ESE | GATE | PSUs

MECHANICAL ENGINEERING

ENGINEERING MECHANICS

Text Book & Work Book :
Theory with worked out Examples and Practice Questions

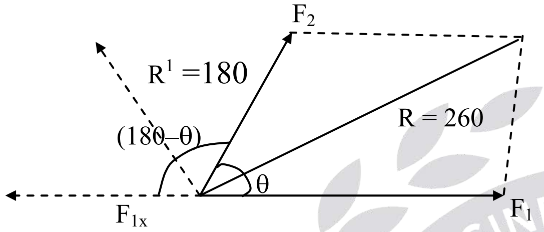
Engineering Mechanics

(Solutions for Text Book Objective & Conventional Practice Questions)

Chapter 1 **Force and Moment Systems**

01. Ans: (b)

Sol:



Assume $F_1 = 2F_2$ $(F_1 > F_2)$

$F_{1x} = 2F_2$

$$R = \sqrt{F_1^2 + F_2^2 + 4F_2^2\cos\theta}$$

$$260 = \sqrt{4F_2^2 + F_2^2 + 4F_2^2\cos\theta}$$

$$260^2 = 5F_2^2 + 4F_2^2\cos\theta \text{ ------ (1)}$$

$$R^1 = \sqrt{F_{1x}^2 + F_2^2 + 2F_{1x}F_2\cos\theta}$$

$$180 = \sqrt{4F_2^2 + F_2^2 + 2.F_2.F_2\cos(180-\theta)}$$

$$180^2 = 5F_2^2 - 4F_2^2\cos\theta \text{ ------ (2)}$$

$$260^2 = 5F_2^2 + 4F_2^2\cos\theta$$

$$180^2 = 5F_2^2 - 4F_2^2\cos\theta$$

$$260^2 + 180^2 = 10F_2^2$$

$\Rightarrow$ $F_2 = 100$N,

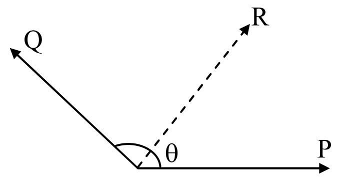
$260^2 = 5(100)^2 + 4(100)^2\cos\theta$

$\Rightarrow \theta = 63.89$

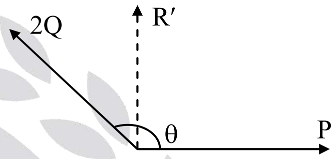
Where θ angle between two forces.

02. Ans: (b)

Sol: Let the angle between the forces be θ



Where, R is the resultant of the two forces.



If Q is doubled i.e., 2Q then resultant (R′) is perpendicular to P.

$$\tan 90 = \frac{2Q\sin\theta}{P + 2Q\cos\theta}$$

$\Rightarrow$ $P + 2Q\cos\theta = 0$

$P = -2Q\cos\theta$ ------(i)

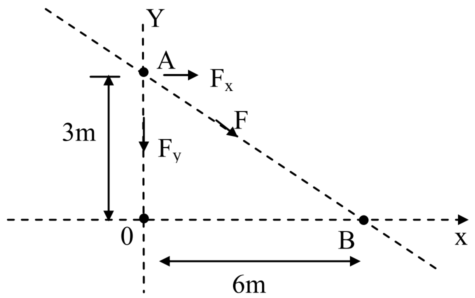
Also, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = Q$ [using eq.(i)]

03. Ans: (b)

Sol: Since moment of F about point A is zero.

$\therefore$ F passes through point A,



$M_0^F = 180 N - m$

$M_B^F = 90 N - m$

$M_A^F = 0$

$M_0^F = 180 = F_x \times 3 + F_y \times 0$

$F_x = 60N$ ……. (1)

$M_B^F = F_x \times 3 - F_y \times 6 = -90$

$60\times3-6F_y = -90$

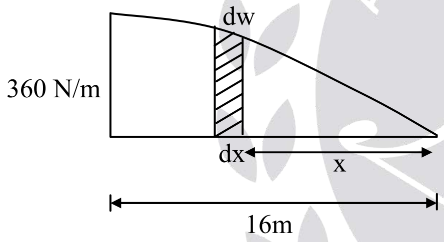
$\Rightarrow F_y = \frac{270}{6}$

$F_y = 45N$

$\therefore\ F = \sqrt{F_x^2 + F_y^2} = \sqrt{60^2 + 45^2} = 75$

04. Ans: (a)

Sol:



$$\int_0^w dw = \int_0^{16} w dx$$

$$w = \int_0^{16} 90\sqrt{x}dx = 90\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^{16}$$

$$= 90\times\frac{2}{3}\left[x^{3/2}\right]_0^{16} = 60\ (16)^{3/2}$$

$w = 3840$ N

The moment due to average force should be equal to the variable force

$R \times d = \Sigma dw \times x$

$$3840\times d = \int_0^{16} 90\sqrt{x}.dx.x$$

$$= 90\int_0^{15} x^{1.5}dx$$

$$3840d = 90\left[\frac{x^{2.5}}{2.5}\right]_0^{16}$$

$\Rightarrow\ d = 9.6$ m

05. Ans: (c)

Sol: Moment about 'O'

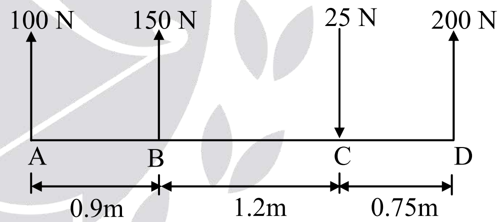
$M_0 = 100\sin 60\times3$

$= 300\times\frac{\sqrt{3}}{2} = 150\sqrt{3}$

$= 259.8 \cong 260$ N

06. Ans: (a)

Sol:



$F_R = \Sigma F_y$

$F_R = 100+150-25+200$ (upward force Positive downward force negative)

$R = 425$ N

For equilibrium

$\Sigma M_A = 0$ (since R = resultant)

Let R is acting at a distance of 'd'

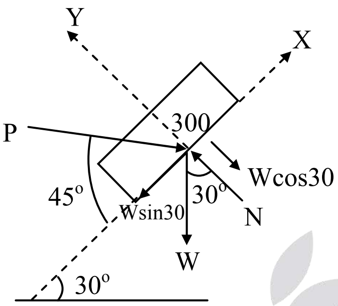
$425\times d = 150\times0.9+25\times2.1-200\times2.85$

$\Rightarrow\ d = 1.535$m (from A)

Chapter 2
Equilibrium of Force System

01. Ans: (d)

Sol:



Resolve the forces along the inclined surface

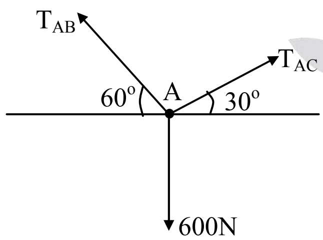
$\sum F_x = 0$

$P\cos 45 - W\sin 30 = 0$

$$P = \frac{300\sin 30}{\cos 45} \Rightarrow P = 212.13\text{ N}$$

02. Ans: (a)

Sol:



$T_{AB}\cos 60° = T_{AC}\cos 30°$

$T_{AB} = \sqrt{3}\ T_{AC}$

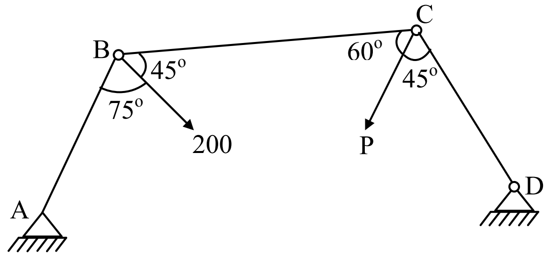
$T_{AB}\sin 60° + T_{AC}\sin 30° = 600\text{ N}$

$$\frac{3}{2}T_{AC} + \frac{1}{2}T_{AC} = 600$$

$\Rightarrow T_{AB} = 520\text{ N}$; $T_{AC} = 300\text{ N}$

03. Ans: (c)

Sol:



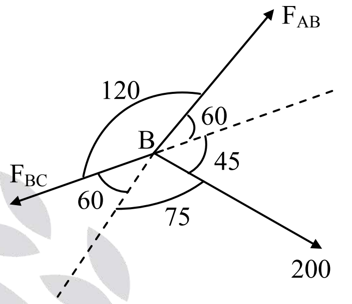


Fig: Free body diagram at 'B'

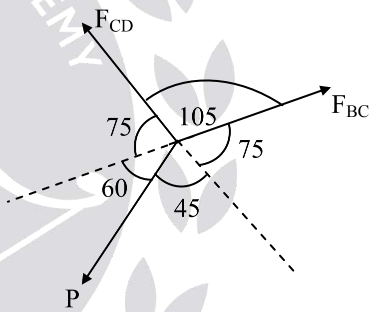


Fig: Free body diagram at 'C'

For Equilibrium of Point 'B'

$$\frac{F_{AB}}{\sin(60+75)} = \frac{F_{BC}}{\sin(60+45)} = \frac{200}{\sin(120)}$$

$F_{BC} = 223.07\text{ N}$

From Sine rule at "C".

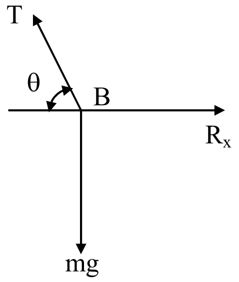
$$\frac{F_{CD}}{\sin(75+45)} = \frac{F_{BC}}{\sin(60+75)} = \frac{P}{\sin 105}$$

$$P = \frac{223.07\times\sin 105}{\sin 135}$$

$P = 304.71\text{ N}$

04. Ans: (d)

Sol:



$$\tan\theta = \frac{125}{275} \Rightarrow \theta = 24.45^0$$

$T\sin\theta = mg.$

$T\sin 24.45 = (35\times 9.81)$

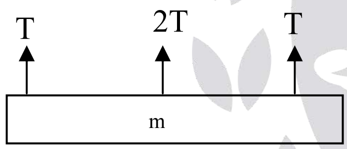
$T = 829.5$ N

$R_x = T\cos 24.45 = 755.4$ N

$R_y = 0$

05. Ans: (c)

Sol:



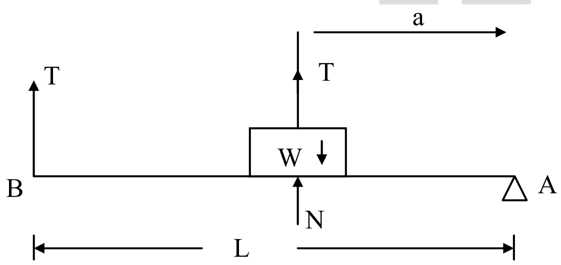
$T+2T+T = mg$

$4T = mg$

$m = 4T/g$

06. Ans: (b)

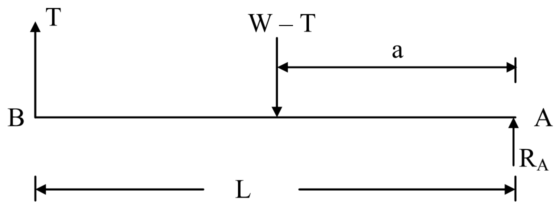
Sol:



For body, $\sum F_y = 0$

$N - W + T = 0$

$\Rightarrow N = W - T$



$\sum F_y = 0$ for entire system

$R_A + T - (W - T) = 0$

$R_A = W - 2T$ ------- (1)

For equilibrium

$\sum M_A = 0$

$T\times L = (W - T)\, a$

$TL = Wa - Ta$

$TL + Ta = Wa$

$T(L + a) = Wa$

$$\Rightarrow T = \frac{Wa}{L+a}$$

T substitute in equation (1)

$$R_A = W - 2\left(\frac{Wa}{L+a}\right)$$

$$= \frac{W(L+a) - 2Wa}{L+a}$$

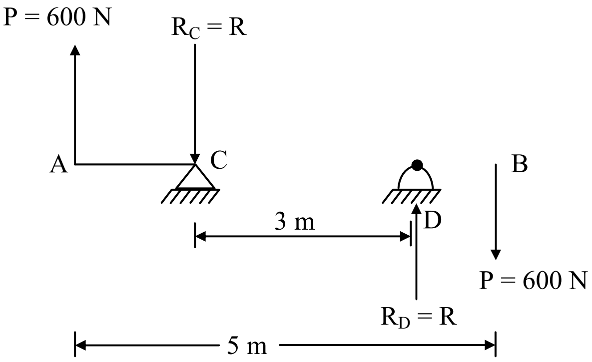
$$= \frac{WL + Wa - 2Wa}{L+a}$$

$$= \frac{WL - Wa}{L+a}$$

$$R_A = \frac{W(L-a)}{L+a}$$

07. Ans: (c)

Sol:



$\sum F_y = 0$

$600 - R_C + R_D - 600 = 0$

$\Rightarrow\ R_C = R_D = R$

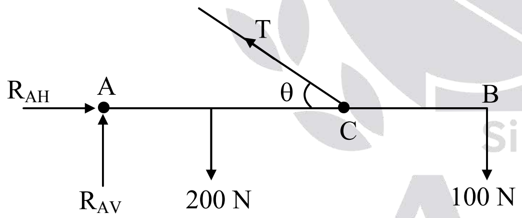
$\sum M = 0$

$600 \times 5 = R \times 3$

$\Rightarrow\ R = 1000\ N = R_C = R_D$

08. Ans: (a)

Sol: F.B.D



$\sum M_A = 0$

$\text{Tan}\theta = \dfrac{8}{4}$

$\theta = 63.43$

$T\sin\theta \times 4\ (\circlearrowleft) - 200 \times 2\ (\circlearrowright) - 100 \times 6\ (\circlearrowright) = 0$

$\Rightarrow T = 279.5\ N$

Now, $\sum F_x = 0$,

$R_{AH} - T\cos\theta = 0$

$R_{AH} = 125\ N$

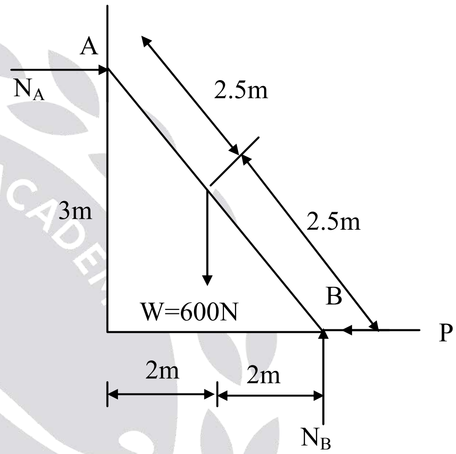
$\sum F_y = 0$

$R_{AV} - 200 - 100 + T\sin\theta = 0$

$\Rightarrow R_{VA} = 50\ N$

09. Ans: 400 N

Sol:



$\sum F_y = 0$

$N_B - W = 0$

$N_B = 600\ N$

$\Sigma M_A = 0$

$P \times 3 + W \times 2 - N_B \times 4 = 0$

$P = \dfrac{4N_B - 2W}{3}$

$P = \dfrac{4 \times 600 - 2 \times 600}{3} = 400N$

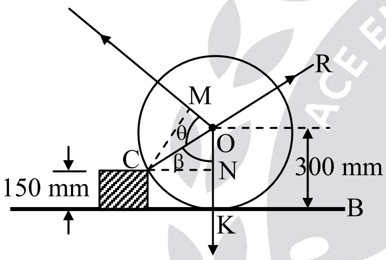
Conventional Practice Solutions

01.

Sol:

(a) When the wheel is about to turn, its contact with the ground will be lost. Hence the wheel has to be in equilibrium under the action of its weight W and the force P.

With reference to figure, CM and CN are the perpendiculars dropped on the lines of action of forces P and W respectively.



OC = OK = 300 mm,

ON = OK = NK = 300 – 150 = 150 mm

$CN = \sqrt{OC^2 - ON^2}$

$= \sqrt{300^2 - 150^2} = 259.81\text{mm}$

Taking moments about point C,

P × CM – W × CN = 0

P × OC sin θ - 20 × 259.81 = 0

$$P = \frac{20\times 259.81}{OC\sin\theta} = \frac{20\times 259.81}{300\sin\theta} = \frac{17.32}{\sin\theta}$$

The force P will be minimum when sinθ is maximum.

For that sinθ = 1 or θ = 90°

Hence, P_{min} = 17.32 kN when pull is applied perpendicular to OC.

(b) The reaction R can be determined by resolving W along OC

R = W cosβ [Where, β = ∠CON]

$$\cos\beta = \frac{ON}{OC} = \frac{150}{300} = \frac{1}{2}$$

$$R = 20\times\frac{1}{2} = 10\,\text{kN}$$

The force P acts perpendicular to OC and as such its resolved part along OC is zero.

02.

Sol: Resolving the forces in the horizontal and vertical directions, we get

ΣF_x=80–100 cos30°+120 cos45°–60 cos30°

= 80 – 86.6 + 84.85 – 51.96

= + 26.29 N →

ΣF_y=–100sin30°+120sin45°+100 +60sin30°

= –50+84.45+100+30=164.85 N↑

Resultant force $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)}^2$

$= \sqrt{(26.69)^2 + (164.85)^2} = \sqrt{27866.68}$

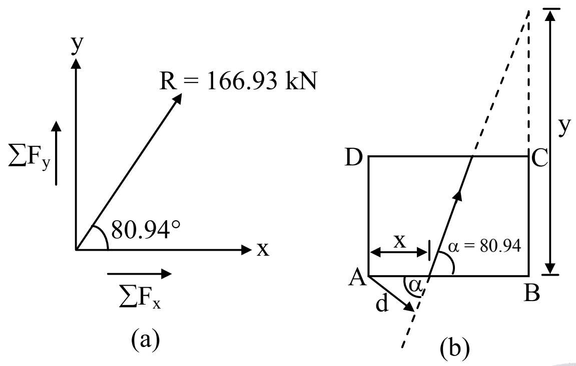
= 166.93 N

Inclination α of the resultant with the horizontal,

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{164.85}{26.29} = 6.27;\ \alpha = 80.94°$$

Since both ΣF_x and ΣF_y are positive, the resultant lies in first quadrant at an angle

80.94° with horizontal, i.e., with x-axis as shown in figure.



(a) (b)

Position of resultant force:

Taking moments about point A (clockwise +ve), we get

$$\Sigma M_a = (100\sin 30°)\times 6 - (120\sin 45°)\times 6 + (120\cos 45°)\times 4 - 100\times 3 - (60\cos 30°)\times 4$$

= 300 – 509.1 + 339.4 – 300 –207.8

= –377.5 Nm (anticlockwise)

The resultant R should then lie as shown in figure so that it can produce anticlockwise (negative) moment about point A.

