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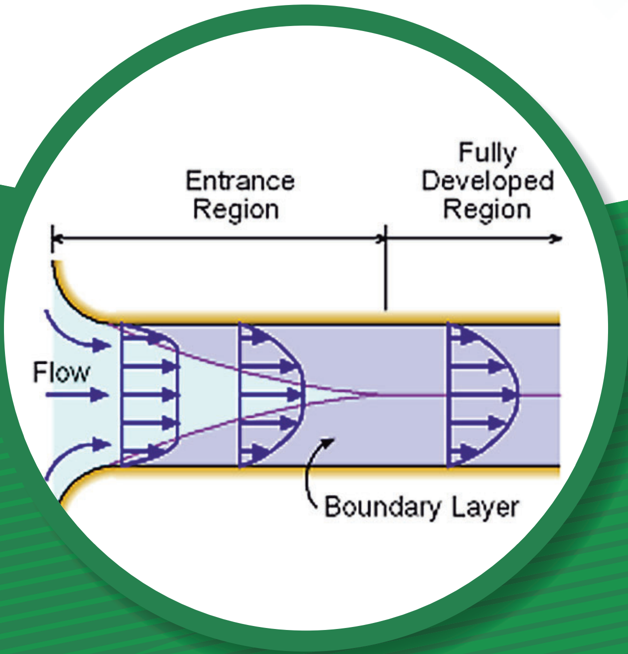


MECHANICAL ENGINEERING

FLUID MECHANICS

Text Book : Theory with worked out Examples and Practice Questions

Fluid Mechanics

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 Properties of Fluids

01. Ans: (c)

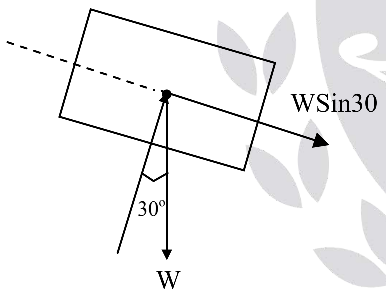
Sol: For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).

02. Ans: 100

Sol: $\tau = \dfrac{\mu V}{h} = \dfrac{0.2 \times 1.5}{3 \times 10^{-3}} = 100 \text{ N/m}^2$

03. Ans: 1

Sol:



$$F = \tau \times A$$

$$W \sin 30 = \frac{\mu A V}{h}$$

$$\frac{100}{2} = \frac{1 \times 0.1 \times V}{2 \times 10^{-3}}$$

$$V = 1 \text{m/s}$$

Common data Q. 04 & 05

04. Ans: (c)

Sol: $D_1 = 100$ mm , $D_2 = 106$ mm

$$\text{Radial clearance, } h = \frac{D_2 - D_1}{2}$$

$$= \frac{106 - 100}{2} = 3\text{mm}$$

L = 2m

μ = 0.2 pa.s

N = 240 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$

$$\omega = 8\pi$$

$$\tau = \frac{\mu \omega r}{h} = \frac{0.2 \times 8\pi \times 50 \times 10^{-3}}{3 \times 10^{-3}}$$

$$= 83.77 \text{N/m}^2$$

05. Ans: (b)

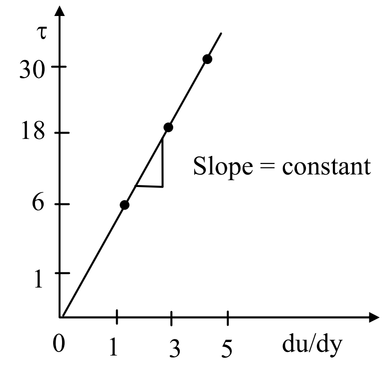
Sol: $\text{Power, } P = \dfrac{2\pi \omega^2 \mu L r^3}{h}$

$$= \frac{2\pi \times (8\pi)^2 \times 0.2 \times 2 \times (0.05)^3}{3 \times 10^{-3}}$$

= 66 Watt

06. Ans: (c)

Sol:



$\therefore$ Newtonian fluid

07. Ans: (a)

Sol: $\tau = \mu \frac{du}{dy}$

$u = 3\sin(5\pi y)$

$\frac{du}{dy} = 3\cos(5\pi y) \times 5\pi = 15\pi\cos(5\pi y)$

$\tau\big|_{y=0.05} = \mu \frac{du}{dy}\Big|_{y=0.05}$

$= 0.5 \times 15\pi\cos(5\pi \times 0.05)$

$= 0.5 \times 15\pi \times \cos\left(\frac{\pi}{4}\right) = 0.5 \times 15\pi \times \frac{1}{\sqrt{2}}$

$= 7.5 \times 3.14 \times 0.707 \approx 16.6\text{N/m}^2$

08. Ans: (d)

Sol:

- Ideal fluid $\to$ Shear stress is zero.
- Newtonian fluid $\to$ Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid $\to$ Shear stress does not vary linearly with the rate of strain.
- Bingham plastic $\to$ Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

09. Ans: (b)

Sol: $V = 0.01 \text{ m}^3$

$\beta = 0.75 \times 10^{-9} \text{ m}^2/\text{N}$

$dP = 2\times10^7 \text{ N/m}^2$

$K = \frac{1}{\beta} = \frac{1}{0.75\times10^{-9}} = \frac{4}{3}\times10^9$

$K = \frac{-dP}{dV/V}$

$dV = \frac{-2\times10^7 \times 10^{-2} \times 3}{4\times10^9} = -1.5\times10^{-4}$

10. Ans: 320 Pa

Sol: $\Delta P = \frac{8\sigma}{D} = \frac{8\times0.04}{1\times10^{-3}} = \frac{32\times10^{-2}}{10^{-3}}$

$\Delta P = 320 \text{ N/m}^2$

11. Ans: (d)

Sol:

- As the temperature is increased, the viscosity of a liquid decreases due to the reduction in intermolecular cohesion.
- In gases, the viscosity increases with the rise in temperature due to increased molecular activity causing an increase in the change of momentum of the molecules, normal to the direction of motion.
- Thus, statement (I) is wrong but statement (II) is correct.

12. Ans: (c)

Sol: The surface energy is given by

$$E = \sigma \times \text{area}$$

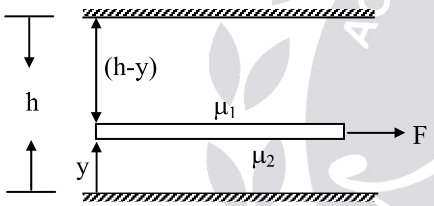
As area increases, surface energy will increase. Thus, statement (I) is correct.

- Surface tension, σ is the property of fluid. Hence, it is independent of the size of the bubble. Thus, statement (II) is wrong.

Conventional Practice Solutions

01.

Sol:



Assumptions:

- Thin plate has negligible thickness.
- Velocity profile is linear because of narrow gap.
- Given fluid is a Newtonian fluid which obeys Newton's law of viscosity.

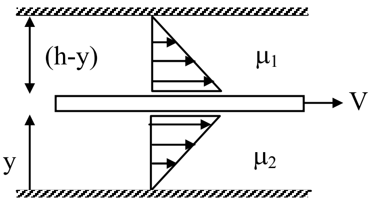
The force required to pull it is proportional to the total shear stress imposed by the two oil layers.

$$F = F_1 + F_2\ ,$$

Where F_1 = Force on top sides of plate,

F_2 = Force on bottom side of plate

The plate moves with velocity V



From Newton's law of viscosity,

$\tau = \dfrac{\mu du}{dy}$ Let A be area of plate

$\therefore$ $F_1 = \tau_1 \times$ Area of plate

$$F_1 = \mu_1 \times \frac{V}{h-y} \times A$$

$$F_2 = \mu_2 \times \frac{V}{y} \times A$$

(i) Shear force on two sides of the plate are equal:

$$F_1 = F_2$$

$$\frac{\mu_1 \times VA}{h-y} = \frac{\mu_2 VA}{y}$$

$$\frac{\mu_1}{\mu_2} = \frac{h-y}{y}$$

$$\frac{h}{y} = \frac{\mu_1}{\mu_2} + 1$$

$$\frac{h}{y} = \frac{\mu_1 + \mu_2}{\mu_2}$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2}$$

(ii) The position of plate so that pull required to drag the plate is minimum.

$$F = \frac{\mu_1 VA}{h-y} + \frac{\mu_2 VA}{y}\ ,$$

[V, A, μ_1 & μ_2, h are constant]

For minimum force, $\dfrac{dF}{dy} = 0$

$$-\mu_1 VA(h-y)^{-2}(-1) - \mu_2 VAy^{-2} = 0$$

$$\frac{\mu_2 VA}{y^2} = \frac{\mu_1 VA}{(h-y)^2}$$

$$\frac{(h-y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h-y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$\dfrac{h}{y} = 1 + \sqrt{\dfrac{\mu_1}{\mu_2}}$ where y is the distance of the thin flat plate from the bottom flat surface.

$$y = \frac{h}{1+\sqrt{\dfrac{\mu_1}{\mu_2}}}$$

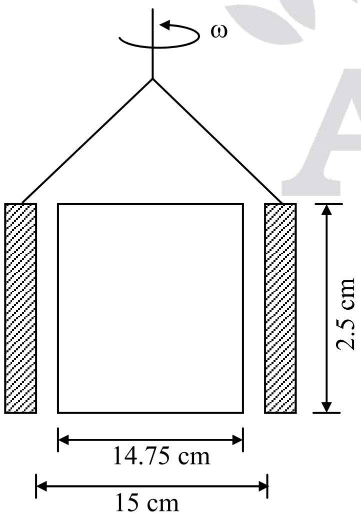
02. Ans: 0.372 Pa.S

Sol: Torque = 1.2 N-m

Speed, N = 600 rpm

Diameter, D_1 = 15 cm , D_2 =14.75 cm

H = 2.5cm



Assumptions:

- The gap between two cylinders is narrow and hence velocity profile in the gap is assumed linear.
- No change in properties

Torque = Tangential force × radius

Force = shear stress×Area

$$= \frac{\mu \times VA}{h}$$

Where h is the clearance (radial)

$$h = \frac{15-14.75}{2}$$

$$= 0.125 \text{ cm} = 1.25\times10^{-3} \text{ m}$$

Area = πDL

$$= \pi\times0.15\times2.5\times10^{-2}$$

$$= 11.781\times10^{-3} \text{ m}^2$$

$$F_s = \frac{\mu\times\omega r\times A}{h}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times600}{60} = 20\pi \text{ rad/s}$$

Torque = $F_s \times r$

$$= \frac{\mu\omega rA}{h}\times r$$

$$= \frac{\mu\omega r^2}{h}\times A$$

$$1.2 = \frac{\mu\times20\pi\times(0.07375)^2\times11.781\times10^{-3}}{1.25\times10^{-3}}$$

μ = 0.3726 Pa.s

Chapter 2

Pressure Measurement & Fluid Statics

01. Ans: (a)

Sol: 1 millibar = $10^{-3}\times10^{5}$ = 100 N/m^2

One mm of Hg = $13.6\times10^{3}\times9.81\times1\times10^{-3}$

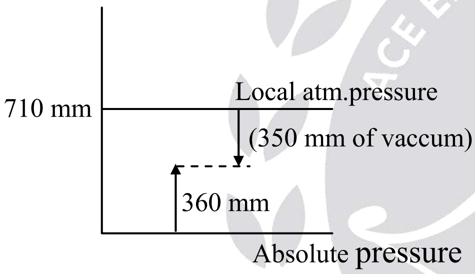
= 133.416 N/m^2

1 N/mm^2 = 1×10^{6} N/m^2

1 kgf/cm^2 = 9.81×10^{4} N/m^2

02. Ans: (b)

Sol:



03. Ans: (c)

Sol: Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure P_A and P_B are same.

04. Ans: (b)

Sol:

- The manometer shown in Fig.1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.
- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.

