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FLUID MECHANICS

Text Book : Theory with worked out Examples and Practice Questions

5 Fluid Mechanics

(Solutions for Text Book Practice Questions)

01. Properties of Fluids

01. Ans: (c)

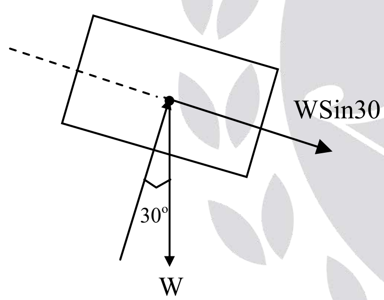
Sol: For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).

02. Ans: 100

Sol: $\tau = \dfrac{\mu V}{h} = \dfrac{0.2 \times 1.5}{3 \times 10^{-3}} = 100\ \text{N/m}^2$

03. Ans: 1

Sol:



$$F = \tau \times A$$

$$W \sin 30 = \frac{\mu A V}{h}$$

$$\frac{100}{2} = \frac{1 \times 0.1 \times V}{2 \times 10^{-3}}$$

$$V = 1\text{m/s}$$

Common data Q. 04 & 05

04. Ans: (c)

Sol: $D_1 = 100$ mm , $D_2 = 106$ mm

$$\text{Radial clearance, } h = \frac{D_2 - D_1}{2}$$

$$= \frac{106 - 100}{2} = 3\text{mm}$$

$L = 0.15\text{m}$

$\mu = 0.2$ pa.s

$N = 240$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$

$$\omega = 8\pi$$

$$\tau = \frac{\mu \omega r}{h} = \frac{0.2 \times 8\pi \times 50 \times 10^{-3}}{3 \times 10^{-3}}$$

$$= 83.77\text{N/m}^2$$

05. Ans: (b)

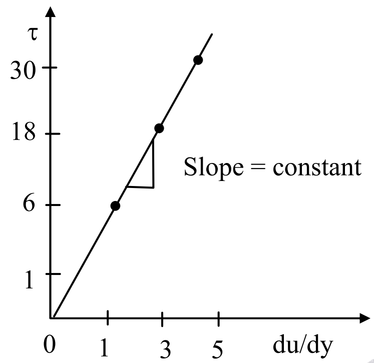
Sol: $$\text{Power, } P = \frac{2\pi \omega^2 \mu L r^3}{h}$$

$$= \frac{2\pi \times (8\pi)^2 \times 0.2 \times 0.15 \times (0.05)^3}{3 \times 10^{-3}}$$

$$= 4.96\ \text{Watt}$$

06. Ans: (c)

Sol:



∴ Newtonian fluid

07. Ans: (a)

Sol: $\tau = \mu \dfrac{du}{dy}$

$u = 3\sin(5\pi y)$

$\dfrac{du}{dy} = 3\cos(5\pi y)\times 5\pi = 15\pi\cos(5\pi y)$

$\tau\big|_{y=0.05} = \mu \dfrac{du}{dy}\Big|_{y=0.05}$

$= 0.5\times 15\pi\cos(5\pi\times 0.05)$

$= 0.5\times 15\pi\times\cos\left(\dfrac{\pi}{4}\right) = 0.5\times 15\pi\times\dfrac{1}{\sqrt{2}}$

$= 7.5\times 3.14\times 0.707 \approx 16.6\text{N/m}^2$

08. Ans: (d)

Sol:

- Ideal fluid → Shear stress is zero.
- Newtonian fluid → Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid → Shear stress does not vary linearly with the rate of strain.
- Bingham plastic → Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

09. Ans: (b)

Sol: $V = 0.01\text{ m}^3$

$\beta = 0.75\times 10^{-9}\text{ m}^2/\text{N}$

$dP = 2\times 10^7\text{ N/m}^2$

$K = \dfrac{1}{\beta} = \dfrac{1}{0.75\times 10^{-9}} = \dfrac{4}{3}\times 10^9$

$K = \dfrac{-dP}{dV/V}$

$dV = \dfrac{-2\times 10^7\times 10^{-2}\times 3}{4\times 10^9} = -1.5\times 10^{-4}$

10. Ans: 320 Pa

Sol: $\Delta P = \dfrac{8\sigma}{D} = \dfrac{8\times 0.04}{1\times 10^{-3}} = \dfrac{32\times 10^{-2}}{10^{-3}}$

$\Delta P = 320\text{ N/m}^2$

11. Ans: (d)

Sol:

- As the temperature is increased, the viscosity of a liquid decreases due to the reduction in intermolecular cohesion.
- In gases, the viscosity increases with the rise in temperature due to increased molecular activity causing an increase in the change of momentum of the molecules, normal to the direction of motion.

- Thus, statement (I) is wrong but statement (II) is correct.

12. Ans: (c)

Sol: The surface energy is given by

$$E = \sigma \times \text{area}$$

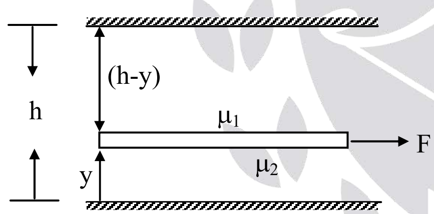
As area increases, surface energy will increase. Thus, statement (I) is correct.

- Surface tension, σ is the property of fluid. Hence, it is independent of the size of the bubble. Thus, statement (II) is wrong.

Conventional Practice Solutions

01.

Sol:



Assumptions:

- Thin plate has negligible thickness.
- Velocity profile is linear because of narrow gap.
- Given fluid is a Newtonian fluid which obeys Newton's law of viscosity.

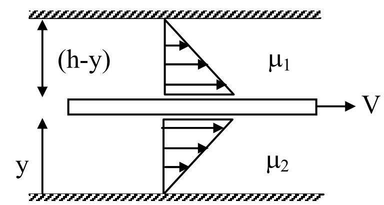
The force required to pull it is proportional to the total shear stress imposed by the two oil layers.

$$F = F_1 + F_2 ,$$

Where F_1 = Force on top sides of plate,

F_2 = Force on bottom side of plate

The plate moves with velocity V



From Newton's law of viscosity,

$$\tau = \frac{\mu du}{dy}$$ Let A be area of plate

$$\therefore \quad F_1 = \tau_1 \times \text{Area of plate}$$

$$F_1 = \mu_1 \times \frac{V}{h-y} \times A$$

$$F_2 = \mu_2 \times \frac{V}{y} \times A$$

(i) Shear force on two sides of the plate are equal:

$$F_1 = F_2$$

$$\frac{\mu_1 \times VA}{h-y} = \frac{\mu_2 VA}{y}$$

$$\frac{\mu_1}{\mu_2} = \frac{h-y}{y}$$

$$\frac{h}{y} = \frac{\mu_1}{\mu_2} + 1$$

$$\frac{h}{y} = \frac{\mu_1 + \mu_2}{\mu_2}$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2}$$

(ii) The position of plate so that pull required to drag the plate is minimum.

$$F = \frac{\mu_1 VA}{h-y} + \frac{\mu_2 VA}{y},$$

[V, A, μ_1 & μ_2, h are constant]

For minimum force, $\frac{dF}{dy} = 0$

$$-\mu_1 VA(h-y)^{-2}(-1) - \mu_2 VAy^{-2} = 0$$

$$\frac{\mu_2 VA}{y^2} = \frac{\mu_1 VA}{(h-y)^2}$$

$$\frac{(h-y)^2}{y^2} = \frac{\mu_1}{\mu_2} \Rightarrow \frac{h-y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$\frac{h}{y} = 1 + \sqrt{\frac{\mu_1}{\mu_2}}$ where y is the distance of the thin flat plate from the bottom flat surface.

$$y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$$

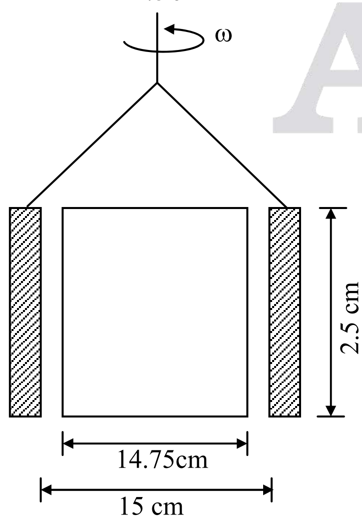
02. Ans: 0.372 Pa. s

Sol: Torque = 1.2 N-m

Speed, N = 600 rpm

Diameter, D_1 = 15 cm , D_2 = 14.75 cm

H = 2.5cm



Assumptions:

- The gap between two cylinders is narrow and hence velocity profile in the gap is assumed linear.
- No change in properties

Torque = Tangential force × radius

Force = shear stress×Area

$$= \frac{\mu \times VA}{h}$$

Where h is the clearance (radial)

$$h = \frac{15 - 14.75}{2}$$

$$= 0.125\text{cm} = 1.25\times10^{-3}\text{m}$$

Area = πDL

$$= \pi\times0.15\times2.5\times10^{-2}$$

$$= 11.781\times10^{-3}\text{m}^2$$

$$F_s = \frac{\mu \times \omega r \times A}{h}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times600}{60} = 20\pi \text{ rad/s}$$

Torque = $F_s \times r$

$$= \frac{\mu\omega rA}{h}\times r$$

$$= \frac{\mu\omega r^2}{h}\times A$$

$$1.2 = \frac{\mu\times20\pi\times(0.07375)^2\times11.781\times10^{-3}}{1.25\times10^{-3}}$$

μ = 0.3726 Pa.s

02. Pressure Measurement & Fluid Statics

01. Ans: (a)

Sol: 1 millibar = $10^{-3}\times10^{5}$ = 100 N/m^2

One mm of Hg = $13.6\times10^{3}\times9.81\times1\times10^{-3}$

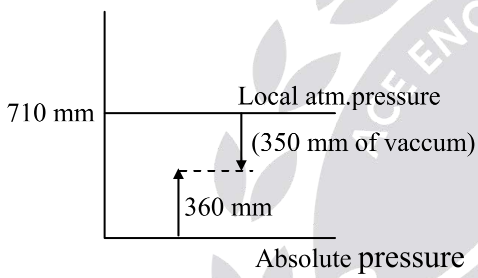
= 133.416 N/m^2

1 N/mm^2 = 1×10^{6} N/m^2

1 kgf/cm^2 = 9.81×10^{4} N/m^2

02. Ans: (b)

Sol:



03. Ans: (c)

Sol: Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure P_A and P_B are same.

04. Ans: (b)

Sol:

- The manometer shown in Fig.1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.
- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.

05. Ans: 2.2

Sol: h_p in terms of oil

$s_o h_o = s_m h_m$

$0.85\times h_0 = 13.6\times0.1$

$h_0 = 1.6$m

$h_p = 0.6+1.6$

$\Rightarrow$ $h_p = 2.2$m of oil

(or) $P_p - \gamma_{oil}\times0.6 - \gamma_{Hg}\times0.1 = P_{atm}$

$$\frac{P_p - P_{atm}}{\gamma_{oil}} = \left(\frac{\gamma_{Hg}}{\gamma_{oil}}\times0.1+0.6\right)$$

$$= \frac{13.6}{0.85}\times0.1+0.6 = 2.2 \text{ m of oil}$$

Gauge pressure of P in terms of m of oil

= 2.2 m of oil

06. Ans: (b)

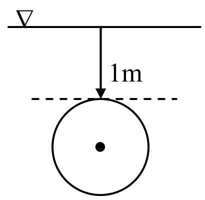
Sol: $$h_M - \frac{s_w}{s_0}h_{w_1} = h_N - \frac{s_w h_{w_2}}{s_0} - h_0$$

$$h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3$$

$$h_M - h_N = -13.843 \text{ cm of oil}$$

07. Ans: 2.125

Sol:



$$h_P = \bar{h} + \frac{I}{A\bar{h}}$$

$$= 2 + \frac{\pi D^4 \times 4}{64 \times D^2 \times 2 \times \pi}$$

$$= 2 + \frac{2^2 \times 4}{64 \times 2} = 2.125\text{m}$$

08. Ans: 10

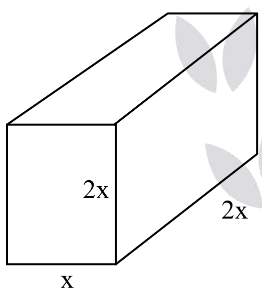
Sol: $F = \rho g \bar{h} A$

$$= 9810 \times 1.625 \times \frac{\pi}{4}\left(1.2^2 - 0.8^2\right)$$

$F = 10\text{kN}$

09. Ans: 1

Sol:



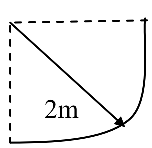
$F_{bottom} = \rho g \times 2x \times 2x \times x$

$F_V = \rho g x \times 2x \times 2x$

$$\frac{F_B}{F_V} = 1$$

10. Ans: 10

Sol:



$F_V = x \times \pi$

$$F_V = \rho g V = 1000 \times 10 \times \frac{\pi \times 2^2}{4}$$

$F_V = 10\pi$ kN

$\therefore$ x = 10

11. Ans: (d)

Sol: $F_{net} = F_{H1} - F_{H2}$

$$F_{H1} = \gamma \times \frac{D}{2} \times D \times 1 = \frac{\gamma D^2}{2}$$

$$F_{H2} = \gamma \times \frac{D}{4} \times \frac{D}{2} \times 1 = \frac{\gamma D^2}{8}$$

$$= \gamma D^2\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\gamma D^2}{8}$$

12. Ans: 2

Sol: Let P be the absolute pressure of fluid f3 at mid-height level of the tank. Starting from the open limb of the manometer (where pressure = P_{atm}) we write :

$$P_{atm} + \gamma \times 1.2 - 2\,\gamma \times 0.2 - 0.5\,\gamma \times \left(0.6 + \frac{h}{2}\right) = P$$

or $P - P_{atm} = P_{gauge}$

$$= \gamma(1.2 - 2\times0.2 - 0.5\times 0.6 - 0.5 \times \frac{h}{2})$$

For P_{gauge} to be zero, we have,

$$\gamma(1.2 - 0.4 - 0.3 - 0.25\,h) = 0$$

or $$h = \frac{0.5}{0.25} = 2$$

13. Ans: (b)

Sol: The depth of centre of pressure from the free liquid surface is given by

$$h_{cp} = \bar{h} + \frac{I_{xx,c}}{A\bar{h}} \qquad \text{--------(1)}$$

Or, $$h_{cp} - \bar{h} = \frac{I_{xx,c}}{A\bar{h}}$$

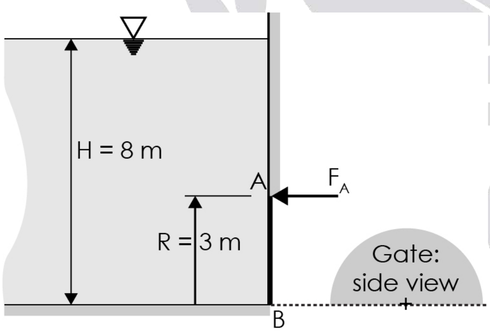
From the above relationship, as $\bar{h}$ increases, $\frac{I_{xx,c}}{A\bar{h}}$ decreases. Thus, at great depth, the difference (h_{cp} – $\bar{h}$) becomes negligible. Hence, statement (I) is correct.

Also, it is clear from equation (1) that h_{cp} is independent of the density of the liquid.

Conventional Practice Solutions

01.

Sol:



$$\bar{h} = 5 + \left(3 - \frac{4\times R}{3\pi}\right)$$

$$= 5 + \left(3 - \frac{4\times 3}{3\pi}\right) = 5 + 1.727 = 6.727 \text{ m}$$

$F_H = \gamma_w \times 6.727 \times$ Area (projected)

$$= \gamma_w \times 6.727 \times \frac{\pi\times 3^2}{2}$$

$$= \gamma_w \times 6.727 \times 4.5\pi$$

$= 932.94$ kN

$$h_{cp} = 6.727 + \frac{0.10976\,R^4}{\frac{\pi R^2}{2}\times 6.727}$$

$$= 6.727 + \frac{0.10976\times 3^2 \times 2}{\pi\times 6.727}$$

$= 6.727 + 0.0935$

$= 6.8205$ m from free liquid surface

$= (8 – 6.8205)$ m from base B

$= 1.1795$ m from base B.

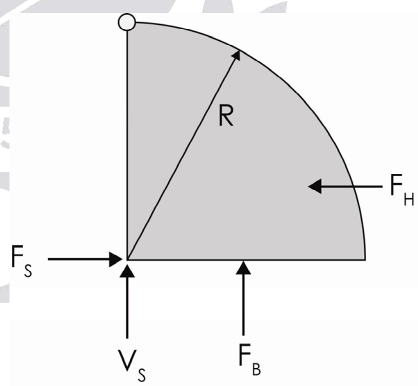
Taking moment about B

$F_A \times 3 = 932.94 \times 1.1795$

$\Rightarrow F_A = 366.8$ kN

02.

Sol:



$$\bar{h} = \left(1.5 + \frac{R}{2}\right)$$

$$F_H = \rho g\bar{h}A_{projected}$$

$$= \rho g\left(1.5 + \frac{R}{2}\right)(R\times 3)$$

$= \gamma(1.5+1.5)(3\times 3)$

$= 27\,\gamma$ N

$$h_{cp} = 3 + \frac{\frac{1}{12}(3\times 3^3)}{(3\times 3)(3)}$$

= 3.25 m from free liquid surface

= 3.25 – 1.5 = 1.75 m from A

$$F_B = \gamma\left(\frac{\pi R^2}{4}\right)(3) = \gamma\times\frac{\pi\times 9\times 3}{4} = \frac{27\pi\gamma}{4}\text{ N}$$

F_B will act through the centroid of the quadrant which is at a distance $\frac{4R}{3\pi}$ from the vertical line AB. Now, taking moment of the forces about the hinge A, we write

$$F_s\times 3 + F_B\times\frac{4R}{3\pi} - F_H\times 1.75 = 0$$

where F_s is the force in x-direction on the stop at B & V_s is in y-direction (does not contribute in the moment).

$$3F_s = 27\times 1.75\gamma - \frac{27\pi\gamma}{4}\times\frac{4R}{3\pi} = 10^4(27\times 1.75 - 9\times 3)$$

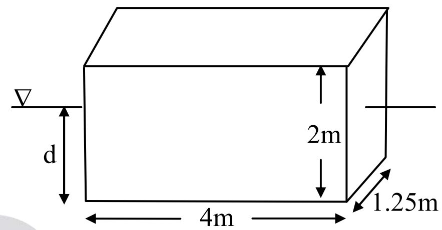
$$= 10^4\times 27\times 0.75 = 202.5\text{ kN.m}$$

$$\Rightarrow F_s = \frac{202.5}{3} = 67.5\text{ kN}$$

03. Buoyancy and Metacentric Height

01. Ans: (d)

Sol:



F_B = weight of body

$\rho_b g V_b = \rho_f g V_f d$

$640\times 4\times 2\times 1.25 = 1025\times(4\times 1.25\times d)$

$d = 1.248$m

$V_{fd} = 1.248\times 4\times 1.25$

$V_{fd} = 6.24\text{ m}^3$

02. Ans: (c)

Sol: Surface area of cube = $6\,a^2$

Surface area of sphere = $4\,\pi r^2$

$4\pi r^2 = 6a^2$

$$\frac{2\pi}{3} = \left(\frac{a}{r}\right)^2$$

$F_{b,s} \propto V_s$

$$= \frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{4}{3}\frac{\pi r^3}{\left(r\sqrt{\frac{2\pi}{3}}\right)^3} = \frac{4}{3}\frac{\pi r^3}{\left(\sqrt{\frac{2\pi}{3}}\times\sqrt{\frac{2\pi}{3}}r^3\right)} = \sqrt{\frac{6}{\pi}}$$

03. Ans: 4.76

Sol: $F_B = F_{B,Hg} + F_{B,W}$

$W_B = F_B$

$\rho_b g\forall_b = \rho_{Hg} g\forall_{Hg} + \rho_w g\forall_w$

$\rho_b\forall_b = \rho_{Hg}\forall_{Hg} + \rho_w\forall_w$

$S\times\forall_b = S_{Hg}\forall_{Hg} + S_w\forall_w$

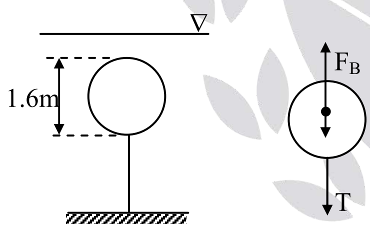
$7.6\times10^3 = 13.6\times10^2(10-x)+10^2\times x$

$-6000 = -1260x$

$x = 4.76$ cm

04. Ans: 11

Sol:



$F_B = W + T$

$W = F_B - T$

$= \rho_f g V_{fd} - T$

$$=10^3\times9.81\times\frac{4}{3}\pi(0.8)^3-\left(10\times10^3\right)$$

$= 21 - 10$

$W = 11$ kN

05. Ans: 1.375

Sol: $W_{water} = 5$ N

$W_{oil} = 7$ N

$S = 0.85$

W – Weight in air

$F_{B1} = W - 5$

$F_{B2} = W - 7$

$W - 5 = \rho_1 g V_{fd}$…..(1)

$W - 7 = \rho_2 g V_{fd}$…..(2)

$V_{fd} = V_b$

$W - 5 = \rho_1 g V_b$

$W - 7 = \rho_2 g V_b$

$2 = (\rho_1 - \rho_2) g V_b$

$$V_b = \frac{2}{(1000-850)9.81}$$

$V_b = 1.3591\times10^{-3}\ m^3$

$W = 5 + (9810\times1.3591\times10^{-3})$

$W = 18.33$ N

$W = \rho_b\, g\, V_b$

$$\frac{18.33}{9.81\times1.3591\times10^{-3}} = \rho_b$$

$\rho_b = 1375.05\ kg/m^3$

$S_b = 1.375$

06. Ans: (d)

Sol: For a floating body to be stable, metacentre should be above its center of gravity. Mathematically GM > 0.

