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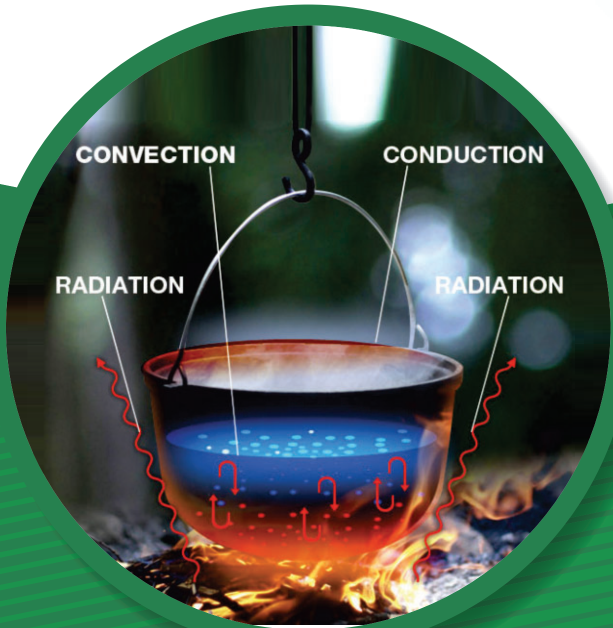


MECHANICAL ENGINEERING

HEAT TRANSFER

Text Book : Theory with worked out Examples and Practice Questions

Heat Transfer

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 Conduction

01. Ans: (b)

Sol: Given data:

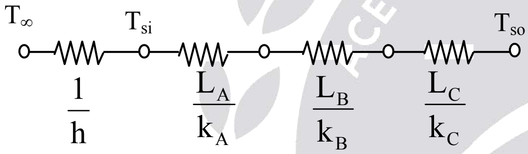
$T_{si} = 600°C$; $T_{so} = 20°C$;

$k_A = 20$ W/mK; $k_C = 50$ W/mK;

$L_A = 0.30$ m $L_B = 0.15$ m;

$L_C = 0.15$ m, $h = 25$ W/m^{2}K

Thermal circuit:



Energy balance:

Convective heat transfer at the wall surface = conductive heat transfer through the wall

$$\frac{T_\infty - T_{si}}{\frac{1}{h}} = \frac{T_{si} - T_{so}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}}$$

$$\frac{800 - 600}{\frac{1}{25}} = \frac{600 - 20}{\frac{0.30}{20} + \frac{0.15}{k_B} + \frac{0.15}{50}}$$

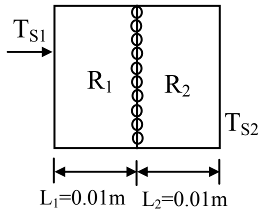
$\Rightarrow$ $k_B = 1.53$ W/mK

02. Ans: (a)

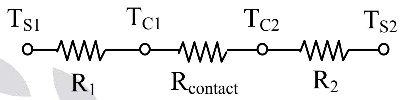
Sol: Given data:

$L_1 = L_2 = 0.01$;

$k_1 = k_2 = 16.6$ W/mK



Thermal circuit:



$$R_1 = R_2 = \frac{L}{k} = \frac{0.01}{16.6}\frac{m^2K}{W}$$

$$q_1 = \frac{T_{S1} - T_{S2}}{2R_1 + R_{constant}} = \frac{100}{2\left[\frac{0.01}{16.6}\right] + 15\times10^{-4}}$$

$q = 36971.046$

$$q = \frac{T_{C1} - T_{C2}}{R_{contact}}$$

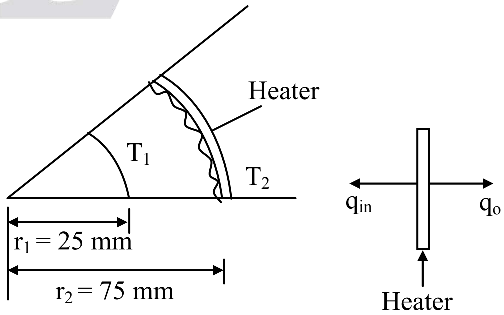
$T_{C1} - T_{C2} = 55.45°C$

03. Ans: (c)

Sol: Given data:

$T_1 = 5°C$, $T_2 = 25°C$,

$k = 10$ W/mK, $R_{contact} = 0.01$ mK/W



$$q_{in} = \frac{T_2 - T_1}{R_{contact} + R_{cond}}$$

$$= \frac{25-5}{0.01 + \frac{\ell n\left(\frac{75}{25}\right)}{2\pi \times 10 \times 1}} = 727.67\ W/m$$

$$q_{out} = \frac{T_2 - T_\infty}{\frac{1}{h_o A_o}}$$

$$= \frac{25+10}{\frac{1}{100 \times 2\pi \times 0.075 \times 1}} = 1649.33\ W/m$$

Heater Power = Total Heat Loss

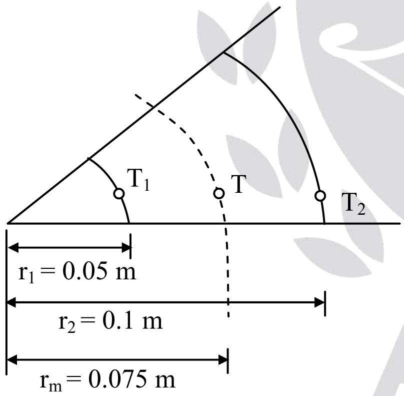
$= q_{in} + q_{out} = 2377$ W/m

04. Ans: (d)

Sol: Given data:

$T_1 = 100°C$, $r_m = 0.075$ m

$T_2 = 45°C$



$$\frac{T_m - T_1}{T_2 - T_1} = \frac{\frac{1}{r_1} - \frac{1}{r_m}}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$\frac{T_m - 100}{45 - 100} = \frac{\frac{1}{0.05} - \frac{1}{0.075}}{\frac{1}{0.05} - \frac{1}{0.1}} \Rightarrow T_m = 63.3°C$$

05. Ans: (485 K)

Sol: Given data:

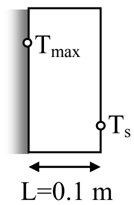
Volumetric heat generation rate

$q_g = 0.3\ MW/m^3$

$k = 25$ W/mK;

$T_\infty = 92°C$;

$h_o = 500\ W/m^2K$



Energy balance:

$Q_{in} + Q_{gen} - Q_{out} = Q_{stored}$

$Q_{gen} = Q_{out}$

$q_g A'L = hA'(T_S - T_\infty)$

$0.3 \times 10^6 \times 0.1 = 500 \times (T_S - 92)$

$\Rightarrow \quad T_S = 152°C$

$$T_{max} - T_S = \frac{q_g L^2}{2k}$$

(q_g = heat generation per unit volume)

$$T_{max} = T_S + \frac{q_g L^2}{2k}$$

$$T_{max} = 152 + \frac{0.3 \times 10^6 \times (0.1)^2}{2 \times 25}$$

$T_{max} = 212°C = 485$ K

06. Ans: (b)

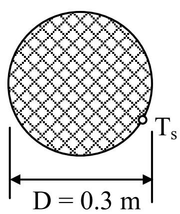
Sol: Given data:

$q_g = 2.6 \times 10^6\ W/m^3$

$k = 45$ W/m°C

$T_\infty = 0°C$

$h = 1200\ W/m^2°C$



Temperature difference between center line and surface of the sphere

$$T_{max} - T_S = \frac{q_q R^2}{6k}$$

$$T_{max} = T_S + \frac{q_q R^2}{6k}$$

$$= 108.33 + \frac{2.6\times10^6 \times (0.15)^2}{6\times 45}$$

T_{max} = 325°C

Energy balance:

$Q_{in} + Q_{gen} - Q_{out} = Q_{stored}$

$Q_{gen} = Q_{out}$

$$q_g \frac{4}{3}\pi R^3 = h4\pi R^2 (T_S - T_\infty)$$

$$T_S - T_\infty = \frac{q_g R}{3h}$$

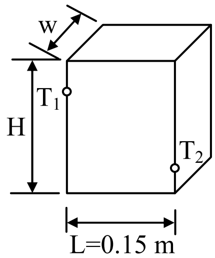
$$T_S = T_\infty + \frac{q_g R}{3h}$$

$$T_S = 0 + \frac{2.6\times10^6 \times 0.15}{3\times1200}$$

T_S = 108.33°C

07. Ans: (b)

Sol:



Given data:

T_1 = 500 K, H = 1.5 m

T_2 = 350 K, W = 0.6 m , L = 0.15 m

$$T_{avg} = \frac{T_1 + T_2}{2} = \frac{500+350}{2} = 425°C$$

$k_T = k_o[1 + \beta T]$

$k_{avg} = k_o[1 + \beta\, T_{avg}]$

$k_{avg} = 25[1+(8.7\times10^{-4})\times425]$

k_{avg} = 34.24 W/mK

$$Q = \frac{T_1 - T_2}{\frac{L}{k_{avg}A}}$$

$$= \frac{500-350}{\frac{0.15}{34.24\times1.5\times0.6}} = 30.816\times10^3 = \omega$$

Q = 30.816 kW

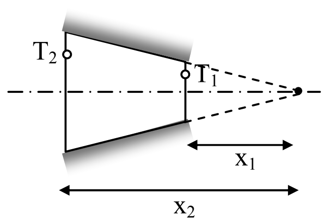
08. Ans: (c)

Sol: Given data:

T_1 = 400 K, T_2 = 600 K

D = ax, a = 0.25

x_1 = 0.05m, x_2 = 0.25 m



$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}a^2x^2$$

$$Q = -kA\frac{dT}{dx}$$

$$Q = -k\frac{\pi}{4}a^2x^2\frac{dT}{dx}$$

$$Q\frac{dx}{x^2} = -\frac{\pi k a^2}{4}dT$$

$$Q\int_{x_1}^{x_2}\frac{dx}{x^2} = \frac{-\pi k a^2}{4}\int_{T_1}^{T_2} dT$$

$$Q\left[\frac{-1}{x}\right]_{x_1}^{x_2} = \frac{-\pi k a^2}{4}(T_2 - T_1)$$

$$Q\left[\frac{-1}{x_2} + \frac{1}{x_1}\right] = \frac{-\pi k a^2}{4}(T_2 - T_1)$$

$$Q = \frac{-\pi k a^2 (T_2 - T_1)}{4\left[\frac{1}{x_1} - \frac{1}{x_2}\right]}$$

$$= \frac{-\pi \times 3.46 \times (0.25)^2 (600 - 400)}{4\left[\frac{1}{0.05} - \frac{1}{0.25}\right]}$$

Q = –2.12 W (– sign indicates the direction of heat transfer)

09. Ans: (d)

Sol: Given data:

Thermal conductivity of insulation

(k_{in}) = 0.5 W/mK

Heat transfer coefficient of surrounding air

(h_o) = 20 W/m^{2}K

Thickness of insulation for maximum heat transfer $= r_c - r = \frac{k_{in}}{h_o} - r$

$$= \frac{0.5}{20} - 0.01 = 15\,\text{mm}$$

10. Ans: (a)

Sol: Given data:

Thermal conductivity of insulation (k_{in}) = 0.1 W/mK

Heat transfer coefficient of surrounding air (h_o) = 10 W/m^{2}K

Radius (r) = 1.5 cm,

Critical radius of insulation (r_c) = $\frac{k_{in}}{h_o}$

$$= \frac{0.1}{10} = 0.01\,\text{m} = 1\,\text{cm}$$

$\because$ $r > r_c$

$\therefore$ Adding the insulation will always reduce the heat transfer rate.

11. Ans: (c)

Sol: Given data:

Radius (r) = 1 mm,

Thermal conductivity of insulation (k_{in}) = 0.175 W/mK

Heat transfer coefficient of surrounding air (h_o) = 125 W/m^{2}K

Thickness = 0.2 mm = $r_{new} - r$

r_{new} = 1.2 mm

Critical radius of insulation $(r_c) = \frac{k_{in}}{h_o}$

$$= \frac{0.175}{125} = 1.4\,\text{mm}$$

$\because$ $r_{new} < r_c$

$\therefore$ Addition of further insulation, heat transfer rate increases first then decreases.

12. Ans: (b)

Sol: Given data:

Thermal conductivity of insulation (k_{in}) = 0.4 W/mK

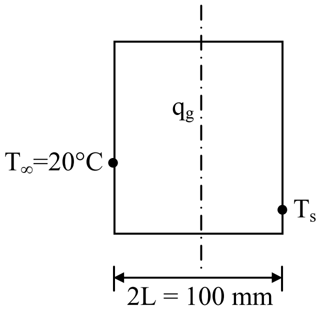
Heat transfer coefficient of surrounding air (h_o) = 10 W/m^{2}K

Critical radius of insulation for the sphere $(r_c) = \frac{2k_{in}}{h_o} = \frac{2 \times 0.04}{10} = 8\,\text{mm}$

Critical diameter (d_c) = $2r_c$ = 16 mm

13. Ans: (b)

Sol:



Volumetric heat generation rate

$(q_g) = 1000\ W/m^3$

$T_x = a(L^2 - x^2) + b$

$T_{x=0.05} = 10\,(10.05^2 - 0.05^2) + 30$

$T_s = 30°C$

$T_\infty = 20°C$

$$\frac{\partial T}{\partial x} = a(0 - 2x)$$

$$\frac{\partial^2 T}{\partial x^2} = -2a = -2\times 10 = -20$$

1-D heat conduction equation with internal heat generation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$-20 + \frac{1000}{K} = 0 \quad \left(\text{for steady state}, \frac{\partial T}{\partial t} = 0\right)$$

$$\frac{1000}{k} = 20 \Rightarrow k = 50\ W/mK$$

Energy balance:

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=+0.05m} = h[T_S - T_\infty]$$

$$-50[a\times(-2x)]_{x=0.05} = h[30-20]$$

$$-50[10\,(-2\times 0.05)] = h\times 10$$

$$\Rightarrow \quad h = 5\ W/m^2K$$

14. Ans: (c)

Sol: Given data:

$\Delta V = 10\ V$, $\rho = 70\times 10^{-8} m$,

$D = 3.2\times 10^{-3} m$, $r = 1.6\times 10^{-3} m$,

$T_s = 93°C$, $T = 22.5\ W/mK$,

$L = 0.3\ m$

Resistance

$$(R) = \frac{\rho L}{A_c} = \frac{70\times 10^{-8}\times(0.3)}{\frac{\pi}{4}\left(3.2\times 10^{-3}\right)^2} = 0.02611\Omega$$

$$I = \frac{\Delta V}{R} = \frac{10}{0.02511} = 382.97\,\text{Amp}$$

$Q_g = \Delta VI = 10\times 383.97$

$Q_g = 3829.75\ W$

$$q_g = \frac{Q_g}{\text{volume}} = 1.587\times 10^9\ W/m^3$$

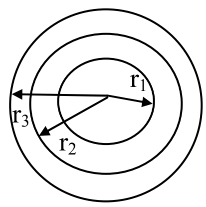
Temperature difference between center line and surface of the cylindrical wire

$$T_{max} - T_S = \frac{q_g R^2}{4k} = 138.14°C$$

15. Ans: (d)

Sol: $\frac{K_1}{K_2} = \frac{1}{2}$, $\frac{r_1}{r_3} = 0.8$

$r_2 - r_1 = r_3 - r_2$



Due to steady state H.T.

