01. Properties of Fluids

01. Ans: (c)
Sol: For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).

02. Ans: 100
Sol: \[ \tau = \frac{\mu V}{h} = \frac{0.2 \times 1.5}{3 \times 10^{-3}} = 100 \text{ N/m}^2 \]

03. Ans: 1
Sol:

\[ W \sin 30 = \frac{\mu AV}{h} \]

\[ \frac{100}{2} = \frac{1 \times 0.1 \times V}{2 \times 10^{-3}} \]

\[ V = 1 \text{ m/s} \]

Common data Q. 04 & 05

04. Ans: (c)
Sol: \[ D_1 = 100 \text{ mm}, \quad D_2 = 106 \text{ mm} \]
Radial clearance, \[ h = \frac{D_2 - D_1}{2} = \frac{106 - 100}{2} = 3 \text{ mm} \]

L = 0.15m
\[ \mu = 0.2 \text{ pa.s} \]
N = 240 rpm

\[ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi \]

\[ \tau = \frac{\mu \omega r}{h} = \frac{0.2 \times 8\pi \times 50 \times 10^{-3}}{3 \times 10^{-3}} = 83.77 \text{ N/m}^2 \]

05. Ans: (b)
Sol: Power, \[ P = \frac{2\pi \omega^2 \mu L r^3}{h} \]

\[ = \frac{2\pi \times (8\pi)^3 \times 0.2 \times 0.15 \times (0.05)^3}{3 \times 10^{-3}} = 4.96 \text{ Watt} \]
06. Ans: (c)
Sol: 
\[ \tau = \mu \frac{du}{dy} \]
\[ u = 3 \sin(5\pi y) \]
\[ \frac{du}{dy} = 3 \cos(5\pi y) \times 5\pi = 15\pi \cos(5\pi y) \]
\[ \tau \bigg|_{y=0.05} = \mu \frac{du}{dy} \bigg|_{y=0.05} \]
\[ = 0.5 \times 15\pi \cos(5\pi \times 0.05) \]
\[ = 0.5 \times 15\pi \times \cos \left( \frac{\pi}{4} \right) = 0.5 \times 15\pi \times \frac{\sqrt{2}}{2} \]
\[ = 7.5 \times 3.14 \times 0.707 \approx 16.6 \text{N/m}^2 \]

07. Ans: (a)
Sol: 
\[ \tau = \frac{\mu}{K} \]
\[ u = 3 \sin(5\pi y) \]
\[ \frac{du}{dy} = 3 \cos(5\pi y) \times 5\pi = 15\pi \cos(5\pi y) \]
\[ \tau \bigg|_{y=0.05} = \frac{\mu}{K} \bigg|_{y=0.05} \]
\[ = 0.5 \times 15\pi \cos(5\pi \times 0.05) \]
\[ = 0.5 \times 15\pi \times \cos \left( \frac{\pi}{4} \right) = 0.5 \times 15\pi \times \frac{\sqrt{2}}{2} \]
\[ = 7.5 \times 3.14 \times 0.707 \approx 16.6 \text{N/m}^2 \]

08. Ans: (d)
Sol: 
- Ideal fluid → Shear stress is zero.
- Newtonian fluid → Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid → Shear stress does not vary linearly with the rate of strain.

- Bingham plastic → Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

09. Ans: (b)
Sol: 
\[ V = 0.01 \text{ m}^3 \]
\[ \beta = 0.75 \times 10^{-9} \text{ m}^2/\text{N} \]
\[ dP = 2 \times 10^7 \text{ N/m}^2 \]
\[ K = \frac{1}{\beta} = \frac{1}{0.75 \times 10^{-9}} = \frac{4}{3} \times 10^9 \]
\[ \frac{dV}{dV} = \frac{-dP}{dV/\sqrt{V}} \]
\[ = -2 \times 10^7 \times 10^{-2} \times \frac{3}{4 \times 10^9} = -1.5 \times 10^{-4} \]

10. Ans: 320 Pa
Sol: 
\[ \Delta P = \frac{8\sigma}{D} = \frac{8 \times 0.04}{1 \times 10^{-3}} = \frac{32 \times 10^{-2}}{10^{-3}} \]
\[ \Delta P = 320 \text{ N/m}^2 \]

11. Ans: (d)
Sol: 
- As the temperature is increased, the viscosity of a liquid decreases due to the reduction in intermolecular cohesion.
- In gases, the viscosity increases with the rise in temperature due to increased molecular activity causing an increase in the change of momentum of the molecules, normal to the direction of motion.
Thus, statement (I) is wrong but statement (II) is correct.

12. Ans: (c)
Sol: The surface energy is given by

\[ E = \sigma \times \text{area} \]

As area increases, surface energy will increase. Thus, statement (I) is correct.

Surface tension, \( \sigma \) is the property of fluid. Hence, it is independent of the size of the bubble. Thus, statement (II) is wrong.

**Conventional Practice Solutions**

01. Sol:

\[ \begin{align*}
F &= F_1 + F_2,
\end{align*} \]

Where \( F_1 = \text{Force on top sides of plate}, \)
\( F_2 = \text{Force on bottom side of plate} \)

The plate moves with velocity \( V \)

\[ \begin{align*}
\tau &= \frac{\mu du}{dy} \\
F_1 &= \tau_1 \times \text{Area of plate} \\
F_1 &= \mu_1 \times \frac{V}{h-y} \times A \\
F_2 &= \mu_2 \times \frac{V}{y} \times A
\end{align*} \]

(i) Shear force on two sides of the plate are equal:

\[ \begin{align*}
F_1 &= F_2 \\
\mu_1 \times VA &= \mu_2 VA \\
\frac{h-y}{y} &= \frac{\mu_1}{\mu_2}
\end{align*} \]

(ii) The position of plate so that pull required to drag the plate is minimum.

\[ \begin{align*}
h &= \frac{\mu_1}{\mu_2} + 1 \\
\frac{h}{y} &= \frac{\mu_1 + \mu_2}{\mu_2} \\
y &= \frac{\mu_2 h}{\mu_1 + \mu_2}
\end{align*} \]

Assumptions:

- Thin plate has negligible thickness.
- Velocity profile is linear because of narrow gap.
- Given fluid is a Newtonian fluid which obeys Newton’s law of viscosity.

The force required to pull it is proportional to the total shear stress imposed by the two oil layers.
\[ F = \mu_1 VA + \frac{\mu_2 VA}{h - y} + \frac{\mu_2 VA}{y}, \]

\[ [V, A, \mu_1 & \mu_2, h \text{ are constant}] \]

For minimum force, \( \frac{dF}{dy} = 0 \)

\[ -\mu_1 VA(h - y)^2 (-1) - \mu_2 VAy^{-2} = 0 \]

\[ \frac{\mu_2 VA}{y^2} = \frac{\mu_1 VA}{(h - y)^2} \]

\[ \frac{(h - y)^2}{y^2} = \frac{\mu_1}{\mu_2} \implies \frac{h - y}{y} = \sqrt{\frac{\mu_1}{\mu_2}} \]

\[ h = 1 + \sqrt{\frac{\mu_1}{\mu_2}} \text{ where } y \text{ is the distance of the thin flat plate from the bottom flat surface.} \]

\[ y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}} \]

**02. Ans: 0.372 Pa.s**

**Sol:** Torque = 1.2 N-m

Speed, \( N = 600 \text{ rpm} \)

Diameter, \( D_1 = 15 \text{ cm, } D_2 = 14.75 \text{ cm} \)

\( H = 2.5 \text{ cm} \)

Assumptions:

- The gap between two cylinders is narrow and hence velocity profile in the gap is assumed linear.
- No change in properties

Torque = Tangential force \( \times \) radius

Force = shear stress \( \times \) Area

\[ \text{Area} = \pi DL \]

\( h = \frac{15 - 14.75}{2} = 0.125 \text{ cm} = 1.25 \times 10^{-3} \text{ m} \)

\[ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 20\pi \text{ rad/s} \]

\[ \text{Torque} = F_s \times r = \frac{\mu \omega r}{h} \times A = \frac{\mu \omega r^2}{h} \times A \]

\[ 1.2 = \frac{\mu \times 2\pi \times (0.07375)^2 \times 11.781 \times 10^{-3}}{1.25 \times 10^{-3}} \]

\[ \mu = 0.3726 \text{ Pa.s} \]
02. Pressure Measurement & Fluid Statics

01. Ans: (a)
Sol: 1 millibar = $10^{-3} \times 10^5 = 100$ N/m²
One mm of Hg = $13.6 \times 10^3 \times 9.81 \times 10^{-3}$
= 133.416 N/m²
1 N/mm² = $1 \times 10^6$ N/m²
1 kgf/cm² = $9.81 \times 10^4$ N/m²

02. Ans: (b)
Sol:

\begin{align*}
\text{710 mm} & \quad \text{Local atm. pressure} \\
\phantom{710} & \quad (350 \text{ mm of vaccum}) \\
\phantom{710} & \quad \text{360 mm} \\
\phantom{710} & \quad \text{Absolute pressure}
\end{align*}

03. Ans: (c)
Sol: Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure $P_A$ and $P_B$ are same.

04. Ans: (b)
Sol:
- The manometer shown in Fig. 1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.

05. Ans: 2.2
Sol: $h_p$ in terms of oil

\[ s_0 h_0 = s_m h_m \]

\[ 0.85 h_0 = 13.6 \times 0.1 \]

\[ h_0 = 1.6 \text{ m} \]

\[ h_p = 0.6 + 1.6 \]

\[ \Rightarrow h_p = 2.2 \text{ m of oil} \]

(Or) \[
\frac{P_p - P_{atm}}{\gamma_{oil}} = \left( \frac{\gamma_{Hg}}{\gamma_{oil}} \times 0.1 + 0.6 \right) \\
= \frac{13.6}{0.85} \times 0.1 + 0.6 = 2.2 \text{ m of oil} \]

Gauge pressure of $P$ in terms of m of oil $= 2.2 \text{ m of oil}$

06. Ans: (b)
Sol:

\[ h_M - \frac{s_w h_w}{s_0} = h_N - \frac{s_w h_w}{s_0} - h_0 \]

\[ h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3 \]

\[ h_M - h_N = -13.843 \text{ cm of oil} \]

- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.
07. Ans: 2.125
Sol:
\[ h_p = \bar{h} + \frac{1}{A_h} \]
\[ = 2 + \frac{\pi D^4 \times 4}{64 \times D^2 \times 2 \times \pi} \]
\[ = 2 + \frac{2^2 \times 4}{64 \times 2} = 2.125 \text{m} \]

08. Ans: 10
Sol:
\[ F = \rho g \bar{h}A \]
\[ = 9810 \times 1.625 \times \frac{\pi}{4} (1.2^2 - 0.8^2) \]
\[ F = 10 \text{kN} \]

09. Ans: 1
Sol:
\[ F_{\text{bottom}} = \rho g \times 2x \times 2x \times x \]
\[ F_V = \rho g x \times 2x \times 2x \]
\[ \frac{F_B}{F_V} = 1 \]

10. Ans: 10
Sol:
\[ F_V = x \times \pi \]
\[ F_V = \rho g V = 1000 \times 10 \times \frac{\pi \times 2^2}{4} \]
\[ F_V = 10 \pi \text{kN} \]
\[ \therefore \ x = 10 \]

11. Ans: (d)
Sol:
\[ F_{\text{net}} = F_{H1} - F_{H2} \]
\[ F_{H1} = \gamma \times \frac{D}{2} \times D \times 1 = \frac{\gamma D^2}{2} \]
\[ F_{H2} = \gamma \times \frac{D}{4} \times D \times 1 = \frac{\gamma D^2}{8} \]
\[ = \gamma D^2 \left( \frac{1}{2} - \frac{1}{8} \right) = \frac{3 \gamma D^2}{8} \]

12. Ans: 2
Sol:
Let \( P \) be the absolute pressure of fluid \( f3 \) at mid-height level of the tank. Starting from the open limb of the manometer (where pressure = \( P_{\text{atm}} \)) we write:
\[ P_{\text{atm}} + \gamma \times 1.2 - 2 \gamma \times 0.2 - 0.5 \gamma \times \left( 0.6 + \frac{h}{2} \right) = P \]
or
\[ P - P_{\text{atm}} = P_{\text{gauge}} \]
\[ = \gamma (1.2 - 2 \times 0.2 - 0.5 \times 0.6 - 0.5 \times \frac{h}{2}) \]

For \( P_{\text{gauge}} \) to be zero, we have,
\[ \gamma (1.2 - 0.4 - 0.3 - 0.25 h) = 0 \]
or
\[ h = \frac{0.5}{0.25} = 2 \]
13. Ans: (b)
Sol: The depth of centre of pressure from the free liquid surface is given by
\[ h_{cp} = \bar{h} + \frac{I_{xx,c}}{Ah} \] ---------(1)
Or, \[ h_{cp} - \bar{h} = \frac{I_{xx,c}}{Ah} \]
From the above relationship, as \( \bar{h} \) increases, \( \frac{I_{xx,c}}{Ah} \) decreases. Thus, at great depth, the difference \( (h_{cp} - \bar{h}) \) becomes negligible.
Hence, statement (I) is correct.
Also, it is clear from equation (1) that \( h_{cp} \) is independent of the density of the liquid.

Conventional Practice Solutions

01. Sol:
\[ \bar{h} = 5 + \left(3 - \frac{4 \times R}{3\pi}\right) \]
\[ = 5 + \left(3 - \frac{4 \times 3}{3\pi}\right) = 5 + 1.727 = 6.727 \text{ m} \]
\[ F_H = \gamma_w \times 6.727 \times \text{Area (projected)} \]

02. Sol:
\[ \bar{h} = \left(1.5 + \frac{R}{2}\right) \]
\[ F_H = \rho \bar{h} A_{\text{projected}} \]
\[ = \rho g \left(1.5 + \frac{R}{2}\right)(R \times 3) \]
\[ \gamma (1.5 + 1.5)(3 \times 3) = 27 \gamma N \]

\[ h_{cp} = 3 + \frac{1}{12} \left( \frac{3 \times 3}{3 \times 3} \right) = 3.25 \text{ m from free liquid surface} \]

\[ = 3.25 - 1.5 = 1.75 \text{ m from } A \]

\[ F_B = \gamma \left( \frac{\pi R^2}{4} \right) (3) = \gamma \times \frac{\pi \times 9 \times 3}{4} = \frac{27 \pi \gamma}{4} N \]

\( F_B \) will act through the centroid of the quadrant which is at a distance \( \frac{4R}{3\pi} \) from the vertical line AB. Now, taking moment of the forces about the hinge A, we write

\[ F_s \times 3 + F_B \times \frac{4R}{3\pi} - F_H \times 1.75 = 0 \]

where \( F_s \) is the force in x-direction on the stop at B & \( V_s \) is in y-direction (does not contribute in the moment).

\[ 3F_s = 27 \times 1.75 \gamma - \frac{27 \pi \gamma}{4} \times \frac{4R}{3\pi} = 10^4 (27 \times 1.75 - 9 \times 3) = 10^4 \times 27 \times 0.75 = 202.5 \text{ kN.m} \]

\[ \Rightarrow F_s = \frac{202.5}{3} = 67.5 \text{ kN} \]

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**03. Buoyancy and Metacentric Height**

**01. Ans: (d)**

**Sol:**

\[ F_B = \text{weight of body} \]

\[ \rho_b g V_b = \rho_f g V_f d \]

\[ 640 \times 4 \times 2 \times 1.25 = 1025 \times (4 \times 1.25 \times d) \]

\[ d = 1.248 \text{ m} \]

\[ V_{fd} = 1.248 \times 4 \times 1.25 = 6.24 \text{ m}^3 \]

**02. Ans: (c)**

**Sol:**

Surface area of cube = 6 \( a^2 \)

Surface area of sphere = 4 \( \pi r^2 \)

\[ 4\pi r^2 = 6a^2 \]

\[ \frac{2\pi}{3} = \left( \frac{a}{r} \right)^2 \]

\[ F_{b,s} \propto V_s \]

\[ \frac{4}{3} \pi r^3 \]

\[ \frac{4}{3} \left( \frac{2\pi}{3} \right)^3 = \frac{4}{3} \left( \frac{2\pi}{3} \times \frac{2\pi}{3} \right)^3 = \sqrt{\frac{6}{\pi}} \]
03. Ans: 4.76
Sol:  
\[ F_B = F_{B,Hg} + F_{B,W} \]
\[ W_B = F_B \]

\[
\rho_b \forall_b = \rho_{Hg} \forall_{Hg} + \rho_w \forall_w \\
\rho_b \forall_b = \rho_{Hg} \forall_{Hg} + \rho_w \forall_w \\
S \times \forall_b = S_{Hg} \forall_{Hg} + S_w \forall_w \\
7.6 \times 10^3 = 13.6 \times 10^2 (10-x) + 10^2 x \\
-6000 = -1260x \\
x = 4.76 \text{ cm} \\

04. Ans: 11
Sol: 
\[ W = F_B - T \]
\[ F_B = W + T \]
\[ W = \rho g V_{fd} - T \]
\[ = 10^3 \times 9.81 \times \frac{4}{3} \pi (0.8)^3 - (10 \times 10^3) \]
\[ = 21 - 10 \]
\[ W = 11 \text{ kN} \]

05. Ans: 1.375
Sol: 
\[ W_{\text{water}} = 5 \text{ N} \]
\[ W_{\text{oil}} = 7 \text{ N} \]
\[ S = 0.85 \]
\[ W = \text{Weight in air} \]
\[ F_{B1} = W - 5 \]
\[ F_{B2} = W - 7 \]
\[ W - 5 = \rho_1 g V_{fd} \ldots \ldots (1) \]
\[ W - 7 = \rho_2 g V_{fd} \ldots \ldots (2) \]
\[ V_{fd} = V_b \]
\[ \frac{W - 5}{\rho_1 g V_b} = \frac{W - 7}{\rho_2 g V_b} \]
\[ V_b = \frac{2}{(1000 - 850)9.81} \]
\[ V_b = 1.3591 \times 10^3 \text{ m}^3 \]
\[ W = 5 + (9810 \times 1.3591 \times 10^3) \]
\[ W = 18.33 \text{ N} \]
\[ W = \rho_b g V_b \]
\[ 18.33 = \frac{9.81 \times 1.3591 \times 10^3}{\rho_b} \]
\[ \rho_b = 1375.05 \text{ kg/m}^3 \]
\[ S_b = 1.375 \]