If d is the perpendicular distance of the resultant from A, then

$$R \times d = \Sigma M_a$$

$$\therefore \quad d = \frac{\Sigma M_a}{R} = \frac{377.5}{166.93} = 2.261\,m$$

The intercepts x on x-axis and y on y-axis are then given by

$$x = \frac{d}{\sin\alpha} = \frac{2.261}{\sin 80.94} = 2.29\,m$$

$$y = \frac{d}{\cos\alpha} = \frac{2.261}{\cos 80.94} = 14.36\,m$$

The intercepts on x-axis and y-axis can also be worked out as

$$x = \frac{\Sigma M_a}{\Sigma F_y} = \frac{377.5}{164.85} = 2.29\,m$$

$$y = \frac{\Sigma M_a}{\Sigma F_x} = \frac{377.5}{26.29} = 14.36\,m$$

Since intercept y is greater than BC, the resultant meets the arm when produced.

03.

Sol: Force $\vec{F}$ = 30i–20j+16k

Vector joining the points A(1,2,–3) and B(–1, –3, 4) is

$$\overrightarrow{AB} = \vec{r} = (-1-1)i + (-3-2)j + [4-(-3)]k$$

$$= -2i - 5j + 7k$$

Unit vector along

$$\hat{e}_{AB} = \frac{-2i-5j+7k}{\sqrt{(-2)^2+(-5)+(7)^2}} = \frac{-2i-5j+7k}{\sqrt{78}}$$

The component of force F along AB is

$$= \vec{F}\,\hat{e}_{AB}$$

$$= (30i-20j+16k).\left(\frac{-2i-5j+7k}{\sqrt{78}}\right)$$

$\simeq$ 17.224 kN

04.

Sol: **Free-Body Diagram**

There are five unknown force magnitude shown on the free-body diagram figure.

Equations of Equilibrium:

Expressing each force in Cartesian vector form, we have

$F = \{-1000j\}N$

$F_A = A_x i + A_y j + A_z k$

$T_C = 0.707T_C i - 0.707T_C k$

$$T_D = T_D\left(\frac{r_{BD}}{r_{BD}}\right) = -\frac{3}{9}T_D i + \frac{6}{9}T_D j - \frac{6}{9}T_D k$$

Applying the force equation of equilibrium gives

$\sum F = 0;$

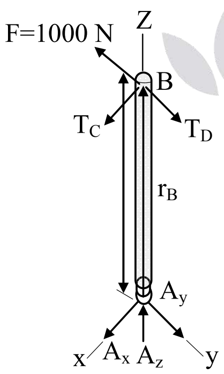
$F + F_A + T_C + T_D = 0$

$$(A_x + 0.707T_C - \frac{3}{9}T_D)i + (-100s0 + A_y + \frac{6}{9}T_D)j + (A_z - 0.7077T_C - \frac{6}{9}T_D)k = 0$$

$$\sum F_x = 0; \quad A_x + 0.707T_C - \frac{3}{9}T_D = 0 \quad \ldots(1)$$

$$\sum F_y = 0; \quad A_y + \frac{6}{9}T_D - 1000 = 0 \quad \ldots\ldots(2)$$

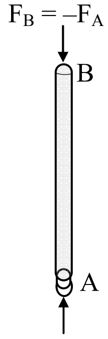
$$\sum F_z = 0; \quad A_z - 0.707T_C - \frac{6}{9}T_D = 0 \quad \ldots..(3)$$



Summing moments about point A, we have

$\sum M_A = 0; \quad r_B \times (F + T_C + T_D) = 0$

$$6k \times (-1000j + 0.707T_C i - 0.707T_C k - \frac{3}{9}T_D i + 69T_D j - \frac{6}{9}T_D k) = 0$$



Evaluating the cross product and combining terms yields

$(-4T_D + 6000)i + (4.24T_C - 2T_D)j = 0$

$\sum M_x = 0; \quad -4T_D + 6000 = 0 \quad \ldots...(4)$

$\sum M_y = 0; \quad 4.24T_C - 2T_D = 0 \quad \ldots\ldots(5)$

The moment equation about the z axis, $\sum M_z = 0$, is automatically satisfied. Why? Solving equations 1 through 5 we have

$T_C = 707$ N

$T_D = 1500$ N

$A_x = 0$ N

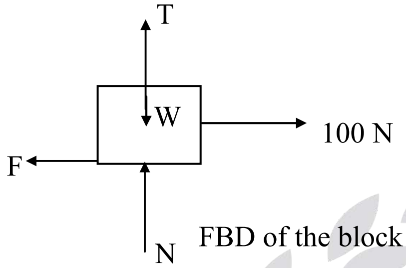
$A_y = 0$ N

$A_z = 1500$ N

Chapter 3 Friction

01. Ans: (c)

Sol: The FBD of the above block shown



FBD of the block

$\Sigma Y = 0 \Rightarrow N + T - W = 0$

$N = W - T = 981 - T$

$F = \mu N = 0.2\,(981 - T)$

$\Sigma X = 0 \Rightarrow 100 - F = 0$

$F = 100 = 0.2\,(981 - T)$

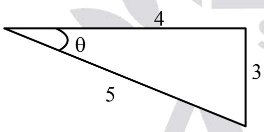
$\Rightarrow T = 481$ N

02. Ans: (c)

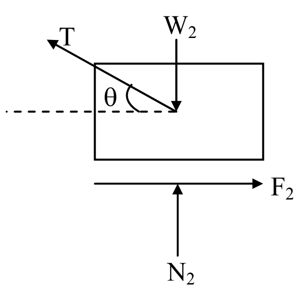
Sol: Given $\tan\theta = \dfrac{3}{4}$

$\sin\theta = 3/5$

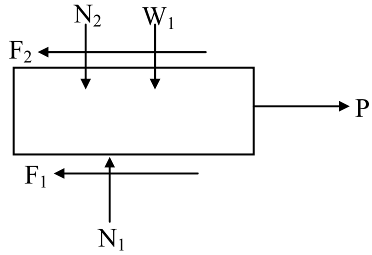
$\cos\theta = 4/5$



Free body diagram for block (2)



Free body diagram for block (1)



From FBD of block (2)

$\Sigma F_x = 0$

$F_2 = T\cos\theta$

$F_2 = \dfrac{4}{5}T = 0.8T$ ------ (1)

$\Sigma F_y = 0$

$N_2 + T\sin\theta - W_2 = 0$

$N_2 = W_2 - T\sin\theta$

$N_2 = 50 - 0.6\,T$

But $F_2 = \mu N_2$

$\Rightarrow F_2 = 0.3(50 - 0.6T)$

$F_2 = 15 - 0.18\,T$ ------ (2)

From (1) & (2)

$0.8T = 15 - 0.18\,T$

$\Rightarrow 0.98T = 15$

$\Rightarrow \quad T = 15.31$ N

$\therefore N_2 = 50 - 0.6T$

$= 50 - 0.6\,(15.31) = 40.81$ N

$F_2 = \mu N_2 = 0.3 \times 40.81 = 12.24$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1 = 40.81 + 200 = 240.81$ N

$F_1 = \mu N_1 \Rightarrow F_1 = 0.3 \times 240.81$

$F_1 = 72.24$ N

$\Sigma F_x = 0$

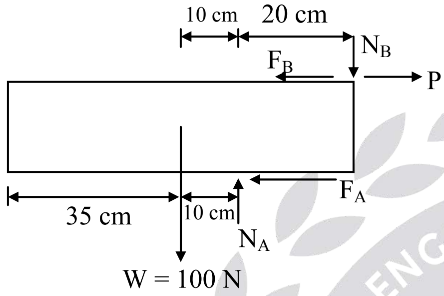
$P - F_1 - F_2 = 0$

$P = F_1 + F_2 = 72.24 + 12.24$

$P = 84.48$ N

03. Ans: (b)

Sol: Free Body Diagram



$$F_A = \mu N_A = \frac{1}{3} N_A$$

$$F_B = \mu N_B = \frac{1}{3} N_B$$

$\Sigma M_B = 0$

$-100\times30(\circlearrowleft) + (N_A\times20)(\circlearrowright) + (F_a \times 12)(\circlearrowright) = 0$

$$-3000 + N_A \times 20 + \frac{1}{3} N_A \times 12 = 0$$

$\Rightarrow N_A = 125$ N

$\Sigma F_y = 0$

$N_A - N_B - 100 = 0$

$\Rightarrow N_B = 25$ N

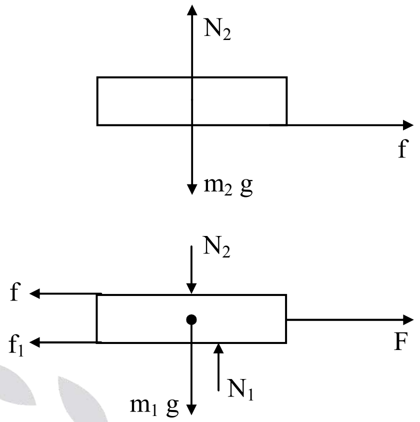
$\Sigma F_x = 0$

$$P = F_A + F_B = \frac{1}{3}(N_A + N_B)$$

$$= \frac{1}{3}(125 + 25) = 50\text{N}$$

04. Ans: (d)

Sol: F.B.D of both the books are shown below.



where, f is the friction between the two books.

f_1 is the friction between the lower book and ground.

Now, maximum possible acceleration of upper book.

$$a_{max} = \frac{f_{max}}{m_2} = \frac{\mu m_2 g}{m_2} = \mu \times g$$

$$= 0.3 \times 9.81 = 2.943 \text{ m/s}^2$$

For slip to occur, acceleration (a_1) of lower book. i.e, $a_1 \geq a_{max}$

$$\frac{F - f - f_1}{m_1} \geq 2.943$$

$F - 2.943 - 0.3 \times 2 \times 9.81 \geq 2.943$

[$\because$ $f = f_{max} = 2.943$ and

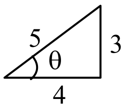
$f_1 = \mu \times (m_1 + m_2) g = 0.3 \times 2 \times 9.81$]

$F \geq 11.77$ N

$F_{min} = 11.77$ N

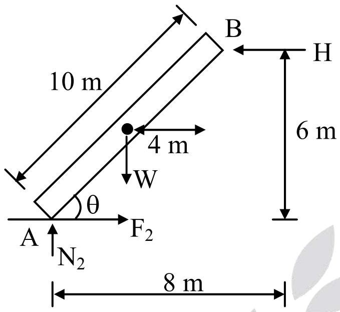
05. Ans: (d)

Sol: $\tan\theta = \dfrac{3}{4} \Rightarrow \sin\theta = \dfrac{3}{5}$

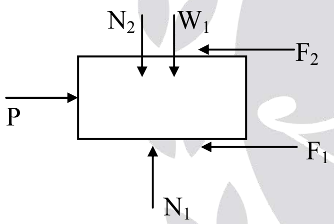


$\cos\theta = \dfrac{4}{5}$

FBD for bar AB (2)



FBD for block (1)



Given $W = 280$ N , $W_1 = 400$ N

Now, $\Sigma M_B = 0$

$-W \times 4\,(\circlearrowright) + N_2 \times 8(\circlearrowright) - F_2 \times 6\,(\circlearrowright) = 0$

$-280 \times 4 + N_2 \times 8 - \mu N_2 \times 6 = 0$

$\Rightarrow N_2 = 200$ N

But, $F_2 = \mu N_2 = 0.4 \times 200 = 80$ N

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1$

$= 200 + 400$

$N_1 = 600$ N

But, $F_1 = \mu N_1 = 0.4 \times 600$

$F_1 = 240$ N

$\Sigma F_x = 0$

$P = F_1 + F_2 = 240 + 80$

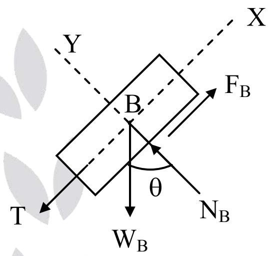
$P = 320$ N

06. Ans: (a)

Sol: Given, $W_A = 200$ N , $\mu_A = 0.2$

$W_B = 300$ N , $\mu_B = 0.5$

FBD for block 'B'.



$\Sigma F_y = 0$

$N_B = W_B\cos\theta$

$N_B = 300\cos\theta$

But, $F_B = \mu N_B = 0.5 \times 300\cos\theta$

$= 150\cos\theta$

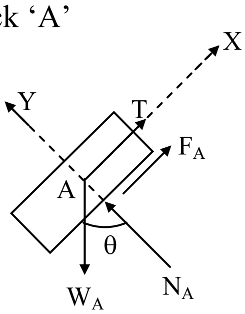
$\Sigma F_x = 0$

$T + W_B\sin\theta - F_B = 0$

$T = F_B - W_B\sin\theta$

$T = 150\cos\theta - 300\sin\theta$ ------ (1)

FBD for block 'A'



$\Sigma F_y = 0$

$N_A - W_A\cos\theta = 0$

$N_A = 200\cos\theta$

$F_A = \mu N_A = 0.2 \times 200\cos\theta$

But, $F_A = 40\cos\theta$

$\Sigma F_x = 0$

$T + F_A - W_A\sin\theta = 0$

$T = W_A\sin\theta - F_A$

$T = 200\sin\theta - 40\cos\theta$

But from equation (1)

$T = 150\cos\theta - 300\sin\theta$

$\therefore 150\cos\theta - 300\sin\theta = 200\sin\theta - 40\cos\theta$

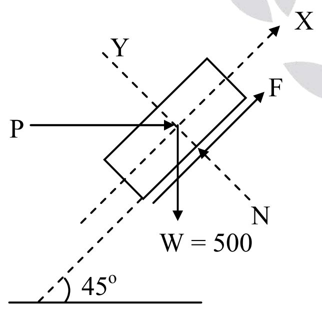
$190\cos\theta = 500\sin\theta$

$\tan\theta = \frac{190}{500}$

$\Rightarrow \theta = 20.8^o$

07. Ans: (d)

Sol: FBD for the block



$\Sigma F_y = 0$

$N - W\sin45 - P\sin45 = 0$

$N = \frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}$

But, $F = \mu N = 0.25\left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}\right)$

$\Sigma F_x = 0$

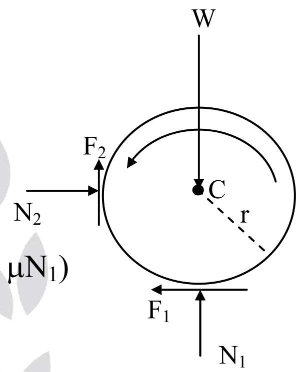
$P\cos45 + F - W\sin45 = 0$

$$P\cos45 + 0.25\left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}\right) - 500 \times \frac{1}{\sqrt{2}} = 0$$

$\Rightarrow P = 300$ N

08. Ans: (a)

Sol: FBD of block



$F_1 = \mu N_1$

$F_2 = \mu N_2$

$\Sigma F_x = 0$

$N_2 - F_1 = 0$

$\Rightarrow N_2 = F_1 \quad (\because F_1 = \mu N_1)$

$N_2 = \mu N_1$

$\Sigma F_y = 0$

$N_1 + F_2 - W = 0$

$N_1 + \mu N_2 - W = 0$

$N_1 + \mu^2 N_1 - W = 0 \qquad (\because N_2 = \mu N_1)$

$N_1(1+\mu^2) = W$

$N_1 = \frac{W}{1+\mu^2}$

$N_2 = \frac{\mu W}{1+\mu^2}$

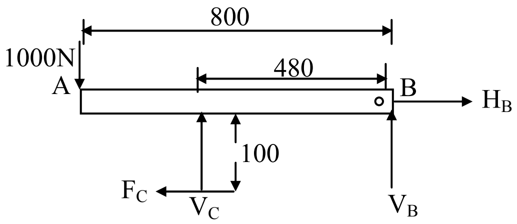
Couple $= (F_1 + F_2) \times r$

$= \mu r(N_1 + N_2)$

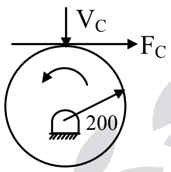
$= \frac{\mu r \times W(1+\mu)}{1+\mu^2} \quad (\because \mu = f)$

09. Ans: 64 N-m

Sol: FBD of shoe bar :



FBD of Drum Brake :



$\Sigma M_B = 0$

$V_C \times 480 + F_C \times 100 - 1000 \times 800 = 0$

$F_C = \mu V_C = 0.2\ V_C$

$480V_C + 0.2V_C \times 100 = 800000$

$500V_C = 800000$

$V_C = 1600$ N

$F_C = 0.2\ V_C = 0.2 \times 1600 = 320$ N

$M = 0.2 \times F_C = 0.2 \times 320 = 64$ N-m

10. Ans: (a)

Sol: $\beta = 2\theta$

$\cos\theta = \dfrac{6}{12}$

$\Rightarrow \theta = 60$

$\beta = 360 - 2\theta$

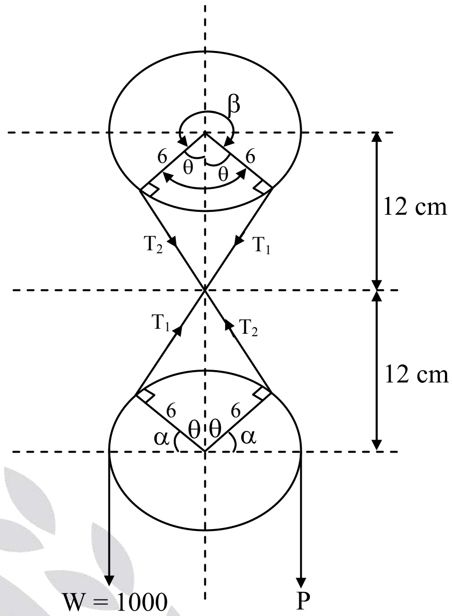
$\beta = 240 = \dfrac{4\pi}{3}$

$2\alpha + 2\theta = 180$

$2\alpha = 180 - 120$

$\alpha = 30 = \dfrac{\pi}{6}$

FBD



(When W moves upwards)

For P_{min} calculation,

$W > T_1$

$\dfrac{W}{T_1} = e^{\mu\alpha}$

$T_1 = \dfrac{1000}{e^{\frac{\pi}{6}\times\frac{1}{\pi}}} = 846.48$ N

$\therefore\ \dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_2 = \dfrac{848.48}{e^{\frac{1}{\pi}\times\frac{4\pi}{3}}} = 223.12$ N

