$$\text{Commutation parameters: } L = 20\mu H,$$

$$C = 50\mu F, T_{ON} = 800\mu sec$$

$$T = 2000\mu sec \text{ \& } I_0 = 80A$$

(i) Effective on period

$$T_{ON}^1 = T_{ON} + 2 \cdot \frac{CV_s}{I_0}$$

$$= (800 \times 10^{-6}) + 2 \left(\frac{50 \times 10^{-6} \times 220}{80} \right)$$

$$= 1075 \mu sec$$

(ii) Peak currents through T_1 and T_A

$$I_{T_1P} = I_0 + V_s \sqrt{\frac{C}{L}}$$

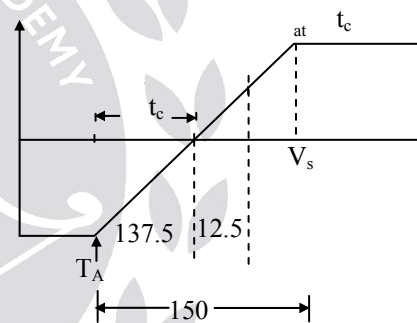
$$= 80 + 220 \sqrt{\frac{50}{20}} = 427.85A$$

$$I_{T_AP} = I_0 = 80A$$

(iii) turn off time for T_1

$$t_c = \frac{CV_s}{I_0} = \frac{50 \times 10^{-6} \times 220}{80} = 137.5 \mu sec$$

(iv) Capacitor voltage 150μsec after T_A , is triggered



$$V_s \text{ at } t_c = 220V$$

$$137.5 \rightarrow 220V$$

$$12.5 \rightarrow ?$$

$$V_c = \frac{220}{137.5} \times 12.5 = 20V$$

19. (i) Ans: (b) (ii) Ans: (b)

Sol: (i) The minimum time is required for change the polarity of capacitor from

$$V_s \text{ to } -V_s$$

$$\text{i.e. } t_1 = \frac{\pi}{\omega_o}$$

$$= \pi \times \sqrt{LC}$$

$$= \pi \times \sqrt{2 \times 10^{-3} \times 1 \times 10^{-6}} = 140 \mu \text{ sec.}$$

This time is turn ON time of thyristor i.e.

$$T_{\text{ON}} = 140 \mu \text{ sec}$$

- (ii) In a voltage commutated chopper, average value of output voltage is given by

$$\begin{aligned} V_o &= \frac{V_s}{T} (T_{\text{ON}} + 2t_c) \\ &= \frac{V_s}{T} \left(T_{\text{ON}} + 2 \cdot \frac{CV_s}{I_o} \right) \\ &= \frac{250}{1 \times 10^{-3}} \left[140 \times 10^{-6} + 2 \times \frac{1 \times 10^{-6} \times 250}{10} \right] \\ &= 47.5 \text{ V} \end{aligned}$$

Conventional Practice Questions with Solutions

20. In a fly-back converter, the required output voltage is 100 V for a nominal input voltage of 12 V. If the switch is operating at $D = 0.5$

- (a) Find the turns ratio of fly-back transformer. Assume voltage drop across switch is 0.8 V and diode is 0.8 V
(b) Find minimum and maximum values of D , if input voltage varies from 10 to 14 V, by maintaining V_o be constant. Assume the switching frequency of 2 kHz
(c) Find the value of L_s on secondary winding so that secondary current is just continuous at the minimum value of D calculated in part (b). Consider load resistance of 100 Ω

20. Ans: (i) $N_2/N_1 = 9$ (ii) $0.46 < D < 0.55$
(iii) 7.35 mH

Sol: Given: $V_{\text{dc}} = 12 \text{ V}$, $D = 0.5$, $V_{\text{SW}} = 0.8$
 $V_d = 0.8 \text{ V}$

- (i) Energy Balance equation

Increase in $d\phi$ = decrease in $d\phi$

$$\frac{(V_{\text{dc}} - V_{\text{sw}})DT}{N_1} = \frac{(V_o + V_d)(1-D)T}{N_2}$$

$$\text{as } D = 0.5 \Rightarrow DT = (1-D)T$$

$$\frac{(V_{\text{dc}} - V_{\text{sw}})DT}{N_1} = \frac{(V_o + V_d)(1-D)T}{N_2}$$

$$\frac{N_2}{N_1} = \frac{V_o + V_d}{V_{\text{dc}} - V_{\text{sw}}} = \frac{100 + 0.8}{12 - 0.8} = 9$$

$$\frac{N_1}{N_2} = \frac{1}{9}$$

- (ii) V_{dc} varies between 10 to 14V

$$\frac{(V_{\text{dc}} - V_{\text{sw}})DT}{N_1} = \frac{(V_o + V_d)(1-D)T}{N_2}$$

$$\frac{D}{1-D} = \frac{V_o + V_d}{V_{\text{dc}} - V_{\text{sw}}} \times \frac{N_1}{N_2}$$

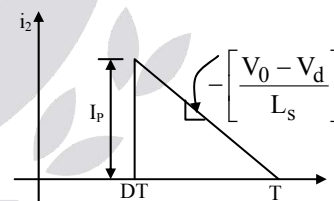
$$\text{find 'D' when } V_{\text{dc}} = 10\text{V} \Rightarrow D = 0.54$$

$$= 12\text{V} \Rightarrow D = 0.5$$

$$= 14\text{V} \Rightarrow D = 0.46$$

$$\therefore 0.46 < D < 0.55$$

- (iii)



$$I_2 = \frac{V_o}{R} = \frac{100}{100} = 1\text{A}$$

But from waveform

$$I_2 = \frac{1}{T} \left[\frac{1}{2} \times I_p \times (T - DT) \right] = 1\text{A}$$

$$I_p = \frac{2}{1 - 0.46} = \frac{2}{0.54} = 3.703$$

$$\frac{V_o + V_d}{L_s} = \frac{I_p}{(1-D)T}$$

$$L_s = \frac{(V_0 + V_d)(1-D)^2 T}{2}$$

$$L_s = \frac{100.8 \times (0.54)^2}{2} \times \frac{1}{2000} = 7.35 \text{ mH}$$

21. A fly-back converter is to be designed to operate in just-continuous conduction mode when the input dc is at its minimum expected voltage of 200 volt and when the load draws maximum power. The load voltage is regulated at 16 volts. What should be the primary to secondary turns ratio (N_1/N_2) of the transformer if the switch duty ratio is limited to 80 %. Neglect ON-state voltage drop across switch and diodes