05. Ans: 2.2

Sol: h_p in terms of oil

$$s_o\, h_o = s_m h_m$$

$$0.85\times h_0 = 13.6\times0.1$$

$$h_0 = 1.6\text{m}$$

$$h_p = 0.6+1.6$$

$$\Rightarrow \quad h_p = 2.2\text{m of oil}$$

(or) $P_p - \gamma_{oil}\times 0.6 - \gamma_{Hg}\times 0.1 = P_{atm}$

$$\frac{P_p - P_{atm}}{\gamma_{oil}} = \left(\frac{\gamma_{Hg}}{\gamma_{oil}}\times 0.1 + 0.6\right)$$

$$= \frac{13.6}{0.85}\times 0.1 + 0.6 = 2.2 \text{ m of oil}$$

Gauge pressure of P in terms of m of oil

= 2.2 m of oil

06. Ans: (b)

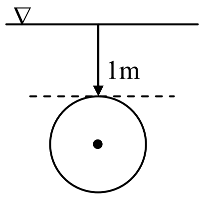
Sol: $h_M - \dfrac{s_w}{s_0}h_{w_1} = h_N - \dfrac{s_w h_{w_2}}{s_0} - h_0$

$$h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3$$

$$h_M - h_N = -13.843\,\text{cm of oil}$$

07. Ans: 2.125

Sol:



$$h_P = \bar{h} + \frac{I}{A\bar{h}}$$

$$= 2 + \frac{\pi D^4 \times 4}{64 \times D^2 \times 2 \times \pi}$$

$$= 2 + \frac{2^2 \times 4}{64 \times 2} = 2.125\text{m}$$

08. Ans: 10

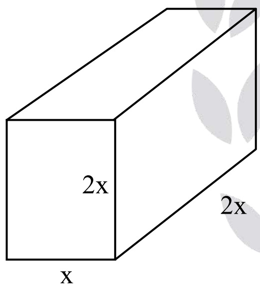
Sol: $F = \rho g \bar{h} A$

$$= 9810 \times 1.625 \times \frac{\pi}{4}\left(1.2^2 - 0.8^2\right)$$

$F = 10\text{kN}$

09. Ans: 1

Sol:



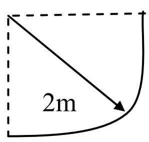
$F_{bottom} = \rho g \times 2x \times 2x \times x$

$F_V = \rho g x \times 2x \times 2x$

$$\frac{F_B}{F_V} = 1$$

10. Ans: 10

Sol:



$F_V = x \times \pi$

$$F_V = \rho g V = 1000 \times 10 \times \frac{\pi \times 2^2}{4}$$

$F_V = 10\pi$ kN

$\therefore x = 10$

11. Ans: (d)

Sol: $F_{net} = F_{H1} - F_{H2}$

$$F_{H1} = \gamma \times \frac{D}{2} \times D \times 1 = \frac{\gamma D^2}{2}$$

$$F_{H2} = \gamma \times \frac{D}{4} \times \frac{D}{2} \times 1 = \frac{\gamma D^2}{8}$$

$$= \gamma D^2\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\gamma D^2}{8}$$

12. Ans: 2

Sol: Let P be the absolute pressure of fluid f3 at mid-height level of the tank. Starting from the open limb of the manometer (where pressure = P_{atm}) we write :

$$P_{atm} + \gamma \times 1.2 - 2\gamma \times 0.2 - 0.5\gamma \times \left(0.6 + \frac{h}{2}\right) = P$$

or $P - P_{atm} = P_{gauge}$

$$= \gamma\left(1.2 - 2\times 0.2 - 0.5 \times 0.6 - 0.5 \times \frac{h}{2}\right)$$

For P_{gauge} to be zero, we have,

$$\gamma(1.2 - 0.4 - 0.3 - 0.25\,h) = 0$$

or $$h = \frac{0.5}{0.25} = 2$$

13. Ans: (b)

Sol: The depth of centre of pressure from the free liquid surface is given by

$$h_{cp} = \bar{h} + \frac{I_{xx,c}}{A\bar{h}} \qquad \text{--------(1)}$$

Or, $\quad h_{cp} - \bar{h} = \frac{I_{xx,c}}{A\bar{h}}$

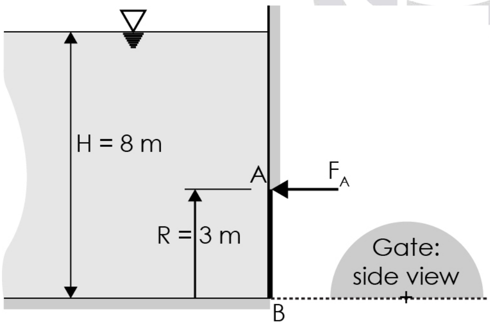
From the above relationship, as $\bar{h}$ increases, $\frac{I_{xx,c}}{A\bar{h}}$ decreases. Thus, at great depth, the difference $(h_{cp} - \bar{h})$ becomes negligible. Hence, statement (I) is correct.

Also, it is clear from equation (1) that h_{cp} is independent of the density of the liquid.

Conventional Practice Solutions

01.

Sol:



$$\bar{h} = 5 + \left(3 - \frac{4 \times R}{3\pi}\right)$$

$$= 5 + \left(3 - \frac{4 \times 3}{3\pi}\right) = 5 + 1.727 = 6.727 \text{ m}$$

$F_H = \gamma_w \times 6.727 \times \text{Area (projected)}$

$= \gamma_w \times 6.727 \times \frac{\pi \times 3^2}{2}$

$= \gamma_w \times 6.727 \times 4.5\pi$

$= 932.94$ kN

$$h_{cp} = 6.727 + \frac{0.10976 R^4}{\frac{\pi R^2}{2} \times 6.727}$$

$$= 6.727 + \frac{0.10976 \times 3^2 \times 2}{\pi \times 6.727}$$

$= 6.727 + 0.0935$

$= 6.8205$ m from free liquid surface

$= (8 - 6.8205)$ m from base B

$= 1.1795$ m from base B.

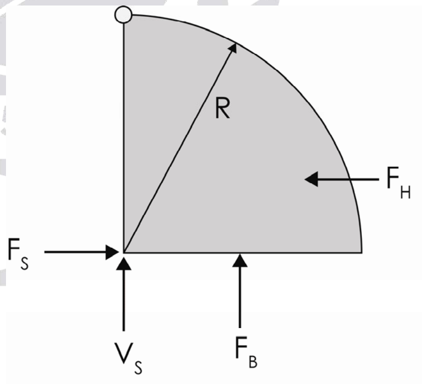
Taking moment about B

$F_A \times 3 = 932.94 \times 1.1795$

$\Rightarrow F_A = 366.8$ kN

02.

Sol:



$$\bar{h} = \left(1.5 + \frac{R}{2}\right)$$

$F_H = \rho g \bar{h} A_{projected}$

$$= \rho g\left(1.5+\frac{R}{2}\right)(R\times 3)$$

$$= \gamma(1.5+1.5)(3\times 3)$$

$$= 27\ \gamma\ N$$

$$h_{cp} = 3+\frac{\frac{1}{12}\left(3\times 3^3\right)}{(3\times 3)(3)}$$

= 3.25 m from free liquid surface

= 3.25 – 1.5 = 1.75 m from A

$$F_B = \gamma\left(\frac{\pi R^2}{4}\right)(3) = \gamma\times\frac{\pi\times 9\times 3}{4} = \frac{27\pi\gamma}{4}\ N$$

F_B will act through the centroid of the quadrant which is at a distance $\frac{4R}{3\pi}$ from the vertical line AB. Now, taking moment of the forces about the hinge A, we write

$$F_s\times 3+F_B\times\frac{4R}{3\pi}-F_H\times 1.75 = 0$$

where F_s is the force in x-direction on the stop at B & V_s is in y-direction (does not contribute in the moment).

$$3F_s = 27\times 1.75\gamma-\frac{27\pi\gamma}{4}\times\frac{4R}{3\pi} = 10^4(27\times 1.75 - 9\times 3)$$

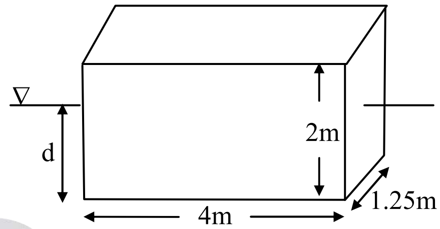
$$= 10^4\times 27\times 0.75 = 202.5\ \text{kN.m}$$

$$\Rightarrow F_s = \frac{202.5}{3} = 67.5\ \text{kN}$$

Chapter 3 Buoyancy and Metacentric Height

01. Ans: (d)

Sol:



$$F_B = \text{weight of body}$$

$$\rho_b g V_b = \rho_f g V_f\, d$$

$$640\times 4\times 2\times 1.25 = 1025\times(4\times 1.25\times d)$$

$$d = 1.248\text{m}$$

$$V_{fd} = 1.248\times 4\times 1.25$$

$$V_{fd} = 6.24\text{m}^3$$

02. Ans: (c)

Sol: Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

$4\pi r^2 = 6a^2$

$$\frac{2\pi}{3} = \left(\frac{a}{r}\right)^2$$

$$F_{b,s} \propto V_s$$

$$= \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\frac{\pi r^3}{\left(r\sqrt{\frac{2\pi}{3}}\right)^3}$$

$$= \frac{4}{3}\frac{\pi r^3}{\left(\sqrt{\frac{2\pi}{3}}\times\sqrt{\frac{2\pi}{3}}r^3\right)} = \sqrt{\frac{6}{\pi}}$$

03. Ans: 4.76

Sol: $F_B = F_{B,Hg} + F_{B,W}$

$W_B = F_B$

$\rho_b g\forall_b = \rho_{Hg} g\forall_{Hg} + \rho_w g\forall_w$

$\rho_b\forall_b = \rho_{Hg}\forall_{Hg} + \rho_w\forall_w$

$S\times\forall_b = S_{Hg}\forall_{Hg} + S_w\forall_w$

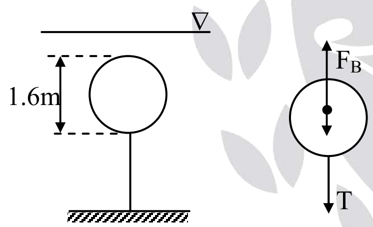
$7.6\times10^3 = 13.6\times10^2(10-x) + 10^2\times x$

$-6000 = -1260x$

$x = 4.76$ cm

04. Ans: 11

Sol:



$F_B = W + T$

$W = F_B - T$

$= \rho_f g V_{fd} - T$

$= 10^3 \times 9.81 \times \frac{4}{3}\pi(0.8)^3 - (10\times10^3)$

$= 21 - 10$

$W = 11$ kN

05. Ans: 1.375

Sol: $W_{water} = 5N$

$W_{oil} = 7N$

$S = 0.85$

W – Weight in air

$F_{B1} = W - 5$

$F_{B2} = W - 7$

$W - 5 = \rho_1 g V_{fd}$…..(1)

$W - 7 = \rho_2 g V_{fd}$…..(2)

$V_{fd} = V_b$

$$\begin{array}{r} W - 5 = \rho_1 g V_b \\ W - 7 = \rho_2 g V_b \\ \hline 2 = (\rho_1 - \rho_2) g V_b \end{array}$$

$$V_b = \frac{2}{(1000-850)9.81}$$

$V_b = 1.3591\times10^{-3} m^3$

$W = 5 + (9810\times1.3591\times10^{-3})$

$W = 18.33N$

$W = \rho_b\, g\, V_b$

$$\frac{18.33}{9.81\times1.3591\times10^{-3}} = \rho_b$$

$\rho_b = 1375.05$ kg/m^3

$S_b = 1.375$

06. Ans: (d)

Sol: For a floating body to be stable, metacentre should be above its center of gravity. Mathematically GM > 0.

07. Ans: (b)

Sol: $W = F_B$

$\rho_b g V_b = \rho_f g V_{fd}$

$\rho_b V_b = \rho_f V_{fd}$

$$0.6\times\frac{\pi}{4}d^2\times 2d = 1\times\frac{\pi}{4}d^2\times x$$

$x = 1.2d$

GM = BM – BG

$$BM = \frac{I}{V} = \frac{\pi d^4}{64\times\frac{\pi}{4}d^2\times 1.2d} = \frac{d}{19.2} = 0.052d$$

BG = d – 0.6d = 0.4d

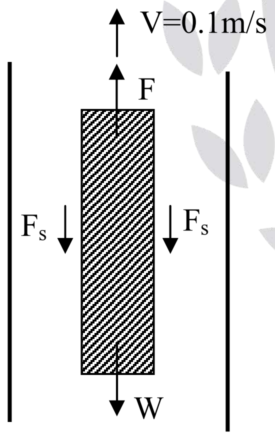
Thus, GM = 0.052d – 0.4d = –0.348 d

GM < 0

$\Rightarrow$ Hence, the cylinder is in unstable condition.

08. Ans: 122.475

Sol:



The thickness of the oil layer is same on either side of plate

y = thickness of oil layer

$$= \frac{23.5-1.5}{2} = 11\text{mm}$$

Shear stress on one side of the plate

$$\tau = \frac{\mu dU}{dy}$$

F_s = total shear force (considering both sides of the plate)

$$= 2A\times\tau = \frac{2A\mu V}{y}$$

$$= \frac{2\times1.5\times1.5\times2.5\times0.1}{11\times10^{-3}}$$

= 102.2727 N

Weight of plate, W = 50 N

Upward force on submerged plate,

$F_v = \rho g V = 900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3}$

= 29.7978 N

Total force required to lift the plate

$= F_s + W - F_v$

= 102.2727 + 50 – 29.7978

= 122.4749 N

09. Ans: (d)

Sol:

- Statement (I) is wrong because the balloon filled with air cannot go up and up, if it is released from the ground.
- However, with increase in elevation, the atmospheric pressure and temperature both decrease resulting into a decrease in air density. Thus, statement (II) is correct.

Conventional Practice Solutions

01. Ans: (i) 0.33, (ii) 0.5 m

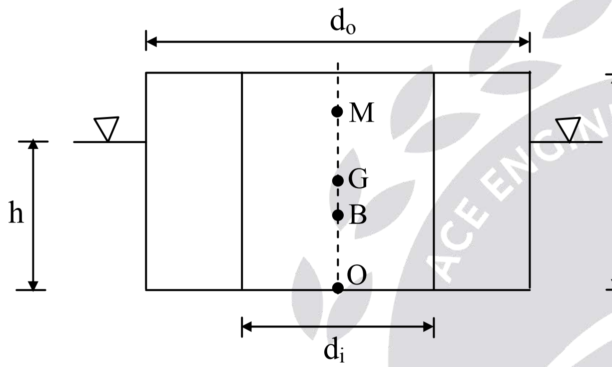
Sol: Given data :

Inner diameter of hollow cylinder, d_i = 300 mm

Outer diameter of hollow cylinder, d_o = 600 mm

S.G. of wooden hollow cylinder = 0.56

S.G. of oil = 0.85



Let 'h' be the depth of immersion of the cylinder in oil and L be the height of the cylinder.

Weight of hollow cylinder = Buoyant force acting on the hollow cylinder

Or, $\gamma_{cyl} \times \frac{\pi}{4}\left(d_0^2 - d_i^2\right) \times L = \gamma_{oil} \times \frac{\pi}{4}\left(d_0^2 - d_i^2\right) \times h$

Or, $h = \frac{\gamma_{cyl}}{\gamma_{oil}} \times L = \frac{0.56}{0.85} L = 0.66\ L$

Let us then calculate the maximum height of the cylinder, L for the stable equilibrium condition.