$\dfrac{T_2}{P_{min}} = e^{\mu\alpha}$

$\Rightarrow P_{min} = \dfrac{223.12}{e^{\frac{1}{\pi}\times\frac{\pi}{6}}}$

$P_{min} = 188.86\ \text{N} \approx 189$ N

For P_{max} calculation

$\dfrac{T_1}{W} = e^{\mu\alpha}$

$T_1 = 1000 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$

$T_1 = 1181.36$ N

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = 1181.36 \times e^{\frac{1}{\pi}\times\frac{4\pi}{3}} = 4481.65$ N

$\frac{P_{max}}{T_2} = e^{\mu\alpha}$

$P_{max} = 4481.68 \times e^{\frac{1}{\pi}\times\frac{\pi}{6}}$

$P_{max} = 5300$ N

11. Ans: (b)

Sol: Given $\mu = 0.2$, $\tan\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

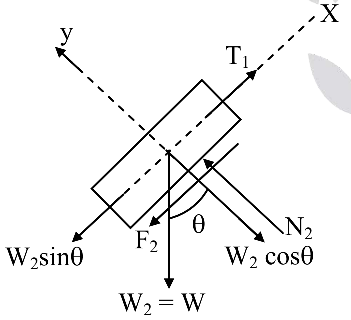


Fig: FBD (1)

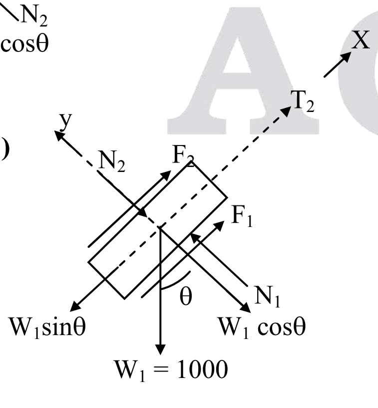


Fig: FBD (2)

From FBD (1)

$\Sigma F_y = 0$

$N_2 - W_2 \cos\theta = 0$

$N_2 = W_2 \cos\theta = W\times 0.8$

$N_2 = 0.8$ W

$\therefore F_2 = \mu N_2 = 0.2 \times 0.8$ W

$F_2 = 0.16$ W

$\Sigma F_x = 0$

$T_1 - W_2\sin\theta - F_2 = 0$

$T_1 = F_2 + W_2\sin\theta = 0.16\text{ W} + 0.6\text{W}$

$T_1 = 0.76$ W

From FBD (2)

$\Sigma F_y = 0$

$N_2 + W_1\cos\theta = N_1$

$N_1 = N_2 + W_1\cos\theta$

$N_1 = 0.8\text{W} + 1000 \times \frac{4}{5}$

$N_1 = 0.8\text{ W} + 800$

$F_1 = \mu N_1 = 0.2\ (0.8\text{ W}+800)$

$= 0.16\text{ W} + 160$

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = T_1\, e^{\mu\beta} = 0.76\text{ W } e^{0.2\times\pi}$

$T_2 = 1.42$ W

$\Sigma F_x = 0$

$T_2 + F_1 + F_2 = W_1 \sin\theta$

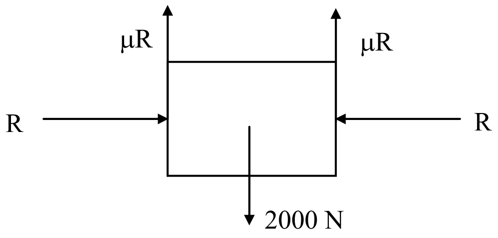
$1.42\text{W}+0.16\text{W}+160+0.16\text{W} = 1000 \times \frac{3}{5}$

$1.74\text{ W} = 440$

$\Rightarrow$ W = 252.87 N

12. Ans: (d)

Sol:



At equilibrium

$2\mu R = 2000$

$\Rightarrow R = \dfrac{2000}{2\times 0.1} = 10{,}000\text{N}$

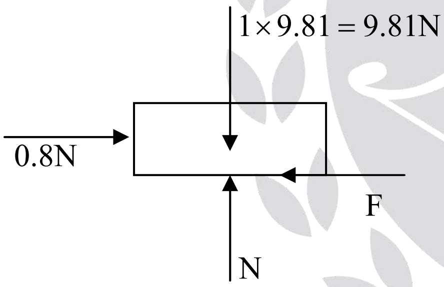
Taking moment about pin

$10{,}000\times 150 = F\times 300$

$\Rightarrow F = 5000$ N

13. Ans: (b)

Sol:



$\Sigma Y = 0$

$\Rightarrow N = 9.81$ N

$F_s = \mu N = 0.1\times 9.81 = 0.98$ N

The External force applied = 0.8 N < F_s

$\Rightarrow$ Frictional force = External applied force = 0.8 N

14. Ans: (b)

Sol:

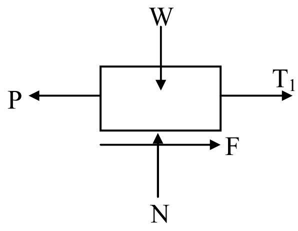


Fig: FBD (1)

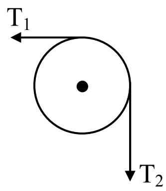


Fig: FBD (2)

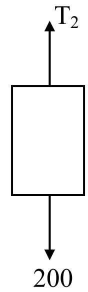


Fig: FBD (3)

From FBD (3)

$\Sigma F_y = 0$

$T_2 - 200 = 0$

$\Rightarrow T_2 = 200$

From FBD (2)

$\dfrac{T_1}{T_2} = e^{\mu\beta}$

$T_1 = T_2\, e^{\mu\beta} = 200\times e^{0.3\times\frac{\pi}{2}}$

$T_1 = 320.39$ N

From FBD (1)

$\Sigma F_y = 0$

$N - W = 0$

$N = 1000$ N

$F = \mu N$

$= 0.3 \times 1000$

$F = 300$ N

$\Sigma F_x = 0,\ T_1 + F - P = 0$

$320.39 + 300 = P$

$\Rightarrow P = 620.39$

$\Rightarrow P = 620.4$ N

Conventional Practice Solutions

01.

Sol: Limiting force of friction between contacting surfaces,

$$F = \mu\, r = \mu\, W$$

∴ Limiting force of friction between A and B,

$F_{ab} = \mu_{ab} \times W_a = 0.3 \times 150 = 45$ N

Limiting force of friction between B and C,

$F_{bc} = \mu_{bc}\,(W_a + W_b) = 0.2(150+50) = 40$ N

Limiting force of friction between B and C,

$F_{cg} = \mu_{cg}(W_a + W_b + W_c)$

$= 0.1\,(150 + 50 + 100) = 30$ N

With gradual increase in applied force P, the frictional force increases till it attains the maximum (limiting) values. Any further increase in P sets the body in motion. Now

(i) When P = 30 N, the block A exerts a force of 30 N on block B. However this force is less than the limiting friction between A and B, and obviously there will be no relative motion between A and B.

(ii) The block B also exerts a force of 30 N on block C. This force is again less the limiting friction between B and C. Obviously no relative motion between B and C.

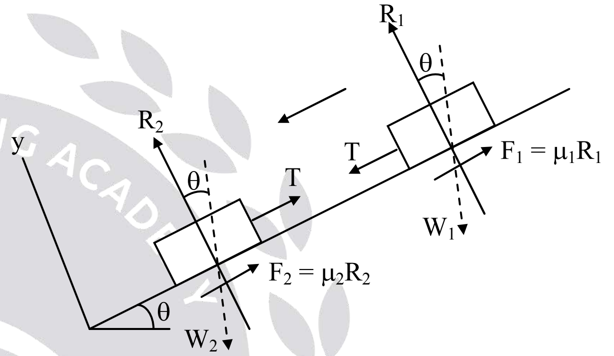
(iii) The block C too is subjected to a pull of 30 N. This equals the limiting friction between block C and the ground. As such the block C will be on the point of motion.

The above remarks tell us that when a horizontal P equals to 30 N is applied, all the three blocks would be in a state of impending motion as a single body.

02.

Sol:

(a) Below figure for the arrangement and the free body diagrams for the two blocks.



Let T be the tension in the string and θ be the inclination of the plane with the horizontal. Considering equilibrium for block of weight W_1

$\sum F_x = 0$(along the plane)

$\mu_1 R_1 - T - W_1 \sin\theta = 0$……… (i)

$\sum F_y = 0$ (perpendicular to the plane)

$R_1 - W_1\cos\theta = 0$; $R_1 = W_1\cos\theta$….(ii)

From identities (i) and (ii)

$\mu_1 W_1 \cos\theta - W_1\sin\theta - T = 0$ ……. (a)

Similarly, considering equilibrium for block of weight W_2, we may write

$\mu_1 R_2 - W_2\sin\theta + T = 0$

$R_2 - W_2\cos\theta = 0$; $R_2 = W_2\cos\theta$

or $\mu_2 W_2\cos\theta - W_2\sin\theta + T = 0$ ……. (b)

Adding expressions (a) and (b), we get

$(\mu_1 W_1 + \mu_2 W_2)\cos\theta - (W_1 + W_2)\sin\theta = 0$

$\therefore\ \tan\theta = \dfrac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}$

which is the required expression.

(b) When $W_1 = W_2 = W$, the above expression reduces to

$$\tan\theta = \frac{\mu_1 + \mu_2}{2}$$

Substituting $\mu_1 = 1/2$ and $\mu_2 = 1/3$,

$$\tan\theta = \frac{\frac{1}{2}+\frac{1}{3}}{2} = \frac{5}{12}$$

$\therefore$ Inclination of plane,

$$\theta = \tan^{-1}\frac{5}{12} = 22.62°$$

03.

Sol: Let F_{ab} be the force in the rod. With reference to free body diagram of block A and for its limiting equilibrium

$\sum F_x = 0$

and $\sum F_y = 0$

$\therefore\ \sum F_x = R - F_{ab}\cos 45° = 0$

or $R = 0.707\ F_{ab}$

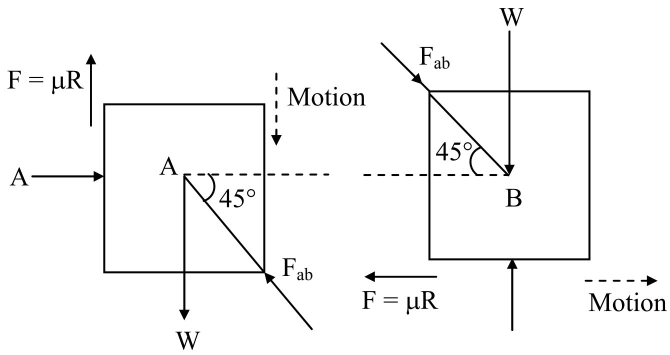
$\sum F_y = F - W - F_{ab}\sin 45° = 0$

Making substitution for $F = \mu R$

$= \mu \times 0.707\ F_{ab}$,

we have $\mu \times 0.707\ F_{ab} - W - 0.707\ F_{ab} = 0$

$$F_{ab} = \frac{W}{0.707(1+\mu)}$$



From free body diagram of block b, the equations for limiting equilibrium are

$\sum F_x = -\mu R + F_{ab}\cos 45° = 0;$

$$R = \frac{0.707\,F_{ab}}{\mu}$$

$\sum F_y = -F_{ab}\sin 45° - W + R = 0$

Making substitution for $r = 0.707F_{ab}/\mu$,

we get $-0.707\ F_{ab} - W + \dfrac{0.707\,F_{ab}}{\mu} = 0$

or $0.707\ F_{ab}\left(\dfrac{1}{\mu} - 1\right) = W$

$$F_{ab} = \frac{W}{0.707}\frac{\mu}{1-\mu} \ \ldots\ldots\ldots \text{(ii)}$$

From expression (i) and (ii)

$$\frac{W}{0.707(1+\mu)} = \frac{W}{0.707}\frac{\mu}{1-\mu}$$

or $(1+\mu)\mu = 1 - \mu$; $\mu^2 + 2\mu - 1 = 0$

Solution of this quadratic equation gives:

$$\mu = \frac{-2 \pm \sqrt{2^2 - 4\times(-1)\times 1}}{2\times 1} = \frac{-2 \pm 2.828}{2}$$

Neglecting –ve value which is impossible, we obtain

$\mu = 0.414$

Chapter 4 Kinematics of Particle Rectilinear and Curvilinear Motion

01. Ans: (d)

Sol: $x = 2t^3 + t^2 + 2t$

$$V = \frac{dx}{dt} = 6t^2 + 2t + 2$$

$$a = \frac{dv}{dt} = 12t + 2$$

At t = 0 ⇒ V = 2 and a = 2

02. Ans: (a)

Sol: $V = kx^3 - 4x^2 + 6x$

$V_{\text{at } x = 2 \text{ if } k = 1} = 2^3 - 4(2)^2 + 6(2) = 4$

$$a = \frac{dV}{dt} = k.3x^2\frac{dx}{dt} - 8x\frac{dx}{dt} + 6\frac{dx}{dt}$$

$a = 3x^2(V) - 8x(V) + 6(V)$

$= 3(2)^2 \times 4 - (8\times2\times4) + 6(4)$

$= 8\ m/s^2$

03. Ans: (d)

Sol: Given, $a = 6\sqrt{V}$

$$\frac{dV}{dt} = 6\sqrt{V}$$

$$\int\frac{dV}{\sqrt{V}} = \int 6\,dt$$

$$2\sqrt{V} = 6t + C_1$$

Given, at t = 2 sec, V = 36

$\Rightarrow 2\sqrt{36} = 6(2) + C_1$

$\Rightarrow C_1 = 0$

$2\sqrt{V} = 6t$

$V = 9t^2$

But $V = \frac{ds}{dt} = 9t^2$

$$\int ds = \int 9t^2\,dt$$

$S = 3t^3 + C_2$

At, t = 2 sec, S = 30 m

$\Rightarrow 30 = 3(2)^3 + C_2$

$\Rightarrow C_2 = 6$

$\therefore S = 3t^3 + 6$

At t = 3 sec

$S = 3(3)^3 + 6$

$S = 87$ m

04. Ans: (a)

Sol: Given $A = -8S^{-2}$

$$\Rightarrow \frac{dV}{dt} = \frac{d^2s}{dt^2} = -8s^{-2} = a$$

We know that, $\int V\,dv = \int a\,ds$

$$\frac{V^2}{2} = \int -8s^{-2}\,ds$$

$$\frac{V^2}{2} = \frac{8}{S} + C_1$$

Given, at S = 4m , V = 2 m/sec

$$\Rightarrow \frac{2^2}{2} = \frac{8}{4} + C_1$$

$\Rightarrow C_1 = 0$

$$\therefore \frac{V^2}{2} = \frac{8}{S}$$

$$V = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \frac{ds}{dt} = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \int \sqrt{s}\, ds = \int 4\ dt$$

$$\frac{2}{3}s^{3/2} = 4t + C_2$$

At t = 1, S = 4

$$\Rightarrow \frac{2}{3}(4)^{3/2} = 4(1) + C_2$$

$$\Rightarrow C_2 = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\therefore \frac{2}{3}s^{3/2} = 4t + C_2$$

$$\Rightarrow \frac{2}{3}s^{3/2} = 4t + \frac{4}{3}$$

At t = 2 sec

$$\frac{2}{3}s^{3/2} = 4(2) + \frac{4}{3}$$

$\Rightarrow$ s = 5.808 m

$$a = \frac{-8}{s^2} = \frac{-8}{5.808^2} = -0.237\ \text{m/sec}^2$$

05. Ans: (c)

Sol: Given, $a = 4t^2 - 2$

$$\frac{dv}{dt} = 4t^2 - 2$$

$$dv = (4t^2 - 2)\, dt$$

$$v = \frac{4t^3}{3} - 2t + C_1$$

$$\frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$$

$$\int dx = \int \left(\frac{4t^3}{3} - 2t + C_1 \right) dt$$

$$x = \frac{4t^4}{3 \times 4} - 2.\frac{t^2}{2} + C_1 t + C_2$$

$$x = \frac{t^4}{3} - t^2 + C_1 t + C_2$$

Given condition,

At t = 0, x = –2 m

$\Rightarrow -2 = C_2$

At t= 2, x = –20 m

$$\Rightarrow -20 = \frac{2^4}{3} - 2^2 + 4(2) + (-2)$$

$$\Rightarrow C_1 = \frac{-29}{3}$$

$$\therefore x = \frac{t^4}{3} - t^2 - \frac{29}{3}t - 2$$

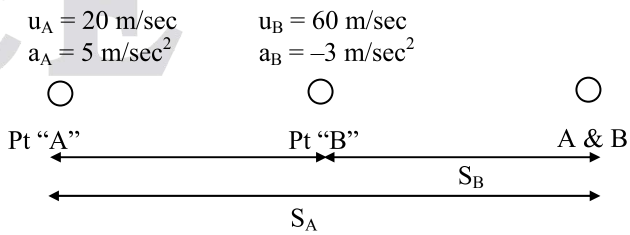
$\therefore$ at t = 4 sec

$$x = \frac{4^4}{3} - 4^2 - \frac{29}{3}(4) - 2$$

= 28.67 m

06. Ans: (b)

Sol:



Let S_A be the distance traveled by "A"

Let S_B be the distance traveled by "B"

$S_A = S_B + 384$

$$u_A t + \frac{1}{2}a_A t^2 = u_B t + \frac{1}{2}a_B t^2 + 384$$

$$20t + \frac{1}{2}5t^2 = 60t - \frac{1}{2}3t^2 + 384$$

$4t^2 - 40t - 384 = 0$

$t = 16$ sec (or) $t = -6$ sec

$\therefore t = 16$ sec

07. Ans: (b)

Sol: Take , $y = x^2 - 4x + 100$

Initial velocity, $V_0 = 4\hat{i} - 16\hat{j}$

If V_x is constant

V_y , a_y at x = 16 m

$$V_x = V_{1x} = \frac{dx}{dt} = 4$$

$$V_y = \frac{dy}{dt} = 2x\frac{dx}{dt} - 4\frac{dx}{dt}$$

$(V_y) = 2x\,(4) - 4(4)$

$V_y = 8x - 16$

$(V_y)_{at\ x = 16} = 8\,(16) - 16 = 112$ m/sec

$$a_y = \frac{dV}{dt} = \frac{d}{dt}(2xV_x - 4V_x)$$

($\because$ V_x = constant)