$Q_1 = Q_2$

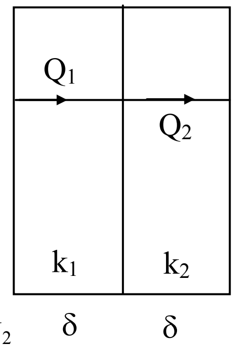
$$\frac{4\pi K_1 r_1 r_2(\Delta T_1)}{(r_2 - r_1)} = \frac{4\pi K_2 r_2 r_3(\Delta T_2)}{(r_3 - r_2)}$$

$$\frac{\Delta T_1}{\Delta T_2} = \frac{K_2 r_3}{K_1 r_1} = \frac{2}{0.8} = 2.5$$

16. Ans: (c)

Sol: $Q_1 = Q_2 = Q$

$R_{eq} = R_1 + R_2$



$$\frac{2\delta}{k_{eq}A} = \frac{\delta}{k_1 A} + \frac{\delta}{k_2 A}$$

$$\frac{2\delta}{k_{eq}} = \frac{\delta}{k_1} + \frac{\delta}{k_2}$$

$$\frac{2}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2}$$

$$k_{eq} = \frac{2k_1 k_2}{k_1 + k_2}$$

17. Ans: (b)

Sol: $r_2 < r_c$, addition of insulation up to r_C will increase the H.T. by Convection largely and reduces the H.T. by Conduction by a small amount and hence the net heat transfer increases and therefore current carrying capacity increases.

18. Ans: (d)

Sol: A good conductor of heat cannot ensure that it will be a good conductor of electricity.

For example, Diamond is a very good conductor of heat but it is not a good conductor of electricity.

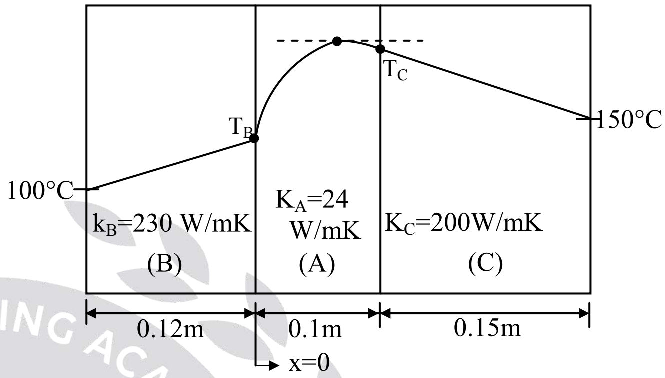
19. Ans: (a)

Sol: A thermal conductivity of diamond is approximately 2300 W/mK. It has one of the highest thermal conductivities among all materials. It is because diamond has highly ordered crystalline structure.

Conventional Practice Solutions

01.

Sol:



Heat generation is only in wall A.

Heat conduction equation 1-D

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$$

Integrate with respect to x

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}.x + C_1$$

Integrate with respect to x

$$T = \frac{-\dot{q}}{k}.\frac{x^2}{2} + C_1 x + C_2 \text{ ----------(1)}$$

Boundary condition, at x = 0, $T = T_B$

$T_B = 0 + 0 + C_2$

$C_2 = T_B$

At the interface AB:

$$-k_A\left(\frac{dT}{dx}\right)_A = -k_B\left(\frac{dT}{dx}\right)_B$$

$$-\dot{q}.x + C_1.k_A = \frac{230(T_B - 100)}{0.12}$$

[at interface AB, x = 0]

$$C_1 \times 24 = \frac{230(T_B - 100)}{0.12}$$

$C_1 = 79.86(T_B - 100)$ ----------(2)

At x = 0.1 m, $T = T_C$

$$T_C = \frac{-\dot{q}}{k_A}.\frac{0.1^2}{2} + C_1 \times 0.1 + T_B$$

$$T_C = \frac{-2.5\times10^5}{24} \times \frac{0.1^2}{2} + 0.1 \times 79.86(T_B - 100) + T_B$$

$T_C = -850.68 + 8.983T_B$ ----------(3)

Total heat generation inside the wall A

= Heat conducted through wall B + Heat conducted through wall C

$$2.5\times10^5 \times 0.1 \times A = \frac{A \times 230(T_B - 100)}{0.12} + \frac{(T_C - 150)\times 200 \times A}{0.15}$$

A = perpendicular area in the direction of heat flow

$34.5T_B + 24T_C = 7500$ ------(4)

From equation number (3) and (4)

$T_B = 111.624°C$

$T_C = 152.04°C$

At maximum temperature, $\frac{dT}{dx} = 0$

$$-\frac{\dot{q}}{k}.x + C_1 = 0$$

and from equation (2)

$C_1 = 79.28(T_B - 100)$

$C_1 = 928.292$ °C/m

$$x = \frac{C_1 \times k_A}{\dot{q}}$$

$$x = \frac{928.292 \times 24}{2.5\times10^5} = 0.089116 \text{ m}$$

at x = 0.089116 m,

$T = T_{max}$

From eq. (1)

$$T_{max} = \frac{-\dot{q}}{k_A}.\frac{x^2}{2} + C_1x + C_2$$

$$T_{max} = \frac{-2.5\times10^5}{24} \times \frac{0.0891^2}{2} + 928.29 \times 0.0891 + 111.624$$

$T_{max} = 152.9868°C$

% of total heat conducted by wall B

$$= \frac{\text{Heat conduction through wall B}}{\text{Total heat generation in wall A}}$$

$$= \frac{\left(\frac{-k_B.A(T_B - 100)}{L_B}\right)}{2.5\times10^5 \times A \times L_A}$$

$$= \frac{\left(\frac{230(111.624 - 100)}{0.12}\right)}{2.5\times10^5 \times 0.1}$$

$$= \frac{22279.33}{25000} = 89.11\%$$

Hence, 89.11% of the total heat transferred through wall B and 10.89% of the total heat transferred through wall C.

Chapter 2

Transient Heat Conduction

01. Ans: (b)

Sol: Given data:

D = 1.2 cm, R = 0.6 cm,

$T_o = 900°C$, $T_\infty = 30°C$,

$h = 125\ W/m^2°C$, $c_p = 480$ J/kg

$L_c = \frac{R}{3} = 0.2\,cm$, T = 850°C,

$\because \quad Bi = \frac{hL_c}{k} < 0.1$

∴ Lumped method can be applied.

$$\ell n\left[\frac{T - T_\infty}{T_0 - T_\infty}\right] = \frac{-ht}{\rho c_p L_c}$$

⇒ t = 3.67 sec

02. Ans: (c)

Sol: Given data:

$\rho = 8500\ kg/m^3$, $c_p = 320$ J/kgK

$h = 65\ W/m^2K$, k = 35 W/mK

d = 1.2 mm

$$\frac{T_o - T}{T_o - T_\infty} = 0.99$$

$\because \quad Bi = \frac{hL_c}{k} < 0.1$

∴ Lumped method can be applied.

$$\frac{T_o - T}{T_o - T_\infty} = 1 - e^{\frac{-ht}{\rho c_p L_c}}$$

$$0.99 = 1 - e^{\frac{-ht}{\rho c_p L_c}}$$

$$e^{\frac{-ht}{\rho c_p L_c}} = 1 - 0.99 = 0.01$$

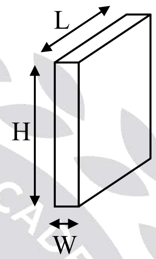
$$\frac{-ht}{\rho c_p L_c} = \ell n(0.01)$$

$$t = \frac{\rho c_p L_c}{h}\ell n(0.01)$$

⇒ t = 38.54 sec

03. Ans: (d)

Sol:



Given data:

$T_o = 25°C$; $T_\infty = 600°C$

$Q_{act} = 0.75\ Q_{max}$

$$L_c = \frac{V}{A_s} = \frac{HWL}{2HU} = \frac{W}{2} = \frac{0.05}{2} = 0.025$$

$$m_c[T - T_o] = 0.75[m_c(T_\infty - T_o)]$$

T – 25 = 0.75 (600–25)

⇒ T = 456.25°C

$$Bi = \frac{hL_c}{k} = \frac{100 \times 0.025}{231} =< 0.1$$

$\because \quad Bi = \frac{hL_c}{k} < 0.1$

∴ Lumped method can be applied.

$$\ell n\left[\frac{T - T_\infty}{T_o - T_\infty}\right] = \frac{-ht}{\rho c_p L_c}$$

⇒ t = 967.34 sec

04. Ans: (b)

Sol: According to lumped capacity analysis:

$$\ell n\left[\frac{T-T_\infty}{T_o-T_\infty}\right]=\frac{-t}{\tau^*}$$

$$\ell n\left[\frac{\frac{T_o+T_\infty}{2}-T_\infty}{T_o-T_\infty}\right]=\frac{-t}{\tau^*}$$

$$\ell n\left[\left(\frac{T_o+T_\infty}{2}\right)\frac{1}{T_o-T_\infty}\right]=\frac{-t}{\tau^*}$$

$$\ell n\left(\frac{1}{2}\right)=\frac{-t}{\tau^*}$$

$$\ell n(2)=\frac{-t}{\tau^*}$$

$$\Rightarrow\quad t=\tau^*\,\ell n(2)$$

05. Ans: (c)

Sol: Given data:

$m = 500$ g $= 0.5$ kg;

$T_o = 530°C$;

$T = 430°C$;

$T_\infty = 30°C$

According to lumped capacity analysis

$$\ell n\left[\frac{T-T_\infty}{T_o-T_\infty}\right]=\frac{-t}{\tau^*},$$

$$\ell n\left[\frac{430-30}{530-30}\right]=\frac{-10}{\tau^*}\quad\ldots\ldots\ldots(1)$$

$$\ell n\frac{400}{500}=\frac{-10}{\tau^*}$$

$$\Rightarrow\quad \tau^* = 44.81\text{ s}$$

Temperature after next 10 s,

$T_o = 430°C$; $t = 10$ sec;

$$\ell n\left[\frac{T-T_\infty}{T_o-T_\infty}\right]=\frac{-t}{\tau^*}$$

$$\frac{T-30}{430-30}=e^{\frac{-10}{\tau^*}}\quad\ldots\ldots\ (2)$$

$$T = 30 + 400\times e^{-10/44.81}$$

$$\Rightarrow\quad T = 350°C$$

06. Ans: 12.05 K/min

Sol: Given data:

$D = 0.05$ m;

$T_o = 900°C$, $T_\infty = 30°C$

$$\rho=\frac{m}{V}$$

$$m=\rho V=\rho\frac{4}{3}\pi R^3$$

$$=7800\times\frac{4}{3}\times\pi(0.025)^3=0.510\,\text{kg}$$

Energy balance:

Decrease in internal energy = Convective heat transfer from the surface

$$-mc\frac{dT}{dt}=hA_s(T_o-T_\infty)$$

$$0.510\times2000\times\frac{dT}{dt}=30\times4\pi(0.025)^2\times(900-30)$$

$$\frac{dT}{dt}=0.2\,K/sec$$

$$\frac{dT}{dt}=12.05\,K/min$$

07. Ans: (c)

Sol: Given data:

$T_o = 350°C$,

$T_\infty = 30°C$,

$T = 100°C$

$c_p = 900$ J/kg.K,

$\rho = 2700$ kg/m^3,

$k = 205$ W/mK,

$h = 60$ W/m^{2}K

$$m = \rho V = \rho \times \frac{4}{3}\pi R^3$$

$$L_C = \frac{R}{3} = 0.02698\,\text{m}$$

$R = 0.0809$ m

$\because$ $$Bi = \frac{hL_c}{k} < 0.1$$

$\therefore$ Lumped method can be applied.

$$\ell n\left[\frac{T - T_\infty}{T_o - T_\infty}\right] = \frac{-ht}{\rho c_p L_C}$$

$$\ell n\left[\frac{100-30}{350-30}\right] = \frac{-60\times t}{2700\times 900\times 0.02698}$$

$\Rightarrow$ t = 1660 sec

08. Ans: (a)

Sol: Temperature distribution for the lumped heat capacity analysis

$$\frac{(T - T_\infty)}{T_i - T_\infty} = \exp\left[\frac{-hA}{\rho Vc}\tau\right]$$

If $\frac{hA\tau}{\rho Vc} \to 0$,

$(T - T_\infty) \to (T_i - T_\infty)$

For the rapid response of the temperature measuring equipment, the value of $\frac{hA\tau}{\rho Vc}$ should be as small as possible.

09. Ans: (a)

Sol: In lumped analysis temp gradient is negligible and B_i is very small due high "k".

10. Ans: (b)

Sol: For the lumped heat capacity analysis, internal conductive resistance is negligible as compared to external convective resistance.

Biot number (Bi) < 0.1

$$\frac{R_{conduction}}{R_{convection}} < 0.1$$

$R_{conduction} <<< R_{convection}$

11. Ans: (d)

Sol: For the lumped heat capacity analysis, the temperature distribution is given by

$$\frac{(T - T_\infty)}{T_i - T_\infty} = \exp\left[\frac{-hA}{\rho Vc}\tau\right],$$

Hence temperature distribution is exponential.

Chapter 3 Extended Surfaces - FINS

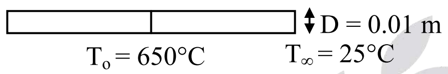
01. Ans: (a)

Sol: Given that:

D = 0.01 m, h = 10 W/m^{2}K,

T_∞ = 25°C,

k = 379 W/mK, T_o = 650°C



For very long fin:

$$Q_{Fin} = kA_c m\theta_o = k\frac{\pi}{4}D^2 \times \sqrt{\frac{4h}{kD}} \times (T_o - T_\infty)$$

$$Q_{Fin} = 379 \times \frac{\pi}{4} \times (0.01)^2 \times \sqrt{\frac{4\times 10}{379 \times 0.01}} \times (650 - 25)$$

Q_{Fin} = 60.43

Power in put = $2Q_{Fin}$ = 120.9 W

02. Ans: (b)

Sol: Given data:

k = 237 W/mK, h = 12 w/m^{2}K,

d = 4 mm, L = 10 cm,

$$mL = \sqrt{\frac{4h}{kd}}\ L$$

$$= \sqrt{\frac{4\times 12}{237\times 4\times 10^{-3}}} \times 0.1 = 0.71156$$

$$\%\,error = \frac{Q_{infinite} - Q_{insulated}}{Q_{insulated}}$$

$$= \frac{kA_C m\theta_o - kA_c m\theta_o \tanh(mL)}{kA_c m\theta_o \tanh(mL)}$$

$$\%\,error = \frac{1-\tanh(mL)}{\tanh h(mL)}$$

$$= \frac{1}{\tanh(mL)} - 1 = 63.48\%$$

03. Ans: (c)

Sol: Given data:

D = 5 mm, L = 50 mm, η = 0.65

$$\frac{\in}{\eta} = \frac{Q_{Fin}}{Q_{without\ fin}} \times \frac{Q_{max}}{Q_{Fin}}$$

$$\frac{\in}{\eta} = \frac{Q_{max}}{Q_{without\ fin}}$$

$$= \frac{hA_s(T_o - T_\infty)}{hA_c(T_o - T_\infty)}$$

Surface area (A_s) = πDL

Cross-sectional area (A_c) = $\frac{\pi}{4}D^2$

$$\frac{\in}{\eta} = \frac{\pi DL}{\frac{\pi}{4}D^2}$$

$$\frac{\in}{\eta} = 4\left(\frac{L}{D}\right)$$

$$\frac{\in}{0.65} = 4\left(\frac{50}{5}\right)$$

$\Rightarrow \in = 26$

04. Ans: 420%

Sol: Heat transfer rate for very long fin:

$Q = kA_c m\theta_o$

$$= \sqrt{hpkA_c}\ \theta_o = \sqrt{h\times \pi D \times k \times \frac{\pi}{4}D^2}\ \theta_o$$