21. Ans: 50 : 1

Sol: $V_{dc} = 200 \text{ V}$

$$V_0 = 16 \text{ V}$$

$$D = 0.8$$

$$\frac{V_0}{V_{dc}} = \frac{D}{1-D} \times \frac{N_2}{N_1}$$

$$\frac{16}{200} = \frac{0.8}{1-0.8} \times \frac{N_2}{N_1}$$

$$\frac{N_1}{N_2} = \frac{50}{1}$$

22. The average output voltage fly back converter is 24 V at a resistive load of 0.8Ω . The duty cycle ratio is 0.5 and switching frequency is 1 kHz. The ON state voltage drops of BJT and Diode are $V_T = 1.2 \text{ V}$ and $V_D = 0.7 \text{ V}$. The turns ratio of transformer is $\frac{N_s}{N_p} = 0.25$. Find the efficiency of the converter

22. Ans: 96%

Sol: $V_0 = 24$

$$R = 0.8 \Omega$$

$$I_0 = \frac{24}{0.8} = 30 \text{ A}$$

voltage across diode = V_0 + voltage across diode
 $(V_d) = 24 + 0.7 = 24.7$

$$\begin{aligned} \text{voltage across primary} &= V_d \times \frac{N_1}{N_2} \\ &= 24.7 \times \frac{1}{0.25} = 98.8 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{input voltage} &= V_{pr} + V_{sw} = 98.8 + 1.2 \\ &= 100 \text{ V} \end{aligned}$$

$$P_0 = V_0 I_0 = 24 \times 30 = 720 \text{ W}$$

$$V_{in} I_p = V_0 I_0 + \text{losses} (V_{sw} \times I_p + V_d I_0)$$

$$100 \times I_p = 720 + 1.2 I_p + 0.7 \times 30$$

$$I_p(100 - 1.2) = 741$$

$$I_p = \frac{741}{98.8} = 7.5$$

$$\eta = \frac{720}{100 \times 7.5} \times 100 = 96\%$$

23. Find maximum voltage stress of the switch in the primary winding and diode in the tertiary winding if the forward converter-transformer has 10 primary turns and 15 tertiary turns and the maximum input dc voltage is 300 V

23. Ans: $V_{sw} = 500 \text{ V}$ and $V_d = 750 \text{ V}$

Sol: $N_1 = 10$; $N_3 = 15$

$$V_{sw} = V_{dc} \left[\frac{N_1}{N_3} + 1 \right]$$

$$= 300 \left[\frac{10}{15} + 1 \right]$$

$$V_{sw} = 500 \text{ V}$$

$$\text{Voltage across } V_{D3} = - \left[V_{dc} + V_{dc} \frac{N_3}{N_1} \right]$$

$$= - \left[300 + 300 \times \frac{15}{10} \right] = -750V$$

24. If the turns ratio of the primary and tertiary windings of the forward transformer are in the ratio of 1:2, what is the maximum duty ratio at which the converter can be operated? Corresponding to this duty ratio, what should be the minimum ratio of secondary to primary turns if the input dc supply is 400 V and the required output voltage is 15 V. Neglect switch and diode conduction voltage drops.

24. Ans: 1/3 and 1/9

Sol: $\frac{N_1}{N_3} = \frac{1}{2}; \quad D_{\max} = \frac{N_1}{N_1 + N_3} = \frac{1}{3}$

$$V_0 = 15$$

$$V_{dc} = 400$$

$$V_0 = D \times V_{dc} \times \frac{N_2}{N_1}$$

$$15 = \frac{1}{3} \times 400 \times \frac{N_2}{N_1}$$

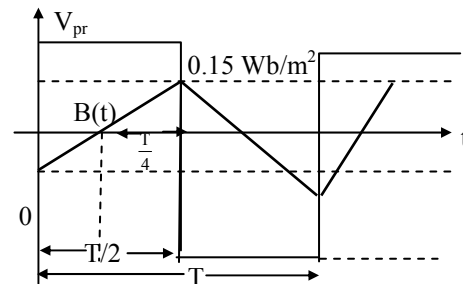
$$\frac{N_2}{N_1} = \frac{9}{80} \approx \frac{1}{9}$$

25. A transformer is wound on a toroidal core. The primary winding is supplied with a square wave voltage with a $\pm 50V$ amplitude and a frequency of 100 kHz. Assuming a uniform flux density in the core, calculate the minimum number of primary winding turns required to keep the peak flux density in the core below 0.15 Wb/m^2 if the core cross sectional area is 0.635 cm^2 . The mean path length equals 3.15 cm and the relative

permeability $\mu_r = 2500$. What is the value of the magnetizing inductance L_m

25. Ans: 1.24 mH

Sol:



$$V_{pr} = N_{pr} \cdot \frac{d\phi}{dt}$$

$$= N_{pr} \times A_c \times \frac{dB(t)}{dt}$$

$$50 = N_{pr} \times 0.635 \times 10^{-4} \times \left(\frac{0.15}{T/4} \right)$$

$$T = 10 \mu s$$

$$N_{pr} = 13.2 \Rightarrow N = 14$$

$$L_m = N^2 \times A_c \times \frac{\mu_0 \mu_r}{\ell_c} = 1.24 \text{ mH}$$

26. A toroidal core has distributed airgaps that make the relative permeability equal to 125. The cross sectional area is 0.113 cm^2 and the mean path length is 3.12 cm. Calculate the number of turns required to obtain an inductance of $25 \mu H$

26. Ans: 21

Sol: $L_m = N^2 \times \frac{A_c (\mu_0 \mu_r)}{\ell_c}$

$$\mu_r = 125; \quad A_c = 0.113 \text{ m}^2; \quad \ell_c = 3.12 \text{ cm};$$

$$L = 25 \mu H$$

$$25 \times 10^{-6} = N^2 \times \frac{0.113 \times (4\pi \times 10^{-7} \times 125)}{3.12 \times 10^{-2}}$$

$$N = 21.$$

4. DC-AC Converters

Solutions for Objective Practice Questions

01. (i) 26.1V (ii) 240W
(iii) 48.43% (iv) 5A
(v) 48V

Sol: i) $V_{o1} = \frac{\sqrt{2}}{\pi} V_{dc} = 0.45 \times 48 = 21.6V$

ii) $P_0 = \frac{\left(\frac{V_{dc}}{2}\right)^2}{R} = \frac{24^2}{2.4} = 240W$

iii) THD = 48.43%

iv) Peak current through switch

$$I_p(\text{sw}) = \frac{V_{dc}}{2R} = 10A$$

$$\text{Average current of diode} = \frac{10}{2} = 5A$$

v) PIV = 48V

With full bridge

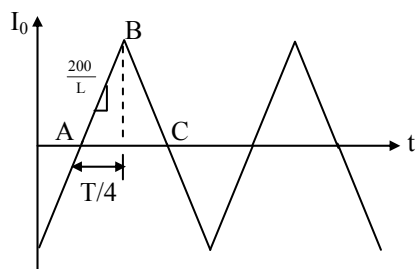
$$V_{o1} = \frac{2\sqrt{2}}{\pi} V_{dc}, \quad P_0 = \frac{V_{dc}^2}{R}$$

$$I_p(\text{sw}) = \frac{V_{dc}}{R}$$

$$I_{avg}(\text{diode}) = \frac{I_p}{2}, \quad \text{PIV} = 48V$$

02. (i) Ans: (b) (ii) Ans: (a)