07. Ans: (b)

Sol: $W = F_B$

$\rho_b g V_b = \rho_f g V_{fd}$

$\rho_b V_b = \rho_f V_{fd}$

$$0.6\times\frac{\pi}{4}d^2\times 2d = 1\times\frac{\pi}{4}d^2\times x$$

$x = 1.2d$

$GM = BM - BG$

$$BM = \frac{I}{V} = \frac{\pi d^4}{64\times\frac{\pi}{4}d^2\times 1.2d} = \frac{d}{19.2} = 0.052d$$

$BG = d - 0.6d = 0.4d$

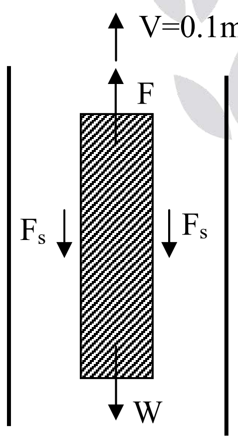
Thus, $GM = 0.052d - 0.4d = -0.348\ d$

$GM < 0$

$\Rightarrow$ Hence, the cylinder is in unstable condition.

08. Ans: 122.475

Sol:



The thickness of the oil layer is same on either side of plate

y = thickness of oil layer

$$= \frac{23.5-1.5}{2} = 11\text{mm}$$

Shear stress on one side of the plate

$$\tau = \frac{\mu dU}{dy}$$

F_s = total shear force (considering both sides of the plate)

$$= 2A\times\tau = \frac{2A\mu V}{y}$$

$$= \frac{2\times1.5\times1.5\times2.5\times0.1}{11\times10^{-3}}$$

$= 102.2727$ N

Weight of plate, W = 50 N

Upward force on submerged plate,

$F_v = \rho g V = 900\times 9.81\times 1.5\times 1.5\times 10^{-3}$

$= 29.7978$ N

Total force required to lift the plate

$= F_s + W - F_v$

$= 102.2727 + 50 - 29.7978$

$= 122.4749$ N

09. Ans: (d)

Sol:

- Statement (I) is wrong because the balloon filled with air cannot go up and up, if it is released from the ground.
- However, with increase in elevation, the atmospheric pressure and temperature both decrease resulting into a decrease in air density. Thus, statement (II) is correct.

Conventional Practice Solutions

01. Ans: (i) 0.33, (ii) 0.5 m

Sol: Given data :

Inner diameter of hollow cylinder,

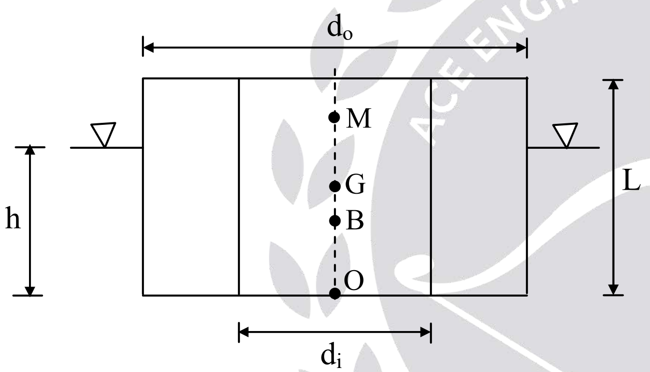
$d_i = 300$ mm

Outer diameter of hollow cylinder,

$d_o = 600$ mm

S.G. of wooden hollow cylinder = 0.56

S.G. of oil = 0.85



Let 'h' be the depth of immersion of the cylinder in oil and L be the height of the cylinder.

Weight of hollow cylinder = Buoyant force acting on the hollow cylinder

Or, $\gamma_{cyl} \times \frac{\pi}{4}\left(d_0^2 - d_i^2\right) \times L = \gamma_{oil} \times \frac{\pi}{4}\left(d_0^2 - d_i^2\right) \times h$

Or, $h = \frac{\gamma_{cyl}}{\gamma_{oil}} \times L = \frac{0.56}{0.85} L = 0.66\ L$

Let us then calculate the maximum height of the cylinder, L for the stable equilibrium condition.

The centre of buoyancy B will be at a distance $\frac{h}{2}$ from O as shown in the figure.

Or, $OB = \frac{h}{2} = 0.33L$

and $OG = \frac{L}{2} = 0.5L$

Now, $BM = \frac{I}{\forall}$

$$= \frac{\pi}{64}\left(d_0^4 - d_i^4\right) \times \frac{4}{\pi \times \left(d_0^2 - d_i^2\right) \times h}$$

$$= \frac{\left(d_0^2 + d_i^2\right)}{16h} = \frac{\left(0.6^2 + 0.3^2\right)}{16 \times 0.66L}$$

$$= \frac{0.0426}{L}$$

Thus, GM = BM – (OG – OB)

$$= \frac{0.0426}{L} - \left(0.5L - 0.33L\right)$$

$$= \frac{0.0426}{L} - 0.17L$$

For stable equilibrium condition, GM ≥ 0. Putting GM = 0 for the maximum height of the cylinder, we get

$$\frac{0.0426}{0.17} = L^2$$

⇒ L = 0.5 m

Thus, h = 0.66 ×0.5 = 0.33 m

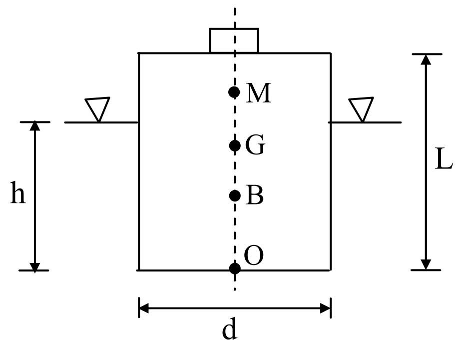
02. Ans: Unstable

Sol: Given data:

d = 1.0 m, L = 1.5 m,

$\rho_{sea\ water} = 1026$ kg/m

$m_{buoy} = 80$ kg m = 10 kg



$$(80 + 10)\times g = \frac{\pi}{4}\times 1^2 \times h \times 1026 \times g$$

where h is the depth of immersion of the buoy.

Thus, $h = \dfrac{4\times 90}{\pi\times 1026} = 0.1117$ m

$$OB = \frac{h}{2} = 0.05585 \text{ m}$$

The position of G due to a mass of 10 kg added to the cylindrical buoy is evaluated as:

$$80 \times 0.75 + 10 \times 1.5 = 90 \times OG$$

Or, $OG = \dfrac{75}{90} = 0.833$m

$$BM = \frac{I}{\forall} = \frac{\pi}{64}\times 1^4 \times \frac{4}{\pi\times 1^2 \times h}$$

$$= \frac{1}{16\times 0.1117} = 0.5595 \text{ m}$$

Thus, GM = BM – (OG – OB)

= 0.5595 – (0.833 – 0.05585)

= – 0.21765 m

Or, GM < 0

Thus, the buoy floats in unstable condition.

04. Fluid Kinematics

01. Ans: (b)

Sol:

- Constant flow rate signifies that the flow is steady.
- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.

Common Data for Questions 02 & 03

02. Ans: 0.94

Sol: $a_{Local} = \dfrac{\partial V}{\partial t}$

$$= \frac{\partial}{\partial t}\left(2t\left(1-\frac{x}{2L}\right)^2\right)$$

$$= \left(1-\frac{x}{2L}\right)^2 \times 2$$

$$(a_{Local})_{at\ x\,=\,0.5,\ L\,=\,0.8} = 2\left(1-\frac{0.5}{2\times 0.8}\right)^2$$

$$= 2(1 - 0.3125)^2 = 0.945 \text{ m/sec}^2$$

03. Ans: –13.68

Sol: $a_{convective} = v.\dfrac{\partial v}{\partial x} = \left[2t\left[1-\dfrac{x}{2L}\right]^2\right]\dfrac{\partial}{\partial x}\left[2t\left(1-\dfrac{x}{2L}\right)^2\right]$

$$= \left[2t\left[1-\frac{x}{2L}\right]^2\right]2t\left[2\left(1-\frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$$

At t = 3 sec; x = 0.5 m; L = 0.8 m

$$a_{convective} = 2\times 3\left[1-\frac{0.5}{2\times 0.8}\right]^2 \times 2\times 3\left[2\left(1-\frac{0.5}{2\times 0.8}\right)\right]\left[\frac{-1}{2\times 0.8}\right]$$

$a_{convective} = -14.62\ m/sec^2$

$a_{total} = a_{local} + a_{convective} = 0.94 - 14.62$

$= -13.68\ m/sec^2$

04. Ans: (d)

Sol: $u = 6xy - 2x^2$

Continuity equation for 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 6y - 4x$$

$$(6y - 4x) + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = (4x - 6y) = 0$$

$\partial v = (4x - 6y)\ dy$

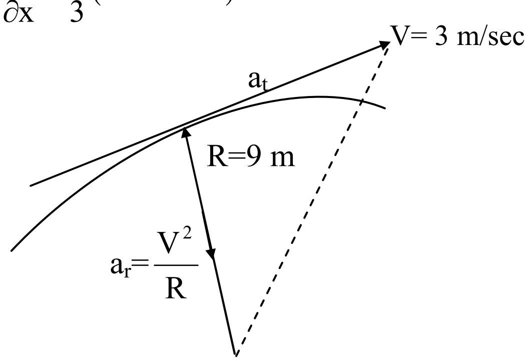
$v = \int 4x dy - \int 6y dy$

$= 4xy - 3y^2 + c$

$= 4xy - 3y^2 + f(x)$

05. Ans: $\sqrt{2}$ = 1.414

Sol: $\frac{\partial V}{\partial x} = \frac{1}{3}(m/sec/m)$



$$a_r = \frac{V^2}{R} = \frac{(3)^2}{9} = \frac{9}{9} = 1\ m/s^2$$

$$a_t = V\frac{\partial V}{\partial x} = 3\times\frac{1}{3} = 1\ m/s^2$$

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}\ m/sec^2$$

06. Ans: 13.75

Sol: $a_{t\ (conv)} = V_{avg} \times \frac{dV}{dx}$

$$a_{t\ (conv)} = \left(\frac{2.5+3}{2}\right)\left(\frac{3-2.5}{0.1}\right) = 2.75 \times 5$$

$a_{t\ (conv)} = 13.75\ m/s^2$

07. Ans: 0.3

Sol: $Q = Au$

$$a_{Local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

$$a_{local} = \frac{1}{A}\frac{\partial Q}{\partial t}$$

$$a_{Local} = \left(\frac{1}{0.4 - 0.1x}\right)\frac{\partial Q}{\partial t}$$

$$(a_{Local})_{at\ x=0} = \frac{1}{0.4}\times 0.12 \quad (\because \frac{\partial Q}{\partial t} = 0.12)$$

$= 0.3\ m/sec^2$

08. Ans: (b)

Sol: $\psi = x^2 - y^2$

$a_{Total} = (a_x)\hat{i} + (a_y)\hat{j}$

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 - y^2) = 2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= (2y)(0) + (2x)(2)$$

$$\therefore a_x = 4x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= (2y)\times(2) + (2x)\times(0)$$

$$a_y = 4y$$

$$\therefore a = (4x)\,\hat{i} + (4y)\,\hat{j}$$

09. Ans: (b)

Sol: Given, The stream function for a potential flow field is $\psi = x^2 - y^2$

$\phi = ?$

$$u = \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial\left(x^2 - y^2\right)}{\partial y}$$

$$u = 2y$$

$$u = -\frac{\partial\phi}{\partial x} = 2y$$

$$\int\partial\phi = -\int 2y\partial x$$

$$\phi = -2xy + c_1$$

Given, ϕ is zero at (0,0)

$\therefore c_1 = 0$

$\therefore \phi = -2xy$

10. Ans: 4

Sol: Given, 2D – flow field

Velocity, V = 3xi + 4xyj

$u = 3x, \quad v = 4xy$

$$\omega_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

$$\omega_z = \frac{1}{2}(4y - 0)$$

$$(\omega_Z)_{at(2,2)} = \frac{1}{2}\times 4(2) = 4 \text{ rad/sec}$$

11. Ans: (b)

Sol: Given, $u = 3x$,

$v = Cy$,

$w = 2$

The shear stress, τ_{xy} is given by

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left[\frac{\partial}{\partial y}(3x) + \frac{\partial}{\partial x}(Cy)\right]$$

$$= \mu\,(0 + 0) = 0$$

12. Ans: (d)

Sol:

- The total acceleration is given as

$$\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}$$

where the first term on the R.H.S is the local acceleration and the second term is the convective acceleration.

- If the flow is steady, then local acceleration will be zero, not the convective acceleration.
- The convective acceleration arises due to the fact that a fluid element experiences different velocities at different locations. Thus, statement (I) is wrong whereas statement (II) is correct.

Conventional Practice Solutions

01. Ans: (ii) $y = \pm x$ (iii) (0, 0)

Sol: Given: $u = c(x^2 - y^2)$ and $v = -2cxy$

The equation of a streamline is given by

$$\frac{dx}{u} = \frac{dy}{v}$$

Or, $$\frac{dy}{dx} = \frac{v}{u} = -\frac{2cxy}{c(x^2 - y^2)} = -\frac{2xy}{x^2 - y^2}$$

(ii) For flow to be parallel to y-axis, $u = 0$

Or, $$\frac{dy}{dx} = \frac{v}{x^2 - y^2} = \infty$$

This is possible when $x = \pm y$

(iii) The fluid is stationary when u & v both are zero.

From the velocity components given, it is possible when (x, y) = (0, 0)

(i) From the equation of streamline

$$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$$

Or, $$\frac{dx}{dy} = -\frac{x^2 - y^2}{2xy} \quad \text{----------(1)}$$

Let $x = fy$ or $dx = fdy + ydf$

Or, $$\frac{dx}{dy} = f + y\frac{df}{dy} \quad \text{---------(2)}$$

Equating (1) with (2),

$$f + y\frac{df}{dy} = -\frac{f^2y^2 - y^2}{2fy \times y} = -\frac{f^2 - 1}{2f} = \frac{1 - f^2}{2f}$$

Or, $$y\frac{df}{dy} = \frac{1 - f^2}{2f} - f = \frac{1 - 3f^2}{2f}$$

Or, $$\frac{2f}{1 - 3f^2}df = \frac{dy}{y}$$

$$\frac{6f}{3f^2 - 1}df = -\frac{3dy}{y}$$

Integrating

$$\ln(3f^2 - 1) + 3\ln y = \ln C$$

Or, $(3f^2 - 1) \times y^3 = C$

Or, $$\left(3\frac{x^2}{y^2} - 1\right)y^3 = C$$

Or, $3x^2y - y^3 = C$

Or, $x^2y - y^3/3 = \text{constant}$, proved

05. Energy Equation and its Applications

01. Ans: (c)

Sol: Applying Bernoulli's equation for ideal fluid

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

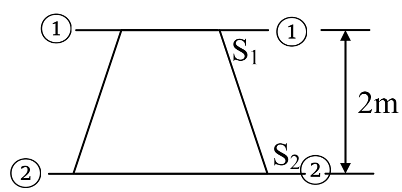
$$\frac{P_1}{\rho g} + \frac{(2)^2}{2g} = \frac{P_2}{\rho g} + \frac{(1)^2}{2g}$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{4}{2g} - \frac{1}{2g}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{3}{2g} = \frac{1.5}{g}$$

02. Ans: (c)

Sol:



$\frac{V_1^2}{2g} = 1.27\text{m}$, $\frac{P_1}{\rho g} = 2.5\text{m}$

$\frac{V_2^2}{2g} = 0.203\text{m}$, $\frac{P_2}{\rho g} = 5.407\text{m}$

$Z_1 = 2$ m , $Z_2 = 0$ m

Total head at (1) – (1)

$$= \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + Z_1$$

= 1.27 + 2.5 + 2 = 5.77 m

Total head at (2) – (2)

$$= \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + Z_2$$

= 0.203 + 5.407 + 0 = 5.61 m

Loss of head = 5.77 – 5.61 = 0.16 m

∴ Energy at (1) – (1) > Energy at (2) – (2)

∴ Flow takes from higher energy to lower energy

i.e. from (S_1) to (S_2)

Flow takes place from top to bottom.

03. Ans: 1.5

Sol: $A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\text{ mm}^2$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\text{ mm}^2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$Z_1 = Z_2$, it is in horizontal position

Since, at outlet, pressure is atmospheric

$P_2 = 0$

Q = 100 lit/sec = 0.1 m^3/sec

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{7.85\times10^{-3}} = 12.73\text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{1.96\times10^{-3}} = 51.02\text{ m/sec}$$

$$\frac{P_{1\text{gauge}}}{\rho_{air}\times g} + \frac{(12.73)^2}{2\times10} = 0 + \frac{(51.02)^2}{2\times10}$$

$$\frac{P_1}{\rho_{air}.g} = 121.53$$

$$P_1 = 121.53\times\rho_{air}\times g$$

= 1.51 kPa

04. Ans: 395

Sol: Q = 100 litre/sec = 0.1 m^3/sec

V_1 = 100 m/sec; $P_1 = 3\times10^5$ N/m^2

V_2 = 50 m/sec; $P_2 = 1\times10^5$ N/m^2

Power (P) = ?