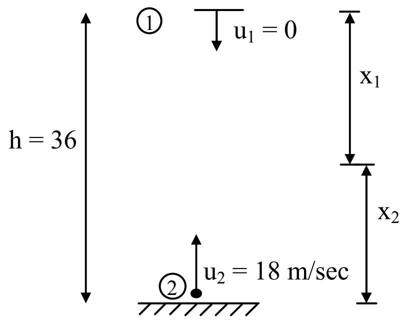
$$= 2V_x\frac{dx}{dt} = 2V_x . V_x$$

$$a_y = 2V_x^2$$

$(a_y)_{\ x = 16} = 2\times4^2 = 32$ m/sec^2

08. Ans: (c)

Sol:



Let at distance of "x_1' ball (1) crossed ball (2)

$\therefore x_1 + x_2 = 36$

$$x_1 = 0(t) + \frac{1}{2}gt^2 \quad (\because s = ut + \frac{1}{2}at^2)$$

$$x_1 = \frac{1}{2}gt^2 \text{ -------- (1)}$$

$$x_2 = 18(t) - \frac{1}{2}gt^2$$

($\because a = -g$ moving upward)

$x_1 + x_2 = 36$

$$\Rightarrow \frac{1}{2}gt^2 + 18t - \frac{1}{2}gt^2 = 36$$

$\Rightarrow 18\,t = 36$

$\Rightarrow t = 2$ sec

$$\therefore x_1 = \frac{1}{2}(9.81).2^2$$

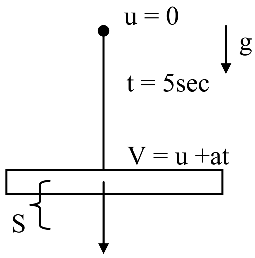
= 19.62 m (from the top)

$x_2 = 36 - 19.62$

= 16.38 m (from the bottom)

09. Ans: (b)

Sol:



$V = u + at$

$V = 0 + 9.81\ (5)$

$V = 49.05$ m/sec

V = velocity with which stone strike the glass

Velocity loss = 20% of V

$$= \frac{49.05 \times 20}{100} = 9.81 \text{ m/sec}$$

∴ Initial velocity for further movement in glass = 49.05 – 9.81 = 39.24 m/sec

Distance traveled for 1 sec of time is given by

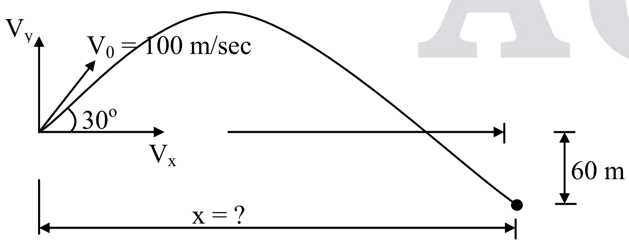
$$S = ut + \frac{1}{2}at^2$$

$$S = 39.24(1) + \frac{1}{2}(9.81)(1)^2$$

$S = 44.145$ m

10. Ans: (a)

Sol:



$a_x = -4$ m/sec^2 , $a_y = -20$ m/sec^2

$$V_x = V_0 \cos 30 = 100 \times \frac{\sqrt{3}}{2} = 86.6 \text{ m/sec}$$

$$V_y = V_0 \sin 30 = 100 \times \frac{1}{2} = 50 \text{ m/sec}$$

$$y = V_{oy}t + \frac{1}{2}a_y t^2$$

$$-60 = 50t + \frac{1}{2}(-20)t^2$$

$10t^2 - 50t - 60 = 0$

$t = 6$ (or) -1 sec

∴ $t = 6$ sec

$$x = V_0 t + \frac{1}{2}a_x t^2$$

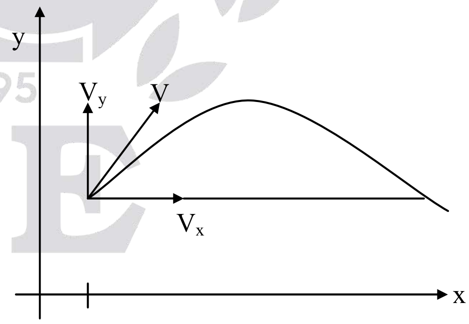
$$x = (86.6 \times 6) + \frac{1}{2}(-4)6^2$$

$x = 447.6$ m $\simeq 448$ m

11. Ans: (a)

Sol: Given, V = 20 m/sec

x = 20 m, y = 8.0 m



$V_x = V\cos\theta$, $V_y = V\sin\theta$

$$x = V_x t + \frac{1}{2}at^2 \ (\because a = 0 \text{ along x direction})$$

$x = V\cos\theta\ t$

$20 = 20 \cos\theta t$

$t = \dfrac{1}{\cos\theta}$ ------- (1)

$y = V_y t - \dfrac{1}{2}gt^2$

$8.0 = V\sin\theta t - \dfrac{1}{2}gt^2$

$8.0 = 20\sin\theta \times \dfrac{1}{\cos\theta} - \dfrac{1}{2}\times 9.81 \times \left(\dfrac{1}{\cos\theta}\right)^2$

$8 = 20\tan\theta - 4.9\sec^2\theta$

$8 = 20\tan\theta - 4.9(1 + \tan^2\theta)$

$4.9\tan^2\theta - 20\tan\theta + 12.9 = 0$

$\tan\theta_1 = 3.28,\ \tan\theta_2 = 0.803$

$\theta_1 = 73.04\ ;\ \theta_2 = 38.76$

12. Ans: (d)

Sol: Range = maximum height

$\dfrac{V_0^2\sin 2\theta}{g} = \dfrac{V_0^2\sin^2\theta}{2g}$

$\sin 2\theta = \dfrac{\sin^2\theta}{2}$

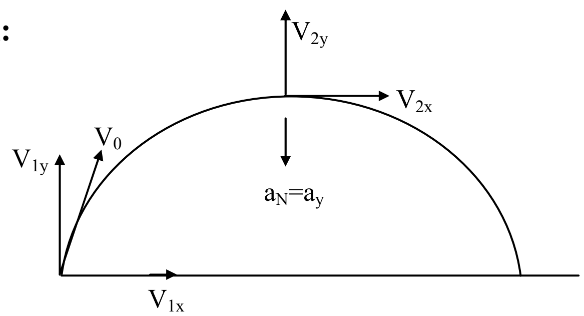
$\Rightarrow 2\sin\theta\cos\theta = \dfrac{\sin^2\theta}{2}$

$\Rightarrow \tan\theta = 4$

$\therefore\ \theta = \tan^{-1}(4) = 76°$

13. Ans: (a)

Sol:



$V_{1x} = 100 - t^{3/2}$

$V_{2y} = 0 \Rightarrow 100 + 10t - 2t^2 = 0$

$(t-10)(t+5) = 0$

$t = 10$ sec

V_{2x} at $t = 10 \Rightarrow V_{2x} = 100 - 10^{3/2}$

$= 68.37$ m/sec

Radius of curvature, $r = \dfrac{V^2}{a_N}$

Where $a_N = a_y = \left(\dfrac{dV_y}{dt}\right)_{at\ t=10\,sec}$

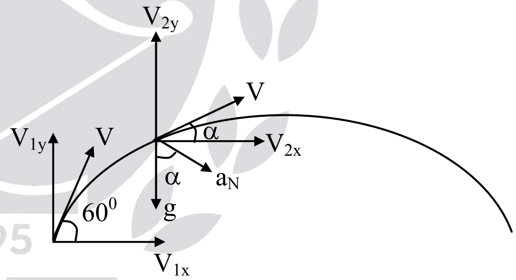
$= (10 - 4t)_{t=10}$

$a_N = -30$ m/sec^2

$r = \dfrac{V_{2x}^2}{a_N} = \dfrac{68.37^2}{30} = 155.8$ m

14. Ans: (a)

Sol:



Given, v = 100 m/sec

$v_{1x} = v\cos 60^0$

$= 100 \times 1/2$

$v_{1x} = 50$ m/sec

$v_{1y} = v\sin 60$

$= 100 \times \dfrac{\sqrt{3}}{2}$

$v_{1y} = 86.6$ m/sec

$v_{2y} = v_{1y} - gt$ (use V = u+at)

$= 86.6 - 9.8(1)$

$v_{2y} = 76.8$ m/sec

$v_{2x} = v_{1x} = 50$ m/sec

$$v_{at\ t=1} = \sqrt{v_{2x}^2 + v_{2y}^2}$$

$$= \sqrt{50^2 + 76.8^2}$$

= 91.6m/sec.

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \text{Tan}^{-1}\left(\frac{76.8}{50}\right)$$

$\alpha = 56.9$

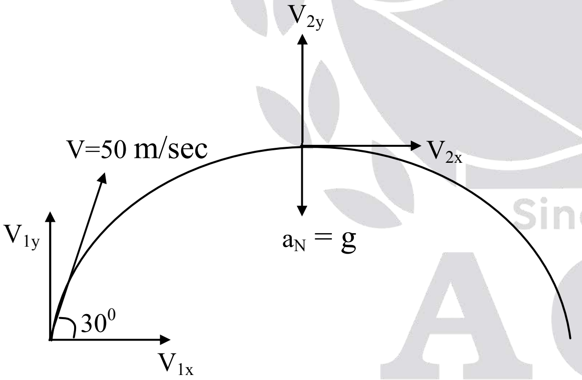
$a_N = g\cos\alpha = 9.81 \times \cos 56.9$

$= 5.35$m/sec^2

$$r = \frac{V^2}{a_N} = \frac{91.6^2}{5.35} = 1568.62 \text{ m}$$

15. Ans: (d)

Sol:



$v_{1x} = v\cos 30 = 43.3$ m/sec

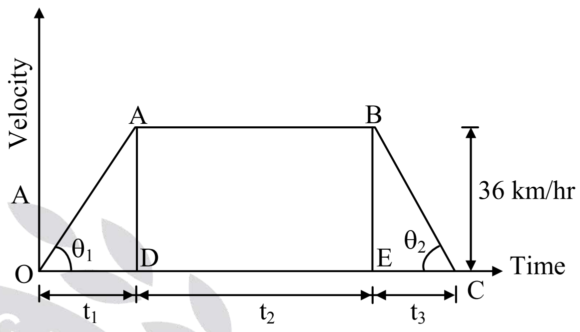
$a_N = g = a$

$$r = \frac{V_{1x}^2}{a_N} = \frac{43.3^2}{9.81} = 191.13 \text{ m}$$

Conventional Practice Solutions

01.

Sol:



Above figure for the velocity time graph.

The slope of velocity-time graph represents acceleration.

$\tan\theta_1 = \tan\theta_2$

$\theta_1 = \theta_2$

Obviously the triangles OAD and BCE are similar in all respects. That gives

OD = EC, i.e., $t_1 = t_3$

The total travel time from start to stop is given to be 6 minutes, i.e., 0.1 hour.

$\therefore \quad t_1 + t_2 + t_3 = 0.1$

or $\quad 2t_1 + t_2 = 0.1$ .....(i)

The are of the velocity-time graph gives the distance travelled during any time interval. Thus

$s = s_1 + s_2 + s_3$

$$= \frac{1}{2}\times 36\times t_1 + 36\times t_2 + \frac{1}{2}\times 36\times t_3$$

$\therefore 2.5 = 18\ t_1 + 36\ t_2 + 18\ t_3$

$= 36\ t_1 + 36\ t_2$ $(\because\ t_1 = t_3)$

or $t_1 + t_2 = 0.0694$ …….(ii)

From expressions (i) and (ii)

$2t_1 + (0.0694 - t_1) = 0.1$

$t_1 = 0.1 - 0.0694 = 0.0360$ hr

From the triangle OAD representing accelerating motion,

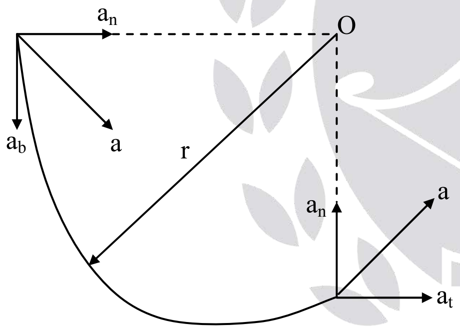
Acceleration a = rate of change of velocity

$$= \frac{V_a - V_0}{t_1} = \frac{36-0}{0.0306} = 1176.47\ \text{km/hr}^2$$

Since retardation is stated to be numerically equal to acceleration, the magnitude of retardation is also 1176.47 km/hr^2.

02.

Sol:



Above the figure, let the automobile travel from A to B.

$$V_a = 30\ \text{km/hr} = \frac{30\times1000}{3600} = 8.33\ \text{m/s}$$

$$V_b = 48\ \text{km/hr} = \frac{48\times1000}{3600} = 13.33\ \text{m/s}$$

Using the kinematic equation

$$v^2 - u^2 = 2as,$$

Tangential acceleration $a_t = \dfrac{13.33^2 - 8.33^2}{2\times400}$

$= 0.1356\ \text{m/s}^2$

If r is the radius of the circular track, then

$$\frac{1}{4}(2\pi r) = 400;$$

$$r = \frac{400\times2}{\pi} = 254.77\ \text{m}$$

At end A:

$$a_n = \frac{V_a^2}{r} = \frac{8.33^2}{254.77} = 0.272\ \text{m/}^2$$

$a_t = 0.135\ \text{m/s}^2$

$\therefore$ Resultant acceleration

$$a = \sqrt{(0.272)^2 + (0.135)^2} = 0.304\ \text{m/}^2$$

If α is the direction of resultant with tangential acceleration, then

$$\tan\alpha = \frac{a_n}{a_t} = \frac{0.272}{0.135} = 2.0148;$$

$\alpha = 63.60$

At end B:

$$a_n = \frac{V_b^2}{r} = \frac{13.33^2}{254.77} = 0.697\ \text{m/s}^2$$

$a_t = 0.135\ \text{m/s}^2$

Resultant acceleration,

$$a = \sqrt{(0.697)^2 + (0.135)^2} = 0.71\ \text{m/s}^2$$

If α is the direction of resultant with tangential acceleration, then

$$\tan\alpha = \frac{a_n}{a_t} = \frac{0.697}{0.135} = 5.163;$$

$\alpha = 79.04°$

03.

Sol: a-s Graph.

Since the equations for segments of the υ-s graph are given, the a-s graph can be determined using a ds = υ dυ.

$0 \le s \le 50$ m; $\upsilon = 0.4s + 5$

$a = \upsilon = 0.4s + 5$

$$a = \upsilon\frac{d\upsilon}{ds} = (0.4s + 5)\frac{d}{ds}(0.4s + 5)$$

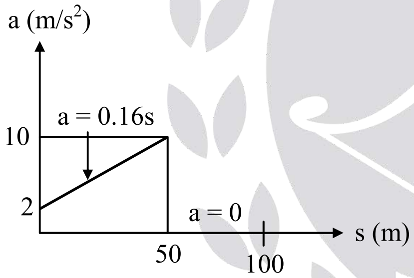
$$= 0.16s + 2$$

$50 \text{ m} < s \le 100$ m;

$\upsilon = 25$;

$$a = \upsilon\frac{d\upsilon}{ds} = (25)\frac{d}{ds}(25) = 0$$

The results are plotted in figure.



Time:

The time can be obtained using the υ-s graph and υ = ds/dt, because this equation relates υ, s, and t. For the first segment of motion, s = 0 at t = 0, so

$0 \le s < 50$ m;

$\upsilon = 0.4s + 5$;

$$dt = \frac{ds}{u} = \frac{ds}{0.4s + 5}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.4s + 5}$$

$t = 2.5\, ln(0.4\, s + 5) + 2.5\, ln\, 5$

At $s = 50$ m,

$t = 2.5\, ln(0.4s + 5) + 2.5\, ln5 = 12.07$ s.

Therefore, for the second segment of motion,

$25 \text{ m} < s \le 100$ m;

$\upsilon = 25$;

$$dt = \frac{ds}{\upsilon} = \frac{ds}{25}$$

$$\int_{12.07}^{t} dt = \int_{50}^{s} \frac{ds}{25}\; ; \; t - 12.07 = \frac{s}{25} - 2$$

$$t = \frac{s}{25} + 10.07$$

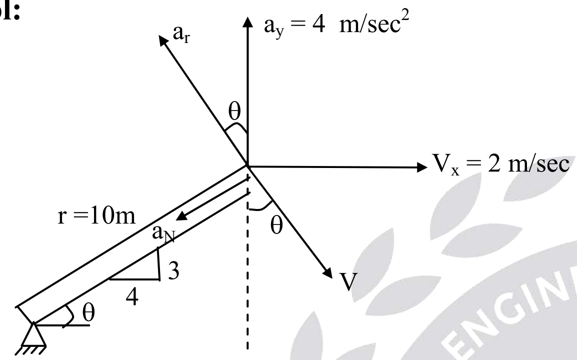
Therefore, at s = 100 m,

$$t = \frac{100}{50} + 10.07 = 12.07$$

Chapter 5
Kinematics of Rigid Bodies Fixed Axis Rotation and General Plane Motion

01. Ans: (a)

Sol:



$\tan\theta = \dfrac{3}{4}$

$\theta = \tan^{-1} 3/4 = 36.6^0$

$a_y = a_T \cos\theta - a_N \sin\theta$

Note: Velocity will always act in the tangential direction

$V_x = V\sin\theta$

$V = \dfrac{2}{\sin 36.6} = 3.33$ m/sec

$\therefore a_N = \dfrac{V^2}{r} = \dfrac{3.33^2}{10}$

$a_N = 1.111$ m/sec^2

$a_y = a_T \cos\theta - a_N \sin\theta$

$4 = a_T \cos 36.6 - 1.111 \sin 36.6$

$\Rightarrow a_T = 5.83$ m/sec^2

$a_T = r\alpha$

$\alpha = \dfrac{a_T}{r} = \dfrac{5.83}{10} = 0.583$ rad/sec^2

02. Ans: (c)

Sol: Given $\omega = 4\sqrt{t}$

$\theta = 2$ radians at t = 1sec

$\theta = ?\ \alpha = ?$ at t = 3sec

$\omega = \dfrac{d\theta}{dt} \Rightarrow \int d\theta = \int \omega dt$

$\theta = \int 4\sqrt{t}dt$

$\theta = \dfrac{8}{3}t^{3/2} + c \ldots(1)$

From given condition, at t = 1, θ = 2rad

$(1) \Rightarrow 2 = \dfrac{8}{3}(1)^{3/2} + c_1 \Rightarrow c_1 = \dfrac{-2}{3}$

