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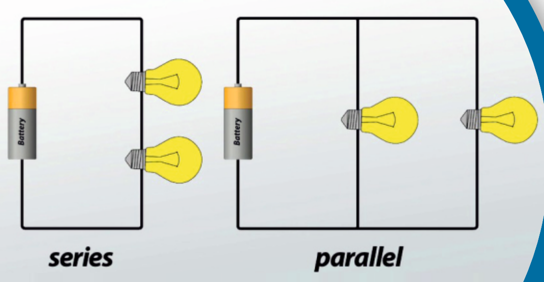


ELECTRONICS & TELECOMMUNICATION ENGINEERING

NETWORK THEORY

Text Book : Theory with worked out Examples and Practice Questions

Chapter 1 Basic Concepts

(Solutions for Text Book Practice Questions)

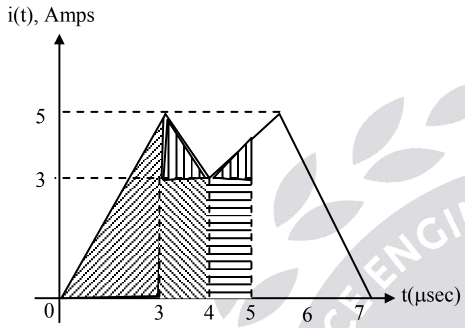
Objective Practice Solutions

01. Ans: (c)

Sol: We know that;

$$i(t) = \frac{dq(t)}{dt}$$

$$dq(t) = i(t).dt$$



$$q = \int_0^{5\mu sec} i(t)dt = \text{Area under } i(t) \text{ upto } 5\ \mu sec$$

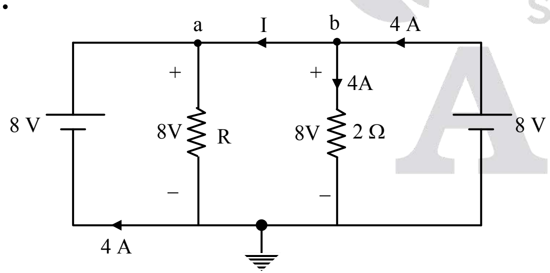
$$q = q_1| + q_2| + q_3|$$

$$= \left(\frac{1}{2}\times3\times5\right)+\left(\frac{1}{2}\times1\times2+(1\times3)\right)+\left(\frac{1}{2}\times1\times1+(1\times3)\right)$$

$$q = 15\mu c$$

02. Ans: (a)

Sol:



Applying KCL at node 'b'

$$I + 4 = 4$$

$$\Rightarrow I = 0A$$

And $\frac{8}{R} = 4$

$$\Rightarrow R = 2\Omega$$

03. Ans: (a)

Sol: The energy stored by the inductor (1Ω, 2H) upto first 6 sec:

$$E_{\text{stored upto 6sec}} = \int P_L dt$$

$$= \int\left(L\frac{di(t)}{dt}.i(t)\right)dt$$

$$= \int_0^2\left(2\left[\frac{d}{dt}(3t)\right]\times3t\right)dt + \int_2^4\left(2\left[\frac{d}{dt}(6)\right]\times6\right)dt$$

$$+\int_4^6\left(2\left[\frac{d}{dt}(-3t+18)\right]\times(-3t+18)\right)dt$$

$$= \int_0^2 18t\,dt + \int_2^4 0\,dt + \int_4^6\left(-6\left[-3t+18\right]\right)dt$$

$$= 36 + 0 - 36 = 0\ J$$

(or)

$$E_{\text{stored upto 6sec}} = E_L|_{t=6sec}$$

$$= \frac{1}{2}L\left(i(t)|_{t=6}\right)^2$$

$$= \frac{1}{2}\times2\times0^2 = 0\ J$$

04. Ans: (d)

Sol: The energy absorbed by the inductor (1Ω, 2H) upto first 6sec:

$$E_{absorbed} = E_{dissipated} + E_{stored}$$

Energy is dissipated in the resistor

$$E_{dissipated} = \int P_R dt = \int (i(t))^2 R\ dt$$

$$= \int_0^2 (3t)^2\times1dt + \int_2^4 (6)^2\times1dt + \int_4^6(-3t+18)^2\times1dt$$