$Q \propto D^3/2$

$$\frac{Q_2}{Q_1}=\frac{(D_2)^{3/2}}{(D_1)^{3/2}}=\frac{(3D_1)^{3/2}}{(D_1)^{3/2}}=5.1962$$

$$\%\,\text{increase in Heat Transfer}=\frac{Q_2-Q_1}{Q_1}$$

$$=\frac{Q_2}{Q_1}-1$$

$$= 5.1962 - 1$$

$$= 4.19 \approx 420\ \%$$

05. Ans: (c)

Sol: Given data:

$k_A = 70$ W/mK,

$x_A = 0.15$ m, $x_B = 0.075$ m

Temperature variation for long fin:

$$\frac{T_o - T_\infty}{T - T_\infty} = e^{mx}$$

$$m = \sqrt{\frac{ph}{kA_c}} = \sqrt{\frac{4h}{kD}}$$

$m \propto \sqrt{\frac{1}{k}}$ (for the same diameter and same environment)

For the same temperatures

$m_A x_A = m_B x_B$

$$\frac{x_2}{x_1} = \frac{m_1}{m_2} = \sqrt{\frac{k_B}{k_A}}$$

$$\frac{k_B}{k_A} = \left(\frac{x_B}{x_A}\right)^2$$

$$\frac{k_B}{70} = \left(\frac{0.075}{0.15}\right)^2$$

$\Rightarrow$ $k_B = 17.5$ W/mK

06. Ans: (d)

Sol: Given data:

$a = 5 \times 10^{-3}$ m = 5 mm,

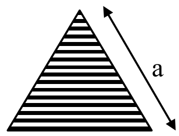
$T_o = 400°C$,

$T_\infty = 50°C$,

k = 54 W/mK,

L = 0.08 m,

h = 90 W/m^{2}K,



$$\frac{P}{A_C} = \frac{3a}{\sqrt{\frac{3}{4}a^2}} = \frac{4\sqrt{3}}{a}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4\sqrt{3}h}{ka}}$$

$$m = \sqrt{\frac{4\sqrt{3}\times 90}{54\times 5\times 10^{-3}}} = 48.05$$

mL = 3.844

$$L_c = L + \frac{A_c}{P} = 0.08 + \frac{a}{4\sqrt{3}}$$

$$= 0.08 + \frac{5\times 10^{-3}}{4\sqrt{3}}$$

$$= 0.08072\text{ m}$$

Heat transfer rate from the fin:

$Q_{Fin} = kA_c m\theta_o \tanh(mL_c)$

$$= 54\times\left(\frac{\sqrt{3}}{4}\times 0.005^2\right)\times 48.05\times(400-50)$$

$$\times\tanh(48.05\times 0.08072)$$

$Q_{Fin} = 9.82$ W

07. Ans: (c)

Sol: Given data:

k = 30 W/mK, D = 0.01 m,

$L = 0.05$ m, $T_\infty = 65°C$,

$h = 50$ W/m^{2}K, $T_o = 98°C$

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4\times 50}{30\times 0.01}}\times 0.05 = 1.2909$$

Temperature variation for insulated fin tip

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh(mL - x)}{\cosh mL}$$

$x = L$, $T = T_L$

$$\frac{T_L - T_\infty}{T_o - T_\infty} = \frac{1}{\cosh mL}$$

$$T_L = T_\infty + \frac{T_o - T_\infty}{\cosh(mL)}$$

$$T_L = 65 + \frac{98 - 65}{\cosh(1.29)}$$

$$T_L = 81.87°C$$

08. Ans: (b)

Sol:
$$\frac{h}{mk} < 1$$

$$\frac{h}{\sqrt{\frac{ph}{kA_c}}\times k} < 1$$

$$\sqrt{\frac{hA_c}{pk}} < 1$$

$$\sqrt{\frac{pk}{hA_c}} > 1$$

Effectiveness ($\in$) > 1

Using the fin will increase the heat transfer rate because effectiveness of the fin is greater than unity.

09. Ans: (a)

Sol: Given data:

$k = 200$ W/m°C, $h = 15$ W/m^2°C, $L = 1$ cm

Cross-sectional area of fin

$(A_c) = 0.5 \times 0.5$ mm^2

$T_o = 80°C$,

$T_\infty = 40°C$

$$m = \sqrt{\frac{ph}{kA_c}}$$

$$= \sqrt{\frac{4\times 0.0005\times 15}{200\times 0.0005\times 0.0005}} = 24.49$$

$mL = 24.49 \times 0.01 = 0.2449$

$\tanh(mL) = 0.240$

Heat transfer rate from fin with insulated tip

$Q_{Fin} = kA_c m\theta_o \tanh(mL)$

$= 200\times(0.5\times10^{-3})^2\times 24.5\times(80–40)\times0.240$

$Q_{Fin} = 0.01176$

$$\text{No.of fin} = \frac{Q_{total}}{Q_{Fin}} = \frac{1}{0.01176} = 85$$

10. Ans: 191.5 W/mK

Sol: Given data:

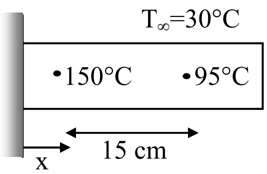
$T_x = 150°C$,

$T_{x+15cm} = 95°C$,

$T_\infty = 30°C$

$D = 25$ mm,

$h = 20$ W/m^2°C



Temperature variation for long fin

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-mx}$$

$$\frac{150 - 30}{T_o - 30} = e^{-mx} \ldots\ldots\ldots\ldots (1)$$

$$\frac{95 - 30}{T_o - 30} = e^{-m(x+15)} \ldots\ldots\ldots\ldots (2)$$

From equation (1) and (2) we get

$$\ell n\left[\frac{150 - 30}{95 - 30}\right] = m\Delta x$$

$$\ell n\left[\frac{150 - 30}{95 - 30}\right] = \sqrt{\frac{4h}{Dk}} \times 0.15$$

$\Rightarrow$ $k = 191.5$ W/mK

11. Ans: (d)

Sol: Fins are always provided at the side where heat transfer coefficient is lowest, the material of the fin should have high thermal conductivity, for high heat transfer to weight ratio, the parabolic shape of fin is preferable and fins must be provide vertical in case of stationary bodies and parallel to the direction of movement in case moving fins.

12. Ans: (c)

Sol: Effectiveness of very long fin is given by

$$\varepsilon = \sqrt{\frac{pk}{hA_c}}$$

13. Ans: (d)

Sol: Fins are always provided at the side where heat transfer coefficient is lowest, the material of the fin should have high thermal conductivity, for high heat transfer to weight ratio the parabolic shape of fin is preferable and fins must be provide vertical in case of stationary bodies and parallel to the direction of movement in case moving fins.

14. Ans: (b)

Sol: Effectiveness of the fin is always highest when it is provided at the lower heat transfer coefficient side whether it is cold or hot side because the effectiveness is inversely proportional to the heat transfer coefficient and temperature of fin is varying along the length of the fin.

Conventional Practice Solutions

01.

Sol: Given:

Length of fin (l) = 30 cm = 0.3 m

Width of fin (b) = 30 cm = 0.3 m

Thickness (t) = 2 mm = 0.002 m

C/s area of fin $A_{c/s} = 0.3 \times 0.002$

$= 6 \times 10^{-4}$ m^2,

Perimeter of the fin P = 0.604 m

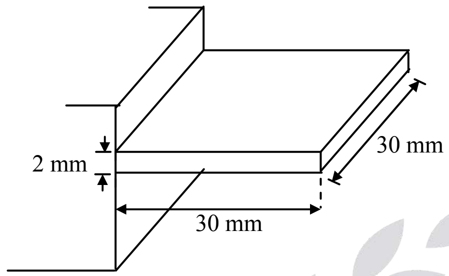
Thermal conductivity (k)= 204 W/mK

Ambient air temperature t_a=30°C

Base temperature t_b=300°C

Convection heat transfer coefficient

(h) = 15 W/m^{2}K



As the tip is not insulated at end, it looses heat from end also, hence the temperature variation for, heat dissipation from a fin loosing heat at tip also is given by

$$\frac{\theta}{\theta_0}=\frac{t-t_a}{t_b-t_a}$$

$$=\frac{cosh[m(l-x)]+\frac{h}{km}[sinh\, m(l-x)]}{cosh(mL)+\frac{h}{km}[sinh(ml)]}$$

From the above equation, for any 'x' from the base, we can find the temperature't'.

$$m=\sqrt{\frac{hp}{kA_{C/s}}}=\sqrt{\frac{15\times 0.604}{204\times 6\times 10^{-4}}}$$

m = 8.603 m^{-1}

$$\frac{t-30}{300-30}=\frac{\cosh[8.603(0.3-0.3)]+\frac{15}{204\times 8.603}[\sinh 8.603(0.3-0.3)]}{\cosh(8.603\times 0.3)+\frac{15}{204\times 8.603}[\sinh(8.603\times 0.3)]}$$

$$=\frac{\cosh(0)+8.547\times 10^{-3}\times\sinh(0)}{\cosh(2.5809)+[8.547\times 10^{-3}\times\sinh(2.5809)}$$

$$=\frac{1}{6.6423+0.0561}$$

$$\frac{t-30}{270}=0.149$$

t = 70.307°C corresponding to l = 30 cm is at the tip end exactly

(b) Rate of heat transfer

$$=\sqrt{PhkA_{cs}}\,(t_b-t_a)\frac{\tanh(m\ell)+\frac{h}{km}}{1+\frac{h}{km}\tanh(m\ell)}$$

$$\frac{h}{km}=\frac{15}{204\times 8.603}=8.547\times 10^{-3}$$

tanh(ml) = tanh(8.603×0.3) = 0.99

$$=\sqrt{0.604\times 15\times 204\times 0.0006}\,(300-30)\left(\frac{0.99+(8.547\times 10^{-3})}{1+(8.547\times 10^{-3})0.99}\right)$$

= 281.53 W

(c) Fin efficiency:

$$\eta_{fin}=\frac{\text{Actual heat transferred by the fin }(Q_{fin})}{\text{Max heat that would be transferred if whole surface of the fin is maintained at the base temperature }(Q_{max})}$$

$$\eta_{fin}=\frac{Q_{fin}}{hP\ell(t_b-t_a)}$$

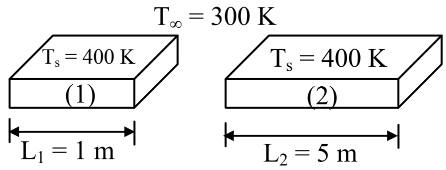
$$\eta_{fin}=\frac{281.53}{15\times 0.604\times 0.3(300-30)}$$

$$\eta_{fin}=0.3836=38.36\ \%$$

Chapter 4 Convection

01. Ans: 40 W/m²K

Sol:



Given that:

$V_1 = 100$ m/s,

$V_2 = 20$ m/s,

$q_1 = 20{,}000$ W/m^2

Heat transfer from object (1) = $h_1 (T_s - T_\infty)$

$$20000 = h_1 (400 - 300)$$

$$h_1 = 200 \text{ W/m}^2\text{K}$$

Reynold's number for object (1)

$$Re_1 = \frac{V_1 L_1}{\nu_1} = \frac{100\times 1}{\nu_1} = \frac{100}{\nu_1}$$

Reynold's number for object (2)

$$Re_2 = \frac{V_2 L_2}{\nu_2} = \frac{20\times 5}{\nu_2} = \frac{100}{\nu_2}$$

Since, $\nu_1 = \nu_2$ (for the same fluid)

∴ $Re_1 = Re_2$

∵ Prandtl number is the property of the fluid.

∴ $Pr_1 = Pr_2$

Nusselt number (Nu) = f [Re.Pr]

$Nu_1 = Nu_2$

$$\frac{h_1 L_1}{k_1} = \frac{h_2 L_2}{k_2}$$

$$\frac{h_2}{h_1} = \frac{L_1}{L_2}$$

$$h_2 = h_1 \times \frac{L_1}{L_2} = 200 \times \frac{1}{5} = 40 \text{ W/m}^2\text{K}$$

02. Ans: (d)

Sol: Given data:

$Pr = 0.7$, $T_\infty = 400$ K

$T_s = 300$ K, $\frac{u_\infty}{\nu} = 5000/\text{m}$

$k = 0.263$ W/mK

$$\frac{T - T_s}{T_\infty - T_s} = 1 - e^{\left(-Pr\frac{u_\infty y}{\nu}\right)}$$

$$T = T_s + (T_\infty - T_s)\left[1 - e^{\left(-Pr\frac{u_\infty y}{\nu}\right)}\right]$$

$$\frac{dT}{dy} = (T_\infty - T_s)\left[0 - e^{\left(-Pr\frac{u_\infty y}{\nu}\right)}\right]\left(-Pr\frac{u_\infty}{\nu}\right)$$

$$\left.\frac{dT}{dy}\right|_{y=0} = (T_\infty - T_s)(-1)\left(-Pr\frac{u_\infty}{\nu}\right)$$

$$= (T_\infty - T_s)\left(Pr\frac{u_\infty}{\nu}\right)$$

Heat transfer rate = Heat conduction just adjacent on the surface (i.e. at y = 0)

$$q = -k\left.\frac{dT}{dy}\right|_{y=0}$$

$$q = -k(T_\infty - T_s)\left(Pr\frac{u_\infty}{\nu}\right)$$

$q = 0.0263\ (300 - 400)\ [0.7 \times 5000]$

$q = 9205$ W/m^2

03. Ans: (c)

Sol: Given data:

$u_{(y)} = Ay + By^2 - cy^3$

$T_{(y)} = D + Ey + Fy^2 - Gy^3$

$$\frac{du}{dy} = A + 2By - 3cy^2$$

$$\left.\frac{du}{dy}\right|_{y=0} = A$$

According to Newton's law of viscosity:

$$\text{Wall shear stress}(\tau_s) = \mu\left.\frac{du}{dy}\right|_{y=0}$$

$$= \mu A$$

$$\text{Skin friction coefficient } (c_f) = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2}$$

$$c_f = \frac{2\mu A}{\rho u_\infty^2}$$

$$c_f = \frac{2\nu A}{u_\infty^2} \qquad \left(\nu = \frac{\mu}{\rho}\right)$$

For the temperature profile:

$$\frac{dT}{dy} = E + 2Fy - 3Gy^2$$

$$\left.\frac{dT}{dy}\right|_{y=0} = E$$

Energy balance:

Conduction heat transfer in the fluid adjacent to the wall (i.e. at y = 0) = convective heat transfer inside the fluid.

$$-k\left.\frac{dT}{dy}\right|_{y=0} = h(T_s - T_\infty)$$

$$h = \frac{-k\left.\frac{dT}{dy}\right|_{y=0}}{T_s - T_\infty} = \frac{-kE}{T_s - T_\infty}$$

$$h = \frac{kE}{T_\infty - D}$$

(T_s = D, from the temperature profile)

04. Ans: (b)

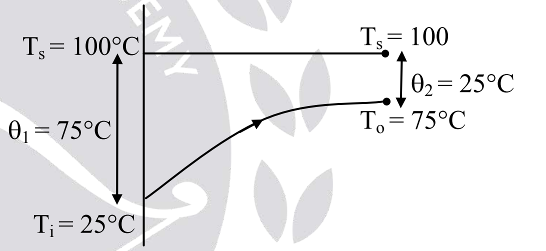
Sol: Given data:

$\dot{m} = 2\,kg/s$, D = 0.04 m, T_i = 25°C,

T_o = 75°C, T_s = 100°C,

h = 6916 W/m^{2}K,

c_p = 4181 J/kg.K.



$$\text{LMTD} = \frac{\theta_1 - \theta_2}{\ell n\left(\frac{\theta_1}{\theta_2}\right)} = \frac{75-25}{\ell n\left(\frac{75}{25}\right)} = 45.51°C$$

Heat transfer rate = h × A × LMTD

$$\dot{m}c_p(T_o - T_i) = 6916 \times \pi \times 0.04 \times L \times 45.51$$

2 × 4181 × (75 – 25) = 39554 L

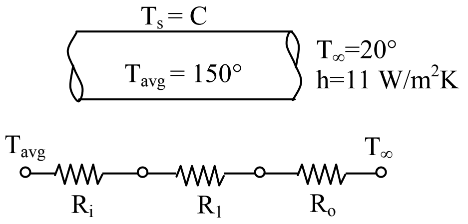
⇒ L ≈ 10.6 m

05. Ans: (b)

Sol: Given data:

D = 30 mm, T_∞ = 20°C,

h = 11 W/m^{2}K, L = 1 m,



For laminar fully developed with constant wall temperature condition:

Nu = 3.66

$$\frac{hD}{k} = 3.66$$

$$h = 3.66\ k/D$$

$$h = 3.66 \times \frac{0.133}{0.03} = 16.22\ W/m^2K$$

$$q = \frac{T_{avg} - T_\infty}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{150-20}{\frac{1}{16.22} - \frac{1}{11}} = 80.3\ W/m^2$$

06. Ans: (c)

Sol: In constant wall temperature condition, mean temperature of the fluid continuously changes in the direction of fluid flow. The temperature difference between surface temperature and mean fluid temperature decreases in the direction of flow.