06. Ans: (d)
Sol: For a floating body to be stable, metacentre should be above its center of gravity. Mathematically \( GM > 0 \).
07. Ans: (b)  
Sol: \[ W = F_B \]
\[ \rho_b g V_b = \rho_f g V_{fd} \]
\[ \rho_b V_b = \rho_f V_{fd} \]
\[ 0.6 \times \frac{\pi}{4} d^2 \times 2d = 1 \times \frac{\pi}{4} d^2 \times x \]
\[ x = 1.2d \]
GM = BM – BG
BM = \[ \frac{I}{V} = \frac{\pi d^4}{64 \times \frac{\pi}{4} d^2 \times 1.2d} = \frac{d}{19.2} = 0.052d \]
BG = d – 0.6d = 0.4d
Thus, GM = 0.052d – 0.4d = –0.348d
GM < 0
\[ \Rightarrow \text{Hence, the cylinder is in unstable condition.} \]

08. Ans: 122.475  
Sol:
\[ \tau = \frac{\mu d U}{dy} \]
\[ F_s = \text{total shear force (considering both sides of the plate)} \]
\[ = 2A \times \frac{2A\mu V}{y} \]
\[ = \frac{2 \times 1.5 \times 1.5 \times 2.5 \times 0.1}{1 \times 10^{-3}} \]
\[ = 102.2727 \text{ N} \]
Weight of plate, W = 50 N
Upward force on submerged plate,
\[ F_v = \rho_g V = 900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3} \]
\[ = 29.7978 \text{ N} \]
Total force required to lift the plate
\[ = F_s + W – F_v \]
\[ = 102.2727 + 50 – 29.7978 \]
\[ = 122.4749 \text{ N} \]

09. Ans: (d)  
Sol:
- Statement (I) is wrong because the balloon filled with air cannot go up and up, if it is released from the ground.
- However, with increase in elevation, the atmospheric pressure and temperature both decrease resulting into a decrease in air density. Thus, statement (II) is correct.
Conventional Practice Solutions

01. Ans: (i) 0.33, (ii) 0.5 m
Sol: Given data:
Inner diameter of hollow cylinder,
\(d_i = 300 \text{ mm}\)
Outer diameter of hollow cylinder,
\(d_o = 600 \text{ mm}\)
S.G. of wooden hollow cylinder = 0.56
S.G. of oil = 0.85

\[
\begin{align*}
\text{Weight of hollow cylinder} &= \text{Buoyant force} \\
\text{acting on the hollow cylinder} \\
\text{Or,} \quad \gamma_{\text{cyl}} \times \frac{\pi}{4} (d_o^2 - d_i^2) \times L &= \gamma_{\text{oil}} \times \frac{\pi}{4} (d_o^2 - d_i^2) \times h \\
\text{Or,} \quad h &= \frac{\gamma_{\text{cyl}} \times L}{\gamma_{\text{oil}}} = \frac{0.56}{0.85} \times 0.66L = 0.33L \\
\text{Let us then calculate the maximum height of the cylinder, } L \text{ for the stable equilibrium condition.}
\end{align*}
\]

The centre of buoyancy B will be at a distance \(\frac{h}{2}\) from O as shown in the figure.

\[
\begin{align*}
\text{Or,} \quad OB &= \frac{h}{2} = 0.33L \\
\text{and} \quad OG &= \frac{L}{2} = 0.5L \\
\text{Now,} \quad BM &= \frac{1}{\forall} \\
&= \frac{\pi}{64} \left(\frac{d_o^4 - d_i^4}{4 \times \pi (d_o^2 - d_i^2) \times h}\right) \\
&= \frac{(d_o^2 + d_i^2)}{16h} = \frac{(0.6^2 + 0.3^2)}{16 \times 0.66L} = 0.0426 \frac{L}{L} \\
\text{Thus,} \quad GM &= BM - (OG - OB) \\
&= \frac{0.0426}{L} - (0.5L - 0.33L) \\
&= \frac{0.0426}{L} - 0.17L \\
\text{For stable equilibrium condition, } GM \geq 0. \\
\text{Putting } GM = 0 \text{ for the maximum height of the cylinder, we get} \\
\frac{0.0426}{0.17} &= L^2 \\
\Rightarrow \quad L &= 0.5 \text{ m} \\
\text{Thus,} \quad h &= 0.66 \times 0.5 = 0.33 \text{ m}
\end{align*}
\]

02. Ans: Unstable
Sol: Given data:
\(d = 1.0 \text{ m}\), \(L = 1.5 \text{ m}\),
\(\rho_{\text{sea water}} = 1026 \text{ kg/m}^3\)
\(m_{\text{buoy}} = 80 \text{ kg}\), \(m = 10 \text{ kg}\)
(80 + 10)\times g = \frac{\pi}{4} \times 1^2 \times h \times 1026 \times g

where h is the depth of immersion of the buoy.

Thus, \[ h = \frac{4 \times 90}{\pi \times 1026} = 0.1117 \text{ m} \]

OB = \frac{h}{2} = 0.05585 \text{ m}

The position of G due to a mass of 10 kg added to the cylindrical buoy is evaluated as:

\[ 80 \times 0.75 + 10 \times 1.5 = 90 \times OG \]

Or, OG = \frac{75}{90} = 0.833 \text{ m}

BM = \frac{1}{V} = \frac{\pi}{64} \times 1^4 \times \frac{4}{\pi \times 1^2 \times h}

= \frac{1}{16 \times 0.1117} = 0.5595 \text{ m}

Thus, GM = BM – (OG – OB)

= 0.5595 – (0.833 – 0.05585)

= – 0.21765 \text{ m}

Or, \ GM < 0

Thus, the buoy floats in unstable condition.

04. Fluid Kinematics

01. Ans: (b)
Sol:
- Constant flow rate signifies that the flow is steady.
- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.

Common Data for Questions 02 & 03

02. Ans: 0.94
Sol: \[ a_{\text{Local}} = \frac{\partial V}{\partial t} \]

\[ = \frac{\partial}{\partial t}\left(2t\left(1 - \frac{x}{2L}\right)^2\right) \]

\[ = \left(1 - \frac{x}{2L}\right)^2 \times 2 \]

\[ (a_{\text{Local}})_{at \ x = 0.5; \ L = 0.8} = 2\left(1 - \frac{0.5}{2 \times 0.8}\right)^2 \]

\[ = 2(1 - 0.3125)^2 = 0.945 \text{ m/sec}^2 \]

03. Ans: –13.68
Sol: \[ a_{\text{convective}} = v \cdot \frac{\partial v}{\partial x} = \left[2t\left(1 - \frac{x}{2L}\right)^2\right] \frac{\partial}{\partial x}\left[2t\left(1 - \frac{x}{2L}\right)^2\right] \]

\[ = \left[2t\left(1 - \frac{x}{2L}\right)^2\right] \cdot 2t\left(1 - \frac{x}{2L}\right)\left(-\frac{1}{2L}\right) \]

At t = 3 sec; x = 0.5 m; L = 0.8 m
\[ a_{\text{convective}} = 2 \times 3 \left[ 1 - \frac{0.5}{2 \times 0.8} \right] \times 2 \times 3 \left[ 2 \times 0.8 \right] = -14.62 \text{ m/sec}^2 \]

\[ a_{\text{total}} = a_{\text{local}} + a_{\text{convective}} = 0.94 - 14.62 = -13.68 \text{ m/sec}^2 \]

04. Ans: (d)
Sol: \( u = 6xy - 2x^2 \)

Continuity equation for 2D flow
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} = 6y - 4x
\]

\[
(6y - 4x) + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial v}{\partial y} = (4x - 6y) = 0
\]

\[
\frac{\partial v}{\partial y} = (4x - 6y) dy
\]

\[
v = \int 4x dy - \int 6y dy
\]

\[
= 4xy - 3y^2 + c
\]

\[
v = 4xy - 3y^2 + f(x)
\]

05. Ans: \( \sqrt{2} = 1.414 \)
Sol: \( \frac{\partial V}{\partial x} = \frac{1}{3} \text{ (m/sec/m)} \)

\[ V = 3 \text{ m/sec} \]

06. Ans: 13.75
Sol: \( a_{t \,(\text{conv})} = V_{avg} \times \frac{dV}{dx} \)

\[
a_{t \,(\text{conv})} = \left( \frac{2.5 + 3}{2} \right) \left( \frac{3 - 2.5}{0.1} \right) = 2.75 \times 5
\]

\[
a_{t \,(\text{conv})} = 13.75 \text{ m/sec}^2
\]

07. Ans: 0.3
Sol: \( Q = Au \)

\[
a_{\text{Local}} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left( \frac{Q}{A} \right)
\]

\[
a_{\text{Local}} = \frac{1}{A} \frac{\partial Q}{\partial t}
\]

\[
(a_{\text{Local}})_{x=0} = \frac{1}{0.4 - 0.1x} \frac{\partial Q}{\partial t}
\]

\[
(\frac{1}{0.4 - 0.1x}) \frac{\partial Q}{\partial t} = 0.12
\]

\[
= 0.3 \text{ m/sec}^2
\]

08. Ans: (b)
Sol: \( \psi = x^2 - y^2 \)

\[
a_{\text{Total}} = (a_x) \hat{i} + (a_y) \hat{j}
\]

\[
u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y} (x^2 - y^2) = 2y
\]

\[
v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} (x^2 - y^2) = 2x
\]
\[ a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \]
\[ = (2y)(0) + (2x)(2) \]
\[ \therefore a_x = 4x \]
\[ a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \]
\[ = (2y)\times(2) + (2x)\times(0) \]
\[ a_y = 4y \]
\[ \therefore a = (4x) \hat{i} + (4y) \hat{j} \]

09. Ans: (b)
Sol: Given, The stream function for a potential flow field is \( \psi = x^2 - y^2 \)
\( \phi = ? \)
\[ u = \frac{-\partial \phi}{\partial x} = \frac{-\partial \psi}{\partial y} \]
\[ u = \frac{-\partial \psi}{\partial y} = \frac{-\partial(x^2 - y^2)}{\partial y} \]
\[ u = 2y \]
\[ u = 2y \]
\[ \int \partial \phi = \int 2y \partial x \]
\[ \phi = -2xy + c_1 \]
Given, \( \phi \) is zero at \((0,0)\)
\[ \therefore c_1 = 0 \]
\[ \therefore \phi = -2xy \]

10. Ans: 4
Sol: Given, 2D – flow field
Velocity, \( V = 3xi + 4xyj \)
\[ u = 3x, \quad v = 4xy \]
\[ \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \]
\[ \omega_z = \frac{1}{2} \left( 4y - 0 \right) \]
\[ (\omega_z)_{at(2,2)} = \frac{1}{2} \times 4(2) = 4 \text{ rad/sec} \]

11. Ans: (b)
Sol: Given, \( u = 3x, \)
\[ v = Cy, \]
\[ w = 2 \]
The shear stress, \( \tau_{xy} \) is given by
\[ \tau_{xy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \mu \left[ \frac{\partial}{\partial y} (3x) + \frac{\partial}{\partial x} (Cy) \right] \]
\[ = \mu (0 + 0) = 0 \]

12. Ans: (d)
Sol:
- The total acceleration is given as
\[
\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{V}) \vec{V}
\]
where the first term on the R.H.S is the local acceleration and the second term is the convective acceleration.
- If the flow is steady, then local acceleration will be zero, not the convective acceleration.
- The convective acceleration arises due to the fact that a fluid element experiences different velocities at different locations. Thus, statement (I) is wrong whereas statement (II) is correct.
Conventional Practice Solutions

01. Ans: (ii) \( y = \pm x \) (iii) \((0, 0)\)

Sol: Given: \( u = c(x^2 - y^2) \) and \( v = -2cxy \)

The equation of a streamline is given by

\[
\frac{dx}{u} = \frac{dy}{v}
\]

Or,

\[
\frac{dy}{dx} = \frac{v}{u} = -\frac{2cxy}{c(x^2 - y^2)} = \frac{-2xy}{x^2 - y^2}
\]

(ii) For flow to be parallel to y-axis, \( u = 0 \)

Or,

\[
\frac{dy}{dx} = \frac{v}{x^2 - y^2} = \infty
\]

This is possible when \( x = \pm y \)

(iii) The fluid is stationary when \( u \) & \( v \) both are zero.

From the velocity components given, it is possible when \((x, y) = (0, 0)\)

(i) From the equation of streamline

\[
\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}
\]

Or,

\[
\frac{dx}{dy} = \frac{x^2 - y^2}{2xy} \quad \text{------(1)}
\]

Let \( x = fy \) or \( dx = fdy + ydf \)

Or,

\[
\frac{dx}{dy} = f + y \frac{df}{dy} \quad \text{------(2)}
\]

Equating (1) with (2),

\[
f + y \frac{df}{dy} = \frac{-f^2y^2 - y^2}{2fy} = \frac{-f^2 - 1}{2f} = \frac{1 - f^2}{2f}
\]

Or,

\[
\frac{df}{dy} = \frac{1 - f^2}{2f} - f = \frac{1 - 3f^2}{2f}
\]

05. Energy Equation and its Applications

01. Ans: (c)

Sol: Applying Bernoulli’s equation for ideal fluid

\[
\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}
\]

\[
\frac{P_1}{\rho g} + (2)^2 = \frac{P_2}{\rho g} + (1)^2
\]

\[
P_2 - P_1 = \frac{4}{\rho g} - \frac{1}{\rho g} = \frac{1.5}{g}
\]

02. Ans: (c)

Sol:

\[
\begin{align*}
\text{Or,} & \quad \frac{2f}{1-3f^2} \frac{df}{dy} = \frac{dy}{y} \\
& \quad \frac{6f}{3f^2 - 1} \frac{df}{dy} = -\frac{3dy}{y}
\end{align*}
\]

Integrating

\[
\ln(3f^2 - 1) + 3 \ln y = \ln C
\]

Or,

\[
(3f^2 - 1)y^3 = C
\]

Or,

\[
3x^2y - y^3 = C
\]

Or,

\[
x^2y - y^3/3 = \text{constant, proved}
\]
\[
\begin{align*}
V_1^2 &= 1.27 \text{m } \frac{2g}{\rho g} , \\
V_2^2 &= 0.203 \text{m } \frac{2g}{\rho g} \\
Z_1 &= 2 \text{ m } , \\
Z_2 &= 0 \text{ m}
\end{align*}
\]

Total head at (1) – (1)
\[
= \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + Z_1 \\
= 1.27 + 2.5 + 2 = 5.77 \text{ m}
\]

Total head at (2) – (2)
\[
= \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + Z_2 \\
= 0.203 + 5.407 + 0 = 5.61 \text{ m}
\]

Loss of head = 5.77 – 5.61 = 0.16 m

\[\therefore\] Energy at (1) – (1) > Energy at (2) – (2)

\[\therefore\] Flow takes from higher energy to lower energy

i.e. from (S_1) to (S_2)

Flow takes place from top to bottom.

03. Ans: 1.5

Sol:
\[
A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4} (0.1)^2 = 7.85 \times 10^{-3} \text{mm}^2
\]
\[
A_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4} (0.05)^2 = 1.96 \times 10^{-3} \text{mm}^2
\]

\[
\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L
\]

\[Z_1 = Z_2,\text{ it is in horizontal position}\]

Since, at outlet, pressure is atmospheric
\[P_2 = 0\]

\[Q = 100 \text{ lit/sec} = 0.1 \text{ m}^3/\text{sec}\]
05. **Ans:** 35

**Sol:**

\[
V = \sqrt{2g\left(h_{stag} - h_{stat}\right)}
\]

\[
= \sqrt{2(9.81)(0.30 - 0.24)} = 1.085 \text{ m/s}
\]

\[
= 1.085 \times 60 = 65.1 \text{ m/min}
\]

07. **Ans:** 81.5

**Sol:**

\[
x = 30 \text{ mm}, \quad g = 10 \text{ m/s}^2
\]

\[
\rho_{air} = 1.23 \text{ kg/m}^3, \quad \rho_{Hg} = 13600 \text{ kg/m}^3
\]

\[
C = 1
\]

\[
V = \sqrt{2gh_D}
\]

\[
h_p = x \left(\frac{S_m}{S} - 1\right)
\]

\[
h_D = 30 \times 10^{-3} \left(\frac{13600}{1.23} - 1\right)
\]

\[
h_D = 331.67 \text{ m}
\]

\[
V = 1 \times \sqrt{2 \times 10 \times 331.67} = 81.5 \text{ m/sec}
\]

08. **Ans:** 140

**Sol:**

\[
Q_a = C_d \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}
\]

\[
C_d \propto \frac{1}{\sqrt{h}}
\]

\[
C_{d_{orifice}} = 0.95 = \frac{h_{orifice}}{h_{venturi}}
\]

\[
h_{venturi} = 140 \text{ mm}
\]

09. **Ans:** (d)

**Sol:**

- For an orifice meter, the fluid re-establishes its flow pattern downstream of the orifice plate. However, the fluid pressure
downstream of the orifice plate is not the same as that at upstream of the orifice plate. Thus, statement (I) is not correct.

- Bernoulli’s equation when applied to any two points (for irrotational, steady and incompressible flow) can be written as

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

If \( V_1 = V_2 \) & \( Z_1 = Z_2 \), we get \( P_1 = P_2 \).

Thus, statement (II) is correct.

### Conventional Practice Solutions

01. Ans: 5.4 cm, 540 Pa

Sol: Air enters into the wind tunnel at \( P_{atm} \) and \( V \approx 0 \). It attains a velocity \( V \) in the test section and the pressure there is \( P \).