$$= \int_0^2 9t^2\,dt + \int_2^4 36dt + \int_4^6(9t^2+324-108t)dt$$

$$= 24 + 72 + 24$$

$$= 120J$$

$\therefore E_{dissipated} = 120$ J

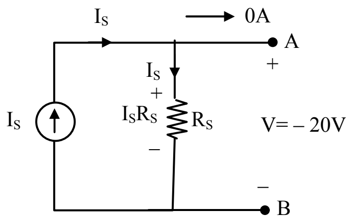
And $E_{stored\ upto\ 6\ sec} = 0\,J$

$\therefore E_{absorbed} = E_{dissipated} + E_{stored}$

$\Rightarrow E_{absorbed} = 120J + 0J = 120J$

05. Ans: (a)

Sol: Point (–20, 0) $\Rightarrow$ V = – 20V and I = 0A

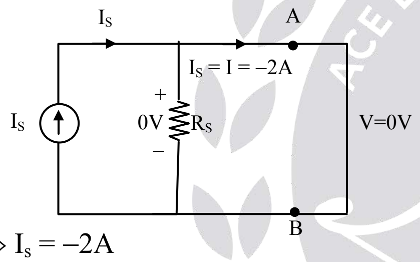


By KVL $\Rightarrow I_S R_S - V = 0$

$\Rightarrow I_SR_S + 20 = 0$

$\Rightarrow I_SR_S = -20V$ ……….. (1)

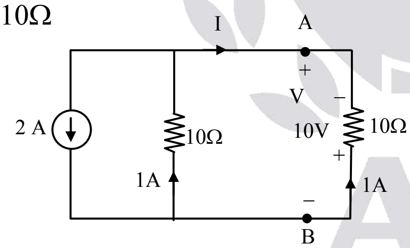
Point: (0, –2) $\Rightarrow$ V = 0V and I = –2A



$\Rightarrow I_s = -2A$

Substituting I_s in eq (1)

$R_S = 10\Omega$

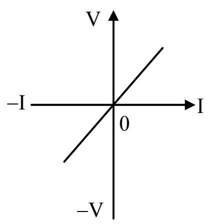


From the diagram;

I = –1A and V = –10V

06. Ans: (a)

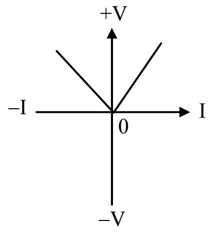
Sol:



* linear
* Passive
* bilateral

07. Ans: (b)

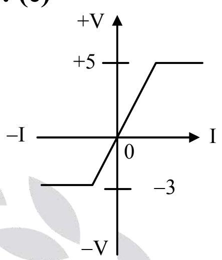
Sol:



* Non linear
* Active
* Unilateral

08. Ans: (e)

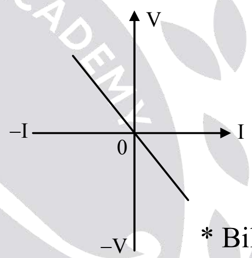
Sol:



* Non linear
* Passive
* Unilateral

09. Ans: (c)

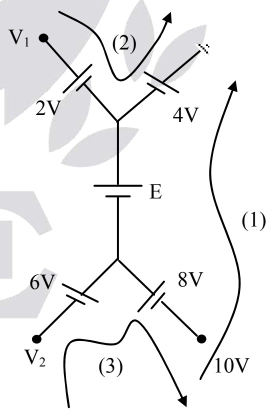
Sol:



* Linear
* Active
* Bilateral

10.

Sol:



(1) By KVL $\Rightarrow + 10 + 8 + E + 4 = 0$

$E = -22V$

(2) By KVL $\Rightarrow + V_1 - 2 + 4 = 0$

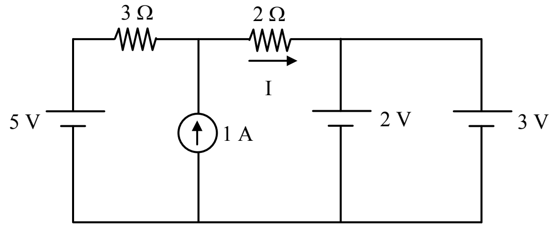
$V_1 = -2V$

(3) By KVL $\Rightarrow + V_2 + 6 - 8 - 10 = 0$

$V_2 = 12V$

11. Ans: (d)

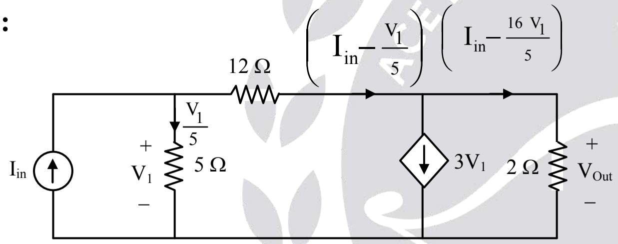
Sol:



Here the 2V voltage source and 3V voltage source are in parallel which violates the KVL. Hence such circuit does not exist. (But practical voltage sources will have some internal resistance so that when two unequal voltage sources are connected in parallel current can flow and such a circuit may exist).

12. Ans: (d)

Sol:



Applying KVL,

$$-V_1 + 12\left(I_{in} - \frac{V_1}{5}\right) + 2\left(I_{in} - \frac{16V_1}{5}\right) = 0$$

$$-V_1 + 12I_{in} - \frac{12V_1}{5} + 2I_{in} - \frac{32V_1}{5} = 0$$

$$14I_{in} = \frac{49}{5}V_1$$

$$\Rightarrow V_1 = \frac{70}{49}I_{in} \qquad \text{……….. (1)}$$

$$\therefore V_{out} = 2\left(I_{in} - \frac{16V_1}{5}\right) \qquad \text{………. (2)}$$

Substitute equation (1) in equation (2)

$$V_{out} = 2\left(I_{in} - \frac{16}{5} \times \frac{70}{49} I_{in}\right)$$

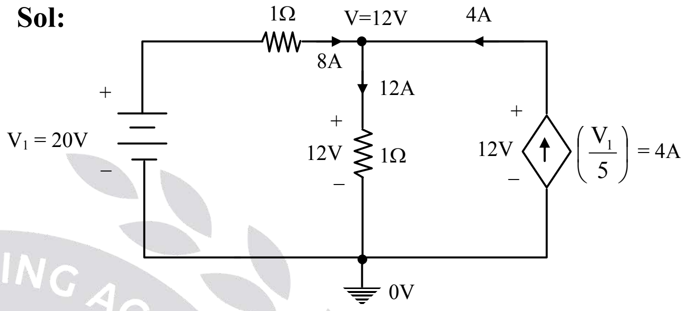
$$= 2\left(\frac{-25}{7}\right) I_{in}$$

$$= \frac{-50}{7} I_{in}$$

$$\therefore V_{out} = -7.143\ I_{in}$$

13. Ans: (c)

Sol:



By nodal ⇒

$V - 20 + V - 4 = 0$

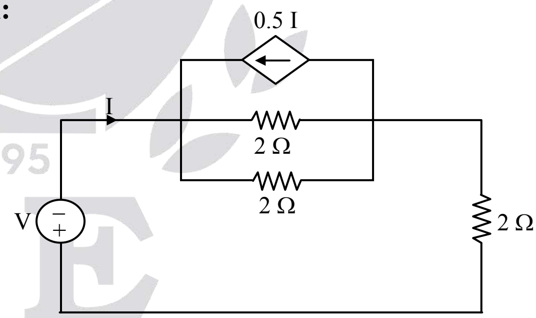
$V = 12$volts

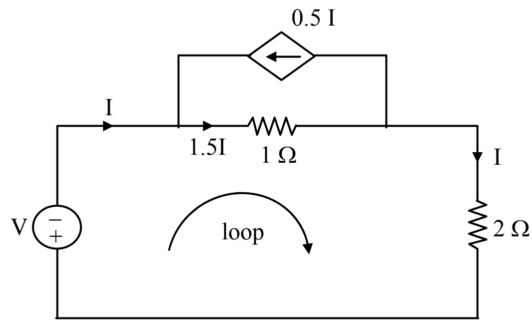
Power delivered by the dependent source is

$P_{del} = (12 \times 4) = 48$ watts

14. Ans: (d)

Sol:





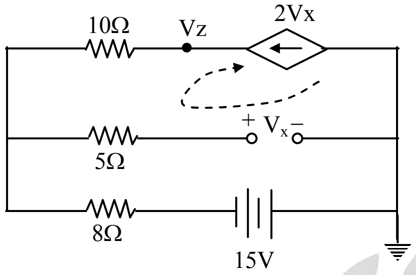
Applying KVL,

$\Rightarrow V + 1.5I + 2I = 0$

$\Rightarrow V = -3.5\ I$

15. Ans: (c)

Sol:



By using KCL

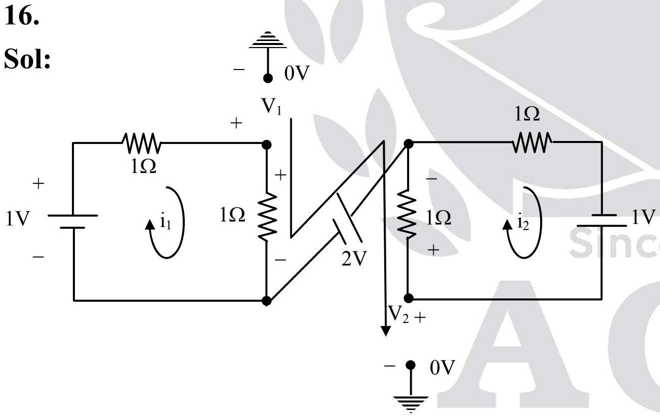
$$\frac{V_x + 15}{8} - 2V_x = 0 \Rightarrow V_x = IV$$

By using nodal Analysis at V_z node

$$\frac{V_z + 15}{18} - 2 = 0 \Rightarrow V_z = +21V$$

16.

Sol:



By KVL $\Rightarrow 1 - i_1 - i_1 = 0$

$i_1 = 0.5A$

By KVL $\Rightarrow - i_2 - i_2 + 1 = 0$

$i_2 = 0.5A$

By KVL $\Rightarrow V_1 - 0.5 + 2 + 0.5 - V_2 = 0$

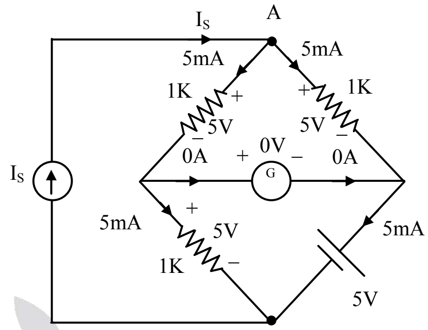
$V_2 = V_1 + 2\ V$

17.

Sol: As the bridge is balanced; voltage across (G) is "0V".

By KCL at node "A" $\Rightarrow - I_s + 5m + 5m = 0$

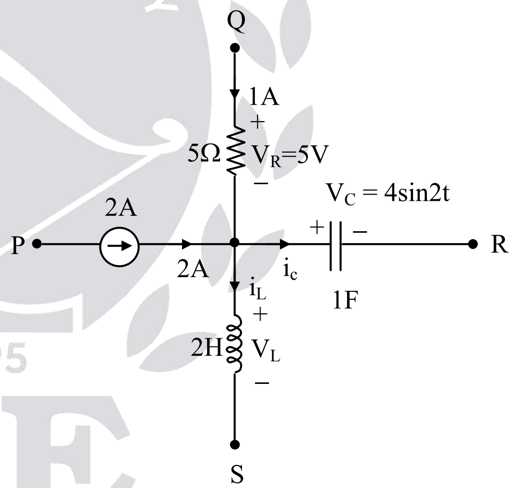
$I_S = 10mA$



18.

Sol: Given data:

$V_R = 5V$ and $V_C = 4\sin 2t$ then $V_L = ?$



$$i_c = \frac{CdV_c}{dt} = \frac{d}{dt}(4\sin 2t) = 8\cos 2t$$

By KCL; $-1 - 2 + i_L + i_c = 0$

$i_L = 3 - 8\cos 2t$

We know that;

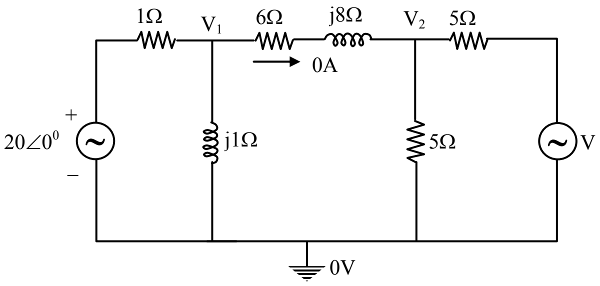
$$V_L = L\frac{di_L}{dt} = 2\frac{d}{dt}(3 - 8\cos 2t)$$

$$= 2(-8)(-2)\sin 2t$$

$V_L = 32\sin 2t$ volt

19.

