MECHANICAL ENGINEERING

Text Book: Theory with worked out Examples and Practice Questions
01. Ans: (a)
Sol: CPM deals with deterministic time durations.

02. Ans: (a)
Sol: Critical Path:
- It is a longest path consumes maximum amount of resources
- It is the minimum time required to complete the project

03. Ans: (a)

04. Ans: (a)
Sol: Gantt chart indicates comparison of actual progress with the scheduled progress.

05. Ans: (c)
Sol:
Critical path = 1 + 3 + 7 + 9 + 10 = 30 days

06. Ans: (c)
Sol:
Critical path (1-3-6-8-9) = 8 + 10 + 13 + 15 = 46 days

07. Ans: (b)
Sol: Rules for drawing Network diagram:
- Each activity is represented by one and only one arrow in the network.
- No two activities can be identified by the same end events.
- Precedence relationships among all activities must always be maintained.
- No dangling is permitted in a network.
- No Looping (or Cycling) is permitted.

08. Ans: (b)
Sol: Activity: Resource consuming and well-defined work element.
Event: Each event is represented as a node in a network diagram and it does not consume any time or resource.
Dummy Activity: An activity does not consume any kind of resource but merely
depicts the technological dependence is called a dummy activity.

**Float:** Permissible delay period for the activity.

9. **Ans:** (b)
   **Sol:**

10. **Ans:** (a)

11. **Ans:** (b)
    **Sol:**
    - Beta Distribution is used to decide the expected duration of an activity.
    - The expected duration of the project can be described by Normal distribution.

12. **Ans:** (b)
    **Sol:**
    \[ T_0 = 8 \text{ min}, \quad T_m = 10 \text{ min}, \quad T_p = 14 \text{ min}, \]
    \[ T_e = \frac{T_o + 4T_m + T_p}{6} \]
    \[ = \frac{8 + 4 \times 10 + 14}{6} = \frac{62}{6} = 10.33 \text{ min} \]

13. **Ans:** (a)
    **Sol:**
    Take 4-3, \( T_e = 6 \text{ days} \)
    Critical path = 1-2-4-3
    \[ = 5 + 14 + 4 = 23 \text{ days} \]
    \[ \sigma_{\text{critical path}} = \sqrt{V_{1-2} + V_{2-4} + V_{4-3}} \]
    \[ = \sqrt{2^2 + 2.8^2 + 2^2} = 3.979 \]
    \[ z = \frac{\text{Due date} - \text{critical path duration}}{\sigma_{\text{critical path}}} \]
    \[ = \frac{27 - 23}{3.979} = 1.005 \]
    \[ \therefore \quad P(z) = 0.841 \]

14. **Ans:** (b)
15. **Ans:** (c)
    **Sol:**
    \( D = 36 \text{ days}, \quad V = 4 \text{ days} \)
    \[ Z = \frac{36 - 36}{\sqrt{4}} = 0 \]
    \[ \Rightarrow P(z) = 50\% \]
16. **Ans:** (c)
    **Sol:**
    \[ \sigma_{\text{path}} = \sqrt{V_{a-b} + V_{b-c} + V_{c-d} + V_{d-e}} \]
    \[ = \sqrt{4 + 16 + 4 + 1} = 5 \]
17. **Ans:** (a)
    **Sol:**
    The latest that an activity can start from the beginning of the project without causing a delay in the completion of the entire project. It is the maximum time up to which an activity can be delayed to start without effecting the project completion duration time. \( \text{LST} = \text{LFT} - \text{duration} \).
18. Ans: (c)
Sol: The earliest expected completion time,
Critical path: A-B-C-D-F-E-H
\[ 5 + 4 + 8 + 5 + 8 = 30 \text{ days} \]

19. Ans: (d)
Sol: Critical path:
1-3-4-6 = 20 days
\[ z = \frac{24 - 20}{\sqrt{4}} = \frac{4}{2} = 2 \]
\[ \Rightarrow P(z) = 97.7\% \]

20. Ans: (d)
Sol: Variance = \[ \left( \frac{t_p - t_c}{6} \right)^2 \]
\[ = \left( \frac{22 - 10}{6} \right)^2 = 4 \]

21. Ans: (a)

22. Ans: (b)

23. Ans: (a)

24. Ans: (b)

25. Ans: (c)
Sol: Range of overall project duration likely to be in \[ 4T + 6K \] and \[ 4T - 6K \]
i.e., \[ 4T \pm 6K \]
Common solutions for Q.29 & Q.30

29. Ans: (b)

30. Ans: (b)
Sol:

<table>
<thead>
<tr>
<th>Paths</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-4-5</td>
<td>8+9+6=23</td>
</tr>
<tr>
<td>1-2-3-4-5</td>
<td>8+9+6=23</td>
</tr>
<tr>
<td>1-3-4-5</td>
<td>6+9+6=21</td>
</tr>
<tr>
<td>1-4-5</td>
<td>16+6=22</td>
</tr>
</tbody>
</table>

\[ \therefore \text{Highest time taken paths are AEF and ADF} \]
\[ \therefore \text{Critical path's are AEF and ADF} \]
Critical paths are ‘2’.
Possible cases to crash
A by 1 day that cost = 80
F by 1 day that cost = 130
E and D by 1 day that cost = 20 + 40 = 60

31. Ans: (c)

32. Ans: (c)
Sol:

<table>
<thead>
<tr>
<th>Path</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>7+5 = 12</td>
</tr>
<tr>
<td>CD</td>
<td>6+6 = 12</td>
</tr>
<tr>
<td>EF</td>
<td>8+4 = 12</td>
</tr>
</tbody>
</table>

Three critical paths, number of activities to be crashed are 3.

33. Ans: (c)
Sol:

\[ (\text{Total Float})_{6-7} = 27 - 9 - 12 = 6 \]
\[ (\text{Free float})_{6-7} = 28 - 9 - 12 = 1 \]

Conventional Practice Solutions

01. Ans: (a-7, b-41)
Sol:

Path      | duration |
----------|----------|
1-2-4-6-7 | 4 + 7 + 15 + 8 = 34 |
1-2-3-5-6-7| 4 + 8 + 9 + 12 + 8 = 41 (days) (critical path) |
1-2-5-6-7 | 4 + 6 + 12 + 8 = 30 |

TF + 7 = 18 – 4
⇒ TF = 14 – 7 = 7
02. Ans: 31 days
Sol:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time estimated</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_e = \frac{T_o + 4T_m + T_p}{6}$</td>
<td>$\sigma = \frac{T_p - T_o}{6}$</td>
</tr>
<tr>
<td>A</td>
<td>$\frac{5 + 4 \times 10 + 15}{6} = 10$</td>
<td>$\frac{15 - 5}{6} = \frac{5}{3}$</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{2 + 4 \times 5 + 8}{6} = 5$</td>
<td>$\frac{8 - 2}{6} = 1$</td>
</tr>
<tr>
<td>C</td>
<td>$\frac{10 + 4 \times 12 + 14}{6} = 12$</td>
<td>$\frac{14 - 10}{6} = \frac{2}{3}$</td>
</tr>
<tr>
<td>D</td>
<td>$\frac{6 + 4 \times 8 + 16}{6} = 9$</td>
<td>$\frac{16 - 6}{6} = \frac{5}{3}$</td>
</tr>
</tbody>
</table>

Critical path:

$1-2-3-4 = 10 + 12 + 9 = 31$ days

$\sigma_{cp} = \sqrt{V_{1-2} + V_{2-3} + V_{3-4}}$

$= \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{6}$

03. Sol:

<table>
<thead>
<tr>
<th>Paths</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>22</td>
</tr>
<tr>
<td>ACE</td>
<td>41 ← CP</td>
</tr>
<tr>
<td>BE</td>
<td>20</td>
</tr>
</tbody>
</table>

04. Sol:

Paths

<table>
<thead>
<tr>
<th>Paths</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-6 → AC</td>
<td>2 + 1 = 3</td>
</tr>
<tr>
<td>1-2-4-5-6 → AEF</td>
<td>2 + 3 + 2 = 7</td>
</tr>
<tr>
<td>1-3-6 → BD</td>
<td>4 + 2 = 6</td>
</tr>
<tr>
<td>1-3-4-5-6 → BEF</td>
<td>4 + 3 + 2 = 9</td>
</tr>
</tbody>
</table>
Highest Duration is ‘9’.

∴ CP is BEF

05.
Sol:

Among all the option the minimum cost slope option is 3-5, which can be reduced by 4 days, at a cost of 50/day

The difference between longest path and next longest path is the maximum duration we can do crashing. Only if the duration is available in the activity taken for crashing.

∴ The Critical path can be crashed for ‘2’ days only

∴ Crash Cost = 2 × 50 = 100

06.
Sol:

Paths Normal duration
1-2-3-6-7-8 25
1-2-3-5-7-8 28
1-2-4-5-7-8 26

∴ 1-2-3-5-7-8 is the critical path

“Crashing on critical path”

<table>
<thead>
<tr>
<th>Possible activities for crashing</th>
<th>No. of day’s can crash</th>
<th>Extra cost/cost saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>4 – 3 = 1</td>
<td>250/day</td>
</tr>
<tr>
<td>2 - 3</td>
<td>5 – 3 = 2</td>
<td>500/day</td>
</tr>
<tr>
<td>3 - 5</td>
<td>8 – 4 = 4</td>
<td>50/day</td>
</tr>
<tr>
<td>5 - 7</td>
<td>7 – 5 = 2</td>
<td>300/day</td>
</tr>
<tr>
<td>7 - 8</td>
<td>4 – 2 = 2</td>
<td>400/day</td>
</tr>
</tbody>
</table>

07.
Sol:

\[
\begin{array}{|c|c|c|}
\hline
\text{Activity} & \text{Cost slope} & \text{Crashing possibility} \\
\text{No.} & \left\{ \frac{C_C - N_C}{N_T - C_T} \right\} \text{(Rs/week)} & \left\{ N_T - N_c \right\} \\
\hline
1-2 & 150 & 1 \text{ week} \\
2-3 & - & - \\
2-4 & 50 & 2 \text{ week} \\
2-5 & - & - \\
3-4 & 30 & 3 \\
4-6 & 40 & 1 \\
5-6 & 25 & 2 \\
\hline
\end{array}
\]

Indirect cost = 100/week

Network diagram

\[
\begin{array}{|c|c|c|}
\hline
\text{Activity} & \text{Cost slope} & \text{Crashing possibility} \\
\text{No.} & \left\{ \frac{C_C - N_C}{N_T - C_T} \right\} \text{(Rs/week)} & \left\{ N_T - N_c \right\} \\
\hline
1-2 & 150 & 1 \text{ week} \\
2-3 & - & - \\
2-4 & 50 & 2 \text{ week} \\
2-5 & - & - \\
3-4 & 30 & 3 \\
4-6 & 40 & 1 \\
5-6 & 25 & 2 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Activity} & \text{Cost slope} & \text{Crashing possibility} \\
\text{No.} & \left\{ \frac{C_C - N_C}{N_T - C_T} \right\} \text{(Rs/week)} & \left\{ N_T - N_c \right\} \\
\hline
1-2 & 150 & 1 \text{ week} \\
2-3 & - & - \\
2-4 & 50 & 2 \text{ week} \\
2-5 & - & - \\
3-4 & 30 & 3 \\
4-6 & 40 & 1 \\
5-6 & 25 & 2 \\
\hline
\end{array}
\]
Path | Duration | Critical path | Sub-critical path
---|---|---|---
1-2-3-4-6 | 13 | | 
1-2-4-6 | 11 | | 
1-2-5-6 | 10 | | 

Crashing possibility from the network = critical path duration – sub critical path = 13 – 11 = 2 weeks

To reduce the project duration by 2 weeks

<table>
<thead>
<tr>
<th>Option</th>
<th>Cost slope</th>
<th>Crashing possibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>150</td>
<td>1 week</td>
</tr>
<tr>
<td>2-3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3-4</td>
<td>30</td>
<td>3 week</td>
</tr>
<tr>
<td>4-6</td>
<td>40</td>
<td>1 week</td>
</tr>
</tbody>
</table>

From the option, crash 3-4 by 2 weeks by crashing 3-4 by 2 weeks the project duration becomes 11 weeks.

Crashing cost = 2 × 30 = Rs. 60
Net savings by means of crashing = 2 × 100 – 60 = Rs. 140

Path | Duration
---|---
1-2-3-4-6 | 10
1-2-4-6 | 10
1-2-5-6 | 10

To reduce project duration by 1 week

<table>
<thead>
<tr>
<th>Option</th>
<th>Cost slope</th>
<th>Crashing possibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>150</td>
<td>1 week</td>
</tr>
<tr>
<td>4-6</td>
<td>40</td>
<td>1 week</td>
</tr>
<tr>
<td>3-4 &amp; 2-4</td>
<td>30+50 = 80</td>
<td>1 week</td>
</tr>
</tbody>
</table>

Among the best option, crash 4-6 by 1 week, the project duration will become 10 weeks

Crashing cost = 1 × 40 = 40
Net savings by crashing (4-6) = 100 – 40 = 60

Path | Duration
---|---
1-2-3-4-6 | 10
1-2-4-6 | 10
1-2-5-6 | 10

To reduce by project duration by 1 week

<table>
<thead>
<tr>
<th>Option</th>
<th>Cost slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>150</td>
</tr>
<tr>
<td>3-4, 2-4, 5-6</td>
<td>30+50+25 = 105</td>
</tr>
</tbody>
</table>

As crashing cost is more than indirect cost/week = further crashing is not economical

Optimum project duration = 10 weeks

Total cost of the project (with crashing) = direct cost + indirect cost/week × project duration + crashing cost

= 945 + 100 × 10 + 30 × 2 + 40 × 1 = 2045
Total cost without crashing
\[ = 945 + 100 \times 13 = 945 + 1300 = 2245 \]

08. Ans:
Sol:

Critical Path:
1-2-3-4-5-6    =   4 + 2 + 1 + 0 + 2 = 9
1-2-4-6           =   4 + 4 + 3 = 11 \rightarrow CP
1-2-3-4-6        =   4 + 2 + 1 + 3 = 10
1-3-5-6           =   3 + 1 + 2 = 6

09. Sol:

10. Sol:

(a) Critical path:

(b) To reduce the project by 1 day the available option is crashing ‘C’ by 1 day.

Option Crashing possibilities \((N_T - C_T)\)
\[
\begin{array}{|c|c|}
\hline
\text{Option} & \text{Crashing possibilities} \\
\hline
A & 8 - 8 = 0 \\
C & 6 - 5 = 1 \\
D & 5 - 5 = 0 \\
\hline
\end{array}
\]

By crashing activity C we can reduce the project duration by 1 day.

Network diagram

Further crashing is not possible due to “A – C – D” critical path.