Energy equation :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{3\times10^5}{1000\times10} + \frac{100^2}{2\times10} + 0 = \frac{1\times10^5}{1000\times10} + \frac{50^2}{2\times10} + 0 + h_L$$

⇒ h_L = 395 m

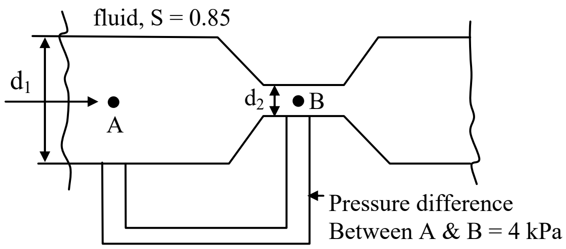
P = $\rho g Q.h_L$

P = 1000 × 10 × 0.10 × 395

P = 395 kW

05. Ans: 35

Sol:



$d_1 = 300$ mm, $d_2 = 120$ mm

$$Q_{Th} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\Delta P}{w}\right)}$$

$$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.30)^2 = 0.07\,m^2$$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.12)^2 = 0.011\,m^2$$

$\Delta P = 4$ kPa,

$$h = \frac{\Delta P}{w} = \frac{\Delta P}{\rho_f .g}$$

$$= \frac{\Delta P}{s_f \rho_w g} = \frac{4\times 10^3}{0.85\times 1000\times 9.81}$$

$$Q_{Th} = \frac{0.07\times 0.011}{\sqrt{(0.07)^2 - (0.011)^2}}\sqrt{\frac{2\times 9.81\times 4\times 10^3}{0.85\times 1000\times 9.81}}$$

$= 0.035\ m^3/sec = 35.15$ ltr/sec

06. Ans: 65

Sol: $h_{stag} = 0.30$ m

$h_{stat} = 0.24$ m

$V = c\sqrt{2gh_{dyna}}$

$V = 1\sqrt{2g(h_{stag} - h_{stat})}$

$= \sqrt{2(9.81)(0.30 - 0.24)} = 1.085$ m/s

$= 1.085 \times 60 = 65.1$ m/min

07. Ans: 81.5

Sol: $x = 30$ mm, $g = 10\ m/s^2$

$\rho_{air} = 1.23\ kg/m^3$; $\rho_{Hg} = 13600\ kg/m^3$

$C = 1$

$V = \sqrt{2gh_D}$

$$h_D = x\left(\frac{S_m}{S} - 1\right)$$

$$h_D = 30\times 10^{-3}\left(\frac{13600}{1.23} - 1\right)$$

$h_D = 331.67$ m

$V = 1\times\sqrt{2\times 10\times 331.67} = 81.5$ m/sec

08. Ans: 140

Sol: $$Q_a = C_d\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$C_d \propto \frac{1}{\sqrt{h}}$$

$$\frac{C_{d_{venturi}}}{C_{d_{orifice}}} = \frac{0.95}{0.65} = \sqrt{\frac{h_{orifice}}{h_{venturi}}}$$

$h_{venturi} = 140$ mm

09. Ans: (d)

Sol:

- For an orifice meter, the fluid re-establishes its flow pattern downstream of the orifice plate. However, the fluid pressure downstream of the orifice plate is not the same as that at upstream of the orifice plate. Thus, statement (I) is not correct.

- Bernoulli's equation when applied to any two points (for irrotational, steady and incompressible flow) can be written as

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

If $V_1 = V_2$ & $Z_1 = Z_2$, we get $P_1 = P_2$.

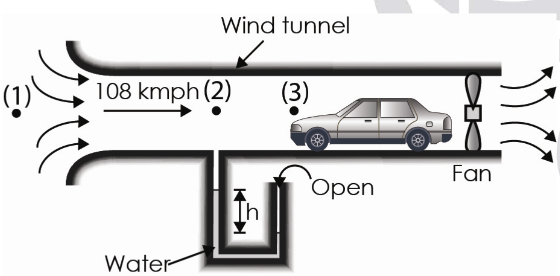
Thus, statement (II) is correct.

Conventional Practice Solutions

01. Ans: 5.4 cm, 540 Pa

Sol: Air enters into the wind tunnel at P_{atm} and $V \approx 0$. It attains a velocity V in the test section and the pressure there is P.

Applying Bernoulli's equation for points (1) and (2) as shown in the figure.



$$\frac{P_1}{\gamma_{air}}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_{air}}+\frac{V_2^2}{2g}+Z_2$$

But $P_1 = P_{atm}$, $V_1 \approx 0$ and $Z_1 = Z_2$

Thus, $\frac{P_{atm}-P_2}{\gamma_{air}}=\frac{V_2^2}{2g}$

$$=\frac{\left(108\times\frac{5}{18}\right)^2}{2\times10}=45 \text{ -----(1)}$$

From manometer,

$P_2 + \gamma_{water}\times h = P_{atm}$

or, $P_{atm} - P_2 = \gamma_{water} \times h$ -------(2)

Hence, equation (1) becomes,

$\frac{\gamma_{water}h}{\gamma_{air}} = 45$ (from (2))

$$h=\frac{45\times\gamma_{air}}{\gamma_{water}}=\frac{45\times1.2\times g}{10^3\times g}=0.054 \text{ m}$$

$\Rightarrow$ h = 5.4 cm

Applying Bernoulli's equation for points (2) and (3)

$$\frac{P_2}{\gamma_{air}}+\frac{V_2^2}{2g}=\frac{P_3}{\gamma_{air}}+\frac{V_3^2}{2g}$$

But point (3) is stagnation point where

$P_3 = P_{stag}$ & $V_3 = 0$

Thus, $\frac{P_{stag}-P_2}{\gamma_{air}}=\frac{V_2^2}{2g}=45$

Or, $P_{stag} - P_2 = 45 \times 1.2 \times 10 = 540$ Pa

06. Momentum equation and its Applications

01. Ans: 1600

Sol: S = 0.80

$A = 0.02\ m^2$

V = 10 m/sec

$F = \rho.A.V^2$

$F = 0.80 \times 1000 \times 0.02 \times 10^2$

F = 1600 N

02. Ans: 6000

Sol: $A = 0.015\ m^2$

$V = 15$ m/sec (Jet velocity)

$U = 5$ m/sec (Plate velocity)

$F = \rho A\ (V + U)^2$

$F = 1000 \times 0.015\ (15 + 5)^2$

$F = 6000$ N

03. Ans: 19.6

Sol: $V = 100$ m/sec (Jet velocity)

$U = 50$ m/sec (Plate velocity)

$d = 0.1$ m

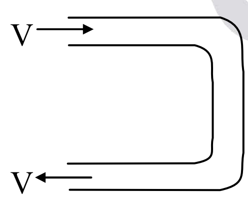
$F = \rho A\ (V - U)^2$

$$F = 1000 \times \frac{\pi}{4} \times 0.1^2 \times (100 - 50)^2$$

$F = 19.6$ kN

04. Ans: (a)

Sol:



$F_x = \rho aV(\ V_{1x} - V_{2x})$

$= \rho aV(\ V - (-V))$

$= 2\ \rho a\ V^2$

$= 2\times\ 1000 \times 10^{-4} \times 5^2 = 5$ N

05. Ans: (d)

Sol: Given, $V = 20$ m/s,

$u = 5$ m/s

$F_1 = \rho A(V - u)^2$

Power $(P_1) = F_1 \times u = \rho A(V - u)^2 \times u$

$F_2 = \rho.A.V \times V_r$

$= \rho.A(V).(V-u)$

Power $(P_2) = F_2 \times u = \rho AV(V-u)u$

$$\frac{P_1}{P_2} = \frac{\rho A(V-u)^2 \times u}{\rho AV(V-u) \times u}$$

$$= \frac{V-u}{V} = 1 - \frac{u}{V}$$

$$= 1 - \frac{5}{20} = 0.75$$

06. Ans: 2035

Sol: Given, $\theta = 30°$, $\dot{m} = 14$ kg/s

$(P_i)_g = 200$ kPa, $(P_e)_g = 0$

$A_i = 113\times10^{-4}\ m^2$, $A_e = 7\times10^{-4}\ m^2$

$\rho = 10^3\ kg/m^3$, $g = 10\ m/s^2$

From the continuity equation :

$\rho A_i\ V_i = 14$

or $$V_i = \frac{14}{10^3 \times 113 \times 10^{-4}} = 1.24\ m/s$$

Similarly, $$V_e = \frac{14}{10^3 \times 7 \times 10^{-4}} = 20\ m/s$$

Let F_x be the force exerted by elbow on water in the +ve x-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write :

$$(P_i)_g A_i + F_x = \dot{m}(V_e \cos 30° - V_i)$$

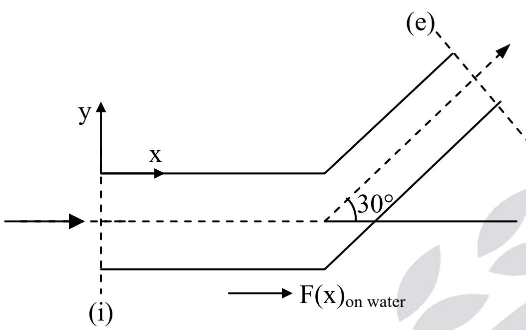
$$F_x = \dot{m}(V_e \cos 30° - V_i) - (P_i)_g A_i$$

$= 14\ (20\times\cos30° - 1.24) - 200\times10^3\times113\times10^{-4}$

$= 225.13 - 2260$

$= -2034.87$ N ≈ -2035 N

The x-component of water force on elbow is $-F_x$ (as per Newton's third law), i.e., $\cong$ 2035 N



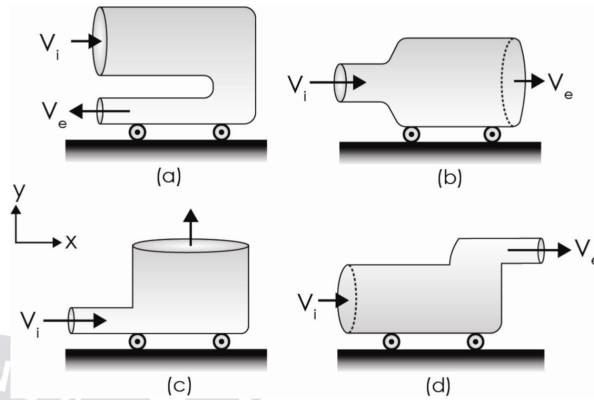
07. Ans: (a)

Sol: In a convergent nozzle, as the area decreases in the direction of flow, the flow velocity will increase (AV = Constant) in the direction of flow. This will result in increase in its momentum. Thus, statement (I) is correct and statement (II) is the correct explanation of statement (I).

Conventional Practice Solutions

01. Ans: Right: a, b, c; Left: d

Sol: Let F_x be the force exerted by the fluid on the device which will be different for different devices. Since inlet and outlet sections of the devices are at atmospheric pressure, there will be no contribution of pressure forces at these sections. Let V_i and V_e be the velocities at inlet and outlet of the devices in x-direction.



Applying linear momentum equation to each of the devices, we write

(a) $F_x = \dot{m}_a[V_i - (-V_e)] = \dot{m}_a[V_i + V_e]$

F_x is acting in +ve x direction.

Therefore, the device (b) will move to the right.

(b) $F_x = \dot{m}_b(V_i - V_e)$

Since $V_i > V_e$, F_x is acting in +ve x direction.

Therefore, the device (a) will move to the right.

(c) $F_x = \dot{m}_c(V_i - 0) = \dot{m}V_i$

F_x is acting in +ve x direction.

Therefore, the device (c) will move to the right.

(d) $F_x = \dot{m}_d(V_i - V_e)$

Since $V_e > V_i$

F_x is acting in –ve x direction. Therefore, the device (d) will move to the left.

02.

Sol: Given data:

$A_j = 0.009\ m^2$

$V_j = 30.5$ m/s

$V_s = 3$ m/s

$A_s + A_j = 0.07\ m^2 = A_T$

$A_s = (0.07 - 0.009) = 0.061\ m^2$

$A_jV_j + A_sV_s = (A_s + A_s)V_e$

(From continuity equation)

$0.009 \times 30.5 + 0.061 \times 3 = 0.07 \times V_e$

Or, $V_e = 6.536$ m/s ≈ 6.54 m/s

Applying linear momentum equation:

$P_1(A_s + A_j) - P_2(A_s + A_j)$

$= \rho A_T \times V_e[V_e] - \rho A_s V_s^2 - \rho A_j V_j^2$

$(P_1 - P_2)A_T = \rho A_T V_e^2 - \rho A_s V_s^2 - \rho A_j V_j^2$

$= \rho\left[0.07\times 6.536^2 - 0.061\times 9 - 0.009\times 30.5^2\right]$

$= -10^3(5.931)$ N

or, $P_2 - P_1 = \dfrac{5.931}{0.07} = 84.73$ kPa

07. Laminar Flow

01. Ans: (d)

Sol: In a pipe, the flow changes from laminar flow to transition flow at Re = 2000. Let V be the average velocity of flow. Then

$$2000 = \frac{V\times 8\times 10^{-2}}{0.4\times 10^{-4}} \Rightarrow V = 1\,m/s$$

In laminar flow through a pipe,

$V_{max} = 2 \times V = 2$ m/s

02. Ans: (d)

Sol: The equation $\tau = \left(-\dfrac{\partial P}{\partial x}\right)\left(\dfrac{r}{2}\right)$ is valid for laminar as well as turbulent flow through a circular tube.

03. Ans: (d)

Sol: $Q = A.V_{avg}$

$Q = A.\dfrac{V_{max}}{2}$ $\quad (\because V_{max} = 2\,V_{avg})$

$$Q = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2 \times \frac{1.5}{2}$$

$$= \frac{\pi}{4}\times(0.04)^2\times 0.75$$

$$= \frac{\pi}{4}\times\frac{4}{100}\times\frac{4}{100}\times\frac{3}{4} = \frac{3\pi}{10000}\ m^3/sec$$

04. Ans: 1.92

Sol: $\rho = 1000\ kg/m^3$

$Q = 800\ mm^3/sec = 800 \times (10^{-3})^3\ m^3/sec$

L = 2 m

D = 0.5 mm

$\Delta P = 2$ MPa $= 2 \times 10^6$ Pa

μ = ?

$$\Delta P = \frac{128.\mu QL}{\pi D^4}$$

$$2\times 10^6 = \frac{128\times\mu\times 800\times\left(10^{-3}\right)^3\times 2}{\pi\left(0.5\times 10^{-3}\right)^4}$$

μ = 1.917 milli Pa – sec

05. Ans: 0.75

Sol: $U_r = U_{max}\left(1-\left(\frac{r}{R}\right)^2\right)$

$$\left[\because \frac{U}{U_{max}} = 1-\left(\frac{r}{R}\right)^2\right]$$

$$= 1\left(1-\left(\frac{50}{200}\right)^2\right)$$

$$= 1\left(1-\frac{1}{4}\right) = \frac{3}{4} = 0.75 \text{ m/s}$$

06. Ans: 0.08

Sol: Given,

$\rho = 0.8 \times 1000 = 800$ kg/m^3

$\mu = 1$ Poise $= 10^{-1}$ N-s/m^2

d = 50 mm = 0.05 m

Velocity = 2 m/s

Reynold's Number, $Re = \frac{\rho VD}{\mu}$

$$= \frac{800\times 2\times 0.05}{10^{-1}} = 800$$

$(\because Re < 2000)$

$\therefore$ Flow is laminar,

For laminar, Darcy friction factor

$$f = \frac{64}{Re} = \frac{64}{800} = 0.08$$

07. Ans: 16

Sol: For fully developed laminar flow,

$$h_f = \frac{32\mu VL}{\rho g D^2} \quad (\therefore Q = AV)$$

$$h_f = \frac{32\mu\left(\frac{Q}{A}\right)L}{\rho g D^2} = \frac{32\mu QL}{AD^2\times\rho g}$$

$$h_f = \frac{32\mu QL}{\frac{\pi}{4}D^2\times D^2\times\rho g}$$

$$h_f \propto \frac{1}{D^4}$$

$$h_{f1}D_1^4 = h_{f_2}D_2^4$$

Given, $D_2 = \frac{D_1}{2}$

$$h_{f1}\times D_1^4 = h_{f2}\times\left(\frac{D_1}{2}\right)^4$$

$$h_{f_2} = 16 h_{f_1}$$

$\therefore$ Head loss, increases by 16 times if diameter is halved.

08. Ans: 5.2

Sol: Oil viscosity, $\mu = 10$ poise $= 10\times 0.1$
$= 1$ N-s/m^2

$y = 50\times 10^{-3}$m

L = 120 cm = 1.20 m

$\Delta P = 3\times 10^3$Pa

Width of plate = 0.2 m

Q = ?

$Q = A.V_{avg}$ = (width of plate$\times$y)V

$$\Delta P = \frac{12\mu VL}{B^2}$$

$$3\times 10^3 = \frac{12\times 1\times V\times 1.20}{(50\times 10^{-3})^2}$$

V = 0.52 m/sec

$Q = AV_{avg} = (0.2\times50\times10^{-3})\ (0.52)$

= 5.2 lit/sec

09. Ans: (a)

Sol: Wall shear stress for flow in a pipe is given by,

$$\tau_o = -\frac{\partial P}{\partial x}\times\frac{R}{2} = \frac{\Delta P}{L}\times\frac{D}{4}$$

$$= \frac{\Delta P\,D}{4L}$$

10. Ans: 72

Sol: Given, $\rho = 800$ kg/m^3 ,

$\mu = 0.1$ Pa.s

Flow is through an inclined pipe.

$d = 1\times10^{-2}$ m,

$V_{av} = 0.1$ m/s,

$\theta = 30°$

$$Re = \frac{\rho V_{av} d}{\mu} = \frac{800\times0.1\times1\times10^{-2}}{0.1} = 8$$

$\Rightarrow$ flow is laminar.

Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+h_f$$

But $V_1 = V_2$,

$(Z_2 - Z_1) = 10\sin30° = 5$ m

and $h_f = \dfrac{32\mu V_{av} L}{\rho g d^2}$

$$\frac{(P_1-P_2)}{\gamma} = (Z_2-Z_1)+\frac{32\mu V_{av}L}{\rho g d^2}$$

$$(P_1-P_2) = \rho g(Z_2-Z_1)+\frac{32\mu V_{av}L}{d^2}$$

$$= 800\times10\times5+\frac{32\times0.1\times0.1\times10}{(1\times10^{-2})^2}$$

$= 40\times10^3 + 32\times10^3 = 72$ kPa

11. Ans: (d)

Sol:

- In hydrodynamic entrance region of the pipe of uniform diameter, the average velocity remains constant in the direction of flow. Thus, statement - I is wrong.
- However, in the above region the centreline velocity increases in the direction of flow as boundary layers grow on the solid surfaces. Thus, statement (II) is correct.

Conventional Practice Solutions

01.

Sol: The velocity profile for fully developed laminar flow between two stationary parallel plates is given by

$$u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(By - y^2)$$

(i) $$\frac{\partial u}{\partial y} = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(B-2y)$$

At the upper surface

$$\left.\frac{\partial u}{\partial y}\right|_{y=B} = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)(B - 2\times B)$$

$$= -\frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)B$$

$$\tau_{y=B} = \mu\left.\frac{\partial u}{\partial y}\right|_{y=B} = -\frac{1}{2}\left(\frac{-\partial P}{\partial x}\right)B$$

$$= \frac{-1}{2}\times 1000\times 5\times 10^{-3} = -2.5\text{ Pa}$$

Thus, the magnitude of the shear stress on the upper plate is 2.5 Pa and its direction is opposite to the direction of flow.

(ii) Discharge per unit length

$$= \int_0^B u(y)(dy\times 1)$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\int_0^B (By - y^2)dy$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left[B\frac{y^2}{2} - \frac{y^3}{3}\right]_0^B$$

$$= \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(\frac{B^3}{2} - \frac{B^3}{3}\right)$$

$$= \frac{B^3}{12\mu}\left(\frac{-\partial P}{\partial x}\right)$$

$$q = \frac{(5\times 10^{-3})^3}{12}\times 1000$$

$$= 20.83\times 10^{-6}\text{ m}^3/\text{s}$$

02.

Sol: This is a problem of Couette flow with pressure gradient. In this case the velocity profile is given by

$$u = \frac{V}{h}y + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)(hy - y^2)$$

$$= \frac{0.1}{0.01}y + \frac{1}{2\times 0.1}(1200)(0.01y - y^2)$$

$$= 10y + 6000(0.01y - y^2)$$

$$= 10y + 60y - 6000y^2$$

$$= 70y - 6000y^2$$

For maximum velocity

$$\frac{\partial u}{\partial y} = 0 = 70 - 12000y$$

Or, $y = \frac{70\times 1000}{12000}\text{ mm} = 5.833\text{ mm}$

and

$$V_{max} = 70\times 5.833\times 10^{-3} - 6000\times(5.833\times 10^{-3})^2$$

$$= 0.204\text{ m/s}$$

08. Flow Through Pipes

01. Ans: (d)

Sol:

- The Darcy-Weisbash equation for head loss in written as:

$$h_f = \frac{fLV^2}{2gd}$$

where V is the average velocity, f is friction factor, L is the length of pipe and d is the diameter of the pipe.

- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as

the nature of pipe surface (smooth or rough)

- For laminar flow, friction factor is a function of Reynolds number.