Sol:



$$y = mx, \quad I_p = \frac{200}{L} \times \frac{T}{4} \quad [T = 20 \text{ msec}]$$

(i) $I_p = \frac{200}{0.1} \times 5 \times 10^{-3} = 10A$

(ii) Each diode will conduct for $\frac{T}{4} \text{ sec}$
= 5 msec

03. Ans: (c)

Sol: $V_1 = \frac{4V_{dc}}{\pi\sqrt{2}} = \frac{4 \times 12}{\pi\sqrt{2}}$

$$V_1 = \frac{48}{\pi\sqrt{2}}$$

Given $V_2 = 240$ and $N_1 = 10$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$N_2 = \frac{240 \times \sqrt{2} \times 3}{4 \times 12} \times 10$$

$$= 150\sqrt{2}$$

04. Ans: (c)

Sol: Given $2d = 150^\circ$

Fundamental component peak value

$$= \frac{4V_s}{\pi} \sin d \cdot \sin \frac{\pi}{2}$$

Fundamental component r.m.s value

$$(V_1) = \frac{4V_s}{\sqrt{2}\pi} \sin d \cdot \sin \frac{\pi}{2}$$

$$= \frac{4 \times V_s \sin(75^\circ)}{\sqrt{2}\pi}$$

$$= \frac{1.22}{\sqrt{2}} V_s = 0.862 V_s$$

and r.ms. value of output voltage

$$V_{r.m.s} = V_s \left(\frac{2d}{\pi} \right)^{1/2}$$

$$= V_s \left(\frac{150}{\pi} \times \frac{\pi}{180} \right)^{1/2} = 0.912 V_s$$

Given

$$\begin{aligned} \text{THD} &= \sqrt{\frac{V_{r.m.s}^2 - V_1^2}{V_1^2}} \times 100 \\ &= \frac{\sqrt{[(0.912)^2 - (0.862)^2]} V_s^2}{V_s 0.862} \times 100 \\ &= \frac{0.2978}{0.862} = 34.55\% \end{aligned}$$

05. Ans: (i) 200.8V, (ii) 24.75°, (iii) 203.64V

Sol: Given data:

$$V_{dc} = 220V, P = 5, \quad 2d = 150$$

$$\begin{aligned} \text{(i)} \quad V_0 &= V_{dc} \sqrt{\frac{2d}{\pi}} \\ &= 220 \sqrt{\frac{150}{180}} \Rightarrow 200.83V \end{aligned}$$

$$\text{(ii)} \quad \text{Now } V_{dc} = V_{dc} + 10\% \text{ of } V_{dc} = 242V$$

$$V_{dc} \sqrt{\frac{5 \times 2d}{\pi}} = 200.8$$

$$(242)^2 \times \frac{5 \times 2d}{180} = (200.8)^2$$

$$2d = 24.78$$

$$\text{(iii)} \quad 200.8 = V_{dc} \times \sqrt{\frac{5 \times 35}{180}} \quad (\text{pulse width } 35)$$

$$V_{dc} = 203.64V$$

06. Ans: (a) (i) 13.2A, (ii) 9.33A,
(iii) 7.84 kW, (iv) 18.67A
(b) (i) 11.43A, (ii) 8.08A,
(iii) 5.88 kW (iv) 14A

Sol: Given data $R = 15\Omega$

In 180° mode

(i) rms value of load current

$$\begin{aligned} I_{or} &= \frac{V_{pn}}{R} = \frac{\frac{\sqrt{2}}{3} V_{dc}}{R} \\ &= \frac{\sqrt{2}}{3} \times \frac{420}{15} = 13.2A \end{aligned}$$

$$\text{(ii)} \quad I_T = I_0 \sqrt{\frac{\pi}{2\pi}} \Rightarrow \frac{I_0}{\sqrt{2}} = 9.33A$$

$$\text{(iii)} \quad \text{Load Power} = 3 \times I_0^2 R$$

$$\begin{aligned} &= 3 \times (13.2)^2 \times 15 \\ &= 7.84 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad 420 \times I_s &= 7.84 \times 10^3 \\ I_s &= 18.67A \end{aligned}$$

120° Operation:

$$\text{i)} \quad I_0 = \frac{V_{ph}}{R} = \frac{\frac{V_{dc}}{\sqrt{6}}}{R} = \frac{\frac{420}{\sqrt{6}}}{15} = 11.43A$$

$$\text{ii)} \quad I_T = \frac{I_0}{\sqrt{3}} = \frac{11.43}{\sqrt{3}} = 6.6A$$

$$\begin{aligned} \text{iii)} \quad P_0 &= 3 \times I_0^2 R \\ &= 3 \times 11.43^2 \times 15 = 5.88 \text{ kW} \end{aligned}$$

$$\text{(iv)} \quad I_s = \frac{5880}{420} = 14A$$

07. Ans: 2.15 μF

Sol: $t_c = F.S \times t_2 = 24 \mu s$

$$\omega t_c = (2\pi) \times 24 \mu = 0.75 \text{ rad}$$

$$\omega t_c = \phi_1 = 0.754$$

$$\tan^{-1} \left(\frac{X_C - X_L}{R} \right) = 0.754$$

$$\frac{X_C - X_L}{3} = 0.013$$

$$X_C - 12 = 0.039 \Rightarrow X_C = 12.039$$

$$\frac{1}{\omega C} = 12.039$$

$$\Rightarrow C = 2.15 \mu\text{F}$$

08. Ans: (i) 500 μs (ii) 750 V

Sol: (i) Circuit turn off time $t_c = \frac{T}{4} = 500 \mu\text{s}$

$$\begin{aligned} \text{(ii) } V &= \frac{I}{C} \times T = \frac{30}{20\mu} \times 500\mu \\ &= 750 \text{ V} \end{aligned}$$

09. Ans: 77.15°

Sol: The given output voltage waveform is having quarter wave symmetry

$$\therefore a_n = 0$$

$$\begin{aligned} \text{And } b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d\omega t \\ &= \frac{2}{\pi} \int_0^{\pi} f(t) \sin(n\omega t) d\omega t \end{aligned}$$