The centre of buoyancy B will be at a distance $\frac{h}{2}$ from O as shown in the figure.

Or, $OB = \frac{h}{2} = 0.33L$

and $OG = \frac{L}{2} = 0.5L$

Now, $BM = \frac{I}{\forall}$

$$= \frac{\pi}{64}\left(d_0^4 - d_i^4\right) \times \frac{4}{\pi \times \left(d_0^2 - d_i^2\right) \times h}$$

$$= \frac{\left(d_0^2 + d_i^2\right)}{16h} = \frac{\left(0.6^2 + 0.3^2\right)}{16 \times 0.66L}$$

$$= \frac{0.0426}{L}$$

Thus, GM = BM – (OG – OB)

$$= \frac{0.0426}{L} - \left(0.5L - 0.33L\right)$$

$$= \frac{0.0426}{L} - 0.17L$$

For stable equilibrium condition, GM ≥ 0. Putting GM = 0 for the maximum height of the cylinder, we get

$$\frac{0.0426}{0.17} = L^2$$

$\Rightarrow$ L = 0.5 m

Thus, h = 0.66 ×0.5 = 0.33 m

02. Ans: Unstable

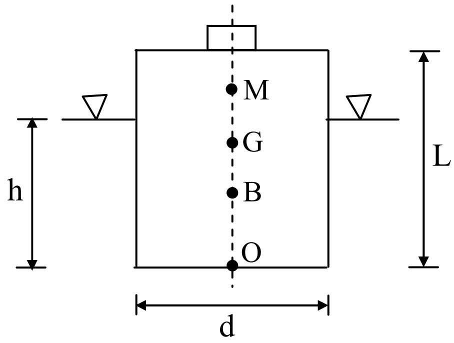
Sol: Given data:

d = 1.0 m, L = 1.5 m,

$\rho_{sea\ water}$ = 1026 kg/m

m_{buoy} = 80 kg

m = 10 kg



$(80 + 10)\times g = \frac{\pi}{4}\times 1^2 \times h \times 1026 \times g$

where h is the depth of immersion of the buoy.

Thus, $h = \frac{4\times 90}{\pi \times 1026} = 0.1117$ m

$OB = \frac{h}{2} = 0.05585$ m

The position of G due to a mass of 10 kg added to the cylindrical buoy is evaluated as:

$80 \times 0.75 + 10 \times 1.5 = 90 \times OG$

Or, $OG = \frac{75}{90} = 0.833$m

$BM = \frac{I}{\forall} = \frac{\pi}{64}\times 1^4 \times \frac{4}{\pi \times 1^2 \times h}$

$= \frac{1}{16 \times 0.1117} = 0.5595$ m

Thus, GM = BM – (OG – OB)

= 0.5595 – (0.833 – 0.05585)

= – 0.21765 m

Or, GM < 0

Thus, the buoy floats in unstable condition.

Chapter 4 Fluid Kinematics

01. Ans: (b)

Sol:

- Constant flow rate signifies that the flow is steady.
- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.

Common Data for Questions 02 & 03

02. Ans: 0.94

Sol: $a_{Local} = \frac{\partial V}{\partial t}$

$= \frac{\partial}{\partial t}\left(2t\left(1-\frac{x}{2L}\right)^2\right)$

$= \left(1-\frac{x}{2L}\right)^2 \times 2$

$(a_{Local})_{at\ x = 0.5,\ L = 0.8} = 2\left(1-\frac{0.5}{2\times 0.8}\right)^2$

$= 2(1 - 0.3125)^2 = 0.945$ m/sec^2

03. Ans: –13.68

Sol: $a_{convective} = v.\frac{\partial v}{\partial x} = \left[2t\left[1-\frac{x}{2L}\right]^2\right]\frac{\partial}{\partial x}\left[2t\left(1-\frac{x}{2L}\right)^2\right]$

$= \left[2t\left[1-\frac{x}{2L}\right]^2\right]2t\left[2\left(1-\frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$

At t = 3 sec; x = 0.5 m; L = 0.8 m

$$a_{convective} = 2\times3\left[1-\frac{0.5}{2\times0.8}\right]^2 \times 2\times3\left[2\left(1-\frac{0.5}{2\times0.8}\right)\right]\left[\frac{-1}{2\times0.8}\right]$$

$a_{convective} = -14.62$ m/sec^2

$a_{total} = a_{local} + a_{convective} = 0.94 - 14.62$

$= -13.68$ m/sec^2

04. Ans: (d)

Sol: $u = 6xy - 2x^2$

Continuity equation for 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 6y - 4x$$

$$(6y-4x) + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = (4x - 6y) = 0$$

$\partial v = (4x-6y)\, dy$

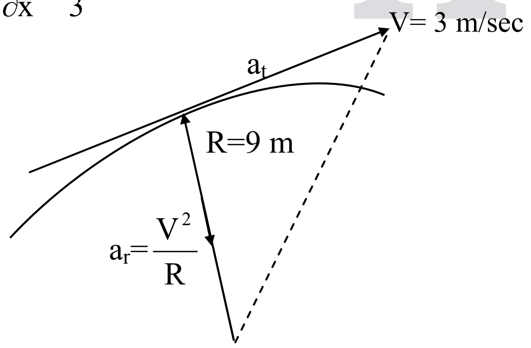
$v = \int 4x dy - \int 6y dy$

$= 4xy - 3y^2 + c$

$= 4xy - 3y^2 + f(x)$

05. Ans: $\sqrt{2}$ = 1.414

Sol: $\frac{\partial V}{\partial x} = \frac{1}{3}(\text{m/sec/m})$



$$a_r = \frac{V^2}{R} = \frac{(3)^2}{9} = \frac{9}{9} = 1 \text{ m/s}^2$$

$$a_t = V\frac{\partial V}{\partial x} = 3\times\frac{1}{3} = 1 \text{ m/s}^2$$

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m/sec}^2$$

06. Ans: 13.75

Sol: $a_{t\,(conv)} = V_{avg} \times \frac{dV}{dx}$

$$a_{t\,(conv)} = \left(\frac{2.5+3}{2}\right)\left(\frac{3-2.5}{0.1}\right) = 2.75 \times 5$$

$a_{t\,(conv)} = 13.75$ m/s^2

07. Ans: 0.3

Sol: $Q = Au$

$$a_{Local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

$$a_{local} = \frac{1}{A}\frac{\partial Q}{\partial t}$$

$$a_{Local} = \left(\frac{1}{0.4-0.1x}\right)\frac{\partial Q}{\partial t}$$

$$(a_{Local})_{at\ x=0} = \frac{1}{0.4}\times 0.12 \quad \left(\because \frac{\partial Q}{\partial t} = 0.12\right)$$

$= 0.3$ m/sec^2

08. Ans: (b)

Sol: $\psi = x^2 - y^2$

$a_{Total} = (a_x)\hat{i} + (a_y)\hat{j}$

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 - y^2) = 2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$= (2y)(0) + (2x)(2)$

$\therefore a_x = 4x$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$= (2y)\times(2) + (2x)\times(0)$

$a_y = 4y$

$\therefore a = (4x)\hat{i} + (4y)\hat{j}$

09. Ans: (b)

Sol: Given, The stream function for a potential flow field is $\psi = x^2 - y^2$

$\phi = ?$

$$u = \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial\left(x^2 - y^2\right)}{\partial y}$$

$u = 2y$

$$u = -\frac{\partial\phi}{\partial x} = 2y$$

$$\int \partial\phi = -\int 2y\partial x$$

$\phi = -2xy + c_1$

Given, ϕ is zero at (0,0)

$\therefore c_1 = 0$

$\therefore \phi = -2xy$

10. Ans: 4

Sol: Given, 2D – flow field

Velocity, V = 3xi + 4xyj

$u = 3x, \quad v = 4xy$

$$\omega_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

$$\omega_z = \frac{1}{2}(4y - 0)$$

$$(\omega_Z)_{at(2,2)} = \frac{1}{2}\times 4(2) = 4 \text{ rad/sec}$$

11. Ans: (b)

Sol: Given, $u = 3x$, $v = Cy$, $w = 2$

The shear stress, τ_{xy} is given by

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left[\frac{\partial}{\partial y}(3x) + \frac{\partial}{\partial x}(Cy)\right]$$

$= \mu\,(0 + 0) = 0$

12. Ans: (d)

Sol:

- The total acceleration is given as
$$\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}$$
where the first term on the R.H.S is the local acceleration and the second term is the convective acceleration.
- If the flow is steady, then local acceleration will be zero, not the convective acceleration.
- The convective acceleration arises due to the fact that a fluid element experiences different velocities at different locations. Thus, statement (I) is wrong whereas statement (II) is correct.

Conventional Practice Solutions

01. Ans: (ii) y = ±x (ii) (0, 0)

Sol: Given: $u = c(x^2 - y^2)$ and $v = -2cxy$

The equation of a streamline is given by

$$\frac{dx}{u} = \frac{dy}{v}$$

Or, $$\frac{dy}{dx} = \frac{v}{u} = -\frac{2Cxy}{C(x^2 - y^2)} = -\frac{2xy}{x^2 - y^2}$$

(ii) For flow to be parallel to y-axis, $u = 0$

Or, $$\frac{dy}{dx} = \frac{v}{x^2 - y^2} = \infty$$

This is possible when $x = \pm y$

(iii) The fluid is stationary when u & v both are zero.

From the velocity components given, it is possible when $(x, y) = (0, 0)$

(i) From the equation of streamline

$$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$$

Or, $$\frac{dx}{dy} = -\frac{x^2 - y^2}{2xy} \quad \text{----------(1)}$$

Let $x = fy$

or $dx = fdy + ydf$

Or, $$\frac{dx}{dy} = f + y\frac{df}{dy} \quad \text{--------(2)}$$

Equating (1) with (2),

$$f + y\frac{df}{dy} = -\frac{f^2y^2 - y^2}{2fy \times y}$$

$$= -\frac{f^2 - 1}{2f} = \frac{1 - f^2}{2f}$$

Or, $$y\frac{df}{dy} = \frac{1 - f^2}{2f} - f = \frac{1 - 3f^2}{2f}$$

Or, $$\frac{2f}{1 - 3f^2}df = \frac{dy}{y}$$

$$\frac{6f}{3f^2 - 1}df = -\frac{3dy}{y}$$

Integrating

$$\ln(3f^2 - 1) + 3\ln y = \ln C$$

Or, $(3f^2 - 1) \times y^3 = C$

Or, $\left(3\frac{x^2}{y^2} - 1\right)y^3 = C$

Or, $3x^2y - y^3 = C$

Or, $x^2y - y^3/3 = \text{constant}$, proved

Chapter 5 Energy Equation and its Applications

01. Ans: (c)

Sol: Applying Bernoulli's equation for ideal fluid

$$\frac{P_1}{\rho g}+Z_1+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+Z_2+\frac{V_2^2}{2g}$$

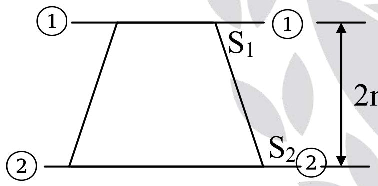
$$\frac{P_1}{\rho g}+\frac{(2)^2}{2g}=\frac{P_2}{\rho g}+\frac{(1)^2}{2g}$$

$$\frac{P_2}{\rho g}-\frac{P_1}{\rho g}=\frac{4}{2g}-\frac{1}{2g}$$

$$\frac{P_2-P_1}{\rho g}=\frac{3}{2g}=\frac{1.5}{g}$$

02. Ans: (c)

Sol:



$$\frac{V_1^2}{2g}=1.27\text{m}\ ,\qquad \frac{P_1}{\rho g}=2.5\text{m}$$

$$\frac{V_2^2}{2g}=0.203\text{m}\ ,\qquad \frac{P_2}{\rho g}=5.407\text{m}$$

$Z_1 = 2$ m , $Z_2 = 0$ m

Total head at (1) – (1)

$$=\frac{V_1^2}{2g}+\frac{P_1}{\rho g}+Z_1$$

$= 1.27 + 2.5 + 2 = 5.77$ m

Total head at (2) – (2)

$$=\frac{V_2^2}{2g}+\frac{P_2}{\rho g}+Z_2$$

$= 0.203 + 5.407 + 0 = 5.61$ m

Loss of head = 5.77 – 5.61 = 0.16 m

∴ Energy at (1) – (1) > Energy at (2) – (2)

∴ Flow takes from higher energy to lower energy

i.e. from (S_1) to (S_2)

Flow takes place from top to bottom.

03. Ans: 1.5

Sol: $A_1=\frac{\pi}{4}d_1^2=\frac{\pi}{4}(0.1)^2=7.85\times10^{-3}\ \text{mm}^2$

$$A_2=\frac{\pi}{4}d_2^2=\frac{\pi}{4}(0.05)^2=1.96\times10^{-3}\ \text{mm}^2$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$Z_1 = Z_2$, it is in horizontal position

Since, at outlet, pressure is atmospheric

$P_2 = 0$

Q = 100 lit/sec = 0.1 m^3/sec

$$V_1=\frac{Q}{A_1}=\frac{0.1}{7.85\times10^{-3}}=12.73\ \text{m/sec}$$

$$V_2=\frac{Q}{A_2}=\frac{0.1}{1.96\times10^{-3}}=51.02\ \text{m/sec}$$

$$\frac{P_{1\,gauge}}{\rho_{air}\times g}+\frac{(12.73)^2}{2\times10}=0+\frac{(51.02)^2}{2\times10}$$

$$\frac{P_1}{\rho_{air}.g}=121.53$$

$$P_1=121.53\times\rho_{air}\times g$$

$= 1.51$ kPa

04. Ans: 395

Sol: Q = 100 litre/sec = 0.1 m^3/sec

V_1 = 100 m/sec; $P_1 = 3 \times 10^5$ N/m^2

V_2 = 50 m/sec; $P_2 = 1 \times 10^5$ N/m^2

Power (P) = ?