Applying Bernoulli’s equation for points (1) and (2) as shown in the figure.

\[
\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} + Z_2
\]

But \( P_1 = P_{atm} \), \( V_1 \approx 0 \) and \( Z_1 = Z_2 \)

Thus, \( \frac{P_{atm} - P_2}{\gamma_{air}} = \frac{V_2^2}{2g} \)

\[
\left( \frac{108 \times \frac{5}{18}}{2 \times 10} \right)^2 = 45 \text{ ------(1)}
\]

From manometer,

\[
P_2 + \gamma_{water} \times h = P_{atm}
\]

or, \( P_{atm} - P_2 = \gamma_{water} \times h \text{ ------(2)} \)

Hence, equation (1) becomes,

\[
\frac{\gamma_{water} \times h}{\gamma_{air}} = 45 \text{ (from (2))}
\]

\[
h = \frac{45 \times \gamma_{air}}{\gamma_{water} \times \gamma_{air}} = \frac{45 \times 1.2 \times 10^3 \times 10^3}{10^3} = 0.054 \text{ m}
\]

\[
\Rightarrow h = 5.4 \text{ cm}
\]

Applying Bernoulli's equation for points (2) and (3)

\[
\frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} = \frac{P_3}{\gamma_{air}} + \frac{V_3^2}{2g}
\]

But point (3) is stagnation point where \( P_3 = P_{stag} \) & \( V_3 = 0 \)

Thus, \( \frac{P_{stag} - P_2}{\gamma_{air}} = \frac{V_2^2}{2g} = 45 \)

Or, \( P_{stag} - P_2 = 45 \times 1.2 \times 10 = 540 \text{ Pa} \)

06. Momentum equation and its Applications

01. Ans: 1600

Sol: \( S = 0.80 \)

\[
A = 0.02 \text{ m}^2
\]

\[
V = 10 \text{ m/sec}
\]

\[
F = \rho \cdot A \cdot V^2
\]

\[
F = 0.80 \times 1000 \times 0.02 \times 10^2
\]

\[
F = 1600 \text{ N}
\]
02. Ans: 6000
Sol: 
A = 0.015 m²
V = 15 m/sec (Jet velocity)
U = 5 m/sec (Plate velocity)
F = ρA (V + U)²
F = 1000 × 0.015 (15 + 5)²
F = 6000 N

03. Ans: 19.6
Sol: 
V = 100 m/sec (Jet velocity)
U = 50 m/sec (Plate velocity)
d = 0.1 m
F = ρA (V – U)²
F = 1000 × π/4 × 0.1² × (100 – 50)²
F = 19.6 kN

04. Ans: (a)
Sol: 
\[ F_x = \rho a V (V_{1x} - V_{2x}) \]
\[ = \rho a V (V - (-V)) \]
\[ = 2 \rho a V^2 \]
\[ = 2 \times 1000 \times 10^{-4} \times 5^2 = 5 \text{ N} \]

05. Ans: (d)
Sol: Given, V = 20 m/s,
u = 5 m/s
\[ F_1 = \rho A (V - u)^2 \]
\[ \text{Power } (P_1) = F_1 \times u = \rho A (V - u)^2 \times u \]
\[ F_2 = \rho A V \times V_r \]
\[ = \rho A V (V - u) \]
\[ \text{Power } (P_2) = F_2 \times u = \rho A V (V - u) \times u \]
\[ \frac{P_1}{P_2} = \frac{\rho A (V - u)^2 \times u}{\rho A V (V - u) \times u} \]
\[ = \frac{V - u}{V} = 1 - \frac{u}{V} \]
\[ = 1 - \frac{5}{20} = 0.75 \]

06. Ans: 2035
Sol: Given, θ = 30°, \( \dot{m} = 14 \text{ kg/s} \)
\( (P_i)_g = 200 \text{ kPa} \), \( (P_e)_g = 0 \)
\( A_i = 113 \times 10^{-4} \text{ m}^2 \), \( A_e = 7 \times 10^{-4} \text{ m}^2 \)
\( \rho = 10^3 \text{ kg/m}^3 \), \( g = 10 \text{ m/s}^2 \)
From the continuity equation:
\[ \rho A_i V_i = 14 \]
or
\[ V_i = \frac{14}{10^3 \times 113 \times 10^{-4}} = 1.24 \text{ m/s} \]
Similarly, \( V_e = \frac{14}{10^3 \times 7 \times 10^{-4}} = 20 \text{ m/s} \)
Let \( F_x \) be the force exerted by elbow on water in the +ve x-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write:
\[ (P_i)_g A_i + F_x = \dot{m} (V_e \cos 30° - V_i) \]
\[ F_x = \dot{m} (V_e \cos 30° - V_i) - (P_i)_g A_i \]
\[ = 14 (20 \times \cos 30° - 1.24) - 200 \times 10^3 \times 113 \times 10^{-4} \]
\[ = 225.13 - 2260 \]
\[ F(x) = -2034.87\, N \approx -2035\, N \]

The x-component of water force on elbow is 
\(-F_x\) (as per Newton’s third law),
i.e., \(\approx 2035\, N\)

07. Ans: (a)
Sol: In a convergent nozzle, as the area decreases in the direction of flow, the flow velocity will increase (AV = Constant) in the direction of flow. This will result in increase in its momentum. Thus, statement (I) is correct and statement (II) is the correct explanation of statement (I).

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Conventional Practice Solutions

01. Ans: Right: a, b, c; Left: d
Sol: Let \(F_x\) be the force exerted by the fluid on the device which will be different for different devices. Since inlet and outlet sections of the devices are at atmospheric pressure, there will be no contribution of pressure forces at these sections. Let \(V_i\) and \(V_e\) be the velocities at inlet and outlet of the devices in x-direction.

Applying linear momentum equation to each of the devices, we write

(a) \[ F_x = \dot{m}_a [V_i - (-V_e)] = \dot{m}_a [V_i + V_e] \]
\(F_x\) is acting in +ve x direction.
Therefore, the device (b) will move to the right.

(b) \[ F_x = \dot{m}_b (V_i - V_e) \]
Since \(V_i > V_e\), \(F_x\) is acting in +ve x direction.
Therefore, the device (a) will move to the right.

(c) \[ F_x = \dot{m}_c (V_i - 0) = \dot{m}V_i \]
\(F_x\) is acting in +ve x direction.
Therefore, the device (c) will move to the right.

(d) \[ F_x = \dot{m}_d (V_i - V_e) \]
Since \(V_e > V_i\)
\(F_x\) is acting in –ve x direction. Therefore, the device (d) will move to the left.
02. **Ans:** (d)

**Sol:** The equation \( \tau = \left( -\frac{\partial P}{\partial x} \right) \left( \frac{r}{2} \right) \) is valid for laminar as well as turbulent flow through a circular tube.

03. **Ans:** (d)

**Sol:**

\[
Q = A \cdot \frac{V_{\text{max}}}{2} \quad (\because V_{\text{max}} = 2 \cdot V_{\text{avg}})
\]

\[
Q = \frac{\pi}{4} \left( \frac{40}{1000} \right)^2 \times 1.5 \times \frac{2}{2}
\]

\[
= \frac{\pi}{4} \times (0.04)^2 \times 0.75
\]

\[
= \frac{\pi}{4} \times \frac{4}{100} \times \frac{4}{100} \times \frac{3}{4} = \frac{3\pi}{10000} \text{ m}^3/\text{sec}
\]

04. **Ans:** 1.92

**Sol:**

\[
\rho = 1000 \text{ kg/m}^3
\]

\[
Q = 800 \text{ mm}^3/\text{sec} = 800 \times (10^{-3})^3 \text{ m}^3/\text{sec}
\]

\[
L = 2 \text{ m}
\]

\[
D = 0.5 \text{ mm}
\]

\[
\Delta P = 2 \text{ MPa} = 2 \times 10^6 \text{Pa}
\]

\[
\mu = ?
\]

\[
\Delta P = \frac{128 \cdot \mu \cdot Q \cdot L}{\pi D^5}
\]

\[
2 \times 10^6 = \frac{128 \times \mu \times 800 \times (10^{-3})^3 \times 2}{\pi (0.5 \times 10^{-3})^4}
\]

\[
\mu = 1.917 \text{ milli Pa} – \text{sec}
\]
05. Ans: 0.75

Sol: \[ U_r = U_{\max} \left(1 - \left(\frac{r}{R}\right)^2\right) \]

\[ \therefore \frac{U}{U_{\max}} = 1 - \left(\frac{r}{R}\right)^2 \]

\[ = 1 - \left(\frac{50}{200}\right)^2 = 0.75 \text{ m/s} \]

06. Ans: 0.08

Sol: Given,
\[ \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3 \]
\[ \mu = 1 \text{ Poise} = 10^{-1} \text{ N-s/m}^2 \]
\[ d = 50 \text{ mm} = 0.05 \text{ m} \]
Velocity = 2 m/s

Reynold’s Number, \[ Re = \frac{\rho V D}{\mu} \]
\[ = \frac{800 \times 2 \times 0.05}{10^{-1}} = 800 \]
\[ (\because Re < 2000) \]

\[ \therefore \text{Flow is laminar,} \]

For laminar, Darcy friction factor
\[ f = \frac{64}{Re} = \frac{64}{800} = 0.08 \]

07. Ans: 16

Sol: For fully developed laminar flow,
\[ h_f = \frac{32\mu V L}{\rho g D^2} \quad (\because Q = AV) \]

08. Ans: 5.2

Sol: Oil viscosity, \[ \mu = 10 \text{ poise} = 10 \times 0.1 = 1 \text{ N-s/m}^2 \]

\[ y = 50 \times 10^{-3} \text{ m} \]
\[ L = 120 \text{ cm} = 1.20 \text{ m} \]
\[ \Delta P = 3 \times 10^3 \text{ Pa} \]

Width of plate = 0.2 m

\[ Q = ? \]

\[ Q = A.V_{\text{avg}} = (\text{width of plate} \times y) \times V \]

\[ \Delta P = \frac{12\mu V L}{B^2} \]

\[ 3 \times 10^3 = \frac{12 \times 1 \times V \times 1.20}{\left(50 \times 10^{-3}\right)^2} \]
V = 0.52 m/sec
Q = AV_{avg} = (0.2 \times 50 \times 10^{-3}) (0.52)
    = 5.2 \text{ lit/sec}

09. Ans: (a)
Sol: Wall shear stress for flow in a pipe is given by,
\[ \tau_o = -\frac{\partial P}{\partial x} \times \frac{R}{2} = \frac{\Delta P}{L} \times \frac{D}{4} \]
\[ = \frac{\Delta P D}{4L} \]

10. Ans: 72
Sol: Given, \( \rho = 800 \text{ kg/m}^3 \), \( \mu = 0.1 \text{ Pa.s} \)
Flow is through an inclined pipe.
\( d = 1 \times 10^{-2} \text{ m}, \)
\( V_{av} = 0.1 \text{ m/s}, \)
\( \theta = 30^\circ \)
\( Re = \frac{\rho V_{av} d}{\mu} = \frac{800 \times 0.1 \times 1 \times 10^{-2}}{0.1} = 8 \)
\Rightarrow \text{flow is laminar.}
Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,
\[ \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_r \]
But \( V_1 = V_2 \),
\( (Z_2 - Z_1) = 10 \sin30^\circ = 5 \text{ m} \)
and \( h_r = \frac{32 \mu V_{av} L}{\rho gd^2} \)
\[ \frac{(P_1 - P_2)}{\gamma} = (Z_2 - Z_1) + \frac{32 \mu V_{av} L}{\rho gd^2} \]
\[ (P_1 - P_2) = \rho g (Z_2 - Z_1) + \frac{32 \mu V_{av} L}{d^2} \]
\[ = 800 \times 10 \times 5 + \frac{32 \times 0.1 \times 0.1 \times 10}{(1 \times 10^{-2})^2} \]
\[ = 40 \times 10^3 + 32 \times 10^3 = 72 \text{ kPa} \]

11. Ans: (d)
Sol:
- In hydrodynamic entrance region of the pipe of uniform diameter, the average velocity remains constant in the direction of flow. Thus, statement - I is wrong.
- However, in the above region the centreline velocity increases in the direction of flow as boundary layers grow on the solid surfaces. Thus, statement (II) is correct.

01. Sol: The velocity profile for fully developed laminar flow between two stationary parallel plates is given by
\[ u = \frac{1}{2\mu} \left( \frac{-\partial P}{\partial x} \right) (By - y^2) \]
\[ \frac{\partial u}{\partial y} = \frac{1}{2\mu} \left( \frac{-\partial P}{\partial x} \right) (B - 2y) \]
At the upper surface
\[ \frac{\partial u}{\partial y} \mid _{y=B} = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) (B-2B) \]
\[ = -\frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) B \]
\[ \tau_y = B = \mu \frac{\partial u}{\partial y} \mid _{y=B} = -\frac{1}{2} \left( -\frac{\partial P}{\partial x} \right) B \]
\[ = -\frac{1}{2} \times 1000 \times 5 \times 10^{-3} = -2.5 \text{ Pa} \]

Thus, the magnitude of the shear stress on the upper plate is 2.5 Pa and its direction is opposite to the direction of flow.

(ii) Discharge per unit length
\[ = \int_0^B u(y)(dy \times 1) \]
\[ = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) \int_0^B (By - y^2)dy \]
\[ = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) \left[ B \frac{y^2}{2} - \frac{y^3}{3} \right]_0^B \]
\[ = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) \left( B^3 \frac{3}{2} - B^3 \frac{3}{3} \right) \]
\[ = \frac{B^3}{12\mu} \left( -\frac{\partial P}{\partial x} \right) \]
\[ q = \frac{(5 \times 10^{-3})^3}{12} \times 1000 \]
\[ = 20.83 \times 10^{-6} \text{ m}^3/\text{s} \]

08. Flow Through Pipes

01. Ans: (d)
Sol:
- The Darcy-Weisbash equation for head loss in written as:
\[ h_f = \frac{f L V^2}{2g d} \]
where V is the average velocity, f is friction factor, L is the length of pipe and d is the diameter of the pipe.
- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as...
the nature of pipe surface (smooth or rough)

- For laminar flow, friction factor is a function of Reynolds number.

02. Ans: 481
Sol: Given data,
\[ \bar{m} = \pi \text{ kg/s}, \quad d = 5 \times 10^{-2} \text{ m}, \]
\[ \mu = 0.001 \text{ Pa.s}, \quad \rho = 1000 \text{ kg/m}^3 \]
\[ V_{av} = \frac{\bar{m}}{\rho A} = \frac{4 \bar{m}}{\rho \pi d^2} = \frac{4 \times \pi}{\rho \pi d^2} = \frac{4}{\rho d^2} \]
\[ \text{Re} = \frac{\rho V_{av} d}{\mu} = \frac{4}{\frac{\rho d^2}{\mu}} = \frac{4}{0.001 \times 5 \times 10^{-2}} = 8 \times 10^4 \]
\[ \Rightarrow \text{Flow is turbulent} \]
\[ f = \frac{0.316}{\text{Re}^{0.25}} = \frac{0.316}{(8 \times 10^4)^{0.25}} = 0.0188 \]
\[ \Delta P = \rho g \frac{f L V_{av}^2}{2gd} = \frac{f \rho L}{\frac{4}{\rho d^2}} \times \frac{1}{2d} \]
\[ \Delta P = f \times \frac{16}{\rho d^3} \times \frac{1}{2} = \frac{8f}{\rho d^3} = \frac{8 \times 0.0188}{10^3 \times (5 \times 10^{-2})^3} \]
\[ = 481.28 \text{ Pa/m} \]

03. Ans: (a)
Sol: In pipes Net work, series arrangement
\[ h_f = \frac{f L V^2}{2gd} = \frac{f L Q^2}{12.1 \times d^2} \]
\[ h_{f_{A}} = \frac{f_{A} L_{A} Q_{A}^2}{12.1 \times d_{B}^3} \]
\[ h_{f_{B}} = \frac{f_{B} L_{B} Q_{B}^2}{12.1 \times d_{A}^3} \]
\[ \text{Given } l_A = l_B, f_A = f_B, Q_A = Q_B \]

\[ \frac{h_{f_{A}}}{h_{f_{B}}} = \left(\frac{d_{B}}{d_{A}}\right)^3 = \left(\frac{d_{B}}{1.2d_{B}}\right)^3 \]
\[ = \left(\frac{1}{1.2}\right)^3 = 0.4018 \approx 0.402 \]

04. Ans: (a)
Sol: Given, \( d_1 = 10 \text{ cm}; d_2 = 20 \text{ cm} \)
\[ f_1 = f_2; \quad l_1 = l_2 = l \]
\[ l_e = l_1 + l_2 = 2l \]
\[ \frac{d_e^5}{d_1^5} + \frac{d_e^5}{d_2^5} \Rightarrow \]
\[ = \frac{2l}{d_e^5} = l^{5} + l^{20} \]
\[ \therefore \quad d_e = 11.4 \text{ cm} \]

05. Ans: (c)
Sol: Given \( d_2 = 2d_1 \)
Losses due to sudden expansion,
\[ h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} \left(1 - \frac{V_2}{V_1}\right)^2 \]
By continuity equation,
\[ Q = A_1 V_1 = A_2 V_2 \]
\[ \therefore \quad \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2 \]
\[ h_L = \frac{V_1^2}{2g} \left(1 - \frac{1}{4}\right)^2 \]
\[ h_L = \frac{9 \times V_1^2}{16 \times 2g} \]

\[ h_b = \frac{9}{V_1^2 \times 2g} \]

06. **Ans: (b)**

**Sol:** Pipes are in parallel

\[ Q_c = Q_A + Q_B \quad \text{------ (i)} \]

\[ h_{Le} = h_{L_A} = h_{L_B} \]

\[ L_e = 175 \text{ m} \]

\[ f_e = 0.015 \]

\[ f_e L_e Q_e^2 = f_A L_A Q_A^2 = f_B L_B Q_B^2 \]

\[ \frac{0.020 \times 150 \times Q_A^2}{12.1 \times (0.1)^3} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^3} \]

\[ Q_A = 1.747 \times Q_B \quad \text{------(ii)} \]

From (i) \[ Q_c = 1.747 \times Q_B + Q_B \]

\[ Q_c = 2.747 \times Q_B \quad \text{------(iii)} \]

\[ D_e = 116.6 \text{ mm} = 117 \text{ mm} \]

07. **Ans: 0.141**

**Sol:**

Given data,

\[ L = 930 \text{ m}, \quad k_{valve} = 5.5 \]

\[ k_{entry} = 0.5, \quad d = 0.3 \text{ m} \]

\[ f = 0.03, \quad g = 10 \text{ m/s}^2 \]

Applying energy equation for points (1) and (2), we write:

\[ \frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2 + h_{L_{entry}} \]

\[ + h_{L_{valve}} + h_{L_{exit}} + h_{e,\text{pipe}} \]

