ELECTRONICS & TELECOMMUNICATION ENGINEERING

ELECTRONIC MEASUREMENTS AND INSTRUMENTATION

Volume - 1: Study Material with Classroom Practice Questions
Objective Practice Solutions

01. Ans: (a)
Sol: For 10V total input resistance
\[ R_v = \frac{V_{fsd}}{I_{mfsd}} = 10/100 \mu A = 10^5 \Omega \]
Sensitivity = \( R_v/V_{fsd} = 10^5/10 \)
= 10 kΩ/V
For 100V \( R_v = 100/100 \mu A = 10^6 \Omega \)
Sensitivity = \( R_v/V_{fsd} = 10^6/100 \)
= 10 kΩ/V
(or)
Sensitivity = \( \frac{1}{I_{mfsd}} = \frac{1}{100 \times 10^{-6}} \)
= 10 kΩ/V

02. Ans: (d)
Sol: Variables are measured with accuracy
\[ x = \pm 0.5\% \text{ of reading 80 (limiting error)} \]
\[ Y = \pm 1\% \text{ of full scale value 100} \]
(Guaranteed error)
\[ Z = \pm 1.5\% \text{ reading 50 (limiting error)} \]
The limiting error for Y is obtained as Guaranteed
Error = 100×( ±1/100) = ±1
Then % L.E in Y meter
\[ 20 \times \frac{x}{100} = \pm 1 \]
\[ x = 5\% \]
Given \( w = xy/z \), Add all %L.E s
Therefore = \( \pm (0.5\% + 5\% + 1.5\%) \)
= \( \pm 7\% \)

03. Ans: (i) 41.97 (ii) 0.224 (iii) \( \pm 0.1513 \)
(Key changed)
Sol: Mean (\( \bar{X} \)) = \( \frac{\sum X}{n} \)
= \( \frac{41.7 + 42 + 41.8 + 42.1 + 41.9 + 42.5 + 42 + 41.9 + 41.8}{10} \)
= 41.97
SD = \( \sqrt{\frac{\sum d^2}{n-1}} \) for \( n < 20 \)
\[ d_n = \bar{X} - X_n \]
= \( \frac{(0.27)^2 + (-0.03)^2 + (-0.17)^2 + (-0.03)^2 + (-0.13)^2 + (0.07)^2 + (-0.53)^2 + (-0.03)^2 + (-0.13)^2 + (0.17)^2}{10-1} \)
= 0.224
Probable error = \( \pm 0.6745 \times SD \)
= \( \pm 0.1513 \)

04. Ans: 150 V (Key changed)
Sol:
\[ 0.05 mA \]
\[ 100 V \]
\[ 100 V \]
\[ V_1 \]
\[ V_2 \]
\[ V_3 \]
\[ I_{fsd} = \frac{1}{S_{dc1}} \]
\[ I_{fsd} = \frac{1}{S_{dc2}} \]

The maximum allowable current in this combination is 0.05 mA, since both are connected in series.
Maximum D.C voltage can be measured as
\[ = 0.05 mA (10 \text{ kΩ/V} \times 100 + 20 \text{ kΩ/V} \times 100) \]
\[ = 3000 \times 0.05 = 150 V \]
05. **Sol:** Internal impedance of 1st voltmeter
\[ = \frac{100\,\text{V}}{5\,\text{mA}} = 20\,\text{k}\Omega \]

Internal impedance of 2nd voltmeter
\[ = 100 \times 250\,\Omega/\text{V} = 25\,\text{k}\Omega \]

Internal impedance of 3rd voltmeters,
\[ = 5\,\text{k}\Omega \]

Total impedance across 120 V
\[ = 20 + 25 + 5 = 50\,\text{k}\Omega \]

Sensitivity = \( \frac{50\,\text{k}\Omega}{120\,\text{V}} \Rightarrow 416.6\,\Omega/\text{V} \)

⇒ Reading of 1st voltmeter
\[ = \frac{20\,\text{k}\Omega}{416.6\,\Omega/\text{V}} = 48\,\text{V} \]

Reading of 2nd voltmeter
\[ = \frac{25\,\text{k}\Omega}{416.6\,\Omega/\text{V}} = 60\,\text{V} \]

Reading of 3rd voltmeter
\[ = \frac{5\,\text{k}\Omega}{4166\,\Omega/\text{V}} = 12\,\text{V} \]

06. **Ans:** (b)

**Sol:** Bridge sensitivity = \( \frac{\text{Change in output}}{\text{Change in input}} \)

\[ = \frac{V_{th}}{10\,\Omega} \]

\[ V_{th} = \frac{1010 \times 100 - 1000 \times 100}{2000} = 0.25\,\text{V} \]

\[ S_B = \frac{0.25\,\text{V}}{10\,\Omega} = 25\,\text{mV/}\Omega \]

07. **Ans:** (d)

**Sol:** \( W_T = W_1 + W_2 = 100 - 50 = 50\,\text{W} \)

\[ \frac{\partial W_T}{\partial W_1} = \frac{\partial W_T}{\partial W_2} = 1 \]

Error in meter 1 = \( \pm \frac{1}{100} \times 100 = \pm 1\,\text{W} \)

Error in meter 2 = \( \pm \frac{0.5}{100} \times 100 = \pm 0.5\,\text{W} \)

\[ W_T = W_1 + W_2 = 50 \pm 1.5\,\text{W} \]

\[ W_T = 50 \pm 3\% \]

08. **Ans:** (b)

**Sol:** Resolution = \( \frac{200}{100} \times \frac{1}{10} = 0.2\,\text{V} \)

09. **Ans:** (b)

**Sol:** \( \%\,LE = \frac{\text{FSV}}{\text{true value}} \times \%\,GAE \)

\[ = \frac{200\,\text{V}}{100\,\text{V}} \times \pm 2\% \]

\[ = \pm 4\% \]

10. **Ans:** (c)

**Sol:** Power = \( P = (2)^2 (100) = 400\,\text{W} \)

\[ \%\,Error = \pm 2 \left( \frac{\partial P}{\partial I} + \frac{\partial P}{\partial R} \right) \times 100 \]

\[ = \pm 2 (1) \pm 0.2 \]

\[ = \pm 2.2\% \]
Error = ± \frac{2.2 \times 400}{100} = ± 8.8 \text{ W}

11. Ans: (c)
Sol: \[ R_T = R_1 + R_2 = 36 + 75 = 111 \Omega \]
\[ \Delta R_T = \left( \frac{5}{100} \times 36 \right) + \left( \frac{5}{100} \times 72 \right) = 5.55 \Omega \]
\[ R_T = 111 ± 5.55 \Omega \]

12. Ans: (a)
Sol: \[ R = \frac{V}{I} \]
\[ \frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 \]
\[ \frac{\Delta R}{R} \times 100 = 2\% + 1\% = 3\% \]

Conventional Practice Solutions

01.
Sol: (a) Sensitivity |_{25^\circ C} = \frac{3.0 - 1.5}{100 - 50} = 30 \mu \text{m/kg}

Sensitivity|_{40^\circ C} = \frac{3.5 - 1.9}{100 - 50} = 32 \mu \text{m/kg}

(b) Zero Drift = 0.3mm – 0mm = 0.3mm

Sensitivity Drift = 32\mu \text{m/kg} – 30\mu \text{m/kg} = 2 \mu \text{m/kg}

Zero drift co-efficient = \frac{0.3 \text{mm}}{40^\circ C - 25^\circ C} = 20.0 \mu \text{m/}^\circ \text{C}

Sensitivity Drift co-efficient = \frac{2 \mu \text{m/kg}}{40^\circ C - 25^\circ C} = 0.133 \mu \text{m/kg/}^\circ \text{C}

02.
Sol:
Using circuit minimizing techniques

\[ 100 \text{ V} \]
\[ 10 \text{ k}\Omega \quad 5 \text{ k}\Omega \quad 5 \text{ k}\Omega \]
\[ 10 \text{ k}\Omega \quad 10 \text{ k}\Omega \quad 15 \text{ k}\Omega \]
\[ R_{th} = 5 \text{ k}\Omega \]

\[ 100 \text{ V} \]
\[ 10 \text{ k}\Omega \quad 10 \text{ k}\Omega \Rightarrow V_{th} = 50 \text{ V} \]

\[ V_{th} = 50 \text{ V} \]
\[ R_{th} = 5 \text{ k}\Omega \quad 5 \text{ k}\Omega \quad 5 \text{ k}\Omega \]
\[ 10 \text{ k}\Omega \quad 15 \text{ k}\Omega \]
Value of current in 15kΩ

\( (I_0) = \frac{25V}{10kΩ + 15kΩ} \)

\( I_0 = 1 \text{ mA} \)

\( (A) \) reading \( (I_L) \)

\( = \frac{25V}{10kΩ + 15kΩ + 2kΩ} \)

\( I_L = 99\% I_0 \) (from the question data)

\( I_L = 0.99 I_0, \ I_L = \frac{1}{1 + \frac{R_a}{10kΩ}} I_0 \)

\( \frac{I_L}{I_0} = \frac{1}{1 + \frac{R_a}{10kΩ + 15kΩ}} \)

\( \Rightarrow 0.99 = \frac{1}{1 + \frac{R_a}{25kΩ}} \)

\( R_a = 250 \Omega \)

03. (a)

**Sol:** **Accuracy** → It indicates degree of closeness of measured value to the True value.

**Precision:** → Indicates the degree of closeness of measured values by an instrument for a fixed input quantity.

Sensitivity (s) → Change in output

\[
\text{Sensitivity (s)} = \frac{\text{Change in output}}{\text{Change in input}}
\]

Unit of sensitivity depends on type of Instrument

For voltmeter sensitivity defined as

\[
\frac{1}{I_{\text{full}} \Omega/V}
\]

03. (b)

**Sol:**

\[
V_{R1} = 200V \times \frac{100kΩ \times 100kΩ}{100kΩ + 100kΩ} = 100V
\]

A voltmeter when connected across either of two 100kΩ resistors (say upper one) acts as shunt for the portion of circuit. The voltmeter will then indicate a lower voltage drop than actually existed before the voltmeter was connected. This happens because of loading effect & mainly occurs with low sensitivity of Instrument. Assume voltmeter resistance as \( R_V \).
Volmeter Indication is ‘90 V’

\[ R_{eq} = 100 \, \text{k}\Omega / R_v = \frac{100\, \text{k}\Omega \times R_v}{100\, \text{k}\Omega + R_v} \]

\[ R_{Total} = (R_{eq} + 100) \, \text{k}\Omega, \]

Voltage division

\[ 90\, \text{V} = 200\, \text{V} \times \frac{R_{eq}}{100 + R_{eq}} \]

\[ R_{eq} = 81.8 \, \text{k}\Omega \]

\[ R_{eq} = \frac{100\, \text{k}\Omega \times R_v}{100\, \text{k}\Omega + R_v} \]

\[ \Rightarrow 81.8 \, \text{k}\Omega = \frac{100\, \text{k}\Omega \times R_v}{100\, \text{k}\Omega + R_v} \]

\[ R_v = 450 \, \text{k}\Omega \]

### 04.

**Sol:** Ranges:

(A) \( \rightarrow \) (0 – 5) A

(V) \( \rightarrow \) (0 – 250 V)

(W) \( \rightarrow \) (0 – 500 W)

**Meter reading**

\[ I = 2.5A \pm 0.5\% \, \text{fsd} \]

\[ \Rightarrow I = 2.5 \pm 1\% \, \text{of reading} \]

\[ V = 115V \pm 0.5\% \, \text{fsd} \]

\[ \Rightarrow V = 115V \pm 1.086\% \, \text{reading} \]

\[ W = 220 \, \text{W} \pm 1\% \, \text{fsd} \]

\[ \Rightarrow W = 220\, \text{W} \pm 2.27\% \, \text{reading} \]

\[ \cos \phi = \frac{P}{VI} \]

\[
\ln \cos \phi = \ln p - \ln v - \ln I
\]

\[
\frac{1}{\cos \phi} = \frac{1}{P} \frac{dp}{d \cos \phi} - \frac{1}{V} \frac{dv}{d \cos \phi} - \frac{1}{I} \frac{dl}{d \cos \phi}
\]

\[
\Delta \cos \phi = \frac{\Delta P}{P} - \frac{\Delta V}{V} - \frac{\Delta I}{I}
\]

\[
\Delta \cos \phi \times 100 = \frac{\Delta P}{P} \times 100 + \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100
\]

\[
= 2.27\% + 1.086\% + 1\%
\]

\[ = 4.356\% \rightarrow \text{max. uncertainty} \]

### 05.

**Sol:** Expected value \( (A_T) = 20\, \text{mA} \)

Measured value \( (A_m) = 18\, \text{mA} \)

i) Absolute error = \( A_m - A_T = 2\, \text{mA} \)

ii) Percentage error = \( \frac{A_m - A_T}{A_T} \times 100 \)

\[ = -10\% \]

iii) Relative Accuracy = \( 1 - | \text{error} | \)

\[ = 1 - |0.1| = 0.9 \]

iv) Percentage Accuracy = \( 0.9 \times 100 = 90\% \)

v) Precision of 6\text{th} measurement

\[ (X_m = X_6) = 18\, \text{mA} \]

\[ \% P = \left( 1 - \frac{X_m - X}{X} \right) \times 100 \]

\[ \text{Avg} X = \frac{(16 + 19 + 20 + 17 + 21 + 18 + 15 + 16 + 18 + 17) \, \text{mA}}{10} \]

\[ = 17.7 \, \text{mA} \]

\[ \% P = \left( 1 - \frac{18\, \text{mA} - 17.7\, \text{mA}}{17.7\, \text{mA}} \right) \times 100 \]

\[ = 98.3\% \]

### 06.

**Sol:** Absolute error = \( 1.14\, \text{k}\Omega - 1.2\, \text{k}\Omega \)

\[ = -0.06\, \text{k}\Omega \]
Absolute error = 1.26 kΩ – 1.2 kΩ
= 0.06 kΩ
Tolerance = ± 0.06 kΩ
Maximum resistance at 25°C is 1.26 kΩ
Temperature coefficient = +500 PPM/°C
Let R₁ = 1.26 kΩ at 25°C
\[ R₁ = \frac{+500}{1,000,000} \times 1.26 \text{kΩ} = +0.63 \Omega/°C \]
Change in temperature (ΔT) = 75°C – 25°C
= 50°C

The maximum resistance at 75°C
= 1.26 kΩ + 0.63 Ω/°C × 50°C
= 1.291 kΩ
Objective Practice Solutions

01. Ans: (d)
Sol: The pointer swings to 1 mA and returns, settles at 0.9 mA i.e., pointer has oscillations. Hence, the meter is under-damped. Now the current in the meter is 0.9 mA.

\[
\begin{align*}
\text{1.8V} & \quad \text{1.8KΩ} \\
\text{S1} & \\
\text{G} & \\
\end{align*}
\]

Applying KVL to circuit,
\[
1.8 \text{ V} - 0.9 \text{ mA} \times \text{R}_m - 0.9 \text{ mA} \times 1.8 \text{ kΩ} = 0
\]
\[
1.8 \text{ V} - 0.9 \times 10^{-3} \text{ R}_m - 1.62 = 0
\]
\[
\text{R}_m = \frac{0.18}{0.9 \times 10^{-3}} = 200 \text{ Ω}
\]

02. Ans: (c)
Sol:

\[
\begin{align*}
\text{S} & = \frac{1}{1000} \Omega / \text{volt} \\
\text{S} & = \frac{1}{I_{\text{fsd}}} \Omega / \text{V} \\
I_{\text{fsd}} & = \frac{1}{S} = \frac{1}{1000} = 1 \text{ mA}
\end{align*}
\]

03. Ans: (c)
Sol:
\[
T_d = \frac{1}{2} I^2 \frac{dL}{dθ}
\]
\[
K_1 \theta = \frac{I^2}{2} \frac{dL}{dθ}
\]
\[
25 \times 10^{-6} \times \theta = \frac{25}{2} \times \left(3 - \frac{0}{2}\right) \times 10^{-6}
\]
\[
\frac{5}{2} \theta = 3
\]
\[
\theta = 1.2 \text{ rad}
\]

04. Ans: (a) (Key changed)
Sol:

\[
\begin{align*}
\text{PMMC meter reads Average value} \\
V_{\text{avg}} & = \frac{\left(\frac{1}{2} \times 10 \times 10 \text{ ms}\right) + \left(-5 \times 2 \text{ ms}\right) + \left(5 \times 8 \text{ ms}\right)}{20 \text{ ms}} \\
& = \frac{50 - 10 + 40}{20} = 4 \text{ V}
\end{align*}
\]
05. Ans: (a)
Sol:

<table>
<thead>
<tr>
<th>Spring stiffness(Kc)</th>
<th>1°C↑</th>
<th>10°C↑</th>
<th>Te</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength of magnet (B)</td>
<td>0.02%↓</td>
<td>0.2%↓</td>
<td>0.2%↓</td>
<td>0.2%↓</td>
</tr>
</tbody>
</table>

Net deflection (θnet) = 0.4%↑ – 0.2%↓ = 0.2%↑

Increases by 0.2%.

06. Ans: 32.4° and 21.1° (Key changed)
Sol: I1 = 5 A, θ1 = 90°; I2 = 3 A, θ2 = ?
θ ∝ I^2 (as given in Question)

(i) Spring controlled
θ ∝ I^2

\[ \frac{θ_2}{θ_1} = \left( \frac{I_2}{I_1} \right)^2 \]

\[ \Rightarrow \frac{θ_2}{90°} = \left( \frac{3}{5} \right)^2 \]

θ2 = 32.4°

(ii) Gravity controlled

\[ \sin θ ∝ I^2 \]

\[ \frac{\sin θ_2}{\sin θ_1} = \left( \frac{I_2}{I_1} \right)^2 \]

\[ \Rightarrow \frac{\sin θ_2}{\sin 90°} = \left( \frac{3}{5} \right)^2 \]

\[ \Rightarrow \frac{\sin θ_2}{1} = 0.36 \]

θ2 = sin^-1(0.36) = 21.1°
In case (i),
\[ I_m = \frac{250 \text{ V}}{\sqrt{R_m^2 + (\omega L_m)^2}} \]
\[ = \frac{250 \text{ V}}{\sqrt{(2500)^2 + (2\pi \times 50 \times 0.6)^2}} \]
\[ = 0.0997 \text{ A} \]
In case (ii),
\[ I_m = \frac{250 \text{ V}}{\sqrt{(R_m + R_{se})^2 + (\omega L_m)^2}} \]
\[ 0.0997 \text{ A} = \frac{500 \text{ V}}{\sqrt{(2500 + R_{se})^2 + (2\pi \times 50 \times 0.6)^2}} \]
\[ \sqrt{(2500 + R_{se})^2 + 35.53 \times 10^3} = \frac{500}{0.0997} \]
\[ \sqrt{(2500 + R_{se})^2 + 35.53 \times 10^3} = 5.015 \times 10^3 \]
\[ R_{se} = 2511.5 \Omega \]

09. Ans: 0.1025 \( \mu \)F
Sol: \[ C = \frac{0.41 \times L_m}{R_{se}^2} \]
\[ C = \frac{0.41 \times 1}{(2 \text{k}\Omega)^2} = 0.1025 \mu \text{F} \]

10. Ans: (c)
Sol: MC – connection

\[ \frac{0.01\Omega}{20A} \]
\[ \frac{1000\Omega}{30V} \]
\[ \text{Error due to current coil} \]
\[ = \frac{20^2 \times 0.01}{(30 \times 20)} \times 100 = 0.667\% \]

11. Ans: (b)
Sol: \[ \phi = \tan \left[ \frac{\sqrt{3(W_e - W_2)}}{(W_e + W_2)} \right] \]
Power factor = \( \cos \phi \)
= 0.917 lag (since load is inductive)

12. Ans: (c)
Sol: \[ R_{load} = \frac{V}{I} \]
\[ = \frac{200}{20} = 10 \Omega \]
For same error \[ R_L = \sqrt{R_C \times R_V} \]
\[ \therefore 100 = 10 \times 10^3 \times R_C \]
\[ \Rightarrow R_C = 0.01 \Omega \]

13. Ans: (a)
Sol: \[ i(t) = 3 + 4 \sqrt{2} \sin 314t \]
PMMC reads average value.
\[ \therefore \text{Average value} = 3A \]
14. Ans: (b)
Sol: Hot wire ammeter reads RMS value
\[ I_{rms} = \sqrt{2^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = 3.46 \text{ A} \]

15. Ans: (d) (Key changed)
Sol: Moving Iron Ammeter, \( \theta \propto I^2 \)
For 1 A dc \( \rightarrow 20^\circ \)
\( I_1 = 1 \text{ A}, \theta_1 = 20^\circ \)
For 3 sin 314 t \( \rightarrow ? \)
MI Ammeter measures the rms value of AC current
\[ I_2 = I_{rms} = \frac{3}{\sqrt{2}} I_1 = \frac{3}{\sqrt{2}} \text{ A}, \quad \theta_2 = ? \]
\[ \theta_2 = \frac{I_2}{I_1} \cdot \theta_1 = \frac{\left(\frac{3}{\sqrt{2}}\right)}{1} \cdot 20^\circ = 90^\circ \]

16. Ans: (a)
Sol: \( V_{dc} = I_{dc} \times 10 \Omega = \left(\frac{12+5}{2}\right) \times 10 = 85 \text{ V} \)

17. Ans: (b)
Sol: Given that,
\[ R_c = 0.03 \Omega, \quad R_p = 6000 \Omega \]
\[ V = 220 \text{ V}, \quad 0.6 \text{ p.f} \]
\[ \% \text{ Error} = \frac{I_L^2 R_c \times 100}{V \cos \phi} \]
\[ = \frac{20^2 \times 0.03}{220 \times 20 \times 0.6} \times 100 = 0.45\% \]

Conventional Practice Solutions

01.
Sol:
\[ R_m = 99\Omega \quad I_m \]
\[ R_s = 1\Omega \]
\[ (a) \quad I_m - I_{fsd} = 0.1\text{mA} \]
* Voltage across instrument = \( I_m R_m \)
\[ = 0.1\text{mA} \times 99\Omega = 9.9\text{mv} \]
* \( I_{sh} = \frac{9.9\text{mV}}{1\Omega} = 9.9\text{mA} \)
* \( I_{total} = I = I_m + I_{sh} = 10\text{mA} \)

(b) \( I_m = 0.5I_{fsd} \)
\[ = 0.5 \times 0.1\text{mA} = 0.05\text{mV} \]
* Voltage across instrument
\[ = I_m R_m = 0.05 \times 99\text{mV} = 4.95 \text{mV} \]
\[ I_{sh} = \frac{4.95\text{mV}}{1\Omega} = 4.95\text{mA} \]
\[ I_{total} = 0.05 + 4.95 = 5\text{mA} \]

(c) \( 0.25 I_{fsd} = I_m \)
\[ = 0.25 \times 0.1 = 0.025\text{mA} \]
Voltage across instrument
\[ = I_m R_m = 2.475 \text{mV} \]
\[ I_{sh} = \frac{2.475\text{mV}}{1\Omega} = 2.475\text{mA} \]
\[ I = I_m + I_{sh} = 2.5\text{mA} \]

02.
Sol: Voltage across instrument for full scale deflection = 100mV.
Current in instrument for full scale deflection, \( I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3} \text{ A} \)
Deflecting torque, \( T_d = B I N A = B I N(\ell \times d) \)
= \( 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3} \)
= \( B \times 375 \times 10^{-6} \)

\[ \therefore \text{Controlling torque for a deflection} \theta = 120^\circ \]
\( T_c = K \theta = 0.375 \times 10^{-6} \times 120 \)

At final steady position, \( T_d = T_c \)
or \( 375 \times 10^{-6} \times B = 45 \times 10^{-6} \)

\[ \therefore \text{Flux density in the air gap,} \ B = \frac{45 \times 10^{-6}}{375 \times 10^{-6}} = 0.12 \text{ Wb/m}^2 \]

Resistance of coil winding, \( R_c = 0.3 \times 20 = 6 \Omega \)

Length of mean turn
\( l = 2 \times (L+d) = 2 \times (30+25) = 110 \text{ mm} \)

Let \( a \) be the area of cross-section of wire and \( P \) be the resistivity

Resistance of coil, \( R_c = N \rho l / a \)

\[ \therefore \text{Area of cross-section of wire} \]
\[ a = \frac{100 \times 1.7 \times 10^{-6} \times 110 \times 10^{-3} \times 10^6}{6} \]
= \( 31.37 \times 10^{-3} \text{ mm}^2 \)

Diameter of wire, \( d = \left[ \frac{4}{\pi} \left( 31.37 \times 10^{-3} \right) \right]^{1/2} \)
= 0.2 mm

**03.**

**Sol:**

\[ \text{(Fig) with normal connections load is kept constant} \]

\[ I_1 = 10 \times 0.05 = 0.5 \text{ mA} \]
\[ I_2 = 10 \times \frac{0.02}{1500 + 0.02} = 1.33 \times 10^{-4} \]
= 0.133 mA

[Diagram with connections]

With shunt connections interchanged

\[ I_1' = 10 \times \frac{0.02}{1000 + 0.05} = 0.199 \text{ mA} \]
\[ I_2' = 10 \times \frac{0.05}{1500 + 0.05} = 0.33 \text{ mA} \]

04.