Energy equation :

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$\frac{3\times10^5}{1000\times10}+\frac{100^2}{2\times10}+0=\frac{1\times10^5}{1000\times10}+\frac{50^2}{2\times10}+0+h_L$$

$\Rightarrow$ h_L = 395 m

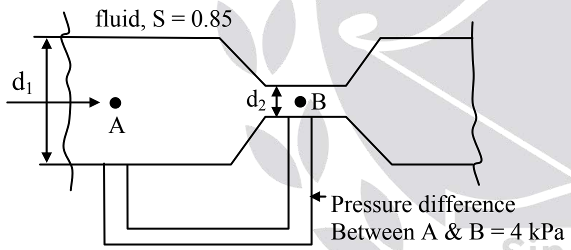
P = $\rho g Q.h_L$

P = 1000 × 10 × 0.10 × 395

P = 395 kW

05. Ans: 35

Sol:



d_1 = 300 mm, d_2 = 120 mm

$$Q_{Th}=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left(\frac{\Delta P}{w}\right)}$$

$$A_1=\frac{\pi}{4}d_1^2=\frac{\pi}{4}(0.30)^2=0.07\,m^2$$

$$A_2=\frac{\pi}{4}d_2^2=\frac{\pi}{4}(0.12)^2=0.011m^2$$

ΔP = 4 kPa,

$$h=\frac{\Delta P}{w}=\frac{\Delta P}{\rho_f.g}$$

$$=\frac{\Delta P}{s_f\rho_w g}=\frac{4\times10^3}{0.85\times1000\times9.81}$$

$$Q_{Th}=\frac{0.07\times0.011}{\sqrt{(0.07)^2-(0.011)^2}}\sqrt{\frac{2\times9.81\times4\times10^3}{0.85\times1000\times9.81}}$$

= 0.035 m^3/sec = 35.15 ltr/sec

06. Ans: 65

Sol: h_{stag} = 0.30 m

h_{stat} = 0.24 m

$V = c\sqrt{2gh_{dyna}}$

$V = 1\sqrt{2g(h_{stag}-h_{stat})}$

$=\sqrt{2(9.81)(0.30-0.24)}$ = 1.085 m/s

= 1.085 × 60 = 65.1 m/min

07. Ans: 81.5

Sol: x = 30 mm, g = 10 m/s^2

ρ_{air} = 1.23 kg/m^3; ρ_{Hg} = 13600 kg/m^3

C = 1

$V=\sqrt{2gh_D}$

$$h_D=x\left(\frac{S_m}{S}-1\right)$$

$$h_D=30\times10^{-3}\left(\frac{13600}{1.23}-1\right)$$

h_D = 331.67 m

$V=1\times\sqrt{2\times10\times331.67}$ = 81.5 m/sec

08. Ans: 140

Sol: $Q_a = C_d \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$

$C_d \propto \dfrac{1}{\sqrt{h}}$

$\dfrac{C_{d_{venturi}}}{C_{d_{orifice}}} = \dfrac{0.95}{0.65} = \sqrt{\dfrac{h_{orifice}}{h_{venturi}}}$

$h_{venturi} = 140$ mm

09. Ans: (d)

Sol:

- For an orifice meter, the fluid re-establishes its flow pattern downstream of the orifice plate. However, the fluid pressure downstream of the orifice plate is not the same as that at upstream of the orifice plate. Thus, statement (I) is not correct.
- Bernoulli's equation when applied to any two points (for irrotational, steady and incompressible flow) can be written as

  $\dfrac{P_1}{\gamma} + \dfrac{V_1^2}{2g} + Z_1 = \dfrac{P_2}{\gamma} + \dfrac{V_2^2}{2g} + Z_2$

  If $V_1 = V_2$ & $Z_1 = Z_2$, we get $P_1 = P_2$.

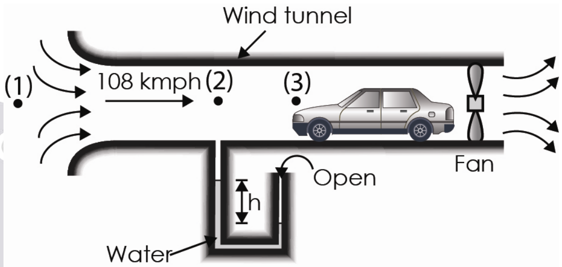
  Thus, statement (II) is correct.

Conventional Practice Solutions

01. Ans: 5.4 cm, 540 Pa

Sol: Air enters into the wind tunnel at P_{atm} and $V \approx 0$. It attains a velocity V in the test section and the pressure there is P.

Applying Bernoulli's equation for points (1) and (2) as shown in the figure.



$$\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} + Z_2$$

But $P_1 = P_{atm}$, $V_1 \approx 0$ and $Z_1 = Z_2$

Thus, $\dfrac{P_{atm} - P_2}{\gamma_{air}} = \dfrac{V_2^2}{2g}$

$$= \frac{\left(108 \times \frac{5}{18}\right)^2}{2 \times 10} = 45 \text{ -----(1)}$$

From manometer,

$P_2 + \gamma_{water} \times h = P_{atm}$

or, $P_{atm} - P_2 = \gamma_{water} \times h$ -------(2)

Hence, equation (1) becomes,

$\dfrac{\gamma_{water} h}{\gamma_{air}} = 45$ (from (2))

$h = \dfrac{45 \times \gamma_{air}}{\gamma_{water}} = \dfrac{45 \times 1.2 \times g}{10^3 \times g} = 0.054$ m

$\Rightarrow h = 5.4$ cm

Applying Bernoulli's equation for points (2) and (3)

$$\frac{P_2}{\gamma_{air}}+\frac{V_2^2}{2g}=\frac{P_3}{\gamma_{air}}+\frac{V_3^2}{2g}$$

But point (3) is stagnation point where

$P_3 = P_{stag}$ & $V_3 = 0$

Thus, $\frac{P_{stag}-P_2}{\gamma_{air}}=\frac{V_2^2}{2g} = 45$

Or, $P_{stag}-P_2 = 45 \times 1.2 \times 10 = 540$ Pa

Chapter 6 Momentum equation and its Applications

01. Ans: 1600

Sol: S = 0.80

$A = 0.02\ m^2$

V = 10 m/sec

$F = \rho.A.V^2$

$F = 0.80 \times 1000 \times 0.02 \times 10^2$

F = 1600 N

02. Ans: 6000

Sol: $A = 0.015\ m^2$

V = 15 m/sec (Jet velocity)

U = 5 m/sec (Plate velocity)

$F = \rho A\ (V + U)^2$

$F = 1000 \times 0.015\ (15 + 5)^2$

F = 6000 N

03. Ans: 19.6

Sol: V = 100 m/sec (Jet velocity)

U = 50 m/sec (Plate velocity)

d = 0.1 m

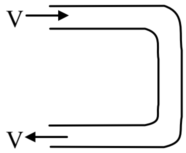
$F = \rho A\ (V - U)^2$

$$F = 1000\times\frac{\pi}{4}\times 0.1^2\times(100-50)^2$$

F = 19.6 kN

04. Ans: (a)

Sol:



$F_x = \rho a V(V_{1x} - V_{2x})$

$= \rho a V(V - (-V))$

$= 2\rho a V^2$

$= 2\times 1000 \times 10^{-4} \times 5^2 = 5$ N

05. Ans: (d)

Sol: Given, V = 20 m/s,

u = 5 m/s

$F_1 = \rho A(V-u)^2$

Power $(P_1) = F_1 \times u = \rho A(V-u)^2 \times u$

$F_2 = \rho.A.V \times V_r$

$= \rho.A(V).(V-u)$

Power $(P_2) = F_2 \times u = \rho AV(V-u)u$

$$\frac{P_1}{P_2} = \frac{\rho A(V-u)^2 \times u}{\rho AV(V-u)\times u}$$

$$= \frac{V-u}{V} = 1 - \frac{u}{V}$$

$$= 1 - \frac{5}{20} = 0.75$$

06. Ans: 2035

Sol: Given, $\theta = 30°$, $\dot{m} = 14$ kg/s

$(P_i)_g = 200$ kPa, $(P_e)_g = 0$

$A_i = 113\times10^{-4}$ m^2 , $A_e = 7\times10^{-4}$ m^2

$\rho = 10^3$ kg/m^3, $g = 10$ m/s^2

From the continuity equation :

$\rho A_i V_i = 14$

or $$V_i = \frac{14}{10^3 \times 113 \times 10^{-4}} = 1.24 \text{ m/s}$$

Similarly, $$V_e = \frac{14}{10^3 \times 7 \times 10^{-4}} = 20 \text{ m/s}$$

Let F_x be the force exerted by elbow on water in the +ve x-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write :

$$(P_i)_g A_i + F_x = \dot{m}(V_e \cos 30^o - V_i)$$

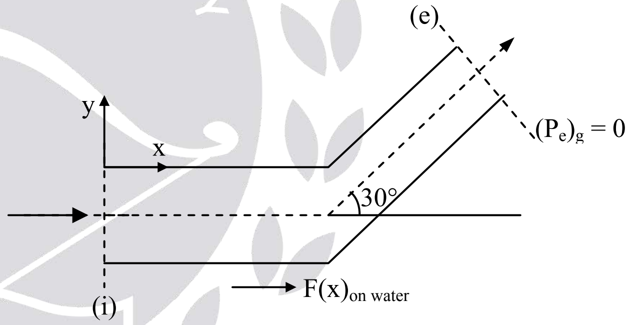
$$F_x = \dot{m}(V_e \cos 30^o - V_i) - (P_i)_g A_i$$

$= 14\,(20\times\cos30° - 1.24) - 200\times10^3\times113\times10^{-4}$

$= 225.13 - 2260$

$= -2034.87$ N ≈ -2035 N

The x-component of water force on elbow is $-F_x$ (as per Newton's third law), i.e., $\cong 2035$ N



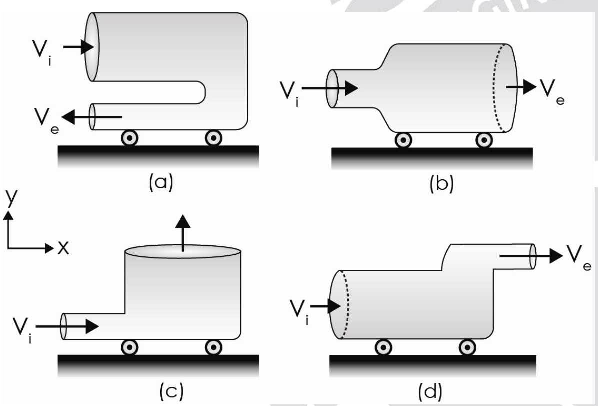
07. Ans: (a)

Sol: In a convergent nozzle, as the area decreases in the direction of flow, the flow velocity will increase (AV = Constant) in the direction of flow. This will result in increase in its momentum. Thus, statement (I) is correct and statement (II) is the correct explanation of statement (I).

Conventional Practice Solutions

01. Ans: Right: a, b, c; Left: d

Sol: Let F_x be the force exerted by the fluid on the device which will be different for different devices. Since inlet and outlet sections of the devices are at atmospheric pressure, there will be no contribution of pressure forces at these sections. Let V_i and V_e be the velocities at inlet and outlet of the devices in x-direction.



Applying linear momentum equation to each of the devices, we write

(a) $F_x = \dot{m}_a\left[V_i - (-V_e)\right] = \dot{m}_a\left[V_i + V_e\right]$

F_x is acting in +ve x direction.

Therefore, the device (b) will move to the right.

(b) $F_x = \dot{m}_b\left(V_i - V_e\right)$

Since $V_i > V_e$, F_x is acting in +ve x direction.

Therefore, the device (a) will move to the right.

(c) $F_x = \dot{m}_c\left(V_i - 0\right) = \dot{m}V_i$

F_x is acting in +ve x direction.

Therefore, the device (c) will move to the right.

(d) $F_x = \dot{m}_d\left(V_i - V_e\right)$

Since $V_e > V_i$

F_x is acting in –ve x direction. Therefore, the device (d) will move to the left.

02.

Sol: Given data:

$A_j = 0.009\ m^2$

$V_j = 30.5$ m/s

$V_s = 3$ m/s

$A_s + A_j = 0.07\ m^2 = A_T$

$A_s = (0.07 - 0.009) = 0.061\ m^2$

$A_jV_j + A_sV_s = (A_s + A_s)V_e$

(From continuity equation)

$0.009 \times 30.5 + 0.061 \times 3 = 0.07 \times V_e$

Or, $V_e = 6.536$ m/s ≈ 6.54 m/s

Applying linear momentum equation:

$P_1(A_s + A_j) - P_2(A_s + A_j)$

$= \rho A_T \times V_e[V_e] - \rho A_s V_s^2 - \rho A_j V_j^2$

$(P_1 - P_2)A_T = \rho A_T V_e^2 - \rho A_s V_s^2 - \rho A_j V_j^2$

$= \rho\left[0.07 \times 6.536^2 - 0.061 \times 9 - 0.009 \times 30.5^2\right]$

$= -10^3(5.931)$ N

Or, $P_2 - P_1 = \dfrac{5.931}{0.07} = 84.73$ kPa

Chapter 7

Laminar Flow

01. Ans: (d)

Sol: In a pipe, the flow changes from laminar flow to transition flow at Re = 2000. Let V be the average velocity of flow. Then

$$2000 = \frac{V \times 8 \times 10^{-2}}{0.4 \times 10^{-4}} \Rightarrow V = 1\,m/s$$

In laminar flow through a pipe,

$V_{max} = 2 \times V = 2$ m/s

02. Ans: (d)

Sol: The equation $\tau = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{r}{2}\right)$ is valid for laminar as well as turbulent flow through a circular tube.