Therefore, mean temperature difference is considered as log mean temperature difference in calculation.

For the temperature profile, refer to the diagram in Solution of Q. No. 04

07. Ans: (b)

Sol: Nusselt number (Nu) = 4.36 for laminar flow through tubes with constant heat flux condition.

Nusselt number (Nu) = 3.36 for laminar flow through tubes with constant wall temperature condition.

For the same tube and fluid,

$$h_{\text{constant heat flux}} > h_{\text{constant wall temperature}}$$

08. Ans: (d)

Sol: Given data:

Pr = 3400,

k = 0.145 W/mK,

$\nu = 288 \times 10^{-6}\ m^2sec$,

$\alpha = 0.847 \times 10^{-7}\ m^2/s$,

$\beta = 0.7 \times 10^{-3}/K$, $T_s = 70°C$,

$T_\infty = 5°C$, D = 0.4 m.

$$\text{Characteristic length } (L_c) = \frac{As}{P} = \frac{\frac{\pi}{4}D^2}{\pi D}$$

$$= \frac{D}{4} = \frac{0.4}{4} = 0.1\ m$$

$$\text{Grashoff number, } (Gr) = \frac{g\beta\Delta T L_c^3}{\nu^2}$$

$$Gr = \frac{9.81 \times 0.70 \times 10^{-3} \times 65 \times (0.1)^3}{(288 \times 10^{-6})^2}$$

Gr = 5381.401

$$Ra = Gr.Pr = 5381.401 \times 3400$$

$$= 18.29 \times 10^6$$

Nusselt number $(Nu) = \frac{\bar{h}L_c}{k} = 0.15(Ra)^{1/3}$

$$= \frac{\bar{h}\times 0.1}{0.145} = 0.15\left(18.29\times 10^6\right)^{1/3}$$

$$\bar{h} = 57.312\,W/m^2K$$

Heat transfer rate, (Q) = hA (T_s–T_∞)

$$Q = 57.312\times\frac{\pi}{4}(0.4)^2\times(70-5)$$

$$Q = 468.13\ W$$

09. Ans: 12.70 W/m^{2}K

Sol: Given data:

ρ = 1.204 kg/m^3,

c_p = 1007 J/kg.K,

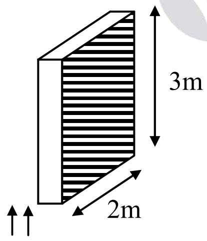
Pr = 0.7309,

F_D = 0.86 N,

T_∞ = 20°C,

u_∞ = 7 m/s

Area (A) = 2[2 × 3] = 12 m^2



Skin friction coefficient (c_f) = $\frac{F_D}{\frac{1}{2}\rho A u_\infty^2}$

$$c_f = \frac{2F_D}{A\rho u_\infty^2} = \frac{2\times 0.86}{12\times 1.204\times 7^2} = 2.43\times 10^{-3}$$

According to Reynold's – Colburn analogy:

$$St.Pr^{2/3} = \frac{c_f}{2}$$

$$St(0.7309)^{2/3} = \frac{2.43\times 10^{-3}}{2}$$

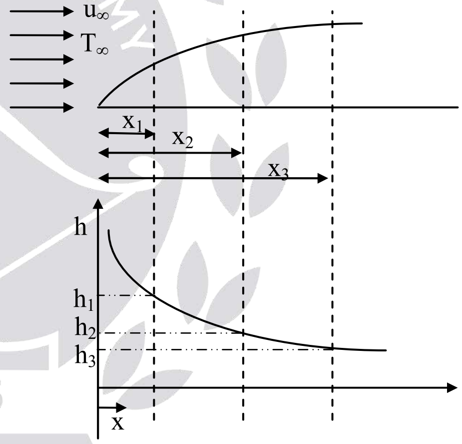
$$St = \frac{h}{\rho u_\infty c_p} = 1.5\times 10^{-3}$$

h = 12.70 W/m^{2}K

10. Ans: (c)

Sol: The variation of heat transfer coefficient (h) in the direction of fluid flow over a flat plate is shown in figure below.

As, $h \propto \frac{1}{\sqrt{x}}$



From the figure $h_1 > h_2 > h_3$

According to Newton's law of cooling,

Heat flux (q) = hΔt

$$q \propto h$$

$$q_1 > q_2 > q_3$$

The maximum local heat flux = q_1

(i.e. at x = x_1)

11. Ans: (a)

Sol: Given data:

L = 3 m,

$h_x = 0.7 + 13.6\,x - 3.4x^2$

Average heat transfer coefficient

$$(\bar{h}) = \frac{1}{L}\int_o^L h_x dx$$

$$\bar{h} = \frac{1}{3}\int_o^3 (0.7 + 13.6x - 3.4x^2)dx$$

$$\bar{h} = \frac{1}{3}\left[0.7x + \frac{13.6x^2}{2} - \frac{3.4x^3}{3}\right]_0^3$$

$$\bar{h} = \frac{1}{3}\left[0.7(3) + \frac{13.6(3)^2}{2} - \frac{3.4(3)^3}{3}\right]$$

$$\bar{h} = 0.7 + \frac{13.6\times 3}{2} - \frac{3.4\times 3^2}{3}$$

$$\bar{h} = 10.9\,W/m^2K$$

Heat transfer coefficient at x = L = 3 m

$h_{x=L=3} = 0.7 + 13.6\,(3) - 3.4\,(3)^2$

$h_{x=L} = 10.9\ W/m^2K$

$$\frac{\bar{h}}{h_{x=L=3m}} = \frac{10.9}{10.9} = 1$$

12. Ans: (a)

Sol: In turbulent flow over cylinder

$$Nu \propto Ra^{\frac{1}{3}} \propto (D^3)^{1/3}$$

$$\frac{hD}{k} \propto D$$

$$h \propto k$$

Nu = Nusselt Number.

Ra = Rayleigh Number = Gr . Pr

Gr = Grashof Number

Pr = Prandtle Number

13. Ans: (a)

Sol: For a fully developed laminar flow through pipe, the Nusselt Number Nu is constant.

$$Nu = \frac{hD}{k}$$

$$h = \frac{Nu \times k}{D}$$

$$h \propto \frac{1}{D}$$

When the Diameter is halved

$$D_2 = \frac{D_1}{2} \Rightarrow \frac{h_2}{h_1} = 2.$$

14. Ans: (b)

Sol: Grashoff's number, $Gr = \frac{g.\beta\Delta TL^3}{\nu^2}$

$Gr \propto L^3$

$Nu = c\,(Gr.Pr)^{1/4}$ → for laminar flow

$Nu \propto (L^3)^{1/4}$

$Nu \propto (L)^{3/4}$

$$\frac{hL}{k} \propto (L)^{3/4}$$

$h \propto (L)^{-1/4}$

Heat transfer,

$Q = h\,A\,\Delta T$

$Q \propto h\,A$

$Q \propto (L)^{-1/4}L \qquad [\because A = \pi dL \Rightarrow A \propto L]$

$Q \propto (L)^{3/4}$

$$\frac{Q_1}{Q_2} = \left(\frac{L_1}{L_2}\right)^{3/4}$$

$$\frac{8}{1} = \left(\frac{320}{L_2}\right)^{3/4}$$

$$(8)^{4/3} = \frac{320}{L_2}$$

$$16 = \frac{320}{L_2}$$

$\Rightarrow L_2 = 20$ cm

15. Ans: (c)

Sol: At the top of the boundary layer the temperature changes are negligible.

16. Ans: (b)

Sol: $\delta = 0.5$ mm, $\mu = 25\times10^{-6}$

C_p= 2000 J/kg.K

k = 0.05 W/mK

$$Pr = \frac{\mu C_p}{K} = \frac{25\times10^{-6}\times2000}{0.05} = 1$$

$$\frac{\delta}{\delta_t} = Pr^{\frac{1}{3}} \Rightarrow \delta = \delta_t = 0.5\text{mm}$$

17. Ans: (d)

Sol: $Q_t = Q_{top} + Q_{bottom} + 4 \times Q_{side}$

$6 \times hA\,\Delta T = h_1A\,\Delta T + h_2A\,\Delta T + 4h_3A\,\Delta T$

$6h = h_1 + h_2 + 4h_3$

$$h = \frac{h_1 + h_2 + 4h_3}{6}$$

18. Ans: (d)

Sol: $\frac{\delta}{\delta_t} = (Pr)^{1/3}$

If $Pr = 1$, $\delta = \delta_t$

$Pr < 1$, $\delta < \delta_t$

$Pr > 1$, $\delta > \delta_t$

19. Ans: (b)

Sol: k = 1.0 W/mK

Re = 1500 $\rightarrow$ means that the flow is laminar

D = 10 cm = 0.1m

For a fully developed laminar flow through pipe

(i) with constant heat flux

$N_u = 4.364 \rightarrow$ constant

$$= \frac{hD}{k}$$

$$h = \frac{4.364\times1.0}{0.1} = 43.64 \text{ W/mk}$$

(ii) With constant wall temp

$$Nu = 3.66 = \frac{hD}{K}$$

$$h = \frac{3.66\times1.0}{0.1} = 36.6 \text{ W/m}^2\text{k}$$

20. Ans: (c)

Sol:

- Transient conduction $\rightarrow$ Biot number
- Mass transfer $\rightarrow$ Sherwood number
- Forced Convection $\rightarrow$ Reynold's number
- Free Convection $\rightarrow$ Grashoff number

21. Ans: (a)

Sol: Stanton number is used in forced convection heat transfer in flow over and flat plate.

According to Reynolds Colburn analogy :

$$St.\ (Pr)^{2/3} = \frac{C_{fx}}{2}$$

where, St = Stanton number,

Pr = Prandtl number,

C_{fx} = Skin friction coefficient

22. Ans: (c)

Sol: Weins law, $\lambda T = 2898$

Fourier law = $Q = -kA\frac{dT}{dx}$

Reynolds No = $Re = \frac{\rho VD}{\mu}$

→ force convection

Stanton No = $St = \frac{Nu}{Re \times Pr}$

→ Forced convection

Fourier No = $Fo = \frac{\alpha\tau}{\ell^2}$

→ Transient conduction

23. Ans: (b)

Sol: $$Nu = C\left[\frac{g\beta\Delta Tx^3}{\nu^2}.\frac{\mu C_p}{k}\right]^{\frac{1}{3}}$$

$$\therefore \frac{hx}{k} = C\left[\frac{g\beta\Delta T}{\nu^2}\frac{\mu C_p}{k}\right]^{\frac{1}{3}}.x$$

⇒ 'h' is independent of the characteristic length .

For natural convection over a vertical flat plate in the turbulent region ($Gr_L > 10^9$) is

$$\overline{Nu_L} = 0.13\left(Gr_L\ Pr\right)^{\frac{1}{3}}$$

24. Ans: (d)

Sol: Nusselt number is greater than 1 in most of cases but not always.

In special case Nu = 1

Nusselt number cannot be less than 1

$$Nu = \frac{R_{cond.fluid}}{R_{conv.fluid}} = \frac{Q_{conv}}{Q_{cond}}$$

25. Ans: (a)

Sol: As the external force is applied on the fluid particles, the velocity of fluid particles is increasing so that many number of cold particles will come in contact with hot surface and carry away more amount of heat and hence heat transfer coefficient for forced convection is higher.

Conventional Practice Solutions

01.

Sol: Given: Velocity of air = 10 m/s

Temperature air (T_a) = 300 K

Diameter of Tube (D) = 11.2 mm = 0.0112 m

Temperature of tube wall (T_w) = 373 K

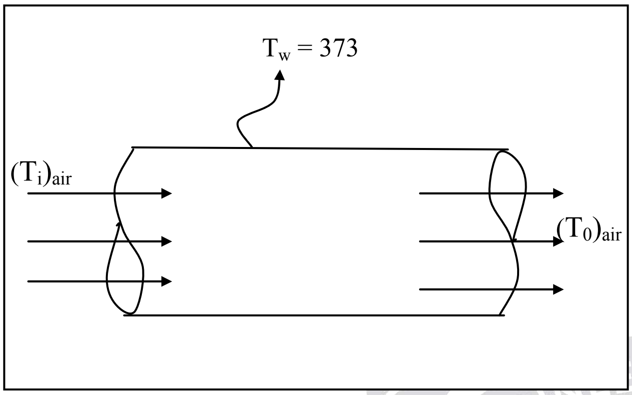
k = 0.02624 W/mK;

$c_p = 1.005 \times 10^3$ J/kg-K

$\rho = 1.174\ kg/m^3$

$\nu = 1.568 \times 10^{-5}\ m^2/sec$

Prandtle No. (Pr) = 0.7



$$Nu = 3.66 + \frac{0.668\left(\frac{d}{2}\right)Re.Pr}{1+0.04(d)Re.Pr}\ ;\ \text{if } Re < 2300$$

$$Nu = 0.023\ Re^{0.8}Pr^{0.4}\ ;\ \text{if } Re > 2300$$

$$Re = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

$$= \frac{10 \times 0.0112}{1.568 \times 10^{-5}}$$

Re = 7142.85 > 2300

Hence flow is turbulent

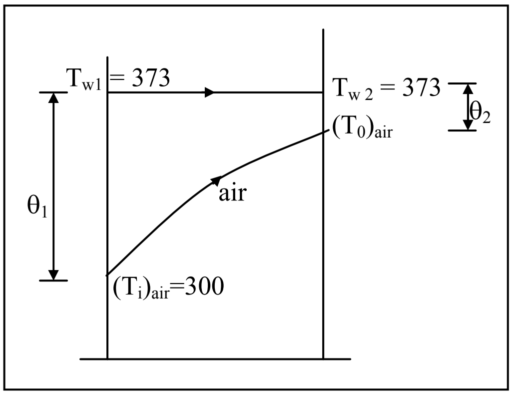
$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$

$$\frac{\bar{h}L}{k} = 0.023 \times (7142.85)^{0.8} \times (0.7)^{0.4}$$

$$\frac{\bar{h} \times D}{k} = 24.46$$

$$\bar{h} = \frac{24.46 \times K}{D} = \frac{24.46 \times 0.02624}{0.0112}$$

$\bar{h} = 56.57\ W/m^2K$



$\theta_1 = 373 - 300 = 73$

$\theta_2 = 373 - t_0$

Energy balance:

Heat transfer = $\dot{m}C_p\Delta T = \bar{h}A\theta_m$

Where $\dot{m} = \rho AV$

$$= 1.174 \times \frac{\pi}{4}(0.0112)^2 \times 10$$

$$= 1.157 \times 10^{-3}\ kg/s$$

$U = \bar{h} = 57.31\ W/m^2K$

$A = \pi DL = \pi(0.0112) \times 2.5 = 0.088\ m^2$

$$\theta_m = \frac{\theta_1 - \theta_2}{\ell n\frac{\theta_1}{\theta_2}} = \frac{73 - (373 - (T_0)_{air})}{\ell n\left(\frac{73}{373 - (T_o)_{air}}\right)}$$

$$\therefore 1.157 \times 10^{-3} \times 1.005 \times 10^3[(T_0)_{air} - 300)]$$

$$= 56.57 \times 0.088 \times \frac{(T_0)_{air} - 300}{ln\left(\frac{73}{373 - (T_0)_{air}}\right)}$$

$$1.163[(T_0)_{air} - 300] = 4.978\left(\frac{(T_0)_{air} - 300}{ln\left(\frac{73}{373 - (T_0)_{air}}\right)}\right)$$

$$(T_0)_{air} - 300 = 4.28\left(\frac{(T_0)_{air} - 300}{ln\left(\frac{73}{373-(T_0)_{air}}\right)}\right)$$

$$\Rightarrow ln\left(\frac{73}{373-(T_0)_{air}}\right) = 4.28$$

$$\Rightarrow (T_0)_{air} = 372\ K$$

02.