$\therefore \theta = \dfrac{8}{3}t^{3/2} - \dfrac{2}{3}$

At t = 3 sec , $\theta = \dfrac{8}{3}(3)^{3/2} - \dfrac{2}{3}$

$\theta_{t=3} = 13.18$rad

$\alpha = \dfrac{d\omega}{dt} = \dfrac{d(4\sqrt{t})}{dt} = \dfrac{2}{\sqrt{t}}$

$\alpha_{t=3} = \dfrac{2}{\sqrt{3}} = 1.15\text{rad/sec}^2$

03. Ans: (b)

Sol: r = 2 cm, ω = 3 rad/sec , a = 30 cm/s^2

$a_N = r\omega^2 = 2(3)^2 = 18$ cm/sec^2

Since total acceleration $a = \sqrt{a_T^2 + a_N^2}$

$\Rightarrow a^2 = a_T^2 + a_N^2$

$30^2 = a_T^2 + 18^2$

$a_T = 24$ cm/sec^2

$a_T = r\alpha = 24$

$\alpha = \dfrac{24}{2} = 12\text{rad/sec}^2$

04. Ans: (d)

Sol: Given angular acceleration, $\alpha = \pi$ rad/sec^2

Angular displacement in time t_1 and t_2

$= \pi$ rad $= \theta_2 - \theta_1$

$\omega_{t2} = 2\pi$ rad/sec

$\omega_{t1} = ?$

$$\omega_{t1}^2 - \omega_0^2 = 2\alpha\theta_1$$

$$\omega_{t2}^2 - \omega_0^2 = 2\alpha\theta_2$$

$$\omega_{t2}^2 - \omega_{t1}^2 = 2\alpha(\theta_2 - \theta_1)$$

$$4\pi^2 - \omega_{t1}^2 = 2\pi^2$$

$$\omega_{t1}^2 = 2\pi^2$$

$$\omega_{t1} = \pi\sqrt{2}$$

05. Ans: (c)

Sol: Given retardation

$\alpha = -3t^2$

$$\frac{d\omega}{dt} = -3t^2$$

$$\int d\omega = \int -3t^2 dt$$

$\omega = -t^3 + c_1$

From given condition at t = 0,

$\omega = 27$ rad/sec

$27 = -0^3 + c_1$

$\Rightarrow c_1 = 27$

$\therefore \omega = -t^3 + 27$

Wheel stops at $\omega = 0$,

$\Rightarrow 0 = -t^3 + 27$

$\Rightarrow t = 3$sec

06. Ans: (c)

Sol: angular speed, $\omega = 5$ rev/sec

$= 5 \times 2\pi$ rad/sec

$\omega = 10\pi$ rad/sec

Radius, r = 0.1m

If ω is constant, $d\omega = 0$

$\Rightarrow \alpha = 0 \Rightarrow a_T = 0$ (since $a_T = r\alpha$)

Since $a_T = 0$

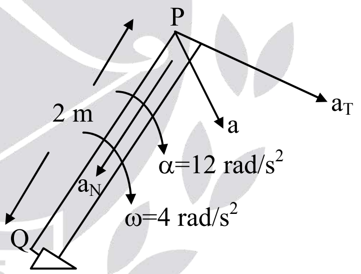
$$a = \sqrt{a_N^2 + a_T^2}$$

$$a = a_N = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$= 0.1 \times (10\pi)^2 = 10\pi^2 \text{ m/sec}^2$$

07. Ans: $a = 40 m/s^2$

Sol:



Tangential acceleration

$a_T = r\,\alpha = 2 \times 12 = 24 m/s^2$

Normal acceleration, $a_N = r\,\omega^2$

$= 2 \times 4^2 = 32$ m/s^2

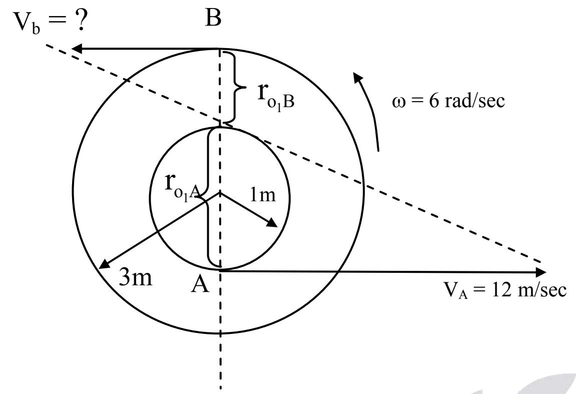
The resultant acceleration

$$a = \sqrt{a_T^2 + a_N^2}$$

$$= \sqrt{24^2 + 32^2} = 40 m/s^2$$

08. Ans: (b)

Sol:



$$V_A = r_{o_1A} \times \omega$$

$$\Rightarrow 12 = r_{o_1A} \times 6$$

$$r_{o_1A} = 2m$$

$$4 = 2 + r_{o_1B}$$

$$r_{o_1B} = 2m$$

$$\therefore V_B = r_{o_1B} \times \omega = 2 \times 6$$

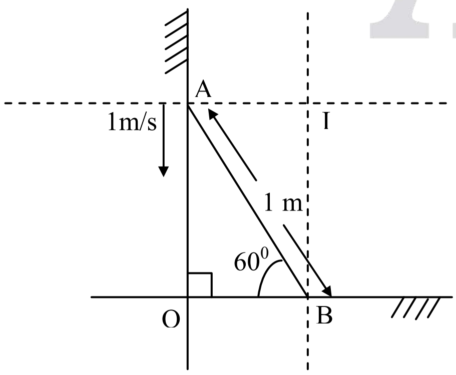
$$V_B = 12 \text{ m/sec}$$

09. Ans: (a)

Sol: Instantaneous centre will have zero velocity because the instantaneous centre is the point of contact between the object and the floor.

10. Ans: (a)

Sol:



V_a = 1 m/s

V_a = along vertical

V_b = along horizontal

So instantaneous center of V_a and V_b will be perpendicular to A and B respectively

$$IA = OB = l \times \cos\theta = 1 \times \cos 60^0 = \frac{1}{2} m$$

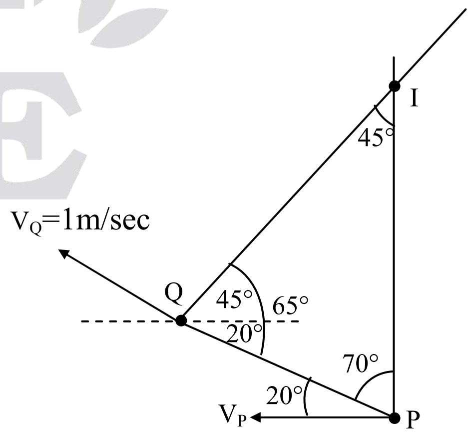
$$IB = OA = l \times \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2} m$$

$$V_a = \omega \times IA$$

$$\Rightarrow \omega = \frac{V_a}{IA} = 2 \text{ rad/sec}$$

11. Ans: (d)

Sol: Refer the figure shown below, by knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P using sine rule.



'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

$$V_Q = IQ \times \omega = 1$$

$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1$$

$$= 0.9645$$

Conventional Practice Solutions

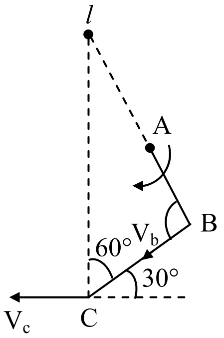
01.

Sol: Velocity of end B,

$$V_b = \omega r = 2.5 \times 1.5 = 3.75 \text{ m/s}$$

The direction of V_b is normal to AB and as BC is perpendicular to AB, it lies along BC. The velocity V_c of end C, at any instant, must be horizontal.

The instantaneous centre I of rod BC is the point of intersection of perpendiculars to V_b and V_c through B and C respectively.



$$IC = \frac{BC}{\cos 60^0} = \frac{3}{0.5} = 6 \text{ m}$$

$IB = IC \sin 60° = 6 \times 0.866 = 5.196$ m

Angular velocity of rod BC,

$$\omega = \frac{V_b}{IB} = \frac{V_C}{IC}$$

$$\omega = \frac{3.75}{5.196} = 0.722 \text{ rad/s (clockwise)}$$

$V_C = \omega \times IC = 0.722 \times 6 = 4.33$ m/s

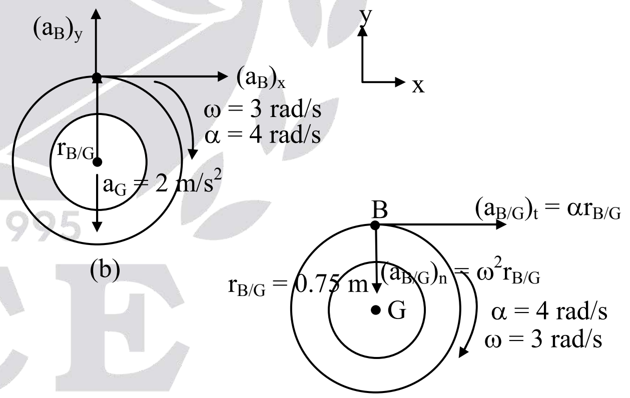
02.

Sol: **Solution I (VECTOR ANALYSIS) :**

The spool "appears" to be rolling downward without slipping at point A.

$a_G = \alpha r = 4 \text{ rad/s}^2 (0.5 \text{ m}) = 2 \text{ m/s}^2$

We will apply the acceleration equation at points G and B.



(b)

(c)

Kinematic Diagram.

Point B moves along a curved path having an unknown radius of curvature. Its acceleration will be represented by its unknown x and y components as shown in figure.

Acceleration Equation.

$$a_B = a_G + \alpha \times r_{B/G} - \omega^2 r_{B/C}$$

$$(a_B)_x i + (a_B)_y j = -2j + (-4k)\times(0.75j) - (3)^2(0.75j)$$

Equating the i and j terms, the component equations are

$$(a_B)_x = 4(0.75) = 3\ \text{m/s}^2 \ \ldots\ldots (1)$$

$$(a_B)_y = -2 - 6.75 = -8.75\ \text{m/s}^2$$

$$= 8.75\ \text{m/s}^2 \downarrow \ \ldots. (2)$$

The magnitude and direction of a_B are therefore

$$a_B = \sqrt{(3)^2 + (8.75)^2} = 9.25\ \text{m/s}^2$$

$$\theta = \tan^{-1}\frac{8.75}{3} = 71.1^\circ \quad \measuredangle\theta$$

Solution II (SCALAR ANALYSIS)

This problem may be solved by writing the scalar component equations directly. The kinematic diagram in figure shown the relative-acceleration components $(a_{B/G})_t$ and $(a_{B/G})_n$. Thus,

$$a_B = a_G + (a_{B/G})_t + (a_{B/G})_n$$

$$\left[\underset{\rightarrow}{(a_B)_x}\right] + \left[\left(\underset{\uparrow}{a_B}\right)_y\right]$$

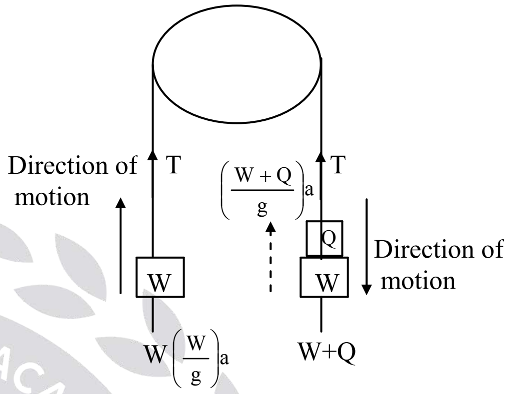
$$= \left[\underset{\downarrow}{2\,\text{m/s}^2}\right] + \left[\underset{\rightarrow}{4\,\text{rad/s}^2(0.75\text{m})}\right] + \left[\underset{\downarrow}{3^2\,\text{rad/s}^2(0.75\text{m})}\right]$$

The x and y components yield equations 1 and 2 above.

Chapter 6 Kinetics of Particle and Rigid Bodies

01. Ans: (a)

Sol:



For the left cord,

$$\Sigma F_y = 0$$

$$T = \left(\frac{W}{g}\right)a + W \ \ldots\ldots\ldots..(1)$$

For the right cord

$$\Sigma F_y = 0$$

$$T + \left(\frac{W+Q}{g}\right)a = (W+Q) \ldots(2)$$

From (1) & (2)

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W+Q}{g}\right)a$$

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W}{g}\right)a - \left(\frac{Q}{g}\right)a$$

$$Q - \frac{Qa}{g} = \frac{2Wa}{g}$$

$$Q\left(\frac{g-a}{g}\right) = \frac{2Wa}{g} \Rightarrow Q = \frac{2Wa}{g-a}$$

02. Ans: (b)

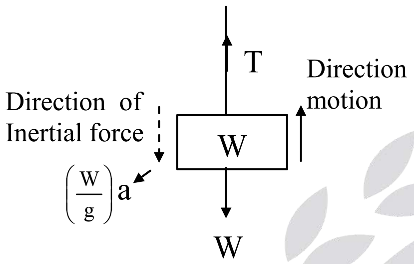
Sol: u = 0, v = 1.828 m/sec, S = 1.825 m,

$v^2 - u^2 = 2as$

$1.828^2 - 0 = 2a \times 1.828$

$a = \frac{1.828}{2}$

$a = 0.914 \text{ m/sec}^2$



For equilibrium, $\Sigma F_y = 0$

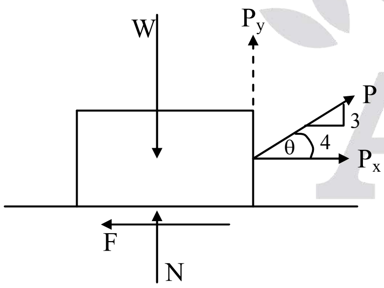
$T = W + \left(\frac{W}{g}\right)a$

$= 4448 + \frac{4448}{9.81} \times 0.194$

T = 4862.42 N

03. Ans: (a)

Sol:



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(3/4) = 36.86$

$(F_{net})_x = ma$

$P_x - F = \left(\frac{W}{g}\right)a$

$P\cos 36.86 - F = \left(\frac{W}{g}\right)a$

$0.8P - F = \left(\frac{2224}{g}\right)(0.2g)$

0.8P – F = 444.8

0.8P – F = 444.8 + F

P = 556+1.25F ……(1)

$\Sigma F_y = 0$

$N + P_y - W = 0$

$N = W - P_y$ (since $\mu = \frac{F}{N}$)

$F = \mu N$

$F = \mu (W - P_y)$

= 0.2(2224 – P sin 36.86)

F = 444.8 – 0.12P …..(2)

From (1) & (2)

P = 556+1.25(444.8 – 0.12P)

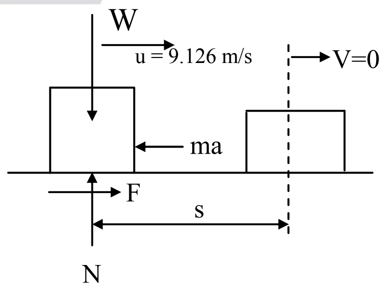
1.15P = 1112

P = 966.95

P = 967 N

04. Ans: (d)

Sol:



From static equilibrium condition

$\Sigma F_y = 0$

$N - W = 0$

$N = W = 44.48N$

From dynamic equilibrium condition

$\Sigma F_x = 0$

$F = ma$

$\mu N = \frac{W}{g} a$

$\mu = \frac{a}{g}$

$a = \mu g$ ….(1)

Since $v^2 - u^2 = 2as$

$0 - (9.126)^2 = 2(-a) \times 13.689$

$a = 3.042$ ….(2)

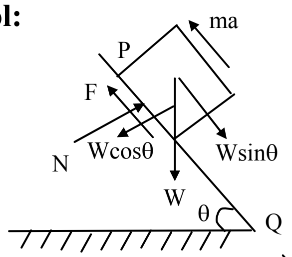
From (1) & (2)

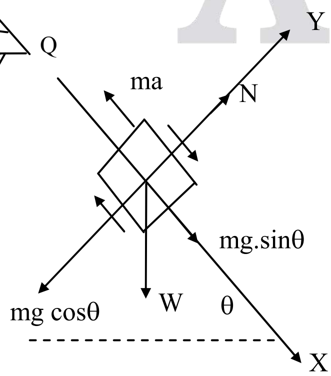
$3.042 = \mu(9.81)$

$\Rightarrow \mu = 0.31$

05. Ans: (a)

Sol:





$\Sigma F_y = 0$ (static equilibrium)

$N - W\cos\theta = 0$

$N = W\cos\theta = mg\cos\theta$

Since $F = \mu N = \mu\, mg\cos\theta$….(1)

$\Sigma F_x = 0$ (Dynamic equilibrium)

$F + ma - W\sin\theta = 0$

$F = -ma + mg\sin\theta$

$F = mg\sin\theta - ma$…(2)

From (1) & (2)

$\mu mg\cos\theta = mg\sin\theta - ma$

$\Rightarrow a = g\sin\theta - \mu g\cos\theta$

$\Rightarrow a = g\cos\theta(\tan\theta - \mu)$

Given PQ = s

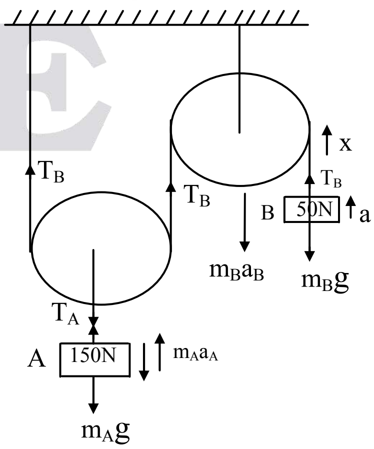
$s = ut + \frac{1}{2} at^2$

$s = 0(t) + \frac{1}{2} at^2 \quad \Rightarrow t = \sqrt{\frac{2s}{a}}$

$= \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$

06. Ans: (a)

Sol:



$T_A = 2T_B$ ….(1)

Work done by A & B equal

$T_A S_A = T_B S_B$

$2T_B S_A = T_B S_B$

$2S_A = S_B$

$2a_A = a_B$ ….(2)

For 'B' body

$T_B = m_B a_B + m_B g$ ….(3)

For 'A' body

$T_A = m_A g - m_A a_A$ ….(4)

(2), (3) & (4) sub in (1)

$m_A g - m_A a_A = 2(m_B(2a_A) + m_B g)$

$m_A g - m_A a_A = 4m_B a_A + 2m_B g$

$m_A a_A + 4m_B a_A = m_A g - 2m_B g$

$$a_A = \frac{m_A g - 2m_B g}{m_A + 4m_B}$$

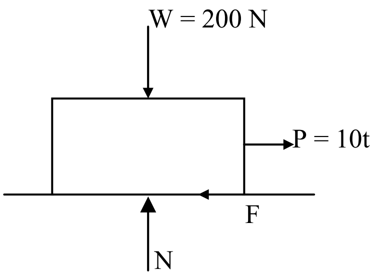
$$= \frac{150 - 2(50)}{\frac{150}{10} + 4\left(\frac{50}{10}\right)}$$

$$= \frac{50}{15 + 20} = \frac{50}{35} = 1.42$$

07. Ans: 4.905 m/s

Sol: $\mu_S = 0.4$; $\mu_K = 0.2$

FBD of the block



W.r.t free body diagram of the block:

$F_S = \mu_S N$;

$F_K = \mu_K N$

$\Sigma F_y = 0$

$N - W = 0$

$N = W = 200$ N

Limiting friction or static friction

$(F_S) = 0.4 \times 200 = 80$ N

Kinetic Friction

$(F_K) = 0.2 \times 200 = 40$ N

The block starts moving only when the force, P exceeds static friction, F_S

Thus, under static equilibrium

$\Rightarrow \sum F_x = 0$

$\Rightarrow P - F_S = 0 \Rightarrow 10t = 80$

$$t = \frac{80}{10} = 8 \text{ sec}$$

$\therefore$ The block starts moving only when t >8seconds

During 8 seconds to 10 seconds of time:

According to Newton's second law of motion

Force = mass × acceleration

$$(P - F_K) = m \times \frac{dv}{dt} \Rightarrow (10t - 40) = \frac{200}{9.81} \times \frac{dv}{dt}$$

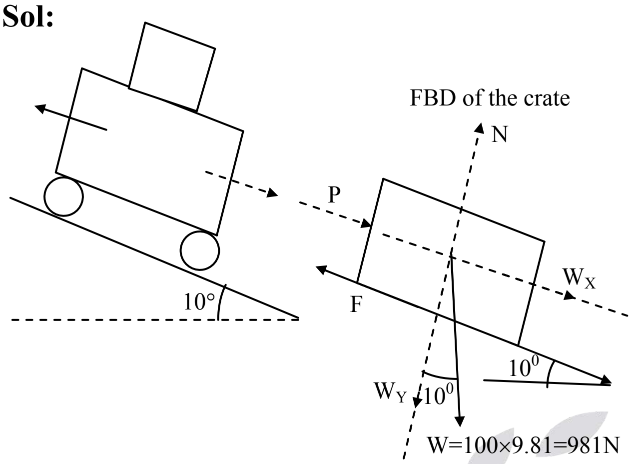
$$\int_8^{10} (10t - 40)dt = \frac{200}{9.81} \int_0^V dv$$

$$\left[5t^2 - 40t\right]_8^{10} = 20.387 \times V \Rightarrow (180 - 80) = 20.387 \times V$$

Velocity (V) = 4.905 m/s

08. Ans: 1.198 m/s²

Sol:



W.r.t. FBD of the crate:

$W_X = W\sin 10^0 = 981 \times \sin 10^0$

$= 170.34N$

$W_Y = W\cos 10^0 = 981 \times \cos 10^0 = 966.09$ N

$\sum F_Y = 0 \Rightarrow N - W_Y = 0$

$N = W_Y = 966.09N;$

$F = \mu N = 0.3 \times 966.09 = 289.828$ N

$\sum F_X = 0 \Rightarrow P + W_X - F = 0$

$\Rightarrow P + 289.828 - 170.34 = 0$

$P = 119.488$ N

$P = ma = 119.488$ N

$$\Rightarrow a = \frac{119.488}{100} = 1.198\,m/s^2$$

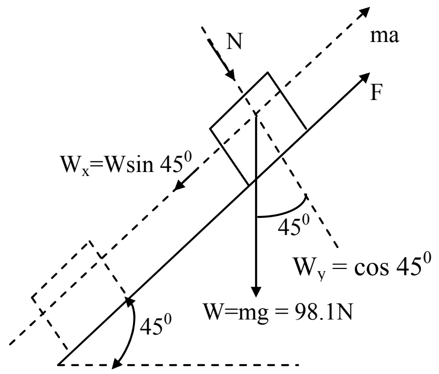
09. Ans: 57.67 m

Sol:

$W_x = W \sin 45$

$= 98.1 \times \sin 45 = 69.367$ N

$W_y = W \cos 45 = 69.367$ N



$\sum F_Y = 0$

$N - W_Y = 0$

$N = W_Y = 69.367$ N

$F = \mu_K N = 0.5 \times 69.367 = 34.683N$

$\sum F_x = 0$ (Dynamic Equilibrium

D′ Alembert principle)

$W_x - F - ma = 0$

$69.367 - 34.683 - 10 \times a = 0$

$a = 3.468 m/s^2$

$$S = ut + \frac{1}{2}at^2$$

$\because$ t is unknown we can not use this equation

So use $V^2 - u^2 = 2as$

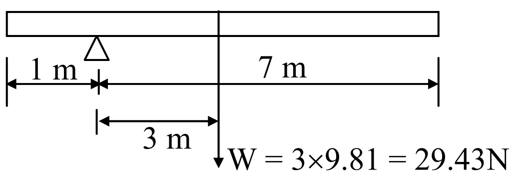
$V = 20m/s^2$; $u = 0$; $a = 3.468 m/s^2$

$V^2 = 2as$

$$S = \frac{V^2}{2 \times a} = \frac{20^2}{2 \times 3.468} = 57.67m$$

10. Ans: 2.053 rad/s²

Sol:



$M = I\alpha$

$M = 29.43 \times 3 = 88.29$N-m

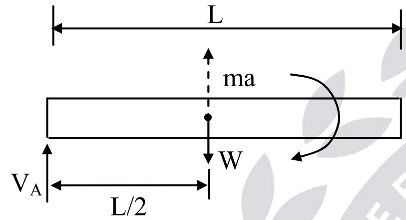
$$I = I_0 + Ad^2 = \frac{m\ell^2}{12} + md^2 = \frac{3\times 8^2}{12} + 3\times 3^2$$

$$= 16 + 27 = 43 kg - m^2$$

$$\alpha = \frac{M}{I} = \frac{88.29}{43} = 2.053\ rad/s^2$$

11. Ans: (d)

Sol:



$\Sigma F_y = 0$

$V_A + ma = W$

$V_A = m(g-a)$…(1)

Where, $a = \frac{L}{2}\alpha$

Since, $M = I\alpha$

$$W\times\frac{L}{2} = \left(\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2\right)\alpha$$

$$mg\times\frac{L}{2} = \frac{4mL^2}{12}\times\frac{2a}{L}$$

$$a = \frac{3}{4}g \ldots(2)$$

from (1) & (2)

$$V_A = m\left(g - \frac{3}{4}g\right) = \frac{mg}{4}$$

$$V_A = \frac{W}{4}$$

12. Ans: (d)

Sol: $I = 5 kg.m^2$

$R = 0.25m$

$F = 8$ N

Mass moment of inertia, $I_x = I_y = \frac{mr^2}{4}$

$I_z = \frac{mr^2}{2}$

$M = I\alpha$

$8\times 0.25 = 5\times\alpha$

$\alpha = 0.4$

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\omega^2 - 0^2 = 2(0.4)\times\pi$ (since for half revolution $\theta = \pi$)

$\omega = 1.58$ rad/sec

13. Ans: 4.6 seconds

Sol: $M = 60$ N – m

$L = 2m$, $\omega_0 = 0$,

$\omega = 200$ rpm $= \frac{200\times 2\pi}{60}$

$\omega = 20.94\ \frac{rad}{sec}$

Moment, $M = I\alpha$

$$60 = \frac{mL^2}{12}\times\alpha$$

$$\Rightarrow 60 = \frac{40\times 2^2}{12}\times\alpha$$

$\alpha = 4.5 rad/sec^2$

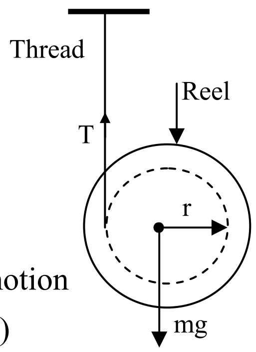
$\omega = \omega_0 + \alpha t$

$20.94 = 4.5t$

$\Rightarrow\ t = 4.65$ sec

14. Ans: (a)

Sol:



a = linear acceleration,

k = radius of gyration

For vertical translation motion

mg – T = ma ------ (1)

For rotational motion

$T \times r = I\alpha$

$Tr = mk^2\,\alpha = mk^2 \times \frac{a}{r}$

$\Rightarrow T = \frac{mk^2}{r^2} \times a$ ------(2)

$mg - \frac{mk^2}{r^2} \times a = ma \Rightarrow a = \frac{gr^2}{(k^2 + r^2)}$

Conventional Practice Solutions

01.

Sol: $\omega_1 = \frac{2\pi \times 1500}{60} = 157$ rad/s and $\omega_2 = 0$

Since the bearing friction is independent of the speed of disc, the angular acceleration is constant and it is given by

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t} = \frac{0 - 157}{145}$$

$= -1.083$ rad/s

Mass moment of inertia $I = \frac{1}{2} mr^2$

$= \frac{1}{2} \times 25 \times 0.5^2 = 3.125$ kg m^2

(a) Bearing torque $T = I\alpha = 3.125 \times (-1.083)$

$= -3.384$ Nm

The negative sign implies that the frictional torque of the bearing resists the rotates of the disc.

(b) From the kinematic relation $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

and noting that $\omega_2 = 0$, we have

Total angular displacement

$$\theta = -\frac{\omega_1^2}{2\alpha} = \frac{-157^2}{2 \times (-1.083)}$$

= 11380 radians

1 revolution = 2π radians

∴ Number of revolutions turned by the disc

$= \frac{11380}{2\pi} = 1812$

02.

Sol: Work done = force × distance moved

= 25 × 6 = 150 Nm

Let the plank move with velocity V m/s towards right and the disc move clockwise with angular ω rad/s.

Initially: The kinetic energy of the plank and discs is zero since initially the system is at rest.

Finally: Kinetic energy of plank $= \frac{1}{2} mV^2$

$= \frac{1}{2} \times 12 \times V^2 = 6\,V^2$ Nm

Kinetic energy of two discs

$$= 2\left[\frac{1}{2} m \times V_G{}^2 + \frac{1}{2} I_G\,\omega^2\right]$$

$$\omega = \frac{V}{0.2} = 5\text{ V};$$

$V_G = \omega r = 5\text{ V} \times 0.1 = 0.5\text{V}$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\times 5\times 0.1^2 = 0.025\text{ kgm}^2$$

$\therefore$ KE of two discs

$$= 2\left[\frac{1}{2}\times 5\times (0.5\text{V})^2 + \frac{1}{2}\times 0.025\times (5\text{V})^2\right]$$

$= [0.625\text{ V}^2 + 0.3125\text{ V}^2]$

$= 1.875\text{ V}^2$

Total kinetic energy of the system

$= 6\text{ V}^2 + 1.875\text{ V}^2 = 7.875\text{ V}^2$

This also equals the change in kinetic energy of the system as initial kinetic energy of the system is zero.

Since work done equals the change in kinetic energy, we have

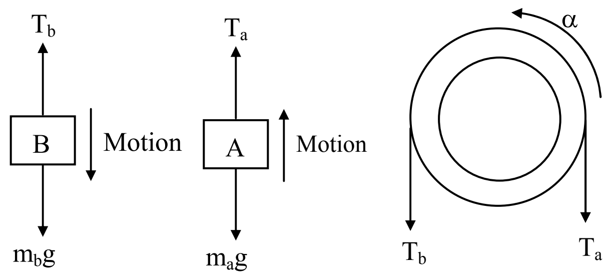
$150 = 7.875\text{ V}^2$

$V = \sqrt{19.05} = 4.36\text{ m/s}$

Thus the plank moves with 4.36 m/s velocity towards right.

03.

Sol: Since block B is heavier than block A, the cylinder would rotate in the anti-clockwise direction and the block A would get lifted upwards



Then with reference to free body diagrams shown in figure.

For block A:

$T_a - m_a g = m_a a_a;$

$T_a = m_a(a_a + g)$

$\therefore T_a = 80\,(1\times\alpha + 9.81) = 80\alpha + 784.8$

For block B:

$T_b - m_b g = -m_b a_b;$

$T_b = m_b(g - a_b)$

$\therefore T_b = 100(9.81 - 0.5\alpha) = 981 - 50\alpha$

For cylinder:

$I = mk^2 = 125\times(0.55)^2 = 37.81\text{ kgm}^2$

From Newton's second law; $T = I\alpha$

$T_bR_1 - T_aR_2 = I\alpha$

Substituting the appropriate values

$(981-50\alpha)\times 0.5-(80\alpha + 784.8)\times 1 = 37.81\alpha$

$(490.5 - 25\alpha) - (80\,\alpha + 784.8) = 37.81\alpha$

$\therefore$ Angular acceleration of the cylinder

$\alpha = -2.06\text{ rad/s}^2$

Corresponding to 2.5 m upward movement of block A, the angular displacement is $2.5/R_2 = 2.5$ radian

Then from the kinematic relation

$\theta = \omega t + \frac{1}{2}\alpha t^2$, we have

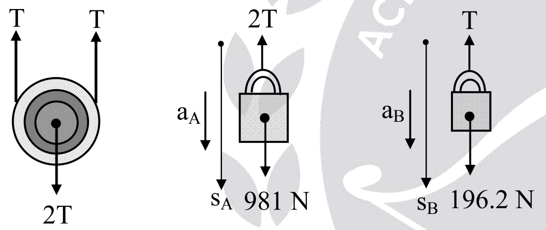
$2.5 = 0\times t + \frac{1}{2}\times(2.06)\times t^2\ (\omega=0)$

$\therefore$ Time required $t = \sqrt{\frac{2.5\times 2}{2.06}} = 1.56$ sec

04.

Sol: Free-Body Diagrams.

Since the mass of the pulleys is neglected, then for pulley C, ma= 0 and we can apply $\sum F_y = 0$ as shown in figure. The free body diagrams for blocks A and B are shown in figure. One can see that for A to remain static requires T = 490.5 N, whereas for B to remain static requires T = 196.2 N. Hence A will move down while B moves up. Here we will assume both blocks accelerated downward, in the direction of $+s_A$ and $+s_B$. The three unknowns are T, a_A, and a_B.



Equations of Motion :

Block A in figure:

$+\downarrow \sum F_y = ma_y$

$981 - 2T = 100a_A$ …. (1)

Block B in figure:

$+\downarrow \sum F_y = ma_y$

$196.2 - T = 20a_B$ …. (2)

Kinematics :

The necessary third equation is obtained by relating a_A to a_B using a dependent motion analysis. The coordinates s_A and s_B measure the position of A and B from the fixed datum, figure.

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \quad …… (3)$$

Notice that in writing equations 1 to 3, the positive direction was always assumed downward. The solution yields

$T = 327.0$ N

$a_A = 3.27 \text{ m/s}^2$,

$a_B = -6.54 \text{ m/s}^2$

Hence when block A accelerates downward, block B accelerates upward. Since a_B is constant, the velocity of block b in 2 s is thus

$(+\downarrow)$ $\upsilon = \upsilon_0 + a_B t$

$= 0 + (-6.54)(2)$

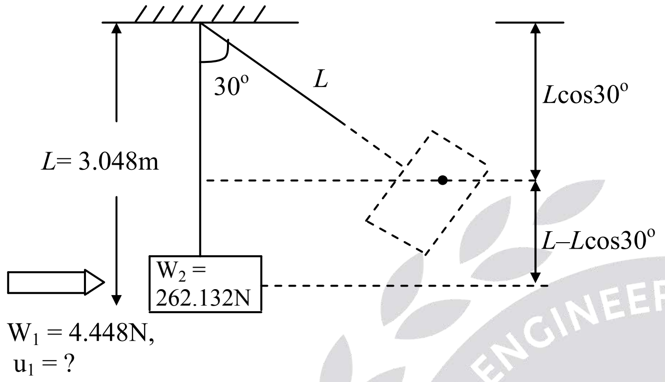
$= -13.1$ m/s

The negative sign indicates that block B is moving upward.