**Sol:**

(a) The internal resistance of 5\( \Omega \) is due to copper only

(b) A 4\( \Omega \) manganin swamping resistor is used in series with a copper resistor of 1\( \Omega \). Assume the temperature coefficient of copper as 0.004 ohm/\( ^\circ \text{C} \) and that of manganin 0.00015 ohm/\( ^\circ \text{C} \)
(c) When the whole of the $5\Omega$ constitute the resistance in the copper of the instrument coil and lead as shown in the figure. Instrument current = 0.015A

\[
\therefore \text{ shunt current } = 100 - 0.015 = 99.985 \text{ A}
\]

![Diagram of circuit](Diagram)

PD across the shunt = $0.015 \times 5$

\[
= 0.075 \text{ V}
\]

\[
\therefore \text{ Shunt resistance } = \frac{0.075}{99.985}
\]

\[
= 0.00075 \Omega \text{ (manganin wire)}
\]

Then the shunt resistance at 10°C temperature rise

\[
= 0.00075 \times (1 + 10 \times 0.00015)
\]

\[
= 0.000751 \Omega
\]

The instrument resistance after 10°C rise in temperature

\[
= 5 \times (1 + 10 \times 0.004)
\]

\[
= 5.2 \Omega
\]

The instrument current corresponding to 100 A input. In inverse proportion to the resistance forming the parallel circuit, A and B will be.

\[
= \frac{0.000751}{5.20075} \times 100 = 0.01444 \text{ A}
\]

and the instrument reading

\[
= \frac{100}{0.015} \times 0.01444 = 96.27 \text{ A}
\]

\[
\therefore \text{ Percentage error due to temperature rise}
\]

\[
= 100 - 96.27 \text{ or } 3.73\% \text{ (10 W)}
\]

(b) The connections in this case are as shown resistance of the instrument → 100A

Circuit after 10°C rise in temperature

![Diagram of circuit](Diagram)

\[
\text{PS Lead}
\]

\[
\text{Swamping } = 4\Omega
\]

The shunt resistance after 10°C rise in temperature is still the same, that is 0.000751 Ω

Instrument current corresponding to 100 A input

\[
= \frac{0.000751}{5.046751} \times 100 = 0.01488 \text{ A}
\]

And the instrument reading

\[
= \frac{100}{0.015} \times 0.01488 = 99.2 \text{ A}
\]

\[
\therefore \text{ Percentage error due to the temperature rise}
\]

\[
= 100 - 99.2 \text{ or } 0.8\% \text{ (10W)}
\]

[This shows the great advantage of using a swamping resistance in parallel with the shunt]
05.
Sol: \( R_m = 40 \Omega \), \( I_{FSD} = 1 \text{mA} \)

**(0 - 10mA)**

\[
R_{sh1} = \frac{R_m}{m-1} = \frac{40}{\left(\frac{10mA}{1mA} - 1\right)} = 4.44 \Omega
\]

\( R_{sh1} \) is the Ayrton shunt

\( R_m = 40 \Omega \)

* (0–50mA)

\[
R_{sh} = \frac{R_m + R_{sh1}}{M_2} = \frac{40\Omega + 4.44}{50mA/1mA} = 0.88 \Omega
\]

06.
Sol: \( L = (10 + 50 - 2\theta^2) \mu \text{H} \)

\[
\frac{dL}{d\theta} = (5-40) \mu \text{H per radian}
\]

and also \( \frac{dL}{d\theta} = \frac{2K\theta}{I^2} \)

\[
\therefore (5-40) \times 10^{-6} = \frac{2K\theta}{I^2}
\]

Substituting \( \theta = 30^\circ \) or \( \frac{\pi}{6} \) radian and \( I = 5A \) in above expression, we have

\[
\left[5-4\times\frac{\pi}{6}\right] \times 10^{-6} = \frac{2\times\pi}{6}\left(5\right)^2
\]

Or \( K = 69.36 \times 10^{-6} \text{N} \cdot \text{m/radian} \)

i.e., spring constant

\( = 69.36 \times 10^{-6} \text{Nm/radian} \text{ Ans} \)

Substituting \( I = 10A \) and \( K = 69.36 \times 10^{-6} \) in Equation (i) we have

\[
(5-40) \times 10^{-6} = \frac{2 \times 69.36 \times 10^{-6}}{10^2} \theta
\]

Or \( \theta = 0.928 \text{ radian or } 53.2^\circ \text{ Ans} \)

07.
Sol: It is given that

\( I = 40^\circ \)

And we know that

\[
\frac{dL}{d\theta} = \frac{2K\theta}{I^2} = \frac{2K\theta}{4^2\theta^2} = \frac{1}{8} k0^{1-2n}
\]

Integrating above expression we have

\[
L = \frac{k0^{2-2n}}{8(2-2n)} + A = \frac{k0^{2-2n}}{16(1-n)} + A \quad \text{(i)}
\]

Where \( A \) is constant of integration

When \( I = 0, \ \text{deflection} \ 0 = 0 \)

and \( L = 10 \times 10^{-3} \text{ or } 0.01\text{H} \)

Substituting \( I = 0, \ \theta = 0 \)

And \( L = 0.01 \text{ H} \) in Equation (1) we have

\( 0.01 = 0 + A \)

or \( A = 0.01 \)

Substituting \( A = 0.01 \) in Equation (i) we have

\[
L = \frac{k}{16(1-n)}0^{2-2n} + 0.01
\]

\[
\therefore K = 0.6 \text{ N} \cdot \text{m/rad}
\]

(ii) substituting \( n = 0.75 \) in above expression we have

\[
L = \frac{0.6}{16(1-0.75)}0^{2-2\times0.75} + 0.01
\]

\[
= 0.150^{0.5+0.01}
\]

Substituting \( L = 60 \times 10^{-3} \text{ or } 0.06 \text{ in above expression we have} \)

\( 0.06 = 0.150^{0.5} + 0.01 \)
or deflection, \( \theta = \left[ \frac{0.05}{0.15} \right]^2 = 0.111 \) radian
or 6.37° Ans
Current, I = 4\( \theta \) = 4(0.111)^0.75
= 0.769 A

08.
**Sol:** Current dawn by instrument when connected across 300 V ac
\( I_{ac} \) = 100mA = 0.1A
At 50 Hz supply Instrument reactance,
\[ X_L = 2\pi fL = 2\pi \times 50 \times 0.8 = 251.33 \Omega \]
\[ = \frac{200.7 - 200}{200} \times 100 = 0.35\% \text{ Ans} \]

09.
**Sol:** The deflecting torque, \( T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \)
\[ I = 1.4 \text{ and } 1.6, \text{ about } 1.5 \text{ on either side,} \]
then \[ \frac{dL}{d\theta} = \frac{577.8 - 576.6}{61.5 - 49.5} = \frac{1.2}{12} = 0.1\times10^{-4} \text{ H/deg} \]
Or \[ 0.1 \times 57.3 = 5.73 \times 10^{-6} \text{ H/ rad} \]
and \[ T_d = \frac{1}{2} \times 1.5^2 \times 5.73 \times 10^{-6} \]
= 6.44 \times 10^{-6} \text{ Nm} 

10.
**Sol:** The rate of change of mutual inductance is
\[ \frac{dM}{d\theta} = \frac{k\theta}{I^2} = \frac{0.1 \times 10^{-6} \times 110}{(10)^2} \]
= 0.11 \times 10^{-6} \text{ H/ rad}
= 0.00192 \mu \text{H/ degree} 

Final inductance
\[ = \text{initial inductance} + \text{change in inductance} \]
\[ = 2.0 + 0.00192 \times 110 = 2.21 \mu \text{H} \]

11
**Sol:** (i) **Frictional Error (Torque/weight):**
- Potential coil has more number of turns so weight of moving coil is more.
- Then it has more frictional error as compared to M.C and M.I

(ii) **Temperature Error:**
- Because of presence of two coils net heat developed is more. Then temperature error is more as compared to other Instruments.

(iii) **Frequency error:**
- When it is used as a voltmeter (or) ammeter Net Inductance is more then the frequency error is more
- But when used as a wattmeter, inductance is very less, so frequency error is low.

**Hystersis & Eddy current error:**
- Because of the use of air core, the hysteresis and eddy current errors are almost zero.
- Both fixed coil & moving coils are electro magnet so poor magnetic field is present
- The external magnetic field can easily distort the inside operating field. Hence stray magnetic field is more as compared to other types of metets.
**Note:** For portable wattmeter, to reduce the stray magnetic field error; iron shielding is used.
- For precision wattmeters, to reduce stray magnetic field error, static system is used. In this, two set of fixed coil and moving coils are used.

(ii). \(I(t) = 80 - 60\sqrt{2}\ \sin (\omega t + 30^\circ) \ A\)

Dynamo Ammeter → rms

\[I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta}\]

\[i^2 = 80^2 - (60\sqrt{2})^2 \sin^2 \left(\theta + \frac{\pi}{6}\right)\]

\[= 6400 - 7200 \sin^2 \left(\theta + \frac{\pi}{6}\right)\]

\[\Rightarrow DA = I_{\text{rms}}^2 = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta}\]

\[= 100 \ A\]
01. Ans: (b)
Sol:
Potential coil voltage = 200 V
C.T. primary current \( (I_p) \)
\[
I_p = I_L = \frac{200 \text{ V}}{\sqrt{4^2 + 3^2 \tan^{-1}\left(\frac{3}{4}\right)}}
\]
\[
I_p = \frac{200 \text{ V}}{5 \angle 36.86^\circ}
\]
\[
I_p = 40 \angle -36.86^\circ
\]
\[
\frac{I_p}{I_s} = \frac{50}{5} = 10 \angle 36.86^\circ
\]
\[
\frac{40}{I_s} = \frac{50}{5} = 10
\]
\[
I_s = \frac{50}{5} \times 40 = 4 \text{ A}
\]
C.T secondary \( (I_s) \) = 4\( \angle -36.86^\circ \)
Wattmeter current coil \( (I_c) \) = 4\( \angle -36.86^\circ \)
Wattmeter reading
\[
= 200 \text{ V} \times 4 \times \cos (36.86^\circ)
\]
\[
= 640.08 \text{ W}
\]

02. Ans: (a)
Sol: Energy consumed in 1 minute
\[
= \frac{240 \times 10 \times 0.8 \times \frac{1}{60}}{1000} = 0.032 \text{ kWh}
\]
Speed of meter disc
\[
= \text{Meter constant in rev/kWhr} \times \text{Energy consumed in kWh/minute}
\]
\[
= 400 \times 0.032
\]
\[
= 12.8 \text{ rpm (revolutions per minute)}
\]

03. Ans: (a)
Sol: Energy consumed (True value)
\[
= \frac{230 \times 5 \times 1}{1000} \times \frac{3}{60} = 0.0575 \text{ kWh}
\]
Energy recorded (Measured value)
\[
= \frac{\text{No. of rev (N)}}{\text{meter constant (k)}}
\]
\[
= \frac{90 \text{ rev}}{1800 \text{ rev/kWh}} = 0.05 \text{ kWh}
\]
\[
\% \text{Error} = \frac{0.05 - 0.0575}{0.0575} \times 100
\]
\[
= -13.04\% = 13.04\% (slow)
\]

04. Ans: (c)
Sol: \( W = \frac{E_1}{\sqrt{2}} \times \frac{I_1}{\sqrt{2}} \cos \phi_1 + \frac{E_3}{\sqrt{2}} \times \frac{I_3}{\sqrt{2}} \cos \phi_3 \)
\[
W = \frac{1}{2} \left[ E_1 I_1 \cos \phi_1 + E_3 I_3 \cos \phi_3 \right]
\]

05. Ans: (c)
Sol: \( V = 220 \text{ V}, \Delta = 85^\circ, I = 5 \text{ A} \)
Error = VI [\sin(\Delta - \phi) - \cos \phi]
(1) \( \cos \phi = UPF, \phi = 0^\circ \)
Error = \( 220 \times 5 \sin(85 - 0) - \cos 0 \)  
\[ = - 4.185 \text{ W} = - 4.12 \text{ W} \]

(2) \( \cos \phi = 0.5 \text{ lag}, \phi = 60^\circ \)
Error = \( 220 \times 5 \sin(85 - 60) - \cos 60 \)  
\[ = - 85.12 \text{ W} \]

\[ \begin{align*}
06. \text{ Ans: (c)} \\
\text{Sol:}
\end{align*} \]

Based on R-Y-B
Assume abc phase sequence
\( V_{ab} = 400 \angle 0^\circ \); \( V_{bc} = 400 \angle -120^\circ \)
\( V_{ca} = 400 \angle -240^\circ \) or \( 400 \angle 120^\circ \)

Current coil current (I\(_c\)) = \( \frac{V_{ca}}{Z_2} \)
\[ = \frac{400 \angle 120^\circ}{100 \Omega} = 4 \angle 120^\circ \]

Potential coil voltage (V\(_{bc}\)) = \( 400 \angle -120^\circ \)
\( W = 400 \times 4 \times \cos(240) = - 800 \text{ W} \)

\[ \begin{align*}
07. \text{ Ans: (d)} \\
\text{Sol:} \quad V_L = 400 \text{ V}, \quad I_L = 10 \text{ A} \\
\cos \phi = 0.866 \text{ lag}, \phi = 30^\circ 
\end{align*} \]

\[ \begin{align*}
08. \text{ Ans: } W &= 519.61 \text{ VAR} \\
\text{Sol:}
\end{align*} \]

\[ \begin{align*}
W_1 &= V_L I_L \cos(30 - \phi) \\
W_2 &= V_L I_L \cos(30 + \phi) \\
W_1 &= 400 \times 10 \times \cos(30 - 30) = 4000 \text{ W} \\
W_2 &= 400 \times 10 \times \cos(30 + 30) = 2000 \text{ W}
\end{align*} \]

\[ \begin{align*}
09. \text{ Ans: 0} & \text{ & 1000 W} \\
\text{Sol:}
\end{align*} \]

Y-phase is made common.
Hence wattmeter readings are
\( W_1 = V_L I_L \cos(30 + \phi) \)
\( W_2 = V_L I_L \cos(30 - \phi) \)
In star-connection
\( I_L = I_{ph}; \quad V_{ph} = \frac{V_L}{\sqrt{3}} \)
\[ I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}} \]
\[ I_L = I_{ph} = \frac{(100/\sqrt{3})}{5} = \frac{20}{\sqrt{3}} = 11.54 A \]

\[ V_L = 100 V, I_L = 11.54 A, \phi = 60^\circ \]
\[ W_1 = 100 \times 11.54 \times \cos(30 + 60) = 0 W \]
\[ W_2 = 100 \times 11.54 \times \cos(30 - 60) = 999.393 W \approx 1000 W \]
\[ W_1 = 0 W, W_2 = 1000 W \]

10. Ans: \(-596.46 W\)

Sol:

\[ V_L = 415 V, V_{BN} = \frac{415}{\sqrt{3}} V \]

Current coil is connected in ‘R phase’, it reads ‘\(I_R\)’ current.

Potential coil reads phase voltage i.e., \(\vec{V}_{BN}\)

\[ W = \vec{V}_{BN} \times I_R \times \cos(\vec{V}_{BN} \cdot I_R) \]

\[ I_R = \frac{V_{RV}}{Z} = \frac{415}{100} = 4.15 A \]
\[ \cos \phi = 0.8 \]

\[ \Rightarrow \phi = 36.86^\circ \text{ between } \vec{V}_{RV} \text{ & } I_R \]
\[ \theta = 36.86^\circ - 30^\circ = 6.86^\circ \]

Now angle between \(\vec{V}_{BN}\) and \(I_R\)
\[ = 120 + 6.86 = 126.86^\circ \]
\[ W = \frac{415}{\sqrt{3}} \times 4.15 \times \cos(126.86) \]
\[ \approx 596.467 W \]

11. Ans: (c) 

Sol:

\[ \text{Meter constant} = 14.4 \text{ A-sec/rev} \]
\[ = 14.4 \times 250 \text{ W-sec/rev} \]
\[ = \frac{14.4 \times 250}{1000} \text{ kW - sec/rev} \]
\[ = \frac{14.4 \times 250}{1000} \times 3600 \text{ kWh/rev} \]

\[ \text{Meter constant} = \frac{1}{1000} \text{ kwh/rev} \]

\[ \text{Meter constant in terms of rev/kWhr} = 1000 \]

12. Ans: (d) 

Sol:

\(R_p = 1000 \Omega, L_p = 0.5 \text{ H, } f = 50 \text{ Hz,}\)
\[ \cos \phi = 0.7, \]
\[ X_{L_p} = 2 \times \pi \times f \times L, \tan \phi = 1 \]
\[ = 2 \times \pi \times 50 \times 0.5 \]
\[ = 157 \Omega \]

\[ \% \text{ Error} = \pm (\tan \phi \tan \beta) \times 100 \]
\[ = \pm \left(1 \times \frac{157}{1000}\right) \times 100 \]
\[ = 15.7\% \approx 16\% \]
13. Ans: (d)  
Sol:  
\[ P = W_1 + W_2 + W_3 = 1732.05 \]  
Power factor, \( \cos \phi = \frac{1732.05}{3464} = 0.5 \text{ lag} \)  
\[ \sqrt{3} \times 400 \times I_L \times 0.5 = 1732.05 \]  
\[ I_L = \frac{1732.05}{\sqrt{3} \times 400 \times 0.5} = 5 \text{ A} \]  
When switch is in position N  
\[ W_1 = W_2 = W_3 = 577.35 \text{ W} \Rightarrow \text{balanced load} \]  
\( \therefore \) total power consumed by load is  
\[ W = W_1 + W_2 + \omega_3 \]  
\[ W = 1732.05 \text{ W} \]  
Given load is inductive  
And VA draw from source = 3464 VA  
\( \therefore \) power factor  
\[ = \frac{W}{VA} \]  
\[ = \frac{1732.05}{3464} = 0.5 \text{ lag} \]  
\( \Rightarrow \) Power factor angle = \(-60^\circ\) (\(\vdots\) lag)  
When switch is connected in \(Y\) position pressure coil of \(W_2\) is shorted  
So \(W_2 = 0\) and phasor diagrams for other two are as follows  
\[ W_1 = V_{RY} I_R \cos( \text{angle between } \nabla_{RY} \text{ and } I_R ) \]  
\[ = 400 \times 5 \times \cos(90^\circ) = 0 \text{ W} \]  
\[ W_3 = V_{BY} I_R \cos( \text{angle between } \nabla_{BY} \text{ and } I_R ) \]  
\[ = 400 \times 5 \times \cos(30^\circ) \]  
\[ = 400 \times 5 \times \frac{\sqrt{3}}{2} = 1732 \text{ W} \]  
\( W_1 = 0 \), \( W_2 = 0 \), \( W_3 = 1732 \text{ W} \)  

14. Ans: (c)  
Sol:  
Energy recorded (kWhr)  
\[ = \frac{5 \text{ rev}}{1200 \text{ rev/kwhr}} = 4.1667 \times 10^{-3} \text{ kwhr} \]  
Energy = 4.1667 Whr  
Load power = \(\frac{4.1667 \text{ Whr}}{75 \text{ sec}} = \frac{4.1667 \text{ Whr}}{3600 \text{ hr}}\)  
Load power = 200 W  

15. Ans: (d)  
Sol:  
Energy recorded (measured value)  
\[ = \frac{51 \text{ rev}}{360 \text{ rev/kwhr}} = 0.141667 \text{ kwhr} \]  
Energy consumed (True value)  
\[ = \frac{10 \text{ kw} \times 50}{3600} = 0.13889 \text{ kwhr} \]  
Error  
\[ = \frac{0.141667 - 0.13889}{0.13889} \times 100 \]  
\[ = 1.999\% \approx +2\% \]
Conventional Practice Solutions

