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Text Book : Theory with worked out Examples and Practice Questions

Chapter 1 Introduction

(Solutions for Text Book Practice Questions)

Objective Practice Solutions

01. Ans: (b)

Sol: We know that

$$e^{-at}u(t) \overset{F.T}{\leftrightarrow} \frac{1}{a+j\omega}$$

$$e^{at}u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{a-j\omega}$$

$$e^{-at}u(t) - e^{at}u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{a+j\omega} - \frac{1}{a-j\omega}$$

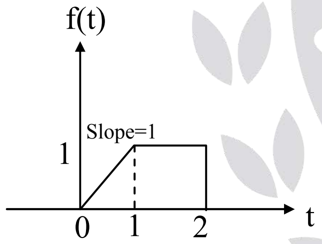
Put a = 0

$$u(t) - u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{j\omega} - \frac{1}{-j\omega}$$

$$\text{sgn}(t) \overset{F.T}{\leftrightarrow} \frac{2}{j\omega}$$

02. Ans: (a)

Sol:



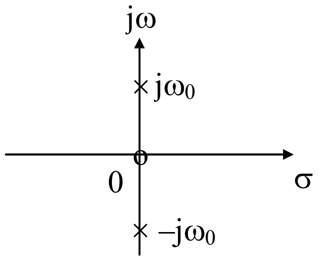
f(t) = r(t) – r(t – 1) – u(t – 2)

03. Ans: (a)

Sol: The convergence of Fourier transform is along the jω-axis in s-plane.

04. Ans: (a)

Sol:

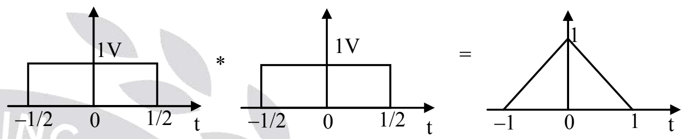


$$F(s) = \frac{s}{s^2+\omega_0^2} \overset{I.L.T}{\leftrightarrow} f(t) = \cos\omega_0 t$$

$$f(t) = \cos\omega_0 t \overset{I.L.T}{\leftrightarrow} F(\omega) = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

05. Ans: (d)

Sol:



06. Ans: (c)

Sol: Given $x(t) = e^{-at^2}$

Fourier transform of x(t) is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2} e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} e^{-\left(at^2+j\omega t\right)}dt$$

$$= e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-\left[\sqrt{a}t+\frac{j\omega}{2\sqrt{a}}\right]^2} dt$$

Let $p = \sqrt{a}t + \frac{j\omega}{2\sqrt{a}}$

$dp = \sqrt{a}dt$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2}dp$$

$$\int_{-\infty}^{\infty} e^{-p^2}dp = \sqrt{\pi}$$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}}\sqrt{\pi}$$

$$X(\omega) = \sqrt{\frac{\pi}{a}}e^{-\frac{\omega^2}{4a}}$$

07. Ans: (d)

Sol: The EFS expression of a periodic signal x(t) is $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$

where, 'c_n' is EFS coefficient.

Apply F.T on both sides

$$X(\omega) = \sum_{n=-\infty}^{\infty} c_n \, FT\left[e^{jn\omega_0 t}\right]$$

$1 \leftrightarrow 2\pi\delta(\omega)$

$e^{jn\omega_0 t} \leftrightarrow 2\pi\delta(\omega - n\omega_0)$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

So, it is a train of impulse.

08. Ans: (a)

Sol: $V(j\omega) = e^{-j2\omega}; \ |\omega| \le 1$

$$\text{Energy} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |V(j\omega)|^2 . d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \left|e^{-j2\omega}\right|^2 . d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} 1 d\omega$$

$$= \frac{2}{2\pi}$$

$$= \frac{1}{\pi}$$

09. Ans: (b)

Sol: Parseval's theorem is used to find the energy of the signal in frequency domain.

$$\therefore \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

10. Ans: (a)

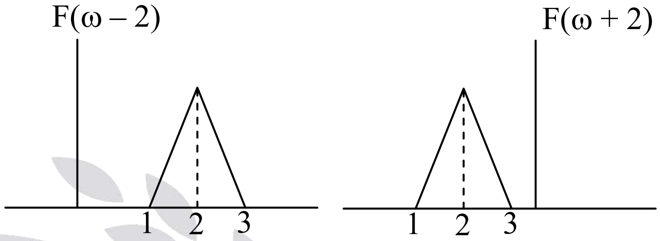
Sol: $f(t) = A.e^{-a|t|} \overset{F.T}{\leftrightarrow} F(j\omega) = \frac{2Aa}{a^2 + \omega^2}$

11. Ans: (d)

Sol: m(t) = f(t) cos2t

Apply Fourier transform

$$M(f) = \frac{1}{2}[F(\omega - 2) + F(\omega + 2)]$$



12. Ans: (b)

Sol: For band limited signals,

$S(f) \ne 0; \ |f| < W$

$S(f) = 0; \ |f| > W$

13. Ans: (a)

Sol: In a communication system, antenna is used to convert voltage variations to field variation and vice-versa.

14. Ans: (d)

Sol: Hilbert transform of f(t) is

$$H.T\{f(t)\} = f(t) * \frac{1}{\pi t}$$

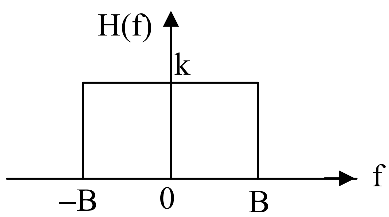
It is in the terms of 't'.

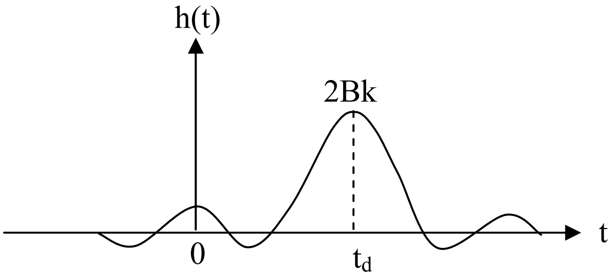
15. Ans: (a)

Sol: For an ideal LPF

$$H(f) = k \ e^{-j\omega t_0} \ \text{ for } -B < f < B$$

$$h(t) = F^{-1}[H(f)] = 2Bk \ \text{sinc} \ 2B\,(t - t_d)$$





$h(t) \neq 0$ for $t < 0$

Output exists before input is applied i.e. non-causal, which is physically impossible.

16. Ans: (b)

Sol: $\delta(at) = \frac{1}{|a|}\delta(t)$

$\delta(2t) = \frac{1}{2}\delta(t)$

17. Ans: (a)

Sol: By modulation we are translating the low frequency spectrum into high frequency spectrum.

18. Ans: (a)

Sol: We know that

$P(dBm) = 10\log(P\times10^3)$

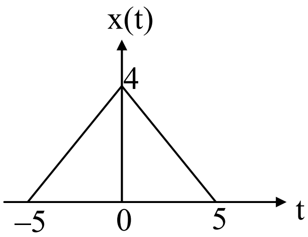
$-10 = 10\log(P\times10^3)$

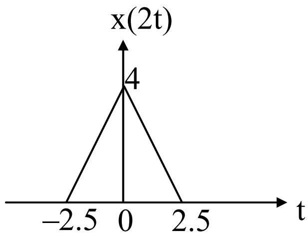
$P\times10^3 = 10^{-1}$

$P = 10^{-4} = 100\ \mu W$

19. Ans: (a)

Sol: $x(2t)$ means signal time axis is compressed by 2





20. Ans: (b)

Sol: Audio frequency is between 20Hz to 20kHz

21. Ans: (d)

Sol: Telephone channel carries voice. Voice frequency is between 300 Hz to 3500 Hz. So bandwidth is 3200Hz. So we approximately consider 4kHz is the bandwidth requirement of a telephone channel.

22. Ans: (c)

Sol: From the signal spectrum f_H = 530 kHz, f_L = 50 kHz

Bandwidth = $f_H - f_L$

= 530 kHz – 50 kHz

= 480 kHz

Chapter 2 Amplitude Modulation

Objective Practice Solutions

01. Ans: (a)

Sol: $V(t) = A_c.\cos\omega_c t + 2\cos\omega_m t\,.\,\cos\omega_c t.$

Comparing this with the AM-DSB-SC signal

$A\cos\omega_c t + m(t).\cos\omega_c t$, it implies that

$m(t) = 2\cos\omega_m t \Rightarrow E_m = 2$

To implement Envelope detection,

$A_c \geq E_m$

$\therefore (A_c)_{min} = 2$

02. Ans: (d)

Sol: $m(t) = (A_c + A_m\cos\omega_m t)\cos\omega_c t.$

$$= A_c\left(1 + \frac{A_m}{A_c}\cos\omega_m t\right)\cos\omega_c t.$$

Given

$A_c = 2A_m$

$$= A_c\left(1 + \frac{1}{2}\cos\omega_m t\right)\cos\omega_c t.$$

$$P_T = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right],\ P_s = \frac{A_c^2}{2}\left[\frac{\mu^2}{4}\right]$$

$$\frac{P_T}{P_S} = \frac{1 + \frac{\mu^2}{2}}{\frac{\mu^2}{4}} = \frac{1 + \frac{1}{8}}{\frac{1}{16}} = \frac{9}{8} \times 16$$

$P_T = 18\,P_s$

03. Ans: (a)

Sol: $m(t) = 2\cos 2\pi f_1 t + \cos 2\pi f_2 t$

$C(t) = A_c\cos 2\pi f_c t$

$S(t) = [A_c + m(t)]\cos 2\pi f_c t$

$$S(t) = A_c\left[1 + \frac{1}{A_C}m(t)\right]\cos 2\pi f_c t$$

$$K_a = \frac{1}{A_c}$$

$A_{m1} = 2,\ A_{m2} = 1$

$$\mu_1 = K_a A_{m1} = \frac{2}{A_C},\ \mu_2 = K_a A_{m2} = \frac{1}{A_C}$$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\Rightarrow 0.5 = \sqrt{\frac{4}{A_c^2} + \frac{1}{A_c^2}}$$

$$\Rightarrow A_C = \sqrt{20}$$

04. Ans: (c)

Sol: $m(t) = -0.2 + 0.6\sin\omega_1 t,\ k_a = 1,\ A_c = 100$

$S(t) = A_c[1 - 0.2 + 0.6\sin\omega_1 t]\cos\omega_c t$

$= 100[0.8 + 0.6\sin\omega_1 t]\cos\omega_c t$

$V_{max} = A_c[1 + \mu] = 100[0.8 + 0.6] = 140\ V$

$V_{min} = A_c[1 - \mu] = 100[0.8 - 0.6] = 20\ V$

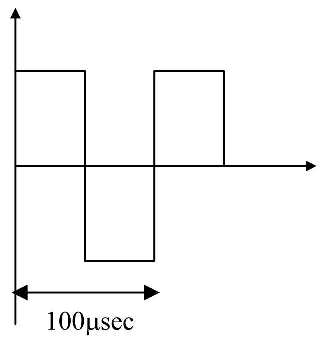
$= 20V$ to $140\ V$

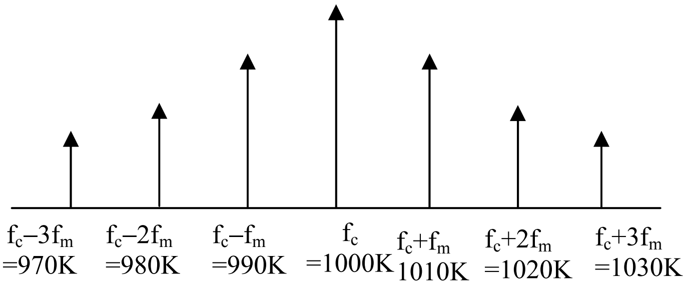
05. Ans: (c)

Sol: $f_C = 1\ MHz = 1000\ kHz$

The given m(t) is symmetrical square wave of period T = 100 μsec

$$f_m = \frac{1}{T_0} = 10\ kHz$$



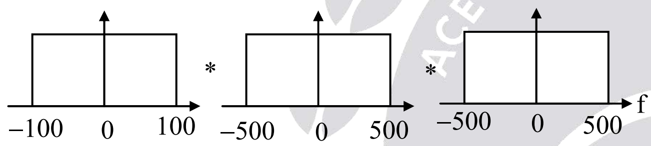


These frequencies 980k, 1020k are not present because the symmetrical square wave it consists of half wave symmetries only odd harmonics are present, even harmonics are dismissed

06. Ans: (d)

Sol: $m(t) = \text{sinc}(200t)\text{sinc}^2(1000t)$

$= \text{sinc}(200t)\text{sinc}(1000t)\text{sinc}(1000t)$

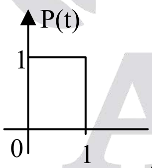


So, highest frequency component in the signal m(t) is 100 + 500 + 500 = 1100

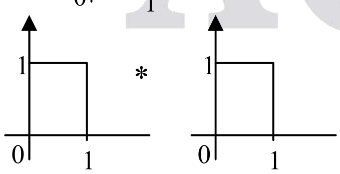
BW = 2 × 1100
BW = 2200 Hz

07. Ans: (a)

Sol: $P(t) = u(t) - u(t-1) \Rightarrow$

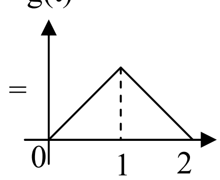


$g(t) = P(t) * P(t) =$



g(t)

=



$x(t) = 100(P(t) + 0.5g(t))\cos\omega_c t$

$= 100(1 + 0.5t)\cos\omega_c t$

$= A_c(1 + K_a m(t))\cos\omega_c t$

$k_a = 0.5,\ m(t) = t$

$\mu = k_a[m(t)]_{max}$

$\mu = 0.5 \times 1 = 0.5$

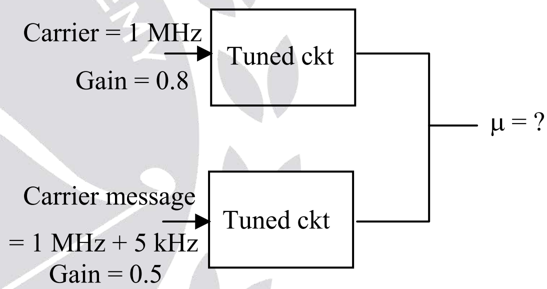
08. Ans: (d)

Sol: $R_L C \le \dfrac{\sqrt{1-\mu^2}}{2\pi f_m \mu}$

So it depends on depth of modulation and the highest modulation frequency.

09. Ans: (b)

Sol: $S(t) = 10\cos 2\pi 10^6 t + 8\cos 2\pi 5\times 10^3 t\cos 2\pi 10^6 t$

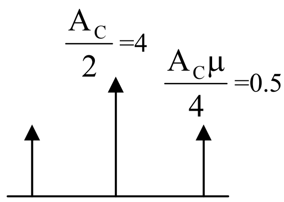


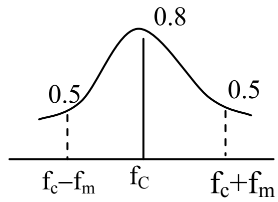
$S(t) = 0.8 \times 10\cos 2\pi 10^6 t$

$+ 0.5\times 8\cos 2\pi 5000t\cos 2\pi 10^6 t$

$= 8(1+\frac{4}{8}\cos 2\pi 5000t)\cos 2\pi 10^6 t$

$\mu = \frac{4}{8} = \frac{1}{2} = 0.5$





10. Ans: (d)

Sol: $A_{max} = 10V$

$A_{min} = 5V$

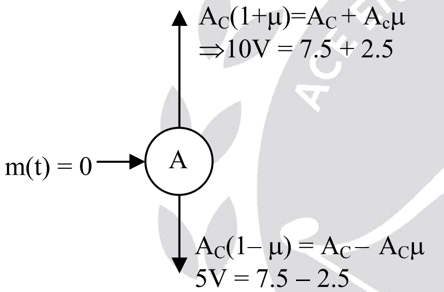
$\mu = 0.1$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$= \frac{1}{3} = 0.33$$

$$A_C = \frac{A_{max} + A_{min}}{2}$$

$$= \frac{10+5}{2} = 7.5\text{ V}$$



Amplitude deviation $A_C\mu = 7.5\times\frac{1}{3} = 2.5\text{ V}$

$\mu_2 = 0.1 \Rightarrow A_{c2}\mu_2 = 2.5$

$A_{c2} = 25\text{ V}$

Which must be added to attain = 17.5

11. Ans: (d)

Sol: Modulation index

$\mu = k_a\,|m(t)|_{max}$

$$k_a = \frac{2b}{a} = \frac{2(\text{square term coefficient})}{\text{linear term coefficient}}$$

$|m(t)|_{max} = 1$

$$\mu = 2\left(\frac{b}{a}\right)$$

$$P_{SB} = \frac{1}{2}P_C \Rightarrow P_C\frac{\mu^2}{2} = \frac{1}{2}P_C$$

$$\mu^2 = 1 \Rightarrow \left(2\frac{b}{a}\right)^2 = 1$$

$$\Rightarrow 2\frac{b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

12. Ans: 0.125

Sol: $s(t) = \cos(2000\pi t) + 4\cos(2400\pi t)$
$+ \cos(2000\pi t)$

Here $4\cos(2400\pi t)$ is the carrier signal.

$\cos(2000\pi t)$ and $\cos(2000\pi t)$ are the sideband message signals.

$$P_c = \frac{4^2}{2} = 8\text{ W}$$

$$P_m = \frac{1}{2} + \frac{1}{2} = 1\text{ W}$$

$$\frac{P_m}{P_c} = \frac{1}{8} = 0.125$$

Conventional Practice Solutions

01.

Sol: μ or m = 60% = 0.6

$m(t) = 5\cos(200\pi t)$ $A_m = 5$; $f_m = 100$Hz

$c(t) = 50\cos(10^4\pi t)$ $A_c = 50$; $f_c = 5\times10^3$Hz

$S(t) = A_c[1+K_a m(t)]c(t)$

$S(t) = 50[1+K_a 5\cos(200\pi t)]\cos(10^4\pi t)$

$|K_a m(t)|_{max} = \mu$

$0.6 = K_a(5)$

$$K_a = \frac{0.6}{5}$$

$K_a = 0.12\ V^{-1}$

$S(t) = 50[1+0.6\cos(200\pi t)]\cos(10^4\pi t)$

$S(t) = 50\cos(10^4\pi t)+30\cos(200\pi t)\cos(10^4\pi t)$

$$\therefore F[\cos 2\pi f_c t] = \frac{\delta(f-f_c)+\delta(f+f_c)}{2}$$

Apply Fourier transform

$$\therefore \cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$S(f) = \frac{50}{2}[\delta(f-5\times10^3)+\delta(f+5\times10^3)] + \frac{30}{4}\begin{bmatrix}\delta(f-5\times10^3-100)+\delta(f+5\times10^3+100)\\ +\delta(f-5\times10^3+100)+\delta(f+5\times10^3-100)\end{bmatrix}$$

$$P_C = \frac{A_C^2}{2} = \frac{(50)^2}{2} = \frac{2500}{2} = 1250\ W$$

$$P_{SB} = P_C.\frac{\mu^2}{2} = 1250\times\frac{(0.6)^2}{2}$$

$$=1250\times\frac{0.36}{2}$$

$P_{SB} = 225;W$

Efficiency

$$\eta = \frac{\mu^2}{\mu^2+2}\times100$$

$$= \frac{(0.6)^2}{2+(0.6)^2}\times100$$

$$= \frac{0.36}{2.36}\times100$$

$\%\eta = 15.25\%$

02.

Sol: $f_m = 10^3$Hz

$f_c = 10^6$ Hz

m = 0.4

$A_c = 15$V

1. $$\frac{1}{R_L C} \ge \frac{\omega_m m}{\sqrt{1-m^2}}$$

$$\tau = R_L C \le \frac{\sqrt{1-m^2}}{\omega_m m}$$

$$\tau = R_L C \le \frac{\sqrt{1-(0.4)^2}}{2\pi\times10^3\times0.4} = \frac{0.9165}{0.8\pi\times10^3}$$

$\tau \le 0.365$msec

2. $\tau \ge R_L C$

$$R_L \le \frac{\tau}{C} = \frac{0.365\times10^{-3}}{100\times10^{-12}}$$

$R_L \le 3.65M\Omega$

$R_L = 3M\Omega$

03.

Sol: Modulation index = 30% = 0.3

$f_c + f_m = 4.928MHz = 4928kHz$

$f_c - f_m = 4.914MHz = 4914kHz$

$2f_c = 9842kHz$

$f_c = 4921kHz$

Amplitude of side band = $\frac{\mu A_c}{2}$

$$75 = \frac{0.3\times A_c}{2}$$

$$\frac{150}{0.3} = A_c$$

$$A_c = \frac{1500}{3} = 500V$$

$A_c = 500V$

04.

Sol: Amplitude modulated voltage equation is

$S(t) = A_c(1+\mu\cos 2\pi f_m t)\cos(2\pi f_c t) \rightarrow (1)$

Where A_c = carrier Amplitude

μ = modulation Index

f_c = carrier frequency

f_m = modulating frequency

The given expression

$V = 100[1+0.2\cos(2\pi\times 10^3 t)]\cos(2\pi\times 10^6 t) \rightarrow (2)$

Compare equation (1) & (2) we get

$A_c = 100$, $\mu = 0.2$, $f_m = 10^3$ Hz, $f_c = 10^6$ Hz

Modulation index = 0.2

Expanding equation $\rightarrow$ (1)

$$S(t) = A_c\cos(2\pi f_c t) + \frac{1}{2}A_c\mu[\cos 2\pi(f_c+f_m)t] + \frac{1}{2}\mu A_c[\cos 2\pi(f_c - f_m)t]$$

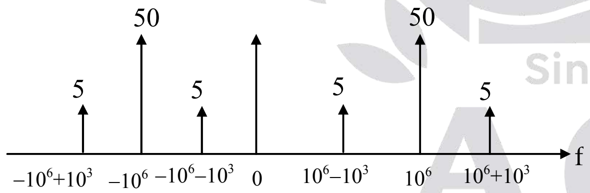
$$f_c + f_m = (10^6 + 10^3)\text{Hz} = 1001\text{kHz}$$

$$f_c - f_m = (10^6 - 10^3)\text{Hz} = 999\text{kHz}$$

$$\frac{1}{2}\mu A_c = \frac{1}{2}\times 0.2\times 100 = 10$$

Amplitude of the side band $\frac{1}{2}\mu A_c = 10$

$\Rightarrow$ The amplitude of sideband spectral impulse = 5



05.

Sol: Time domain equation for single tone AM signal can be written as

$S_{AM}(t) = A_C[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t$

$$= A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c\mu}{2}\cos 2\pi(f_c - f_m)t$$

(Carrier) (USB) (LSB)

Carrier power $P_c = \frac{A_c^2}{2R}$

Sideband power

$$P_{USB} = \frac{\left(\frac{A_c\mu}{2\sqrt{2}}\right)^2}{R} = \frac{A_c^2\mu^2}{8R}$$

$$P_{LSB} = \frac{\left(\frac{A_c\mu}{2\sqrt{2}}\right)^2}{R} = \frac{A_c^2\mu^2}{8R}$$

$$P_T = P_C + P_{USB} + P_{LSB}$$

$$= \frac{A_C^2}{2R} + \frac{A_C^2\mu^2}{8R} + \frac{A_C^2\mu^2}{8R}$$

$$= \frac{A_C^2}{2R} + \frac{A_C^2\mu^2}{4R} = \frac{A_C^2}{2} + \frac{A_C^2\mu^2}{4} = \frac{A_C^2}{2}\left[1+\frac{\mu^2}{2}\right]$$

$$P_T = P_C\left(1+\frac{\mu^2}{2}\right) = P_C + \frac{P_C\mu^2}{2}$$

Total power = carrier power + sideband power

$$\text{Efficiency}(\eta) = \frac{P_{SB}}{P_T} = \frac{P_C\frac{\mu^2}{2}}{P_C\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2}$$

If 80% modulation is used, then the ratio of the total sideband power to the total power in the modulating signal is

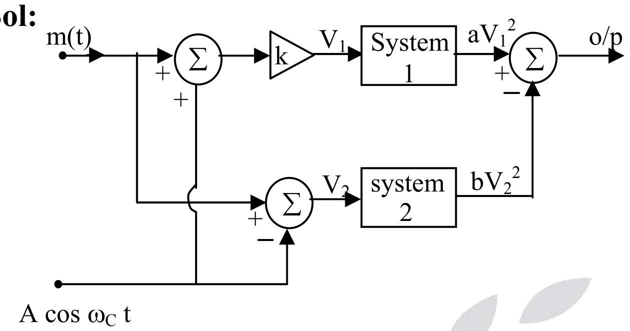
$$\frac{P_{SB}}{P_T} = \frac{\mu^2}{2+\mu^2} = \frac{0.8^2}{2+0.8^2} = 0.2424$$

Chapter 3 Sideband Modulation Techniques

Objective Practice Solutions

01. Ans: (c)

Sol:



$V_1 = k\,[m(t) + c(t)]$

$V_2 = [m(t) - c(t)]$

$V_0 = aV_1^2 - b\,V_2^2$

$= ak^2[m(t) + c(t)]^2 - b[m(t) - c(t)]^2$

$= ak^2\,[m^2(t) + c^2(t) + 2m(t)c(t)]$

$\quad - b[m^2(t) + c^2(t) - 2m(t)c(t)]$

$= [ak^2 - b]m^2(t) + [ak^2 - b]c^2(t)$

$\quad + 2[ak^2 + b][m(t)c(t)]$

on verification if $k = \sqrt{\dfrac{b}{a}}$

$S(t) = 4bm(t)c(t) \rightarrow$ DSBSC Signal

02. Ans: (d)

Sol: Given A = 10

$m(t) = \cos 1000\pi t$

$b = 1$

B.W = ? and power = ?

