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MECHANICAL ENGINEERING

IM & OR

Text Book : Theory with worked out Examples and Practice Questions

IM & OR

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 PERT & CPM

01. Ans: (a)

Sol: CPM deals with deterministic time durations.

02. Ans: (a)

Sol: Critical Path :

- It is a longest path consumes maximum amount of resources
- It is the minimum time required to complete the project

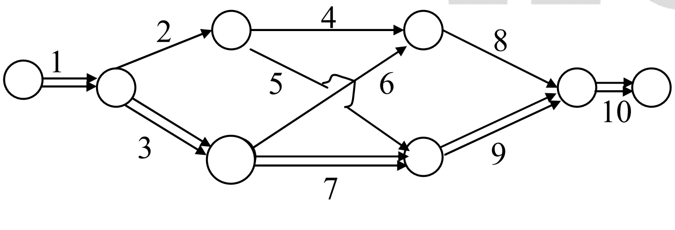
03. Ans: (a)

04. Ans: (a)

Sol: Gantt chart indicates comparison of actual progress with the scheduled progress.

05. Ans: (c)

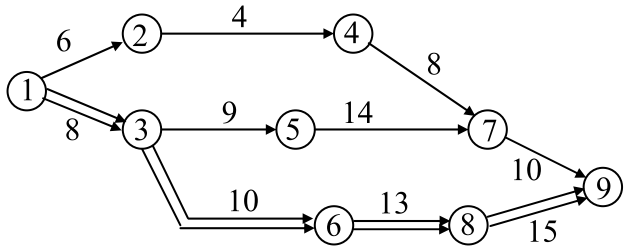
Sol:



Critical path = 1 + 3 + 7 + 9 + 10 = 30 days

06. Ans: (c)

Sol:



Critical path (1-3-6-8-9) = 8 + 10 + 13 + 15
= 46 days

07. Ans: (b)

Sol: Rules for drawing Network diagram:

- Each activity is represented by one and only one arrow in the network.
- No two activities can be identified by the same end events.
- Precedence relationships among all activities must always be maintained.
- No dangling is permitted in a network.
- No Looping (or Cycling) is permitted.

08. Ans: (b)

Sol: **Activity:** Resource consuming and well-defined work element.

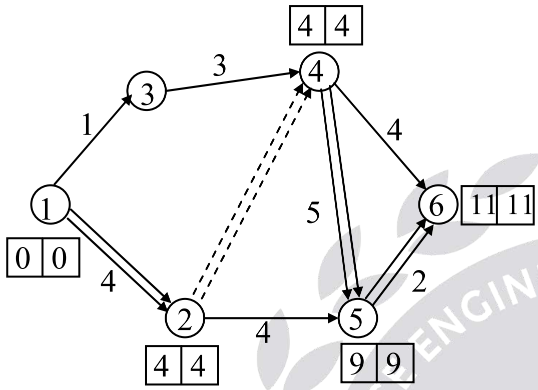
Event: Each event is represented as a node in a network diagram and it does not consume any time or resource.

Dummy Activity: An activity does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.

Float: Permissible delay period for the activity.

09. Ans: (b)

Sol:



10. Ans: (a)

11. Ans: (b)

Sol:

- Beta Distribution is used to decide the expected duration of an activity.
- The expected duration of the project can be described by Normal distribution.

12. Ans: (b)

Sol: $T_0 = 8$ min, $T_m = 10$ min, $T_p = 14$ min,

$$T_e = \frac{T_o + 4T_m + T_p}{6}$$

$$= \frac{8 + 4 \times 10 + 14}{6} = \frac{62}{6} = 10.33 \text{ min}$$

13. Ans: (a)

Sol: Take 4-3, $T_e = 6$ days

Critical path = 1-2-4-3

= 5 + 14 + 4 = 23 days

$$\sigma_{\text{critical path}} = \sqrt{V_{1-2} + V_{2-4} + V_{4-3}}$$

$$= \sqrt{2^2 + 2.8^2 + 2^2} = 3.979$$

$$z = \frac{\text{Due date} - \text{critical path duration}}{\sigma_{\text{critical path}}}$$

$$z = \frac{27 - 23}{3.979} = 1.005$$

$\therefore$ P(z) = 0.841

14. Ans: (b)

15. Ans: (c)

Sol: D = 36 days, V = 4 days

$$Z = \frac{36 - 36}{\sqrt{4}} = 0$$

$\Rightarrow$ P(z) = 50%

16. Ans: (c)

Sol: $\sigma_{cp} = \sqrt{V_{a-b} + V_{b-c} + V_{c-d} + V_{d-e}}$

$$= \sqrt{4 + 16 + 4 + 1} = 5$$

17. Ans: (a)

Sol: The latest that an activity can start from the beginning of the project without causing a delay in the completion of the entire project. It is the maximum time up to which an activity can be delayed to start without effecting the project completion duration time. (LST = LFT – duration).

18. Ans: (c)

Sol: The earliest expected completion time,

Critical path : A-B-C-D-F-E-H

$\Rightarrow 5 + 4 + 8 + 5 + 8 = 30$ days

19. Ans: (d)

Sol: *Critical path :*

1-3-4-6 = 20 days

$$z = \frac{24-20}{\sqrt{4}} = \frac{4}{2} = 2$$

$\Rightarrow P(z) = 97.7\%$

20. Ans: (d)

Sol: $$\text{Variance} = \left(\frac{t_p - t_o}{6}\right)^2$$

$$= \left(\frac{22-10}{6}\right)^2 = 4$$

21. Ans: (a)

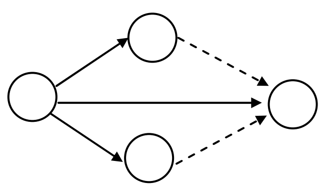
22. Ans: (b)

23. Ans: (a)

24. Ans: (b)

25. Ans: (c)

Sol:

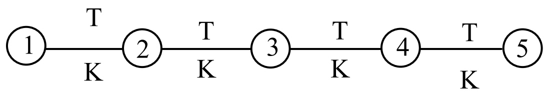


26. Ans: (c)

27. Ans: (b)

28. Ans: (d)

Sol:



Given each activity having time mean duration 'T' and standard deviation 'K'.

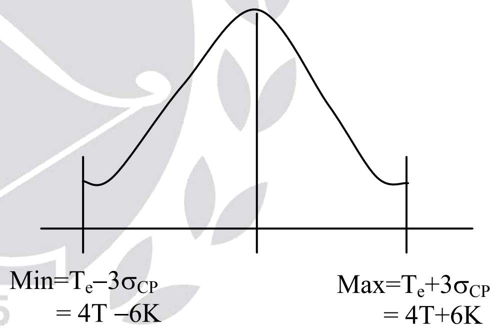
Total time estimate $T_e = 4T$

Variance of the path

$(\Sigma var)_{CP} = R^2 + R^2 + R^2 + R^2$

$= 4R^2$

Standard deviation of CP= $\sqrt{\sum (var)_{CP}}$



$\sigma_{CP} = \sqrt{4K^2}$

$\sigma_{CP} = \pm 2K$

Range of overall project duration likely to be in 4T + 6K and 4T – 6K

i.e., $4T \pm 6K$

Common solutions for Q.29 & Q.30

29. Ans: (b)

30. Ans: (b)

Sol:

Paths	Duration
1-2-4-5 = (AEF)	8+9+6=23
1-2-3-4-5=(ADF)	8+9+6=23
1-3-4-5 (BDF)	6+9+6 = 21
1-4-5 (CF)	16+6=22

∴ Highest time taken paths are AEF and ADF

∴ Critical path's are AEF and ADF

Critical paths are '2'.

Possible cases to crash

A by 1 day that cost = 80

F by 1 day that cost = 130

E and D by 1 day that cost = 20 + 40 = 60

31. Ans: (c)

32. Ans: (c)

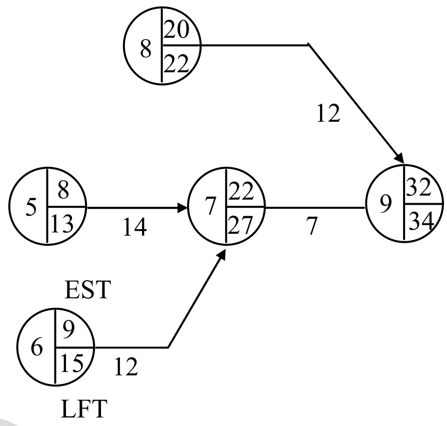
Sol:

Path	Duration
AB	7+5 = 12
CD	6+6 = 12
EF	8+4 = 12

Three critical paths, number of activities to be crashed are 3.

33. Ans: (c)

Sol:



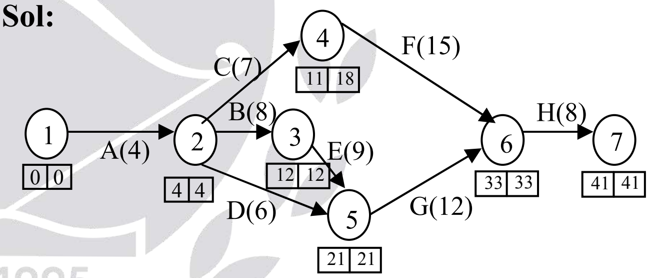
$(\text{Total Float})_{6\text{-}7} = 27 - 9 - 12 = 6$

$(\text{Free float})_{6\text{-}7} = 28 - 9 - 12 = 1$

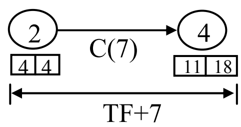
Conventional Practice Solutions

01. Ans: (a-7, b-41)

Sol:



Path	**duration**
1-2-4-6-7	= 4 + 7 + 15 + 8 = 34
1-2-3-5-6-7	= 4 + 8 + 9 + 12 + 8
	= **41 (days) (critical path)**
1-2-5-6-7	= 4 + 6 + 12 + 8 = 30



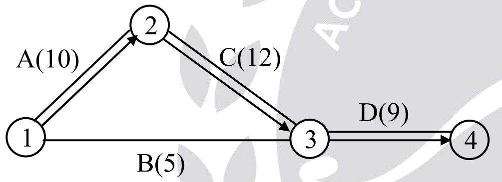
TF + 7 = 18 – 4

⇒ TF = 14 – 7 = **7**

02. Ans: 31 days

Sol:

Activity	Time estimated	Standard deviation
	$T_e = \frac{T_o + 4T_m + T_p}{6}$	$\sigma = \frac{T_p - T_o}{6}$
A	$\frac{5+4\times10+15}{6}=10$	$\frac{15-5}{6}=\frac{5}{3}$
B	$\frac{2+4\times5+8}{6}=5$	$\frac{8-2}{6}=1$
C	$\frac{10+4\times12+14}{6}=12$	$\frac{14-10}{6}=\frac{2}{3}$
D	$\frac{6+4\times8+16}{6}=9$	$\frac{16-6}{6}=\frac{5}{3}$



Critical path :

1-2-3-4 = 10 + 12 + 9 = 31 days

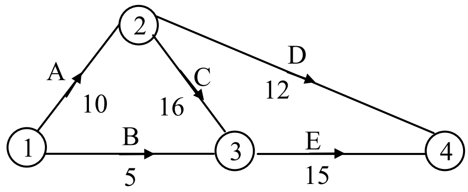
$$\sigma_{cp} = \sqrt{V_{1-2} + V_{2-3} + V_{3-4}}$$

$$= \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{6}$$

03.

Sol:

Paths	Duration
AD	22
ACE	41←CP
BE	20



$$(\text{Var})_{cp} = (\text{Var})_A + (\text{Var})_C + (\text{Var})_E$$

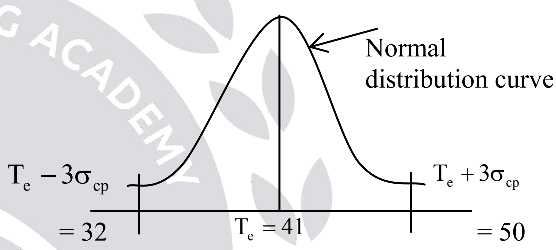
$$= \sigma_A^2 + \sigma_C^2 + \sigma_E^2$$

$$= 2^2 + 2^2 + 1^2 = 4 + 4 + 1 = 9$$

$$\sigma_{CP} = \sqrt{(\text{Var})_{CP}} = \sqrt{9} = 3$$

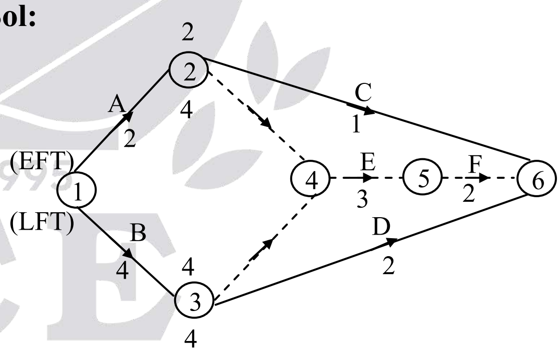
Minimum completion time = 32 days

Maximum completion time = 50 days



04.

Sol:



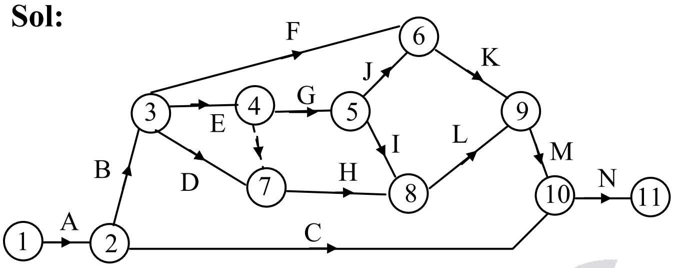
Paths	Duration
1-2-6 → AC	2 + 1 = 3
1-2-4-5-6→ AEF	2+3+2 = 7
1-3-6→ BD	4+2 = 6
1-3-4-5-6 → BEF	4+3+2 = 9

Highest Duration is '9'.

∴ CP is BEF

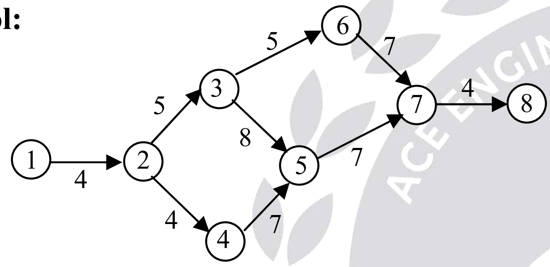
05.

Sol:



06.

Sol:



Paths	Normal duration
1-2-3-6-7-8	25
1-2-3-5-7-8	28
1-2-4-5-7-8	26

∴ 1-2-3-5-7-8 is the critical path

"Crashing on critical path"

Possible activities for crashing	No. of day's can crash	Extra cost/cost saved
1 - 2	4–3 = 1	250/day
2 - 3	5 –3 = 2	500/day
3 - 5	8 – 4 = 4	50/day
5 - 7	7 – 5 = 2	300/day
7 - 8	4 – 2 = 2	400/day

Among all the option the minimum cost slope option is 3-5, which can be reduced by 4 days, at a cost of 50/day

The difference between longest path and next longest path is the maximum duration we can do crashing. Only if the duration is available in the activity taken for crashing.

∴ The Critical path can be crashed for '2' days only

∴ Crash Cost = 2 × 50 = 100

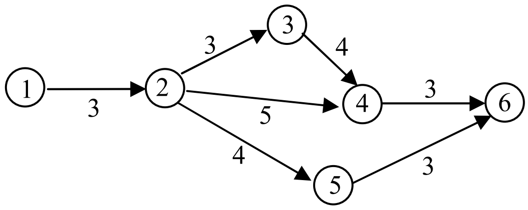
07.

Sol:

Activity	Cost slope $= \frac{C_C - N_C}{N_T - C_T}$ (Rs/week)	Crashing possibility ($N_T - N_C$)
1-2	150	1 week
2-3	-	-
2-4	50	2 week
2-5	-	-
3-4	30	3
4-6	40	1
5-6	25	2

Indirect cost = 100/week

Network diagram



Path	Duration	
1-2-3-4-6	**13**	**Critical path**
1-2-4-6	11	Sub-critical path
1-2-5-6	10	

Crashing possibility from the network = critical path duration – sub critical path

= 13 – 11 = 2 weeks

To reduce the project duration by 2 weeks

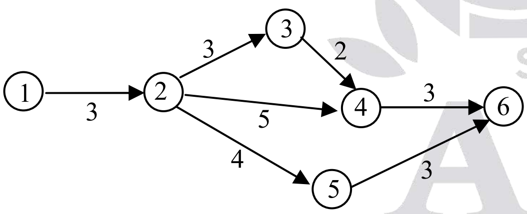
Option	Cost slope	Crashing possibility
1-2	150	1 week
2-3	-	-
3-4	30	3 week
4-6	40	1 week

From the option crash 3-4 by 2weeks by crashing 3-4 by 2 weeks the project duration becomes 11 weeks.