01. Sol: Power calculated by freshman

\[ P = I^2 R = (30.4)^2 \times 0.0105 \]
\[ = 9.70368 \text{ W} \]

True value of current (\(I_T\)) =

\[ 30.4A + \left( \frac{1.2}{100} \times 30.4A \right) = 30.77 \text{ A} \]

True value of resistance (\(R_T\)) =

\[ 0.0105\Omega + \left( \frac{0.3}{100} \times 0.0105\Omega \right) = 0.010532\Omega \]

True power (\(P_T\)) = (30.77)^2 \times 0.010532\Omega
\[ = 9.971\text{ W} \]

\[ \text{True value of power calculated by freshman} \times 100 \]
\[ \frac{9.971\text{ W}}{9.70368} \times 100 \]
\[ = 102.75\% \]

02. Sol:

\[ V = 240\text{ V} \]
\[ 50 \text{ Hz AC} \]
\[ 1-\phi, 100V_{\text{rms}} \]

\[ I_{dc} = I_{cc} = \frac{24V}{5\Omega} = 4.8\text{ A} \]

\[ V_{\text{rms}} = 100\text{ V}, V_{\text{max}} = 100 \times \sqrt{2} \]

Wattmeter reading (W)
\[ W = \frac{1}{2\pi} \int (V_{pc} \times I_{dc}) \, dt \]
\[ = \frac{1}{2\pi} \left[ \int (100\sqrt{2} \sin \omega t \times 4.8) \, dt + \int (0 \times 4.8) \, dt \right] \]
\[ = \frac{4.8 \times 100\sqrt{2}}{2\pi} \left[ -\cos \omega t \right]_0^\pi \]
\[ = 216.07 \text{ W} \]

03. Sol: \(W_m = (1 + \tan \phi \tan \beta) W_T\)

700W = (1 + tan \phi tan 2) W_T ------- (1)

620W = (1 + tan \phi tan 1) W_T ------- (2)

\[ \frac{(1)}{(35)} = \frac{(1 + \tan \phi \times 0.034)}{31} \]
\[ \Rightarrow (1 + \tan \phi \times 0.0174) 35 \]
\[ = (1 + \tan \phi \times 0.03492) 31 \]
\[ \Rightarrow 35 + 35 (\tan \phi \times 0.03942) \]
\[ = 31 + 31 \tan \phi \times 0.03492 \]
\[ \Rightarrow 35 - 31 = 1.0852 \tan \phi - 0.609 \tan \phi \]
\[ \Rightarrow 4 = 0.4762 \tan \phi \]
\[ \Rightarrow \phi = 83.2108 \]
\[ \cos \phi = 0.1182 \]

\[ W_T = \frac{700}{(1 + \tan 83.2108 \times \tan 2)} \]
\[ = 541.2407 \]
\[ V = 240\text{ V} \]
\[ P = V I \cos \phi \]
\[ \Rightarrow I = \frac{P}{V \cos \phi} = \frac{541.2407}{240 \times 0.1182} = 19.079 \text{ A} \]
04.
Sol: Consider RYB phase sequence

\[ V_R = \frac{440V}{\sqrt{3}} \angle 0^\circ = 254.03 \angle 0^\circ V \]
\[ V_Y = 254.03 \angle -120^\circ V \]
\[ V_B = 254.03 \angle -240^\circ V \]

\[ \Rightarrow I_R = \frac{V_R}{10\Omega} = \frac{254.03 \angle 0^\circ}{10\Omega} = 25.403 \angle 0^\circ A \]
\[ I_Y = \frac{V_Y}{15\Omega} = \frac{254.03 \angle -120^\circ}{15\Omega} = 16.935 \angle -120^\circ A \]
\[ I_B = \frac{V_B}{20\Omega} = \frac{254.03 \angle -240^\circ}{20\Omega} = 12.701 \angle -240^\circ A \]

According to connections

\[ W = P.C \text{ voltage } \times C.C \text{ current } \times \text{ cost of angle between (P.C Voltage & CC current)} \]

\[ W_1 = V_{RY} \times I_R \cos (V_{RY} \& I_R) \]
\[ W_1 = 440V \times 25.403 \times \cos 30 \]
\[ = 9679.84 W \]

\[ W_2 = V_{BY} \times \cos (V_{BY} \& I_B) \times I_B \]
\[ = 440V \times 12.701 \times \cos 30 \]
\[ = 4839.92 W \]

3φ Total P = W_1 + W_2
\[ = 14.519 kW \]

05.
Sol: (i) \( P_1 = 5000W, P_2 = -1000W \)

Total power \( (P_T) = P_1 + P_2 \Rightarrow \)
\[ 5000 -1000 = 4000W \]

Power Factor Angle \( (\phi) \)
\[ = \tan^{-1} \left[ \sqrt{3} \frac{P_1 - P_2}{P_1 + P_2} \right] \]
\[ \Rightarrow \tan^{-1} \left[ \sqrt{3} \frac{5000 - (-1000)}{5000 - 1000} \right] \]
\[ \Rightarrow \tan^{-1} \left[ \frac{3 \sqrt{3}}{2} \right] \phi \]
\[ = 68.94^\circ \]

\[ \therefore \text{ Power Factor } \cos \phi = \cos (68.94^\circ) \]
\[ \Rightarrow 0.3593 \]
\[ \approx 0.36 \text{ Lag} \]

(ii) Ans: Power consumed By each phase
\[ \frac{P_{Total}}{3} = \frac{4000}{3} = 1333.33W \]

In \( \Delta \) connected system, voltage of each phase
\[ V_{ph} = V_L = 440V \]

\[ = \text{ supply voltage} \]

\[ \rightarrow \text{ current in each phase} \]
\[ = \frac{1333.33}{440 \times 0.36} = 8.41748 \text{ Amp} \]

\[ \rightarrow \text{ Impedance of each phase} \]
\[ = \frac{440}{8.41748} = 52.27217 \Omega \]

\[ \rightarrow \text{ Resistance of each phase} \]
\[ = \frac{1333.33}{(8.41748)^2} = 18.818 \Omega \]

\[ \rightarrow \text{ Reactance (X) of each phase} \]
\[ = \sqrt{(52.27217)^2 - (18.818)^2} = 48.7674 \Omega \]
In order that one of the wattmeter’s should read zero, the power factor should be 0.5
\[ \cos \phi = 0.5, \quad \tan \phi = 1.73 \]
\[ \therefore \text{Reactance of circuit } X = RTan \phi \]
\[ X = 18.818 \times 1.73 \Rightarrow X = 32.55514 \Omega \]
\[ \therefore \text{Capacitive Reactance Required} \]
\[ = 48.7674 - 32.55514 \]
\[ = 16.21226 \]
\[ \therefore \text{Capacitive Reactance} \]
\[ = \frac{1}{2\pi \times 50 \times 16.21226} \]
\[ C = 196.33 \mu F \]

06.

**Sol:** Load power = 230 \times 5 \times 0.1 = 115W

Load power factor \( \cos \phi = 0.1 \)

\[ \therefore \phi = 84.26^\circ \text{ and } \sin \phi = 0.995 \text{ and } \tan \phi = 9.95 \]

\[ \Rightarrow \text{Resistance of pressure coil circuit} \]
\[ R_p = 10000 \Omega \]

\[ \Rightarrow \text{Reactance of pressure coil circuit} \]
\[ = 2\pi \times 50 \times 100 \times 10^{-3} = 31.416 \Omega \]

As the phase Angle of pressure coil circuit is small,

\[ \beta = \tan \beta \approx \frac{31.416}{10000} = 3.1416 \times 10^{-3} \text{ rad} \]

\[ = 0.0031416 \text{ rad} \]

\[ \Rightarrow \text{When the pressure coil is connected on the load side the wattmeter measures} \]

power loss in pressure coil circuit in addition to load power

True power = VIcos\( \phi \) = 230 \times 5 \times 0.1

\[ = 115W \]

Let us consider only the effect of Inductance

Reading of wattmeter = true power
\[ [1 + \tan \phi \tan \beta] \]
\[ = 115 \times [1 + 9.95 \times 0.00314] = 118.593W \]

\[ \Rightarrow \text{Power loss in pressure coil circuit} \]
\[ \Rightarrow \frac{V^2}{R_p} = \frac{230^2}{10000} = 5.29 \text{ W} \]

\[ \therefore \text{Reading of wattmeter considering} \]

The power loss in pressure coil circuit

\[ = 118.593 + 5.29 = 123.883 \]

\[ \text{Percentage error} = \frac{123.883 - 115}{115} \times 100 \]

\[ = 7.724\% \]

07.

**Sol:**

\[ E_{RY} = 400 \angle 0^\circ, \quad E_{RB} = 400 \angle -120^\circ, \]

\[ E_{BR} = 400 \angle 120^\circ, \]

\[ I_{gy} = \frac{400}{30 - j40} = 400 \angle 50^\circ = 8 \angle 53^\circ = 4.8 + j 6.4 \]

\[ I_{RB} = \frac{400 \angle 120^\circ}{40} \]

\[ \Rightarrow 10 \angle -60^\circ = I_{ML} \]

[\text{wattmeter current coil current}]

For the wattmeter pressure coil circuit

\[ 20 I_{RB} + V_1 V_2 - (\text{j}40 I_{RY}) = 0 \]

(i.e) \[ 0 = 200 \angle -60^\circ + 40 \angle 90^\circ \times 8 \angle 53^\circ + V_1 V_2 \]

\[ \Rightarrow -V_1 V_2 = 100 - j173 + 320 \angle 143 \]

\[ = 100 - j173 - 255.6 + j195.2 \]

\[ \Rightarrow -V_1 V_2 = -155.6 + j20 \]

\[ V_1 V_2 = 155.6 - j20 = 156.9 \angle -7.3^\circ \]

\[ \therefore \text{the reading of wattmeter} = I_{ML} \times V_1 V_2 \]

\[ = 10 \times 156.9 \times 10^{-3} \cos (52.7^\circ) \text{ kW} = 0.94 \text{ kW} \]

08.

**Sol:** CBA → Phase sequence same as ACB phase sequence
→ supply Line voltage = 230 V

\[ V_{AC} \angle 0^\circ = V_L \angle 0^\circ = 230 \angle 0^\circ \]

\[ V_{CB} \angle -120^\circ = V_L \angle 120^\circ = 230 \angle -120^\circ \]

\[ V_{BA} \angle -240^\circ = V_L \angle 240^\circ = 230 \angle -240^\circ \]

Phase current \( (i_b) = \frac{V_{AC} \angle 0^\circ}{20 \angle 0^\circ} = 20 \angle 0^\circ \)
\[ = 11.5 \angle 0^\circ \text{ A} \]

\[ i_b = \frac{V_{BA} \angle 240^\circ}{25 \angle 90^\circ} = 9.2 \angle 330^\circ \text{ A} \]

\[ i_c = \frac{V_{CB} \angle 120^\circ}{15 \angle 30^\circ} = 15.33 \angle 150^\circ \text{ A} \]

Line current \( (I_A) = i_b - i_c \)
\[ = [11.5 \angle 0^\circ - 9.2 \angle 330^\circ] \]

\[ (I_A) = 11.5 - 9.2 \left( \frac{\sqrt{3}}{2} + \frac{j}{2} \right) \]

\[ (I_A) = 11.5 - 7.96743 - j4.6 \]

\[ (I_A) = 3.53257 - j4.6 = 5.8 \angle -52.47^\circ \]

\[ W_1 = V_{PC} I_{CC} \cos (\angle V_{PC} & I_{CC}) \]
\[ = V_{AC} \times I_A \cos (\angle V_{AC} & I_A) \]
\[ = 230 \times 5.8 \times \cos [52.47^\circ] \]
\[ = 812.512 \text{ W} \]

For \( W_2 \) Reading: \[ I_B = i_b - i_c \]
\[ = 9.2 \angle 150^\circ - 15.33 \angle 150^\circ \]

\[ I_B = 9.2 \left( \frac{\sqrt{3}}{2} + \frac{j}{2} \right) - 15.33 \left( \frac{-\sqrt{3}}{2} - \frac{j}{2} \right) \]
\[ = 21.2436 + j12.265 \]

\[ I_B = 24.53 \angle 30^\circ \]

\[ V_{BC} = V_{CB} \angle 0^\circ + 180^\circ = 230 \angle -120^\circ + 180^\circ \]

\[ V_{PC} = V_{BC} = 230 \angle 60^\circ \]
\[ = 230 \times 24.53 \cos (60^\circ - 30^\circ) \]
\[ = 5641.9 \times 0.866 \]
\[ = 4886.6 \text{ W} \]

09. Sol:

\[ W_A = V_{RB} \times I_R \times \cos (\angle V_{RB} & I_R) \]

\[ V_{RB} = 400 \angle -120^\circ \], But \( V_{YB} \angle 0^\circ \)

\[ V_{PC} = V_{RB} = 400 \angle 120^\circ + 180^\circ \]

\[ V_{RB} = 400 \angle 60^\circ = V_{PC} \]

\[ W_A = 400 \times 3 \times \cos (60^\circ - 30^\circ) \]
\[ = 1039.23 \text{ W} \]

\[ W_B = V_{PC} \times I_{CC} \times \cos (\angle V_{PC} & I_{CC}) \]

\[ V_{PC} = V_{YB} = 400 \angle 0^\circ, I_{CC} = I_Y = 4 \angle 300^\circ \]

\[ W_B = 400 \times 4 \times \cos (-300^\circ) = 800 \text{ W} \]
10. Data not sufficient

11. Sol: Phantom loading: when the current rating of a meter under test is high, a test with actual loading assignments would involve a considerable waste of power. In order to avoid this, phantom loading is done. Phantom loading consists of supplying the pressure coil circuit from a circuit of required normal voltage, and the current coil circuit from a separate low voltage supply. It is possible to circulate the rated current through the current circuit with a low voltage supply as the impedance of this circuit is very low. With this arrangement, the total power supplied for the test is that due to small pressure coil current at normal voltage, plus that due to the current with low voltage. Therefore, power required for testing the meter with phantom loading is comparatively very small.

**Given data:**

\[ V = 230V, \quad I = 12A \]
\[ \cos\phi = 0.8\text{lag}, \quad K = 1200 \text{ rev/kWh}, \]
\[ N_1 = 1150 \text{ rev} \]

(i) Error in registration,

Actual kWh consumed

\[ = 230 \times 12 \times 0.8 \times \frac{1}{2} \times 10^{-3} \]
\[ = 1.104 \text{ kWh}. \]

⇒ Actual revolutions registered

\[ = k \times \text{kWh} \]
\[ = 1200 \times 1.104 \]
\[ N_2 = 1324.8 \text{ Revolutions} \]

\[ \text{Error registration} = \frac{N_2 - N_1}{N_2} \times 100 \]
\[ = \frac{1324.8 - 1150}{1324.8} \times 100 \]
\[ = 13.2\% \]

(ii) Revolutions per minute

\[ \frac{1324.8}{60} = 22.08 \text{ rev/min} \]

\[ \text{Revolutions registered/min} = \frac{1150}{30} \]

(half an hour measuring)

\[ = 38.33 \text{ rev/min} \]

\[ \% \text{ Error} = \frac{38.33 - 22.08}{22.08} \times 100 \]
\[ = 73.6\% \text{ (fast)} \]

**Rectification:** In energy meter, error in rpm can be rectified by bringing the braking magnet near to the centre of disc.
01. Ans: (a)  
Sol: It is Maxwell Inductance Capacitance bridge  
\[ R_x R_4 = R_2 R_3 \]  
\[ R_x = \frac{R_2 R_3}{R_4} \]  
\[ R_x = \frac{750 \times 2000}{4000} \]  
\[ R_x = 375 \, \Omega \]  
\[ L_x = R_2 R_3 \]  
\[ L_x = C_4 R_2 R_3 \]  
\[ L_x = 0.05 \times 10^{-6} \times 750 \times 2000 \]  
\[ L_x = 75 \, \text{mH} \]

02. Ans: (d)  
Sol:  
\[ V = V_+ - V_- \]  
\[ = 10 \times \frac{20}{30} - 10 \times \frac{10}{30} \]  
\[ = 6.66 - 3.33 = 3.33 \, \text{V} \]

03. Ans: (c)  
Sol: The voltage across \( R_2 \) is  
\[ = E \frac{R_2}{R_1 + R_2} = \frac{E}{2} \]  
The voltage across \( R_1 \) is  
\[ = E \frac{R_1}{R_1 + R_2} = \frac{E}{2} \]  
Now,  
\[ \frac{E}{2} = IR_3 + V \]  
\[ I = E - 2V \]  
\[ \frac{2}{2} = \frac{1}{2} R_3 \]  
\[ I = \frac{E - 2V}{2R_3} \]  
and  
\[ \frac{E}{2} = 1R_4 \]  
\[ \frac{E}{2} = \left( E - 2V \right) \left( R + \Delta R \right) \]  
\[ E = \frac{E - 2V}{2R} \]  
\[ \frac{E}{2} = \left( E - 2V \right) \left( R + \Delta R \right) \]  
\[ R + \Delta R = \frac{ER}{E - 2V} - R \]  
\[ \Delta R = \frac{ER}{E - 2V} - R \]  
\[ \Delta R = \frac{2VR}{E - 2V} \]  
\[ \Delta R = \frac{ER - ER + 2VR}{E - 2V} \]

04. Ans: (a)  
Sol: The deflection of galvanometer is directly proportional to current passing through circuit, hence inversely proportional to the total resistance of the circuit.  
Let \( S \) = standard resistance
05. Ans: (a)
Sol: Thevenin’s equivalent of circuit is

\[
\begin{align*}
R_0 &= \text{Resistance of circuit looking into terminals b & d with a & c short circuited.} \\
\frac{R}{R_0} &= \frac{1 \times 5}{1 + 5} + \frac{1 \times Q}{1 + Q} \\
&= 0.833 + \frac{Q}{1 + Q} \text{KΩ}
\end{align*}
\]

Now, \( R_0 + G = \frac{2.4 \times 10^{-3}}{13.6 \times 10^{-6}} = 1765 \text{ kΩ} \)

(or) \( R_0 = 1765 - 100 = 1665 \text{ Ω} \)

\[
Q = 4.95 \text{ kΩ}
\]

06. Ans: (c)
Sol: \[
R = \frac{0.4343 T}{C \log_{10} \left( \frac{E}{V} \right)}
\]
\[
= \frac{0.4343 \times 60}{600 \times 10^{-2} \times \log_{10} \left( \frac{250}{92} \right)}
\]
\[
= \frac{26.058}{260.49 \times 10^{-12}}
\]
\[
R = 100.03 \times 10^{9} \text{ Ω}
\]

07. Ans: 0.118 \( \mu \)F, 4.26kΩ
Sol: Given: \( R_3 = 1000 \text{ Ω} \)
\[
C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d}
\]
\[
= \frac{2.3 \times 4 \pi \times 10^{-7} \times 314 \times 10^{-4}}{0.3 \times 10^{-2}}
\]
\[
C_1 = 30.25 \text{ µF}
\]
\( \delta = 9^\circ \text{ for 50 Hz} \)
\[
\tan \delta = \frac{\omega C_1 r_1}{\omega L_4 R_4}
\]
\[
r_1 = 16.67 \text{ Ω}
\]
Variable resistor \( R_4 = R_3 \left( \frac{C_1}{C_2} \right) \)
\[
R_4 = 4.26 \text{ kΩ}, C_4 = 0.118 \text{ µF}
\]

08.
Sol: Resistance of unknown resistor required for balance
\[
R = (P/Q)S = \frac{1000}{100} \times 200 = 2000 \text{ Ω}
\]
In the actual bridge the unknown resistor has a value of 2005 Ω or the deviation from the balance conditions is \( \Delta R = 2005 - 2000 = 5 \text{ Ω} \).
Thevenin source generator emf
\[ E_0 = E \left[ \frac{R}{R+S} - \frac{P}{P+Q} \right] \]
\[ = 5 \left[ \frac{2005}{2005 + 200} - \frac{1000}{1000 + 100} \right] \]
\[ = 1.0307 \times 10^{-3} V. \]

Internal resistance of bridge looking into terminals b and d.
\[ R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q} \]
\[ = \frac{2005 \times 200}{2005 + 200} + \frac{1000 \times 100}{1000 + 100} \]
\[ = 272.8 \Omega \]

Hence the current through the galvanometer
\[ I_g = \frac{E_0}{R_0 + G} \]
\[ = \frac{1.0307 \times 10^{-3}}{272.8 + 100} \]
\[ = 2.77 \mu A. \]

Deflection of the galvanometer
\[ \theta = S_0 I_g = 10 \times 2.77 \]
\[ = 27.7 \text{ mm/}\Omega. \]

Sensitivity of bridge
\[ S_B = \frac{\theta}{\Delta R} \]
\[ = \frac{27.7}{5} = 5.54 \text{ mm/}\Omega. \]

09. Ans: (d)
Sol: \( R_1 R_4 = R_2 R_3 \)
\[ R_4 = \frac{R_2 R_3}{R_1} \]
\[ = \frac{5 \times 100}{10} \pm (2\% + 5\% + 3\%) \]
\[ = 50 \pm 10\% \Omega. \]

10. Ans: (c)
Sol: Under balanced condition, current through Galvanometer is zero, then
\[ R_{eq} = (120\Omega + 80\Omega) || (120\Omega + 80\Omega) \]
\[ = 100 \Omega \]
\[ I_B = \frac{1V}{100\Omega} = 10 \text{ mA} \]

11. Ans: (c)
Sol: Sensitivity = \frac{\text{Change in output}}{\text{Change in input}} \]
\[ = \frac{3 \text{ mm}}{6 \Omega} = 0.5 \text{ mm/}\Omega \]

12. Ans: (d)
Sol: Changing the arms resistance by same value will not affect the Balance condition.

13. Ans: (a)
Sol: \[ V_1 = \sqrt{2} \cos(1000t) \text{ V} \]
\[ V_2 = 2 \cos(1000t + 45^\circ) \text{ V} \]

Under balanced condition,
\[ V_1 = I_2 R \]
\[ I_2 = \frac{V_1}{R} = \sqrt{2} \cos 1000t \]
\[ I_2 = 10^{-2} \times \sqrt{2} \cos(1000t) \]
\[ V_2 = 2 \cos(1000t + 45^\circ) \text{ At } Z_x \]
‘I₂’ lags ‘V₂’ by 45°. So, Zx has ‘R’ and ‘L’ in series.

\[ R = Z \cos \theta = \frac{2}{10^{-2} \times \sqrt{2}} \cos 45° = 100 \Omega \]

\[ X_L = Z \sin \theta = \frac{2}{\sqrt{2} \times 10^{-2}} \sin 45° = 100 \Omega \]

\[ X_L = \omega L \]

\[ L = \frac{X_L}{\omega} = \frac{100}{1000} = 0.1 \text{ H} = 100 \text{ mH} \]

14. Ans: (b)

Sol:

[The circuit diagram is shown below.]

\[ Z_1 = 100 \text{ nF} \]

\[ Z_2 = 10k\Omega \]

\[ Z_3 = 500\Omega \]

\[ Z_4 = 1k\Omega \]

\[ R = 10\cos314t \]

\[ R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC} \]

\[ \left( \frac{R}{1 + j\omega RC} \right) Z = 1k\Omega \times 500\Omega \]

\[ \left( \frac{10k\Omega}{1 + j\omega \times 10k\Omega \times 10^9} \right) Z = 1k\Omega \times 500\Omega \]

\[ \frac{10^4 \times Z}{1 + j\omega \times 10^9} = 50 \times 10^4 \]

\[ Z = 50 + j0 \times 0.05 \]

\[ Z = R + j \omega L \]

\[ R = 50 \Omega \text{ in series with } L = 0.05 \text{ H} \]

Conventional Practice Solutions

01.