$s(t) = 4b.A\cos 2\pi f_c t.\cos 2\pi\,(500)t$

$= 40.\cos 2\pi f_c t.\cos 2\pi\,(500)t$

$B.W = 2\,f_m$

$= 2\,(500)$

$= 1$ kHz

$$\text{Power} = \frac{A_c^2 A_m^2}{4}$$

$$= \frac{1600 \times 1}{4}$$

$$= 400\text{W}$$

03. Ans: (c)

Sol: Carrier $= \cos 2\pi\,(100 \times 10^6)t$

Modulating signal $= \cos(2\pi \times 10^6)t$

Output of Balanced modulator

$= 0.5[\cos 2\pi\,(101 \times 10^6)t + \cos 2\pi(99\times10^6)t]$

The Output of HPF is $0.5\cos 2\pi(101 \times 10^6)t$

Output of the adder is

$= 0.5\cos 2\pi\,(101\times10^6)\,t + \sin 2\pi\,(100\times10^6)t$

$= 0.5\cos 2\pi[(100+1)10^6 t] + \sin 2\pi(100\times10^6)t$

$= 0.5[\cos 2\pi\,(100\times10^6)t.\cos 2\pi\,(10^6)t$

$\quad - \sin 2\pi(100 \times 10^6)t.\sin 2\pi\,(10^6)t]$

$\quad + \sin 2\pi(100\times10^6)t]$

$= 0.5\cos 2\pi\,(100\times10^6)t.\cos 2\pi\,(10^6)t$

$\quad + \sin 2\pi(100\times10^6)t\,[1 - 0.5\sin 2\pi\,(10^6)t]$

Let $0.5\cos 2\pi\,(10^6)t = r(t)\cos\theta(t)$

$1 - 0.5\sin 2\pi\,(10^6)t = r(t).\sin\theta(t)$

The envelope is

$r(t) = [\,0.25\cos^2 2\pi\,(10^6)t$

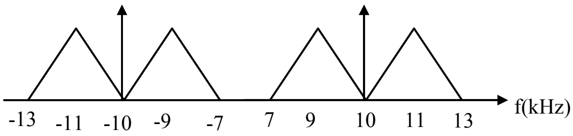
$\quad + \{1 - 0.5\sin 2\pi\,(10^6)t\}^2]^{1/2}$

$= [1.25 - \sin 2\pi(10^6)t]^{1/2}$

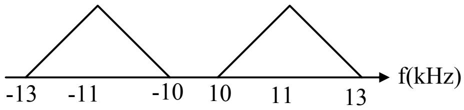
$= [\dfrac{5}{4} - \sin 2\pi\,(10^6)t]^{1/2}$

04. Ans: (b)

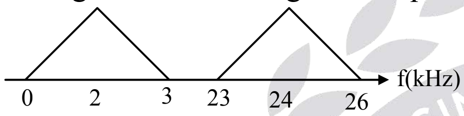
Sol: Output of 1st balanced modulator is



Output of HPF is



The Output of 2nd balanced modulator is consisting of the following +ve frequencies.



Thus, the spectral peaks occur at 2 kHz and 24 kHz

05. Ans: (c)

Sol: Given

$f_{m_1} = 100\text{Hz},\ \ f_{m_2} = 200\text{Hz},\ \ f_{m_3} = 400\text{Hz},$

$f_c = 100\text{KHz},\ f_{c_{L0}} = 100.02\text{KHz}$

$$S(t)/_{T_X} = \frac{A_cA_m}{2}[\cos(f_c + f_{m_1})t + \cos(f_c + f_{m_2})t + \cos(f_c + f_{m_3})t]$$

$$S(t)/R_x = [S(t)/T_x]A_c \cos 2\pi f_{c_{L0}} t$$

$$\Rightarrow \frac{A_c^2A_m}{4}[\cos(f_c + f_{c_{Lo}} + f_{m_1}) + \cos(f_{m_1} - 20) + \cos(f_c + f_{c_{Lo}} + f_{m_2}) + \cos(f_{m_2} - 20) + \cos(f_c + f_{c_{Lo}} + f_{m_3}) + \cos(f_{m_3} - 20)]$$

Detector output frequencies:

80Hz, 180Hz, 380Hz

06. Ans: (b)

Sol: Given

SSB AM is used, LSB is transmitted

$f_{LO} = (f_c + 10)$

$$S(t)/T_X = \frac{A_cA_m}{2}\cos 2\pi[f_c - f_m]t$$

$$S(t)/R_X = \left[\frac{A_cA_m}{2}\cos 2\pi(f_c - f_m)t\right]\cos 2\pi(f_c + 10)t$$

$$\Rightarrow \frac{A_cA_m}{4}[\cos 2\pi(2f_c + 10 - f_m)t + \cos 2\pi(10 + f_m)t]$$

i.e., from 310 Hz to 1010 Hz

07. Ans: (b)

Sol: BW of Basic group = 12×4 = 48 kHz

BW of super group = 5×48 = 240 kHz

08. Ans: (d)

Sol: Given 11 voice signals

B.W. of each signals = 3 kHz

Guard Band Width = 1 kHz

Lowest f_c = 300 kHz

Highest f_c =

$$\Rightarrow f_{c_H} + f_{m_{lost}} = 300\text{kHz} + 11(3\text{kHz}) + 10(1\text{kHz})$$

$$= 343 \text{ kHz}$$

$$f_{c_H} = 343\,\text{kHz} - 3\,\text{kHz}$$

$$= 340\,\text{kHz}$$

09. Ans: (b)

Sol: f_{m1} = 5 kHz → AM

f_{m2} = 10 kHz → DSB

f_{m3} = 10kHz → SSB

f_{m4} = 2kHz → SSB

f_{m5} = 5kHz → AM

f_g = 1kHz

$$BW = (2f_{m1} + 2f_{m2} + f_{m3} + f_{m4} + 2f_{m5} + 4f_g)$$

$$= 2\times 5 + 2\times 10 + 10 + 2 + 2\times 5 + 4\times 1$$

$$= 10 + 20 + 10 + 10 + 6$$

$$= 56 \text{ kHz}$$

$\therefore$ BW = 56 kHz

Conventional Practice Solutions

01.

Sol: m(t) contains {100, 200, 400} Hz.

The transmitted SSB signal is

$$\frac{A_C}{2}\{m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\}$$

$[\therefore\ f_c = 100\text{ kHz}]$

Demodulation is done using a product mod ulator & multiplying by

$A_c'\cos(2\pi f_c' t)\ \ [\therefore f_c' = 100.02\,\text{kHz}]$

(1) $$V_0(t) = \frac{1}{2}A_c A_c'\cos(2\pi f_c' t)\begin{bmatrix} m(t)\cos(2\pi f_c t) \\ -\hat{m}(t)\sin(2\pi f_c t)\end{bmatrix}$$

Higher frequency term will be filtered out and so can be ignored for the purpose of determining the output of the detector

$$V_0(t) = \frac{1}{4}A_c A_c'[m(t)\cos(2\pi\Delta f t) - \hat{m}(t)\sin(2\pi\Delta f t)]$$

When the upper side band is transmitted, $\Delta f > 0$ the frequencies are shifted inward by Δf.

$V_0(f)$ contains {99.98, 199.98, 399.98} Hz.

(2) For lower side band transmission $\Delta f < 0$ the frequencies are shifted outward by Δf $V_0(f)$ contains {100.02, 200.02, 400.02}Hz.

02.

Sol: Given:

Number of Voice inputs = 24

Bandwidth of each voice input f_m = 4 KHz

FDM system using AM-SSB so band width (f_m) = 4 kHz

Transmission Bandwidth = 24×4 = 96 kHz

N = 24; n = 8

$f_s = 2\times f_m$ = 8 kHz

$$\text{Bandwidth} = \frac{R_b}{2} = \frac{Nnf_s}{2} = \frac{24\times 8\times 2\times 4}{2}$$

$$= 768\,\text{kHz}$$

So, PCM - TDM requires more bandwidth than the AM-SSB-FDM system.

Chapter 4 Angle Modulation

Objective Practice Solutions

01. Ans: (a)

Sol: $s(t) = 10\cos(20\pi t + \pi t^2)$

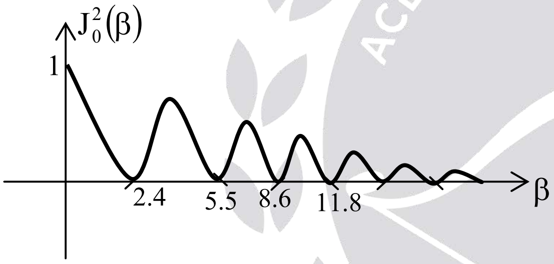
$$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$f_i = \frac{1}{2\pi}[20\pi + 2\pi t]$$

$$\frac{df_i}{dt} = \frac{1}{2\pi}\times 2\pi\times 1 = 1\text{Hz/sec}$$

02. Ans: (d)

Sol: $P_{fc} = \dfrac{A_c{}^2 J_0{}^2(\beta)}{2}$



So, $J_0{}^2(\beta)$ is decreasing first, becoming zero and then increasing so power is also behave like $J_0{}^2(\beta)$.

03. Ans: (a)

Sol: In an FM signal, adjacent spectral components will get separated by $f_m = 5$ kHz

Since BW $= 2(\Delta f + f_m) = 1$MHz

$= 1000\times 10^3$

$\Delta f + f_m = 500$ kHz

$\Delta f = 495$ kHz

The n^{th} order non-linearity makes the carrier frequency and frequency deviation increased by n-fold, with the base-band signal frequency (f_m) left unchanged since n = 3,

$\therefore (\Delta f)_{New} = 1485$ kHz &

$(f_c)_{New} = 300$ MHz

New BW $= 2(1485 + 5)\times 10^3$

$= 2.98$ MHz

$= 3$ MHz

04. Ans: (d)

Sol: $S(t) = A_c\sum_{n=-\infty}^{\infty} J_n(\beta)\cos 2\pi(f_c + nf_m)t$

$\Delta f = 3(2f_m) = 12$ kHz

$\beta = \dfrac{\Delta f}{f_m} = 6$

$\therefore S(t) = \sum_{n=-\infty}^{\infty} 5.J_n(6)\cos 2\pi(f_c + nf_m)t$

$f_c = 1000$kHz, $f_m = 2$ kHz

$= \cos 2\pi(1008\times 10^3)t$

$= \cos 2\pi(1000 + 4\times 2)\times 10^3 t$

i.e., n = 4

The required coefficient is $5.J_4(6)$

05. Ans: (c)

Sol: $2\pi f_m = 4\pi\ 10^3$

$\Rightarrow f_m = 2k$

$J_0(\beta) = 0$ at $\beta = 2.4$

$\beta = \dfrac{k_f\ A_m}{f_m} \Rightarrow 2.4 = \dfrac{k_f\times 2}{2k}$

$k_f = 2.4$ KHz /V

at $\beta = 5.5$

$$5.5 = \frac{2.4\,k \times 2}{f_m}$$

$\Rightarrow f_m = 872.72$ Hz

06. Ans: (c)

Sol: $\beta = 6$

$J_0(6) = 0.1506$; $J_3(6) = 0.1148$

$J_1(6) = 0.2767$; $J_4(6) = 0.3576$

$J_2(6) = 0.2429$;

$$\frac{P_{f_c \pm 4f_m}}{P_T} = ? \qquad P_T = \frac{{A_c}^2}{2R}$$

$$P_{f_c \pm 4f_m} = \frac{A_C^2}{R}\left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_3^2(\beta) + J_4^2(\beta)\right]$$

$$P_{f_c \pm 4f_m} = \frac{A_C^2}{R}\left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_4^2(\beta)\right]$$

$$\frac{P_{f_c \pm 4f_m}}{P_T} = \frac{0.2879}{1/2} = 0.5759 = 57.6\ \%$$

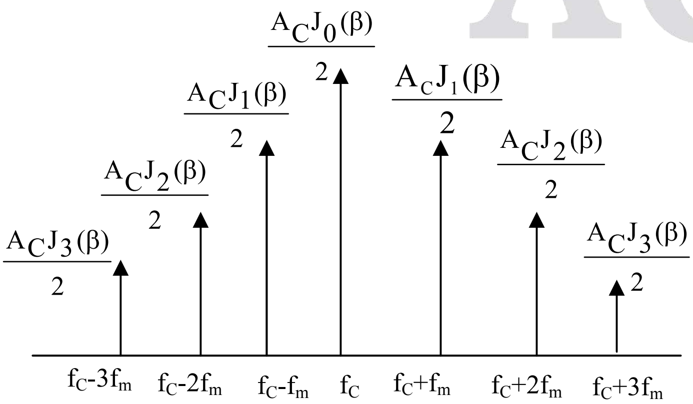
07. Ans: (c)

Sol: $m(t) = 10\cos 20\pi t$

$f_m = 10$ Hz

inserting correct signal and frequency

$$\beta = \frac{k_f A_m}{f_m} = \frac{5 \times 10}{10} = 5$$



From f_c to $f_c + 4f_m$ pass through ideal BPF

Powers in these frequency components

$$P = \frac{A_C^2}{2R} J_0^2(\beta) + 2\frac{A_C^2}{2R} J_1^2(\beta) + 2\frac{A_C^2}{2R} J_2^2(\beta)$$

$$+ 2\frac{A_C^2}{2R} J_3^2 \beta + 2\frac{A_C^2}{1R} J_4^2(\beta)$$

$$= \frac{A_C^2}{2R}\left[\begin{array}{l}(-0.178)^2 + 2(-0.328)^2 + 2(0.049)^2 \\ + 2(0.365)^2 + 2(0.391)^2\end{array}\right]$$

$= 41.17$ Watts

08. Ans: (d)

Sol: $P_t = \dfrac{A_C^2}{2R}$ $(R = 1\Omega)$

$$= \frac{100}{2} = 50\text{ W}$$

$$\%\text{ Power} = \frac{\text{Power in components}}{\text{total power}} \times 100$$

$$= \frac{41.17}{50} \times 100$$

$= 82.35\%$

09. Ans: (d)

Sol: In frequency modulation the spectrum contains $f_c \pm nf_1 \pm mf_2$, where n & m = 0, 1, 2, 3………

10. Ans: (c)

Sol: Given $f_c = 1$MHz

$f_{max} = f_c + k_f A_m$

$k_p = 2\pi\, k_f$

$$k_f = \frac{k_p}{2\pi} = \frac{\pi}{2\pi}$$

$$= \frac{1}{2}$$

$$= \left(10^6 + \frac{1}{2}\times 10^5\right) = \left(10^6 + 0.5\times 10^5\right)$$

$$= \left(10^6 + 5\times 10^4\right)$$

$$= \left(10^3 + 50\right)10^3$$

$$= (10^3 + 50)\text{ k}$$

$$= 1050 \text{ kHz.}$$

$$f_{min} = f_c - k_f A_m$$

$$= \left(10^6 - \frac{1}{2}\times 10^5\right)$$

$$= \left(10^6 - 0.5\times 10^5\right)$$

$$= \left(10^6 - 5\times 10^4\right)$$

$$= \left(10^3 - 50\right)10^3$$

$$= (10^3 - 50)\text{ k}$$

$$= 950 \text{ kHz}$$

11. Ans: (d)

Sol: $\beta = \dfrac{\Delta f}{f_m}$

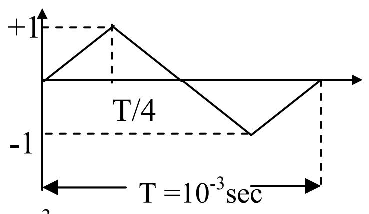
$$\Delta\phi = \frac{\Delta f}{f_m}$$

$$\Delta f = \Delta\phi\ f_m$$

$$= k_p A_m f_m$$

12. Ans: (c)

Sol: Given



$f_c = 100\times 10^3$ Hz

$k_f = 10\times 10^3$Hz

$m(t)|_{max} = +1$, $m(t)|_{min} = -1$

$$f_i = f_c \pm \Delta f$$

$$= f_c \pm k_f A_m$$

$$= 100\times 10^3 \pm 10\times 10^3\ (m(t))$$

$$= 110 \text{ kHz } \& \text{ } 90 \text{ kHz}$$

13. Ans: (c)

Sol: $S(t) = A_c \cos\underbrace{(2\pi f_c t + k_p m(t))}_{\theta_i(t)}$

$$f_i = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$$

$$= \frac{1}{2\pi}\frac{d}{dt}\ (2\pi f_c t + k_p m(t))$$

$$= f_c + \frac{1}{2\pi}k_p\frac{d}{dt}m(t)$$

$$f_{max} = f_c + \frac{k_p}{2\pi}\frac{1}{\left(\frac{10^{-3}}{4}\right)} = f_c + \frac{k_p}{2\pi}\times 4\times 10^3$$

$$= 100\text{ kHz} + \frac{\pi}{2\pi}\times 4\times 10^3$$

$$= 102 \text{ kHz}$$

$$f_{min} = f_c - k_p\frac{1}{\left(\frac{10^{-3}}{4}\right)}$$

$$= f_c - 2 \text{ kHz}$$

$$f_{min} = 98\text{kHz}$$

14. Ans: (c)

Sol: Given,

$$S(t) = A_c \cos(\theta_i(t))$$

$$= A_c \cos(\omega_c t + \phi(t))$$

$$m(t) = \cos(\omega_m t)$$

$$f_i(t) = f_c + 2\pi k (f_m)^2 \cos\omega_m t$$

$$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$\theta_i(t) = \int 2\pi\, f_i(t)dt$

$\theta_i(t) = \int 2\pi[f_c + 2\pi k(f_m)^2 \cos\omega_m t]dt$

$\theta_i(t) = 2\pi f_c t + (2\pi f_m)^2\, k\, \dfrac{\cos\omega_m t}{\omega_m t}$

$\theta_i(t) = \omega_c t + \omega_m k \sin\omega_m t$

15. Ans: (b)

Sol: $\Delta f_{max} = K_f\, |m(t)|_{max}$

$= \dfrac{100}{2\pi}\times[10]$

$\Delta f_{max} = \left(\dfrac{500}{\pi}\right)\text{Hz}$

16. Ans: (b)

Sol: Given that

$s(t) = \cos[\omega_c t + 2\pi m(t)]$volts

$f_i = \dfrac{1}{2\pi}\dfrac{d}{dt}[\omega_c t + 2\pi m(t)]$

$= \dfrac{1}{2\pi}\dfrac{d}{dt}[2\pi f_c t + 2\pi m(t)]$

$f_i = f_c + \dfrac{d}{dt}[m(t)]$

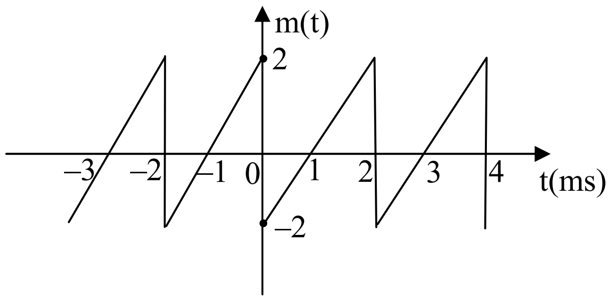
we know that $f_i = f_c + k_f m(t)$

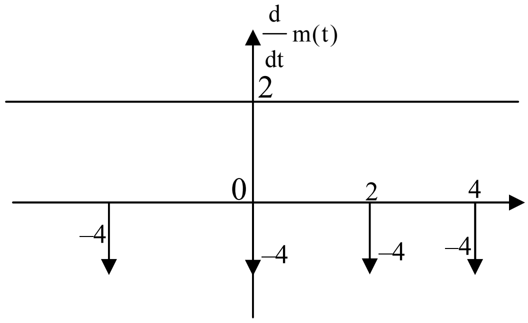
Here $k_f m(t) = \dfrac{d}{dt}[m(t)]$

$\Delta f = \max\{k_f m(t)\}$

$\Delta f = \max\left[\dfrac{d}{dt}m(t)\right]$

$\Delta f = 2\text{kHz}$





17. Ans: (a)

Sol: $\beta_p = k_p \max[|m(t)|] = 1.5\times 2 = 3$

$\beta_f = \dfrac{k_f \max[|m(t)|]}{f_m}$

$= \dfrac{3000\times 2}{1000}$

$= 6$

18. Ans: (a)

Sol: Using Carson's rule we obtain

$BW_{PM} = 2(\beta_p + 1)f_m = 8\times 1000 = 8000\text{Hz}$

$BW_{FM} = 2(\beta_f + 1)f_m = 14\times 1000 = 14000\text{Hz}$

19. Ans: 70 kHz

Sol: $s(t) = A_c\cos[2\pi f_c t + k_p m(t)]$

$f_i = f_c + \dfrac{k_p}{2\pi}\dfrac{d}{dt}x(t)$

$= 20k + \dfrac{5}{2\pi}\times 5\dfrac{d}{dt}\left(\sin 4\pi 10^3 t - 10\pi\cos 2\pi 10^3 t\right)$

$= 20k + \dfrac{25}{2\pi}\times\begin{bmatrix}\cos(4\pi 10^3 t - 10\pi\cos 2\pi 10^3 t)\\ \left(4\pi 10^3 + 10\pi\sin 2\pi 10^3 t\times 2\pi 10^3\right)\end{bmatrix}$

$f_{i(t=0.5ms)} = 20k + \dfrac{25}{2\pi}\times\cos(4\pi + 10\pi)\times 4\pi\times 10^3$

$= 20k + \dfrac{25}{2\pi}\times 4\pi\times 10^3$

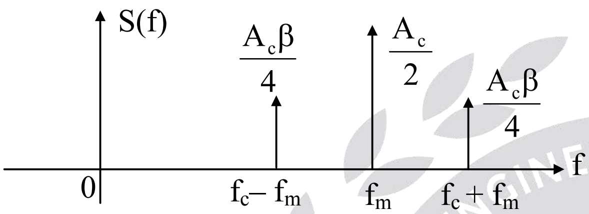
$= 20k + 50k$

$f_{i(t=0.5ms)} = 70\text{kHz}$

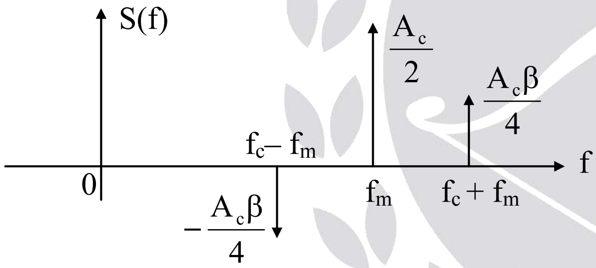
Conventional Practice Solutions

01.
Sol:
(a) The spectrum of AM and NBFM are identical except that the spectral component of NBFM at frequency $f_c - f_m$ is 180° out of phase. The difference in AM and NBFM can be shown through the following spectrums:

a. Spectrum for AM



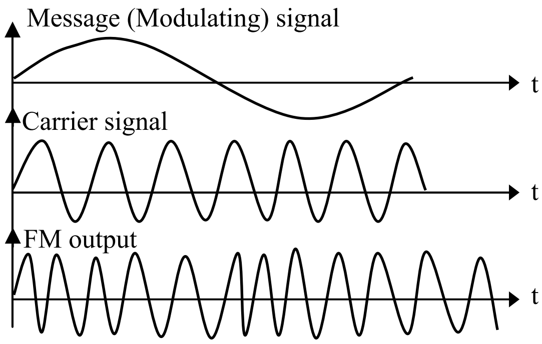
b. Spectrum for NBFM



Frequency Modulation:
Changing the frequency of the carrier according to the message signal amplitude variations is called Frequency Modulation.

Instantaneous frequency,
$f_i(t) = f_c + K_f\, m(t)$
K_f = Frequency sensitivity (Hertz / Volt)

For single tone modulation
$f_i(t) = f_c + K_f A_m \cos 2\pi f_m t$
$f_{i,\max} = f_c + K_f A_m$
$f_{i,\min} = f_c - K_f A_m$
$\Delta f = K_f A_m$ = Frequency deviation
Carrier Swing (Total variation of carrier frequency) = $2\,\Delta f$



Let $s(t) = A_c \cos\theta(t)$ be the FM wave
$\theta(t) = 2\pi f_i t$
$d\theta(t)/dt = 2\pi f_i$

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_i$$

$$\theta(t) = 2\pi\int_0^t f_i(t)\,dt$$

$$\theta(t) = 2\pi\int_0^t [f_c + K_f\, m(t)]\,dt$$

$$\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t)\,dt$$

$$\therefore S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t)\,dt\right]$$
$\rightarrow$ FM signal

$S(t) = A_c \cos[2\pi f_c t + K_p\, m(t)]$
$\rightarrow$ PM signal

For a single tone frequency modulation
$m(t) = A_m \cos 2\pi f_m t$

$$S(t) = A_c \cos\left(2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m}\sin 2\pi f_m t\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{K_f A_m}{f_m}\sin 2\pi f_m t\right)$$

Modulation index of FM

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

$\beta << 1$, Narrow band FM

$\beta >> 1$, Wide band FM

Narrow Band FM : (β < < 1)

$S(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$

$= A_c [\cos 2\pi f_c t \,.\, \cos(\beta \sin 2\pi f_m t)$

$- \sin 2\pi f_c t \,.\, \sin(\beta \sin 2\pi f_m t)]$

When $\beta << 1$

$\cos(\beta \sin 2\pi f_m t) \cong 1$

$\sin(\beta \sin 2\pi f_m t) \cong \beta \sin 2\pi f_m t$

$\cong A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin(2\pi f_m t)$

$\cong A_c \cos 2\pi f_c t - \frac{A_c \beta}{2}[\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t]$

$S(t) = A_c \cos 2\pi f_c t + \frac{A_c\beta}{2}\cos 2\pi(f_c + f_m)t - \frac{A_c\beta}{2}\cos 2\pi(f_c - f_m)t$

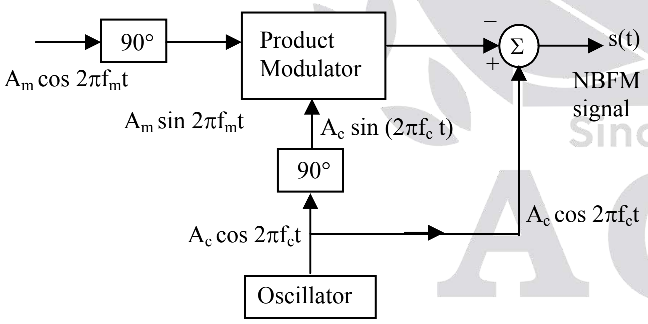
$$S(f) = \frac{A_c}{2}[\delta(f - f_c) - \delta(f + f_c)]$$
$$+ \frac{A_c\beta}{4}[\delta(f - f_c - f_m) - \delta(f + f_c + f_m)]$$
$$- \frac{A_c\beta}{4}[\delta(f - f_c + f_m) - \delta(f + f_c - f_m)]$$

B.W of NBFM $= 2 f_m$

The spectrum of AM and FM are identical except that the spectral component at $f_c - f_m$ is 180^0 out of phase.

$$P_t = P_c \left(1 + \frac{\beta^2}{2}\right)$$

Generation of NBFM Signal:



Wideband FM : (β > > 1)

$S(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$

Bessel function of order 'n' is given by

$$J_n(x) = \left(\frac{1}{2\pi}\right)\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)}d\theta$$

$$S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$$

$n = 0, \pm1, \pm2, \ldots\ldots \pm\infty$

$S(t) = A_c J_0(\beta)\cos 2\pi f_c t$

$+ A_c J_1(\beta)\cos 2\pi(f_c + f_m)t$

$+ A_c J_{-1}(\beta)\cos 2\pi(f_c - f_m)t$

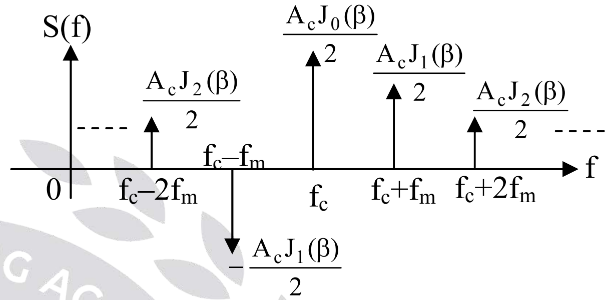
$+ A_c J_2(\beta)\cos 2\pi(f_c + 2f_m)t$

$+ A_c J_{-2}(\beta)\cos 2\pi(f_c - 2f_m)t + \ldots\ldots$

$S(t) = A_c J_0(\beta)\cos 2\pi f_c t$

$+ A_c J_1(\beta)[\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t]$

$+ A_c J_2(\beta)[\cos 2\pi(f_c + 2f_m)t + \cos 2\pi(f_c - 2f_m)t]$



∴ Theoretical bandwidth of a WBFM is '∞'.