Crashing cost = 2 × 30 = Rs. 60

Net savings by means of crashing

= 2×100 – 60 = Rs. 140



Path	Duration
1-2-4-6	11
1-2-3-4-6	11
1-2-5-6	10

Crashing possibility from the network

= 11 – 10 = 1 week

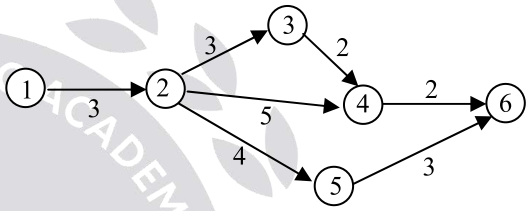
To reduce project duration by 1 week

Option	Cost slope	Crashing possibility
1-2	150	1 week
4-6	40	1 week
3-4 & 2-4	30+50 = 80	1 week

Among the best option, crash 4-6 by 1 week, the project duration will become 10 weeks

Crashing cost = 1×40 = 40

Net savings by crashing (4-6) = 100 – 40 = 60



Path	Duration
1-2-3-4-6	10
1-2-4-6	10
1-2-5-6	10

To reduce by project duration by 1 week

Option	Cost slope
1-2	150
3-4, 2-4 , 5-6	30+50+25 = 105

As crashing cost is more than indirect cost/week = further crashing is not economical

Optimum project duration = 10 weeks

Total cost of the project (with crashing) = direct cost + indirect cost/week × project duration + crashing cost

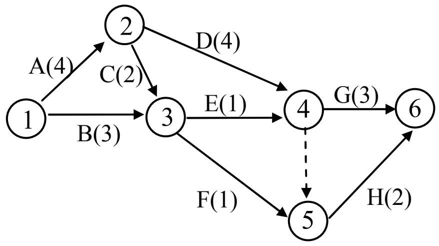
= 945 + 100×10 + 30×2 + 40×1 = 2045

Total cost without crashing

$= 945 + 100 \times 13 = 945 + 1300 = 2245$

08. Ans:

Sol:



Critical Path :

1-2-3-4-5-6 = 4 + 2 + 1 + 0 + 2 = 9

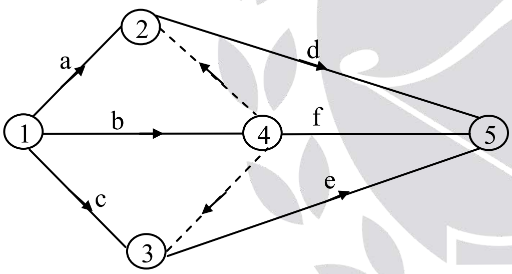
1-2-4-6 = 4 + 4 + 3 = **11 → CP**

1-2-3-4-6 = 4 + 2 + 1 + 3 = 10

1-3-5-6 = 3 + 1 + 2 = 6

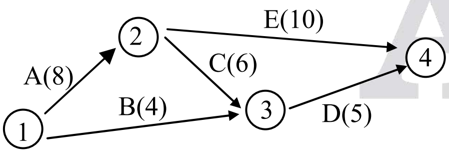
09.

Sol:



10.

Sol:



(a) Critical path :

Path	Duration
A-E	8+10 = 18
A-C-D	8+6+5 = 19
B-D	4+5 = 9

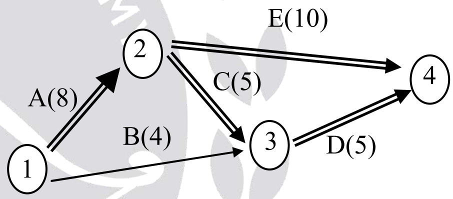
∴ Minimum duration of project = 19

(b) To reduce the project by 1 day the available option is crashing 'C' by 1 day

Option	**Crashing possibilities ($N_T – C_T$)**
A	8 – 8 = 0
C	6 – 5 = 1
D	5 – 5 = 0

By crashing activity C we can reduce the project duration by 1 day.

Network diagram



Path	**Duration**
A-E	8+10 = 18
A-C-D	8+5+5 = 18
B-D	4+5 = 9

Further crashing is not possible due to "A – C – D" critical path.

Chapter 2 Network Models

01. Ans: (c)

Sol:

d_{ij} → "Distance from any node i to next node j"

s_j → "Denotes shortest path from node P to any node j".

$d_{ij} = d_{QG}$ (Adjacent nodes)

$d_{ij} = d_{RG}$ (Adjacent from node R to G)

$S_j = S_Q$ (Shortest path from node P to node Q)

$S_j = S_R$ (Shortest path from node P to node R)

We can go from P to G via Q or via R.

P to G via Q

$$S_G = S_Q + d_{QG}$$

P to G via R.

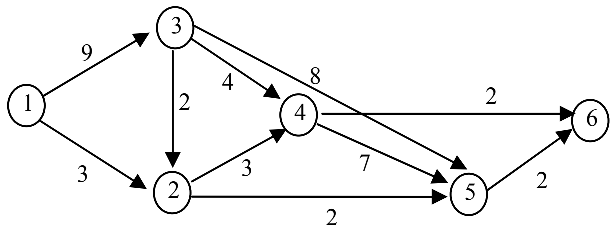
$$S_G = S_R + d_{RG}$$

Optimum answer is minimum above two answers.

$S_G = \text{MIN}\ [S_Q + d_{QG}\ ;\ S_R + d_{RG}]$

02. Ans: (c)

Sol:



Path	Cost
1-3-4-6	9+4+2 = 15
1-3-2-4-6	9+2+3+2 = 16
1-3-4-5-6	9+4+7+2 = 22
1-3-2-5-6	9+2+2+2 =15
1-3-2-4-5-6	9+2+3+7+2 = 23
1-2-4-6	3+3+2 = 8
1-2-5-6	3+2+2 = 7
1-2-4-5-6	3+3+7+2 = 15
1-3-5-6	9+8+2 = 19

From the given statement, we got shortest path (least total cost) is 1-2-5-6 and a path which does not have 1-2, 2-5, 5-6 activities should be considered.

The next path which does not have the above activities is 1-3-4-6 = 15

and 1-3-2-4-6 = 16.

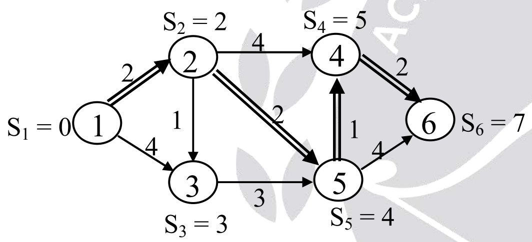
∴ In this second least total cost is 15.

03. Ans: 7

Sol:

Path	Arc length
1-2-4-6	8
1-2-5-4-6	7
1-2-5-6	8
1-2-3-5-4-6	9
1-3-5-4-6	10
1-3-5-6	11

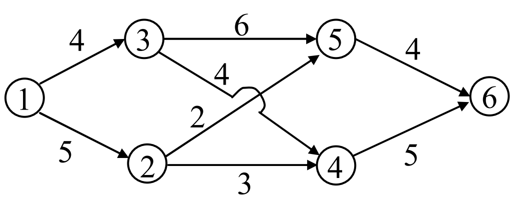
Shortest path length from node 1 to node 6 is 7.



Conventional Practice Solutions

01.

Sol:

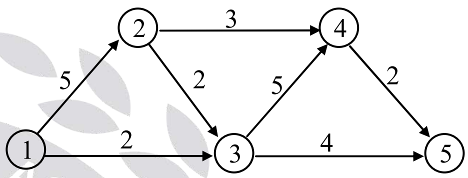


Path	Duration
1 – 3 – 4 – 6	4 + 4 + 5 = 13
1 – 2 – 5 – 6	5 + 2 + 4 = 11
1 – 3 – 5 – 6	4 + 6 + 4 = 14

∴ Shortest path from node 1 to node 6 is 11.

02.

Sol:



Path	Duration
1 – 2 – 4	5 + 3 = 8
1 – 3 – 4	2 + 5 = 7
1 – 2 – 3 – 4	5 + 2 + 5 = 12

∴ Shortest path from node 1 to node 4 is 7.

Chapter 3 Linear Programming

01. Ans: (d)

Sol: A restriction on the resources available to a firm (stated in the form of an inequality or an equation) is called constraint.

02. Ans: (d)

03. Ans: (c)

04. Ans: (d)

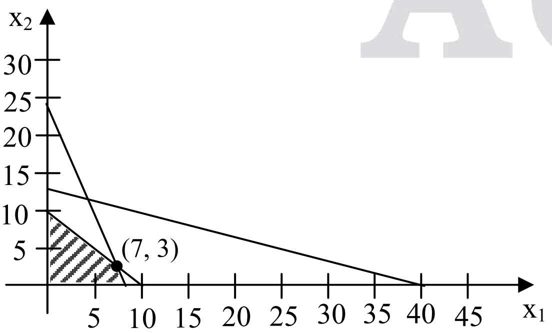
Sol: The theory of LP states that the optimal solution must lie at one of the corner points.

05. Ans: (b)

Sol: The feasible region of a linear programming problem is convex. The value of the decision variables, which maximize or minimize the objective function, is located on the extreme point of the convex set formed by the feasible solutions.

06. Ans: (a)

Sol:



$Z(7, 3) = 2\times7 + 5\times3 = 29$

07. Ans: (a)

Sol: $Z_{max} = x+2y$,

Subjected to

$4y - 4x \geq -1$……… (1)

$5x + y \geq -10$ ……….. (2)

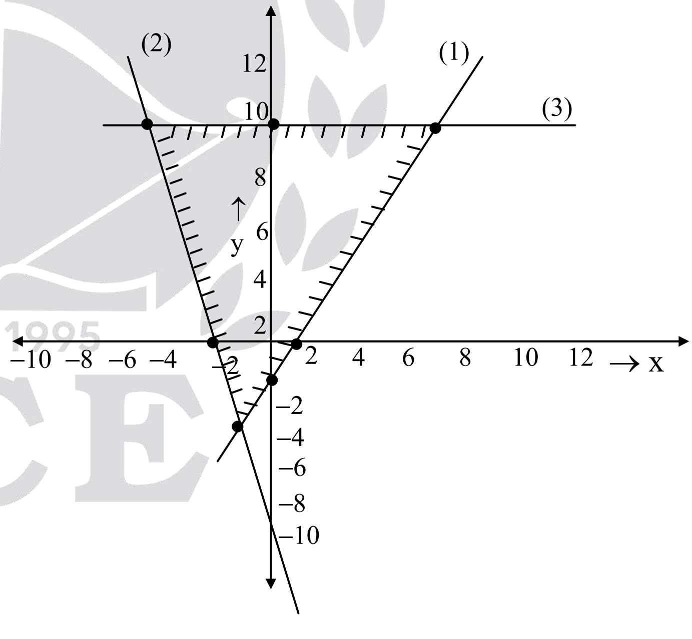
$y \leq 10$ …………… (3)

x and y are unrestricted in sign

$$(1) \Rightarrow \frac{x}{\left(\frac{1}{4}\right)} + \frac{y}{\left(\frac{-1}{4}\right)} \leq 1$$

$$(2) \Rightarrow \frac{x}{(-2)} + \frac{y}{(-10)} \leq 1$$

$$(3) \Rightarrow \frac{y}{10} \leq 1$$



Only one value gives max value, then solution is unique.

08. Ans: (b)

Sol: $Z_{max} = 3x_1+2x_2$

Subjected to

$4x_1+x_2 \le 60$ ....(1)

$8x_1+x_2 \le 90$ ....(2)

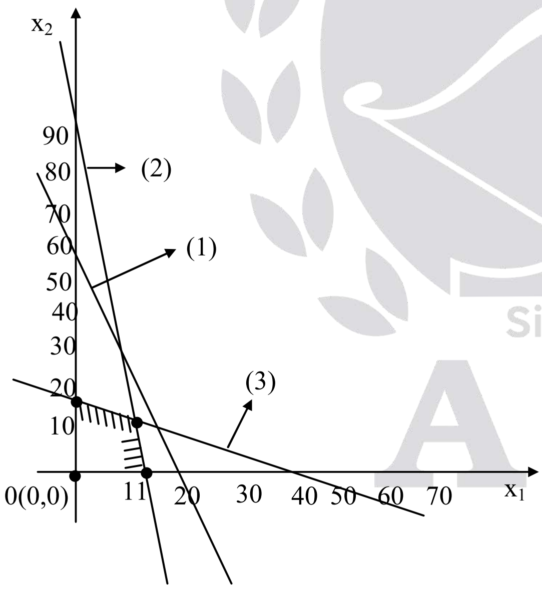
$2x_1+5x_2 \le 80$ .... (3)

$x_1, x_2 \ge 0$

$$(1) \Rightarrow \frac{x_1}{15}+\frac{x_2}{60} \le 1$$

$$(2) \Rightarrow \frac{x_1}{11.25}+\frac{x_2}{90} \le 1$$

$$(3) \Rightarrow \frac{x_1}{40}+\frac{x_2}{16} \le 1$$



From the above graph the No. of corner points for feasible solutions are 4

09. Ans: (c)

Sol: Let, P type toys produced = x ,

Q type toys produced = y

	P	Q	
Time	1	2	2000
Raw material	1	1	1500
Electric switch	-	1	600
Profit	**3**	**5**	
	x	**y**	

$Z_{max} = 3x + 5y$

$x+2y \le 2000$; $\frac{x}{2000}+\frac{y}{1000} \le 1$

$x+y \le 1500$; $\frac{x}{1500}+\frac{y}{1500} \le 1$

$y \le 600$; $\frac{y}{600} \le 1$

$x, y \ge 0$

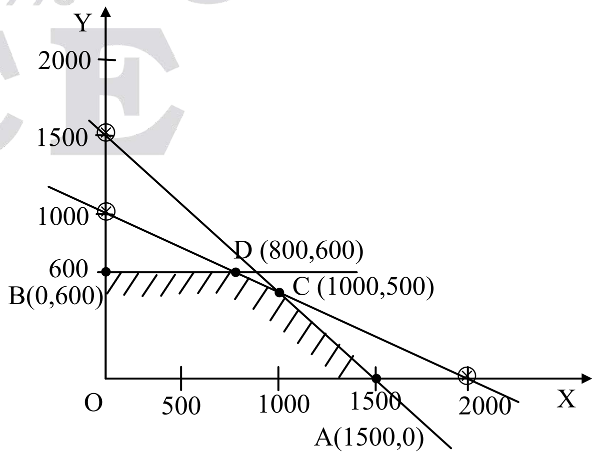
$Z_{max} = 3x + 5y$

$Z_A = 3 \times 1500 + 5 \times 0 = 4500$

$Z_B = 3 \times 0 + 5 \times 600 = 3000$

$Z_C = 3 \times 1000 + 5 \times 500 = 5500$

$Z_D = 3 \times 800 + 5 \times 600 = 5400$



C does not exist in answer.

Hence, Z_{max} is at D, i.e., Z_{max} @ D = 5400

10. Ans: (c)

Sol: $Z_{max} = x_1 + 1.5\, x_2$

Subject to

$2x_1 + 3x_2 \le 6$ ------- (1)

$x_1 + 2x_2 \le 4$ -------- (2)

$x_1,\ x_2 \ge 0$

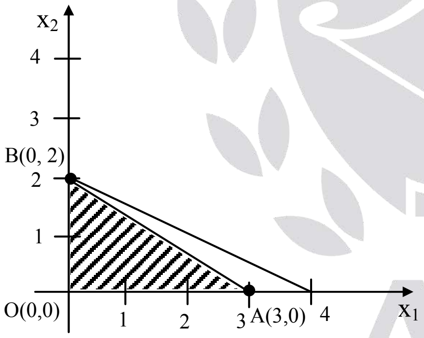
$$\frac{x_1}{3} + \frac{x_2}{2} \le 1$$

$$\frac{x_1}{4} + \frac{x_2}{2} \le 1$$

Let, "c" in the intersection of (1) and (2)

Solve (1) & (2) for 'c'.

It follows, $x_1 = \frac{12}{5};\ x_2 = \frac{2}{5}$



$Z_{max} = x_1 + 1.5x_2$

$Z_0 = 0$

$Z_A = 3 + 1.5 \times 0 = 3$

$Z_B = 3 \times 0 + 1.5 \times 2 = 3$

Problem is having multiple solutions and it is Optimal at (A) and (B).

11. Ans: (a)

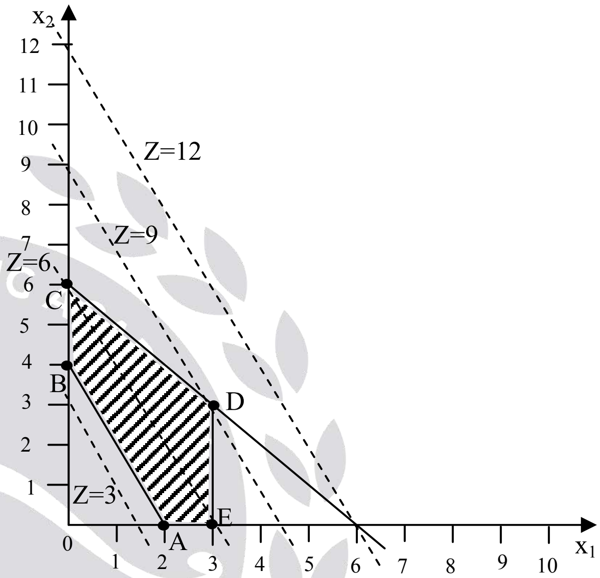
Sol: $Z_{max} = 2x_1 + x_2$

Subjected $x_1 + x_2 \le 6$

$x_1 \le 3$

$2x_1 + x_2 \ge 4$

$x_1,\ x_2 \ge 0$



But feasible region is ABCDEA

($\because x_1,\ x_2 > 0$)

A(2,0) B(0,4) C(0,6) E(3,0)

D can be obtained by solving

$x_1 \le 3$ & $x_1 + x_2 \le 6$

$\Rightarrow x_1 = 3$ and $x_2 = 3$ and D (3,3)

Z_{max}	A(2,0)	$2\times2+1\times0 = 4$
	B (0,4)	$0\times2+1\times4 = 4$
	C(0,6)	$0\times2+1\times6 = 6$
	E(3,0)	$3\times2+0\times1 = 6$
	D(3,3)	$3\times2+1\times3 = 9$

$Z_{max} = 9$ at D (3,3)

12. Ans: (d)

13. Ans: (a)

Sol: $Z_{max} = 4\,x_1 + 6\,x_2 + x_3$

s.t

$2\,x_1 - x_2 + 3x_3 \le 5$

$x_1, x_2, x_3 \ge 0$

$2\,x_1 - x_2 + 3x_3 + s_1 = 5$

$Z_{max} = 4x_1 + 6x_2 + x_3 + 0\,s_1$

$c_j \rightarrow$ s v↓	4 x_1	6 x_2	1 x_3	0 s_1	B_0	Min Ratio
0 s_1	2	-1	3	1	5	– 5
z_j c_j - z_j	0 4	0 (6) ↑ EV	0 1	0 0	0	

Entering vector exists but leaving vector doesn't exist as minimum ratio column is having negative values. It is a case of unbounded solution space and unbounded optimal solution to problem.