Sol: The circuit diagram is shown below.

\[ Z_1 = 303.3 \angle -72.4° \]

\[ Z_3 = 1049.5 \angle -17.7° \]

\[ Z_4 = 274.8 \angle 43.3° \]

Using balance conditions,

\[ Z_1 Z_4 = Z_2 Z_3 \quad \rightarrow (1) \]

\[ \theta_1 + \theta_4 = \theta_2 + \theta_3 \quad \rightarrow (2) \]

\[ Z_2 = 79.4 \text{ (from (1))} \]

\[ \theta_2 = -11.4° \quad \text{(from (2))} \]

\[ \therefore Z_2 = 79.4 \angle -11.4° = 77.8 - j15.7 \]

\[ R = 77.8 \]

\[ X_C = 15.7 \]

\[ C = 10.2 \mu F \]

So, resistance of value 77.8 Ω in series with C of value 10.2 μF

02.

Sol: Given data: \( f = 50 \text{ Hz}, R_3 = 300\Omega, \)

\[ C_2 = 500 \mu F, R_4 = 72.6 \Omega, C_4 = 0.148 \mu F \]

\[ f = 50\text{Hz} \]

\[ Z_1Z_4 = Z_2Z_3 \]

\[ Z_1 = Z_2Z_3Y_4 \]
If we consider in bushing $R_1$ in series

$$\frac{R_1 R_4 - j R_4}{\omega C_1} = \frac{j R_3}{\omega C_2} + \frac{R_3 R_4 C_4}{C_2}$$

$$R_1 = \frac{R_4 C_4}{C_2}, \quad C_1 = C_2 \frac{R_4}{R_3}$$

Loss angle $\tan \delta = \omega C_1 R_1$

$$= \frac{\omega R_2 C_4}{C_2} \times \frac{R_4}{R_3}$$

$$= \omega C_4 R_4$$

$$= 2\pi \times 50 \times 72.6 \times 10^{-6} \times 0.148$$

$$\delta = 0.1934^{\circ}$$

03.

Sol: We know that $\varepsilon_r = \frac{c_1 d}{\varepsilon_0 A}$

$$\Rightarrow \frac{\varepsilon_r \varepsilon_0 A}{d} = c_1$$

$$\Rightarrow \frac{2.3 \times 8.854 \times 10^{-12} \times 314 \times 10^{-4}}{0.3 \times 10^{-2}} = c_1$$

$$\therefore c_1 = 213.145 \text{pF}$$

$$r_1 = \frac{c_4}{c_2} \frac{R_3}{c_2}$$

$$R_4 = \frac{c_1}{c_2} \frac{R_3}{c_2} = \frac{213.145 \text{pF}}{50 \text{pF}} \times 1000 = 4.269 \text{k}\Omega$$

$$\tan\delta = 9^\circ$$

$$\delta = \tan^{-1}(9^\circ) = 1.46^\circ$$

$$\tan \delta = \omega c_4 R_4$$

$$0.1583 = \omega c_4 R_4$$

$$c_4 = \frac{0.1583}{2 \times \pi \times f \times R_4} = 0.1180 \mu\text{F}$$

$$r_1 = \frac{c_4}{c_2} \frac{R_3}{c_2} = \frac{0.1180 \mu\text{F}}{50 \text{pF}} \times 1000$$

$$= 2.36 \text{M}\Omega$$

04.

Sol: Given

$$R_1 = 800 \Omega$$

$$C_1 = 0.4 \mu\text{F}$$

$$R_4 = 1200 \Omega$$

$$R_2 = 500 \Omega$$

$$C_2 = 1 \text{pF}$$
This is Wien’s bridge under balanced condition, \( f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}} \) Hz

\[
R_4 = \frac{R_2 + C_1}{R_1 + C_2}
\]

\[
\Rightarrow f = \frac{1}{2\pi \sqrt{800 \times 500 \times 0.4 \times 10^{-18}}} \text{ Mz}
\]

\( f = 397.9 \text{ kHz} \)

Also, \( \frac{1200}{R_3} = \frac{500}{800} + \frac{0.4}{10^{-6}} \)

\( \Rightarrow R_3 = 3 \, \text{m\Omega} \)

**05.**

**Sol: Hay Bridge:**

At balance condition, \( Z_1 Z_4 = Z_2 Z_3 \) and phase angles are also equal.

Here, phase null condition means phase angles are equal.

\[
\theta_1 = \tan^{-1} \left( \frac{\omega L_1}{R_1} \right)
\]

(Or)

\[
\tan \theta_i = \frac{\omega L_1}{R_1} = \text{Quality factor}
\]

\[
\tan \theta_c = \tan \theta_3 = \frac{1}{\omega C_3 R_3}
\]

\( \theta_1 = \theta_3 \) (because \( \theta_3 \) is negative

\[
\frac{\omega L_1}{R_1} = \frac{1}{\omega C_3 R_3}
\]

\( Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_3 R_3} \)

Here \( Q = \frac{1}{\omega C_3 R_3} \)

Take ‘I’ is the current flowing in \( R_3 \) and ‘\( C_3 \)’.

So, \( Q = \frac{1}{\omega C_3 (IR_3)} \)

\( \Rightarrow \frac{1}{\omega C_3} \times \frac{1}{IR_3} \)

The potential drop in \( C_3 = \frac{1}{\omega C_3} = V_c \)

\( \Rightarrow \) Voltage across ‘\( C_3 \)’

\( IR_3 = \) voltage across resistor ‘\( R_3 \)’

\( Q = \frac{\text{Voltage across } C_3}{\text{Voltage across } R_3} \)
Constructional features of moving iron synchroscope:
- Figure above shows the construction of a moving iron synchroscope, which is due to Lip man.

Principle of working:
- When the Frequency of incoming machine is the same as that of the bus bars, the instrument behaves exactly like the corresponding form of the power factor meter.
- The deflection of the pointer from the plane of reference is equal to phase difference between the two voltages.
• However, if the frequencies of the two voltages are different, the pointer rotates continuously at a speed corresponding to difference in frequency of the two voltages.
• The direction of rotation depends whether the incoming machine is too fast or too slow.
• Let \( v_1 \) and \( v_2 \) be the voltages of bus bar and incoming machine respectively.
• Let the frequencies of the two voltages be equal. Torques produced by coils \( P_1 \) and \( P_2 \) are:

\[
T_1 = kv_1v_2\sin\theta \cos(\pm \alpha)
\]

\[
T_2 = kv_1v_2\sin(90^\circ-\theta) \cos(90^\circ\pm \alpha)
= kv_1v_2\cos\theta \sin(\pm \alpha)
\]

Where
\( \theta \) = deflection of the pointer from plane of reference. This plane of reference is the vertical position of the pointer.
\( \alpha \) = phase angle between the two voltages.
• The two torques are in opposite direction, therefore, under equilibrium

\[
T_1 = T_2
\]

\[
kv_1v_2\sin\theta \cos(\pm \alpha) = kv_1v_2\cos\theta \sin(\pm \alpha)
\]

(or) \( \theta = \pm \alpha \)

Thus the pointer is stationary and its deflection from plane of reference equals the phase difference between the two voltages.
• If the frequency of the incoming machine differs from that of the bus bars, the torques are

\[
T_1 = kv_1v_2\sin\theta \cos(\pm \pi f'\pm \alpha)
\]

And

\[
T_2 = kv_1v_2\sin(90^\circ-\cos(90^\circ-(\pm 2\pi f'\pm \alpha))
= kv_1v_2\cos\theta \sin(\pm 2\pi f'\pm \alpha)
\]

Where
\( f' = f_2 - f_1 \) = difference in two frequencies.
Hence at equilibrium

\[
\sin \theta \cos (\pm 2\pi f'\pm \alpha) = \cos \theta \sin (\pm 2\pi f'\pm \alpha)
\]

(or)

\[
\theta = 2\pi f'\pm \alpha
\]

Thus, the moving system rotates with a frequency corresponding to difference in the two frequencies.
• The direction of rotation depends upon whether the frequency of incoming machine is higher (\( f' \) positive) or lower (\( f' \) negative) than the frequency at the bus bars.
• The instant of synchronizing is when the pointer is stationary (\( f' = 0 \)) and when it is at its vertical position.
• Moving iron synchroscopes are more common in use. They are cheap and their operation is simple and also they have a 360° scale.

02. Sol:
Construction and working principle of a single phase electro-dynamic power factor meter:

• The construction of a single phase electrodynamometer type power factor is shown in below figure. It consists of a fixed coil, which acts as the current coil.
This coil is split up into two parts and carries the current of the circuit under test. Therefore, the magnetic field produced by this coil is proportional to the main current.

Two identical pressure coils A and B pivoted on a spindle constitute the moving system.

Pressure coil A has a non-inductive resistance R connected in series with it, and coil B has highly inductive choke coil L connected in series with it.

The two coils are connected across the voltage of the circuit. The values of R and L are so adjusted that the two coils carry the same value of current at normal frequency, i.e. \( R = \omega L \).

The current through coil A is in phase with the circuit voltage, while that through coils B lags the voltage by an angle \( \Delta \) which is nearly equal to \( 90^0 \).

The angle between the planes of coils is also made equal to \( \Delta \). There is no controlling device. Connections to moving coils are made through thin silver or gold ligaments, which are extremely flexible and thus, give a minimum control effect on the moving system.

In order to simplify the problem, we assume that the current through coil B lags the voltage by exactly \( 90^0 \). Also that the angle between planes of coils is exactly \( 90^0 \) (i.e. \( \Delta = 90^0 \)).

Now, there will be two deflecting torques, one acting on coil A and the other on coil B. The coil windings are so arranged that the torques due to the two coils are opposite in direction.

Therefore, the pointer will take up a position where these two torques are equal.

Let us consider the case of a lagging power factor of \( \cos \phi \).

**Deflecting torque acting on coil A is:**

\[
T_A = KVIM_{\text{max}} \cos \phi \sin \theta
\]

\( \theta \rightarrow \) Angular deflection from the plane of reference,

\( M_{\text{max}} \rightarrow \) maximum value of mutual inductance between the two coils

This torque acts in the clockwise direction.

**Deflecting torque acting on coil B is:**

\[
T_B = KVIM_{\text{max}} \cos (90^0 - \phi) \sin (90^0 + \theta)
\]

\[= KVIM_{\text{max}} \sin \phi \cos \theta\]
• This torque acts in the anticlockwise direction. The value of $M_{\text{max}}$ is the same in the two expressions, owing to similar constructions of the coils.

• The coils will take up such a position that the two torques are equal. Hence at equilibrium $T_A = T_B$

\[ KVIM_{\text{max}} \cos \phi \sin \theta = KVIM_{\text{max}} \sin \phi \cos \theta \]

\[ \theta = \phi \]

\[ \therefore \text{The deflection of the instrument is a measure of phase angle of the circuit. The scale of the instrument can be calibrated directly in terms of power factor.} \]

**Comparison with Moving Iron type power factor Meter:**

1. The working forces are very large in moving iron type power factor meter, where as in electrodynamometer type, working forces are very small
2. All coils in a moving iron instrument are fixed and therefore, the use of ligaments is eliminated.
3. The scale in moving iron instrument extends up to $360^\circ$
4. Moving iron instruments are simple and robust in construction, and also comparatively cheap
5. These instruments are less accurate than electrodynamometer type.
01. Ans: (d)  
Sol: Under null balanced condition the current flow in through unknown source is zero. Therefore the power consumed in the circuit is ideally zero.

02. Ans: (d)  
Sol: Potentiometer is used for measurement of low resistance, current and calibration of ammeter.

03. Ans: (a)  
Sol: Since the instrument is a standardized with an emf of 1.018 V with sliding contact at 101.8 cm, it is obvious that a length 101.8 cm represents a voltage of 1.018. 
Resistance of 101.8 cm length of wire = \((101.8/200) \times 400\) = 203.6 \(\Omega\)  
\[\therefore\] Working current \(I_w\) = 1.018/203.6 = 0.005 A  
= 5 mA  
Total resistance of the battery circuit  
= resistance of rheostat  
+ resistance of slide wire  
\[\therefore\] Resistance of rheostat \(R_h\) = total resistance  
− resistance of slide wire  
\[\frac{3}{5 \times 10^{-3}} - 400\]  
= 600 – 400 = 200\(\Omega\)  

04. Ans: (b)  
Sol: Voltage drop per unit length  
\[= \frac{1.45 V}{50cm} = 0.029 V/cm\]  
Voltage drop across 75 cm length  
\[= 0.029 \times 75 = 2.175 V\]  
Current through resistor (I)  
\[= \frac{2.175 V}{0.1 \Omega} = 21.75 A \quad \text{(or)}\]  
75 cm \(\rightarrow\) 0.1 \(\Omega\)  
50 cm \(\rightarrow\)?  
Slide wire resistance with standard cell  
\[= \frac{50}{70} \times 0.1 = 0.067 \Omega\]  
Then 0.067 \(\times\) \(I_w\) = 1.45 V  
\[I_w = \frac{1.45}{0.067} = 21.75 A\]  

05. Ans: (a)  
Sol: Under balanced, \(I_g = 0\)  
\[E_x \times \frac{200}{(200 + 200 + 2800)} = 0.2 V\]  
\[E_x = 200 mV\]
06. Ans: (a)
Sol:

\[ V_h = 2 \text{ V} \]

\[ R_n \]

\[ I_w \]

Resistance 1 \( \Omega /\text{cm} \)

For 11 m \( \rightarrow 11 \) \( \Omega \)

For 10m + 18cm \( \rightarrow 10.8 \) \( \Omega \)

\[ I_w \times 10.8 \Omega = 1.018 \text{ V} \]

\[ I_w = \frac{V_n}{R_n + I_i} \]

\[ \Rightarrow 0.1 = \frac{2}{R_n + 11 \Omega} \]

\[ R_n = \frac{2}{0.1} - 11 \]

\[ = 9 \) \( \Omega \]

07. Ans: (a)
Sol: It is closed loop inverting amplifier

\[ V_h = -\frac{R_f}{R_i} V_{in} \]

\[ = -\frac{15 \text{ k}\Omega}{10 \text{ k}\Omega} \times 1 \text{ V} \]

\[ = -1.5 \text{ V} \]

08. Ans: (a)
Sol:

Dial resistor has 15 steps and each step is

10 \( \Omega = 15 \times 10 \Omega = 150 \Omega \)

Slide wire resistance = 10 \( \Omega \)

Total resistance = 150+10 = 160 \( \Omega \)

Working current (Iw) = 10 mA

Range of potentiometer

10mA \times 160 \Omega = 1.6 \text{ V} \)

Resolution of potentiometer

\[ = \frac{\text{working current} \times \text{slide wire resistance}}{\text{slide wire length}} \]

\[ = \frac{10 \text{ mA} \times 10 \Omega}{100 \text{ cm}} \]

\[ = 0.001 \text{ V/cm} \]

(1 div = 1 cm)

One fifth of a division can be read certainly.

Resolution \[ = \frac{1}{5} \times 0.001 = 0.2 \text{ mV} \]
09. Ans: (d)
Sol:

Write KVL for loop 1
\[ 2V - 900I_w - 900(I_w - 0.2mA) = 0 \]
\[ I_w = 1.211mA \]

Write KVL for loop 2
\[ V_x + 0.2mA \cdot R_g - 900(I_w - 0.2mA) = 0 \]
\[ V_x + 0.2 \times 10^{-3} R_g - 900(1.211 \times 10^{-3} - 0.2 \times 10^{-3}) = 0 \]
\[ V_x = 0.909 - 0.2 \times 10^{-3} R_g \] (1)

When \( V_x \) is reversed, the circuit is

Write KVL for loop (1)
\[ 2V - 900I_w - 900(I_w - 3.8mA) = 0 \]
\[ I_w = 3.011mA \]

Write KVL for loop (2)
\[ V_x - 900(3.8mA - I_w) - 3.8mA \cdot R_g = 0 \]
\[ V_x - 900(3.8 \times 10^{-3} - 3.011 \times 10^{-3}) - 3.8 \times 10^{-3} R_g = 0 \]
\[ V_x = 0.710 + 3.8 \times 10^{-3} R_g \] (2)

Substitute (2) in eqn (1)
\[ 0.710 + 3.8 \times 10^{-3} R_g = 0.909 - 0.2 \times 10^{-3} R_g \]
\[ R_g = 49.72 \Omega \approx 50 \Omega \]

Substitute ‘\( R_g \)’ value in eqn (2)
\[ V_x = 0.710 + 3.8 \times 10^{-3} \times 50 \]
\[ = 0.9001V \]

10. Ans: (a)
Sol:

Apply KVL in loop
\[ -V + I \alpha R + V_x = 0 \]
\[ -V + \frac{2V}{R} \alpha R + V_x = 0 \]
\[ V - 2V \alpha = V_x \]
\[ V_x = [1 - 2\alpha] V \]
**Conventional Practice Solutions**

01.

**Sol:** The burden of secondary winding is purely resistive and therefore secondary winding PF is unity.

PF of exciting current = 0.4

\[ \cos(90 - \alpha) = 0.4 \]

\[ \alpha = 23.57 \]

Exciting current \( I_0 = 1 \)A

Nominal ratio \( K_n = \frac{1000}{5} = 200 \)

Since there is no turn compensation, the turns ratio equal to the nominal ratio

\( n = K_n = 200 \)

Rated secondary winding current, \( I_s = 5 \)A

\( nI_s = 200 \times 5 = 1000 \)A

Actual transformation Ratio

\[
R = n + \frac{I_0}{I_c} \sin(\delta + \alpha)
\]

\[
= 200 + \frac{1}{5} \sin(0 + 23.57) = 200.08
\]

Ratio error = \[
= \frac{\text{Nominal Ratio} - \text{actual ratio}}{\text{actual ratio}} \times 100
\]

\[
= \frac{200 - 200.8}{200.8} \times 100 = -0.04\%
\]

Phone angle error

\[
\theta = \frac{180^\circ}{\pi} \left[ \frac{I_0 \cos(\delta + \alpha)}{nI_s} \right]
\]

\[
= \frac{180^\circ}{\pi} \left[ \frac{\cos(23.57)}{1000} \right]
\]

\[ = 0.0525 \]

02.

**Sol:** Primary winding turns, \( N_p = 1 \)

Secondary winding turns, \( N_s = 300 \)

Turns ratio, \( n = \frac{N_s}{N_p} = 300 \)

Secondary circuit burden impedance

\[
= \sqrt{(1.5)^2 + (1.0)^2} = 1.8 \text{ } \Omega
\]

Secondary winding circuit

\[
\cos\delta = \frac{1.5}{1.8} \text{ and } \sin\delta = \frac{1.0}{1.8} = 0.554
\]

\[ = 0.833 \]

Secondary induced voltage

\( (E_s) = 5 \times 1.8 = 9.0 \text{V} \)

Primary induced voltage

\[
E_p = \frac{E_s}{300} = \frac{9}{300} = 0.03 \text{V}
\]

Loss component of current referred to primary winding.

\[
I_c = \frac{\text{iron loss}}{E} = \frac{1.2}{0.03} = 40 \text{A}
\]

Magnetizing current, \( I_m \)

\[
= \frac{\text{magnetizing mmf}}{\text{Primary winding turns}}
\]

\[ = \frac{100}{1} = 100 \text{A} \]
Actual ratio, \( R = n + \frac{I_n}{I_s} \sin (\delta + \alpha) \)
\[= n + \frac{I_c \cos \delta + I_m \sin \delta}{I_s} \]
\[= 300 + \frac{40 \times 0.833 + 100 \times 0.555}{5} \]
\[= 317.6 \]

\( K_n = n = 300 \)

Percentage Ratio = \( \frac{K_n - R}{R} \times 100 \)
\[= \frac{300 - 317.6}{317.6} \times 100 = -5.54\% \]

Phase angle error \( \theta = \frac{180^\circ}{\pi} \left( \frac{I_m \cos \delta - I_n \sin \delta}{nl} \right) \)
\[= \frac{180^\circ}{\pi} \left( \frac{100 \times 0.833 - 40 \times 0.555}{300 \times 5} \right) \]
\[= 2.34^\circ \]

**03.**

**Sol:**

(a)

(b) To keep error within 3\% \( V_{\text{new}} \) is the voltage developed across the common resistance of \( R_{\text{eq}} \) and \( R_{\text{new}} \).

\[3 = \frac{(0.33 - V_{\text{new}})E}{0.33E} \times 100 \]
\[V_{\text{new}} = 0.3233E \text{ V} \]

now \[0.323 = \frac{R_{\text{new}}}{16 + R_{\text{new}}} E \]
\[\Rightarrow R_{\text{new}} = 7.64k\Omega \]

now, \( R_{\max} = \frac{8R_{\text{new}}}{8 - R_{\text{new}}} \)
\[8 \times 7.6442 = 172k\Omega \]

**04.**

**Sol:** Phase angle of pressure coil circuit (\( \beta \)) = 30\(^\circ\)
Phase angle of load = \( \cos^{-1}(0.5) = 60^\circ \)

Where

- \( V \) = voltage across the load = 11 kV
- \( I \) = load current = 100A
- \( \phi \) = Phase angle between current and voltage = 60\(^\circ\)
- \( \alpha \) = phase angle bandwidth

Currents in the current and pressure coils of watt meter
\( V_s = \) voltage across secondary of the potential transformer.