03. Ans: (d)

Sol: $Q = A.V_{avg}$

$Q = A.\frac{V_{max}}{2}$ $(\because V_{max} = 2\,V_{avg})$

$$Q = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2 \times \frac{1.5}{2}$$

$$= \frac{\pi}{4} \times (0.04)^2 \times 0.75$$

$$= \frac{\pi}{4} \times \frac{4}{100} \times \frac{4}{100} \times \frac{3}{4} = \frac{3\pi}{10000}\ m^3/sec$$

04. Ans: 1.92

Sol: $\rho = 1000$ kg/m^3

$Q = 800$ mm^3/sec $= 800 \times (10^{-3})^3$ m^3/sec

L = 2 m

D = 0.5 mm

$\Delta P = 2$ MPa $= 2 \times 10^6$ Pa

$\mu = ?$

$$\Delta P = \frac{128.\mu QL}{\pi D^4}$$

$$2 \times 10^6 = \frac{128 \times \mu \times 800 \times (10^{-3})^3 \times 2}{\pi(0.5 \times 10^{-3})^4}$$

$\mu = 1.917$ milli Pa – sec

05. Ans: 0.75

Sol: $U_r = U_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$

$$\left[\because \frac{U}{U_{max}} = 1 - \left(\frac{r}{R}\right)^2\right]$$

$$= 1\left(1 - \left(\frac{50}{200}\right)^2\right)$$

$$= 1\left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75\ m/s$$

06. Ans: 0.08

Sol: Given,

$\rho = 0.8 \times 1000 = 800$ kg/m^3

$\mu = 1$ Poise $= 10^{-1}$ N-s/m^2

d = 50 mm = 0.05 m

Velocity = 2 m/s

Reynold's Number, $Re = \frac{\rho VD}{\mu}$

$$= \frac{800 \times 2 \times 0.05}{10^{-1}} = 800$$

$(\because Re < 2000)$

$\therefore$ Flow is laminar,

For laminar, Darcy friction factor

$$f = \frac{64}{Re} = \frac{64}{800} = 0.08$$

07. Ans: 16

Sol: For fully developed laminar flow,

$$h_f = \frac{32\mu VL}{\rho g D^2} \quad (\therefore Q = AV)$$

$$h_f = \frac{32\mu\left(\frac{Q}{A}\right)L}{\rho g D^2} = \frac{32\mu QL}{AD^2 \times \rho g}$$

$$h_f = \frac{32\mu QL}{\frac{\pi}{4}D^2 \times D^2 \times \rho g}$$

$$h_f \propto \frac{1}{D^4}$$

$$h_{f1} D_1^4 = h_{f_2} D_2^4$$

Given, $D_2 = \frac{D_1}{2}$

$$h_{f1} \times D_1^4 = h_{f2} \times \left(\frac{D_1}{2}\right)^4$$

$$h_{f_2} = 16 h_{f_1}$$

$\therefore$ Head loss, increases by 16 times if diameter is halved.

08. Ans: 5.2

Sol: Oil viscosity, μ = 10 poise = 10×0.1

= 1 N-s/m^2

$y = 50 \times 10^{-3}$m

L = 120 cm = 1.20 m

$\Delta P = 3 \times 10^3$Pa

Width of plate = 0.2 m

Q = ?

$Q = A.V_{avg}$ = (width of plate×y)V

$$\Delta P = \frac{12\mu VL}{B^2}$$

$$3\times10^3 = \frac{12\times1\times V\times1.20}{\left(50\times10^{-3}\right)^2}$$

V = 0.52 m/sec

$Q = AV_{avg} = (0.2\times50\times10^{-3})$ (0.52)

= 5.2 lit/sec

09. Ans: (a)

Sol: Wall shear stress for flow in a pipe is given by,

$$\tau_o = -\frac{\partial P}{\partial x}\times\frac{R}{2} = \frac{\Delta P}{L}\times\frac{D}{4}$$

$$= \frac{\Delta P\, D}{4L}$$

10. Ans: 72

Sol: Given, ρ = 800 kg/m^3 ,

μ = 0.1 Pa.s

Flow is through an inclined pipe.

$d = 1 \times 10^{-2}$ m,

V_{av} = 0.1 m/s,

$\theta = 30°$

$$Re = \frac{\rho V_{av} d}{\mu} = \frac{800 \times 0.1 \times 1 \times 10^{-2}}{0.1} = 8$$

$\Rightarrow$ flow is laminar.

Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f$$

But $V_1 = V_2$,

$(Z_2 - Z_1) = 10 \sin 30° = 5$ m

and $\quad h_f = \dfrac{32\mu V_{av} L}{\rho g d^2}$

$$\frac{(P_1 - P_2)}{\gamma} = (Z_2 - Z_1) + \frac{32\mu V_{av} L}{\rho g d^2}$$

$$(P_1 - P_2) = \rho g (Z_2 - Z_1) + \frac{32\mu V_{av} L}{d^2}$$

$$= 800 \times 10 \times 5 + \frac{32 \times 0.1 \times 0.1 \times 10}{(1 \times 10^{-2})^2}$$

$$= 40 \times 10^3 + 32 \times 10^3 = 72 \text{ kPa}$$

11. Ans: (d)

Sol:

- In hydrodynamic entrance region of the pipe of uniform diameter, the average velocity remains constant in the direction of flow. Thus, statement - I is wrong.
- However, in the above region the centreline velocity increases in the direction of flow as boundary layers grow on the solid surfaces. Thus, statement (II) is correct.

Conventional Practice Solutions

01.

Sol: The velocity profile for fully developed laminar flow between two stationary parallel plates is given by

$$u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(By - y^2)$$

(i) $\quad \dfrac{\partial u}{\partial y} = \dfrac{1}{2\mu}\left(\dfrac{-\partial P}{\partial x}\right)(B - 2y)$

At the upper surface

$$\left.\frac{\partial u}{\partial y}\right|_{y=B} = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(B - 2 \times B)$$

$$= -\frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)B$$

$$\tau_{y=B} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=B} = -\frac{1}{2}\left(\frac{-\partial P}{\partial x}\right)B$$

$$= \frac{-1}{2} \times 1000 \times 5 \times 10^{-3} = -2.5 \text{ Pa}$$

Thus, the magnitude of the shear stress on the upper plate is 2.5 Pa and its direction is opposite to the direction of flow.

(ii) Discharge per unit length

$$= \int_0^B u(y)(dy \times 1)$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\int_0^B (By - y^2)dy$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left[B\frac{y^2}{2} - \frac{y^3}{3}\right]_0^B$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(\frac{B^3}{2} - \frac{B^3}{3}\right)$$

$$= \frac{B^3}{12\mu}\left(\frac{-\partial P}{\partial x}\right)$$

$$q = \frac{\left(5\times10^{-3}\right)^3}{12}\times1000$$

$$= 20.83\times10^{-6}\ \text{m}^3/\text{s}$$

02.

Sol: This is a problem of Couette flow with pressure gradient. In this case the velocity profile is given by

$$u = \frac{V}{h}y + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(hy - y^2\right)$$

$$= \frac{0.1}{0.01}y + \frac{1}{2\times0.1}(1200)\left(0.01y - y^2\right)$$

$$= 10y + 6000(0.01y - y^2)$$

$$= 10y + 60y - 6000y^2$$

$$= 70y - 6000y^2$$

For maximum velocity

$$\frac{\partial u}{\partial y} = 0 = 70 - 12000y$$

Or, $$y = \frac{70\times1000}{12000}\ \text{mm} = 5.833\ \text{mm}$$

and

$$V_{max} = 70\times5.833\times10^{-3} - 6000\times(5.833\times10^{-3})^2$$

$$= 0.204\ \text{m/s}$$

Chapter 8 Flow through Pipes

01. Ans: (d)

Sol:

- The Darcy-Weisbash equation for head loss in written as:

$$h_f = \frac{f\,L\,V^2}{2g\,d}$$

where V is the average velocity, f is friction factor, L is the length of pipe and d is the diameter of the pipe.

- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as the nature of pipe surface (smooth or rough)
- For laminar flow, friction factor is a function of Reynolds number.

02. Ans: 481

Sol: Given data,

$\dot{m} = \pi$ kg/s, $d = 5\times10^{-2}$ m,

$\mu = 0.001$ Pa.s , $\rho = 1000$ kg/m^3

$$V_{av} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho\pi d^2} = \frac{4\times\pi}{\rho\pi d^2} = \frac{4}{\rho d^2}$$

$$Re = \frac{\rho V_{av} d}{\mu} = \rho\times\frac{4}{\rho d^2}\times\frac{d}{\mu} = \frac{4}{\mu d}$$

$$= \frac{4}{0.001\times5\times10^{-2}} = 8\times10^4$$

$\Rightarrow$ Flow is turbulent

$$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{\left(8\times10^4\right)^{0.25}} = 0.0188$$

$$\Delta P = \rho g \frac{f L V_{av}^2}{2gd} = f\rho L \times \left(\frac{4}{\rho d^2}\right)^2 \times \frac{1}{2d}$$

$$\frac{\Delta P}{L} = f \times \frac{16}{\rho d^5} \times \frac{1}{2} = \frac{8f}{\rho d^5} = \frac{8\times0.0188}{10^3\times\left(5\times10^{-2}\right)^5}$$

$$= 481.28 \text{ Pa/m}$$

03. Ans: (a)

Sol: In pipes Net work, series arrangement

$$\therefore\ h_f = \frac{f\times\ell\times V^2}{2gd} = \frac{f\times\ell\times Q^2}{12.1\times d^5}$$

$$\frac{h_{f_A}}{h_{f_B}} = \frac{f_A\times\ell_A\times Q_a^2}{12.1\times d_A^5}\times\frac{12.1\times d_B^5}{f_B\times\ell_B\times Q_B^2}$$

Given $l_A = l_B$, $f_A = f_B$, $Q_A = Q_B$

$$\frac{h_{f_A}}{h_{f_B}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2d_B}\right)^5$$

$$= \left(\frac{1}{1.2}\right)^5 = 0.4018 \approx 0.402$$

04. Ans: (a)

Sol: Given, $d_1 = 10$ cm;

$d_2 = 20$ cm

$f_1 = f_2$;

$l_1 = l_2 = l$

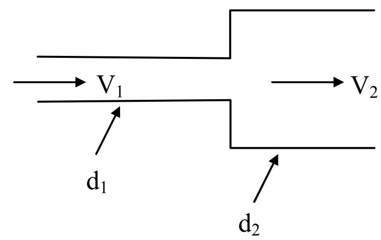
$l_e = l_1 + l_2 = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \Rightarrow \frac{2l}{d_e^5} = \frac{l}{10^5} + \frac{l}{20^5}$$

$\therefore$ $d_e = 11.4$ cm

05. Ans: (c)

Sol:



Given $d_2 = 2d_1$

Losses due to sudden expansion,

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

By continuity equation,

$Q = A_1V_1 = A_2V_2$

$$\therefore\ \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{1}{4}\right)^2$$

$$h_L = \frac{9}{16}\times\frac{V_1^2}{2g}$$

$$\frac{h_L}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

06. Ans: (b)

Sol: Pipes are in parallel

$Q_e = Q_A + Q_B$ ------- (i)

$h_{Le} = h_{L_A} = h_{L_B}$

$L_e = 175$ m

$f_e = 0.015$

$$\frac{f_e L_e Q_e^2}{12.1 D_e^5} = \frac{f_A . L_A Q_A^2}{12.1 D_A^5} = \frac{f_B L_B Q_B^2}{12.1 D_B^5}$$

$$\frac{0.020 \times 150 \times Q_A^2}{12.1 \times (0.1)^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$Q_A = 1.747\ Q_B$ -------(ii)

From (i) $Q_e = 1.747\ Q_B + Q_B$

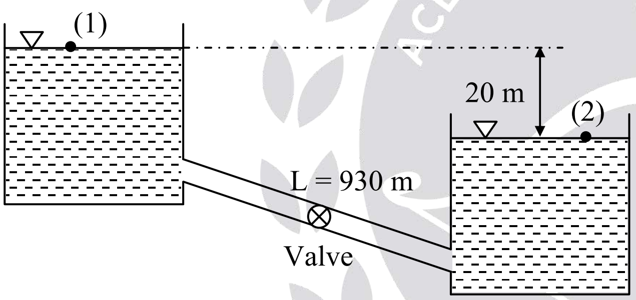
$Q_e = 2.747\ Q_B$ -------(iii)

$$\frac{0.015 \times 175 (2.747 Q_B)^2}{12.1 \times D_e^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$D_e = 116.6$ mm $\simeq 117$ mm

07. Ans: 0.141

Sol:



Given data,

$L = 930$ m , $k_{valve} = 5.5$

$k_{entry} = 0.5$, $d = 0.3$ m

$f = 0.03$, $g = 10$ m/s^2

Applying energy equation for points (1) and (2), we write :

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2 + h_{L,entry} + h_{L,valve} + h_{L,exit} + h_{f,pipe}$$