For fundamental $n = 1$

$$\begin{aligned} \therefore b_1 &= \frac{2V_{dc}}{\pi} \left[\int_0^{\alpha} \sin \omega t d\omega t - \int_{\alpha}^{\pi-\alpha} \sin \omega t d\omega t + \int_{\pi-\alpha}^{\pi} \sin \omega t d\omega t \right] \\ &= \frac{2V_{dc}}{\pi} \left[-\cos \omega t \Big|_0^{\alpha} + \cos \omega t \Big|_{\alpha}^{\pi-\alpha} - \cos \omega t \Big|_{\pi-\alpha}^{\pi} \right] \\ &= \frac{2V_{dc}}{\pi} [2 - 4 \cos \alpha] \end{aligned}$$

RMS value of fundamental output voltage can be

$$\Rightarrow \frac{2V_{dc}}{\pi\sqrt{2}} [2 - 4 \cos \alpha] = 50 \text{ V}$$

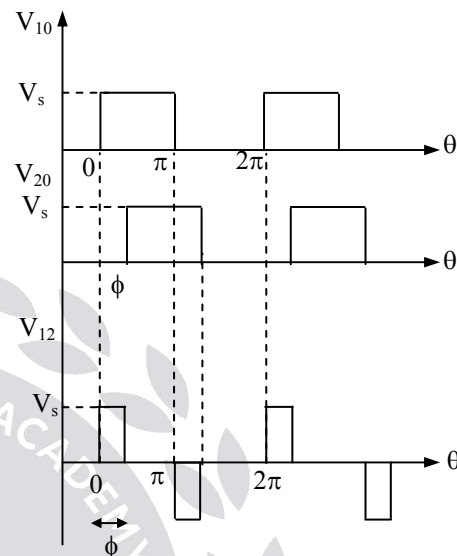
$$\Rightarrow \cos \alpha = 0.2223$$

$$\Rightarrow \alpha = 77.15^\circ$$

10. Ans: $V_s \sqrt{\frac{\phi}{\pi}}$

Sol: $V_{12} = V_{10} - V_{20}$

From Fig. (b)



Explanation: At instant 0, $V_{10} = V_s$ and $V_{20} = 0$

$$\Rightarrow V_{12} = V_{10} - V_{20} = V_s - 0 = V_s$$

After period of ϕ , $V_{10} = V_s$

and $V_{20} = V_s$

$$\Rightarrow V_{12} = 0$$

At instant of π , $V_{10} = 0$ and $V_{20} = V_s$

$$\Rightarrow V_{12} = V_{10} - V_{20} = 0 - V_s = -V_s$$

R.M.S value of V_{12} .

$$\begin{aligned} (V_{12})_{\text{r.m.s}} &= \left[\frac{1}{\pi} \int_0^{\phi} V_s^2 d\theta \right]^{1/2} \\ &= \left[\frac{V_s^2}{\pi} (\theta)_0^{\phi} \right]^{1/2} \\ &= V_s \sqrt{\frac{\phi}{\pi}} \end{aligned}$$

11. Ans: 9.9 to 10.1

Sol: Modulation index, $m_a = \frac{\hat{V}_m}{\hat{V}_{tri}}$

$$= \frac{0.8}{1} = 0.8$$

Amplitude of the fundamental output voltage,

$$\begin{aligned} (\hat{V}_{AO})_1 &= m_a \times \frac{V_{dc}}{2} \\ &= 0.8 \times 250 = 200 \text{ V} \end{aligned}$$

From the given modulating voltage equation, it can be understood that $\omega_1 = 200\pi$ means, fundamental component frequency = 100 Hz

Load impedance at 100 Hz frequency,

$$Z_1 = \sqrt{R^2 + X^2} = \sqrt{12^2 + 16^2} = 20 \Omega$$

$$\therefore \hat{I}_{L1} = \frac{\hat{V}_{AO1}}{Z_1} = \frac{200}{20} = 10 \text{ A}$$

12. Ans: 24 (Range: 23 to 25)

Sol: By converting the load into equivalent star,

$$R_{ph} = \frac{30}{3} = 10 \Omega$$

In 180° conduction mode, rms value of each phase voltage,

$$\begin{aligned} V_{ph} &= \frac{\sqrt{2}}{3} V_{dc} \\ &= \frac{\sqrt{2}}{3} \times 600 = 200\sqrt{2} \text{ V} \end{aligned}$$

Power consumed by the load,

$$\begin{aligned} P_o &= 3 \times \frac{V_{ph}^2}{R} \\ &= 3 \times \frac{(200\sqrt{2})^2}{10} = 24 \text{ kW} \end{aligned}$$

13. Ans: 60 to 64

Sol: $m_a = 0.7$

$$V_{in} = 100 \text{ V}$$

$$L = 9.55 \text{ mH}$$

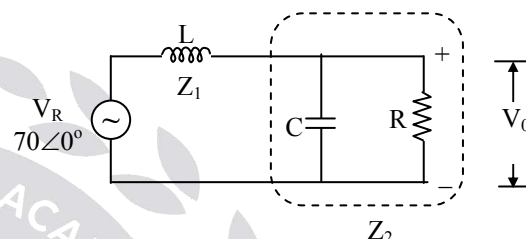
$$C = 63.66 \mu\text{F}$$

$$R = 5 \Omega, f = 50 \text{ Hz}$$

$$V_R = \hat{V}_R \sin(2\pi ft)$$

$$= (m_a \times V_{in}) \sin(100\pi t)$$

$$= (0.7 \times 100) \sin(100\pi t) = 70 \sin(100\pi t)$$



$$\begin{aligned} V_0 &= 70\angle 0^\circ \times \frac{Z_2}{Z_1 + Z_2} \\ &= 70\angle 0^\circ \times \frac{5 \times (-j50)}{5 - j50} \\ &= 70\angle 0^\circ \times \frac{5 \times (-j50)}{j3 + \frac{5 \times (-j50)}{5 - j50}} \\ &= 70\angle 0^\circ \times \left[\frac{-j250}{15j + 150 - j250} \right] \\ &= 70\angle 0^\circ \times \left[\frac{-j250}{150 - j235} \right] \\ V_0 &= 62.77 \text{ V} \end{aligned}$$