But \( P_1 = P_2 = P_{\text{atm}} = 0 \)

\[ V_1 = 0 = V_2 \]

\[ Z_1 - Z_2 = 20 \text{ m}, \quad k_{exit} = 1 \]

\[ \begin{aligned} 
Z_1 - Z_2 &= 0.5 \frac{V_2^2}{2g} + 5.5 \frac{V_2^2}{2g} + 1 \times \frac{V_2^2}{2g} + \frac{f L V^2}{2gd} \\
&= 7 \frac{V_2^2}{2g} + f L \frac{V^2}{2g} = V_2^2 \left( 7 + \frac{f L}{d} \right) \\
\text{or} \quad 20 &= \frac{V_2^2}{2g} \left( 7 + \frac{0.03 \times 930}{0.3} \right) = 100 \frac{V_2^2}{2g} \\
\text{or} \quad V_2 &= \frac{20 \times 2g}{100} = \frac{20 \times 2 \times 10}{100} \\
\Rightarrow \quad V &= 2 \text{ m/s} 
\end{aligned} \]

Thus, discharge,

\[ Q = \frac{\pi}{4} \times 0.3^2 \times 2 = 0.1414 \text{ m}^3/\text{s} \]

08. **Ans: (c)**

**Sol:** Given data:

Fanning friction factor, \( f = m \cdot \text{Re}^{-0.2} \)

For turbulent flow through a smooth pipe.

\[ \Delta P = \frac{\rho f_{\text{Darcy}} L V^2}{2d} = \frac{\rho (4f) L V^2}{2d} \]
\[
\frac{1}{\sqrt{f}} \approx -1.8 \log \left[ \frac{6.9}{\text{Re}} + \left( \frac{\varepsilon}{D_h} \right)^{1.11} \right]
\]

\[
\frac{1}{\sqrt{f}} = -1.8 \log \left[ \frac{6.9}{72495.5} + \left( \frac{0.045 \times 10^{-3}}{0.1714} \right)^{1.11} \right]
\]

\[
= -1.8 \log[9.518 \times 10^{-5} + 2.48 \times 10^{-5}]
\]

\[
= -1.8 \log(11.998 \times 10^{-5})
\]

\[
= 7.058
\]

\[
f = 0.02
\]

The pressure drop in the duct is,

\[
\Delta P = \frac{\rho f L V^2}{2D_h}
\]

\[
= \frac{1.145 \times 0.02 \times 10 \times 7^2}{2 \times 0.1714} = 32.73 \text{ Pa}
\]

The required pumping power will be

\[
P_{\text{pumping}} = Q \Delta P = AV_{av} \times \Delta P
\]

\[
= (0.15 \times 0.2) \times 7 \times (32.73)
\]

\[
= 6.87 \text{ W} \approx 7 \text{ W}
\]

09. Ans: (b)
Sol: Given data:
- Rectangular duct, L = 10 m,
- X-section of duct = 15 cm \times 20 cm
- Material of duct - Commercial steel, \( \varepsilon = 0.045 \text{ mm} \)
- Fluid is air (\( \rho = 1.145 \text{ kg/m}^3 \), \( v = 1.655 \times 10^{-5} \text{ m}^2/\text{s} \))
- \( V_{av} = 7 \text{ m/s} \)
- \( \text{Re} = \frac{V_{av} \times D_h}{v} \)

where, \( D_h = \text{Hydraulic diameter} = 4 \times \text{Crosssectional area} \div \text{Perimeter} \)

\[
= \frac{4 \times 0.15 \times 0.2}{2(0.15 + 0.2)} = 0.1714 \text{ m}
\]

\[
\text{Re} = \frac{7 \times 0.1714}{1.655 \times 10^{-5}} = 72495.5
\]

\( \Rightarrow \) Flow is turbulent.

Using Haaland equation to find friction factor,

\[
\frac{1}{\sqrt{f}} = 7.058
\]

\[
f = 0.02
\]

The pressure drop in the duct is,

\[
\Delta P_2 = 3.482 \times 10 = 34.82 \text{ kPa}
\]

10. Ans: 26.5
Sol:

Case I: Without additional pipe,
Let Q be the discharge through the pipe.
Then
\[ \frac{P_p + V_p^2}{\gamma} + Z_p = \frac{P_s + V_s^2}{\gamma} + Z_s + f \frac{L Q^2}{12.1 d^4} \]

But \( V_p = V_s \) and \( Z_p = Z_s \)

\( P_p \) and \( P_s \) are the pressures at sections \( P \) and \( S \), respectively.

Thus,
\[ \frac{P_p - P_s}{\gamma} = \frac{f L Q^2}{12.1 d^4} \quad \text{(1)} \]

**Case II:** When a pipe \((\frac{L}{2})\) is connected in parallel.

In this case, let \( Q' \) be the total discharge.

\[ Q_{Q-R} = \frac{Q'}{2} \quad \text{and} \quad Q_{R-S} = Q' \]

Then,
\[ \frac{P_p' + V_p'^2}{\gamma} + Z_p' = \frac{P_s' + V_s'^2}{\gamma} + Z_s' + \frac{f (L/4)Q'^2}{12.1 d^4} \]
\[ + \frac{f (L/2)(Q'/2)^2}{12.1 d^4} + \frac{f (L/4)Q'^2}{12.1 d^4} \]

\( P_p' \) and \( P_s' \) are the pressures at sections \( P \) and \( S \) in the second case.

But \( V_p' = V_s' \); \( Z_p' = Z_s' \)

So,
\[ \frac{P_p' - P_s'}{\gamma} = \frac{f L Q'^2}{12.1 d^4} \left[ \frac{1}{4} + \frac{1}{8} + \frac{1}{4} \right] \]
\[ = \frac{5}{8} \times \frac{f L Q'^2}{12.1 d^4} \quad \text{(2)} \]

Given that end conditions remain same.

i.e.,
\[ \frac{P_p - P_s}{\gamma} = \frac{P_p' - P_s'}{\gamma} \]

Hence, equation (2) becomes,
\[ \frac{f L Q'^2}{12.1 d^4} = \frac{5}{8} \times \frac{f L Q'^2}{12.1 d^4} \quad \text{from eq.(1)} \]

or \( \frac{Q'^2}{Q^2} = \frac{8}{5} \)

or \( \frac{Q'}{Q} = 1.265 \)

Hence, percentage increase in discharge is
\[ = \frac{Q' - Q}{Q} \times 100 \]
\[ = (1.265 - 1) \times 100 \]
\[ = 26.5 \% \]

11. **Ans:** 20%

**Sol:** Since, discharge decrease is associated with increase in friction.
\[ \frac{df}{f} = -2 \times \frac{dQ}{Q} = 2 \left[ -\frac{dQ}{Q} \right] \]
\[ = 2 \times 10 = 20\% \]

12. **Ans:** (c)

**Sol:** As compared to sharp entrance, the rounded entrance will give less energy loss in flow through a pipe. For sharp entrance, the flow gets separated and there will be recirculation zone till the fluid stream gets attached to the surface. Thus, the rounded entrance increases the flow rate when everything else remains constant. Hence, statement (I) is correct. However, statement (II) is wrong as discussed above.
13. Ans: (d)  
Sol: The surge tanks are provided on upstream side of the valve in order to offset the effect of water hammer mainly due to the pressure rise which may damage the pipe. Thus, statement (I) is wrong. However, statement (II) is correct.

Conventional Practice Solutions

01. Sol:

Applying Energy equation for two points, just upstream and downstream of the fan in the pipe loop.

\[
\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} + Z_2 + \frac{fL}{2gD}V_{total}^2 + 4\times K_{elbow}V_{elbow}^2
\]

where \( V_1 = V_2 = V \) ; \( Z_1 = Z_2 \)

\( f = 0.01, \quad D = 3 \text{ m}, \)

\( V = 40 \text{ m/s}, \quad L = 60 \text{ m}, \)

\( K_{elbow} = 0.3 \) (Given)

\[
\frac{P_1 - P_2}{\gamma_{air}} = \frac{V^2}{2g} \left[ \frac{fL}{D} + 4 \times K_{elbow} \right]
\]

\[
= \frac{40^2}{2g} \times 1.4
\]

\( \Delta P = \rho_{air} \times \frac{40^2}{2} \times 1.4 \)

\[
= 1.2 \times \frac{40^2}{2} \times 1.4 = 1,344 \text{ Pa}
\]

Power added to air by fan, \( P = Q\Delta P \)

\[
= \frac{\pi}{4} \times 3^2 \times 40 \times 1,344 = 380 \text{ kW}
\]

02. Sol:

Applying energy equation between points (1) and (2)

\[
\frac{P_1}{\gamma_f} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_f} + \frac{V_2^2}{2g} + Z_2 + \left(h_f\right)_{pipe}
\]

But \( P_1 = P_2 = P_{atm}, Z_1 = H, Z_2 = 0, V_1 = 0 \)

Thus,

\[
H = \frac{V_2^2}{2g} + \left(h_f\right)_{pipe}
\]

For maximum power transmission, \( H = 3h_f \)

Or,

\[
3 \times \frac{fL V_p^2}{2gD} = \frac{V_2^2}{2g} + \frac{fL V_p^2}{2gD}
\]

Or,

\[
\frac{2fL V_p^2}{2gD} = \frac{V_2^2}{2g}
\]

Or,

\[
\left( \frac{V_2}{V_p} \right)^2 = 2fL \quad \text{(1)}
\]

From equation of continuity,
\[
\frac{\pi}{4} D^2V_p = \frac{\pi}{4} d^2V_2
\]

Or
\[
\frac{V_2}{V_p} = \frac{D^2}{d^2}
\]

Thus, substituting in equation (1), we get
\[
\left(\frac{D^2}{d^2}\right)^2 = \frac{2fL}{D}
\]

\[d = \left(\frac{D^5}{2fL}\right)^{\frac{1}{2}}\]...... Proved

---

### 09. Elementary Turbulent Flow

**01. Ans: (b)**

**Sol:** The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by
\[
u = \frac{yV^*}{v}
\]
where \(V^*\) is the shear velocity.

**02. Ans: (d)**

**Sol:**
\[
\Delta P = \rho g h_f
\]
\[
= \frac{\rho f L V^2}{2D} = \frac{\rho g f L Q^2}{12.1D^5}
\]
For \(Q = \text{constant}\)
\[
\Delta P \propto \frac{1}{D^5}
\]
or
\[
\frac{\Delta P_2}{\Delta P_1} = \left(\frac{D_1}{2D_1}\right)^5 = \frac{1}{32}
\]

---

**03. Ans: 2.4**

**Sol:**

Given: \(V = 2\) m/s
\(f = 0.02\)

\(V_{\text{max}} = ?\)

\(V_{\text{max}} = V(1 + 1.43 \sqrt{f})\)
\(= 2(1 + 1.43 \sqrt{0.02})\)
\(= 2 \times 1.2 = 2.4\) m/s

**04. Ans: (c)**

**Sol:**

Given data:
\(D = 30\) cm = 0.3 m
\(\text{Re} = 10^6\)
\(f = 0.025\)

Thickness of laminar sub layer, \(\delta' = ?\)

\[
\delta' = \frac{11.6v}{V^*}
\]
where \(V^* = \text{shear velocity} = V \sqrt{\frac{f}{8}}\)
\(v = \text{Kinematic viscosity}\)

\[
\text{Re} = \frac{VD}{v}
\]
\[
\therefore v = \frac{VD}{\text{Re}}
\]
\[
\frac{11.6 \times VD}{\text{Re}}
\]
\[
\delta' = \frac{11.6 \times D}{\text{Re} \sqrt{8}}
\]
\[ \tau = \frac{11.6 \times 0.3}{10^6 \times \sqrt{\frac{0.025}{8}}} = 6.22 \times 10^{-5} \text{ m} = 0.0622 \text{ mm} \]

05. Ans: 25
Sol: Given:

\[ L = 100 \text{ m} \]
\[ D = 0.1 \text{ m} \]
\[ h_L = 10 \text{ m} \]
\[ \tau = ? \]

For any type of flow, the shear stress at wall/surface \( \tau = -\frac{dP}{dx} \times \frac{R}{2} \)

\[ \tau = \frac{\rho g h_L \times R}{L} \times \frac{D}{4} \]

\[ = \frac{1000 \times 9.81 \times 10^{10}}{100} \times \frac{0.1}{4} \]

\[ = 24.525 \text{ N/m}^2 = 25 \text{ Pa} \]

06. Ans: 0.905
Sol: \( k = 0.15 \text{ mm} \)
\( \tau = 4.9 \text{ N/m}^2 \)
\( \nu = 1 \text{ centi-stoke} \)

\[ V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/sec} \]
\( \nu = 1 \text{ centi-stoke} \)

\[ = \frac{1}{100} \times \frac{10^{-4}}{100} = 10^{-6} \text{ m}^2 / \text{sec} \]

\[ \frac{k}{\delta} = \frac{0.15 \times 10^{-3}}{11.6 \times \nu} \left( \frac{V^*}{V^*} \right) \]

\[ = \frac{0.15 \times 10^{-3}}{11.6 \times 10^{-6}} = 0.905 \]

07. Ans: (a)
Sol: The velocity profile in the laminar sublayer is given as

\[ \frac{u}{V^*} = \frac{yV^*}{\nu} \]

or

\[ \nu = \frac{y(V^*)^2}{u} \]

where, \( V^* \) is the shear velocity.

Thus,

\[ \nu = \frac{0.5 \times 10^{-3} \times (0.05)^2}{1.25} = 1 \times 10^{-6} \text{ m}^2 / \text{s} \]

\[ = 1 \times 10^{-2} \text{ cm}^2 / \text{s} \]

08. Ans: 47.74 N/m²
Sol: Given data:
\( d = 100 \text{ mm} = 0.1 \text{ m} \)
\( u_r = 0 = u_{\text{max}} = 2 \text{ m/s} \)

Velocity at \( r = 30 \text{ mm} = 1.5 \text{ m/s} \)

Flow is turbulent.

The velocity profile in turbulent flow is

\[ \frac{u_{\text{max}} - u}{V^*} = 5.75 \log \left( \frac{R}{y} \right) \]

where \( u \) is the velocity at \( y \) and \( V^* \) is the shear velocity.

For pipe, \( y = R - r \)

\[ = (50 - 30) \text{ mm} = 20 \text{ mm} \]
Thus,
\[
\frac{2 - 1.5}{V^*} = 5.75 \log \left( \frac{50}{20} \right) = 2.288
\]

or \( V^* = \frac{0.5}{2.288} = 0.2185 \text{ m/s} \)

Using the relation,
\[
V^* = \sqrt{\frac{\tau_w}{\rho}} \quad \text{or} \quad \tau_w = \rho (V^*)^2
\]

\[
\tau_w = 10^3 \times (0.2185)^2 = 47.74 \text{ N/m}^2
\]

09. Ans: (a)

Sol:
- In turbulent flow, shear stress is given by
  \[
  \tau = \mu \left( \frac{d\bar{u}}{dy} \right) + \eta \left( \frac{d\bar{u}}{dy} \right)
  \]
  where \( \mu \) is dynamic viscosity and \( \eta \) is the eddy viscosity which is not a fluid proper but it is a flow property which depends upon turbulence condition of the flow.
- From the above expression we say that the shear stress in turbulent flow is more than that predicted by Newton's law of viscosity. Thus, statement - I is correct.
- Statement (II) is also correct statement and it is the correct explanation of statement (I).

01. Conventional Practice Solutions

Sol: Given data:
\[
r = 0, \quad u = 1.5 \text{ m/s at } y = R - 0 = R
\]
\[
r = \frac{R}{2}, \quad u = 1.35 \text{ m/s at } y = R - \frac{R}{2} = \frac{R}{2}
\]
\[
D = 0.2 \text{ m or } R = 0.1 \text{ m}
\]
Centreline velocity 1.5 m/s = \( u_{\text{max}} \)

Using the logarithmic velocity profile as:
\[
\frac{u_{\text{max}} - u}{V^*} = 5.75 \log \left( \frac{R}{y} \right)
\]

where \( V^* \) is the shear velocity, we can find \( V^* \).

\[
\frac{1.5 - 1.35}{V^*} = 5.75 \log \left( \frac{R}{y} \right) = 5.75 \log(2)
\]

\[
\Rightarrow V^* = 0.0867 \text{ m/s}
\]

Similarly using the logarithmic velocity profile in terms of \( u, V \) and \( V^* \) (where \( V \) is the average velocity) we can find \( V \) as:
\[
\frac{u - V}{V^*} = 5.75 \log \left( \frac{y}{R} \right) + 3.75
\]
at \( y = R \),
\[
u = u_{\text{max}}
\]
\[
\frac{1.5 - V}{0.0867} = 5.75 \log \left( \frac{R}{R} \right) + 3.75 = 0 + 3.75
\]

\[
\Rightarrow V = 1.5 - 0.0867 \times 3.75 = 1.175 \text{ m/s}
\]
(i) Thus, discharge \( d = \frac{\pi \times 0.2^2 \times 1.175}{4} \)
\[ = 0.0369 \text{ m}^3/\text{s} \]

(ii) We know that \( V^* = V \sqrt{\frac{f'}{2}} \)
where, \( f' \) is the coefficient of friction.
Thus, \( f' = 2 \times \left( \frac{V^*}{V} \right)^2 \)
\[ = 2 \times \left( \frac{0.0867}{1.175} \right)^2 \]
\[ = 0.011 \]
The friction factor, \( f = 4f' = 0.044 \)

(iii) The relationship between height of roughness projections, \( K \) and friction factor is given by
\[ \frac{1}{\sqrt{f}} = 2.0 \log \left( \frac{R}{K} \right)+1.74 \]
Substituting the values, we get
\[ \sqrt{0.044} = 2.0 \log \left( \frac{R}{K} \right)+1.74 \]
\[ \log \left( \frac{R}{K} \right) = 1.5136 \]
\[ \frac{R}{K} = 32.629 \]
\[ K = \frac{R}{32.629} = 0.1 \times 10^3 \text{ mm} \]
\[ = 3.065 \text{ mm} \]