02. Ans: 481

Sol: Given data,

$\dot{m} = \pi$ kg/s, $\quad$ d = 5×10^{-2} m,

$\mu = 0.001$ Pa.s , $\quad \rho = 1000$ kg/m^3

$$V_{av} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho\pi d^2} = \frac{4\times\pi}{\rho\pi d^2} = \frac{4}{\rho d^2}$$

$$Re = \frac{\rho V_{av} d}{\mu} = \rho\times\frac{4}{\rho d^2}\times\frac{d}{\mu} = \frac{4}{\mu d}$$

$$= \frac{4}{0.001\times5\times10^{-2}} = 8\times10^4$$

$\Rightarrow$ Flow is turbulent

$$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{\left(8\times10^4\right)^{0.25}} = 0.0188$$

$$\Delta P = \rho g\frac{f L V_{av}^2}{2gd} = f\rho L\times\left(\frac{4}{\rho d^2}\right)^2\times\frac{1}{2d}$$

$$\frac{\Delta P}{L} = f\times\frac{16}{\rho d^5}\times\frac{1}{2} = \frac{8f}{\rho d^5} = \frac{8\times0.0188}{10^3\times\left(5\times10^{-2}\right)^5}$$

$$= 481.28 \text{ Pa/m}$$

03. Ans: (a)

Sol: In pipes Net work, series arrangement

$$\therefore h_f = \frac{f.lV^2}{2gd} = \frac{f.lQ^2}{12.1\times d^5}$$

$$\frac{h_{f_A}}{h_{f_B}} = \frac{f_A.\ell_A.Q_a^2}{12.1\times d_A^5}\times\frac{12.1\times d_B^5}{f_B.\ell_B.Q_B^2}$$

Given $l_A = l_B$, $f_A = f_B$, $Q_A = Q_B$

$$\frac{h_{f_A}}{h_{f_B}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2d_B}\right)^5$$

$$= \left(\frac{1}{1.2}\right)^5 = 0.4018 \approx 0.402$$

04. Ans: (a)

Sol: Given, $d_1 = 10$ cm; $d_2 = 20$ cm

$f_1 = f_2$;

$l_1 = l_2 = l$

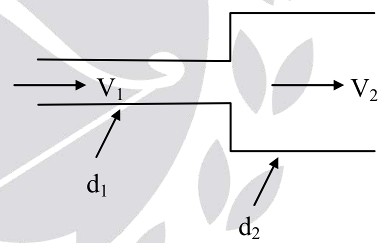
$l_e = l_1 + l_2 = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \Rightarrow \frac{2l}{d_e^5} = \frac{l}{10^5} + \frac{l}{20^5}$$

$\therefore d_e = 11.4$ cm

05. Ans: (c)

Sol:



Given $d_2 = 2d_1$

Losses due to sudden expansion,

$$h_L = \frac{\left(V_1 - V_2\right)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

By continuity equation,

$$Q = A_1V_1 = A_2V_2$$

$$\therefore \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{1}{4}\right)^2$$

$$h_L = \frac{9}{16}\times\frac{V_1^2}{2g}$$

$$\frac{h_L}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

06. Ans: (b)

Sol: Pipes are in parallel

$Q_e = Q_A + Q_B$ ------- (i)

$h_{Le} = h_{L_A} = h_{L_B}$

$L_e = 175$ m

$f_e = 0.015$

$$\frac{f_e L_e Q_e^2}{12.1 D_e^5} = \frac{f_A . L_A Q_A^2}{12.1 D_A^5} = \frac{f_B L_B Q_B^2}{12.1 D_B^5}$$

$$\frac{0.020\times150\times Q_A^2}{12.1\times(0.1)^5} = \frac{0.015\times200\times Q_B^2}{12.1\times(0.08)^5}$$

$Q_A = 1.747\ Q_B$ -------(ii)

From (i) $Q_e = 1.747\ Q_B + Q_B$

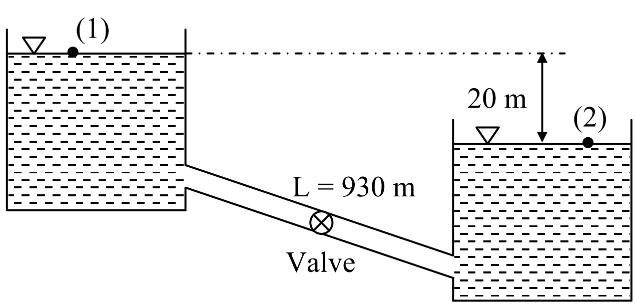
$Q_e = 2.747\ Q_B$ -------(iii)

$$\frac{0.015\times175(2.747Q_B)^2}{12.1\times D_e^5} = \frac{0.015\times200\times Q_B^2}{12.1\times(0.08)^5}$$

$D_e = 116.6$ mm $\simeq 117$ mm

07. Ans: 0.141

Sol:



Given data,

$L = 930$ m, $k_{valve} = 5.5$

$k_{entry} = 0.5$, $d = 0.3$ m

$f = 0.03$, $g = 10\ m/s^2$

Applying energy equation for points (1) and (2), we write :

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2 + h_{L,entry} + h_{L,valve} + h_{L,exit} + h_{f,pipe}$$

But $P_1 = P_2 = P_{atm} = 0$

$V_1 = 0 = V_2$

$Z_1 - Z_2 = 20$ m, $k_{exit} = 1$

$$Z_1 - Z_2 = 0.5\frac{V^2}{2g} + 5.5\frac{V^2}{2g} + 1\times\frac{V^2}{2g} + \frac{fLV^2}{2gd}$$

$$= 7\frac{V^2}{2g} + \frac{fLV^2}{2gd} = \frac{V^2}{2g}\left(7 + \frac{fL}{d}\right)$$

or $$20 = \frac{V^2}{2g}\left[7 + \frac{0.03\times930}{0.3}\right] = 100\frac{V^2}{2g}$$

or $$V^2 = \frac{20\times2g}{100} = \frac{20\times2\times10}{100}$$

$\Rightarrow$ $V = 2$ m/s

Thus, discharge, $Q = \frac{\pi}{4}\times0.3^2\times2$

$= 0.1414\ m^3/s$

08. Ans: (c)

Sol: Given data :

Fanning friction factor, $f = m\,Re^{-0.2}$

For turbulent flow through a smooth pipe.

$$\Delta P = \frac{\rho f_{Darcy} L V^2}{2d} = \frac{\rho(4f)LV^2}{2d}$$

$$= \frac{2\rho m Re^{-0.2} L V^2}{d}$$

or $\Delta P \propto V^{-0.2}V^2 \propto V^{1.8}$ (as all other parameters remain constant)

We may thus write :

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{V_2}{V_1}\right)^{1.8} = \left(\frac{2}{1}\right)^{1.8} = 3.4822$$

or $\Delta P_2 = 3.4822 \times 10 = 34.82$ kPa

09. Ans: (b)

Sol: Given data :

Rectangular duct, L = 10 m,

X-section of duct = 15 cm × 20 cm

Material of duct - Commercial steel,

$\varepsilon = 0.045$ mm

Fluid is air ($\rho = 1.145$ kg/m^3,

$\nu = 1.655\times10^{-5}$ m^2/s)

$V_{av} = 7$ m/s

$$Re = \frac{V_{av} \times D_h}{\nu}$$

where, D_h = Hydraulic diameter

$$= \frac{4 \times \text{Cross sectional area}}{\text{Perimeter}}$$

$$= \frac{4 \times 0.15 \times 0.2}{2(0.15 + 0.2)} = 0.1714\,\text{m}$$

$$Re = \frac{7 \times 0.1714}{1.655 \times 10^{-5}} = 72495.5$$

$\Rightarrow$ Flow is turbulent.

Using Haaland equation to find friction factor,

$$\frac{1}{\sqrt{f}} \simeq -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D_h}{3.7}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{72495.5} + \left(\frac{0.045\times10^{-3}}{0.1714\times3.7}\right)^{1.11}\right]$$

$$= -1.8\log[9.518\times10^{-5} + 2.48\times10^{-5}]$$

$$= -1.8\log(11.998\times10^{-5})$$

$$\frac{1}{\sqrt{f}} = 7.058$$

$f = 0.02$

The pressure drop in the duct is,

$$\Delta P = \frac{\rho f L V^2}{2D_h}$$

$$= \frac{1.145\times0.02\times10\times7^2}{2\times0.1714} = 32.73 \text{ Pa}$$

The required pumping power will be

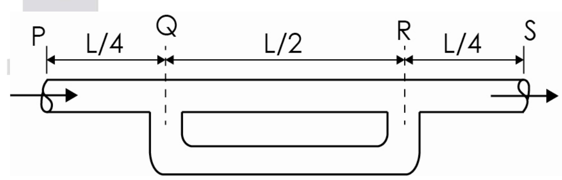
$P_{pumping} = Q\,\Delta P = A\,V_{av} \times \Delta P$

$= (0.15 \times 0.2) \times 7 \times (32.73)$

$= 6.87 \text{ W} \simeq 7 \text{ W}$

10. Ans: 26.5

Sol:



Case I: Without additional pipe,

Let Q be the discharge through the pipe. Then

$$\frac{P_P}{\gamma}+\frac{V_P^2}{2g}+Z_P=\frac{P_S}{\gamma}+\frac{V_S^2}{2g}+Z_S+\frac{f\,L\,Q^2}{12.1\,d^5}$$

But $V_P = V_S$ and $Z_P = Z_S$

P_P and P_S are the pressures at sections P and S, respectively.

Thus,

$$\frac{P_P}{\gamma}-\frac{P_S}{\gamma}=\frac{f\,L\,Q^2}{12.1d^5} \quad \text{-------(1)}$$

Case II: When a pipe (L/2) is connected in parallel.

In this case, let Q′ be the total discharge.

$Q_{Q-R}=\frac{Q'}{2}$ and $Q_{R\text{-}S} = Q'$

Then,

$$\frac{P'_P}{\gamma}+\frac{V_P'^2}{2g}+Z'_P=\frac{P'_S}{\gamma}+\frac{V_S'^2}{2g}+Z'_S+\frac{f(L/4)Q'^2}{12.1\,d^5}+\frac{f(L/2)(Q'/2)^2}{12.1\,d^5}+\frac{f(L/4)Q'^2}{12.1\,d^5}$$

P_P' and P_S' are the pressures at sections P and S in the second case.

But $V_P' = V_S'$; $Z_P' = Z_S'$

So, $$\frac{P'_P}{\gamma}-\frac{P'_S}{\gamma}=\frac{f\,L\,Q'^2}{12.1d^5}\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{4}\right]$$

$$=\frac{5}{8}\times\frac{f\,L\,Q'^2}{12.1d^5} \quad \text{-------(2)}$$

Given that end conditions remain same.

i.e., $$\frac{P_P}{\gamma}-\frac{P_S}{\gamma}=\frac{P'_P}{\gamma}-\frac{P'_S}{\gamma}$$

Hence, equation (2) becomes,

$$\frac{f\,L\,Q^2}{12.1d^5}=\frac{5}{8}\frac{f\,L\,Q'^2}{12.1d^5} \quad \text{from eq.(1)}$$

or $$\left(\frac{Q'}{Q}\right)^2=\frac{8}{5}$$

or $$\frac{Q'}{Q}=1.265$$

Hence, percentage increase in discharge is

$$=\frac{Q'-Q}{Q}\times 100$$

$= (1.265 – 1) \times 100$

$= 26.5\ \%$

11. Ans: 20%

Sol: Since, discharge decrease is associated with increase in friction.

$$\frac{df}{f}=-2\times\frac{dQ}{Q}=2\left[-\frac{dQ}{Q}\right]$$

$$=2\times 10=20\%$$

12. Ans: (c)

Sol: As compared to sharp entrance, the rounded entrance will give less energy loss in flow through a pipe. For sharp entrance, the flow gets separated and there will be recirculation zone till the fluid stream gets attached to the surface. Thus, the rounded entrance increases the flow rate when everything else remains constant. Hence, statement (I) is correct. However, statement (II) is wrong as discussed above.

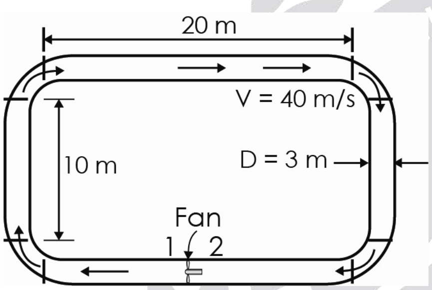
13. Ans: (d)

Sol: The surge tanks are provided on upstream side of the valve in order to offset the effect of water hammer mainly due to the pressure rise which may damage the pipe. Thus, statement (I) is wrong. However, statement (II) is correct.

Conventional Practice Solutions

01.

Sol:



Applying Energy equation for two points, just upstream and downstream of the fan in the pipe loop.

$$\frac{P_1}{\gamma_{air}}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_{air}}+\frac{V_2^2}{2g}+Z_2+\frac{fL_{total}V^2}{2gD}+4\times\frac{K_{elbow}V^2}{2g}$$

where $V_1 = V_2 = V$; $Z_1 = Z_2$

$f = 0.01$, $D = 3m$,

$V = 40$ m/s, $L = 60$ m,

$K_{elbow} = 0.3$ (Given)

$$\frac{P_1-P_2}{\gamma_{air}}=\frac{V^2}{2g}\left[\frac{fL}{D}+4\times K_{elbow}\right]$$

$$=\frac{40^2}{2g}\left[\frac{0.01\times 60}{3}+4\times 0.3\right]$$

$$=\frac{40^2}{2g}\times 1.4$$

$$\Delta P=\rho_{air}\times\frac{40^2}{2}\times 1.4$$

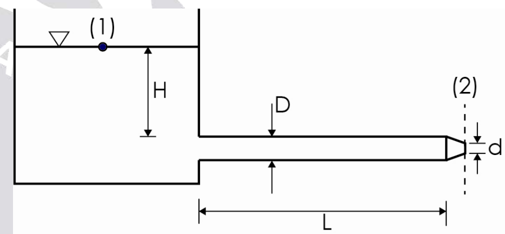
$$=1.2\times\frac{40^2}{2}\times 1.4=1{,}344 \text{ Pa}$$

Power added to air by fan, $P = Q\Delta P$

$$=\frac{\pi}{4}\times 3^2\times 40\times 1{,}344=380 \text{ kW}$$

02.

Sol:



Applying energy equation between points (1) and (2)

$$\frac{P_1}{\gamma_f}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma_f}+\frac{V_2^2}{2g}+Z_2+(h_f)_{pipe}$$

But $P_1 = P_2 = P_{atm}$, $Z_1 = H$, $Z_2 = 0$, $V_1 = 0$

Thus,

$$H=\frac{V_2^2}{2g}+(h_f)_{pipe}=\frac{V_2^2}{2g}+\frac{fLV_p^2}{2gD}$$

For maximum power transmission, $H = 3h_f$

Or, $$3\times\frac{fLV_p^2}{2gD}=\frac{V_2^2}{2g}+\frac{fLV_p^2}{2gD}$$

Or, $$\frac{2fLV_p^2}{2gD}=\frac{V_2^2}{2g}$$

Or, $$\left(\frac{V_2}{V_p}\right)^2=\frac{2fL}{D} \qquad \text{---------------(1)}$$

From equation of continuity,

$$\frac{\pi}{4}D^2V_p = \frac{\pi}{4}d^2V_2$$

Or $$\frac{V_2}{V_p} = \frac{D^2}{d^2}$$

Thus, substituting in equation (1), we get

$$\left(\frac{D^2}{d^2}\right)^2 = \frac{2fL}{D}$$

$$\Rightarrow \quad d = \left(\frac{D^5}{2fL}\right)^{\frac{1}{4}} \text{ .... Proved}$$

09. Elementary Turbulent Flow

01. Ans: (b)

Sol: The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

where V* is the shear velocity.

02. Ans: (d)

Sol: $\Delta P = \rho g h_f$

$$= \frac{\rho f L V^2}{2D} = \frac{\rho g f L Q^2}{12.1 D^5}$$

For Q = constant

$$\Delta P \propto \frac{1}{D^5}$$

or $$\frac{\Delta P_2}{\Delta P_1} = \frac{D_1^5}{D_2^5} = \left(\frac{D_1}{2D_1}\right)^5 = \frac{1}{32}$$

03. Ans: 2.4

Sol: Given: V = 2 m/s

f = 0.02

V_{max} = ?

$$V_{max} = V(1 + 1.43\sqrt{f})$$

$$= 2\left(1 + 1.43\sqrt{0.02}\right)$$

$$= 2 \times 1.2 = 2.4 \text{ m/s}$$

04. Ans: (c)

Sol: Given data:

D = 30 cm = 0.3 m

Re = 10^6

f = 0.025

Thickness of laminar sub layer, δ' = ?

$$\delta' = \frac{11.6\nu}{V^*}$$

where V^* = shear velocity = $V\sqrt{\frac{f}{8}}$

ν = Kinematic viscosity

$$Re = \frac{V.D}{\nu}$$

$$\therefore \nu = \frac{V.D}{Re}$$

$$\delta' = \frac{11.6 \times \frac{VD}{Re}}{V\sqrt{\frac{f}{8}}}$$

$$\delta' = \frac{11.6 \times D}{Re\sqrt{\frac{f}{8}}}$$

$$= \frac{11.6 \times 0.3}{10^6 \times \sqrt{\frac{0.025}{8}}}$$

$= 6.22 \times 10^{-5}$ m $= 0.0622$ mm

05. Ans: 25

Sol: Given:

L = 100 m

D = 0.1 m

h_L = 10 m

τ = ?

For any type of flow, the shear stress at wall/surface $\tau = \frac{-dP}{dx} \times \frac{R}{2}$

$$\tau = \frac{\rho g h_L}{L} \times \frac{R}{2}$$

$$\tau = \frac{\rho g h_L}{L} \times \frac{D}{4}$$

$$= \frac{1000 \times 9.81 \times 10}{100} \times \frac{0.1}{4}$$

$= 24.525$ N/m^2 = 25 Pa

06. Ans: 0.905

Sol: k = 0.15 mm

$\tau = 4.9$ N/m^2

ν = 1 centi-stoke

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/sec}$$

ν = 1 centi-stoke

$$= \frac{1}{100} \text{stoke} = \frac{10^{-4}}{100} = 10^{-6} \text{m}^2/\text{sec}$$

$$\frac{k}{\delta'} = \frac{0.15 \times 10^{-3}}{\left(\frac{11.6 \times \nu}{V^*}\right)}$$

$$= \frac{0.15 \times 10^{-3}}{\frac{11.6 \times 10^{-6}}{0.07}} = 0.905$$

07. Ans: (a)

Sol: The velocity profile in the laminar sublayer is given as

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

or $\nu = \frac{y(V^*)^2}{u}$

where, V* is the shear velocity.

Thus, $\nu = \frac{0.5 \times 10^{-3} \times (0.05)^2}{1.25}$

$= 1 \times 10^{-6}$ m^2/s

$= 1 \times 10^{-2}$ cm^2/s

08. Ans: 47.74 N/m^2

Sol: Given data :

d = 100 mm = 0.1 m

$u_{r=0} = u_{max} = 2$ m/s

Velocity at r = 30 mm = 1.5 m/s

Flow is turbulent.

The velocity profile in turbulent flow is

$$\frac{u_{max} - u}{V^*} = 5.75 \log\left(\frac{R}{y}\right)$$

where u is the velocity at y and V* is the shear velocity.

For pipe, y = R – r

= (50 – 30) mm = 20 mm

Thus,

$$\frac{2-1.5}{V^*} = 5.75\log\left(\frac{50}{20}\right) = 2.288$$

or $V^* = \frac{0.5}{2.288} = 0.2185\,\text{m/s}$

Using the relation,

$V^* = \sqrt{\frac{\tau_w}{\rho}}$ or $\tau_w = \rho\,(V^*)^2$

$\tau_w = 10^3 \times (0.2185)^2 = 47.74\ \text{N/m}^2$

09. Ans: (a)

Sol:

- In turbulent flow, shear stress is given by

  $$\tau = \mu\left(\frac{d\bar{u}}{dy}\right) + \eta\left(\frac{d\bar{u}}{dy}\right)$$

  = Viscous stress + Reynolds stress

  where μ is dynamic viscosity and η is the eddy viscosity which is not a fluid properly but it is a flow property which depends upon turbulence condition of the flow.
- From the above expression we say that the shear stress in turbulent flow is more than that predicted by Newton's law of viscosity. Thus, statement - I is correct.
- Statement (II) is also correct statement and it is the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: Given data:

$r = 0$, $u = 1.5$ m/s at $y = R - 0 = R$

$r = \frac{R}{2}$, $u = 1.35$ m/s at $y = R - \frac{R}{2} = \frac{R}{2}$

D = 0.2 m or R = 0.1 m

Centreline velocity 1.5 m/s = u_{max}

Using the logarithmic velocity profile as:

$$\frac{u_{max} - u}{V^*} = 5.75\log\left(\frac{R}{y}\right)$$

where V^* is the shear velocity, we can find V^*.

$$\frac{1.5-1.35}{V^*} = 5.75\log\left(\frac{R}{R/2}\right) = 5.75\log(2)$$

$\Rightarrow$ $V^* = 0.0867$ m/s.