Path Duration
\[
\begin{array}{|c|c|}
\hline
\text{Path} & \text{Duration} \\
\hline
A-E & 8+10 = 18 \\
A-C-D & 8+6+5 = 19 \\
B-D & 4+5 = 9 \\
\hline
\end{array}
\]
01. Ans: (c)
Sol:
\[ d_{ij} \rightarrow "\text{Distance from any node i to next node j}" \]
\[ s_j \rightarrow "\text{Denotes shortest path from node P to any node j}". \]
\[ d_{ij} = d_{QG} \text{ (Adjacent nodes)} \]
\[ d_{ij} = d_{RG} \text{ (Adjacent from node R to G)} \]
\[ s_j = S_Q \text{ (Shortest path from node P to node Q)} \]
\[ s_j = S_R \text{ (Shortest path from node P to node R)} \]

We can go from P to G via Q or via R.
P to G via Q
\[ S_G = S_Q + d_{QG} \]
P to G via R.
\[ S_G = S_R + d_{RG} \]
Optimum answer is minimum above two answers.
\[ S_G = \text{MIN} [S_Q + d_{QG} ; S_R + d_{RG}] \]

02. Ans: (c)
Sol:
\[
\begin{array}{c|c|c}
\text{Path} & \text{Cost} \\
\hline
1-3-4-6 & 9+4+2 = 15 \\
1-3-2-4-6 & 9+2+3+2 = 16 \\
1-3-4-5-6 & 9+4+7+2 = 22 \\
1-3-2-5-6 & 9+2+2+2 = 15 \\
1-3-2-4-5-6 & 9+2+3+7+2 = 23 \\
1-2-4-6 & 3+3+2 = 8 \\
1-2-5-6 & 3+2+2 = 7 \\
1-2-4-5-6 & 3+3+7+2 = 15 \\
1-3-5-6 & 9+8+2 = 19 \\
\end{array}
\]

From the given statement, we got shortest path (least total cost) is 1-2-5-6 and a path which does not have 1-2, 2-5, 5-6 activities should be considered. The next path which does not have the above activities is 1-3-4-6 = 15
and 1-3-2-4-5-6 = 16.
\[ \because \text{In this second least total cost is 15.} \]
03. Ans: 7
Sol:

<table>
<thead>
<tr>
<th>Path</th>
<th>Arc length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-4-6</td>
<td>8</td>
</tr>
<tr>
<td>1-2-5-6</td>
<td>8</td>
</tr>
<tr>
<td>1-2-3-5-4-6</td>
<td>9</td>
</tr>
<tr>
<td>1-3-5-4-6</td>
<td>10</td>
</tr>
<tr>
<td>1-3-5-6</td>
<td>11</td>
</tr>
</tbody>
</table>

Shortest path length from node 1 to node 6 is 7.

\[ S_1 = 0 \]
\[ S_2 = 2 \]
\[ S_4 = 5 \]
\[ S_5 = 4 \]
\[ S_3 = 3 \]
\[ S_6 = 7 \]

\[ \therefore \text{Shortest path from node 1 to node 6 is 11.} \]

02. Sol:

<table>
<thead>
<tr>
<th>Path</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 3 – 4 – 6</td>
<td>4 + 4 + 5 = 13</td>
</tr>
<tr>
<td>1 – 2 – 5 – 6</td>
<td>5 + 2 + 4 = 11</td>
</tr>
<tr>
<td>1 – 3 – 5 – 6</td>
<td>4 + 6 + 4 = 14</td>
</tr>
</tbody>
</table>

\[ \therefore \text{Shortest path from node 1 to node 4 is 7.} \]

Conventional Practice Solutions

01. Sol:

<table>
<thead>
<tr>
<th>Path</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2 – 4</td>
<td>5 + 3 = 8</td>
</tr>
<tr>
<td>1 – 3 – 4</td>
<td>2 + 5 = 7</td>
</tr>
<tr>
<td>1 – 2 – 3 – 4</td>
<td>5 + 2 + 5 = 12</td>
</tr>
</tbody>
</table>

\[ \therefore \text{Shortest path from node 1 to node 4 is 7.} \]
Chapter 3

Linear Programming

01. Ans: (d)
Sol: A restriction on the resources available to a firm (stated in the form of an inequality or an equation) is called constraint.

02. Ans: (d)

03. Ans: (c)

04. Ans: (d)
Sol: The theory of LP states that the optimal solution must lie at one of the corner points.

05. Ans: (b)
Sol: The feasible region of a linear programming problem is convex. The value of the decision variables, which maximize or minimize the objective function, is located on the extreme point of the convex set formed by the feasible solutions.

06. Ans: (a)
Sol: 

Only one value gives max value, then solution is unique.

Z(7, 3) = 2×7 + 5×3 = 29

07. Ans: (a)
Sol: 

Subjected to

\[ 4y - 4x \geq -1 \]  \[ 5x + y \geq -10 \]  \[ y \leq 10 \]

x and y are unrestricted in sign

\[ \min Z = x + 2y \]

Subjected to

\[ \frac{x}{4} + \frac{y}{-1} \leq 1 \]

(2) \[ \frac{x}{-2} + \frac{y}{10} \leq 1 \]

(3) \[ \frac{y}{10} \leq 1 \]
08. Ans: (b)
Sol: \[ Z_{\text{max}} = 3x_1 + 2x_2 \]
Subjected to
\[ 4x_1 + x_2 \leq 60 \quad \text{………(1)} \]
\[ 8x_1 + x_2 \leq 90 \quad \text{………(2)} \]
\[ 2x_1 + 5x_2 \leq 80 \quad \text{……… (3)} \]
\[ x_1, x_2 \geq 0 \]

\[ (1) \Rightarrow \frac{x_1}{15} + \frac{x_2}{60} \leq 1 \]
\[ (2) \Rightarrow \frac{x_1}{11.25} + \frac{x_2}{90} \leq 1 \]
\[ (3) \Rightarrow \frac{x_1}{40} + \frac{x_2}{16} \leq 1 \]

From the above graph the No. of corner points for feasible solutions are 4

09. Ans: (c)
Sol: Let, P type toys produced = x ,
Q type toys produced = y

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Raw material</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Electric switch</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Profit</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ Z_{\text{max}} = 3x + 5y \]
\[ x + 2y \leq 2000 \; ; \frac{x}{2000} + \frac{y}{1000} \leq 1 \]
\[ x + y \leq 1500 \; ; \frac{x}{1500} + \frac{y}{1500} \leq 1 \]
\[ y \leq 600 \; ; \frac{y}{600} \leq 1 \]
\[ x, y \geq 0 \]

\[ Z_{\text{max}} = 3x + 5y \]
\[ Z_A = 3 \times 1500 + 5 \times 0 = 4500 \]
\[ Z_B = 3 \times 0 + 5 \times 600 = 3000 \]
\[ Z_C = 3 \times 1000 + 5 \times 500 = 5500 \]
\[ Z_D = 3 \times 800 + 5 \times 600 = 5400 \]
C does not exist in answer.
Hence, \( Z_{\text{max}} \) is at D, i.e., \( Z_{\text{max}} @ D = 5400 \)

10. Ans: (c)
Sol: \( Z_{\text{max}} = x_1 + 1.5x_2 \)
Subject to
\[ 2x_1 + 3x_2 \leq 6 \quad \text{(1)} \]
\[ x_1 + 2x_2 \leq 4 \quad \text{(2)} \]
\[ x_1, x_2 \geq 0 \]
\[ \frac{x_1}{3} + \frac{x_2}{2} \leq 1 \]
\[ \frac{x_1}{4} + \frac{x_2}{2} \leq 1 \]

Let, “c” in the intersection of (1) and (2)

Solve (1) & (2) for ‘c’.

It follows,
\[ x_1 = \frac{12}{5}, \quad x_2 = \frac{2}{5} \]

\[ Z_{\text{max}} = x_1 + 1.5x_2 \]
\[ Z_0 = 0 \]
\[ Z_A = 3 + 1.5 \times 0 = 3 \]
\[ Z_B = 3 \times 0 + 1.5 \times 2 = 3 \]

Problem is having multiple solutions and it is Optimal at (A) and (B).

11. Ans: (a)
Sol: \( Z_{\text{max}} = 2x_1 + x_2 \)
Subjected \( x_1 + x_2 \leq 6 \)
\[ x_1 \leq 3 \]
\[ 2x_1 + x_2 \geq 4 \]
\[ x_1, x_2 \geq 0 \]

But feasible region is ABCDEA
(∵ \( x_1, x_2 > 0 \))

A(2,0) B(0,4) C(0,6) E(3,0) D can be obtained by solving \( x_1 \leq 3 \) & \( x_1 + x_2 \leq 6 \)

\Rightarrow \( x_1 = 3 \) and \( x_2 = 3 \) and D (3,3)

\[ \begin{array}{c|c}
\text{Z}_{\text{max}} & \text{A(2,0)} & 2\times2+1\times0 = 4 \\
& \text{B (0,4)} & 0\times2+1\times4 = 4 \\
& \text{C(0,6)} & 0\times2+1\times6 = 6 \\
& \text{E(3,0)} & 3\times2+0\times1 = 6 \\
& \text{D(3,3)} & 3\times2+1\times3 = 9 \\
\end{array} \]

\[ Z_{\text{max}} = 9 \text{ at D (3,3)} \]
12. Ans: (d)

13. Ans: (a)
Sol: 
\[ Z_{\text{max}} = 4x_1 + 6x_2 + x_3 \]
\[ s.t \]
\[ 2x_1 - x_2 + 3x_3 \leq 5 \]
\[ x_1, x_2, x_3 \geq 0 \]
\[ 2x_1 - x_2 + 3x_3 + s_1 = 5 \]
\[ Z_{\text{max}} = 4x_1 + 6x_2 + x_3 + 0s_1 \]

<table>
<thead>
<tr>
<th>( c_j \rightarrow )</th>
<th>( s \downarrow )</th>
<th>4</th>
<th>6</th>
<th>1</th>
<th>0</th>
<th>( B_0 )</th>
<th>Min Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0s_1</td>
<td>2</td>
<td>-1</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>( z_j )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_j - z_j )</td>
<td>4</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>EV</td>
</tr>
</tbody>
</table>

Entering vector exists but leaving vector doesn’t exist as minimum ratio column is having negative values. It is a case of unbounded solution space and unbounded optimal solution to problem.

14. Ans: (d)
Sol: Number of zeros in \( Z \) row = 4
Number of basic variable = 3
As the number of zeros in \( Z \) row is greater than number of basic variable so it has multiple optimal solutions.

15. Ans: (b)
Sol: Solution is optimal; but number of zeros are greater than the number of basic Variables in \( C_j - Z_j \) (net evaluation row) hence multiple optimal solutions.

16. Ans: (b)
Sol: If all the elements in the objective row are non-negative incidence of maximization, then the solution is said to be optimal.
Here, the solution is optimal, \( Z_{\text{max}} = 1350 \).

17. Ans: (a)
Sol:
- A tie for leaving variable in simplex procedure implies degeneracy.
- If in a basic feasible solution, one of the basic variables takes on a zero value then it is case of degenerate solution

Common Data Solutions
18. Ans: (d)

19. Ans: (a)
Sol: As the No. of zeros greater than No. of basic variables hence it is a case of multiple solutions or alternate optimal solution exists.
From the table gives the optimum $x_2 = 0, x_1 = 8$, $Z_{\text{max}} = 48$

Look at the coefficient of the non basic variable in the $z$-equation of iterations. The coefficient of non basic $x_2$ is zero, indicating that $x_2$ can enter the basic solution without changing the value of $Z$, but causing a change in the values of the variables.

Alternate optimal solution:
Here $x_2$ is the entering variable.

<table>
<thead>
<tr>
<th>Basic</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>48</td>
</tr>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>5/3</td>
<td>1</td>
<td>-2/3</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>-1/3</td>
<td>0</td>
<td>1/3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$x_1$</td>
<td>1</td>
<td>2/3</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

$$
\begin{array}{l}
\text{Row} & \text{Basic} & x_1 & x_2 & S_1 & S_2 & S_3 & \text{RHS} & \text{Ratio} \\
R_1 & z & 0 & 0 & 0 & 2 & 0 & 48 & \\
R_2 & s_1 & 0 & 5/3 & 1 & -2/3 & 0 & 14 & 14/(5/3)=8.4 \\
R_3 & s_3 & 0 & -1/3 & 0 & 1/3 & 1 & 5 & \\
R_4 & x_1 & 1 & 2/3 & 0 & 1/3 & 0 & 8 & 8/(2/3)=12 \\
\end{array}
$$

In the above table
$$x_1 = \frac{12}{5}, \quad x_2 = \frac{42}{5}, \quad s_3 = \frac{39}{5}$$
20. Ans: (c)

21. Ans: (a)

22. Ans: (c)

Sol: \[ Z_{\text{min}} = 10x_1 + x_2 + 5x_3 + 0S_1 \]

Dual, \[ W_{\text{min}} = 50y_1 \]

subjected to

\[ 5y_1 \leq 10, \quad y_1 \leq 2, \quad W_{\text{max}} = 100 \]
\[ 3y_1 \leq 5, \quad y_1 \leq 5/3, \quad W_{\text{max}} = 250/3 \]

\( y_1, y_2 \geq 0 \)

\( \Rightarrow Z_{\text{max}} = 250/3 \)

Common Data for Questions

23. Ans: (c)

Sol: Given, \[ Z_{\text{max}} = 5x_1 + 10x_2 + 8x_3 \]

Subjected to

\[ 3x_1 + 5x_2 + 2x_3 \leq 60 \rightarrow \text{Material} \]
\[ 4x_1 + 4x_2 + 4x_3 \leq 72 \rightarrow \text{Machine hours} \]
\[ 2x_1 + 4x_2 + 5x_3 \leq 100 \rightarrow \text{Labour hours} \]

\( x_1, x_2, x_3 \geq 0 \)

\[ 3x_1 + 5x_2 + 2x_3 + s_1 = 60 \]
\[ 4x_1 + 4x_2 + 4x_3 + s_2 = 73 \]
\[ 2x_1 + 4x_2 + 5x_3 + s_3 = 100 \]
\[ Z_{\text{max}} = 5x_1 + 10x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3 \]

<table>
<thead>
<tr>
<th>( C_j \rightarrow )</th>
<th>( 5 )</th>
<th>( 1 )</th>
<th>( 8 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Min} ) Ratio</td>
<td>( B_0 )</td>
<td>( s_1 )</td>
<td>( s_2 )</td>
<td>( s_3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>( S )</td>
<td>( V )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_1 )</td>
<td>( x_2 )</td>
<td>( x_3 )</td>
<td>( s_1 )</td>
<td>( s_2 )</td>
<td>( s_3 )</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>( x_2 )</td>
<td>( 1/3 )</td>
<td>1</td>
<td>0</td>
<td>1/3</td>
<td>1/6</td>
</tr>
</tbody>
</table>

In \( C_j - Z_j \) row all elements are negatives or zeros, hence the solution is optimal and unique.

Basic variables are:

\( x_2 = 8, \quad x_3 = 10, \quad s_3 = 18 \)

i.e., production of B = 8 units, C = 10 units

18 labours hours remained unutilized

Non Basic variable

\( x_2 = 0, \quad s_1 = 0, \quad s_2 = 0 \)

Resource materials and resource machine hours are fully utilized. In \( (C_j - Z_j) \) row at optimality, the values under \( s_1, s_2 \) and \( s_3 \) columns represents the shadow prices.

So, If 1 kg material increases, contribution increases by \( \frac{2}{3} \).
If 1 kg material decreases, contribution decreases by \( \frac{2}{3} \).