14. Ans: (d)

Sol: Number of zeros in Z row = 4

Number of basic variable = 3

As the number of zeros in Z row is greater than number of basic variable so it has multiple optimal solutions.

15. Ans: (b)

Sol: Solution is optimal; but Number of zeros are greater than the number of basic Variables in $C_j - Z_j$(net evaluation row) hence multiple optimal solutions.

16. Ans: (b)

Sol: If all the elements in the objective row are non-negative incase of maximization, then the solution is said to be optimal.

Here, the solution is optimal, Z_{max} = 1350.

17. Ans: (a)

Sol:

- A tie for leaving variable in simplex procedure implies degeneracy.
- If in a basic feasible solution, one of the basic variables takes on a zero value then it is case of degenerate solution

Common Data Solutions

18. Ans: (d) &

19. Ans: (a)

Sol: As the No. of zeros greater than No. of basic variables hence it is a case of multiple solutions or alternate optimal solution exists.

Basic	x_1	x_2	S_1	S_2	S_3	RHS
z	0	0	0	2	0	48
s_1	0	5/3	1	–2/3	0	14
s_3	0	–1/3	0	1/3	1	5
x_1	1	2/3	0	1/3	0	8

From the table gives the optimum $x_2 = 0$,
$x_1 = 8$, $Z_{max} = 48$

Look at the coefficient of the non basic variable in the z-equation of iterations. The coefficient of non basic x_2 is zero, indicating that x_2 can enter the basic solution without changing the value of Z, but causing a change in the values of the variables.

Alternate optimal solution :

Here x_2 is the entering variable.

Row	Basic	x_1	x_2	S_1	S_2	S_3	RHS	Ratio
R_1	z	0	0	0	2	0	48	
R_2	s_1	0	5/3	1	–2/3	0	14	14/(5/3)=8.4 → Leaving variable
R_3	s_3	0	–1/3	0	1/3	1	5	–
R_4	x_1	1	2/3	0	1/3	0	8	8/(2/3)=12

↑ Entering variable

Row	Basic	x_1	x_2	S_1	S_2	S_3	RHS
R_1	z	0	0	0	2	0	48
$R_2' = \frac{R_2}{(5/3)}$	x_2	0	1	3/5	–2/5	0	42/5
$R_3' = R_3' + \frac{R_2'}{3}$	s_3	0	0	1/5	1/5	1	39/5
$R_4' = R_4 - \frac{2}{3}R_2'$	x_1	1	0	–3/5	3/5	0	12/5

In the above table $x_1 = \frac{12}{5}$, $x_2 = \frac{42}{5}$, $s_3 = \frac{39}{5}$

20. Ans: (c)

21. Ans: (a)

22. Ans: (c)

Sol: $Z_{min} = 10x_1 + x_2 + 5x_3 + 0S_1$

Dual , $W_{min} = 50y_1$

subjected to

$5y_1 \le 10, \quad y_1 \le 2, \quad W_{max} = 100$

$3y_1 \le 5, \quad y_1 \le 5/3, W_{max} = 250/3$

$y_1, \; y_2 \ge 0$

$\Rightarrow Z_{max} = 250 / 3$

Common Data for Questions

23. Ans: (c)

Sol: Given, $Z_{max} = 5x_1 + 10x_2 + 8x_3$

Subjected to

$3x_1 + 5x_2 + 2x_3 \le 60 \rightarrow$ Material

$4x_1 + 4x_2 + 4x_3 \le 72 \rightarrow$ Machine hours

$2x_1 + 4x_2 + 5x_3 \le 100 \rightarrow$ Labour hours

$x_1, x_2, x_3 \ge 0$

$3x_1 + 5x_2 + 2x_3 + s_1 = 60$

$4x_1 + 4x_2 + 4x_3 + s_2 = 73$

$2x_1 + 4x_2 + 5x_3 + s_3 = 100$

$Z_{max} = 5x_1 + 10x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$

$C_j \rightarrow$		5	10	8	0	0	0	B_0	Min Ratio
C_B	SV	x_1	x_2	x_3	s_1	s_2	s_3		
10	x_2	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{-1}{6}$	0	8	
8	x_3	$\frac{2}{3}$	0	1	$\frac{-1}{3}$	$\frac{5}{12}$	0	10	
0	S_3	$\frac{-8}{3}$	0	0	$\frac{1}{3}$	$\frac{-17}{12}$	1	18	
Z_j		$\frac{26}{3}$	10	8	$\frac{2}{3}$	$\frac{5}{3}$	0	160	
$C_j - Z_j$		$\frac{-11}{3}$	0	0	$\frac{-2}{3}$	$\frac{-5}{3}$	0		
$\frac{C_j - Z_j}{x_2}$		−11	0	0	−2	10	0	LL=2 UL=10	10−2=8 10+10=20
$\frac{C_j - Z_j}{x_3}$		$\frac{-11}{2}$	0	0	2	−4	0	LL=4 UL=2	8−4=4 8+2=10

In $C_j - Z_j$ row all elements are negatives or zeros, hence the solution is optimal and unique..

Basic variables are:

$x_2 = 8$, $x_3 = 10$, $s_3 = 18$

i.e., production of B = 8 units, C = 10 units

18 labours hours remained unutilized

Non Basic variable

$x_1 = 0$, $s_1 = 0$, $s_2 = 0$

Resource materials and resource machine hours are fully utilized. In $(C_j - Z_j)$ row at optimality, the values under s_1, s_2 and s_3 columns represents the shadow prices.

So, If 1 kg material increases, contribution increases by $\frac{2}{3}$.

If 1 kg material decreases, contribution decreases by $\frac{2}{3}$.

If 1 kg material increases, then production B increases by $\frac{1}{3}$ and production C decreases by $\frac{1}{3}$

If m/c hr increases by 1 units, contribution increases by 5/3.

If m/c hr decreases by 1 units, contribution decreases by $\frac{5}{3}$

If m/c hr increases by 1 units, production B decreases by $\frac{1}{6}$ and production increases by $\frac{5}{12}$.

If m/c hr decreases by 1 units, production B increases by $\frac{1}{6}$ and production C decreases by $\frac{5}{12}$

If 1 unit of A produces, contribution decreases by $\frac{11}{3}$, production B decreases by $\frac{1}{3}$, production C decreases by $\frac{2}{3}$.

24. Ans: (a)

Sol: If 3 kg material increases, contribution increases by $3\times\frac{2}{3}$ = Rs. 2

25. Ans: (a)

Sol: Present profit = 160 ⇒ $160 - \frac{5}{3}\times 12 = 140/-$

26. Ans: (b)

Sol: New production of B

$$= 8 - \left(12\times\frac{-1}{6}\right) = 8 + \left(12\times\frac{1}{6}\right)$$

$$= 8 + 2 = 10 \text{ units}$$

27. Ans: (c)

Sol: If materials are increased by 3kgs then the new production of C is $= 10 + \left(3\times\frac{-1}{3}\right)$

$$= 10 - \left(3\times\frac{1}{3}\right) = 10 - 1 = 9$$

28. Ans: (a)

Sol: If 1 unit of A produces, contribution decreases by $\frac{11}{3}$

29. Ans: (a)

Sol: If 6 units of A are produced then the new profit is,

$$160 - \left(6\times\frac{11}{3}\right) = 138$$

30. Ans: (a)

Sol: Production of B , $3\times\frac{1}{3} = 1$

Production of C, $3\times\frac{2}{3} = 2$

Common data 35 & 36

31. Ans: (b) , 32. Ans: (b)

Sol: Basic variables

$x_1 = 20$, $x_2 = 10$

Non-basic variables

$s_1 = 0 \Rightarrow$ first constraint is fully consumed.

$s_2 = 0 \Rightarrow$ second constraint is fully consumed.

$x_3 = 0$ (unwanted variable)

	x_1	x_2	x_3	s_1	s_2	RHS
z-row	0	0	2	1	2	110
x_1	1	0	1	1	-1	20
x_2	0	0	0	–1	2	10

	s_1
z-row	1
x_1	1
x_2	–1

If RHS value of 1^{st} constraint increases by 1 unit then

From the table

z increases by 1 unit, x_1 increases by 1 unit,

x_2 decreases by 1 unit,

If RHS value of 2nd constraint increases by 1 unit then

	s_2
z-row	2
x_1	-1
x_2	2

From the table

z increases by 2 units, x_1 decreases by 1 unit

x_2 decreases by 2 units,

If RHS value of 1st constraint decreases by 10 units then z decreases by 10 units,

The new objective value ,

$$Z_{max} = 110 – 10 = 100$$

33. Ans: (c)

Sol:

	X_1	X_2	S_1	S_2	RHS	Ratio
z-row	–3	–5	0	0	0	0
S_1	2	1	1	0	2	2/1=2
S_2	3	2	0	1	4	4/2=2

↑

Entering variable X_2

Minimum ratio = min(2/1, 4/2) = 2^*

*Tie w.r.t leaving variables S_1 and S_2

Thus it has degenerate solution.

34. Ans: (d)

Sol:

	X_1	X_2	S_1	S_2	RHS
z-row	–2	–1	0	0	0
S_1	–2	1	1	0	4
S_2	0	1	0	1	3

↑

Entering variable X_1

Ratio = Min{4/–2, 3/0}

As there is no least positive ratio, there is no leaving variable which results the problem has unbounded solution.

35.

Sol:

Demand	Products		Maximum available
	Chairs (x_1)	Tables (x_2)	
Wood	1	2	200
Chairs	1	–	150
Tables	–	1	80
Profit/loss	100	300	

$Z_{max} = 100x_1 + 300x_2$

Subject to

$x_1 + 2x_2 \le 200$

$x_1 \le 150$ and $x_2 \le 80$

36.

Sol:

Demand	Products		Maximum available
	A (x_1)	B (x_2)	
Raw material	1	1	850
Special type of buckle	1	—	500
Ordinary buckle	–	1	700
Time	1	½	500
Profits/unit	10/-	5/-	

Constraints :

x_1 = No. of belts of type 'A'

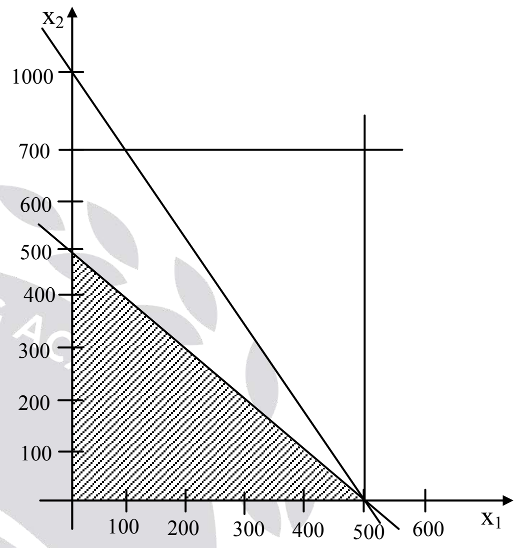
x_2 = No. of belts of type 'B'

$Z_{max} = 10x_1 + 5x_2$

s.t $x_1 + x_2 \le 850$

$x_1 \le 500$, $x_2 \le 700$

$x_1 + \frac{1}{2}x_2 \le 500$, $x_1, x_2 \ge 0$



$Z_{max} = (10\times0) + (5\times500) = 2500$ /-

Conventional Practice Solutions

01.

Sol: Let, x_1 be the number of ash trays

x_2 be the number of tea trays

Production to be maximized

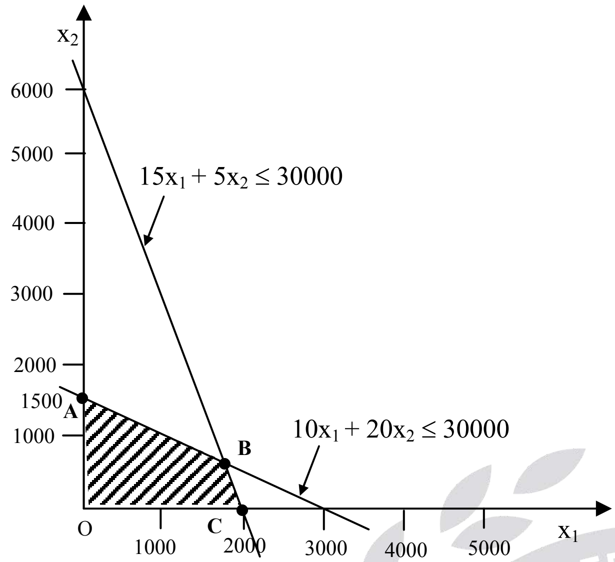
$Z = 20x_1 + 30x_2$

From the table given, constrained are

$10x_1 + 20x_2 \le 30000$

$15x_1 + 5x_2 \le 30000$

Fixed daily cost = Rs. 45000



From the graph, common feasible region is OABC O(0,0) , A(0,1500), C(2000,0)

B would be obtained by solving the constraints. B(1800 , 600)

Z	A(0,1500)	20×0+30×1500 = Rs.45000
	B(1800,600)	20×1800+30×600 = Rs.54000
	C(2000,0)	20×2000+30×0 = Rs.40000

Z_{max} = Rs. 54000 at B

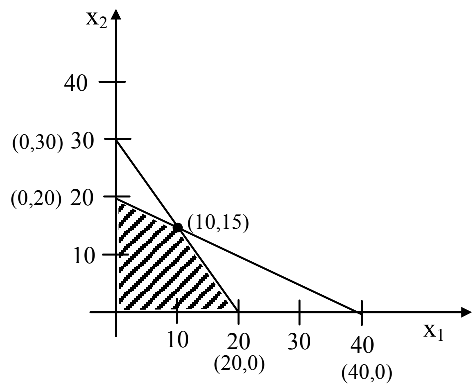
02.

Sol: $Z_{max} = 60x_1 + 50x_2$

s.t $x_1 + 2x_2 \le 40$

$3x_1 + 2x_2 \le 60$

$\frac{x_1}{40} + \frac{x_2}{20} \le 1$, $\frac{x_1}{20} + \frac{x_2}{30} \le 1$



$(Z_{max})_{(10,15)} = 60\times10+50\times15 = 1350$ /-

03.

Sol:

Type of machine	Products		Total time available
	A	B	
P	10	7.5	75
Q	6	9	54
R	5	13	65

Profit for product, A = Rs. 60 per unit

Profit for product, B = Rs. 70 per unit

Let, x = number of A type products

y = number of B type products

∴ Maximization problem

$Z_{max} = 60x + 70y$

Constraints are, (in times)

$10x + 7.5y \le 75$

$6x + 9y \le 54$

$5x + 13y \le 65$

Common feasible region is OABCDO

O(0,0), A(0,5), D(7.5,0)

B is point of intersection of lines

$6x + 9y \le 54$,

$5x + 13y \le 65$

Solving this B = (3.55 , 3.64)

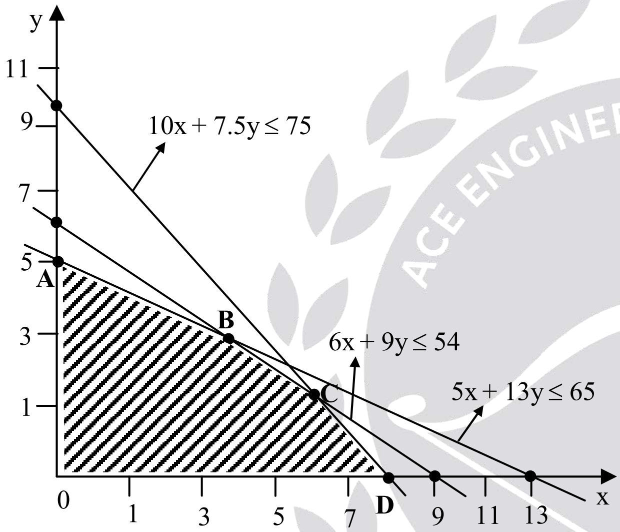
C is the point of intersection of the lines

$6x + 9y \le 54$,

$10x + 7.5y \le 75$

Solving these, C = (6,2)

Graphically solving :



Points	Z = 60x+70y
A (0,5)	60×0+70×5 = 350
B (3.53,3.64)	3.55×60+70×3.64 = 464.8
C (6,2)	60×6+70×2 = 500
D (7.5,0)	7.5×60+0×70 = 450
O (0,0)	0×60+0×70 = 0

$\therefore Z_{max} = 500$ at C(6,2)

$\therefore$ A type products = 6 ,

B type products = 2

04.