10. **Boundary Layer Theory**

01. **Ans: (c)**

Sol: \( \text{Re}_{\text{Critical}} = \frac{U_{\infty} x_{\text{critical}}}{v} \)
Assume water properties
\[ 5 \times 10^5 = 6 \times x_{\text{critical}} \]
\[ x_{\text{critical}} = 0.08333 \text{ m} = 83.33 \text{ mm} \]

02. **Ans: 1.6**

Sol: \( \delta \propto \frac{1}{\sqrt{\text{Re}}} \) (At given distance ‘x’)
\[ \frac{\delta_1}{\delta_2} = \frac{\text{Re}_2}{\text{Re}_1} \]
\[ \frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6 \]

03. **Ans: 80**

Sol:
\[ \delta \propto \sqrt{x} \]
\[ \frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1 + 1)}} \]
\[ x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1 + 1}} \]
\[ 4 = \frac{x_1}{x_1 + 1} \]
\[ 5x_1 = 4 \Rightarrow x_1 = 80 \text{ cm} \]
04. Ans: 2
Sol: \( \tau \propto \frac{1}{\delta} \)
\[ \tau \propto \frac{1}{\sqrt{x}} \quad \therefore \delta \propto \sqrt{x} \]
\[ \tau_1 = \frac{x_2}{x_1} \quad \tau_2 = \frac{\tau_1}{\sqrt{4}} = 2 \]

05. Ans: 3
Sol:
\[ \frac{U}{U_\infty} = \frac{y}{\delta} \]
\[ \frac{\delta^*}{\theta} = \text{Shape factor} = ? \]
\[ \delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy \]
\[ = \int_0^\delta \left(1 - \frac{y}{8}\right) dy \]
\[ = \frac{y^2}{2\delta} \quad \delta - \frac{\delta}{2} = \frac{\delta}{2} \]
\[ \theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy \]
\[ = \int_0^\delta \frac{y}{8} \left(1 - \frac{y}{\delta}\right) dy \]
\[ = \frac{y^2}{2\delta} - \frac{y^3}{3\delta} \bigg|_0^\delta = \frac{\delta}{2} = \frac{\delta}{3} = \frac{\delta}{6} \]
Shape factor = \( \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3 \)

06. Ans: 22.6
Sol: Drag force,
\[ F_D = \frac{1}{2} C_D \rho A_{Proj}. U_\infty^2 \]
\[ B = 1.5 \text{ m}, \quad \rho = 1.2 \text{ kg/m}^3 \]
\[ L = 3.0 \text{ m}, \quad \nu = 0.15 \text{ stokes} \]
\[ U_\infty = 2 \text{ m/sec} \]
\[ Re = \frac{U_\infty L}{\nu} = \frac{2 \times 3}{0.15 \times 10^{-4}} = 4 \times 10^5 \]
\[ C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{4 \times 10^5}} = 2.09 \times 10^{-3} \]
Drag force,
\[ F_D = \frac{1}{2} \times 2.09 \times 10^{-3} \times 1.2 \times (1.5 \times 3)^2 \]
\[ = 22.57 \text{ milli-Newton} \]

07. Ans: 1.62
Sol:
Given data,
\[ U_\infty = 30 \text{ m/s}, \quad \rho = 1.2 \text{ kg/m}^3 \]
Velocity profile at a distance x from leading edge,
\[ \frac{u}{U_\infty} = \frac{\delta}{\delta^*} \quad \delta = 1.5 \text{ mm} \]
Mass flow rate of air entering section ab,
\[ (\dot{m}_{in})_{ab} = \rho U_\infty (\delta \times 1) = \rho U_\infty \delta \text{ kg/s} \]
Mass flow rate of air leaving section cd,
\[ (\dot{m}_{out})_{cd} = \rho \int_0^\delta u(dy \times 1) = \rho \int_0^\delta U_\infty \left(\frac{y}{\delta}\right) dy \]
\[ = \frac{\rho U_\infty}{\delta} \left[ \frac{y^2}{2} \right]_0^\delta = \frac{\rho U_\infty \delta}{2} \]
From the law of conservation of mass:

\[(m_{in})_{ab} = (m_{out})_{cd} + (m_{out})_{bc}\]

Hence, \[(m_{out})_{bc} = (m_{in})_{ab} - (m_{out})_{cd}\]

\[= \rho U_x \delta - \frac{\rho U_x \delta}{2}\]

\[= \frac{\rho U_x \delta}{2}\]

\[= \frac{1.2 \times 30 \times 1.5 \times 10^{-3}}{2}\]

\[= 27 \times 10^{-3} \text{ kg/s}\]

\[= 27 \times 10^{-3} \times 60 \text{ kg/min}\]

\[= 1.62 \text{ kg/min}\]

08. Ans: (b)

Sol: For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in y-direction, \(\frac{\partial u}{\partial y}\) only.

The correct option is (b).

09. Ans: 28.5

Sol: Given data,

Flow is over a flat plate.

\(L = 1 \text{ m},\)

\(U_x = 6 \text{ m/s}\)

\(\nu = 0.15 \text{ stoke} = 0.15 \times 10^{-4} \text{ m}^2/\text{s}\)

\(\rho = 1.226 \text{ kg/m}^3\)

\(\delta(x) = \frac{3.46x}{\sqrt{Re_x}}\)

Velocity profile is linear.

Using von-Karman momentum integral equation for flat plate.

\[\frac{d\delta}{dx} = \frac{\tau_w}{\rho U_x^2}\]

we can find out \(\tau_w\).

From linear velocity profile, \(\frac{u}{U_x} = \frac{y}{\delta}\), we evaluate first \(\theta\), momentum thickness as

\[\theta = \int_{0}^{\delta} \frac{u}{U_x} \left(1 - \frac{u}{U_x}\right) dy\]

\[= \int_{0}^{\delta} \frac{y}{\delta} \left(1 - \frac{y}{\delta}\right) dy = \int_{0}^{\delta} \left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right) dy\]

\[= \left(\frac{y^3}{6\delta} - \frac{y^4}{3\delta^2}\right)_0^\delta = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}\]

\[\Rightarrow \theta = \frac{\delta}{6} = \frac{1}{6} \times \frac{3.46x}{\sqrt{Re_x}}\]

\[= \frac{3.46}{6} \left(\frac{U_x}{\nu}\right)^{1/2}\]

Differentiating \(\theta\) w.r.t \(x\), we get:

\[\frac{d\theta}{dx} = \frac{3.46}{6} \left(\frac{U_x}{\nu}\right)^{1/2} = 0.2883 \frac{1}{\sqrt{\frac{U_x x}{\nu}}}\]

\[\frac{d\theta}{dx}_{x=0.5m} = 0.2883 \times \frac{1}{\sqrt{\frac{6 \times 0.5}{0.15 \times 10^{-4}}}} = 0.2883 \frac{1}{447.2}\]

\[= 0.02845 \text{ N/m}^2 \approx 28.5 \text{ mN/m}^2\]
10. Ans: (c)
Sol:
- For laminar boundary layer over a flat plate, the velocity gradient at the surface decreases in the direction of flow.
- This results in the decrease in shear stress and hence, the decrease in skin friction coefficient in the direction of flow.
- Thus, statement (I) is correct but the statement (II) is wrong.

11. Ans: (b)
Sol:
- The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free stream velocity. This results in higher skin friction drag in turbulent boundary layer. Thus, statement (I) is correct.
- The separation of turbulent boundary is late as compared to laminar boundary layer. Thus, statement (II) is also correct but it is not the correct explanation of statement (I).

01. Given data:
Test section dia = 40 cm
Test section length = 60 cm
Velocity of air at inlet = 2 m/s
and \( \delta^* = \frac{1.72x}{\sqrt{Re_x}} \)
\( Re_x = \frac{2 \times 0.6}{10^{-3}} = 1.2 \times 10^5 \)
So, \( \delta^* \) at \( x = 0.6m = \frac{1.72 \times 0.6}{\sqrt{12 \times 10^5}} = 2.979 \times 10^{-3} \) m

From equation of continuity
\( A_{in} V_{in} = A_{exit} V_{exit} \)
But \( d_{exit} = 0.4 - 2 \delta^* \)
\( = (0.4 - 2 \times 2.979 \times 10^{-3}) \) m
Thus, \( V_{exit} = \left( \frac{0.4}{0.4 - 2 \times 2.979 \times 10^{-3}} \right)^2 \times 2 \)
\( = 2.061 \) m/s

02. Given data:
Flow over a flat plate
Fluid is water.
\( U_\infty = 1 \) m/s
\( L = 1 \) m

Case I: Flow is turbulent
At \( x = 1 \) m
\( Re_x = \frac{U_\infty x}{v_{water}} = \frac{1 \times 1}{10^{-6}} = 10^6 \)
\[ \frac{\delta_{\text{tur}}}{x} = \frac{0.376}{(\text{Re}_x)^{\frac{1}{5}}} = \frac{0.376}{(10^6)^{\frac{1}{5}}} \]

\[ \delta_{\text{tur}} = \frac{0.376 \times 1}{(10^6)^{\frac{1}{5}}} = 0.0237 \text{ m} \approx 24 \text{ mm} \]

\[ \frac{\tau_w}{\frac{1}{2} \rho U_\infty^2} = C_{i,x} = \frac{0.059}{(\text{Re}_x)^{\frac{1}{5}}} \]

\[ \tau_w = \frac{0.059 \times 1}{2} \times 10^3 \times 1^2 = 1.86 \text{ N/m}^2 \]

**Case 2: If the flow is laminar**

For the comparison purpose, consider the same Reynolds number:

\[ \frac{\delta_{\text{lam}}}{x} = \frac{5}{\sqrt{\text{Re}_x}} \]

\[ \delta_{\text{lam}} = \frac{5 \times 1}{\sqrt{10^6}} = 5 \text{ mm} \]

\[ \text{and } \tau_w = \frac{0.664}{\sqrt{\text{Re}_x}} \times \frac{1}{2} \rho U_\infty^2 \]

\[ = \frac{0.664}{\sqrt{10^6}} \times \frac{1}{2} \times 10^3 \times 1^2 = 0.332 \text{ N/m}^2 \]

**02. Ans: 4.56 m**

**Sol:**

\[ F_D = C_D \cdot \frac{\rho A V^2}{2} \]

\[ W = 0.8 \times 1.2 \times \frac{\pi (D)^2 \times V^2}{2} \]

(Note: \( A = \text{Normal (or) projected Area} = \frac{\pi D^2}{4} \))

\[ 784.8 = 0.8 \times 1.2 \times \frac{\pi (D)^2 \times 10^2}{2} \]

\[ \therefore D = 4.56 \text{ m} \]

**03. Ans: 4**

**Sol:**

Given data:

\( l = 0.5 \text{ km} = 500 \text{ m} \)

\( d = 1.25 \text{ cm} \)

\( V_{\text{Wind}} = 100 \text{ km/hr} \)

\( \gamma_{\text{Air}} = 1.36 \times 9.81 = 13.4 \text{ N/m}^3 \)

\( v = 1.4 \times 10^{-5} \text{ m}^2/\text{s} \)

\( C_D = 1.2 \text{ for Re > 10000} \)

\( C_D = 1.3 \text{ for Re < 10000} \)
Note: The characteristic dimension for electric power transmission tower wire is “d”

Re = \( \frac{V_L}{v} = \frac{100 \times 5}{18} \left( 0.0125 \right) \)

\( \Rightarrow \) Re = 24801 > 10,000

\( \therefore \) \( C_D = 1.2 \)

\[
F_D = C_D \times \frac{\rho AV^2}{2}
\]

\[
= 1.2 \times \frac{\left( \frac{13.4}{9.81} \right) (L \times d) V^2}{2}
\]

\[
= 1.2 \times \frac{\left( \frac{13.4}{9.81} \right) (500 \times 0.0125) (100 \times \frac{5}{18})}{2}
\]

\( = 3952.4 \text{ N} = 4 \text{ kN} \)

04. Ans: 0.144 & 0.126

Sol: Given data:

\( W_{Kite} = 2.5 \text{ N} \)

\( A = 1 \text{ m}^2 \)

\( \theta = 45^\circ \)

\( T = 25 \text{ N} \)

\( V_{Wind} = 54 \text{ km/hr} \)

\( = 54 \times \frac{5}{18} = 15 \text{ m/s} \)

Resolving forces horizontally

\( F_D = T \cos 45^\circ \)

\( C_D \times \frac{\rho AV^2}{2} = 25 \times \cos 45^\circ \)

\( = 25 \times \frac{1}{\sqrt{2}} \)

\( \therefore \) \( C_D = 0.126 \)

Resolving forces vertically

\( F_L = W_{Kite} + T \sin 45^\circ \)

\( C_L \times \frac{\rho AV^2}{2} = 2.5 + 25 \sin 45^\circ \)

\( = 2.5 + \frac{25}{\sqrt{2}} \)

\( \therefore \) \( C_L = 0.144 \)
05. Ans: (a)
Sol: Given data:
\[ C_{D_2} = 0.75 C_{D_1} \ (25\% \text{ reduced}) \]
Drag power = Drag force \times \text{Velocity}
\[ P = F_D \times V = \frac{C_D \rho AV^2}{2} \times V \]
\[ P = C_D \times \frac{\rho AV^3}{2} \]
Keeping \( \rho \), \( A \) and power constant
\[ C_D V^3 = \text{constant} = C \]
\[ C_{D_2} \left( \frac{V_2}{V_1} \right)^3 = \frac{V_2}{V_1} \]
\[ \frac{C_{D_2}}{C_{D_1}} = \left( \frac{V_2}{V_1} \right)^{\frac{3}{2}} \]
\[ \left( \frac{C_{D_2}}{0.75C_{D_1}} \right)^{\frac{3}{2}} = \frac{V_2}{V_1} \]
\[ \therefore V_2 = 1.10064V_1 \]
% Increase in speed = 10.064%

06. Ans: (c)
Sol: When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid:
Weight of sphere = Buoyant force + Drag force

07. Ans: 0.62
Sol: Given data,
Diameter of dust particle, \( d = 0.1 \text{ mm} \)
Density of dust particle,
\[ \rho = 2.1 \text{ g/cm}^3 = 2100 \text{ kg/m}^3 \]
\[ \mu_{\text{air}} = 1.849 \times 10^{-5} \text{ Pa.s}, \]

At suspended position of the dust particle,
\[ W_{\text{particle}} = F_D + F_B \]
where \( F_D \) is the drag force on the particle and \( F_B \) is the buoyancy force.
From Stokes law:
\[ F_D = 3\pi \mu V d \]
Thus,
\[ \frac{4}{3} \pi r^3 \rho = 3\pi \mu V d \]
or,
\[ \frac{4}{3} \pi r^3 (\rho - \rho_{\text{air}}) = 3\pi \mu_{\text{air}} V (2r) \]
or
\[ V = \frac{2}{9} r^2 \left( \frac{\rho - \rho_{\text{air}}}{\mu_{\text{air}}} \right) \]
\[ = \frac{2}{9} \times (0.05 \times 10^{-3})^2 \times 9.81 \times \frac{2100 - 1.2}{1.849 \times 10^{-5}} \]
\[ = 0.619 \text{ m/s} \approx 0.62 \text{ m/s} \]

08. Ans: (b)
Sol: Since the two models \( M_1 \) and \( M_2 \) have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.
From their shapes, we can say that \( M_2 \) reaches the bottom earlier than \( M_1 \).
09. Ans: (a)
Sol:
- Drag of object \( A_1 \) will be less than that on \( A_2 \). There are chances of flow separation on \( A_2 \) due to which drag will increase as compared to that on \( A_1 \).
- Drag of object \( B_1 \) will be more than that of object \( B_2 \). Because of rough surface of \( B_2 \), the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but \( C_2 \) is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on \( C_2 \) will be more than that on \( C_1 \) because of flow becoming turbulent due to roughness. Hence, drag of object \( C_2 \) will be more than that of object \( C_1 \).
- Thus, the correct answer is option (a).