Chapter 7 Work-Energy Principle and Impulse Momentum Equation

01. Ans: (a)

Sol:



The loss of KE of shell converted to do the work in lifting the sand box and shell to a height of "$L - L\cos 30^o$"

i.e., $Wd = \frac{1}{2}mV^2$

Where $d = L - L\cos 30^o$

$= 3.048 - 3.048\times\cos 30 = 0.41$ m

$$266.58\times 0.41 = \frac{1}{2}\left(\frac{266.58}{9.81}\right)\times V^2$$

$\Rightarrow V = 2.83$ m/sec

Where V is the velocity of block & shell

By momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Where $v_1 = v_2 = V$ & $u_1 = ?$, $u_2 = 0$

$$\frac{4.448}{9.81}\times u_1 = \frac{4.448 + 262.132}{9.81}\times 2.83$$

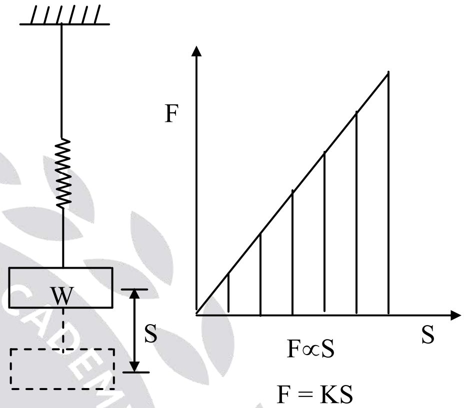
$\Rightarrow u_1 = 169.6$ m/sec

u_1 & u_2 = Initial velocity of shell and block respectively

V_1 & V_2 = Final velocity of block & shell

02. Ans: (b)

Sol:



Strain energy in spring = Area under the force displacement curve.

$$= \frac{1}{2}F\times s = \frac{1}{2}(ks)\times s = \frac{1}{2}ks^2$$

$$\frac{1}{2}ks^2 = \text{Gain of KE}$$

$$\frac{1}{2}ks^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{ks^2}{m} = \frac{ks^2}{w}g$$

$$v = \sqrt{\frac{kg}{w}}.s \qquad \left(\because m = \frac{w}{g}\right)$$

03. Ans: (a)

Sol: Given, m = 2 kg

Position at any time is given as

$$x = t + 5t^2 + 2t^3$$

At t = 0, x = 0,

At t = 3sec,

$x = 3 + 5(3^2) + 2(3^3) = 102m$

Velocity, $V = \frac{dx}{dt} = 1 + 10t + 6t^2$

Initial velocity i.e., t = 0, is v_i = 1m/s

Final velocity i.e., at t = 3sec,

is $v_f = 1 + 10(3) + 6(3)^2 = 85$m/s

Work done = change in KE

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2} \times 2\left(85^2 - 1^2\right) = 7224 \text{ J}$$

04. Ans: (a)

Sol: Given force $F = e^{-2x}$

Work done = $\int_{x_1}^{x_2} F dx$

$$= \int_{0.2}^{1.5} e^{-2x} dx = \left[\frac{e^{-2x}}{-2}\right]_{0.2}^{1.5} = 0.31\text{J}$$

05. Ans: (b)

Sol: $F = 4x - 3x^2$

Potential Energy at x = 1.7 = work required to move object from 0 to 1.7m

$$PE = \int_0^{1.7} F dx$$

$$= \int_0^{1.7} \left(4x - 3x^2\right) dx$$

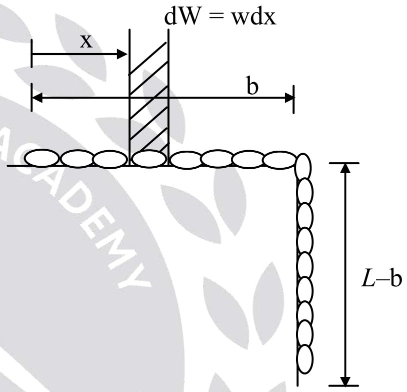
$$= \left[4\left(\frac{x^2}{2}\right) - 3\left(\frac{x^3}{3}\right)\right]_0^{1.7}$$

$$= \left[2x^2 - x^3\right]_0^{1.7}$$

$$= 2(1.7)^2 - (1.7)^3 = 0.867 \text{ J}$$

06. Ans: (c)

Sol:



Where w = weight per unit meter

dw = a small work done in moving small elemental "dx" of chain through a d/s "x"

Work done = change in KE

$$\left(\int_0^b dw \times x\right) + \left(w(L-b) \times b\right) = \frac{1}{2}\left(\frac{wL}{g}\right)v^2$$

$$\int_0^b wdx.x + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + w(L-b)b = \frac{1}{2}\frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + wLb - wb^2 = \frac{1}{2}\frac{wLv^2}{g}$$

$$wLb - \frac{wb^2}{2} = \frac{1}{2}\frac{wLv^2}{g}$$

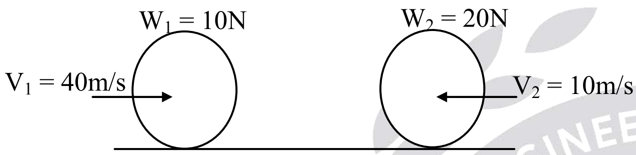
$$b\left(L-\frac{b}{2}\right)=\frac{1}{2}\frac{Lv^2}{g}$$

$$v^2 = 2gb\left(1-\frac{b}{2L}\right)$$

$$v = \sqrt{gb\left(2-\frac{b}{L}\right)}$$

07. Ans: (d)

Sol:



$m_1 = 1$kg , $m_2 = 2$kg ,(since $g = 10$m/sec^2)

Velocities before impact

$v_1 = 40$ m/sec, $v_2 = -10$m/s

Velocities after impact

$u_1 = ?$ $u_2 = ?$

Coefficient of restitution e = 0.6

From momentum equation

$$m_1v_1+m_2 v_2 = m_1u_1+m_2u_2$$

$\Rightarrow 1(40) + 2(-10) = 1(u_1) + 2(u_2)$

$\Rightarrow u_1 + 2u_2 = 20$…………………..(1)

$$e=\frac{u_2-u_1}{v_1-v_2}=\frac{\text{relative velocity of Seperation}}{\text{relative velocity of approach}}$$

$$0.6=\frac{u_2-u_1}{40-(-10)}$$

$\Rightarrow u_2 - u_1 = 30$………………………(2)

From 1 & 2

$u_1 = -13.33$ m/sec

$u_2 = 16.66$ m/sec

08. Ans: (b)

Sol: Given, $m_1 = 3$ kg, $m_2 = 6$ kg

Velocities before impact

$u_1 = 4$ m/s, $u_2 = -1$ m/s

Velocities after impact

$v_1 = 0$m/s , $v_2 =?$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3(4) + 6(-1) = 3(0) + 6(v_2)$

$\Rightarrow 6 = 6v_2$

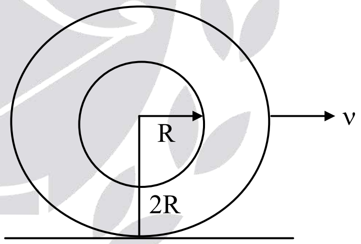
$\Rightarrow v_2 = 1$m/s

Coefficient of restitution, $e=\frac{v_2-v_1}{u_1-u_2}$

$$e=\frac{1-0}{4-(-1)}=\frac{1}{5}$$

09. Ans: (c)

Sol:



$$KE=\frac{1}{2}mV^2+\frac{1}{2}I\omega^2$$

Where, $\omega=\frac{V}{2R}$

$$I=\frac{1}{2}m\left((2R)^2+R^2\right)=\frac{5}{2}mR^2$$

$$\therefore\ KE=\frac{1}{2}mV^2+\frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2$$

$$KE = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2$$

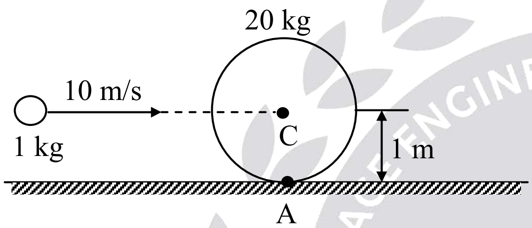
$$= \frac{1}{2}mV^2 + \frac{5}{4}mR^2 \times \frac{V^2}{4R^2}$$

$$= \frac{1}{2}mV^2 + \frac{5}{16}mV^2$$

$$KE = \frac{13mV^2}{16}$$

10. Ans: (a)

Sol:



Method I **:**

By conservation of linear momentum ,we get

$1\times10 = (20 + 1) \times V_{cm}$ (where, V_{cm} = velocity of centre of mass)

$$\Rightarrow V_{cm} = \frac{10}{21} m/s$$

Applying angular momentum conservation about an axis passing through the contact point (A) and perpendicular to the plane of paper, we get

$$1\times10\times1 = I_{cm}\omega + 21\times\frac{10}{21}\times1$$

[Angular momentum about any axis passing through A can be written as, $\vec{L}_A = \vec{L}_{cm} + m\left(\vec{r}\times\vec{V}_{cm}\right)$]

$\Rightarrow \omega = 0$ rad/sec

Method II **:**

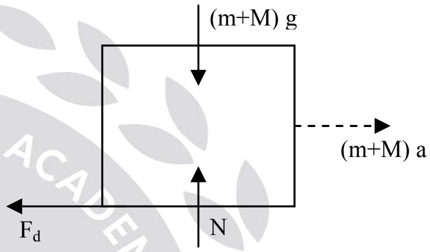
Applying angular momentum conservation about an axis passing through centre of wheel and perpendicular to the plane of paper.

$\therefore\ 0 = I_{cm}\,\omega$

$\Rightarrow \omega = 0$ rad/sec

11. Ans: (a)

Sol:



$m_1 = m \rightarrow$ mass of bullet

$m_2 = M \rightarrow$ mass of block

$u_1 = V \rightarrow$ bullet initial velocity

$u_2 = 0 \rightarrow$ block initial velocity

$v_1 = v_2 = v \rightarrow$velocity of bullet and block after impact.

$$F_d = \mu N$$

$(M+m)a = \mu(M+m)g$

$\Rightarrow a = \mu g$

From momentum equation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mV + m(0) = (m + M)V$

$$v = \frac{mV}{m+M}$$

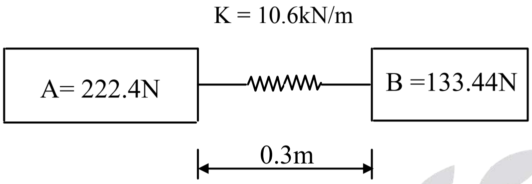
Now from $v^2 - u^2 = 2as$

$$0 - \left(\frac{mV}{m+M}\right)^2 = 2\mu gs$$

$$V = \frac{m+M}{m}\sqrt{2\mu gs}$$

12. Ans: (a)

Sol:



$u_A = 0$, $u_B = 0$

From momentum equation

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$0 = 222.4V_A + 133.44V_B$…………..(1)

$$\frac{1}{2}ks^2 = \frac{1}{2}m_A v_A{}^2 + \frac{1}{2}m_B v_B{}^2$$

$$10.6\times10^3\times0.15^2 = \frac{222.4}{9.81}v_A^2 + \frac{133.44}{9.81}v_B^2$$

…………..(2)

From 1 & 2

$v_A = -1.98$ m/s , $v_B = 3.3$ m/s

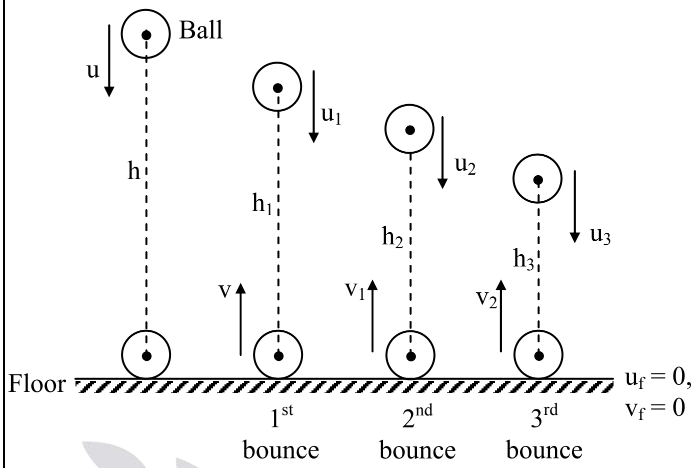
Conventional Practice Solutions

01.

Sol: Let u and v be the velocity of ball at first impact (Strike and bounce) with the floor.

Then $u = \sqrt{2gh}$ ($\downarrow$)

and $v = \sqrt{2gh_1}$ ($\uparrow$)



The velocity of floor is zero before and after every impact, i.e., $u_f = 0$ and $v_f = 0$

From the relation for coefficient of restitution,

$$e = \frac{v_f - v}{u - u_f} = \frac{-v}{u} \qquad (\because u_f = v_f = 0)$$

since u and v are in opposite direction

$$e = \frac{v}{u} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}} = \left(\frac{h_1}{h}\right)^{1/2}$$

or $h_1 = e^2 h$

Like wise: $h_2 = e^2 h_1 = e^2 \times e^2 h = e^4 h$

$h_3 = e^2 h_2 = e^2 \times e^4 h = e^6 h$

substituting the given data:

$h_3 = 10$ m and $e = (0.5)^{1/3}$

$$h = \frac{h_3}{e^6} = \frac{10}{(0.5)^2} = 40 \text{ m}$$

Thus the ball must be dropped from a height of 40 m

02.

Sol: Let u_1 and v_1 = velocity of ball A before and after the impact.

u_2 and v_2 = velocity of ball B before and after the impact.

From the given data: $u_1 = \sqrt{2gh}$

$= 4.43\sqrt{h}$; $u_2 = 0$

The ball B, after the impact, should attain velocity v_2 just sufficient to rise to a height of 30 cm and leave the container.

$$v_2 = \sqrt{2g\times 0.3} = 2.43 \text{ m/s}$$

From the principle of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $0.1 \times 4.43\sqrt{h} + 0.025 \times 0$

$$= 0.1\, v_1 + 0.025 \times 2.43$$

or $0.443\sqrt{h} = 0.1v_1 + 0.0607$ ….. (i)

From the expression for the coefficient for restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 0.8 = \frac{2.43 - v_1}{4.43\sqrt{h}}$$

$$3.544\sqrt{h} = 2.43 - v_1$$

Multiplying both sides by 0.1, we get

$0.3544\sqrt{h} = 0.243 - 0.1\, v_1$ …. (ii)

Adding expression (i) and (ii)

$$(0.443 + 0.3544)\sqrt{h} = 0.243 + 0.0607$$

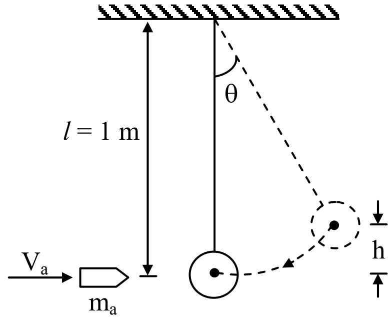
or $0.7974\sqrt{h} = 0.3037$

$$\sqrt{h} = \frac{0.3037}{0.7974} = 0.3809$$

$\therefore$ $h = (0.3809)^2 = 0.1451 \text{ m} = 14.51 \text{ cm}$

03.

Sol:



Above figure which shows the given system with the various parameters inserted.

Applying the principle of conservation of momentum, momentum before impact = momentum after impact

$$m_aV_a + m_bV_b = (m_a + m_b)V$$

where V is the common velocity with which the body and the bullet together after impact.

$$\frac{30}{1000}\times 450 + 10\times 0 = \left(\frac{30}{1000} + 10\right)V$$

or $13.5 = 10.03$ V

$\therefore$ $V = 1.346$ m/s

Applying the principle of conservation of energy, we have loss of kinetic energy = gain of potential energy

$$\frac{1}{2}(m_a + m_b)V^2 = (m_a + m_b)gh$$

Where h is the height to which the body rises

$$h = \frac{V^2}{2g} = \frac{1.346^2}{2\times 9.81} = 0.0923 \text{ m}$$

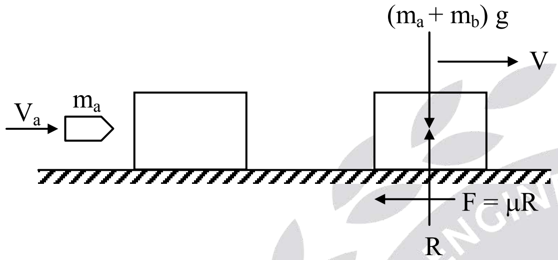
From the geometry of figure,

$$\cos\theta = \frac{\ell - h}{\ell} = \frac{1-0.0923}{1} = 0.9077$$

$$\theta = 24.81°$$

04.

Sol: Below figure which shows the block with various parameters inserted.



Applying the principle of conservation of momentum,

Momentum before impact = momentum after impact

$$m_aV_a + m_bV_b = (m_a + m_b)\,V$$

$$\frac{0.25}{9.81}\times 750 + \frac{40}{9.81}\times 0 = \frac{0.25+40}{9.81}\times V$$

$$19.11 = 4.103\,V$$

$$V = 4.65 \text{ m/s}$$

Normal reaction $R = (m_a + m_b)g$

$$= \frac{0.25+40}{g}\times g = 40.25 \text{ N}$$

Force of friction $F = \mu R = 0.35 \times 40.25$

$$= 14.087 \text{ N}$$

Applying the work – energy correlation,

Work done to overcome = kinetic energy lost by the block the frictional force with bullet embedded

$$F \times s = \frac{1}{2}(m_a + m_b)V^2$$

$$14.087 \times s = \frac{1}{2}\left(\frac{0.25+40}{9.81}\right)\times 4.657^2$$

$$= 43.632$$

$\therefore$ Displacement of block

$$s = \frac{43.632}{14.087} = 3.097 \text{ m}$$

05.

Sol: The figure for the various forces acting on the block.

Normal reaction $R = W\cos\theta$

$= (50 \times 9.81)\cos 30° = 424.78$ N

Force of friction $F = \mu R = 0.2 \times 424.78$

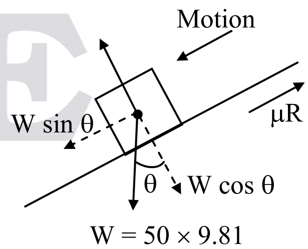
$= 84.96$ N

This frictional force acts in a direction opposite to that of motion

Then the net force on the block when it starts moving downwards the plane is

$= W\sin\theta - F = 50\times 9.81\times \sin 30° - 84.96$

$= 160.29$ N



If x is the deformation (compression) of the spring, then distance moved by the block

$= (1.5 + x)$

Work done by the block = 160.29×(1.5+x)

Work done by the spring = $-\frac{1}{2}kx^2$

$= -\frac{1}{2}\times(25\times10^3)x^2$

$= -12.5 \times 10^3x^2$

∴ Total work done by the system

$= 160.29 \times (1.5 + x) - 12.5 \times 10^3x$

Accordingly from the work-energy principle,

work done = change in kinetic energy

$160.29 \times (1.5 + x) - 12.5 \times 10^3x^2 = 0$

or $12.5x^2 - 0.16x - 0.24 = 0$

$$x = \frac{-(-0.16)\pm\sqrt{(-0.16)^2-4(12.5)(-0.24)}}{2\times12.5}$$

$$= \frac{0.16\pm3.47}{25} = 0.145 \text{ m or } -0.132 \text{ m}$$

The value x = 0.145 m is positive and hence acceptable.

(b) According to Newton's second law F = ma

$160.29 = 50 \times a$

$a = 3.21 \text{ m/s}^2$

From the kinematic relation $v^2 - u^2 = 2as$, we have

$v^2 = 2 \times 3.21 \times 1.5 \quad (\because u = 0)$

∴ Maximum velocity of block v

$= \sqrt{2\times3.21\times1.5} = 3.10$ m/s

(c) When the block moves up the plane due to rebound, both the component of weight of block along the plane and the frictional force act in the same direction.

Therefore force on the block = W sin θ + frictional force

$= 50 \times 9.81 \sin 30° + 84.96$

$= 330.21$ N

If s is the distance moved up the block, then work done by forces acting on the block

$= 330.21 \times s$

The work done equals the energy stored in the spring.

Accordingly $\frac{1}{2}kx^2 = 330.21\times s$

$\frac{1}{2}\times(25\times10^3)\times(0.145)^2 = 330.21 \times s$

$\Rightarrow s = 0.796$ m

Thus the distance of rebound of block on the plane is 0.796 m.

06.

Sol: The problem involves oblique impact. In order to seek a solution, we have established the x and y axes along the line of impact and the plane of contact, respectively, figure.