If 1 kg material increases, then production B increases by \( \frac{1}{3} \) and production C decreases by \( \frac{1}{3} \).

If m/c hr increases by 1 units, contribution increases by \( \frac{5}{3} \).

If m/c hr decreases by 1 units, contribution decreases by \( \frac{5}{3} \).

If m/c hr increases by 1 units, production B decreases by \( \frac{1}{6} \) and production increases by \( \frac{5}{12} \).

If m/c hr decreases by 1 units, production B increases by \( \frac{1}{6} \) and production C decreases by \( \frac{12}{5} \).

If 1 unit of A produces, contribution decreases by \( \frac{3}{11} \), production B decreases by \( \frac{3}{1} \), production C decreases by \( \frac{3}{2} \).

24. Ans: (a)
Sol: If 3 kg material increases, contribution increases by \( 3 \times \frac{2}{3} = \text{Rs. 2} \)

25. Ans: (a)
Sol: Present profit = 160 \( \Rightarrow 160 - \frac{5}{3} \times 12 = 140/- \)

26. Ans: (b)
Sol: New production of B
\[
= 8 - \left( 12 \times \frac{-1}{6} \right) = 8 + \left( 12 \times \frac{1}{6} \right) \\
= 8 + 2 = 10 \text{ units}
\]

27. Ans: (c)
Sol: If materials are increased by 3kgs then the new production of C is
\[
= 10 + \left( 3 \times \frac{-1}{3} \right) \\
= 10 - \left( 3 \times \frac{1}{3} \right) = 10 - 1 = 9
\]

28. Ans: (a)
Sol: If 1 unit of A produces, contribution decreases by \( \frac{11}{3} \)

29. Ans: (a)
Sol: If 6 units of A are produced then the new profit is,
\[
160 - \left( 6 \times \frac{11}{3} \right) = 138
\]

30. Ans: (a)
Sol: Production of B, \( 3 \times \frac{1}{3} = 1 \)
Production of C, \( 3 \times \frac{2}{3} = 2 \)
Common data 35 & 36
31. Ans: (b) , 32. Ans: (b)
Sol: Basic variables
\[ x_1 = 20, \quad x_2 = 10 \]
Non-basic variables
\[ s_1 = 0 \Rightarrow \text{first constraint is fully consumed.} \]
\[ s_2 = 0 \Rightarrow \text{second constraint is fully consumed.} \]
\[ x_3 = 0 \text{ (unwanted variable)} \]

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>z-row</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>110</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>20</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

From the table
- \( z \) increases by 2 units, \( x_1 \) decreases by 1 unit,
- \( x_2 \) decreases by 2 units,
If RHS value of 1st constraint decreases by 10 units then \( z \) decreases by 10 units,
The new objective value,
\[ Z_{\text{max}} = 110 - 10 = 100 \]
33. Ans: (c)
Sol:
From the table
- \( Z \) increases by 1 unit, \( x_1 \) increases by 1 unit,
- \( x_2 \) decreases by 1 unit,
If RHS value of 2nd constraint increases by 1 unit then

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>RHS</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>z-row</td>
<td>-3</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2/1=2</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4/2=2</td>
</tr>
</tbody>
</table>

Entering variable \( X_2 \)
Minimum ratio = \( \min(2/1, 4/2) = 2^* \)
*Tie w.r.t leaving variables \( S_1 \) and \( S_2 \)
Thus it has degenerate solution.

34. Ans: (d)
Sol:
From the table
- \( Z \) increases by 1 unit, \( x_1 \) increases by 1 unit,
- \( x_2 \) decreases by 1 unit,
If RHS value of 2nd constraint increases by 1 unit then

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>z-row</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Entering variable \( X_1 \)
Ratio = \( \text{Min}\{4/-2, 3/0\} \)
As there is no least positive ratio, there is no leaving variable which results the problem has unbounded solution.
35. 
Sol:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Products</th>
<th>Maximum available</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chairs (x₁)</td>
<td>Tables (x₂)</td>
</tr>
<tr>
<td>Wood</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Chairs</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Tables</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>Profit/loss</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>

\[ Z_{\text{max}} = 100x_1 + 300x_2 \]
Subject to
\[ x_1 + 2x_2 \leq 200 \]
\[ x_1 \leq 150 \quad \text{and} \quad x_2 \leq 80 \]

36. 
Sol:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Products</th>
<th>Maximum available</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A (x₁)</td>
<td>B (x₂)</td>
</tr>
<tr>
<td>Raw material</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Special type of</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>buckle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary buckle</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>½</td>
</tr>
<tr>
<td>Profits/unit</td>
<td>10/-</td>
<td>5/-</td>
</tr>
</tbody>
</table>

Constraints:
\[ x_1 = \text{No. of belts of type 'A'} \]
\[ x_2 = \text{No. of belts of type 'B'} \]

\[ Z_{\text{max}} = 10x_1 + 5x_2 \]
Subject to
\[ x_1 + x_2 \leq 850 \]
\[ x_1 \leq 500, \quad x_2 \leq 700 \]
\[ x_1 + \frac{1}{2}x_2 \leq 500, \quad x_1, x_2 \geq 0 \]

\[ Z_{\text{max}} = (10\times0) + (5\times500) = 2500 /- \]

Conventional Practice Solutions

01. 
Sol: Let, \( x_1 \) be the number of ash trays
\( x_2 \) be the number of tea trays
Production to be maximized
\[ Z = 20x_1 + 30x_2 \]
From the table given, constrained are
\[ 10x_1 + 20x_2 \leq 30000 \]
\[ 15x_1 + 5x_2 \leq 30000 \]
Fixed daily cost = Rs. 45000
From the graph, common feasible region is OABC O(0,0), A(0,1500), C(2000,0)

B would be obtained by solving the constraints. B(1800, 600)

\[ Z = 20 \times 0 + 30 \times 1500 = \text{Rs.45000} \]

\[ Z = 20 \times 1800 + 30 \times 600 = \text{Rs.54000} \]

\[ Z = 20 \times 2000 + 30 \times 0 = \text{Rs.40000} \]

\[ Z_{\text{max}} = \text{Rs. 54000 at B} \]

02.

Sol: \[ Z_{\text{max}} = 60x_1 + 50x_2 \]

s.t \[ x_1 + 2x_2 \leq 40 \]

\[ 3x_1 + 2x_2 \leq 60 \]

\[ \frac{x_1}{40} + \frac{x_2}{20} \leq 1 \quad , \quad \frac{x_1}{20} + \frac{x_2}{30} \leq 1 \]

\[ (Z_{\text{max}})_{(10,15)} = 60 \times 10 + 50 \times 15 = 1350 \text{ / -} \]

03.

Sol:

<table>
<thead>
<tr>
<th>Type of machine</th>
<th>Products</th>
<th>Total time available</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>P</td>
<td>10</td>
<td>7.5</td>
</tr>
<tr>
<td>Q</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>R</td>
<td>5</td>
<td>13</td>
</tr>
</tbody>
</table>

Profit for product, A = Rs. 60 per unit
Profit for product, B = Rs. 70 per unit

Let, \( x = \) number of A type products
\( y = \) number of B type products

.: Maximization problem

\[ Z_{\text{max}} = 60x + 70y \]

Constraints are, (in times)

\[ 10x + 7.5y \leq 75 \]

\[ 6x + 9y \leq 54 \]

\[ 5x + 13y \leq 65 \]

Common feasible region is OABCDO

O(0,0), A(0,5), D(7.5,0)
B is point of intersection of lines
\[ 6x + 9y \leq 54, \]
\[ 5x + 13y \leq 65 \]
Solving this, B = (3.55, 3.64)

C is the point of intersection of the lines
\[ 6x + 9y \leq 54, \]
\[ 10x + 7.5y \leq 75 \]
Solving these, C = (6,2)

**Graphically solving:**

\[ 10x + 7.5y \leq 75 \]
\[ 6x + 9y \leq 54 \]
\[ 5x + 13y \leq 65 \]

<table>
<thead>
<tr>
<th>Points</th>
<th>[ Z = 60x + 70y ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (0,5)</td>
<td>60x0+70x5 = 350</td>
</tr>
<tr>
<td>B (3.53,3.64)</td>
<td>3.55x60+70x3.64 = 464.8</td>
</tr>
<tr>
<td>C (6,2)</td>
<td>60x6+70x2 = 500</td>
</tr>
<tr>
<td>D (7,5)</td>
<td>7.5x60+0x70 = 450</td>
</tr>
<tr>
<td>O (0,0)</td>
<td>0x60+0x70 = 0</td>
</tr>
</tbody>
</table>

\[ Z_{\text{max}} = 500 \text{ at } C(6,2) \]

\[ Z_{\text{max}} = 8x + 6y \]

\[ 30x + 20y \leq 300, \quad \frac{x}{10} + \frac{y}{15} \leq 1 \quad \text{(1)} \]
\[ 5x + 10y \leq 110, \quad \frac{x}{22} + \frac{y}{11} \leq 1 \quad \text{(2)} \]
\[ x, y \geq 0 \]

“C” is the intersection of (1) and (2)
Solve equation (1) & (2) for \( x, y \)
We will get \( x = 4, y = 9 \)
\[ Z = 8x + 6y \]
\[ Z_A = 0 \]
\[ Z_B = 8x0 + 6x11 = 66 \]
\[ Z_C = 8x4 + 6x9 = 86 \]

Solution is optimal at (c)
\[ Z_{\text{max}} = 86 \text{ at } x = 4, y = 9 \]
01. Ans: (b)
Sol: 
\[ EOQ = \sqrt{\frac{2AS}{CI}} \]
\[ EOQ_1 = \sqrt{2} \times \frac{2AS}{CI} \]
\[ EOQ_1 = \sqrt{2} \times EOQ \]

02. Ans: (c)
Sol: 
\[ EOQ = \sqrt{\frac{2DC_0}{C_c}} \]

03. Ans: (b)
Sol: 
A = 900 unit
S = 100 per order
CI = 2 per unit per year

\[ EOQ = ELS = \sqrt{\frac{2AS}{CI}} \]
\[ = \sqrt{\frac{2 \times 900 \times 100}{2}} = 300 \]

04. Ans: (c)
Sol: 
Inventory carrying cost:
It involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

05. Ans: (b)
Sol: 
At EOQ, Carrying cost = Ordering cost

06. Ans: (d)
Sol: 
Inventory carrying cost involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

07. Ans: (a)
Sol: 
A = 800, S = 50/-,
\[ C_1 = 2 \text{ per unit} = CI \]
\[ (TIC)_{EOQ} = \sqrt{2ASCi} \]
\[ = \sqrt{2 \times 800 \times 50 \times 2} = 400 \]

08. Ans: (c)
Sol: 
\[ TC(Q_1) = TC(Q_2) \]
\[ \frac{kd}{Q_1} + \frac{hQ_1}{2} = \frac{kd}{Q_2} + \frac{hQ_2}{2} \]
\[ kd\left(\frac{Q_2 - Q_1}{Q_1Q_2}\right) = \frac{h}{2}(Q_2 - Q_1) \]
\[ 2kd = Q_1Q_2 \]
\[ h \]
\[ (Q^*)^2 = Q_1 \times Q_2 \]
\[ Q^* = \sqrt{Q_1 \times Q_2} = \sqrt{300 \times 600} = 424.264 \]

09. Ans: (c)
Sol: 
\[ \frac{EOQ_1}{EOQ_2} = \sqrt{\frac{2AS}{CI}} \times \sqrt{\frac{CI}{2AS_B}} \]
\[ = \sqrt{\frac{2 \times 100 \times 100}{4}} \times \sqrt{\frac{1}{2 \times 400 \times 100}} \]
\[ (EOQ)_{A} : (EOQ)_{B} = 1:4 \]
10. Ans: (d)
Sol: (No of orders = \( \frac{A}{Q} \) = \( \frac{12 \text{ months}}{45 \text{ days}} \) = 12 = 8)

\[ Q = 100 \]

\[ \text{TVC} = \frac{A}{Q} \times \frac{S}{2} + \frac{Q}{2} \times \text{CI} \]

\[ = 8 \times 100 + \frac{100}{2} \times 120 = \text{Rs. 6800} \]

11. Ans: (b)
Sol: Average inventory

\[ = \frac{Q}{2} = \frac{6000}{2} = 3000 \text{ per year} \]

\[ = 250 \text{ per month} \]

12. Ans: (b)
Sol: \( P = 1000, \ r = 500, \ Q = 1000 \)

\[ I_{\text{max}} = \frac{1000}{1000} (1000 - 500) = 500 \]

13. Ans: (c)
Sol: \( D = 1000 \text{ units}, \ C_0 = \text{Rs.100/order}, \ C_c = 100/\text{unit/year}, \ C_s = 400/\text{unit/year} \)

\[ Q_{\text{max}} = \text{EOQ} \times \frac{C_s}{C_c + C_s} \]

\[ = \sqrt{\frac{2DC_0}{C_c}} \sqrt{\frac{C_c + C_s}{C_s}} \times \left( \frac{C_s}{C_c + C_s} \right) \]

\[ = 40 \text{ units} \]

14. Ans: (d)
Sol: Re-order level = 1.25[\( \sum x p(x) \)]

\[ = 1.25 [80 \times 0.2 + 100 \times 0.25 + 120 \times 0.3 + 140 \times 0.25] \]

\[ = 140 \text{ units} \]

<table>
<thead>
<tr>
<th>Demand</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
<td><strong>0.25</strong></td>
</tr>
</tbody>
</table>

| Cumulative probability (Service level) | 0.2 | 0.45 | 0.75 | **1.0** |

Service Level = 100 %

15. Ans: (b)

16. Ans: (b)

17. Ans: (d)
Sol: \( C \) – Class means these class items will have very less consumption values. – least consumption values

- \( B \rightarrow 300 \times 0.15 = 45 \)
- \( F \rightarrow 300 \times 0.1 = 30 \)
- \( C \rightarrow 2 \times 200 = 400 \)
- \( E \rightarrow 5 \times 0.3 = 1.5 \)
- \( J \rightarrow 5 \times 0.2 = 1.0 \)
- \( G \rightarrow 10 \times 0.05 = 0.5 \)
- \( H \rightarrow 7 \times 0.1 = 0.7 \)

G, H items are classified as C class items because they are having least consumption values.