Sol:

	Tables	Chairs	Availability
Wood	30	20	300
Labour	5	10	110
Profit/unit	8	6	
	x	y	

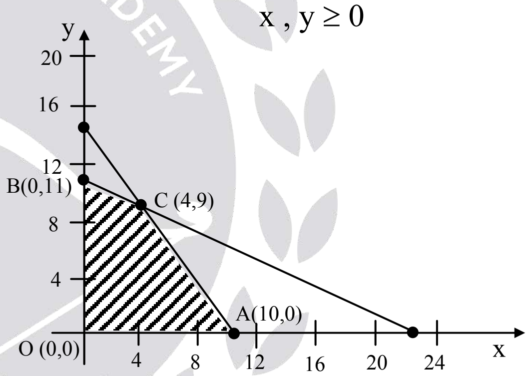
$Z_{max} = 8x + 6y$

Subject to

$30x + 20y \le 300$, $\frac{x}{10} + \frac{y}{15} \le 1$ ----- (1)

$5x + 10y \le 110$, $\frac{x}{22} + \frac{y}{11} \le 1$ ------ (2)

$x , y \ge 0$



"C' is the intersection of (1) and (2)

Solve equation (1) & (2) for x,y

We will get x = 4, y = 9

$Z = 8x + 6y$

$Z_0 = 0$

$Z_A = 8\times10 + 6\times0 = 80$

$Z_B = 8\times0 + 6\times11 = 66$

$Z_C = 8\times4 + 6\times9 = 86$

Solution is optimal at (c)

$Z_{max} = 86$ at x = 4 , y = 9

Chapter 4

Inventory Control

01. Ans: (b)

Sol: $EOQ = \sqrt{\frac{2AS}{CI}}$

$EOQ_1 = \sqrt{2} \times \sqrt{\frac{2AS}{CI}}$

$EOQ_1 = \sqrt{2} \times EOQ$

02. Ans: (c)

Sol: $EOQ = \sqrt{\frac{2DC_o}{C_c}}$

03. Ans: (b)

Sol: A = 900 unit

S = 100 per order

CI = 2 per unit per year

$EOQ = ELS = \sqrt{\frac{2AS}{CI}}$

$= \sqrt{\frac{2 \times 900 \times 100}{2}} = 300$

04. Ans: (c)

Sol: **Inventory carrying cost:**

It involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

05. Ans: (b)

Sol: At EOQ, Carrying cost = Ordering cost

06. Ans: (d)

Sol: Inventory carrying cost involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

07. Ans: (a)

Sol: A = 800 , S = 50/- ,

C_s = 2 per unit = CI

$(TIC)_{EOQ} = \sqrt{2ASCI}$

$= \sqrt{2 \times 800 \times 50 \times 2} = 400$

08. Ans: (c)

Sol: $TC(Q_1) = TC(Q_2)$

$\frac{kd}{Q_1} + \frac{hQ_1}{2} = \frac{kd}{Q_2} + \frac{hQ_2}{2}$

$kd\left(\frac{Q_2 - Q_1}{Q_1 Q_2}\right) = \frac{h}{2}(Q_2 - Q_1)$

$\frac{2kd}{h} = Q_1 Q_2$

$(Q^*)^2 = Q_1 \times Q_2$

$Q^* = \sqrt{Q_1 \times Q_2} = \sqrt{300 \times 600} = 424.264$

09. Ans: (c)

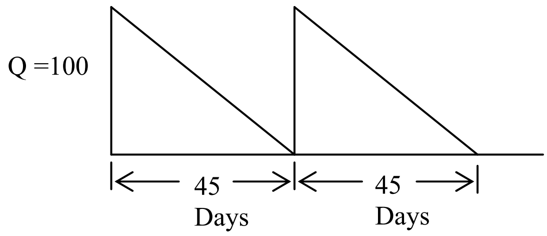
Sol: $\frac{EOQ_1}{EOQ_2} = \sqrt{\left(\frac{2AS}{CI}\right)_A} \times \sqrt{\left(\frac{CI}{2AS}\right)_B}$

$= \sqrt{\left(\frac{2 \times 100 \times 100}{4}\right)} \times \sqrt{\left(\frac{1}{2 \times 400 \times 100}\right)}$

$(EOQ)_A : (EOQ)_B = 1:4$

10. Ans: (d)

Sol: (No of orders $=\frac{A}{Q}=\frac{12\,\text{months}}{45\,\text{days}}=\frac{12}{1.5}=8$)



$$TVC=\frac{A}{Q}S+\frac{Q}{2}CI.$$

$$=8\times100+\frac{100}{2}\times120=\text{Rs. }6800$$

11. Ans: (b)

Sol: Average inventory

$$=\frac{Q}{2}=\frac{6000}{2}=3000\text{ per year}$$

= 250 per month

12. Ans: (b)

Sol: P = 1000 , r = 500 , Q = 1000

$$I_{max}=\frac{1000}{1000}(1000-500)=500$$

13. Ans: (c)

Sol: D = 1000 units, C_0 = Rs.100/order,

C_c = 100/unit/year, C_s = 400/unit/year

$$Q_{max}=EOQ_s\times\frac{C_S}{C_C+C_S}$$

$$=\sqrt{\frac{2DC_0}{C_c}}\sqrt{\frac{C_c+C_s}{C_s}}\times\left(\frac{C_s}{C_c+C_s}\right)$$

= 40 units

14. Ans: (d)

Sol: Re-order level = 1.25[Σx p(x)]

= 1.25 [80×0.2+100×0.25+120×0.3+140×0.25]

= 140 units

Demand	80	100	120	**140**
Probability	0.20	0.25	0.30	**0.25**
Cumulative probability (Service level)	0.2	0.45	0.75	**1.0**

Service Level = 100 %

15. Ans: (b) **16. Ans: (b)**

17. Ans: (d)

Sol: C – Class means these class items will have very less consumption values. – least consumption values

B → 300 × 0.15 = 45

F → 300 × 0.1 = 30

C → 2 × 200 = 400

E → 5 × 0.3 = 1.5

J → 5 × 0.2 = 1.0

G → 10 × 0.05 = 0.5

H → 7 × 0.1 = 0.7

∴ G, H items are classified as C class items because they are having least consumption values.

18. Ans: (b)

Sol: In ABC analysis :

Category "A" = Low safety stock

Category "B" = Medium safety stock

Category "C" = High safety stock

Conventional Practice Solutions

01.

Sol: Given, A = 5000 units/year,

S = 16/-

I = 0.02 + 0.12 + 0.06 = 0.2,

C = 20/-

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$= \sqrt{\frac{2\times5000\times16}{20\times0.2}} = 200\,\text{units}$$

$$(TVC)_{EOQ} = \sqrt{2ASCI}$$

$$= \sqrt{2\times5000\times16\times20\times0.2}$$

= Rs. 800 /-

02.

Sol: Given, A = 1000 units/year, S = 40/-

I = 0.1, C = 500/-

a) $EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times1000\times40}{500\times0.1}} = 40\,\text{units}$

b) No. of annual orders = $\frac{A}{Q} = \frac{1000}{40} = 25$

c) $(TAC)_{EOQ} = AC + \sqrt{2ACSI}$

$$= 1000\times500 + \sqrt{2\times1000\times500\times40\times0.1}$$

= 5,02,000/-

Order per month $= \frac{1000}{12} = 83.33\text{units.}$

$$(TAC)_Q = AC + \frac{A}{Q}.S + \frac{Q}{2}.CI$$

$$(TAC)_{83.38} = 1000\times500 + \frac{1000}{83.33}\times40 + \frac{83.33}{2}\times500\times0.1$$

$$= 5{,}02{,}563/-$$

Savings $= (TAC)_Q - (TAC)_{EOQ}$

= 502563 – 502000 = Rs. 563 /-

03.

Sol: Simultaneous consumption producing Model

A = 15,000 units, C.I = 5/ units/year

S = 25 /–, P = 100 units/day

No. of working days = 250 /year

Consumption rate = r = $\frac{15{,}000}{250} = 60\,\text{units/day}$

EBQ = EPQ = ELS

$$EPQ = \sqrt{\frac{2AS}{CI}\left(\frac{P}{P-r}\right)}$$

$$Q = \sqrt{\frac{2\times15000\times25}{5}\left(\frac{100}{100-60}\right)}$$

Q = 612.37 units

$$(TVC)_{EPQ} = \sqrt{2ASCI\frac{(P-r)}{P}}$$

$$= \sqrt{2\times15000\times25\times5\times\left(\frac{100-60}{100}\right)}$$

= 1225/-

No of production runs $= \frac{A}{Q}$

$$= \frac{15000}{612.37} = 24.5 \approx 25$$

04.

Sol: D = 192000 units,

A = Rs. 1080 /set-up,

$h = 0.3 \times 12 = 3.60$ /pack/year,

$d = \frac{192000}{240} = 800$ packs per day

$p = \frac{20000}{20} = 1000$ packs/day.

(a) Optimum lots size = $\sqrt{\frac{2DA}{h}\left(\frac{p}{p-d}\right)}$

$= \sqrt{\frac{2\times192000\times1080}{3.60}\left(\frac{1000}{1000-800}\right)}$

= 24000 packs

(b) Optimum number of production runs

$= \frac{\text{Annual demand}}{\text{Optimum lot size}}$

$= \frac{192000}{24000} = 8$

(c) Time interval between successive production runs

$= \frac{\text{No. of working days}}{\text{No. of runs}}$

$= \frac{240}{8}$

= 30 working days

(d) Total variable cost = $\sqrt{2DAh\left(\frac{p-d}{p}\right)}$

$= \sqrt{2\times192000\times1080\times3.60\times\left(\frac{1000-800}{1000}\right)}$

= Rs. 17,280 /-

05.

Sol: A = 10,000 units

S = 200/order

CI = 4/unit/year

C = 20/-

(a) $EOQ = \sqrt{\frac{2AS}{CI}}$

$= \sqrt{\frac{2\times10000\times200}{4}}$ = 1000 units.

Total annual cost at EOQ,

$(TAC)_{EOQ} = AC + \sqrt{2ACSI}$

$= 10000(20) + \sqrt{2(10000)4(200)}$

= 2,04,000/-

(b) $(EOQ)_{shortage} = \sqrt{\frac{2AS}{CI}\times\frac{C_s+CI}{C_s}}$

$= \sqrt{\frac{2\times10000\times200}{4}\times\frac{20+4}{20}}$

= 1095.45 units

Optimal level of shortages

$S^* = Q^* \times \left(\frac{C_s}{C_s+CI}\right)$

$= 1095.45 \times \frac{20}{20+4}$

= 912.87 units

Maximum inventory level = Q* – S*

= 1095.45 – 912.87

= 182.58

06.

Sol: Given :

C = Rs. 5/unit ,

A = 4000 units

S = Rs. 30/order ,

CI = Rs. 1.5

$$EOQ = \sqrt{\frac{2\times4000\times30}{1.5}} = 400 \text{ units}$$

$$\text{no. of order per year} = \frac{4000}{400} = 10 \text{ runs}$$

$$(\text{Total yearly cost})_{EOQ} = AC + \sqrt{2ASCI}$$

$$= (4000\times5) + \sqrt{2\times4000\times30\times1.5}$$

= Rs. 20600/-

$$(TC)_{Q_1 @ R_1\%} = AC\left(1-\frac{R_1}{100}\right) + \frac{A}{Q_1}S + \frac{Q_1}{2}CI\left(1-\frac{R_1}{100}\right)$$

$$= (4000\times5)\left(1-\frac{2}{100}\right) + \frac{4000}{1000}\times30 + \frac{1000}{2}\times1.5\left(1-\frac{2}{100}\right)$$

= Rs. 20455/-

$$(TC)_{Q_2 @\%} = 4000\times5\left(1-\frac{3}{100}\right)+\frac{4000}{2000}\times30+\frac{2000}{2}\times1.5\times\left(1-\frac{3}{100}\right)$$

= Rs. 20915/-

Among all 2% discount for ordering quantities of 1000 or more

07.

Sol: Given:

A = 2000 units/year ,

S = Rs. 20/-,

I = 25%

C_u = Rs. 8/- (Lowest with unit price)

$$EOQ\,|_{C_u=8\%} = \sqrt{\frac{2\times2000\times20}{8\times0.25}} = 200 \text{ units}$$

The $EOQ_{at\ Cu\ =\ Rs.\ 8/-}$ is satisfying the Quantity range hence it is declared as an optimal order quantity.

08.

Sol:

Daily sales	No. of days	Probability P_i	SL	SOR
10	15	0.15	0.15	1
11	20	0.20	0.35	0.85
12	40	0.40	0.75	0.65
13	25	0.25	1	0.25

Cus = SP – CP = 5 – 2 = 3

Cos = CP = 2

$$SL = \frac{Cus}{Cus + Cos}$$

$$= \frac{3}{3+2} = 0.6$$

SOR = 1 – SL = 1 – 0.6 = 0.4

As SL = 0.6 falling in the range 11 to 12 sales, hence order 12 for 40 days.

(Cus) = Cost of under stock

(Cos) = Cost of over stock

(SL) = Service levels

(SOR) = Stock out risk

SP = selling price, CP = cost price

09.

Sol: Cus = SP −CP = 2 − 0.8 = 1.2

Cos = CP − Salvage value = 0.8 − 0 = 0.8

$$SL = \frac{Cus}{Cus + +Cos} = \frac{1.2}{1.2+0.8} = 0.6$$

For 60% − Service levels

$$Q_{Optimum} = I_{min} + SL\,(I_{max} - I_{min})$$

$$= 20000 + 0.6(24000 - 20000)$$

$$= 22400$$

10.

Sol:

Stage – I:

Let C = Rs. 185 /-

$$EOQ\big|_{C=185} = \sqrt{\frac{2AS}{C\times I}}$$

$$= \sqrt{\frac{2\times 8000\times 1800}{185\times 0.1}} = 1247.7 \text{ units}$$

EOQ does not satisfy the quantity range. Hence we calculate

$$TC\big|_{\substack{Q=2000\\C=185}} = \frac{Q}{2}\times C.I + \frac{A}{Q}S + AC$$

$$= \left(\frac{2000}{2}\times 185\times 0.1\right) + \left(\frac{8000}{2000}\times 1800\right) + (8000\times 185)$$

= Rs 1505700/-

Stage -II:

$$EOQ\big|_{C=190} = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times 8000\times 1800}{190\times 0.1}}$$

$$= 1231.17 \text{ units}$$

EOQ does not satisfy the quantity range. Hence we calculate

$$TC\big|_{\substack{Q=1500\\C=190}} = \frac{Q}{2}\times CI + \frac{A}{Q}S + AC$$

$$= \left(\frac{1500}{2}\times 190\times 0.1\right) + \left(\frac{8000}{1500}\times 1800\right) + (8000\times 190)$$

= Rs 1543850 /-

Stage – III:

$$EOQ\big|_{C=200} = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times 8000\times 1800}{200\times 0.1}}$$

$$= 1200 \text{ units}$$

EOQ satisfy the quantity range. Hence we calculate

$$TC\big|_{EOQ=1200} = \sqrt{2ASCI} + AC$$

$$= \sqrt{2\times 8000\times 1800\times 2000\times 0.1} + 8000\times 200$$

= Rs 1675894.66 /-

Among all the total cost, the minimum in

$$TC\big|_{\substack{Q=2000\\C=185}}$$

So the best order size is 2000 units

11.

Sol: Annual demand (A) = 2000 units

Cost per item (C) = 20/-

Ordering cost = 50/-

Inventory carrying cost (I) = 0.25

$$EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times 2000\times 50}{20\times 0.25}} = 200 \text{ units}$$

$$(TAC)_{EOQ} = AC + \sqrt{2ACSI}$$

$$= (2000\times 20) + \sqrt{2\times 2000\times 20\times 50\times 0.25}$$

$$= 41{,}000/-$$

Now, TAC at Q_1 with discount r%

$$(TAC)_{Q1} = AC\left(1-\frac{r_1}{100}\right)+\frac{A}{Q_1}S+\frac{Q_1}{2}CI\left(1-\frac{r_1}{100}\right)$$

$$= 2000\times 20\left(1-\frac{3}{100}\right)+\frac{2000}{1000}\times 50+\frac{1000}{2}20\times 0.25\left[1-\frac{3}{100}\right]$$

= 41325 /-

As the total annual cost (TAC) with discount r% is greater than (TAC) at EOQ, hence reject the discount and order 200 at a time.

12.

Sol: $EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times 25\times 25}{0.4}}$

= 55.9 units ≈ 56 units

$$\text{Re-order point} = \begin{pmatrix}\text{Daily}\\ \text{demand}\end{pmatrix}\times \text{Lead Time}$$

= 25 × 16 = 400 units

13.