Sol: V = ? If power dissipated in 6Ω resistor is zero.



$P_{6\Omega} = 0$ W (Given)

$\Rightarrow i_{6\Omega}^2 .6 = 0$

$\Rightarrow \quad i_{6\Omega} = 0 \ (V_{6\Omega} = 0)$

$$\frac{V_1 - V_2}{6 + j8} = 0;\ V_1 = V_2$$

By Nodal $\Rightarrow$

$$\frac{V_1 - 20\angle 0^0}{1} + \frac{V_1}{j1} + 0 = 0$$

$V_1 = 10\sqrt{2}\ \angle 45^0 = V_2$

By Nodal $\Rightarrow$

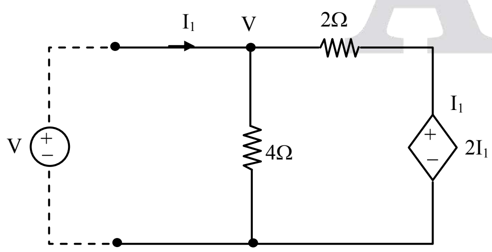
$$0 + \frac{V_2}{5} + \frac{V_2 - V}{5} = 0$$

$V = 2V_2 = 2(10\sqrt{2}\ \angle 45^0)$

$\therefore V = 20\sqrt{2}\ \angle 45^0$

20. Ans: (d)

Sol:



Note: Since no independent source in the network, the network is said to be unenergised, so called a DEAD network".

The behavior of this network is a load resistor behavior.

By Nodal $\Rightarrow$

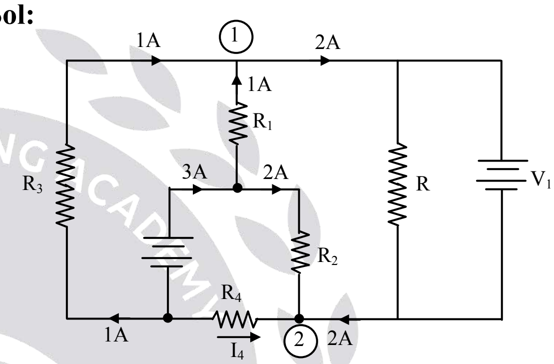
$$-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$3V = 8I_1$

$$R_{eq} = \frac{V}{I_1} = \frac{8}{3}\Omega$$

21. Ans: (a)

Sol:



Apply KCL at Node – 1,

$I = I_{R1} + I_{R3} = 1 + 1 = 2A$

Apply KCL at Node – 2,

$I_4 = -I_2 - I = -2 - 2 = -4A$

22.

Sol:

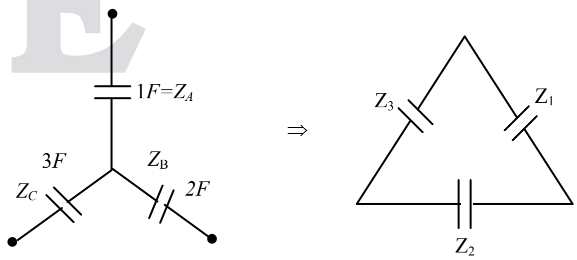


Fig.1

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A Z_B}{Z_C}\right)$$

$$= \frac{1}{s} + \frac{1}{2s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{2s}\right)}{\left(\frac{1}{3s}\right)}$$

$$Z_1 = \frac{1}{s\left(\frac{1}{3}\right)}\ ; \qquad C = \frac{1}{3}F$$

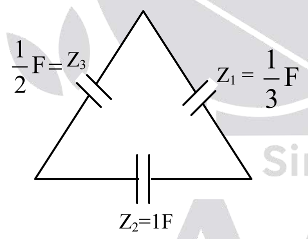
$$Z_2 = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} = \frac{1}{2s} + \frac{1}{3s} + \frac{\left(\frac{1}{2s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{s}\right)}$$

$$Z_2 = \frac{1}{S(1)}\ ; C = 1F$$

$$Z_3 = Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$$

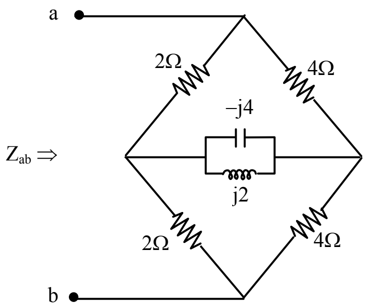
$$= \frac{1}{s} + \frac{1}{3s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{2s}\right)}$$