18. Ans: (b)
Sol: In ABC analysis:

Category “A” = Low safety stock
Category “B” = Medium safety stock
Category “C” = High safety stock
Conventional Practice Solutions

01.
Sol: Given, \( A = 5000 \) units/year,
\[
S = 16/\text{-},
\]
\[
I = 0.02 + 0.12 + 0.06 = 0.2,
\]
\[
C = 20/\text{-}
\]
\[
EOQ = \sqrt{\frac{2AS}{CI}}
\]
\[
= \sqrt{\frac{2 \times 5000 \times 16}{20 \times 0.2}} = 200 \text{ units}
\]
\[
(TVC)_{EOQ} = \sqrt{2ASC}I
\]
\[
= \sqrt{2 \times 5000 \times 16 \times 20 \times 0.2} = \text{Rs. } 800 /\text{-}
\]

02.
Sol: Given, \( A = 1000 \) units/year, \( S = 40/\text{-} \)
\[
I = 0.1, \quad C = 500/\text{-}
\]
a) \( EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2 \times 1000 \times 40}{500 \times 0.1}} = 40 \text{ units} \)
b) No. of annual orders = \( \frac{A}{Q} = \frac{1000}{40} = 25 \)
c) \( (TAC)_{EOQ} = AC + \sqrt{2ACSI} \)
\[
= 1000 \times 500 + \sqrt{2 \times 1000 \times 500 \times 40 \times 0.1}
\]
\[
= 5,02,000 /\text{-}
\]
Order per month = \( \frac{1000}{12} = 83.33 \) units.
\( (TAC)_Q = AC + \frac{A}{Q}S + \frac{Q}{2}CI \)

\[
(TAC)_{Q_{13.33}} = 1000 \times 500 + \frac{1000}{83.33} \times 40 + \frac{83.33}{2} \times 500 \times 0.1
\]
\[
= 5,02,563 /\text{-}
\]
Savings = \( (TAC)_Q - (TAC)_{EOQ} \)
\[
= 502563 - 502000 = \text{Rs. } 563 /\text{-}
\]

03.
Sol: Simultaneous consumption producing Model
\[
A = 15,000 \text{ units}, \quad C.I = 5/ \text{ units/year}
\]
\[
S = 25 /\text{-}, \quad P = 100 \text{ units/day}
\]
No. of working days = 250 /year
Consumption rate = \( r = \frac{15,000}{250} = 60 \text{ units/day} \)
\[
EBQ = EPQ = ELS
\]
\[
EPQ = \sqrt{\frac{2AS}{CI} \left( \frac{P}{P-r} \right)}
\]
\[
Q = \sqrt{\frac{2 \times 15000 \times 25 \times 100}{100 - 60}}
\]
\[
= 612.37 \text{ units}
\]
\[
(TVC)_{EPQ} = \sqrt{2ASC}I \left( \frac{P-r}{P} \right)
\]
\[
= \sqrt{2 \times 500 \times 60 \times 100 \left( \frac{100}{100 - 60} \right)}
\]
\[
= 1225 /\text{-}
\]
No of production runs = \( \frac{A}{Q} = \frac{15000}{612.37} = 24.5 \approx 25 \)
04.

Sol: D = 192000 units,
A = Rs. 1080 /set-up,
h = 0.3 × 12 = 3.60 /pack/year,
\( d = \frac{192000}{240} = 800 \) packs per day
\( p = \frac{20000}{20} = 1000 \) packs/day.

(a) Optimum lots size = \( \sqrt{\frac{2AD}{h}} \left( \frac{p}{p-d} \right) \)
= \( \sqrt{\frac{2 \times 192000 \times 1080}{3.60}} \left( \frac{1000}{1000-800} \right) \)
= 24000 packs

(b) Optimum number of production runs
= \( \frac{\text{Annual demand}}{\text{Optimum lot size}} \)
= \( \frac{192000}{24000} = 8 \)

(c) Time interval between successive production runs
= \( \frac{\text{No. of working days}}{\text{No. of runs}} \)
= \( \frac{240}{8} = 30 \) working days

(d) Total variable cost = \( \sqrt{2DAh} \left( \frac{p-d}{p} \right) \)
= \( \sqrt{2 \times 192000 \times 1080 \times 3.60 \times \frac{1000-800}{1000}} \)
= Rs. 17,280 /-

05.

Sol: A = 10,000 units
S = 200/order
CI = 4/unit/year
C = 20/-

(a) \( \text{EOQ} = \sqrt{\frac{2AS}{CI}} \)
= \( \sqrt{\frac{2 \times 10000 \times 200}{4}} = 1000 \) units.

Total annual cost at EOQ,
\( (TAC)_{EOQ} = AC + \sqrt{2ACSI} \)
= 10000(20) + \sqrt{2(10000)4(200)}
= 2,04,000/-

(b) \( (EOQ)_{shortage} = \sqrt{\frac{2AS}{CI} \left( \frac{C_s + CI}{C_s} \right)} \)
= \( \sqrt{\frac{2 \times 10000 \times 200 \times 20 + 4}{4}} \)
= 1095.45 units

Optimal level of shortages
\( S^* = Q^* \times \left( \frac{C_s}{C_s + CI} \right) \)
= 1095.45 \times \frac{20}{20 + 4}
= 912.87 units

Maximum inventory level = \( Q^* - S^* \)
= 1095.45 – 912.87
= 182.58
06.
**Sol:** Given:
- \( C = \text{Rs. 5/unit} \)
- \( A = 4000 \text{ units} \)
- \( S = \text{Rs. 30/order} \)
- \( \text{CI} = \text{Rs. 1.5} \)

\[
\text{EOQ} = \sqrt{\frac{2 \times 4000 \times 30}{1.5}} = 400 \text{ units}
\]

Number of orders per year = \( \frac{4000}{400} = 10 \) runs

\[
\text{(Total yearly cost)}_{EOQ} = AC + \sqrt{2AC\text{CI}}
\]

\[
= (4000 \times 5) + \sqrt{2 \times 4000 \times 30 \times 1.5}
\]

\[
= \text{Rs. 20600/-}
\]

\[
(TC)_{OEQ} = AC\left(1 - \frac{R_1}{100}\right) + \frac{A}{Q_1}S + \frac{Q_1}{2} \times \text{CI}\left(1 - \frac{R_1}{100}\right)
\]

\[
= (4000 \times 5)\left(1 - \frac{2}{100}\right) + \frac{4000}{1000} \times 30 + \frac{1000}{2} \times 1.5 \left(1 - \frac{2}{100}\right)
\]

\[
= \text{Rs. 20455/-}
\]

\[
(TC)_{OEQ} = 4000 \times 5 \left(1 - \frac{3}{100}\right) + \frac{4000}{2000} \times 30 + \frac{2000}{2} \times 1.5 \left(1 - \frac{3}{100}\right)
\]

\[
= \text{Rs. 20915/-}
\]

Among all 2% discount for ordering quantities of 1000 or more

07.
**Sol:** Given:
- \( A = 2000 \text{ units/year} \)
- \( S = \text{Rs. 20/-} \)
- \( I = 25\% \)
- \( C_u = \text{Rs. 8/-} \) (Lowest with unit price)

\[
\text{EOQ}_{C_u=8\%} = \sqrt{\frac{2 \times 2000 \times 20}{8 \times 0.25}} = 200 \text{ units}
\]

The \( \text{EOQ}_{C_u=8\%} = \text{Rs. 8/-} \) is satisfying the Quantity range hence it is declared as an optimal order quantity.

08.
**Sol:**

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Daily sales} & \text{No. of days} & \text{Probability} & \text{SL} & \text{SOR} \\
\hline
10 & 15 & 0.15 & 0.15 & 1 \\
11 & 20 & 0.20 & 0.35 & 0.85 \\
12 & 40 & 0.40 & 0.75 & 0.65 \\
13 & 25 & 0.25 & 1 & 0.25 \\
\hline
\end{array}
\]

\[
\begin{align*}
\text{Cus} &= \text{SP} - \text{CP} = 5 - 2 = 3 \\
\text{Cos} &= \text{CP} = 2 \\
\text{SL} &= \frac{\text{Cus}}{\text{Cus} + \text{Cos}} \\
&= \frac{3}{3+2} = 0.6 \\
\text{SOR} &= 1 - \text{SL} = 1 - 0.6 = 0.4
\end{align*}
\]

As \( SL = 0.6 \) falling in the range 11 to 12 sales, hence order 12 for 40 days.

- \( (\text{Cus}) = \text{Cost of under stock} \)
- \( (\text{Cos}) = \text{Cost of over stock} \)
- \( (\text{SL}) = \text{Service levels} \)
- \( (\text{SOR}) = \text{Stock out risk} \)
- \( \text{SP} = \text{selling price}, \text{CP} = \text{cost price} \)
09. 
**Sol:** Cus = SP – CP = 2 – 0.8 = 1.2  
\[ \text{Cos} = \frac{\text{Cus}}{\text{Cus} + \text{+Cos}} = \frac{1.2}{1.2 + 0.8} = 0.6 \]
For 60% – Service levels  
\[ Q_{\text{Optimum}} = I_{\text{min}} + \text{SL} \times (I_{\text{max}} - I_{\text{min}}) \]
\[ = 20000 + 0.6(24000 - 20000) \]
\[ = 22400 \]

10. 
**Sol:**  
Stage – I:  
Let C = Rs. 185 /-
\[ \text{EOQ}_{C=185} = \sqrt{\frac{2\times \text{AS}}{\text{C} \times \text{I}}} = \sqrt{\frac{2 \times 8000 \times 1800}{185 \times 0.1}} = 1247.7 \text{ units} \]
EOQ does not satisfy the quantity range.
Hence we calculate  
\[ \text{TC}_{Q=2000} = \frac{Q}{2} \times \text{C.I} + \frac{A}{Q} \times \text{S} + \text{AC} \]
\[ = \left( \frac{2000}{2} \times 185 \times 0.1 \right) + \left( \frac{8000}{2000} \times 1800 \right) + (8000 \times 185) \]
\[ = 1505700/- \]

Stage – II:  
\[ \text{EOQ}_{C=190} = \sqrt{\frac{2\times \text{AS}}{\text{C} \times \text{I}}} = \sqrt{\frac{2 \times 8000 \times 1800}{190 \times 0.1}} \]
\[ = 1231.17 \text{ units} \]
EOQ does not satisfy the quantity range.
Hence we calculate  
\[ \text{TC}_{Q=2000} = \frac{Q}{2} \times \text{C.I} + \frac{A}{Q} \times \text{S} + \text{AC} \]
\[ = \left( \frac{2000}{2} \times 190 \times 0.1 \right) + \left( \frac{8000}{2000} \times 1800 \right) + (8000 \times 190) \]
\[ = \text{Rs 1543850} /- \]

Stage – III:  
\[ \text{EOQ}_{C=200} = \sqrt{\frac{2\times \text{AS}}{\text{C} \times \text{I}}} = \sqrt{\frac{2 \times 8000 \times 1800}{200 \times 0.1}} \]
\[ = 1200 \text{ units} \]
EOQ satisfy the quantity range. Hence we calculate  
\[ \text{TC}_{Q=2000} = \frac{Q}{2} \times \text{C.I} + \frac{A}{Q} \times \text{S} + \text{AC} \]
\[ = \left( \frac{2000}{2} \times 2000 \times 0.1 \right) + 8000 \times 200 \]
\[ = \text{Rs 1675894.66} /- \]
Among all the total cost, the minimum in  
\[ \text{TC}_{Q=2000} = \text{Rs 1543850} /- \]
So the best order size is 2000 units

11. 
**Sol:** Annual demand (A) = 2000 units  
Cost per item (C) = 20/-  
Ordering cost = 50/-  
Inventory carrying cost (I) = 0.25  
\[ \text{TAC}_{\text{EOQ}} = \text{AC} + \sqrt{2\text{ACSI}} \]
\[ = (2000 \times 20) + \sqrt{2 \times 2000 \times 20 \times 50 \times 0.25} \]
\[ = 41,000/- \]
Now, TAC at Q, with discount r%
\[
(TAC)_{oi} = AC\left(1 - \frac{r_1}{100}\right) + \frac{A}{Q_1}S + \frac{Q_1}{2}CI\left(1 - \frac{r_1}{100}\right)
\]
\[
= 2000 \times 20\left(1 - \frac{3}{100}\right) + \frac{2000}{1000} \times 50 + \frac{1000}{2} \times 20 \times 0.25\left[1 - \frac{3}{100}\right]
\]
\[
= 41325 
\]
As the total annual cost (TAC) with discount \( r\% \) is greater than (TAC) at EOQ, hence reject the discount and order 200 at a time.

12.
Sol: \[
EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2 \times 25 \times 25}{0.4}} = 55.9 \text{ units} \approx 56 \text{ units}
\]
Re-order point = \( \text{Daily demand} \times \text{Lead Time} \)
\[
= 25 \times 16 = 400 \text{ units}
\]

13. Sol: Given,

Daily demand – D, D,
Lead Time – L.T
Re-order Level – ROL

For Item A
EOQ = \[
\sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2 \times 8000 \times 15}{0.06}} = 2000 \text{ units}
\]
R.O.L = daily demand \( \times \) Lead Time
\[
= \frac{8000}{250} \times 10 = 320 \text{ units}
\]

For Item B
ROL = D.D \times L.T
216 = \( \frac{A}{250} \times 6 \)
A = 9000 units
EOQ = \[
\sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2 \times 9000 \times 40}{0.18}} = 2000 \text{ units}
\]

For Item C
EOQ = \[
\sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2 \times 7500 \times S}{30}} = 300 \text{ units}
\]
S = Rs. 180/order
ROL = D.D \times L.T
210 = \( \frac{7500 \times L.T}{250} \)
Lead Time = 7 days

14. Sol:

\begin{align*}
\mu - 3\sigma & = 120 - 3(20) = 60 \\
\mu + 3\sigma & = 120 + 3(20) = 180
\end{align*}

a) SOR = 2%,
For service level (SL) = 98% to be safety factor on \( \sigma \) basis, \( SF_\sigma = 2.05 \)
Safety stock (SS) = \( SF_\sigma \times \sigma \)
\[
= 2.05 \times 20 = 41
\]
Re-order point (ROP)

\[ \text{ROP} = \text{Avg lead time demand} + \text{SS} \]
\[ = 120 + 41 = 161 \]

b) Given, ROP = 140 units, \( \text{SF}_\sigma = ? \)
\[ 140 = 120 + \text{SF}_\sigma \times 20 \]
\[ \text{SF}_\sigma = 1 \]

i.e., as \( \text{SF}_\sigma \) basis is 1 will achieve service levels (SL) 84.13%.

Stock out risk (SOR) = 100 – SL
\[ (\because \text{SOR} + \text{SL} = 100\%) \]
\[ = 100 - 84.13 \]
\[ \text{SOR} = 15.87\% \]

Stock out = 140 – 100 = 40 units

15.

Sol: \( \sigma = 60 \) units, \( \text{SL} = \frac{51}{52} = 98\% \)
(Consider 52 weeks/year)

\[ \text{SS} = \text{SF}_\sigma \times \sigma = 2.05 \times 60 = 123 \]

\[ \text{ROL} = \text{ALTd} + \text{SS} \]
\[ = \text{ALT} \times \text{CR} + \text{SF}_\sigma \sigma \]
\[ = 500 \times 1 + 123 = 623 \text{ units} \]

Where, \( \text{CR} \) = consumption rate
\( \text{ALT} \) = Average lead time

16.

Sol: Lead Time > order cycle
\[ \sigma_{OC} = \sqrt{n \sigma^2} = \sqrt{6 \times 5^2} = 12.21 \]

Safety stock (SS) = \( \text{SF}_\sigma \times \sigma \)
\[ = 1.28 \times 12.21 = 15.67 \approx 16 \]

\[ (\because \text{For 90\% SL} \rightarrow \text{SF}_\sigma \approx 1.28) \]

\[ \text{ROL} = \text{ALTd} + \text{SS} = 40 + 16 = 56 \]

17.