\( I_s = \) secondary current of current transformer.

\( I_p = \) current in the wattmeter pressure coil.

\( \beta = \) angle by which \( I_s \) lags \( V_s \) on account of inductance of pressure coil = 30° = 1/2°

\( \delta = \) Phase angle of potential transformer = 45° = \( \frac{3}{4} \)

\( \theta = \) phase angle of current transformer = 90° = \( \frac{1}{2} \)

Phase angle between pressure coil current \( I_p \) and current \( I_s \) of wattmeter current coil

\[
\alpha = \phi - \theta - \beta - \delta = 60° - \frac{3°}{2} - \frac{1°}{2} - \frac{3°}{4} = 57.25°
\]

Correction factor \( k = \frac{\cos \phi \cos \beta \cos \alpha}{\cos 60° \left( \cos(0.5°) \times \cos(57.25°) \right)} = 0.924 \)

Percentage ratio error \( = \frac{k_n - R \times 100}{R} \)

Actual ratio \( R = \frac{k_n \times 100}{(100 + \text{percentage ratio error})} \)

Actual ratio of C.T = \( \frac{20 \times 100}{(100 - 0.2)} = 20.04 \)

Actual ratio of P.T = \( \frac{100 \times 100}{(100 + 0.8)} = 99.2 \)

Power of lead = \( k \times \text{actual ratio of P.T} \times \text{actual ratio of C.T} \times \text{wattmeter} \times \text{wattmeter reading} \)

(2) Power of lead = 11kV \times 0.5 \times 100

\( = 0.924 \times 20.4 \times 99.2 \times \text{wattmeter reading} \)

(3) Wattmeter reading = 294.18 Watts
01. Ans: (b)
Sol: Time period of one cycle = \( \frac{8.8}{2} \times 0.5 \)  
    = 2.2 msec  
Therefore frequency = \( \frac{1}{T} = \frac{1}{2.2 \times 10^{-3}} \)  
    = 454.5 Hz  
The peak to peak Voltage = 4.6×100  
    = 460 mV  
Therefore the peak voltage \( V_m = 230 \) mV  
R.M.S voltage = \( \frac{230}{\sqrt{2}} = 162.6 \) mV

02. Ans: (c)
Sol: In channel 1  
The peak to peak voltage is 5V and peak to peak divisions of upper trace voltage = 2  
Therefore for one division voltage is 2.5V  
In channel 2, the no. of divisions for unknown voltage = 3  
Divisions = 3, voltage/division = 2.5  
∴ voltage = 2.5 × 3 = 7.5 V  
Similarly frequency of upper trace is 1kHz  
So the time period T  
(for four divisions) = \( \frac{1}{f} \)  
T = \( \frac{1}{10^3} \)  ⇒ 1 msec  
i.e for four divisions time period = 1m sec  
In channel 2, for eight divisions of unknown waveform time period = 2m sec.

03. Ans: (c)
Sol: No. of cycles of signal displayed  
\[ = f_{\text{signal}} \times T_{\text{sweep}} \]  
\[ = 200\text{Hz} \times \left(10\text{cm} \times \frac{0.5\text{ms}}{\text{cm}}\right) = 1 \]  
i.e, one cycle of sine wave will be displayed.
We know \( V_{\text{rms}} = \frac{V_{p-p}}{2\sqrt{2}} \)  
\[ V_{\text{rms}} = \frac{N_v \times \text{Volt/div}}{2\sqrt{2}} \]  
\[ \Rightarrow N_v = \frac{2\sqrt{2} \times V_{\text{rms}}}{\text{Volt/div}} \]  
\[ \Rightarrow N_v = \frac{2\sqrt{2} \times 300\text{mV}}{100\text{mv/cm}} \]  
\[ \Rightarrow N_v = 8.485 \text{cm} \]  
i.e 8.485cm required to display peak to peak of signal. But screen has only 8cm (vertical)  
As such, peak points will be clipped.

04. Ans: (b)
Sol:  
→ Given data: Y input signal is a symmetrical square wave  
\[ f_{\text{signal}} = 25\text{KHz}, \ V_{pp} = 10\text{V} \]  
→ Screen has 10 Horizontal divisions & 8 vertical divisions  
which displays 1.25 cycles of Y-input signal.
5. **Ans:** (a)

**Sol:** Frequency ratio is 2

\[
\begin{align*}
\text{Ans:} (a) \\
\text{Sol:} \\
\text{Frequency ratio is 2}
\end{align*}
\]

\[\therefore \text{Two cycles of sine wave displayed on vertical time base}\]

6. **Ans:** (a)

**Sol:**

\[\begin{align*}
\text{Ans:} (a) \\
\text{Sol:} \\
\text{Vertical straight line}
\end{align*}\]

7. **Ans:** (a)

**Sol:** Since the coupling mode is set to DC the capacitance effect at the input side is zero. Therefore the waveform displayed on the screen is both DC and AC components.

\[\begin{align*}
\text{Ans:} (a) \\
\text{Sol:} \\
\text{Since the coupling mode is set to DC the capacitance effect at the input side is zero. Therefore the waveform displayed on the screen is both DC and AC components.}
\end{align*}\]

8. **Ans:** (d)

**Sol:**

\[\begin{align*}
\text{Ans:} (d) \\
\text{Sol:} \\
\text{Vertical straight line}
\end{align*}\]

9. **Ans:** (b)

**Sol:**

\[\begin{align*}
\text{Ans:} (b) \\
\text{Sol:} \\
\text{Frequency ratio is 2}
\end{align*}\]

10. **Ans:** (d)

**Sol:**

\[\begin{align*}
\text{Ans:} (d) \\
\text{Sol:} \\
\text{Frequency ratio is 2}
\end{align*}\]
11. Ans: (d)
Sol: Voltage signal = $5 \sin \left(314t + 45^\circ\right)$

$$f_{\text{signal}} = \frac{314}{2\pi} \text{ Hz}$$
$$= 50 \text{ Hz}$$

No. of cycles of signal displayed
$$= f_{\text{signal}} \times T_{\text{sweep}}$$
$$= 50 \times 10 \times 5 \text{ ms/div}$$
$$= 2.5$$

12. Ans: (d)
Sol:

By using these points draw the line which is a diagonal line inclined at $45^\circ$ w.r.t the x-axis.

13. Ans: (a)
Sol: Lissajous figures are used for measurement of frequency and phase difference.

14. Ans: (d)
Sol:

Because of phase difference only figures changes from ellipse to circle and back to ellipse.

15. Ans: (d)
Sol: A trigger setting that ensures a stationary display is with trigger voltage level as 1.8V and trigger slope as $-Ve$. 
01.

**Sol:** We know: \( t_m = \sqrt{t_s^2 + t_0^2} \) or \( t_s = \sqrt{t_m^2 - t_0^2} \)

Where, \( t_m \) = measured rise time
\( t_s \) = actual or true rise time of signal
\( t_0 \) = oscilloscope rise time

Given that: \( t_0 = 15 \text{ ns} \) & \( t_m = 20 \text{ ns} \)

\[ t_s = \sqrt{t_m^2 - t_0^2} = \sqrt{(20\text{ ns})^2 - (15\text{ ns})^2} = 13.23\text{ ns} \]

Therefore, the actual rise time of signal is 13.23 ns.

02.

**Sol:**

\[ v_x = \sqrt{\frac{2eV_s}{m}} = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \cdot C \cdot 1000V}{9.11 \times 10^{-31} \text{ kg}}} = 1.87 \times 10^7 \text{ m/s} \]

03.

**Sol:** given that:

**Probe resistance**, \( R_p = 4 \text{ M}\Omega \)
**Probe capacitance**, \( C_p = C \)
**Cable capacitance**, \( C_c = 90 \text{ pF} \)
**CRO input resistance**, \( R_i = 2 \text{ M}\Omega \)

**CRO input resistance**, \( C_i = 10 \text{ pF} \)

The equivalent circuit

![Equivalent Circuit Diagram]

We know \( R_p C_p = R_i (C_c + C_i) \)

\[ C_p = \frac{2 \text{ M}\Omega \cdot (90 \text{ pF} + 10 \text{ pF})}{4 \text{ M}\Omega} \Rightarrow C_p = 50 \text{ pF} \]

\[ V_i = K \cdot V_s \text{ where } K \text{ is attenuation factor} \]

\[ K = \frac{V_i}{V_s} = \frac{R_i}{R_p + R_i} \text{ (or) } \frac{C_p}{C_p + (C_i + C_c)} \]

\[ \therefore K = \frac{2 \text{ M}\Omega}{6 \text{ M}\Omega} \text{ or } \frac{50 \text{ pF}}{150 \text{ pF}} \]

\[ K = \frac{1}{3} \]

i.e., The test signal voltage will be attenuated by 3 times.

04.

**Sol:**

Vertical amplifier sensitivity = 5V/cm
Sweep speed = 50\( \mu \)s/cm
Peak – to Peak amplitude = 5.4 cm
Distance for two complete cycles = 8.4 cm

From the above given data wave form can be constructed as shown below.

![Waveform Diagram]

From the above wave form one cycle of wave form can be represented in 4.2 cm.
Voltage (Peak – Peak) = Vertical amplifier sensitivity \times Peak to Peak amplitude.

\[ = 5.4 \times \frac{5 \text{ V}}{\text{ cm}} = 27 \text{ V} \]

Rms voltage = \( \frac{V_{p-p}}{2\sqrt{2}} \)

\[ = \frac{27}{2\sqrt{2}} = 9.54 \text{ V} \]
Time period for one cycle
   = sweep speed × distance for one cycle
   = 50μs/cm × 4.2
   = 210μs

Frequency = \( \frac{1}{\text{Time Period}} = \frac{1}{210 \times 10^{-6}} \)
   = 4.76 kHz

(b) Voltage applied to Horizontal axis
   = \( V_m1 \) cos \( \omega t \)

Voltage applied to vertical axis = \( V_m2 \) cos \( \omega t \)

When \( \omega t = 0 \) then ,
   \( V_y = V_m2 \sin(0) = 0; V_x = V_m1\cos(0) = V_m1 \)

When \( \omega t = 30^\circ \) then,
   \( V_x = 0.866V_m1; V_y = 0.5 V_m2 \)

When \( \omega t = 60^\circ \) then,
   \( V_x = 0.5V_m1; V_y = 0.866 V_m2 \)

When \( \omega t = 90^\circ \) then,
   \( V_x = -0.5V_m1; V_y = 0.866 V_m2 \)

When \( \omega t = 120^\circ \) then,
   \( V_x = -0.866V_m1; V_y = 0.5 V_m2 \)

When \( \omega t = 150^\circ \) then,
   \( V_x = -0.5V_m1; V_y = 0.966 V_m2 \)

When \( \omega t = 180^\circ \) then,
   \( V_x = -V_m1; V_y = 0 \)

When \( \omega t = 210^\circ \) then,
   \( V_x = -0.866 V_m1; V_y = -0.5 V_m2 \)

When \( \omega t = 240^\circ \) then,
   \( V_x = -0.5V_m1; V_y = 0.966 V_m2 \)

When \( \omega t = 270^\circ \) then,
   \( V_x = 0; V_y = - V_m2 \)

When \( \omega t = 300^\circ \) then,
   \( V_x = 0.5V_m1; V_y = 0.966 V_m2 \)

When \( \omega t = 330^\circ \) then,
   \( V_x = 0.966V_m1; V_y = -0.5 V_m2 \)

When \( \omega t = 360^\circ \) then,
   \( V_x = 1V_m1; V_y = 0 \)

If we plot the above obtained \( V_y \) and \( V_x \) values on x-y plot for different angles then it can be represented as.

From the above plot we can conclude that

i) When \( V_m1 \neq V_m2 \) the trace will be an ellipse with 'x' axis as major axis when \( V_m1 > V_m2 \) and with 'y' axis as major axis when \( V_m2 > V_m1 \).

ii) When \( V_m1 = V_m2 \) the trace will be a circle.

05.

Sol: Assume peak amplitude of saw tooth waveform is 2V; the screen dimensions are 8 cm × 8 cm. The horizontal dial setting is 2 volt-cm. Given that y input saw tooth signal is leading x-input saw tooth signal by 90°.

<table>
<thead>
<tr>
<th>t</th>
<th>( V_x )</th>
<th>( V_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 )</td>
<td>-2V</td>
<td>-1V</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>-1V</td>
<td>0</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>0</td>
<td>+1V</td>
</tr>
<tr>
<td>( t_3 )</td>
<td>+1V</td>
<td>+2V</td>
</tr>
<tr>
<td>( t_4 )</td>
<td>+2V</td>
<td>-1V</td>
</tr>
</tbody>
</table>
06.
Sol: \( V_a = 2000 \text{V} \)

Length of deflecting plates \( l_d = 2 \text{ cm} \)
Distance between the plates \( d = 0.5 \text{ cm} \)
Distance between centre of plates to the screen = 30 cm
Deflection height of the beam \( D = 3 \text{ cm} \)
Gain of the amplifier \( A = 100 \)

\[
D = \frac{L \cdot V_a}{2d} \quad (\therefore V_d = \text{deflection voltage})
\]

\[
V_d = \frac{2d \cdot V_a \times D}{L \cdot l_d}
\]

\[
V_d = \frac{2 \times 0.5 \times 2000 \times 3}{30 \times 2}
\]

\[
V_d = 100 \text{ V}
\]

\[
\frac{V_d}{V_i} = 100
\]

\[
V_i = 1 \text{ V}
\]

Velocity of electron beam \( V_{ox} = \sqrt{\frac{2qV_a}{m}} \)

\( q \rightarrow \text{Charge of an electron} = 1.65 \times 10^{-9} \text{C} \)
\( V_a \rightarrow \text{anode voltage required} = 2000 \text{ V} \)
\( m \rightarrow \text{mass of an electron} = 9.1 \times 10^{-31} \text{ kg} \)

\[
V_{ox} = \sqrt{\frac{2qV_a}{m}}
\]

\[
= \sqrt{\frac{2 \times 1.65 \times 10^{-10} \times 2000}{9.1 \times 10^{-31}}}
\]

\[
= 26.93 \times 10^6 \text{ m/s}
\]

07.
Sol:

(a) \( f = 100 \text{ kHz} \):

\[
X_C = \frac{1}{2\pi \times 100 \times 10^3 \times 50 \times 10^{-12}} = 32000 \Omega
\]

Impedance of oscilloscope

\[
(Z_L) = \frac{R \times (-jX_c)}{R - jX_c}
\]

\[
Z_L = \frac{10^6 \times (-j32000 \Omega)}{10^6 - j32000 \Omega}
\]

\[
= 32000 \Omega \angle -90^\circ
\]

Voltage indicated by CRO \( (E_L) \)

\[
= \frac{E_0}{1 + Z_0 / Z_L}
\]

\[
E_L = \frac{1 \text{ V} \angle 0^\circ}{1 + \frac{10 \text{k} \angle 0^\circ}{32000 \angle -90^\circ}}
\]

\[
= 0.954 \text{ V} \angle -17.4^\circ \text{ peak}
\]
(b) \( f = 1 \text{MHZ} \)

\[
X_C = \frac{1}{2\pi \times 1 \times 10^6 \times 50 \text{pF}} = 3200 \Omega
\]

Impedance of oscilloscope \((Z_L)\)

\[
Z_L = \frac{10^6 (-j3200 \Omega)}{10^6 - j3200 \Omega} = 3.2 \times 10^3 \angle -90^\circ
\]

Voltage indicated by CRO \((E_L)\)

\[
E_L = \frac{V_{\text{ind}}}{1 + Z_0/Z_L}
\]

\[
E_L = \frac{1 \text{V} \angle 0^\circ}{1 + \frac{10 \text{k}\Omega \angle 0^\circ}{3.2 \times 10^3 \angle -90^\circ}} = 0.304 \text{V} \angle -72.3^\circ
\]
01. Ans: (a)
Sol: The type of A/D converter normally used in a 3½ digit multimeter is Dual-slope integrating type since it offers highest Accuracy, Highest Noise rejection and Highest Stability than other A/D converters.

02. Ans: (d)
Sol: DVM measures the average value of the input signal which is 1 V. :

03. Ans: (c)
Sol: 0.2% of reading +10 counts \(\rightarrow\) (1)
\[
\begin{align*}
0.2\% \text{ of reading} & = 0.2 \times \frac{100}{100} + 10 \text{(sensitivity \times range)} \\
& = 0.2 \times \frac{100}{100} + 10 \left(\frac{1}{2 \times 10^4} \times 200\right) \\
& = 0.2 + 0.1 = \pm 0.3 \text{ V} \\
\% \text{error} & = \pm \frac{0.3}{100} \times 100 = 0.3\%
\end{align*}
\]

04. Ans: (d)
Sol: When \(\frac{1}{2}\) digit is present voltage range becomes double. Therefore 1V can read upto 1.9999 V.

05. Ans: (d)
Sol: Resolution = \[
\frac{\text{full – scale reading}}{\text{maximaum count}} = \frac{9.999 \text{ V}}{9999} = 1 \text{ mV}
\]

06. Ans: (b)
Sol: Sensitivity = resolution \times lowest voltage range
\[
= \frac{1}{10^4} \times 100 \text{ mV} = 0.01 \text{ mV}
\]

07. Ans: (c)
Sol: The DVM has 3½ digit display
Therefore, the count range is from 0 to 1999 i.e., 2000 counts.
Resolution = \[
\frac{\text{given voltage range}}{\text{Maximum count}} = \frac{200 \text{ mV}}{2000} = 0.1 \text{ mV}
\]

08. Ans: (a)
Sol: Resolution = \[
\frac{\text{max. voltage}}{\text{max. count}} = \frac{3.999 \text{ V}}{3999} = 1 \text{ mV}
\]

09. Ans: (b)
Sol: A and R are true, but R is not correct explanation for A.

10. Ans: (c)
Sol: When \(\frac{1}{2}\) digit switched ON, then DVM will be able to read more than the selected range.
11. Ans: (b)
Sol: Given, $3\frac{1}{2}$ digit, FSD value of 200 mV

Resolution = \( \frac{200 \text{ mV}}{2000} = 0.1 \text{ mV} \)

\[ \therefore \text{Error} = \frac{0.5}{100} \times 100 \text{ mV} + 5 \times 0.1 \text{ mV} \]

\[ = \pm 1 \text{ mV} \]

\[ \therefore \text{The value lies between 99.0 mV & 101.0 mV} \]

12. Ans: (d)
Sol: The DVM has $3\frac{1}{2}$ digit display.

Therefore, its scale resolution is 0.001

- Its resolution in 200mV range is 100mV
- The maximum voltage that can be measured in this 200mV lowest Range: 199.9mV.

13. Ans: (b)
Sol: For N-decade counter

Pulse width (max) = \( \frac{10^N}{f_{\text{clk}}} \)

Resolution \( \Rightarrow \) 1 count \( \Rightarrow \) 1.T_{\text{clk}}

Resolution = \( \frac{1}{f} \)

Range of pulse width

\[ \Rightarrow \frac{1}{f} \text{ to } \left( \frac{10^N - 1}{f} \right) \]

Conventional Practice Solutions

01. 
Sol: The count range of $3\frac{1}{2}$ digit DVM is from 0 to 1999, i.e., 2000 counts.

Due to adding $\frac{1}{2}$ digit, the 1V range of this DVM extended to 2V and 10V range extended to 20V.

resolution of $3\frac{1}{2}$ digit DVM = \( \frac{1}{2 \times 10^3} \times 2V \)

in 1V range of operation

\[ = \frac{2V}{2000 \text{ counts}} = 1 \text{ mV} \]

resolution of $3\frac{1}{2}$ digit DVM = \( \frac{1}{2 \times 10^3} \times 20V \)

in 10V range of operation

\[ = \frac{20V}{2000 \text{ counts}} = 10 \text{ mV} \]

02. 
Sol: We know: \( V_m T_1 = V_{\text{ref}} T_2 \)

\[ \Rightarrow V_m N_F = V_{\text{ref}} n \]

\[ \Rightarrow V_m = \frac{V_{\text{ref}} \times n}{N_F} \]

\[ = \frac{1000V}{1000} \times 762 = 762V. \]

03. 
Sol: For analog multimeter

\[ V_m = 10V, \ V_{\text{FS}} = 20V, \]

G.A.E = \( \pm 2\% \text{ of } V_{\text{FS}} \)
In question asking amount of error in% means limiting error.
% Limiting error =

\[
\text{Measured value} \times \frac{x}{100} = \text{G.A.E value form}
\]
Where \( x \rightarrow \% \) limiting error

\[
= 10 \times \frac{x}{100} = \frac{2}{100} \times (20)
\]

\( \Rightarrow x = 4\% \)

\( \therefore \% \) error = 4\%

\( \Rightarrow \) For \( 3 \frac{1}{2} \) digital multimeter, accuracy of

\[ \pm [0.5\% \text{ of reading} + 1 \text{ count}] \]

\( \Rightarrow \) For 1 count = \( \frac{V_{\text{FS}}}{\text{Maximum value of Display}} \)

\( \Rightarrow \) For \( 3 \frac{1}{2} \) DMM,

\[
\text{Maximum Display}
\]

\[
1 \begin{array}{c} 9 \end{array} \begin{array}{c} 9 \end{array} 9
\]

1 count = \( \frac{20}{1999} = 0.01 \)

\[ = \pm \left[ \frac{0.5}{100} \times (10) + 0.01 \right] = 0.06 \]

This is absolute error

\( \therefore \% \) error = \( \frac{0.06}{10} \times 100 = 0.6\% \)
### Objective Practice Solutions

**01. Ans: (d)**
**Sol:**
The output of FWR is fed as input to DC voltmeter. As such, the DC voltmeter measures average value of $V_{\text{out}}$.

- $V_{\text{avg}}$ of $V_{\text{out}} = 100\text{V}$
- $V_{\text{rms(ind)}} = 1.11 \times V_{\text{avg}} = 1.11 \times 100 \text{V}$
  
  
  
  
  
  $= 111 \text{V}$

**02. Ans: (b)**
**Sol:**
$V_{\text{measured \ (rms)}} = 1.11 \times \text{average value}$

- $= 1.11 \times 75$
- $= 83.25 \text{V}$

$V_{\text{True \ (rms)}} = \frac{V_{\text{m}}}{\sqrt{3}} = \frac{150}{\sqrt{3}} = 86.6 \text{V}$

% error $= \frac{V_{\text{measured}} - V_{\text{true}}}{V_{\text{true}}} \times 100$

- $= \frac{83.25 - 86.6}{86.6} \times 100$
- $= -3.87\%$

**03. Ans: (a)**
**Sol:**
The full wave Rectifier type electronic AC voltmeter has a scale calibrated to read r.m.s value for square wave inputs. As such, the scale calibration factor used for deriving rms volt scale from DC volt scale is 1.

