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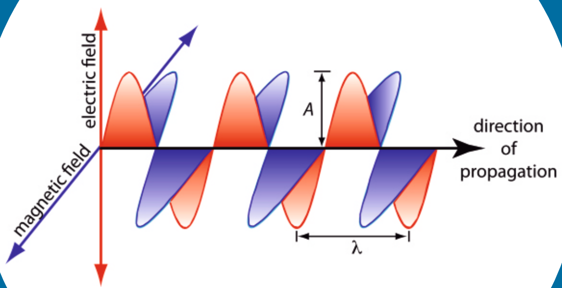


ELECTRONICS & TELECOMMUNICATION ENGINEERING

ELECTROMAGNETICS

Text Book : Theory with worked out Examples and Practice Questions

Chapter 1 Static Fields
(Solutions for Text Book Practice Questions)

Objective Practice Solutions

01. Ans: 1

Sol: $\vec{V} = x\cos^2 y\hat{i} + x^2e^z\hat{j} + z\sin^2 y\hat{k}$

$$= x\cos^2 y\hat{a}_x + x^2e^z\hat{a}_y + z\sin^2 y\hat{a}_z$$

From divergence theorem

$$\oiint \overline{V}.\hat{n}\,ds = \int_v (\nabla.\overline{D})dv \text{ ....1}$$

$$\nabla.\overline{D} = \frac{\partial}{\partial x}(x\cos^2 y) + \frac{\partial}{\partial y}(x^2e^z) + \frac{\partial}{\partial z}(z\sin^2 y)$$

$$= \cos^2 y + \sin^2 y = 1$$

dv = dxdydz

Putting these value in equation 1 we have

$$\oiint \overline{V}.\hat{n}\,ds = \int_0^1\int_0^1\int_0^1 1\times dx\,dy\,dz$$

$$= \int_0^1 dx \int_0^1 dy \int_0^1 dz = 1$$

02. Ans: (c)

Sol: Given $\vec{A} = xy\vec{a}_x + x^2\vec{a}_y$

Let $I = \oint \vec{A}.d\vec{\ell}$, I is evaluated over the path shown in the Fig., as follows

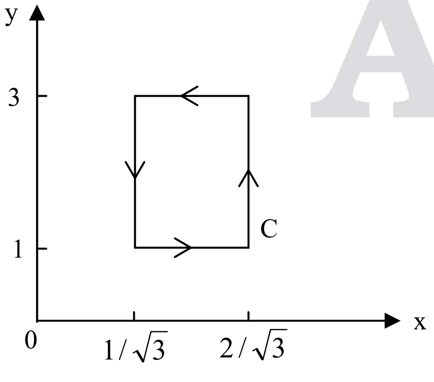


Fig.

$$I = \oint \vec{A}.dx\,\vec{a}_x,\ y = 1,\ x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$+\int \vec{A}.dy\,\vec{a}_y,\ x = \frac{2}{\sqrt{3}},\ y = \text{from 1 to 3}$$

$$-\int \vec{A}.dx\,\vec{a}_x,\ y = 3, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$-\int \vec{A}.dy\,\vec{a}_y,\ x = 1/\sqrt{3},\ y = \text{from 1 to 3}$$

$$= \int xy\,dx + \int x^2\,dy - \int xy\,dx - \int x^2\,dy$$

$$= y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2 y\Big|_1^3 - y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2 y\Big|_1^3$$

at $y = 1 \quad x = 2/\sqrt{3} \quad y = 3 \quad x = 1/\sqrt{3}$

$$= \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) + \frac{4}{3}(3-1) - \frac{3}{2}\left(\frac{4}{3} - \frac{1}{3}\right) - \frac{1}{3}(3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\overline{F} = \rho a_\rho + \rho\sin^2\phi\, a_\phi - z a_z$

$$= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$$

$$\nabla.\overline{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(F_\phi) + \frac{\partial}{\partial z}(F_Z)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho\sin^2\phi) + \frac{\partial}{\partial z}(-z)$$

$$= 2 + 2\sin\phi\cos\phi - 1$$

$$= 1 + 2\sin\phi\cos\phi$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2,\ \nabla.F\big|_{\phi=0} = 1$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2\nabla.F\big|_{\phi=0}$$

04. Ans: (c)

Sol: $\overline{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_Z \qquad \overline{D} = |\overline{D}|\overline{a}_n$

$|\overline{D}| = \sqrt{16} = 4 \qquad = \rho_s\hat{a}_n$

$$\therefore \overline{D} = 4\left\{\frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_Z}{4}\right\}$$

$$= \rho_s \hat{a}_n \qquad \therefore \rho_s = 4\,C/m^2$$

05. Ans: (d)

Sol: $V = 10y^4 + 20x^3$

$$E = -\nabla V = -60x^2\hat{a}_x - 40y^3\hat{a}_y$$

$$D = \varepsilon_0 E = -60x^2\varepsilon_0\hat{a}_x - 40y^3\varepsilon_0\hat{a}_y$$

$$\nabla . D = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x}(-60x^2\varepsilon_0) + \frac{\partial}{\partial y}(-40y^3\varepsilon_0)$$

$$= -120\, x\varepsilon_0 - 120\, y^2\varepsilon_0$$

$$\rho_v(\text{at } 2, 0) = -120\times 2\varepsilon_0 - 120\times 0^2\,\varepsilon_0$$

$$= -240\,\varepsilon_0$$

06. Ans: (d)

Sol: Given

$$V(x, y, z) = 50\,x^2 + 50\,y^2 + 50\,z^2$$

$\vec{E}\,(x, y, z)$ in free space $= -\text{grad}\,(V)$

$$= -\nabla V$$

$$= -\left[\frac{\partial}{\partial x}V\vec{a_x} + \frac{\partial}{\partial y}V\vec{a_y} + \frac{\partial}{\partial z}V\vec{a_z}\right]$$

$$= -\left[100x\,\vec{a_x} + 100y\,\vec{a_y} + 100z\,\vec{a_z}\right]V/m$$

$$\vec{E}\,(1, -1, 1) =$$

$$-\left[100\,\vec{a_x} - 100\,\vec{a_y} + 100\,\vec{a_z}\right]V/m$$

$$E(1, -1, 1) = 100\sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of $\vec{E}$.

$$\vec{a}_E = \frac{\vec{E}(1, -1, 1)}{\left|E(1, -1, 1)\right|} = \frac{1}{\sqrt{3}}\left[-\vec{a_x} + \vec{a_y} - \vec{a_z}\right]$$

or in i, j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}}\left[-i + j - k\right]$

07. Ans: (b)

Sol: For valid B, $\nabla.B = 0$

$$\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)\left(x^2a_x - xya_y - Kxza_z\right) = 0$$

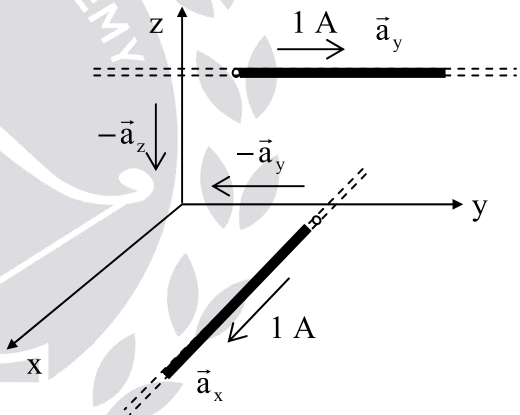
$$2x - x - Kx = 0$$

$$\Rightarrow 2 - 1 - K = 0$$

$$\therefore K = 1$$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the $\vec{a}_y$ direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the $\vec{a}_x$ direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors along the 'x', 'y' and 'z' axes respectively.

$\therefore$ x and z components of magnetic field are non-zero at the origin.

09. Ans: (a)

Sol: $\nabla.\overline{B} = 0$

A divergence less vector may be a curl of some other vector

$\overline{B} = \nabla \times \overline{A}$

$\nabla \times \overline{A} = \overline{B}$

$\oint_l \overline{A}\ .\overline{dl} = \int_s \overline{B}\,.\,\overline{ds}$

$\int_s \overline{B}\,.\,\overline{ds}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

Sol: In general, for an infinite sheet of current density K A/m

$H = \frac{1}{2} K \times a_n$

$H = \frac{1}{2}\left(8\overline{a}_x \times \overline{a}_z\right)$

$= -4\ \overline{a}_y \quad \left(\because \overline{a}_x \times \overline{a}_z = -\overline{a}_y\right)$

11. Ans: (b)

Sol:

$\varepsilon_r = 1 \qquad \overline{E}_2 = a_x$

$\varepsilon_r = 2 \qquad \overline{E}_1 = 2a_x$

$D_{n_2} - D_{n_1} = \rho_S \rightarrow$ (a)

$D_{n_2} = \in E_{n_2} = \in_0 a_x$

$D_{n_1} = \in_0 2\times 2a_x = 4\in_0 a_x$

From (a)

$(\in_0 - 4\in_0)\, a_x = \rho_s \Rightarrow \rho_s = -3\in_0$

12. Ans: (a)

Sol:

$\mu_{r_1} = 2 \qquad \mu_{r_2} = 1$

$z = 0$

$B_1 = 1.2\overline{a}_x + 0.8\overline{a}_y + 0.4\overline{a}_z$

$B_{n_1} = 0.4\ \overline{a}_z$

(Since z = 0 has normal component a_x)

$B_{t_1} = 1.2\ \overline{a}_x + 0.8\ \overline{a}_y$

We know magnetic flux density is continuous

$B_{n_1} = B_{n_2}$

$B_{n_2} = 0.4\ \overline{a}_z$

Surface charge, $\overline{k} = 0$

$H_{t_2} - H_{t_1} = 0$

$H_{t_2} = H_{t_1}$

$\mu_1 B_{t_2} = \mu_2 B_{t_1}$

$B_{t_2} = \frac{1}{2}(1.2a_x + 0.8a_y)$

$B_2 = B_{t_2} + B_{n_2}$

$= 0.6\ \overline{a}_x + 0.\overline{4}a_y + 0.4\overline{a}_z$

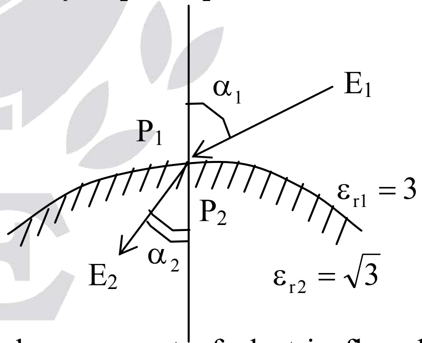
$\mu_0\mu_{r_2} H_2 = 0.6\ \overline{a}_x + 0.\overline{4}a_y + 0.4\overline{a}_z$

$H_2 = \frac{1}{\mu_0}[0.6\ \overline{a}_x + 0.\overline{4}a_y + 0.4\overline{a}_z]$ A/m

13. Ans: (b)

Sol: Tangential components of electric fields are continuous $\left(E_{t_1} = E_{t_2}\right)$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$ ----(1)



Normal component of electric flux densities are continuous across a charge free interface

$D_{n_1} = D_{n_2}$

$3E_1\cos\alpha_1 = \sqrt{3}E_2\cos\alpha_2$ ----(2)

$\alpha_1 = 60^0$

$\frac{(1)}{(2)} \Rightarrow \frac{\tan\alpha_1}{3} = \frac{\tan\alpha_2}{\sqrt{3}} \Rightarrow \tan\ \alpha_2 = 1$

$\alpha_2 = 45^0$

14. Ans: (b)

Sol: $E = -\nabla V$

Statement (I) is true

Statement (II) is true

But Statement (II) is not the correct explanation of Statement (I).

15. Ans: (a)

Sol: $\psi = Q_{enc}$ (from Gauss's law)

Statement (I) is true.

$\psi = Q_{enc} = \oint_s \overline{D}.d\overline{s}$

Statement (II) is true.

Statement (II) is true and correct explanation of Statement (I).

16. Ans: (b)

Sol: Statement (I) is true

W= Force × displacement

Statement (II) is true

$E = \underset{Q_t \to 0}{Lt} \frac{F}{Q_t}$

Statement (II) is true but not the correct explanation of Statement (I).

17. Ans: (c)

Sol: $\nabla^2 V = -\frac{\rho_V}{\varepsilon}$ (Poisson's equation)

For charge free region, $\rho_V = 0$

$\nabla^2 V = 0$ (Laplace's equation)

∴ Laplace's equation is a special case of Poisson's equation. So, statement (I) is true.

In case of charge free region Poisson's equation becomes Laplace's equation. So, statement (II) is false.

18. Ans: (d)

Sol: $E_{tan} = 0$

On the surface of conductor, tangential components of E-field does not exists

Statement (I) is false.

For a conductor to dielectric interface, normal components of electric flux densities are equal to surface change densities.

$$\overline{D} = \rho_s \hat{a}_n$$

Statement (II) is true

19. Ans: (c)

Sol: $H_{1t} - H_{2t} = \hat{a}_n \times \overline{K}$

Statement (I) is true.

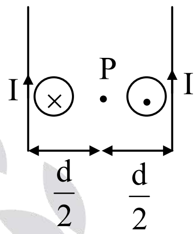
$B_{1n} = B_{2n}$

Statement (II) is false.

20. Ans: (d)

Sol: Magnetic field is always tangential to the conductor.

Statement (I) is false.



$\overline{H}_p = \overline{H}_1 + \overline{H}_2$

Here, $\overline{H}_1 = -\overline{H}_2$

$\Rightarrow \overline{H}_p = 0$

Statement (II) is true.

21. Ans: (b)

Sol: In static fields E and H are independent

Statement (I) is true.

$\nabla \times \overline{E} = 0$

$\nabla \times H = \overline{J}$

In time varying fields E & H are depends on each other

$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$

$\nabla \times \overline{H} = J + \frac{\partial \overline{D}}{\partial t}$

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

22. Ans: (d)

Sol: The solution of Poisson's equation and solution of Laplac's equation are not same.

Statement (I) is false

Statement (II) is true.

Chapter 2 Maxwell Equations & EM Waves

Identify polarization of following (Page number 75 in Volume-I booklet)

01. $\overline{E} = 20\sin(\omega t - \beta x)\hat{a}_y$ V/m

Sol: At x = 0

$\overline{E} = 20\sin(\omega t)\hat{a}_y$ V/m

Let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 0$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 20\hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = 0$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -20\hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = 0$

i.e., linear polarization and also vertical polarization with respect to $\hat{x}$ – axis

02. $\overline{H} = 45\cos(\omega t - \beta z)\hat{a}_x$ A/m

Sol: This is linear polarization

03. $\overline{E} = 20\sin(\omega t - \beta z)\hat{a}_x + 30\sin(\omega t - \beta z)\hat{a}_y$

Sol: phase difference between $\hat{a}_x$ component and $\hat{a}_y$ component is 0°

So that it is linear polarization

Note: For phase difference 0° & 180°, irrespective of their amplitudes it must be in linear polarization.

04. $\overline{E} = 55\cos(\omega t - \beta z)\hat{a}_x + 55\sin(\omega t - \beta z)\hat{a}_y$

Sol: Phase difference between $\hat{a}_x$ component and $\hat{a}_y$ component is $\frac{\pi}{2}$

Amplitudes are same.

So it is circular polarization

at z = 0 and let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 55\hat{a}_x + 0\hat{a}_y$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_x + 55\hat{a}_y$

It is CCW direction i.e. RHCP

05. $\overline{E} = 40\sin(\omega t - \beta y)\hat{a}_x + 50\cos(\omega t - \beta y)\hat{a}_z$

Sol: Phase difference = $\frac{\pi}{2}$

Amplitudes = not same

So it is elliptical polarization. To decide direction of rotation follow below procedure.

At y = 0, and Let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 0\hat{a}_x + 50\hat{a}_z$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 40\hat{a}_x + 0\hat{a}_z$

$\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x - 50\hat{a}_z$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -40\hat{a}_x + 0\hat{a}_z$

It is Anti clock wise direction i.e., Right Hand Elliptical Polarization.

06.

Sol: $\overline{E} = \text{Re}\left\{\left[\hat{a}_x + j\hat{a}_y\right]e^{j(\omega t - \beta Z)}\right\}$

$$\overline{E} = \text{Re}\begin{bmatrix}(\cos(\omega t - \beta z) + j\sin(\omega t - \beta z))\hat{a}_x + \\ j(\cos(\omega t - \beta z) + j^2\sin(\omega t - \beta z)\hat{a}_y)\end{bmatrix}$$

$\overline{E} = (\cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y)$

Magnitudes of amplitudes are same, phase difference is $\frac{\pi}{2}$; So it is circular polarization. Now we proceed to decide direction of rotation.

Here

$\overline{E} = \cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y$

At z = 0 & let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = \hat{a}_x - 0\hat{a}_y$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_x - \hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = -\hat{a}_x + 0\hat{a}_y$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_x - \hat{a}_y$

i.e., we get clock wise rotation i.e., Left Hand Circular Polarization

07. not a valid EM wave representation

08.

Sol: $\overline{E} = 5\cos(\omega t - \beta r)\hat{a}_\theta$

Let r = 0 & θ = ωt

at $\theta = 0 \Rightarrow \overline{E} = 5\hat{a}_\theta$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_\theta$

$\theta = \pi \Rightarrow \overline{E} = -5\hat{a}_\theta$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_\theta$

i.e., linear polarization

09.

Sol: $\overline{E} = \text{Im}\{[\hat{a}_x + 2j\hat{a}_z]e^{j(\omega t - \beta y)}\}$

$= \text{Im}\begin{Bmatrix} [\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_x + \\ 2j[\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_z \end{Bmatrix}$

$= \sin(\omega t - \beta y)\hat{a}_x + 2\cos(\omega t - \beta y)\ \hat{a}_z$

Let y = 0 & θ = ωt

$\theta = 0 \Rightarrow \overline{E} = 0\hat{a}_x + 2\hat{a}_z$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = \hat{a}_x + 0\hat{a}_z$

$\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x - 2\hat{a}_z$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -\hat{a}_x + 0\hat{a}_z$

So it is Right Hand Elliptical Polarization

10. $\overline{E} = 20\sin(\omega t - \beta y)\hat{a}_x + 30\sin(\omega t - \beta y + 45^\circ)\hat{a}_z$

Sol: let y = 0 & θ = ωt

At θ = 0

$\Rightarrow \overline{E} = 0\hat{a}_x + 30\sin 45^\circ\hat{a}_z$

$= 0\hat{a}_x + \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 20\hat{a}_x + 30\sin(135^\circ)\hat{a}_z$

$= 20\hat{a}_x + \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x + 30\sin(225^\circ)\hat{a}_z$

$= 0\hat{a}_x - \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -20\hat{a}_x + 30\sin(315^\circ)\hat{a}_z$

$= -20\hat{a}_x - \frac{30}{\sqrt{2}}\hat{a}_z$

Note: θ = 62.76° is the maximum values direction obtained by

$\frac{d\overline{E}}{d\theta} = 0$ at y = 0 & ωt = θ

at $\theta = -\frac{\pi}{4} \Rightarrow \overline{E} = \frac{-20}{\sqrt{2}}\hat{a}_x + 0\hat{a}_z$

at $\theta = \frac{\pi}{4} \Rightarrow \overline{E} = \frac{20}{\sqrt{2}}\hat{a}_x + 30\hat{a}_z$

So it is RHEP

11. $\overline{E} = 20\sin(\omega t - \beta z)\hat{a}_x + 20\sin(\omega t - \beta z + 45^\circ)\hat{a}_y$

Sol: Valid EM wave but polarization can not defined.

This is a valid EM wave representation but it is not satisfy anyone of the polarization principle

Objective Practice Solutions

01. Ans: (c)

Sol: Given flux $\phi = (t^3 - 2t)$mWb

Magnitude of inducted emf $|e'| = \left|\frac{d\phi}{dt}\right|_{t=4\,sec}$

$$|e'| = 3t^2 - 2\Big|_{t=4\,sec}$$

$$= 3(4)^2 - 2$$

$$= 46\text{mWb}$$

This 'e' for one turn; but for 100 turns

$$|e| = N|e'| = 100 \times 46\text{mWb}$$

$$|e| = 4.6 \text{ volts}$$

02. Ans: (d)

Sol: Given,

$$E = 120\,\pi \cos(10^6\,\pi\, t - \beta x)\,\hat{a}_y \text{ V/m}$$

$$H = A\cos(10^6\,\pi\, t - \beta x)\,\hat{a}_z \text{ A/m}$$

$\varepsilon_r = 8$; $\mu_r = 2$

We know that, $\frac{E_y}{H_z} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$H_z = \frac{E_y}{120\pi\sqrt{\frac{2}{8}}} = \frac{2E_y}{120\pi} = 2\text{A/m}$$

$$H_z = 2\cos(10^6\,\pi\, t - \beta x)\,\hat{a}_z \text{ A/m}$$

$$\therefore A = 2$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{10^6\pi \times \sqrt{2\times 8}}{3\times 10^8}$$

$$= 0.0418\text{rad/m}$$

03. Ans: (b)

Sol: This question relates to normal incidence of a UPW on the air (medium 1) to glass (medium 2) interface as shown in Fig.