Sol: Raking of items according to their usage values

<table>
<thead>
<tr>
<th>Part code</th>
<th>Price per unit Rs</th>
<th>Units /year</th>
<th>Total cost (Rs)</th>
<th>% of total cost</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>P01</td>
<td>100</td>
<td>100</td>
<td>10000</td>
<td>0.2</td>
<td>X</td>
</tr>
<tr>
<td>P02</td>
<td>200</td>
<td>300</td>
<td>60000</td>
<td>1.2</td>
<td>VI</td>
</tr>
<tr>
<td>P03</td>
<td>50</td>
<td>700</td>
<td>35000</td>
<td>0.7</td>
<td>IV</td>
</tr>
<tr>
<td>P04</td>
<td>300</td>
<td>400</td>
<td>120000</td>
<td>2.4</td>
<td>IV</td>
</tr>
<tr>
<td>P05</td>
<td>500</td>
<td>1000</td>
<td>500000</td>
<td>10</td>
<td>III</td>
</tr>
<tr>
<td>P06</td>
<td>3000</td>
<td>30</td>
<td>60000</td>
<td>1.2</td>
<td>VII</td>
</tr>
<tr>
<td>P07</td>
<td>1000</td>
<td>100</td>
<td>100000</td>
<td>2</td>
<td>V</td>
</tr>
<tr>
<td>P08</td>
<td>7000</td>
<td>500</td>
<td>3500000</td>
<td>70.5</td>
<td>I</td>
</tr>
<tr>
<td>P09</td>
<td>5000</td>
<td>105</td>
<td>525000</td>
<td>10.6</td>
<td>II</td>
</tr>
<tr>
<td>P10</td>
<td>60</td>
<td>1000</td>
<td>60000</td>
<td>1.2</td>
<td>VIII</td>
</tr>
<tr>
<td>Total</td>
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<td>4970000</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

ABC PLAN

<table>
<thead>
<tr>
<th>RANK</th>
<th>Part code</th>
<th>% of total cost%</th>
<th>Cumulative percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>P08</td>
<td>70.5</td>
<td>70.5</td>
</tr>
<tr>
<td>II</td>
<td>P09</td>
<td>10.6</td>
<td>81.1</td>
</tr>
<tr>
<td>III</td>
<td>P05</td>
<td>10</td>
<td>91.1</td>
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<tr>
<td>IV</td>
<td>P04</td>
<td>2.4</td>
<td>93.5</td>
</tr>
<tr>
<td>V</td>
<td>P07</td>
<td>2</td>
<td>95.5</td>
</tr>
<tr>
<td>VI</td>
<td>P02</td>
<td>1.2</td>
<td>96.7</td>
</tr>
<tr>
<td>VII</td>
<td>P06</td>
<td>1.2</td>
<td>97.9</td>
</tr>
<tr>
<td>VIII</td>
<td>P10</td>
<td>1.2</td>
<td>99.1</td>
</tr>
<tr>
<td>IX</td>
<td>P03</td>
<td>0.7</td>
<td>99.8</td>
</tr>
<tr>
<td>X</td>
<td>P01</td>
<td>0.2</td>
<td>100</td>
</tr>
</tbody>
</table>

Class A items → Nil
Class B items → I, II
Class C items → III, IV, V, VI, VII, VIII, IX, X
01. Ans: (d)

02. Ans: (d)

Sol:
- A simple moving average is a method of computing the average of a specified number of the most recent data values in a series.
- This method assigns equal weight to all observations in the average.
- Greater smoothing effect could be obtained by including more observations in the moving average.

03. Ans: (a)

Sol: 3 period moving avg = \(\frac{100 + 99 + 101}{3} = 100\)

4 period moving average
\[= \frac{102 + 100 + 99 + 101}{4} = 100.5\]

5 period moving average
\[= \frac{99 + 102 + 100 + 99 + 101}{5} = 100.2\]

Arithmetic Mean
\[= \frac{101 + 99 + 102 + 100 + 99 + 101}{6} = 100.33\]

04. Ans: (a)

Sol: \(D_t = 100\) units, \(F_t = 105\) units
\[\alpha = 0.2\]
\[F_{t+1} = 105 + 0.2(100 - 105) = 104\]

05. Ans: (c)

Sol: \(D_t = 105\), \(F_t = 97\), \(\alpha = 0.4\)
\[F_{t+1} = 97 + 0.4(105 - 97) = 100.2\]

06. Ans: (c)

Sol: \(F_{t+1} = F_t + \alpha(X_t - F_t)\)

07. Ans: (c)

Sol: Another form of weighted moving average is the exponential smoothed average. This method keeps a running average of demand and adjusts if for each period in proportion to the difference between the latest actual demand and the latest value of the forecast.

08. Ans: (a)

09. Ans: (b)

Sol:

<table>
<thead>
<tr>
<th>Period</th>
<th>(D_t)</th>
<th>(F_t)</th>
<th>((D_t - F_t)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>100</td>
<td>75</td>
<td>625</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>87.5</td>
<td>156.25</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>93.75</td>
<td>39.0625</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\Sigma(D_t - F_t)^2 = 820.31)</td>
</tr>
</tbody>
</table>

\[F_{15} = F_{14} + \alpha(D_{14} - F_{14})\]
\[= 75 + 0.5(100 - 75) = 87.5\]
\[ F_{16} = F_{15} + \alpha(D_{15} - F_{15}) = 87.5 + 0.5(100 - 87.5) = 93.75 \]

Mean square error (MSE) = \[ \frac{\sum(D_i - F_i)^2}{n} = \frac{820.31}{3} = 273.13 \]

10. Ans: (a)
Sol:

| Period | D_i | F_i | |D_i–F_i|| \[
| 1 | 10 | 9.8 | 0.2 |
| 2 | 13 | 12.7 | 0.3 |
| 3 | 15 | 15.6 | 0.6 |
| 4 | 18 | 18.5 | 0.5 |
| 5 | 22 | 21.4 | 0.6 |

\[ \Sigma|D_i–F_i| = 2.2 \]

11. Ans: (d)
Sol:

\[ m_1 = \text{moving average periods give forecast } F_1(t) \]
\[ m_2 = \text{moving average periods give forecast } F_2(t) \]
\[ m_1 > m_2 \]

\( F_1(t) \) is a stable forecast has less variability. \( F_2(t) \) is a sensitive (inflationary) forecast and has high variability.

12. Ans: (d)
Sol: Following are the purposes of long term forecasting:
- To plan for the new unit of production.
- To plan for the long-term financial requirement.
- To make the proper arrangement for training the personal.
- Budgetary allegations are not done in the beginning of a project. So, deciding the purchase program is not the purpose of long term forecasting.

13. Ans: (d)
Sol:

- Time horizon is less for a new product and keeps increasing as the product ages. So, statement (I) is correct.
- Judgemental techniques apply statistical method like random sampling to a small population and extrapolate it on a larger scale. So, statement (II) is correct.
- Low values of smoothing constant result in stable forecast. So statement (3) is correct.

14. Ans: (i) 50, (ii) 52.5, (iii) (42.5, 40)
Sol:

\( F_7 = \frac{60 + 50 + 40}{3} = 50 \)

\( F_7 = \frac{60 \times 0.5 + 50 \times 0.25 + 40 \times 0.25}{0.5 + 0.25 + 0.25} = 52.5 \)

\( 2 \text{ period moving average} = \frac{60 + 50}{2} = 55 \)

\( 4 \text{ period moving average} = \frac{60 + 50 + 40 + 20}{4} = 42.5 \)

\( 5 \text{ period moving average} = \frac{60 + 50 + 40 + 20 + 30}{5} = 40 \)
15. Ans: (114.8 units, 9 periods)

Sol: At $\alpha = 0.2$

$F_{\text{May}} = 100 + 0.2 \times (200 - 100) = 120$

$F_{\text{June}} = 120 + 0.2 \times (50 - 120) = 106$

$F_{\text{July}} = 106 + 0.2 \times (150 - 106) = 114.8$

<table>
<thead>
<tr>
<th>Time</th>
<th>Demand</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>May</td>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>June</td>
<td>150</td>
<td>106</td>
</tr>
<tr>
<td>July</td>
<td>-</td>
<td>114.8</td>
</tr>
</tbody>
</table>

$\alpha = \frac{2}{n + 1}$

$n + 1 = \frac{2}{\alpha} \Rightarrow n = \frac{2}{0.2} - 1 = 9$ periods

02.

Sol: Simple exponential method

$\alpha = 0.2$, $D_{\text{Jan}} = 200$

$F_{\text{Jan}} = 175$, $D_{\text{Feb}} = 170$

$F_{\text{Feb}} = F_{\text{Jan}} + \alpha (D_{\text{Feb}} - F_{\text{Jan}})$

$= 175 + 0.2 \times (200 - 175) = 180$

$F_{\text{March}} = F_{\text{Feb}} + \alpha (D_{\text{Feb}} - F_{\text{Feb}})$

$= 180 + 0.2 \times (170 - 180) = 178$

03.

Sol: Linear Regression model:

<table>
<thead>
<tr>
<th>(x)</th>
<th>y (Rs)</th>
<th>xy</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>450</td>
<td>450</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>550</td>
<td>1110</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>625</td>
<td>1875</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>650</td>
<td>2600</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>750</td>
<td>3750</td>
<td>25</td>
</tr>
</tbody>
</table>

$\sum x = 15$, $\sum y = 3025$, $\sum xy = 9775$, $|\sum x^2 - 55|$

$y = a + bx \Rightarrow \Sigma y = na + b\Sigma x$

$xy = ax + bx^2 \Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

$3025 = 5a + 15b \ldots \ldots (1)$

$9775 = 15a + 55b \ldots \ldots (2)$

Now, solve (1) and (2) for a, b

$a = 395$, $b = 70$

Forecast equ. $y_c = a + bx$

$y_c = 395 + 70x$

Forecast for month – 6,

$y_6 = 395 + 70(6) = 815$

Forecast For month – 7

$y_7 = 395 + 70(7) = 885$
04.
Sol: Deviation = \( D_i - F_i \)

\[
\text{MAD} = \frac{\sum_{i=1}^{n} |D_i - F_i|}{n} = \frac{7.5 + 18 + 0 + 28.12}{6} = \frac{70}{6} = 11.66
\]

Tracking signal = \( \frac{\text{Cumulative deviation}}{\text{MAD}} \)

\[
= \frac{-24}{11.66} = 2.05 < 4
\]
If tracking signal < 4 – No significant deviation in data
If tracking signal > 4 – significant deviation in data

05.
Sol: \( n = 20, \)
\( \Sigma (y - \bar{y})^2 = 2800 \)
\( \Sigma x = 80, \)
\( \Sigma y = 1200, \)
\( \Sigma x^2 = 340, \)
\( \Sigma y^2 = 74,800, \)
\( \Sigma xy = 5000 \)

\( y = a + bx \)

\[
\Rightarrow \Sigma y = na + b\Sigma x
\]
\( 1200 = 20a + b(80) \ldots (1) \)

\[
\Sigma xy = ax + bx^2
\]

\[
5000 = a(80) + b(340) \ldots (2)
\]

Solve (1) and (2) for a, b

\( a = 20, \quad b = 10 \)

Standard error

\[
S_{yx} = \sqrt{\frac{\Sigma y^2 - a\Sigma y - b\Sigma xy}{n-2}}
\]
\[
= \sqrt{\frac{74800 - (20 \times 1200) - (10 \times 5000)}{20 - 2}}
\]
\[
= 6.67
\]

Correlation coefficient,

\[
r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{(n\Sigma x^2 - (\Sigma x)^2)(n\Sigma y^2 - (\Sigma y)^2)}}
\]
\[
= \frac{20 \times 500 - 80 \times 1200}{\sqrt{(20 \times 340 - (80)^2)(20 \times 74800 - (1200)^2)}}
\]
\[
= 0.84
\]
As ‘r’ closer to ‘1’ i.e., good correlation
01. **Ans:** (a)

**Sol:**
\[ \lambda = 3 \text{ per day} \]
\[ \mu = 6 \text{ per day} \]
\[ W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{3}{6(6 - 3)} = \frac{1}{6} \text{ day} \]

02. **Ans:** (c)

**Sol:**
\[ \lambda = 0.35 \text{ min}^{-1}, \]
\[ \mu = 0.5 \text{ min}^{-1} \]
\[ P_n = \left[ 1 - \frac{\lambda}{\mu} \right]^{\frac{\lambda}{\mu}} = \left[ 1 - 0.35 \times \frac{0.35}{0.5} \right]^{0.35} = 0.0173 \]

03. **Ans:** (a)

**Sol:**
\[ \lambda = 10 \text{ hr}^{-1}, \]
\[ \mu = 15 \text{ hr}^{-1} \]
\[ L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{15(15 - 10)} = 1.33 \]

04. **Ans:** (b)

**Sol:**
\[ \lambda = 4 \text{ hr}^{-1}, \mu = \frac{60}{12} = 5 \text{ hr}^{-1} \]
\[ L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{4^2}{5(5 - 4)} = \frac{16}{5} = 3.2 \]

05. **Ans:** (b)

**Sol:**
\[ L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\lambda^2}{\mu^2 \left( 1 - \frac{\lambda}{\mu} \right)} = \frac{\rho^2}{1 - \rho} \]

06. **Ans:** (d)

**Sol:**
\[ \lambda = \frac{1}{4} = 0.25 \text{ min}^{-1} \]
\[ \mu = \frac{1}{3} = 0.33 \text{ min}^{-1} \]
\[ \rho = \frac{\lambda}{\mu} = \frac{0.25}{0.33} = 0.75 \]

07. **Ans:** (b)

**Sol:**
\[ \lambda = \frac{1}{10} = 0.1 \text{ min}^{-1} \]
\[ \mu = \frac{1}{4} = 0.25 \text{ min}^{-1} \]
\[ \text{System busy } \Rightarrow (\rho) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4 \]

08. **Ans:** (c)

**Sol:**
\[ \lambda = 4 \text{ hr}^{-1}, \mu = 6 \text{ hr}^{-1} \]
\[ P(Qs \geq 2) = \left( \frac{\lambda}{\mu} \right)^2 = \left( \frac{4}{6} \right)^2 = \frac{4}{9} \]

09. **Ans:** (c)
Conventional Practice Solutions

01.
Sol: $\lambda = 8 \text{ hr}^{-1}$; $\mu = \frac{60}{5} = 12 \text{ hr}^{-1}$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{12 - 8} = \frac{1}{4}$$

02.
Sol: $\lambda = 100 \text{ h}^{-1}$; $\mu = 120 \text{ h}^{-1}$

$$\rho = \frac{\lambda}{\mu} = \frac{100}{120} = \frac{10}{12}$$

$P_0$ (no customer in the system)

$$= 1 - \rho = 1 - \frac{10}{12} = \frac{2}{12} = \frac{1}{6}$$

03.
Sol: $\lambda = 8 \text{ h}^{-1}$

$$\mu = \frac{60}{5} h^{-1} = 12 \text{ h}^{-1}$$

(a) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(8)^2}{12 \times 4} = 1.33$

(b) $L_s = \frac{\lambda}{(\mu - \lambda)} = \frac{8}{12 - 8} = 2$

(c) $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{8}{12 \times 4} = 0.1666$