Reading $= 1 \times V_{dc}$ Where $V_{dc}$ is Average voltage of output of full wave Rectifier for given input.

- This voltmeter is used to measure a sinusoidal voltage

**04. Ans: (b)**
**Sol:** A rectifier moving coil instrument consists of a rectifier at primary stage whose output is fed to PMMC meter.

As such, it measures average value but indicates rms value since scale is calibrated in terms of rms.

---

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05. Ans: (d)
Sol: Assertion is wrong since Fullwave rectifier AC voltmeter is a derived rms meter and reads true rms of only one waveform which meter scale is calibrated.

06. Ans: (b) (Key changed)
Sol: \[ S_{AC} = 0.9 \times S_{dc} \]
\[ = 0.9 \times 10 \, \text{k}\Omega/\text{V} \]
\[ = 9 \, \text{k}\Omega/\text{V} \]
Input resistance in 10V range
\[ = 9 \, \text{k}\Omega/\text{V} \times 10 \]
\[ = 90 \, \text{k}\Omega \]

07. Ans: (c)
Sol: Given data: Full wave Bride Rectifier AC voltmeter’s AC voltmeter’s AC volt range is 0-100V. The PMMC ammeter used in the design has full scale current rating of 1mA and internal resistance of 100Ω & diodes are ideal
\[ R_s = 0.9 \times \frac{V_{rmsFSD}}{I_{dcFSD}} - 2R_d - R_m \]
\[ = 0.9 \times \frac{100 \, \text{V}}{1\, \text{mA}} - 100\Omega \]
\[ = 90\, \text{k}\Omega - 100\Omega \]
\[ = 89.9 \, \text{k}\Omega \]

08. Ans: (a)
Sol: We know, for FWBR type AC voltmeter
\[ V_{rms} = 1.11 \times (R_{VFw}) \times I_{dc} \]
\[ \Rightarrow 100\, \text{kV} = 1.11 \times [X_c] \times 45 \times 10^{-3} \, \text{A} \]
\[ \Rightarrow X_c = \frac{100 \times 10^3 \, \text{V}}{1.11 \times 45 \times 10^{-3} \, \text{A}} \]

\[ \Rightarrow \frac{1}{2\pi f_c} = \frac{100 \times 10^3 \, \text{V}}{1.11 \times 45 \times 10^{-3} \, \text{A}} \]
\[ \Rightarrow C = \frac{1.11 \times 45 \times 10^{-3} \, \text{A}}{2\pi \times 50\, \text{Hz} \times 100 \times 10^3 \, \text{V}} \]
\[ \approx 15.9 \times 10^{-10} \, \text{F} \]

09. Ans: (d)
Sol: Multimeter measure voltage (ac & dc), current (ac & dc), resistance.

10. Ans: (c)
Sol: The meter reads full scale with 12V at M and range switch at B.
Now range switch is at D and meter reads full scale value.
Voltage at M is ?
At position B:
\[ V_0 = V_s \left( \frac{1.2 + 0.6 + 0.12 + 0.06 + 0.02}{2 + 6 + 1.2 + 0.6 + 0.12 + 0.06 + 0.02} \right) \]
\[ = 12 \times 0.2 = 2.4 \]
At position D:
\[ V_0 = V_s \left( \frac{0.12 + 0.06 + 0.02}{2 + 6 + 1.2 + 0.6 + 0.12 + 0.06 + 0.02} \right) \]
\[ 2.4 = V_s \times 0.02 \]
\[ V_s = 120\, \text{V} \]

11. Ans: (b)
Sol: Given data: Voltmeter sensitivity is 20kΩ/V
Reading of 4.5V on its 5V full scale
Reading of 6V on its 10V full scale
• Say, voltage source is \( V_s \) and its internal resistance is \( R_s \).
5V range:
\[ R_v = \frac{20 \, \text{k}\Omega}{V} \times 5 = 100 \, \text{k}\Omega \]
→ Reading = \( V \times \frac{100k\Omega}{R_s + 100k\Omega} \)

4.5V = \( V \times \frac{100k\Omega}{R_s + 100k\Omega} \)

\[ V = \frac{4.5V}{100k\Omega} (R_s + 100k\Omega) \] \[ \ldots \ldots (1) \]

10V Range:

→ \( R_v = 20 \frac{k\Omega}{V} \times 10V \)

= 200k\Ω

→ reading = \( V \times \frac{200k\Omega}{R_s + 200k\Omega} \)

6V = \( V \times \frac{200k\Omega}{R_s + 200k\Omega} \)

\[ V = \frac{6V}{200k\Omega} (R_s + 200k\Omega) \] \[ \ldots \ldots (2) \]

Solving eq (1) & (2)

\[ 6V \]

\[ 200k\Omega \]

\[ (R_s + 200k\Omega) \]

\[ = \frac{4.5V}{100k\Omega} (R_s + 100k\Omega) \]

\[ R_s + 200k\Omega = 1.5(R_s + 100k\Omega) \]

0.5\( R_s = 50k\Omega \)

\( R_s = 100k\Omega \)

Putting the value of \( R_s \) in eq (1)

\[ V = \frac{4.5V}{100k\Omega} (100k\Omega + 100k\Omega) \]

\[ = 4.5V \times 2 = 9V \]

Therefore, the voltage source is 9V and its internal resistance is 100k\Ω.

12. Ans: (b)

Sol: For the measurement of the voltage of the order of mv, an amplifier-rectifier type VTVM is best suited where the low magnitude input signal (AC) is first amplified and then rectified and then driven to PMMC meter.

**Conventional Practice Solutions**

01.

Sol: (i) Reading of true rms meter:

A true rms meter measures true rms value of input

\[ V_{rms(true)} = \sqrt{\frac{1}{T_0} \int_{-T_0}^{T_0} V^2(t)dt} \]

\[ = \sqrt{\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} V^2(t)dt} \]

\[ = \sqrt{\frac{9}{2\pi} \times (t)^{\pi/2} + \frac{1}{2\pi} \times (t)^{2\pi} \times (-1)^2 dt} \]

\[ = \sqrt{\frac{9}{2\pi} \times \frac{\pi}{2} + \frac{1}{2\pi} \times \frac{3\pi}{2} \times 2} \]

\[ = \sqrt{\frac{12}{4} V} \]

⇒ Reading of true rms meter is \( \sqrt{3} \) V, i.e.,

(ii) Reading of average measuring – rms indicating meter:

This meter consists of a full wave rectifier at the primary stage, whose output is fed to DC voltmeter and the scale is calibrated to read rms of sine wave.

\[ \therefore \text{ Reading} = 1.11V_{dc} \]

Where \( V_{dc} \) is average voltage of output of FWR for given input.
(iii) **Reading of “peak measuring” – “rms indicating” meter:**

This meter consists of a peak detector at the primary stage whose output is fed to DC voltmeter and the scale is calibrated to read rms of sine wave.

\[ V_{dc} = \frac{1}{2\pi} \left[ \int_{0}^{\pi/2} 3V \, dt + \int_{\pi/2}^{\pi} 1V \, dt \right] \]

\[ = \frac{1}{2\pi} \left[ 3V \times \frac{\pi}{2} + 1V \times \frac{\pi}{2} \right] \]

\[ = \frac{3V}{4} + \frac{3V}{4} = \frac{6}{4}V = 1.5V \]

\[ V_{rms(ind)} = 1.11 \times 1.5V = 1.665V \]

DC voltmeter measures “V\(_{dc}\) of output of peak detector”

\[ V_{dc} = 3V \]

\[ V_{rms(ind)} = 0.707 \times 3V = 2.121V \]

### Sol.

**Advantages of Electronic Voltmeters:**

1. Electronic voltmeters offer very high input resistance in the order of MΩ.
2. Electronic voltmeters cause minimum loading on the circuit under test because of their high input resistance and in turn provide accurate reading.
3. Electronic voltmeters offer better accuracy.
4. Electronic voltmeters offer very high sensitivity.
5. Electronic voltmeters can detect or sense low level input signals because of their high sensitivity.
6. Electronic voltmeters offer improved dynamic input range
7. Electronic voltmeter take less power from circuit under test as the power required for deflection is provided from the external circuit using amplifiers but not from circuit under test. As such, their power consumption is low.
8. Electronic voltmeters offer faster response
9. Frequency range of operation of electronic voltmeters is high
10. Electronic voltmeters are compact in size and are more portable.

Electronic Voltmeter using Bridge Circuit for Full Wave Rectification

A full wave bridge rectifier type AC voltmeter consists of full wave bridge rectifier at primary stage whose output is fed to PMMC indicating meter and the scale of PMMC is calibrated to read rms voltage of input sine wave

Circuit diagram:

Available PMMC ammeter ratings:
0 - \( I_{\text{dcFSD}} \), \( R_{\text{m}} \).

Required AC voltmeter range: 0 - \( V_{\text{rmsFSD}} \)

The sinusoidal input voltage whose rms value is to be measured is first fed to full wave bridge rectifier. During the half cycle of input, \( D_2 \) & \( D_4 \) conduct and current passes through ammeter. Similarly during –ve half cycle of input, \( D_1 \) & \( D_3 \) conduct and current passes through ammeter (in same direction)

But the design requirement is “ \( V_{\text{rms}} \) of \( V_{\text{in}} \)”

\[
V_{\text{rms}} = \frac{1}{\sqrt{2}} V_m = \frac{1}{\sqrt{2}} R_v \times I_m
\]

\[
= \frac{1}{\sqrt{2}} R_v \times \frac{\pi}{2} I_{\text{avg}}
\]

\[
\Rightarrow V_{\text{rms}} = \frac{\pi}{2\sqrt{2}} R_v \times I_{\text{avg}}
\]

\[
\Rightarrow V_{\text{rms}} = 1.11 R_v \times I_{\text{avg}} \text{--scale calibration relation}
\]

Where,

\( R_v \) = input resistance of FWBR voltmeters
\( = 2R_d + R_{\text{se}} + R_{\text{m}} \)
This relation is used to calibrate the DC current scale in terms of AC volts (rms volts)

\[ V_{\text{rms(FSD)}} = V_{\text{rms(ind)}} \times 1.11R_v \]

\[ \therefore \text{This voltmeter internally measures average value but indicates rms value.} \]

**Solution to the problem**
For given voltage waveform,

\[ V_{\text{rms(true)}} = \sqrt{\frac{1}{3.6} \int_0^{3.6} (160 \text{V})^2 dt} = 92.3 \text{V} \]

This voltage is input to FWBR. As such, the output of FWBR:

\[ V_{\text{rms(ind)}} = 1.11 \times 80 \text{V} = 88.8 \text{V} \]

% error in measurement
\[ = \frac{88.8 \text{V} - 92.3 \text{V}}{92.3 \text{V}} \times 100 = -3.8\% \]
## Q-Meter

### Objective Practice Solutions

**01. Ans: (a)**

**Sol:**

\[ C_1 = 300\text{pF} \quad C_2 = 200\text{ pF} \]

\[ Q = \frac{1}{(\omega C_1 R)} \]

\[ = 120 = \frac{1}{( C_2 + C_x)R} \]

\[ C_1 = C_2 + C_x \]

\[ \therefore C_x = 100\text{ pF} \]

**02. Ans: (b)**

**Sol:**

\[ \%\text{error} = \frac{r}{r + R} \times 100 \]

\[ = \frac{0.02}{0.02 + 10} \times 100 = -0.2\% \]

**03. Ans: (c)**

**Sol:**

Q-meter consists of R, L, C connected in series.

\[ \therefore \text{Q-meter works on the principle of series resonance.} \]

**04. Ans: (b)**

**Sol:**

Given data: \( C_d = 820\text{ pF}, \)
\[ \omega = 10^6\text{rad/sec} \quad \text{&} \quad C = 9.18\text{nF} \]

We know, \[ L = \frac{1}{\omega^2[C + C_d]} \]

\[ = \frac{1}{(10^6)^2[9.18\text{nF} + 820\text{pF}]} = 100\text{\mu H} \]

The inductance of coil tested with a Q-meter is \(100\text{\mu H}.\)

**05. Ans: (b)**

**Sol:**

A series RLC circuit exhibits voltage magnification property at resonance. i.e., the voltage across the capacitor will be equal to \(Q\)-times of applied voltage.

Given that \( V \) = applied voltage and \( V_0 = \) Voltage across capacitor

There fore, \[ Q = \frac{V_{\text{max}}}{V_{\text{in}}} \]

\[ \Rightarrow Q = \frac{V_0}{V} \]

**06. Ans: (b)**

**Sol:**

\[ f_1 = 500\text{ kHz} \quad ; \quad f_2 = 250\text{kHz} \]

\[ C_1 = 36\text{ pF} \quad ; \quad C_2 = 160\text{ pF} \]

\[ n = \frac{250\text{kHz}}{500\text{kHz}} \]

\[ \Rightarrow n = 0.5 \]

\[ C_d = \frac{36\text{pF} - (0.5)^2160\text{pF}}{(0.5)^2 - 1} \]

\[ = 5.33\text{pF} \]

**07. Ans: (c)**

**Sol:**

\[ Q = \frac{\text{capacitor voltmeter reading}}{\text{Input voltage}} \]

\[ = \frac{10}{500 \times 10^{-3}} \]

\[ = 20 \]

**08. Ans: i \rightarrow (c), \ii \rightarrow (a)**

**Sol:**

(i) \[ C_d = \frac{C_1 - n^2C_2}{n^2 - 1} \]

\[ = \frac{360 - 288}{3} \]

\[ = 24\text{ pF} \]
(ii) \[ L = \frac{1}{\omega^2 [C_1 + C_d]} \]
\[ = \frac{1}{\left[2\pi \times 500 \times 10^3\right] \left[24 + 360\right] \times 10^{-6}} = 264\mu H \]

09. Ans: (b)
Sol: \[ Q_{true} = Q_{meas} \left(1 + \frac{r}{R_{coil}}\right) \]
\[ Q_{actual} = Q_{observed} \left[1 + \frac{R}{R_1}\right] \]

10. Ans: (c)
Sol: \[ 1 + \frac{C_d}{C} = \frac{Q_{true}}{Q_{measured}} \]
\[ \Rightarrow \frac{C_d}{C} = \frac{245}{244.5} - 1 \]
\[ = 2.044 \times 10^{-3} \]
\[ \Rightarrow \frac{C}{C_d} = 489 \]

11. Ans: (b)
Sol: Q-meter works on the principle of series resonance
\[ V_s = Q \times V_{in} \]
\[ \therefore \] Both A & R are individually true but R is not the correct explanation of A.

Conventional Practice Solutions

01.
Sol: Given that: \( f_1 = 3 \) MHz & \( C_1 = 251pF \)
\( f_2 = 6 \) MHz & \( C_2 = 50pF \)
\[ \therefore n = \frac{f_2}{f_1} = \frac{6MHz}{3MHz} = 2 \]
After inserting the test coil into socket of Q-meter, the resonance is obtained for the first time at 3 MHz with tuning capacitance set to 251pF. Then, the frequency is doubled (i.e., \( n = 2 \)) and, the resonance is obtained for the second time at 6 MHz with tuning capacitance set to 50pF.
We know: \( C_d = \frac{C_1 - n^2 C_2}{n^2 - 1} \)
\[ = \frac{251pF - (2)^2 \times 50pF}{(2)^2 - 1} \]
\[ = \frac{251pF - 200pF}{3} \]
\[ \Rightarrow C_d = 17pF. \]
\[ \therefore \] The self capacitance or distributed capacitance of coil is found to be 17pF.

02.
Sol: The effective Q-of the coil, \( Q_t = \frac{1}{\omega CR} \)
\[ = \frac{1}{2\pi \times 1 \times 10^6 \times 60 \times 10^{-12} \times 10} \]
\[ = 265.258 \]
The indicated or calculated Q-of the coil
\[ Q_{ind} = \frac{1}{2\pi \times 1 \times 10^6 \times 60 \times 10^{-12} \times (10 + 0.02)} \]
\[ = 264.729 \]
% error = \frac{Q_{\text{ind}} - Q_t}{Q_t} \times 100 \\
= \frac{264.729 - 265.258}{265.258} \times 100 \\
= -0.19943\% 

03. 
Sol: Refer previous Q for working of Q-meter 
given that: \( f_1 = 1\text{MHz} \) & \( C_1 = 1530 \text{ pF} \) 
\( f_2 = 3 \text{ MHz} \) & \( C_2 = 162 \text{ pF} \) 
\( n = \frac{f_2}{f_1} = \frac{3\text{MHz}}{1\text{MHz}} = 3 \) 

After inserting the test coil into socket of Q-meter, the resonance is obtained for the first time at 1MHz with tuning capacitor adjusted to 1530PF. Then the frequency is tripled (i.e., \( n = 3 \)) and the resonance is obtained for the second time at 3MHz with tuning capacitor adjusted to 162 pF.

We know: 
\[ C_d = \frac{C_1 - n^2C_2}{n^2 - 1} \]
\[ = \frac{1530\text{pF} - (3)^2 \times 162\text{pF}}{(3)^2 - 1} \]
\[ = \frac{1530\text{pF} - 1458\text{pF}}{8} \]
\[ = 9 \text{ pF} \]

:\text{ The self capacitance of the coil is 9 pF.}

04. 
Sol: Given distributed capacitance 
\( C_d = 20\text{PF} \) 
Tuning capacitance at first resonance = \( C_1 = 200\text{pF} \) 
Tuning capacitance at second resonance = \( C_2 = ? \)

First resonance frequency occurs at 
\[ f_1 = \frac{1}{2\pi \sqrt{L(C_d + C_1)}} \]
\[ = \frac{1}{2\pi \sqrt{L(220)}} \]

Second resonance frequency will be double the first resonance frequency 
\[ f_2 = 2f_1 \]
\[ f_2 = \frac{1}{2\pi \sqrt{L(C_2 + C_d)}} = 2 \times \frac{1}{2\pi \sqrt{L(C_1 + C_d)}} \]
\[ \Rightarrow \frac{1}{2\pi \sqrt{L(C_2 + C_d)}} = \frac{2 \times 1}{2\pi \sqrt{L(C_1 + C_d)}} \]

By cross multiplying and squaring 
\((C_1 + C_d) = 4 \times (C_2 + C_d)\)

By solving 
\[ C_2 = \frac{C_1 - 3C_d}{4} \] .... (1)

By substituting values of \( C_d \) and \( C_1 \) in (1) then 
\[ C_2 = \frac{200 - 3 \times 20}{4} = \frac{200 - 60}{4} = 35\text{pF} \]

Tuning capacitance at second resonance 
\( C_2 = 35\text{pF} \) and 
Second resonance frequency will be double the first resonance frequency 
\( f_2 = 2f_1 \)

05. 
Sol: 
\[ Q = \frac{omegaL}{R} \]
\[ Q = \frac{X_L}{R} \]
\[ Q = \frac{100}{10} = 10 \]
06.

Sol: Given that: $f_1 = 1\text{MHz}$, $C_1 = 210\ \text{pF}$, $Q_1 = 100$ and $f_2 = 2\ \text{MHz}$, $C_2 = 45\ \text{pF}$

\[ n = \frac{2\text{MHz}}{1\text{MHz}} = 2 \]

\[ C_d = \frac{210\text{pF} - 2^2 \times 45\text{pF}}{2^2 - 1} \]
\[ = \frac{210\text{pF} - 4 \times 45\text{pF}}{3} = 10\ \text{pF} \]

\[ L = \frac{1}{(2\pi \times 1\text{MHz})^2[210\text{pF} + 10\text{pF}]} \]
\[ = 1.15 \times 10^{-4}\ \text{H} \]

\[ R_{\text{coil}} = \frac{2\pi \times 1\text{MHz} \times 1.1513 \times 10^{-4}}{100} \]
\[ = 7.245\ \Omega \]
Objective Practice Solutions

01. Ans: (d)
Sol: Piezo electric transducer is an active transducer.

02. Ans: (c)
Sol: Active transducers do not require an auxiliary power source to produce their output. From given options thermocouple & solar cell pair transducers are active transducers as they produce output with no auxiliary power source.

03. Ans: (d)
Sol: Pressure → Piezoelectric crystal
Temperature → Thermistor
Displacement → Capacitive transducer
Stress → Resistance strain gauge

04. Ans: (d)
Sol: Thermocouple → Cold junction compensation
Strain gauge → DC bridge
Piezoelectric crystal → Charge amplifier
LVDT → Phase sensitive detector

05. Ans: (b)
Sol: Usually from transducers we get small output which is not sufficient for further processing, so in order to amplify that output we require signal conditioning circuit. Finally to read the output we require recorder.

06. Ans: (d)
Sol: Bolometer
→ measurement of power at 500 MHz
Hot wire anemometer
→ measurement of flow of air around an aeroplane.
C-type bourdon tube
→ measurement of high pressure
Optical pyrometer
→ measurement of temperature of furnace.

07. Ans: (d)
Sol: Charge amplifier with very low bias current and high input impedance → piezoelectric sensor for measurement of static force
Voltage amplifier with low bias current and very high input impedance → Glass electrode PH sensor
Voltage amplifier with very high CMRR sensing applications → strain gauge in unipolar DC wheatstone bridge.

08. Ans: (b)
Sol: Mcleod gauge → Pressure
Turbine meter → Flow
Pyrometer → Temperature
Synchros → Displacement

09. Ans: (a)
Sol: Variable capacitance device
→ Pressure transducer
Orifice meter → Flow measurement
Thermistors → Temperature measurement
10. Ans: (b)
Sol: From given options diaphragm and pivot torque are employed for displacement measurement while thermistor and thermocouple not related to displacement measurement.

11. Ans: (a)
Sol: Instrumentation amplifier is used to amplify signals from transducer.

12. Ans: (a)
Sol: LVDT gives linear output & also very accurate compare to any other transducer given in options.

13. Ans: (b)
Sol: The lower limit of useful working range of a transducer is determined by transducer error and noise.

14. Ans: (d)
Sol: From given options, thermocouple and thermopile, piezoelectric pick-up, photovoltaic cell are a self generating type transducers.