14. Ans: (d)

Sol: For 180° mode, $P = 3 \frac{V_{ph \text{ RMS}}^2}{R}$

$$10 \text{ kW} = \frac{3}{R} \left(\frac{V_{dc} \sqrt{2}}{3} \right)^2$$

$$\frac{V_{dc}^2}{R} = 15 \text{ kW}$$

For 120° mode, $P = \frac{3V_{ph\ RMS}^2}{R}$

$$= \frac{3}{R} \left(\frac{V_{dc}}{\sqrt{6}} \right)^2$$

$$= 3 \frac{V_{dc}^2}{R} \cdot \frac{1}{6}$$

$$= 3 \times 15 \times \frac{1}{6}$$

$P = 7.5\text{kW}$

15. Ans: (c)

Sol: Modulation index $M = \frac{\text{reference voltage}}{\text{carrier voltage}}$

$$= \frac{1}{5} = 0.2$$

Number of cycles (N)

$$= \frac{f_c}{2f_r} - 1$$

$$= \frac{1000}{2 \times 50} - 1 = 10 - 1 = 9$$

Order of harmonics = $2N \pm 1$

$$= 18 \pm 1$$

$$= 17, 19$$

16. Ans: (c)

Sol: $R = 40\ \Omega$ and $X_L = 100\pi \times \left(\frac{0.3}{\pi} \right) = 30\ \Omega$

Load impedance, $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{40^2 + 30^2} = 50\ \Omega$$

$P_o = 1440$

$$\Rightarrow I_{o1}^2 \times 40 = 1440$$

$$\Rightarrow I_{o1} = 6\text{ A}$$

RMS value of fundamental output voltage,

$$V_{o1} = \frac{M \times V_{DC}}{\sqrt{2}}$$

$$= \frac{0.6 \times V_{DC}}{\sqrt{2}}$$

But, $I_{o1} = \frac{V_{o1}}{Z_1}$

$$= \frac{0.6 \times V_{DC}}{\sqrt{2} \times 50} = 6$$

$$\Rightarrow V_{DC} = \frac{50 \times 6 \times \sqrt{2}}{0.6}$$

$$= 500\sqrt{2}\text{ V}$$

17. Ans: 244.8 (Range 244 to 246)

Sol: The line to line voltage at fundamental frequency can be written as, $\hat{v}_{ph1} \propto m_a$

$$\hat{v}_{ph1} = m_a \times \frac{V_{dc}}{2}$$

$$V_{LL1(rms)} = \frac{\sqrt{3}}{\sqrt{2}} \times m_a \times \frac{V_{ds}}{2}$$

$$= 0.612 \times m_a \times V_{dc}$$

$$V_{LL1(rms)} = 0.612 \times m_a \times V_{dc}$$

$$= 0.612 \times 0.8 \times 500$$

$$= 244.8\text{V}$$

18. Ans: 47.325 V, 78.03V, 58 V

Sol: The Fourier series of function given in above figure is

$$V_{on} = \frac{4V_s}{n\pi} (1 - 2\cos n\alpha_1 + 2\cos n\alpha_2)$$

Given $V_s = 150\text{ V}$, $\alpha_1 = 23.62^\circ$, $\alpha_2 = 33.3^\circ$

$$V_{o7,m} = \frac{4V_s}{7\pi} [1 - 2\cos(7 \times 23.62) + 2\cos(7 \times 33.3)]$$

$$= 0.3155 V_s = 47.325\text{ V}$$

$$\begin{aligned}
 V_{o9,m} &= \frac{4V_s}{9\pi} [1 - 2\cos(9 \times 23.62) + 2\cos(9 \times 33.3)] \\
 &= 0.5202 V_s = 78.03 \text{ V} \\
 V_{o11,m} &= \frac{4V_s}{11\pi} [1 - 2\cos(11 \times 23.62) + 2\cos(11 \times 33.3)] \\
 &= 0.3867 V_s = 57.85 \text{ V}
 \end{aligned}$$

19. Ans: 9.1 to 9.3

Sol: Input power = Output power

$$\Rightarrow V_{rms} I_{rms} = 5 \text{ kW}$$

$$(220) I_{rms} = 5000$$

$$I_{rms} = \frac{5000}{220} = 22.72 \text{ A}$$

$$\begin{aligned}
 \tan \delta &= \frac{I_s X_s}{V_s} = \frac{\left(\frac{500}{22}\right) \times (2\pi f L)}{V_{rms}} \\
 &= \frac{\frac{500}{22} \times 100\pi \times 5 \times 10^{-3}}{220} \\
 &= 0.1621
 \end{aligned}$$

$$\Rightarrow \delta = \tan^{-1}(0.1621) = 9.2^\circ$$

Solutions for Conventional Practice Questions
01.

Sol: $V_{dc} = 230 \text{ V}$; $R = 6 \Omega$, $L = 20 \text{ mH}$; $C = 100 \mu\text{F}$;
 $f = 100 \text{ Hz}$; $\omega = 200\pi = 628$

$$V_o(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \sin n\omega t$$

$$\begin{aligned}
 \text{(i) THD} &= \sqrt{\frac{V_{rms}^2 - V_{o1}^2}{V_{o1}^2}} \\
 &= \sqrt{\frac{V_{dc}^2}{\left(\frac{2\sqrt{2}}{\pi}\right)^2 V_{dc}^2} - 1} = \sqrt{\frac{\pi^2}{8} - 1}
 \end{aligned}$$

$$\therefore \text{THD} = 48.43\%$$

$$\text{(ii) } i_o(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi Z_n} \sin(n\omega t - \phi_n)$$

$$Z_n = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2};$$

$$\phi_n = \tan^{-1}\left(\frac{n\omega L - 1/n\omega C}{R}\right)$$

$$Z_1 = 6.8714 \Omega; \quad \phi_1 = -29.1696^\circ$$

$$Z_3 = 32.9449 \Omega; \quad \phi_3 = 79.50^\circ$$

$$Z_5 = 59.9498 \Omega; \quad \phi_5 = 84.256^\circ$$

$$Z_7 = 85.90 \Omega; \quad \phi_7 = 85.994^\circ$$

$$\therefore I_{m1} = \frac{4V_{dc}}{(\pi)Z_1} = \frac{292.845}{6.8714} = 42.617 \text{ A}$$

$$I_{m3} = \frac{4V_{dc}}{3\pi Z_3} = 2.9629 \text{ A}$$

$$I_{m5} = \frac{4V_{dc}}{5\pi Z_5} = 0.9769 \text{ A}$$

$$I_{m7} = \frac{4V_{dc}}{7\pi Z_7} = 0.487 \text{ A}$$

$$\begin{aligned}
 i(t) &= 42.617 \sin(\omega t + 29.169^\circ) + 2.9629 \\
 &\sin(3\omega t - 79.5^\circ) + 0.9769 \sin(5\omega t - 84.256^\circ) \\
 &+ 0.487 \sin(7\omega t - 85.994^\circ)
 \end{aligned}$$

$$\text{(iii) THD of load current} = \sqrt{\frac{I_{rms}^2}{I_{o1}^2} - 1}$$

$$\begin{aligned}
 I_{rms}^2 &= I_{o1}^2 + I_{o3}^2 + I_{o5}^2 + I_{o7}^2 \\
 &= \frac{I_{m1}^2 + I_{m3}^2 + I_{m5}^2 + I_{m7}^2}{2}
 \end{aligned}$$

$$\Rightarrow I_{rms} = 30.21737; \quad I_{o1} = 30.1347;$$

$$I_m = 42.734$$

$$\begin{aligned}
 \therefore \text{THD} &= \sqrt{\left(\frac{30.217}{30.1347}\right)^2 - 1} \\
 &= 7.409\%
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) Load power} &= V_{dc1} I_{O1} \cos \phi_1 \\
 &= \left(\frac{2\sqrt{2}}{\pi} \right) V_{dc} \times \left(\frac{2\sqrt{2} V_{dc}}{\pi Z_1} \right) \cos \phi_1 \\
 &= 207.07 \times 30.134 \times \cos(-29.169) \\
 &= 5448.72 \text{ W}
 \end{aligned}$$