(d) $W_s = \frac{1}{(\mu - \lambda)} = \frac{1}{4} = 0.25$

(e) $\rho = \frac{\lambda}{\mu} = \frac{8}{12} = 0.666$

04.
Sol: $\lambda = 20 \text{ h}^{-1}$; $\mu = \frac{60}{2} \text{ h}^{-1} = 30 \text{ h}^{-1}$

(a) $P_0 = \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{20}{30}\right) = \frac{1}{3}$

(b) $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{20}{30(30 - 20)} = 0.066$

(c) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(20)^2}{30(30 - 20)} = 1.33$

(d) $\rho = \frac{\lambda}{\mu} = \frac{20}{30} = 0.66$

05.
Sol: $\lambda = 2 \text{ hr}^{-1}$; $\mu = 5 \text{ hr}^{-1}$

a) Traffic intensity ($\rho$) = $\frac{\lambda}{\mu} = \frac{2}{5} = 0.4$

b) No customer $\Rightarrow$ service facility idle

$$P_0 = 1 - \rho = 1 - 0.4 = 0.6$$

c) The probability that there is no customer waiting to be served = Probability that atmost 1 customer at the counter who is getting the service or no one in the counter $= P_0 + P_1$

$$P_0 + P_1 = \left(1 - \frac{\lambda}{\mu}\right) + \left(1 - \frac{\lambda}{\mu}\right) \frac{\lambda}{\mu}$$

$$= \left(1 - \frac{\lambda}{\mu}\right) \left(1 + \frac{\lambda}{\mu}\right) = 1 - \left(\frac{\lambda}{\mu}\right)^2$$

$$= 1 - 0.16 = 0.84$$

As $\mu > \lambda \Rightarrow L_q$ is finite

If $\mu = \lambda \Rightarrow L_q$ is infinite
06. 
Sol: 
\[
\begin{align*}
\lambda &= 3 \text{ hr}^{-1} \\
\mu &= 6 \text{ hr}^{-1} \\
\text{NPC/hr} &= 15 \text{ Rs} \\
\text{LC/hr} &= 20 \\
\end{align*}
\]

L<sub>S</sub> represents non productive machining
\[
L_S = \frac{\lambda}{\mu - \lambda} = \frac{3}{6 - 3} = 1 \\
L_S = \frac{\lambda}{\mu - \lambda} = \frac{3}{4 - 3} = 3 \text{ m/c}
\]
NPC/hr = 1×15Rs NPC/hr = 3×15 = Rs. 45
LC/hr = 20/-
“A” should be hired

01. Ans: (a) 
Sol: SPT rule

<table>
<thead>
<tr>
<th>Job</th>
<th>Process time (days)</th>
<th>Completion time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>42</td>
</tr>
</tbody>
</table>

\[\Sigma C_i = 125\]

Average Flow Time = \[
\frac{n}{n} \sum C_i = \frac{125}{6} = 20.83
\]

02. Ans: (a) 
Sol: According to SPT rule total inventory cost is minimum.

03. Ans: (d) 
Sol: EDD rule can minimize maximum lateness. The job sequence is R – P – Q – S

04. Ans: (d) 
Sol: Johnson’s rule:

Optimum job sequence III – I – IV – II 
Do the job 1<sup>st</sup> if the minimum time happens to be on the machine (M) and do it on the end if .it is on second machine (N). Select either in case of a tie.
05. Ans: (b)
Sol:

<table>
<thead>
<tr>
<th>Job</th>
<th>M</th>
<th>N</th>
<th>Idle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In</td>
<td>PT</td>
<td>Out</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>IV</td>
<td>4</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>II</td>
<td>11</td>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

Total idle time on machine (N) = 3

06. Ans: (a)
Sol: Optimum sequence of jobs

2 3 1 4

07. Ans: (b)
Sol: Optimum sequence is

R T S Q U P

<table>
<thead>
<tr>
<th>Job</th>
<th>M1</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In</td>
<td>PT</td>
</tr>
<tr>
<td>R</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>T</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>S</td>
<td>19</td>
<td>27</td>
</tr>
<tr>
<td>Q</td>
<td>46</td>
<td>32</td>
</tr>
<tr>
<td>U</td>
<td>78</td>
<td>16</td>
</tr>
<tr>
<td>P</td>
<td>94</td>
<td>15</td>
</tr>
</tbody>
</table>

The optimal make-span time = 115 days

08. Ans: (c)

Conventional Practice Solutions

01.
Sol: SPT rule is used for minimizing mean flow time

<table>
<thead>
<tr>
<th>Job</th>
<th>t_i</th>
<th>C_i</th>
<th>d_i</th>
<th>C_i – d_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>-7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
<td>12</td>
<td>-7</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>16</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>24</td>
<td>20</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ \sum C_i = 57 \]

EJ - EARLY JOB, OS - ON SCHEDULE, TJ - TARDY JOB

Minimum total cost = 57 × 60 = 3,420

Number of jobs which fail to meet due date are 2.

02.
Sol: SPT – rule minimizes average flow time

<table>
<thead>
<tr>
<th>Job</th>
<th>T_i</th>
<th>C_i</th>
<th>D_i</th>
<th>C_i – D_i</th>
<th>T_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>15</td>
<td>-13</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>21</td>
<td>-17</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>7</td>
<td>17</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>11</td>
<td>12</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>15</td>
<td>24</td>
<td>-9</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>24</td>
<td>5</td>
<td>19</td>
<td>19</td>
</tr>
</tbody>
</table>

\[ \sum C_i = 63 \]
\[ \sum C_i – D_i = 49 \]
Mean Flow Time, \( \text{MFT} = \frac{63}{6} = 10.5 \)

Mean Tardiness, \( \text{MT} = \frac{19}{6} = 3.17 \)

No. of tardy job = 1

**EDD** – rule minimizes mean tardiness

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Job} & T_i & C_i & D_i & C_i - D_i & T_i \\
\hline
3 & 9 & 9 & 5 & 4 & 4 \\
4 & 4 & 13 & 12 & 1 & 1 \\
5 & 2 & 15 & 15 & -- & 0 \\
1 & 3 & 18 & 17 & 1 & 1 \\
2 & 2 & 20 & 21 & -1 & 0 \\
6 & 4 & 24 & 24 & 0 & 0 \\
\hline
\text{\( \sum C_i = 99 \)} & \text{\( \sum (C_i - D_i) = 6 \)} & & & & \\
\end{array}
\]

\[
\text{MFT} = \frac{\sum C_i}{n} = \frac{99}{6} = 16.5 \\
\text{MT} = \frac{\sum (C_i - D_i)}{n} = \frac{6}{6} = 1 \\
\]

\( T_i = \) Process Time, \( C_i = \) Completion Time

\( D_i = \) Due Date

No. of tardy job = 3

04. **Ans:** F-C-G-B-E-D-A

**Sol:** Calendar date required (CDR)

<table>
<thead>
<tr>
<th>Job</th>
<th>CDR</th>
<th>PT</th>
<th>( \text{Critical ratio} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>190</td>
<td>5</td>
<td>( \frac{(190-175)}{5} = 3 ) → Ahead of schedule</td>
</tr>
<tr>
<td>B</td>
<td>178</td>
<td>2</td>
<td>( \frac{(178-175)}{2} = 1.5 ) → Ahead of schedule</td>
</tr>
<tr>
<td>C</td>
<td>184</td>
<td>10</td>
<td>( \frac{(184-175)}{10} = 0.9 ) → Behind schedule</td>
</tr>
<tr>
<td>D</td>
<td>181</td>
<td>3</td>
<td>( \frac{(181-175)}{3} = 2 ) → Ahead of schedule</td>
</tr>
<tr>
<td>E</td>
<td>205</td>
<td>17</td>
<td>( \frac{(205-175)}{17} = 1.76 ) → Ahead of schedule</td>
</tr>
<tr>
<td>F</td>
<td>187</td>
<td>15</td>
<td>( \frac{(187-175)}{15} = 0.8 ) → Behind schedule</td>
</tr>
<tr>
<td>G</td>
<td>184</td>
<td>9</td>
<td>( \frac{(184-175)}{9} = 1 ) → on schedule</td>
</tr>
</tbody>
</table>

If critical ratio is one job will be on schedule.
If critical ratio is less than one job will be behind schedule.
If critical ratio is greater than one job will be ahead of schedule.

Note:

\( \text{Stack = Due Date (DD) – Processing time (P.T)} \)
05.
Sol:

<table>
<thead>
<tr>
<th>Job</th>
<th>T_j</th>
<th>F_j</th>
<th>D_j</th>
<th>L_j</th>
<th>T_j = max of (0, L_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>7</td>
<td>15</td>
<td>18</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>9</td>
<td>24</td>
<td>21</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>d</td>
<td>12</td>
<td>36</td>
<td>38</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>14</td>
<td>50</td>
<td>41</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>f</td>
<td>10</td>
<td>60</td>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(i) Make-span time = 60 days  
(ii) Mean flow time = \( \frac{\sum F_y}{n} = \frac{193}{6} = 32.16 \)  
(iii) No. of tardy jobs = 2 (c & e)  
(iv) Mean tardiness,  
\[ T = \frac{\sum T_j}{n} = \frac{12}{6} = 2 \]

06.
Sol: Sequence by Johnson’s Rule is:  
6, 3, 4, 1, 2, 5  

Minimum Make Span = 30

07.  
Sol: Optimum sequence:  
A B C E D

TABULAR METHOD:

<table>
<thead>
<tr>
<th>Job</th>
<th>M/C -I</th>
<th>M/C - II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T_0</td>
<td>T_0</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>E</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

Processing time: 27, 28
Idle time: 30–27 = 3, (30–28 = 2)

%utilization: 27/30 \times 100, 28/30 \times 100

GANTT CHART

08.
Sol: Optimum Sequence:  
A C D B E

PT = processing time
Minimum time for completion of all jobs = 31

09.

Sol: Condition : \( \text{Max} (t_{2j}) \leq \text{Min} (t_{ij} \text{ or } t_{3j}) \)
\[
4 \leq 4 \text{ or } 4
\]

Since the condition is satisfied, we can create two virtual Machines ‘G’ & ‘H’.
\( X = t_{1j}, M = t_{2j}, W = t_{3j} \)

<table>
<thead>
<tr>
<th>Comp</th>
<th>X</th>
<th>M</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>8</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>A</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Comp Machine X Machine M Idle Machine W Idle
<table>
<thead>
<tr>
<th>In</th>
<th>PT</th>
<th>Out</th>
<th>In</th>
<th>PT</th>
<th>Out</th>
<th>In</th>
<th>PT</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>L</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>4</td>
<td>13</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>O</td>
<td>9</td>
<td>7</td>
<td>16</td>
<td>16</td>
<td>3</td>
<td>19</td>
<td>3</td>
<td>22</td>
</tr>
<tr>
<td>N</td>
<td>16</td>
<td>8</td>
<td>24</td>
<td>24</td>
<td>3</td>
<td>27</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>E</td>
<td>24</td>
<td>6</td>
<td>30</td>
<td>30</td>
<td>4</td>
<td>34</td>
<td>3</td>
<td>34</td>
</tr>
</tbody>
</table>

Optimum sequence : A L O N E

Gantt Chart :
(iii) % utilization:

Machine X = \( \frac{30}{38} \times 100 = 78.94\% \)

Machine m = \( \frac{38 - 20}{38} \times 100 = 47.73\% \)

Machine W = \( \frac{38 - 8}{38} \times 100 = 78.94\% \)

10.
Sol: Optimum Sequence:

D C E F G B A

Machines Polish Idle

<table>
<thead>
<tr>
<th>In</th>
<th>PT</th>
<th>Out</th>
<th>In</th>
<th>PT</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>E</td>
<td>9</td>
<td>6</td>
<td>15</td>
<td>21</td>
<td>9</td>
</tr>
<tr>
<td>F</td>
<td>15</td>
<td>9</td>
<td>24</td>
<td>30</td>
<td>11</td>
</tr>
<tr>
<td>G</td>
<td>24</td>
<td>7</td>
<td>31</td>
<td>41</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>31</td>
<td>6</td>
<td>37</td>
<td>47</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
<td>37</td>
<td>10</td>
<td>47</td>
<td>50</td>
<td>2</td>
</tr>
</tbody>
</table>

Minimum flow time = 52

11.
Sol: The given machine sequence is ‘ACB’ hence, we need to re-arrange the given data

<table>
<thead>
<tr>
<th>Job</th>
<th>A</th>
<th>C</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Max \{ \( t_{2j} \) ≤ min\{\( t_{1j} \) or \( t_{3j} \)\} \}

5 ≤ 5 or 3

Optimum sequence 1

Machine G Machine H

5 4 3 2 1

Optimum sequence 2

Machine G Machine H

2 5 4 3 1
12.
Sol:

<table>
<thead>
<tr>
<th>Job</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In PT Out</td>
<td>In PT Out</td>
<td>In PT Out Idle</td>
</tr>
<tr>
<td>5</td>
<td>0 5 5</td>
<td>5 3 8</td>
<td>8 7 15 8</td>
</tr>
<tr>
<td>4</td>
<td>5 9 14</td>
<td>14 5 19</td>
<td>6 19 6 25 4</td>
</tr>
<tr>
<td>3</td>
<td>14 6 20</td>
<td>20 4 24</td>
<td>1 25 5 30 –</td>
</tr>
<tr>
<td>2</td>
<td>20 7 27</td>
<td>27 1 28</td>
<td>3 30 7 37 –</td>
</tr>
<tr>
<td>1</td>
<td>27 5 32</td>
<td>32 2 34</td>
<td>4 37 3 40 –</td>
</tr>
</tbody>
</table>

Job 1 is idle

21+3 = 24 → Job 2

21+2 = 23 → Job 1

- Job 1 is idle

- Job 2 is idle
01. Ans: (c)  
Sol: A no. of allocations : \( m + n - 1 \)  
\[ \Rightarrow 5 + 3 - 1 = 7 \]

02. Ans: (a)  
Sol: For degeneracy in transportations, number of allocations < \((m + n) - 1\)  
where \( m = \) no. of rows, \( n = \) no. of columns

03. Ans: (b)  
Sol: In Transportation problem for solving the initial feasible solution for total cost, Vogel’s approximation methods are employed for obtaining solutions which are faster than LPP due to the reduced number of equations for solving. Optimality is reached using MODI/ U-V method or stepping stone method.

04. Ans: (b)  
Sol: It generates the best initial basic feasible solution. This method is the best choice in order to get an optimal solution within minimum number of iterations. The Vogel’s approximation method is also known as the penalty method.