$f_c = 5$ kHz

$k_f = 10$ Hz/V

$f(t) = 100 \cos 200\pi t$

$f_m = \frac{200\pi}{2\pi} = 100$ Hz

$A_m = 100$

$\Delta f = k_f A_m = 10 \times 100 = 1000$ Hz

β = modulation index $= \frac{\Delta f}{f_m} = \frac{1000}{100} = 10$

$BW = 2(\beta + 1)f_m = 2(10+1)(100)$

$= 2 \times 11 \times 100$

$BW = 2.2$ kHz

(b)

	Envelope detection	**Synchronous Detection**
1.	It is a simple circuit. Therefore it is used in broadcasting receivers	Complex circuitry is required. Therefore it is used in point to point communication.
2.	No synchronization of carrier	Synchronization of carrier is compulsory
3.	No phase and frequency errors	Phase and frequency errors occurs

Costas Receiver:

- One of the method to obtain practical synchronous receiving system is costas receiver, suitable for demodulating DSBSC signals
- This system consists of two coherent detectors supplied with the same input signal, but with individual local oscillator signals that are in phase and quadrature to each other
- The detector in the upper path is referred to as the in-phase coherent detector or I-channel and lower path referred as quadrature-phase coherent detector or Q-channel
- These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave as shown in figure.

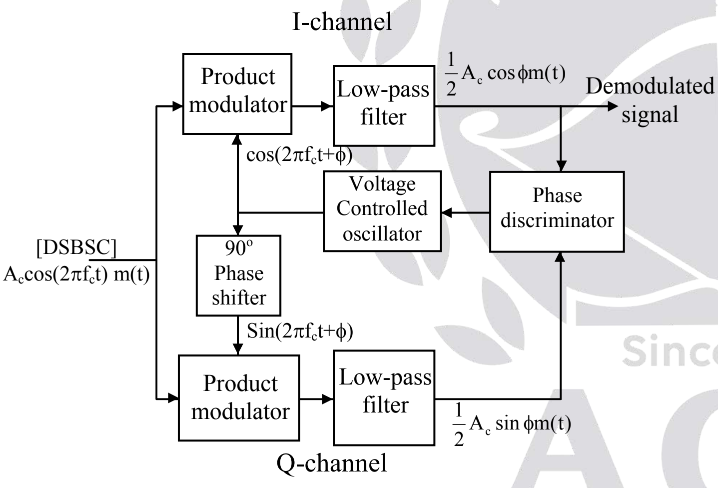
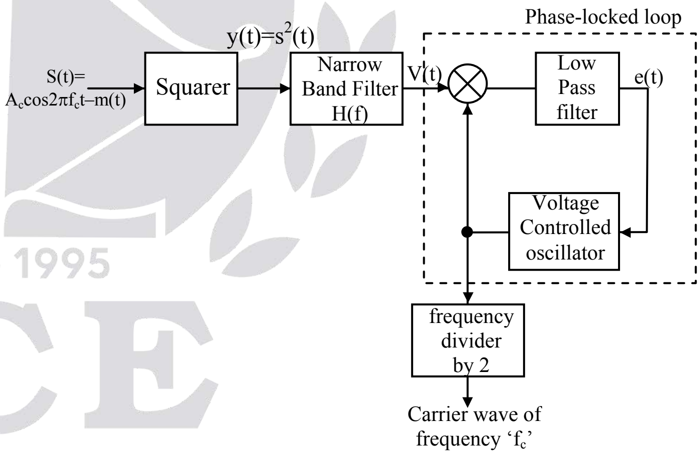


Figure: Costas receiver block diagram

- Suppose that the local oscillator signal is of the same phase as the carrier wave, $A_c\cos(2\pi f_c t)$ used to generate the incoming DSBSC wave. Under these conditions, we find the I-channel output contains the desired demodulated signal m(t), whereas Q-channel output, is zero due to the quadrature null effect of the Q-channel.
- Suppose next the local oscillator phase drifts from its proper value by a small amount ϕ radians. The I-channel output will remain essentially unchanged, but there will now be some signal appearing at the Q-channel output, which is proportional to $\sin\phi \approx \phi$. This Q-channel output will have the same polarity as the I-channel output for one duration of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift. Thus, by combining the I and Q-channel outputs in a phase discriminator (which consist of a multiplier followed by a low pass filter). A dc control signal is obtained which automatically corrects for local oscillator phase errors.

Squaring loop circuit:

Another method for generating a reference carrier from a DSBSC wave is to use a squaring loop as shown in figure.



$\therefore$ The DSBSC wave is given by

$s(t) = A_c\cos(2\pi f_c t)\,m(t)$

applied to the input of the square circuit, we obtain

$$y(t) = A_c^2\cos^2(2\pi f_c t)m^2(t)$$

$$= \frac{A_c^2}{2}m^2(t)[1+\cos(4\pi f_c t)]$$

The signal y(t) is next applied to a narrow band filter centered about $2f_c$. The output is approximately sinusoidal as given by

$$v(t) \simeq \frac{A_c^2}{2} E\Delta f \cos(4\pi f_c t)$$

where E is the energy of the message signal m(t) and Δf is bandwidth. This signal is given to phase locked loop, which consists of multiplier, LPF and voltage controlled oscillator (VCO). In the PLL tracking is taken place and error signal e(t) becomes zero using VCO. The final output of PLL is divided by '2' using frequency divider, which is a carrier signal with frequency 'f_c'. This carrier is used for the synchronous detector and it will not produce any phase and frequency errors.

02.

Sol: $\beta = 0.2$

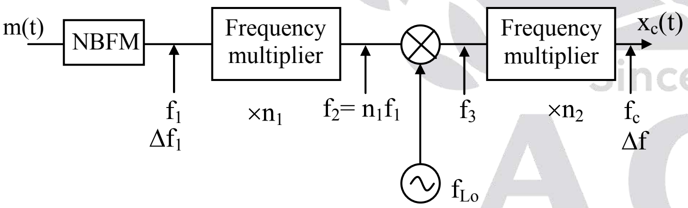
$f_m = 40Hz$

$\Delta f = 80kHz = 80\times10^3$ Hz

$f_{L0} = ?$

$f_1 = 200kHz = 2\times10^5$ Hz

$f_c = 108\times10^6$ Hz



$\Delta f_1 = \beta f_m = (0.2)(40) = 8Hz$

$$\frac{\Delta f}{\Delta f_1} = \frac{80\times10^3}{8} = 10^4 = n_1 n_2$$

$$n_1 n_2 = 10^4$$

Assuming down conversion

$$f_2 - f_{L0} = \frac{f_c}{n_2}$$

Where, $f_2 = n_1 f_1 = n_1 \times 2\times10^5$

$= 2n_1 \times 10^5$ Hz

$$f_{L0} = f_2 - \frac{f_c}{n_2}$$

$$= n_1\, 2\times10^5 - \frac{108\times10^6}{n_2}$$

$$f_{L0} = \frac{n_1 n_2 \times 2\times10^5 - 108\times10^6}{n_2}$$

$$= \frac{2\times10^9 - 108\times10^6}{n_2}$$

$$= \frac{(2000-108)\times10^6}{n_2}$$

$$f_{L0} = \frac{1892\times10^6}{n_2}$$

By assumption let, $n_2 = 200$

$$n_1 = \frac{10^4}{n_2} = \frac{10^4}{200} = 50$$

$$f_{L0} = \frac{1892}{200}\times10^6$$

$$f_{L0} = 9.46MHz$$

$n_1 = 50$, $n_2 = 200$, $f_{L0} = 9.46$ MHz

03.

Sol: Given Audio frequency $(f_m) = 500Hz$

Audio frequency voltage $(A_m) = 2.4V$

Deviation $(\Delta f) = 4.8KHz$

We know that

$$\text{Modulation index } (\beta) = \frac{\Delta f}{f_m} = \frac{4.8\times10^3}{500}$$

$= 9.6$

$$K_f = \frac{\Delta f}{A_m}$$

Frequency sensitivity $(K_f) = 2\times10^3$ Hz/V

i) Now $A_m = 7.2V$

Deviation $(\Delta f) = K_f A_m$

$= 2\times10^3\times7.2$

$\Delta f = 14.4KHz$

$$\text{Modulation index } (\beta) = \frac{\Delta f}{f_m}$$

$\beta = 28.8$

ii) $A_m = 10V$, $f_m = 200Hz$

Deviation (Δf) = $K_f A_m$

$= 2 \times 10^3 \times 10$

$\Delta f = 20\times10^3 = 20KHz$

Modulation Index (β) = $\dfrac{\Delta f}{f_m}$

$= \dfrac{20\times10^3}{200} = 100$

Disadvantage of FM over AM:

1. A much wider channel is required by FM, up to 10 times as large as that needed by AM. This is the most significant disadvantage of FM
2. FM transmitting and receiving equipment tends to be more complex, particularly for modulation and demodulation. So Circuit is more complex than AM.
   Threshold effect is more in FM than AM.

04.

Sol: Given: carrier $c(t) = 10 \cos \omega_c t$

$A_C = 10$

Modulating message signal $m(t) = 3\cos\omega_m(t)$

$A_m = 3$

$f_c = 100$ kHz

$f_m = 4$ kHz

Amplitude modulation

1. Modulation index $\mu = \dfrac{A_m}{A_C} = \dfrac{3}{10} = 0.3$

2. Channel BW = $2f_m = 2 \times 4K = 8$ kHz

Frequency modulation

Given sensitivity of the frequency modulator to be 5 kHz/volt

$\beta = \dfrac{\Delta f}{f_m}$

Frequency deviation $\Delta f = K_f A_m$

$= 5 \times 10^3 \times 3$

$= 15 \times 10^3$

$\beta = \dfrac{15\times10^3}{4\times10^3} = \dfrac{15}{4} = 3.75$

As $\beta > 1$. This is WBFM

The channel Bandwidth $B = 2(\beta+1) f_m$

$= 2(3.75+1)4K$

$= 38$ kHz

05.

Sol: $f_c = 105$ MHz ; $f_{max} = 105.03$ MHz

$f_m = 5$ kHz

(i) Frequency deviation

$f_{max} = f_c + \Delta f$

$\Delta f = f_{max} - f_c$

$= 105.03 - 105 = 0.03$ MHz $= 30$ kHz

(ii) Carrier swing = $2\Delta f = 60$ kHz

(iii) Modulating index $\beta = \dfrac{\Delta f}{f_m} = \dfrac{30\times10^3}{5\times10^3} = 6$

(iv) Percentage of modulation

$\dfrac{\Delta f}{(\Delta f)_{max}}\times100\% = \dfrac{30kHz}{75kHz}\times100\% = 40\%$

Chapter 5 Radio Receivers

Objective Practice Solutions

01. Ans: (d)

Sol: The image channel selectivity of super heterodyne receiver depends upon Pre selector and RF amplifier only.

02. Ans: (b)

Sol: The image (second) channel selectivity of a super heterodyne communication receiver is determined by the pre selector and RF amplifier.

03. Ans: (d)

Sol: Given $f_s = 4$ to 10 MHz

$IF = 1.8$ MHz

$f_{si} = ?$

$f_{si} = f_s + 2\times IF$

$= 7.6$ MHz to 13.6 MHz

04. Ans: (a)

Sol: Image frequency $f_{si} = f_s + 2\times IF$

$= 700\times10^3 + 2(450\times10^3)$

$= 1600$ kHz

Local oscillator frequency, $f_l = f_s + IF$

$(f_l)_{max} = (f_s)_{max} + IF = 1650 + 450$

$= 2100$ kHz

$(f_l)_{min} = (f_s)_{min} + IF = 550 + 450$

$= 1000$ kHz

$$R = \frac{C_{max}}{C_{min}} = \left(\frac{f_{l\,max}}{f_{l\,min}}\right)^2 = \left(\frac{2100}{1000}\right)^2 = 4.41$$

05. Ans: (a)

Sol: f_s(range) = 88 - 108MHz

Given condition $f_{IF} < f_{LO},\ f_{si} > 108$ MHz

$f_{si} = f_s + 2\times IF$

$f_{si} > 108$ MHz

$f_s + 2IF > 108$ MHz

$88\text{MHz} + 2\times IF > 108$ MHz

$IF > 10$MHz

Among the given options IF = 10.7 MHz

06. Ans: (a)

Sol: Range of variation in local oscillator frequency is

$f_{Lmin} = f_{smin} + IF$

$= 88 + 10.7$

$f_{Lmin} = 98.7$ MHz

$f_{Lmax} = f_{smax} + IF$

$= 108 + 10.7$

$f_{Lmax} = 118.7$ MHz

07. Ans: 5

Sol: $f_s = 58$ MHz – 68 MHz

When $f_s = 58$ MHz

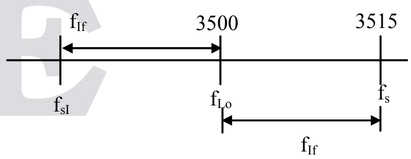
$f_{si} = f_s + 2IF > 68$ MHz

$2IF > 10$ MHz

$IF \geq 5$ MHz

08. Ans: 3485 MHz

Sol:



$f_{If} = 15$ MHz

$f_{L\,o} = 3500$ MHz

$f_s - f_{Lo} = f_{IF}$

$f_s = f_{Lo} + f_{IF} = 3515$ MHz

f_{si} = image frequency $= f_s - 2\,f_{IF}$

$= 3515 - 2\times15$

$= 3485$ MHz

Conventional Practice Solutions

01.

Sol:

(a) **Selectivity:** Selectivity is a receiver parameter that is used to measure the ability of the receiver to accept a given band of frequencies and reject all others.

Selectivity determines the adjacent-channel rejection of a Receiver. It is also determined by the response of the IF section, with the mixer and RF amplifier input circuits. It varies with the receiving frequency if ordinary tuned circuits are used in the IF section.

Selectivity can be measured experimentally by attenuation factor and frequency of the same signal.

The attenuation can be given as the ratio of the voltage required for resonance to the voltage required when the generator is tuned to the receiver's frequency. Generally attenuation is expressed in decibels (dB).

In the commercial AM broad cast band, each station's transmitter is allocated a 10- kHz band width. For a receiver to select only those frequencies which are assigned to a single channel, the receiver must limit its Bandwidth to 10 kHz. If the pass band is greater than 10 kHz, more than one channel may be received and demodulated simultaneously .If the pass band is less than 10 kHz a portion of the modulating signal information of that channel is rejected or blocked from entering the demodulator.

Experimental technique: one common way is to simply give the bandwidth of the receiver at the –3dB points. This Bandwidth is not necessarily a good means of determining how well the receiver will reject unwanted frequencies. It is common to give the receiver bandwidth at two levels of attenuation .For example -3dB & -60 dB. The ratio of these two bandwidth's is called the "Shape Factor"

$$SF = \frac{\text{Bandwidth}\,(-60\text{dB})}{\text{Bandwidth}\,(-3\text{dB})}$$

SF = shape factor (unit less)

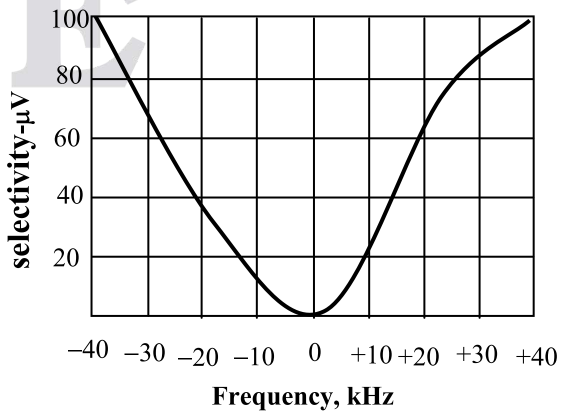
Bandwidth (–60dB) = Bandwidth 60dB below maximum signal level

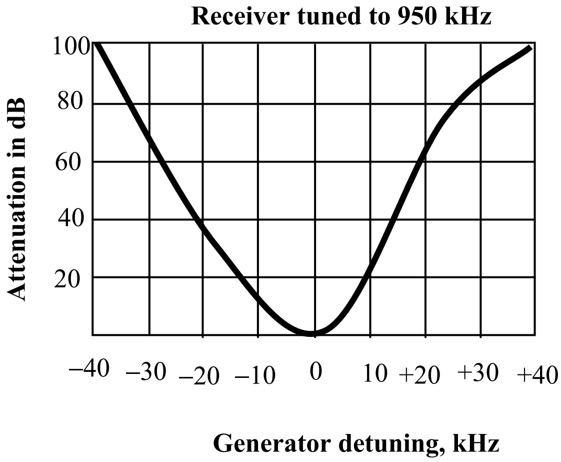
Bandwidth (–3dB) = Bandwidth 3dB below maximum signal level

Ideally the Bandwidth at the –3 dB and –60 dB points would be equal, and shape factor = 1. but, this is impossible receiver might have a –3 dB bandwidth of 10 kHz and –60 dB bandwidth of 20 kHz which gives a SF of 2.

A radio receiver must be capable of separating the desired channel's signals without allowing interference from an adjacent channel to spill over into the desired channel's pass band.

Curve for selectivity of the receiver:





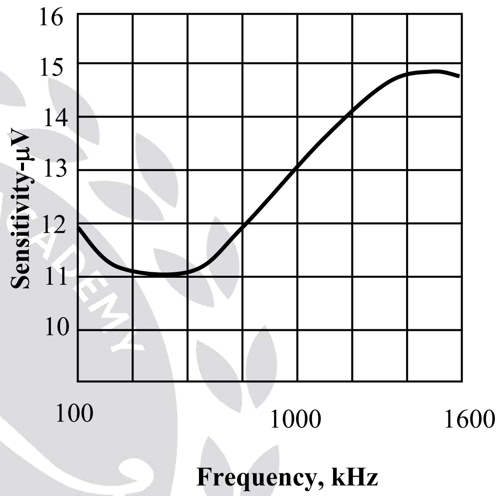
Sensitivity:
It is defined as the minimum signal strength that should be maintained at the input of a receiver to get a standard output. Sensitivity depends up on the over all gain of an amplifier. If the gain of the amplifier is high, the sensitivity is also high. The sensitivity of a receiver is usually stated in micro volts of received signal. The signal to noise ratio and the power of the signal at the output of the audio section are used to determine the quality of a received signal.

In commercial AM broadcast band receivers, a 10-dB or more signal to noise ratio with 1/2 W (27 dBm) of power at the output of the audio section is used.

For broadband microwave receivers, a 40dB or more signal to noise ratio with approximately 5mW (7dBm) of signal power is the minimum acceptable value.

For a typical sensitivity of a commercial broadcast band AM receivers is $50\mu v$ and a two way mobile radio receivers generally has a sensitivity bandwidth of 0.1 μV and $10\mu V$.

Curve for sensitivity of the receiver:
The receiver's sensitivity is also called receivers threshold. The sensitivity of an AM receivers depends on the noise power present at the input to the receivers, the receiver's noise figure, the sensitivity of the AM detector, and the bandwidth improvement factor of the receivers. The best way to improve the sensitivity of receivers is to reduce the noise level. This can be accomplished by reducing either the temperature or the bandwidth of the receivers or improving the receiver's noise figure.



Fidelity: Fidelity is a measure of an audio signal quality. It is defined as the ability of the receivers to reproduce all audio frequencies equally in the entire tuning range at the output of receivers High fidelity or (Hi- fi) systems are used for high quality output.

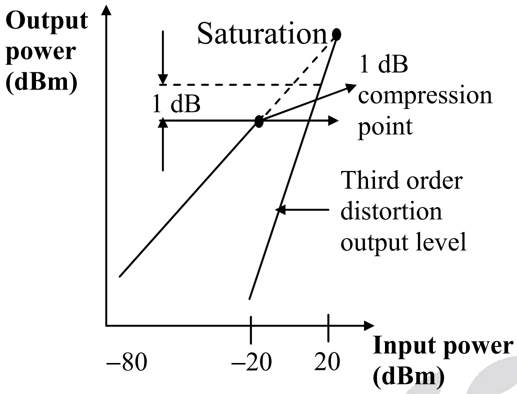
Ex: CD, DVD players.

It will produce an exact replica of the original source information. Any frequency phase or amplitude variations that represent in the demodulated waveform that were not in the original information signal are considered as distortions.

There are three forms of distortion that can deteriorate the fidelity of a communications systems

1. Amplitude 2. Phase 3. Frequency

Curve for the Fidelity of a receiver:

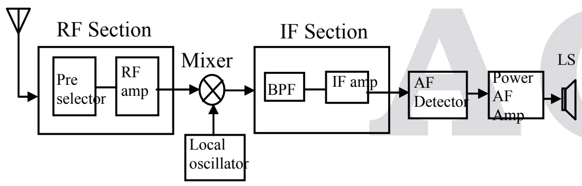


The super heterodyne receiver has a good gain, selectivity, sensitivity and hence it finds practical use.

TRF receiver has a non uniform selectivity hence super heterodyne receiver is preferred The Block diagram of Super Heterodyne Receiver is shown in figure.

It consists of five sections namely, the RF section, the mixer section, the IF section, the audio detector and audio amplifier section.

In Super Heterodyne Receiver the RF signal $\rightarrow$ IF signal $\rightarrow$ AF signal

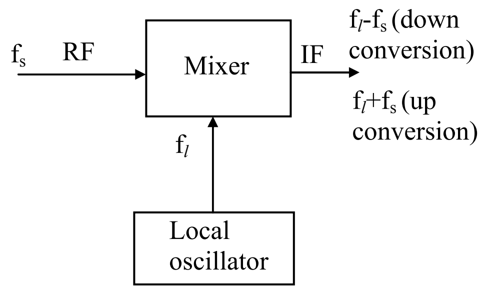


1. **RF Section:** The RF amplifier internally consists of a pre selector and an amplifier stage in a single combined circuit.
   The pre selector is a broad-tuned BPF with an adjustable center frequency that is tuned to the desired carrier frequency.
   The RF amplifier must be a low noise amplifier which reduces the noise bandwidth of the receiver and provides the initial step reducing the overall receiver bandwidth to the minimum bandwidth required to pass the information signals. The RF amplifier determines the sensitivity of the Receiver. It is also called as tuned RF amplifier. By tuning arrangement, we are making the resonant frequency of the tuned circuit equal to the carrier frequency of the required channel.
   The advantages of including RF amplifiers in a Receiver are
   1. Greater gain $\Rightarrow$ better sensitivity
   2. Improved image frequency rejection
   3. Better signal to noise ratio
   4. Better selectivity

   Heterodyne means to mix two frequencies together in a non linear device or to translate one frequency to another using non linear mixing.

2. **Mixer section:** The mixer stage is a non linear device and its purpose is to convert radio frequencies to intermediate frequency (RF-to-IF) frequency translation. Heterodyning takes place in the mixer stage and radio frequencies are down converted to intermediate frequencies. Always the local oscillator frequency should be greater than the signal frequency.

$$f_l >> f_s$$



A constant frequency difference is maintained between the local oscillator and the RF circuits. The down conversion is done with respect to the tuned circuit. Tuning means changing the local oscillator frequency. Mixer will change the carrier frequency from f_s to f_{IF}. The most common intermediate frequency used in AM broad cast receiver is 455 kHz

3. **IF Section:** IF section consists of an IF amplifier and BPF. Most of the receiver gain and selectivity is achieved in the IF section. IF is always lower in frequency than the RF because it is easier and less expensive to construct high-gain stable amplifiers for the low-frequency signals.

$$\mathbf{IF = f_l - f_s}$$

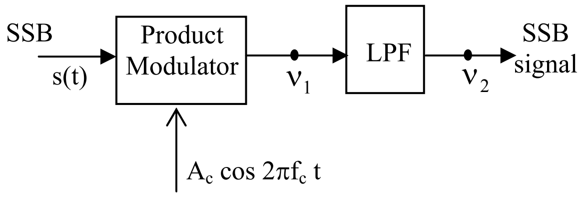
Choice of IF:

1. If the IF is too high, poor selectivity and poor adjacent channel rejection.
2. A high value of IF increases tracking difficulties
3. If IF is very low, image frequency rejection becomes poorer.
4. If IF is very low, the frequency stability of the local oscillator should be very high.
5. The IF must not fall with in the tuning range of the Receiver.

4. **Detector section:** The purpose of the detector section is to convert the IF signals back to the original source information. The detector is generally called an audio detector in the broadcast receiver because the information signals are audio-frequencies.

5. **Audio Amplifier Section:** The audio section comprises several cascaded audio amplifiers. The number of amplifiers used depends upon the audio signal power.