But $P_1 = P_2 = P_{atm} = 0$

$V_1 = 0 = V_2$

$Z_1 - Z_2 = 20$ m , $k_{exit} = 1$

$$Z_1 - Z_2 = 0.5\frac{V^2}{2g} + 5.5\frac{V^2}{2g} + 1 \times \frac{V^2}{2g} + \frac{f L V^2}{2gd}$$

$$= 7\frac{V^2}{2g} + \frac{f L V^2}{2gd} = \frac{V^2}{2g}\left(7 + \frac{f L}{d}\right)$$

or $$20 = \frac{V^2}{2g}\left[7 + \frac{0.03 \times 930}{0.3}\right] = 100\frac{V^2}{2g}$$

or $$V^2 = \frac{20 \times 2g}{100} = \frac{20 \times 2 \times 10}{100}$$

$\Rightarrow$ $V = 2$ m/s

Thus, discharge, $Q = \frac{\pi}{4} \times 0.3^2 \times 2$

$= 0.1414$ m^3/s

08. Ans: (c)

Sol: Given data :

Fanning friction factor, $f = m\ Re^{-0.2}$

For turbulent flow through a smooth pipe.

$$\Delta P = \frac{\rho f_{Darcy} L V^2}{2d} = \frac{\rho (4f) L V^2}{2d}$$

$$= \frac{2\rho m Re^{-0.2} L V^2}{d}$$

or $\Delta P \propto V^{-0.2} V^2 \propto V^{1.8}$ (as all other parameters remain constant)

We may thus write :

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{V_2}{V_1}\right)^{1.8} = \left(\frac{2}{1}\right)^{1.8} = 3.4822$$

or $\Delta P_2 = 3.4822 \times 10 = 34.82$ kPa

09. Ans: (b)

Sol: Given data :

Rectangular duct, L = 10 m,

X-section of duct = 15 cm × 20 cm

Material of duct - Commercial steel,

$\varepsilon = 0.045$ mm

Fluid is air ($\rho = 1.145$ kg/m^3,

$\nu = 1.655\times10^{-5}$ m^2/s)

$V_{av} = 7$ m/s

$$Re = \frac{V_{av}\times D_h}{\nu}$$

where, D_h = Hydraulic diameter

$$= \frac{4\times \text{Cross sectional area}}{\text{Perimeter}}$$

$$= \frac{4\times0.15\times0.2}{2(0.15+0.2)} = 0.1714\,\text{m}$$

$$Re = \frac{7\times0.1714}{1.655\times10^{-5}} = 72495.5$$

$\Rightarrow$ Flow is turbulent.

Using Haaland equation to find friction factor,

$$\frac{1}{\sqrt{f}} \simeq -1.8\log\left[\frac{6.9}{Re}+\left(\frac{\varepsilon/D_h}{3.7}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{72495.5}+\left(\frac{0.045\times10^{-3}}{0.1714\times3.7}\right)^{1.11}\right]$$

$$= -1.8\log[9.518\times10^{-5} + 2.48\times10^{-5}]$$

$$= -1.8\log(11.998\times10^{-5})$$

$$\frac{1}{\sqrt{f}} = 7.058$$

$$f = 0.02$$

The pressure drop in the duct is,

$$\Delta P = \frac{\rho f L V^2}{2D_h}$$

$$= \frac{1.145\times0.02\times10\times7^2}{2\times0.1714} = 32.73 \text{ Pa}$$

The required pumping power will be

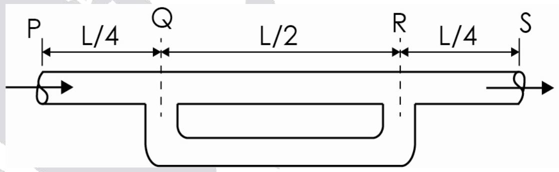
$P_{pumping} = Q\,\Delta P = A\,V_{av}\times\Delta P$

$= (0.15\times0.2)\times7\times(32.73)$

$= 6.87 \text{ W} \simeq 7 \text{ W}$

10. Ans: 26.5

Sol:



Case I: Without additional pipe,

Let Q be the discharge through the pipe. Then

$$\frac{P_P}{\gamma}+\frac{V_P^2}{2g}+Z_P = \frac{P_S}{\gamma}+\frac{V_S^2}{2g}+Z_S+\frac{fLQ^2}{12.1\,d^5}$$

But $V_P = V_S$

and $Z_P = Z_S$

P_P and P_S are the pressures at sections P and S, respectively.

Thus,

$$\frac{P_P}{\gamma}-\frac{P_S}{\gamma} = \frac{fLQ^2}{12.1d^5} \quad \text{-------(1)}$$

Case II: When a pipe (L/2) is connected in parallel.

In this case, let Q′ be the total discharge.

$$Q_{Q-R} = \frac{Q'}{2} \text{ and } Q_{R\text{-}S} = Q'$$

Then,

$$\frac{P'_P}{\gamma} + \frac{V_P'^{2}}{2g} + Z'_P = \frac{P'_S}{\gamma} + \frac{V_S'^{2}}{2g} + Z'_S + \frac{f(L/4)Q'^2}{12.1\,d^5} + \frac{f(L/2)(Q'/2)^2}{12.1\,d^5} + \frac{f(L/4)Q'^2}{12.1\,d^5}$$

P_P' and P_S' are the pressures at sections P and S in the second case.

But $V_P' = V_S'$; $Z_P' = Z_S'$

So, $$\frac{P'_P}{\gamma} - \frac{P'_S}{\gamma} = \frac{f\,L\,Q'^2}{12.1d^5}\left[\frac{1}{4} + \frac{1}{8} + \frac{1}{4}\right]$$

$$= \frac{5}{8} \times \frac{f\,L\,Q'^2}{12.1d^5} \text{ -------(2)}$$

Given that end conditions remain same.

i.e., $$\frac{P_P}{\gamma} - \frac{P_S}{\gamma} = \frac{P'_P}{\gamma} - \frac{P'_S}{\gamma}$$

Hence, equation (2) becomes,

$$\frac{f\,L\,Q^2}{12.1d^5} = \frac{5}{8}\frac{f\,L\,Q'^2}{12.1d^5} \text{ from eq.(1)}$$

or $$\left(\frac{Q'}{Q}\right)^2 = \frac{8}{5}$$

or $$\frac{Q'}{Q} = 1.265$$

Hence, percentage increase in discharge is

$$= \frac{Q' - Q}{Q} \times 100$$

= (1.265 – 1) × 100

= 26.5 %

11. Ans: 20%

Sol: Since, discharge decrease is associated with increase in friction.

$$\frac{df}{f} = -2 \times \frac{dQ}{Q} = 2\left[-\frac{dQ}{Q}\right]$$

$$= 2 \times 10 = 20\%$$

12. Ans: (c)

Sol: As compared to sharp entrance, the rounded entrance will give less energy loss in flow through a pipe. For sharp entrance, the flow gets separated and there will be recirculation zone till the fluid stream gets attached to the surface. Thus, the rounded entrance increases the flow rate when everything else remains constant. Hence, statement (I) is correct. However, statement (II) is wrong as discussed above.

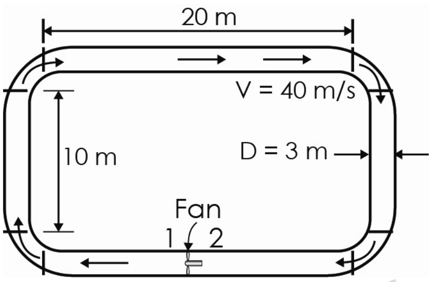
13. Ans: (d)

Sol: The surge tanks are provided on upstream side of the valve in order to offset the effect of water hammer mainly due to the pressure rise which may damage the pipe. Thus, statement (I) is wrong. However, statement (II) is correct.

Conventional Practice Solutions

01.

Sol:



Applying Energy equation for two points, just upstream and downstream of the fan in the pipe loop.

$$\frac{P_1}{\gamma_{air}}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_{air}}+\frac{V_2^2}{2g}+Z_2+\frac{fL_{total}V^2}{2gD}+4\times\frac{K_{elbow}V^2}{2g}$$

where $V_1 = V_2 = V$; $Z_1 = Z_2$

$f = 0.01$, $D = 3m$,

$V = 40$ m/s, $L = 60$ m,

$K_{elbow} = 0.3$ (Given)

$$\frac{P_1-P_2}{\gamma_{air}}=\frac{V^2}{2g}\left[\frac{fL}{D}+4\times K_{elbow}\right]$$

$$=\frac{40^2}{2g}\left[\frac{0.01\times 60}{3}+4\times 0.3\right]$$

$$=\frac{40^2}{2g}\times 1.4$$

$$\Delta P=\rho_{air}\times\frac{40^2}{2}\times 1.4$$

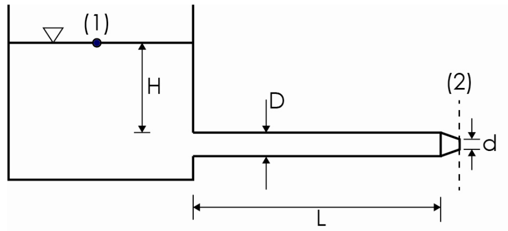
$$=1.2\times\frac{40^2}{2}\times 1.4=1{,}344 \text{ Pa}$$

Power added to air by fan, $P = Q\Delta P$

$$=\frac{\pi}{4}\times 3^2\times 40\times 1{,}344=380 \text{ kW}$$

02.

Sol:



Applying energy equation between points (1) and (2)

$$\frac{P_1}{\gamma_f}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_f}+\frac{V_2^2}{2g}+Z_2+(h_f)_{pipe}$$

But $P_1 = P_2 = P_{atm}$, $Z_1 = H$, $Z_2 = 0$, $V_1 = 0$

Thus,

$$H=\frac{V_2^2}{2g}+(h_f)_{pipe}=\frac{V_2^2}{2g}+\frac{fLV_p^2}{2gD}$$

For maximum power transmission, $H = 3h_f$

Or, $$3\times\frac{fLV_p^2}{2gD}=\frac{V_2^2}{2g}+\frac{fLV_p^2}{2gD}$$

Or, $$\frac{2fLV_p^2}{2gD}=\frac{V_2^2}{2g}$$

Or, $$\left(\frac{V_2}{V_p}\right)^2=\frac{2fL}{D} \qquad \text{---------------(1)}$$

From equation of continuity,

$$\frac{\pi}{4}D^2V_p=\frac{\pi}{4}d^2V_2$$

Or $$\frac{V_2}{V_p}=\frac{D^2}{d^2}$$

Thus, substituting in equation (1), we get

$$\left(\frac{D^2}{d^2}\right)^2=\frac{2fL}{D}$$

$$\Rightarrow \quad d=\left(\frac{D^5}{2fL}\right)^{\frac{1}{4}} \text{ .... Proved}$$

Chapter 9 Elementary Turbulent Flow

01. Ans: (b)

Sol: The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

where V* is the shear velocity.

02. Ans: (d)

Sol: $\Delta P = \rho g h_f$

$$= \frac{\rho f L V^2}{2D} = \frac{\rho g f L Q^2}{12.1 D^5}$$

For Q = constant

$$\Delta P \propto \frac{1}{D^5}$$

or $\dfrac{\Delta P_2}{\Delta P_1} = \dfrac{D_1^5}{D_2^5} = \left(\dfrac{D_1}{2D_1}\right)^5 = \dfrac{1}{32}$

03. Ans: 2.4

Sol: Given: V = 2 m/s

f = 0.02

V_{max} = ?

$V_{max} = V(1 + 1.43\sqrt{f})$

$= 2\left(1+1.43\sqrt{0.02}\right)$

$= 2 \times 1.2 = 2.4$ m/s

04. Ans: (c)

Sol: Given data:

D = 30 cm = 0.3 m

Re = 10^6

f = 0.025

Thickness of laminar sub layer, δ' = ?

$$\delta' = \frac{11.6\nu}{V^*}$$

where V^* = shear velocity = $V\sqrt{\frac{f}{8}}$

ν = Kinematic viscosity

$$Re = \frac{V.D}{\nu}$$

$$\therefore \nu = \frac{V.D}{Re}$$

$$\delta' = \frac{11.6 \times \frac{VD}{Re}}{V\sqrt{\frac{f}{8}}}$$

$$\delta' = \frac{11.6 \times D}{Re\sqrt{\frac{f}{8}}}$$

$$= \frac{11.6 \times 0.3}{10^6 \times \sqrt{\frac{0.025}{8}}}$$

$= 6.22 \times 10^{-5}$ m = 0.0622 mm

05. Ans: 25

Sol: Given:

L = 100 m

D = 0.1 m

h_L = 10 m

τ = ?

For any type of flow, the shear stress at wall/surface $\tau = \frac{-dP}{dx} \times \frac{R}{2}$

$$\tau = \frac{\rho g h_L}{L} \times \frac{R}{2}$$

$$\tau = \frac{\rho g h_L}{L} \times \frac{D}{4}$$

$$= \frac{1000 \times 9.81 \times 10}{100} \times \frac{0.1}{4}$$

$$= 24.525\ \text{N/m}^2 = 25\ \text{Pa}$$

06. Ans: 0.905

Sol: k = 0.15 mm

$\tau = 4.9\ \text{N/m}^2$

ν = 1 centi-stoke

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07\ \text{m/sec}$$

ν = 1 centi-stoke

$$= \frac{1}{100}\text{stoke} = \frac{10^{-4}}{100} = 10^{-6}\ \text{m}^2/\text{sec}$$

$$\frac{k}{\delta'} = \frac{0.15 \times 10^{-3}}{\left(\frac{11.6 \times \nu}{V^*}\right)}$$

$$= \frac{0.15 \times 10^{-3}}{\frac{11.6 \times 10^{-6}}{0.07}} = 0.905$$

07. Ans: (a)

Sol: The velocity profile in the laminar sublayer is given as

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

or $\nu = \frac{y(V^*)^2}{u}$

where, V* is the shear velocity.