05. Ans: (a)  
Sol: No. of allocations = 5  
\( \therefore \) no. of allocations = \( m + n - 1 \)  
\[ m + n - 1 = 4 + 3 - 1 \]  
\( \therefore \) It is a degenerate solution

06. Ans: (a)  
Sol:  
\[
\begin{array}{cccc|c}
\text{A} & 
\begin{array}{ccccc}
10 & 2 & 20 & 11 & 15 \\
5 & 7 & 9 & 10 & 20 \\
12 & 10 & 15 & 20 & 25 \\
5 & 14 & 16 & 18 & 10 \\
\end{array} \\
\text{B} & \\
\text{C} & \\
\text{Demand} & 5 & 15 & 15 & 15 & 50 \\
\end{array}
\]

Evaluation of empty cells:  
Cell (A1) Evaluation = \( C_{A1} - C_{A4} + C_{C4} - C_{C1} \)  
\[ = 10 - 11 + 18 - 5 = 12 \]  
Cell (A3) Evaluation = \( C_{A3} - C_{A2} + C_{B2} - C_{B3} \)  
\[ = 20 - 9 + 7 - 2 = 16 \]  
Cell (B1) Evaluation = \( 12 - 7 + 2 - 11 + 18 - 4 = 10 \)  
Cell (B4) Evaluation = \( 20 - 7 + 2 - 11 = 4 \)  
Cell (C2) Evaluation = \( 14 - 2 + 11 - 18 = 5 \)  
Cell (C3) Evaluation = \( 16 - 9 + 7 - 2 - 18 = 5 \)  
If cell cost evaluation value is ‘\(-ve\)’, indicates further unit transportation cost is decreasing and if cost evaluation value is ‘\(+ve\)’ indicates further unit transportation cost is increases. If cost evaluation value is zero, unit transportation cost doesn’t change.
As for A3 cell cost evaluation is +16, means that, if we transport goods to A3 the unit transportation cost is increased by 16/-.

Common Data for Questions Q07, Q08 & Q09:

07. Ans: (b)  
08. Ans: (a)  
09. Ans: (b)

Sol:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>5</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>55</td>
<td>12</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

No. of allocations = 6  
\[R + C - 1 = 6\]

As No. of allocations = \(R + C - 1\)  
Hence the problem is not degeneracy case.

Opportunity cost of cell \((i, j)\) is  
\[C_{ij} - (U_i + V_j)\]

If \(C_{ij} - (U_i + V_j) \geq 0\) \(\Rightarrow\) problem is optimal.

Empty cell evaluation (or) Opportunity cost of cells:

- \(A_1 = -12\), \(A_2 = -19\), \(B_2 = -8\)
- \(B_4 = 12\), \(C_3 = 3\), \(C_4 = 12\)

From the above as A2 has opportunity cost ‘−19’ indicates unit transportation cost is decreased by 19/-.

By forming loop A2, A3, B2, B3 it is observed that to transport minimum quantity is 25 among 25, 30, 35.

\[\text{The reduction in the transportation cost is } 25 \times 19 = 475\]

10. Ans: (c)

Sol:

![Transportation Table]

By stepping stone method,  
Cell evaluation of B – 1 cell  
\[= +7 - 5 + 8 - 10 + 14 - 12\]
\[= 2\]

11. Ans: (c)

Sol: To find the number units shifted to A2 cell.

![Transportation Table]

\(0 = \text{minimum value of } |15-0, 20-0| = 0\)
\(0 = 15 \text{ units}\)
Conventional Practice Solutions

01.

Sol: Total supply = 80 + 60 + 40 + 20 = 200 & Total demand = 60 + 60 + 30 + 40 + 10 = 200

.: Total supply = Total demand

The problem is balanced

<table>
<thead>
<tr>
<th>Source</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Required</td>
<td>60</td>
<td>60</td>
<td>30</td>
<td>40</td>
<td>10</td>
<td>200</td>
</tr>
</tbody>
</table>

(i) By North West Corner rule:

<table>
<thead>
<tr>
<th>Source</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60</td>
<td>20</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>80 / 20 / 0</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>40</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>30</td>
<td>2</td>
<td>40 / 30 / 0</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>10</td>
<td>20 / 0</td>
</tr>
</tbody>
</table>

Demand 60 / 60 / 30 / 40 / 10

Total transportation cost = 4×60 + 3×20 + 2×40 + 3×20 + 6×10 + 3×30 + 5×10 + 3×10 = 670 /-

02.

Sol: Total supply = 14 + 16 + 5 = 35

Total demand = 6 + 10 + 15 + 4 = 35

.: Total supply = Total demand

It is a balanced transportation model
(i) By North West corner rule

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>15</td>
<td>22</td>
<td>26</td>
<td>25</td>
<td>14/8/0</td>
</tr>
<tr>
<td>W2</td>
<td>36</td>
<td>38</td>
<td>18</td>
<td>40</td>
<td>16/14/0</td>
</tr>
<tr>
<td>W3</td>
<td>45</td>
<td>35</td>
<td>60</td>
<td>52</td>
<td>5/0</td>
</tr>
</tbody>
</table>

Demand

/ 0 / 2 / 1 / 0 / 35
/ 0 / 0

Transportation cost = 15 \times 6 + 22 \times 8 + 26 \times 2 + 25 \times 14 + 38 \times 1 + 52 \times 4 = 862 /-

(ii) Least Cost Method:

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>15</td>
<td>22</td>
<td>26</td>
<td>25</td>
<td>14/8/0</td>
</tr>
<tr>
<td>W2</td>
<td>36</td>
<td>38</td>
<td>18</td>
<td>40</td>
<td>16/1/0</td>
</tr>
<tr>
<td>W3</td>
<td>45</td>
<td>35</td>
<td>60</td>
<td>52</td>
<td>5/3/0</td>
</tr>
</tbody>
</table>

Demand

/ 0 / 2 / 1 / 0 / 35
/ 0 / 0

Transportation cost = 15 \times 6 + 22 \times 8 + 18 \times 15 + 40 \times 1 + 35 \times 2 + 52 \times 3 = Rs. 802 /-

(iii) VAM

**Step 1:** Find out the difference between least and next highest numbers for rows and columns. Which is called as the penalty.

**Step 2:** Select the maximum penalty row and column and allocate the maximum possible amount to the box with least cost.

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>15</td>
<td>22</td>
<td>26</td>
<td>25</td>
<td>14/8/3/0</td>
</tr>
<tr>
<td>W2</td>
<td>36</td>
<td>38</td>
<td>18</td>
<td>40</td>
<td>16/1/0</td>
</tr>
<tr>
<td>W3</td>
<td>45</td>
<td>35</td>
<td>60</td>
<td>52</td>
<td>5/0</td>
</tr>
</tbody>
</table>

Demand

/ 0 / 2 / 1 / 0 / 35
/ 0 / 0

Transportation cost = 15 \times 6 + 22 \times 5 + 25 \times 3 + 18 \times 15 + 40 \times 1 + 35 \times 5 = 760 /-
Chapter 9  Assignment Model

01. Ans: (a)
Sol: Let $C_{ij} = \text{unit assignment cost}$

$X_{ij} = \text{Decision variable (allocation)}$

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to:

$\sum_{j=1}^{n} X_{ij} = 1$

$\sum_{i=1}^{n} X_{ij} = 1$

$X_{ij} = 1 \text{ (when assigned)}$

$X_{ij} = 0 \text{ (when not assigned)}$

- Number of decision variables = $n^2$ (or) $m^2$
- Number of basic variables = Number of assignments = $n$ (or) $m$

02. Ans: (c)

03. Ans: (a)

04. Ans: (c)
Sol:

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>110</td>
<td>120</td>
<td>130</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Q$</td>
<td>115</td>
<td>140</td>
<td>140</td>
<td>0</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>$R$</td>
<td>125</td>
<td>145</td>
<td>165</td>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

P-S$_2 = 120$
Q-S$_3 = 140$
R-S$_1 = 125$
Total = 385

05. Ans: (1-B, 2-D, 3-C, 4-A)
Sol: Step-1:

Take the row minimum of subtract it from all elements of corresponding row.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Step – 2:

Take the column minimum & subtract it from all elements of corresponding column.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Step – 3:

Select single zero row or column and assign at the all where zero exists. If there is no single zero row or column. Then use straight line method.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
1 – B : 7  
2 – D : 8  
3 – C : 2  
4 – A : 5  
Total cost = 22

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>9</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>11</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

**Step – 1 :**

\[
\begin{array}{cccc}
5 & 0 & 10 & 8 \\
0 & 6 & 5 & 15 \\
8 & 5 & 0 & 1 \\
0 & 6 & 2 & 4 \\
\end{array}
\]

**Step – 2 :**

\[
\begin{array}{cccc}
5 & 0 & 10 & 7 \\
0 & 6 & 5 & 14 \\
8 & 5 & 0 & 0 \\
0 & 6 & 2 & 3 \\
\end{array}
\]

**Step – 3**

It may be noted there are no remaining zeroes and row – 4 and column – 4 each has no assignment. Thus optimal solution is not reached at this stage. Therefore, proceed to following important steps.

**Step – 4 :**

Draw the minimum number of horizontal and vertical lines necessary to cover all zeroes at least once.

Take the above Table

\[
\begin{align*}
J_1 & : 0 \quad 10 \quad 7 \\
J_2 & : 0 \quad 6 \quad 5 \quad 14 \\
J_3 & : 0 \quad 5 \quad 0 \quad 0 \\
J_4 & : 0 \quad 6 \quad 2 \quad 3 \\
\end{align*}
\]

(i) Mark row – 4 in which there is no assignment  
(ii) Mark column 1 which have zeroes in marked column.  
(iii) Next mark row 2 because this row contains assignment in marked column 1.  
No further rows or columns will be required to mark during this procedure.  
(iv) Draw the required lines as follows.  
(a) Draw L₁ through marked column 1  
(b) Draw L₂ and L₃ through unmarked row (1 and 3)
Step – 5 :
Select the smallest element (2).
Among all the uncovered elements of the above table and substract this value from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines L₁, L₂ and L₃ and leaving the remaining element unchange.

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>7</td>
<td>0</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>C₂</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>C₃</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C₄</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

It may be added that there are no remaining zeroes and every row and column has an assignment.
Since, the no. of assignment = no. of row or column
∴ The solution is optimal

The pattern of assignment at which job has been assigned to each contractor.

<table>
<thead>
<tr>
<th>Contractor</th>
<th>Job</th>
<th>Amount (Rs)×1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>J₂</td>
<td>5</td>
</tr>
<tr>
<td>C₂</td>
<td>J₁</td>
<td>3</td>
</tr>
<tr>
<td>C₃</td>
<td>J₄</td>
<td>3</td>
</tr>
<tr>
<td>C₄</td>
<td>J₃</td>
<td>7</td>
</tr>
</tbody>
</table>

18×1000=18000

Minimum amount = Rs. 18,000/-
03.

Sol: Here no. of rows ≠ no. of column

\[ \therefore \text{The algorithm is not balanced so add one dummy column.} \]

<table>
<thead>
<tr>
<th>Operates</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

Step – 1:

\[ \begin{array}{cccc}
9 & 26 & 15 & 0 \\
13 & 27 & 6 & 0 \\
35 & 20 & 15 & 0 \\
18 & 30 & 20 & 0 \\
\end{array} \]

Here the operator – 4 is assigned to dummy column.

\[ \therefore \text{He is the idle worker.} \]

TC = 9 + 6 + 20 + 0 = 35
01. Ans: (d) 02. Ans: (b)

03. Ans: (b)
Sol:

<table>
<thead>
<tr>
<th>Months</th>
<th>Month 1</th>
<th>Month 2</th>
<th>Month 3</th>
<th>Unused capacity</th>
<th>Capacity Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 RT</td>
<td>90</td>
<td>10</td>
<td>22</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>1 OT</td>
<td>24</td>
<td>24</td>
<td>28</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>2 RT</td>
<td>100</td>
<td>20</td>
<td>22</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>2 OT</td>
<td>20</td>
<td>24</td>
<td>26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 RT</td>
<td>80</td>
<td></td>
<td>20</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>3 OT</td>
<td></td>
<td></td>
<td>30</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>RT</td>
<td>90</td>
<td>130</td>
<td>110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Level of planned production in overtimes in 3\textsuperscript{rd} period is ‘30’.
RT = Regular time
OT = Over time
### 04. Ans: (b)  
Sol:

<table>
<thead>
<tr>
<th>Month</th>
<th>Cumulative Production</th>
<th>Cumulative Demand</th>
<th>Inventory</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>End Inventory</td>
<td>Stock out cost</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>80</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>180</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>260</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
<td>300</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total</td>
<td>180</td>
</tr>
</tbody>
</table>

### 05. Ans: (b)  
06. Ans: (d)

**Conventional Practice Solutions**

### 01. Ans: (b)  
Sol:

<table>
<thead>
<tr>
<th>Supply from</th>
<th>Demand for</th>
<th>Total Capacity Available (supply)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning inventory</td>
<td>Period 1</td>
<td>Period 2</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>1 Regular</td>
<td>700</td>
<td>60</td>
</tr>
<tr>
<td>1 Overtime</td>
<td>70</td>
<td>75</td>
</tr>
<tr>
<td>2 Regular</td>
<td>500</td>
<td>60</td>
</tr>
<tr>
<td>2 Overtime</td>
<td>70</td>
<td>75</td>
</tr>
<tr>
<td>3 Regular</td>
<td>200</td>
<td>60</td>
</tr>
<tr>
<td>3 Overtime</td>
<td>70</td>
<td>200</td>
</tr>
<tr>
<td>4 Regular</td>
<td>700</td>
<td>60</td>
</tr>
<tr>
<td>4 Overtime</td>
<td>300</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>900</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total cost = (700 × 60) + (500 × 60) + (200 × 70) + (200 × 60) + (500 × 65) + (200 × 75)  
+ (700 × 60) + (300 × 70) = Rs 2,08,500/-
## 02. Ans:

**Sol:**

<table>
<thead>
<tr>
<th>Supply from</th>
<th>Demand for</th>
<th>Total Capacity Available (supply)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period1</td>
<td>Period2</td>
</tr>
<tr>
<td>Beginning Inventory</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td>Regular</td>
<td>900</td>
<td>25</td>
</tr>
<tr>
<td>Overtime</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>Subcontract</td>
<td>200</td>
<td>35</td>
</tr>
<tr>
<td>Regular</td>
<td>660</td>
<td>25</td>
</tr>
<tr>
<td>Overtime</td>
<td>125</td>
<td>30</td>
</tr>
<tr>
<td>Subcontract</td>
<td>175</td>
<td>35</td>
</tr>
<tr>
<td>Regular</td>
<td>700</td>
<td>25</td>
</tr>
<tr>
<td>Overtime</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td>Subcontract</td>
<td>35</td>
<td>-</td>
</tr>
<tr>
<td>Regular</td>
<td>800</td>
<td>25</td>
</tr>
<tr>
<td>Overtime</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>Subcontract</td>
<td>250</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>1400</td>
<td>900</td>
</tr>
</tbody>
</table>

Total cost = (900 \times 25) + (150 \times 30) + (200 \times 35) + (600 \times 25) + (125 \times 30) + (175 \times 35) + (700 \times 25) + (100 \times 30) + (50 \times 32) + (800 \times 25) + (200 \times 30) + (250 \times 35) = Rs 1,15,725/-
01. Ans: (b)

02. Ans: (c)
Sol: Based on master production schedule, a material requirements planning system:
- Creates schedules, identifying the specific parts and materials required to produce end items.
- Determines exact unit numbers needed.
- Determines the dates when orders for those materials should be released, based on lead times.