Medium, 1 Ai	Medium, 2 Glass slab
$n_1 = 1$	$n_2 = 1.5$
$\mu_1 = \mu_0$	$\mu_2 = \mu_0$
$\epsilon_1 = \epsilon_0$	$\epsilon_2 = \epsilon_0\,\epsilon_r$

Fig.

If n_1 and n_2 are the refractive indices and v_1 and v_2 are the velocities

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\sqrt{\mu_1\,\epsilon_1}}{\sqrt{\mu_2\,\epsilon_2}}$$

$$= \sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad \text{for } \mu_1 = \mu_2 = \mu_0$$

For $n_1 = 1$, $n_2 = 1.5$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{1.5} = \frac{2}{3}$$

Reflection coefficient,

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1} = -\frac{1}{5}$$

$$\therefore \frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{25} = 4\%$$

04. Ans: (d)

Sol: Normal incidence is shown in Fig.

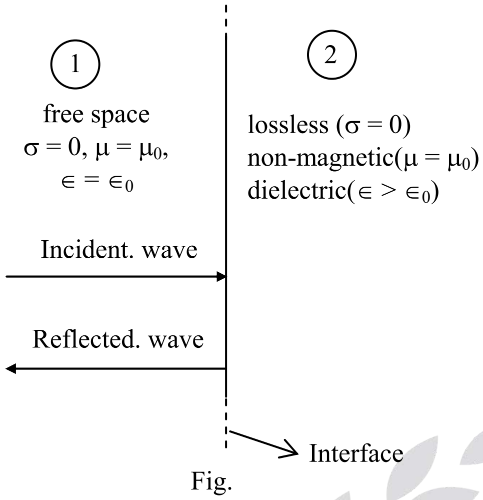


Fig.

Given: $E_{max} = 5\ E_{min}$ in medium 1.

$\therefore$ VSWR, $S = \dfrac{E_{max}}{E_{min}} = 5$

$$|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

Reflection coefficient,

$$K = \frac{E_r}{E_i} = \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1} = \frac{-2}{3}$$

$$-3\frac{\eta_2}{\eta_1} + 3 = 2\frac{\eta_2}{\eta_1} + 2$$

$$\therefore \frac{\eta_2}{\eta_1} = \frac{1}{5}, \quad \eta_2 = \frac{1}{5}\eta_1$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \sqrt{4\pi\times10^{-7}\times36\pi\times10^{9}}$$

$$= (120\pi)\ \Omega$$

$\therefore$ Intrinsic impedance of the dielectric medium, $\eta_2 = \dfrac{1}{5}\times120\pi = 24\pi$

05. Ans: (a)

Sol: Given:

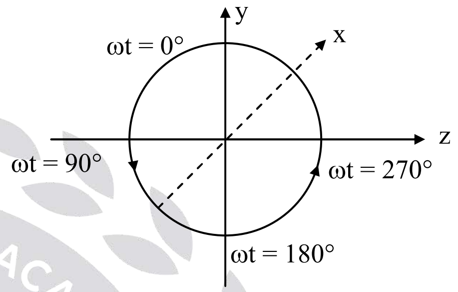
$\vec{E} = 10(\hat{a}_y + j\hat{a}_z)\ e^{-j25x}$ in free space.

$\vec{E} = (E_y\,\vec{a}_y + E_z\,\vec{a}_z)\ e^{-j\beta x}$

$\beta = 25 = \dfrac{\omega}{c} \Rightarrow$

$\omega = 25\ c = 25\times3\times10^{8}$ rad/s

$f = 1.19$ GHz ≈ 1.2 GHz



$E_y = 10$, $E_z = j\ 10$

E_z leads E_y by 90°

At x = 0

Let $E_y = 10\cos(\omega t)$

then $E_z = 10\cos(\omega t + 90°)$

A Left Hand screw is to be turned in the direction along the circle as time increases so that the screw moves in the direction of propagation, 'x'.

$\therefore$ The wave is left circularly polarized.

06. Ans: (b)

Sol: $\overline{H} = 0.2\cos(\omega t - \beta x)\,\hat{a}_z$

Wave is progressing along + X direction $\to (+X)$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

$$\therefore \overline{E} = 0.2\eta\cos(\omega t - \beta x)\,\hat{a}_y$$

$$\overline{E}_s = 0.2\eta e^{-j\beta x}\,\hat{a}_y \qquad \overline{H}_s = 0.2e^{-j\beta x}\,\hat{a}_z$$

$$\overline{P}_{avg} = \frac{1}{2}\overline{E}_s\times\overline{H}_s^*$$

$$= \frac{1}{2}(0.2)^2\,\eta\,\hat{a}_x$$

$$= \frac{1}{2}(0.2)^2 (120\pi)\,\hat{a}_x \text{ w/m}^2$$

x = 1 plane $\Rightarrow \overline{ds} = dydz\hat{a}_x$

$$W_{avg} = \int_S \overline{P}_{avg}.\overline{ds} \text{ watts}$$

$$= \frac{1}{2}(0.2)^2(120\pi)\iint dydz$$

$$= \left[\frac{1}{2}\left((0.2)^2(120\pi)\right)\right]\left[\pi(5)^2\right]\times10^{-4}$$

= 0.0592 Watts

= 59.2 mW $\simeq$ 60 mW

07. Ans: (a)

Sol: $P \propto \frac{1}{r^2}$

$$\frac{P_Q}{P_p} = \frac{r_p^2}{r_Q^2} = \frac{(R)^2}{\left(\frac{R}{2}\right)^2}$$

$$\frac{P_Q}{P_P} = \frac{4}{1} = 4:1$$

08. Ans: (b)

Sol: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$

$$\delta \,\alpha\, \sqrt{\frac{1}{f}} \Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{1.5}{\delta} = \sqrt{\frac{8\times10^9}{2\times10^9}}$$

$$\delta = \frac{1.5}{2} = 0.75\ \mu\text{m}$$

Similarly

$$\frac{1.5}{\delta} = \sqrt{\frac{18\times10^9}{2\times10^9}} = 3$$

$$\delta = \frac{1.5}{3} = 0.5\mu\text{m}$$

09. Ans: (b)

Sol: $\frac{\sigma}{\omega\varepsilon} = \frac{5}{2\times\pi\times25\times10^3\times80\times8.854\times10^{-12}}$

$= 44938.7$

Since $\frac{\sigma}{\omega\varepsilon} >> 1$ hence sea water is a good conductor

Where attenuation is 90%, transmission is 10%, then $e^{-\alpha x} = 0.1$

Where α is attenuation constant

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2\times\pi\times25\times10^3\times4\pi\times10^{-7}\times5}{2}}$$

$\alpha = 0.7025$

$-\alpha x = ln(0.1)$

$-0.7025x = -2.3$

$x = 3.27$m

10. Ans: (b)

Sol: $\delta = \frac{1}{\alpha} = \frac{1}{2\pi} = 0.159$

11. Ans: (c)

Sol: E is minimum

H is maximum

i.e., 'c' is the option

$E_{Tan_1} = E_{Tan_2} = 0$

[perfect conductor $E_{Tan_2} = 0$]

$H_{Tan_1} = J_S \times a_n + H_{Tan_2}$

$H_{Tan_1} = J_S \times a_n$

[perfect conductor $H_{Tan_2} = 0$]

12. Ans: (d)

Sol: $\vec{H} = 0.5e^{-0.1x}\cos(10^6t - 2x)\,\hat{a}_z$ A/m $\rightarrow$ (+X)

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

Wave frequency = 10^6 radians/s

Phase constant β = 2 rad/m

$\beta = \frac{2\pi}{\lambda} = 2 \text{ rad/m}$

$\lambda = \pi = 3.14$m.

The wave is traveling along +X direction, Given wave is polarized along Y.

$\because$ It has Y-component of electric field

13. Ans: (a)

Sol: The normal incidence of a plane wave traveling in positive y – direction is shown at the interface y = 0 in Fig.

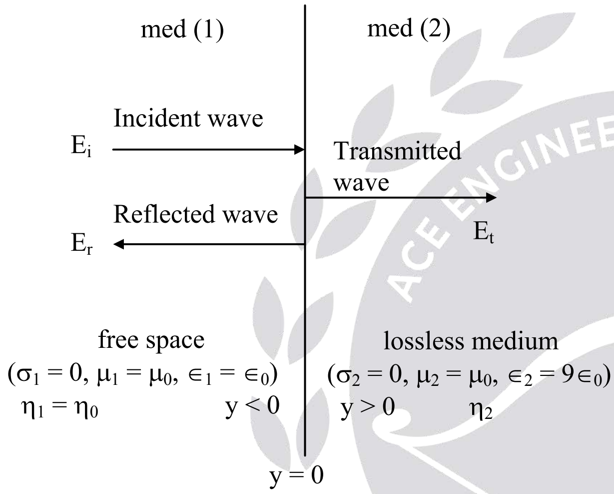


Fig.

Given: $\vec{E}_i = E_{iz}\,\vec{a}_z$

where $E_{iz} = 24\cos(3\times10^8 t - \beta y)$ V/m

$\omega = 3\times10^8$ rad/s, $\beta = \frac{\omega}{v}$,

For free space, $v = v_0 = 3\times10^8$ m/s

$\therefore \beta = 1$ rad/m

$$\eta_1 = \eta_0 = \frac{E_{iz}}{H_{ix}}$$

$$\therefore H_{ix} = \frac{E_{iz}}{\eta_0} = \frac{24\cos(3\times10^8 t - \beta y)}{120\pi}$$

$$\vec{H}_i = H_{ix}\,\vec{a}_x$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\frac{\eta_1}{\eta_2} - 1}{\frac{\eta_1}{\eta_2} + 1},$$

Where $\frac{\eta_1}{\eta_2} = \frac{\sqrt{\mu_1\epsilon_2}}{\sqrt{\epsilon_1\mu_2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{9\epsilon_0}{\epsilon_0}} = 3$

$$\therefore \frac{H_r}{H_i} = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\therefore \vec{H}_r = \frac{1}{2}\,\frac{24}{120\pi}\cos(3\times10^8 t + 1y)\,\vec{a}_x$$

$$= \frac{1}{10\pi}\cos(3\times10^8 t + 1y)\,\vec{a}_x \quad \text{A/m}$$

Note that $\vec{H}_r$ is reflected wave which travels in negative y direction, which corresponds to + βy term with β = 1 in the expression for $\vec{H}_r$.

14. Ans: (b)

Sol: Brewster's angle $\theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$$\theta_B = \tan^{-1}\sqrt{\frac{1}{3}} = 30^o$$

At this angle there is no reflected wave when wave is parallel polarized.

$$n_1\sin\theta_i = n_2\sin\theta_t$$

$$\sqrt{\epsilon_1}\sin\theta_i = \sqrt{\epsilon_2}\sin\theta_t$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$$

$$\sin\theta_t = \sqrt{3}\,\frac{1}{2}\left(\theta_i = 30^o\right)$$

$$\theta_t = 60^\circ$$

15. Ans: (d)

Sol: Given that

$E_t = -2E_r$

Where

E_t is electric field of transmitted wave

E_r is electric field of reflected wave

$$\frac{E_t}{E_r} = -2$$

If E_i is electric field of incident wave.

But $-\frac{2E_r}{E_i} = \frac{2\eta_2}{\eta_2+\eta_1}$

and $\frac{E_r}{E_i} = \frac{-\eta_2}{\eta_1+\eta_2}$

and also $\frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$

so $\frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{-\eta_2}{\eta_2+\eta_1}$

$\eta_1 = 2\eta_2$

$\frac{\eta_1}{\eta_2} = 2 \Rightarrow \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 2 \Rightarrow \frac{\varepsilon_2}{\varepsilon_1} = 4$

16. Ans: (b)

Sol: Solutions of wave equations represents a wave.

Statement (I) is true

An EM wave is a function of both space & time.

Statement (II) is true

But Statement (II) is not the correct explanation of Statement (I).

17. Ans: (d)

Sol: The direction of Poynting vector is same as the direction of wave propagation

So, Statement (I) is false.

Polarization of a wave is defined as orientation of electric field vector

Statement (II) is true.

18. Ans: (a)

Sol: Skin depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

For perfect conductor, conductivity $(\sigma) = \infty \Rightarrow \delta = \sqrt{\frac{2}{\infty}} = 0$

Statement (I) is true

$\alpha = \frac{1}{\delta} = \infty$

Statement (II) is true

Statement (II) is the correct explanation of Statement (I).

19. Ans: (d)

Sol: For TEM wave, electric and magnetic fields does not exist along the direction of propagation.

Statement (I) is false.

$\overline{E}, \overline{H}$ and $\overline{K}$ are always orthogonal to each other.

$\overline{E} \perp \overline{H} \perp \overline{K}$

Statement (II) is true.

20. Ans: (b)

Sol: Intrinsic impedance

$\eta = \frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

So, it will depends on medium properties.

Statement (I) is true.

For good conductor

$\eta = \frac{E}{H} = \sqrt{\frac{\omega\mu}{\sigma}} e^{j45^\circ} \Rightarrow$ Electric field leads magnetic field by 45°

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

21. Ans: (d)

Sol: For oblique the wave vector will makes some angle to the normal

Statement (I) is false.

For perpendicular polarization electric fields is normal total plane incidence

Statement (II) is true.

22. Ans: (b)

Sol: $\overline{K}$ - wave number

β - phase shift constant

$\beta = |\overline{K}|$

Statement (I) is true.

Suppressing time variations will gives phasor form

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

23. Ans: (b)

Sol: Brewster angle is that angle of incidence for which no reflection takes place

Statement (I) is true.

Critical angle is the maximum angle of incidence, for reflections will exists

$\theta_i > \theta_c \rightarrow$ TIR occurs

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

24. Ans: (b)

Sol: For good conductor

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

Statement (I) is true.

Skindepth $\delta = \frac{1}{\alpha}$

Statement (II) is true.

But Statement (II) is not the correct explanation Statement (I).

25. Ans: (d)

Sol: To achieve elliptical polarization the phase difference between transverse fields would not be 0 (or) 180

Statement (I) is false.

VP and HP are the special case in LP

Statement (II) is true.

26. Ans: (c)

Sol: Displacement current is the out come of Maxwell

Statement (I) is true.

Existence of magnetic charges would result displacement current

Statement (II) is false.

27. Ans: (b)

Sol: $V_{EMF} = -\frac{d\lambda}{dt}$

λ - magnetic flux linkage

Statement (I) is true

According to lenz's law the induced voltage in a loop is always so directed of to produce a flux opposing the change in the flux density.

S_2 - true

But Statement (II) is not the correct explanation of Statement (I).

Conventional Practice Solutions

01.

Sol: Assuming the wave is coming from free space i.e. medium 1, $\eta_1 = 120\pi$, at frequency f = 2GHz.

Medium 2 is a conductor and

$\sigma_2 = 5.8\times10^7 \text{S/m}$

We have,

$$\frac{\sigma}{\omega\varepsilon} = \frac{5.8\times10^7\times36\pi}{2\pi\times2\times10^9\times10^{-9}} >> 1$$

i.e at given frequency medium 2 is a conductor

$\gamma = \alpha + j\beta$ in conductor, $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$= \sqrt{\frac{2\pi\times2\times10^9\times4\pi\times10^{-7}\times5.8\times10^7}{2}}$$

$= 0.67\times10^6$/m

So, $\alpha = \beta = 0.67\times10^6$ /m

$\gamma = 0.6\times10^6\,(1+j1)$

Skin depth $\delta = \frac{1}{\alpha} = \frac{10^{-6}}{0.6} \cong 0 \Rightarrow$ i.e no exiting wave

so, $\tau = 0$, $\eta_2 = 0$, $\Gamma = -1$

02.

Sol:

We have, conduction current density $(J_C) = \sigma E$. and displacement current density $(J_D) = j\omega\varepsilon E$

$\left|\frac{J_c}{J_d}\right| = \tan\theta = \frac{\sigma}{\omega\varepsilon}$ is known as loss tangent

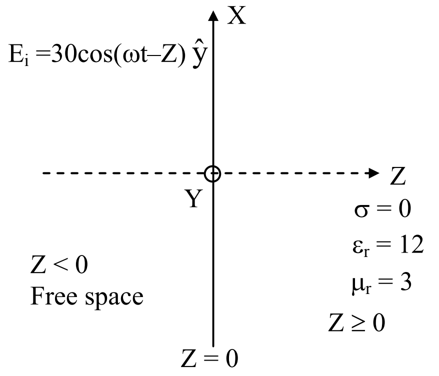
If $\frac{\sigma}{\omega\varepsilon} >> 1 \Rightarrow \theta = 90^\circ$, then the medium is behaving as conductor.

$\frac{\sigma}{\omega\varepsilon} << 1 \Rightarrow \theta = 0°$ medium behaves as dielectric

$\frac{\sigma}{\omega\varepsilon} = 1 \Rightarrow \theta = 45°$ medium behaves as a Quasi conductor or semi conductor

03.

Sol:



(i) Standing wave ratio:

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|},\ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$\eta_1 = \eta_0 = 120\pi$,

$$\eta_2 = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\times\sqrt{\frac{3}{12}} = 60\pi$$

$$\therefore \Gamma = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -\frac{1}{3} \Rightarrow \rho = \frac{1+1/3}{1-1/3} = 2$$

(ii) Reflected magnetic field:

$$\overline{H}_R = \hat{a}_{PR}\times\frac{\overline{E}_R}{\eta}$$

$$\overline{E}_R = \Gamma\overline{E}_i$$

$$= -\frac{1}{3}\times30\cos(\omega t + \hat{z})\hat{y}$$

$$\hat{a}_{PR} = -\hat{z}$$

$$\therefore\ \overline{H}_R = -\hat{z}\times-\hat{y}\frac{10}{120\pi}\cos(\omega t + z)$$

$$\overline{H}_R = -\hat{x}\frac{1}{12\pi}\cos(\omega t + z)$$

04.

Sol: Given: $E = E_0\cos(\omega t - \beta x)\ \hat{y}$,

$$H = \frac{E_0}{\eta}\cos(\omega t - \beta x)\,\hat{z} \text{ in free space.}$$

In free space ,

$$v_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\omega}{v_p} = \frac{\omega}{1/\sqrt{\mu\varepsilon}} =$$

$$\omega\sqrt{\mu_0\varepsilon_0} \Rightarrow \beta = \frac{\omega}{c}$$

Intrinsic impedance $\frac{E}{H} = \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

As $E_y \neq 0$, based on Maxwell equation

$$\nabla \times \overline{E} = -\mu\frac{\partial \overline{H}}{\partial t}$$

$$\Rightarrow \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{bmatrix} = -\mu\frac{\partial}{\partial t}\left(H_x\hat{x} + H_y\hat{y} + H_z\hat{z}\right)$$

$$= \left[0 - \frac{\partial E_y}{\partial z}\right]\hat{x} - [0-0]\hat{y} + \left[\frac{\partial E_y}{\partial x} - 0\right]\hat{z}$$

$$= -\mu\frac{\partial}{\partial t}H_x\hat{x} + -\mu\frac{\partial}{\partial t}H_y\hat{y} + -\mu\frac{\partial}{\partial t}H_z\hat{z}$$

$$= (0)\hat{x} - (0)\hat{y} + \frac{\partial E_y}{\partial x}\hat{z}$$

$$= -\mu\frac{\partial}{\partial t}H_x\hat{x} + -\mu\frac{\partial}{\partial t}H_y\hat{y} + -\mu\frac{\partial}{\partial t}H_z\hat{z}$$

$H_x = H_y = 0$

So, $\frac{\partial E_y}{\partial x} = -\mu\frac{\partial H_z}{\partial t}$

$$\Rightarrow \frac{\partial H_z}{\partial t} = -\frac{1}{\mu}\frac{\partial}{\partial x}\left(E_0\cos(\omega t - \beta x)\right)$$

$$= -\frac{E_0}{\mu} \times -\sin(\omega t - \beta x) \times -\beta$$

$$= -\frac{E_0\beta}{\mu} \times \sin(\omega t - \beta x)$$

$$H_z = \int -\frac{E_0\beta}{\mu} \times \sin(\omega t - \beta x)dt$$

$$= \frac{E_0\beta}{\omega\mu} \times \cos(\omega t - \beta x)$$

$$\therefore H = \frac{E_0\beta}{\omega\mu}\cos(\omega t - \beta x)\hat{z}$$

$$\therefore \frac{E_y}{H_z} = \frac{\omega\mu}{\beta} = \mu.v_p = \mu_0 \times \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0$$

$$\therefore \frac{E}{H} = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\ ,\ \beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$$

at 10 MHz, $\beta = \frac{2\pi \times 10^7}{3 \times 10^8} = \frac{2\pi}{30}$

$$= \frac{\pi}{15}\text{rad/m},$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

As E is only along y-direction, it is a linearly polarized wave. The wave is propagating along + x direction.

05.