$$Z_3 = \frac{1}{s\left(\frac{1}{2}\right)}\ ; C = \frac{1}{2}F$$

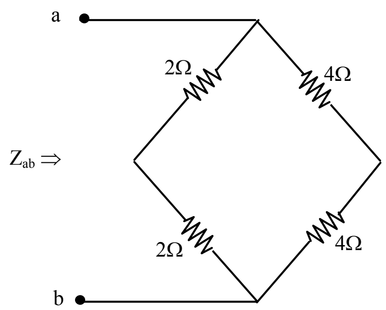


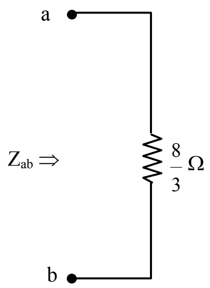
23.

Sol: $Z_{ab} = ?$



Since 2 * 4 = 4 * 2; the given bridge is balanced one, therefore the current through the middle branch is zero. The bridge acts as below :

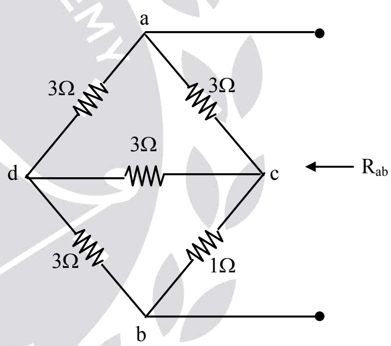




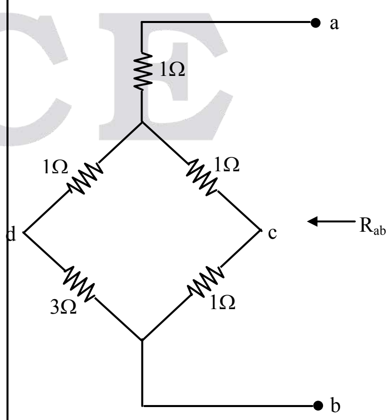
$$Z_{ab} = \frac{4\times 8}{4+8} = \frac{8}{3}\,\Omega$$

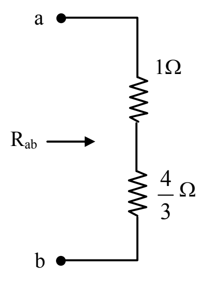
24.

Sol: Redraw the circuit diagram as shown below:



Using Δ to star transformation:

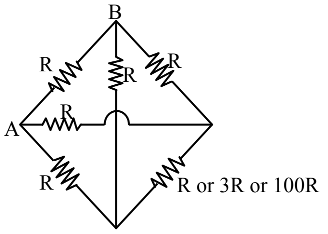


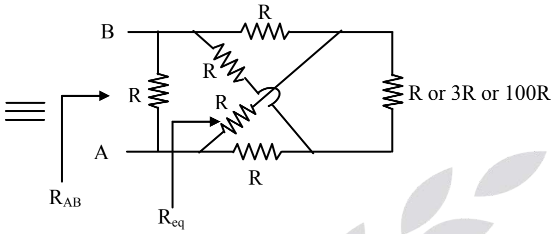


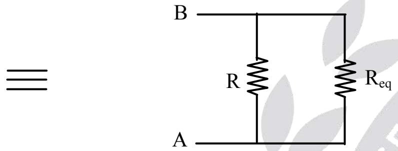
$$\therefore\ R_{ab} = 1 + \frac{4}{3} = \frac{7}{3}\,\Omega$$

25.