Sol: Given:

Viscosity of brine (μ) $=16.5\times10^{-6}$ N-s/m^2

Thermal conductivity (k) = 0.85 W/m-K

Inner diameter of pipe = 0.025 m

Velocity of fluid V_1 = 6.1 m/s

Heat transfer coefficient, h_1=1135 W/m^{2}K

Initial brine temperature $t_{b1} = -1^oC$

Pipe temperature $t_p = 18.3^oC$

Rise in prime temperature Δt = ?

Velocity doubled $V_2 = 2V_1$

Specific heat of brine (C_p) = 3768 J/kg-K

Density (ρ) = 1000 kg/m^3

Assume length of pipe (l) = 1 m

$$\text{Reynold No. (Re)} = \frac{\rho VD}{\mu}$$

$$= \frac{1000\times6.1\times0.025}{16.5\times10^{-6}}$$

$$= 9.2424\times10^6 > 4000$$

$\therefore$ Hence flow is turbulent.

For turbulent flow

Nusselt's Number (Nu) = $0.023(Re)^{0.8}(Pr)^n$

Where, n = 0.4 for heating

n = 0.3 for cooling

As brine solution getting heated, consider n = 0.4.

$\therefore$ Nu = $0.023(Re)^{0.8}(Pr)^{0.4}$

But for same fluid Pr no. is constant

$$\frac{(Nu)_2}{(Nu)_1} = \left(\frac{Re_2}{Re_1}\right)^{0.8}$$

$$\frac{h_2}{h_1} = \left(\frac{V_2}{V_1}\right)^{0.8}$$

$$h_2 = h_1\times\left(\frac{V_2}{V_1}\right)^{0.8}$$

$$= 1135\times2^{0.8}$$

$$= 1976.149 \approx 1976\ W/m^2K$$

$$\left(\because Re = \frac{\rho VD}{\mu}\quad \rho, D, \mu \text{ are constant}\quad Re = f(V)\right)$$

$$\left(\because N = \frac{hL}{K}\quad L, K \text{ are constant}\quad Nu = f(h)\right)$$

Where suffix '1' and '2' denotes at section (1) and (2)

Energy balance:

Heat gained by brine solution = Heat lost by convection from the pipe surface.

$$\dot{m}C_p(T_{b2} - T_{b1}) = h_2A(T_p - (T_m)_b)$$

$$\dot{m} = \rho AV = 1000\times\frac{\pi}{4}(0.025)^2\times12.2$$

($\because V_2 = 2V_1 = 2\times 6.1 = 12.2$)

$\dot{m} = 5.99$ kg/sec

Let mean bulk temperature of brine

$(T_m)_b = \dfrac{T_{b1} + T_{b2}}{2}$

$(T_m)_b = \dfrac{T_{b2} - 1}{2}$

$\Rightarrow 5.99\times3768\times(T_{b2}+1)$

$= 1976\times(\pi\times0.025\times L)\times\left(18.3 - \dfrac{T_{b2}-1}{2}\right)$

(Assume L = 1m)

$\Rightarrow 22570.32(T_{b2} + 1) = 77.6(37.6 - T_{b2})$

$\Rightarrow 22647.92\ T_{b2} = -19652.56$

$\Rightarrow T_{b2} = -0.8677°C$

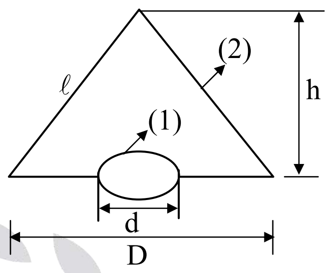
$\therefore$ Rise temperature of brine solution per meter length of pipe is $= -0.8677 - (-1)$

$= 0.1323°C/m$

Chapter 5 Radiation

01. Ans: (c)

Sol:



$A_1F_{12} = A_2F_{21}$ & $F_{12} = 1$

$F_{21} = \dfrac{A_1}{A_2}$

$$F_{21} = \frac{\frac{\pi}{4}d^2}{\left(\frac{\pi\times D\times \ell}{2}\right)} = \frac{\frac{\pi}{4}d^2}{\frac{\pi\times D}{2}\times\sqrt{\frac{D^2}{4}+h^2}}$$

$$= \frac{d^2}{2D\frac{\sqrt{D^2+4h^2}}{2}} = \frac{d^2}{D\sqrt{D^2+4h^2}}$$

02. Ans: (a)

Sol: Given data:

$\in_1 = 0.5$, $\in_2 = 0.9$,

$T_1 = 600$ K, $T_2 = 400$ K

Net heat exchange between two long parallel plates,

$$\frac{Q}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\in_1}+\frac{1}{\in_2}-1}$$

$$= \frac{5.67\times10^{-8}\left(600^4 - 400^4\right)}{\frac{1}{0.5}+\frac{1}{0.9}-1} = 2.79\,kW/m^2$$

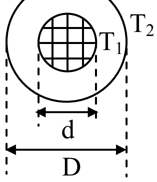
03. Ans: 792.16 K

Sol: Given data:

$d = 0.05$ m, $k = 15$ W/mK

$D = 0.06$ m, $q_g = 20 \times 10^3$ W/m^3

$T_2 = 773$ K, $\epsilon_1 = \epsilon_2 = 0.2$



Total heat generated

$$(Q_g) = q_g \frac{\pi}{4} d^2 L = 12.5\pi L$$

$$Q_g = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \left(\frac{D_1}{D_2}\right)\left(\frac{1}{\epsilon_2} - 1\right)}$$

$$12.5\pi L = \frac{\pi d L \times \sigma\left(T_1^4 - 773^4\right)}{\frac{1}{0.2} + \left(\frac{50}{60}\right)\left(\frac{1}{0.2} - 1\right)}$$

$$12.5 = \frac{0.05 \times \sigma\left(T_1^4 - 773^4\right)}{\frac{1}{0.2} + \left(\frac{50}{60}\right)\left(\frac{1}{0.2} - 1\right)}$$

$\Rightarrow$ $T_1 = 792.16$ K

04. Ans: (c)

Sol: $D_1 = 0.8$m,

$D_2 = 1.2$ m,

$\epsilon_1 = \epsilon_2 = 0.05$,

$T_1 = 95$ K,

$T_2 = 280$ K,

$h_{fg} = 2.13 \times 10^5$ J/kg

Net heat transfer,

$$Q = -\dot{m}h_{fg} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \left(\frac{D_1}{D_2}\right)^2\left(\frac{1}{\epsilon_2} - 1\right)}$$

$$-\dot{m} \times 2.13 \times 10^5 = \frac{\pi(0.8)^2 \times 5.67 \times 10^{-8} \times \left(95^4 - 280^4\right)}{\frac{1}{0.05} + \left(\frac{0.8}{1.2}\right)^2\left(\frac{1}{0.05} - 1\right)}$$

$$\therefore \quad \dot{m} = 1.1913 \times 10^{-4}\,\text{kg/s} = 0.4108\,\text{kg/hr}$$

05. Ans: (d)

Sol: Given data:

$\epsilon_1 = \epsilon_2 = 0.8$,

$Q_{\text{without shield}} = 10\ Q_{\text{with shield}}$

$$\frac{Q_{\text{with shield}}}{Q_{\text{without shield}}} = \frac{\dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2N}{\epsilon_s} - (N+1)}}{\dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}}$$

$$\frac{Q_{\text{with shield}}}{Q_{\text{without shield}}} = \frac{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{2N}{\epsilon_s} - (N+1)}$$

$$10 = \frac{\frac{1}{0.8} + \frac{1}{0.8} - 1}{\frac{1}{0.8} + \frac{1}{0.8} + \frac{2}{\epsilon_S} - 2}$$

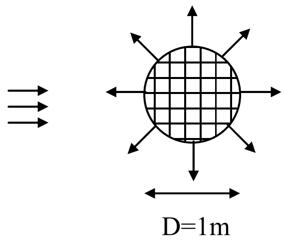
(Number of shield (N) = 1)

$\epsilon_S = 0.138$

06. Ans: (c)

Sol: Given data:

G = 300 W/m^2, $\in = 0.4$, $\alpha = 0.3$



$\alpha G A_{projected} = \in E_b A$

$$0.3\times300\times\frac{\pi}{4}D^2 = 0.04\times\sigma\times T^4\times\pi D^2$$

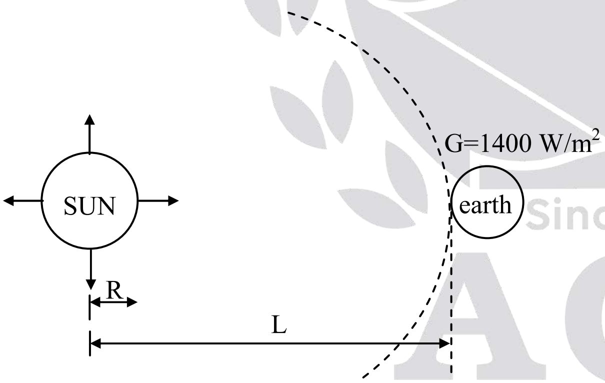
$$0.3\times300\times\frac{\pi}{4}(1)^2 = 0.04\times5.67\times10^{-8}\times T^4\times\pi(1)^2$$

$\Rightarrow$ T = 315.6 K

07. Ans: (c)

Sol: Given data:

L = 1.5 × 10^{11} m, R_{SUN} = 7 × 10^8 m,



Energy balance:

$E_b \times A_{SUN} = G \times A_{Hemisphere}$

$$\sigma T_{SUN}^4 \times 4\pi R^2 = G\times4\pi L^2$$

$$T_{SUN}^4 = \left(\frac{L}{R}\right)^2\frac{G}{\sigma}$$

T_{SUN} = 5802.634 ≈ 5800 K

08. Ans: (a)

Sol: Given data:

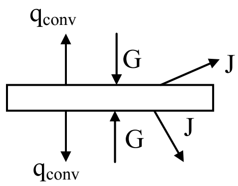
J = 5000 W/m^2,

T_1 = 350 K,

T_∞ = 300 K,

h = 40 W/m^{2}K,

α = 0.4



Convective heat transfer (q_{conv}) = h($T_s - T_\infty$)

= 40 (350 – 300)

= 2000 W/m^2

Energy balance:

$Q_{in} + Q_{gen} - Q_{out} = Q_{stored}$

$Q_{in} - Q_{out} = 0$ (Q_{stored} = 0 and Q_{gen} = 0)

2G – [2J + 2 q_{conv}] = 0

2G – [2 × 5000 + 2 × 2000] = 0

G = 7000 W/m^2

Leaving energy (J) = ρG + E + τG

J = ($\rho + \tau$) G + E

J = (1 – α) G + E

J = (1 – 0.40) × 7000 + $\in E_b$

5000 = 0.6×7000 + $\in$ ×5.67 × 10^{-8}×(350)^4

$\Rightarrow \in$ = 0.940

09. Ans: (d)

Sol: Black body emission does not depend on the size of the object.

10. Ans: (b)

Sol: Given data:

$T_w = 533$ K, $T_{tc} = 1066$ K,

$\in = 0.5$, $\bar{h} = 114\,W/m^2K$

Energy balance:

Heat transfer by convection = Heat transfer by radiation

$q_{conv} = q_{rad}$

$$\bar{h}(T_{air} - T_{tc}) = \in \sigma (T_{tc}^4 - T_w^4)$$

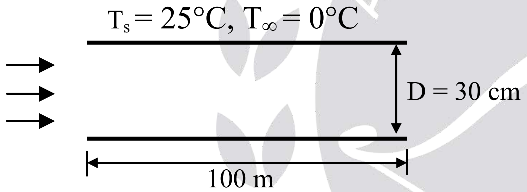
$$114(T_{air} - 1066) = 0.5\times5.67\times10^{-8}(1066^4 - 533^4)$$

$$\Rightarrow \quad T_{air} = 1367\text{ K}$$

11. Ans: (a)

Sol: $T_{sky} = -30°C$,

$h = 4.36$ W/m^{2}K



Power required by resistance heater = Heat loss by convection from the surface + Heat loss by radiation from surface

$P = hA\,(T_s - T_\infty) + \in\sigma\,A_s\,(T_s^4 - T_{sky}^4)$

$= 4.36\times\pi \times D \times L\,(25 - 0) + 0.8 \times 5.67 \times 10^{-8} \times \pi \times D \times L\,(298^4 - 243^4)$

$= 4.36\times\pi \times 0.3 \times 100\,(25 - 0) + 0.8 \times 5.67 \times 10^{-8} \times \pi \times 0.3 \times 100\,(298^4 - 243^4)$

$= 29080.64$ W

$P = 29.08$ kW

12. Ans: (b)

Sol: $F_{11} + F_{12} + F_{13} + F_{14} = 1$ (Summation rule)

$F_{14} = 1 - (0.1 + 0.4 + 0.25)$

$= 1 - 0.75 = 0.25$

$A_1 F_{14} = A_4 F_{41}$ (Reciprocity theorem)

$$F_{41} = \frac{A_1}{A_4} \times F_{14} = \frac{4}{2} \times 0.25 = 0.5$$

13. Ans: (d)

Sol:

- Infinite parallel plates $\Rightarrow \dfrac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$
- Body 1 completely enclosed by body 2 but body 1 is very small $\Rightarrow \varepsilon_1$
- Radiative exchange with two small gray bodies $\Rightarrow \varepsilon_1\varepsilon_2$
- Two concentric cylinders with large length $\Rightarrow \dfrac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$

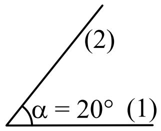
14. Ans: (c)

Sol:

- *Window glass:* Transparency to short wavelength and becomes opaque for long wavelength.
- *Gray surface:* emissivity independent of wavelength.
- *Carbondioxide:* emission and absorption limited to certain bands and wavelength.
- *Radiosity:* Rate at which radiation leaves a surface (Total leaving energy from a surface).