(b) Demodulation of SSB signals:



$$s(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \mp \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t$$

$$\nu_1(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t \;\; A_c \cos 2\pi f_c t$$
$$\mp \frac{A_c}{2}\hat{m}(t) \sin 2\pi f_c t \;\; A_c \cos 2\pi f_c t$$

$$= \frac{A_c^2}{2} m(t) \cos^2 2\pi f_c t$$
$$\mp \frac{A_c^2}{2}\hat{m}(t) \sin 2\pi f_c t \;\; \cos 2\pi f_c t$$

$$\nu_1(t) = \frac{A_c^2}{4}[1 + \cos 4\pi f_c t] m(t) \mp \frac{A_c^2}{4}\hat{m}(t)\sin 4\pi f_c t$$

$$\therefore \nu_2(t) = \frac{A_c^2}{4} m(t)$$

Consider the locally generated signal as $A_c \cos(2\pi f_c t + \phi)$,

Suppose locally generated carrier is not in phase with incoming SSB carrier.

$$\nu_2(t) = \frac{A_c^2}{4}[\cos\phi\, m(t) \mp \hat{m}(t)\sin\phi]$$

If $\phi = 0^\circ$, $\nu_2(t) = \frac{A_c^2}{4} m(t)$

$\Rightarrow$ Information is not lost

Again if $\phi = 90^0,\ v_2(t) = \left(\frac{A_c^{\,2}}{4}\right)\hat{m}(t) \neq 0$

$\Rightarrow$ Information is not lost

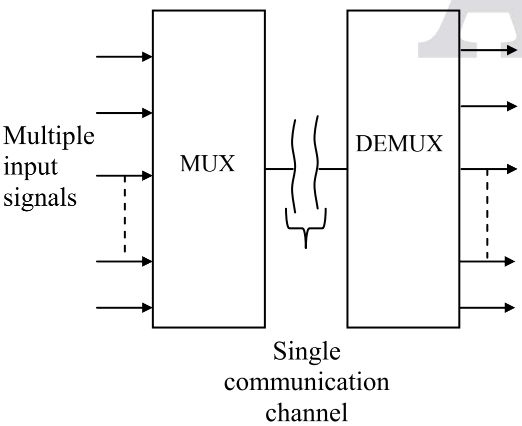
So no Quadrature Null effect in the case of SSB, which is a major advantage over DSB.

(c) Multiplexing

Multiplexing is the process of simultaneously transmitting two or more individual signals over a single communication channel. Due to multiplexing, it is possible to increase the number of communication channels so that more information can be transmitted.

The typical applications of multiplexing are in telemetry, telephony satellite communication.

The concept of multiplexing is shown in fig. The multiplexer receives a large number of different input signals. Multiplexer has only one output which is connected to the single communication channel. The multiplexer combines all input signals into a single composite signal and transmits it over the communication medium. Sometimes, the composite signal is used to modulate a carrier before transmission. At the receiving end of communication link, a demultiplexer is used to sort out the signals into their original form.



S.No.	TDM	FDM
1.	It is a technique for transmitting several messages on one channel by dividing time domain slots. One slot for each message	It is a technique to transmit several messages on one channel, message signals are distributed in frequency spectrum such that they do not overlap.
2.	It requires commutator at the transmitting end and a decommutator at the receiving end	FDM requires modulator, filters and demodulators.
3	Perfect synchronization between transmitter and receiver is required.	Synchronization between transmitter and receiver is not required.
4.	Crosstalk problem is not severe in TDM.	FDM suffers from crosstalk problem due to imperfect band pass filter.
5.	It is usually preferred for digital signal transmission	It is usually preferred for analog signal transmission.
6.	It does not require very Complex circuitry.	It requires complex circuitry at transmitter and receiver.

02.

Sol: Given that,

$Q = 40$

Local oscillator frequency $f_{LO} = 1010$ kHz

Input signal frequency $f_s = 555$ kHz

$f_{LO} = f_s + f_i$

$1010k = 555k + f_i$

$\therefore\ f_i = 455$ kHz

$\therefore$ Image frequency $f_{si} = f_s + 2f_i$

$f_{si} = 555 + 2\times455 = 555 + 910 = 1465$ kHz

Rejection ratio $\alpha = \sqrt{1+Q^2\rho^2}$

Where

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1465}{555} - \frac{555}{1465} = 2.261$$

$$\therefore \alpha = \sqrt{1+40^2\times(2.261)^2}$$

$\alpha = 90.4$

03.

Sol:

IF = Intermediate frequency = 450 kHz

Q = 65

(a) f_s = tuned frequency of circuit = 1200 kHz

$f_s = 1.2\text{MHz}$

$f_{si} = f_s + 2\text{ IF} = 1200 + 2\times 450 = 2.1\text{ MHz}$

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$\rho = \frac{2.1}{15} - \frac{1.2}{2.1} = 1.178$$

$$\text{IRR} = \sqrt{1+\rho^2 Q^2}$$

$$\text{IRR} = \sqrt{1+(65)^2(1.178)^2}$$

IRR = 76.61

(b) $f_s = 20\text{ MHz}$

$f_{si} = f_s + 2\text{ IF}$

$= 20\text{ MHz} + 2\times 0.45$

$f_{si} = 20.9\text{ MHz}$

$$\rho = \frac{20.9}{20} - \frac{20}{20.9} = 0.088$$

IRR = 5.81

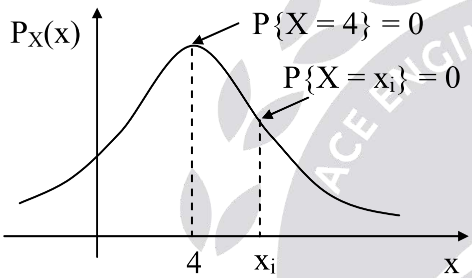
Chapter 6 Random Variables & Noise

Objective Practice Solutions

01. Ans: (c)

Sol: A continuous Random variable X takes every value in a certain range, the probability that X = x, is zero for every x in that range.

Given $P_X(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}}$ is a continuous Random variable therefore probability of the event {X = 4} is zero.



02. Ans: (b)

Sol: Given,

X & Y are two Random Variables

Y = cosπx

$f(x) = 1 \quad \frac{-1}{2} < x < \frac{1}{2}$

$\quad = 0$ else where

f(y) = ?

$$f(y) = f(x)\left|\frac{dx}{dy}\right|$$

$$x = \frac{1}{\pi}\cos^{-1}(y)$$

$$dx = \frac{1}{\pi} \times \frac{-1}{\sqrt{1-y^2}} dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{\pi\sqrt{1-y^2}}$$

$$f(y) = \frac{1}{\pi\sqrt{1-y^2}}$$

$$\sigma_y{}^2 = E[y^2] - [E[y]]^2$$

03. Ans: (d)

Sol: The probability density function of the envelope of a sinusoidal plus narrrow band noise is Rician.

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right)$$

04. Ans: (a)

Sol: Given,

Differential equation of a system is

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$$

Applying Fourier transform,

$$\Rightarrow Y(f)(1 + jf) = X(f)(jf - 1)$$

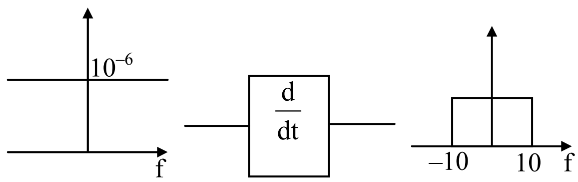
$$\frac{Y(f)}{X(f)} = \frac{-1 + jf}{1 + jf}$$

The transform function of system is a All pass filter

$\therefore S_y(f) = S_x(f)$

05. Ans: (a)

Sol:



$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$

H(f) = j2πf

$|H(f)|^2 = 4\pi^2 f^2$

$S_{YY}(f) = 4\pi^2 f^2 S_{XX}(f)$

The Noise power at the output of the LPF is

$$N_o = \int_{-10}^{10} S_{YY}(f)df$$

$$N_o = \int_{-10}^{10} 4\pi^2 f^2 \times 10^{-6} df$$

$$= 2\times 4\pi^2 \times 10^{-6} \int_0^{10} f^2 df$$

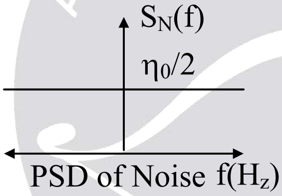
$$= 2\times 4\pi^2 \times 10^{-6} \times \frac{10^3}{3}$$

$$\boxed{\therefore\ N_o = 0.0263W}$$

06. Ans: (a)

Sol: Given,

PSD of Noise $= \frac{\eta_0}{2}$



$T = 27^\circ C \Rightarrow 300K$

$P_n = K.T.B$

$\eta_0 = KT$

$= 1.38\times10^{-23}\times300$

$$PSD = \frac{\eta_0}{2}$$

$$= 1.38\times10^{23}\times150$$

$$= \frac{207}{10^{23}}$$

07. Ans: (b)

Sol: $P_n = K.T.B$

$$= \left(\frac{1}{2}\times1.38\times10^{-23}\times300\right)\times2\times10^6\times2$$

$$= 8.28\times10^{-15}\ W$$

08. Ans: (b)

Sol: $E(X) = \int_{-1}^{3} x.p(x)dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{3} = 1$

$$E(X^2) = \int_{-1}^{3} x^2 p(x)dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{3} = 7/3$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{7}{3} - 1 = \frac{4}{3}$$

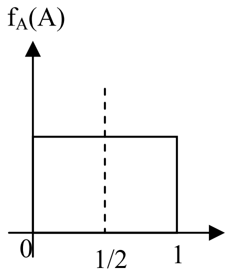
09. Ans: (d)

Sol: $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$

$= E[A\cos\omega t_1 A\cos\omega t_2]$

$= \cos\omega t_1 \cos\omega t_2\, E[A^2]$ $[\because E[A^2] = 1/3]$

$= \frac{1}{3}\cos\omega t_1 \cos\omega t_2$



$\sigma^2 = \frac{(1)^2}{12} \rightarrow$ variance

$E[A^2] = \sigma^2 + [E[A]]^2$

$= \frac{1}{12} + \frac{1}{4}$

$E[A^2] = \frac{4}{12} = \frac{1}{3}$

10. Ans: (b)

Sol: $R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)]$

Let $t_2 - t_1 = \tau$

$E[(A\cos\omega t_1 + B\sin\omega t_1)(B\cos\omega t_2 - A\sin\omega t_2)]$

$\because E[AB] = E[A]\,E[B]$

$E[AB] = 0$

$E[BA] = 0$

$E[A^2] = \sigma^2$

$E[B^2] = \sigma^2$

$= \cos\omega t_1.\cos\omega t_2 E[AB] - \sin\omega t_1 \sin\omega t_2 E[BA]$

$\quad - E[A^2]\cos\omega t_1 \sin\omega t_2$

$\quad + E[B^2]\sin\omega t_1 \cos\omega t_2]$

$= 0 - 0 - \sigma^2\cos\omega t_1\sin\omega t_2 + \sigma^2\sin\omega t_1\cos\omega t_2$

$= -\sigma^2(\cos\omega t_1\sin\omega t_2 + \sin\omega t_1\cos\omega t_2)$

$= -\sigma^2\sin\omega(t_2-t_1)\,(\because \tau = (t_2-t_1))$

$= -\sigma^2\sin\omega\tau$

11. Ans: (b)

Sol: X(t) = positive frequencies required

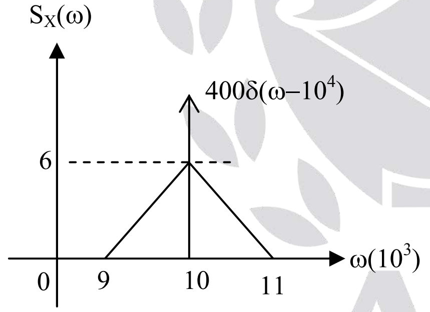
$E[X^2(t)]$ and $E[X(t)]$

$$E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)d\omega$$

$$= \frac{1}{\pi}\left(400 + \frac{1}{2}(2000)\times 6\right) = \frac{6400}{\pi}$$

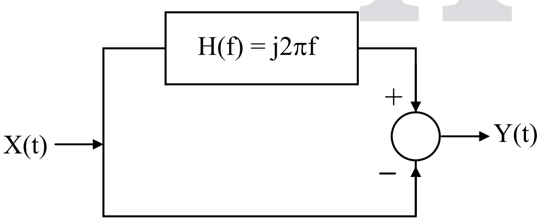
$E[X(t)] = 0$

[$\because$ The given function is periodic function]



12. Ans: (a)

Sol:



Overall $H(f) = j2\pi f - 1$

$R_X(\tau) = e^{-\pi\tau^2}$

$Y(t) = X(t)*h(t)$

$|H(f)|^2 = (4\pi^2 f^2+1)$

$R_{XX}(T) \xleftrightarrow{FT} S_{XX}(f)$

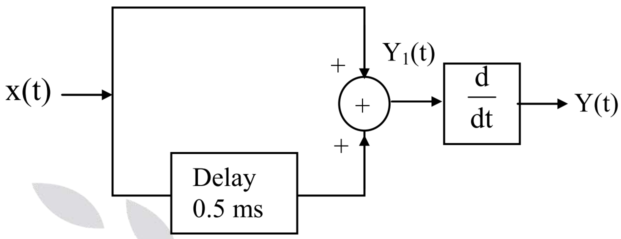
$e^{-\pi\tau^2} \xleftrightarrow{FT} e^{-\pi f^2}$

Normalized Gaussian function

$S_{YY}(f) = |H(f)|^2 S_{XX}(f) = (4\pi^2 f^2 + 1)\,e^{-\pi f^2}$

13. Ans: (d)

Sol:



$$Y(t) = \frac{d}{dt}\left(X(t) + X(t-t_d)\right)$$

$$Y(f) = j2\pi f\left(1 + e^{-j2\pi f t_d}\right)X(f)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

$$= j2\pi f(1 + e^{-j2\pi f t_d})$$

$|H(f)|^2 = 4\cos^2\pi f t_d$

$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$

$= 4\pi^2 f^2(2\cos(\pi f t_d))^2 S_{XX}(f)$

At $S_{YY}(f) = 0$

$$\pi f t_d = (2n+1)\frac{1}{2t_d}$$

$$f = (2n+1)\frac{1}{2\times 0.5\times 10^{-3}}$$

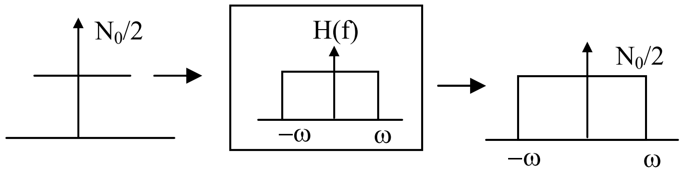
$f = (2n+1)10^3$

$f = (2n+1)f_0$

$f_0 = 1$ kHz

14. Ans: (b)

Sol:



Uncorrelated $\Rightarrow$ cov(τ) $\Rightarrow R_{XX}(\tau) - \mu^2 \times (\tau)$

cov(τ) = $R_{XX}(\tau)$ $\Rightarrow R_{n_0}(\tau) = 0$

$\Rightarrow N\omega_0 \sin(2\omega\tau) = 0$, sinCx = 0; x is an integer

$2\omega\tau = m$

$\tau = \frac{m}{2\omega}$, integer m = 1, 2, 3 …….

15. Ans: (b)

Sol: We know that,

$$\text{ACF} \xleftrightarrow{\text{F.T}} S_x(f)$$

Taking Inverse Fourier Transform

$$F^{-1}\left[S_y(t)\right] = \int_{-\infty}^{\infty} S_y(t)\, e^{j2\pi f\tau} df$$

$$R_y(\tau) = \int_{-B_0}^{B_0} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2}\left[\frac{e^{j2\pi f\tau}}{j2\pi\tau}\right]_{-B_0}^{B_0}$$

$$= \frac{N_0}{2\pi\tau}\left[\frac{e^{j2\pi B_0\tau} - e^{-j2\pi B_0\tau}}{2j}\right]$$

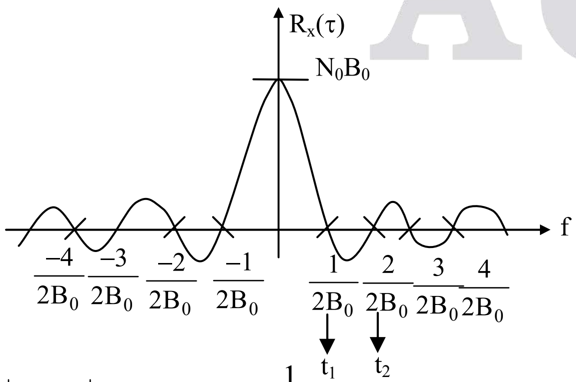
$$= \frac{N_0}{2\pi\tau}\sin(2\pi B_0\tau)$$

$$= N_0 B_0 \frac{\sin(2\pi B_0\tau)}{2\pi B_0\tau}$$

$$R_y(\tau) = N_0 B_0 \operatorname{sinc}(2B_0\tau)$$

16. Ans: (b)

Sol:



$$|t_1 - t_2| = \text{multiple of } \frac{1}{2B}$$

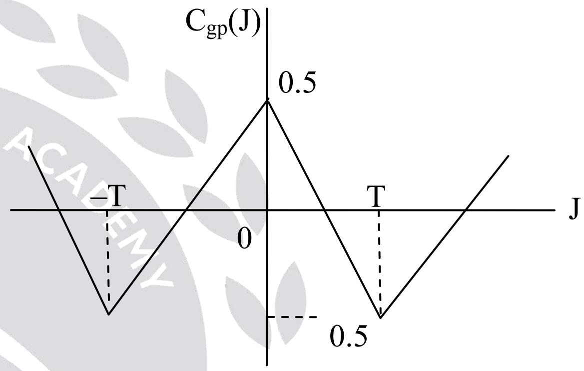
17.

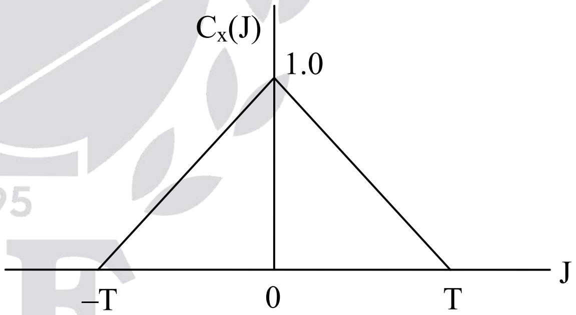
Sol: Since

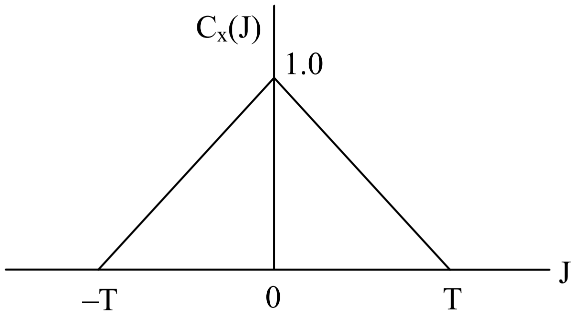
$y(t) = g_p(t) + X(t) + \sqrt{3/2}$

and g_p (t) and X (t) are uncorrelated, then $C_Y(\tau) = C_{g_p}(\tau) + C_X(\tau)$.

Where $C_{gp}(\tau)$ is the auto covariance of the periodic component and $C_x(\tau)$ is the auto covariance of the random component $C_Y(\tau)$ is the plot figure shifted down by 3/2, removing the DC component $C_{gp}(\tau)$ and $C_x(\tau)$ are plotted below







Both $g_p(t)$ and X(t) have zero mean, Average

(a) The power of the periodic component $g_p(t)$ is therefore,

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p^2(t)dt = C_{g_p}(0) = \frac{1}{2}$$

(b) The average power of the random component x(t) is

$E[X^2(t)] = C_x(0) = 1$

18.

Sol:

(a) The power spectral density consists of two components:

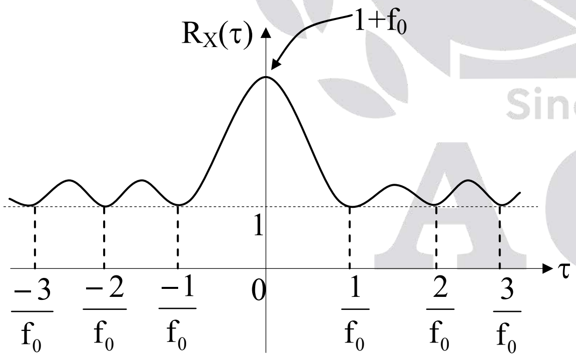
(1) A delta function δ(t) and the origin, whose inverse Fourier transform is one.

(2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0\ \text{sinc}^2(f_0\tau)$

Therefore, the autocorrelation function of X(t) is

$R_X(\tau) = 1 + f_0\ \text{sinc}^2(f_0\tau)$

Which is sketched below:



(b) Since $R_X(\tau)$ contains a constant component of amplitude 1. It follows that the dc power contained in X(t) is 1.

(c) The mean-square value of X(t) is given by

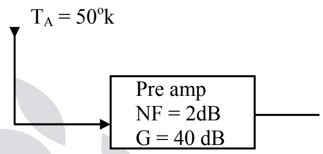
$$E[X^2(t)] = R_X(0)$$
$$= 1 + f_0$$

The ac power contained in X(f) is therefore equal to f_0.

(d) If the sampling rate is f_0/n, where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if X(t) were a Gaussian process.

19. Ans: (a)

Sol:



$10 \log_{10} NF = 2dB$

$\log_{10} NF = 0.2$

$NF = 10^{0.2}$

Noise temperature $= (F - 1)\ T_o$

$= (10^{0.2} - 1)\ 290o$

$= 169.36$ K

Noise power i/p $= k\ T_e B$

$= 1.38 \times 10^{-23} \times (169.36 + 50) \times 12 \times 10^6$

Noise power at o/p $= (3.632 \times 10^{-14}) \times 10^4$

$= 3.73 \times 10^{-10}$ watts

20. Ans: 100 W

Sol: $E[x^2(t)] = E[(3V(t) - 8)^2]$

$= E[(9V(t)^2 + 64 - 2\times3V(t)\times8]$

$= E[(9V^2(t) + 64 - 48V(t)]$

$= 9E[V^2(t)] + E[64] - 48E[V(t)]$

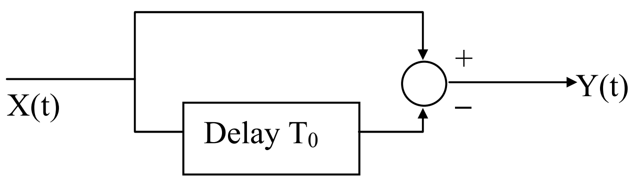
$[E[V(t)] = 0,\ E[V^2(t)] = MS = R(0) = 4e^{-5(0)} = 4,$
$E[\text{constant}] = \text{constant}]$

$E[x^2(t)] = 9\times4 + 64 = 36 + 64$

$= 100$

21. Ans: (b)

Sol:



$Y(t) = X(t) - X(t - T_o)$

ACf of o/p $= R_y(\tau) = E[y(t)\, Y(t+\tau)]$

$R_y(\tau) = E[(X(t) - X(t - T_o)]\,[X(t+\tau) - X(t+\tau-T_o)]$

$R_y(\tau) = E[(X(t)\,X(t+\tau) - X(t)X(t+\tau-T_o) - X(t-T_o)\,X(t+\tau) + X(t-T_o)\,X(t+\tau-T_o)]$

$R_y(\tau) = [R_x(\tau) - R_x(\tau - T_o) - R_x(\tau + T_o) + R_x(\tau)]$

$R_y(\tau) = 2\,R_x(\tau) - R_x(\tau - T_o) - R_x(\tau + T_o)$

Conventional Practice Solutions

01.

Sol: The auto correlation function

$$R_{XX}(\tau) = e^{\left(-\tau^2/2\sigma^2\right)} \quad -\infty \le \tau \le \infty$$

From the wiener-khinchin theorem auto correlation function and power spectral density form a fourier transform pair

$$R_{xx}(\tau) \xleftrightarrow{F} S_{xx}(f)$$

Where $S_{xx}(f)$ is PSD (Power spectral density) of the signal.

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j2\pi f\tau} d\tau$$

$$S_{xx}(f) = \int_{-\infty}^{\infty} e^{-\tau^2/2\sigma^2} e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\left(\frac{1}{2\sigma^2}\tau^2 + j2\pi f\tau\right)} d\tau$$

$$= \sqrt{\frac{\pi}{\frac{1}{2\sigma^2}}}\; e^{\frac{(j2\pi f)^2 - 4\left(\frac{1}{2\sigma^2}\right)}{4.\frac{1}{2\sigma^2}}}$$

$$\therefore \int_{-\infty}^{\infty} e^{-\left(at^2+bt+c\right)} dt = \sqrt{\frac{\pi}{a}} e^{\frac{b^2-4ac}{4a}}$$

$$= \sqrt{2\pi\sigma^2}\, e^{\frac{-4\pi^2 f^2}{4.\frac{1}{2\sigma^2}}}$$

$$S_{xx}(f) = \sqrt{2\pi\sigma^2}\, e^{-2\sigma^2\pi^2 f^2}$$

Normalized average power

$$P_{avg} = \int_{-\infty}^{\infty} S_{xx}(f) d_f = \sqrt{2\pi\sigma^2} \int_{-\infty}^{\infty} e^{-2\sigma^2\pi^2 f^2} df$$

$$= \sqrt{2\pi\sigma^2} \sqrt{\frac{\pi}{2\sigma^2\pi^2}} = 1\,W$$

02.