Resolving each of the initial velocities into x and y components, we have

$(\upsilon_{Ax})_1 = 3 \cos 30° = 2.60$ m/s

$(\upsilon_{Ay})_1 = 3 \sin 30° = 1.50$ m/s

$(\upsilon_{Bx})_1 = -1 \cos 45° = -0.707$ m/s

$(\upsilon_{By})_1 = -1 \sin 45° = -0.707$ m/s

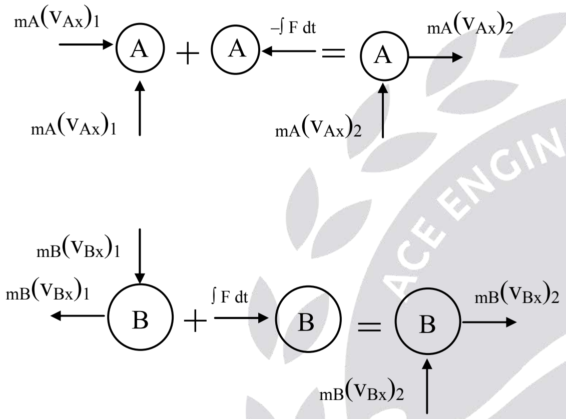
Conservation of "x" Momentum:

In reference to the momentum diagrams, we have

$$\left(\overset{+}{\rightarrow}\right) m_A(\upsilon_{Ax})_1 + m_B(\upsilon_{Bx})_1 = m_A(\upsilon_{Ax})_2 + m_B(\upsilon_{Bx})_2$$

1 kg (2.60 m/s) + 2kg(–0.707 m/s) = 1 kg $(\upsilon_{Ax})_2$ + 2kg$(\upsilon_{Bx})_2$

$(\upsilon_{Ax})_2 + 2(\upsilon_{Bx})_2 = 1.18$ …… (1)



Coefficient of Restitution (x).

Both disks are assumed to have components of velocity in the +x direction after collision, figure.

$$\left(\overset{+}{\rightarrow}\right) e = \frac{(\upsilon_{Bx})_2 - (\upsilon_{Ax})_2}{(\upsilon_{Ax})_1 - (\upsilon_{Bx})_1} = 0.75$$

$$= \frac{(\upsilon_{Bx})_2 - (\upsilon_{Ax})_2}{2.60\,m/s - (-0.707\,m/s)}$$

$(\upsilon_{Bx})_2 - (\upsilon_{Ax})_2 = 2.48$…….. (2)

Solving equations 1 and 2 $(\upsilon_{Ax})_2$ yields

$(\upsilon_{Ax})_2 = -1.26$ m/s = 1.26 m/s ←

$(\upsilon_{Bx})_2 = 1.22$ m/s →

Conservation of "y" Momentum:

The momentum of each disk is conserved in the y direction (plane of contact), since the disks are smooth and therefore no external impulse acts in this direction.

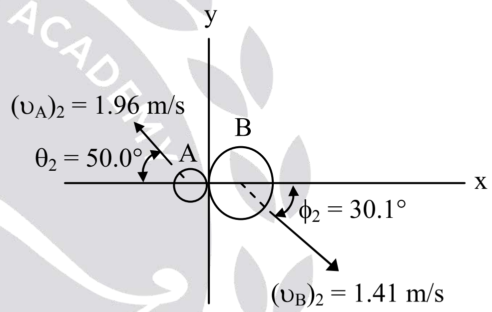
From figure (c)

$(+\uparrow)$ $m_A(\upsilon_{Ay})_1 = m_A(\upsilon_{Ay})_2$;

$(\upsilon_{Ay})_2 = 1.50$ m/s ↑

$(+\uparrow)$ $m_B(\upsilon_{By})_1 = m_B(\upsilon_{By})_2$;

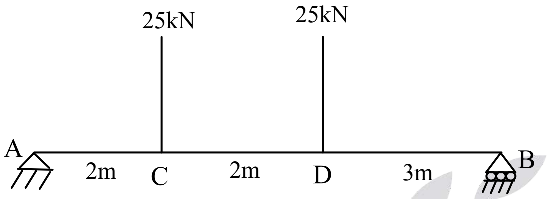
$(\upsilon_{By})_2 = 0.707$ m/s = 0.707 m/s ↓



Chapter 8 Virtual Work

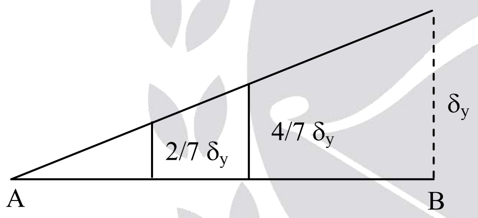
01.

Sol:



Let R_A & R_B be the reactions at support A & B respectively.

Let δ_y displacement be given to the beam at B without giving displacement at 'A'



The corresponding displacement at C & D are $\frac{2}{7}\delta_y$ and $\frac{4}{7}\delta_y$

By virtual work principle,

$$R_A\times0-25\times\frac{2}{7}\delta_y-25\times\frac{4}{7}\delta_y+R_B\times\delta_y=0$$

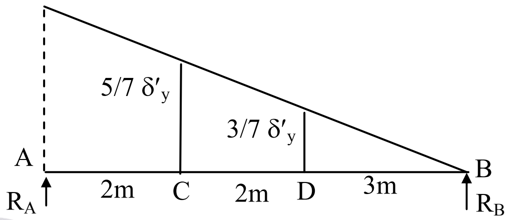
$$\Rightarrow\left(\frac{-150}{7}+R_B\right)\delta_y=0$$

Since $\delta_y\neq0$, $R_B-\frac{150}{7}=0$

$$R_B=\frac{150}{7}\text{ kN}$$

Now let us give virtual displacement at A as δ_y',

Therefore corresponding displacement at C & D are $\frac{5}{7}\delta_y'$ & $\frac{3}{7}\delta_y'$



$\therefore$ By virtual work principle,

$$R_A\times\delta'_y-25\times\frac{5}{7}\delta'_y-25\times\frac{3}{7}\delta'_y+R_B\times0=0$$

$$\left(R_A-\frac{125}{7}-\frac{75}{7}\right)\delta_y'=0$$

$\delta_y'\neq0$,

$$R_A-\frac{200}{7}=0$$

$$R_A=\frac{200}{7}\text{ kN}$$

02. Ans: 750 N

Ans: For equilibrium total virtual work = 0

Let us displace point A by 'dx' the displacement of point B is '3dx'

Work by force P = – Pdx

Work by force 250 N = 250 × 3 dx

250 × 3dx – Pdx = 0

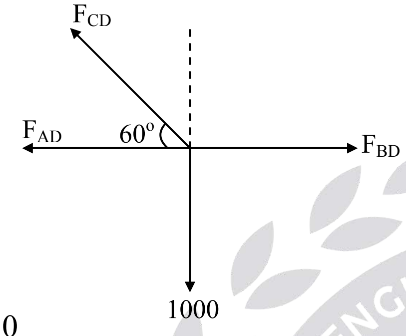
$\Rightarrow$ P = 750 N

Chapter 9

Analysis of Trusses

01. Ans: (b)

Sol: At joint



$\Sigma F_y = 0$

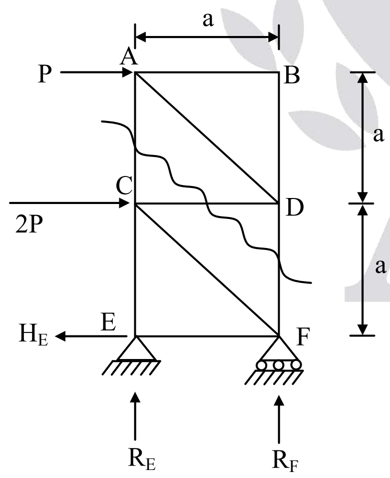
$F_{CD} \sin 60 = 1000$

$F_{CD} = \dfrac{1000}{\sin 60}$

$F_{CD} = 1154$ N

02. Ans: (d)

Sol:



$\Sigma F_x = 0$

$\Rightarrow H_E - P - 2P = 0$

$H_E = 3P$

$\Sigma F_y = 0$

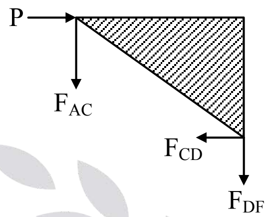
$R_E + R_F = 0$

$\Sigma M_F = 0$

$P \times 2a + 2P \times a + R_E \times a = 0$

$R_E = -4P$ (downward)

$R_F = 4P$ (upward)



$\Sigma F_x = 0$

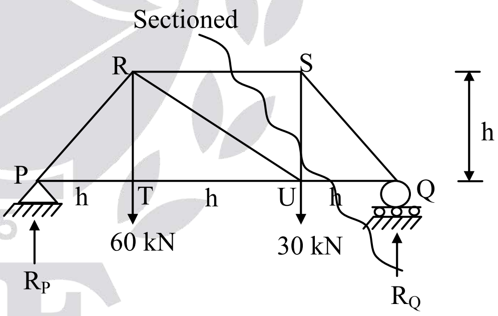
$P - F_{CD} = 0$

$P = F_{CD}$

(Positive indicate CD in tension)

03. Ans: (d)

Sol:

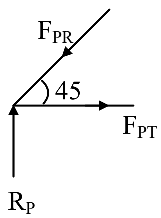


Taking moments about point 'P'

$R_Q \times 3h - 30 \times 2h - 60 \times h = 0$

$R_Q \times 3h = 120\ h$

$R_Q = 40$ kN



$\therefore R_P + R_Q = 60 + 30$

$R_P = 90 - 40$

$R_P = 50$ kN

At joint 'P'

$\sum F_y = 0$

$R_p = F_{PR}\sin 45^o$

$F_{PR} = \dfrac{R_p}{\sin 45}$

$= \dfrac{50}{1/\sqrt{2}}$

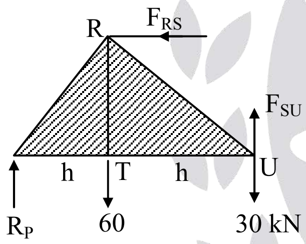
$F_{PR} = 50\sqrt{2}$ (compression)

$\sum F_x = 0$

$F_{PT} = F_{PR}\cos 45$

$F_{PT} = 50\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$F_{PT} = 50$ kN (Tension)



$\sum M_u = 0$

$F_{RS} \times h\,(\circlearrowleft) + 60 \times h\,(\circlearrowleft) - R_P \times 2h(\circlearrowright) = 0$

$F_{RS} \times h + 60\,h - 100\,h = 0$

$F_{RS}\,h = 40\,h$

$F_{RS} = 40$ kN (Compression)

$\sum F_y = 0$

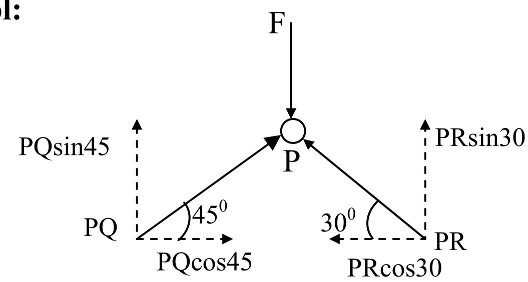
$F_{SU} + R_P - 60 = 0$

$F_{SU} + 50 - 60 - 30 = 0$

$F_{SU} = 40$ kN (Tension)

04. Ans: (b)

Sol:



Force in member PQ considering joint **P**

PQ cos45 = PR cos30

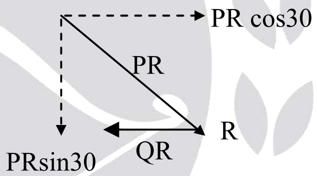
PQ = 1.224 PR

PQ sin45 + PR sin30 = F

$1.224PR \times 0.707 + 0.5PR = F$

PR = 0.732 F

Now, considering joint **R**



QR = PR cos30 = $0.732F \times \cos 30$

= 0.63F (Tensile)

05. Ans: (a)

Sol: $\sum F_y = 0 \Rightarrow R_A + R_B = P \times L$

$\sum M_B = 0 \Rightarrow R_A \times 3L = PL \times \dfrac{3L}{2}$

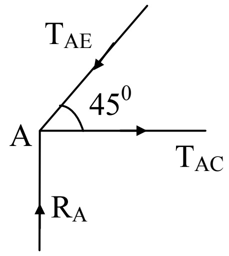
$\Rightarrow R_A = \dfrac{PL}{2}, \; R_B = \dfrac{PL}{2}$

FBD at Point A:

$\sum F_y = 0$

$\Rightarrow T_{AE}\sin 45 = R_A = \dfrac{PL}{2}$

$\Rightarrow T_A = \dfrac{PL}{\sqrt{2}}$



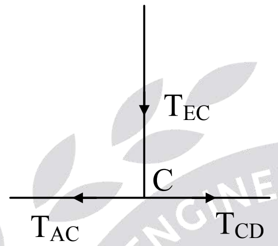
$$\sum F_x = 0 \Rightarrow T_{AC} = T_{AE}\cos 45 = \frac{PL}{2}$$

FBD at Point C:

$\sum F_y = 0$

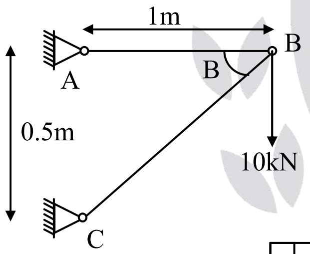
$\Rightarrow T_{EC} = 0$

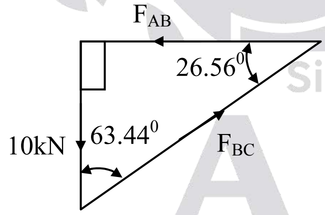
$T_{AC} = T_{CD} = \dfrac{PL}{2}$



06. Ans : 20 kN

Sol:





$$\tan\theta = \frac{0.5}{1.0} \Rightarrow \theta = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.56^0$$

From the Lami's triangle

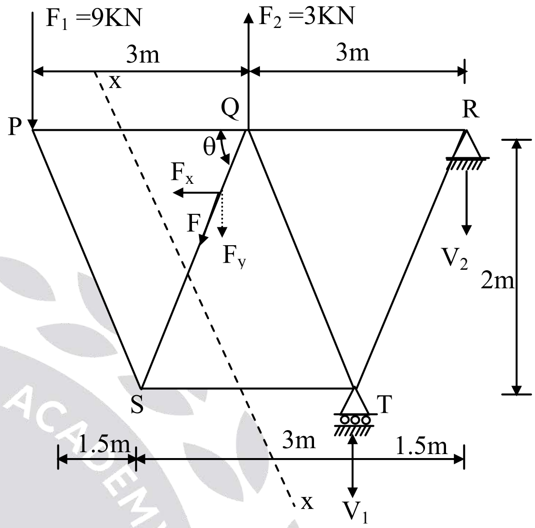
$$\frac{10}{\sin 26.56^0} = \frac{F_{BC}}{\sin 90^0} = \frac{F_{AB}}{\sin 63.44^0}$$

$$F_{AB} = \frac{10}{\sin 26.56}\times \sin 63.44 = 20\,kN$$

$$F_{BC} = \frac{10}{\sin 26.56}\times \sin 90 = 22.36kN$$

07. Ans: (a)

Sol:



$\Sigma F_y = 0$

$V_1 + V_2 - 9 + 3 = 0$

$\Sigma M_R = 0$

$\Rightarrow V_1 \times 1.5 + 3\times 3 - 9\times 6 = 0$

$\Rightarrow V_1 = 30$ kN (↑)

$V_2 = -30 + 9 - 3 = -24$ kN (↓)

Adopting method of sections–section x-x adopted and RHS taken

$$\theta = \tan^{-1}\left(\frac{2.0}{1.5}\right) = 53.13^0$$

$\Sigma F_y = 0$ (W.r.t. RHS of the section x-x)

$V_1 + F_2 - V_2 - F_y = 0$

$\Rightarrow F\sin 53.13 = 30 + 3 - 24$

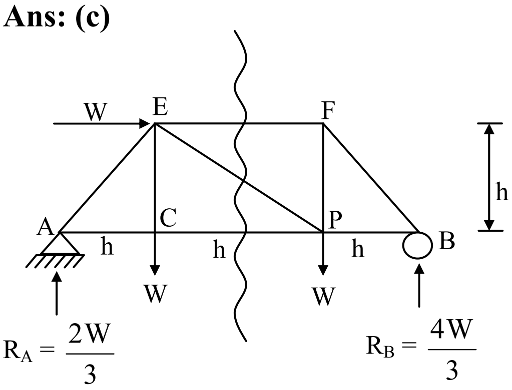
$F = 11.25$ kN (Tension)

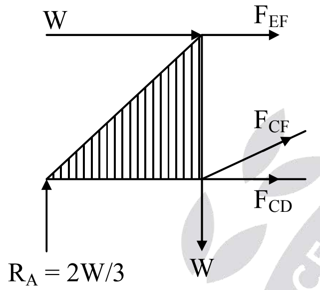
∴ Force in member

QS = 11.25 kN (Tension)

08. Ans: (c)

Sol:





$\sum M_B = 0$

W ×h (↻) –W×h (↺) –W(2h)(↺) +R_A×3h(↻) = 0

$Wh - Wh - 2Wh + 3hR_A = 0$

$3hR_A = 2Wh$

$R_A = \frac{2W}{3}$

$\therefore R_A + R_B = 2W$

$R_B = 2W - \frac{2W}{3} = \frac{4W}{3}$

$\sum F_y = 0$ (at the joint C)

$F_{CF} \sin 45 - W + R_A = 0$

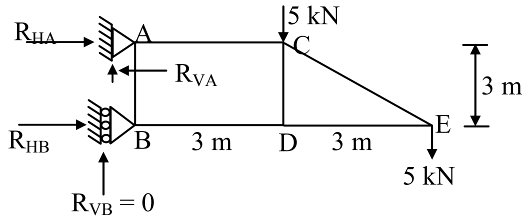
$F_{CF} \sin 45 - W + \frac{2W}{3} = 0$

$F_{CF} \times \frac{1}{\sqrt{2}} = \frac{W}{3}$

$\Rightarrow F_{CF} = \frac{W\sqrt{2}}{3}$

09. Ans: (c)

Sol:



$\sum M_A = 0$

5×3 (↻) +5 ×6 (↻) – R_{HB} × 3 = 0

$15 + 30 = R_H \times 3$

$R_{HB} = \frac{45}{3}$

$R_{HB} = 15$ kN

$\sum F_X = 0$

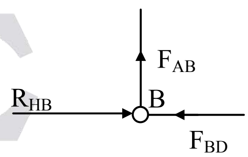
$\therefore R_{HA} + R_{HB} = 0$

$R_{HA} = -R_{HB}$

$R_{HA} = -15$ kN

(Negative indicate R_{HA} is left side)

At joint 'B'



$\sum F_x = 0$

$F_{BD} = 15$ kN

$\sum F_y = 0$

$F_{AB} = 0$