Sol:

$E_i = 110\cos(\omega t - 4\pi x)\,\hat{z}$ V/m

$\varepsilon_{r1} = 4$ $\qquad$ $\varepsilon_{r2} = 9$

$\eta_1 = \frac{120\pi}{\sqrt{4}} = 60\pi$ $\qquad$ $\mu_{r2} = 4$

$\sigma = 0$

$$\eta_2 = 120\pi \times \sqrt{\frac{4}{9}}$$

$$= 120\pi \times \frac{2}{3}$$

$$= 80\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{80\pi - 60\pi}{80\pi + 60\pi} = \frac{1}{7}$$

Phase constant $\beta_1 = \frac{\omega}{c}\sqrt{\varepsilon_r} = 4\pi$

$E_R = \Gamma E_i = \frac{1}{7} \times 110 \cos(\omega t + 4\pi x)\hat{z}$

$|E_T| = \tau E_i = (1+\Gamma)E_i = \left(1+\frac{1}{7}\right)110 = \frac{8}{7} \times 110$

$= 125.7$

$\beta_2 = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} \quad \left[\because \omega = \frac{4\pi \times c}{\sqrt{4}}\right]$

$= \frac{4\pi \times c}{2c}\sqrt{9 \times 4} = 12\pi$

$\therefore E_T = 125.7 \cos(6\pi \times 10^8 t - 12\pi x)\ \hat{z}$

06.

Sol: Given: $H = 2e^{-j0.1\pi z}\hat{y}$, $v_p = 2\times 10^8$ m/s and $\mu_r = 1.8$ .We have $\beta = 0.1\pi$

$\omega = 2\pi f = v_p \times \beta = 2\times 10^8 \times 0.1\pi$

$\Rightarrow f = 10$ MHz

$v_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = 2\times 10^8$

$\Rightarrow \frac{3\times 10^8}{\sqrt{1.8}\times\sqrt{\varepsilon_r}} = 2\times 10^8$

$\Rightarrow \sqrt{\varepsilon_r} = \frac{1.5}{\sqrt{1.8}} = 1.118$

$\Rightarrow \varepsilon_r = (1.118)^2 = 1.25$

$\lambda = \frac{v_p}{f} = \frac{2\times 10^8}{10^7} = 20\,\text{m}$

$|\overline{E}| = \eta|\overline{H}|$

$= 120\pi \times \sqrt{\frac{1.8}{1.25}} \times 2$

$= 904.778$ V/m

07.

Sol: Given: $\mu_r = 4$, $\varepsilon_r = 9$, $f = 10\text{MHz} = 10^7\text{Hz}$

$\hat{a}_k = \hat{a}_y$ .given $E_{xo} = 400$V/m.

We have,

$\nu_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3\times 10^8}{\sqrt{9\times 4}} = 5\times 10^7\,\text{m/sec}$

$\beta = \frac{\omega}{\nu_p} = \frac{2\pi \times 10^7}{5\times 10^7} = \frac{2\pi}{5}\,\text{rad/m}$

$\lambda = \frac{\nu_p}{f} = \frac{5\times 10^7}{10^7} = 5\text{m}$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi \times \sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi \times \sqrt{\frac{4}{9}} = 80\pi\Omega$

$\overline{E}(t) = E_{xo}\cos(\omega t - \beta y)\hat{x}$

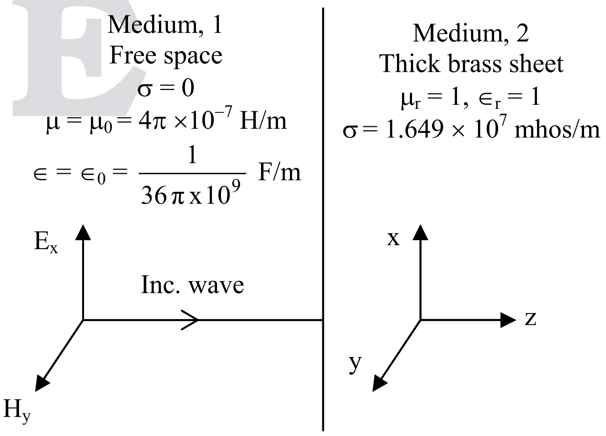
$= 400\cos\left(2\pi \times 10^7 t - \frac{2\pi}{5}y\right)\hat{x}$

and $\overline{H}(t) = \frac{400}{80\pi}\cos\left(2\pi \times 10^7 t - \frac{2\pi}{5}y\right)(-\hat{z})$

$= -1.591\cos\left(2\pi \times 10^7 t - \frac{2\pi}{5}y\right)\hat{z}$

08.

Sol: Normal incidence is shown in Fig.



In medium, 1

Fig.

Given : $\vec{E} = E_x \, \vec{a}_x$

$E_x = 1225 \cos(5.89 \times 10^{10} t - \beta z)$ V/m,

where $\omega = 5.89 \times 10^{10}$ r/sec.

$\therefore \; H_y = \dfrac{E_x}{\eta_1}$, where $\eta_1 = \eta_0 = (120\,\pi)\;\Omega$

For medium, 2 :

$$\frac{\sigma}{\omega\in} = \frac{1.649 \times 10^7 \times 36\pi \times 10^9}{5.89 \times 10^{10}} >> 1$$

$\therefore$ Brass sheet can be taken as almost perfect conductor with E and H equal to zero inside it.

$\therefore$ H_y in the first medium gives rise to a surface current of linear current density, J_s

$$J_s = H_y \text{ A/m}$$

Power that causes heating of the brass sheet = Power dissipated in the brass sheet

$= P = J^2_{s,\,rms}\; R_s$

where $J_{s,\,rms}$ is the rms value of J_s and R_s is the surface resistance given by $R_s = \eta_{real}$ (for good conductor) $= \sqrt{\dfrac{\omega\,\mu}{2\sigma}}$

$$R_S = \sqrt{\frac{5.89 \times 10^{10} \times 4\pi \times 10^{-7}}{2 \times 1.649 \times 10^7}}\;\Omega$$

$$= 4.74 \times 10^{-2}\;\Omega$$

$$P = \frac{(1225)^2}{2(120\pi)^2} \times 4.74 \times 10^{-2}\text{ W}$$

$$= 25 \times 10^{-2}\text{ W} = 0.25\text{ W}$$

09.
Sol:

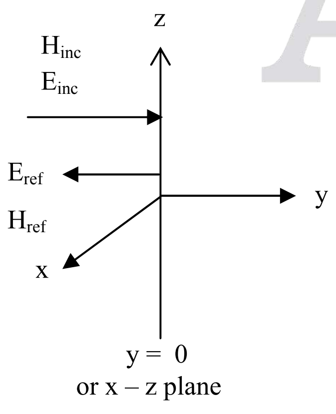


Fig.

In a charge free medium, electric field is given by $\vec{E} = A \sin(\beta y) \sin(\omega t)\, \vec{a}_x$

$$\vec{E} = \frac{A}{2}\left[\cos(\omega t - \beta y) - \cos(\omega t + \beta y)\right]\vec{a}_x \;\ldots\ldots (1)$$

$\vec{E}$ can be written as

$$\vec{E} = E_x^+\,\vec{a}_x + E_x^-\,\vec{a}_x \;\ldots\ldots\ldots\ldots (2),$$

$$\left.\begin{aligned} &\text{where} \quad E_x^+ = \frac{A}{2}\cos(\omega t - \beta y) \\ &\text{and} \quad E_x^- = -\frac{A}{2}\cos(\omega t + \beta y) \end{aligned}\right\}\ldots\ldots (3)$$

E has only x component. The y and z components are zero.

(b) The 1[st] term E_x^+ in equation (2) represents a plane wave traveling in the positive y - direction with velocity $v = \dfrac{\omega}{\beta}$ and the 2[nd] term E_x^- in equation (2) represents a plane wave traveling in the negative y-direction with the same velocity 'v' as shown in Fig., giving rise to a standing wave with reflection coefficient, -1 and standing wave ratio, ∞ in the y - direction which does not progress.

(a) From equation (3) $E_x^- = -E_x^+$ so that E at y = 0 is equal to $E_x^- + E_x^+ = 0$.

$\therefore$ y = 0 plane can be taken as a perfect reflecting surface with no transmission for $y > 0$

Let the intrinsic impedance of the medium be η. For free space

$\eta = \eta_0 = (120\,\pi)\Omega$.

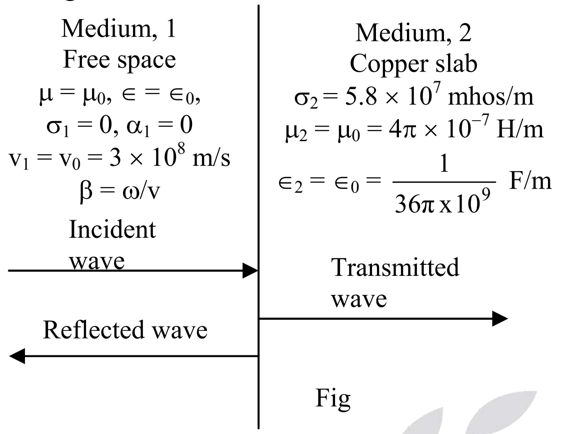
Then the associated magnetic field $\vec{H}$ is given by $\vec{H} = H_z^+\,\vec{a}_z + H_z^-\,\vec{a}_z$,

where $H_z^+ = \dfrac{-E_x^+}{\eta_0}$ and $H_z^- = \dfrac{E_x^-}{\eta_0}$

$$\therefore \vec{H} = \frac{-A}{240\,\pi}\left[\cos(\omega t - \beta y) + \cos(\omega t + \beta y)\right]\text{A/m}$$

10.

Sol: Normal incidence of an EM wave is shown in Fig.



Fig

Given f = 300 MHz

β in free space,

$$\beta_1 = \frac{\omega}{v_1} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \ \text{r/m}$$

as $\beta_1 = \dfrac{2\pi}{\lambda_1}$, $\lambda_1 = 1$ m

In medium, 2:

$$\frac{\sigma_2}{\omega \in_2} = \frac{5.8 \times 10^7 \times 36\pi \times 10^9}{2\pi \times 3 \times 10^8}$$

$$= 34.8 \times 10^8 \ >> 1$$

(a) Attenuation constant, α_2 = Phase shift constant, $\beta_2 \approx \sqrt{\dfrac{\omega\mu\sigma}{2}}$

$$\alpha_2 = \sqrt{\pi f \mu \sigma}$$

$$= \sqrt{\pi \times 3\times10^8 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}$$

$$= 2\pi\times10^4 \sqrt{3 \times 5.8} = 2.62\times10^5 \text{ Np/m}$$

(b) $\beta_2 = 2.62 \times 10^5$ r/m

(c) δ = Skin depth $= \dfrac{1}{\alpha_2}$

$$= \frac{1}{2.62 \times 10^5} \text{ m} = \frac{1}{2.62} \times 10^{-2} \text{ mm}$$

$$= 0.38 \times 10^{-2} \text{ mm}$$

(d) Phase velocity, $v_p = v_2 = \dfrac{\omega}{\beta_2}$

$$= \frac{2\pi \times 3 \times 10^8}{2.62 \times 10^5} \text{ m/s}$$

$$= 7.2 \times 10^3 \text{ m/s}$$

(e) Group velocity, v_g is given by

$$v_g = \frac{v_0^2}{v_p} = \frac{9\times10^{16}}{7.2\times10^3} = 1.25 \times 10^{13} \text{ m/s}$$

11.

Sol: Normal incidence at the interface between medium 1 and 2 is shown in Fig.

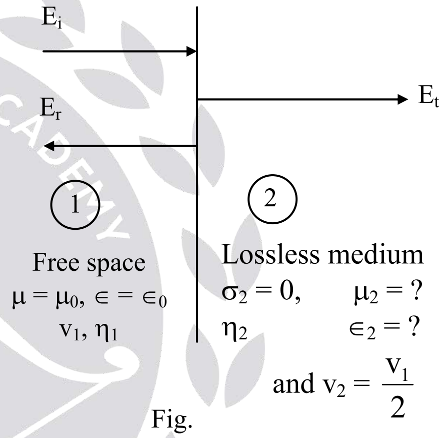


Fig.

$$v_1 = \frac{1}{\sqrt{\mu_1 \in_1}} = \frac{1}{\sqrt{\mu_0 \in_0}} \ ; \ v_2 = \frac{1}{\sqrt{\mu_2 \in_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\mu_0 \in_0}{\mu_2 \in_2}}$$

Given: $\dfrac{v_2}{v_1} = \dfrac{1}{2}$

$$\sqrt{\mu_2 \in_2} = 2\sqrt{\mu_0 \in_0}$$

or $\mu_2 \in_2 = 4 \mu_0 \in_0 = \dfrac{4}{9 \times 10^{16}}$ ………..(1)

It is also given that a standing wave is set up in medium 1 (free space) with reflection coefficient $K = \dfrac{E_r}{E_i} = -0.125$

In terms of intrinsic impedances:

$$K = \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1} \;;\; K\frac{\eta_2}{\eta_1} + K = \frac{\eta_2}{\eta_1} - 1$$

$$(K-1)\frac{\eta_2}{\eta_1} = -(K+1)$$

$$\frac{\eta_2}{\eta_1} = \frac{K+1}{1-K} \text{ or } = \frac{0.875}{1.125} = \frac{7}{9}$$

$$\text{As } \eta = \sqrt{\frac{\mu}{\in}},\ \frac{\eta_2}{\eta_1} = \sqrt{\frac{\mu_2}{\in_2}}\frac{\sqrt{\in_1}}{\sqrt{\mu_1}} = \frac{7}{9}$$

$$\sqrt{\frac{\mu_2 \in_0}{\in_2 \mu_0}} = \frac{7}{9} \text{ or } \frac{\mu_2 \in_0}{\in_2 \mu_0} = \frac{49}{81},$$

$$\frac{\mu_2}{\in_2} = \frac{49}{81}(120\,\pi)^2 \text{ ……………….. (2)}$$

From (1) and (2),

$$\mu_2^2 = \frac{4}{9\times10^{16}}\frac{49}{81}(120\pi)^2$$

$$\mu_2 = \frac{2}{3\times10^{8}}\frac{7}{9}120\pi$$

$$= \frac{14}{9}(4\pi\times10^{-7})\text{ H/m}$$

$$= \left(\frac{14}{9}\mu_0\right)\text{ H/m}$$

From (1),

$$\in_2 = \frac{4}{9\times10^{16}}\ \frac{9}{14\times4\pi\times10^{-7}}\text{ F/m}$$

$$= \frac{18}{7}\left(\frac{1}{36\pi x10^{9}}\right) = \frac{18}{7}\in_0\text{ F/m}$$

12.

Sol: Given

$$\vec{E} = E_0\ e^{j(\omega t + 3x - 4y)}\ \frac{8\vec{a}_x + 6\vec{a}_y + 5\vec{a}_z}{\sqrt{125}}\text{ V/m}$$

$\rightarrow$(1)

f = 10 GHz

The equation of the plane wave traveling in the direction of unit vector $\hat{n}$ normal to the plane of constant phase is given by

$$\vec{E} = E_0\ e^{j(\omega t - \beta\hat{n}.\hat{r})}\ \hat{n} \rightarrow(2)$$

where $\omega = 2\pi f$ is the radian frequency

and β = phase shift constant in the direction of $\hat{n}$.

$\vec{r}$ is the position vector in x, y, z coordinates

$$\vec{r} = x\,\vec{a}_x + y\,\vec{a}_y + z\,\vec{a}_z \text{ …………… (3)}$$

$$\hat{n} = \cos(A)\,\vec{a}_x + \cos(B)\,\vec{a}_y + \cos(C)\,\vec{a}_z \text{ ……....(4)}$$

where cos A, cos B, cos C are known as direction cosines.

A, B and C are the angles which the unit vector $\hat{n}$ makes with the positive x, y and z axis.

$$\hat{n}.\vec{r} = x\cos(A) + y\cos(B) + z\cos(C) \text{ ……....(5)}$$

$$\vec{E} = E_0\ e^{j[\omega t - (\beta_x)x - (\beta_y)y - (\beta_z)z]}\ \hat{n} \text{ ………….(6)}$$

where $\beta_x = \beta\cos(A)$,

$\beta_y = \beta\cos(B)$ ……………. (7)

and $\beta_z = \beta\cos(C)$

β_x, β_y and β_z are the phase shift constants in x, y and z directions respectively

Similarly

$$\left.\begin{aligned} \lambda_x &= \frac{2\pi}{\beta_x} = \frac{2\pi}{\beta\cos(A)} \\ \lambda_y &= \frac{2\pi}{\beta\cos(B)} \\ \lambda_z &= \frac{2\pi}{\beta\cos(C)} \end{aligned}\right\} \text{ ………….. (8)}$$

$$\left.\begin{aligned} v_x &= \frac{\omega}{\beta_x} = \frac{\omega}{\beta\cos(A)} \\ v_y &= \frac{\omega}{\beta\cos(B)} \\ v_z &= \frac{\omega}{\beta\cos(C)} \end{aligned}\right\} \text{ ………….. (9)}$$

(i) Comparing (1) and (6), it can be concluded that the field vector $\vec{E}$ given by equation (1) represents a plane wave in the direction of the unit vector $\hat{n}$ where

$$\hat{n} = \frac{8}{\sqrt{125}}\vec{a}_x + \frac{6}{\sqrt{125}}\vec{a}_y + \frac{5}{\sqrt{125}}\vec{a}_z$$

$$\text{and } \cos(A) = \frac{8}{\sqrt{125}},$$

$$\cos(B) = \frac{6}{\sqrt{125}}$$

$$\cos(C) = \frac{5}{\sqrt{125}}$$

$\beta_x = \beta \cos(A) = -3,$

$\beta_y = \beta \cos(B) = 4,$

$\beta_z = \beta \cos(C) = 0$

(ii) The propagation constant γ is given by

$$\gamma = 0 + j\beta = j\beta$$

$$\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2(\cos^2 A + \cos^2 B + \cos^2 C)$$

$$= \beta^2\left(\frac{64}{125} + \frac{36}{125}\right) = \frac{4}{5}\beta^2$$

$$\frac{4}{5}\beta^2 = (-3)^2 + (4)^2 = 25$$

$$\beta^2 = \frac{5 \times 25}{4}$$

$$\beta = \frac{5\sqrt{5}}{2} = 2.5\sqrt{5} \text{ r/m} = 5.59 \text{ r/m}$$

$$\therefore \gamma = j\,5.59$$

(iii) Phase velocity in the y-direction

$$v_y = \frac{\omega}{\beta_y} = \frac{\omega}{\beta\cos(B)}$$

$$= \frac{2\pi \times 10 \times 10^9}{4}$$

$$= 5\pi \times 10^9 \text{ m/sec}$$

$$= 15.7 \times 10^9 \text{ m/sec}$$

13.

Sol: Normal incidence is shown in Fig.

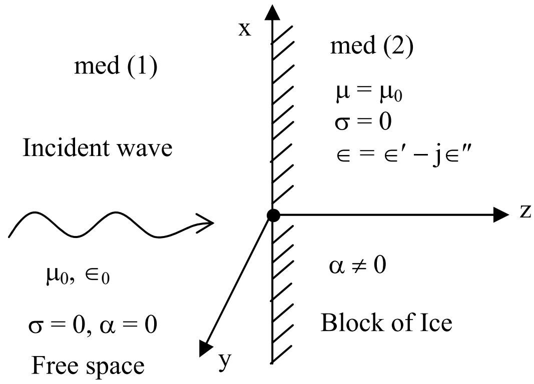


Fig.

In free space, given:

$$\vec{E} = \left[E_x\vec{a}_x + E_y\vec{a}_y\right]e^{j(\omega t - kz)} \quad ...(1)$$

where $E_x = 10\sqrt{\pi}$ V/m,

$E_y = 11.8\sqrt{\pi}$ V/m

$\omega = 4\pi \times 10^8$ r/s

Phase shift constant

$$k = \beta = \omega\sqrt{\mu \in} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Magnitude of the electric field in the incident wave, E_1 is

$$E_1^2 = E_x^2 + E_y^2$$

$$= (10^2 + 11.8^2)\pi = 239.24\,\pi$$

Intrinsic impedance in free space

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\in_0}} = (120\,\pi)\ \Omega$$

(a) Let the Average power associated with the incident wave be P_1

$$P_1 = \frac{E_1^2}{2\eta_0}$$

$$= \frac{239.24\,\pi}{2 \times 120\,\pi} = 0.997 \text{ W/m}^2$$

(b) For ice given: $\in = \in' - j\in''$

Where $\in' = 9\in_0$ and

$\in'' = 0.001 \times 9\in_0 = 10^{-3}\in'$

The loss mechanism in a non-magnetic dielectric medium different from free space is modeled by $\mu = \mu_0$ and a complex permittivity

$$\epsilon = \epsilon' - j\,\epsilon''$$

This gives rise to values of α, β and η different from free space values

α = attenuation constant

$$= \text{Re}\{jk\}$$
$$= \text{Re}\left\{j\omega\sqrt{\mu_0(\epsilon' - j\epsilon'')}\right\}$$

β = phase shift constant

$$= \text{Im}\{jk\}$$
$$= \text{Im}\left\{j\omega\sqrt{\mu_0(\epsilon' - j\epsilon'')}\right\}$$

η = intrinsic impedance

$$= \sqrt{\frac{j\omega\mu_0}{[j\omega(\epsilon' - j\epsilon'')]}}$$

For the given values of ϵ' and ϵ'',

$$\frac{\epsilon''}{\epsilon'} = 10^{-3} \;\; (<<1)$$

Under this condition,

$$\alpha \approx \frac{\omega}{2}\epsilon''\sqrt{\frac{\mu_0}{\epsilon'}}$$
$$= \frac{4\pi\times10^8}{2}\times 9\epsilon_0 \times 10^{-3}\times\sqrt{\frac{\mu_0}{9\epsilon_0}}$$
$$= 2\pi \text{ x } \frac{10^5 \text{ x} 9 \text{ x} 10^{-9}}{36\pi} \text{ x } 40\pi$$
$$= 2\pi \text{ x } 10^{-3} \text{ n/m} = 6.284 \text{ x } 10^{-3} \text{ n/m}$$

$$\beta = \beta_2 \approx \omega\sqrt{\mu_0\,\epsilon'} = \omega\sqrt{\mu_0\,(9\epsilon_0)}$$
$$= 4\pi\times10^8\times3\times\frac{1}{3\times10^8}$$
$$= 4\pi \text{ r/m} = 12.56 \text{ r/m}$$

and $\eta = \eta_2 \approx \sqrt{\frac{\mu_o}{\epsilon'}} = \sqrt{\frac{\mu_o}{9\epsilon_0}}$

$$= \frac{120\pi}{3} = (40\pi)\Omega = 125.66\ \Omega$$

Skin depth in ice,

$$\delta = \frac{1}{\alpha} = \frac{10^3}{6.284} = 159 \text{ m}$$

(c) In ice as $\alpha \neq 0$, the amplitude of the field decreases exponentially according to the factor $e^{-\alpha z}$.