15. Ans: (a)

Sol:



View factor $F_{1-2} = F_{2-1} = 1 - \sin(\alpha/2)$

$= 1 - \sin(10)$

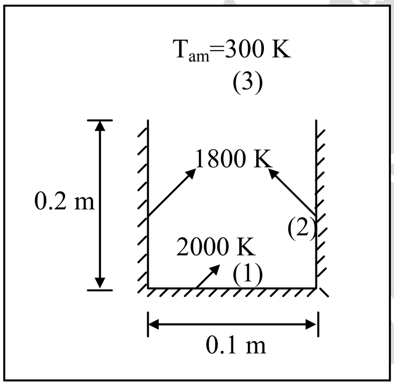
$= 0.83$

16. Ans: (a)

Conventional Practice Solutions

01.

Sol:



A_2 = sides surface area

A_1 = Bottom surface area

Given shape factor from the bottom surface to surroundings is 0.06.

i.e., $F_{13} = 0.06$

As we know $F_{11} + F_{12} + F_{13} = 1$

($\because F_{11} = 0$ since flat surface)

$F_{12} = 1 - 0.06 = 0.94$

Similarly $F_{21} + F_{22} + F_{23} = 1$

By reciprocity theorem

$A_1F_{12} = A_2F_{21}$

$$F_{21} = \frac{A_1}{A_2}F_{12}$$

$$= \frac{\pi\left(\frac{0.1}{2}\right)^2}{\pi(0.1)(0.2)}\times 0.94 = \frac{0.1}{4\times 0.2}\times 0.94$$

$F_{21} = 0.1175$

$\therefore F_{23} = 0.1175$

($\because F_{21} = F_{23}$ by symmetry)

$F_{22} = 1 - 2(0.1175)$

$F_{22} = 0.765$

Power required to the furnace

$= A_2F_{21}\sigma_b(T_2^4 - T_1^4) + A_2F_{23}\sigma_b(T_2^4 - T_3^4)$

$= \pi(0.1)(0.2)\times 0.1175\times 5.67\times 10^{-8}\times(1800^4 - 2000^4) + \pi(0.1)(0.2)\times 0.1175\times 5.67\times 10^{-8}(1800^4 - 300^4)$

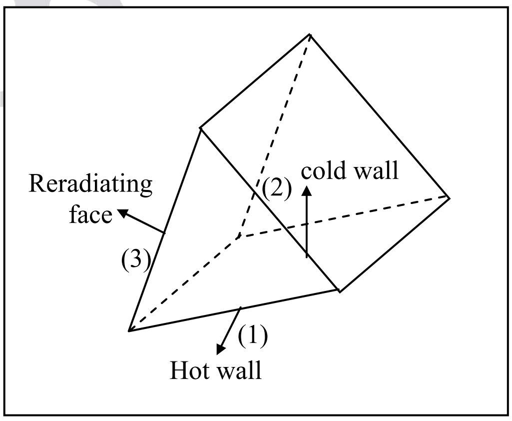
Power = 2087.60

Power required = 2.0876 kW

≈ heat lost from surface to surroundings

02.

Sol:



Given:

$T_1 = 1000$ K, $T_2 = 500$ K

$\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.8$,

$\varepsilon_3 = 1$ for reradiating surfaces & $Q_3 = 0$

Since all surface are flat, shape factors w.r.t itself are 'zero'.

i.e. $F_{11} = F_{22} = F_{33} = 0$

since $Q_3 = 0$

$$Q_3 = A_3(F_g)_{31}\sigma_b\left(T_3^4 - T_1^4\right) + A_3(F_g)_{32}\sigma_b\left(T_3^4 - T_2^4\right) \quad \text{-------- (1)}$$

$F_{11} + F_{12} + F_{13} = 1$

$F_{12} + F_{13} = 1$ $(\because F_{11} = 0)$

$F_{12} = F_{13} = 0.5$ $(\because F_{12} = F_{13})$

$F_{21} + F_{22} + F_{23} = 1$ $(\because F_{22} = 0)$

$F_{21} = F_{23} = 0.5$ $(\because F_{21} = F_{23})$

Similarly $F_{31} = F_{32} = 0.5$, $F_{33} = 0$

$$(F_g)_{31} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{31}} + \frac{1-\varepsilon_3}{\varepsilon_3}}$$

$$= \frac{1}{\frac{1-0.8}{0.8} + \frac{1}{0.5} + \frac{1-1}{1}} = 0.444$$

$(F_g)_{31} = (F_g)_{13} = 0.444$ ……… (2)

$(F_g)_{32} = (F_g)_{23} = 0.444$

$$(F_g)_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2}}$$

$$= \frac{1}{\frac{1-0.8}{0.8} + \frac{1}{0.5} + \frac{1-0.8}{0.8}}$$

$(F_g)_{12} = 0.4$

$(F_g)_{12} = (F_g)_{13} = (F_g)_{21} = (F_g)_{31} = 0.4$

$$\therefore (1) \Rightarrow T_3 = \left(\frac{T_1^4 + T_2^4}{2}\right)^{\frac{1}{4}}$$

$$= \left(\frac{1000^4 + 500^4}{2}\right)^{\frac{1}{4}}$$

$T_3 = 853.74$ K

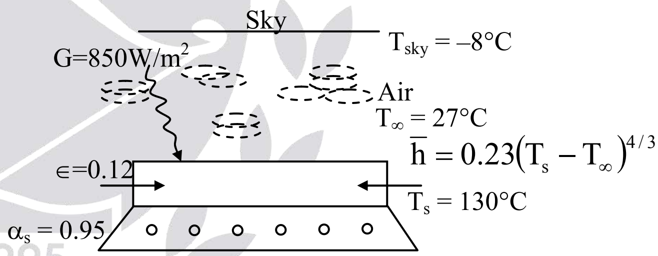
$$\therefore Q_1 = A_1(F_g)_{12}\sigma_b\left(T_1^4 - T_2^4\right) + A_1(F_g)_{13}\sigma_b\left(T_1^4 - T_3^4\right)$$

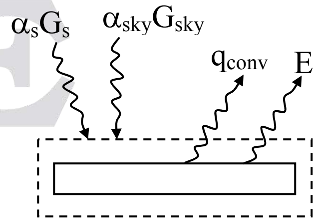
$$\frac{Q_1}{A_1} = 0.4 \times 5.67 \times 10^{-8}(1000^4 - 500^4) + 0.444 \times 5.67 \times 10^{-8}(1000^4 - 853.74^4)$$

$$\frac{Q}{A} = 33.063 \text{ kW/m}^2$$

03.

Sol:





Sky is a black body (infinite area)

$G_{sky} = \sigma T_{sky}^4$

$\therefore q_{conv} = \bar{h}\left(T_s - T_\infty\right)$

$= 0.23(T_s - T_\infty)^{4/3}(T_s - T_\infty)$

$= 0.23(T_s - T_\infty)^{7/3}$

$\therefore \quad E = \in\sigma T_s^4$

$\therefore$ Since the atmospheric irradiation is concentrated in approximately the same spectral region as that of surface emission so its is reasonable to assume that

$\alpha_{sky} = \in = 0.12$

(Kirchoff's law = $\alpha = \in$)

Energy balance on absorber

$Q_{in} + Q_{gen} - Q_{out} = Q_{useful}$

$\alpha_s G_s + \alpha_{sky}\sigma T_{sky}^4 - 0.23(T_s - T_\infty)^{7/3} - \in\sigma T_s^4 = q_{useful}$

$\Rightarrow \quad q_{useful} = -10776.35\ W/m^2$ (–ve sign indicates that collector is loosing heat)

The collector efficiency, defined as the fraction of the solar irradiation extracted as useful energy is then

$$\eta = \frac{q_{useful}}{G_{solar}} = \frac{10776.35}{850} = 12.678 = 1267.8\%$$

(Absurd value)

Comment:

Since the spectral range of G_{sky} is entirely different from that at Gs at would be incorrect to assume that $\alpha_{sky} = \alpha_s$

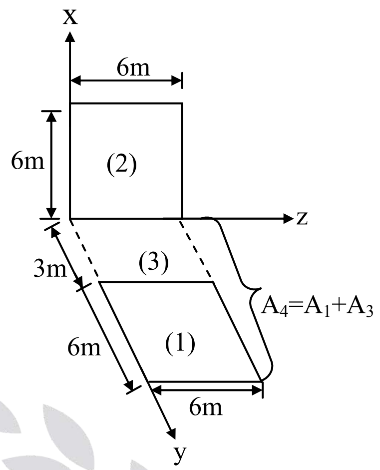
Note: If $\bar{h} = 0.23(T_s - T_\infty)^{1/3}$

Then, $q_{useful} = 550.53\ W/m^2$

$$\eta = \frac{550.53}{850} = 0.6476 = 64.76\%$$

04.

Sol:



Given,

$F_{12} = ?$

Let, $A_4 = A_1 + A_3$

$= (6 \times 6) + (6 \times 3) = 54$

$A_4F_{4-2} = A_3F_{3-2} + A_1F_{1-2}$

(*Note:* The above table will be given question paper)

$\therefore \quad 54 \times 0.125 = (3\times6) \times 0.28 + (6\times6) \times F_{1-2}$

$\Rightarrow \quad F_{1-2} = 0.0475$

Chapter 6 Heat Exchangers

01. Ans: (d)

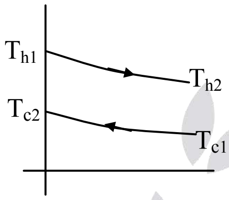
Sol: Given that:

$T_{h1} = 70°C$, $T_{c1} = 30°C$

$T_{h2} = 40°C$, $T_{c2} = 50°C$

$\Delta T_1 = T_{h1} - T_{c2} = 20$

$\Delta T_2 = T_{h2} - T_{c1} = 10$

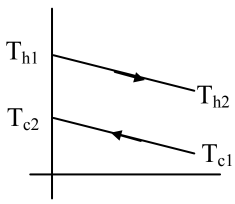


Log Mean Temperature Difference

$$(\text{LMTD}) = \frac{\Delta T_1 - \Delta T_2}{\ell n\left[\frac{\Delta T_1}{\Delta T_2}\right]} = \frac{20-10}{\ell n\left(\frac{20}{10}\right)} = 14.42°C$$

02. Ans: (c)

Sol:



LMTD = 20°C, $T_{c1} = 20°C$, $T_{h1} = 100°C$

$\dot{m}_c = 2\dot{m}_h$ $c_h = 2c_c$,

$C_h = \dot{m}_h c_h = 2\dot{m}_h c_c$

$C_c = \dot{m}_c c_c = 2\dot{m}_h c_c$

When $C = \frac{C_{min}}{C_{max}} = 1$, Temperature profile will be linear for the counter flow heat exchanger and the mean temperature difference between hot fluid and cold fluid will be same at every section.

$\text{LMTD} = \Delta T_1 = \Delta T_2$

$\text{LMTD} = \Delta T_1 = T_{h1} - T_{c2}$

$20 = 100 - T_{c2}$

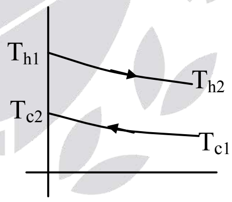
$T_{c2} = 100 - 20 = 80°C$

03. Ans: 0.9

Sol: This is the counter flow type of heat exchanger because exit temperature of cold fluid is greater than that of hot fluid.

$T_{h1} - T_{h2} = 200 - 110 = 90°C$

$T_{c2} - T_{c1} = 125 - 100 = 25°C$



Energy balance:

Heat released by hot fluid = heat received by cold fluid

$\dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$

$\dot{m}_h c_h \times 90 = \dot{m}_c c_c \times 25$

From the above equation $\dot{m}_c c_c > \dot{m}_h c_h$

$$\text{Effectiveness } (\in) = \frac{Q_{act}}{Q_{max}} = \frac{\dot{m}_h c_h (T_{h1} - T_{h2})}{\dot{m}_h c_h (T_{h1} - T_{c1})}$$

$$= \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} = \frac{90}{200 - 100}$$

$$= \frac{90}{100} = 0.9$$

04. Ans: (c)

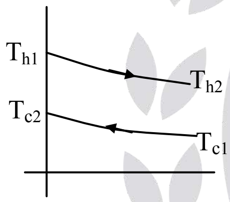
Sol: Given data:

$\dot{m}_h = 3.5\,kg/s$, $T_{h1} = 80°C$,

$c_c = 4180$ J/kg°C, $U_i = 250$ W/m^2°C

$c_h = 2560$ J/kg°C, $T_{h2} = 40°C$,

$T_{c1} = 20°C$, $T_{c2} = 55°C$



$\Delta T_1 = T_{h1} - T_{c2} = 25$

$\Delta T_2 = T_{h2} - T_{c1} = 20°$

Log Mean Temperature Difference

$$(LMTD) = \frac{\Delta T_1 - \Delta T_2}{\ell n\left[\frac{\Delta T_1}{\Delta T_2}\right]} = \frac{25 - 20}{\ell n\left(\frac{25}{20}\right)} = 22.40°C$$

Heat transfer rate

$$(Q) = \dot{m}_h c_h (T_{h1} - T_{h2}) = U_i \times A_i \times LMTD$$

$$35 \times 2560\ (80–40) = 250 \times A_i \times 22.4$$

$$A_i = 64\ m^2$$

05. Ans: (a)

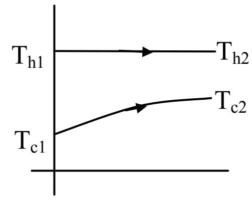
Sol: Given data:

$T_{h1} = T_{h2} = 75°C$,

$\dot{m}_h = 2.7\,kg/s$

$T_{c1} = 21°C$,

$T_{c2} = 28°C$,



$A = 24$ m^2, $h_{fg} = 255.7$ kJ/kg

$\Delta T_1 = T_{h1} - T_{c1} = 54°C$

$\Delta T_2 = T_{h2} - T_{c2} = 47°C$

Log Mean Temperature Difference (LMTD)

$$= \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{54 - 47}{\ell n\left(\frac{54}{47}\right)} = 50.149°C$$

Heat transfer rate

$$(Q) = \dot{m}_h \times h_{fg} = U \times A \times LMTD$$

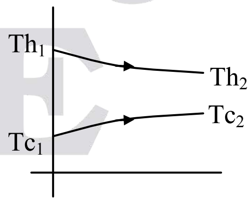
$$2.7 \times 255.7 \times 10^3 = U \times 24 \times 50.419$$

$$\Rightarrow \quad U = 571\ W/m^2°C$$

06. Ans: (c)

Sol: $T_{h1} = 150°C$, $T_{c1} = 25°C$

$T_{h2} = 80°C$, $T_{c2} = 60°C$



$\Delta T_1 = T_{h1} – T_{c1} = 125$

$\Delta T_2 = T_{h2} - T_{c2} = 20$

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{125 - 20}{\ell n\left(\frac{125}{20}\right)} = 57.29°C$$

Energy balance:

Heat released by hot fluid = heat received by cold fluid

$$\dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

$$\dot{m}_h c_h (150 - 80) = \dot{m}_c c_c (60 - 25)$$

$$\dot{m}_h c_h \times 70 = \dot{m}_c c_c \times 35$$

From the above equation

$$\dot{m}_c c_c > \dot{m}_h c_h \Rightarrow C_{min} = \dot{m}_h c_h$$

Heat transfer rate (Q)

$$\dot{m}_h c_h (T_{h1} - T_{h2}) = U \times A \times LMTD$$

$$C_{min}(T_{h1} - T_{h2}) = U \times A \times LMTD$$

$$NTU = \frac{UA}{C_{min}} = \frac{T_{h1} - T_{h2}}{LMTD} = \frac{70}{57.29} = 1.22$$