Sol: Given,

Daily demand – D. D ,

Lead Time – L.T

Re-order Level – ROL

For Item A

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$= \sqrt{\frac{2\times 8000\times 15}{0.06}} = 2000 \text{ units}$$

R.O.L = daily demand × Lead Time

$$= \frac{8000}{250}\times 10 = 320 \text{ units}$$

For Item B

ROL = D.D × L.T

$$216 = \frac{A}{250}\times 6$$

A = 9000 units

$$EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times 9000\times 40}{0.18}} = 2000 \text{ units}$$

For Item C

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$300 = \sqrt{\frac{2\times 7500\times S}{30}}$$

S = Rs. 180/order

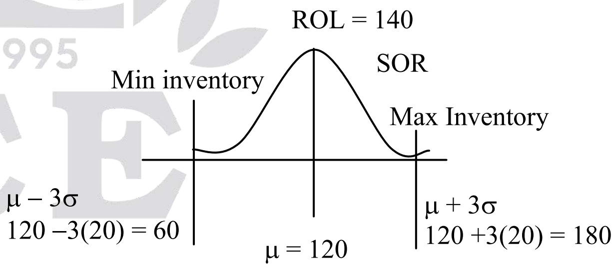
ROL = D.D × L.T

$$210 = \frac{7500}{250}\times LT$$

Lead Time = 7 days

14.

Sol:



a) SOR = 2%,

For service level (SL) = 98% to be safety factor on σ basis, $SF_\sigma = 2.05$

Safety stock (SS) = $SF_\sigma \times \sigma$

= 2.05 × 20 = 41

Re-order point (ROP)

= Avg lead time demand + SS

= 120 + 41 = 161

b) Given, ROP = 140 units, $SF_\sigma = ?$

$140 = 120 + SF_\sigma\, 20$

$SF_\sigma = 1$

i.e., as SF_σ basis is 1 will achieve service levels (SL) 84.13%.

Stock out risk (SOR) = 100 – SL

($\because$ SOR + SL =100%)

= 100 – 84.13

SOR = 15.87%

Stock out = 140 –100 = 40 units

15.

Sol: $\sigma = 60$ units , $SL = \frac{51}{52} = 98\%$

(Consider 52 weeks/year)

$SS = SF_\sigma \times \sigma = 2.05 \times 60 = 123$

ROL = ALTd + SS

$= ALT \times CR + SF_\sigma \sigma$

$= 500 \times 1 + 123 = 623$ units

Where, CR = consumption rate

ALT = Average lead time

16.

Sol: Lead Time > order cycle

$\sigma_{OC} = \sqrt{n\sigma^2} = \sqrt{6 \times 5^2} = 12.21$

Safety stock (SS) = $SF_\sigma \times \sigma$

$= 1.28 \times 12.21 = 15.67\ m \approx 16.$

($\because$ For 90% SL $\rightarrow SF_\sigma = 1.28$)

ROL = ALTd + SS = 40 + 16 = 56

17.

Sol: Raking of items according to their usage values

Part code	Price per unit Rs	Units /year	Total cost (Rs)	% of total cost	Ranking
P01	100	100	10000	0.2	X
P02	200	300	60000	1.2	VI
P03	50	700	35000	0.7	IV
P04	300	400	120000	2.4	IV
P05	500	1000	500000	10	III
P06	3000	30	60000	1.2	VII
P07	1000	100	100000	2	V
P08	7000	500	3500000	70.5	I
P09	5000	105	525000	10.6	II
P10	60	1000	60000	1.2	VIII
Total			**4970000**	**100**	

ABC PLAN

RANK	Part code	% of total cost%	Cumulative percentage
I	P08	70.5	70.5
II	P09	10.6	81.1
III	P05	10	91.1
IV	P04	2.4	93.5
V	P07	2	95.5
VI	P02	1.2	96.7
VII	P06	1.2	97.9
VIII	P10	1.2	99.1
IX	P03	0.7	99.8
X	P01	0.2	100

Class A items $\rightarrow$ Nil

Class B items $\rightarrow$ I, II

Class C items $\rightarrow$ III,IV,V,VI,VII,VIII,IX,X

Chapter 5

Forecasting

01. Ans: (d)

02. Ans: (d)

Sol:

- A simple moving average is a method of computing the average of a specified number of the most recent data values in a series.
- This method assigns equal weight to all observations in the average.
- Greater smoothing effect could be obtained by including more observations in the moving average.

03. Ans: (a)

Sol: 3 period moving avg = $\frac{100+99+101}{3}$

$= 100$

4 period moving average

$= \frac{102+100+99+101}{4} = 100.5$

5 period moving average

$= \frac{99+102+100+99+101}{5} = 100.2$

Arithmetic Mean

$= \frac{101+99+102+100+99+101}{6}$

$= 100.33$

04. Ans: (a)

Sol: $D_t = 100$ units , $F_t = 105$ units

$\alpha = 0.2$

$F_{t+1} = 105 + 0.2\,(100 - 105) = 104$

05. Ans: (c)

Sol: $D_t = 105$, $F_t = 97$, $\alpha = 0.4$

$F_{t+1} = 97 + 0.4\,(105 - 97) = 100.2$

06. Ans: (c)

Sol: $F_{t+1} = F_t + a\,(X_t - F_t)$

07. Ans: (c)

Sol: Another form of weighted moving average is the exponential smoothed average. This method keeps a running average of demand and adjusts if for each period in proportion to the difference between the latest actual demand and the latest value of the forecast.

08. Ans: (a)

09. Ans: (b)

Sol:

Period	D_i	F_i	$(D_i - F_i)^2$
14	100	75	625
15	100	87.5	156.25
16.	100	93.75	39.0625
		$\Sigma(D_i - F_i)^2 = 820.31$	

$F_{15} = F_{14} + \alpha(D_{14} - F_{14})$

$= 75 + 0.5(100 - 75) = 87.5$

$F_{16} = F_{15} + \alpha(D_{15} - F_{15})$

$= 87.5 + 0.5(100 - 87.5) = 93.75$

$$\text{Mean square error (MSE)} = \frac{\sum (D_i - F_i)^2}{n}$$

$$= \frac{820.31}{3} = 273.13$$

10. Ans: (a)

Sol:

Period	D_i	F_i	$\lvert(D_i - F_i)\rvert$
1	10	9.8	0.2
2	13	12.7	0.3
3	15	15.6	0.6
4	18	18.5	0.5
5	22	21.4	0.6

$\Sigma|D_i - F_i| = 2.2$

11. Ans: (d)

Sol:

m_1 = moving average periods give forecast $F_1(t)$

m_2 = moving average periods give forecast $F_2(t)$

$m_1 > m_2$

$F_1(t)$ is a stable forecast has less variability.

$F_2(t)$ is a sensitive (inflationary) forecast and has high variability.

12. Ans: (d)

Sol: Following are the purposes of long term forecasting :

- To plan for the new unit of production.
- To plan for the long-term financial requirement.
- To make the proper arrangement for training the personal.
- Budgetary allegations are not done in the beginning of a project. So, deciding the purchase program is not the purpose of long term forecasting.

13. Ans: (d)

Sol:

- Time horizon is less for a new product and keeps increasing as the product ages. So, statement (I) is correct.
- Judgemental techniques apply statistical method like random sampling to a small population and extrapolate it on a larger scale. So, statement (II) is correct.
- Low values of smoothing constant result in stable forecast. So statement (3) is correct.

14. Ans: (i) 50, (ii) 52.5, (iii) (42.5, 40)

Sol:

(i) $F_7 = \dfrac{60 + 50 + 40}{3} = 50$

(ii) $F_7 = \dfrac{60 \times 0.5 + 50 \times 0.25 + 40 \times 0.25}{0.5 + 0.25 + 0.25} = 52.5$

(iii) 2 period moving average $= \dfrac{60 + 50}{2} = 55$

4 period moving average

$$= \frac{60 + 50 + 40 + 20}{4} = 42.5$$

5 period moving average

$$= \frac{60 + 50 + 40 + 20 + 30}{5} = 40$$

15. Ans: (114.8 units, 9 periods)

Sol: At $\alpha = 0.2$

$F_{may} = 100 + 0.2\,(200 - 100) = 120$

$F_{june} = 120 + 0.2\,(50 - 120) = 106$

$F_{july} = 106 + 0.2\,(150 - 106) = 114.8$

Time	Demand	Forecast
April	200	100
May	50	120
June	150	106
July	-	114.8

$$\alpha = \frac{2}{n+1}$$

$$n+1 = \frac{2}{\alpha} \Rightarrow n = \frac{2}{0.2} - 1 = 9 \text{ period}$$

Conventional Practice Solutions

01.

Sol: In, Jun, July, Aug, Sep demand is Stable

In Oct, Nov, Dec – demand is Fluctuating

$$F_{Jan} = \frac{327+339+355}{3} = 340.33 \text{ units.}$$

Last '3' months average is forecast for next month

The inflation start only from October hence considering last 3 months data was highly significant

Simple exponential $\alpha = 0.1$

$F_{Jan} = F_{Dec} + \alpha(D_{Dec} - F_{Dec})$

$= 307 + 0.1(355 - 307)$

$= 311.8$

02.

Sol: Simple exponential method

$\alpha = 0.2$, $D_{Jan} = 200$

$F_{Jan} = 175$, $D_{Feb} = 170$

$$F_{feb} = F_{Jan} + \alpha(D_{Jan} - F_{Jan})$$

$= 175 + 0.2\,(200 - 175) = 180$

$F_{march} = F_{Feb} + \alpha(D_{Feb} - F_{Feb})$

$= 180 + 0.2(170 - 180) = 178$

03.

Sol: Linear Regression model:

(x)	y (Rs)	xy	x^2
1	450	450	1
2	550	1110	4
3	625	1875	9
4	650	2600	16
5.	750	3750	25
$\Sigma x = 15$	$\Sigma y = 3025$	$\Sigma xy = 9775$	$\lvert\Sigma x^2 = 55\rvert$

$y = a + bx \Rightarrow \Sigma y = na + b\Sigma x$

$xy = ax + bx^2 \Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

$3025 = 5a + 15b$ …….. (1)

$9775 = 15a + 55b$…….. (2)

Now, solve (1) and (2) for a, b

$a = 395$, $b = 70$

Forecast equ. $y_c = a + bx$

$y_c = 395 + 70x$

Forecast for month – 6,

$y_6 = 395 + 70(6) = 815$

Forecast For month – 7

$y_7 = 395 + 70(7) = 885$

04.

Sol: Deviation = $D_i - F_i$

$$MAD = \sum_{i=1}^{n} |D_i - F_i|$$

$$MAD = \frac{7.5 + 18 + 0 + 28. + 12}{6}$$

$$= \frac{70}{6} = 11.66$$

$$\text{Tracking signal} = \left|\frac{\text{Cumulative deviation}}{\text{MAD}}\right|$$

$$= \left|\frac{-24}{11.66}\right| = 2.05 < 4$$

If tracking signal < 4 – No significant deviation in data

If tracking signal > 4 – significant deviation in data

05.

Sol: n = 20,

$\Sigma(y - \bar{y})^2 = 2800$

$\Sigma x = 80$,

$\Sigma y = 1200$,

$\Sigma x^2 = 340$,

$\Sigma y^2 = 74,800$,

$\Sigma xy = 5000$

$y = a + bx$

$\Rightarrow \Sigma y = na + b\Sigma x$

$1200 = 20a + b(80)$….. (1)

$xy = ax + bx^2$

$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

$5000 = a(80) + b(340)$……(2)

Solve (1) and (2) for a, b

$a = 20, \quad b = 10$

Standard error

$$S_{yx} = \sqrt{\frac{\Sigma y^2 - a\Sigma y - b\Sigma xy}{n-2}}$$

$$= \sqrt{\frac{74800 - (20\times 1200) - (10\times 5000)}{20-2}}$$

$= 6.67$

Correlation coefficient,

$$r = \frac{n\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{\left(n\Sigma x^2 - (\Sigma x)^2\right)\left(\left(n\Sigma y^2 - (\Sigma y)^2\right)\right)}}$$

$$= \frac{20\times 500 - 80\times 1200}{\sqrt{\left(20\times 340 - (80)^2\right)\left(20\times 74800 - (1200)^2\right)}}$$

$= 0.84$

As ‘r’ closer to '1' i.e., good correlation

Chapter 6 Queuing Theory

01. Ans: (a)

Sol: $\lambda = 3$ per day

$\mu = 6$ per day

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{3}{6(6-3)} = \frac{1}{6}\text{day}$$

02. Ans: (c)

Sol: $\lambda = 0.35\ \text{min}^{-1}$,

$\mu = 0.5\ \text{min}^{-1}$

$$P_n = \left[1-\frac{\lambda}{\mu}\right]\left[\frac{\lambda}{\mu}\right]^n$$

$$= \left[1-\frac{0.35}{0.5}\right]\left[\frac{0.35}{0.5}\right]^8 = 0.0173$$

03. Ans: (a)

Sol: $\lambda = 10\ \text{hr}^{-1}$,

$\mu = 15\ \text{hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{15(15-10)} = 1.33$$

04. Ans: (b)

Sol: $\lambda = 4\ \text{hr}^{-1}$, $\mu = \frac{60}{12} = 5\ \text{hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{4^2}{5(5-4)} = \frac{16}{5} = 3.2$$

05. Ans: (b)

Sol: $$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu^2\left(1-\frac{\lambda}{\mu}\right)} = \frac{\rho^2}{(1-\rho)}$$

06. Ans: (d)

Sol: $\lambda = \frac{1}{4} = 0.25\text{min}^{-1}$

$\mu = \frac{1}{3} = 0.33\ \text{min}^{1}$

$\rho = \frac{\lambda}{\mu} = \frac{0.25}{0.33} = 0.75$

07. Ans: (b)

Sol: $\lambda = \frac{1}{10} = 0.1\text{min}^{-1}$

$\mu = \frac{1}{4} = 0.25\,\text{min}^{-1}$

System busy $\Rightarrow$ $(\rho) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$

08. Ans: (c)

Sol: $\lambda = 4\text{hr}^{-1}$, $\mu = 6\ \text{hr}^{-1}$

$$P(Q_S \geq 2) = \left(\frac{\lambda}{\mu}\right)^2$$

$$= \left(\frac{4}{6}\right)^2 = \frac{4}{9}$$

09. Ans: (c)

Conventional Practice Solutions

01.

Sol: $\lambda = 8\ hr^{-1}$; $\mu = \frac{60}{5} = 12 hr^{-1}$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{12-8} = \frac{1}{4}$$

02.

Sol: $\lambda = 100\ h^{-1}$; $\mu = 120\ h^{-1}$

$$\rho = \frac{\lambda}{\mu} = \frac{100}{120} = \frac{10}{12}$$

P_0 (no customer in the system)

$$= 1 - \rho = 1 - \frac{10}{12} \Rightarrow \frac{2}{12} = \frac{1}{6}$$

03.

Sol: $\lambda = 8\ h^{-1}$

$$\mu = \frac{60}{5} h^{-1} = 12\ h^{-1}$$

(a) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(8)^2}{12 \times 4} = 1.33$

(b) $L_s = \frac{\lambda}{(\mu-\lambda)} = \frac{8}{12-8} = 2$

(c) $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{8}{12 \times 4} = 0.1666$

(d) $W_s = \frac{1}{(\mu-\lambda)} = \frac{1}{4} = 0.25$

(e) $\rho = \frac{\lambda}{\mu} = \frac{8}{12} = 0.666$

04.

Sol: $\lambda = 20\ h^{-1}$; $\mu = \frac{60}{2} h^{-1} = 30\ h^{-1}$

(a) $P_0 = \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{20}{30}\right) = \frac{1}{3}$

(b) $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20}{30(30-20)} = 0.066$

(c) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(20)^2}{30(30-20)} = 1.33$

(d) $\rho = \frac{\lambda}{\mu} = \frac{20}{30} = 0.66$

05.

Sol: $\lambda = 2\ hr^{-1}$, $\mu = 5\ hr^{-1}$

a) Traffic intensity (ρ) = $\frac{\lambda}{\mu} = \frac{2}{5} = 0.4$

b) No customer $\Rightarrow$ service facility idle

$$P_0 = 1 - \rho = 1 - 0.4 = 0.6$$

c) The probability that there is no customer waiting to be served = Probability that atmost 1 customer at the counter who is getting the service or no one in the counter = $P_0 + P_1$

$$P_0 + P_1 = \left(1 - \frac{\lambda}{\mu}\right) + \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(1 + \frac{\lambda}{\mu}\right) = 1 - \left(\frac{\lambda}{\mu}\right)^2$$

$$= 1 - 0.16 = 0.84$$

As $\mu > \lambda \Rightarrow L_q$ is finite

If $\mu = \lambda \Rightarrow L_q$ is infinite

06.

Sol: **A** **B**

$\lambda = 3\ hr^{-1}$ $\lambda = 3\ hr^{-1}$

$\mu = 6\ hr^{-1}$ $\mu = 4\ hr^{1}$

NPC/hr = 15 Rs NPC/hr = 15

LC/hr = 20 LC/hr = 12

L_S represents non productive machining

$L_S = \dfrac{\lambda}{\mu - \lambda}$ $L_S = \dfrac{\lambda}{\mu - \lambda}$

$= \dfrac{3}{6-3} = 1$ $= \dfrac{3}{4-3} = 3$ m/c

NPC/hr =1×15Rs NPc/hr =3×15 = Rs. 45

LC/hr = 20/-

"A" should be hired

Chapter 7	**Sequencing & Scheduling**

01. Ans: (a)

Sol: SPT rule

Job	Process time (days)	Completion time
1	4	4
3	5	9
5	6	15
6	8	23
2	9	32
4	10	42
	$\Sigma C_i =$	**125**

Average Flow Time $= \dfrac{\sum C_i}{n}$

$= \dfrac{125}{6} = 20.83$

02. Ans: (a)

Sol: According to SPT rule total inventory cost is minimum.