$\therefore$ E_2 in ice = E_1 in free space $\times\, e^{-\alpha z}$

$\therefore$ E_2 at a distance = 5δ is given by

$E_2 = E_1\, e^{-\alpha 5\delta} = E_1\, e^{-5}$ V/m

$\therefore$ Average power density at z = 5 δ from the interface is

$$P_2 = \frac{E_2^2}{2\eta_2} = \frac{239.24\,\pi}{2\times40\,\pi}\times e^{-10}$$
$$= 3\times e^{-10}\ \text{W/m}^2$$

Chapter 3 Transmission Lines

Objective Practice Solutions

01. Ans: (b)

Sol: $Z_{in} = Z_0 \dfrac{Z_R + jZ_0 \tan \beta\ell}{Z_0 + jZ_R \tan \beta\ell}$

Phase velocity

$$\upsilon_p = \frac{\omega}{\beta}$$

$$\upsilon_p = \frac{2\pi f}{\beta}$$

$$\beta = \frac{2\pi f}{\upsilon_p} = \frac{2\times\pi\times10^8}{2\times10^8} = \pi$$

$$\beta\ell = \pi.\ell \Rightarrow \pi \text{ (Given } l\text{=1m)}$$

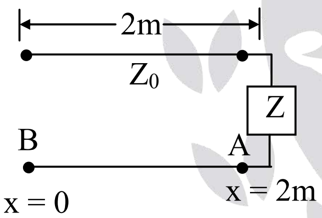
$$\tan\beta\ell = 0$$

$$Z_{in} = Z_R$$

$$= (30 - j40)\Omega$$

02. Ans: (a)

Sol:



$$K_x = \frac{C_2}{C_1} e^{2j\beta X}$$

$$K_A = \frac{C_2}{C_1} e^{j4\beta} \text{ at } (x = 2)$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(0)} \text{ at } (x = 0)$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta(0)}}{\frac{C_2}{C_1} e^{j4\beta}} = e^{-j4\beta}$$

$$\upsilon_P = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\pi}{2}$$

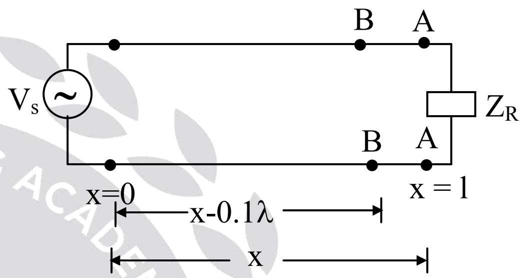
Given f = 50 MHz

$$\upsilon_p = 2 \times 10^8 \text{ m/s}$$

$$\frac{K_B}{K_A} = e^{-j4\left(\frac{\pi}{2}\right)} = e^{-j2\pi} = 1 \text{(or) } \frac{\Gamma_i}{\Gamma_R} = 1$$

03. Ans: (b)

Sol:



$$V = C_1 e^{-j\beta x} + C_2 e^{+j\beta x}$$

$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = 0.3e^{-j30^0} = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(x-0.1\lambda)}$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta x}\, e^{-j4\frac{\pi}{\lambda}0.1\lambda}}{\frac{C_2}{C_1} e^{2j\beta x}}$$

$$K_B = K_A . e^{-j.4\pi}$$

$$= 0.3e^{-j30^0} e^{-72^0}$$

$$= 0.3\, e^{-j102^0}$$

Note: In the options $0.3\, e^{j102^0}$ is given. But correct answer is $0.3\, e^{-j102^0}$

04. Ans: (c)

Sol: From the voltage SW pattern,

$V_{min} = 1$, $V_{max} = 4$, VSWR = S = 4

$Z_0 = R_0 = 50\ \Omega$

Let the resistive load be R_L

For Resistive loads

$$S = \frac{R_L}{R_0} \quad \text{for } R_L > R_0$$

$$= \frac{R_0}{R_L} \quad \text{for } R_0 > R_L$$

$\therefore$ $R_L = S\,R_0 = 4 \times 50 = 200\ \Omega$ for $R_L > R_0$

$R_L = R_0/S = 50/4 = 12.5\ \Omega$ for $R_0 > R_L$

As voltage minimum is occurring at the load point, $R_L = 12.5\ \Omega$.

05. Ans: (a)

Sol: Reflection coefficient:

$$\Gamma = \frac{R_L - R_0}{R_L + R_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

06. Ans: (d)

Sol: The interconnection of TL's is shown in Fig.

$$Z_{i1} = \frac{(50)^2}{100} = 25\Omega$$

$$Z_{i2} = \frac{(50)^2}{200} = 12.5\Omega$$

$$Z_L = 25 \parallel 12.5 = \frac{25}{3}\Omega$$

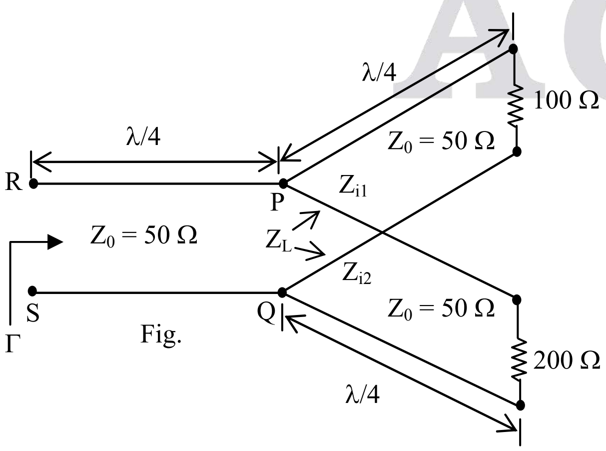


Fig.

Reflection coefficient at PQ $= \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = -\frac{125}{175} = -\frac{5}{7}$$

$\therefore$ At the input RS,

Reflection coefficient, $\Gamma = -\dfrac{5}{7} e^{-j2\beta\ell}$

$$\text{As } \beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Gamma = -\frac{5}{7} e^{-j\pi} = \frac{5}{7}$$

07. Ans: (d)

Sol: $Z_{in} = Z_0\left[\dfrac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}\right]$

i) For a shorted line,

$Z_L = 0$

$\ell = \lambda/8$

$$\beta\ell = \frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0\left[\frac{0 + jZ_0}{Z_0 + 0}\right]$$

$Z_{in} = j\,Z_0$

ii) For a shorted line means $Z_L = 0$

Given that $\ell = \dfrac{\lambda}{4}$

$$\beta\ell = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_0^{\,2}}{Z_L} = \frac{Z_0^{\,2}}{0}$$

$Z_{in} = \infty$

iii) Open line means $Z_L = \infty$,

Given that $\ell = \dfrac{\lambda}{2}$

$$\therefore \beta\ell = \frac{2\pi}{\lambda}.\frac{\lambda}{2} = \pi \;\Rightarrow \tan\pi = 0$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\pi}{Z_0 + jZ_L \tan\pi}\right]$$

$Z_{in} = Z_L$

iv) For a matched line of any length

$Z_L = Z_0$

$$Z_{in} = Z_0\left[\frac{Z_0 + jZ_0 \tan\beta\ell}{Z_0 + jZ_0 \tan\beta\ell}\right] = Z_0$$

08. Ans: (c)

Sol: The line is matched as $Z_L = Z_0 = 50\ \Omega$ and hence reflected wave is absent.

For the traveling wave, given:

Phase difference for a length of 2 mm = $\pi/4$ rad

Frequency of excitation = 10 GHz

Phase velocity, $v_p = \dfrac{\omega}{\beta}$

$\omega = 2\pi \times 10 \times 10^9$ rad/sec

β = Phase-shift per unit length

$$= \frac{\pi}{4\times2\times10^{-3}} \text{ rad/m}$$

$$v_p = \frac{2\pi \times 10^{10} \times 8}{\pi \times 10^3} = 1.6 \times 10^8 \text{ m/s}$$

09. Ans: (b)

Sol: $[S] = \begin{bmatrix} 0.3\angle0^0 & 0.9\angle90^0 \\ 0.9\angle90^0 & 0.2\angle0^0 \end{bmatrix}$

For reciprocal; $S_{12} = S_{21}$

It is satisfied.

For lossless line $|S_{11}|^2 + |S_{12}|^2 = 1$

$$(0.3)^2 + (0.9)^2 = 0.9 \neq 1$$

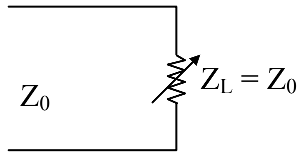
$\therefore$ It is a lossy line

10. Ans: (b)

Sol: If we connect infinite number at transmission lines, the input impedance is same as characteristic impedance.

Statement (I) is true.

An infinite line is equal to finite line when the finite line is terminated by Z_0



Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

11. Ans: (b)

Sol: For a Transmission line

The propagation constant,

$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$ ------ (1)

The distortion less condition for transmission line is

$\dfrac{R}{L} = \dfrac{G}{C}$ ------ (2)

From equation (1) and (2)

$\alpha + j\beta = R\sqrt{\dfrac{C}{L}} + j\omega\sqrt{LC}$ (or)

$\alpha + j\beta = \sqrt{\dfrac{L}{C}} + j\omega\sqrt{LC}$

$\therefore \alpha = R\sqrt{\dfrac{C}{L}}$ (or) $G\sqrt{\dfrac{L}{C}}$ ----- (3)

$\beta = \omega\sqrt{LC}$

$v_p = \dfrac{\omega}{\beta} = \dfrac{\omega}{\omega\sqrt{LC}} = \dfrac{1}{\sqrt{LC}}$ ------- (4)

$\therefore$ A distortion less condition is same as the condition for minimum attenuation. So, statement (I) is true.

From equation (3) and (4), it is clear that attenuation constant (α) and phase velocity (v_p) are independent of frequency in a distortion less transmission line. So, statement (II) is true but not the correct explanation for statement (I).

12. Ans: (b)

Sol: $Z(x) = Z_0\left[\dfrac{Z_R + jZ_0 \tan\beta x}{Z_0 + jZ_R \tan\beta x}\right]$

If x-changes, Z(x) - changes

So, impedance is not same

Statement (I) is true.

The reason for reflections is the impedance discontinuity

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

13. Ans: (a)

Sol: $Z_L = Z_0$

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = 0$

$S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = 1$

Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

14. Ans: (c)

Sol: The successive distance between two minimas is $\dfrac{\lambda}{2}$

Statement (I) is true.

At the location of voltage minima, Voltage is minimum and current is maximum (V_{min},I_{max}).

$\therefore Z = \dfrac{V_{min}}{I_{max}} = Z_{min}$

Statement (II) is false.

But Statement (II) is not the correct explanation of Statement (I).

15. Ans: (c)

Sol: Impedance transformers are used for matching purpose.

Statement (I) is true

$Z_{in}\left(l = \dfrac{\lambda}{4}\right) = \dfrac{Z_0^2}{Z_L}$

Statement (II) is false.

16. Ans: (b)

Sol: Transmission line are used as circuit elements

Length	Short Circuited Line	Open Circuited Line
1. $0 \le l \le \dfrac{\lambda}{4}$	1. Inductor	1. Capacitor
2. $\dfrac{\lambda}{4} \le l \le \dfrac{\lambda}{2}$	2. Capacitor	2. Inductor
3. $l = \dfrac{\lambda}{4}$	3. Parallel Resonator	3. Series Resonator
4. $l = \dfrac{\lambda}{2}$	4. Series Resonator	4. Parallel Resonator

Stubs are used for matching purpose.

Both Statement (I) and Statement (II) are individually true but statement (II) is not the correct explanation of statement (I).

17. Ans: (d)

Sol: $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$Z_L = \pm jX$

$|\Gamma| = 1$

$s = \infty$

Statement (I) is false but Statement (II) is true.

18. Ans: (b)

Sol: Both Statement (I) and Statement (II) are individually true but statement (II) is not the correct explanation of statement (I).

19. Ans: (b)

Sol: $Z_{in}\left(l = \frac{\lambda}{2}\right) = Z_L$

Statement (I) is true

$Z_{in}\left(l = (2n+1)\frac{\lambda}{4}\right) = \frac{Z_0^2}{Z_L}$

Statement (II) is true

Both Statement (I) and Statement (II) are individually true but statement (II) is not the correct explanation of statement (I).

Conventional Practice Solutions

01.

Sol: Lossless co-axial cable diameter ratio = 2

$= \frac{b}{a}$

$\varepsilon_r = 2.025$

For co-axial cable

Inductance = $L = \frac{\mu}{2\pi}\ln(b/a)$

$= \frac{4\pi\times10^{-7}}{2\pi}\ln(2) = 13.86\,\mu H$

Capacitance $C = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)}$

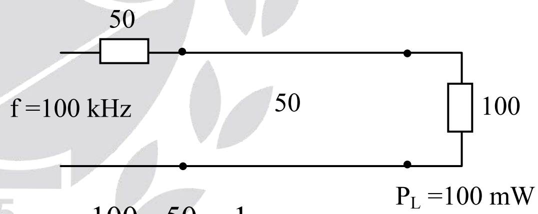
$= 2\pi\times\frac{10^{-9}}{36\pi}\times\frac{2.025}{\ln(2)} = \frac{2.025}{18\ln(2)}\times10^{-9}$

$= 0.16 nF$

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{13.86\times10^{-6}}{0.16\times10^{-9}}} = 294.3\,\Omega$

02.

Sol:



(i) $\Gamma = \frac{100-50}{100+50} = \frac{1}{3}$

(ii) VSWR (ρ) = $\frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2$

(iii) Position of 1st V_{max} at l = 0 (at load)

i.e., $R_L > R_0$

and 1st V_{min} at $l = \lambda/4$ i.e., V_{max} and V_{min} separated by $\lambda/4$

(iv) Impedance at $V_{max} = R_{max} = R_0\,\rho$

$= 50 \times 2 = 100$

and at $V_{min} = R_{min} = \frac{R_0}{\rho} = \frac{50}{2} = 25\,\Omega$

$$P_L = \frac{V_{max}^2}{Z_L} = 100\text{mW}$$

$$\Rightarrow V_{max} = \sqrt{100 \times 10^{-3} \times 100} = 3.162$$

$$\text{VSWR} = \frac{V_{max}}{V_{min}}$$

$$\Rightarrow V_{min} = \frac{3.162}{2} = 1.581\text{Volt}$$

Attenuation loss: Reduction in power carried by the wave due to imperfection of the structure (or) medium.

Reflection loss: Amount of reduction in delivered power at load due to mismatching at the load.

Transmission loss: Amount of reduction in available power at the load due to transmission of wave from input to load end.

Return loss: Amount of reduction in available power at the input due to mismatching at the input.

03.

Sol: The $\frac{\lambda}{2}$ – TL consisting of two $\left(\frac{\lambda}{4}\right)$ – sections is shown in Fig.1

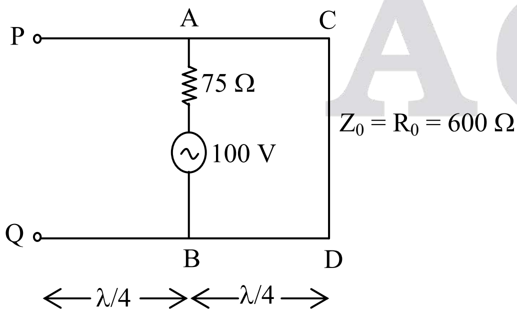


Fig.1

The short – circuit at CD gives rise to open – circuit at AB and the open – circuit at PQ gives rise to short – circuit at AB.

Therefore the effective load for 100 V / 75 Ω generator is a short-circuit.

∴ The current through the generator, $I_g = \frac{100}{75} = \frac{4}{3}$ A as shown in Fig.2

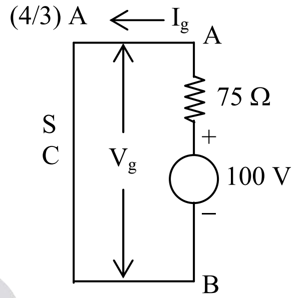


Fig. 2

For a lossless transmission line the voltage, V_l at any distance 'l' as shown in Fig. 3 is given by $V_l = V_g \cos \beta l + j\, I_g R_0 \sin \beta l$

For $l = \lambda/4$, $\beta l = \pi/2$

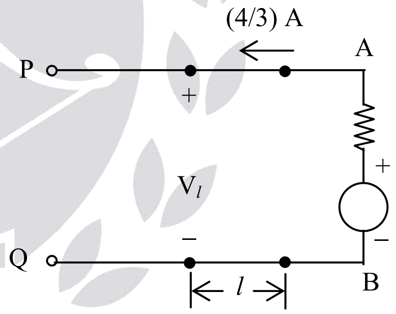


Fig.3

∴ Voltage at PQ is given by

$V_{PQ} = j\, I_g R_0 = j\,(4/3)\,(600)$

$\quad = j\,(800)\text{ V} = 800\angle 90°\text{ V}$

04.

Sol: Given : $V_g = 200$ V (rms)

Internal resistance $R_g = 200\ \Omega$

Characteristic impedance, $Z_0 = 200\ \Omega$

Length of TL, $l = 10$ m

Load resistance R_L (or) $Z_L = 100\ \Omega$

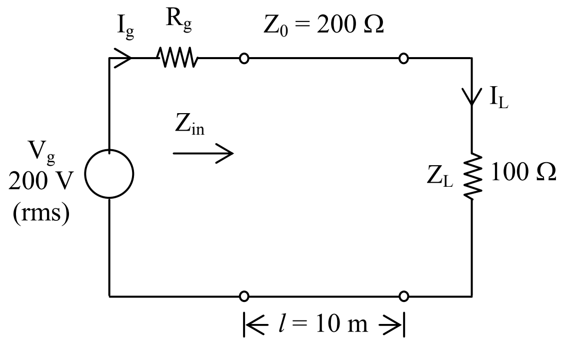


Fig.

For a loss-less ($\alpha = 0$) transmission line input impedance Z_{in} is given by

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan(\beta \ell)}{Z_0 + j Z_L \tan(\beta \ell)}$$

The wavelength λ for transmission line is given by $\lambda = \frac{v}{f}$

Given operating frequency, $f = 37.5$ MHz
$= 37.5 \times 10^6$ Hz

$$\therefore \lambda = \frac{3\times10^8}{37.5\times10^6} = \frac{300}{37.5} = 8 \text{ m}$$

The angle (βl) is given

$$\beta\ell = \frac{2\pi}{\lambda}\ell = \frac{2\pi}{8}\times10$$

$$= 2\pi\ (1.25) = \left(2\pi + \frac{\pi}{2}\right)$$

and $\tan(\beta l) = \tan(2\pi + \pi/2) = \tan(\pi/2)$

Therefore the input impedance Z_{in} becomes

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{200 \times 200}{100} = 400\ \Omega$$

(a) Current drawn from the generator, I_g is given by

$$I_g = \frac{V_g}{(R_g + Z_{in})} = \frac{200}{(200 + 400)}$$

$$= \frac{200}{600} = \frac{1}{3} \text{ A (rms)}$$

(b) The current drawn from the generator will also incident on the load resistance at a phase shift of $\pi/2$ radians

$$\therefore I_L = I_g\, e^{-j\frac{\pi}{2}} = \frac{1}{3} e^{-j\frac{\pi}{2}} \text{ A}$$

$\therefore$ Magnitude and phase of the current flowing in the load are 1/3 and −90°

(c) Power incident at the load, P_{inci} is given by

$$P_{inci} = I_{inci}^2\, Z_L$$

$$= \left(\frac{1}{3}\right)^2 \times 100 = 11.11 \text{ W}$$

Reflection coefficient at the load is given by

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-200}{100+200} = -\frac{1}{3}$$

Power delivered to the load $= P_{inci}(1 - |\Gamma|^2)$

$$= 11.11\left(1 - \frac{1}{9}\right) = 9.875\text{W}$$

Chapter 4 Waveguides

Objective Practice Solutions

01. Ans: (b)

Sol: Evanescent modes means no wave propagation.
Dominant mode means, the guide has lowest cut-off frequency.
TM_{01} and TM_{10} not possible, the minimum values of m, n for TM are at least 1, 1 respectively.