07. Ans: (c)

Sol: Given data:

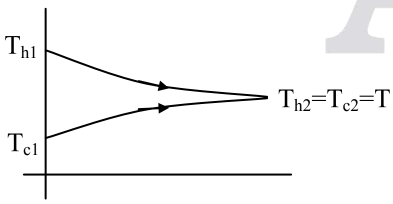
$T_{c1} = 20°C$, $T_{h1} = 80°C$,

$\dot{m}_c = 20$ kg/s, $\dot{m}_h = 10$ kg/s,

$c_h = c_c = 4.2 \times 10^3$ J/kg.K,

$\dot{m}_h c_h = C_{min}$

Case – I, For parallel flow heat exchanger:



Energy balance:

Heat released by hot fluid = heat received by cold fluid

$$\dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

$$10(80 – T) = 20(T – 20)$$

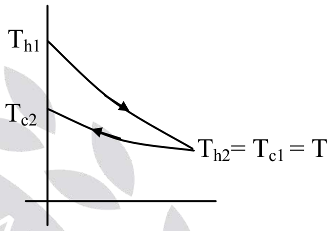
$$80 – T = 2(T – 20)$$

$$80 – T = 2T – 40$$

$$120 = 3T$$

$$\Rightarrow T = 40°C$$

Case – II, For counter flow heat exchanger:



Energy balance:

Heat released by hot fluid = heat received by cold fluid

$$\dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

$$10(80 – T_{c1}) = 20\,(T_{c2} – T_{c1})$$

$$10(80 – 20) = 20\,(T_{c2} – 20)$$

$$\Rightarrow T_{c2} = 50°C$$

08. Ans: (b)

Sol: Given data:

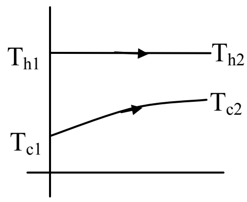
$Q = 23.07 \times 10^6$ W, $T_{h1} = T_{h2} = 50°C$,

$T_{c1} = 15°C$, $T_{c2} = 25°C$,

D = 0.0225 m, $c_c = 4180$ J/kg.K

$u_{avg} = 2.5$ m/s, $U = 3160.07$ W/m^{2}K,

LMTD = 29.72°C



Heat transfer rate $(Q) = \dot{m}_c c_c (T_{c2} - T_{c1})$

$$23.07 \times 10^6 = \dot{m}_c \times 4180(25 - 15)$$

$$\dot{m}_c = 551.91 \text{kg/sec}$$

$$\dot{m}_{\text{each tube}} = \rho A u_{avg} = \rho \frac{\pi}{u_{avg}} D^2 u_{avg}$$

$$= 998.8 \times \frac{\pi}{4}(0.0225)^2 \times 2.5$$

$$\dot{m}_{\text{each tube}} = 0.7942 \text{kg/sec}$$

$$\text{No.of tube} \times \dot{m}_{\text{each tube}} = \dot{m}_c$$

$$\text{No. of tube} = \frac{551.91}{0.7942} = 695$$

Heat transfer rate (Q) = U × A × LMTD

$$23.07 \times 10^6 = 3160.17 \times A \times 29.72$$

$$\Rightarrow \quad A = 245.64 \text{ m}^2$$

$$A = \pi DL \times \text{No. of tube} \times \text{No. of pass}$$

$$245.64 = \pi \times 0.0225 \times 2.5 \times 695 \times \text{No. of pass}$$

No. of pass = 2

09. Ans: (d)

Sol: Effectiveness ($\in$) of heat exchanger will be minimum when C $\left(\frac{C_{min}}{C_{max}}\right) = 1$

Effectiveness of parallel flow heat exchanger $= \frac{1 - e^{-(1+C)NTU}}{1 + C}$

When C = 1

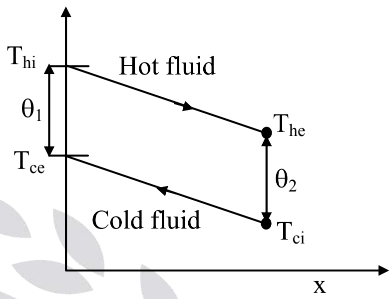
$$\in = \frac{1 - e^{-2NTU}}{2},$$

$$\in = \frac{1 - e^{-2\times 2.5}}{2} = 0.4966 = 50\%$$

10. Ans: (d)

Sol: Counter flow with equal heat capacities rates:

$$C = \frac{C_{min}}{C_{max}} = 1 \quad \text{(Linear temperature profile)}$$

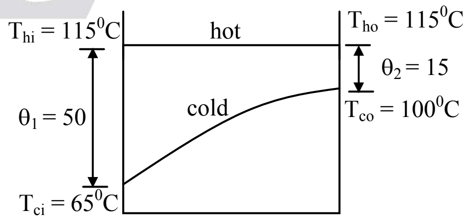


- When C = 1, LMTD = $\theta_1 = \theta_2$
  LMTD is possible to determine in counter flow heat exchanger when C = 1
- Temperature difference between hot and cold fluid is same at every section. i.e. temperature difference is invariant along the length as shown in figure.

Conventional Practice Solutions

01.

Sol:



Given $h_i = 5$ kW/m^{2}K,

$h_o = 10$ kW/m^{2}K

Over all heat transfer is given by

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{1}{5} + \frac{1}{10}$$

$U = 3.3\times10^3$ J/kg K

Given C_{Pc} = 4.6 kJ/kg = 4600 J/kg K

$\rho_h = 1100$ kg/m^3

$\dot{m}_c = 11.8$ kg/s, velocity "V" = 1.2 m/s

Diameter of pipe, d = 0.025 m

$T_{hi} = T_{ho} = 115^oC$,

$T_{ci} = 65^oC$,

$T_{ho} = 100^oC$

$$Q_{\text{lost cold fluid}} = \dot{m}_c C_{pc}\times \Delta T$$
$$= 11.8\times4600\times(100-65)$$
$$= 1900 \text{ kW}$$

But, $Q = UA\ \theta_m$

$$1900\times10^3 = 3.3\times10^3\times A\times\left(\frac{\theta_1-\theta_2}{ln\left(\frac{\theta_1}{\theta_2}\right)}\right)$$

$$1900\times10^3 = 3.3\times10^3\times A\times\left(\frac{50-15}{ln\left(\frac{50}{15}\right)}\right)$$

$\Rightarrow$ A = 19.805 m^2

But $\dot{m}_c = \frac{\pi}{4}d^2\times V\times\rho\times N_p$

Where

N_p = Number of tubes required per pass.

$$11.8 = \frac{\pi}{4}(0.025)^2\times1.2\times1100\times N_p$$

N_p = 18.2 = 19 tubes

A = π.d.L.n.p

19.805 = π×0.025×3.5×19×p

$\Rightarrow$ p = 3.91

Number of passes, (p) = 4

02.

Sol: Given

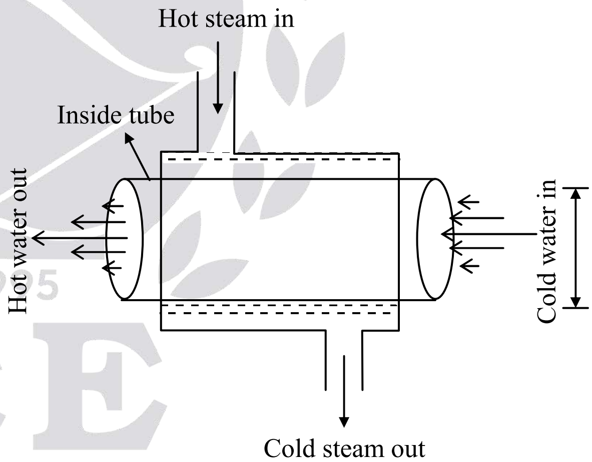
Inlet temperature of water

(T_i) = 10°C = 283 K

Outlet temperature of water

(T_o) = 40°C = 313 K

Water mass rate ($\dot{m}$) = 0.5 kg/s



Condensing side i.e. Tube outside heat transfer coefficient (h_o) = 10000 W/m^{2}K

Density (ρ) = 990 kg/m^3,

Thermal conductivity (k) = 0.57 W/m^{2}K

Specific heat (C_p) = 4180 J/kg.K

Dynamic viscosity (μ) = 0.8 × 10^{-3} Passes

$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$

$$Re = \frac{\rho V d}{\mu} = \frac{4\dot{m}}{\pi d \mu} = \frac{4 \times 0.5}{\pi \times 0.025 \times 0.8 \times 10^{-3}}$$

$$\approx 31831$$

$$Pr = \frac{\mu C_p}{k} = \frac{0.8 \times 10^{-3} \times 4180}{0.57} = 5.867$$

$Nu = 0.023 \times (31831)^{0.8} \times (5.87)^{0.4}$

$Nu = 186.8$

But, $Nu = \frac{hd}{k}$

(For pipe L_C = diameter of pipe)

$$\therefore \frac{h_i d}{K} = 186.8$$

$$h_i = \frac{186 \times 0.57}{0.025} = 4259.04$$

Since, $\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o}$ $(\because A_0 = A_i)$

$$= \left(\frac{1}{h_i} + \frac{1}{h_o}\right) \times \frac{1}{A}$$

$$= \left(\frac{1}{4259.04} + \frac{1}{10000}\right) \times \frac{1}{\pi \times 0.025 \times 10}$$

$\Rightarrow UA = 2345.9$

$$NTU = \frac{UA}{C_{min}} = \frac{2345.9}{4180 \times 0.5} = 1.122$$

$(\because C_{min} = \dot{m}_w \times C_w)$

Effectiveness, $\varepsilon = 1 - e^{-NTU}$

$$= 1 - e^{-1.122} = 0.67$$

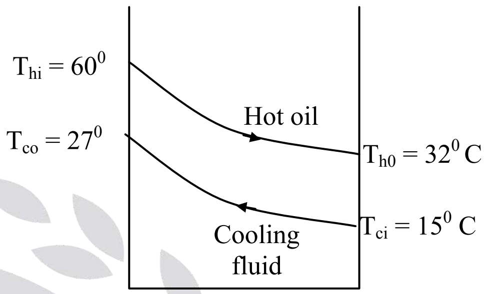
03.

Sol: *Given:* $\dot{m}_h$ = 900 kg/hr,

$T_{ho} = 32°C$, $T_{hi} = 60°C$

$T_{co} = 27°C$, $T_{ci} = 15°C$

$C_{ph} = 0.5$ kJ/kg-K, $C_{PC} = 1$ kJ/kg-K



Energy balance:

Heat lost by hot oil

= heat gained by cooling fluid

$$\dot{m}_h C_{ph}(T_{hi} - T_{h0}) = \dot{m}_c C_{pc}(T_{c0} - T_{ci})$$

$$900 \times 0.5(60 - 32) = \dot{m}_c \times 1(27 - 15)$$

$\Rightarrow \dot{m}_c = 1050$ kg/hr

$\dot{m}_c C_{pc} = 1050 \times 1 = 1050$ kJ/hr

$\dot{m}_h C_{ph} = 900 \times 0.5 = 450$ kJ/hr

$C_{min} = 450$ kJ/hr $= 0.125$ kJ/sec

Heat capacity ratio $R = \frac{C_{min}}{C_{max}}$

$$R = \frac{450}{1050} = 0.4285$$

Effectiveness $(\varepsilon) = \frac{\dot{m}_h Cp_h (T_{hi} - T_{ho})}{C_{min}(T_{hi} - T_{co})}$

$(\because C_{min} = \dot{m}_h C_{ph})$

$$\varepsilon = \frac{T_{hi} - T_{h0}}{T_{hi} - T_{ci}} = \frac{60 - 32}{60 - 15} = 0.622$$

But for counter flow heat exchanger

$$\varepsilon = \frac{1-e^{-NTU(1-R)}}{1-R.e^{-NTU(1-R)}}$$

$$0.622 = \frac{1-e^{-NTU(1-0.4285)}}{1-0.4285.e^{-NTU(1-0.4285)}}$$

$$0.622 = \frac{1-e^{-0.5715\ NTU}}{1-0.4285e^{-0.5715\ NTU}}$$

$\Rightarrow$ NTU = 1.159 $\approx$ 2

But, $NTU = \frac{UA}{C_{min}}$

$$\Rightarrow \ 2 = \frac{200\times\pi DL}{0.125\times10^3} \Rightarrow L = 19.89$$

As the length of the H.E is limited to 12 m.

Hence No. of tubes required is $\frac{19.89}{12} \approx 2$

04.

Sol: Given diameter of pipe = d = 0.023 m

Velocity of water V = 2.1 m/s

Inlet temperature (T_i) = 15°C

Outlet temperature (T_o) = 25°C

Mean temperature $(T_m) = \frac{T_i + T_0}{2}$ = 20°C

All values takes at 20°C

$\therefore$ At 20°C, $Re = \frac{\rho V d}{\mu} = \frac{Vd}{\nu}$

$$= \frac{2.1\times0.023}{1.006\times10^{-6}}$$

= 48012 > 2300

As Re > 2300, fluid flow is turbulent

For turbulent flow,

$$Nu = 0.023\ (Re)^{0.8}(Pr)^n$$

Where, n = 0.4 for heating

n = 0.3 for cooling

As in the given case water is heated up take

n = 0.4

Hence $Nu = 0.023\ (Re)^{0.8}(Pr)^{0.4}$

Prandtle number (Pr) = 7.02

(At 20°C from table)

But, $Nu = \frac{hd}{k}$

$$\therefore \frac{hd}{k} = 0.023 \times (48012)^{0.8} \times (7.02)^{0.4}$$

$$\Rightarrow h_i = \frac{59.78\times10^{-2}}{0.023}\times0.023\times(48012)^{0.8}\times(7.02)^{0.4}$$

$\Rightarrow h_i$ = 7247.2 W/m^2 K

Heat transfer (Q) = $hA_s\Delta T = h(\pi d l)\Delta T$

Q = 7247.205 ×(π×0.023)×10

= 5236.58 W/m

Q/L = 5.236 kW/m

05.

Sol: Given:

Inner diameter of the copper tube,

$(d_i)_c$ = 16 mm = 0.016 m

Outer diameter of copper tube,

$(d_0)_c$ = 19 mm = 0.019 m

Inner diameter of steel tube,

$(d_i)_s$ = 26 mm = 0.026 m

Outer diameter of steel tube,

$(d_0)_s$ = 30 mm = 0.03 m

Hot field oil inlet temperature, (T_{h1}) = 65°C

Hot fluid oil out let temperature, (T_{h2}) = 50°C

Cold fluid water inlet temperature,

$(T_{c1}) = 32°C$

Cold fluid water outlet temperature, (T_{c2}) = ?