Thus, $\nu = \frac{0.5 \times 10^{-3} \times (0.05)^2}{1.25}$

$$= 1 \times 10^{-6}\ \text{m}^2/\text{s}$$

$$= 1 \times 10^{-2}\ \text{cm}^2/\text{s}$$

08. Ans: 47.74 N/m²

Sol: Given data :

d = 100 mm = 0.1 m

$u_{r=0} = u_{max} = 2$ m/s

Velocity at r = 30 mm = 1.5 m/s

Flow is turbulent.

The velocity profile in turbulent flow is

$$\frac{u_{max} - u}{V^*} = 5.75 \log\left(\frac{R}{y}\right)$$

where u is the velocity at y and V* is the shear velocity.

For pipe, y = R – r

= (50 – 30) mm = 20 mm

Thus,

$$\frac{2 - 1.5}{V^*} = 5.75 \log\left(\frac{50}{20}\right) = 2.288$$

or $V^* = \frac{0.5}{2.288} = 0.2185\ \text{m/s}$

Using the relation,

$V^* = \sqrt{\frac{\tau_w}{\rho}}$ or $\tau_w = \rho\,(V^*)^2$

$$\tau_w = 10^3 \times (0.2185)^2 = 47.74\ \text{N/m}^2$$

09. Ans: (a)
Sol:

- In turbulent flow, shear stress is given by

$$\tau = \mu\left(\frac{d\bar{u}}{dy}\right) + \eta\left(\frac{d\bar{u}}{dy}\right)$$

= Viscous stress + Reynolds stress

where μ is dynamic viscosity and η is the eddy viscosity which is not a fluid properly but it is a flow property which depends upon turbulence condition of the flow.

- From the above expression we say that the shear stress in turbulent flow is more than that predicted by Newton's law of viscosity. Thus, statement - I is correct.
- Statement (II) is also correct statement and it is the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: Given data:

$r = 0$, $u = 1.5$ m/s at $y = R - 0 = R$

$r = \frac{R}{2}$, $u = 1.35$ m/s at $y = R - \frac{R}{2} = \frac{R}{2}$

D = 0.2 m or R = 0.1 m

Centreline velocity 1.5 m/s = u_{max}

Using the logarithmic velocity profile as:

$$\frac{u_{max} - u}{V^*} = 5.75\log\left(\frac{R}{y}\right)$$

where V^* is the shear velocity, we can find V^*.

$$\frac{1.5 - 1.35}{V^*} = 5.75\log\left(\frac{R}{R/2}\right) = 5.75\log(2)$$

$\Rightarrow \quad V^* = 0.0867$ m/s.

Similarly using the logarithmic velocity profile in terms of u, V and V^* (where V is the average velocity) we can find V as:

$$\frac{u - V}{V^*} = 5.75\log\left(\frac{y}{R}\right) + 3.75$$

at y = R,

$u = u_{max}$

$$\frac{1.5 - V}{0.0867} = 5.75\log\left(\frac{R}{R}\right) + 3.75 = 0 + 3.75$$

$\Rightarrow \quad V = 1.5 - 0.0867\times3.75 = 1.175$ m/s

(i) Thus, discharge $= \frac{\pi}{4}\times 0.2^2 \times 1.175$

$= 0.0369\ m^3/s$

(ii) We know that $V^* = V\sqrt{\frac{f'}{2}}$

where, f' is the coefficient of friction.

Thus, $f' = 2\times\left(\frac{V^*}{V}\right)^2$

$$= 2\times\left(\frac{0.0867}{1.175}\right)^2$$

$= 0.011$

The friction factor, $f = 4f' = 0.044$

(iii) The relationship between height of roughness projections, K and friction factor is given by

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{R}{K}\right) + 1.74$$

Substituting the values, we get

$$\frac{1}{\sqrt{0.044}} = 2.0\log\left(\frac{R}{K}\right) + 1.74$$

$$\log\left(\frac{R}{K}\right) = 1.5136$$

$$\frac{R}{K} = 32.629$$

$$K = \frac{R}{32.629} = \frac{0.1\times10^3}{32.629}\text{mm}$$

$$= 3.065 \text{ mm}$$

Chapter 10 Boundary Layer Theory

01. Ans: (c)

Sol: $\text{Re}_{\text{Critical}} = \frac{U_\infty x_{\text{critical}}}{\nu}$

Assume water properties

$$5\times10^5 = \frac{6\times x_{\text{critical}}}{1\times10^{-6}}$$

$x_{\text{critical}} = 0.08333$ m = 83.33 mm

02. Ans: 1.6

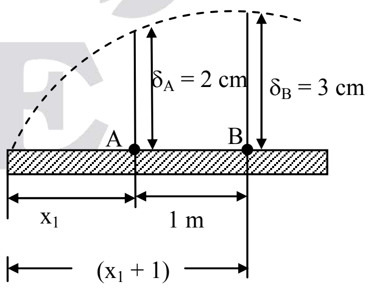
Sol: $\delta \propto \frac{1}{\sqrt{\text{Re}}}$ (At given distance 'x')

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{\text{Re}_2}{\text{Re}_1}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6$$

03. Ans: 80

Sol:



$$\delta \propto \sqrt{x}$$

$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1+1)}}$$

$$x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1+1}}$$

$$\frac{4}{9} = \frac{x_1}{x_1+1}$$

$5x_1 = 4 \Rightarrow x_1 = 80$ cm

04. Ans: 2

Sol: $\tau \propto \frac{1}{\delta}$

$$\tau \propto \frac{1}{\sqrt{x}} \quad \because \delta \propto \sqrt{x}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{4} = 2$$

05. Ans: 3

Sol: $\frac{U}{U_\infty} = \frac{y}{\delta}$

$$\frac{\delta^*}{\theta} = \text{Shape factor} = ?$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \left(1 - \frac{y}{8}\right) dy$$

$$= y - \frac{y^2}{2\delta}\Big|_0^\delta$$

$$= \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \frac{y}{8}\left(1 - \frac{y}{\delta}\right) dy$$

$$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta}\Big|_0^\delta$$

$$= \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\text{Shape factor} = \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$$

06. Ans: 22.6

Sol: Drag force,

$$F_D = \frac{1}{2} C_D.\rho.A_{Proj}.U_\infty^2$$

$B = 1.5$ m, $\rho = 1.2$ kg/m^3

$L = 3.0$ m, $\nu = 0.15$ stokes

$U_\infty = 2$ m/sec

$$Re = \frac{U_\infty L}{\nu} = \frac{2\times 3}{0.15\times 10^{-4}} = 4\times 10^5$$

$$C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{4\times 10^5}} = 2.09\times 10^{-3}$$

Drag force,

$$F_D = \frac{1}{2}\times 2.09\times 10^{-3}\times 1.2\times(1.5\times 3)\times 2^2$$

$= 22.57$ milli-Newton

07. Ans: 1.62

Sol: Given data,

$U_\infty = 30$ m/s,

$\rho = 1.2$ kg/m^3

Velocity profile at a distance x from leading edge,

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

$$\delta = 1.5 \text{ mm}$$

Mass flow rate of air entering section ab,

$$(\dot{m}_{in})_{ab} = \rho U_\infty(\delta \times 1) = \rho U_\infty \delta \text{ kg/s}$$

Mass flow rate of air leaving section cd,

$$(\dot{m}_{out})_{cd} = \rho\int_0^\delta u(dy \times 1) = \rho\int_0^\delta U_\infty\left(\frac{y}{\delta}\right)dy$$

$$= \frac{\rho U_\infty}{\delta}\left[\frac{y^2}{2}\right]_0^\delta = \frac{\rho U_\infty \delta}{2}$$

From the law of conservation of mass :

$$(\dot{m}_{in})_{ab} = (\dot{m}_{out})_{cd} + (\dot{m}_{out})_{bc}$$

Hence, $(\dot{m}_{out})_{bc} = (\dot{m}_{in})_{ab} - (\dot{m}_{out})_{cd}$

$$= \rho U_\infty \delta - \frac{\rho U_\infty \delta}{2}$$

$$= \frac{\rho U_\infty \delta}{2}$$

$$= \frac{1.2 \times 30 \times 1.5 \times 10^{-3}}{2}$$

$$= 27\times10^{-3} \text{ kg/s}$$

$$= 27\times10^{-3}\times60 \text{ kg/min}$$

$$= 1.62 \text{ kg/min}$$

08. Ans: (b)

Sol: For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in y-direction, $\frac{\partial u}{\partial y}$ only.

The correct option is (b).

09. Ans: 28.5

Sol: Given data,

Flow is over a flat plate.

L = 1 m,

$U_\infty = 6$ m/s

$\nu = 0.15$ stoke $= 0.15\times10^{-4}$ m^2/s

$\rho = 1.226$ kg/m^3

$$\delta(x) = \frac{3.46x}{\sqrt{Re_x}}$$

Velocity profile is linear.

Using von-Karman momentum integral equation for flat plate.

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2} \text{ --------(1)}$$

we can find out τ_w .

From linear velocity profile, $\frac{u}{U_\infty} = \frac{y}{\delta}$, we evaluate first θ, momentum thickness as

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy = \int_0^\delta\left(\frac{y}{\delta}-\frac{y^2}{\delta^2}\right)dy$$

$$= \left(\frac{y^2}{2\delta}-\frac{y^3}{3\delta^2}\right)_0^\delta = \frac{\delta}{2}-\frac{\delta}{3} = \frac{\delta}{6}$$

$$\Rightarrow \theta = \frac{\delta}{6} = \frac{1}{6}\times\frac{3.46x}{\sqrt{Re_x}}$$

$$= \frac{3.46}{6}\frac{x^{1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}}$$

Differentiating θ w.r.t x, we get :

$$\frac{d\theta}{dx} = \frac{3.46}{6\times 2}\frac{x^{-1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}} = 0.2883\frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0.5\,m} = 0.2883\times\frac{1}{\sqrt{\frac{6\times 0.5}{0.15\times 10^{-4}}}} = \frac{0.2883}{447.2}$$

-----(2)

From equation (1)

$$\tau_w\big|_{x=0.5m} = \left.\frac{d\theta}{dx}\right|_{x=0.5\,m}\times\rho U_\infty^2$$

$$= \frac{0.2883}{447.2}\times 1.226\times 6^2$$

$= 0.02845$ N/m^2

$\simeq$ 28.5 mN/m^2

10. Ans: (c)

Sol:

- For laminar boundary layer over a flat plate, the velocity gradient at the surface decreases in the direction of flow.
- This results in the decrease in shear stress and hence, the decrease in skin friction coefficient in the direction of flow.
- Thus, statement (I) is correct but the statement (II) is wrong.

11. Ans: (b)

Sol:

- The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free stream velocity. This results in higher skin friction drag in turbulent boundary layer. Thus, statement (I) is correct.
- The separation of turbulent boundary is late as compared to laminar boundary layer. Thus, statement (II) is also correct but it is not the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: Given data:

Test section dia = 40 cm

Test section length = 60 cm

Velocity of air at inlet = 2 m/s

and $\delta^* = \frac{1.72x}{\sqrt{Re_x}}$

$$Re_L = \frac{2\times 0.6}{10^{-5}} = 1.2\times 10^5$$

So, δ^* at x = 0.6m = $\frac{1.72\times 0.6}{\sqrt{1.2\times 10^5}}$

$= 2.979\times 10^{-3}$ m

From equation of continuity

$A_{in}V_{in} = A_{exit}V_{exit}$

But $d_{exit} = 0.4 - 2\delta^*$

$= (0.4 - 2\times 2.979\times 10^{-3})$ m

Thus, $V_{exit} = \left(\frac{0.4}{0.4 - 2\times 2.979\times 10^{-3}}\right)^2 \times 2$

= 2.061 m/s

02.

Sol: Given data:

Flow over a flat plate

Fluid is water.