02. Ans: (a)

Sol: The mode which has lowest cutoff frequency is called dominant mode TE_{10}.
At 4GHz all modes are evanescent.
At 7GHz degenerate modes are possible
TE_{11} and TM_{11} are degenerate.

$$f_{c\,TE_{10}} = \frac{c}{2a} = \frac{3\times10^{8}}{2\times3\times10^{-2}} = 5\text{ GHz.}$$

At 6 GHz dominant mode will propagate.
At 11 GHz higher order modes are possible

03. Ans: (a)

Sol: Given: In a rectangular WG of cross-section : $(a \times b)$

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{2\pi}{a}x\right)\sin(\omega t - \beta z)\,\hat{y}$$

The wave is traveling in the z-direction having E_y component only as function of 'x'. As there is no component of $\vec{E}$ in the direction of propagation, $\vec{a}_z$ the wave is Transverse Electric (TE). Comparing the 'sin' term in $\vec{E}$ with the general expression: $\sin\left(\frac{m\pi}{a}x\right)$

$$m = 2$$

As there is no function of 'y' in $\vec{E}$, n = 0
$\therefore$ The mode of propagation in the WG is TE_{20}

04. Ans: (d)

Sol: Given
$a = 4.755$, $b = 2.215$,
$f = 12$ GHz, $c = 3 \times 10^8$ m/s
Cut off frequency

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For $TE_{10,}$ mode

$$f_c = \frac{c}{2a} = 3.15\text{ GHz}$$

$f > f_c$ (TE_{10} mode) so it propagates
For TE_{20} mode

$$f_C(TE_{20}) = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2}$$

$$= 2\,[f_c(TE_{10})] = 6.30\text{ GHz}$$

$f > f_c$ [TE_{20}] so it propagates
For TE_{01} mode

$$f_{C\,(TE01)} = \frac{c}{2}\sqrt{\frac{1}{b^2}}$$

$$= \frac{c}{2b} = 6.77\text{GHz}$$

$\therefore$ $f > f_c$ (TE_{01}] so it propagate
For TE_{11} mode

$$f_{c[TE11]} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 7.47\text{ GHz}$$

$f > f_c$ (TE_{11}) so it propagate
So, all modes are possible to propagate.

05. Ans: (a)

Sol: Given a = 6cm, b = 4 cm f = 3 GHz
Cut off frequency

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

TE_{10}: $f_c = \dfrac{c}{2a} = 2.5$ GHz

TE_{01}: $f_c = \dfrac{c}{2b} = 3.75$ GHz

TE_{11}: $f_c = \dfrac{c}{2}\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}} = 4.50$ GHz

TM_{11}: $f_c = \dfrac{c}{2}\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}} = 4.50$ GHz

06. Ans: (a)

Sol: $\dfrac{m\pi}{a} = \dfrac{2\pi}{a} \Rightarrow m = 2$

$\dfrac{n\pi}{b} = \dfrac{3\pi}{b} \Rightarrow n = 3$

For TM wave propagating along z-direction
$E_z \neq 0$ and $H_z = 0$
TM_{23}

$$TM_{23} \Rightarrow f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Substitute $c = 3 \times 10^{10}$ cm/sec
$m = 2$, $a = 6$ cm
$n = 3$, $b = 3$ cm
we get $f_c = 15.811$ GHz

$$\eta_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\omega = 10^{12} \Rightarrow f = \frac{10^{12}}{2\pi} = \frac{10^3}{2\pi}\text{GHz}$$

and $\eta = 120\,\pi$. & $f_c = 15.811$ GHz
Substitute all the above values and we get $\eta_{TM} = 375\ \Omega$.

07. Ans: (c)

Sol: $W_{avg} = \dfrac{1}{4}\dfrac{E_{yo}^2}{\eta_{TE_{10}}}a.b$; $\eta_{TE_{10}} = \dfrac{\eta}{\sqrt{1-(\lambda/\lambda_c)^2}}$

$\eta = 120\pi$, $\lambda = \dfrac{c}{f} = \dfrac{3\times10^{10}}{11\times10^9} = 2.72$cm

$\lambda_c = 2a = 2\times2.29 = 4.58$cm

So we get $\eta_{TE_{10}} = 469.52\Omega$

Putting all the values
$\therefore\ W_{avg} = 31.32$kW

08. Ans: (a)

Sol: $f_{c_{10}} = \dfrac{c}{2a} = \dfrac{3\times10^{10}}{2\times2} = 7.5$GHz

For b = a/2 , the next high order mode is TE_{01} or TE_{20}.

$$\therefore f_{c_{01}} = f_{c_{20}} = \frac{3\times10^{10}}{2} = 15\text{GHz}.$$

So the range of single mode (dominant mode propagation) is
$7.5 < f < 15$GHz.

09. Ans: (a)

Sol: $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_g^2} + \dfrac{1}{\lambda_c^2}$

$f_c = 0.908$GHz

$$\Rightarrow \lambda_c = \frac{3\times10^{10}}{0.908\times10^9} = 33.03\text{cm}$$

Substitute $\lambda_g = 40$ cm, $\lambda_c = 33.03$ cm
We get, $\lambda = 25.47$ cm

$$\Rightarrow f = \frac{3\times10^{10}}{25.47}$$
$$= 1.18 \text{ GHz}$$

10. Ans: (a)

Sol: $\dfrac{c}{2a} = 0.908$GHz

$$\Rightarrow a = \frac{3\times10^{10}}{2\times(0.908)\times10^9}$$

$$= 16.51\text{cm}$$

$$\Rightarrow b = \frac{a}{2} = 8.26\text{cm}$$

11. Ans: (a)

Sol: $\overline{\beta} = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$= \frac{2\pi}{25.47}\sqrt{1-\left(\frac{0.908}{1.18}\right)^2}$$

= 0.157 rad/cm

= 15.7 rad/m

12. Ans: (a)

Sol: Waveguides are used as transmission lines at microwave frequencies
Statement (I) is true.
At microwave frequencies two wire lines offers high attenuation
Statement (II) is true.
Statement (II) is the correct explanation of Statement (I).

13. Ans: (b)

Sol: Wave propagation inside the waveguide is by means of total internal reflection between the walls
Statement (I) is true.
The propagating modes inside the wave guides depends on excitation.
Statement (II) is true.
But Statement (II) is not the correct explanation of Statement (I).

14. Ans: (b)

Sol: The mode which has lowest cut-off frequencies or highest cut-off wavelength is called dominant mode.
Statement (I) is true.
Dominant mode is recommended to have maximum transfer of energy through the waveguide.
Statement (II) is true.
But Statement (II) is not the correct explanation of Statement (I).

15. Ans: (d)

Sol: Rectangular waveguide does not support TEM waves
Statement (I) is false.
Waveguide has no central conductors
Statement (II) is true.

16. Ans: (b)

Sol: $\lambda_g = \dfrac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$\lambda_g \geq \lambda$

Statement (I) is true.

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

f_c depends on dimension of the waveguide ($a\times b$), medium inside the waveguide and mode of propagation.
Statement (II) is true.

17. Ans: (b)

Sol: For Evanescent mode, $\alpha \neq 0$, $\beta = 0$ and γ - real. So, Statement (I) is true.
Evanescent waves are not propagating through the waveguide.
Statement (II) is true.
But Statement (II) is not the correct explanation of Statement (I).

18. Ans: (c)

Sol: If two different modes have same cut-off frequencies then those modes are called degenerative modes.
Statement (I) is true.
Degenerate modes are possible in the waveguides.
Statement (II) is false.

19. Ans: (d)

Sol: TM waves should not have magnetic field along the direction of propagation.

Statement (I) is false.
For TE waves, electric fields lie entirely in the transverse plane.
Statement (II) is true.

20. Ans: (b)

Sol: Waveguides are in cylindrical structure
Statement (I) is true.
The preferred cross section of the waveguide are circular, rectangular or elliptical.
Statement (II) is true.
But Statement (II) is not the correct explanation of Statement (I).

21. Ans: (b)

Sol: $\beta_g = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ at $f = f_c$

$\beta_g = 0$
Statement (I) is true.
Above cut-off frequency (i.e. $f > f_c$), the propagation constant (γ) is imaginary
i.e. $\gamma = j\beta$ (for lossless medium)
Statement (II) is true.
But Statement (II) is not the correct explanation of Statement (I).

Conventional Practice Solutions

01.

Sol: Spacing between plates a = 8 cm, f = 6GHz

For TE_{10} mode $f_c = \frac{c}{2a} = \frac{3\times10^{10}}{2\times8} = 1.8\text{GHz}$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \lambda = \frac{c}{f} = \frac{3\times10^{10}}{6\times10^9} = 5\text{cm}$$

$$\lambda_g = \frac{5}{\sqrt{1-\left(\frac{1.8}{6}\right)^2}} = \frac{5}{0.95} = 5.2\text{cm}$$

$$\eta_g = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{1.8}{6}\right)^2}} = \frac{377}{0.95} = 396.84\Omega$$

02.

Sol: Given:
Cross section for rectangular WG is (5 cm × 3 cm)
Relative permittivity, $\in_r = 3$

(i) Cutoff frequency for mode numbers, m and n is given by

$$f_C = \frac{v_0}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$v_0 = \frac{1}{\sqrt{\mu\in}} = \frac{1}{\sqrt{\mu\in_0\in_r}}$$

$$= \frac{3\times10^8}{\sqrt{3}} = \sqrt{3}\times10^8\ m/s$$

For TE_{11} mode,

$$f_C = \frac{\sqrt{3}}{2}\times10^8\sqrt{\frac{1}{(0.05)^2} + \frac{1}{(0.03)^2}}$$

$$f_C = \frac{\sqrt{3}}{2}\times10^8\times38.87 = 3.366\text{ GHz}$$

(ii) Given:
Attenuation constant,
$\alpha = (3\pi)$ Np/m for TE_{11} mode
For $f < f_C$, wave does not exist.
For rectangular WG with $(x \times y)$ cross-section, the fields are attenuated with the factor, $e^{-\alpha z}$

where $\alpha = \sqrt{\mu \in (\omega_C^2 - \omega^2)}$

$$\alpha = 2\pi\sqrt{\mu\in}\sqrt{f_C^2 - f^2} = \frac{2\pi}{v_0}\sqrt{f_C^2 - f^2}$$

$$f_C^2 - f^2 = \frac{\alpha^2 v_0^2}{4\pi^2}$$

$$f^2 = f_C^2 - \frac{\alpha^2 v_0^2}{4\pi^2}$$

$$f = \sqrt{f_C^2 - \frac{\alpha^2 v_0^2}{4\pi^2}}$$

$$\frac{\alpha^2 v_0^2}{4\pi^2} = \frac{9\pi^2 \times 3 \times 10^{16}}{4\pi^2} = \frac{27}{4}\times 10^{16}$$

$$f_C^2 = (3.366)^2 \times 10^{18}$$

$$f_C^2 - \frac{\alpha^2 v_0^2}{4\pi^2} = 10^{16}\left[(33.66)^2 - \frac{27}{4}\right]$$

$$= 10^{16} \times 1126.25$$

$$f = 10^8 \times 33.56 \text{ Hz} = 3.356 \text{ GHz}$$

03.

Sol: For hollow rectangular WG given
Free space wavelength, $\lambda_0 = 3.2$ cm
Conditions to be satisfied
(i) For TE_{10} mode, $\bar{\lambda} = 1.4\,\lambda_0$
(ii) λ_C for TM_{11} mode = $0.4\,\lambda_C$ for TE_{10} mode
To design rectangular WG i.e., to find the cross section $a \times b$.
Cut-off wavelength,
λ_C for TE_{10} mode = 2a …………… (1)
From the condition (i):
$\bar{\lambda} = 1.4\,\lambda_0$ ………………… (2)
The relation between $\bar{\lambda}$, λ_C and λ_0 is given by

$$\frac{1}{\bar{\lambda}^2} + \frac{1}{\lambda_C^2} = \frac{1}{\lambda_0^2}$$

or $$\bar{\lambda} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_C}\right)^2}}$$

Using (2), $$1.4\,\lambda_0 = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_C}\right)^2}}$$

or $$\sqrt{1-\left(\frac{\lambda_0}{\lambda_C}\right)^2} = \frac{1}{1.4} = \frac{10}{14}$$

$$1-\left(\frac{10}{14}\right)^2 = \frac{\lambda_0^2}{\lambda_C^2}$$

$$\therefore\ \lambda_C^2 = \lambda_0^2\left(\frac{14^2}{14^2-10^2}\right)$$

$$\lambda_C = \lambda_0\left(\frac{14}{\sqrt{96}}\right) = 3.2\times 1.429 = 4.5724 \text{ cm}$$

Using (1), 2a = 4.5724, a = 2.2862 cm

$$\lambda_C \text{ for } TM_{11} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$$

$= 0.4\,[\lambda_C$ for $TE_{10}] = 0.4\,(2a)$

$$\frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}} = 0.8\,a$$

Squaring on both sides

$$\frac{1}{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2} = 0.16\,a^2$$

$$0.16 + 0.16\left(\frac{a}{b}\right)^2 = 1$$

$$\left(\frac{a}{b}\right)^2 = \frac{0.84}{0.16}$$

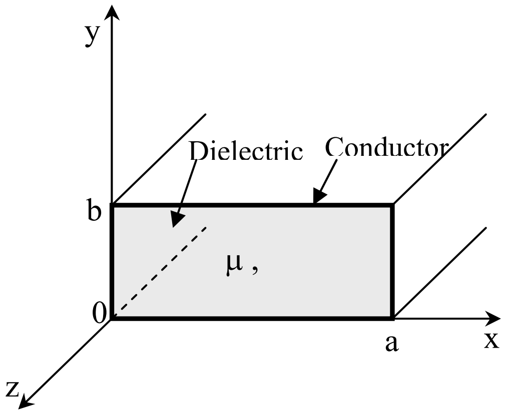
$$\left(\frac{a}{b}\right) = 2.2913$$

$$b = \frac{2.2862}{2.2913} = 0.9978 \text{ cm}$$

Therefore Cross-section of the given rectangular waveguide = 2.2862cm × 0.9978 cm

Q. NO. 4 and 5 solution

Sol:

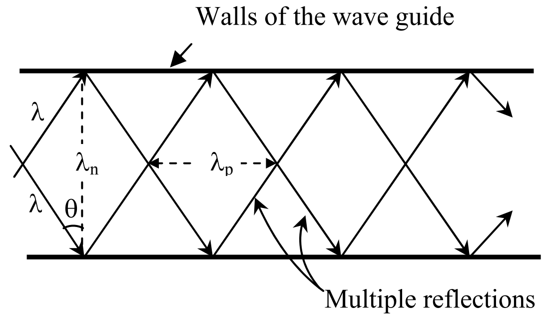


Rectangular wave guide

- The geometry of a rectangular waveguide is shown in fig.
- Where it is assumed that the guide is filled with a dielectric material of permittivity ε and permeability μ.
- Consider a > b Where a = length of the waveguide, b = breadth of the waveguide.
- Waveguide is a single conductor hollow structure.
- The walls of the waveguide are usually made of "Copper alloy (Brass)" and its inside surface is coated with a thin layer of either gold or silver in order to
  - i) Improve the conductivity of the walls of the waveguide.
  - ii) To ensure that the inside surface is smooth which reduces the losses inside the waveguide.

Properties and Characteristics of Waveguide

1) The conducting walls of the guide confine the electromagnetic fields and there by guide the electromagnetic wave through "multiple reflections" as shown in fig below Thus a number of distinct field configurations or modes can exist in waveguides.



λ = wave length of signal in unbounded medium.

$\lambda_n = \dfrac{\lambda}{\cos\theta} \Rightarrow$ is in the direction normal to the reflecting plane

$\lambda_p = \dfrac{\lambda}{\sin\theta} \Rightarrow$ is parallel to the plane.

2) When the waves travel longitudinally down the guide. The plane waves are reflected from wall to wall as shown in fig. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave. Thus only TM & TE waves can propagate through the waveguide.
3) TEM waves can't propagate through the waveguide since it requires an axial conductor for axial current flow (or) an axial displacement current to support a transverse magnetic field.
4) The wave length inside the wave guide (called guide wavelength λ_g) is quite different from the free space wave length λ_0. Because of multiple reflections from the walls of the guide, "λ_g will always be greater than λ_0".
5) When the wave length inside the waveguide differs from that outside the waveguide, the velocity of the wave propagation inside the waveguide must also be different from that through free space.
6) If one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from the other end, then there

will be complete reflection of the waves resulting in standing waves. If the other end is also closed using shorting plate, then the hallow space so formed can support a signal which can bounce back and forth between the two shorting plates. This results in "**Resonance**". The hollow space so formed is called "**Cavity**" and the closed waveguide then becomes a "**Cavity Resonator**".

7) In a two-line lossless transmission line system, all the frequency signals are allowed to propagate. But in a waveguide, there exist a cut off frequency (f_c) below which propagation is not possible. i.e., all the frequencies above f_c are allowed to propagate and hence waveguide acts as a "**high pass filter**".

- A Rectangular waveguide made of metallic of high conductivity with perfect dielectric, such as air of magnetic permeability μ and permittivity ε inside the guide.
- For the conductor to dielectric interface, the boundary conditions are

Conductor
(High loss)

Boundary

Dielectric
(Low loss)

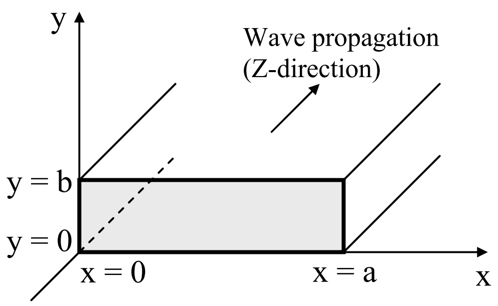
$E_{tan} = 0$

$D_n = \rho_S$

$H_t = J$

$B_{normal} = 0 \Rightarrow \mu H_n = 0$

$H_n = 0$



- Consider the wave is propagating in z-direction.
- There are four boundaries i.e. x = 0, x = a, y = 0, y = b

Boundary Conditions are

i) The tangential component of electric field must be zero $E_t = E_{tangential} = 0$

ii) The Normal component of magnetic field must be zero $H_n = H_{normal} = 0$

At y = 0 and y = b(XZ-plane)

$E_t = 0 \Rightarrow E_x = 0,\ E_z = 0$

$H_n = 0 \Rightarrow H_y = 0$

At x = 0 and x = a (YZ-plane)

$E_t = 0 \Rightarrow E_y = 0, E_z = 0$

$H_n = 0 \Rightarrow H_x = 0$

From the above boundary conditions we conclude that

- Electromagnetic waves do not pass through conductors, but rather, they are reflected.
- Any electric field that touches a conductor must be perpendicular to it.
- Any magnetic field close to a conductor must be parallel to it.
- Fields associated with a propagating wave inside the waveguide are expected to satisfy Maxwell equations, wave equations & boundary conditions.
- The Maxwell equations in time domain are expressed as

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} = J_D$$

$$\nabla . D = \rho_v$$

$$\nabla . B = 0$$

Here the dielectric occupy the hollow region of waveguide is either low loss (or) loss less $\left(\sigma = 0 (\text{or}) \frac{J_c}{J_D} << 1\right)$

The electric & magnetic field components are assumed to vary sinusoidally with respect to time.