Input power = output power

$$\Rightarrow V_s I_s = \text{load power} = 5448.72 \text{ W}$$

$$\begin{aligned}
 \Rightarrow \text{Source current } (I_s) &= \frac{5448.72}{V_s} \\
 &= \frac{5448.72}{230} = 23.69 \text{ A}
 \end{aligned}$$

(v) The current leads the fundamental voltage component by 29.169° . This means diode conducts for 29.169° and thyristor conducts for 150.831° .

$$\begin{aligned}
 \text{Conduction time of diode} &= 29.169^\circ \times \frac{5}{180^\circ} = 0.9025 \text{ ms}
 \end{aligned}$$

Conduction time of thyristor = 4.18975 ms

$$\begin{aligned}
 \text{(vi) Peak thyristor current } I_m &= I_{rms} \times \sqrt{2} \\
 &= 42.734 \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 \text{RMS value of thyristor} &= \frac{42.734}{2} \\
 &= 21.367 \text{ A}
 \end{aligned}$$

5. AC-AC Converters

Solutions for Objective Practice Questions

01. (i) 0.486 (ii) 0.687 lag

Sol: $R_L = 5 \Omega$, $V = 230 \text{ V}$, 50 Hz ; $P_0 = 5 \text{ kW}$

$$V_{rms}^2 = P_0 \times R = 25000$$

$$(i) V_{rms}^2 = \frac{V_m^2}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)$$

$$\Rightarrow 25000 = \left(\frac{1}{2\pi} \right) (230\sqrt{2})^2 \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]$$

$$\Rightarrow 1.4846 = \pi - \alpha + \frac{\sin 2\alpha}{2}$$

$$\Rightarrow \alpha - \frac{\sin 2\alpha}{2} = 1.6569$$

$$\Rightarrow \alpha = 92.45^\circ$$

$$\begin{aligned}
 \therefore \text{Duty cycle} &= \left(\frac{\pi - \alpha}{\pi} \right) = \frac{180 - 92.45}{180} \\
 &= 0.486
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Power factor} &= \frac{1}{\sqrt{\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2} \\
 &= 0.687 \text{ lag}
 \end{aligned}$$

02. Ans: (c)

03. Ans: (a)

04. (i) 4255.8 W (ii) 2127.9W

$$\begin{aligned}
 \text{Sol: (i) } V_{rms}^2 &= \frac{V_m^2}{2\pi} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right] \\
 &= \frac{(230\sqrt{2})^2}{2\pi} \left[\pi - \pi/3 + \frac{\sin 120^\circ}{2} \right] \\
 &= 42557.99 \\
 P &= \frac{V_{rms}^2}{R} \\
 &= 4255.8 \text{ W}
 \end{aligned}$$

(ii) As D_1 gets open circuited, the positive half of waveform becomes zero. So, the new rms voltage is reduced by $\sqrt{2}$ times and power is reduced to half of original value.

$$\begin{aligned}
 \therefore P &= \frac{4255.8}{2} \\
 &= 2127.9 \text{ W}
 \end{aligned}$$

05. Ans: (c)

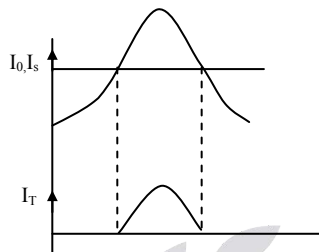
Solutions for Conventional Practice Questions

01. Ans: (a) $90^\circ \leq \alpha \leq 180^\circ$, (b) 57.5 A

(c) 25.88 A, 40.66 A

(d) 2.5546×10^4 A/s (e) 180°

Sol: $V_s = 230$ V $X_L = 4 \Omega$



(a) $90^\circ < \alpha < 180^\circ$

$$I_{T,avg} = \frac{V_m}{\pi}$$

$$I_{T,rms} = \frac{V_m}{2}$$

(b) $I_{or} = \frac{V_s}{X_L} = \frac{230}{4} = 57.5$ A

(c) $I_{T,av} = \frac{V_m}{X_L \times \pi}$

$$= \frac{230\sqrt{2}}{4 \times \pi} = 25.88$$
 A

$$\text{Max } I_T = \frac{V_m}{X_L \times 2} = 40.6$$
 A

(d) $i_0 = \frac{V_m}{X_L} \cos\left(\omega t - \frac{\pi}{2}\right) \times \omega$

$$= \frac{V_m}{X_L} \omega \sin(\omega t)$$

$$\text{Max } \frac{di}{dt} = \frac{V_m}{X_L} \times \omega$$

$$= \frac{230\sqrt{2}}{4} \times 2\pi \times 50$$

$$= 2.5546 \times 10^4 \text{ A/sec}$$

(e) $\gamma = \pi$

02.

Sol: $V = 120$ V, 50 Hz; $R_L = 15 \Omega$

(i) $P_0 = 500$ W

$$V_{rms}^2 = P_0 \times R = 500 \times 15 = 7500$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)$$

$$\Rightarrow 7500 = \frac{(120\sqrt{2})^2}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)$$

$$\Rightarrow \frac{75\pi}{144} = \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right) \dots\dots\dots(1)$$

$$\Rightarrow \alpha - \frac{1}{2} \sin 2\alpha = 1.5053$$

$$\alpha = 88.125^\circ$$

(ii) rms source current = $\left(\frac{V_{rms}}{R} \right)$

$$= \frac{\sqrt{7500}}{15} = 5.7735$$
 A

(iii) IPF = $\frac{1}{\sqrt{\pi}} \left[\left(\pi - \alpha \right) + \frac{1}{2} \sin 2\alpha \right]^{1/2}$

$$= \frac{1}{\sqrt{\pi}} \times \sqrt{\frac{75\pi}{144}} \dots\dots\dots(\text{From (1)})$$