Similarly using the logarithmic velocity profile in terms of u, V and V^* (where V is the average velocity) we can find V as:

$$\frac{u - V}{V^*} = 5.75\log\left(\frac{y}{R}\right) + 3.75$$

at y = R,

$u = u_{max}$

$$\frac{1.5 - V}{0.0867} = 5.75\log\left(\frac{R}{R}\right) + 3.75 = 0 + 3.75$$

$\Rightarrow$ $V = 1.5 - 0.0867 \times 3.75 = 1.175$ m/s

(i) Thus, discharge = $\frac{\pi}{4}\times 0.2^2 \times 1.175$

$= 0.0369\ m^3/s$

(ii) We know that $V^* = V\sqrt{\frac{f'}{2}}$

where, f' is the coefficient of friction.

Thus, $f' = 2\times\left(\frac{V^*}{V}\right)^2$

$= 2\times\left(\frac{0.0867}{1.175}\right)^2$

$= 0.011$

The friction factor, $f = 4f' = 0.044$

(iii) The relationship between height of roughness projections, K and friction factor is given by

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{R}{K}\right)+1.74$$

Substituting the values, we get

$$\frac{1}{\sqrt{0.044}} = 2.0\log\left(\frac{R}{K}\right)+1.74$$

$$\log\left(\frac{R}{K}\right) = 1.5136$$

$$\frac{R}{K} = 32.629$$

$$K = \frac{R}{32.629} = \frac{0.1\times 10^3}{32.629}\text{mm}$$

$= 3.065$ mm

10. Boundary Layer Theory

01. Ans: (c)

Sol: $Re_{\text{Critical}} = \frac{U_\infty x_{\text{critical}}}{\nu}$

Assume water properties

$$5\times 10^5 = \frac{6\times x_{\text{critical}}}{1\times 10^{-6}}$$

$x_{\text{critical}} = 0.08333$ m = 83.33 mm

02. Ans: 1.6

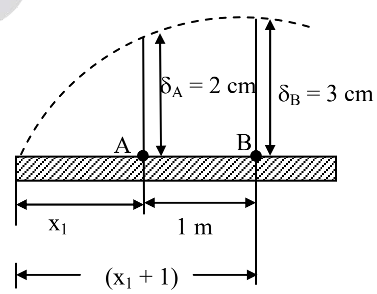
Sol: $\delta \propto \frac{1}{\sqrt{Re}}$ (At given distance 'x')

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{Re_2}{Re_1}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6$$

03. Ans: 80

Sol:



$\delta \propto \sqrt{x}$

$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1+1)}}$$

$$x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1+1}}$$

$$\frac{4}{9} = \frac{x_1}{x_1+1}$$

$5x_1 = 4 \Rightarrow x_1 = 80$ cm

04. Ans: 2

Sol: $\tau \propto \frac{1}{\delta}$

$\tau \propto \frac{1}{\sqrt{x}} \quad \because \delta \propto \sqrt{x}$

$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}} \qquad \frac{\tau_1}{\tau_2} = \sqrt{4} = 2$

05. Ans: 3

Sol: $\frac{U}{U_\infty} = \frac{y}{\delta}$

$\frac{\delta^*}{\theta} = \text{Shape factor} = ?$

$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

$= \int_0^\delta \left(1 - \frac{y}{8}\right) dy$

$= y - \frac{y^2}{2\delta}\Big|_0^\delta \qquad = \delta - \frac{\delta}{2} = \frac{\delta}{2}$

$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

$= \int_0^\delta \frac{y}{8}\left(1 - \frac{y}{\delta}\right) dy$

$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta}\Big|_0^\delta \qquad = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$

Shape factor $= \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

06. Ans: 22.6

Sol: Drag force,

$F_D = \frac{1}{2} C_D.\rho.A_{Proj}.U_\infty^2$

B = 1.5 m, $\rho = 1.2$ kg/m^3

L = 3.0 m, $\nu = 0.15$ stokes

$U_\infty = 2$ m/sec

$Re = \frac{U_\infty L}{\nu} = \frac{2 \times 3}{0.15 \times 10^{-4}} = 4 \times 10^5$

$C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{4 \times 10^5}} = 2.09 \times 10^{-3}$

Drag force,

$F_D = \frac{1}{2} \times 2.09 \times 10^{-3} \times 1.2 \times (1.5 \times 3) \times 2^2$

= 22.57 milli-Newton

07. Ans: 1.62

Sol: Given data,

$U_\infty = 30$ m/s,

$\rho = 1.2$ kg/m^3

Velocity profile at a distance x from leading edge,

$\frac{u}{U_\infty} = \frac{y}{\delta}, \qquad \delta = 1.5$ mm

Mass flow rate of air entering section ab,

$(\dot{m}_{in})_{ab} = \rho U_\infty (\delta \times 1) = \rho U_\infty \delta$ kg/s

Mass flow rate of air leaving section cd,

$(\dot{m}_{out})_{cd} = \rho \int_0^\delta u(dy \times 1) = \rho \int_0^\delta U_\infty \left(\frac{y}{\delta}\right) dy$

$= \frac{\rho U_\infty}{\delta}\left[\frac{y^2}{2}\right]_0^\delta = \frac{\rho U_\infty \delta}{2}$

From the law of conservation of mass :

$$(\dot{m}_{in})_{ab} = (\dot{m}_{out})_{cd} + (\dot{m}_{out})_{bc}$$

Hence, $(\dot{m}_{out})_{bc} = (\dot{m}_{in})_{ab} - (\dot{m}_{out})_{cd}$

$$= \rho U_\infty \delta - \frac{\rho U_\infty \delta}{2}$$

$$= \frac{\rho U_\infty \delta}{2}$$

$$= \frac{1.2\times 30\times 1.5\times 10^{-3}}{2}$$

$$= 27\times10^{-3}\ \text{kg/s}$$

$$= 27\times10^{-3}\times 60\ \text{kg/min}$$

$$= 1.62\ \text{kg/min}$$

08. Ans: (b)

Sol: For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in y-direction, $\frac{\partial u}{\partial y}$ only.

The correct option is (b).

09. Ans: 28.5

Sol: Given data,

Flow is over a flat plate.

L = 1 m,

U_∞ = 6 m/s

ν = 0.15 stoke = 0.15×10^{-4} m^2/s

ρ = 1.226 kg/m^3

$$\delta(x) = \frac{3.46x}{\sqrt{Re_x}}$$

Velocity profile is linear.

Using von-Karman momentum integral equation for flat plate.

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2} \text{ --------(1)}$$

we can find out τ_w .

From linear velocity profile, $\frac{u}{U_\infty} = \frac{y}{\delta}$, we evaluate first θ, momentum thickness as

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy = \int_0^\delta\left(\frac{y}{\delta}-\frac{y^2}{\delta^2}\right)dy$$

$$= \left(\frac{y^2}{2\delta}-\frac{y^3}{3\delta^2}\right)_0^\delta = \frac{\delta}{2}-\frac{\delta}{3} = \frac{\delta}{6}$$

$$\Rightarrow \theta = \frac{\delta}{6} = \frac{1}{6}\times\frac{3.46\,x}{\sqrt{Re_x}}$$

$$= \frac{3.46}{6}\frac{x^{1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}}$$

Differentiating θ w.r.t x, we get :

$$\frac{d\theta}{dx} = \frac{3.46}{6\times2}\frac{x^{-1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}} = 0.2883\frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0.5\,m} = 0.2883\times\frac{1}{\sqrt{\frac{6\times0.5}{0.15\times10^{-4}}}} = \frac{0.2883}{447.2} \text{ -----(2)}$$

From equation (1)

$$\tau_w\big|_{x=0.5m} = \left.\frac{d\theta}{dx}\right|_{x=0.5\,m}\times\rho U_\infty^2$$

$$= \frac{0.2883}{447.2}\times1.226\times6^2$$

$$= 0.02845\ \text{N/m}^2 \simeq 28.5\ \text{mN/m}^2$$

10. Ans: (c)

Sol:

- For laminar boundary layer over a flat plate, the velocity gradient at the surface decreases in the direction of flow.
- This results in the decrease in shear stress and hence, the decrease in skin friction coefficient in the direction of flow.
- Thus, statement (I) is correct but the statement (II) is wrong.

11. Ans: (b)

Sol:

- The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free stream velocity. This results in higher skin friction drag in turbulent boundary layer. Thus, statement (I) is correct.
- The separation of turbulent boundary is late as compared to laminar boundary layer. Thus, statement (II) is also correct but it is not the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: Given data:

Test section dia = 40 cm

Test section length = 60 cm

Velocity of air at inlet = 2 m/s

and $\delta^* = \frac{1.72x}{\sqrt{Re_x}}$

$$Re_x = \frac{2\times 0.6}{10^{-5}} = 1.2\times 10^5$$

So, δ^* at $x = 0.6\text{m} = \frac{1.72\times 0.6}{\sqrt{1.2\times 10^5}}$

$= 2.979\times 10^{-3}$ m

From equation of continuity

$A_{in}V_{in} = A_{exit}V_{exit}$

But $d_{exit} = 0.4 - 2\delta^*$

$= (0.4 - 2\times 2.979\times 10^{-3})$ m

Thus, $V_{exit} = \left(\frac{0.4}{0.4 - 2\times 2.979\times 10^{-3}}\right)^2 \times 2$

$= 2.061$ m/s

02.

Sol: Given data:

Flow over a flat plate

Fluid is water.

$U_\infty = 1$ m/s

$L = 1$ m

Case I: *Flow is turbulent*

At x = 1 m

$$Re_x = \frac{U_\infty x}{\nu_{water}} = \frac{1\times 1}{10^{-6}} = 10^6$$

$$\frac{\delta_{tur}}{x} = \frac{0.376}{(Re_x)^{\frac{1}{5}}} = \frac{0.376}{(10^6)^{\frac{1}{5}}}$$

$$\delta_{tur} = \frac{0.376 \times 1}{(10^6)^{\frac{1}{5}}} = 0.0237\,m \approx 24 \text{ mm}$$

$$\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = C_{f,x} = \frac{0.059}{(Re_x)^{\frac{1}{5}}}$$

$$\tau_w = \frac{0.059}{(10^6)^{\frac{1}{5}}} \times \frac{1}{2} \times 10^3 \times 1^2 = 1.86 \text{ N/m}^2$$

Case 2: *If the flow is laminar*

For the comparison purpose, consider the same Reynolds number.

$$\frac{\delta_{lam}}{x} = \frac{5}{\sqrt{Re_x}}$$

$$\delta_{lam} = \frac{5 \times 1}{\sqrt{10^6}} = 5 \text{ mm}$$

and $\tau_w = \frac{0.664}{\sqrt{Re_x}} \times \frac{1}{2}\rho U_\infty^2$

$$= \frac{0.664}{\sqrt{10^6}} \times \frac{1}{2} \times 10^3 \times 1^2 = 0.332 \text{ N/m}^2$$

11. Force on Submerged Bodies

01. Ans: 8

Sol: Drag power = Drag Force × Velocity

$P = F_D \times V$

$$P = C_D \times \frac{\rho A V^2}{2} \times V$$

$P \propto V^3$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^3$$

$$\frac{P_1}{P_2} = \left(\frac{V}{2V}\right)^3$$

$P_2 = 8P_1$

Comparing the above relation with XP,

We get, X = 8

02. Ans: 4.56 m

Sol: $F_D = C_D.\frac{\rho A V^2}{2}$

$$W = 0.8 \times 1.2 \times \frac{\frac{\pi}{4}(D)^2 \times V^2}{2}$$

(Note: A = Normal (or) projected Area = $\frac{\pi}{4}D^2$)

$$784.8 = 0.8 \times 1.2 \times \frac{\pi}{4}(D)^2 \times \frac{10^2}{2}$$

$\therefore$ D = 4.56 m

03. Ans: 4

Sol: Given data:

l = 0.5 km = 500 m

d = 1.25 cm

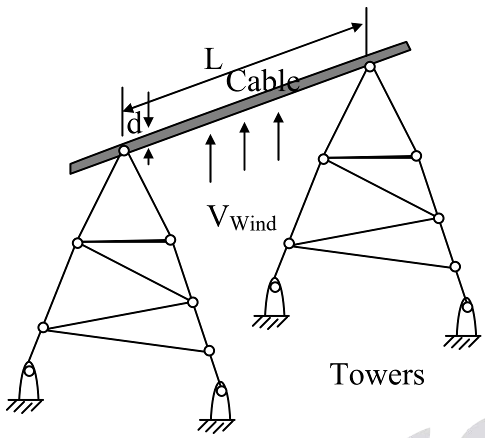
V_{Wind} = 100 km/hr

$\gamma_{Air} = 1.36 \times 9.81 = 13.4 \text{ N/m}^3$

$\nu = 1.4 \times 10^{-5} \text{ m}^2/\text{s}$

C_D = 1.2 for Re > 10000

C_D = 1.3 for Re < 10000



$$Re = \frac{V.L}{\nu} = \frac{\left(\frac{100\times5}{18}\right)(0.0125)}{1.4\times10^{-5}}$$

Note: The characteristic dimension for electric power transmission tower wire is "d"

Re = 24801 > 10,000

$\therefore$ $C_D = 1.2$

$$F_D = C_D\times\frac{\rho AV^2}{2}$$

$$= 1.2\times\frac{\left(\frac{13.4}{9.81}\right)(L\times d)V^2}{2}$$

$$= \frac{1.2\times\left(\frac{13.4}{9.81}\right)(500\times0.0125)\left(100\times\frac{5}{18}\right)^2}{2}$$

= 3952.4 N = 4 kN

04. Ans: 0.144 & 0.126

Sol: Given data:

$W_{Kite} = 2.5$ N

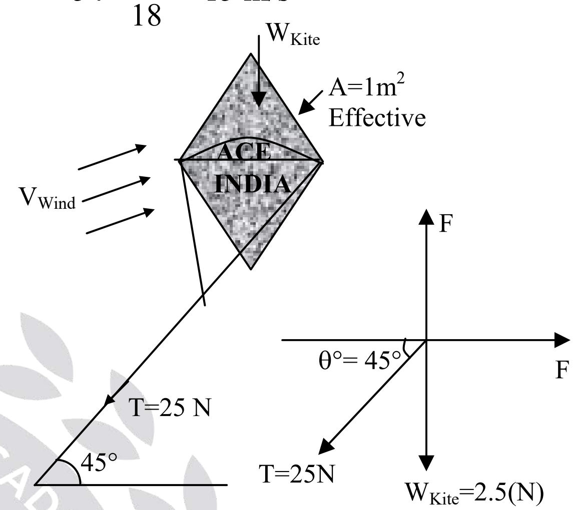
$A = 1$ m^2

$\theta = 45^o$

T = 25 N

$V_{Wind} = 54$ km/hr

$$= 54\times\frac{5}{18} = 15 \text{ m/s}$$



Resolving forces horizontally

$$F_D = T\cos45^o$$

$$C_D\times\frac{\rho AV^2}{2} = 25\times\cos45^o$$

$$\frac{C_D\times\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 25\times\frac{1}{\sqrt{2}}$$

$\therefore$ $C_D = 0.126$

Resolving forces vertically

$$F_L = W_{Kite} + T\sin45^o$$

$$\frac{C_L\rho AV^2}{2} = 2.5 + 25\sin45^o$$

$$\frac{C_L\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 2.5 + \frac{25}{\sqrt{2}}$$

$\therefore C_L = 0.144$

05. Ans: (a)

Sol: Given data:

$C_{D_2} = 0.75\,C_{D_1}$ (25% reduced)

Drag power = Drag force × Velocity

$$P = F_D \times V = \frac{C_D \rho A V^2}{2} \times V$$

$$P = C_D \times \frac{\rho A V^3}{2}$$

Keeping ρ, A and power constant

$C_D V^3$ = constant = C

$$\frac{C_{D_1}}{C_{D_2}} = \left(\frac{V_2}{V_1}\right)^3$$

$$\left(\frac{C_{D_1}}{0.75 C_{D_1}}\right)^{1/3} = \frac{V_2}{V_1}$$

$$\therefore V_2 = 1.10064 V_1$$

% Increase in speed = 10.064%

06. Ans: (c)

Sol: When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid :

Weight of sphere = Buoyant force + Drag force

07. Ans: 0.62

Sol: Given data,

Diameter of dust particle, d = 0.1 mm

Density of dust particle,

$\rho = 2.1$ g/cm^3 = 2100 kg/m^3

$\mu_{air} = 1.849 \times 10^{-5}$ Pa.s,

At suspended position of the dust particle,

$$W_{particle} = F_D + F_B$$

where F_D is the drag force on the particle and F_B is the buoyancy force.

From Stokes law:

$$F_D = 3\pi\mu\,V\,d$$

Thus,

$$\frac{4}{3} \times \pi r^3 \times \rho \times g = 3\pi\mu V d + \frac{4}{3}\pi r^3 \rho_{air} g$$

$$\text{or, } \frac{4}{3}\pi r^3 g(\rho - \rho_{air}) = 3\pi\mu_{air} V(2r)$$

$$\text{or } V = \frac{2}{9} r^2 g\left(\frac{\rho - \rho_{air}}{\mu_{air}}\right)$$

$$= \frac{2}{9} \times (0.05 \times 10^{-3})^2 \times 9.81 \times \frac{(2100 - 1.2)}{1.849 \times 10^{-5}}$$

$$= 0.619 \text{ m/s} \approx 0.62 \text{ m/s}$$

08. Ans: (b)

Sol: Since the two models M_1 and M_2 have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.

From their shapes, we can say that M_2 reaches the bottom earlier than M_1.

09. Ans: (a)

Sol:

- Drag of object A_1 will be less than that on A_2. There are chances of flow separation on A_2 due to which drag will increase as compared to that on A_1.
- Drag of object B_1 will be more than that of object B_2. Because of rough surface of B_2, the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but C_2 is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on C_2 will be more than that on C_1 because of flow becoming turbulent due to roughness. Hence, drag of object C_2 will be more than that of object C_1.
- Thus, the correct answer is option (a).