$U_\infty = 1$ m/s

$L = 1$ m

Case I: *Flow is turbulent*

At x = 1 m

$$Re_x = \frac{U_\infty x}{\nu_{water}} = \frac{1\times 1}{10^{-6}} = 10^6$$

$$\frac{\delta_{tur}}{x} = \frac{0.376}{(Re_x)^{\frac{1}{5}}} = \frac{0.376}{(10^6)^{\frac{1}{5}}}$$

$$\delta_{tur} = \frac{0.376\times 1}{(10^6)^{\frac{1}{5}}} = 0.0237\,\text{m} \approx 24\text{ mm}$$

$$\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = C_{f,x} = \frac{0.059}{(Re_x)^{\frac{1}{5}}}$$

$$\tau_w = \frac{0.059}{(10^6)^{\frac{1}{5}}}\times\frac{1}{2}\times 10^3\times 1^2 = 1.86\text{ N/m}^2$$

Case 2: *If the flow is laminar*

For the comparison purpose, consider the same Reynolds number.

$$\frac{\delta_{lam}}{x} = \frac{5}{\sqrt{Re_x}}$$

$$\delta_{lam} = \frac{5\times 1}{\sqrt{10^6}} = 5\text{ mm}$$

and $$\tau_w = \frac{0.664}{\sqrt{Re_x}}\times\frac{1}{2}\rho U_\infty^2$$

$$= \frac{0.664}{\sqrt{10^6}}\times\frac{1}{2}\times 10^3\times 1^2 = 0.332\text{ N/m}^2$$

Chapter 11 Force on Submerged Bodies

01. Ans: 8

Sol: Drag power = Drag Force × Velocity

$$P = F_D \times V$$

$$P = C_D \times \frac{\rho A V^2}{2}\times V$$

$$P \propto V^3$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^3$$

$$\frac{P_1}{P_2} = \left(\frac{V}{2V}\right)^3$$

$$P_2 = 8P_1$$

Comparing the above relation with XP,

We get, X = 8

02. Ans: 4.56 m

Sol: $F_D = C_D.\frac{\rho A V^2}{2}$

$$W = 0.8\times 1.2\times\frac{\frac{\pi}{4}(D)^2\times V^2}{2}$$

(Note: A = Normal (or) projected Area = $\frac{\pi}{4}D^2$)

$$784.8 = 0.8\times 1.2\times\frac{\pi}{4}(D)^2\times\frac{10^2}{2}$$

$\therefore$ D = 4.56 m

03. Ans: 4

Sol: Given data:

$l = 0.5$ km = 500 m

d = 1.25 cm

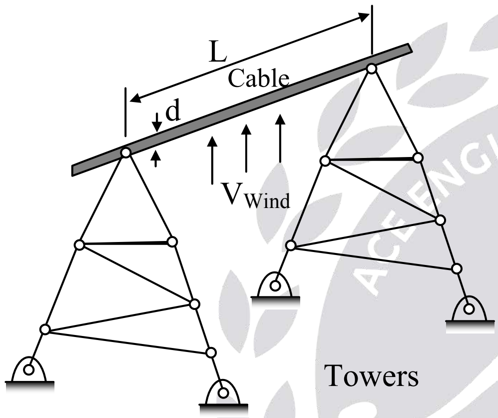
$V_{Wind} = 100$ km/hr

$\gamma_{Air} = 1.36 \times 9.81 = 13.4\ N/m^3$

$\nu = 1.4 \times 10^{-5}\ m^2/s$

$C_D = 1.2$ for Re > 10000

$C_D = 1.3$ for Re < 10000



$$Re = \frac{V.L}{\nu} = \frac{\left(\frac{100 \times 5}{18}\right)(500)}{1.4 \times 10^{-5}}$$

Note: The characteristic dimension for electric power transmission tower wire is "L"

$Re = 992 \times 10^6 > 10{,}000$

$\therefore C_D = 1.2$

$$F_D = C_D \times \frac{\rho A V^2}{2}$$

$$= 1.2 \times \frac{\left(\frac{13.4}{9.81}\right)(L \times d)V^2}{2}$$

$$= \frac{1.2 \times \left(\frac{13.4}{9.81}\right)(500 \times 0.0125)\left(100 \times \frac{5}{18}\right)^2}{2}$$

= 3952.4 N

= 4 kN

04. Ans: 0.144 & 0.126

Sol: Given data:

$W_{Kite} = 2.5$ N

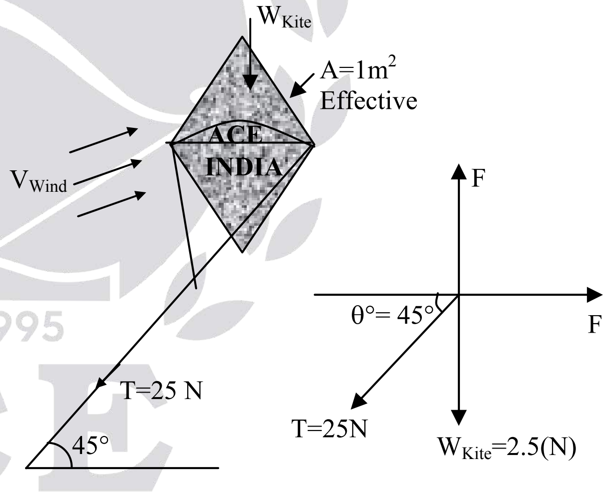
$A = 1\ m^2$

$\theta = 45^o$

T = 25 N

$V_{Wind} = 54$ km/hr

$= 54 \times \frac{5}{18} = 15$ m/s



Resolving forces horizontally

$F_D = T\cos 45^o$

$$C_D \times \frac{\rho A V^2}{2} = 25 \times \cos 45^o$$

$$\frac{C_D \times \left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 25 \times \frac{1}{\sqrt{2}}$$

$\therefore \quad C_D = 0.126$

Resolving forces vertically

$F_L = W_{Kite} + T\sin 45^o$

$$\frac{C_L \rho A V^2}{2} = 2.5 + 25\sin 45^o$$

$$\frac{C_L\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 2.5 + \frac{25}{\sqrt{2}}$$

$\therefore C_L = 0.144$

05. Ans: (a)

Sol: Given data:

$C_{D_2} = 0.75\, C_{D_1}$ (25% reduced)

Drag power = Drag force × Velocity

$$P = F_D \times V = \frac{C_D \rho A V^2}{2} \times V$$

$$P = C_D \times \frac{\rho A V^3}{2}$$

Keeping ρ, A and power constant

$C_D V^3$ = constant = C

$$\frac{C_{D_1}}{C_{D_2}} = \left(\frac{V_2}{V_1}\right)^3$$

$$\left(\frac{C_{D_1}}{0.75 C_{D_1}}\right)^{1/3} = \frac{V_2}{V_1}$$

$\therefore V_2 = 1.10064 V_1$

% Increase in speed = 10.064%

06. Ans: (c)

Sol: When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid :

Weight of sphere = Buoyant force + Drag force

07. Ans: 0.62

Sol: Given data,

Diameter of dust particle, d = 0.1 mm

Density of dust particle,

$\rho = 2.1\ g/cm^3 = 2100\ kg/m^3$

$\mu_{air} = 1.849\times10^{-5}$ Pa.s,

At suspended position of the dust particle,

$W_{particle} = F_D + F_B$

where F_D is the drag force on the particle and F_B is the buoyancy force.

From Stokes law:

$F_D = 3\pi\mu\, V\, d$

Thus,

$$\frac{4}{3} \times \pi r^3 \times \rho \times g = 3\pi\mu V d + \frac{4}{3}\pi r^3 \rho_{air} g$$

$$\text{or, } \frac{4}{3}\pi r^3 g(\rho - \rho_{air}) = 3\pi\mu_{air} V(2r)$$

$$\text{or } V = \frac{2}{9} r^2 g\left(\frac{\rho - \rho_{air}}{\mu_{air}}\right)$$

$$= \frac{2}{9} \times (0.05\times10^{-3})^2 \times 9.81 \times \frac{(2100 - 1.2)}{1.849\times10^{-5}}$$

$= 0.619$ m/s ≈ 0.62 m/s

08. Ans: (b)

Sol: Since the two models M_1 and M_2 have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.

From their shapes, we can say that M_2 reaches the bottom earlier than M_1.

09. Ans: (a)

Sol:

- Drag of object A_1 will be less than that on A_2. There are chances of flow separation on A_2 due to which drag will increase as compared to that on A_1.
- Drag of object B_1 will be more than that of object B_2. Because of rough surface of B_2, the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but C_2 is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on C_2 will be more than that on C_1 because of flow becoming turbulent due to roughness. Hence, drag of object C_2 will be more than that of object C_1.
- Thus, the correct answer is option (a).

10. Ans: (a)

Sol:

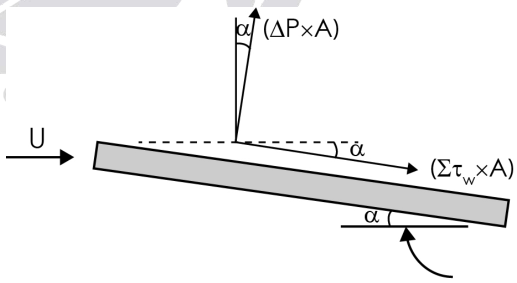
- Dimples on a golf ball are intentionally provided to make its surface rough so that flow becomes turbulent.
- A turbulent boundary layer, having more momentum than a laminar boundary layer, can better resist an adverse pressure gradient, thus avoiding early separation.
- Thus, both statements are correct and statement (II) is the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: The lift force on the given plate is

$F_L = (\Delta P \times A)\cos\alpha - (\Sigma\tau_w \times A)\sin\alpha$



$F_D = (\Delta P \times A)\sin\alpha + (\Sigma\tau_w \times A)\cos\alpha$

Substituting the values given:

$F_D = [2.3 - (-1.2)](1)\sin7^\circ + (7.6\times10^{-2} + 5.8\times10^{-2})\times1\times\cos7^\circ$

$= 426.5\text{ N} + 133\text{ N} = 559.5\text{ N} \approx 560\text{ N}$

$F_L = [2.3 - (-1.2)]\times1\times\cos7^o - (7.6\times10^{-2} + 5.8\times10^{-2})\times1\times\sin7^o$

$= 3474 \text{ N} - 16.3 \text{ N}$

$= 3457.7 \text{ N} \approx 3458 \text{ N}$

When the shear stress is neglected, then

$F_D = (\Delta P \times A)\cos\alpha = 426.5 \text{ N} \approx 427 \text{ N}$

and $F_L = (\Delta P \times A)\sin\alpha = 3474 \text{ N}$

Chapter 12 Dimensional Analysis

01. Ans: (c)

Sol: Total number of variables,

n = 8 and m = 3 (M, L & T)

Therefore, number of π's are = 8 – 3 = 5

02. Ans: (b)

Sol:

1. $\dfrac{T}{\rho D^2V^2} = \dfrac{MLT^2}{ML^{-3}\times L^2\times L^2\times T^{-2}} = 1$.

$\rightarrow$ It is a non-dimensional parameter.

2. $\dfrac{VD}{\mu} = \dfrac{LT^{-1}\times L}{ML^{-1}T^{-1}} \neq 1$.

$\rightarrow$ It is a dimensional parameter.

3. $\dfrac{D\omega}{V} = 1$.

$\rightarrow$ It is a non-dimensional parameter.

4. $\dfrac{\rho VD}{\mu} = Re$.

$\rightarrow$ It is a non-dimensional parameter.

03. Ans: (b)

Sol: T = f (l, g)

Total number of variable,

n = 3, m = 2 (L & T only)

Hence, no. of π terms = 3 – 2 = 1

04. Ans: (c)

Sol:

- Mach Number → Launching of rockets
- Thomas Number → Cavitation flow in soil
- Reynolds Number → Motion of a submarine
- Weber Number → Capillary flow in soil

05. Ans: (b)

Sol: According to Froude's law

$$T_r = \sqrt{L_r}$$

$$\frac{t_m}{t_p} = \sqrt{L_r}$$

$$t_p = \frac{t_m}{\sqrt{L_r}} = \frac{10}{\sqrt{1/25}}$$

t_p = 50 min

06. Ans: (a)

Sol: L = 100 m

$V_p = 10\,m/s$,

$$L_r = \frac{1}{25}$$

As viscous parameters are not discussed, follow Froude's law.

According to Froude,

$$V_r = \sqrt{L_r}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{1}{25}}$$

$$V_m = \frac{1}{5} \times 10 = 2 \text{ m/s}$$

07. Ans: (d)

Sol: Froude number = Reynolds number.

ν_r = 0.0894

If both gravity & viscous forces are important then

$$\nu_r = (L_r)^{3/2}$$

$$\sqrt[3]{(\nu_r)^2} = L_r$$

L_r = 1:5

08. Ans: (c)

Sol: For distorted model according to Froude's law

$$Q_r = L_H L_V^{3/2}$$

L_H = 1:1000 ,

L_V = 1:100

Q_m = 0.1 m^3/s

$$Q_r = \frac{1}{1000} \times \left(\frac{1}{100}\right)^{3/2} = \frac{0.1}{Q_p}$$

Q_P = 10^5 m^3/s

09. Ans: (c)

Sol: For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

$$\frac{\rho_w \times V_w \times d_w}{\mu_w} = \frac{\rho_a \times V_a \times d_a}{\mu_a}$$

or $$V_w = \frac{\rho_a}{\rho_w} \times \frac{d_a}{d_w} \times \frac{\mu_w}{\mu_a} \times V_a$$

(where suffixes w and a stand for water and air respectively)

Substituting the values given, we get

$$V_w = \frac{1.2}{10^3}\times\frac{4}{0.1}\times\frac{10^{-3}}{1.8\times10^{-5}}\times 1 = \frac{8}{3}\,\text{m/s}$$

To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.

i.e, $\left(\frac{F_D}{\rho A V^2}\right)_P = \left(\frac{F_D}{\rho A V^2}\right)_m$

Hence, $(F_D)_p = (F_D)_m \times \frac{\rho_a}{\rho_w}\times\frac{A_a}{A_w}\times\left(\frac{V_a}{V_w}\right)^2$

$$= 4\times\frac{1.2}{10^3}\times\left(\frac{4}{0.1}\right)^2\times\left(\frac{1}{8/3}\right)^2$$

$= 1.08$ N

10. Ans: 47.9

Sol: Given data,

	Sea water (Prototype testing)	Fresh water (model testing)
V	0.5	?
ρ	1025 kg/m^3	10^3 kg/m^3
μ	1.07×10^{-3} Pa.s	1×10^{-3} Pa.s

For dynamic similarity, Re should be same in both testing.

i.e., $\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p}$

$$V_m = V_P\times\frac{\rho_p}{\rho_m}\times\frac{d_p}{d_m}\times\frac{\mu_m}{\mu_p}$$

$$= 0.5\times\frac{1025}{10^3}\times100\times\frac{10^{-3}}{1.07\times10^{-3}}$$

$= 47.9$ m/s