Sol: (i) Given values

$$\frac{\eta}{2} = 10^{-12}\, V^2/Hz$$

3dB bandwidth of RC filter is ω_0

$= 2\pi(1000)$

The frequency response of an RC low pass filter is $H(\omega)$ with a 3dB cutoff frequency is given by

$$H(\omega) = \frac{1}{1 + j\omega/\omega_0}$$

Now output Noise power N_0 is given by

$$N_0 = E[n_0^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\eta}{2}|H(\omega)|^2\, d\omega$$

Substituting value of $H(\omega)$

$$N_0 = \frac{\eta}{2}\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^2}d\omega$$

$$= \frac{\eta}{2}\frac{1}{2\pi}\left.\frac{\tan^{-1}\left(\frac{\omega}{\omega_0}\right)}{\left(\frac{1}{\omega_0}\right)}\right|_{-\infty}^{\infty}$$

Simplifying, we get $= \frac{\eta}{2}\frac{1}{2\pi}(\pi\times\omega_0)$

$$= \frac{\eta}{2}\frac{1}{2}\omega_0$$

$$N_0 = \frac{1}{2}\frac{\eta}{2}\omega_0$$

$$= \frac{1}{2}\times 10^{-12}\times 2\pi\times 1000\, W$$

$$[\because \omega_0 = 2\pi(1000)]$$

$= 3.14 \times 10^{-9}$ Watts

$= 3140\mu$Watts

(ii) $R_s(\tau) = 2e^{-|\tau|}$

$R_n(\tau) = e^{-2|\tau|}$

$R_r(\tau) = E(r(t)\, r(t+\tau))$

$= R[(s(t) + n(t)]\, [s(t+\tau) + n(t+\tau]]$

$= E[s(t)s(t+\tau)] + E[s(t)n(t+\tau))]$

$+ E[s(t+\tau)n(t+\tau)] + E[s(t+\tau)n(\tau)]$

Since s(t) & n(t) are statistically independence

$E[s(t+\tau)n(t)] = 0$

$E[s(t)n\,(t+\tau)] = 0$

$R_r(\tau) = E[s(t)\,(t+\tau)] + E[n(t)n(t+\tau)]$

$= R_s(\tau) + R_n(\tau)$

$R_r(\tau) = 2e^{-|\tau|} + e^{-2|\tau|}$

$R_r(0)$ gives total power of r(t)

$\therefore$ Total power $= R_r(0) = 2e^{-0} + e^{-0}$

$= 3W$

Chapter 7 Noise in Analog Communication

Objective Practice Solutions

01. Ans: (d)

Sol: Output of the multiplier

$= m(t).\cos\omega_o t\cos(\omega_o t+\theta)$

$= \dfrac{m(t)}{2}[\cos(2\omega_o t+\theta)+\cos\theta]$

Output of LPF $V_0(t) = \dfrac{m(t)}{2}\cos\theta$

$= \dfrac{1}{2}\cos\theta\ m(t)$

Power of o/p signal $= \underset{T\to\infty}{\text{Lt}}\dfrac{1}{T}\int_{<T>} v_0^2(t)\,dt$

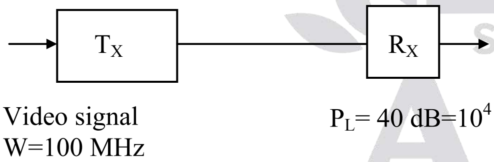
$= \underset{T\to\infty}{\text{Lt}}\dfrac{1}{T}\int_{<T>}\left(\dfrac{1}{2}\cos\theta\, m(t)\right)^2 dt$

$= \dfrac{1}{4}\cos^2\theta\left[\underset{T\to\infty}{\text{Lt}}\dfrac{1}{T}\int_{<T>} m^2(t)\,dt\right]$

$= \dfrac{1}{4}\cos^2\theta\ P_m$

02. Ans: (a)

Sol:



$n_i = n_0$

$n_i = n_0 \times W = 10^{-20}\times 100\times 10^6$

$S_i = \dfrac{P_t}{P_L} = \dfrac{1\text{mw}}{10^4} = 1\times 10^{-7}$

$n_i = 10^{-20}\times 100\times 10^6$

$\dfrac{S_i}{n_i} = \dfrac{10^{-7}}{10^{-12}} = 10^5 = 50\text{ dB}$

$\dfrac{S_i}{n_i} = 50\text{ dB}$

03. Ans: (b)

Sol: $\Delta f = 75$ kHz

$f_m = 15$ kHz

$\left(\dfrac{S}{N}\right)_0 = 40\text{dB} = 10^4$

$\text{FOM} = \dfrac{3}{2}\beta^2;\ \beta = \dfrac{\Delta f}{f_m}$

$\dfrac{\left(\dfrac{S_0}{N_0}\right)}{\left(S_i/N_i\right)} = \dfrac{3}{2}\beta^2$

$\left(S/N\right)_i = \left(\dfrac{S}{N}\right)_0\times\dfrac{2}{3}\times\dfrac{1}{\beta^2}$

$\left(S/N\right)_{i(\text{dB})} = 24\text{dB}$

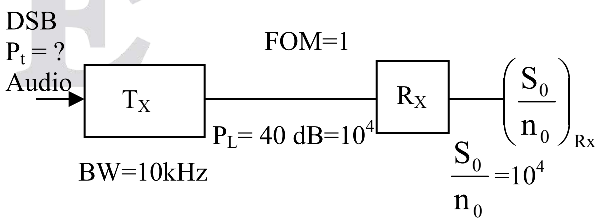
04. Ans: (c)

Sol: $\left(\dfrac{S}{N}\right)_i = 10\text{ dB}$; FOM $= \dfrac{1}{3}$

$\left(\dfrac{S}{N}\right)_0 = \dfrac{1}{3}\times 10 = 3.33$

05. Ans: (a)

Sol:



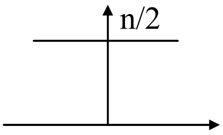
06. Ans: (a)

Sol: For SSB modulation

$\Rightarrow \dfrac{S_0}{N_0} = \dfrac{S_i}{N_i} = 10^4$

(Only SSB modulation in one sided n/2)

$P_t = ?$



$\dfrac{S_i}{n_i} = \dfrac{S_0}{n_0} = 10^4$

$S_i = 10^4 \times 10 \times 10^3 \times 2 \times 10^{-9}$ w/Hz

$S_i = 20 \times 10^{-2}$

$(S_i)_{dB} = (P_t)_{dB} - (P_t)_{dB}$

$(P_t)_{dB} = (S_i)_{dB} + (P_L)_{dB}$

$P_t = S_i P_L = 20 \times 10^{-2} \times 10^4$

$P_L = 2$ kW

07. Ans: (c)

Sol: For AM

FOM $= \dfrac{1}{3}$ (if $\mu = 1$)

$\dfrac{S_0}{N_0} = \left(\dfrac{1}{3}\right)\dfrac{S_i}{N_i}$

$\Rightarrow S_i = 3\left(\dfrac{S_0}{N_0}\right) \times N_i$

$= 3 \times 10^4 \times 2 \times 10^{-9} \times 10\text{kHz}$

$= 0.6$

$\therefore P_t = S_i \times P_L$

$= 0.6 \times 10^4$

$= 6\text{KW}$

08. Ans: (b)

Sol: Noise figure $= \dfrac{(SNR)_{I/P}}{(SNR)_{O/P}}$

$Nf_{,dB} = SNR_{i,dB} - SNR_{o/p,dB}$

$SNR_{o/p,dB} = SNR_{I/P,dB} - Nf_{dB}$

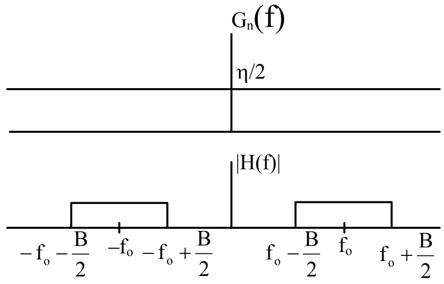
$= 37 - 3$

$= 34$ dB

Conventional Practice Solutions

01.

Sol: White noise PSD is,



Output PSD = input PSD × $|H(f)|^2$

$|H(f)| = 1; \quad f_o - \dfrac{B}{2} \le |f| \le f_o + \dfrac{B}{2}$

$\Rightarrow$ Output PSD = input PSD ;

$f_o - \dfrac{B}{2} \le |f| \le f_o + \dfrac{B}{2}$

Output power

$= 2\int_{f_o - \frac{B}{2}}^{f_o + \frac{B}{2}} (\text{output PSD})\,df = \left[\int_{f_o - \frac{B}{2}}^{f_o + \frac{B}{2}} \left(\dfrac{\eta}{2}\right) df\right] \times 2$

$= 2 \times \dfrac{\eta}{2}[B] = \eta B$ Watts.

02.

Sol: Resistance of antenna $R_a = 50\Omega$

Equivalent noise resistance

$R_{eq} = 30\ \Omega$

$f = 1 + \dfrac{R_{eq}}{R_a} = 1 + \dfrac{30}{50}$

$f = 1.6$

f (dB) = 2.04 dB

= 10 log 1.6

f (dB) = 2.04 dB

$t_e = T_o (f - 1)$

But $T_o = 290 = 17^oC$

03.

Sol: Given data

Noise resistance (R_{eq}) = 220Ω

Input resistance (R) = 300Ω

Bandwidth of Amplifier (B) = 6MHz

Temperature (T) = 18^0C = 273+18 = 291^0K

$\therefore R_T = R_{eq} + R = 220 + 300v = 520\Omega$

Noise voltage (V) = $\sqrt{4KTBR_T}$

$= \sqrt{4\times1.38\times10^{-23}\times291\times520\times6\times10^6}$

= 6.89 μV

04.

Sol: $\frac{1}{(S/N)_{overall}} = \frac{1}{(S/N)_1} + \frac{1}{(S/N)_2} + \frac{1}{(S/N)_3}$

Given that $\left(\frac{S}{N}\right)_1 = \left(\frac{S}{N}\right)_2 = \left(\frac{S}{N}\right)_3$.

$\left(\frac{S}{N}\right)_1 = 60\text{dB}$

$\Rightarrow \left(\frac{S}{N}\right)_1 = 10^6$

$\frac{1}{(S/N)_{overall}} = \frac{1}{10^6} + \frac{1}{10^6} + \frac{1}{10^6}$

$\Rightarrow \left(\frac{S}{N}\right)_{overall} = \frac{10^6}{3}$

$\left(\frac{S}{N}\right)_{overall} = 10\log\left(\frac{10^6}{3}\right) = 10\log10^6 - 10\log3$

$\left(\frac{S}{N}\right)_{overall} = 60 - 4.771 = 55.228\text{dB}.$

Chapter 8 Baseband Data Transmission

Objective Practice Solutions

01. Ans: (d)

Sol: $\Delta = \dfrac{V_{max} - V_{min}}{2^n}$

$\Delta \alpha \dfrac{1}{2^n}$; $\dfrac{\Delta_1}{\Delta_2} = \dfrac{2^{n_2}}{2^{n_1}}$

$\dfrac{0.1}{\Delta_2} = \dfrac{2^{n+3}}{2^n}$

$\Delta_2 = 0.1 \times \dfrac{1}{8}$

$= 0.0125$

02. Ans: (3)

Sol: $(BW)_{PCM} = \dfrac{n f_s}{2}$

Where 'n' is the number of bits to encode the signal and $L = 2^n$, where 'L' is the number of quantization levels.

$L_1 = 4 \Rightarrow n_1 = 2$

$L_2 = 64 \Rightarrow n_2 = 6$

$\dfrac{(BW)_2}{(BW)_1} = \dfrac{n_2}{n_1} = \dfrac{6}{2} = 3$

$(BW)_2 = 3\ (BW)_1$

03. Ans: (c)

Sol: Given,

Two signals are sampled with $f_s = 44100$s/sec and each sample contains '16' bits

Due to additional bits there is a 100% overhead.

Out put bit rate =?

$R_b = n^| f_s^|$

$f_s^| = 2f_{s|} = 2\ [44100]$

($\because$ two signals sampled simultaneously)

$n^| = 2n$

($\because$ due to overhead by additional bits)

$R_b = 4\ (nf_s) = 2.822$Mbps

04. Ans (c)

Sol: Number of bits recorded over an hour

$= R_b \times 3600 = 10.16$ G.bits

05. Ans: (c)

Sol: $p(t) = \dfrac{\sin(4\pi W t)}{4\pi W t\ (1-16W^2t^2)}$

At $t = \dfrac{1}{4W}$; $P\left(\dfrac{1}{4W}\right) = \dfrac{0}{0}$

Use L-Hospital Rule

$$\underset{t\to\frac{1}{4W}}{\text{Lt}}\ p(t) = \underset{t\to\frac{1}{4W}}{\text{Lt}} \frac{4\pi W\ \cos(4\pi W t)}{4\pi W - 64\pi W^3(3t^2)}$$

$$= \frac{4\pi W(-1)}{4\pi W - 64\pi W^3 3\left(\dfrac{1}{16W^2}\right)}$$

$$= \frac{-4\pi W}{-8\pi W} = 0.5$$

06. Ans: 35

Sol: Given bit rate $R_b = 56$ kbps, Roll of factor $\alpha = 0.25$

BW required for base band binary PAM system

$BW = \dfrac{R_b}{2}[1+\alpha] = \dfrac{56}{2}[1+0.25]$kHz

$= 35$kHz

07. Ans: 16

Sol: $R_b = nf_s = 8$bit/sample $\times$ 8kHz $= 64$ kbps

$(B_T)_{min} = \dfrac{R_b}{2\log_2 M}$

$= \dfrac{R_b}{2\log_2 4} = \dfrac{R_b}{2\times 2}$

$= \dfrac{R_b}{4} = \dfrac{64}{4}$

$= 16$kHz

08. Ans: (b)

Sol: Given $f_s = 1/T_s = 2k$ symbols/sec

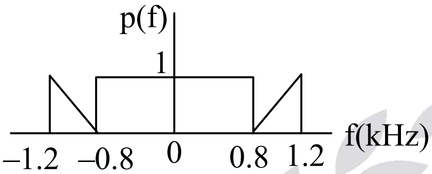
If $P(f) \overset{F.T}{\leftrightarrow} p(t)$,

Condition for zero ISI is given by

$$\frac{1}{T_s}\sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)T_s$$

p(0) = area under P(f)

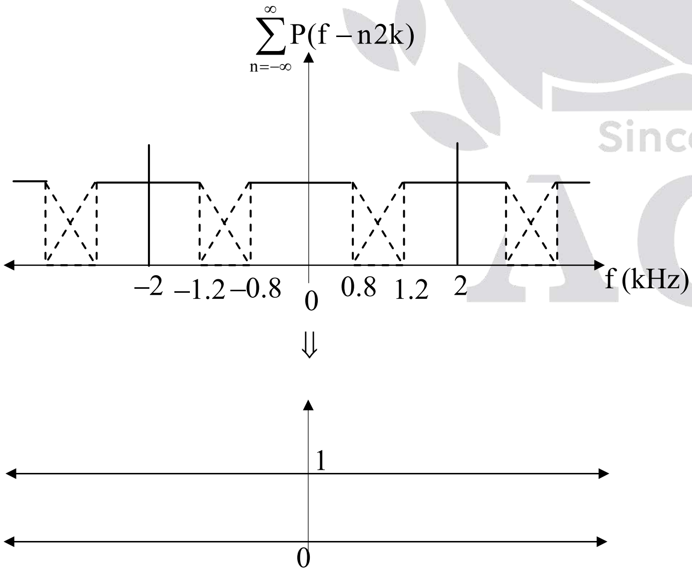


$$\text{Area} = 2\times\frac{1}{2}(1)(0.4)k + 2\times 0.8k = 2k$$

$$p(0)\ T_s = 2k \times \frac{1}{2k} = 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = 1$$

The above condition is satisfied by only option (b)



$$\therefore \sum_{n=-\infty}^{\infty} P(f - n2k) = 1$$

Option (a) is correct if pulse duration is from –1 to + 1

Option (c) is correct if the transition is from 0.8 to 1.2, –0.8 to –1.2

Option (d) is correct if the triangular duration is from –2 to +2

09. Ans: 200

Sol: $m(t) = \sin 100\pi t + \cos 100\pi t$

$= \sqrt{2}\cos[100\pi t + \phi]$

$$\Delta = 0.75 = \frac{V_{max} - V_{min}}{L} = \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

$$L = \frac{2\sqrt{2}}{0.75} \approx 4 = 2^n$$

So n = 2

f = 50 Hz so Nyquist rate = 100

So, the bit rate = 100 × 2 = 200 bps

10. Ans: (b)

Sol: Given

$f_{m_1} = 3.6\text{kHz} \Rightarrow f_{s_1} = 7.2\text{kHz}$

$f_{m_2} = f_{m_3} = 1.2\text{kHz} \Rightarrow f_{s_2} = f_{s_3} = 2.4\text{kHz}$

$f_s = f_{s_1} + f_{s_2} + f_3$

$= 12\text{kHz}$

No. of Levels used = 1024

$\Rightarrow$ n = 10bits

$\therefore$ Bit rate = nf_s

$= 10 \times 12$ kHz

$= 120$ kbps

11. Ans: (a)

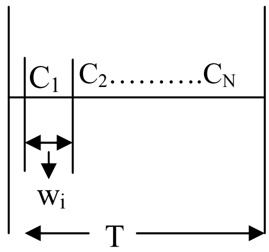
Sol: $(f_s)_{min} = (f_{s_1})_{min} + (f_{s_2})_{min} + (f_{s_3})_{min} + (f_{s_4})_{min}$

$= 200 + 200 + 400 + 800$

$= 1600$ Hz

12. **Ans: (c)**

Sol:



Minimum B.W of TDM is $\sum_{i=1}^{N} w_i$

13. **Ans: (b)**

Sol: Number of patients = 10

ECG signal B.W = 100Hz

$(Q_e)_{max} \le (0.25)\ \%V_{max}$

$$\frac{2V_{max}}{2\times 2^n} \le \frac{0.25}{100} V_{max}$$

$2^n \ge 400$

$n \ge 8.64$

$n = 9$

Bit rate of transmitted data = 10×9×200

= 18kbps

14. **Ans: (a)**

Sol: Peak amplitude $\rightarrow A_m$

Peak to peak amplitude A_m

$$\frac{-\Delta}{2} \le Q_e \le \frac{\Delta}{2}$$

PCM maximum tolerable $\frac{\Delta}{2} = 0.2\%\ A_m$

$$\Delta = \frac{\text{Peak to peak}}{L} \Rightarrow \frac{2A/m}{2L} = \frac{0.2}{100} A_m$$

$(\because \Delta = \frac{2A_m}{L})$

$\Rightarrow L = 500$

$2^n = 500$

$n = 9$

$R_b = n(f_S)_{TDM} + 9$

$f_S = R_N + 20\%R_N = R_N + 0.2R_N$

$f_S = 1.2R_N = 1.2 \times 2 \times \omega$

$f_S = 2.4$ K samples/sec

$(f_S)_{TDM} = 5(f_S)$

$= 5 \times 2.4$ K

= 12 K sample/sec

$R_b = (nf_S) + 0.5\%(nf_S)$

$$= (9 \times 12k) + \frac{0.5}{100}(9\times 12k)$$

= 108540 bps

15. **Ans: (b)**

Sol: To avoid slope over loading, rate of rise of the o/p of the Integrator and rate of rise of the Base band signal should be the same.

$\therefore \Delta f_s$ = slope of base band signal

$\Delta \times 32 \times 10^3 = 125$

$\Delta = 2^{-8}$ Volts.

16. **Ans: (b)**

Sol: $x(t) = E_m \sin 2\pi f_m(t)$

$\frac{\Delta}{T_S} < \left|\frac{dm(t)}{dt}\right| \rightarrow$ slope overload distortion takes place

$\Delta f_S < E_m 2\pi f_m$

$$\Rightarrow \frac{\Delta f_S}{2\pi} < E_m f_m \qquad (\because \Delta = 0.628)$$

$$\Rightarrow \frac{0.628\times 40K}{2\pi} < E_m f_m$$

$f_S = 40$ kHz $\Rightarrow 4$ kHz $< E_m f_m$

Check for options

(a) $E_m \times f_m = 0.3 \times 8\text{ K} = 2.4\text{ kHz}$

$(4K \nless 2.4\text{ K})$

(b) $E_m \times f_m = 1.5 \times 4K = 6\text{ kHz}$

$(4K < 6\text{ K})$ correct

(c) $E_m \times f_m = 1.5 \times 2\text{ K} = 3\text{ kHz}$

$(4K \nless 3K)$

(d) $E_m \times f_m = 30 \times 1\text{ K} = 3\text{ kHz}$

$(4K \nless 3K)$

17. Ans: (a)

Sol: Given

$m(t) = 6\sin(2\pi\times10^3 t) + 4\sin(4\pi\times10^3 t)$

$\Delta = 0.314\text{ V}$

Maximum slope of $m(t) = \frac{d}{dt}(m(t))/t = \frac{\pi}{2}$

$= 2\pi\times10^3(6) + 4\pi\times10^3[4] = 28\pi\times10^3$

18. Ans: (c)

Sol: Pulse rate which avoid distortion

$$\Delta f_s = \frac{d}{dt}m(t)$$

$$f_s = \frac{28\pi\times10^5}{0.314}$$

$f_s = 280\times10^3$ pulses/sec

Conventional Practice Solutions

01.
Sol
(i) A PCM Generator or Transmitter

The elements of a PCM system (i.e., transmitter, transmission path and receiver). Here we shall discuss the PCM generator (i.e., transmitter) from a practical point of view. Figure shows a practical block diagram of a PCM generator.

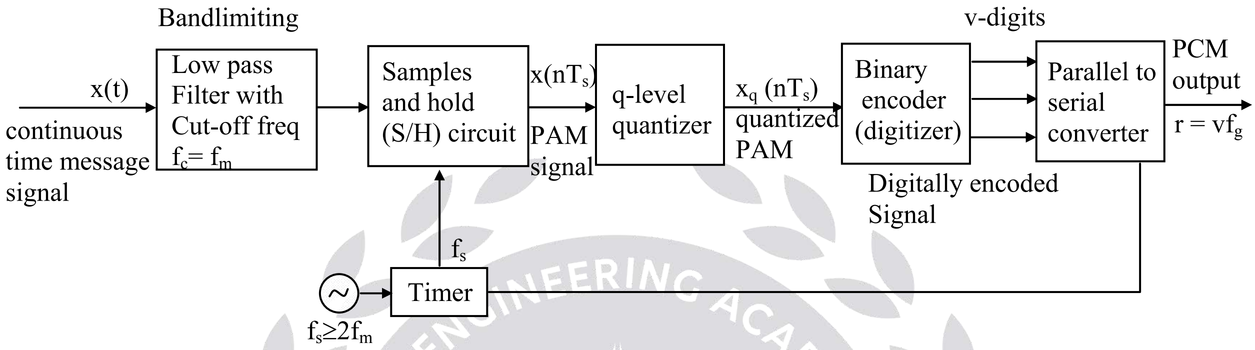


Figure (1): A Practical PCM generator

In PCM generator of figure (1) the signal x(t) is first passed through the low-pass filter of cut-off frequency f_m Hz. This low pass filter blocks all the frequency components which are lying above f_m Hz.

This means that now the signal x(t) is band limited to f_m Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above nyquist rate to avoid aliasing. i.e., $f_s \geq 2f_m$

In figure (1) the output of sample and hold circuit is denoted by $x(nT_s)$. This signal $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called quantization error. Thus output of quantizer is a digital level called $x_q(nT_s)$.

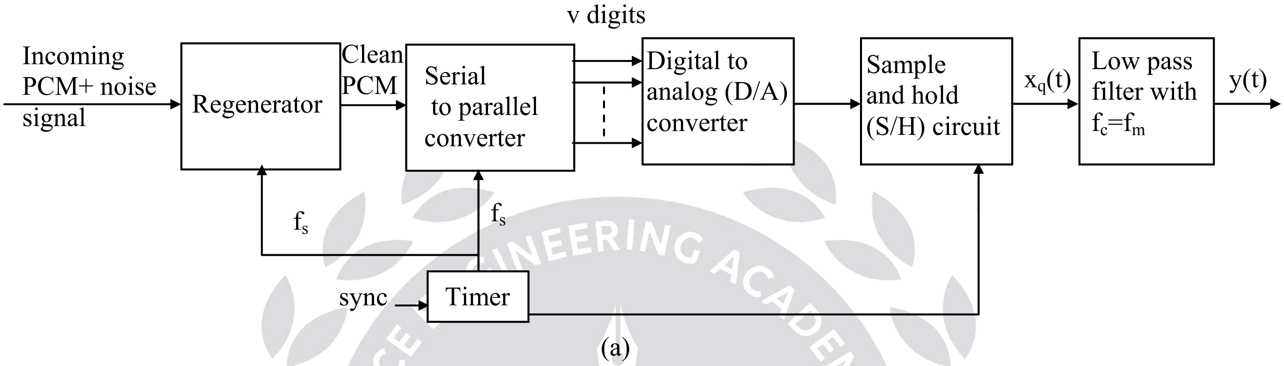
Now, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'v' binary bits. This encoder is also known as digitizer.

Also, an oscillator generates the clocks for sample and hold circuit and parallel to serial converter. In the pulse code modulation generator discussed above, sample and hold, quantizer and encoder combinely form an analog to digital converter (ADC).

PCM Receiver:

Here we shall discuss a PCM receiver from practical point of view. Figure 2(a). shows the block diagram of PCM receiver and figure 2(b). shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes pulses and removes the noise. This signal is then converted to parallel digital words for each sample.

Now, the digital word is converted to its analog value denoted as $x_q(t)$ with the help of a sample and hold circuit. This signal, at the output of sample and hold circuit is allowed to pass through a lowpass reconstruction filter to get the appropriate original message signal denoted as y(t).



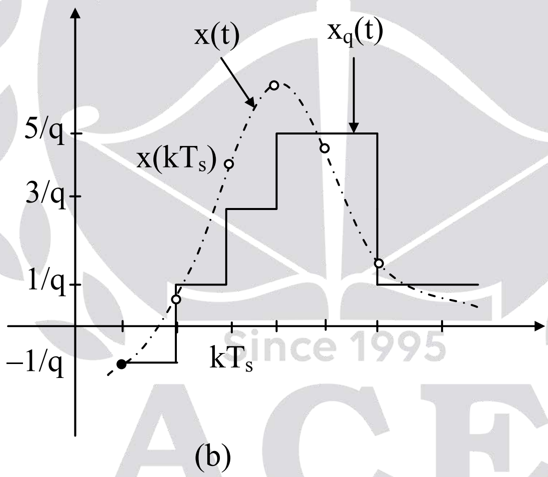


Figure (2) : (a) PCM Receiver (b) Reconstructed waveform

Applications of PCM

Some of the applications of PCM are as under:

(i) In telephony (with the advent of fibre optic cables)

(ii) In the space communication, space craft transmits signals to earth. Here, the transmitted power is very low (10 or 15W) and the distances are huge (a few million km). Still due to the high noise immunity, only PCM systems can be used in such applications

Advantages of PCM

Following are the advantages of a PCM system:

(i) Very high noise immunity

(ii) Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise

(iii) It is possible to store the PCM signal due to its digital nature.

(iv) It is possible to use various coding techniques so that only the desired person can decode the received signal

Disadvantages of PCM:

A PCM system has few drawbacks as under:

(i) The encoding, decoding and quantizing circuitry of PCM is complex

(ii) PCM requires a large bandwidth as compared to the other systems

Quantizer: A q-level quantizer compares the discrete –time input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called quantization error. Thus, the output of a quantizer is a digital level called $x_q(nT_s)$.

Classification of Quantization process:

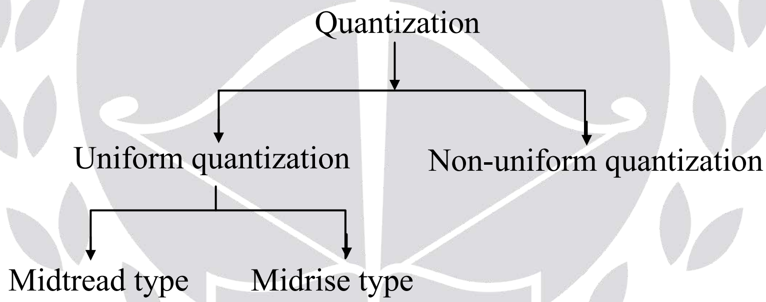


Figure (3) : Classification of quantization process

The quantization process can be classified into two types as under:

(i) Uniform quantization

(ii) Non-uniform quantization

This classification is based on the step size.