10. Ans: (a)

Sol:

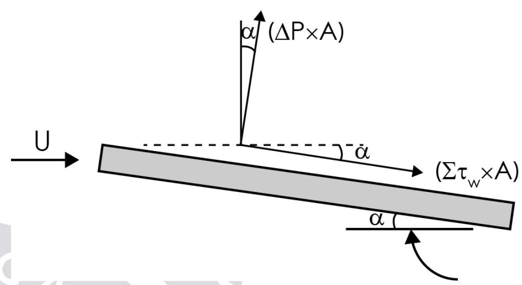
- Dimples on a golf ball are intentionally provided to make its surface rough so that flow becomes turbulent.
- A turbulent boundary layer, having more momentum than a laminar boundary layer, can better resist an adverse pressure gradient, thus avoiding early separation.
- Thus, both statements are correct and statement (II) is the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: The lift force on the given plate is

$F_L = (\Delta P \times A)\cos\alpha - (\Sigma\tau_w \times A)\sin\alpha$



$F_D = (\Delta P \times A)\sin\alpha + (\Sigma\tau_w \times A)\cos\alpha$

Substituting the values given:

$F_D = [2.3 - (-1.2)](1)\sin7° + (7.6\times10^{-2} + 5.8\times10^{-2})\times1\times\cos7°$

$= 426.5 \text{ N} + 133 \text{ N} = 559.5 \text{ N} \approx 560 \text{ N}$

$F_L = [2.3 - (-1.2)]\times1\times\cos7° - (7.6\times10^{-2} + 5.8\times10^{-2})\times1\times\sin7°$

$= 3474 \text{ N} - 16.3 \text{ N}$

$= 3457.7 \text{ N} \approx 3458 \text{ N}$

When the shear stress is neglected, then

$F_D = (\Delta P \times A)\cos\alpha = 426.5 \text{ N} \approx 427 \text{ N}$

and $F_L = (\Delta P \times A)\sin\alpha = 3474 \text{ N}$

12. Open Channel Flow

02. Ans: (b)

Sol: $Q_1 = 15 m^3/sec$, $y = 1.5$ m

$S_1 = \frac{1}{1690}$, if $S_2 = \frac{1}{1000}$

Then $Q_2 = ?$

$Q \propto \sqrt{S}$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{\frac{1}{1000}}{\frac{1}{1690}}}$$

$Q_2 = 1.3 \times 15 = 19.5\ m^3/s$

03. Ans: (d)

Sol: $Q = AV$

$= B \times y \times \frac{1}{n} R^{2/3} S^{1/2}$

$= B \times y \times \frac{1}{n} y^{2/3} S^{1/2}$

$= R \approx y \rightarrow$ For wide rectangular channel $Q \propto y^{5/3}$

$$\frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{\frac{5}{3}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{1.25 y_1}{y_1}\right)^{\frac{5}{3}}$$

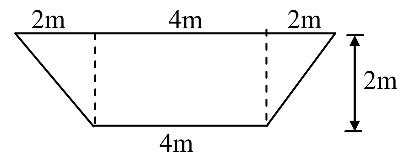
$$\frac{Q_2}{Q_1} = 1.45$$

$Q_2 = 1.45\ Q_1$

It is increased by 45%

05. Ans: 24.33

Sol:



$\tau_{avg} = \gamma_w RS$

$R = \frac{A}{P}$

$A = 2 \times \left(\frac{1}{2} \times 2 \times 2\right) + 4 \times 2$

$= 2 \times 2 + 4 \times 2 = 12\ m^2$

$P = 4 + 2\sqrt{2^2 + 2^2}$

$= 9.66$ m

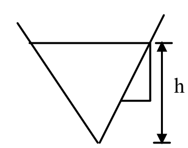
$R = \frac{12}{9.66} = 1.24\,m$

$\tau_{avg} = 9810 \times 1.24 \times 0.002$

$= 24.33\ N/m^2$

06. Ans: (d)

Sol: **Triangular:**



Triangle

$P = 2$ (Inclined portion)

$P = 2I = 2h\sqrt{1+m^2} \quad \left(\because I = h\sqrt{1+m^2}\right)$

$$= 2h\sqrt{1+1^2}$$

$$= 2h\sqrt{2}$$

$$\frac{P}{h} = 2\sqrt{2} = 2.83$$

Trapezoidal: Efficient trapezoidal section is half of the Hexagon for which all sides are equal

Trapezoidal

$$I = h\sqrt{1+m^2}$$

$$P = I = h\sqrt{(1)+\left(\frac{1}{\sqrt{3}}\right)^2} = h(1.15)$$

$$\frac{P}{h} = 1.15\times 3 = 3.46 \quad \text{(3 sides are equal)}$$

Rectangular:

$$P = b + 2h = 2h + 2h = 4h \;(\;b = 2y\;)$$

$$\frac{P}{h} = 4$$

07. Ans: 0.37

Sol: A = y (b + my)

$$A = \frac{Q}{V} = \frac{5}{1.25} = 4\,m^2$$

$$4 = \left(b + \frac{y}{\sqrt{3}}\right)y \;\ldots\ldots.(I) \quad \left(\because m = \frac{1}{\sqrt{3}}\right)$$

But b = I ($\because$ Efficient trapezoidal section)

$$b = y\sqrt{1+m^2}$$

$$b = \frac{2y}{\sqrt{3}} \;\ldots\ldots\ldots\ldots\ldots(II)$$

From (I) & (II)

y= 1.519 m

$$\therefore\; D = \frac{(b+my)y}{b+2my} = 1.14\,m$$

$$\therefore F_r = \frac{V}{\sqrt{gD}}$$

$$F_r = 0.37$$

08. Ans: (a)

Sol: Alternate depths

$y_1 = 0.4$ m

$y_2 = 1.6$ m

Specific energy at section =?

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2g\,y_2^2}$$

$$0.4 + \frac{q^2}{2\times 9.81\times 0.4^2} = 1.6 + \frac{q^2}{2\times 9.81\times 1.6^2}$$

$$q^2\left(\frac{1}{3.1392} - \frac{1}{50.22}\right) = 1.6 - 0.4$$

$$q^2(0.298) = 1.2$$

$$q^2 = 4.02$$

$$q = 2\ m^3\ /s/m$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2}$$

$$E_1 = 0.4 + \frac{2^2}{2\times 9.81\times 0.4^2} = 1.68\ m$$

09. Ans: (b)

Sol: Depth = 1.6 m

Specific energy = 2.8 m

$$E_1 = \left[y_1 + \frac{v^2}{2g}\right] \Rightarrow 2.8 = 1.6 + \frac{V^2}{2\times 9.81}$$

V = 4.85 m/s

$$F_r = \frac{V}{\sqrt{gy}}$$

$$F_r = \frac{4.85}{\sqrt{9.81\times 1.6}} = 1.22 > 1 \text{(Supercritical)}$$

10. Ans: (c)

Sol: $F_r = 5.2$ (uniform flow)

The ratio of critical depth to normal depth $\frac{y_c}{y_n} = ?$

Note: The given two depths y_c & y_n are not alternate depths as they will have different specific energies.

$$F_r = \frac{V}{\sqrt{gy}} \Rightarrow F_r^2 = \frac{V^2}{gy} = \frac{q^2}{gy^3}\left(\because v = \frac{q}{y}\right)$$

$$\frac{(F_{rn})^2}{(F_{rc})^2} = \frac{q^2}{gy_n^3}\times\frac{gy_c^3}{q^2} = \frac{y_c^3}{y_n^3}$$

$$\frac{y_c^3}{y_n^3} = \frac{(F_{rn})^2}{(F_{rc})^2} \Rightarrow \frac{y_c}{y_n} = \frac{(F_{rn})^{2/3}}{(F_{rc})^{2/3}}$$

$$\frac{y_c}{y_n} = (5.2)^{2/3} = 3$$

11. Ans: (c)

Sol: Rectangular channel

Alternate depths $y_1 = 0.2$, $y_2 = 4$m

$E_1 = E_2$ ($\because$ alternate depths), $F_r = \frac{V}{\sqrt{gD}}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left[1 + \frac{Fr_2^2}{2}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{Fr_2^2}{2}}{1 + \frac{Fr_1^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{4^2}{2}}{1 + \frac{0.2^2}{2}}\right]$$

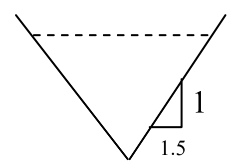
$$\frac{y_1}{y_2} = \left(\frac{2+16}{2+0.04}\right) = 8.8$$

12. Ans: (d)

Sol: Triangular channel

H:V = 1.5:1

Specific energy = 2.5 m



$$E_C = \frac{5}{4}y_c$$

$$\frac{4}{5}E_C = y_c$$

$y_c = 2$m

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} \Rightarrow 2 = \left(\frac{2\times Q^2}{9.81\times 1.5^2}\right)^{1/5}$$

Q = 18.79 m^3/sec

13. Ans: 0.47

Sol: $E_1 = E_2 + (\Delta z)$

$$V_1 = \frac{Q}{A_1} = \frac{12}{2.4\times 2} = 2.5\,\text{m/sec}$$

$A_2 = (b_2 + my_2)y_2 = (1.8 + 1\times 1.6)\ 1.6$

$= 5.44\ m^2$

$$V_2 = \frac{Q}{A_2} = \frac{12}{5.44} = 2.2\,m/sec$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{(2.5)^2}{2\times 9.81} = 2.318\,m$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 1.6 + \frac{2.2^2}{2\times 9.81} = 1.846\ m$$

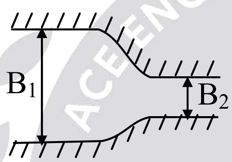
$2.318 = 1.846 + \Delta Z \Rightarrow \Delta Z = 0.47\ m$

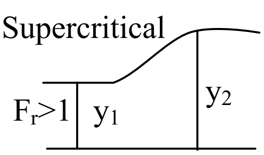
14. Ans: (c)

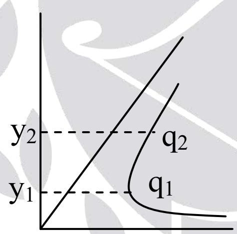
Sol: $F_r > 1$

$B_2 < B_1$

$q_2 > q_1$







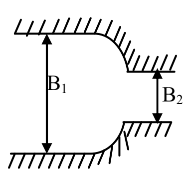
As Potential energy (y) increases then kinetic energy (v) decreases

$\therefore$ 'y' increases and 'v' decreases.

15. Ans: (a)

Sol: $Q = 3m^3/s$

$B_1 = 2m,\ D = 1.2\ m$



Width reduce d to 1.5 m (B_2)

Assume channel bottom as horizontal

$\therefore E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{B_1 y_1} = \frac{3}{2\times 1.2} = 1.25\,m/sec$$

$$V_2 = \frac{Q}{B_2 y_2} = \frac{3}{1.5\times y_2} = \frac{2}{y_2}$$

$$1.2 + \frac{(1.25)^2}{2\times 9.81} = y_2 + \frac{\left(\frac{2}{y_2}\right)^2}{2\times 9.81}$$

$$1.27 = y_2 + \frac{4}{y_2^2\times 19.62}$$

$$1.27 = y_2 + \frac{0.2}{y_2^2}$$

$$y_2^2(1.27) = y_2^3 + 0.2$$

$$y_2^3 - 1.27y_2^2 + 0.2 = 0$$

$y_2 = 1.12\ m$

$$F_{r_1} = \frac{1.25}{\sqrt{9.81\times 1.2}}\left[\frac{V}{\sqrt{gD}} < 1\right] = 0.364 < 1$$

Approaching flow is sub critical. If approaching flow is sub critical the level of water falls in the throat portion.

16. Ans: (d)

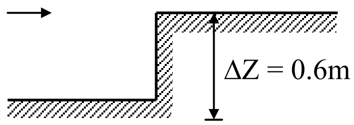
Sol: Rectangular Channel

$y_1 = 1.2m$

$V_1 = 2.4m/s$

$\Delta Z = 0.6m$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{(2.4)^2}{2\times 9.81} = 1.49m$$



$Q = 2.4 \times 1.2 = 2.88\ m^3/s/m$

Assuming channel width as constant, the critical depth

$$y_c = \left[\frac{Q^2}{gB^2}\right]^{\frac{1}{3}} = 0.94m$$

Critical specific energy for rectangular channel $E_C = \frac{3}{2} y_c$

$$E_c = \frac{3}{2}(0.94) = 1.41$$

We know for critical flow in the hump portion $E_1 = E_2 + (\Delta Z) = E_C + (\Delta Z)_C$

$\Rightarrow 1.49 = 1.41 + (\Delta Z)_C$

$\therefore (\Delta Z)_C = 0.08m$

If the hump provided is more than the critical hump height the u/s flow gets affected.

(or)

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{2.4}{\sqrt{9.81\times 1.2}} = 0.69 < 1$$

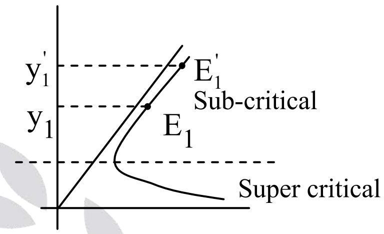
$\Rightarrow$ Hence sub-critical.

If the approaching flow is sub-critical the level of water will fall in the hump portion.

Option (b) is correct if the hump height provided is less than critical hump height.

As the hump height provided is more than critical, the u/s flow gets affected with the increase of the specific energy from E_1 to E_1^1.

In the sub-critical region as the specific energy increases, the level of water rises from y_1 to y_1^1 in the form of a surge.



$$E_1^1 = y_1^1 + \frac{v^{1^1}}{2g}$$

$$E_1^1 = y_1^1 + \frac{q^2}{2gy_1^{1^2}} \quad \ldots (1)$$

Also $E_1^1 = E_c + (\Delta Z)$ provided.

$= 1.41 + 0.6$

$= 2.01m$

$$\therefore 2.01 = y_1^1 + \frac{2.88^2}{2\times 9.81\times y_1^2}$$

Solve by trial & error

for $y_1^1 > 1.2m$

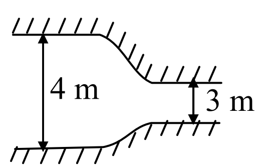
17. Ans: (c)

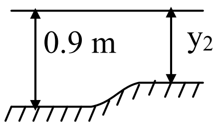
Sol: $B_1 = 4$ m

$B_2 = 3$ m

(U/S) $y_1 = 0.9$ m

$E_1 = E_2 + \Delta Z$





$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta Z$$

$V_1 = V_2$

According to continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2 V_2$

$A_1 = A_2$

$B_2y_1 = B_2 y_2$

$4\times0.9 = 3\times y_2$

$y_2 = 1.2$ m

$y_1 = y_2+\Delta Z$

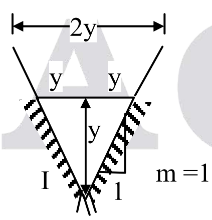
$0.9 = 1.2+\Delta Z$

$\Delta Z = -0.3$ m

Negative indicates that the hump assumed is wrong infact it is a drop.

18. Ans: (a)

Sol: Given :

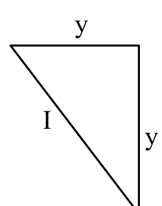


Top width = 2y

$$\text{Area} = \frac{1}{2}\times b\times h$$

$$= \frac{1}{2}\times 2y\times y$$

$A = y^2$



Wetted perimeter

$$I^2 = \sqrt{y^2 + y^2} = y\sqrt{2}$$

(Both sides) total wetted perimeter

$(P) = \sqrt{2}.y + \sqrt{2}.y = 2\sqrt{2}.y$

Hydraulic mean depth

$$(R) = \frac{A}{P} = \frac{y^2}{2\sqrt{2}y} = \frac{y}{2\sqrt{2}}$$

$y = y_n$(say)

Using Mannings formula

$$Q = A.\frac{1}{n}.(R)^{2/3}.(S)^{1/2}$$

$$0.2 = y_n^2\frac{1}{0.015}\left[\frac{y_n}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$\frac{1}{y_n^{8/3}} = \frac{1}{0.015\times0.2}\times\left[\frac{1}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$y_n^{8/3} = 0.2\times0.015\times\left(2\sqrt{2}\right)^{2/3}\left[\frac{1}{0.001}\right]^{1/2}$$

$$(y_n)^{8/3} = 0.189$$

$$y_n = (0.189)^{3/8}$$

$y_n = 0.54$ m

$$\text{critical depth}(y_c) = \left[\frac{2Q^2}{g}\right]^{1/5}$$

(for triangle)

$$y_c = \left[\frac{2\times0.2^2}{9.81}\right]^{1/5} = 0.382\,\text{m}$$

$y_n > y_c$ (0.54>0.38)

$\therefore$ mild slope

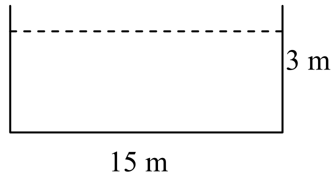
If (actual) depth at flow = 0.4m = y

$Y_n > y > y_c$ [0.54 > 0.4 > 0.38]

$\therefore$ Profile is M_2

19. Ans: 4.36×10^{-5}

Sol:



$\therefore$ Discharge, Q = 29 m^3/sec

Area of rectangular channel, A = 15 × 3 = 45 m^2

Perimeter, P = 15 + 2 × 3 = 21 m

Hydraulic radius, $R = \frac{A}{P} = \frac{45}{21} = 2.142$ m

$\therefore$ The basic differential equation governing the gradually varied flow is

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2T}{gA^3}}$$

$\frac{dy}{dx}$ = Slope of free water surface w.r.t to channel bottom

Velocity of flow $V = \frac{Q}{A} = \frac{29}{45}$

$= 0.644$ m/sec

$\therefore$ By Chezy's equation

Velocity, $V = C\sqrt{RS_f}$

$$0.644 = 65\sqrt{2.142 \times S_f}$$

$$S_f = 4.589 \times 10^{-5}$$

$$S_o = \frac{1}{5000} = 2 \times 10^{-4}$$

$$\frac{Q^2T}{gA^3} = \frac{29^2 \times 15}{9.81 \times 4^3} = 0.0141$$

$$\therefore \frac{dy}{dx} = \frac{2 \times 10^{-4} - 4.589 \times 10^{-5}}{1 - 0.0141}$$

$$= 1.5631 \times 10^{-4}$$

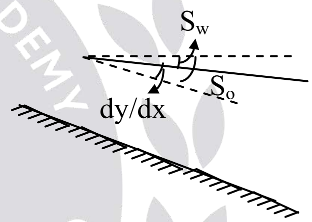
$$\therefore S_o = S_w + \frac{dy}{dx}$$

S_w water surface slope with respect to horizontal

$$S_w = S_o - \frac{dy}{dx}$$

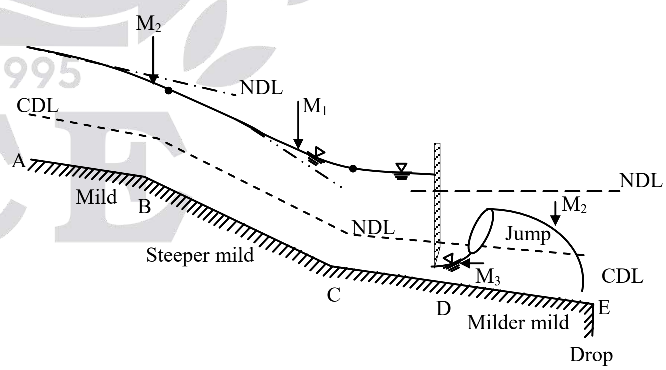
$$= 2 \times 10^{-4} - 1.563 \times 10^{-4}$$

$$S_w = 4.36 \times 10^{-5}$$



20. Ans: (a)

Sol:



22. Ans: 0.74

Sol: Free fall $\rightarrow$ 2nd profile

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

$$y_c = \left(\frac{2^2}{9.81}\right)^{\frac{1}{3}} = 0.74 \text{ m}$$

$$V = \frac{q}{y_n}$$

$$\frac{2}{y_n} = \frac{1}{n} y_n^{2/3} S^{1/2}$$

$$\frac{2}{y_n} = \frac{1}{0.012} \times y_n^{2/3} (0.0004)^{1/2}$$

$y_n = 1.11$ m

$y_n > y_c$

Hence the water surface will have a depth equal to y_c

$y_c = 0.74$ m

23. Ans: (d)

Sol: $q = 2 \text{ m}^2/\text{sec}$

$y_A = 1.5$ m; $y_B = 1.6$ m

$\Delta E = 0.09$

$$S_o = \frac{1}{2000}$$

$$\overline{S}_f = 0.003$$

$$\Delta x = \frac{\Delta E}{S_o - \overline{S}_f} = \frac{0.09}{\frac{1}{2000} - 0.003} = -36 \text{ m}$$

24. Ans: (d)

Sol: Given $q_1 = Q/B = 10 \text{ m}^3/\text{s}$

$v_1 = 20$ m/s

$$\therefore y_1 = \frac{q_1}{v_1} = \frac{10}{20} = 0.5 \text{ m}$$

We know that relation between y_1 and y_2 for hydraulic jump is

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.81 \times 0.5}} = 9.03$$

$$\therefore \frac{y_2}{0.5} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 \times (9.03)^2}\right]$$

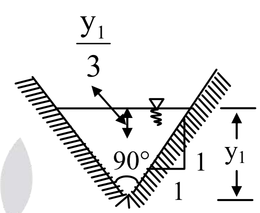
$y_2 = 6.14$ m

25. Ans: (c)

Sol: $Q = 1 \text{ m}^3/\text{s}$

$y_1 = 0.5$ m

$y_2 = ?$



As it is not a rectangular channel, let us work out from fundamentals by equating specific force at the two sections.

$$\left[\frac{Q^2}{gA} + A\bar{z}\right]_1 = \left[\frac{Q^2}{gA} + A\bar{z}\right]_2$$

$$\frac{1^2}{9.81 \times y_1^2} + y_1^2 \times \frac{y_1}{3} = \frac{1^2}{9.81 y_2^2} + y_2^2 \times \frac{y_2}{3}$$

$$0.449 = \frac{1}{9.81 y_2^2} + \frac{y_2^3}{3}$$

$y_2 = 1.02$ m

26. Ans: (b)

Sol: **Given:**

Head = 5 m = (ΔE)

Froud number = 8.5

Approximate sequent depths =?