10. Ans: (a)
Sol:
- Dimples on a golf ball are intentionally provided to make its surface rough so that flow becomes turbulent.
- A turbulent boundary layer, having more momentum than a laminar boundary layer, can better resist an adverse pressure gradient, thus avoiding early separation.
- Thus, both statements are correct and statement (II) is the correct explanation of statement (I).

\[ F_D = (\Delta P \times A) \sin \alpha + (\Sigma \tau_w \times A) \cos \alpha \]

Substituting the values given:
\[ F_D = [2.3 - (-1.2)](1)\sin 7^\circ + (7.6 \times 10^{-2} + 5.8 \times 10^{-2}) \times 1 \times \cos 7^\circ \]
\[ = 426.5 \text{ N} + 133 \text{ N} = 559.5 \text{ N} \approx 560 \text{ N} \]
\[ F_L = [2.3 - (-1.2)] \times 1 \times \sin 7^\circ - (7.6 \times 10^{-2} + 5.8 \times 10^{-2}) \times 1 \times \sin 7^\circ \]
\[ = 3474 \text{ N} - 16.3 \text{ N} \]
\[ = 3457.7 \text{ N} \approx 3458 \text{ N} \]

When the shear stress is neglected, then
\[ F_D = (\Delta P \times A) \cos \alpha = 426.5 \text{ N} \approx 427 \text{ N} \]
and \[ F_L = (\Delta P \times A) \sin \alpha = 3474 \text{ N} \]
12. Open Channel Flow

02. Ans: (b)
Sol: \( Q_1 = 15 \text{ m}^3/\text{sec}, \ y = 1.5 \text{ m} \)
\[
S_1 = \frac{1}{1690}, \text{if } S_2 = \frac{1}{1000}
\]
Then \( Q_2 = ? \)
\[
Q \propto \sqrt{S}
\]
\[
\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}
\]
\[
\frac{Q_2}{Q_1} = \sqrt{\frac{1}{1000} / \frac{1}{1690}} = 1.3
\]
\[
Q_2 = 1.3 \times 15 = 19.5 \text{ m}^3/\text{sec}
\]

03. Ans: (d)
Sol: \( Q = AV \)
\[
B \times y \times \frac{1}{n} R^{2/3} S^{1/2}
\]
\[
= B \times y \times \frac{1}{n} y^{2/3} S^{1/2}
\]
\[
= R \approx y \rightarrow \text{For wide rectangular channel} \quad Q \propto y^{5/3}
\]
\[
\frac{Q_2}{Q_1} = \left( \frac{y_2}{y_1} \right)^{5/3}
\]
\[
\frac{Q_2}{Q_1} = \left( \frac{1.25 y_1}{y_1} \right)^{5/3} = 1.45
\]

04. Ans: 24.33
Sol:
\[
\tau_{avg} = \gamma_w RS
\]
\[
R = \frac{A}{P}
\]
\[
A = 2 \times \left( \frac{1}{2} \times 2 \times 2 \right) + 4 \times 2 = 12 \text{ m}^2
\]
\[
P = 4 + 2 \sqrt{2^3} + 2^2 = 9.66 \text{ m}
\]
\[
\tau_{avg} = 9810 \times 1.24 \times 0.002 = 24.33 \text{ N/m}^2
\]

05. Ans: 24.33
Sol:
\[
\tau_{avg} = \gamma_w RS
\]
\[
R = \frac{A}{P}
\]
\[
\tau_{avg} = \frac{A}{P} \gamma_w RS
\]
\[
A = 2 \times \left( \frac{1}{2} \times 2 \times 2 \right) + 4 \times 2 = 12 \text{ m}^2
\]
\[
P = 4 + 2 \sqrt{2^3} + 2^2 = 9.66 \text{ m}
\]
\[
\tau_{avg} = 9810 \times 1.24 \times 0.002 = 24.33 \text{ N/m}^2
\]

06. Ans: (d)
Sol: Triangular:
\[
P = 2 (\text{Inclined portion})
\]
\[
P = 2(1 + h \sqrt{1 + m^2}) (\therefore 1 = h \sqrt{1 + m^2})
\]
\[ \frac{P}{h} = 2\sqrt{2} = 2.83 \]

**Trapezoidal:** Efficient trapezoidal section is half of the Hexagon for which all sides are equal.

\[ I = h \sqrt{1 + m^2} \]

\[ P = I = h \sqrt{(1) + \left(\frac{1}{\sqrt{3}}\right)^2} = h(1.15) \]

\[ \frac{P}{h} = 1.15 \times 3 = 3.46 \text{ (3 sides are equal)} \]

**Rectangular:**

\[ P = b + 2h = 2h + 2h = 4h \text{ (} b = 2y \text{)} \]

\[ \frac{P}{h} = 4 \]

07. **Ans:** 0.37

**Sol:**

\[ A = y (b + my) \]

\[ A = \frac{Q}{V} = \frac{5}{1.25} = 4 \text{ m}^2 \]

\[ 4 = \left(b + \frac{y}{\sqrt{3}}\right)y \text{ .......(I) } \left(\therefore m = \frac{1}{\sqrt{3}}\right) \]

But \( b = I \) (\because Efficient trapezoidal section)

\[ b = y \sqrt{1 + m^2} \]

\[ b = \frac{2y}{\sqrt{3}} \text{ .............(II)} \]

From (I) & (II)

\[ y = 1.519 \text{ m} \]

\[ \therefore D = \frac{b + my}{b + 2my} = 1.14 \text{ m} \]

\[ \therefore F_r = \frac{V}{\sqrt{gD}} \]

\[ F_r = 0.37 \]

08. **Ans:** (a)

**Sol:** Alternate depths

\[ y_1 = 0.4 \text{ m} \]

\[ y_2 = 1.6 \text{ m} \]

Specific energy at section =?

\[ y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} \]

\[ 0.4 + \frac{q^2}{2 \times 9.81 \times 0.4^2} = 1.6 + \frac{q^2}{2 \times 9.81 \times 1.6^2} \]

\[ q^2 \left(\frac{1}{3.1392} - \frac{1}{50.22}\right) = 1.6 - 0.4 \]

\[ q^2 (0.298) = 1.2 \]

\[ q^2 = 4.02 \]

\[ q = 2 \text{ m}^3 /\text{s/m} \]

\[ E_1 = y_1 + \frac{q^2}{2gy_1^2} \]

\[ E_1 = 0.4 + \frac{2^2}{2 \times 9.81 \times 0.4^2} = 1.68 \text{ m} \]

09. **Ans:** (b)

**Sol:** Depth = 1.6 m

Specific energy = 2.8 m
Fluid Mechanics

10. Ans: (c)
Sol: \( F_r = 5.2 \) (uniform flow)
The ratio of critical depth to normal depth \( \frac{y_c}{y_n} = ? \)

Note: The given two depths \( y_c \) & \( y_n \) are not alternate depths as they will have different specific energies.

\[
F_r = \frac{V}{\sqrt{g y}} \Rightarrow F_r^2 = \frac{V^2}{g y} = \frac{q^2}{g y} \quad \therefore \frac{q}{y} = \frac{q_c}{y_c}
\]

\[
\frac{(F_m)^3}{(F_n)^3} = \frac{q^2}{g y_c^3} \times \frac{g y_c^3}{q^2} = \frac{y_c^3}{y_n^3}
\]

\[
\frac{y_c^3}{y_n^3} = \frac{(F_m)^2}{(F_n)^2} \Rightarrow \frac{y_c}{y_n} = \left(\frac{F_m}{F_n}\right)^{2/3}
\]

\[
\frac{y_c}{y_n} = (5.2)^{2/3} = 3
\]

11. Ans: (c)
Sol: Rectangular channel
Alternate depths \( y_1 = 0.2, y_2 = 4 \) m

\[
E_1 = E_2 \quad (\therefore \text{alternate depths}), \quad F_r = \frac{V}{\sqrt{g D}}
\]

12. Ans: (d)
Sol: Triangular channel
\( H:V = 1.5:1 \)
Specific energy = 2.5 m

\[
E_c = \frac{5}{4} y_c
\]

\[
E_c = \frac{4}{5} y_c = 2 \text{ m}
\]

\[
y_c = \left(\frac{2Q^2}{9.81 \times 1.5^2}\right)^{1/5} \Rightarrow 2 = \left(\frac{2 \times Q^2}{9.81 \times 1.5^2}\right)^{1/5}
\]

\[
Q = 18.79 \text{ m}^3/\text{sec}
\]

13. Ans: 0.47
Sol: \( E_1 = E_2 + (\Delta z) \)

\[
V_1 = \frac{Q}{A_1} = \frac{12}{2.4 \times 2} = 2.5 \text{ m/sec}
\]
14. Ans: (c)
Sol: $F_r > 1$
$B_2 < B_1$
$q_2 > q_1$

As Potential energy $(y)$ increases then kinetic energy $(v)$ decreases

$\therefore \; y$ increases and $v$ decreases.

15. Ans: (a)
Sol: $Q = 3 \text{m}^3/\text{s}$
$B_1 = 2 \text{m}, D = 1.2 \text{ m}$

Width reduce $d$ to $1.5 \text{ m} (B_2)$
Assume channel bottom as horizontal

16. Ans: (d)
Sol: Rectangular Channel
$y_1 = 1.2 \text{ m}$
$V_1 = 2.4 \text{ m/s}$
$\Delta Z = 0.6 \text{ m}$
E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{(2.4)^2}{2 \times 9.81} = 1.49\text{m}

\[ \Delta Z = 0.6\text{m} \]

Q = 2.4 \times 1.2 = 2.88\text{m}^3/\text{s/m}

Assuming channel width as constant, the critical depth

\[ y_c = \left[ \frac{Q^2}{gB^2} \right]^{\frac{1}{3}} = 0.94\text{m} \]

Critical specific energy for rectangular channel \( E_c = \frac{3}{2} y_c \).

\[ E_c = \frac{3}{2}(0.94) = 1.41 \]

We know for critical flow in the hump portion \( E_1 = E_2 + (\Delta Z) = E_c + (\Delta Z)_c \)

\[ 1.49 = 1.41 + (\Delta Z)_c \]

\[ \therefore (\Delta Z)_c = 0.08\text{m} \]

If the hump provided is more than the critical hump height the u/s flow gets affected.

(or)

\[ F_r_1 = \frac{v_1}{\sqrt{g y_1}} = \frac{2.4}{\sqrt{9.81 \times 1.2}} = 0.69 < 1 \]

\[ \Rightarrow \text{Hence sub-critical.} \]

If the approaching flow is sub-critical the level of water will fall in the hump portion.

Option (b) is correct if the hump height provided is less than critical hump height.

As the hump height provided is more than critical, the u/s flow gets affected with the increase of the specific energy from \( E_1 \) to \( E_1' \).

In the sub-critical region as the specific energy increases, the level of water rises from \( y_1 \) to \( y_1' \) in the form of a surge.

\[ E_1' = y_1' + \frac{v_1'^2}{2g} \]

\[ E_1' = y_1' + \frac{q_1'^2}{2gy_1'^2} \ldots \text{(1)} \]

Also \( E_1' = E_c + (\Delta Z) \) provided.

\[ 1.41 + 0.6 = 2.01\text{m} \]

\[ \therefore 2.01 = y_1' + \frac{2.88^2}{2 \times 9.81 \times y_1'^2} \]

Solve by trial & error for \( y_1' > 1.2\text{m} \)

\[ 17. \textbf{Ans: (c)} \]

\[ \textbf{Sol:} \]

\[ B_1 = 4\text{ m} \]
\[ B_2 = 3\text{ m} \]

\[ (U/S) y_1 = 0.9\text{ m} \]

\[ E_1 = E_2 + \Delta Z \]
According to continuity equation
\[ Q_1 = Q_2 \]
\[ A_1V_1 = A_2V_2 \]
\[ A_1 = A_2 \]
\[ B_2y_1 = B_2y_2 \]
\[ 4 \times 0.9 = 3 \times y_2 \]
\[ y_2 = 1.2 \text{ m} \]
\[ y_1 = y_2 + \Delta Z \]
\[ 0.9 = 1.2 + \Delta Z \]
\[ \Delta Z = -0.3 \text{ m} \]
Negative indicates that the hump assumed is wrong in fact it is a drop.

**Ans:** (a)

**Sol:**

- **Top width:** 2
- **Area:**
  \[ A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 2y \times y = y^2 \]
- **Wetted perimeter:**
  \[ I^2 = \sqrt{y^2 + y^2} = y\sqrt{2} \]
  (Both sides) total wetted perimeter

Hydraulic mean depth
\[ (R) = \frac{A}{P} = \frac{y^2}{2\sqrt{2}y} = \frac{y}{2\sqrt{2}} \]
\[ y = y_n \text{(say)} \]

Using Mannings formula
\[ Q = A \frac{1}{n} (R)^{2/3} (S)^{1/2} \]
\[ 0.2 = y_n^2 \times 0.015 \times \left[ \frac{y_n}{2\sqrt{2}} \right]^{2/3} (0.001)^{1/2} \]
\[ \frac{1}{y_n^{8/3}} = \frac{1}{0.015 \times 0.2} \times \left[ \frac{1}{2\sqrt{2}} \right]^{2/3} (0.001)^{1/2} \]
\[ y_n^{8/3} = 0.2 \times 0.015 \times (2\sqrt{2})^{2/3} \left[ \frac{1}{0.001} \right]^{1/2} \]
\[ (y_n)^{8/3} = 0.189 \]
\[ y_n = (0.189)^{3/8} \]
\[ y_n = 0.54 \text{ m} \]

Critical depth \( y_c \)
\[ y_c = \left[ \frac{20^2}{g} \right]^{1/5} \]
(for triangle)
\[ y_c = \left[ \frac{2 \times 0.2 \times 2^{1/5}}{9.81} \right] = 0.382 \text{ m} \]
\[ y_n > y_c \quad (0.54 > 0.38) \]
∴ Mild slope

If (actual) depth at flow = 0.4 m = \( y \)
\[ Y_n > y > y_c \quad [0.54 > 0.4 > 0.38] \]
∴ Profile is M2
19. Ans: $4.36 \times 10^{-5}$
Sol:

\[
\begin{align*}
\text{Discharge, } Q &= 29 \text{ m}^3/\text{sec} \\
\text{Area of rectangular channel, } A &= 15 \times 3 = 45 \text{ m}^2 \\
\text{Perimeter, } P &= 15 + 2 \times 3 = 21 \text{ m} \\
\text{Hydraulic radius, } R &= \frac{A}{P} = \frac{45}{21} = 2.142 \text{ m} \\
\end{align*}
\]

\therefore \text{The basic differential equation governing the gradually varied flow is}
\[
\frac{dy}{dx} = \frac{S_o - S_r}{1 - \frac{Q^2T}{gA^3}}
\]

\[
\frac{dy}{dx} = \text{Slope of free water surface w.r.t to channel bottom}
\]

Velocity of flow \( V = \frac{Q}{A} = \frac{29}{45} \)

\[
= 0.644 \text{ m/sec}
\]

\therefore \text{By Chezy’s equation}

Velocity, \( V = C \sqrt{RS_f} \)

\[
0.644 = 65 \sqrt{2.142 \times S_f} \\
S_f = 4.589 \times 10^{-5}
\]

\[
S_o = \frac{1}{5000} = 2 \times 10^{-4}
\]

\[
\frac{Q^2T}{gA^3} = \frac{29^2 \times 15}{9.81 \times 4^3} = 0.0141
\]

\[\begin{align*}
\therefore \frac{dy}{dx} &= \frac{2 \times 10^{-4} - 4.589 \times 10^{-5}}{1 - 0.0141} \\
&= 1.5631 \times 10^{-4} \\
\therefore S_w &= S_o + \frac{dy}{dx}
\end{align*}\]

\( S_w \) water surface slope with respect to horizontal

\[
S_w = S_o - \frac{dy}{dx} = 2 \times 10^{-4} - 1.563 \times 10^{-4} \\
S_w = 4.36 \times 10^{-5}
\]

20. Ans: (a)
Sol:

22. Ans: 0.74
Sol: Free fall → 2\textsuperscript{nd} profile

Critical depth, \( y_c = \left( \frac{q^2}{g} \right)^{\frac{1}{3}} \)
\[ y_c = \left( \frac{2^2}{9.81} \right)^{\frac{1}{3}} = 0.74 \text{ m} \]

\[ V = \frac{q}{y_n} \]

\[ \frac{2}{y_n} = \frac{1}{n} y_n^{2/3} S_{1/2}^{1/2} \]

\[ \frac{2}{y_n} = \frac{1}{0.012} \times y_n^{2/3} (0.0004)^{1/2} \]

\[ y_n = 1.11 \text{ m} \]

\[ y_n > y_c \]

Hence the water surface will have a depth equal to \( y_c \)

\[ y_c = 0.74 \text{ m} \]

23. Ans: (d)

Sol: \[ q = 2 \text{ m}^2/\text{sec} \]
\[ y_A = 1.5 \text{ m}; y_B = 1.6 \text{ m} \]
\[ \Delta E = 0.09 \]
\[ S_o = \frac{1}{2000} \]
\[ S_t = 0.003 \]
\[ \Delta x = \frac{\Delta E}{S_o - S_t} = \frac{0.09}{\frac{1}{2000} - 0.003} = -36 \text{ m} \]

24. Ans: (d)

Sol: Given \( q_1 = Q/B = 10 \text{ m}^3/\text{s} \)
\[ v_1 = 20 \text{ m/s} \]
\[ \therefore y_1 = \frac{q_1}{v_1} = \frac{10}{20} = 0.5 \text{ m} \]

We know that relation between \( y_1 \) and \( y_2 \) for hydraulic jump is

\[ \frac{y_2}{y_1} = \frac{1}{2} \left[ 1 - \sqrt{1 + 8 Fr_1^2} \right] \]

\[ Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{20}{\sqrt{9.81 \times 0.5}} = 9.03 \]

\[ \therefore \frac{y_2}{0.5} = \frac{1}{2} \left[ 1 - \sqrt{1 + 8 \times (9.03)^2} \right] \]

\[ y_2 = 6.14 \text{ m} \]

25. Ans: (c)

Sol: \[ Q = 1 \text{ m}^3/\text{s} \]
\[ y_1 = 0.5 \text{ m} \]
\[ y_2 = ? \]

As it is not a rectangular channel, let us work out from fundamentals by equating specific force at the two sections.

\[ \frac{Q^2}{gA} + Az_1 = \frac{Q^2}{gA} + Az_2 \]

\[ \frac{1^2}{9.81 \times y_1^2} + y_1 \times y_1 = \frac{1^2}{9.81 y_2^2} + y_2 \times y_2 \]

\[ 0.449 = \frac{1}{9.81 y_2^2} + \frac{y_2^3}{3} \]

\[ y_2 = 1.02 \text{ m} \]

26. Ans: (b)

Sol: Given:

Head = 5 m = (\Delta E)

Froude number = 8.5

Approximate sequent depths =?
\[
\frac{y_2}{y_1} = \frac{1}{2} \left[ 1 + \sqrt{1 + 8F_{yl}^2} \right]
\]

\[
= \frac{1}{2} \left[ 1 + \sqrt{1 + 8(8.5)^2} \right]
= 11.5 \text{ m}
\]

\(y_2 = 11.5 \ y_1\)

(a) \(y_2 = 11.5(0.3) = 3.45\) from options

(b) \(y_2 = 11.5(0.2) = 2.3 \text{ m}\)

\(y_1 = 0.2, \ y_2 = 2.3 \text{ m}\)

(or)

\(\Delta E = 5 \text{ m}\)

\(\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}\)

\(\frac{(11.5y_1 - y_1)^3}{4(11.5y_1)y_1} = 5\)

\((10.5y_1)^3 = 230y_1^2\)

\(1157.625 \ y_1 = 230\)

\(y_1 = 0.2 \text{ m}\)

\(y_2 = 11.5(0.2)\)

\(y_2 = 2.3 \text{ m}\)

27. Ans: 1.43

Sol: \(y_1 = 1.2 \text{ m}\)

\(V_w + V_i = \sqrt{gy_i}\)

\(V_i = \sqrt{9.81 \times 1.2 - 2}\)

\(V_i = 1.43 \text{ m/s}\)

In this problem if the wave moves downstream the velocity of wave is

\(V_w - V_i = \sqrt{gy_i}\)

\(V_w = \sqrt{gy_i} + V_i\)

\(= \sqrt{9.81 \times 1.2 + 2}\)

\(= 5.43 \text{ m/s}\)

28. Ans: (b)

Refer previous ESE-Obj-(Vol-2) solutions
Book (Cha-12, 79th Question -pg: 154)

29. Ans: (c)

Refer previous ESE-Obj-(Vol-2) solutions
Book (Cha-12, 87th Question -pg: 155)
Hydraulic Radius

\[ R_1 = \frac{A_1}{P_1} = \frac{10}{9} = 1.11 \text{ m} \]

Discharge,

\[ Q = A_1V_1 = 10 \times \left( \frac{1}{n} R^{2/3} S^{1/2} \right) \]

\[ = 10 \times \left( \frac{1}{0.015} \times (1.1)^{2/3} \times \left( \frac{1}{1600} \right) \right)^{1/2} \]

\[ = 10 \times (1.79) \]

\[ = 17.8675 \text{ m}^3/\text{s} \]

**Case - 2:**

Consider hydraulically efficient rectangular channel so that discharge is maximum.