$$= 0.7217 \text{ lag}$$

(iv) THD of source current = $\sqrt{\frac{I_{rms}^2 - I_{s1}^2}{I_{s1}^2}}$

$$I_{s1} = I_{01} = \frac{V_{01}}{R}$$

$$V_{01,max} = \sqrt{a_1^2 + b_1^2}$$

$$a_1 = \frac{2}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cos \omega t d\omega t$$

$$= \frac{V_m}{2\pi} (-1 + \cos 2\alpha) = -53.9611$$

$$b_1 = \frac{2}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cdot \sin \omega t d\omega t$$

$$\begin{aligned}
 &= \frac{V_m}{\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right] \\
 &= 88.39 \\
 V_{01, \max} &= \sqrt{(-53.96)^2 + 88.39^2} \\
 &= 103.56 \text{ V} \\
 V_{01} &= \frac{V_{01, \max}}{\sqrt{2}} = 73.28 \text{ V} \\
 I_{01} &= \frac{V_{01}}{R} = 4.881; I_{\text{rms}} \\
 &= 5.7735 \\
 \text{THD} &= \sqrt{\frac{(5.7735)^2}{(4.8817)^2} - 1} \\
 &= 63.14\%
 \end{aligned}$$

6. Fundamentals of Drives

Solutions for Conventional Practice Questions

01. Ans: (i) 62.84A, (ii) 118.46Nm

Sol: Given Data, 15 Hp, 220V, 1000rpm

$$R_a + R_f = 0.2\Omega$$

$$V_0 = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$= \frac{250 \times \sqrt{2}}{\pi} (1 + \cos 30^\circ) = 210\text{V}$$

$$E_b = K_b I_a \omega_m = 0.03 \times I_a \times \frac{2\pi N}{60} = 3.14 I_a$$

$$V_f = (R_a + R_f) I_a + E_b$$

$$210 = 0.2 I_a + 3.14 I_a \Rightarrow I_a = 62.87\text{A}$$

$$\begin{aligned}
 \text{Torque } T &= K_b I_a^2 \\
 &= 0.03 \times (62.87)^2 = 118.5\text{Nm}
 \end{aligned}$$

02. Ans: (i) 1254 rpm, (ii) 8.54Nm

Sol: Given 220 V, 1500 r.p.m, 10 A motor.

(i) Motor constant K_m can be evaluated from the rating of motor as follows:

$$V_t = E_a + I_a R_a$$

$$220 = K_m \omega_m + I_a R_a$$

$$K_m \omega_m = -10(1) + 220 \Rightarrow 210$$

$$K_m = \frac{210 \times 60}{2\pi \times 1500} = 1.337 \text{ V-s/rad}$$

$$\alpha_1 = 30^\circ, T_e = 5 \text{ Nm}$$

For the torque of 5 Nm, armature current

$$I_a = \frac{5}{1.337} = 3.74 \text{ A}$$

The equation for the operation of converter motor is

$$V_0 = V_t = E_a + I_a R_a$$

$$\frac{2V_m}{\pi} \cos \alpha = K_m \omega_m + I_a R_a$$

$$\frac{2 \times \sqrt{2} \times 230}{\pi} \cos 30^\circ = 1.337 \times \omega_m + (3.74)1$$

$$\omega_m = 131.31 \text{ rad/sec}$$

$$N = 1253.92 \text{ r.p.m}$$

(ii) $\alpha = 45^\circ$, $N = 1000 \text{ r.p.m}$, $T_e = ?$

$$\frac{2\sqrt{2} \times 230}{\pi} \cos 45^\circ = 1.337 \times \frac{2\pi \times 1000}{60} + I_a \times 1$$

$$146.4 = 140.01 + I_a \times 1$$

$$I_a = 6.39 \text{ A}$$

$$\begin{aligned}
 T_e &= K_m I_a \\
 &= 1.337 \times 6.39 = 8.543 \text{ N-m}
 \end{aligned}$$

03. Ans: (i) 45.12A, 45.12Nm, (ii) 0.92lag

Sol: Given Data,

$$I_f = 2\text{A}, R_a = 0.8\Omega, K_2 = 0.5 \text{ Vs/rad}$$

$$V_s = 230\text{V}, N = 1500 \text{ rpm}, \alpha = 30^\circ$$

$$V_0 = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$= \frac{230 \times \sqrt{2}}{\pi} (1 + \cos 30^\circ) \Rightarrow 193.2 \text{ V}$$

$$V_0 = V_t = E_b + I_a R_a$$

$$193.2 = K_2 I_f \omega_m + I_a R_a$$

$$193.2 = 0.5(2) \left(\frac{2\pi \times 1500}{60} \right) + I_a (0.8)$$

$$I_a = 45.12 \text{ A}$$

$$\text{Torque: } T_e = K_2 I_f I_a$$

$$= 0.5 \times 2 \times 45.12 \Rightarrow 45.12 \text{ N-m}$$

$$\text{Power factor: } P_f = \frac{V_t I_a}{V_s I_{sr}}$$

$$= \frac{193.2 \times 45.12}{230 \times 45.12 \sqrt{\frac{180-30}{180}}} = 0.92 \text{ lag}$$

04. Ans: 0.608

Sol: Given data: $R_a = 2.5 \Omega$, $I_a = 20 \text{ A}$

Chopper frequency = 1 kHz, $V_{dc} = 250 \text{ V}$

Duty cycle (D) = ?

At rated conditions (i.e., at 1000 rpm), back emf,

$$E_{b1} = 220 - 20 \times 2.5 = 170 \text{ V}$$

At 600 rpm, the back emf (E_{b2}) can be determined as,

$$E_{b2} = \frac{N_2}{N_1} \times E_{b1} = \frac{600}{1000} \times 170 = 102 \text{ V}$$

$$DV_{dc} = 102 + (20)(2.5)$$

$$D = \frac{152}{250} = 0.608$$

05. Ans: $\alpha = 39.2^\circ$ and $\mu = 8.3^\circ$

Sol: $V_t = E_b + I_a R_a$

$$\frac{3V_{m\ell}}{\pi} \cos \alpha - \frac{3\omega L_s}{\pi} I_a = E_b + I_a R_a$$

$$\frac{3 \times 400\sqrt{2}}{\pi} \cos \alpha - \frac{3 \times 100\pi}{\pi} \times \frac{0.5}{1000} \times 175$$

$$= (0.25 \times 1500) + 175 \times 0.1$$

$$540.18 \cos \alpha - 26.25 = 375 + 17.5$$

$$\alpha = 39.17^\circ$$

$$\cos(\alpha + \mu) = \cos \alpha - \frac{2\omega L_s}{V_{m\ell}} I_0$$

$$\cos(39.17^\circ + \mu) =$$

$$\cos(39.15^\circ) - 2 \times \frac{2\pi \times 50 \times 0.5 \times 10^{-3}}{400\sqrt{2}} \times 175$$