$E = E_0 e^{-\bar{\gamma} z} e^{j\omega t}$ then

$\frac{\partial E}{\partial t} = E_0(j\omega)e^{-\bar{\gamma}z}e^{j\omega t} = j\omega E$ and $\frac{\partial E}{\partial z}$

$= E_0(-\bar{\gamma})e^{-\bar{\gamma}z}e^{j\omega t} = -\bar{\gamma}E$

$\frac{\partial^2 E}{\partial t^2} = E_0(j\omega)^2 e^{-\bar{\gamma}z}e^{j\omega t} = -\omega^2 E$ and $\frac{\partial^2 E}{\partial z^2}$

$= E_0(-\bar{\gamma})^2 e^{-\bar{\gamma}z}e^{j\omega t} = \bar{\gamma}^2 E$

$H = H_0 e^{-\bar{\gamma} z} e^{j\omega t}$ then

$\frac{\partial H}{\partial t} = H_0(j\omega)e^{-\bar{\gamma}z}e^{j\omega t} = j\omega H$ and $\frac{\partial H}{\partial z}$

$= H_0(-\bar{\gamma})e^{-\bar{\gamma}z}e^{j\omega t} = -\bar{\gamma}H$

$\frac{\partial^2 H}{\partial t^2} = H_0(j\omega)^2 e^{-\bar{\gamma}z}e^{j\omega t} = -\omega^2 H$ and $\frac{\partial^2 H}{\partial z^2}$

$= H_0(-\bar{\gamma})^2 e^{-\bar{\gamma}z}e^{j\omega t} = \bar{\gamma}^2 H$

- For time varying fields the Maxwell equations are

$\nabla \times H = j\omega\varepsilon E \rightarrow$ ME – 1 (Maxwell eq – 1)

$\nabla \times E = -j\omega\mu H \rightarrow$ ME – 2 (Maxwell eq – 2)

Consider Maxwell eq – (1) (ME-1)

$\nabla \times H = j\omega\varepsilon E$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right]$$

Equating the components on both sides

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

Rearranging the above equations

$$\frac{\partial H_z}{\partial y} + \bar{\gamma}H_y = j\omega\varepsilon E_x \quad --(i)$$

$$-\bar{\gamma}H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad --(ii)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad ---(iii)$$

Consider Maxwell eq – (2) (ME-2)

$\nabla \times E = -j\omega\mu H$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z\right]$$

Equating the components on both sides

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Rearranging the above equations

$$\frac{\partial E_z}{\partial y} + \bar{\gamma}E_y = -j\omega\mu H_x \quad --(i)$$

$$-\bar{\gamma}E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad ---(ii)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad ---(iii)$$

Combining (i) of ME-1 and (ii) of ME-2

From (ii) of ME-2

$$H_y = \frac{1}{j\omega\mu}\left[\bar{\gamma}E_x + \frac{\partial E_z}{\partial x}\right]$$

Substituting H_y in (1) of ME-1

$$\frac{\partial H_z}{\partial y} + \bar{\gamma}\frac{1}{j\omega\mu}\left[\bar{\gamma}E_x + \frac{\partial E_z}{\partial x}\right] = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y}+\frac{\bar{\gamma}^2}{j\omega\mu}E_x+\frac{\bar{\gamma}}{j\omega\mu}\frac{\partial E_z}{\partial x}=j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y}+\frac{\bar{\gamma}}{j\omega\mu}\frac{\partial E_z}{\partial x}=j\omega\varepsilon E_x-\frac{\bar{\gamma}^2}{j\omega\mu}E_x$$

$$\frac{\partial H_z}{\partial y}+\frac{\bar{\gamma}}{j\omega\mu}\frac{\partial E_z}{\partial x}=\left[\frac{-\omega^2\mu\varepsilon-\bar{\gamma}^2}{j\omega\mu}\right]E_x$$

$$\frac{\partial H_z}{\partial y}+\frac{\bar{\gamma}}{j\omega\mu}\frac{\partial E_z}{\partial x}=\frac{-h^2}{j\omega\mu}E_x$$

$$\therefore E_x=\frac{-\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x}-\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

Where $h^2=\bar{\gamma}^2+\omega^2\mu\varepsilon$

Similarly,

$$H_y=\frac{-\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y}-\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_x=\frac{-\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x}+\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$E_y=-\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y}+\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

The field components of the waveguide are

$$E_x=\frac{-\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x}-\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y=-\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y}+\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x=-\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x}+\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y=-\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y}-\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Where $h^2=\bar{\gamma}^2+\omega^2\mu\varepsilon$

We know for a TEM wave

$$E_z=0 \ \ and\ H_z=0$$

Substituting these values in above equations. The field components along x and y directions i.e. E_x, E_y, H_x, H_y vanish. "**Hence a TEM wave can't exist inside a wave guide**.

- Inspecting the above set of equations it can be concluded. That wave propagating inside the waveguide is either

  $TM(H_z=0\,\&\,E_z\neq 0)$ (or) $TE(H_z\neq 0\ \ E_z=0)$
- In other words there is no possibility of TEM wave propagating inside the waveguide in other words for a wave propagating inside the waveguide supporting by transverse electric & magnetic fields there must be one of the longitudinal existing i.e. when a wave propagates along the waveguide in z-direction either E_z field is present or H_z field is present.
- This implies that to support wave propagation inside the waveguide when $H_z = 0$, E_z field is present which is termed. TM-wave, where as when E_z fields is zero H_z field is present the wave is TE wave.

For TM wave $\Rightarrow H_z=0,\ E_z\neq 0$

For TE wave $\Rightarrow H_z\neq 0,\ E_z=0$

TM Wave (or) E-Wave (or) Transverse Magnetic Wave: $\Rightarrow H_z=0,\ E_z\neq 0$

The field equations are

$$E_x=-\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_y=-\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_x=\frac{j\omega\varepsilon}{h^2}\frac{\partial E_x}{\partial y}$$

$$H_y=-\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Where $h^2=\bar{\gamma}^2+\omega^2\mu\varepsilon$

$$\frac{E_x}{H_y}=\eta_{TM}=-\frac{E_y}{H_x}=\frac{\bar{\gamma}}{j\omega\varepsilon}\text{ for TM wave}$$

TE Wave (or) H-ware (or) Transverse Electric Wave: $\Rightarrow H_z \neq 0,\ E_z = 0$

The field equations are

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y}$$

Where $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$

$$\frac{E_x}{H_y} = \eta_{TE} = -\frac{E_y}{H_x} = \frac{j\omega\mu}{\bar{\gamma}} \text{ for TE wave}$$

From the above relationships

$$\eta_{TE}\eta_{TM} = \frac{\mu}{\varepsilon} = \left(\sqrt{\frac{\mu}{\varepsilon}}\right)^2 = (\eta_{TEM})^2$$

TM Wave solution:

For TM (Transverse magnetic)waves the magnetic field exists only along transverse directions and no component along the direction of propagation but Electric field components present in all directions.

The wave equations for waves propagating along the z-direction are given by

$$\nabla^2 E_z = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} \text{ and } H_z = 0 \text{ for TM wave}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2\mu\varepsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \bar{\gamma}^2 E_z = -\omega^2\mu\varepsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\bar{\gamma}^2 + \omega^2\mu\varepsilon)E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$$

Where $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$

The above equation solved by using "separation of variables" method.

Let us assume $E_z = XY$

X = a pure function of x only

Y = a pure function of y only.

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} + h^2 XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

Dividing both sides XY

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2\right] + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = 0$$

Let us assume

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = A^2 \text{ then } \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + A^2 = 0$$

Rearranging

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0 \Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0$$

Where $B^2 = h^2 - A^2$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \Rightarrow \frac{\partial^2 X}{\partial x^2} + B^2 X = 0$$

The general solutions are

$$X = C_1\cos Bx + C_2\sin Bx$$

Where C_1 and C_2 are constants.

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + A^2 = 0 \Rightarrow \frac{\partial^2 Y}{\partial y^2} + A^2 Y = 0$$

The general solutions are

$$Y = C_3\cos Ay + C_4\sin Ay$$

Where C_3 and C_4 are constants.

$\therefore E_z = XY =$

$(C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay + C_4\sin Ay)$

The above general solution is required to satisfy boundary conditions.

$$E_z = 0 \text{ at} \begin{Bmatrix} x=0 & x=a \\ y=0 & y=b \end{Bmatrix}$$

$$E_z\big|_{x=0} = C_1C_3\cos Ay + C_1C_4 \sin Ay$$

$E_z\big|_{x=0} = 0$ only when $\mathbf{C_1 = 0.}$

$$E_z\big|_{y=0} = C_1C_3\cos Bx + C_2C_3 \sin Bx$$

$E_z\big|_{y=0} = 0$ only when $\mathbf{C_3 = 0.}$

$$\therefore E_z = C_2C_4 \sin Bx \sin Ay$$

$$E_z\big|_{x=a} = C_2C_4 \sin Ba \ \sin Ay$$

$E_z\big|_{x=a} = 0$ only when

$\sin Ba = 0$

$Ba = m\pi$

$\therefore B = \dfrac{m\pi}{a}$ **w**here $m = 1,2,3,......$

$$E_z\big|_{x=b} = C_2C_4 \sin Bx \ \sin Ab$$

$E_z\big|_{x=b} = 0$ only when

$\sin Ab = 0$

$Ab = n\pi$

$\therefore A = \dfrac{n\pi}{b}$ **w**here $n = 1,2,3,......$

$$\therefore \mathbf{E_z = C_2C_4 \sin\left(\frac{m\pi}{a}\right)x \ \sin\left(\frac{n\pi}{b}\right)y}$$

TE wave solution:

For TE(Transverse electric)waves the electric field exists only along transverse directions and no component along the direction of propagation. But magnetic field components present in all directions.

The wave equations for waves propagating along the z-direction are given by

$\nabla^2 H_z = \mu\varepsilon \dfrac{\partial^2 H_z}{\partial t^2}$ and $E_z = 0$ for TE wave.

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2\mu\varepsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \bar{\gamma}^2 H_z = -\omega^2\mu\varepsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\bar{\gamma}^2 + \omega^2\mu\varepsilon) H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$

Where $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$

The above equation solved by using "separation of variables" method.

Let us assume $H_z = XY$

X= a pure function of x only

Y= a pure function of y only.

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} + h^2 XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

Dividing both sides XY

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2\right] + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = 0$$

Let us assume

$\dfrac{1}{X}\dfrac{\partial^2 X}{\partial x^2} + h^2 = A^2$ then $\dfrac{1}{Y}\dfrac{\partial^2 Y}{\partial y^2} + A^2 = 0$

Rearranging

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0 \Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0$$

Where $B^2 = h^2 - A^2$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \Rightarrow \frac{\partial^2 X}{\partial x^2} + B^2 X = 0$$

The general solutions are

$$X = C_5 \cos Bx + C_6 \sin Bx$$

Where C_5 and C_6 are constants.

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+A^2=0 \Rightarrow \frac{\partial^2 Y}{\partial y^2}+A^2 Y=0$$

The general solutions are

$$Y = C_7 \cos Ay + C_8 \sin Ay$$

Where C_7 and C_8 are constants.

$$\therefore H_z = XY$$

$$= (C_5 \cos Bx + C_6 \sin Bx)(C_7 \cos Ay + C_8 \sin Ay)$$

$$H_z = C_5C_7 \text{CosBxCosAy} + C_5C_8\text{CosBxSinAy} + C_6C_7\text{SinBxCosAy} + C_6C_8\text{SinBxSinAy}$$

The above general solution must be satisfy the boundary conditions are

$E_y = 0, E_z = 0, H_x = 0$ at $x = 0$ and $x = a$

$E_x = 0, E_z = 0, H_y = 0$ at $y = 0$ and $y = b$

TE Waves are

i) $E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$ $\quad H_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y}$

$$H_z = C_5C_7 \text{CosBxCosAy} + C_5C_8\text{CosBxSinAy} + C_6C_7\text{SinBxCosAy} + C_6C_8\text{SinBxSinAy}$$

$$\frac{\partial H_z}{\partial y} = 0 \text{ at } y = 0 \,\&\, y = b$$

$\Rightarrow$ Satisfies boundary conditions

$E_x \,\&\, H_y = 0$ at $y = 0 \,\&\, y = b$

Similarly,

$E_y = 0\, H_x = 0$ at $x = 0 \,\&\, x = a$

To satisfy this $\frac{\partial H_z}{\partial x} = 0$ at $x = 0 \,\&\, x = a$

$$\frac{\partial H_z}{\partial x} = C_5C_7(-B)\text{SinBxCosAy} + C_5C_8(-B)\text{SinBxSinAy} + C_6C_7B\,\text{CosBxCosAy} + C_6C_8B\,\text{CosBxSinAy}$$

$$\frac{\partial H_z}{\partial y} = C_5C_7(-A)\text{SinBxCosAy} + C_5C_8A\text{CosBxCosAy} + C_6C_7(-A)\text{SinBSinAy} + C_6C_8A\text{SinBxCosAy}$$

$$\left.\frac{\partial H_z}{\partial x}\right|_{x=0} = C_6C_7B\text{CosAy} + C_6C_8B\text{SinAy}$$

$$\left.\frac{\partial H_z}{\partial x}\right|_{x=0} = 0 \text{ only when } C_6 = 0$$

$$\left.\frac{\partial H_z}{\partial y}\right|_{y=0} = C_5C_8\text{CosBx} + C_6C_8A\text{SinBx}$$

$$\left.\frac{\partial H_z}{\partial y}\right|_{y=0} = 0 \text{ only when } C_8 = 0$$

$$\therefore \mathbf{H_z = C_5C_7 CosBxCosAy}$$

$$\frac{\partial H_z}{\partial x} = C_5C_7(-B)\text{SinBxCosAy}$$

$$\frac{\partial H_z}{\partial y} = C_5C_7(-A)\text{CosBxSinAy}$$

$$\left.\frac{\partial H_z}{\partial x}\right|_{x=a} = C_5C_7(-B)\text{SinBaCosAy}$$

$$\left.\frac{\partial H_z}{\partial x}\right|_{x=a} = 0 \text{ only when SinBa} = 0$$

$$\text{SinBa} = 0$$

$$Ba = m\pi$$

$$B = \frac{m\pi}{a} \quad \text{Where } m = 1,2,3,.........$$

$$\left.\frac{\partial H_z}{\partial y}\right|_{y=b} = C_5C_7(-A)\text{CosBxSinAb}$$

$$\left.\frac{\partial H_z}{\partial y}\right|_{y=b} = 0 \text{ only when SinAb} = 0$$

$\text{SinAb} = 0$

$\text{Ab} = n\pi$

$\therefore A = \frac{n\pi}{b}$ Where $n = 1,2,3.....$

$$\therefore \mathbf{H_z = C_5C_7Cos\left(\frac{m\pi}{a}x\right)Cos\left(\frac{n\pi}{b}y\right)}$$

The Field Equations are:

$$E_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x} = -\frac{\bar{\gamma}}{h^2}C_2C_4BCosBxSinAy$$

$$E_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial y} = -\frac{\bar{\gamma}}{h^2}C_2C_4ASinBxCosAy$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} = \frac{j\omega\varepsilon}{h^2}C_2C_4ASinBxCosAy$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} = -\frac{j\omega\varepsilon}{h^2}C_2C_4BCosBxSinAy$$

Where $B = \frac{m\pi}{a}$, $A = \frac{n\pi}{b}$

Depending on the values of m and n, we have varies modes in TM waves. In general we represent the modes are TM_{mn}.

Various TM_{mn} Modes:

1) **TM_{00} Mode:**
For which m = 0 and n = 0, we observe that all field components E_x, E_y, H_x, H_y are vanish inside the waveguide.
Hence **TM_{00} Mode can't exist inside the waveguide.**

2) **TM_{01} Mode:**
For m = 0, n = 1 then all the fields are vanish "**Hence TM_{01} does not exist"**

3) **TM_{10} Mode:**
i.e. m=1 and n = 0. In this mode also all the fields are vanish
$\therefore TM_{10}$ mode does not exist.

4) **TM_{11} Mode:**
m =1 and n = 1. In this mode all the fields are not vanish. This mode is exist in rectangular waveguide.
$\therefore$ TM_{11} mode exist

- **In a rectangular waveguide TM_{00}, TM_{01}, TM_{10} modes does not exist.**
- **The lowest TM mode that can exist in a rectangular waveguide is TM_{11}.**
$\therefore TM_{m\neq 0, n\neq 0}$ is the propagating TM wave in the rectangular waveguide.

The field Equations are:

$$E_x = -\frac{j\omega\varepsilon}{h^2}\frac{\partial H_z}{\partial y} = -\frac{j\omega\mu}{h^2}C_5C_7(-A)CosBxSinAy$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} = \frac{j\omega\mu}{h^2}C_5C_7(-B)SinBxCosAy.$$

$$H_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} = -\frac{\bar{\gamma}}{h^2}C_5C_7(-B)SinBxCosAy$$

$$H_y = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial y} = -\frac{\bar{\gamma}}{h^2}C_5C_7(-A)CosBxSinAy$$

Where $B = \frac{m\pi}{a}$, $A = \frac{n\pi}{b}$, $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$

Depending on the values of m and n, we have varies modes in TE waves. In general we represent the modes are TE_{mn}.

Various TE_{mn} Modes:

1) **TE_{00} Mode:**
m = 0 and n = 0 all field components Vanish inside the waveguide.
Hence TE_{00} doesn't exist.

2) **TE_{01} Mode:**
Form = 0 and n =1 $\Rightarrow$ $E_y = 0$, $H_x = 0$, E_x and H_y are exist.
$\therefore$ TE_{01} mode exists inside the waveguide.

3) **TE_{10} Mode:**
For m =1 & n =0 $\Rightarrow$ $E_x = 0$, $H_y = 0$, E_y&H_x are exist
$\therefore$ TE_{10} mode exist

4) TE_{11} Mode and all other higher modes Can exist inside the waveguide
$\therefore TE_{00}$ mode doesn't exist in rectangular wave guide.

- TE_{00} mode $\Rightarrow$ H_z component is constant. Then all E_x, H_y, H_x, E_y are Zero.
- TE_{m0} modes does exist for all values of m except m = 0. i.e. TE_{10}, TE_{20}, …… are exist

- TE_{on} does exist for all values of n except n = 0. i.e. TE_{01}, TE_{02}, …… are exist
- The lowest values of n for TE_{on} exist is n = 1
- **In Rectangular waveguide TM_{00}, TM_{0n}, TM_{m0}, TE_{00} modes doesn't exist.**

Propagation Characteristics

Note: The cut off frequency, cut off wave length, and phase and group velocities are same for TE and TM modes.

We know that

$$h^2 = \bar{\gamma}^2 + w^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Where m, n are integers.
a = width of the wave guide
b = height of the wave guide

$$\bar{\gamma}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

The Propagation constant of the waveguide is

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \bar{\alpha} + j\bar{\beta}$$

Case (i): Cut-off frequency:

We observe that all the lower frequencies are attenuated completely and higher frequencies are propagated. Thus there must exist a frequency at which the propagation just begins. This frequency is called "**Cutoff Frequency**" or "**Threshold Frequency**" denoted by **fc**.

At cutoff frequency $f = f_c$ (or) $\omega_c = 2\pi f_c$.

There is no wave propagation.

i.e.at $f = f_c, \bar{\gamma} = 0 (\text{or}) \bar{\alpha} = 0 = \bar{\beta}$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon} = 0$$

The value of ω that causes this is called the **cutoff angular frequency** (ω_c) that is

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Cutoff frequency is

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$\therefore$ Cutoff wavelength

$$\boxed{\lambda_c = \frac{v}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$

$\therefore$ Note that the cutoff frequency for a particular rectangular waveguide mode depends on the dimensions of the waveguide (a, b), the material inside the waveguide (ε,μ), and the indices of the mode (m, n).

- The dominant mode in a particular guide is the mode having the lowest cutoff frequency. All the frequencies greater than f_c is allowed to propagate inside the waveguide and those less than f_c are attenuated.
- All wave lengths greater than λ_c are attenuated and those less than λ_c are allowed to propagate inside the waveguide.

Case (ii) (Evanescent):

The Propagation constant of the waveguide is

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \bar{\alpha} + j\bar{\beta}$$

When a wave guide is exited at frequencies less than cutoff the behavior is entirely different from the behavior at frequencies greater than cutoff.

- At low frequencies i.e.

$f < f_c$ or $\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, $\bar{\gamma}$ is real and positive equal to attenuation constant $\bar{\alpha}$. Therefore the wave is completely attenuated. Also there is no phase shift and hence the wave can't propagate, i.e. $\bar{\gamma} = \bar{\alpha}$; $\bar{\beta} = 0$

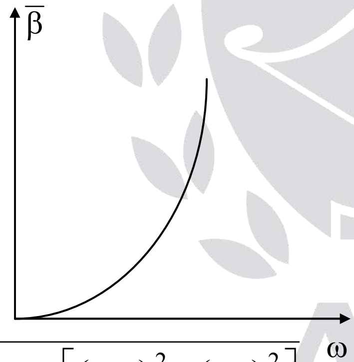
- In this case no propagation at all. These non-propagating (or) attenuating modes are said to be "**Evanescent**".

Case (iii) (propagation)

- At high frequencies i.e. $f > f_c$ or $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, $\bar{\gamma}$ become imaginary equal to $j\bar{\beta}$ phase shift occurs with respect to some reference and hence the wave propagates with some wave lengths inside the waveguide.
- In a two-line lossless transmission line system, all the frequency signals are allowed to propagate. But in a waveguide, there exist a cut off frequency (f_c) below which propagation is not possible. i.e., all the frequencies above f_c are allowed to propagate and hence waveguide acts as a "**high pass filter**".

$$\bar{\gamma} = j\bar{\beta} \quad \bar{\alpha} = 0$$

The phase constant $\bar{\beta}$ becomes



$$\bar{\beta} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$= \sqrt{\omega^2\mu\varepsilon - \omega_c^{\,2}\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

- The phase constant of the wave propagating inside the waveguide that is $\bar{\beta}$ is a non-linear function of frequency. This implies that wave propagation is dispersive type inside the waveguide i.e. the wave changes non-linearly with the frequency.

$$\bar{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

Phase velocity

$$v_p = \frac{\omega}{\bar{\beta}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{v}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \text{ and } v_p > v$$

$v =$ velocity of the wave in freespace

- The phase velocity is defined as the rate at which the wave changes its phase in terms of guide wave length. or The velocity at which a point of constant phase moves.
- The velocity at which energy is transported down the length of the waveguide is defined as the *group velocity*.
- In the waveguide phase velocity is not equal to the velocity of energy transport along the waveguide (group velocity).
- The information in a wave guide generally does not travel at the phase velocity. Information travels at the group velocity, which must be less than the speed of light.
- Note: The velocity of propagation for a TEM wave (plane wave or transmission line wave) is referred to as the *phase velocity* (the velocity at which a point of constant phase moves). The phase velocity of a TEM wave is equal to the velocity of energy transport.