(i) Uniform Quantizer

A uniform quanitzer is that type of quantizer in which the 'step size' remains same throughout the input range.

(ii) Non Uniform Quantizer

A non-uniform quantizer is that type of quantizer in which the step-size varies according to the input signal values.

A uniform Quantizer

A quantizer is called as an uniform quantizer if the step size remains constant through out the input range.

Types of uniform quantizer

There are two types of uniform quantizer as under:

(i) symmetric quantizer of the mid-tread type

(ii) symmetric quantizer of the midrise type.

Basically, quantizer can be of a uniform or non-uniform type. In a uniform quantizer, the representation levels are uniformly spaced. Otherwise, the quantizer is non-uniform. Now, let us consider only uniform quantizers.

The quantizer characteristics can also mid-tread or midrise type. figure 4(a). shows the input-output characteristics of a uniform quantizer of the mid-tread type, which is so called because.

The origin lies in the middle of tread of the staircase like graph. Figure 4(b). shows the corresponding input-output characteristics of a uniform quantizer of the midrise type, in which the origin lies in the middle of a rising part of the stair case like graph. It may be noted that both the mid-tread and mid-rise types of uniform quantizer illustrated in figure (4). are symmetric about the origin.

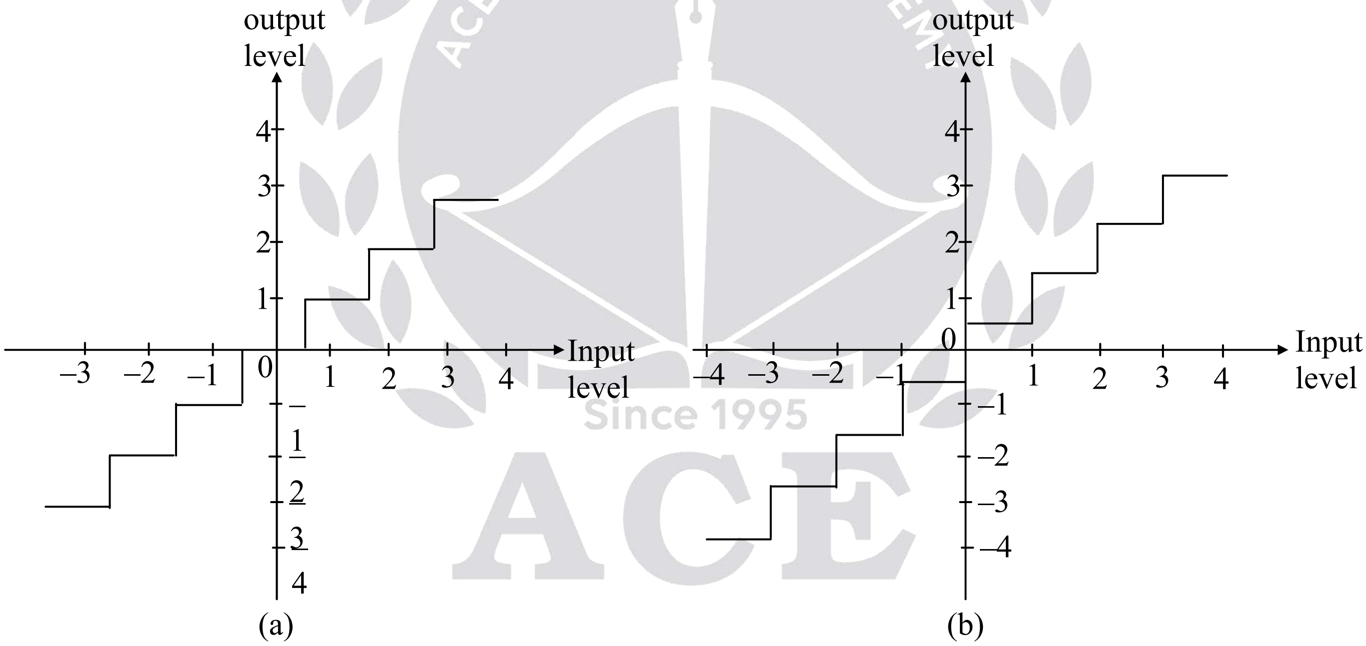


figure (4) Two types of uniform quantization: (a) Mid-tread, and (b) Midrise

$$\text{Quantization error} = \frac{\Delta^2}{12} = \frac{(\text{step size})^2}{12}$$

$$\Delta = \frac{2x_{max}}{q}$$

It can be reduced.

(1) By reducing the step size

(2) By increasing the level of quantizer

(3) By reducing the amplitude range.

(ii) $\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right) = 20$

$$\frac{S}{N} = 100$$

(i) $x_{max} = 3.8V$

$P = 30$ mW

$$\frac{S}{N} = \frac{3P.2^{2V}}{x_{max}^2}$$

$$P = \left(\frac{S}{N}\right)\frac{x_{max}^2}{3.2^{2V}}$$

$$30\times10^{-3} = P = 100\times\frac{(3.8)^2}{3.2^{2V}}$$

$$2^{2V} = \frac{100\times(3.8)^2}{3\times30\times10^{-3}}$$

$$(2^V)^2 = \left[\frac{10\times3.8}{3\times10^{-1}}\right]^2$$

$$2^V = \frac{380}{3}$$

$V = \log_2 126.66$

$V = 6.98 \approx 7$ bits

$V = 7$ bits

02.

Sol: The samples of a signal are highly correlated with each other. This is because the signal does not change fast. This means that its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resulting encoded signal contains some redundant information. To overcome this redundancy, DPCM is used.

Differential Pulse Code Modulation (DPCM) Transmitter

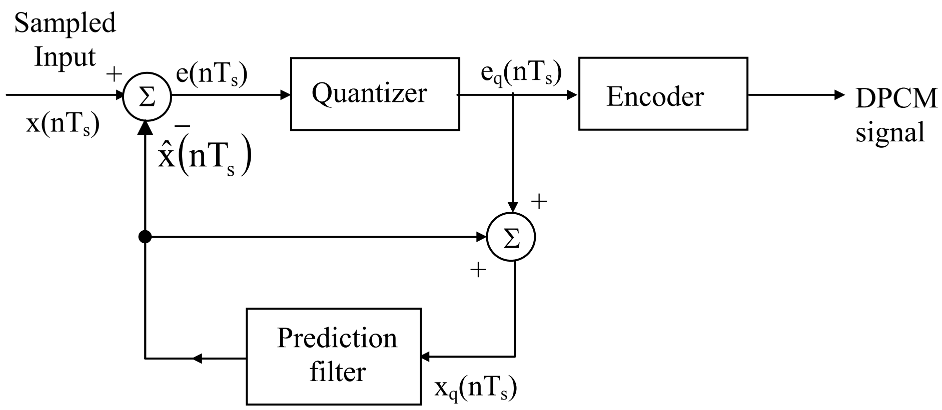


Fig: A Differential PULSE CODE MODULATION Transmitter.

Working Principle:
The DPCM works on the principle of prediction, the value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is known as Prediction error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \text{ ..... (1)}$$

The error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. The prediction is more close to the actual sampled signal. The quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.
The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \text{ ..... (2)}$$

Here, $q(nT_s)$ is the quantization error. The prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output i.e.,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \text{ ..... (3)}$$

substituting the value of $e_q(nT_s)$ from equation 2 in equation 3, we get,

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

Equation 1 is written as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore \; e(nT_s) + \hat{x}(nT_s) = x(nT_s)$$

$$x_q(nT_s) = x(nT_s) + q(nT_s)$$

This equation does not depend on the prediction filter characteristics.
Hence, the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error $q(nT_s)$. The quantization error can be positive or negative.

DPCM receiver:

The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal.

Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

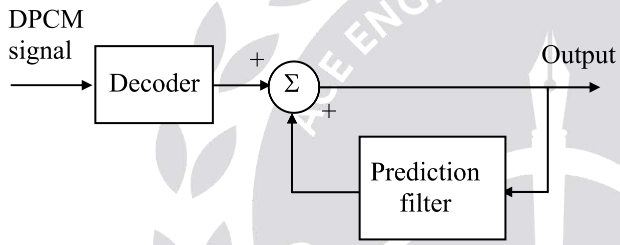


Fig: DPCM receiver

Table: Comparison between PCM and Differential Pulse Code Modulation

S.No.	Parameter of comparison	Pulse Code Modulation (PCM)	Differential Pulse Code Modulation (DPCM)
1.	Number of bits.	It can use 4, 8 or 16 bits per sample	Bits can be more than one but less than PCM
2.	Levels and step size	The number of levels depend on number of bits. Level size is kept fixed	Here, fixed number of levels are used.
3.	Transmission Band width	Highest bandwidth is required since number of bits are high	Bandwidth required is lower than PCM.

03.

Sol: Here an accuracy is given an $\pm 0.1\%$. this means that the quantization error must be $\pm 0.1\%$ or the maximum quantization error must be $\pm 0.1\%$.

Thus, $\varepsilon_{max} = \pm 0.1\% = \pm 0.001$

We know that the maximum quantization error for an uniform quantizer is expressed as

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$

$$\left|\frac{\Delta}{2}\right| = 0.001$$

Therefore, $\Delta = 0.002$

We know that the step size, number of quantization levels and maximum value of the signal are related as

$$\Delta = \frac{2x_{max}}{Q} \text{ ------------------- (1)}$$

Given $|x_{max}| = 10\text{volts}$

From equation (1)

$$0.002 = \frac{2\times10}{Q}$$

Q = 10000

(i) The maximum frequency in the signal is given as 100Hz, i.e,

$f_m = 100\text{Hz}$

by sampling theorem minimum sampling frequency should be

$f_s \geq 2f_m$

$f_s \geq 2\times100$

$f_s \geq 200\text{Hz}$

(ii) We know that minimum 10,000 levels should be used to quantize the signal. If binary PCM is used, then number of bits for each samples may be calculated as under i.e.,

$q = 2^V$

Here, V = bits in PCM

Q = number of levels.

$10000 = 2^V$

$\log_{10}10000 = V \log_{10}2$

$$V = \frac{\log_{10}^{10000}}{\log_{10}^{2}} = 13.288$$

V = 14 bits

(iii) The bit rate or signalling rate is expressed as

$r \geq vf_s$

$r \geq 14\times200$

$r \geq 2800$ bps

(iv) the transmission bandwidth for PCM is expressed as

$$BW \geq \frac{r}{2}$$

$$BW \geq \frac{2800}{2}$$

$$BW \geq 1400\text{Hz}$$

04.

Sol: Regenerative Repeater:

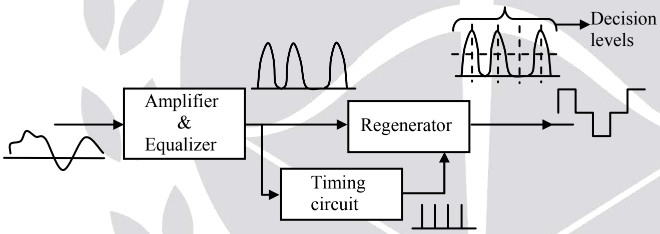
A digital communication system is more robust to the noise and distortion. The main reason for the superiority of digital systems over analog is the viability of regenerative repeaters. Repeater stations are placed along the communication path of a digital system at distances short enough to ensure that noise & distortion remain within limit.

At each repeater station, the incoming pulses are detected & new clean pulses are transmitted to the next repeater station with repeaters, we can transmit signals over longer distances with higher accuracy.

On the other hand, the distance in analog communication is limited by the transmitted power.

An amplifier strengthen the applied signal upto some extent i.e., an amplifier multiplier the signal with a fixed gain, where as a repeater, the values are saturated i.e., the output values can be logic 1 or logic 0 respective levels.

Below figure shows the block diagram of regenerative repeater.



Repeaters are actually amplifiers with suitable frequency response characteristics, placed at the regular intervals along the transmission channel.

Repeater:

The repeaters are used in a digital link are necessarily regenerative, i.e., they amplify as well as regenerate the error free transmitted signal from distorted and attenuated received signal through a decision device at the receiver end.

Three basic functions are performed by regenerative repeater as follows.

(i) Reshaping of the incoming pulse train using an equalizing fitter.

(ii) Extracting necessary timing information for sampling

(iii) Decision making based on the state of transmission from sampled values.

Equalizer:

The function of equalizer is to shape the received pulse so as to compensate the phase and amplitude distortion by the transmission channel.

An equalizer is a filter that compensates for the dispersion effect of the transmission channel.

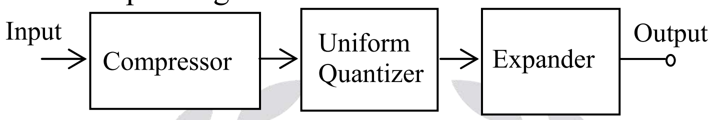
Timing circuit:

Timing circuit produces train of pulses in order to take decision at regular intervals i.e., bit duration (t_b).

05.

Sol: Companding is non uniform quantization. It is required to improve the signal to quantization noise ratio of weak (or) low level signals.

The quantization noise is given by $N_q = \frac{\Delta^2}{12}$. In the uniform quantization, once the step size is fixed, the quantization noise power remains constant but the signal power is not constant which is proportional to the square of signal amplitude. Hence signal power will be small for weak signal, but quantization noise power remains constant. Therefore, the signal to quantization noise for the weak signals is very poor. This will affect the quality of signal. The remedy is to use companding. Companding is a term derived from two words i.e., Compression and expansion. It is the process of compressing and then expanding.



With companded systems, the higher amplitude analog signals are compressed before the transmission and then expanded in the receiver. Companding is a means of improving the dynamic range of a communication systems and increasing the SQNR for low level signals.

Analog Companding:

In the transmitter of PCM, the dynamic range of the analog signal is compressed, sampled and then converted to a linear PCM code. In the receiver of PCM, the PCM code is converted to a PAM signal, filtered and then expanded back to its original dynamic range.

There are two methods of analog companding being used that closely approximate a logarithmic function and are often called log-PCM codes.

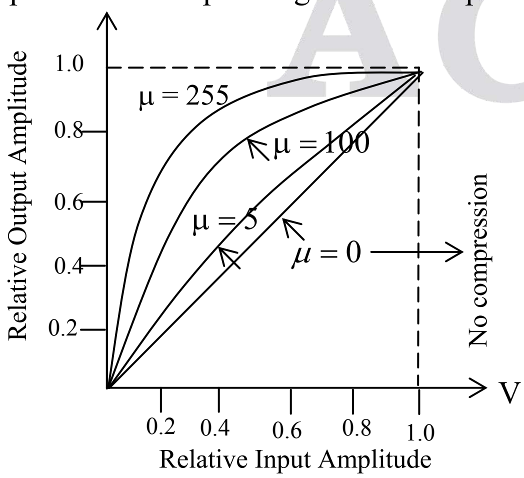
The two methods are

1. μ - law. 2. A-law.

1. **μ - law:** The compression characteristics for $\mu -$ law is

$$\frac{V_{out}}{V_{max}} = \frac{\ln\left(1+\mu\frac{V_{in}}{V_{max}}\right)}{\ln(1+\mu)} : 0 \le \frac{V_{in}}{V_{max}} \le 1$$

It is the process of compressing and then expanding.



V_{max} = maximum uncompressed analog input amplitude (V)
V_{in} = amplitude of the input signal at a particular instant of time (V)
μ = Parameter used to define the amount of compression (unit less).
V_{out} = compressed output amplitude.

It is mainly used in USA and JAPAN.

For example 8 bit code
$2^8 = 256(0-255) \quad \therefore \mu = 255$
For Early Bell System 7 bit code $\mu = 100$

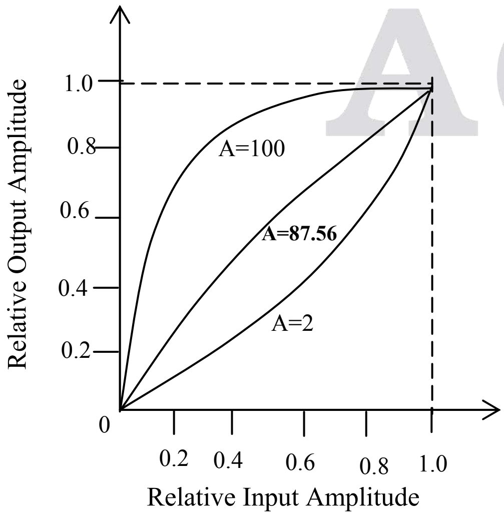
The minimum dynamic range of voice transmission is 40dB. For a relatively constant SQNR and 40dB dynamic range,
a $\mu \geq 100$ is required.

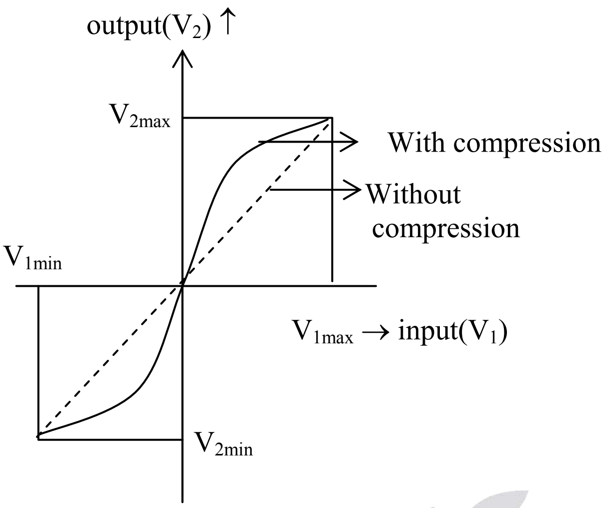
μ law is linear at low levels, $\mu|V_{out}| << 1$ and logarithmic at high input levels $\mu|V_{out}| >> 1$.

2. A-Law Companding: The compression characteristics for A-Law Companding is

$$\frac{V_{out}}{V_{max}} = \begin{cases} \dfrac{A\dfrac{V_{in}}{V_{max}}}{1+\ln A} & ; \quad 0 \leq \dfrac{V_{in}}{V_{max}} \leq \dfrac{1}{A} \\[2ex] \dfrac{1+\ln\left(A\dfrac{V_{in}}{V_{max}}\right)}{1+\ln A} & ; \quad \dfrac{1}{A} \leq \dfrac{V_{in}}{V_{max}} \leq 1 \end{cases}$$

'A' is the compression coefficient and the practical value of A = 87.56.
For a special dynamic range, A law companding has a slightly flatter SQNR than μ - law
A-law companding is inferior to μ - law in terms of small signal quality (idle channel noise).
It is mainly used in INDIA & EUROPE, the ITU - T has established an approximate true logarithmic companding.





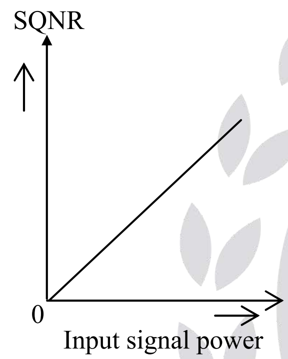


Fig: Without Compression

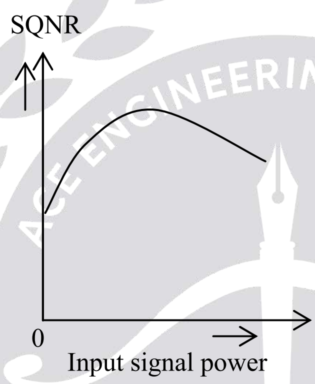


Fig: With Compression

Advantages of Companding

1. Companding improves the S/N ratio in voice circuits up to 25 dB
2. SQNR is more uniform for all signal levels.
3. The process of non-uniform quantization is based on the method of companding
4. The use of non-uniform quantization leads to increase in SNR for low level signal.
5. A non-uniform quantizer is basically a compressor followed by uniform quantizer

Chapter 9 Bandpass Data Transmission

Objective Practice Solutions

01. Ans: (c)

Sol: $(BW)_{BPSK} = 2f_b = 20$ kHz

$(BW)_{QPSK} = f_b = 10$ kHz

02. Ans: (b)

Sol: $f_H = 25$ kHz ; $f_L = 10$ kHz

$\therefore$ Center frequency

$$= \left(\frac{25+10}{2}\right) \text{kHz}$$

$= 17.5$ kHz

$\therefore$ Frequency offset,

$\Omega = 2\pi\,(25 - 17.5) \times 10^3$

$= 2\pi\,(7.5) \times 10^3$

$= 15 \times 10^3\pi$ rad/sec.

The two possible FSK signals are orthogonal, if $2\Omega T = n\pi$

$\Rightarrow 2(15\pi) \times 10^3 \times T = n\pi$

$\Rightarrow 30 \times 10^3 \times T = n$ (integer)

This is satisfied for, T = 200µsec.

03. Ans: (a)

Sol: $r_b = 8$ kbps

Coherent detection

$$\Delta f = \frac{nr_b}{2}$$

Best possible n = 1

$$\Delta f = \frac{8K}{2} = 4K$$

To verify the options $\Delta f = 4k$

i.e. $f_{C2} - f_{C1} = 4K$

(a) 20 K – 16 K = 4 K

(b) 32 K – 20 K = 12 K

(c) 40 K – 20 K = 20 K

(d) 40 K – 32 K = 8 K

04. Ans: (a) & (c)

Sol: Non coherent detection of PSK is not possible. So to overcome that, DPSK is implemented. A coherent carrier is not required to be generated at the receiver.

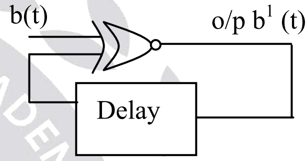
05. Ans: (c)

Sol: In QPSK baud rate $= \dfrac{\text{bit rate}}{2} = \dfrac{34}{2}$

$= 17$ Mbps

06. Ans: (d)

Sol:



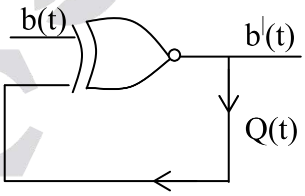
b(t)	0	1	0	0	1
$b^1(t)_{(\textbf{Ref.bit})}$	0	0	1	0	0
Phase	π	π	0	π	π

07. Ans: (b)

Sol: Given

Bit stream 110 111001

Reference bit = 1



$b^|(t) = b(t) \odot Q(t)$

1 1 0 1 1 1 0 0 1

1

1 1 0 0 0 0 1 0 0

0 0 π π π π 0 π π

08. Ans: (d)

Sol: $r_b = 1.544 \times 10^6$

$\alpha = 0.2$

$$BW = \frac{r_b}{\log_2^M}(1+\alpha)$$

$$= \frac{1.544\times10^6}{2}(1+0.2) \quad (\because M = 4)$$

$BW = 926.4 \times 10^3$ Hz

09. Ans: 0.25

Sol: BW = 1500 Hz

BW required for M-ary PSK is

$$\frac{R_b[1+\alpha]}{\log_2 16} = 1500\text{Hz}$$

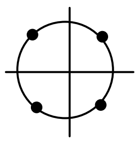
$\Rightarrow R_b\,[1 + \alpha] = 1500 \times 4 = 6000$

$$\Rightarrow (1+\alpha) = \frac{6000}{4800}$$

Roll off factor $\Rightarrow \alpha = \dfrac{6000}{4800} - 1 = 0.25$

10. Ans: (d)

Sol:



Here only phase is changing.
From options (d) is the optimum answer.

11. Ans: (b)

Sol: Here 16-points are available in constellation which are varying in both amplitude and phase. So, it 16QAM.

12. Ans: (d)

Sol: $BW = \dfrac{r_b}{\log_2 M}(1+\alpha)$

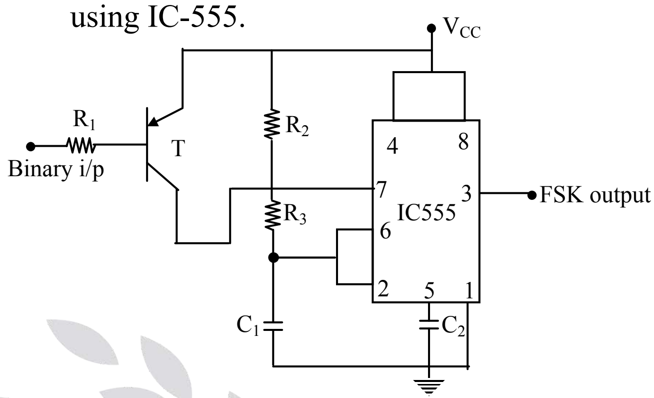
$$36\times10^6 = \frac{r_b}{2}(1+0.2)(\because M = 4, \text{QPSK})$$

$r_b = 60\times10^6$ bps

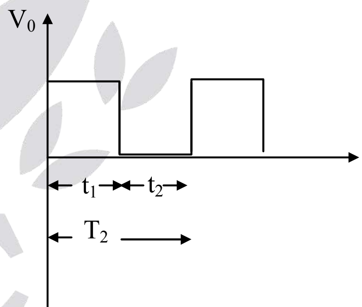
Conventional Practice Solutions

01.

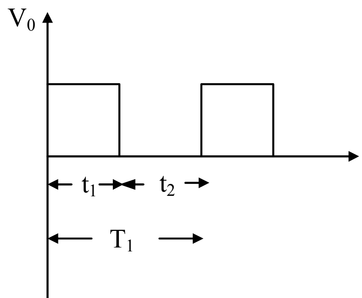
Sol: The below circuit shows FSK generator using IC-555.



- Basically the above circuit is an astable multivibrator, whose frequency is controlled by the transistor T.
- Since a carrier is shifted between two different frequency and this frequency depends on logic state of digital input, hence the data transmission is said to be frequency shift keying technique.
- The below figure shows wave forms when the binary input '1'& '0' is applied



For $V_{in} = 0$ T_1 = ON



For $V_{in} = 1$ T_1 = OFF

When Digital 1 is applied

$$T_1 = t_1 + t_2 = \frac{1}{f_m}$$

$$\Rightarrow f_m = \frac{1.45}{(R_2 + 2R_3)C_1} = 1070 \;(\because \text{Given})$$

$$(R_2 + 2R_3)C_1 = 1.35\times10^{-3}\text{ sec} \rightarrow (1)$$

When ‘0’ is applied

$$T_2 = t_1^1 + t_2^1 = \frac{1}{t_s}$$

$$\because f_s = \frac{1.45}{2R_3C_1} = 1270\,(\text{Given})$$

$$\text{Let } R_3C_1 = \frac{1.45}{2\times1270} = 0.57\times10^{-3}\text{ sec} \rightarrow (2)$$

Let $C_1 = 1\mu F$

$$\therefore R_3 = \frac{0.57\times10^{-3}}{C_1} = \frac{0.57\times10^{-3}}{1\times10^{-6}}$$

$R_3 \simeq 570\Omega$

Substituting the value of R_3 in eq (1)

$$R_2 = \frac{1.35\times10^{-3}}{1\times10^{-6}} - 2\times570$$

$R_2 = 210\Omega$

$\therefore$ The required value of R_2& R_3 are 210Ω & 570Ω respectively and R_1 is the current limiting resistor.