$$\mu = 8.14^\circ$$

06. Ans: (i) 69.7° (ii) 0.82 (iii) 70.5%

Sol: Given

$$V = 200 \text{ V}$$

$$N = 1450 \text{ rpm}$$

$$I_0 = 100 \text{ A}$$

$$R_a = 0.04 \Omega$$

3- ϕ half controlled converter

3- ϕ 220 V, 50 Hz

$$V_0 = E + I_a R_a$$

$$200 = \frac{K \times 2 \times \pi \times 1450}{60} + (100 \times 0.04)$$

$$K = 1.29 \frac{\text{V-sec}}{\text{rad}}$$

$$(a) \frac{3 \times V_{m\ell}}{2\pi} (1 + \cos \alpha) = 200 \text{ V}$$

$$\frac{3 \times \sqrt{2} \times 220}{2\pi} (1 + \cos \alpha) = 200 \text{ V}$$

$$\alpha = 69.79^\circ$$

(b) Fundamental power factor = DPF

$$= \cos \frac{\alpha}{2}$$

$$= \cos \frac{69.72}{2}$$

$$= 0.82055$$

$$(c) THD = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1}$$

I_s = rms value of source current
= rms value of fundamental source current

$$I_s = I_0 \sqrt{\frac{\pi - \alpha}{\pi}} \quad \alpha \geq \frac{\pi}{3}$$

$$I_{s1} = \frac{\sqrt{6}}{\pi} I_0 \cos \frac{\alpha}{2}$$

$$I_s = 100A \sqrt{\frac{180 - 69.79^\circ}{180}}$$

$$= 78.25 A$$

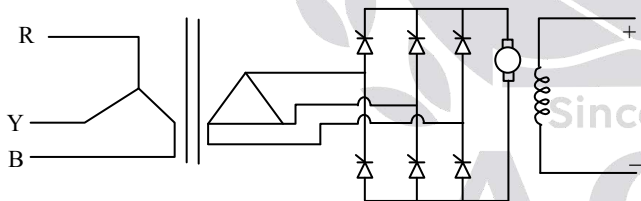
$$I_{s1} = \frac{\sqrt{6}}{\pi} \times 100 \times \cos\left(\frac{69.79}{2}\right)$$

$$THD = \sqrt{\left[\frac{78.25}{63.935}\right]^2 - 1}$$

$$= 70.56 \%$$

07. Ans: (a) 1.559: 1 (b) (i) 34.65° (ii) 104.2°

Sol:



(a) Given data motor rated terminal voltage equals the rated voltage when converter firing angle is zero.

$$\frac{3V_{m\ell}}{\pi} \cos \alpha = 220 V \quad (\alpha = 0)$$

$$\frac{3V_{m\ell}}{\pi} \cos 0^\circ = 220 V$$

$$V_{m\ell} = \frac{220 \times \pi}{3}$$

$$V_{m\ell} = 230.383$$

$$V_{L-L} = \frac{230.383}{\sqrt{2}} = 162.905 V$$

In Δ connection

$$V_{ph} = V_{L-L} = 162.905 V$$

Given

$$V_{LL} = 440 V$$

In Star connection

$$V_{ph} = \frac{440}{\sqrt{3}} = 254.0341 V$$

Transformer phase turns ratio from primary

$$\text{to secondary} = \frac{254.0341}{162.905V} = 1.559: 1$$

(b) At rated conditions

$$(i) V_0 = E + I_a R_a$$

$$V_0 = K\omega + I_a R_a$$

$$220 = K \times \frac{2 \times \pi \times 1500}{60} + 50 \times 0.5$$

$$K = 1.2414 \frac{V \cdot \text{sec}}{\text{rad}}$$

'E' at 120 rpm

$$E = 1.2414 \times \frac{2 \times \pi \times 1200}{60}$$

$$E = 156 V$$

$$\frac{3V_{m\ell}}{\pi} \cos \alpha = 156 + 50 \times 0.5$$

$$V_{m\ell} = 230.383 \text{ [from (a)]}$$

$$\cos \alpha = 0.8277$$

$$\alpha = 34.64^\circ$$

(ii) Find E at rated condition

$$V_0 = E + I_a R_a$$

$$220 = E + 50 \times 0.5$$

$$E = 195 V$$

$$\phi = \text{constant}$$

$$E \propto N$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = E_1 \frac{N_2}{N_1} = 195 \frac{(-800)}{1500}$$

$$E_2 = -104 \text{ V}$$

$$\frac{3 \times V_{m\ell}}{\pi} = E + I_a R_a$$

Given torque is twice the rated torque

$$\frac{I_{a2}}{I_{a1}} = \frac{T_2}{T_1} \Rightarrow I_{a2} = 50 \times 2$$

$$I_{a2} = 100 \text{ A}$$

$$\frac{3 \times 230.383}{\pi} \cos \alpha = -104 + (100 \times 0.5)$$

$$\cos \alpha = -0.245$$

$$\alpha = 104.21^\circ$$

08. Ans: $0.08\sqrt{2}$ A

Sol: Given,

Total harmonic distortion,

$$\text{THD} = 4\%$$

Maximum value of function component of Load current = 4A.

RMS value of net harmonic current =?

$$\therefore \text{THD} = \sqrt{\frac{I_r^2 - I_{lr}^2}{I_{lr}^2}} \times 100$$

$$\text{THD} = \sqrt{\frac{1}{g^2} - 1}, \text{ where } g = \left[\frac{I_{lr}}{I_r} \right]$$

$$\therefore 0.04 = \sqrt{\frac{1}{g^2} - 1} \Rightarrow g = 0.9992$$

$$\therefore g = \frac{I_{lr}}{I_r} \Rightarrow I_r = \left(\frac{2\sqrt{2}}{0.9992} \right) \text{ A}$$

$$\begin{aligned} \therefore I_{\text{harm}} &= \sqrt{I_{\text{rms}}^2 - I_{lr}^2} \\ &= 0.08 \sqrt{2} \text{ A.} \end{aligned}$$

09. Ans: 1.16 to 1.22

Sol: Given, 3 ϕ , 50Hz Induction motor,

Slip at fundamental frequency, $s = 0.04$

$$\begin{aligned} \text{Synchronous speed, } N_s &= (120 \times f)/P \\ &= 1500 \text{ rpm} \end{aligned}$$

5th harmonic component rotates in opposite direction to that of fundamental, Hence

$$\text{Speed of 5th harmonic} \Rightarrow 5N_s$$

$$\Rightarrow 7500 \text{ rpm}$$

Slip of 5th harmonic component

$$= \left(\frac{N_s + N_r}{N_s} \right)$$

$$= \frac{7500 + 1440}{7550}$$

$$= 1.192.$$