Group Velocity $v_g = \dfrac{1}{\left(\dfrac{d\bar{\beta}}{d\omega}\right)}$

$$\frac{d\bar{\beta}}{d\omega} = \frac{d}{d\omega}\left[\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}\right] = \sqrt{\mu\varepsilon}\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$v_g = \frac{1}{\left(\frac{d\bar{\beta}}{d\omega}\right)} = \frac{1}{\sqrt{\mu\varepsilon}\,\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$$

$$= \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = v\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$v_g < v$ $\quad v =$ velocity of wave in free space.

From the above we conclude that

$v_p > v. > v_g$ (or) $v_g < v < v_p$ and $v_g v_p = v^2$

v = velocity of wave in unbounded dielectric medium.

v_p = Phase velocity of the wave in waveguide.

v_g = Group velocity of the wave in waveguide.

$$\therefore \bar{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \Rightarrow \frac{2\pi}{\bar{\lambda}} = 2\pi f\frac{1}{v}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\Rightarrow \frac{1}{\bar{\lambda}} = \frac{f}{v_p} \Rightarrow v_p = \bar{\lambda} f$$

$$\frac{1}{\bar{\lambda}} = \frac{1}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2} \Rightarrow \bar{\lambda}$$

$$= \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

and

$\bar{\lambda} > \lambda$

$\lambda_c < \lambda$

$\therefore \bar{\lambda}$ = guided wave length $\Rightarrow$ propagating wave length inside the waveguide.

$$\left(\frac{1}{\bar{\lambda}}\right)^2 = \left(\frac{1}{\lambda}\right)^2\left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right] = \left(\frac{1}{\lambda}\right)^2 - \left(\frac{1}{\lambda_c}\right)^2$$

$$\left(\frac{1}{\lambda}\right)^2 = \left(\frac{1}{\bar{\lambda}}\right)^2 + \left(\frac{1}{\lambda_c}\right)^2$$

Where

λ_c = cutoff wavelength

λ = free space wave length

$\bar{\lambda}$ = guide wave length.

- We conclude that the wave length inside the waveguide is greater than the wave length outside the waveguide i.e. $\bar{\lambda} > \lambda$
- Wave propagate through the waveguide only when $\lambda_c < \lambda$

The relation between phase velocity and guided wavelength of waveguide is

$$v_p = \left(\frac{\bar{\lambda}}{\lambda}v\right) \Rightarrow \frac{v_p}{v} = \frac{\bar{\lambda}}{\lambda} \Rightarrow \frac{v}{v_g} = \frac{\bar{\lambda}}{\lambda}.$$

- We already know that

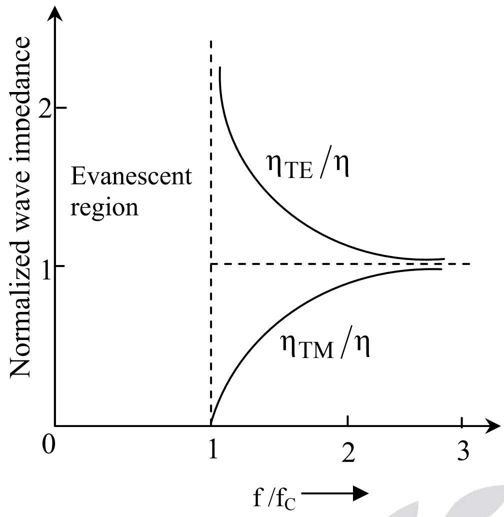
$$\eta_{TE} = \frac{j\omega\mu}{\bar{\gamma}} = \frac{j\omega\mu}{j\bar{\beta}} = \frac{\omega\mu}{\bar{\beta}}$$

$$\eta_{TE} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \text{i.e.} \therefore \eta_{TE} > \eta_{TEM}$$

$$\eta_{TM} = \frac{\bar{\gamma}}{j\omega\varepsilon} = \frac{j\bar{\beta}}{j\omega\varepsilon} = \frac{\bar{\beta}}{\omega\varepsilon} = \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \eta_{TEM}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$



i.e. $\therefore \eta_{TM} < \eta_{TEM}$

From the above we conclude that

$\eta_{TM} < \eta_{TEM} < \eta_{TE}$ $\qquad \eta_{TE}\eta_{TM} = (\eta_{TEM})^2$

Chapter 5 Elements of Antennas

Objective Practice Solutions

01. Ans: (c)

Sol: Antenna receives 2 μW of power: $P_r = 2\ \mu W$

RMS value of incident E field $= 20$ mV/m

Power density, P_d

$$= \frac{E^2}{\eta} = \frac{(20\times10^{-3})^2}{377} W/m^2$$

Effective aperture area, $A_e = \frac{P_r}{P_d}$

$$= \frac{2\times10^{-6}}{\frac{(20\times10^{-3})^2}{377}} = \frac{377\times 2}{400} = 1.885\ m^2$$

02. Ans: (b)

Sol: Lossless antenna directive gain = 6 dB = 4

Input power to the antenna = 1 mW

for lossless we get 100% efficiency

$$\frac{W_{rad}}{W_{in}} = \frac{G_o}{D_o} = 1$$

$W_{rad} = W_{in}$

$W_{rad} = 1mW$

03. Ans: (c)

Sol: $P_{rad} = \frac{A_0 \sin^2\theta}{r^2}\hat{a}_r \quad W/m^2$

$$W_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{A_0\sin^2\theta}{r^2} r^2 \sin\theta d\theta d\phi$$

$$= A_0\, 2\pi \int_{\theta=0}^{\pi} \sin^3\theta d\theta$$

$$= A_0\, 2\pi \frac{4}{3}$$

$$W_{rad} = A_0 \frac{8\pi}{3}$$

$$U = r^2 P_{rad} = r^2 \frac{A_0 \sin^2\theta}{r^2} = A_0 \sin^2\theta$$

$$D_{max} = \frac{U_{max}}{W_{rad}} 4\pi = \frac{|A_0 \sin^2\theta|_{max}}{\frac{8\pi}{3}A_0} \times 4\pi$$

$$= \frac{4\pi A_0}{8\pi A_0}\times 3$$

$$= \frac{3}{2} = D_{max} = 1.5$$

04. Ans: (d)

Sol: Where $W_{rad} = \oiint \overline{P}_{rad}.\overline{ds}$

$$\overline{P}_{rad} = \frac{W_{rad}}{2\pi r^2}.\hat{a}_r = \frac{40}{\pi}\hat{a}_r\ \mu W/m^2$$

05. Ans: (b)

Sol: $R_{rad} = 30\ \Omega$, $R_l = 10\Omega$

$G_D = 4$, $G_p = ?$

$$\eta = \frac{R_{rad}}{R_{rad} + R_\ell} = \frac{30}{40} = 0.75$$

$$G_p = \eta\ G_D$$

$$= 0.75 \times 4 = 3$$

06. Ans: (c)

Sol: $D_g = 30$ dB $= 1000$

$P_T = 7.5$ kW

$$D_g = \frac{4\pi \times \text{Radiation intensity}}{\text{Radiated Power}}$$

$D_g = 4\pi \dfrac{U}{W_{rad}}$

$\therefore\ U = \dfrac{7.5\times10^3 \times 1000}{4\pi}$

$\Rightarrow U = r^2 P_{rad}$

P_{rad} : Power density we have to find

P_{rad} at $r = 40\times10^3$ m

$$P_{rad} = \frac{U}{r^2}$$
$$= \frac{7.5\times10^3\times1000}{4\pi\times(40\times10^3)^2}\ \text{W/m}^2$$

07. Ans: (d)

Sol: $W_{rad} = 10$kW

$E_{max} = 120$ mV/m

R = 20km

$\eta = 98\%$

$$P_{rad} = \frac{E_0^2}{2\eta_0}$$
$$= \frac{(120\times10^{-3})^2}{2\times120\pi}$$
$$= 1.909\times10^{-5}$$

$U_{max} = (20\times10^3)^2\times1.909\times10^{-5}$

$= 7636$

$$D_0 = 4\pi\frac{U_{max}}{W_{rad}}$$

$$D_0 = 4\pi\frac{7639.43}{10\times10^3} = 9.59$$

$$\eta = \frac{G_0}{D_0} = 0.98$$

$G_0 = 0.98\times9.59$

$= 9.407$

08. Ans: 0.21

Sol: Given:

Antenna length, l = 1cm

Frequency, f = 1 GHz

Distance, r = 100λ

Wave length, $\lambda = \dfrac{C}{f}$

$$= \frac{3\times10^8}{10^9}$$

$= 30$ cm

$\dfrac{d\ell}{\lambda} = \dfrac{1}{30}$, hence the given antenna is Hertzian dipole.

In the far field, the tangential electric field is given by, $E_\theta = \dfrac{j\eta I d\ell \sin\theta}{4\pi}\dfrac{\beta}{r}$

$$= \frac{j377\times100\times10^{-3}\times2\pi\times10^{-2}\times1}{30\times10^{-2}\times4\pi\times100\times30\times10^{-2}}$$

$$\therefore |E_\theta| = 0.21\text{V/cm}$$

09. Ans: (c)

Sol: Given:

Length of dipole, $\ell = 0.01\lambda$

As it is very small, compared with wavelength, hence it can be approximated to Hertzian dipole

$$R_{rad} = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2$$

$= 80\ \pi^2\ (0.01)^2$

$R_{rad} = 0.08\ \Omega$

10. Ans: (d)

Sol: $AF = \dfrac{\sin\dfrac{n\varphi}{2}}{\sin\dfrac{\varphi}{2}}$

take limit

$$\frac{\underset{\frac{n\varphi}{2}\to 0}{Lt}\dfrac{\sin\frac{n\varphi}{2}}{\frac{n\varphi}{2}}.\dfrac{n\varphi}{2}}{\underset{\frac{\varphi}{2}\to 0}{Lt}\dfrac{\sin\frac{\varphi}{2}}{\frac{\varphi}{2}}.\dfrac{\varphi}{2}} = n$$

11. Ans: (b)

Sol: In broad side array the BWFN is given by

$BWFN = \dfrac{2\lambda}{L}(rad)$

Where, L = length of the array

L = (n–1) d

Given: n = 9

Spacing, $d = \dfrac{\lambda}{4}$

$BWFN = \dfrac{2\lambda}{(9-1)\dfrac{\lambda}{4}}$

$= \dfrac{2\lambda}{2\lambda}\times\dfrac{180}{\pi}$

$\therefore$ BWFN = 57.29°

12. Ans: (d)

Sol: The directivity of n-element end fire array is given by

$D = \dfrac{4L}{\lambda}$

Where, L = (n–1)d

$L \cong nd$ ($\because n = 1000$, very large)

$D = \dfrac{4\times nd}{\lambda}$

$= \dfrac{4\times 1000\lambda}{\lambda\times 4}$

$\therefore D \simeq 1000$

Directivity, (in dB) = 30

13. Ans: 7.78

Sol: Directivity, $D = 4\pi\dfrac{U_{max}}{P_{rad}}$

Given: $U(\theta, \phi) = 2\sin\theta\sin^3\phi$; $0 \le \theta \le \pi$, $0 \le \phi \le \pi$

$U_{max} = 2$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi} 2\sin\theta\sin^3\phi\sin\theta d\theta d\phi$$

$$= 2\int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi}\sin^2\theta\sin^3\phi d\theta d\phi$$

$$= 2\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right)$$

$$= \frac{4\pi}{3}$$

$$D = 4\pi\times\frac{2}{\left(\frac{4\pi}{3}\right)}$$

D = 6

Directivity, (in dB) = 10log6 = 7.7815

14. Ans: 2793

Sol: For Hertzian dipole the directivity, D is given by D = 1.5

$$D = \left(\frac{4\pi}{\lambda^2}\right)A_e$$

$$A_e = 1.5 \times \frac{\lambda^2}{4\pi}$$

$A_e = 0.119\ \lambda^2$

Wavelength, $\lambda = \frac{3\times10^8}{10^8} = 3\text{m}$

$\therefore A_e = 0.119 \times 9$

$A_e = 1.074\ \text{m}^2$

Aperture area of antenna is given by

$$A_e = \frac{P_r}{P}$$

Where, P_r = power received at the antenna load terminals.

P = power density of incident wave

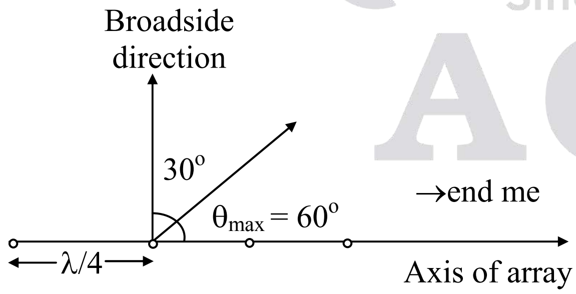
$$P = \frac{P_r}{A_e}$$

$$= \frac{3\times10^{-6}}{1.074}$$

$\therefore$ P = 2.793 μW/m^2 (or) 2793 nW/m^2

15. Ans: (c)

Sol:



Given: No. of elements, n = 4

Spacing, $d = \frac{\lambda}{4}$

Direction of main beam (or) principal lobe, $\theta_{max} = 60^o$

Array phase function, ψ is given by

$\psi = \beta d\cos\theta + \alpha$

To form a major lobe. $\psi = 0$

$\alpha = -\beta d\cos\theta_{max}$

$$\alpha = -\frac{2\pi}{\lambda}\times\frac{\lambda}{4}\cos 60$$

$$\alpha = -\frac{\pi^C}{4}$$

The phase shaft between the elements required is $\alpha = -\frac{\pi^C}{4}$

16. Ans: (b)

Sol: Quarter wave monopole radiates in the upper hemisphere only

Statement (I) is true.

All dipole antennas are half wave dipole antennas

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

17. Ans: (b)

Sol: Isotropic radiator radiates uniformly in the all the directions. So it is a non-directional antenna.

Statement (I) is true.

Isotropic radiator is taken as reference antenna

Statement (II) is true.

Statement (II) is not the correct explanation of Statement (I).

18. Ans: (b)

Sol: $A_e \propto D$

$$A_e = \frac{\lambda^2}{4\pi}D$$

If A_e - high

D - high

Statement (I) is true.

$$D = \frac{4\pi}{\Omega_A}$$

$D \propto \frac{1}{\Omega_A}$

Ω_A - Beam area

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

19. Ans: (a)

Sol: Omni directional antennas will radiates uniform radiations in azimuthal planes and non-uniform radiation in the elevation planes.

Statement (I) is true.

Hertzian dipole antenna is omni directional antenna.

Statement (II) is true and correct explanation for Statement (I).

20. Ans: (b)

Sol: Polarization of antenna is one of the design parameters of an antenna.

Statement (I) is true.

Polarization of wave is the property of the wave.

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

21. Ans: (b)

Sol: Antenna array would result high directivity

Statement (I) is true.

High directivity antennas are used for point to point communications.

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

22. Ans: (b)

Sol: Fields in Fraunhofer near field zone are reactive fields.

Statement (I) is true.

Antennas are operated in the Fraunhofer far field zone only.

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

23. Ans: (b)

Sol: For lossless antennas directivity and power gain are same

Statement (I) is true.

Radiation intensities is defined as power radiated per unit solid angle

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

24. Ans: (a)

Sol: For broad side antennas the maximum radiation is normal to the array axis.

Statement (I) is true.

For maximum radiation normal to array axis the antennas are excited with uniform amplitudes and no progressive phase shift.

Statement (II) is true.

Statement (II) is the Correct explanation of Statement (I).

25. Ans: (b)

Sol: The maximum value of directive gain are called directivity

Statement (I) is true.

Isotropic radiator (non-directional) antenna.

Directivity is unity.

Statement (II) is true.

But Statement (II) is not the correct explanation of Statement (I).

26. Ans: (b)

Sol: The array factor is unique for a particularly geometry of antenna array

Statement (I) is true.

Over all radiation can not be obtained by array factor.

Statement (II) is true but not the correct explanation for statement (I).

Conventional Practice Solutions

01.

Sol: A 2 - element array is shown in Fig. 1. where the elements are marked as 1 and 2 with $I_2 = k\, I_1 \angle\alpha$ spacing = d and direction of radiation with line of antennas = ϕ

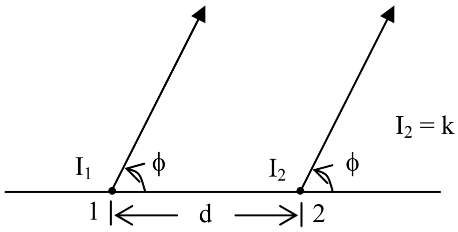


Fig. 1

In the present problem,

$I_2 = I_1 = I\angle 0$, $k = \dfrac{|I_2|}{|I_1|} = 1$

$\alpha = 0$ and $d = 0.5\,\lambda$

General formula for n-elements with k = 1

$$\frac{E_T}{E_1} = \left|\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

$$\psi = \beta d \cos\phi + \alpha,\quad \beta = \frac{2\pi}{\lambda}$$

where E_1 is the field strength due to antenna 1 alone and E_T is the magnitude of the total field strength due to both the antennas.

For n = 2, $\alpha = 0$, $d = 0.5\,\lambda$

$$\psi = \frac{2\pi}{\lambda}\, 0.5\lambda\, \cos(\phi) = \pi\cos\phi$$

$$\frac{E_T}{E_1} = \left|\frac{2\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

$$= \left|2\cos\left(\frac{\psi}{2}\right)\right| = \left|2\cos\left(\frac{\pi}{2}\cos\phi\right)\right|$$

The radiation pattern is shown in

ϕ	E_T / E_0
0°	0
90°	2
180°	0
–90°	2
360°	0

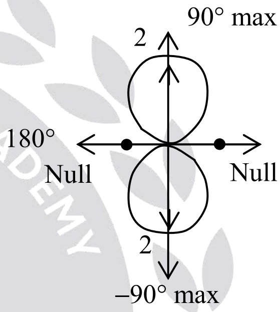


Fig. 2

Directions of maximum radiation are $\phi = \pm 90°$ (Broadside array)

By turning the direction of maximum radiation by 90° either clockwise or anticlockwise, the radiation pattern is as shown in Fig.3.

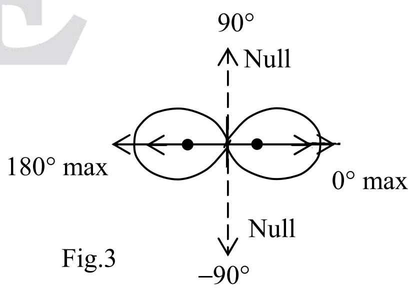


Fig.3

Direction of maximum radiation are 0° and 180° (End-fire array)

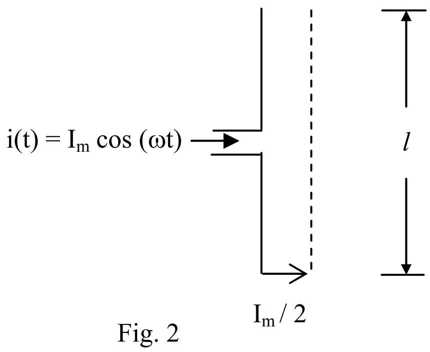
≡ 

Fig. 2

Let $\alpha = \alpha_x$ be the phase shift required for this change in the pattern

$$\frac{E_T}{E_0} = \left|2\cos\left(\frac{\pi}{2}\cos\phi + \frac{\alpha_x}{2}\right)\right|$$

$$2 = \left|2\cos\left(\frac{\pi}{2}\cos\left(0^\circ \text{ or } 180^\circ\right) + \frac{\alpha_x}{2}\right)\right|$$

$$1 = \left|\cos\left(\pm 90^\circ + \frac{\alpha_x}{2}\right)\right|$$

$$1 = \left|\mp\sin\left(\frac{\alpha_x}{2}\right)\right|$$

$\alpha_x = \pm 180°$

Note that the requirement of $\alpha = -\beta d$ (= ±180°) is satisfied for the end-fire array in Fig. 3.

02.

Sol: The 2 – element array of antennas A and B is shown in Fig. 1, where ϕ is the direction of radiation from the line of antennas and $\theta = 90° - \phi$.