Given that lining area constant w.r.to original channel.

\[ P_2 = P_1 = 9 \text{ m} \]

\[ b_2 + 2y_2 = 9 \text{ m} \]

For efficient rectangular channel we know

\[ b = 2y, \]

\[ 2y_2 + 2y_2 = 9 \]

\[ y_2 = \frac{9}{4} = 2.25 \text{ m} \]

\[ b_2 = 2 \times 2.25 = 4.5 \text{ m} \]

**By Manning’s formula:**

\[ V_2 = \frac{1}{n} R^{2/3} S^{1/2} \]

\[ = \frac{1}{0.015} \times \left( \frac{2.25}{2} \right)^{2/3} \left( \frac{1}{1600} \right)^{1/2} \]

\[ V_2 = 1.80 \text{ m/s} \]

\[ Q = A_2V_2 \]

\[ = (4.5 \times 2.25) (1.80) \left( \frac{R = \frac{y}{2} = \frac{b}{4}}{2} \right) \]

\[ = 18.25 \text{ m}^3/\text{sec} \]

% increase in discharge \[ Q = \frac{Q_2 - Q_1}{Q_1} \times 100 \]

\[ = \frac{18.25 - 17.8675}{17.9} \times 100 \]

\[ = 2.14\% \]

By Froude’s number

\[ F_n = \frac{V_1}{\sqrt{gy_1}} = \frac{1.8}{\sqrt{9.81 \times 2}} \]

\[ F_n = 0.40 < 1 \]

It is sub critical flow.

\[ F_e = \frac{V_2}{\sqrt{gy_2}} = \frac{1.8}{\sqrt{9.81 \times 2.25}} \]

\[ F_e = 0.38 < 1 \]

So it also sub critical flow.

∴ The sub critical flow is not changing into super critical flow.

**02.**

Sol: Say, \[ q = \text{discharge per meter width}, \]

according to the continuity equation for constant width

\[ q = V_1 y_1 = V_2 y_2 \]

As \( y_1 \) and \( y_2 \) are alternative depths, the specific energy is same at both the sections.

\[ E_1 = E_2 \]
\[ y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \]
\[ y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} \]

Hence,
\[ (y_1 - y_2) = \frac{\frac{q^2}{2g} \left( \frac{1}{y_2^2} - \frac{1}{y_1^2} \right)}{2} \]
\[ 2(y_1 - y_2) = \frac{(y_1^2 - y_2^2)}{y_1y_2} q^2 \]

For a rectangular channel \( \frac{q^2}{g} = \frac{y_c^3}{y_1} \)

Hence,
\[ y_c^3 = \frac{2(y_1 - y_2)y_1y_2}{(y_1^2 - y_2^2)} \]
\[ y_c^3 = \frac{2y_1y_2}{y_1 + y_2} \]

Specific energy, \( E = y_1 + \frac{1}{2} \left( \frac{q^2}{g} \right) \frac{1}{y_1} \)

Substituting the value of \( \frac{q^2}{g} \) in the above equation
\[ E = y_1 + \frac{1}{2} \times \frac{2y_1^2y_2^2}{(y_1 + y_2)} \times \frac{1}{y_1} \]
\[ = y_1 + \frac{y_2^2}{y_1 + y_2} \]
\[ = \frac{y_1^2 + y_1y_2 + y_2^2}{(y_1 + y_2)} \]

Hence proved

### 03.
#### Sol:

At upstream section 1
\[ A_1 = 1.6 \times 4.0 = 6.4 \text{ m}^2 \]
\[ Q = A_1V_1 = 1.10 \times 6.4 = 7.04 \text{ m}^3/\text{s} \]

Discharge intensity,
\[ q_1 = \frac{Q}{b_1} = \frac{7.04}{4.0} = 1.76 \text{ m}^3/\text{s/m} \]

\[ \frac{V_1}{\sqrt{gy_1}} = \frac{(1.10)^2}{2 \times 9.81} = 0.06167 \text{ m} \]

Specific energy, \( E_1 = y_1 + \frac{V_1^2}{2g} \)
\[ = 1.60 + 0.06167 \]
\[ = 1.66167 \text{ m} \]

Froude number, \( F_1 = \frac{V_1}{\sqrt{gy_1}} \)
\[ = \frac{1.10}{\sqrt{9.81 \times 1.6}} = 0.2776 \]

As \( F_1 < 1.0 \), upstream flow is subcritical. The water surface will drop down at the contracted section.
Contracted section: (Section 2)

\[ q_2 = \frac{Q}{b_2} = \frac{7.04}{3.50} = 2.0114 \text{ m}^3/\text{s/m} \]

Critical depth, \( y_{c2} = \left( \frac{q_2^2}{g} \right)^{1/3} \)

\[ = \left( \frac{(2.0114)^2}{9.81} \right)^{1/3} = 0.7444 \text{ m} \]

Minimum specific energy at section 2 = \( E_{c2} \)

\[ E_{c2} = y_{c2} + \frac{V_{c2}^2}{2g} = 1.5y_{c2} = 1.5 \times 0.7444 = 1.1165 \text{ m} \]

At critical conditions

\[ E_1 = E_2 + \Delta Z \]

\[ 1.66167 = 1.1165 + \Delta Z \]

\[ \therefore \Delta Z = 0.545 \]

\( \Delta Z < \Delta Z_c \) given

So up stream level would not get disturbed

By energy equation,

\[ E_1 = 1.66167 = y_2 + \frac{V_{c2}^2}{2g} + \Delta z \]

\[ = y_2 + \frac{q_2^2}{2gy_2} + \Delta z \]

\[ 1.66167 = y_2 + \frac{(2.0114)^2}{2 \times 9.81 \times y_2} + 0.35 \]

\[ y_2 + \left( \frac{0.2062}{y_2} \right) = 1.3117 \]

By trail and error method

Value of \( y_2 \) is found as

\[ y_2 = 1.158 \text{ m} \]

The upstream depth will remain unaffected at \( y_1 = 1.60 \text{ m} \)

Hence, with the bed level of the section 1 as datum

Elevation of upstream water surface = 1.60 m

Elevation of water section at the contracted section

\[ = y_2 + \Delta z = 1.158 + 0.35 \]

\[ = 1.508 \]

---

04.

Sol:

Froude number at vena contracta = \( \frac{V_{vc}}{\sqrt{gy_{vc}}} \)

\[ V_{vc} = \frac{q}{y_{vc}} = \frac{2}{0.15} = 13.33 \text{ m/s} \]

\[ Fr_{vc} = \frac{13.33}{\sqrt{9.81 \times 0.15}} = 10.98 \]

We know

\[ y_2 = \frac{y_{1(VC)}}{2} \left[ -1 + \sqrt{1 + 8Fr_{vc}^2} \right] \]

\[ y_2 = \frac{0.15}{2} \left[ -1 + \sqrt{1 + 8(10.98)^2} \right] = 2.255 \text{ m} \]

If hydraulic jump starts at vena contracta the tail water depth shall be 2.255 m given tail.
Water depth is 1.8 m. It means due to practical situation the jump is repelling in such a case the jump will not start at vena contracta but slightly ahead of the vena contracta towards tail water.

\[ V = \frac{q}{y} = y + \frac{V^2}{2g} \]

For horizontal flow \( S_o = 0 \)

\[ \Delta x = \frac{\Delta E}{S_o - S_f} = \frac{-\Delta E}{S_f} \]

\[ SF = \text{Energy slope} \]

\[ V = \frac{1}{n} R^{2/3} \sqrt{S_f} \]

\[ S_f = \frac{V^2n^2}{R^{4/3}} = \frac{V^2n^2}{y^{4/3}} \]

\[ \overline{S} = \frac{S_f + S_f}{2} \]

<table>
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<tr>
<th>Step</th>
<th>( y )</th>
<th>( V )</th>
<th>( E )</th>
<th>( \Delta E )</th>
<th>SF</th>
<th>( \overline{S} )</th>
<th>( \Delta x ) (m)</th>
</tr>
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<tbody>
<tr>
<td>0.15</td>
<td>13.33</td>
<td>9.20</td>
<td>4.13</td>
<td>0.5016</td>
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<td>0.387</td>
<td>7.054</td>
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<td>0.148</td>
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<td>1.64</td>
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<tr>
<td>3</td>
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<td>0.148</td>
<td>3.51</td>
<td>0.133</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Actual initial depth

\[ y_1 = \frac{y + \sqrt{1 + 8F_y^2}}{2} \]

\[ = \frac{1.8 + \sqrt{1 + 8 \times 0.264^2}}{2} \]

\[ = 0.223 \text{ m} \]

The distance between vena contracta and starting of jump is \( \Delta x \) calculated by direct step method.

\[ \Delta x = \Delta x \]

\[ 0.15 \]

\[ 13.33 \]

\[ 9.20 \]

\[ 4.13 \]

\[ 0.5016 \]

\[ 0.218 \]

\[ 7.52 \]

\[ 0.133 \]

\[ 18.08 \text{ m} \]

05.

Sol: Given discharge \( Q = 4.8 \text{ m}^3/\text{sec} \)

Width of the channel \( b = 4 \text{ m} \)

Initial velocity of channel \( V_1 = 1 \text{ m/sec} \)

\[ \therefore \text{Discharge per meter width} \]

\[ q = \frac{Q}{4} = 1.2 \text{ m}^3/\text{sec/m} \]

\[ y_1 = \frac{q}{V} = 1.2 = 1.2 \text{ m} \]
By sudden increase of discharge the channel depth is rised by 50%.

\[ y_2 = 1.2 \times 1.5 = 1.8 \text{ m} \]

If discharge is suddenly increased surge will develop which will move downstream with a velocity ‘V_w’ as shown in figure.

The surge is unsteady rapidly varied flow. This unsteady flow case can be transformed into a steady one by superimposing flow with velocity ‘V_w’ in the opposite direction shown in figure.

The continuity equation may be written as

\[ A_1 V_1 = A_2 V_2 \]

For unit width of the channel

\[ y_1 (V_w - V_1) = y_2 (V_w - V_2) \]
\[ 1.2 (V_w - 1) = 1.8 (V_w - V_2) \]
\[ V_w - 1 = 1.5 (V_w - V_2) \]
\[ V_w - 1 = 1.5 V_w - 1.5 V_2 \]
\[ V_w = 3 V_2 - 2 \rightarrow (i) \]

A positive surge moving downstream applying momentum equation

\[
\begin{align*}
F_2 - F_1 &= \rho Q (V_d/s - V_u/s) \\
F_2 - F_1 &= \frac{\gamma y_2^2}{2} - \frac{\gamma y_1^2}{2} = \rho y_1 (V_w - V_1) [(V_w - V_1) - (V_w - V_2)] \\
&= \left( \frac{y_2^2 - y_1^2}{2} \right) \rho g = \rho y_1 (V_w - V_1)(V_2 - V_1) \\
&= \left( \frac{1.8^2 - 1.2^2}{2} \right) 9.81 = 12(V_w - 1) (V_2 - V_1) \\
&= 7.3575 = (V_w - 1) (V_2 - 1) \\
7.3575 &= (V_w - 1) (V_2 - 1) \\
\end{align*}
\]

From equation (i)

\[ V_w = 3 V_2 - 2 \]
\[ 7.3575 = (3V_2 - 2 - 1)(V_2 - 1) \]
\[ 7.3575 = (3V_2 - 3)(V_2 - 1) \]
\[ 7.357 = 3(V_2 - 1)^2 \]
\[ V_2 = 2.566 \]

\[ \therefore \text{ From equation (i) } V_w = 3(2.566) - 2 \]
\[ V_w = 5.698 \text{ m/sec} \]

\[ \therefore \text{ New flow rate } = by_2 V_2 \]
\[ = 4 \times 1.8 \times 1.5 \]
\[ Q_2 = 18.4752 \text{ m}^3/\text{sec} \]
13. Dimensional Analysis

01. Ans: (c)
Sol: Total number of variables,
\[ n = 8 \text{ and } m = 3 \] (M, L & T)
Therefore, number of \( \pi \)'s are \( 8 - 3 = 5 \)

02. Ans: (b)
Sol:
\[ \frac{T}{\rho D^2 V^2} = \frac{ML^2}{ML^3 \times L^2 \times L^2 \times T^2} = 1. \]
\[ \rightarrow \text{It is a non-dimensional parameter.} \]
\[ \frac{VD}{\mu} = \frac{LT^{-1} \times L}{ML^{-1}T^{-1}} \neq 1. \]
\[ \rightarrow \text{It is a dimensional parameter.} \]
\[ \frac{D\omega}{V} = 1. \]
\[ \rightarrow \text{It is a non-dimensional parameter.} \]
\[ \frac{\rho V D}{\mu} = \text{Re}. \]
\[ \rightarrow \text{It is a non-dimensional parameter.} \]

03. Ans: (b)
Sol: \( T = f(l, g) \)
Total number of variable,
\[ n = 3, m = 2 \] (L & T only)
Hence, no. of \( \pi \) terms = \( 3 - 2 = 1 \)

04. Ans: (c)
Sol:
- Mach Number \( \rightarrow \) Launching of rockets
- Thomas Number \( \rightarrow \) Cavitation flow in soil
- Reynolds Number \( \rightarrow \) Motion of a submarine
- Weber Number \( \rightarrow \) Capillary flow in soil

05. Ans: (b)
Sol: According to Froude’s law
\[ T_r = \sqrt{\frac{L_r}{t_p}} \]
\[ \frac{t_m}{t_p} = \frac{\sqrt{L_r}}{10} \]
\[ t_p = 50 \text{ min} \]

06. Ans: (a)
Sol: \( L = 100 \text{ m} \)
\( V_p = 10 \text{ m/s} \),
\[ L_r = \frac{1}{25} \]
As viscous parameters are not discussed, follow Froude’s law.
According to Froude,
\[ V_r = \sqrt{L_r} \]
\[ \frac{V_m}{V_p} = \sqrt{\frac{1}{25}} \]
\[ V_m = \frac{1}{5} \times 10 = 2 \text{ m/s} \]
07. Ans: (d)
Sol: Froude number = Reynolds number.

\[ \nu_r = 0.0894 \]

If both gravity & viscous forces are important then

\[ \nu_r = \left( \frac{L_r}{L} \right)^{1/2} \]

\[ \frac{3}{2} \left( \nu_r \right)^2 = L_r \]

\[ L_r = 1:5 \]

08. Ans: (c)
Sol: For distorted model according to Froude’s law

\[ Q_r = L_H L_V^{3/2} \]

\[ L_H = 1:1000 \]

\[ L_V = 1:100 \]

\[ Q_m = 0.1 \text{ m}^3/\text{s} \]

\[ Q_r = \frac{1}{1000} \times \left( \frac{1}{100} \right)^{3/2} = \frac{0.1}{Q_p} \]

\[ Q_p = 10^5 \text{ m}^3/\text{s} \]

09. Ans: (c)
Sol: For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

\[ \frac{\rho_w \times V_w \times d_w}{\mu_w} = \frac{\rho_a \times V_a \times d_a}{\mu_a} \]

or

\[ V_w = \frac{\rho_a \times d_a \times \mu_w}{\rho_w \times \mu_a} \times V_a \]

(where suffixes w and a stand for water and air respectively)

Substituting the values given, we get

\[ V_w = \frac{1.2}{10^3} \times \frac{4}{0.1} \times \frac{10^{-3}}{1.8 \times 10^{-5}} \times 1 = \frac{8}{3} \text{ m/s} \]

To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.

i.e,

\[ \left( \frac{F_D}{\rho A V^2} \right)_p = \left( \frac{F_D}{\rho A V^2} \right)_m \]

Hence,

\[ (F_D)_p = (F_D)_m \times \frac{\rho_a A_a}{\rho_w A_w} \times \left( \frac{V_a}{V_w} \right)^2 \]

\[ = 4 \times \frac{1.2}{10^3} \times \left( \frac{4}{0.1} \right)^2 \times \left( \frac{1}{8/3} \right)^2 \]

\[ = 1.08 \text{ N} \]

10. Ans: 47.9
Sol: Given data,


<table>
<thead>
<tr>
<th>Sea water (Prototype testing)</th>
<th>Fresh water (model testing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho )</td>
<td>1025 \text{ kg/m}^3</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.07 \times 10^{-3} \text{ Pa.s}</td>
</tr>
</tbody>
</table>

For dynamic similarity, Re should be same in both testing.

i.e., \[ \frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p} \]

\[ V_m = \frac{\rho_p}{\rho_m} \times \frac{d_p}{d_m} \times \frac{\mu_m}{\mu_p} \]

\[ = 0.5 \times \frac{1025}{10^3} \times 100 \times \frac{10^{-3}}{1.07 \times 10^{-3}} \]

\[ = 47.9 \text{ m/s} \]
11. Refer previous GATE solutions Book (Cha-8, One marks 5th Question -pg: 575)

12. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 5th Question -pg: 205)

13. Ans: (a)
Sol: \( V_p = 10 \text{ m/s} \) \( \text{dia} = 3 \text{m} \)
\( V_m = 5 \text{ m/s} \), \( F_m = 50 \text{ N} \), \( F_p = ? \)
Acc to Froude’s law:- \( F_r = L_r^3 \)
(But \( L_r \) is not given)
\[ P \propto \rho V^2 = \frac{F}{A} \]
\[ \rho AV^2 = F \]
Reynolds law

Now scale ratio:
\[ \frac{F_m}{F_p} = \frac{V_m^2}{V_p^2} \times \frac{A_m}{A_p} \times \frac{\rho_m}{\rho_p} \]
\[ 50 \times \left( \frac{1}{10} \right)^2 \times \left( \frac{5}{10} \right)^2 (A = L_r^2) \] (\( \because \) same fluid)
\( F_p = 20000 \text{ N} \)

14. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 4th Question -pg: 205)

15. Repeated (Same as 13th Question)

16. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 21st Question -pg: 208)

17. Ans: (a)
Sol: \( L_r = \frac{1}{100} \)
\( a_m = 0.013 \)
\[ \frac{a_m}{a_p} = \left( \frac{1}{100} \right)^{1/6} \]
\[ a_p = 0.028 \]

18. Ans: (a)
Sol: \( L_r = \frac{1}{9} \)
\( y_{p1} = 0.5 \text{ m}, \quad y_{p2} = 1.5 \text{ m} \)
\( q_m = ? \), \( q_p = ? \)
\[ \frac{2q_p^2}{g} = y_{p1} \cdot y_{p2} (y_{p1} + y_{p2}) \]
\[ \frac{2q_p^2}{9.81} = 0.5 \times 1.5 \times (0.5 + 1.5) \]

\[ \frac{2q_p^2}{9.81} = (0.5)(1.5)(2) \]
\( q_p = 2.71 \)
\[ q_r = \frac{q_m}{q_p} = L_r^{3/2} \]
\[ q_m = \left( \frac{1}{9} \right)^{3/2} \times q_p = 0.1 \text{ m}^3/\text{s/m} \]

19. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 03rd Question -pg: 205)
01. Conventional Practice Solutions

Sol: Buckingham π-theorem is stated as:

If there are \( n \) variables (dependent and independent variables) in a dimensionally homogeneous equation and if these variables contain \( m \) fundamental dimensions (such as \( M, L, T, \) etc.) then the variables are arranged into \((n–m)\) dimensionless terms. These dimensionless terms are called \( \pi \)-terms.