03. Ans: (d)
Sol: Refer to the solution of Q.No. 02

04. Ans: (c)
Sol: MRP has three major input components:
1. Master production Schedule of end items required. It dictates gross or projected requirements for end items to the MRP system.
2. Inventory status file of on-hand and on-order items, lot sizes, lead times etc.
3. Bill of materials (BOM) or Product structure file what components and sub assemblies go into each end product.

05. Ans: (c)

06. Ans: (c)

07. Ans: (b)

08. Ans: (b)

09. Ans: (c)
Sol:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>Q</td>
<td>T</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LT=2</td>
<td>LT=3</td>
<td>LT=10</td>
<td>LT=6</td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LT=5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Maximum Lead time = 12 weeks

Conventional Practice Solutions

01. Sol: A → 1 × 10 = 10
B → 2 × 10 = 20
C → (1 × 2 × 10) + (3 × 4 × 2 × 10) = 260
D → (4 × 2 × 10) = 80
E → (3 × 4 × 2 × 10) + (2 × 2 × 10) + (4×10) = 320
02.
Sol:

<table>
<thead>
<tr>
<th>Order Quantity = 200</th>
<th>Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT = 3 Weeks</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Project required</td>
<td>40</td>
</tr>
<tr>
<td>Receipts</td>
<td>85</td>
</tr>
<tr>
<td>On hand inventory</td>
<td>10</td>
</tr>
<tr>
<td>Planned order release</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
</tbody>
</table>

(On hand inventory),
1st week = 140 + 0 – 40 = 100
3rd week = 15 + 0 – 10 = 5
5th week = 145 + 0 – 130 = 15
7th week = 105 + 0 – 50 = 55
∴ Order before 3-weeks

03.
Sol:
A = (1 \times 100) = 100
B = (1 \times 100) = 100
C = (1 \times 100) = 100
D = (2 \times 1 \times 100) = 200
E = (2 \times 1 \times 100) = 100
F = (1 \times 1 \times 100 + 1 \times 1 \times 100) = 200
G = (1 \times 1 \times 100) = 100
H = (3 \times 1 \times 100) = 300
J = (2 \times 1 \times 100 + 2 \times 2 \times 1 \times 100) = 600
K = (1 \times 2 \times 1 \times 100) = 200

**Chapter 12**

**Break Even Analysis**

01. **Ans:** (c)

**Sol:**
- Total fixed cost, TFC = Rs 5000/-
- Sales price, SP = Rs 30/-
- Variable cost, VC = Rs 20/-

Break even production per month,

\[
Q^* = \frac{TFC}{SP - VC} = \frac{5000}{30 - 20} = 500 \text{ units}
\]

02. **Ans:** (a)

**Sol:**
- Total cost = 20 + 3X \quad \quad \quad \quad \quad \quad \quad (1)
- Total cost = 50 + X \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (2)

By solving eq. (1) and (2)

\[2X = 30\]
\[\therefore \quad X = 15 \text{ units}\]

When X = 10 units

- TC_1 = 20 + (3 \times 10) = Rs 50/-
- TC_2 = 50 + (1 \times 10) = Rs 60/-

Among both, total cost for process is less

So process-1 is choose.

03. **Ans:** (c)

**Sol:**
In automated assembly there are less labour, so variable cost is less, but fixed is more because machine usage is more. In job shop production, labour is more but machine is less. So variable cost is more and fixed cost is less.
### 04. Ans: (c)
**Sol:**
\[
\text{TC} = \text{Total cost} \\
TCA = \text{Total cost for jig-A} \\
TCB = \text{Total for jig-B} \\
TCA = TCB \\
800 + 0.1X = 1200 + 0.08X \\
0.02X = 400 \\
\therefore X = \frac{400}{0.02} = 20,000 \text{ units}
\]

### 05. Ans: (d)
**Sol:**
Sales price – Total cost = Profit
\[
(CP \times 14000) - (47000 + 14000 \times 15) = 23000 \\
\therefore CP = 20
\]

### 06. Ans: (b)
### 07. Ans: (a)
### 08. Ans: (c)
### 09. Ans: 1500
**Sol:**
\[
X \quad Y \\
S_1 = 100 \quad S_2 = 120 \\
F_1 = 20,000 \quad F_2 = 8000 \\
V_1 = 12 \quad V_2 = 40 \\
\]
\[
P = q(S - V) - F \\
P_1 = q(100 - 12) - 20,000 \\
P_2 = q(120 - 40) - 80,000 \\
P_1 = P_2 \\
88q - 20,000 = 80q - 80,000 \\
12000 = 8q \\
\Rightarrow q = 1500
\]

### 10. Ans: (b)
### 11. Ans: (c)
**Sol:**
At breakeven point
Total cost = Total revenue
\[
FC + VC \times Q = SP \times Q \\
Q = \frac{FC}{(SP - VC)} \\
FC = 1000/-, \quad VC = 3/-, \quad SP = 4/- \\
Q = \frac{1000}{(4 - 3)} = 1000 \text{ units}
\]
If sales price is increased to 25%
\[
SP = 4 + \frac{1}{4} \times 4 = 5/- \\
Q' = \frac{1000}{(5 - 3)} = 500 \text{ units} \\
\therefore \text{Breakeven quantity decreases by} \\
\frac{100 - 500}{100} \times 100 = 50\%
\]

### Conventional Practice Solutions

<table>
<thead>
<tr>
<th>Sol:</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{Standard machine tool}</td>
</tr>
<tr>
<td>\text{F}_1 = \text{F.C.}</td>
</tr>
<tr>
<td>\frac{30}{60} \times 200 = \text{Rs.100}</td>
</tr>
<tr>
<td>\text{V.C}</td>
</tr>
</tbody>
</table>

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q = \frac{1600 - 100}{73.33 - 66.67} = 225 \text{ volts}

If greater than 225 units then automatic machine tool is economic.

02. Ans: 16
Sol: Preparation cost for
Conventional lathe = 30,
CNC lathe = 150
Production time of
Conventional lathe = 30 min,
Variable cost per hour
Conventional lathe = 75 per hour
\[ = 75 \times 30 \text{ per product} \]
CNC lathe = 120 per hour
\[ = 120 \times 15 \text{ per product} \]
Total cost for Q products
Conventional lathe = 30 + 37.5 Q
CNC lathe = 150 + 30 Q
At break even quantities
\[(TC)_1 = (TC)_2\]
\[\Rightarrow 30 + 37.5 Q = 150 + 30 Q\]
\[\Rightarrow 7.5 Q = 120\]
\[\Rightarrow Q = 16\]
\[\therefore \text{ CNC lathe is economical when production per day is above 16.}\]
03. Ans: (c)
Sol: Assembly line balancing:
Line balancing is done to meet the production rate for a given time, minimizing the idle time and maximizing the work output. As the time is minimized, the idle time at the stations decreases, decreasing the in-process inventory.
Statements 1, 3, 4 apply to the benefits of assembly line balancing.

04. Ans: (c)
Sol: Cycle Time = \( \frac{\text{Total time}}{\text{Total production}} \)
= \( \frac{8 \times 60}{320} = 1.5 \text{ min} \)

Time to assemble one unit
= \( 1.3 + 1.5 + 1.4 + 1.5 + 1.3 = 7 \text{ min} \)

No. of work station
= \( \frac{\text{Time to assemble one unit}}{\text{CycleTime}} \)
= \( \frac{7}{1.5} = 4.67 \approx 5 \)

\( \eta = \frac{\text{Time to assemble one unit}}{\text{No.of work stations} \times \text{CycleTime}} \)
= \( \frac{7}{5 \times 1.5} = 0.93 \)

05. Ans: (d)
Sol: Cycle Time = \( \frac{480 \times 60}{1450} = 19.87 \text{ sec} \)
No. of work station = \( \frac{310}{19.87} = 15.6 \approx 16 \)
\( \eta = \frac{310}{16 \times 19.87} \times 100 = 97.5\% \)

06. Ans: (a)
Sol: Cycle time is equal to the time of the bottleneck operation or the maximum station time.

### Conventional Practice Solutions

<table>
<thead>
<tr>
<th>Work stations</th>
<th>Work elements</th>
<th>Work element times resp</th>
<th>Total time/w,</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A, B</td>
<td>4, 3</td>
<td>7</td>
</tr>
<tr>
<td>II</td>
<td>C</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>III</td>
<td>D, F</td>
<td>4, 4</td>
<td>8</td>
</tr>
<tr>
<td>IV</td>
<td>E</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>V</td>
<td>G</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>VI</td>
<td>H</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Cycle time = 8 minutes
No. of unit produced = \( \frac{60 \times 8}{8} = 60 \)
Line efficiency $\eta = \frac{40}{8 \times 6} \times 100 = 23.33$

If cycle time = 11 min
By combining G&H ⇒ no. of work station can be reduced by two.

**Sol:**
Available time for production
$$= 5 \times 7 \times 3600 \text{ sec.}$$
No. of units produced = 8400 units
Cycle time $C = \frac{\text{Available time for production}}{\text{No. of units produced}}$
$$= \frac{5 \times 7 \times 3600}{8400} = 15 \text{ sec/unit}$$
Time to assemble one unit = 130 sec (sum of all elements operation times)
Theoretical no. of work stations
$$= \frac{a}{c} = \frac{130}{15} = 8.67 \approx 9$$
Theoretical efficiency
$$= \frac{\text{Time to assemble one unit}}{\text{cycle time} \times \text{theoretical no. of work stations}} \times 100$$
$$= \frac{130}{9 \times 15} \times 100 = 96.29\%$$

Theoretical balance delay = $100 - \eta_{th}$
$$= 100 - 96.29$$
$$= 3.71\%$$

Technological precedence diagram:

Actual no. of work stations are 10
Actual efficiency $\eta = \frac{\text{Time to assemble one unit}}{\text{Actual no. of work station} \times \text{cycle time}} \times 100$
$$= \frac{130}{10 \times 15} \times 100 = 86.67\%$$
Actual balance delay = $100 - \eta_{act}$
$$= 100 - 86.67 = 13.33\%$$

<table>
<thead>
<tr>
<th>Work Station</th>
<th>Element allotted</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>a</td>
<td>14</td>
</tr>
<tr>
<td>II</td>
<td>b</td>
<td>10</td>
</tr>
<tr>
<td>III</td>
<td>b, e</td>
<td>8</td>
</tr>
<tr>
<td>IV</td>
<td>c</td>
<td>15</td>
</tr>
<tr>
<td>V</td>
<td>c</td>
<td>15</td>
</tr>
<tr>
<td>VI</td>
<td>f</td>
<td>13</td>
</tr>
<tr>
<td>VII</td>
<td>g</td>
<td>14</td>
</tr>
<tr>
<td>VIII</td>
<td>h</td>
<td>14</td>
</tr>
<tr>
<td>IX</td>
<td>i, j</td>
<td>13</td>
</tr>
<tr>
<td>X</td>
<td>k, l, m</td>
<td>14</td>
</tr>
</tbody>
</table>
Smoothness index (SI) = \sqrt{\frac{1}{n} \sum (T - T_i)^2}

Where T = cycle time
T_i = Time allotted to the highest work station

\[ SI = \sqrt{(15-14)^2 + (15-10)^2 + (15-8)^2 + (15-15)^2 + (15-15)^2 + (15-13)^2 + (15-14)^2 + (15-13)^2 + (15-14)^2} \]

\[ SI = \sqrt{1 + 25 + 289 + 4 + 1 + 1 + 4 + 1} \]

SI = \sqrt{326}

If SI is zero then it indicates 100% line efficiency

03.
Sol:
(i)

(ii) Given,
Available production Time, T = 8 hours
No. of units to be produced, N = 400 units
Cycle time = \frac{T}{N} = \frac{(8 \times 60) - 40}{400 \text{ units}}
= 1.1 \text{ min/unit station}

The sum of the work element times
= 1.1 + 0.4 + 0.5 + 1.1 + 0.3 + 0.4 + 3.2 + 0.8 + 0.7 + 0.3
= 8.8 = 528 \text{ sec/units}

Theoretical no. of work stations
\[ \frac{\sum t_i}{\text{Cycle time}} = \frac{528 \text{ sec/unit}}{72 \text{ sec/unit – station}} \]
\[ = 7.33 \approx 8 \text{ stations} \]

(iii) Cycle time = 72 sec/unit – station
\[ = 1.2 \text{ min/unit-station} \]
No. of work stations = 8

04.
Sol:
(a) By using Kil bridge and Wester method:
For all line balancing problems, we use activity one node.

Stage-1 Stage-2 Stage-3 Stage-4 Stage-5 Stage-6 Stage-7
1 5 2 2 2 5 7
10 10 4 7 5 7 7
6 6 5 8 8 10 10

WS = work station

Given cycle time = 10 minutes

\[ \frac{10 \text{ minutes}}{WS-i} = \frac{10}{WS-1} = \frac{2+7=9}{WS-2} = \frac{5+2=7}{WS-3} = \frac{5+2=7}{WS-4} = \frac{10}{WS-5} \]

\[ \frac{5}{WS-6} = \frac{7}{WS-7} \]}
Line efficiency, \( \eta_{\text{line}} = \frac{\text{Total work time}}{n \times \text{cycle time}} \times 100 \)

\[
= \frac{5 + 10 + 5 + 2 + 7 + 5 + 2 + 10 + 5 + 7}{7 \times 10} \times 100
\]

\[
= \frac{58}{70} \times 100 = 82.85\%
\]

<table>
<thead>
<tr>
<th>Work station</th>
<th>Idle time</th>
<th>((\text{Idle time})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10–10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>10–10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>10–9 = 1</td>
<td>1</td>
</tr>
<tr>
<td>IV</td>
<td>10–7 = 3</td>
<td>9</td>
</tr>
<tr>
<td>V</td>
<td>10–10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>10–5 = 5</td>
<td>25</td>
</tr>
<tr>
<td>VII</td>
<td>10–7 = 3</td>
<td>9</td>
</tr>
</tbody>
</table>

\[\sum(\text{Idle time})^2 = 44\]

\[\therefore \text{Smoothing index} = \sqrt{44} = 6.63\]

(b) Let’s use Helgeron and Birnie method, which is also called as ranked position method

<table>
<thead>
<tr>
<th>Element</th>
<th>Positional weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>

Arrange the elements in the decreasing order

<table>
<thead>
<tr>
<th>Element</th>
<th>Positional weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

Line balance efficiency

\[
(\eta_{\text{line}}) = \frac{10 + 10 + 7 + 10 + 9 + 5 + 7}{7 \times 10} \times 100
\]

\[
= \frac{58}{70} \times 100 = 82.85\%
\]
## Workstation Idle Time Table

<table>
<thead>
<tr>
<th>Workstation</th>
<th>Idle Time</th>
<th>(Idle Time)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10 – 10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>10–10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>10–7 = 3</td>
<td>9</td>
</tr>
<tr>
<td>IV</td>
<td>10–10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>10–9 = 1</td>
<td>1</td>
</tr>
<tr>
<td>VI</td>
<td>10–5 = 5</td>
<td>25</td>
</tr>
<tr>
<td>VII</td>
<td>10–7 = 3</td>
<td>9</td>
</tr>
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</table>

\[ \sum (\text{idle time})^2 = 44 \]

\[ \therefore \text{Smoothing Index} = \sqrt{44} = 6.63 \]