03. Ans: (d)

Sol: EDD rule can minimize maximum lateness.

The job sequence is **R – P – Q – S**

04. Ans: (d)

Sol: Johnson's rule :

Optimum job sequence **III – I – IV – II**

Do the job 1st if the minimum time happens to be on the machine (M) and do it on the end if .it is on second machine (N). Select either in case of a tie.

05. Ans: (b)

Sol:

Job	M			N			Idle
	In	PT	Out	In	PT	Out	
III	0	1	1	1	2	3	-
I	1	3	4	4	6	10	1
IV	4	7	11	11	5	16	1
II	11	5	16	16	2	18	-

Total idle time on machine (N) = 3

06. Ans: (a)

Sol: Optimum sequence of jobs

2	3	1	4

07. Ans: (b)

Sol: Optimum sequence is

R	T	S	Q	U	P

Job	M_1			M_2		
	In	PT	Out	In	PT	Out
R	0	8	8	8	13	21
T	8	11	19	21	14	35
S	19	27	46	46	20	66
Q	46	32	78	78	19	97
U	78	16	94	97	7	104
P	94	15	109	109	6	115

The optimal make-span time = 115 days

08. Ans: (c)

Conventional Practice Solutions

01.

Sol: SPT rule is used for minimizing mean flow time

Job	t_i	C_i	d_i	$C_i - d_i$	
4	2	2	9	-7	→ E J
2	3	5	12	-7	→ E J
1	5	10	10	--	→ OS
5	6	16	8	8	→ T J
3	8	24	20	4	→ T J
		$\sum C_i = 57$			

EJ - EARLY JOB ,

OS - ON SCHEDULE

TJ - TARDY JOB

Minimum total cost = 57 × 60 = 3,420

Number of jobs which fail to meet due date are 2.

02.

Sol: **SPT** – rule minimizes average flow time

Job	T_i	C_i	D_i	$C_i - D_i$	T_j
5	2	2	15	-13	0
2	2	4	21	-17	0
1	3	7	17	-10	0
4	4	11	12	-1	0
6	4	15	24	-9	0
3	9	24	5	19	19
		$\sum C_i = 63$		$\sum C_i - D_i = 49$	19

Mean Flow Time, MFT = $\frac{63}{6} = 10.5$

Mean Tardiness, MT = $\frac{19}{6} = 3.17$

No. of tardy job = 1

EDD – rule minimizes mean tardiness

Job	T_i	C_i	D_i	$C_i - D_i$	T_j
3	9	9	5	4	4
4	4	13	12	1	1
5	2	15	15	--	0
1	3	18	17	1	1
2	2	20	21	-1	0
6	4	24	24	0	0
		$\sum C_i = 99$	$\sum C_i - D_i = 6$		6

MFT= $\frac{\Sigma C_i}{n} = \frac{99}{6} = 16.5$

MT = $\frac{\Sigma C_i - D_i}{n} = \frac{6}{6} = 1$

T_i = Process Time, C_i = Completion Time

D_i = Due Date ,

No. of tardy job = 3

03.

Sol:

FCFS	EDD		SPT	LPT	STACK	STACK	
	(or)					(or)	
A	A	F	C	A	1–10=–9	A	A
B	F	A	F	B	9 – 7=2	E	F
C	E	E	E	D	7 – 2 =5	F	E
D	C	C	D	E	7 – 6 =1	D	D
E	D	D	B	F	2 – 5= –3	B	B
F	B	B	A	C	1 – 4= –3	C	C

Note:

Stack = Due Date (DD) – Processing time (P.T)

04. Ans: F-C-G-B-E-D-A

Sol: Calendar date required (CDR)

Processing time (PT)

Process time remained (PTR)

Job	CDR	PT	Critical ratio $= \frac{\text{CDR} - \text{Todays date}}{\text{PTR}}$
A	190	5	(190–175)/5 = 3 $\rightarrow$ Ahead of schedule
B	178	2	(178–175)/2 = 1.5 $\rightarrow$ Ahead of schedule
C	184	10	(184–175)10 = 0.9 $\rightarrow$ Behind schedule
D	181	3	(181–175)/3 = 2 $\rightarrow$ Ahead of schedule
E	205	17	(205–175)/17 = 1.76 $\rightarrow$ Ahead of schedule
F	187	15	(187–175)/15 = 0.8 $\rightarrow$ Behind schedule
G	184	9	(184–175)/9 = 1 $\rightarrow$ on schedule

If critical ratio is one job will be on schedule.

If critical ratio is less than one job will be behind schedule.

If critical ratio is greater than one job will be ahead of schedule.

05.

Sol:

Job	T_j	F_j	D_j	L_j	T_j = max of (0, L_j)
a	8	8	9	-1	0
b	7	15	18	-3	0
c	9	24	21	3	3
d	12	36	38	-2	0
e	14	50	41	9	9
f	10	60	60	0	0

(i) Make-span time = 60 days

(ii) Mean flow time = $\frac{\Sigma F_y}{n} = \frac{193}{6} = 32.16$

(iii) No. of tardy jobs = 2 (c & e)

(iv) Mean tardiness, $\overline{T} = \frac{\Sigma T_j}{n} = \frac{12}{6} = 2$

06.

Sol: Sequence by Johnson's Rule is:

6, 3, 4, 1, 2, 5

Job	DENTER T_{in}	DENTER T_{out}	PAINTER T_{in}	PAINTER T_{out}
6	0	1	1	7
3	1	3	7	12
4	3	8	12	16
1	8	12	16	19
2	12	22	22	24
5	22	28	28	30

Minimum Make Span = 30

07.

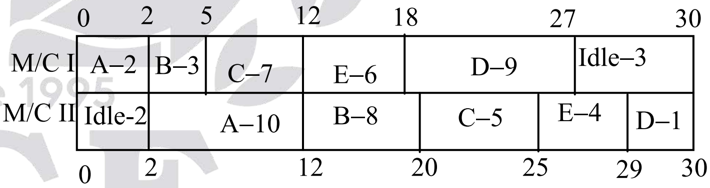
Sol: Optimum sequence :

A	B	C	E	D

TABULAR METHOD :

Job	M/C -I		M/C - II	
	T_i	T_0	T_i	T_0
A	0	2	2	12
B	2	5	12	20
C	5	12	20	25
E	12	18	25	×29
D	18	27	29	(30)
Processing time	27		28	
Idle time	30–27=3		(30–28=2)	
%utilization	$\frac{27}{30}\times 100$		$\frac{28}{30}\times 100$	

GANTT CHART



08.

Sol: Optimum Sequence :

A	C	D	B	E

PT = processing time

Job	Machine – 1			Machine – 2			Idle Time
	In	PT	Out	In	PT	Out	
A	0	2	2	2	4	6	-
C	2	5	7	7	6	13	1
D	7	6	13	13	7	20	-
B	13	7	20	20	8	28	-
E	20	5	25	28	3	31	-

Minimum time for completion of all jobs = 31

09.

Sol: *Condition :* Max (t_{2j}) ≤ Min (t_{ij} or t_{3j})

4 ≤ 4 or 4

Comp	X	M	W
N	8	3	5
A	4	4	6
O	7	3	7
L	5	4	8
E	6	4	4

Since the condition is satisfied, we can create two virtual Machines 'G' & 'H'.

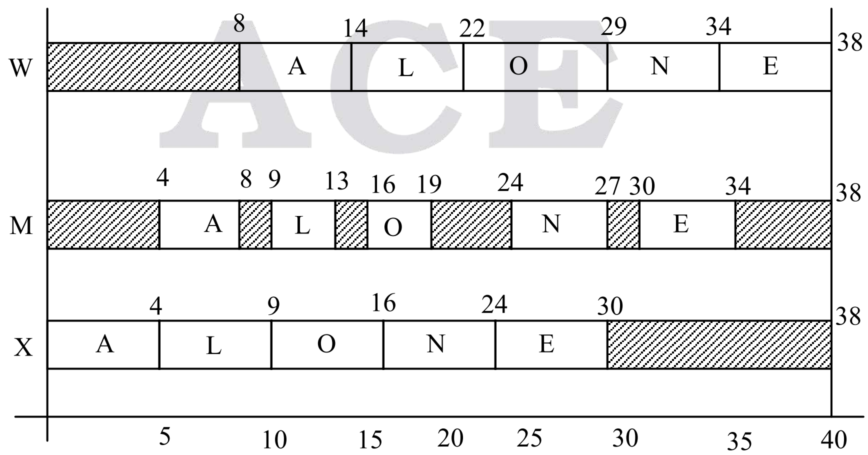
$X = t_{1j}$, $M = t_{2j}$, $W = t_{3j}$

Comp	Machine G (X+M)	Machine H (M+W)
N	11	8
A	8	10
O	10	10
L	9	12
E	10	8

Optimum sequence :	A	L	O	N	E

Comp	Machine X			Machine M			Idle	Machine W			Idle
	In	PT	Out	In	PT	Out		In	PT	Out	
A	0	4	4	4	4	8	4	8	6	14	8
L	4	5	9	9	4	13	1	14	8	22	-
O	9	7	16	16	3	19	3	22	7	29	-
N	16	8	24	24	3	27	5	29	5	34	-
E	24	6	30	30	4	34	3	34	4	38	-

Gantt Chart :



(iii) % utilization :

$$\text{Machine X} = \frac{30}{38}\times 100 = 78.94\%$$

$$\text{Machine m} = \frac{38-20}{38}\times 100 = 47.73\%$$

$$\text{Machine W} = \frac{38-8}{38}\times 100 = 78.94\%$$

10.

Sol: Optimum Sequence :

D	C	E	F	G	B	A

	Machines			**Polish**			**Idle**
	In	**PT**	**Out**	**In**	**PT**	**Out**	
D	0	4	4	4	5	9	4
C	4	5	9	9	12	21	-
E	9	6	15	21	9	30	-
F	15	9	24	30	11	41	-
G	24	7	31	41	6	47	-
B	31	6	37	47	3	50	-
A	37	10	47	50	2	52	-

Minimum flow time = 52

11.

Sol: The given machine sequence is 'ACB' hence, we need to re-arrange the given data

Job	**A**	**C**	**B**
1	5	2	3
2	7	1	7
3	6	4	5
4	9	5	6
5	5	3	7

$\text{Max}\{t_{2j}\} \le \min\{t_{1j} \text{ or } t_{3j}\}$

$5 \le 5 \text{ or } 3$

Job	**Machine G (A+C)**	**Machine H (C+B)**
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

<u>Optimum sequence 1</u>

Machine G				**Machine H**
5	4	3	2	1

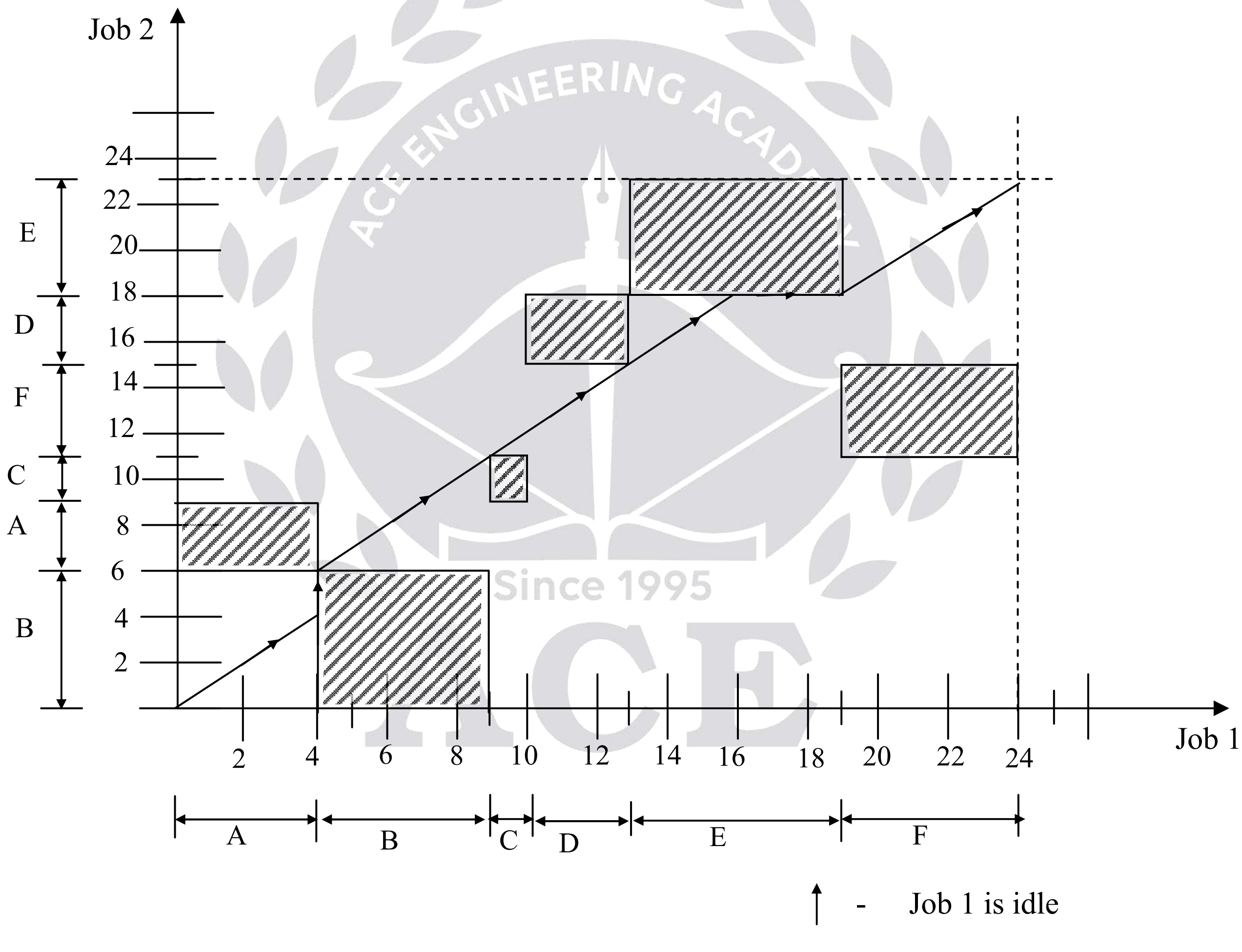
<u>Optimum sequence 2</u>

Machine G				**Machine H**
2	5	4	3	1

Job	A			B				C			
	In	PT	Out	In	PT	Out	Idle	In	PT	Out	Idle
5	0	5	5	5	3	8	5	8	7	15	8
4	5	9	14	14	5	19	6	19	6	25	4
3	14	6	20	20	4	24	1	25	5	30	–
2	20	7	27	27	1	28	3	30	7	37	–
1	27	5	32	32	2	34	4	37	3	40	–

12.

Sol:



21+3 = 24 → Job 2

21+2 = 23 → Job 1

Chapter 8 Transportation Model

01. Ans: (c)

Sol: A no. of allocations : $m + n - 1$

$\Rightarrow 5 + 3 - 1 = 7$

02. Ans: (a)

Sol: For degeneracy in transportations, number of allocations < $(m + n) - 1$

where m = no. of rows,

n = no. of columns

03. Ans: (b)

Sol: In Transportation problem for solving the initial feasible solution for total cost, Vogel's approximation methods are employed for obtaining solutions which are faster than LPP due to the reduced number of equations for solving.

Optimality is reached using MODI/ U-V method or stepping stone method.

04. Ans: (b)

Sol: It generates the best initial basic feasible solution. This method is the best choice in order to get an optimal solution within minimum number of iterations**.**

The Vogel's approximation method is also known as the penalty method.

05. Ans: (a)

Sol: No. of allocations = 5

$\therefore$ no. of allocations = $m + n - 1$

$m + n - 1 = 4 + 3 - 1$

$\therefore$ It is a degenerate solution

06. Ans: (a)

Sol:

	1	2	3	4	Supply
A	[10]	[2] 5	[20]	[11] 10	15
B	[12]	[7] 10	[9] 15	[20]	25
C	[5] 5	[14]	[16]	[18] 5	10
Demand	5	15	15	15	50 / 50

Evaluation of empty cells:

Cell (A1) Evaluation = $C_{A1} - C_{A4} + C_{C4} - C_{C1}$

$= 10 - 11 + 18 - 5 = 12$

Cell (A3) Evaluation = $C_{A3} - C_{A2} + C_{B2} - C_{B3}$

$= 20 - 9 + 7 - 2 = 16$

Cell (B1) Evaluation $= 12 - 7 + 2 - 11 + 18 - 4 = 10$

Cell (B4) Evaluation $= 20 - 7 + 2 - 11 = 4$

Cell (C2) Evaluation $= 14 - 2 + 11 - 18 = 5$

Cell (C3) Evaluation $= 16 - 9 + 7 - 2 - 18 = 5$

If cell cost evaluation value is '–ve', indicates further unit transportation cost is decreasing and if cost evaluation value is '+ve' indicates further unit transportation cost is increases. If cost evaluation value is zero, unit transportation cost doesn't change.