Given that drag force on partially submerged body is a function of

\[ F_D = f(V, \nu, K, \rho, g, L) \]

Thus, \( n = 7 \)

and \( m = 3 \) (\( M, L \& T \))

Hence, no. of \( \pi \)-terms \( = n – m = 7 – 3 = 4 \)

Out of 4- \( \pi \)-terms, one of the obvious \( \pi \)-term will be, say \( \pi_1 = \frac{K}{L} \)

Let us choose \( \rho, V \) and \( L \) as the repeating variables. Then,

\[ \pi_2 = F_D \rho^a V^b L^c \]

\[ M^a L^b T^c = MLT^{-2} \left( ML^{-3} \right)^a \left( LT^{-1} \right)^b \left( L \right)^c \]

\[ = M^{1+a} L^{1-3a+b+c} T^{-2-b} \]

Equating the indices of \( M, L \) and \( T \):

For \( M \): \( 1 + a_1 = 0 \) \( \Rightarrow a_1 = -1 \)

For \( T \): \(-2 - b_1 = 0 \) \( \Rightarrow b_1 = -2 \)

For \( L \): \(-3a_1 + b_1 + c_1 = 0 \)

Or, \( 1 + 3 - 2 + c_1 = 0 \)

\[ \Rightarrow c_1 = -2 \]

Thus, \( \pi_2 = F_D \rho^{-1} V^{-2} L^{-2} = \frac{F_D}{\rho L^2 V^2} \)

Similarly,

\[ \pi_3 = \nu \rho^a V^b L^c \]

\[ M^a L^b T^c = ML^{-2} \left( ML^{-3} \right)^a \left( LT^{-1} \right)^b \left( L \right)^c \]

\[ = M^{a} L^{1-3a+b+c} T^{-2-b} \]

For \( M \): \( a_2 = 0 \)

For \( T \): \(-1 - b_2 = 0 \) \( \Rightarrow b_2 = -1 \)

For \( L \): \(-2 + 3a_2 + b_2 + c_2 = 0 \)

Or, \( 2 - 0 - 1 + c_2 = 0 \)

\[ \Rightarrow c_2 = -1 \]

Thus, \( \pi_3 = \nu \rho^{-1} V^{-1} L^{-1} = \frac{V}{\nu L} = \frac{1}{Re} \)

Similarly,

\[ \pi_4 = g \rho^a V^b L^c \]

\[ M^a L^b T^c = LT^{-2} \left( ML^{-3} \right)^a \left( LT^{-1} \right)^b \left( L \right)^c \]

\[ = M^{a} L^{1-3a+b+c} T^{-2-b} \]

For \( M \): \( a_3 = 0 \)

For \( T \): \(-2 - b_3 = 0 \) \( \Rightarrow b_3 = -2 \)

For \( L \): \(-3a_3 + b_3 + c_3 = 0 \)

Or, \( 1 - 0 - 2 + c_3 = 0 \)

\[ \Rightarrow c_3 = 1 \]

So, \( \pi_4 = g \rho^{-1} V^{-2} L = \frac{gL}{V^2} = \frac{1}{F_r^2} \)

Thus, we can write:

\[ \pi_2 = f(\pi_1, \pi_3, \pi_4) \]

Or, \( \frac{F_D}{\rho L^2 V^2} = f\left( \frac{K}{L}, \frac{1}{Re}, \frac{1}{Fr^2} \right) \)
02.

Sol: Given:

River Rectangular Pier (Prototype)  Model

\[ W_p = 1.5 \text{ m}, \quad L_r = 1/25 \]
\[ L_p = 4.5 \text{ m}, \quad V_m = 0.65 \text{ m/s} \]
\[ F_m = 3.92 \text{ N} \]
\[ H_m = 3.5 \text{ cm} \]

where \( H \) is the height of standing wave.

(i) The corresponding speed in the prototype \( V_p \):

As the flow in a river is a free surface flow affected by gravity, the dynamic similarity between the model and its prototype will be achieved by equating the Froude's number.

\[ \frac{V_p}{\sqrt{gL_p}} = \frac{V_m}{\sqrt{gL_m}} \]

Or,
\[ \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = 25 \]

\[ V_p = V_m \times 5 = 0.65 \times 5 = 3.25 \text{ m/s} \]

(ii) The force acting on the prototype, \( F_p \):

Force = mass \( \times \) acceleration = \( \rho L^3 \times \frac{V}{T} \)

\[ = \rho L^3 \times \frac{V}{(L/V)} \quad [\because V = \frac{L}{T} \text{ or } T = \frac{L}{V}] \]

\[ = \rho L^3 \times \frac{V^2}{L} = \rho L^2 V^2 \]

Force ratio, \( F_r = \frac{F_m}{F_p} \)

\[ = \frac{\rho_m L^2}{\rho_p L^2} \times \frac{V_m^2}{V_p^2} = \rho_r L^2 V^2 \]

\[ = L_r^2 \left( \frac{V_m}{V_p} \right)^2 = L_r^2 \]

\[ \therefore F_p = \frac{F_m}{L_r^2} = 3.92 \times (25)^3 \]

\[ = 61,250 \text{ N} = 61.25 \text{ kN} \]

(\( \because \rho_r = 1 \), fluid being same in model and prototype)

(iii) The height of the standing wave in the prototype, \( H_p \):

\[ H_p = H_m \times 25 = 3.5 \times 25 = 87.5 \text{ cm} \]

(iv) The co-efficient of drag resistance:

The co-efficient of drag resistance is defined by

\[ F = C_D \rho A \frac{V^2}{2} \]

where \( F \) is the drag force.

\[ \therefore \quad C_D = \frac{F}{\frac{1}{2} \rho A V^2} \]

Or,
\[ (C_D)_p = \frac{F_p}{\frac{1}{2} \rho_p A_p V_p^2} \]

where,

\( F_p = \) Force acting on the prototype (= 61,250 N),
\( \rho_p = \) Density of water (= 1000 kg/m\(^3\))
\( A_p = \text{width of the pier} \times \text{depth of water in the river} = 1.5 \times 3 = 4.5 \text{ m}^2 \), and
\( V_p = \text{velocity of flow in the prototype} \ (= 3.25 \text{ m/s}). \)

\[
(C_D)_p = \frac{61,250}{\frac{1}{2} \times 1000 \times 4.5 \times (3.25)^2} = 2.58
\]

The drag co-efficient will be same for model and prototype, i.e.,
\( (C_D)_m = (C_D)_p = 2.58 \)

### 14. Flow Through Orifices, Mouth Pieces, Notches and Weirs

01. **Ans:** (c)

**Sol:**

\[
C_v = \frac{V_{act}}{C_{th}}
\]

\[
V_{th} = \sqrt{2gh}
\]

\[
= \sqrt{2 \times 9.81 \times 1.25}
\]

\[
= 4.952 \text{ m/s}
\]

\[
V_{act} = \sqrt{2 \times 9.81 \times 1.2}
\]

\[
= 4.852 \text{ m/s}
\]

\[
C_v = 0.98
\]

02. **Ans:** (d)

**Sol:**

\[
t = \frac{2A}{C_d \alpha \sqrt{2g}}(H_1^{1/2} - H_2^{1/2})
\]

\( C_d, \alpha, H_1, H_2 \) are constant

\[
t \propto \frac{1}{d^2}
\]

\[
t_2 \left( \frac{d_1}{d_2} \right)^2 = \frac{200}{4} = 50
\]

05. **Ans:** (a)

**Sol:**

\[
Q \propto H^{3/2}
\]

\[
= \frac{Q_2 - Q_1}{Q_1}
\]

\[
= \frac{H_2^{3/2} - H_1^{3/2}}{H_1^{3/2}}
\]

\[
= \left( \frac{H_2}{H_1} \right)^{3/2} - 1
\]

\[
= \left( \frac{31}{3D} \right)^{3/2} - 1 = 5.041\%
\]

06. **Ans:** (b)

**Sol:**

\[
Q = C_d \frac{2}{3} \sqrt{2gh(L(H))^{1/2}}
\]

\[
Q \propto H^{3/2}
\]

\[
\frac{dQ}{Q} = \frac{dL}{L} + \frac{2}{3} \frac{dH}{H}
\]

\[
= -1.5 + \frac{3}{2} \times 1
\]

\[
= -1.5 + 1.5 = 0
\]

07. **Ans:** 0.792

**Sol:**

\[
a = 0.0003 \text{ m}^2
\]

\[
H = 1 \text{ m}
\]

\[
C_d = 0.60
\]

\[
\sqrt{2g} = 4.4
\]
\[ Q = C_d \sqrt{2gH} \]

\[ Q = 0.60 \times 0.0003 \times 0.44 \times \sqrt{1} \]

\[ Q = 7.92 \times 10^{-4} \text{ m}^3/\text{sec} \]

\[ = 0.792 \text{ ltr/sec} \]

**08. Ans: 16 : 1**

Discharge through an orifice, \( Q = A. V \)

\[ Q_{\text{Actual}} = C_d \cdot \frac{\pi}{4} d^2 \sqrt{2gH} \]

\[ Q = d^2 \sqrt{h} \]

\[ \frac{Q_1}{Q_2} = \left( \frac{d_1}{d_2} \right)^2 \left( \frac{h_1}{h_2} \right) \]

For same discharges \( Q_1 = Q_2 \)

\[ \left( \frac{d_2}{d_1} \right)^2 = \left( \frac{h_1}{h_2} \right) \]

\[ \frac{h_1}{h_2} = \left( \frac{d_2}{d_1} \right)^4 \Rightarrow \frac{h_1}{h_2} = \left( \frac{2}{1} \right)^4 = \frac{h_1}{h_2} = 16 : 1 \]

**09. Ans: (d)**

Sol: \( Q = 1.418H^{\frac{5}{2}} \)

\[ Q \propto H^{\frac{5}{2}} \]

\[ \frac{Q_2}{Q_1} = \left( \frac{H_2}{H_1} \right)^{\frac{5}{2}} \quad \frac{Q_2}{Q_1} = \left( \frac{0.3}{0.15} \right)^{\frac{5}{2}} \]

\[ \frac{Q_2}{Q_1} = 5.657 \]

**10. Ans: (d)**

Sol: \( Q = \frac{8}{15} C_d \sqrt{2g} \tan \left( \frac{\theta}{2} \right) H^{\frac{5}{2}} \)

\[ Q \propto H^{\frac{5}{2}} \]

\[ \frac{Q_2}{Q_1} = \left( \frac{H_2}{H_1} \right)^{\frac{5}{2}} \]

\[ \frac{Q_2}{Q_1} = \left( \frac{0.2}{0.1} \right)^{\frac{5}{2}} = 5.66 \]

**11. Ans: (d)**

Sol: \( Q \propto f \left( \tan \frac{\theta}{2} \right) \)

\[ Q = K \tan \frac{\theta}{2} \]

\[ \frac{dQ}{Q} = K \sec^2 \frac{\theta}{2} \frac{d\theta}{2} \]

\[ \frac{d\theta}{\theta} = 2\% \quad \text{(Given)} \]

% Error in discharge,

\[ \frac{dQ}{Q} \times 100 = \frac{K \sec^2 \frac{\theta}{2} \frac{d\theta}{2} \times 100}{K \tan \frac{\theta}{2}} \]

\[ = \frac{1}{2} \times \frac{1}{\cos^2 \frac{\theta}{2} \sin \frac{\theta}{2}} \times \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} \times 100 \]

\[ = \frac{1}{\sin \theta} \times d\theta \times 100 \]

\[ = \cosec 90 = \pi \]
**Conventional Practice Solutions**

**01.**

**Sol:** Volume of water falling down = discharge × time

\[ \text{Adh} = Qdt \]

\[ A = 0.93 \text{ m}^2 \]

\[ Q = \frac{8}{15} C_d \sqrt{2g} H^{5/2} \]

\[ H = 0.075 \text{ m} \]

\[ \frac{dh}{dt} = 2.54 \text{ mm} = 2.54 \times 10^{-3} \text{ m/s} \]

Thus substitution

\[ 0.93 \times 2.54 \times 10^{-3} = \frac{8}{15} C_d \sqrt{2 \times 9.81 \times (0.075)^{5/2}} \]

\[ C_d = 0.649 \]

**02.**

**Sol:** Given:

Width of river = crest length

\[ L = 30 \text{ m} \]

Depth of flow, \( y = 3 \text{ m} \)

\[ \therefore \text{Area of flow section} = (30 \times 3) = 90 \text{ m}^2 \]

Mean velocity of flow

\[ V = 1.2 \text{ m/sec} \]

Discharge \( Q = AV \)

\[ = (90 \times 1.2) = 108 \text{ m}^3/\text{sec} \]

Since the anicut (Weir) is constructed to raise the water level by 1m, the depth of flow on the upstream of the anicut becomes

\[ (3+1) = 4 \text{ m} \]

Velocity of approach

\[ V_a = \frac{Q}{A_a} = \frac{108}{30 \times 4} = 0.9 \text{ m/s} \]

Head due to velocity of approach

\[ h_a = \frac{V_a^2}{2g} = \frac{(0.9)^2}{2 \times 9.81} = 0.0413 \text{ m} \]

Assuming that the weir is discharging free, then

\[ Q = \frac{2}{3} C_{d1} L \sqrt{2g} \left[ (H + h_a)^{3/2} - h_a^{3/2} \right] \]

Thus by substitution,

\[ 108 = \frac{2}{3} \times 0.58 \times 30 \times \sqrt{2 \times 9.81} \left[ (H + 0.0413)^{3/2} - (0.0413)^{3/2} \right] \]

\[ \Rightarrow H = 1.604 \text{ m} \]

The height of the weir is then

\[ Z = (4 - 1.604) = 2.396 \text{ m} \]

Since the depth of water in the channel on the downstream of the weir will also be 3m, the anicut will be submerged.
For a submerged weir the discharge is given by

\[ Q = Q_1 + Q_2 \]

\[ Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g \left[ \left( (H_1 - H_2) + h_a \right)^{3/2} - h_a^{3/2} \right]} \]

\[ Q_2 = C_{d2} \times (L \times H_2) \sqrt{2g \left( H_2 - H_1 \right) + V_a^2} \]

\( (H_1 - H_2) = 1 \text{ m} \)

Given \( C_{d1} = 0.58 \); \( C_{d2} = 0.80 \)

Thus by substitution, we get

\[ 108 = \frac{2}{3} \times 0.58 \times 30 \times \sqrt{2 \times 9.81 \times \left[ \left( 1 + 0.0413 \right)^{1/2} - (0.0413)^{1/2} \right] + 0.80 \times (0.30 \times H_2) \sqrt{2 \times 9.81 \times 1 + (0.9)^2}} \]

Or, \( 108 = 54.17 + 108.5H_2 \)

\[ H_2 = \frac{108 - 54.17}{108.5} = 0.496 \text{ m} \]

Therefore the height of the anicut

\[ = (3 - 0.496) = 2.504 \text{ m} \]

We know for the orifices, the coefficient of velocity is related as

\[ C_v = \sqrt{\frac{x^2}{4yH}} \]

\( \therefore \) The trajectory is given by

\[ x^2 = 4yHC_v^2 \]..............(i)

At the point of intersection of the two jets

\[ x_a^2 = 4y_aH_aC_v^2 \]..............(ii)

\[ x_b^2 = 4y_bH_bC_v^2 \]..............(iii)

But \( x_a = x_b = x \)

Equating (ii) and (iii)

\[ \frac{y_a}{y_b} = \frac{H_b}{H_a} \]..............(iv)

Also \( H_a + y_a = H_b + y_b \)

\[ y_a - y_b = H_b - H_a \]..............(v)

Solving for \( y_a \) from Eqs. (iv) and (v)

From Equation (iv)

\[ y_a = \frac{H_b}{H_a} \times y_b \]..............(vi)

substitute the value of \( y_a \) in equation of (v)
\[ \frac{H_b}{H_a} \times y_b - y_b = H_b - H_a \]

\[ y_b \left( \frac{H_b}{H_a} - 1 \right) = H_b - H_a \]

\[ y_b \left( \frac{H_b - H_a}{H_a} \right) = H_b - H_a \]

\[ \Rightarrow y_b = H_a \]

Substituting the value of \( y_b \) in equation (vi)

\[ y_a = H_b \]

Now, Substituting in Eq. (i),

\[ x^2 = 4y_aH_aC_v^2 = 4y_bH_bC_v^2 \]

\[ x = \sqrt{4y_aH_aC_v^2} = 2C_v\sqrt{H_aH_b} \]