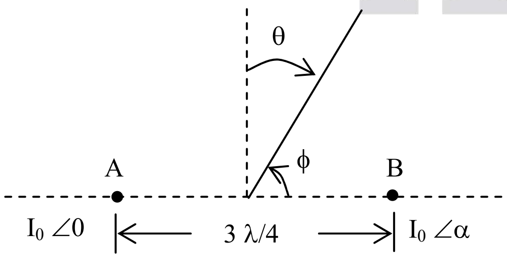


Fig. 1

General formula for $\frac{|I_B|}{|I_A|} = k = 1$

$$\frac{E_T}{E_A} = 2\cos\left(\frac{\psi}{2}\right),\ \psi = \beta d\cos(\phi) + \alpha$$

$$\beta = \frac{2\pi}{\lambda},\ d = \frac{3\lambda}{4},\ \beta d = \frac{3\pi}{2}$$

$$\psi = \frac{3\pi}{2}\cos(\phi) + \alpha$$

$$\frac{E_T}{E_A} = 2\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\alpha}{2}\right)$$

For the null at $\theta = 30°$ or $\phi = 60°$, α = ?

$$0 = 2\cos\left(\frac{3\pi}{8} + \frac{\alpha}{2}\right),$$

$$\frac{3\pi}{8} + \frac{\alpha}{2} = \frac{\pi}{2},\ \alpha = \frac{\pi}{4}$$

With this value of α,

$$\frac{E_T}{E_A} = 2\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\pi}{8}\right) \quad \text{…. (I)}$$

For maximum radiation, $\left|\frac{E_T}{E_A}\right| = 2$

$$\frac{3\pi}{4}\cos(\phi_m) + \frac{\pi}{8} = 0 \text{ or } \pm\pi$$

$$\cos(\phi_m) = -\frac{1}{6},\ \phi_m = \pm 99.6^0$$

or $\theta_m = -9.6^0$ and -170.4^0

The radiation pattern is shown in Fig. 2. according to equation (I) with typical values of ϕ.

ϕ	$\psi/2$	E_T/E_A
0°	$\frac{7\pi}{8}$ or $\frac{-\pi}{8}$	1.85
60°	$\frac{\pi}{2}$	0
90°	$\frac{\pi}{8}$	1.85
99.6°	0	2
180°	$\frac{-5\pi}{8}$	0.77

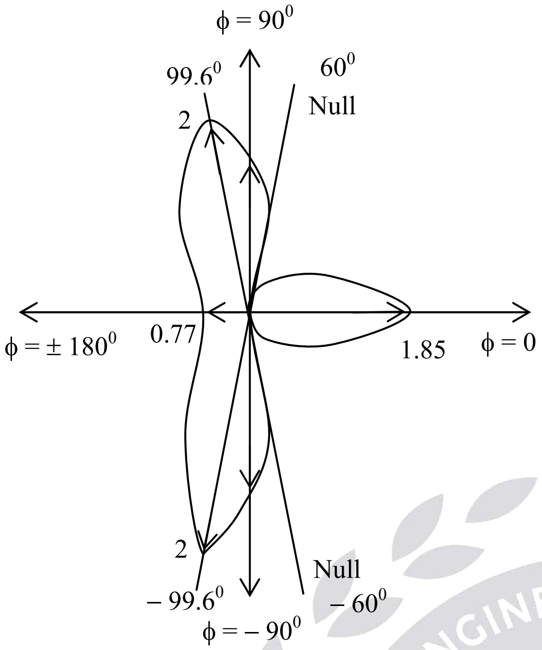


Fig. 2

03.

Sol: A dipole antenna having a sin θ radiation pattern can be considered as an Hertzian dipole or elementary dipole (I dl element, dl < < λ). Such a dipole 'A' vertically located above the ideal ground plane is shown in Fig.1

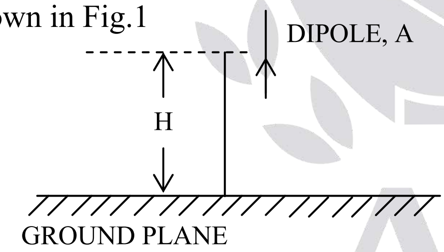


Fig. 1

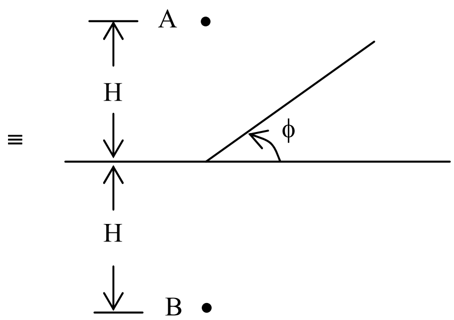


Fig. 2

Because of the perfect reflection from the ideal ground, this dipole configuration is equivalent to 2-element array as shown in Fig. 2, where 'B' is the mirror image antenna. Spacing between the antennas d = 2 H, k = 1, α = 0, $\beta d = \frac{2\pi}{\lambda}\times 2H = \frac{4\pi H}{\lambda}$

$$\therefore\ \frac{E_T}{E_A} = 2\cos\left(\frac{\psi}{2}\right)$$

$$\psi = \beta d\cos\phi = \frac{4\pi H}{\lambda}\cos\phi$$

$= 2$ (max.) , if $\psi = 0$

$= 0$ (min.) , if $\psi = \pi$

For Null at $\phi = 45°$

$$\psi = \pi = \frac{4\pi H}{\lambda}\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{H}{\lambda} = \frac{1}{2\sqrt{2}}\ ,\quad H = \frac{\lambda}{2\sqrt{2}}$$

For maximum radiation

$\psi = 0$, $\beta d\cos\phi = 0$, $\phi = 90°$

04.

Sol: (a) Linear array of two half – wave dipoles A and B is shown in Fig. 1.

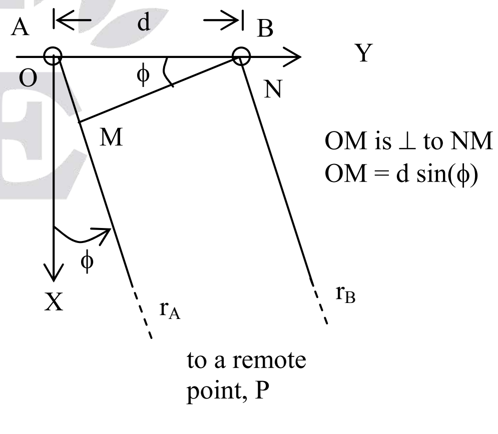


Fig. 1

Given: $d = \frac{\lambda}{4}$, $I_B = k\, I_A \angle\alpha$, $k = 1, \alpha = -90°$

The radius vectors $\vec{r}_A$ and $\vec{r}_B$ to a remote point, P can be considered parallel.
Then $r_B = r_A - OM = r_A - d\sin(\phi)$
$\therefore$ Path difference $= r_B - r_A$
$= -d\sin(\phi)$
Phase difference due to this path difference
$= -\beta d\sin(\phi)$, $\beta = \frac{2\pi}{\lambda}$

$\therefore$ Phase difference between radiations from the two antennas,
$\psi = \beta d\sin(\phi) + \alpha$ ……............ (I)
$r_B \approx r_A$ may be used as far as the magnitudes of the fields from the two antennas are concerned.
$\therefore$ Phasor sum of the fields will be
$E = E_A(1 + k e^{j\psi})$
where E_A is the field strength due to A alone.

The magnitude of the field strength
$E_T = |E_A(1 + k e^{j\psi})|$
$= E_A\sqrt{(1+k\cos\psi)^2 + k^2\sin^2(\psi)}$
For k = 1 as in the present problem
$\frac{E_T}{E_A} = 2\cos\left(\frac{\psi}{2}\right)$ …………… (II)

Radiation pattern for $\frac{E_T}{E_A}$ in the XY plane is given by equations (II) and (I).

(b) For $d = \frac{\lambda}{4}$, $\beta d = \frac{\pi}{2}$ and $\alpha = \frac{-\pi}{2}$
From (I)
$\frac{\psi}{2} = \frac{\pi}{4}\sin\phi - \frac{\pi}{4}$ …………… (III)
From (II) and (III)
$\frac{E_T}{E_A} = 2\cos\left[\left(\frac{\pi}{4}\sin\phi - \frac{\pi}{4}\right)\right]$ ………. (IV)

For different values of ϕ, values of $\frac{E_T}{E_A}$ are given below and the sketch of radiation pattern is shown in Fig. 2, with these values

ϕ	$\frac{\psi}{2}$	$\frac{E_T}{E_A}$
0	$\frac{-\pi}{4}$	$\sqrt{2}$
90°	0	2 (max)
180°	$\frac{-\pi}{4}$	$\sqrt{2}$
– 90°	$\frac{-\pi}{2}$	0 (null)

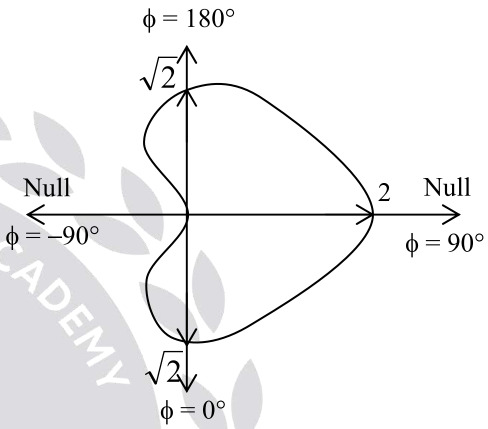


Fig. 2

05.
Sol: The three element array with elements 0, 1, 2 is shown in Fig. 1

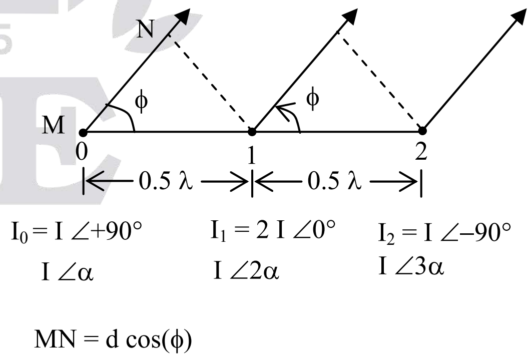


Fig.1

Element spacing d = 0.5 λ
Path difference = MN = d cos(ϕ)
Phase difference due to this path difference
$= \beta d\cos(\phi)$

$\beta d = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

Progressive Phase difference between the currents

$\alpha = \angle I_1 - \angle I_0 = \angle I_2 - \angle I_1$

$= -90° = -\frac{\pi}{2}$ rad

$k_1 = \left|\frac{I_1}{I_0}\right| = 2, \quad k_2 = \left|\frac{I_2}{I_0}\right| = 1$

Total progressive phase shift between successive radiations from the elements,

$\psi = \beta d \cos\phi + \alpha = \pi\cos\phi - \frac{\pi}{2}$

$$\frac{E_T}{E_0} = 1 + k_1 e^{j\psi} + k_2 e^{j2\psi}$$
$$= 1 + 2e^{j\psi} + e^{j2\psi}$$
$$= (1 + 2\cos\psi + 1\cos 2\psi) + j(2\sin\psi + \sin 2\psi)$$

$$\left|\frac{E_T}{E_0}\right|^2 = (1 + 2\cos\psi + \cos 2\psi)^2 + (2\sin\psi + \sin 2\psi)^2$$
$$= [2\cos\psi + 2\cos^2\psi]^2 + [2\sin\psi\,(1+\cos\psi)]^2$$
$$= [2\cos\psi\,(1+\cos\psi)]^2 + [2\sin\psi\,(1+\cos\psi)]^2$$
$$= (1+\cos\psi)^2\,(4\cos^2\psi + 4\sin^2\psi)$$
$$= 4\left[2\cos^2\frac{\psi}{2}\right]^2$$

$$\left|\frac{E_T}{E_0}\right|^2 = 16\cos^4\left(\frac{\psi}{2}\right)$$

$$\left|\frac{E_T}{E_0}\right| = 4\cos^2\left(\frac{\psi}{2}\right)$$

Differentiate w.r.t ψ

$\left|\frac{E_T}{E_0}\right|$ is maximum or minimum when :

$$4 \times 2\cos\left(\frac{\psi}{2}\right)\left(-\sin\frac{\psi}{2}\right)\frac{1}{2} = 0$$

$\sin\frac{\psi}{2}\cos\frac{\psi}{2} = 0$ or $\sin\psi = 0$

$\psi = 0$ (for max), $\psi = \pi$ (for min)

ψ	$\left\|\frac{E_T}{E_0}\right\|^2$
0	16 (max)
π or $-\pi$	0 (min)

$\psi = \pi\cos\phi - \frac{\pi}{2} = \pi\left(\cos\phi - \frac{1}{2}\right)$

$= 0$, when $\phi = \pm 60°$ (for max)

$= -\pi$, when $\cos\phi = -\frac{1}{2}$

$\phi = \pm 120°$ (for min)

$\therefore$ The direction of major lobe is $\pm 60°$

<u>Half - Power Beam width</u>

In terms of ψ, half power angles are given by

$16\cos^4\frac{\psi}{2} = \frac{1}{2} \times 16$

$\cos^2\frac{\psi}{2} = \frac{1}{\sqrt{2}}$

$\cos\frac{\psi}{2} = \sqrt{\frac{1}{\sqrt{2}}} = 0.841$

$\frac{\psi}{2} = \pm 32.8°, \quad \psi = \pm 65.6°$

In terms of ϕ, half power angles are given by

$\pi\cos\phi - \frac{\pi}{2} = \pm 65.6°$

$\cos\phi - \frac{1}{2} = \pm\frac{65.6}{180} = \pm 0.3644$

$\cos\phi = 0.8644$ or 0.1356

$\phi = \pm 30.2°$ or $\pm 82.2°$

$\therefore$Half - Power Beam width $= 52°$

$$\left.\begin{array}{l} \text{When } \phi = 0,\ \psi = \frac{\pi}{2} \\ \phi = \pm\frac{\pi}{2},\ \psi = -\frac{\pi}{2} \\ \phi = \pi,\ \psi = +\frac{\pi}{2} \end{array}\right\} \left|\frac{E_T}{E_0}\right| = 4$$

The power pattern or $\left|\frac{E_T}{E_0}\right|^2$ pattern is shown in Fig.2.

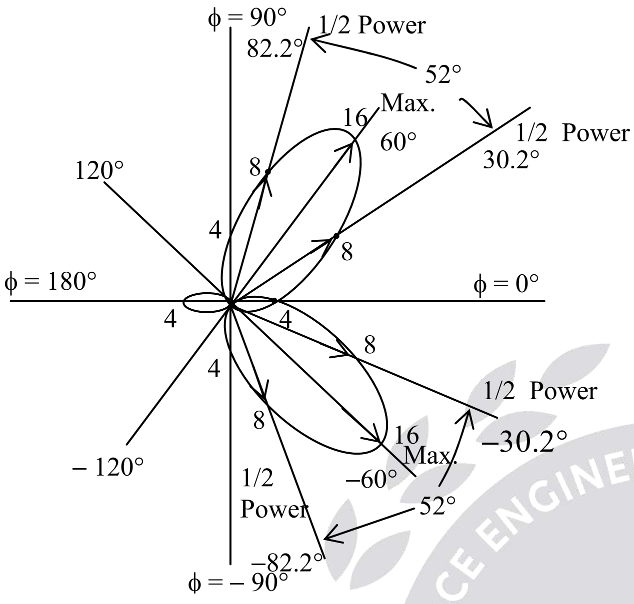


Fig .2

06.
Sol:

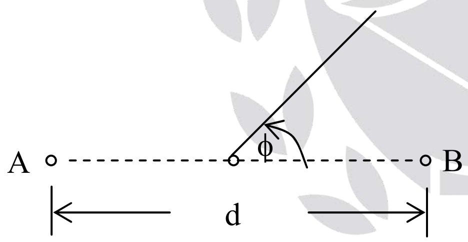


Fig. 1

The array of 2 half wave dipoles A and B is shown in Fig.1

Given : $d = 1.5\,\lambda$, $\beta d = \frac{2\pi}{\lambda} 1.5\,\lambda = 3\pi$

(a) $I_B = I_A \angle\alpha$, $|I_A| = |I_B|$

$k = 1$, $\alpha = 0$, (Broadside array)

$$\frac{E_T}{E_A} = 2\cos\left(\frac{\psi}{2}\right)$$

$\psi = \beta d \cos(\phi) + \alpha$

For $\alpha = 0$, $\beta d = 3\pi$

$\psi = 3\pi\cos(\phi)$

$$\frac{E_T}{E_A} = 2\cos\left(\frac{3\pi}{2}\cos(\phi)\right)$$

$\frac{E_T}{E_A}$ is maximum = 2, if $\phi = 90°$

For half-power beam width, BW = $2\,\Delta\phi$

$$2\cos\left[\frac{3\pi}{2}\cos(90 \pm \Delta\varphi\right] = \frac{1}{\sqrt{2}}2$$

$$\therefore \frac{3\pi}{2}\sin(\Delta\phi) = \frac{\pi}{4}$$

$$\sin(\Delta\phi) = \frac{1}{6},\quad \Delta\phi = 9.6°$$

Half power Beam width = $2\,\Delta\phi = 19.2°$

(b) $|I_A| = |I_B|$, $k = 1$

$\alpha = 540° = 3\pi$ rad $= \pm 3\pi = \pm\pi$

$$\psi = \beta d\cos(\phi) + \alpha$$
$$= 3\pi\cos(\phi) \pm 3\pi$$

$\alpha = -\beta d$ (End fire array)

$$\frac{E_T}{E_A} = 2\cos\left(\frac{\psi}{2}\right)$$
$$= 2\cos\left[\frac{3\pi}{2}\cos\phi \pm \frac{3\pi}{2}\right]$$

$\left|\frac{E_T}{E_A}\right|$ is maximum = 2

if $\frac{3\pi}{2}\cos\phi \pm \frac{3\pi}{2} = 0$

$\therefore \phi = 0$ and $180°$

Two principal maxima occur.

For half-power beam width = $2\,\Delta\phi$

$$2\cos\left[\frac{3\pi}{2}\cos(\Delta\varphi) - \frac{3\pi}{2}\right] = \frac{1}{\sqrt{2}}2$$

$$\frac{3\pi}{2}[\cos(\Delta\phi) - 1] = \pm\frac{\pi}{4}$$

$$[\cos(\Delta\phi) - 1] = \pm\frac{1}{6}$$

$$\cos(\Delta\phi) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\Delta\phi = 33.6°$$

Half-power beam width = $2\Delta\phi = 67.2°$

07.

Sol: The three element array is shown in Fig. 1.

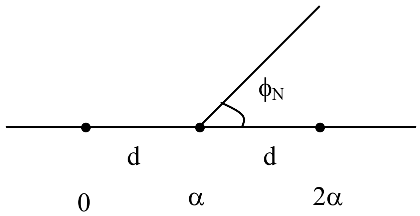


Fig. 1

$$d = \frac{\lambda}{4},\ \phi_N = 33.56^\circ$$

$$\beta d = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\psi = \beta d\cos\phi + \alpha$$

$$\frac{E_T}{E_0} = \left|\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right|,\quad n = 3$$

Nulls at $\frac{3\psi}{2} = \pm\pi, \pm 2\pi$, etc

$$\psi = \pm\frac{2}{3}\pi, \pm\frac{4}{3}\pi \text{ etc}$$

$$\beta d\cos\phi_N + \alpha = \pm\frac{2}{3}\pi$$

$$\alpha = \pm\frac{2}{3}\pi - \frac{\pi}{2}\cos 33.56^\circ$$

$$\alpha = \pm\frac{2}{3}\pi - \frac{\pi}{2}\times 0.8333$$

$$= \pm 120 - 75^\circ$$

$$= -195^\circ\ (165^\circ)\ or\ 45^\circ$$

Principal (main) beam at $\psi = 0$

$$\frac{E_T}{E_0} = n = 3$$

$$\beta d\cos\phi + \alpha = 0$$

$$\frac{\pi}{2}\cos\phi = -\alpha = -\frac{\pi}{4}$$

$$\cos\phi = -\frac{1}{2},\quad \phi = \pm 120^\circ$$

$\alpha = +165^\circ$, gives $\cos\phi > 1$, ϕ is imaginary (Not possible)

The radiation pattern is shown in Fig. 2. for the sake of practice.

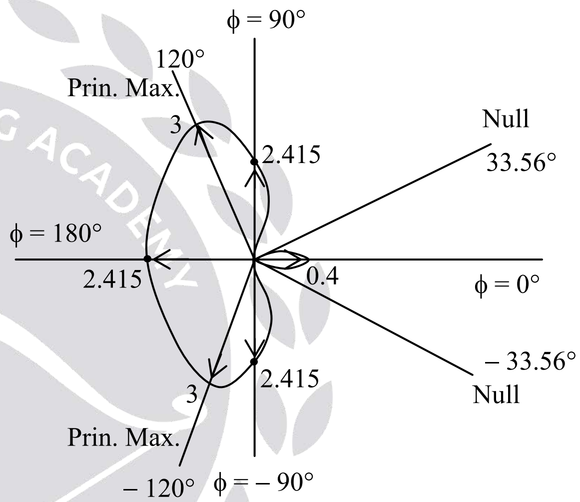


Fig. 2 Radiation Pattern