∴ As for A3 cell cost evaluation is +16, means that, if we transport goods to A3 the unit transportation cost is increased by 16/-.

Common Data for Questions Q07, Q08 & Q09 :

07. Ans: (b) **08. Ans: (a)**

09. Ans: (b)

Sol:

	1	2	3	4
A	[6]	[1]	[9] 25	[3] 45
B	[11] 30	[5]	[2] 25	[8]
C	[10] 55	[12] 35	[4]	[7]

No. of allocations = 6

R + C – 1 = 6

As No. of allocations = R + C – 1

Hence the problem is not degeneracy case.

Opportunity cost of cell (i, j) is

$C_{ij} - (U_i + V_j)$

If $C_{ij} - (U_i + V_j) \geq 0 \Rightarrow$ problem is optimal,

Empty cell evaluation (or) Opportunity cost of cells:

$A_1 = -12$, $A_2 = -19$, $B_2 = -8$

$B_4 = 12$, $C_3 = 3$, $C_4 = 12$

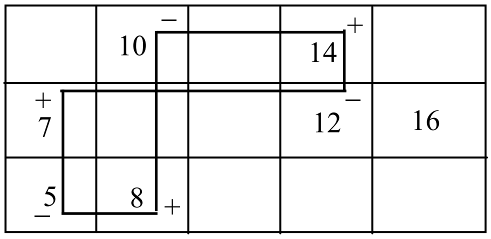
From the above as A2 has opportunity cost '–19' indicates unit transportation cost is decreased by 19/-

By forming loop A2, A3, B2, B3 it is observed that to transport minimum quantity is 25 among 25, 30, 35.

∴ The reduction in the transportation cost is 25 × 19 = 475

10. Ans: (c)

Sol:

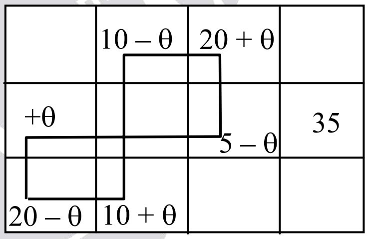


By stepping stone method,

Cell evaluation of B – 1 cell

= +7 – 5 + 8 – 10 + 14 – 12

= 2/-



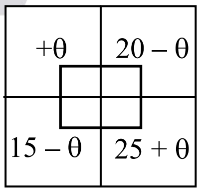
θ = minimum of |10 – θ, 5 – θ, 20 – θ| = 0

θ = 5 units

Increase in cost = 5 × 2 = 10/-

11. Ans: (c)

Sol: To find the number units shifted to A_2 cell.



θ = minimum value of |15–θ, 20–θ| = 0

θ = 15 units

Conventional Practice Solutions

01.

Sol: Total supply = 80+ 60 + 40 + 20 = 200 & Total demand = 60 + 60 + 30 + 40 +10 = 200

∴ Total supply = Total demand

The problem is balanced

Source \ Destination	1	2	3	4	5	Available
A	4	3	1	2	6	80
B	5	2	3	4	5	60
C	3	5	6	3	2	40
D	2	4	4	5	3	20
Required	60	60	30	40	10	**200**

(i) By North West Corner rule :

	1	2	3	4	5	Supply
A	[60] 4	[20] 3	1	2	6	80 / 20 /0
B	5	[40] 2	[20] 3	4	5	60 / 20 / 0
C	3	5	[10] 6	[30] 3	2	40 / 30 / 0
D	2	4	4	[10] 5	[10] 3	20 / 0
Demand	60 / 0	60 / 40	30 / 10	40 / 10	10 / 0	

Total transportation cost = 4×60 + 3×20 + 2×40 + 3×20 + 6×10 + 3×30 + 5×10 + 3×10 = 670 /-

02.

Sol: Total supply = 14 + 16 + 5 = 35

Total demand = 6 + 10 + 15 + 4 = 35

∴ Total supply = Total demand

It is a balanced transportation model

(i) By North West corner rule

	S_1	S_2	S_3	S_4	Supply
W_1	[6] 15	[8] 22	26	25	14 / 8 /0
W_2	36	[2] 38	[14] 18	40	16 / 14 / 0
W_3	45	35	[1] 60	[4] 52	5/ 0
Demand	6 / 0	10 / 2 / 0	15 / 1 / 0	4 / 0	35 / 35

Transportation cost = 15 × 6 + 22 × 8 + 38 ×2 + 18 × 14 + 60 × 1 + 52 × 4 = 862 /-

(ii) Least Cost Method :

	S_1	S_2	S_3	S_4	Supply
W_1	[6] 15	[8] 22	26	25	14 / 8 /0
W_2	36	38	[15] 18	[1] 40	16 / 1 / 0
W_3	45	[2] 35	60	[3] 52	5/ 3 / 0
Demand	6 / 0	10 / 2 / 0	15 / 0	4 / 3 / 0	35 / 35

Transportation cost = 15×6 + 22×8 + 18×15 + 40×1 + 35×2 + 52×3 = Rs. 802 /-

(iii) **VAM**

Step 1: Find out the difference between least and next highest numbers for rows and columns. Which is called as the penalty.

Step 2: Select the maximum penalty row and column and allocate the maximum possible amount to the box with least cost.

	S_1	S_2	S_3	S_4	Supply				
W_1	[6] 15	[5] 22	26	[3] 25	14 / 8 / 3 / 0	7	3	3	1
W_2	36	38	[15] 18	[1] 40	16 / 1 / 0	18	10	10	22
W_3	45	[5] 35	60	52	5 / 0	10	17	-	-
Demand	6 / 0	10 / 5 / 0	15 / 0	4 / 0	35 / 35				
	21	13	8	15					
	-	13	8	15					
	-	16	6	15					
	-	-	8	15					

Transportation cost = 15×6 + 22×5+ 25 × 3 + 18×15 + 40×1 + 35 ×5 = **760 /-**

Chapter 9 Assignment Model

01. Ans: (a)

Sol: Let C_{ij} = unit assignment cost

X_{ij} = Decision variable (allocation)

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{J=1}^{n} C_{ij} X_{ij}$$

$$\text{Subject to : } \sum_{i=1}^{n} X_{ij} = 1$$

$$\sum_{j=1}^{n} X_{ij} = 1$$

$X_{ij} = 1$ (when assigned)

$X_{ij} = 0$ (when not assigned)

- Number of decision variables = n^2 (or) m^2
- Number of basic variables = Number of assignments = n (or) m

02. Ans: (c)

03. Ans: (a)

04. Ans: (c)

Sol:

	S_1	S_2	S_3
P	110	120	130
Q	115	140	140
R	125	145	165

Row Transaction

	S_1	S_2	S_3
P	0	10	20
Q	0	25	25
R	0	20	40

Column Transaction

S_1	S_2	S_3
~~0~~	~~0~~	~~0~~
0	15	5
0	10	20

~~5~~	~~0~~	~~0~~
~~0~~	~~10~~	~~0~~
~~0~~	~~5~~	~~15~~

P-S_2 – 120

Q-S_3 – 140

R-S_1 – 125

Total = 385

05. Ans: (1-B, 2-D, 3-C, 4-A)

Sol: Step-1:

Take the row minimum of subtract it from all elements of corresponding row.

1	0	2	3
0	2	2	1
8	5	0	1
0	6	2	4

Step – 2 :

Take the column minimum & substract it from all elements of corresponding column.

1	0	2	2
0	2	2	0
8	5	0	0
0	6	2	3

Step – 3 :

Select single zero row or column and assign at the all where zero exists. If there is no single zero row or column. Then use straight line method.

	A	B	C	D
1	1	**[0]**	2	2
2	0	2	2	**[0]**
3	8	5	**[0]**	0
4	**[0]**	6	2	3

1 – B : 7
2 – D : 8
3 – C : 2
4 – A : 5

Total cost = 22

Conventional Practice Solutions

01.

Sol:

	A	B	C	D
1	10	5	15	13
2	3	9	8	18
3	10	7	2	3
4	5	11	7	9

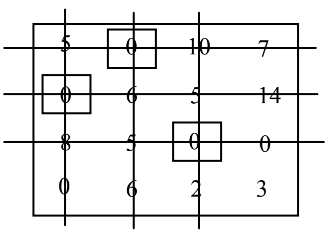
Step – 1 :

5	0	10	8
0	6	5	15
8	5	0	1
0	6	2	4

Step – 2 :

5	0	10	7
0	6	5	14
8	5	0	0
0	6	2	3

Step – 3

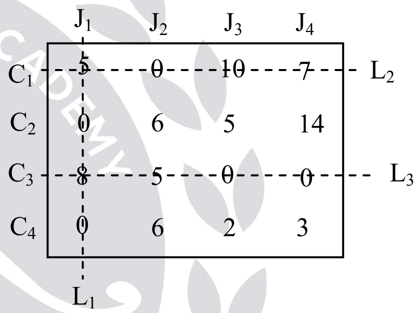


It may be noted there are no remaining zeroes and row – 4 and column – 4 each has no assignment. Thus optimal solution is not reached at this stage. Therefore, proceed to following important steps.

Step – 4 :

Draw the minimum number of horizontal and vertical lines necessary to cover all zeroes at least once.

Take the above Table



(i) Mark row – 4 in which there is no assignment

(ii) Mark column 1 which have zeroes in marked column.

(iii) Next mark row 2 because this row contains assignment in marked column 1.

No further rows or columns will be required to mark during this procedure.

(iv) Draw the required lines as follows.

(a) Draw L_1 through marked column 1

(b) Draw L_2 and L_3 through unmarked row (1 and 3)

Step – 5 :

Select the smallest element (2).

Among all the uncovered elements of the above table and substract this value from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines L_1, L_2,and L_3 and leaving the remaining element unchange.

	J_1	J_2	J_3	J_4
C_1	7	[0]	10	7
C_2	[0]	4	3	12
C_3	10	5	0	[0]
C_4	0	4	[0]	5

It may be added that there are no remaining zeroes and every row and column has an assignment.

Since, the no. of assignment = no. of row or column

∴ The solution is optimal

The pattern of assignment at which job has been assigned to each contractor.

Contractor	Job	Amount (Rs)×1000
C_1	J_2	5
C_2	J_1	3
C_3	J_4	3
C_4	J_3	7
		18×1000=18000

Minimum amount = Rs. 18,000/-

02.

Sol:

	Job 1	Job 2	Job 3	Job 4	
A	20	36	31	27	
B	24	34	45	22	
C	22	45	38	18	
D	37	40	35	28	
A	0	16	11	7	Row Transaction
B	2	12	23	0	
C	4	27	20	0	
D	9	12	7	0	
A	[0]	4	4	7	Column Transaction
B	2	[0]	16	0	
C	4	15	13	[0]	
D	9	0	[0]	0	
	$A - J_1 \rightarrow 20$				
	$B - J_2 \rightarrow 34$				
	$C - J_4 \rightarrow 18$				
	$D - J_3 \rightarrow 35$				
	107				

03.

Sol: Here no. of rows ≠ no. of column

∴ The algorithm is not balanced so add one dummy column.

Operates	Machine			
	A	**B**	**C**	**Dummy**
1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

Step – 1:

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Step – 2:

[0]	6	9	0
4	7	[0]	0
26	[0]	9	0
9	10	14	[0]

Here the operator – 4 is assigned to dummy column.

∴ He is the idle worker.

TC = 9 + 6 + 20 + 0 = 35

Chapter 10 PPC & Aggregate Planning

01. Ans: (d) **02. Ans: (b)**

03. Ans: (b)

Sol:

Months		Month 1	Month 2	Month 3	Unused capacity	Capacity Available
1	RT	90 [20]	10 [22]	[24]	10	100
	OT	[24]	[26]	[28]		20
2	RT		100 [20]	[22]		100
	OT		20 [24]	[26]		20
3	RT			80 [20]		80
	OT			30 [24]	10	40
	RT / OT	90	130	110		

Level of planned production in overtimes in 3^{rd} period is '30'.

RT = Regular time

OT = Over time

04. Ans: (b)

Sol:

Month	Cumulative Production	Cumulative Demand	Inventory		Cost	
			End	Stock out	End inventory	Stock out cost
1	100	80	20	-	40	-
2	180	180	-	-	-	-
3	250	260	-	10	-	100
4	320	300	20	-	40	-
					80	100
				Total	180	

05. Ans: (b) **06. Ans: (d)**

Conventional Practice Solutions

01. Ans:

Sol:

Supply from		Demand for					Total Capacity Available (supply)
		Period 1	Period 2	Period 3	Period 4	Un used capacity	
Beginning inventory		[200] 0	5	10	15	-	200
1	Regular	[700] 60	65	70	75	0	700
	Overtime	70	75	80	85	300	300
2	Regular		[500] 60	65	[200] 70	0	700
	Overtime		70	75	80	300	300
3	Regular			[200] 60	[500] 65	0	700
	Overtime			70	[200] 75	100	300
4	Regular				[700] 60	0	700
	Overtime				[300] 70	0	300
		900	500	200	1900	700	4200 / 4200

Total cost = (700 ×60) + (500 ×60) + (200 ×70) + (200 × 60) + (500 ×65) + (200 ×75) + (700 ×60)+ (300 ×70) = Rs 2,08,500/-

02. Ans:

Sol:

Supply from		Demand for Period1	Period2	Period3	Period4	Unused capacity	Total Capacity Available (supply)
Beginning Inventory		150 0	2	4	6	-	150
1	Regular	900 25	27	29	31	-	900
	Overtime	150 30	32	34	36	-	150
	Subcontract	200 35	-	-	-	100 -	300
2	Regular		600 25	27	29	-	600
	Overtime		125 30	32	34	-	125
	Subcontract		175 35	-	-	125 -	300
3	Regular			700 25	27	-	700
	Overtime			100 30	50 32	-	150
	Subcontract			35	-	300 -	300
4	Regular				800 25	-	800
	Overtime				200 30	-	200
	Subcontract				250 35	50 -	300
		1400	900	800	1200+100	575	4975 / 4975

Total cost = (900 ×25) + (150 × 30) + (200 ×35) + (600 ×25) + (125 ×30) + (175 ×35) + (700 ×25) + (100 ×30)+(50 ×32) + (800 × 25) + (200 ×30) + (250 ×35) = Rs 1,15,725/-

Chapter 11 Material Requirement & Planning

01. Ans: (b)

02. Ans: (c)

Sol: Based on master production schedule, a material requirements planning system :

- Creates schedules, identifying the specific parts and materials required to produce end items.
- Determines exact unit numbers needed.
- Determines the dates when orders for those materials should be released, based on lead times.

03. Ans: (d)

Sol: *Refer to the solution of Q.No. 02*

04. Ans: (c)

Sol: MRP has three major input components:

1. Master production Schedule of end items required. It dictates gross or projected requirements for end items to the MRP system.
2. Inventory status file of on-hand and on-order items, lot sizes, lead times etc.
3. Bill of materials (BOM) or Product structure file what components and sub assemblies go into each end product.

05. Ans: (c)

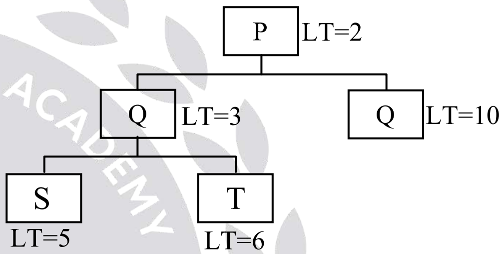
06. Ans: (c)

07. Ans: (b)

08. Ans: (b)

09. Ans: (c)

Sol:



Maximum Lead time = 12 weeks

Conventional Practice Solutions

01.

Sol: A → 1 × 10 = 10

B → 2 × 10 = 20

C → (1 × 2 ×10) + (3 × 4 × 2 ×10) = 260

D → (4 × 2 ×10) = 80

E → (3 × 4 × 2 ×10) + (2 × 2 ×10) + (4×10) = 320

02.

Sol:

Order Quantity = 200 LT = 3 Weeks	Week							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Project required	40	85	10	60	130	110	50	170
Receipts				200		200		200
On hand inventory	100	15	5	145	15	105	55	85
Planned order release	200		200		200			

$(\text{On hand inventory})_t$

1^{st} week = 140 + 0 – 40 = 100

2^{nd} week = 100 + 0 – 85 = 15

3^{rd} week = 15 + 0 – 10 = 5

4^{th} week = 5 + 200 – 60 = 145

5^{th} week = 145 + 0 – 130 = 15

6^{th} week = 15 + 200 – 110 = 105

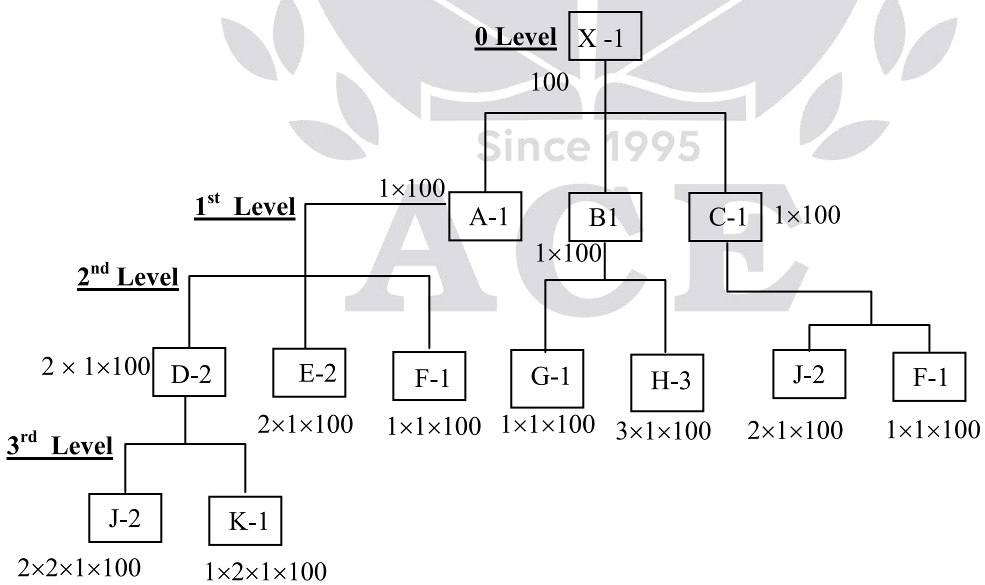
7^{th} week = 105 + 0 – 50 = 55

8^{th} week = 55 + 200 – 170 = 85

∴ Order before 3-weeks

03.