Conventional Practice Solutions

01.
Sol: Transducers can be classified
   - Based upon transduction principle
   - as primary and secondary transducers
   - as passive and active transducers
   - as analog and digital transducers
   - as transducers and inverse transducers

Based upon transduction principle
The transducers can be classified on the basis of principle of transduction as resistive, inductive, capacitive etc., depending upon how they convert the input quantity into resistance, inductance or capacitance respectively.

02.
Sol: Primary and Secondary Transducers:
The first transducer which converts physical phenomenon into displacement, pressure, velocity etc. which is to be accepted by next stage is known as “Primary Transducer”. The output of the primary transducer is converted subsequently into a usable output by a device called “Secondary Transducer”

Passive and Active Transducers:
Passive transducers: They derive the power required for transduction from an auxiliary power source.
Eg: Resistive, inductive and capacitive transducers.
Active transducers: They do not require an auxiliary power source to produce their
output. They are also known as self-generating type since they develop their own voltage or current output.

Eg: piezoelectric, photovoltaic etc.

**Analog and digital Transducers:**

Analog transducers: These transducers convert the input quantity into an analog output which is a continuous function of time.

Eg: LVDT, thermocouple etc.

Digital Transducers:- These transducers convert the input quantity into an electrical output which is in the form of pulses.

---

**Transducers & Inverse Transducers**

**Transducer:** A transducer can be broadly defined as a device which converts a non-electrical quantity into an electrical quantity.

**Inverse transducer:** An inverse transducer is defined as a device which converts an electrical quantity into a non-electrical quantity.

---

**03.**

**Sol:** In the direct method of measurement, the physical quantities like length or mass are measured directly by the measuring instruments. The indirect method of measurement comprises of various stages for the measurement of the physical quantity like temperature, pressure, force etc, since they cannot be measured by the direct instruments. In this method, the transducer is used which is connected to a host of other instruments to convert one form of energy that cannot be measured into the other form that can be measured easily. The input and the output values are calibrated so that for all the value of output the value of the input can be calculated.

**Transducer:**

The transducers that convert the mechanical input signals into electrical output signals are called as electrical transducers. The output obtained from the electrical transducers can be read by the humans or it can given as input to the controllers. The input given to the electrical transducers can be in the form of displacement, strain, velocity, temperature, flow etc and the output obtained from them can be in the form of current, voltage and change in resistance, inductance and capacitance. The output can be measured easily and it is calibrated against the input, thus enabling the measurement of the value of the input.

1. **Potentiometers:** They convert the change in displacement into change in the resistance, which can be measured easily.

2. **Variable Capacitance Transducers:**
   These comprise of the two parallel plates between which there is dielectric material like air. The change in distance between the two plates produced by the displacement results in change in capacitance, which can be easily measured.

3. **Variable Resistance Transducers:** There is change in the resistance of these sensors when certain physical quantity is applied
to it. It is most commonly used in resistance thermometers or thermistors for measurement of temperature.

4. **Magnetic sensors:** The input given to these sensors is in the form of displacement and the output obtained is in the form of change in inductance or reluctance and production of the eddy currents.

5. **Piezoelectric transducers:** When force is applied to these transducers, they produce voltage that can be measured easily. They are used for measurement of pressure, acceleration and force.

6. **Strain gauges:** When strain gauges are strained or stretched there is change in their resistance. They consist of the long wire and are able to detect very small displacements produced by the applied force or pressure.

7. **Photo electric transducers:** When the light is applied to these transducers, they produce voltage.

8. **Linear variable differential transformer (LVDT):** LVDT is the transformer consisting of the primary and the secondary coil. It converts the displacement into the change in inductance.

9. **Ultrasonic Transducers:** These transducers use the ultrasonic or ultrasound waves to measure parameters like fluid level, flow rate etc.

Apart from these, there are some more electrical type of transducers like moving coil type, changing dielectric type, changing core positions type etc.

i) **Inductive transducer:**

The inductive transducers work on the principle of the magnetic induction of magnetic material. Just as the resistance of the electric conductor depends on number of factors, the induction of the magnetic material depends on a number of variables like the number of turns, of the coil on the material, the size of the magnetic material, and the permeability of the flux path. In the inductive transducers, the magnetic materials are used in the flux path and there are one or more air gaps. The change in the air gap also results in change in the inductance of the circuit and in most of the inductive transducers; it is used for the working of the instrument.

ii) **Capacitive transducer.** The capacitive transducer or sensor is nothing but the capacitor with variable capacitance. The capacitive transducer comprises of two parallel metal plates that are separated by the material such as air, which is called as the dielectric material. In the typical capacitor, the distance between the two plates is fixed, but in variable capacitance transducers, the distance between the two plates is variable.

In the instruments using capacitance transducers, the value of the capacitance changes due to change in the value of the
input quantity that is to be measured. This change in capacitance can be measured easily and it is calibrated against the input quantity, thus the value if the input quantity can be measured directly.

The capacitance $C$ between the two plates of capacitive transducers is given by:

$$C = \varepsilon_o \times \varepsilon_r \times \frac{A}{d}$$

Where $C$ is the capacitance of the capacitor or the variable capacitance transducer

- $\varepsilon_o$ is the absolute permittivity
- $\varepsilon_r$ is the relative permittivity

The product of $\varepsilon_o$ and $\varepsilon_r$ is also called as the dielectric constant of the capacitive transducer.

$A$ is the area of the plates

$d$ is the distance between the plates

It is clear from the above formula that capacitance of the capacitive transducer depends on the area of the plates and the distance between the plates. The capacitance of the capacitive transducer also changes with the dielectric constant of the dielectric material used in it.

Thus, the capacitance of the variable capacitance transducer can change with the change of the dielectric material, change in the area of the plates and the distance between the plates.

**iii) Strain gauges:** Strain gauges are devices whose resistance changes under the application of force or strain. They can be used for measurement of force, strain, stress, pressure, displacement, acceleration etc. It is often easy to measure the parameters like length, displacement, weight etc that can be felt easily by some senses. However, it is very difficult to measure the dimensions like force, stress and strain that cannot be really sensed directly by any instrument. For such cases, special devices called strain gauges are very useful.

There are some materials whose resistance changes when strain is applied to them or when they are stretched and this change in resistance can be measured easily. For applying the strain you need force, thus the change in resistance of the material can be calibrated to measure the applied force. Thus the devices whose resistance changes due to applied strain or applied force are called as the strain gauges.

When force is applied to any metallic wire its length increases due to the strain. The more is the applied force, more is the strain and more is the increase in length of the wire. If $L_1$ is the initial length of the wire and $L_2$ is the final length after application of the force, the strain is given as:

$$\varepsilon = \frac{L_2 - L_1}{L_1}$$

Further, as the length of the stretched wire increases, its diameter decreases. Now, we know that resistance of the conductor is the inverse function of the length. As the length of the conductor increases, its resistance decreases. This change in resistance of the conductor can be measured easily and calibrated against the applied force. Thus, strain gauges can be used to measure force and related parameters like displacement.
and stress. The input and output relationship of the strain gauges can be expressed by the term gauge factor or gauge gradient, which is defined as the change in resistance $R$ for the given value of applied strain $\varepsilon$.

iv) **Piezoelectric transducers**

![Piezoelectric transducer diagram]

Piezo-electricity represents the property of a number of crystalline materials that cause the crystal to develop an electric charge or potential difference when subjected to mechanical forces or stresses along specific planes. Conversely, the crystal would undergo change in thickness (and thus produce mechanical forces) when charged electrically by a potential difference applied to its proper axis. Elements exhibiting piezo-electric qualities are sometimes known as electro restrictive elements.

A typical mode of operation of a piezo electric device for measuring varying force applied to a simple plate is shown in the fig. Metal electrodes are attached to the selected faces of a crystal in order to detect the electrical charge developed. The magnitude and polarity of the induced charge on the crystal surface is proportional to the magnitude and direction of the applied force and is given by

$$Q = KF$$

Where $Q$ is the charge in coulomb, $F$ is the impressed force in Newton’s and $K$ is the crystal sensitivity in C/N;
Chapter 13: Resistive, Inductive & Capacitive Transducers

Objective Practice Solutions

01. Ans: (b)
Sol: For resistive potentiometer
output \( E_0 = K \times E_i \)
Where \( E_0 = \) output of the potentiometer
\( E_i = \) input of the potentiometer

02. Ans: (b)
Sol: In a resistance potentiometer, non linearity decreases with increase of load to potentiometer resistance because the output equation for potentiometer under loading condition is
\[ E_0 = \frac{K}{1 + K(1 - K) \times \left( \frac{R_p}{R_m} \right)} \]
\( R_p = \) resistance of the potentiometer
\( R_m = \) resistance of the meter

03. Ans: (a)
Sol: In a resistance potentiometer high value of resistance lease to high value of sensitivity.

04. Ans: (c)
Sol: Total temperature of POT = \( T_{\text{ambient}} + T_{\text{pot}} \)
\[
T_{\text{pot}} = 30 \left( ^\circ \text{C} \right) \times P
\]
\[
P = \frac{V^2}{R} = \frac{(10)^2}{100} = 1 \text{W}
\]
\[
T_{\text{pot}} = 30 \left( ^\circ \text{C} \right) \times 1 \text{W}
\]
\[
= 30 ^\circ \text{C}
\]

05. Ans: (a)
Sol: Semiconductor strain gauge has a much higher gauge factor then that of a metal wire strain gauge because of piezo resistive effect.

06. Ans: (b)
Sol: We for quarter bridge strain measuring circuit the output \( \bar{V} = \frac{V}{4} \times G_f \times \varepsilon \)
Here, \( \bar{V} = 1 \text{ mV} \)
Strain (\( \varepsilon \)) = \( 500 \times 10^{-6} \)
\( V_i = 4 \text{ V} \)
Now \( G_f = 2 \)

07. Ans: (a)
Sol: For the given bending force we can say that configuration P is subjected to more tension compare to other two configurations.

08. Ans: (d)
Sol: \( G_f = 1 + 2 \times 0.35 = 1.70 \)

09. Ans: (c)
Sol: For sinusoidal input output should have to be sinusoidal.

10. Ans: (c)
Sol: Gauge factor \( \frac{0.150}{250} / \frac{1.5 \times 10^{-4}}{4} = 4 \)
11. Ans: (b)
Sol: A gauge factor is a ratio of per unit change in resistance to per unit change in length
\[ \text{G.F} = \frac{\Delta R}{R} \frac{L}{\Delta L} \]

12. Ans: (b)
Sol: To receive the optimum output signal for shear strain, all the gauges should be placed at a position that is 45° in with respect to the longitudinal axis.

13. Ans: (c)
Sol: A strain gauge bridge sometimes excited with ac to avoid the power frequency pick-up.

14. Ans: (b)
Sol: \[ \text{GF} = 2 \]
\[ \varepsilon = 1 \times 10^{-6} \]
\[ R = 120 \Omega \]
We know \[ \frac{\Delta R}{R} = \text{GF}\varepsilon \]
\[ \Delta R = \text{GF}\varepsilon R \]
\[ = 2 \times 1 \times 10^{-6} \times 120 \]
\[ \Delta R = 240 \times 10^{-6} \Omega \]

15. Ans: (c)
Sol: The piezoresistive effect is a change in the electrical resistivity of a semi conductor or metal when mechanical strain is applied. Semiconductor strain gauges have higher piezoresistive coefficient so higher gauge factor.

16. Ans: (a)
Sol: In given diagram of LVDT, two secondary coils are so connected that output will always be added. So output is constant voltage graph.

17. Ans: (d)
Sol: LVDT has one primary coil and two secondary coils are connected in opposition, so that output must difference between two secondary output voltage.

18. Ans: (a)
Sol: LVDT is an inductive transducer which translates the linear motion into electrical signals.

19. Ans: (b)
Sol: Air cored inductive transducers are suitable to use at higher frequencies.

20. Ans: (d)
Sol: Inductive transducer in differential configuration of output is unaffected by External magnetic field temperature changes, variation of supply voltage & frequency.

21. Ans: (b)
Sol:
For push-pull arrangement for L change of inductance exhibited at output
= \( L + \Delta L - (L - \Delta L) = 2\Delta L \)

22. **Ans:** (d)
**Sol:** An LVDT exhibits linear characteristics up to a displacement of ±5 mm, linearly of 0.05 % has an infinite resolution and high sensitivity of the order of 40 V/mm.

23. **Ans:** (a)
**Sol:** To avoid the effect of fringing, the potential of the guard ring of a capacitance transducer holds at circuit potential. Ground is supposed to be a conduit to remove extraneous noise from the circuit.

24. **Ans:** (b)
**Sol:** The transfer function of capacitive transducer is given as
\[
\frac{X_o(s)}{X_i(s)} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega \tau}\right)^2}}
\]
So this resembles a high pass filter.

25. **Ans:** (a)
**Sol:** A strain gauge is an example of an electromechanical transducer in which displacement is used to vary the resistance. So we can say that both statement I & II are true as well as related.

26. **Ans:** (c)
**Sol:** Incase of strain gauge
Statement –I is true but
Statement –II is false.

---

### Conventional Practice Solutions

01. **Question not clear.**

02. **Sol:** Given data:
- Gauge factor of given soft iron wire = 5.2
- We know the relationship between gauge factor and Poisson’s ratio
  \[
  G_f = 1 + 2\nu
  \]
  \[
  \nu = \frac{G_f - 1}{2} = \frac{5.2 - 1}{2} = \frac{4.2}{2} = 2.1
  \]

03. **Sol:** Given data:
- Conductor length \( l = 24 \text{ mm} \)
- Charge in length of conductor \( \Delta l = 1 \text{ mm} \)
- Diameter of the conductor \( D = 1.5 \text{ mm} \)
- Charge in diameter of conductor \( \Delta D = 0.02 \text{ mm} \)
- Poisson’s ratio \( \nu = \frac{\Delta D/D}{\Delta l/l} \)
  \[
  \Delta D/D = \frac{0.02}{1.5} \text{ mm/mm} = 1/24
  \]
- \( \nu = 0.32 \)
- Gauge factor = \( 1 + 2\nu \)
  \[
  = 1 + (2 \times 0.32) = 1.64
  \]

04. **Sol:** Given data:
- \( G_f = 2 \)
- Stress = 1000 (kg/cm²)
- \( E = 2 \times 10^6 \) (kg/cm²)
Percentage change in Resistance of the strain gauge

\[
\Delta R \times \frac{100}{R} = \frac{G_f \times \text{stress}}{E} \times 100
\]

\[
= \frac{2 \times 1000}{2 \times 10^6} \times 100
\]

\[
= 0.1\%\]

We know the relationship between gauge factor and Poisson’s ratio

\[
G_f = 1 + 2\nu
\]

\[
\nu = \frac{G_f - 1}{2} = \frac{2 - 1}{2} = 0.5
\]

05.

Sol: Air gap after displacement of the armature

\[
= 1.00 - 0.025 = 0.975 \text{ mm}
\]

Since inductance is inversely proportional to the length of air gap, inductance with new gap would be

\[
= \frac{1950}{0.975} = 2000 \text{ }\mu\text{H}
\]

Change in inductance = 2000 – 1950 = 50 \text{ }\mu\text{H}

\[
\text{Ratio} = \frac{\text{change in inductance}}{\text{original inductance}} = \frac{50}{1950} = 0.0256
\]

\[
\text{Ratio} = \frac{\text{displacement}}{\text{original gap}} = \frac{0.025}{1.0} = 0.025
\]

Since the two ratios are equal, we can say that the change in inductance is linearly proportional to the displacements.

06.

Sol: Given data:

LVDT rms output voltage = 2.6 V

LVDT displacement = 0.4 \mu\text{m}

Sensitivity of LVDT is

\[
S = \frac{\text{LVDT rms output voltage}}{\text{LVDT displacement}}
\]

\[
= \frac{2.6}{0.4} \left( \frac{\text{V}}{\mu\text{m}} \right)
\]

\[
= 6.5 \left( \frac{\text{V}}{\mu\text{m}} \right)
\]

07.

Sol: Given data:

Sensitivity of given LVDT = 60 \left( \frac{\text{V}}{\text{mm}} \right)

Bellow deflection = 0.15 mm for given pressure of \(1.2 \times 10^6 \text{ (N/m}^2)\)

Given output voltage = 4.5 V

Sensitivity of LVDT in

\[
= 60 \left( \frac{\text{V}}{\text{mm}} \right) \times 0.15(\text{mm}) \times \frac{1}{1.2 \times 10^6(\text{N/m}^2)}
\]

\[
= 7.5 \left( \frac{\text{mV}}{\text{N/m}^2} \right)
\]

Pressure when output voltage is 4.5 V is

\[
= \frac{\text{output voltage}}{\text{sensitivity of LVDT}}
\]

\[
= \frac{4.5(\text{V})}{7.5 \left( \frac{\text{mV}}{\text{N/m}^2}} \right)
\]

\[
= 6 \times 10^5(\text{N/m}^2)
\]

08.

Sol: Given data:

LVDT output voltage in rms = 6(V) for displacement of \(0.4 \times 10^{-3}(\text{mm})\)
Voltmeter specification: 10 V voltmeter
Scale divisions = 100
Division can be estimated = \( \frac{2}{10} \) th division

Sensitivity of the given LVDT
\[
= \frac{\text{Voltage output}}{\text{input displacement}}
= \frac{6}{0.4 \times 10^{-3}} \text{(V)} = \frac{15000}{\text{(V/mm)}}
\]

Resolution of voltmeter
\[
= \frac{\text{voltmeter range}}{\text{S \times scale division}} \times \frac{2}{10}
= \frac{10 \text{(V)}}{15000 \text{(V/mm)} \times 100 \text{ (mm)}} \times \frac{2}{10}
R = 1.333 \times 10^{-6} \text{(mm)}
\]

09.

Sol: Given data:
Plate separation (x) under static condition
= 0.05 mm

\( C_{\text{static}} \) capacitance under static condition
= \( 5 \times 10^{-12} \) (F)

Change in capacitance due to axial displacement = \( 0.75 \times 10^{-12} \) (F)

We know the relationship between capacitance (C) and plate separation (x)
\[
C \propto \frac{1}{x}
\]

\[
\frac{C_{\text{static}}}{C_{\text{static}} + \Delta C} = \frac{x_1}{x}
\]

\[
x_1 = x \times \frac{C_{\text{static}}}{C_{\text{static}} + \Delta C}
= 0.05 \times 10^{-3} \times \frac{5 \times 10^{-12}}{(5 \times 10^{-12}) + (0.75 \times 10^{-12})}
\]

\[
x_1 = 43.478 \mu\text{m}
\]

Axial displacement = \( x - x_1 \)
= \( (0.05 \times 10^{-3}) - (43.478 \times 10^{-6}) \)
= 0.006522 mm

10.

Sol: Given data:
Area of quartz diaphragm = 750 mm\(^2\)
\( t_1 \) = separation distance between quartz diaphragm = 3.5 mm
\( C = 370 \) pF with no pressure with separation distance ‘\( t_1 \)’
\( \Delta t_2 \) = deflection due to applied pressure
\( \Delta t_2 = 0.6 \) mm for the applied pressure of 900 kN/m\(^2\)

We know the relationship between capacitance (C) and separation distance (t) between the plates is
\[
C \propto \frac{1}{t}
\]

\[
t_2 = t_1 - \Delta t_2 = 3.5 \text{ mm} - 0.6 \text{ mm}
= 2.9 \text{ (mm)}
\]

\[
C_1 = \frac{t_2}{t_1}
\]

\[
C_2 = C_1 \times \frac{t_1}{t_2} = 370 \times 10^{-12} \times \frac{3.5 \times 10^{-3}}{2.9 \times 10^{-3}}
\]

\[
C_2 = 446.552 \text{ pF}
\]
11. Sol: Given data:

Capacitor overlapping area = 5 \times 10^{-4} (m^2)

\( C = 9.5 \) (pF)

\( \varepsilon_r = 81 \)

\( \varepsilon_0 = 8.854 \times 10^{-12} \) (pF/m)

We know that

\[ C = \frac{\varepsilon_0 \varepsilon_r A}{d} \]

\[ d = \frac{\varepsilon_0 \varepsilon_r A}{C} \]

\[ = \frac{8.854 \times 10^{-12} \times 81 \times 5 \times 10^{-4}}{9.5 \times 10^{-12}} = 0.03775 \text{ (m)} \]

\[ d = 37.75 \text{ (mm)} \]

Sensitivity \( S = \frac{\partial C}{\partial d} = -\frac{\varepsilon_0 \varepsilon_r A}{d^2} \]

\[ = -\frac{8.854 \times 10^{-12} \times 81 \times 5 \times 10^{-4}}{(37.75 \times 10^{-3})^2} \]

\[ S = -0.025 \times 10^{-8} \text{ (F/m)} \]

12. Sol: Given data:

Separation distance \( (t_1) \) between diaphragm = 4 mm

For separation distance \( t_1 \) capacitance is 300 pF & oscillator frequency of 100 kHz.

\( P = \text{applied pressure} = 500 \text{ (kN/m^2)} \)

\( \Delta t_2 = \text{average deflection for pressure} \)

\( P = 0.28 \text{ mm} \)

\( t_2 = \text{separation distance after application of pressure} \)

\( t_2 = t_1 - \Delta t_2 \)

\[ = 4 \text{ mm} - 0.28 \text{ mm} \]

\[ = 3.72 \text{ mm} \]
Chapter 14  
Piezo Electric Transducers

### Objective Practice Solutions

01. **Ans:** (a)  
**Sol:** For piezo electric transducer  
\[
\text{frequency } \propto \frac{1}{\text{cable length}}
\]
\[
f_{\text{new}} = \frac{f_{\text{old}}}{2} = \frac{1000}{2} = 500 \text{Hz}
\]

02. **Ans:** (a)  
**Sol:** The output piezo electric transducer is a zero for static pressure.

03. **Ans:** (a)  
**Sol:** For signal conditioning of piezo electric type transducer we require a charge amplifier.

04. **Ans:** (d)  
**Sol:** Piezoelectric transducers is used to measure dynamic pressure measurement while for static its output is zero millivolts.

05. **Ans:** (a)  
**Sol:**  
\[
G = 0.05 \times \frac{V_m}{N}
\]
\[
P = 1.6 \times 10^5 \text{ N/m}^2
\]
  
We know  
\[
\varepsilon = gtp = 0.05 \times 2.5 \times 10^{-3} \times 1.6 \times 10^6
\]
\[
e_0 = 200 \text{ V}
\]

06. **Ans:** (c)  
**Sol:** The piezoelectric transducers vibrate at ultrasonic frequencies. Piezoelectric material is a type of electro acoustic transducer that converts electrical energy into mechanical and vice versa.