Mass flow rate of oil $(\dot{m}_h)$ = 0.4 kg/sec

Mass flow rate of water

$$(\dot{m}_c) = \frac{\pi}{4}(0.016)^2 \times 1.48 \times 995$$

$$= 0.296 \text{ kg/sec} \qquad (\because \dot{m} = \rho A V)$$

Rate of heat transfer is given by

$$Q = \dot{m}_h C_{ph}(T_{h1} - T_{h2}) = \dot{m}_c C_{P_c}(T_{c2} - T_{c1})$$

$$Q = 0.4 \times 1.89 \times (65-50) = 11.34 \text{ kW}$$

$$Q = \dot{m}_c C_{p_c}(T_{c2} - T_{c1})$$

$$11.34 = 0.296 \times 4.187(T_{c2} - 32)$$

$$\Rightarrow T_{c2} = 41°C$$

Reynolds number for copper tube

$$Re = \frac{\rho V d_i}{\mu} = \frac{\rho d_i}{\mu} \times \frac{\dot{m}}{\frac{\pi}{4} d_i^2 \times \rho}$$

$$Re = \frac{4\dot{m}_c}{\pi(d_i)\mu} \qquad (\because \dot{m} = \rho A V \Rightarrow V = \frac{\dot{m}}{\rho A})$$

$$Re = \frac{4 \times 0.296}{\pi \times 0.016(995 \times 4.18 \times 10^{-7})} \qquad (\because \mu = \rho \nu)$$

Re = 56826 > 2300

$\therefore$ As Re > 2300, flow is turbulent

$$Pr = \frac{\mu C_p}{k} = \frac{995 \times 4.18 \times 10^{-7} \times 4.187 \times 10^3}{0.615}$$

Pr = 2.83

For turbulent flow

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.3}$$

$$Nu = 0.023(56826)^{0.8}(2.83)^{0.3} = 199.94$$

But, $Nu = \frac{h_i (d_i)_c}{k} = 199.94$

$$\Rightarrow h_i = \frac{199.94 \times 0.615}{0.016} = 7685.22 \text{ W/m}^2\text{K}$$

As the oil flows in annulus, hydraulic diameter has to be consider to find Re for annulus flow

Hydraulic diameter $(d_h) = (d_i)_s - (d_0)_c$

$= 0.026 - 0.019$

$= 0.007$ m

$$Re = \frac{\rho U_m D_m}{\mu} = \frac{4\dot{m}}{\pi d_h \mu}$$

$$= \frac{4 \times 0.4}{\pi(0.007) \times (850 \times 7.44 \times 10^{-6})}$$

$$= 11504.83$$

As Re > 2300, flow is turbulent in annulus

$$\therefore Nu = 0.023(Re)^{0.8}(Pr)^{0.4}$$

$$Pr = \frac{\mu C_p}{k} = \frac{850 \times 7.44 \times 10^{-6} \times 1890}{0.138} = 86.61$$

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.3}$$

$$Nu = \frac{h_0 D_h}{K} = 0.023(11504.83)^{0.8} \times (86.61)^{0.3}$$

$$= 155.48$$

$$h_0 = \frac{155.48 \times 0.138}{0.007} = 3065.37 \text{ W/m}^2\text{K}$$

Over all heat transfer coefficient is given by

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_o} \times \frac{(r_i)}{(r_o)} + R_{fi} + R_{fo} \times \frac{(r_i)}{(r_o)}$$

$$= \frac{1}{7685.22} + \frac{1}{3065.37}\left(\frac{0.008}{0.0095}\right) + 0.0005 + 0.0008\left(\frac{0.008}{0.0095}\right)$$

$$U_i = 633.5 \text{ W/m}^2\text{K}$$

$$LMTD\ (\theta)_m = \frac{\theta_1-\theta_2}{ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{24-18}{ln\frac{24}{18}} = 20.86°C$$

Heat transfer, $Q = U_i A_i \theta_m$

$$11.34\times10^3 = 633.5(\pi\times d_i\times L)\times20.86$$

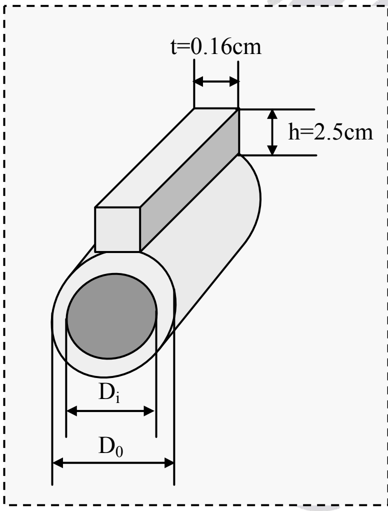
$$11340 = 633.5\times\pi\times0.016\times L\times20.86$$

$$\Rightarrow L = 17.07\ m$$

06.

Sol: Tube inner diameter, D_i = 2.118 cm

Tube outer diameter, D_0 = 0.0254 cm



No. of fins (n) = 20

Height of fins (h) = 2.5 cm = 0.025 m

Thickness of fin (t) = 0.16 cm = 0.0016 m

Heat transfer coefficient inside

(h_i) = 113 W/m^2°C

Heat transfer coefficient outside

(h_0) = 255 W/m^2°C

Total surface area which is exposed to heat transfer

$$A_0 = (\pi D0\times L) + 2(h\times L)\times n$$

Where, L = length of Heat Exchanger (m)

n = No. of fins

$$A_0 = [\pi(0.0254) + 2(0.025)]\times L$$

$$= 1.0797L \approx 1.08\ L\ m^2$$

Overall heat transfer coefficient with fins is given by

$$U_0 = \frac{1}{\frac{A_0}{A_i}\times\left(\frac{1}{h_i}\right)+\frac{A_0}{2\pi kL}.ln\frac{r_0}{r_i}+\frac{1}{h_0}}$$

$$=\frac{1}{\left(\frac{1.08L}{\pi D_i L}\times\frac{1}{1130}\right)+\left(\frac{1.08L}{2\pi(45)L}\times ln\left(\frac{0.0254}{0.02118}\right)+\frac{1}{255}\right.}$$

$$=\frac{1}{0.01436+6.94\times10^{-4}+3.9215\times10^{-3}}$$

U_0 = 52.699 W/m^2°C

$(\because \text{where } \pi D_i L = \pi(0.02118L))$

Overall heat transfer coefficient without fins.

$$U_0 = \frac{1}{\frac{1}{h_0}+\frac{\pi D_0 L}{2\pi kL}.ln\frac{r_0}{r_i}+\frac{r_0}{h_i r_i}}$$

$$=\frac{1}{\frac{1}{255}+\frac{\pi(0.0254)L}{2\pi(45)L}ln\left(\frac{0.0254}{0.02118}\right)+\left(\frac{0.0254}{1130\times0.02118}\right)}$$

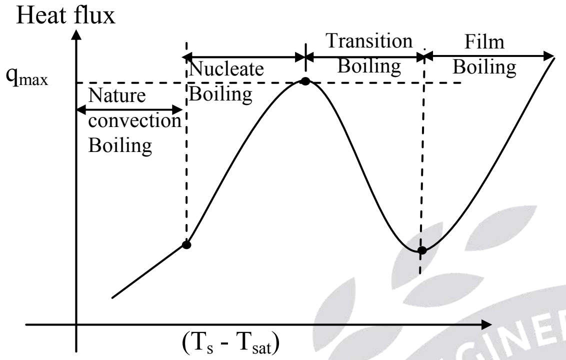
U_0 = 198.706 W/m^2°C

Chapter 7

Boiling and Condensation

01. Ans: (a)

Sol:



Temperature in nucleate boiling region is less than that of film boiling region as shown in figure.

02. Ans: (d)

Sol: If a surface pock-marked with a number of cavities, the surface area would increase. And as a result nucleate boiling, conduction and convection will increase.

03. Ans: (d)

Sol: The convective coefficients for boiling and condensation usually lie in the range of 2500 – 10,000 W/m^{2}K.

04. Ans: (c)

Sol: Dropwise condensation usually occurs on oily surface. The heat transfer coefficient for the dropwise condensation is more as compared to filmwise condensation.

05. Ans: (a)

Sol: When saturated steam is condensed over the flat plate, the temp of steam remains constant hence the heat transfer coefficient along the length does not vary. Therefore local and average heat transfer coefficients are equal. **Correct answer is option (a).**

06. Ans: (c)

Sol: Condensation heat transfer coefficient for vertical surface is given by

$$h_v = 0.943\left[\frac{k^3\rho^2 g h_{fg}}{\mu\ell(t_{sat} - t_s)}\right]^{\frac{1}{4}}$$

$h_v \propto (k^3)^{1/4}$

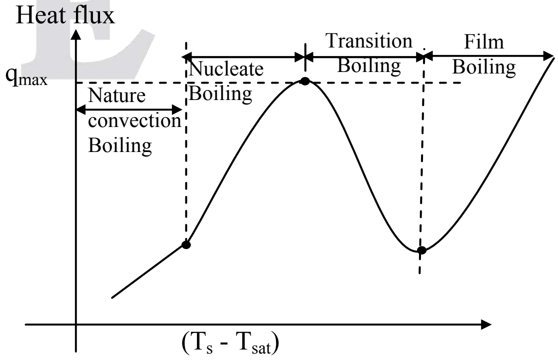
$h_v \propto k^{3/4}$

07. Ans: (c)

Sol: Milk spills over when it is boiled in an open vessel. The boiling of milk at this instant considered as film boiling.

08. Ans: (d)

Sol:



From above figure, with increase in temperature, the heat flux first increases upto critical point and then decreases and again it increases.

09. Ans: (c)

Sol: The excess temperature range between 50°C and 200°C indicates partial film boiling.

10. Ans: (d)

Sol: The heat flux in nucleate boiling (Rohsenow Correlation)

$$q_s = \mu_\ell h_{fg}\left(\frac{g(\rho_\ell - \rho_v)}{\sigma}\right)^{\frac{1}{2}}\left(\frac{c_{p,\ell} \times \Delta T_{ex}}{C_{s,f} \times h_{fg} \times Pr_\ell^n}\right)^3$$

$$q_s \propto h_{fg} \times \left(\frac{1}{h_{fg}}\right)^3$$

$$q_s \propto \left(\frac{1}{h_{fg}}\right)^2$$

Correct answer is option (d).

11. Ans: (d)

Sol: Heat flux in nucleate pool boiling depends on following parameters :

- Properties of liquid (Ex: h_{fg}, Pr, etc)
- material
- surface condition, etc

12. Ans: (a)

Sol: **Boiling:** It occurs when a heated surface is exposed to a liquid and maintained at a temperature higher than the saturation temperature of the liquid.

13. Ans: (b)

Sol: $q_{max} = 0.149 h_{fg} \times \rho_v \left(\frac{\sigma g(\rho_\ell - \rho_v)}{\rho_v^2}\right)^{\frac{1}{4}}$

Critical (burnout) heat flux depends on

- Heat of evaporation (h_{fg})
- Density of vapour (ρ_v)
- Density of liquid (ρ_l)
- Surface tension at the vapour-liquid Interface (σ)

14. Ans: (a)

Sol: *Refer to Solution of Q.No. 01*

15. Ans: (d)

16. Ans: (b)

Sol: Film boiling gets promoted on a smooth surface.

Conventional Practice Solutions

01.

Sol: Nusselt's analysis of film condensation makes the following assumptions.

1. The film of the liquid formed flows under the action of gravity.
2. The condensate flow is laminar and the fluid properties are constant
3. The liquid film is in good thermal contact with the cooling surface and, therefore, temperature at the inside of the film is taken equal to the surface temperature t_s. Further, the temperature at the liquid vapour interface is equal to the saturation temperature t_{sat} at the prevailing pressure.
4. Viscous shear and gravitational forces are assumed to act on the fluid; thus normal viscous force and inertia forces are neglected.
5. The shear stress at the liquid vapor interface is negligible. This means there is no velocity gradient at the liquid-vapour interface [i.e., $\left(\frac{\partial u}{\partial y}\right)_{y=\delta} = 0$].
6. The heat transfer across the condensate layer is by pure conduction & temperature distribution is linear.
7. The condensing vapour is entirely clean and free from gases, air and non-condensing impurities.
8. Radiation between vapour and liquid film; horizontal component of velocity at any point in the liquid film; and curvature of the film are considered negligibly small.

Film heat transfer coefficient:

The heat flow from the vapour to the surface by conduction through the liquid film is given by

$$dQ = \frac{k(b.dx)}{\delta}(t_{sat} - t_s) \text{ …………...(1)}$$

The heat flow can also be expressed as

$$dQ = h_x(b.dx)(t_{sat} - t_s) \text{ ……….......(2)}$$

Where,

h_x = the local heat transfer coefficient.

From (1) and (2), we get

$$\frac{k(b.dx)}{\delta}(t_{sat} - t_s) = h_x(b.dx)(t_{sat} - t_s)$$

$$h_x = \frac{k}{\delta} \text{ …………………….(3)}$$

Equation (3) depicts that at a definite point on the heat transfer surface, the film coefficient (h_x) is directly proportional to thermal conductivity (k) and inversely proportional to thickness of film (δ) at that point.

Substituting the value of δ from equation

$$\delta = \left[\frac{4k\mu(t_{sat} - t_s)x}{\rho_\ell(\rho_\ell - \rho_v)gh_{fg}}\right]^{\frac{1}{4}}, \text{ we get}$$

$$h_x = \left[\frac{\rho_\ell(\rho_\ell - \rho_v)k^3 gh_{fg}}{4\mu x(t_{sat} - t_s)}\right]^{\frac{1}{4}}$$

Local heat transfer coefficient at the lower end of the plate, i.e., x = L

$$h_L = \left[\frac{k^3\rho^2 g h_{fg}}{4\mu L(t_{sat} - t_s)}\right]^{\frac{1}{4}}$$

Evidently the rate of condensation heat transfer is higher at the upper end of the plate than that at the lower end.

The average value of heat transfer can be obtained by integrating the local value of coefficient equation as follows:

$$\bar{h} = \frac{1}{L}\int_0^L h_x dx$$

$$= \frac{1}{L}\int_0^L \left[\frac{\rho_\ell(\rho_\ell - \rho_v)k^3 g h_{fg}}{4\mu x(t_{sat} - t_s)}\right]^{\frac{1}{4}} dx$$

$$= \frac{1}{L}\left[\frac{\rho_\ell(\rho_\ell - \rho_v)k^3 g h_{fg}}{4\mu(t_{sat} - t_s)}\right]^{\frac{1}{4}} \int_0^L x^{-\frac{1}{4}} dx$$

$$= \frac{1}{L}\left[\frac{\rho_\ell(\rho_\ell - \rho_v)k^3 g h_{fg}}{4\mu(t_{sat} - t_s)}\right]^{\frac{1}{4}} \left[\frac{x^{\left(\frac{-1}{4}+1\right)}}{\frac{-1}{4}+1}\right]_0^L$$

$$\bar{h} = \frac{4}{3}\left[\frac{\rho_\ell(\rho_\ell - \rho_v)k^3 g h_{fg}}{4\mu L(t_{sat} - t_s)}\right]^{\frac{1}{4}} \quad \text{.......(4)}$$

$$\bar{h} = \left(\frac{4}{3}h_L\right) = \left(\frac{4}{3}\times\frac{k}{\delta_L}\right)$$

Where, h_L= local heat transfer coefficient at the lower edge of the plate.

This shows that the average heat transfer coefficient is $\frac{4}{3}$ times the local heat transfer coefficient at the trailing edge of plate.

Equation (4) is usually written in the form

$$\bar{h} = 0.943\left(\frac{\rho_l(\rho_l - \rho_v)k^3 g h_{fg}}{\mu L(t_{sat} - t_s)}\right)^{\frac{1}{4}}$$

The Nusselt solution derived above is an approximate one because experimental results have shown that it yields results which are approximately 20 percent lower than the measured values. Mc Adams proposed to use a value of 1.13 in place of coefficient 0.943. Hence

$$\bar{h} = 1.13\left(\frac{\rho_l(\rho_l - \rho_v)k^3 g h_{fg}}{\mu L(t_{sat} - t_s)}\right)^{\frac{1}{4}}$$

Where,

ρ_l = Density of liquid film

ρ_v = Density of vapour

h_{fg} = latent heat of condensation

k = thermal conductivity of liquid film

μ = absolute viscosity of liquid film

t_s = surface temperature

t_{sat} = saturation temperature of vapour at prevailing pressure