Sol:



A = (1 ×100) = 100

B = (1 ×100) = 100

C = (1 × 100) = 100

D = (2 ×1 × 100) = 200

E = (2 ×1 × 100) = 100

F = (1 ×1 × 100 + 1 ×1 × 100) = 200

G = (1 ×1 × 100) = 100

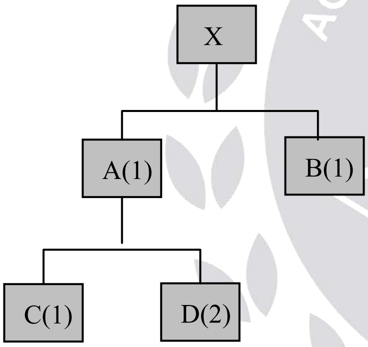
H = (3 ×1 × 100) = 300

J = (2 ×1 × 100 + 2 ×2× 1× 100) = 600

K = (1 ×2 ×1 × 100) = 200

04.

Sol:



Net required

A = (1 × 1 × 20 −10) = 10

B = 1 × 20 −1×5 = 15

C = (1 × 1 × 20–1×10 −10) = 0

D = 2 × 1 × 20 – 2 × 10 −10 = 10

Chapter 12 Break Even Analysis

01. Ans: (c)

Sol: Total fixed cost, TFC = Rs 5000/-

Sales price, SP = Rs 30/-

Variable cost, VC = Rs 20/-

Break even production per month,

$$Q^* = \frac{TFC}{SP - VC} = \frac{5000}{30 - 20} = 500 \text{ units}$$

02. Ans: (a)

Sol: Total cost = 20 + 3X --------------(1)

Total cost = 50 + X ------------(2)

By solving equ. (1) and (2)

2X = 30

∴ X = 15 units

When X = 10 units

$TC_1 = 20 + (3 \times 10) = $ Rs 50/-

$TC_2 = 50 + (1 \times 10) = $ Rs 60/-

Among both, total cost for process is less So process-1 is choose.

03. Ans: (c)

Sol: In automated assembly there are less labour, so variable cost is less, but fixed is more because machine usage is more. In job shop production, labour is more but machine is less. So variable cost is more and fixed cost is less.

04. Ans: (c)

Sol: TC = Total cost

TC_A = Total cost for jig-A

TC_B = Total for jig-B

$$TC_A = TC_B$$

$$800 + 0.1X = 1200 + 0.08X$$

$$0.02X = 400$$

$$\therefore X = \frac{400}{0.02} = \frac{400}{2}\times 100 = 20{,}000 \text{ units}$$

05. Ans: (d)

Sol: Sales price – Total cost = Profit

$$(C_P \times 14000) - (47000 + 14000\times 15) = 23000$$

$$\therefore C_P = 20$$

06. Ans: (b)

07. Ans: (a) **08. Ans: (c)**

09. Ans: 1500

Sol:

X	Y
$S_1 = 100$	$S_2 = 120$
$F_1 = 20{,}000$	$F_2 = 8000$
$V_1 = 12$	$V_2 = 40$

$$P = q(S - V) - F$$

$$P_1 = q(100 - 12) - 20{,}000$$

$$P_2 = q(120 - 40) - 80{,}000$$

$$P_1 = P_2$$

$$88q - 20{,}000 = 80q - 80{,}000$$

$$12000 = 8q$$

$$\Rightarrow q = 1500$$

10. Ans: (b)

11. Ans: (c)

Sol: At breakeven point

Total cost = Total revenue

$$FC + VC \times Q = SP \times Q$$

$$Q = \frac{FC}{(SP - VC)}$$

FC = 1000/- , VC = 3/-, SP = 4/-

$$Q = \frac{1000}{(4-3)} = 1000 \text{ units}$$

If sales price is increased to 25%

$$SP = 4 + \frac{1}{4}\times 4 = 5/-$$

$$Q^* = \frac{1000}{(5-3)} = 500 \text{ units}$$

$\therefore$ Breakeven quantity decreases by

$$\frac{100-500}{100}\times 100 = 50\%$$

Conventional Practice Solutions

01. Ans: (d)

Sol:

	Standard machine tool	Automatic machine tool
F_1 = F.C.	$\frac{30}{60}\times 200 = \text{Rs.}100$	2×800 = Rs.1600 = F_2
V.C	$= \frac{20}{60}\times 200$ = Rs. 73.33	$= \frac{5}{60}\times 800$ = Rs. 66.67

$$q = \frac{1600-100}{73.33-66.67} = 225\,\text{volts}$$

If greater than 225 units then automatic machine tool is economic.

02. Ans: 16

Sol: Preparation cost for

Conventional lathe = 30,

CNC lathe = 150

Production time of

Conventional lathe = 30 min,

Variable cost per hour

Conventional lathe = 75 per hour

$$= \frac{75}{60}\times 30 \text{ per product}$$

CNC lathe = 120 per hour

$$= \frac{120}{60}\times 15 \text{ per product}$$

Total cost for Q products

Conventional lathe = 30 + 37.5 Q

CNC lathe = 150 + 30 Q

At break even quantities

$$(TC)_1 = (TC)_2$$

$\Rightarrow$ 30 + 37.5 Q = 150 + 30 Q

$\Rightarrow$ 7.5 Q = 120

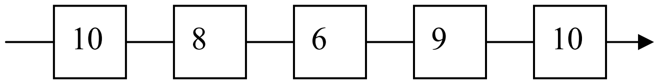
$\Rightarrow$ Q = 16

$\therefore$ CNC lathe is economical when production per day is above 16.

Chapter 13 Line Balancing

01. Ans: (c)

Sol:



$\sum t_i = 43$; n = 5; C = 10

$$\text{Balance delay} = 1 - \frac{\sum t_i}{nC}$$

$$= 1 - \frac{43}{5\times 10}$$

= 0.14 or 14 %

02. Ans: (d)

Sol: $$\text{Cycle Time} = \frac{\text{Total time}}{\text{Total production}}$$

$$= \frac{8\times 60\times 60}{3000}$$

$$CT = \frac{48}{5} = 9.6 \text{ seconds}$$

Time to assemble one unit

= 10+20+15+5+30+15+5 = 100 sec

No. of work station

$$= \frac{\text{Time to assemble one unit}}{\text{Cycle Time} \times \eta}$$

$$= \frac{100}{9.6} = 11 \quad (\text{consider } \eta = 100\%)$$

03. Ans: (c)

Sol: Assembly line balancing :

Line balancing is done to meet the production rate for a given time, minimizing the idle time and maximizing the work output. As the time is minimized, the idle time at the stations decreases, decreasing the in-process inventory.

Statements 1, 3, 4 apply to the benefits of assembly line balancing.

04. Ans: (c)

Sol: $\text{Cycle Time} = \frac{\text{Total time}}{\text{Total production}}$

$$= \frac{8 \times 60}{320} = 1.5\,\text{min}$$

Time to assemble one unit

= 1.3 + 1.5 + 1.4 + 1.5 + 1.3 = 7 min

No. of work station

$$= \frac{\text{Time to assemble one unit}}{\text{Cycle Time}}$$

$$= \frac{7}{1.5} = \frac{14}{3} \approx 5$$

$$\eta = \frac{\text{Time to assemble one unit}}{\text{No. of work stations} \times \text{Cycle Time}}$$

$$= \frac{7}{5 \times 1.5} = 0.93$$

05. Ans: (d)

Sol: $\text{Cycle Time} = \frac{480 \times 60}{1450} = 19.87\ \text{sec}$

$$\text{No. of work station} = \frac{310}{19.87} = 15.6 \approx 16$$

$$\eta = \frac{310}{16 \times 19.87} \times 100 = 97.5\%$$

06. Ans: (a)

Sol: Cycle time is equal to the time of the bottleneck operation or the maximum station time.

Conventional Practice Solutions

01.

Sol:

Work stations	**Work elements**	**Work element times resp**	**Total time/w_s**
I	A, B	4, 3	7
II	C	8	8
III	D, F	4, 4	8
IV	E	6	6
V	G	5	5
VI	H	6	6

Cycle time = 8 minutes

$$\text{No. of unit produced} = \frac{60 \times 8}{8} = 60$$

Line efficiency $\eta = \frac{40}{8\times 6}\times 100 = 23.33$

If cycle time = 11 min

By combining G&H ⇒ no. of work station can be reduced by two.

02.

Sol: Available time for production

$= 5\times 7\times 3600$ sec.

No. of units produced = 8400 units

$$\text{Cycle time } C = \frac{\text{Available time for production}}{\text{No. of units produced}}$$

$$= \frac{5\times 7\times 3600}{8400}$$

= 15 sec/unit

Time to assemble one unit = 130 sec (sum of all elements operation times)

Theoretical no. of work stations

$$= \frac{a}{c} = \frac{130}{15} = 8.67 \cong 9$$

Theoretical efficiency

$$= \frac{\text{Time to assemble one unit}}{\text{cycle time} \times \text{theoritical no. of work stations}}\times 100$$

$$= \frac{130}{9\times 15}\times 100$$

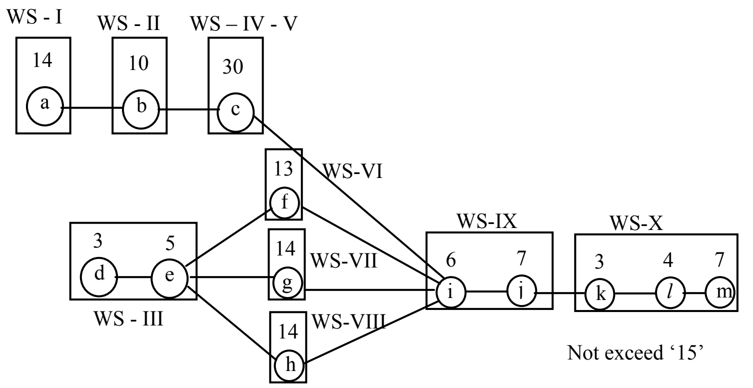
= 96.29%

Theoretical balance delay = $100 - \eta_{th}$

$= 100 - 96.29$

$= 3.71\%$

Technological precedence diagram :



Actual no. of work stations are 10

$$\text{Actual efficiency} = \eta = \frac{\text{Time to assemble one unit}}{\text{Actual no. of work station} \times \text{cycle time}}\times 100$$

$$= \frac{130}{10\times 15}\times 100 = 86.67\%$$

Actual balance delay = $100 - \eta_{act}$

$= 100 - 86.67 = 13.33\ \%$

Work Station	Element allotted	Time (sec)
I	a	14
II	b	10
III	b , e	8
IV	c	15
V	c	15
VI	f	13
VII	g	14
VIII	h	14
IX	i, j	13
X	k, *l*, m	14

Smoothness index (SI) = $\sqrt{\sum_{i+1}^{n}(T-T_i)^2}$

Where T = cycle time

T_i = Time allotted to the highest work station

$$SI=\sqrt{\begin{aligned}&(15-14)^2+(15-10)^2+(15-8)^2+(15-15)^2\\&+(15-15)^2+(15-13)^2+(15-14)^2\\&+(15-14)^2+(15-13)^2+(15-14)^2\end{aligned}}$$

$$SI=\sqrt{1+25+289+4+1+1+4+1}$$

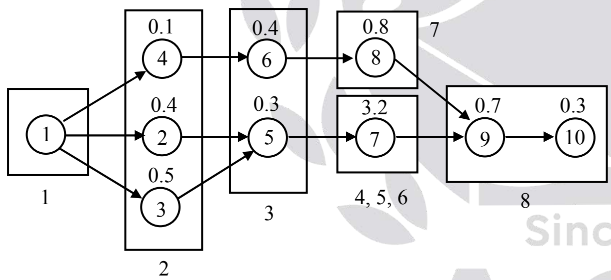
$$SI=\sqrt{326}$$

If SI is zero then it indicates 100% line efficiency

03.

Sol:

(i)



(ii) Given,

Available production Time, T = 8 hours

No. of units to be produced, N = 400 units

$$\text{Cycle time} = \frac{T}{N} = \frac{(8\times 60)-40}{400\,\text{units}}$$

= 1.1 min/unit station

The sum of the work element times

=1.1+0.4+0.5+1.1+0.3+0.4+3.2+0.8+0.7+0.3

= 8.8 = 528 sec/units

Theoretical no. of work stations

$$=\frac{\sum t_i}{\text{Cycle time}}=\frac{528\,\text{sec/unit}}{72\,\text{sec/unit}-\text{station}}$$

= 7.33 ≈ 8 stations

(iii) Cycle time = 72 sec/ unit – station

= 1.2 min/unit-station

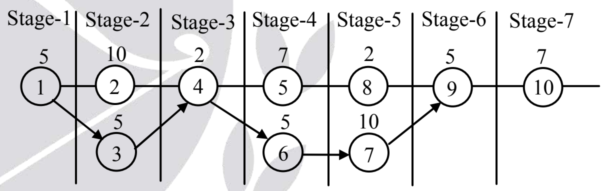
No. of work stations = 8

04.

Sol:

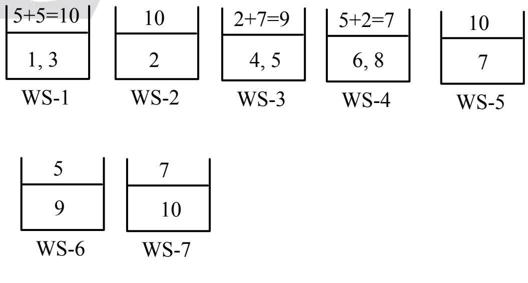
(a) By using Kil bridge and Wester method :

For all line balancing problems, we use activity one node.



Given cycle time = 10 minutes

WS = work station



Line efficiency, $\eta_{line} = \dfrac{\text{Total work time}}{n \times \text{cycle time}} \times 100$

$$= \frac{5+10+5+2+7+5+2+10+5+7}{7 \times 10} \times 100$$

$$= \frac{58}{70} \times 100 = 82.85\%$$

Work station	Idle time	(Idle time)^2
I	10–10 = 0	0
II	10–10 = 0	0
III	10–9 = 1	1
IV	10–7 = 3	9
V	10–10 = 0	0
VI	10–5 = 5	25
VII	10–7 = 3	9
		$\sum$(Idle time)^2=44

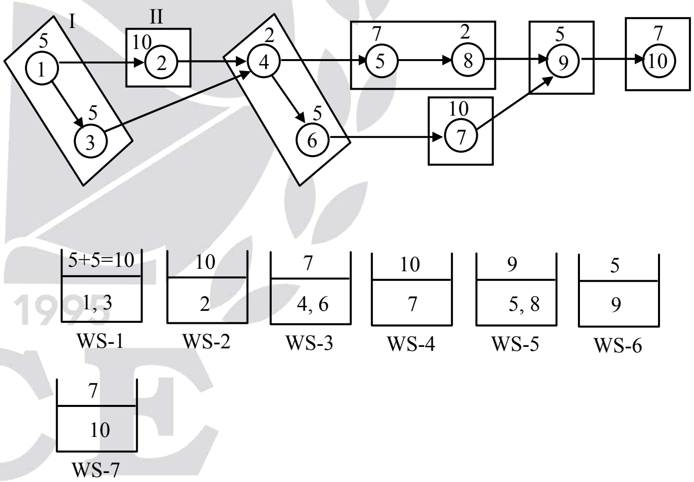
$\therefore$ Smoothing index = $\sqrt{44} = 6.63$

(b) Let's use Helgeron and Birnie method, which is also called as ranked position method

Element	Positional weights
10	7
9	12
7	22
8	14
5	21
6	27
4	38
3	43
2	48
1	58

Arrange the elements in the decreasing order

Element	Positional weights
1	58
2	48
3	43
4	38
6	27
5	21
7	22
8	14
9	12
10	7



Line balance efficiency

$$(\eta_{line}) = \frac{10+10+7+10+9+5+7}{7 \times 10} \times 100$$

$$= \frac{58}{70} \times 100 = 82.85\%$$

Work station	Idle time	(Idle time)^2
I	10 – 10 = 0	0
II	10–10 = 0	0
III	10–7 = 3	9
IV	10–10 = 0	0
V	10–9 = 1	1
VI	10–5 = 5	25
VII	10–7 = 3	9
		Σ (idle time)^2 = 44

$\therefore$ Smoothing Index = $\sqrt{44} = 6.63$