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_{r1}^2}\right]$$

$$= \frac{1}{2}\left[-1+\sqrt{1+8(8.5)^2}\right]$$

$= 11.5$ m

$y_2 = 11.5\ y_1$

$$\left.\begin{array}{l}(a)\ y_2 = 11.5(0.3) = 3.45 \\ (b)\ y_2 = 11.5\,(0.2) = 2.3\text{ m}\end{array}\right\}\text{from options}$$

$y_1 = 0.2$, $y_2 = 2.3$ m

(or)

$\Delta E = 5$ m

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\frac{(11.5y_1 - y_1)^3}{4(11.5y_1)y_1} = 5$$

$$(10.5y_1)^3 = 230y_1^2$$

$$1157.625\ y_1 = 230$$

$y_1 = 0.2$ m

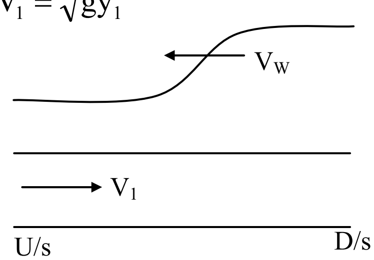
$y_2 = 11.5(0.2)$

$y_2 = 2.3$ m

27. Ans: 1.43

Sol: $y_1 = 1.2$ m

$$V_w + V_1 = \sqrt{gy_1}$$



$$V_1 = \sqrt{9.81\times1.2} - 2$$

$V_1 = 1.43$ m/s

In this problem if the wave moves downstream the velocity of wave is

$$V_w - V_1 = \sqrt{gy_1}$$

$$V_w = \sqrt{gy_1} + V_1$$

$$= \sqrt{9.81\times1.2} + 2$$

$= 5.43$ m/s

28. Ans: (b)

Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-12, 79th Question -pg: 154)

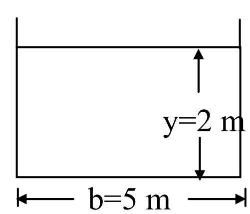
29. Ans: (c)

Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-12, 87th Question -pg: 155)

Conventional Practice Solutions

01.

Sol: Given data:



$n = 0.015$

$b = 5$ m, $y = 2$ m

$S_1 = 1: 1600$

Discharge calculation in

Case - 1:

Wetted area $A_1 = 5 \times 2 = 10\ m^2$

Wetted perimeter $P_1 = b + 2y$

$= 5+2\ (2) = 9$ m

Hydraulic Radius

$$R_1 = \frac{A_1}{P_1} = \frac{10}{9} = 1.11\,m$$

Discharge,

$$Q = A_1V_1 = 10 \times \left(\frac{1}{n} R^{2/3} . S^{1/2}\right)$$

$$= 10 \times \left(\frac{1}{0.015} \times (1.11)^{2/3} \times \left(\frac{1}{1600}\right)^{1/2}\right)$$

$$= 10 \times (1.79)$$

$$= 17.8675\ m^3/s$$

Case - 2:

Consider hydraulically efficient rectangular channel so that discharge is maximum.

Given that lining area constant w.r.to original channel.

$$P_2 = P_1 = 9\ m$$

$$b_2 + 2y_2 = 9\ m$$

For efficient rectangular channel we know

$$b = 2y,$$

$$2y_2 + 2y_2 = 9$$

$$y_2 = 9/4 = 2.25\ m$$

$$b_2 = 2 \times 2.25 = 4.5\ m$$

By Manning's formula:

$$V_2 = \frac{1}{n} R^{2/3} . S^{1/2}$$

$$= \frac{1}{0.015} \times \left(\frac{2.25}{2}\right)^{2/3} \left(\frac{1}{1600}\right)^{1/2}$$

$$V_2 = 1.80\ m/s$$

$$Q = A_2V_2$$

$$= (4.5 \times 2.25)(1.80) \qquad \left(R = \frac{y}{2} = \frac{b}{4}\right)$$

$$= 18.25\ m^3/sec$$

% increase in discharge $Q = \frac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{18.25 - 17.8675}{17.9} \times 100$$

$$= 2.14\%$$

By Froude's number

$$F_{r_1} = \frac{V_1}{\sqrt{gy_1}} = \frac{1.8}{\sqrt{9.81 \times 2}}$$

$$F_{r_1} = 0.40 < 1$$

It is sub critical flow.

$$F_{r_2} = \frac{V_2}{\sqrt{gy_2}} = \frac{1.8}{\sqrt{9.81 \times 2.25}}$$

$$F_{r_2} = 0.38 < 1$$

So it also sub critical flow.

$\therefore$ The sub critical flow is not changing into super critical flow.

02.

Sol: Say, q = discharge per meter width, according to the continuity equation for constant width

$$q = V_1\, y_1 = V_2\, y_2$$

As y_1 and y_2 are alternative depths, the specific energy is same at both the sections.

$E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

Hence,

$$(y_1 - y_2) = \frac{q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right)$$

$$2(y_1 - y_2) = \frac{(y_1^2 - y_2^2)}{y_1^2 y_2^2} \cdot \frac{q^2}{g}$$

For a rectangular channel $\frac{q^2}{g} = y_c^3$

Hence, $$y_c^3 = \frac{2(y_1 - y_2)y_1^2 y_2^2}{(y_1^2 - y_2^2)}$$

$$y_c^3 = \frac{2y_1^2 y_2^2}{(y_1 + y_2)}$$

Specific energy, $E = y_1 + \frac{1}{2}\cdot\left(\frac{q^2}{g}\right)\frac{1}{y_1^2}$

Substituting the value of $\frac{q^2}{g}$ in the above equation

$$E = y_1 + \frac{1}{2} \times \frac{2y_1^2 y_2^2}{(y_1 + y_2)} \times \frac{1}{y_1^2}$$

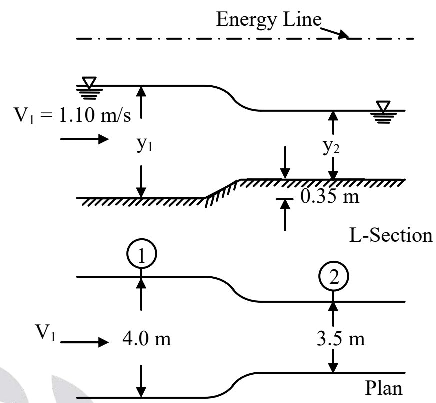
$$= y_1 + \frac{y_2^2}{y_1 + y_2}$$

$$= \frac{y_1^2 + y_1 y_2 + y_2^2}{(y_1 + y_2)}$$

Hence proved

03.

Sol:



At upstream section 1

$A_1 = 1.6 \times 4.0 = 6.4 \text{ m}^2$

$Q = A_1V_1 = 1.10 \times 6.4 = 7.04 \text{ m}^3/\text{s}$

Discharge intensity,

$$q_1 = \frac{Q}{b_1} = \frac{7.04}{4.0}$$

$$= 1.76 \text{ m}^3/\text{s/m}$$

$V_1 = 1.10$ m/s, $\frac{V_1^2}{2g} = \frac{(1.10)^2}{2\times 9.81} = 0.06167$ m

Specific energy, $E_1 = y_1 + \frac{V_1^2}{2g}$

$= 1.60 + 0.06167$

$= 1.66167$ m

Froude number, $F_1 = \frac{V_1}{\sqrt{gy_1}}$

$$= \frac{1.10}{\sqrt{9.81 \times 1.6}} = 0.2776$$

As $F_1 < 1.0$, upstream flow is subcritical. The water surface will drop down at the contracted section.

Contracted section: (Section 2)

$$q_2 = \frac{Q}{b_2} = \frac{7.04}{3.50} = 2.0114 \text{ m}^3\text{/s/m}$$

Critical depth, $y_{c2} = \left(\frac{q_2^2}{g}\right)^{1/3}$

$$= \left(\frac{(2.0114)^2}{9.81}\right)^{1/3} = 0.7444 \text{ m}$$

Minimum specific energy at section 2 = E_{c2}

$$E_{c2} = y_{c2} + \frac{V_{c2}^2}{2g} = 1.5y_{c2} = 1.5 \times 0.7444 = 1.1165 \text{ m}$$

At critical at conditions

$E_1 = E_2 + \Delta Z_c$

$1.66167 = 1.1165 + \Delta Z_c$

$\therefore \Delta Z_c = 0.545$

$\Delta Z < \Delta Z_c$ given

So up stream level would not get disturbed

By energy equation,

$$E_1 = 1.66167 = y_2 + \frac{V_2^2}{2g} + \Delta z$$

$$= y_2 + \frac{q_2^2}{2gy_2^2} + \Delta z$$

$$1.66167 = y_2 + \frac{(2.0114)^2}{2 \times 9.81 \times y_2^2} + 0.35$$

$$y_2 + \frac{0.2062}{y_2^2} = 1.3117$$

By trail and error method

Value of y_2 is found as

$y_2 = 1.158$ m

The upstream depth will remain unaffected at

$y_1 = 1.60$ m

Hence, with the bed level of the section 1 as datum

Elevation of upstream water surface = 1.60 m

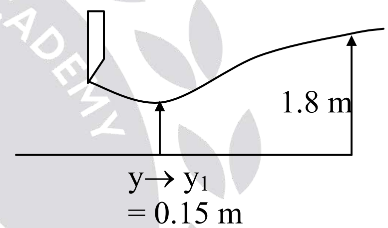
Elevation of water section at the contracted section

$= y_2 + \Delta z = 1.158 + 0.35$

$= 1.508$

04.

Sol:



Froud number at vena contracta = $\frac{V_{VC}}{\sqrt{gy_{VC}}}$

$$V_{VC} = \frac{q}{y_{vc}} = \frac{2}{0.15} = 13.33 \text{ m/s}$$

$$Fr_{(VC)} = \frac{13.33}{\sqrt{9.81 \times 0.15}} = 10.98$$

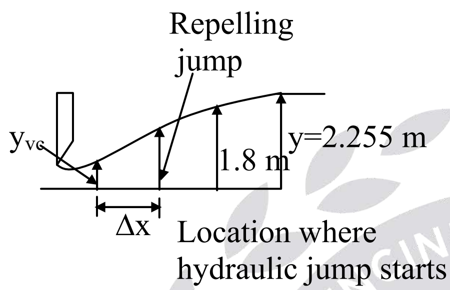
We know

$$y_2 = \frac{y_{1(VC)}}{2}\left[-1 + \sqrt{1 + 8F_{r(VC)}^2}\right]$$

$$y_2 = \frac{0.15}{2}\left[-1 + \sqrt{1 + 8(10.98)^2}\right] = 2.255 \text{ m}$$

If hydraulic jump starts at vena contracta the tail water depth shall be 2.255 m given tail.

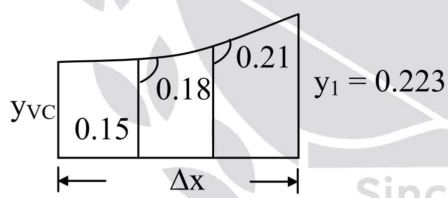
Water depth is 1.8 m. It means due to practical situation the jump is repelling in such a case the jump will not start at vena contracta. but slightly ahead of the vena contracta towards tail water.



$y_{vc} = 0.15$

$$V_2 = \frac{q}{y_2} = \frac{2}{1.8} = 1.11 \text{ m/sec}$$

$$F_{r_2} = \frac{V_2}{\sqrt{gy_2}} = \frac{1.11}{\sqrt{9.81\times 1.8}} = 0.264$$



Actual initial depth

$$y_1 = \frac{y_2}{2}\left[-1+\sqrt{1+8F_{r_2}^2}\right]$$

$$= \frac{1.8}{2}\left[-1+\sqrt{1+8\times 0.264^2}\right]$$

$= 0.223$ m

The distance between vena contracta and starting of jump is Δx calculated by direct step method.

$$V = \frac{q}{y}; E = y + \frac{V^2}{2g}$$

For horizontal flow $S_o = 0$

$$\Delta x = \frac{\Delta E}{S_o - S_f} = (-)\frac{\Delta E}{\overline{S}_f}$$

SF = Energy slope

$$V = \frac{1}{n}R^{2/3}\sqrt{S_f}$$

$$S_f = \frac{V^2 n^2}{R^{4/3}} = \frac{V^2 n^2}{y^{4/3}}$$

$$\overline{S}_f = \frac{S_{f_1} + S_{f_2}}{2}$$

Step	**y**	**V**	**E**	**ΔE**	**SF**	$\overline{S}F$	**Δx (m)**
	0.15	13.33	9.20		0.5016		
1				2.73		0.387	7.054
	0.18	11.11	6.47		0.273		
2				1.64		0.218	7.52
	0.21	9.52	4.83		0.163		
3				0.52		0.148	3.51
	0.223	8.96	4.31		0.133		
							18.08 m

05.

Sol: Given discharge $Q = 4.8$ m^3/sec

Width of the channel b = 4 m

Initial velocity of channel $V_1 = 1$ m/sec

∴ Discharge per meter width

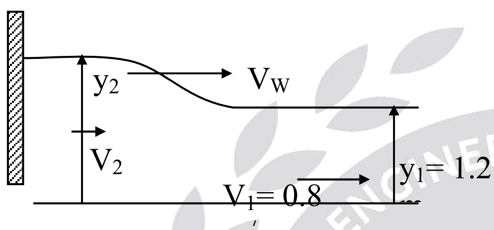
$$q = \frac{Q}{4} = 1.2\,\text{m}^3/\text{sec/m}$$

$$y_1 = \frac{q}{V} = 1.2 = 1.2\,\text{m}$$

By sudden increase of discharge the channel depth is rised by 50%.

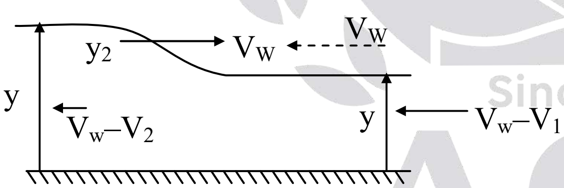
$$y_2 = 1.2 \times 1.5 = 1.8 \text{ m}$$

If discharge is suddenly increased surge will develop which will move downstream with a velocity 'V_w' as shown in figure.



The surge is unsteady rapidly varid flow.

This unsteady flow case can be transformed into a steady one by superimposing flow with velocity 'V_w' in the opposite direction shown in figure.



The continuity equation may be written as

$$A_1V_1 = A_2V_2$$

For unit width of the channel

$y_1(V_w - V_1) = y_2 (V_w - V_2)$

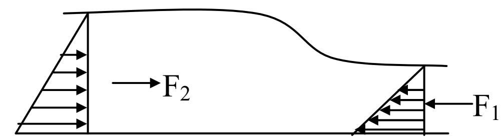
$1.2(V_w - 1) = 1.8 (V_w - V_2)$

$V_w - 1 = 1.5 (V_w - V_2)$

$V_W - 1 = 1.5 V_W - 1.5V_2$

$V_w = 3V_2 - 2 \quad \rightarrow \text{(i)}$

A positive surge moving downstream applying momentum equation



In rectangular channels per unit width the force $F_1 = \dfrac{\gamma y_1^2}{2}, F_2 = \dfrac{\gamma y_2^2}{2}$

$$F_2 - F_1 = \rho Q (V_{d/s} - V_{u/s})$$

$$\frac{\gamma y_2^2}{2} - \frac{\gamma y_1^2}{2} = \rho\, y_1 (V_W - V_1)[(V_w - V_1) - (V_W - V_2)]$$

$$\left(\frac{y_2^2 - y_1^2}{2}\right)\rho g = \rho y_1 (V_W - V_1)(V_2 - V_1)$$

$$\left(\frac{1.8^2 - 1.2^2}{2}\right) 9.81 = 1.2(V_w - 1)\ (V_2 - V_1)$$

$+ 7.3575 = (V_W - 1)(V_2 - 1)$

$7.3575 = (V_w - 1)(V_2 - 1)$

From equation (i)

$$V_w = 3V_2 - 2$$

$7.3575 = (3V_2 - 2 - 1)(V_2 - 1)$

$7.3575 = (3V_2 - 3)(V_2 - 1)$

$7.357 = 3(V_2 - 1)^2$

$V_2 = 2.566$

$\therefore$ From equation (i) $V_w = 3(2.566) - 2$

$V_w = 5.698$ m/sec

$\therefore$ New flow rate $= by_2V_2$

$= 4 \times 1.8 \times 2.566$

$Q_2 = 18.4752 \text{ m}^3\text{/sec}$

13. Dimensional Analysis

01. Ans: (c)

Sol: Total number of variables,

n = 8 and m = 3 (M, L & T)

Therefore, number of π's are = 8 – 3 = 5

02. Ans: (b)

Sol:

1. $\frac{T}{\rho D^2 V^2} = \frac{MLT^2}{ML^{-3} \times L^2 \times L^2 \times T^{-2}} = 1$.

   → It is a non-dimensional parameter.

2. $\frac{VD}{\mu} = \frac{LT^{-1} \times L}{ML^{-1}T^{-1}} \neq 1$.

   → It is a dimensional parameter.

3. $\frac{D\omega}{V} = 1$.

   → It is a non-dimensional parameter.

4. $\frac{\rho VD}{\mu} = Re$.

   → It is a non-dimensional parameter.

03. Ans: (b)

Sol: T = f (l, g)

Total number of variable,

n = 3, m = 2 (L & T only)

Hence, no. of π terms = 3 – 2 = 1

04. Ans: (c)

Sol:

- Mach Number → Launching of rockets
- Thomas Number → Cavitation flow in soil
- Reynolds Number → Motion of a submarine
- Weber Number → Capillary flow in soil

05. Ans: (b)

Sol: According to Froude's law

$$T_r = \sqrt{L_r}$$

$$\frac{t_m}{t_p} = \sqrt{L_r}$$

$$t_p = \frac{t_m}{\sqrt{L_r}} = \frac{10}{\sqrt{1/25}}$$

t_p = 50 min

06. Ans: (a)

Sol: L = 100 m

$V_p = 10\,m/s$,

$L_r = \frac{1}{25}$

As viscous parameters are not discussed, follow Froude's law.

According to Froude ,

$$V_r = \sqrt{L_r}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{1}{25}}$$

$$V_m = \frac{1}{5} \times 10 = 2 \text{ m/s}$$

07. Ans: (d)

Sol: Froude number = Reynolds number.