Chapter 10 Noise in Digital Communication

Objective Practice Solutions

Noise Ratio

01. Ans: (b)

Sol: Signal to quantization noise ratio only depends on no. of quantization levels (L) and no. of bits per sample(n)

For sinusoidal input SQNR = 1.76+6n dB
$= 1.76 + 6\times 12$
$= 73.76$ dB

For uniform distributed signal = 6ndB
$= 6\times 12$
$= 72$ dB

02. Ans: (a)

Sol: For Bipolar pulses,

$$\text{PSD} = \frac{|P(\omega)|^2}{T_b} \cdot \sin^2\left(\frac{\omega T_b}{2}\right)$$

The zero magnitude occurs for

$f = n/T_b$.

$\therefore$ The width of the major lobe $= 1/T_b$
$= f_b$

$\therefore (B.W)_{min} = f_b$

Here, Data rate $= nf_s$
$= 8(8\text{ kHz}) = 64$ kbps

$\therefore (B.W)_{min} = 64$ kHz

03. Ans: (c)

Sol: Since the signal is uniformly distributed,

$$f(x) = \frac{1}{10} \quad \text{for } -5 \le x \le 5$$
$$= 0 \quad : \text{ else where.}$$

$$\text{Signal Power} = \int_{-5}^{5} x^2 f(x)dx = \frac{25}{3}\text{volts}^2$$

$$\text{Step size} = \frac{V_{p-p}}{L} = \frac{10}{2^8} = 0.039\text{ V}$$

$$N_q = \frac{\Delta^2}{12} = 0.126\text{ mW}$$

Signal to noise ratio, SNR in dB is

$$\text{SNR} = 10\log\left(\frac{\text{signal power}}{\text{Noise power}}\right)$$

$$= 10\log\left(\frac{25/3}{0.126\times 10^{-3}}\right)$$

$$= 48\text{ dB}$$

04. Ans: (b)

Sol: For every one bit increase in data word length, quantization Noise Power becomes $\frac{1}{4}$th of the original. Hence, Data word length for n = 9 bits is,

$\therefore L = 2^n = 2^9 = 512$

05. Ans: (c)

Sol: $V_{P-P} = -5$V to 5V

$20\log L = 43.5$

$L = 10^{2.175}$

$= 149.6$

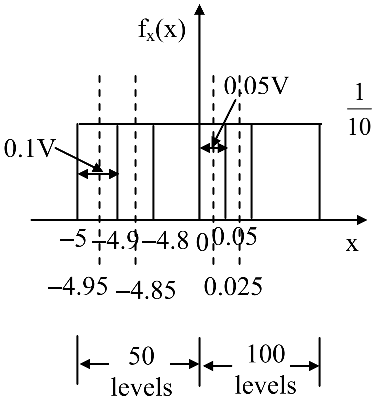
$$\Rightarrow \Delta = \frac{V_H - V_L}{L}$$

$$= \frac{5-(-5)}{10^{2.175}}$$

$\Delta = 0.06683$

06. Ans: (c)

Sol:



Signal power $E[X^2] = \int_{-5}^{5} x^2\left(\frac{1}{10}\right)dx$

$$= \frac{1}{10}\left(\frac{x^3}{3}\right)_{-5}^{5} = \frac{1}{30}(250) = \frac{25}{3}\text{ W}$$

Quantization Noise power

$$= E[[X - Q(X)]^2]$$

$$= \int_{-5}^{5}[x-Q(x)]^2\ f_X(x)\,dx$$

$$= \int_{-5}^{-4.9}[x-(-4.95)]^2\frac{1}{10}dx$$

$$+\int_{-4.9}^{-4.8}[x-(-4.85)^2]\frac{1}{10}dx + ...(50\text{ times})$$

$$+\int_{0}^{0.05}(x-0.025)^2\frac{1}{10}dx$$

$$+\int_{0.05}^{0.1}[(x-0.075)^2]\frac{1}{10}dx + ...(100\text{ times})$$

$$= 50\int_{-5}^{-4.9}(x+4.95)^2\frac{1}{10}dx + 100\int_{0}^{0.05}(x-0.025)^2\frac{1}{10}dx$$

$$= 5\left[\frac{(x+4.95)^3}{3}\right]_{-5}^{-4.9} + 10\left[\frac{(x-0.025)^3}{3}\right]_{0}^{0.05}$$

$$= \frac{5}{3}[(0.05)^3+(0.05)^3] + \frac{10}{3}[(0.025)^3+(0.025)^3]$$

$$= \frac{5}{3}(125\times10^{-6}+125\times10^{-6}) + \frac{10}{3}[(0.025)^3+(0.025)^3]$$

$$= \frac{5}{3}(125\times10^{-6}+125\times10^{-6}) + \frac{10}{3}(3.125\times10^{-5})$$

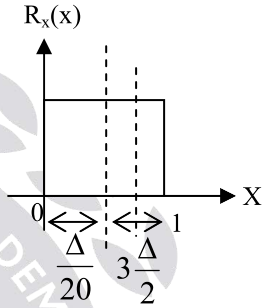
$$= \frac{1250}{3}\times10^{-6} + \frac{312.5}{3}\times10^{-6}$$

$$= 520.83333\times10^{-6}$$

$$(SNR)_{dB} = 10\log\left(\frac{25}{3\times5.2\times10^{-4}}\right)$$

$$= 42.04\,\text{dB}$$

$$\approx 42\text{ dB}$$



07. Ans: (b)

Sol: $E[X-Q(x)]^2$

$$= \int_{0}^{0.3}(X-0)^2(1)dx\int_{0.3}^{1}(X-(0.7))^2(1)dx$$

$$= \left[\frac{x^3}{3}\right]_0^{0.3} + \left[\frac{(x-0.7)^3}{3}\right]_{0.3}^{1}$$

$$= \frac{(0.3)^3}{3} + \frac{(0.3)^3}{3} + \frac{(0.4)^3}{3}$$

$$= 0.198$$

08. Ans: (b)

Sol: Since, all the quantization levels are equiprobable,

$$\int_{-a}^{a}\frac{1}{4}dx = \frac{1}{3} \quad \Rightarrow a = \frac{2}{3}$$

09. Ans: (a)

Sol: $\int_{-2/3}^{2/3} x^2\, f(x).dx = \frac{1}{4}\int_{-2/3}^{2/3} x^2.dx = \frac{4}{81}$

Matched Filter

01. Ans: (d)

Sol: The time domain representation of the o/p of a Matched filter is proportional to Auto correlation function of the i/p signal, except for a time delay

$$R_{ss}(\tau) = \int_0^{10^{-4}} S(t).S(t+\tau)dt$$

$$= \int_0^{10^{-4}} 10\sin(2\pi\times10^6 t).10\sin(2\pi\times10^6(t+\tau))]dt$$

$$= 50\int_0^{10^{-4}}\left[\cos(2\pi\times10^6\tau)-\cos(4\pi\times10^6 t+2\pi\times10^6\tau)\right]dt$$

$$= 50\times10^{-4}\cos(2\pi\times10^6)\tau$$

$\therefore$ The Peak is 5mV

02. Ans: (b)

Sol: The matched filter has maximum value of output at t = T is energy of the signal

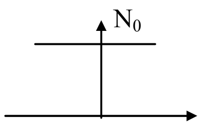
$$\Rightarrow E_s = \int_0^1 A^2 dt + \int_2^3 A^2(1)\,dt$$

$$= A^2 + A^2 = 2A^2$$

03. Ans: (d)

Sol: $$(SNR)_0 = \frac{E_S}{N_0} = \frac{\frac{B^2}{2}.T}{N}$$

$$= \frac{B^2T}{2N}$$



04. Ans: (b)

Sol: Given,

$$\frac{S_{02}(t)}{N} = \frac{S_{01}(t)}{N} \Rightarrow \frac{2E_{s_1}}{N} = \frac{2E_2}{N}$$

$$A^2T = \frac{B^2}{2}T \Rightarrow A = \frac{B}{\sqrt{2}}$$

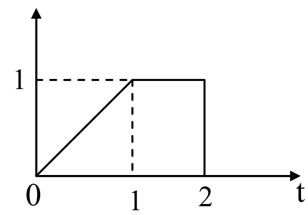
05. Ans: (d)

Sol: Output of the matched filter is maximum which is equal to the energy in the signal

$$E = \int_0^1 1.t^2dt + \int_1^2 (1)dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + 1[t]_1^2$$

$$= \frac{1}{3} + 1 = 4/3$$



The time instant which occurs the maximum value is its time period T = 2

06. Ans: (c)

Sol: Given,

$$H(f) = \frac{1-e^{-j\omega t}}{j\omega}$$

$$H(f) = \frac{1}{j\omega} - \frac{e^{-j\omega t}}{j\omega}$$

Applying I.F.T

$h(t) = 0.5(\mathrm{sgn}(t) - \mathrm{sgn}(t - T_0))$

$$\left(\because F(\mathrm{sgn}(t)) = \frac{2}{j\omega}\right)$$

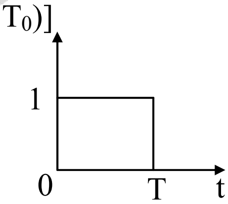
$$= 0.5[2u(t) - 1 - [2u(t-T_0) - 1]]$$

$$= [u(t) - u(t - T_0)]$$

We know that

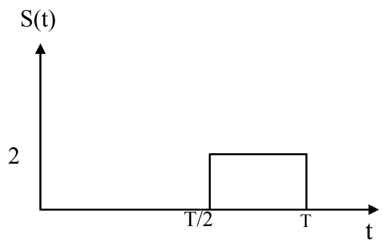
$h(t) = s^*(t - T)$

$\therefore S_i(t)$



07. Ans: (d)

Sol: The maximum value in the output is energy inside the signal



$$\Rightarrow S_0(t)\big|_{max} = \int_{T/2}^{T} 2^2.dt$$

$$= 4\int_{T/2}^{T} 1.dt$$

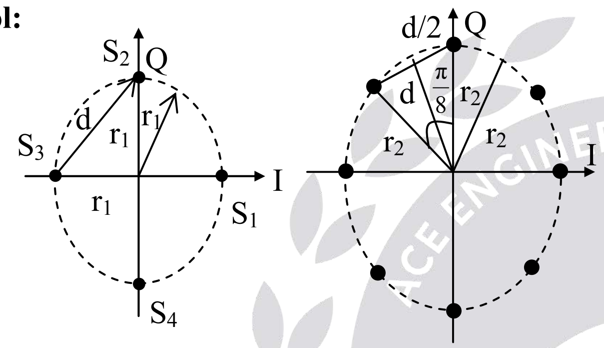
$$= 4[T - T/2]$$

$$= 2T$$

Probability of Error

01. Ans: (d)

Sol:



$d = \sqrt{2}\, r_1$ $\quad\quad \sin\dfrac{\pi}{8} = \dfrac{(d/2)}{r_2}$

$$\Rightarrow r_1 = \frac{d}{\sqrt{2}} = 0.707\,d$$

$$\Rightarrow r_2 = \frac{d}{2\sin\frac{\pi}{8}} = 1.307\,d$$

02. Ans: (d)

Sol: 4-PSK, 8-PSK both have same error probability when both signals have same minimum distance between pairs of signal points.

$$P_e = Q\left(\frac{\sqrt{d_{min}^2}}{2N_0}\right)$$

$$P_e = 2Q\left(\sqrt{\frac{2E_s}{N_0}\sin^2\left(\frac{\pi}{M}\right)}\right)$$

Where E_s is the average symbol energy

Given both constellation d_{min} is same i.e.,'d'

Average Symbol Energy:

$$(E_s)_{4PSK} = \frac{E_{s_1} + E_{s_2} + E_{s_3} + E_{s_4}}{4}$$

Where E_{s_k} is the symbol 'S_k' Energy

= (distance from the origin to the symbol 'S_k')^2

$$(E_s)_{4PSK} = \frac{r_1^2 + r_1^2 + r_1^2 + r_1^2}{4} = r_1^2$$

Similarly, For 8 PSK

$$(E_s)_{8PSK} = r_2^2$$

$$\frac{(E_s)_{8PSK}}{(E_s)_{4PSK}} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1.307d}{0.707d}\right)^2$$

In dB,

$$(E_s)_{8PSK(dB)} - (E_s)_{4PSK(dB)} = 10\log\left(\frac{1.307}{0.707}\right)^2$$

$$= 5.33 \text{ dB}$$

$$(E_s)_{8PSK} = (E_s)_{4PSK} + 5.33 \text{ dB}$$

8 PSK required additional 5.33 dB

03. Ans: (b)

Sol: Constellation 1:

$s_1(t) = 0$;

$s_2(t) = -\sqrt{2}\ a\,\phi_1 + \sqrt{2}\ a\,\phi_2$

$s_3(t) = -2\sqrt{2}\ a.\phi_1$;

$s_4(t) = -\sqrt{2}\ a\,\phi_1 - \sqrt{2}\ a\,\phi_2$

Energy of $S_1(t) = E_{S1} = 0$; $E_{S2} = 4a^2$;

$E_{S3} = 8a^2$; $E_{S4} = 4a^2$

Average Energy of constellation 1

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = 4a^2$$

Constellation 2:

$s_1(t) = a\phi_1 \quad \Rightarrow \quad E_{S1} = a^2$

$s_2(t) = a.\phi_2 \quad \Rightarrow \quad E_{S2} = a^2$

$s_3(t) = -a.\phi_1 \Rightarrow E_{S3} = a^2$

$s_4(t) = -a.\phi_2 \Rightarrow E_{S4} = a^2$

Average Energy of constellation 2

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = a^2$$

The required Ratio is 4

04. Ans: (a)

Sol: The distance between the two closest points in constellation 1 is $d_1 = 2a$.

The same in constellation 2,

$d_2 = \sqrt{2}\, a$

Since $d_1 > d_2$, Probability of symbol error for constellation 1 is lower

05. Ans: (a)

Sol: $S(t) = \sqrt{\frac{2E}{T_b}}\left[\cos\left(\omega_c t + \frac{2\pi}{m}(i-1)\right)\right]$

$$= \sqrt{\frac{2E}{T_b}}\left[\cos\omega_c t..\cos\left(\frac{2T}{m}(i-1)\right) - \sin\omega_c t.\sin\frac{2\pi}{m}(i-1)\right]$$

$$= \sqrt{\frac{2}{T_b}}\cos\omega_t\sqrt{E}\cos\left(\frac{2\pi}{m}(i-1)\right) - \sqrt{\frac{2}{T_b}}\sin\omega_c t\sqrt{E}\sin\frac{2\pi}{m}(i-1)$$

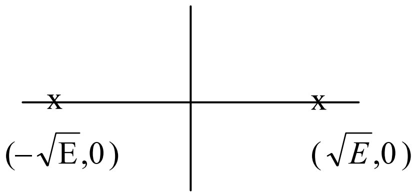
Given binary digital communication m = 2

$$\sqrt{\frac{2}{T_b}}\cos\omega_c t\sqrt{E}\cos\pi$$

$\because$ basic function $= 2\cos\omega_c t$

$\Rightarrow T_b = 1/2$

$$2\cos\omega_c t\left(\sqrt{E}\cos\pi(f-1)\right) - [2\sin\omega_c t]\sqrt{E}\sin\pi(i-1)$$



Distance between two points is:

$\sqrt{(\sqrt{E}+\sqrt{E})^2 + 0}$

$\sqrt{4E} = 2\sqrt{E}$

Energy of the signal:

$$\int_0^{T_b}(A\cos\omega_c t)^2 = \frac{A^2T}{2}$$

$$\Rightarrow d = 2\sqrt{\frac{A^2T_b}{2}} = 2\sqrt{\frac{A^2\times T_b}{2}} = A$$

$\left(\because T_b = 1/2\right) \qquad \therefore\ d = A$

06. Ans: (c)

Sol: $P_e = Q\left[\sqrt{\frac{E_b}{N_o}}\right]$

$$E_b = \frac{\alpha^2 T_b}{2} = \frac{\alpha^2}{2R_b}$$

$\alpha = 4$mV, $R_b = 500$ kbps,

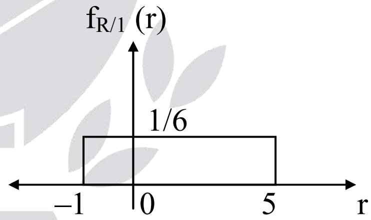
$N_o = 10^{-12}$W/Hz.

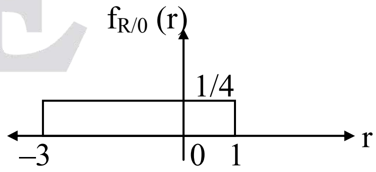
$$\frac{E_b}{N_o} = \frac{16\times10^{-6}}{2\times500\times10^3\times10^{-12}} = 16$$

$$P_e = Q\left[\sqrt{16}\right] = Q[4]$$

07. Ans: (d)

Sol:



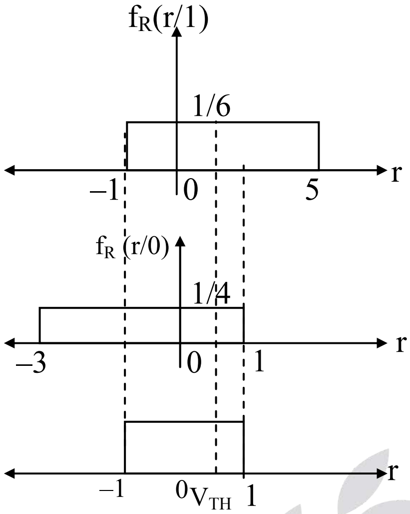


P(0) = 1/3; P(1) = 2/3

The probability of error of the symbols 0 & 1 are not the same.

$\therefore$ The intersection point of the two pdf's is not the threshold of detection.

Assume the threshold value to be V_{TH}



For minimum error the V_{TH} should lie in the area of intersection of the 2 pdf's.

$$P_{e_1} = \int_{-1}^{V_{TH}} \left(\frac{1}{6}\right) dr = \frac{1}{6}(V_{TH}+1)$$

$$P_{e_0} = \int_{V_{TH}}^{1} \left(\frac{1}{4}\right) dr = \frac{1}{4}(1-V_{TH})$$

Decision error probability

$$= P_{e_0}P(0) + P_{e_1}P(1)$$

$$= \frac{1}{4}(1-V_{TH})\left(\frac{1}{3}\right) + \frac{1}{6}(1+V_{TH})\left(\frac{2}{3}\right)$$

$$P_e = \frac{1-V_{TH}}{12} + \frac{2(1+V_{TH})}{18}$$

For minimum decision error probability,

$$-1 \le V_{TH} \le 1$$

For $V_{TH} = -1$

$$BER = \frac{1-(-1)}{12} = \frac{1}{6} \text{ (min value)}$$

$\therefore$ Decision error probability = 1/6

08. Ans: (c)

Sol: The optimum threshold value is

$$\hat{x} = \frac{\sigma^2}{x_1 - x_2}\left[\ell n \frac{P(x_2)}{P(x_1)} + \frac{x_1^2 - x_2^2}{2\sigma^2}\right]$$

$x_1 = 1$, $x_2 = -1$

$P(x_1) = 0.75$, $P(x_2) = 0.25$

$$\hat{x} = \frac{\sigma^2}{2}\left[\ell n \frac{0.25}{0.75}\right] = -\frac{\sigma^2}{2}$$

So $\hat{x}$ should be strictly negative.

09. Ans: (c)

Sol: $Y = X + Z$

Z is Gaussian RV with mean βx

$x \in \{-a, +a\}$

when $\beta = 0$ $E[y] = E[x] + E[z]$

$E[y] = E[x] = +a$

$= a$

$BER = Q(a) = 1 \times 10^{-8}$

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^\infty e^{\frac{-v^2}{2}} du \cong e^{\frac{-v^2}{2}}$$

$$Q(a) = 1 \times 10^{-8} \approx e^{\frac{-a^2}{2}}$$

$a = 6$

when $\beta = -0.3$ mean $= 6 \times -0.3 = -1.8$

so $E(y) = E(x) + E(z)$

$= 6 - 1.8 = 4.2$

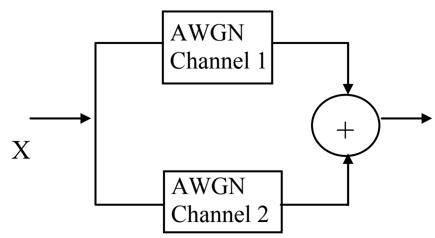
$$\text{so BER} = Q(4.2) \cong e^{\frac{-(4.2)^2}{2}}$$

$\cong 0.0001$

$\cong 10^{-4}$

10. Ans: 1.414

Sol: When the signal is transmitted through a channel BER = $Q[\sqrt{r}]$.



At the input of the receiver signal amplitude is doubled. But when two independent Gaussian Random Variables are added, the resultant random variables is also a Gaussian random. The pdf is the convolution of individual pdf's.

The variance indicates the noise power

But the variance is doubled.

Signal power increased by a factor of 4(mean is doubled).

But the noise increases by a factor of 2

So the signal to noise increases by a factor of 2

So b = $\sqrt{2}$ =1.414

BER = $Q[\sqrt{2r}] = Q[\sqrt{2}\sqrt{r}] = Q[1.414\sqrt{r}]$

So b = 1.414

11. Ans: (a)

Sol: Probability of error for an AWGN channel for binary transmission is given as

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

Where $E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$

Given $s_1(t) = g(t)$

$s_2(t) = -g(t)$

$$E_d = \int_0^T [g(t) - (-g(t))]^2 dt$$

$$= 4\int_0^T g^2(t)dt$$

$$E_{d,a} = 4\int_0^1 (1)^2 dt = 4$$

$$E_{d,b} = 4\left[\int_0^{1/2} (2t)^2 dt + \int_{1/2}^1 (-2t+2)^2\right] dt$$

$$= \frac{4}{6} + \frac{4}{6} = \frac{4}{3}$$

$$E_{d,c} = 4\int_0^1 (1-t)^2 dt = \frac{4}{3}$$

$$E_{d,d} = 4\int_0^1 (t)^2 dt = \frac{4}{3}$$

P_e is minimum when E_d is maximum

E_d of signal (a) is more when compared to E_d of other signals.

$\therefore$ Probability of error is minimum for signal (a).

12. Ans: (b)

Sol: o/p Noise Power = o/p PSD × B.W

$= 10^{-20} \times 2 \times 10^6$

$= 2 \times 10^{-14}$ W

Since mean square value = Power

$\frac{2}{\alpha^2} = 2 \times 10^{-14} \Rightarrow \alpha = 10^7$

13. Ans: (d)

Sol: When a 1 is transmitted:

$Y_k = a + N_k$

Threshold Z = $\frac{a}{2} = 10^{-6}$

$\Rightarrow a = 2\times10^{-6}$

For error to occur, $Y_k < 10^{-6}$

$2\times10^{-6} + N_k < 10^{-6}$

$N_k < -10^{-6}$

$$\therefore P(0/1) = \int_{-\infty}^{-10^{-6}} P(n)\,dn:$$

$$= \int_{-\infty}^{-10^{-6}} (0.5)\alpha.e^{-\alpha n}.dn, \text{with } \alpha = 10^7$$

$= 0.5 \times e^{-10}$

When a '0' is Transmitted:

$Y_k = N_k$

For error to occur, $Y_k > 10^{-6}$

$$\therefore P(1/0) = \int_{10^{-6}}^{\infty} P(n)\,dn = 0.5 \times e^{-10}$$

Since, both bits are equiprobable, the Probability of bit error

$= \frac{1}{2}$ [P(0/1) + P(1/0)]

$= 0.5 \times e^{-10}$

14. Ans: (a)

Sol: $P(0/1) = P(1/0) = p$

$\Rightarrow P(1/1) = P(0/0) = 1 - p$.

Reception with error means getting at most one 1.

$\therefore$ P(reception with error)

$$= P(X = 0) + P(X = 1)$$

$$= 3_{C_0}(1-p)^0 p^3 + 3_{C_1}(1-p)^1 p^2$$

$$= p^3 + 3p^2(1-p)$$

15. Ans: (d)

Sol: p = probability of a bit being in error $= 10^{-3}$

q = probability of the bit not being in error

$= 1 - p = 1 - 10^{-3}$

$= 0.999$

(1) Total number of bits = 10;

P_e = probability of error

$= 1 - P(X = 0)$

$P(X = 0)$ = Probability of no error

$\therefore P_e = 1 - [{}^{10}C_0(10^{-3})^0(1-10^{-3})^{10}] = 0.00995$

(2) Total number of bits = 100

$P_e = 1 - [{}^{100}C_0(10^{-3})^0(1-10^{-3})^{100}]$

$= 0.0952$

(3) Total number of bits = 1000

$P_e = 1 - [{}^{1000}C_0(10^{-3})^0(0.999^{1000})]$

$P_e = 0.632$

(4) If total number of bits = 10, 000

$= 1 - [({}^{10,000}C_0)(1-10^{-3})^0(0.999)^{10,000}]$

$= 0.9999$

Conclusion: As the number of bits increases, the probability of error increases and it approaches unity.

16. Ans: (a)

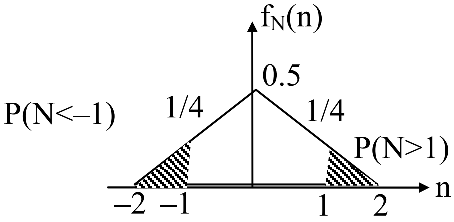
Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

17. Ans: (a)

Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

18. Ans: 0.125

Sol:



$$P(E) = P(x=-1)P\left(\frac{R}{x=-1} > 0\right) + P(x=1)P\left(\frac{R}{x=+1} < 0\right)$$

$= 0.5P(x+N>0) + 0.5\,P(x+N<0)$

$= 0.5\,P(-1+N>0) + 0.5P(1+N<0)$

$= 0.5\,P(N>1) + 0.5P(N<-1)$

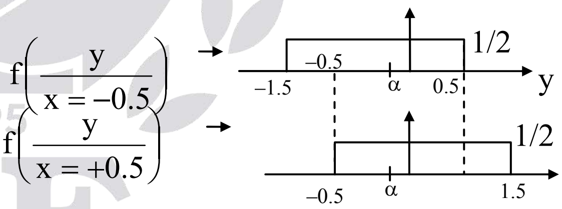
$$= 0.5\left[\frac{1}{2}\frac{1}{4}(1)\right] + 0.5\left[\frac{1}{2}\frac{1}{4}\right]$$

$$= \frac{1}{8} = 0.125$$

19. Ans: –0.5

Sol: $x = \{-0.5, 0.5\}$

$P(x = -0.5) = ¼$, $P(x = 0.5) = 3/4$



P_e in the overlap region $-0.5 < \alpha < 0.5$

$$P_e = \frac{1}{4}\frac{1}{2}(0.5-\alpha) + \frac{3}{4}\left(\frac{1}{2}\right)(\alpha + 0.5)$$

$$= \frac{0.5}{8} + \frac{1.5}{8} + \left(\frac{3}{8} - \frac{1}{8}\right)\alpha$$

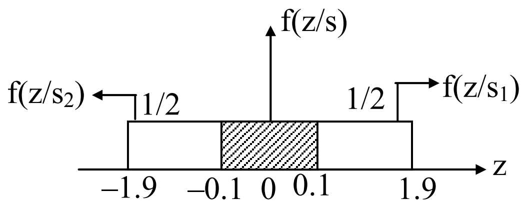
$$= \frac{2}{8} + \frac{2}{8}\alpha$$

$\therefore P_e$ is minimum for $\alpha = -0.5$

Conventional Practice Solutions

01.