07. **Ans:** (a)  
**Sol:** Piezoelectric crystal can be shown as electrical equivalent circuit in terms L and C. Quartz, Rochelle salt, tour maline are piezoelectric crystal. Also piezoelectric crystal exhibits the reverse effect of electrostriction.

08. **Ans:** (c)  
**Sol:** Piezoelectric transducer used for dynamic displacement only and it is useless for static displacement. Piezoelectric materials have low dielectric constant.  
Quartz dielectric constant is 4.2

09. **Ans:** (a)  
**Sol:** BaTiO₃ used in record player. Mechanical stress in piezoelectric material producer on electric polarization and application of electric field produces a mechanical strain.
Conventional Practice Solutions

01.
Sol: Given \( t = 2 \text{ mm} \)

\[
g = 0.05 \text{ Vm/N}
\]

\[
F = 15 \times 10^5 \text{ N/m}^2
\]

We know

\[
g = \frac{E_0}{t F/A}
\]

So \( E_0 = g \times \frac{F}{A} \times t \)

\[
= 0.05 \text{ Vm/N} \times 15 \times 10^5 \text{ N/m}^2 \times 2 \times 10^{-3}
\]

\[
= 150 \text{ V}
\]

02.
Sol: Given data:

Area of given crystal = 36 mm\(^2\)

Thickness of given crystal = 1.5 mm

\[
g = 0.012 \left( \frac{\text{Vm}}{N} \right)
\]

\[
\varepsilon_r = 1400
\]

\[
E = 120 \times 10^{10} \text{ (N/m}^2) \]

\[
F = 10 \text{ N}
\]

(i) Output voltage \( e_0 = g t F/A \)

\[
= 0.012 \times 1.5 \times 10^{-3} \times 10
\]

\[
= 18 \times 10^{-12}
\]

\( e_0 = 5 \text{ V} \)

(ii) Charge sensitivity \( d = g \varepsilon_0 \varepsilon_r \)

\[
= 0.012 \times 8.854 \times 10^{-12} \times 1400
\]

\[
d = 148.75 \text{ (pC/N)}
\]

(iii) Strain \( \varepsilon = \frac{\text{stress}}{E} = \frac{F}{AE} \)

\[
= 0.2315 \times 10^{-6}
\]

= 0.2315 \( \mu \text{strain} \)

(iv) Charge generated \( Q = dF \)

\[
= 148.75 \times 10^{-12} \times 10
\]

\[
Q = 1487.5 \text{ (pC)}
\]

(v) Capacitance of pick up \( C = \frac{Q}{V} \)

\[
= \frac{1487.5 \times 10^{-12}}{5}
\]

= 297.5(pF)

03.
Sol: Charge sensitivity \( d_{33} = d = 150 \times 10^{-12} \text{ C/N} \)

\[
C_{pz} = 25 \times 10^{-12} \text{ F}
\]

\[
R_{pz} = 10^{10} \Omega
\]

Input force = 2N \( u(t) \) (step force)

The Voltage generated

\[
= \text{Charge Generated}
\]

\[
e_{pz} = e_0 = \frac{d \times F}{C} = \frac{q}{\varepsilon}
\]

\[
e_{pz} = \frac{150 \times 10^{-12} \text{ C/N} \times 2 \text{N}}{25 \times 10^{-12}} = 12 \text{V}
\]

The equivalent circuit under the given condition is:

\[
\begin{align*}
\text{DVM} \quad + \\
\text{C}_{pz} \quad R_{pz} \\
\text{R}_{DVM} \quad V_0 \\
\text{DVM} \quad -
\end{align*}
\]
\[
\frac{V_0(S)}{e_{pz}(S)} = \frac{\tau s}{\tau s + 1}
\]

\[
\tau = C_{pz}(R_{pz} || R_{DVM})
\]

\[
\tau = 20\text{PF} \left(10^{10}\Omega \ || \ 10^{13}\Omega\right)
\]

\[
\tau = 25 \times 10^{-12} \times 999 \times 10^9\Omega
\]

\[
\tau = 0.25 \text{ sec}
\]

\[
V_0(t) = e_{pz} \times e^{-t/\tau}
\]

Where \(V_0(t)\) is voltage across the Dvm.

Drop allowed from peak value i.e, 12 v is no more than 0.12v. Time at which \(V_0\) falls to 11.88 V must be calculated.

Assigning \(V_0(t) = 11.88v\)

\[
11.88 = 12 \times e^{-t/\tau}
\]

\[
\frac{11.88}{12} = e^{\frac{-t}{\tau}}
\]

\[
\Rightarrow \ln\left(\frac{11.88}{12}\right) = -\frac{t}{\tau}
\]

\[
\Rightarrow \frac{t}{\tau} = \ln\left(\frac{12}{11.88}\right)
\]

\[
\Rightarrow t = \tau \times \ln\left(\frac{12}{11.88}\right)
\]

\[
= 0.025 \ln\left(\frac{12}{11.88}\right) \text{ seconds}
\]

\[
\Rightarrow t = 2.512 \text{ m Seconds}
\]
Chapter 15 Measurement of Temperature

Objective Practice Solutions

01. Ans: (a)
Sol: Platinum resistance thermometer used to measure temperature in the range –200°C to 1000°C

<table>
<thead>
<tr>
<th>Material</th>
<th>Minimum temperature</th>
<th>Maximum temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platinum</td>
<td>–200°C</td>
<td>1000°C</td>
</tr>
<tr>
<td>Copper</td>
<td>–200°C</td>
<td>150°C</td>
</tr>
<tr>
<td>Nickel</td>
<td>–70°C</td>
<td>150°C</td>
</tr>
<tr>
<td>Tungsten</td>
<td>–200°C</td>
<td>850°C</td>
</tr>
</tbody>
</table>

02. Ans: (d)
Sol: RTD material must have
   (i) High temperature coefficient of resistance
   (ii) Higher resistivity
   (iii) Linear relationship between R and T
   (iv) Stability of the electrical characteristics of the material

03. Ans: (d)
Sol: Platinum has a constant volume of temperature coefficient in 0 to 100°C range. Resistivity of platinum tends to increase less rapidly at higher temperatures. Platinum has stability over higher range of temperature.

04. Ans: (b)
Sol: \( \alpha_{45^\circ C} = \frac{R_2 - R_1}{R_1(T_2 - T_1)} \frac{1}{R_{45^\circ C}} \)

\( R_{45^\circ C} = \frac{R_1 + R_2}{2} = \frac{5 + 6.5}{2} = 5.75 \Omega \)

Here \( R_1 = 5 \Omega \) at \( T_1 = 30^\circ C \)
\( R_2 = 6.5 \Omega \) at \( T_2 = 60^\circ C \)

\( \alpha_{45^\circ C} = \left( \frac{6.5 - 5}{60 - 30} \right) \frac{1}{5.75} = 0.0087 \ (1/{^\circ C}) \)

05. Ans: (d)
Sol: The resistance temperature characteristics of a temperature transducer is related to positive temperature coefficient thermistor.

06. Ans: (d)
Sol: Thermistors are well suited to precision temperature measurement. It is used in range of –100°C to 300°C. It has higher negative temperature coefficient of resistance.

07. Ans: (c)
Sol: A thermistor can exhibit either a negative change of resistance (NTC) or positive change of resistance (PTC) with increase of temperature depending upon the type of material used.

08. Ans: (b)
Sol: \( R = 5000 \Omega \) at \( T = 25^\circ C, \alpha = 0.04 \ (1/{^\circ C}) \)

\( R_{\text{lead}} = R [1 + \alpha (T_{\text{lead}} - T)] \)

\( 10 = 5000 [1 + 0.04 (T - 25)] \)

\( T = 0.05^\circ C \)

09. Ans: (b)
Sol: Thermistors are essentially semiconductor devices that behaves as resistors with high negative temperature coefficient and are atleast 10 times as sensitive as the platinum resistance thermometer.
10. Ans: (b)  
Sol: \( \beta = 3000 \text{ K} \)

\[
R = 1050 \Omega \text{ at } T = 27^\circ \text{C} \\
= 300 \text{ K}
\]

So temperature coefficient of resistances for the thermistor 
\[
\alpha = \frac{-\beta}{T^2} = \frac{-3000}{(300)^2} \\
= -0.033 \left( \frac{\Omega}{\Omega/\circ \text{C}} \right)
\]

11. Ans: (d)  
Sol: In case of thermocouple we required a reference junction compensation to get stable and reliable output. Also thermocouple output is very small.

12. Ans: (a)  
Sol: \( V_0 = \) output of thermocouple = 50 mV  
\( R_i = \) thermocouple internal resistance = 50\( \Omega \)  
\( R_{\text{lead}} = 10\Omega \)  
\( r = \) PMMC internal resistance = 120\( \Omega \)  
so output voltage indicated by PMMC \( V_{\text{PMMC}} \) is

\[
V_{\text{PMMC}} = \frac{r}{r + R_i + R_{\text{lead}}} \times V_0 \\
= \frac{120}{120 + 50 + 10} \times 50 \times 10^{-3} \\
= \frac{120 \times 50 \times 10^{-3}}{180} \\
= 33.33 \text{ mV}
\]

13. Ans: (d)  
Sol:

<table>
<thead>
<tr>
<th>Thermocouple</th>
<th>Temperature range(in°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper-constantan</td>
<td>-200 to 350</td>
</tr>
<tr>
<td>Iron - constantan</td>
<td>-200 to 850</td>
</tr>
<tr>
<td>Alumel-Chromel</td>
<td>-200 to 1100</td>
</tr>
<tr>
<td>Platinum Rhodium</td>
<td>450 to 1500</td>
</tr>
</tbody>
</table>

14. Ans: (b)  
Sol: Iron-constantan thermocouple is most suitable for temperature measurement in the range of 700 °C to 800 °C.

15. Ans: (a)  
Sol: Time to reach equilibrium

Conditions \( 5T = 10 \Rightarrow T = 2 \text{ sec} \)

\[
\theta = \theta_0 \left[ 1 - e^{-t/T} \right] \\
0.5 = \left[ 1 - e^{-t/T} \right] \\
T = 1.39 \text{ sec}
\]
Conventional Practice Solutions

01.

Sol: Temperature of thermistor at 2330 Ω is
Here \( R_T = 2330 \) Ω
\( R_0 = 1050 \) Ω
T = unknown temperature at 2330 Ω
\( T_0 = 27°C = 300°K \)
\[ R_T = R_0 e^{\frac{1}{T} - \frac{1}{T_0}} \]
\[ 0.7971 = 3140 \left( \frac{1}{T} - \frac{1}{300} \right) \]
T = 278.77K
T = 5.77°C
This temperature corresponds to the resistance of 2330 Ω.

Sensitivity \( S = \frac{\text{d}R}{\text{d}T} = R_0 e^{\frac{1}{T} - \frac{1}{T_0}} \times \left( \frac{-\beta}{T^2} \right) \)
= \(-2330 \times 3140\)
278.77²
S = 94.144 (Ω/K)
This is the required sensitivity of the given thermistor at a given operating point (2330 Ω and 278.77°C).

02.

Sol: Given data:
Sensitivity of copper with respect to platinum
\( S_{\text{CuPt}} = 7.4 \left( \frac{\mu V}{°C} \right) \)
Sensitivity of constantan with respect to platinum
\( S_{\text{CuPt}} = 34.4 \left( \frac{\mu V}{°C} \right) \)
Sensitivity of copper with respect to constantan
\( S_{\text{CuCn}} = S_{\text{CuPt}} - S_{\text{CuPt}} \)
= \( 7.4 \left( \frac{\mu V}{°C} \right) - \left( -34.4 \left( \frac{\mu V}{°C} \right) \right) \)
\( S_{\text{CuCn}} = 41.8 \left( \frac{\mu V}{°C} \right) \)

Now corresponding temperature difference in a thermocouple made up of copper and constantan junction for temperature difference of 250°C is
\[ S_{\text{CuCn}} \times 250°C = 41.8 \left( \frac{\mu V}{°C} \right) \times 250°C = 10.45 (mV) \]

03.

Sol: Given data:
\( \alpha = 37.5 \left( \frac{\mu V}{°C} \right) \)
\( \beta = 0.0045 \left( \frac{\mu V}{°C} \right) \)
\( \theta_{\text{hot}} = 200°C \)
\( \theta_{\text{cold}} = 0°C \)

Now we know the thermocouple relationship between thermo-emf set up and temperature difference between hot and cold junction is given by
\[ E = \alpha (\theta_{\text{hot}} - \theta_{\text{cold}}) + \beta (\theta_{\text{hot}}^2 - \theta_{\text{cold}}^2) \]
= \( 37.5 \times 10^{-6} \times (200 - 0) + 0.0045 \times 10^{-6} \times (200^2 - 0^2) \)
= 7.68 mV
04.
Sol:

\[ \text{Thermistor} \]

At 25°C the resistance of thermistor is 10k\(\Omega\)
At 100°C the resistance of thermistor is 1k\(\Omega\)
This thermistor is used in a temperature range of 0-150°C.

\[ P = I^2R = \frac{V^2}{R} \]

\begin{align*}
R_T &= R_0 \\
\ln \left[ \frac{R_T}{R_0} \right] &= \beta \left[ \frac{1}{T} - \frac{1}{T_0} \right] \\
T_0 &= 25°C + 273 = 293\text{K} \quad \text{and} \quad R_0 = 10k\Omega \\
T &= 100 + 273 = 373\text{K} \quad \text{and} \quad R_T = 1k\Omega \\
\ln \left[ \frac{1}{10} \right] &= \beta \left[ \frac{1}{373} - \frac{1}{298} \right] \\
\beta &= 3412.55 \text{K} \\
\text{Now} \\
T_0 &= 100°C + 273 = 373\text{K} \quad \text{and} \quad R_0 = 1K\Omega \\
T &= 150°C + 273 = 473\text{K} \\
\ln \left[ \frac{R_T}{1k\Omega} \right] &= 3412.55 \left[ \frac{1}{423} - \frac{1}{373} \right] \\
R_T &= 339.12 \Omega
\end{align*}

05.
Sol: Given data:

\[ R_1=R_2=R_3=R_4 = 400\Omega; \quad \alpha = 0.042\Omega/\text{°C} \]
\[ T = 30\text{°C}; \quad I = 30\text{mA} \]

Due to temperature rise of 30°C, increase in the resistance of sensor by 

\[ = 0.042 \times 30 \]

\[ = 1.26\Omega \]

So resistance of sensor branch become

\[ = 400 + 1.26 = 401.26\Omega \]

Current through sensor is restricted is 30mA

Voltage across AB terminals 

\[ = 30[400 + 401.26] \times 10^{-3} \]

\[ = 24.038 \approx 24\text{V} \text{ (DC supply)} \]

Thevinin equivalent resistance across AB terminals

\[ = (400 + 401.26)\times(400 + 400) \]

\[ = (801.26)\times(800) \]

\[ = 801.26 \times 800 = 400.3147\Omega \]

\[ = 801.26 + 800 \]
\[ V_{th} = 24 \times \left[ \frac{400}{800} - \frac{400}{801.26} \right] \]
\[ = 24[0.5 -0.499] = 0.0188V \]

\[ \therefore \text{I through the meter} \]

\[ I = \frac{V_{th}}{R_m + R_m} \]
\[ = \frac{0.0188}{400.3147 + 100} = 37.58 \mu\text{A} \]

Deflection of the meter = \( 37.58 \times 2^\circ = 75.16^\circ \)

Sensitivity 10 mV/°C means change in 1°C in RTD corresponding to 10 mV change in Bridge output.

\[ R_T = R_0 [1+\alpha T] \]
\[ = 100[1+0.00392(1-0)] \]
\[ R_T = 100.392\Omega \]

10 mV = \( A_d [V_1 - V_2] \)

\[ V_1 - V_2 = \left( \frac{R_4}{R_2 + R_4} \right)V - \left( \frac{R_3}{R_1 + R_3} \right)V \]
\[ = 10\left[ \frac{150.392}{100.392 + 10k} - \frac{100}{100 + 10k} \right] \]
\[ = 0.3842 \text{ mV} \]

\[ A_d = \frac{10}{0.3842} = 26.02 \]

06.
Sol:
Chapter 16 Measurement of Flow, Viscosity, Humidity

Objective Practice Solutions

01. Ans: (d)
Sol: In case of rotameter with increase in the flow rate, the float rises in the tube and there occurs an increase in the annular area between the float and the tube. So we can say that rotameter is a variable area device.

02. Ans: (a)
Sol: Flow rate in pitot tube is
\[ Q = AV = A \sqrt{\frac{2P_d}{\rho}} \]
\[ \theta \propto \sqrt{P_d} \]

03. Ans: (c)
Sol:

<table>
<thead>
<tr>
<th>Restrictors</th>
<th>Discharge coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orifice tube</td>
<td>0.60</td>
</tr>
<tr>
<td>Venturi tube</td>
<td>0.98</td>
</tr>
<tr>
<td>Flow nozzle</td>
<td>0.80</td>
</tr>
</tbody>
</table>

04. Ans: (b)
Sol: Rotary vane type transducer is an example of positive displacement flow meter.

05. Ans: (b)
Sol: Venturimeter has the lowest pressure drop for a given range of flow because venturimeter has highest coefficient of discharge about 0.98.

06. Ans: (a)
Sol: In Rota meter, flow rate is directly proportional to height.

\[ Q \propto h \]
\[ \therefore \frac{Q_1}{Q_2} = \frac{h_1}{h_2} = \frac{70\text{ mm}}{20\text{ mm}} = \frac{7}{2} = 3.5 \]

07. Ans: (d)
Sol: During the flow through an orifice meter, the fluid jet on leaving the orifice contracts to minimum area at a section called vena-contracta, area of fluid jet at vena contracta is less than areas of the orifice & the two are related as
area at vena contracta = \( C_d \times \) orifice area.

\[ = 0.6 \left( \frac{\pi d^2}{4} \right) \]
\[ C_d \text{ of orifice} = 0.6 \]
\[ \text{Area of orifice} = \frac{\pi d^2}{4} \]

08. Ans: (a)
Sol: Pressure at throat of a venturi tube is lower compare to upstream pressure. While velocity at throat of a venture tube is higher compare to velocity of flow at up stream.

09. Ans: (a)
Sol: In case of rotameter the weight of the float is balanced by the buoyancy and the drag force acting on the float. Volume flow rate sensitive to density changes of the fluid. By using rotameter volume flow rate of gas can be measured.
10. Ans: (c)
Sol: Flowing fluid density affected in orifice plate, rotameter, pitot static tube meter white flowing fluid density is not matter for measurement of flow in non obstruction type meter like electromagnetic flow meter.

11. Ans: (a)
Sol: When an electrically heated wire is placed in a flowing gas stream, heat is transferred from wire to the gas and hence the temperature of the wire reduces and due to this the resistance of the wire also changes. This change in resistance of the wire becomes a measure of flow rate.

12. Ans: (a)
Sol: Hot wire anemometer gives good result when the flowing fluid is exceptionally clean.

13. Ans: (c)
Sol: The turbine type flow meter used to measure totalisation of flow.

14. Ans: (b)
Sol: In case of electromagnetic flow meter the induced voltage is proportional to flow rate as
\[ e = \frac{B/V}{A} = \frac{B/Q}{A} \]
\[ e \propto Q \]
<table>
<thead>
<tr>
<th>Objective Practice Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>01. Ans: (d)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> In seismic vibration sensor for measuring amplitude of vibration ( \omega_n \ll \omega ) &amp; slightly less than 1.</td>
</tr>
<tr>
<td><strong>02. Ans: (d)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> ( \omega_n = \sqrt{\frac{K}{M}} )</td>
</tr>
<tr>
<td>Decreasing the mass in case of a seismic acceleration sensor while keeping all other parameters constant will increase the natural frequency, without affecting steady state sensitivity.</td>
</tr>
<tr>
<td><strong>03. Ans: (a)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> ( M = 100 , \mu g )</td>
</tr>
<tr>
<td>( F_n = 1 , kHz )</td>
</tr>
<tr>
<td><strong>04. Ans: (a)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> ( f = 100 , Hz )</td>
</tr>
<tr>
<td>( X = 10 , mm )</td>
</tr>
<tr>
<td>Peak acceleration of the seismic mass = ( \omega^2 x )</td>
</tr>
<tr>
<td>= ( (2\pi f)^2 x )</td>
</tr>
<tr>
<td>= ( (2\pi \times 100)^2 \times 10 \times 10^{-3} )</td>
</tr>
<tr>
<td>= 3947.84 (m/sec²)</td>
</tr>
<tr>
<td><strong>05. Ans: (a)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> Accelerometer input range</td>
</tr>
<tr>
<td>= 0 m/sec² to 98.1 (m/sec²)</td>
</tr>
<tr>
<td>( F = 30 , Hz )</td>
</tr>
<tr>
<td><strong>06. Ans: (b)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> In dc tachogenerators used for measurement of speed of a shaft, frequent calibration has to be done because the strength of permanent magnet decreases with age.</td>
</tr>
<tr>
<td><strong>07. Ans: (c)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> In a drag up type ac tachogenerator, the output voltage is modulated waveform.</td>
</tr>
<tr>
<td><strong>08. Ans: (b)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> ( n_{\text{teeth}} = 60 )</td>
</tr>
<tr>
<td>( N = \text{speed of shaft} )</td>
</tr>
<tr>
<td>( N = 25 , rps )</td>
</tr>
<tr>
<td>We know</td>
</tr>
<tr>
<td>Speed of shaft in rps = ( \frac{\text{pulse rate}}{n_{\text{teeth}}} )</td>
</tr>
<tr>
<td>( 25 = \frac{\text{pulses per second}}{50} )</td>
</tr>
<tr>
<td>Pulses per second = 25 \times 60</td>
</tr>
<tr>
<td>= 1500</td>
</tr>
<tr>
<td><strong>09. Ans: (b)</strong></td>
</tr>
<tr>
<td><strong>Sol:</strong> ( f_r = ) rotating frequency of motor</td>
</tr>
</tbody>
</table>
10. Ans: (c)
Sol: \[ \text{rpm} = \left( \frac{\text{pulses per second}}{\text{number of teeth}} \right) \times 60 \]
\[ \text{rpm} = \left( \frac{\text{flash per minute}}{\text{number of teeth}} \right) \]
\[ N = \frac{F}{n} \]

So star mark moves at a speed of 30 rpm against the direction of rotation.