$\nu_r = 0.0894$

If both gravity & viscous forces are important then

$$\nu_r = (L_r)^{3/2}$$

$$\sqrt[3]{(\nu_r)^2} = L_r$$

$L_r = 1:5$

08. Ans: (c)

Sol: For distorted model according to Froude's law

$$Q_r = L_H L_V^{3/2}$$

$L_H = 1:1000$,

$L_V = 1:100$

$Q_m = 0.1\ m^3/s$

$$Q_r = \frac{1}{1000} \times \left(\frac{1}{100}\right)^{3/2} = \frac{0.1}{Q_p}$$

$Q_P = 10^5\ m^3/s$

09. Ans: (c)

Sol: For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

$$\frac{\rho_w \times V_w \times d_w}{\mu_w} = \frac{\rho_a \times V_a \times d_a}{\mu_a}$$

or $$V_w = \frac{\rho_a}{\rho_w} \times \frac{d_a}{d_w} \times \frac{\mu_w}{\mu_a} \times V_a$$

(where suffixes w and a stand for water and air respectively)

Substituting the values given, we get

$$V_w = \frac{1.2}{10^3} \times \frac{4}{0.1} \times \frac{10^{-3}}{1.8 \times 10^{-5}} \times 1 = \frac{8}{3} m/s$$

To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.

i.e, $$\left(\frac{F_D}{\rho A V^2}\right)_P = \left(\frac{F_D}{\rho A V^2}\right)_m$$

Hence, $$(F_D)_p = (F_D)_m \times \frac{\rho_a}{\rho_w} \times \frac{A_a}{A_w} \times \left(\frac{V_a}{V_w}\right)^2$$

$$= 4 \times \frac{1.2}{10^3} \times \left(\frac{4}{0.1}\right)^2 \times \left(\frac{1}{8/3}\right)^2$$

$= 1.08$ N

10. Ans: 47.9

Sol: Given data,

	Sea water (Prototype testing)	Fresh water (model testing)
V	0.5	?
ρ	1025 kg/m³	10^3 kg/m³
μ	1.07×10^{-3} Pa.s	1×10^{-3} Pa.s

For dynamic similarity, Re should be same in both testing.

i.e., $$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p}$$

$$V_m = V_P \times \frac{\rho_p}{\rho_m} \times \frac{d_p}{d_m} \times \frac{\mu_m}{\mu_p}$$

$$= 0.5 \times \frac{1025}{10^3} \times 100 \times \frac{10^{-3}}{1.07 \times 10^{-3}}$$

$= 47.9$ m/s

11. Refer previous GATE solutions Book (Cha-8, One marks 5th Question -pg: 575)

12. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 5th Question -pg: 205)

13. Ans: (a)

Sol: $V_P = 10$ m/s dia = 3m

$V_m = 5$ m/s, $F_m = 50$ N, $F_p = ?$

Acc to Froude's law:- $F_r = L_r^3$

(But L_r is not given)

$P \propto \rho V^2 = \frac{F}{A}$

$\boxed{\rho A V^2 = F}$ Reynolds law

Now scale ratio:

$$\frac{F_m}{F_P} = \frac{V_m^2}{V_P^2} \times \frac{A_m}{A_p} \times \frac{\rho_m}{\rho_p}$$

$$\frac{50}{F_P} = \left(\frac{1}{10}\right)^2 \times \left(\frac{5}{10}\right)^2 \left(A = L_r^2\right) \ (\because \text{same fluid})$$

$F_P = 20000$ N

14. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 4th Question -pg: 205)

15. Repeated (Same as 13th Question)

16. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 21st Question -pg: 208)

17. Ans: (a)

Sol: $L_r = \frac{1}{100}$

$a_m = 0.013$

$$\frac{a_m}{a_p} = (L_r)^{\frac{1}{6}}$$

$$a_p = \frac{a_m}{(L_r)^{\frac{1}{6}}} = \frac{0.013}{\left(\frac{1}{100}\right)^{1/6}}$$

$a_p = 0.028$

18. Ans: (a)

Sol: $L_r = \frac{1}{9}$

$y_{p1} = 0.5$ m , $y_{p2} = 1.5$ m

$q_m = ?$, $q_p = ?$

$$\frac{2q_P^2}{g} = y_{1p}.y_{2p}\left(y_{1p} + y_{2p}\right)$$

$$\frac{2q_P^2}{9.81} = 0.5 \times 1.5 \times (0.5 + 1.5)$$

$$\frac{2q_P^2}{9.81} = (0.5)(1.5)(2)$$

$q_p = 2.71$

$$q_r = \frac{q_m}{q_p} = L_r^{3/2}$$

$$q_m = \left(\frac{1}{9}\right)^{3/2} \times q_p = 0.1 \text{ m}^3\text{/s/m}$$

19. Refer previous ESE-Obj-(Vol-2) solutions Book (Cha-14, 03rd Question -pg: 205)

Conventional Practice Solutions

01.

Sol: Buckingham π-theorem is stated as:

If there are n variables (dependent and independent variables) in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such as M, L, T, etc.) then the variables are arranged into (n–m) dimensionless terms. These dimensionless terms are called π-terms.

Given that drag force on partially submerged body is a function of

$$F_D = f(V, \nu, K, \rho, g, L)$$

Thus, $n = 7$

and $m = 3$ (M, L & T)

Hence, no. of π - terms = n – m = 7 – 3 = 4

Out of 4- π terms, one of the obvious π-term will be, say $\pi_1 = \frac{K}{L}$

Let us choose ρ, V and L as the repeating variables. Then,

$$\pi_2 = F_D \rho^{a_1} V^{b_1} L^{c_1}$$

$$M^o L^o T^o = MLT^{-2}\left(ML^{-3}\right)^{a_1}\left(LT^{-1}\right)^{b_1}(L)^{c_1}$$

$$= M^{1+a_1} L^{1-3a_1+b_1+c_1} T^{-2-b_1}$$

Equating the indices of M, L and T:

For M: $1 + a_1 = 0 \Rightarrow a_1 = -1$

For T: $-2-b_1 = 0 \Rightarrow b_1 = -2$

For L: $1-3a_1 + b_1 + c_1 = 0$

Or, $1 + 3 - 2 + c_1 = 0$

$\Rightarrow c_1 = -2$

Thus, $\pi_2 = F_D \rho^{-1} V^{-2} L^{-2} = \frac{F_D}{\rho L^2 V^2}$

Similarly,

$$\pi_3 = \nu \rho^{a_2} V^{b_2} L^{c_2}$$

Or, $M^o L^o T^o = L^2 T^{-1}\left(ML^{-3}\right)^{a_2}\left(LT^{-1}\right)^{b_2}(L)^{c_2}$

$$= M^{a_2} L^{2-3a_2+b_2+c_2} T^{-1-b_2}$$

For M : $a_2 = 0$

For T : $-1 - b_2 = 0 \Rightarrow b_2 = -1$

For L : $2 - 3a_2 + b_2 + c_2 = 0$

Or, $2 - 0 - 1 + c_2 = 0$

$\Rightarrow c_2 = -1$

Thus, $\pi_3 = \nu \rho^o V^{-1} L^{-1} = \frac{\nu}{VL} = \frac{1}{Re}$

Similarly,

$$\pi_4 = g \rho^{a_3} V^{b_3} L^{c_3}$$

$$M^o L^o T^o = LT^{-2}\left(ML^{-3}\right)^{a_3}\left(LT^{-1}\right)^{b_3}(L)^{c_3}$$

$$= M^{a_3} L^{1-3a_3+b_3+c_3} T^{-2-b_3}$$

So,

For M : $a_3 = 0$

For T : $-2 - b_3 = 0 \Rightarrow b_3 = -2$

For L : $1 - 3a_3 + b_3 + c_3 = 0$

Or , $1 - 0 - 2 + c_3 = 0$

$\Rightarrow c_3 = 1$

So, $\pi_4 = g \rho^o V^{-2} L^1 = \frac{gL}{V^2} = \frac{1}{F_r^2}$

Thus, we can write:

$$\pi_2 = f(\pi_1, \pi_3, \pi_4)$$

Or, $\frac{F_D}{\rho L^2 V^2} = f\left(\frac{K}{L}, \frac{1}{Re}, \frac{1}{Fr^2}\right)$

02.

Sol: Given:

River Rectangular

Pier (Prototype)	**Model**
$W_p = 1.5$ m,	$L_r = 1/25$
$L_p = 4.5$ m,	$V_m = 0.65$ m/s
	$F_m = 3.92$ N
	$H_m = 3.5$ cm

where H is the height of standing wave.

(i) **The corresponding speed in the prototype V_p:**

As the flow in a river is a free surface flow affected by gravity, the dynamic similarity between the model and its prototype will be achieved by equating the Froude's number.

$$\therefore \quad \frac{V_p}{\sqrt{L_p g_p}} = \frac{V_m}{\sqrt{L_m g_m}}$$

$$\text{Or, } \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{25} = 5 \qquad (\because\ g_p = g_m)$$

$$\therefore \quad V_p = V_m \times 5 = 0.65 \times 5 = 3.25 \text{ m/s}$$

(ii) **The force acting on the prototype, F_p:**

$$\text{Force} = \text{mass} \times \text{acceleration} = \rho L^3 \times \frac{V}{T}$$

$$= \rho L^3 \times \frac{V}{(L/V)} \left[\because V = \frac{L}{T} \text{ or } T = \frac{L}{V}\right]$$

$$= \rho L^3 \times \frac{V^2}{L} = \rho L^2 V^2$$

$$\text{Force ratio, } F_r = \frac{F_m}{F_p}$$

$$= \frac{\rho_m}{\rho_p} \times \frac{L_m^2}{L_p^2} \times \frac{V_m^2}{V_p^2} = \rho_r.L_r^2.V_r^2$$

$$= L_r^2 \times \left(\sqrt{L_r}\right)^2 = L_r^3$$

$$\therefore \ F_p = \frac{F_m}{L_r^3} = 3.92 \times (25)^3$$

$$= 61{,}250 \text{ N} = 61.25 \text{ kN}$$

($\because$ $\rho_r = 1$, fluid being same in model and prototype)

(iii) **The height of the standing wave in the prototype, H_p:**

$$\frac{H_p}{H_m} = \frac{1}{L_r} = 25$$

$$H_p = H_m \times 25 = 3.5 \times 25 = 87.5 \text{ cm}$$

(iv) **The co-efficient of drag resistance:**

The co-efficient of drag resistance is defined by

$$F = C_D.\rho A \frac{V^2}{2}$$

where F is the drag force.

$$\therefore \quad C_D = \frac{F}{\frac{1}{2} \times \rho A V^2}$$

$$\text{Or, } (C_D)_p = \frac{F_p}{\frac{1}{2} \times \rho_p A_p V_p^2}$$

where,

F_p = Force acting on the prototype (= 61,250 N),

ρ_p = Density of water (= 1000 kg/m^3)

A_p = width of the pier × depth of water in the river = 1.5 × 3 = 4.5 m^2, and

V_p = velocity of flow in the prototype (= 3.25 m/s).

$$(C_D)_p = \frac{61{,}250}{\frac{1}{2}\times 1000\times 4.5\times 3.25^2} = 2.58$$

The drag co-efficient will be same for model and prototype, i.e.,

$$(C_D)_m = (C_D)_p = 2.58$$

14. Flow Through Orifices, Mouth Pieces, Notches and Weirs

01. Ans: (c)

Sol: $C_V = \frac{V_{act}}{C_{th}}$

$V_{th} = \sqrt{2gh}$

$= \sqrt{2\times 9.81\times 1.25}$

= 4.952 m/s

$V_{act} = \sqrt{2\times 9.81\times 1.2}$

= 4.852 m/s

C_v = 0.98

02. Ans: (d)

Sol: $t = \frac{2A}{C_d a\sqrt{2g}}\left(H_1^{1/2} - H_2^{1/2}\right)$

C_d, A, H_1, H_2 are constant

$t \propto \frac{1}{d^2}$

$$\frac{t_2}{t_1} = \left(\frac{d_1}{d_2}\right)^2$$

$$t_2 = 20\times\left(\frac{d_1}{2d_1}\right)^2 = \frac{200}{4} = 50$$

05. Ans: (a)

Sol: $Q \propto H^{3/2}$

$= \frac{Q_2 - Q_1}{Q_1}$

$= \frac{H_2^{3/2} - H_1^{3/2}}{H_1^{3/2}}$

$= \left(\frac{H_2}{H_1}\right)^{3/2} - 1$

$= \left(\frac{31}{3D}\right)^{3/2} - 1 = 5.041\%$

06. Ans: (b)

Sol: $Q = C_D\frac{2}{3}\sqrt{2g}L(H)^{3/2}$

$Q \propto L\,H^{3/2}$

$\frac{dQ}{Q} = \frac{dL}{L} + \frac{2}{3}\frac{dH}{H}$

$= -1.5 + \frac{3}{2}\times 1$

$= -1.5 + 1.5 = 0$

07. Ans: 0.792

Sol: a = 0.0003 m^2

H = 1 m

C_d = 0.60

$\sqrt{2g} = 4.4$

$Q = C_d.a\sqrt{2gH}$

$Q = 0.60 \times 0.0003 \times 0.44 \times \sqrt{1}$

$Q = 7.92 \times 10^{-4}\ m^3/sec$

$= 0.792$ ltr/sec

08. Ans: 16 : 1

Discharge through an orifice, Q = A. V

$$Q_{Actual} = C_d \cdot \frac{\pi}{4} d^2 \sqrt{2gh}$$

$$Q = d^2 \sqrt{h}$$

$$\frac{Q_1}{Q_2} = \left(\frac{d_1}{d_2}\right)^2 \sqrt{\frac{h_1}{h_2}}$$

For same discharges $Q_1 = Q_2$

$$\left(\frac{d_2}{d_1}\right)^2 = \sqrt{\frac{h_1}{h_2}}$$

$$\frac{h_1}{h_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{h_1}{h_2} = \left(\frac{2}{1}\right)^4 = \frac{h_1}{h_2} = 16:1$$

09. Ans: (d)

Sol: $Q = 1.418 H^{\frac{5}{2}}$

$$Q \propto H^{\frac{5}{2}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{\frac{5}{2}} \qquad \frac{Q_2}{Q_1} = \left(\frac{0.3}{0.15}\right)^{\frac{5}{2}}$$

$$\frac{Q_2}{Q_1} = 5.657$$

10. Ans: (d)

Sol: $Q = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{\frac{5}{2}}$

$$Q \propto H^{\frac{5}{2}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{\frac{5}{2}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{0.2}{0.1}\right)^{\frac{5}{2}} = 5.66$$

11. Ans: (d)

Sol: $Q \propto f\left(\tan\frac{\theta}{2}\right)$

$$Q = K \tan\frac{\theta}{2}$$

$$dQ = K \sec^2\frac{\theta}{2} \frac{1}{2} d\theta$$

$$\frac{d\theta}{\theta} = 2\% \qquad \text{(Given)}$$

% Error in discharge,

$$\frac{dQ}{Q} \times 100 = \frac{K \sec^2\frac{\theta}{2} \times \frac{1}{2} d\theta}{K \tan\frac{\theta}{2}} \times 100$$

$$= \frac{1}{2} \times \frac{1}{\cos^2\frac{\theta}{2}} \times \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \times 100$$

$$= \frac{1}{\sin\theta} d\theta \times 100$$

$$= \text{cosec } 90 \quad = \pi$$

Conventional Practice Solutions

01.

Sol: Volume of water falling down= discharge × time

Adh = Qdt

A = 0.93 m^2

$$Q = \frac{8}{15} C_d \sqrt{2g}\ H^{5/2}$$

H = 0.075 m

$$\frac{dh}{dt} = 2.54\text{mm} = 2.54 \times 10^{-3}\ \text{m/s}$$

Thus substitution

$$0.93 \times 2.54 \times 10^{-3} = \frac{8}{15} C_d \times \sqrt{2 \times 9.81} \times (0.075)^{5/2}$$

C_d = 0.649

02.

Sol: ***Given:***

Width of river = crest length

L = 30 m

Depth of flow, y = 3m

∴ Area of flow section = (30×3) = 90 m^2

Mean velocity of flow

V = 1.2 m/sec

Discharge Q = AV

= (90×1.2) = 108m^3/sec

Since the anicut (Weir) is constructed to raise the water level by 1m, the depth of flow on the upstream of the anicut becomes (3+1) = 4 m

Velocity of approach

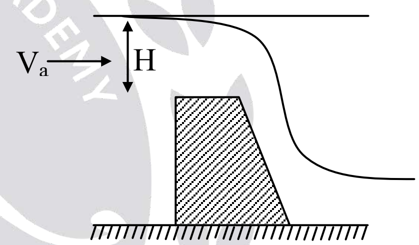
$$V_a = \frac{Q}{A_a} = \frac{108}{30 \times 4} = 0.9\text{m/s}$$

Head due to velocity of approach

$$h_a = \frac{V_a^2}{2g} = \frac{(0.9)^2}{2 \times 9.81} = 0.0413\text{m}$$

Assuming that the weir is discharging free, then

$$Q = \frac{2}{3} C_{d1} L \sqrt{2g}\left[(H + h_a)^{3/2} - h_a^{3/2}\right]$$



Free Weir

Assume

C_{d1} = 0.58

Thus by substitution,

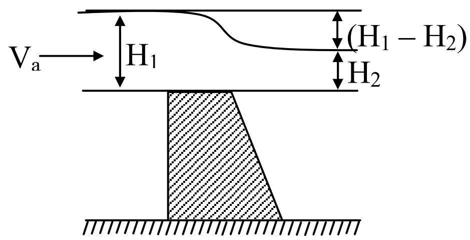
$$108 = \frac{2}{3} \times 0.58 \times 30 \times \sqrt{2 \times 9.81}\left[(H + 0.0413)^{3/2} - (0.0413)^{3/2}\right]$$

⇒ H = 1.604m

The height of the weir is then

Z = (4–1.604) = 2.396m

Since the depth of water in the channel on the downstream of the weir will also be 3m, the anicut will be submerged .



Submerged Weir

For a submerged weir the discharge is given by

$Q = Q_1 + Q_2$

$$Q_1 = \frac{2}{3}\, C_{d1}\, L\sqrt{2g}\left[\left((H_1 - H_2) + h_a\right)^{3/2} - h_a^{3/2}\right]$$

$$Q_2 = C_{d2} \times (L \times H_2)\left[\sqrt{2g(H_1 - H_2) + V_a^2}\right]$$

$(H_1 - H_2) = 1\text{m}$

Given $C_{d1} = 0.58$; $C_{d2} = 0.80$

Thus by substitution, we get

$$108 = \frac{2}{3} \times 0.58 \times 30 \times \sqrt{2 \times 9.81} \times \left[(1 + 0.0413)^{3/2} - (0.0413)^{3/2}\right] + 0.80 \times (0.30 \times H_2)\left[\sqrt{(2 \times 9.81 \times 1) + (0.9)^2}\right]$$

Or, $108 = 54.17 + 108.5H_2$

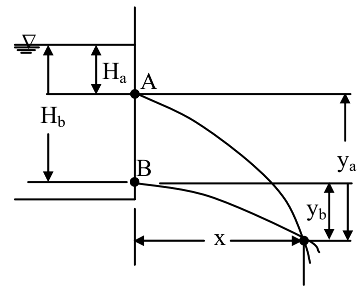
$$H_2 = \frac{108 - 54.17}{108.5} = 0.496\text{ m}$$

Therefore the height of the anicut

$= (3 - 0.496) = 2.504\text{m}$

03.

Sol: The given data is shown in figure below



We know for the orifices, the coefficient of velocity is related as

$$C_v = \sqrt{\frac{x^2}{4yH}}$$

$\therefore$ The trajectory is given by

$$x^2 = 4yHC_v^2 \quad \ldots\ldots\ldots\ldots\text{(i)}$$

At the point of intersection of the two jets

$$x_a^2 = 4y_aH_aC_v^2 \quad \ldots\ldots\ldots\ldots\text{(ii)}$$

$$x_b^2 = 4y_bH_bC_v^2 \quad \ldots\ldots\ldots\ldots\text{(iii)}$$

But $x_a = x_b = x$

Equating (ii) and (iii)

$$\frac{y_a}{y_b} = \frac{H_b}{H_a} \quad \ldots\ldots\ldots\text{(iv)}$$

Also $H_a + y_a = H_b + y_b$

$$y_a - y_b = H_b - H_a \quad \ldots\ldots\ldots\ldots\text{(v)}$$

Solving for y_a from Eqs. (iv) and (v)

From Equation (iv)

$$y_a = \frac{H_b}{H_a} \times y_b \quad \ldots\ldots\ldots\ldots\text{(vi)}$$

substitute the value of y_a in equation of (v)

$$\frac{H_b}{H_a} \times y_b - y_b = H_b - H_a$$

$$y_b\left(\frac{H_b}{H_a} - 1\right) = H_b - H_a$$

$$y_b\left(\frac{H_b - H_a}{H_a}\right) = H_b - H_a$$

$$\Rightarrow \quad y_b = H_a$$

Substituting the value of y_b in equation (vi)

$$y_a = H_b$$

Now, Substituting in Eq. (i),

$$x^2 = 4y_a H_a C_v^2 = 4y_b H_b C_v^2$$

$$x = \sqrt{4y_a H_a C_v^2} = 2C_v\sqrt{H_a H_b}$$