Sol: The pdf's of received symbol are given as



$$p_e = \frac{1}{2}p\left(\frac{s_1}{s_2}\right) + \frac{1}{2}p\left(\frac{s_2}{s_1}\right)$$

$p\left(\frac{s_1}{s_2}\right) =$ probability that s_2 is sent and s_1 is received.

$p\left(\frac{s_2}{s_1}\right) =$ probability that s_1 is sent and s_2 is received.

$$p\left(\frac{s_2}{s_1}\right) = 0.1 \times \frac{1}{2}$$

$$p\left(\frac{s_2}{s_1}\right) = 0.1 \times \frac{1}{2}$$

$$p_e = \frac{1}{2} \times 0.1 \times \frac{1}{2} + 0.1 \times \frac{1}{2} \times \frac{1}{2}$$

$$= 0.1 \times \frac{1}{2} \times 2 \times \frac{1}{2} = 0.05$$

The optimum threshold is considered to be 0

02.

Sol: $a_1 = 1$

$a_2 = -1$

(a) The symbols are given to be equiprobable

$\therefore$ The decision threshold

$$\lambda_0 = \frac{a_1 + a_2}{2} = \frac{1 + (-1)}{2} = 0$$

$\therefore$ According to optimum decision rule:

Threshold value

$\lambda_0 = 0$

(b) $P_e = Q\left[\frac{a_1 + a_2}{2\sigma}\right]$

Given Variance

$\sigma^2 = 0.1$

$\Rightarrow \sigma = 0.3162$

$$P_e = Q\left[\frac{1 - (-1)}{2 \times 0.3162}\right]$$

$P_e = Q[3.162]$

Chapter 11 Information Theory & Coding

Objective Practice Solutions

01. Ans: (b)

Sol: Huffman encoder is the most efficient source encoder

0.5	1	0.5	0
0.25	00	0.5	1
0.25	01		

$\bar{L} = 1\times0.5 + 2\times0.25 + 2\times0.25$

$= 1.5$ bits/symbol

Average bit rate $= 3000 \times 1.5$

$= 4500$ bps

02. Ans: (c)

Sol: Assuming all the 64 levels are equiprobable, $H = \log_2 64 = 6$ bits/pixel

Total No. of pixels $= 625 \times 400 \times 400$

$= 100$ M pixels /sec

Data rate $= 6$ bits/pixel$\times100\times10^6$ pixel/sec

$= 600$ Mbps

03. Ans: (b)

Sol: $C = B\log\left(1+\frac{S}{N}\right)$

Since $\frac{S}{N} >> 1$. $1+\frac{S}{N} \cong \frac{S}{N}$

$\therefore\ C_1 = B\log\frac{S}{N}$

$C_2 = B\log\left(2.\frac{S}{N}\right)$

$= B\log 2 + B\log\left(\frac{S}{N}\right) = C_1 + B$

04. Ans: (b)

Sol: Given

B. W = 3 kHz

SNR = 10dB

$\Rightarrow 10\log_{10}(\text{SNR}) = 10$

$\text{SNR} = 10^1 = 10$

Number of characters = 128

Channel capacity $= B\log_2\left(1+\frac{S}{N}\right)$

$= 3\times10^3\log_2(1+10)$

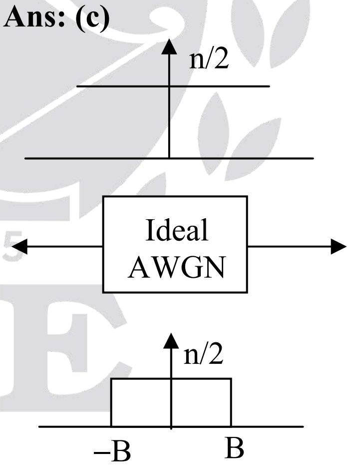
$= 10378$bps

05. Ans: (b)

Sol: Number of characteristics can be sent without any error $= \frac{c}{\log_2 M} = \frac{c}{7} = 1482$.cps

06. Ans: (c)

Sol:



$C = B\log_2\left(1+\frac{S}{N}\right)$

$$\lim_{B\to\infty} C = \lim_{B\to\infty}\frac{S}{n}\times\frac{n}{S}B\log_2\left(1+\frac{S}{nB}\right)$$

$$\lim_{B\to\infty} C = \frac{S}{n}\log_2 e$$

$(\because \lim_{n\to\infty} x\log\left(1+\frac{1}{Q}\right)$ = loge)

$$\lim_{B\to\infty} C = 1.44\frac{S}{n}$$

07. Ans: (b)

Sol: Max. entropy = $512 \times 512 \times \log_2 8$

= 786432 bits

08. Ans: (d)

Sol: Maximum entropy of a binary source:

$$H(x)/_{max} = \log_2 M$$

$H(x)/_{max} = \log_2 2$ = 1 bit/symbol

09. Ans: 0.4

Sol: $P\left(\frac{x=1}{y=0}\right) = \frac{P(x=1, y=0)}{P(y=0)}$

$$= \frac{P(x=1)P\left(\frac{y=0}{x=1}\right)}{P(x=1)P\left(\frac{y=0}{x=1}\right) + P(x=0)P\left(\frac{y=0}{x=0}\right)}$$

$$= \frac{0.8 \times \frac{1}{7}}{0.8 \times \frac{1}{7} + 0.2 \times \frac{6}{7}}$$

= 0.4

Conventional Practice Solutions

01.

Sol: Given k = 20, t = 1

$$\sum_{i=0}^{t}\binom{n}{i}=2^{n-k} \quad \text{(Hamming Bound)}$$

$$2^{n-20}=\sum_{i=0}^{1}\binom{n}{i}$$

$$\sum_{i=0}^{1}\binom{n}{i}=\frac{n!}{(n-0)!0!}+\frac{n!}{(n-1)!1!}$$

$$=\frac{n!}{n!}+\frac{n(n-1)!}{(n-1)!}=1+n$$

$$2^{n-20}=1+n$$

$$\log 2^{(n-20)}=\log(1+n)$$

$$0.3n-6=\log(1+n)$$

Given K = 20 so n should be ≥ 20.

By trial & error method n ≈ 25.

$n-K=$ m (number of parity bits)

m = 25 – 20 = 5

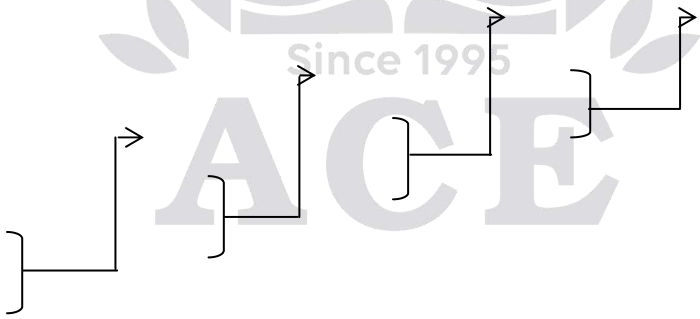
∴ Five Hamming bits are required.

02.

Sol: **Huffman coding:**

$Y_1 = 0$

X
i
P
(
X
i
)
C
O



Entropy H(X) = $\sum_{i=1}^{6} P_i \log_2 \frac{1}{P_i}$

$$= -[0.30\log_2(0.30)+0.25\log_2(0.25)+0.15\log_2(0.15)$$
$$+0.12\log_2(0.12)+0.10\log_2(0.10)+0.08\log_2(0.08)]$$

= 2.42 bits/messages

Average length $(\bar{L}) = \sum_{i=1}^{6} P_i x_i$

$= 0.3 \times 2 + 0.25 \times 2 + 0.15\times3 + 0.12\times3 + 0.10 \times3 + 0.08 \times3. = 2.45$

$\therefore$ Efficiency $(\eta) = \frac{H(X)}{\bar{L}} \times 100 = \frac{2.42}{2.45} \times 100 = 98.7\%$

$\therefore$ Redundancy $= 1-\eta = 1 - 0.9877 = 0.0122$

03.

Sol: Shannon Fano Algorithm:

1) List the source symbols in the order of decreasing probability.

2) Partition the set into two sets, that are as close to equiprobable as possible, and assign '0' to the upper set and a '1' to the lower set.

3) Continue this process, each time partitioning the sets with as nearly equal probabilities as possible, until further partitioning is not possible.

Symbol	**Probability P(x_i)**	**Code word**	**No. of bits**
X_1	1/2	0	1
X_2	1/8	100	3
X_3	1/8	101	3
X_4	1/16	1100	4
X_5	1/16	1101	4
X_6	1/16	1110	4
X_7	1/32	11110	5
X_8	1/32	11111	5

Entropy $H(x) = \sum_{i=1}^{8} P_i \log_2 \frac{1}{P_i}$ bits/symbol

$$= \frac{1}{2}\log_2 2 + \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{32}\log_2 32$$

= 2.3125 bits/ symbol

Average length

$(\bar{L}) = \frac{1}{2}\times1 + \frac{2}{8}\times3 + \frac{3}{16}\times4 + \frac{2}{32}\times5$

= 2.3125 bits/ symbol

$\therefore$ Efficiency $(\eta) = \frac{H(X)}{\bar{L}} \times 100$

= 100%

04.

Sol: Conditional Entropy

The input probabilities - $P(x_i)$,
output probabilities - $P(y_j)$,
transition probabilities - $P\left(\frac{y_j}{x_i}\right)$,
joint probabilities - $P(x_i, y_j)$.

H(x) is the average uncertainty of the channel input. $H(X) = -\sum_{i=1}^{m} P(x_i)\log_2 P(x_i)$

H(y) is the average uncertainty of the channel output

$$H(Y) = -\sum_{j=1}^{n} P(y_j)\log_2 P(y_j)$$

The conditional entropy can be given as

$$H(X/Y) = -\sum_{j=1}^{n}\sum_{i=1}^{m} P(x_i, y_j)\log_2 P\left(\frac{x_i}{y_j}\right)$$

The conditional entropy H(x/y) is a measure of the average uncertainty about the channel input after the channel output has been observed. H(x/y) is sometimes called the equivocation of x w.r.t y.

$$H(Y/X) = -\sum_{i=1}^{m}\sum_{j=1}^{n} P(x_i, y_j)\log_2 P\left(\frac{y_j}{x_i}\right)$$

The conditional entropy H(Y/X) is average uncertainty of the channel output when x was transmitted and y was received.

Redundancy: Redundancy is the number of bits used to transmit a message minus the number of bits of actual information in the message. Informally, it is the amount of wasted "space" used to transmit certain data. Data compression is a way to reduce or eliminate unwanted redundancy, while checksums are a way of adding desired redundancy for purposes of error detection when communicating over a noisy channel of limited capacity

The combined role of the channel encoder and decoder is to provide reliable communication over a noisy channel. This is done by introducing redundancy in the channel encoder and exploiting in the channel decoder to reconstruct the original encoder input as accurately as possible.

In source coding, we remove redundancy, where as in channel coding we introduce controlled redundancy. Because of redundancy, we are able to decode a message accurately without errors in the received message.

For example to the code words 0001 if we add a fifth pulse of positive polarity to make a new code word 00011. Now the number of positive pulses is 2 (even).

If a single error occurs in any position, this parity will be violated. The receiver knows that an error has been made and can request retransmission of the message. It can detect an error, but cannot locate it.

Redundancy =1 – Efficiency

$$\gamma = 1 - \eta = 1 - \frac{H(x)}{\bar{L}}$$

Given $P_0 = \frac{3}{8}$; $P_1 = \frac{5}{8}$;

$$P(X) = \begin{bmatrix} \frac{3}{8} & \frac{5}{8} \end{bmatrix}$$

$$P\left(\frac{1}{0}\right) = \frac{3}{4} \;,\; P\left(\frac{0}{1}\right) = \frac{1}{16};$$

$$P\left(\frac{1}{1}\right) = 1 - P\left(\frac{0}{1}\right) = \frac{15}{16}$$

$$P\left(\frac{0}{0}\right) = 1 - P\left(\frac{1}{0}\right) = \frac{1}{4}$$

$$P\left(\frac{y}{x}\right) = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{16} & \frac{15}{16} \end{bmatrix}_{2\times 2} \end{array}$$

$$H\left(\frac{Y}{X}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} P(x_i, y_j) \log P\left(\frac{y_j}{x_i}\right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} P(x_i) P\left(\frac{y_j}{x_i}\right) \log \frac{1}{P\left(\frac{y_j}{x_i}\right)}$$

$$= P(0).P\left(\frac{1}{0}\right)\log\frac{1}{P\left(\frac{1}{0}\right)} + P(1).P\left(\frac{0}{1}\right)\log\frac{1}{P\left(\frac{0}{1}\right)} + P(0).P\left(\frac{0}{0}\right)\log\frac{1}{P\left(\frac{0}{0}\right)} + P(1).P\left(\frac{1}{1}\right)\log\frac{1}{P\left(\frac{1}{1}\right)}$$

$$= \frac{3}{8}.\frac{3}{4}\log\frac{4}{3} + \frac{5}{8}.\frac{1}{16}\log\frac{16}{1} + \frac{3}{8}.\frac{1}{4}\log\frac{4}{1} + \frac{5}{8}.\frac{15}{16}.\log\frac{16}{15}$$

$$= \frac{3}{8}\left[\frac{3}{4}\log\frac{4}{3} + \frac{1}{4}\log 4\right] + \frac{5}{8}\left[\frac{15}{16}\log\frac{16}{15} + \frac{1}{16}\log 16\right]$$

$$= 0.515$$

$$H(X) = -[P(0)\log P(0) + P(1)\log P(1)]$$

$$= -\left[\frac{3}{8}\log\frac{3}{8} + \frac{5}{8}\log\frac{5}{8}\right]$$

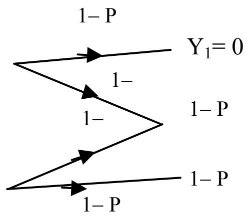
$$= 0.530 + 0.423$$

$$= 0.953$$

Redundancy $\gamma = 1 - \frac{H\left(\frac{Y}{X}\right)}{H(X)} = 1 - \frac{0.515}{0.953} = 45.96\%$

05.
Sol:

i) Amount of Information per symbol $H(X) = \sum_{i=1}^{m} P_i \log_2 \frac{1}{P_i}$ bits / symbol



$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{64}\log_2 64 + \frac{1}{128}\log_2 128 + \frac{1}{128}\log_2 128$$

$$= \left[\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{5}{32} + \frac{6}{64} + \frac{7}{128} + \frac{7}{128}\right]$$

$$= 0.5 + 0.5 + 0.375 + 0.25 + 0.156 + 0.093 + 0.109$$

= 1.983 bits/symbol.

ii) Probability of occurring '0' is

$$P(0) = \frac{1}{3}\left[\frac{1}{2}\times 3 + \frac{1}{4}\times 2 + \frac{1}{8}\times 2 + \frac{1}{16}\times 1 + \frac{1}{32}\times 2 + \frac{1}{64}\times 1 + \frac{1}{128}\times 1 + \frac{1}{128}\times 0\right]$$

$$= \frac{1}{3}\left[\frac{3}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128}\right]$$

$$= \frac{1}{3}(1.5 + 0.5 + 0.25 + 0.0625 + 0.0625 + 0.015625 + 7.8\times 10^{-3})$$

$$= \frac{1}{3}(2.398) = 0.8$$

Probability of occurring '1' is

1 − P(0) = 1 − 0.8

= 0.2

iii) Average length of the code

$$(\bar{L}) = \sum_{i=1}^{8} P_i\, x_i = \frac{765}{256} = 2.988 \text{ bits/symbol}$$

$$= \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{128}\right]\times 3$$

$$= \begin{bmatrix} 0.5 + 0.25 + 0.125 + 0.0625 + 0.03125 \\ + 0.015625 + 7.8\times 10^{-3} + 7.8\times 10^{-3} \end{bmatrix}\times 3$$

= 2.999 ≅ 3

Efficiency of the code $(\eta) = \frac{H(x)}{\bar{L}}\times 100 = \frac{1.983}{3}\times 100 = 66.10\%$

iv) Shannon Fano coding:

Symbol	Probability	Code	No.
A_1	1/2	0	1
A_2	1/4	10	2
A_3	1/8	110	3
A_4	1/16	1110	4
A_5	1/32	11110	5
A_6	1/64	111110	6
A_7	1/128	1111110	7
A_8	1/128	1111111	7

v) Average length of the code:

$$(\bar{L})_{new} = \begin{bmatrix} 1\times\frac{1}{2}+2\times\frac{1}{4}+3\times\frac{1}{8}+4\times\frac{1}{16} \\ +5\times\frac{1}{32}+6\times\frac{1}{64}+7\times\frac{1}{128}+\frac{7}{128} \end{bmatrix}$$

$= 1.98$ bits/symbol.

New Efficiency

$$\eta_{(new)} = \frac{H(x)}{(\bar{L})_{new}}\times 100 = \frac{1.98}{1.98}\times 100 = 100\% .$$

Chapter 12 Error Correcting Codes

Conventional Practice Solutions

01.
Sol:

Data word	Code word
0 0 0	0 0 0 0 0 0
0 0 1	1 1 0 0 0 1
0 1 0	1 1 1 0 1 0
0 1 1	0 0 1 0 1 1
1 0 0	0 1 1 1 0 1
1 0 1	1 0 1 1 0 0
1 1 0	1 0 0 1 1 1
1 1 1	0 1 0 1 1 0

From this code we can see that the distance between any two code words is at least 3.

Hence $d_{min} = 3$

02.
Sol:

(a) $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ \underbrace{0 \quad 0 \quad 1}_{I_k} & & & \underbrace{1 \quad 0 \quad 1}_{P} & & \end{bmatrix}$ & $H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b)

Data word	Code word
0 0 0	0 0 0 0 0 0
0 0 1	0 0 1 1 0 1
0 1 0	0 1 0 1 1 0
0 1 1	0 1 1 0 1 1
1 0 0	1 0 0 1 1 1
1 0 1	1 0 1 0 1 0
1 1 0	1 1 0 0 0 1
1 1 1	1 1 1 1 0 0

(c) The minimum distance between any two code words is 3. Hence, this is a single error correcting code. Since there are 6 single errors and 7 syndromes, we can correct all single errors and one double error.

(d)

E						S		
1	0	0	0	0	0	1	1	1
0	1	0	0	0	0	1	1	0
0	0	1	0	0	0	1	0	1
0	0	0	1	0	0	1	0	0
0	0	0	0	1	0	0	1	0
0	0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	1	1

$s = eH^T$

(e)

r	s	e	c	d
1 0 1 1 0 0	1 1 0	0 1 0 0 0 0	1 1 1 1 0 0	1 1 1
0 0 0 1 1 0	1 1 0	0 1 0 0 0 0	0 1 0 1 1 0	0 1 0
1 0 1 0 1 0	0 0 0	0 0 0 0 0 0	1 0 1 0 1 0	1 0 1

03.
Sol:
(a)

Data word		Code word			
0	0	0	0	0	0
0	1	0	1	1	1
1	0	1	0	1	0
1	1	1	1	0	1

The minimum distance between any two code words is $d_{min} = 4$. Therefore, it can correct all 1-error patterns. Since the code over satisfies hamming bound it can also correct some 2-error and possibly some 3-error patterns.

(b) $H^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $s = eH^T$

	e						s			
6 single- error patterns	1	0	0	0	0	0	1	1	1	0
	0	1	0	0	0	0	1	0	1	1
	0	0	1	0	0	0	1	0	0	0
	0	0	0	1	0	0	0	1	0	0
	0	0	0	0	1	0	0	0	1	0
	0	0	0	0	0	1	0	0	0	1
7double- error patterns	1	1	0	0	0	0	0	1	0	1
	1	0	1	0	0	0	0	1	1	0
	1	0	0	1	0	0	1	0	1	0
	1	0	0	0	1	0	1	1	0	0
	1	0	0	0	0	1	1	1	1	1
	0	1	1	0	0	0	0	0	1	1
	0	1	0	0	1	0	1	0	0	1
2 triple- error patterns	0	0	0	1	1	1	0	1	1	1
	0	0	1	1	0	1	1	1	0	1

Chapter 13 Optical Fiber Communication

Objective Practice Solutions

01. Ans: (d)

Sol: NA = 0.25

$$n_2 = \frac{C}{V}$$

$$n_2 = \frac{C}{\frac{C}{\sqrt{\varepsilon_r}}}$$

$$n_2 = \sqrt{\varepsilon_r} = \sqrt{2.4375}$$

$n_2 = 1.56$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$(NA)^2 = n_1^2 - n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

$$= \sqrt{(0.25)^2 + 2.4375}$$

$$= \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

02. Ans: (d)

Sol: Number of modes $M = \frac{V^2}{2}\frac{\alpha}{\alpha+2}$

$$= \frac{1}{2}\left(\frac{2\pi a}{\lambda}(NA)^2\right)\frac{\alpha}{\alpha+2}$$

Here a = core radius

λ = wavelength

α = refractive index profile.

03. Ans: (b)

Sol: Power loss = 0.25dB/km

For 100km, the power load = 100 × 0.25

= 25 dB

The optical power at 100km

$= 10 \log 0.1\times10^{-3} - 25$

= –65 dB

In dBm

→ –65 dB + 30 = –35 dBm.

04. Ans: (c)

Sol: Numerical Aperture is used to describes light gathering (or) light collecting ability of an optical fiber.

05. Ans: (c)

Sol: The refractive index of the cladding material should be less than that of the core.

06. Ans: (d)

Sol: Fibers with higher numerical aperture exhibit greater losses and lower bandwidth.

Conventional Practice Solutions

01.

Sol: Core diameter (d) = 50 μm

Core radius (a) = 25 μm

NA = 0.2

Wave length λ = 1 μm

$$V = \frac{2\pi a}{\lambda} NA$$

$$= \frac{2\pi \times 25 \times 10^{-6}}{10^{-6}} \times 0.2$$

$$= 31.41$$

For a graded index fiber with a parabolic refractive index profile generally α = 2

The Total number of guided modes propagating in the fiber is

$$M_g = \frac{\alpha}{\alpha+2}\frac{V^2}{2}$$

$$= \frac{V^2}{4} = \frac{(31.41)^2}{4}$$

$$= 246$$

02.

Sol: Given

Core refractive index $n_1 = 1.55$

Cladding refractive index $n_2 = 1.51$

Wave length λ = 0.80 μm

Core diameter d = 50 μm

Core radius a = 25 μm

$$V \text{ number } = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2}$$

$$= \frac{2\pi \times 25 \times 10^{-6}}{0.8 \times 10^{-6}}\sqrt{(1.55)^2 - (1.51)^2}$$

$$= 68.69$$

Approximate number of modes it will propagate

$$M \approx \frac{V^2}{2} = 2359$$

03.
Sol:

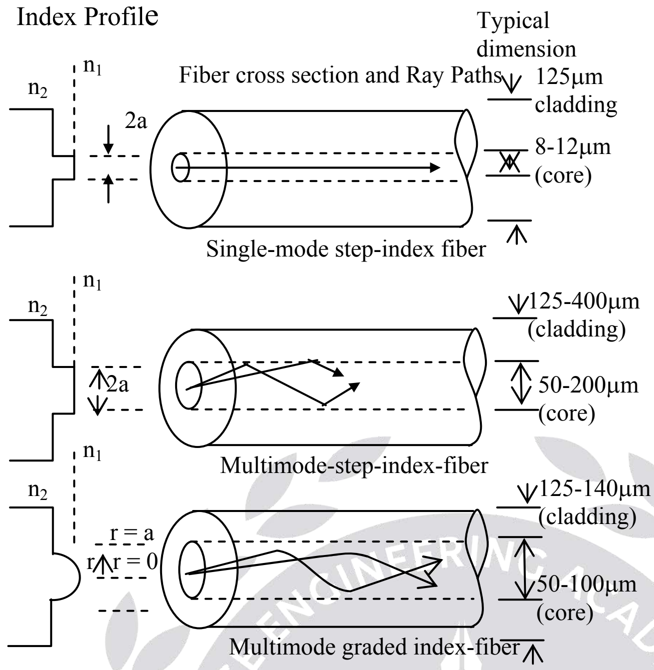


Fig: Comparison of conventional single-mode and multimode step-index and graded index optical fibers with ray transmission

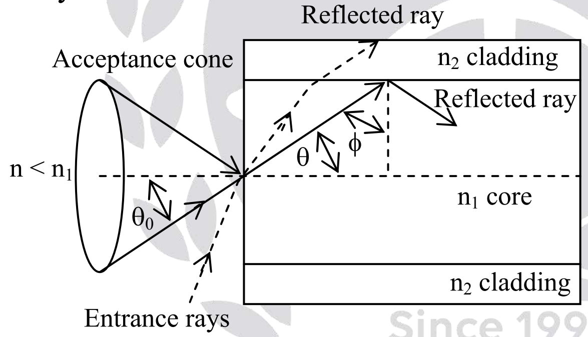


Fig: Meridional ray optics representation of the propagation mechanism in an ideal step-index optical waveguide

Given core refractive index $n_1 = 1.46$

Core radius $a = 4.5$ μm

Relative index difference $\Delta = 0.25$ %

$$\Rightarrow \Delta = \frac{0.25}{100} = 0.0025$$

Cut off wave length λ_c, the V number = 2.405

$$V_c = \frac{2\pi a}{\lambda_c} n_1 \sqrt{2\Delta}$$

$$2.405 = \frac{2\pi \times 4.5 \times 10^{-6}}{\lambda_c} 1.46\sqrt{2 \times 0.0025} \Rightarrow \lambda_c = 1.213\mu m = 1213 nm$$