Sol: On redrawing the circuit diagram







As bridge is balanced

So $R_{AB}=R \| R_{eq}=R \| R=R/2$

26. Ans: (b)

Sol: The equivalent capacitance across a, b is calculated by simplifying the bridge circuit as shown in Fig. 1 to Fig. 5. $[\because C = 0.1\mu F]$

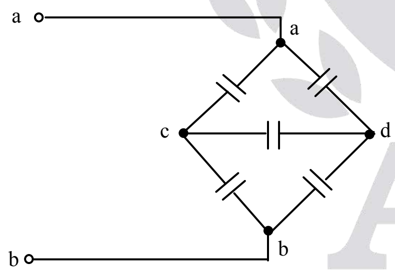
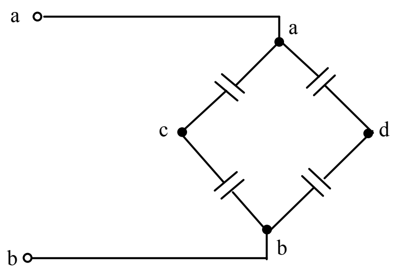
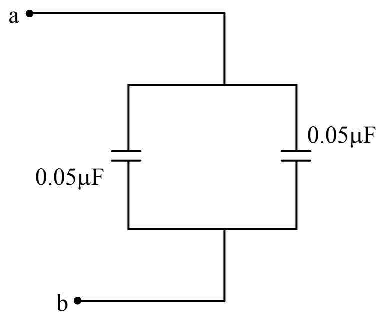


Fig. 1



$$= \frac{0.1\times 0.1}{0.2} = 0.05\mu F$$

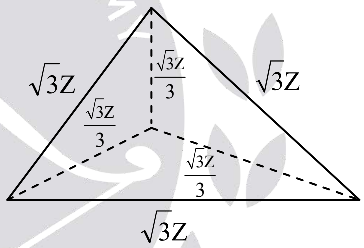


$C_{ab} = 0.1\ \mu F$

Note: The bridge is balanced and the answer is easy to get.

27. Ans:(a)

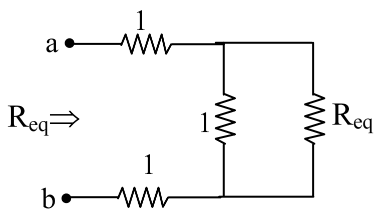
Sol: Consider a Δ connected network



Then each branch of the equivalent λ connected impedance is $\dfrac{\sqrt{3}Z}{3} = \dfrac{Z}{\sqrt{3}}$

28. Ans: (a)

Sol: Network is redrawn as



$$R_{eq} = 1+1+\frac{R_{eq}}{1+R_{eq}}$$

$$= 2 + \frac{R_{eq}}{1+R_{eq}} = \frac{2+2R_{eq}+R_{eq}}{1+R_{eq}}$$

$$R_{eq} + R_{eq}^2 = 2 + 3R_{eq}$$

$$R_{eq}^2 - 2R_{eq} - 2 = 0$$

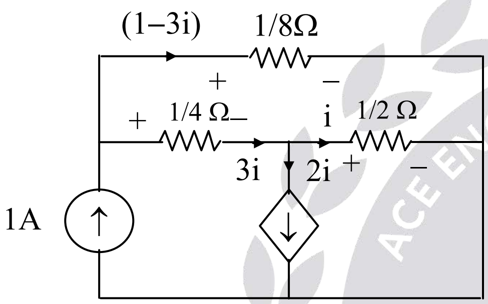
$$R_{eq} = \left(1+\sqrt{3}\right)\Omega$$

29. Ans: (c)

Sol: Applying KCL

$I_{0.25\Omega} = 2i + i = 3i$

$I_{0.125\Omega} = (1 - 3i)$ A



Applying KVL in upper loop.

$$-\frac{(1-3i)}{8} + \frac{i}{2} + \frac{3i}{4} = 0$$

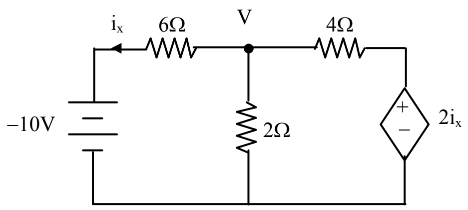
$$\frac{5i}{4} = \frac{1-3i}{8} \Rightarrow 10i = 1 - 3i$$

$$\therefore \qquad i = \frac{1}{13}\text{A}$$

$$V = \frac{3i}{4} = \frac{3}{4} \times \frac{1}{13} = \frac{3}{52}\text{V}$$

30. Ans: (a)

Sol:



Applying KCL at Node V

$$\frac{V}{2} + \frac{V-2i_x}{4} + i_x = 0 \qquad \text{……….. (1)}$$

$$i_x = \frac{V+10}{6} \Rightarrow V = 6i_x - 10$$

Put in equation (1), we get

$3i_x - 5 + i_x - 2.5 + i_x = 0$

$5i_x = 7.5$

$i_x = 1.5$A

$V = -1$V

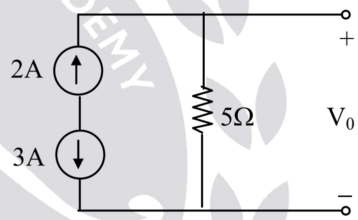
$$I_{\text{dependent souce}} = \frac{V-2i_x}{4} = \frac{-1-3}{4} = -1\text{A}$$

$\therefore$ Power absorbed = ($I_{\text{dependent source}}$) ($2i_x$)

= (–1) (3) = –3W

31. Ans: (d)

Sol: V_0 = ?



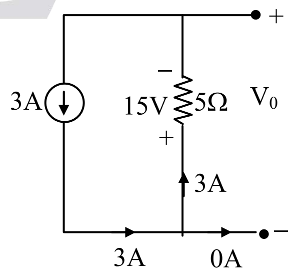
By KCL ⇒ +2 + 3 = 0

+ 5 ≠ 0

Since the violation of KCL in the circuit; physical connection is not possible and the circuit does not exist.

32. Ans: (b)

Sol: Redraw the given circuit as shown below:



By KVL ⇒

$-15 - V_0 = 0$

$V_0 = -15$V

33. Ans: (d)

Sol: Redraw the circuit diagram as shown below:

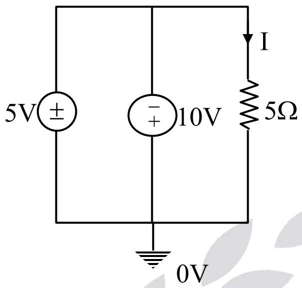
Across any element two different voltages at a time is impossible and hence the circuit does not exist.

Another method:

By KVL ⇒

5 + 10 = 0

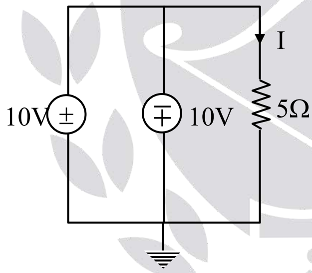
15 ≠ 0



Since the violation of KVL in the circuit, the physical connection is not possible.

34. Ans: (d)

Sol: Redraw the given circuit as shown below:



By KVL ⇒

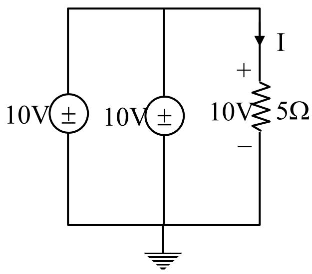
–10 –10 = 0

–20 ≠ 0

Since the violation of KVL in the circuit, the physical connection is not possible.

35. Ans: (b)

Sol: Redraw the given circuit as shown below:



By KVL ⇒

10 – 10 = 0

0 = 0

KVL is satisfied

$I_{5\Omega} = \frac{10}{5} = 2A$

$I_{5\Omega} = 2A$

36. Ans: (d)

Sol:

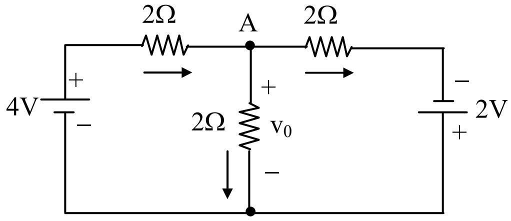


Fig. 1

The diode is forward biased. Assuming that the diode is ideal, the Network is redrawn with node A marked as in Fig. 1.

Apply KCL at node A

$$\frac{4-v_0}{2} = \frac{v_0}{2} + \frac{v_0+2}{2}$$

$$\frac{3v_0}{2} = 1$$

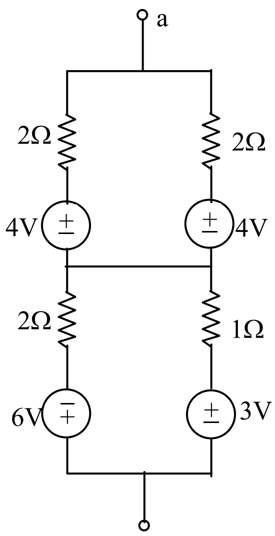
$$v_0 = \frac{2}{3}V$$

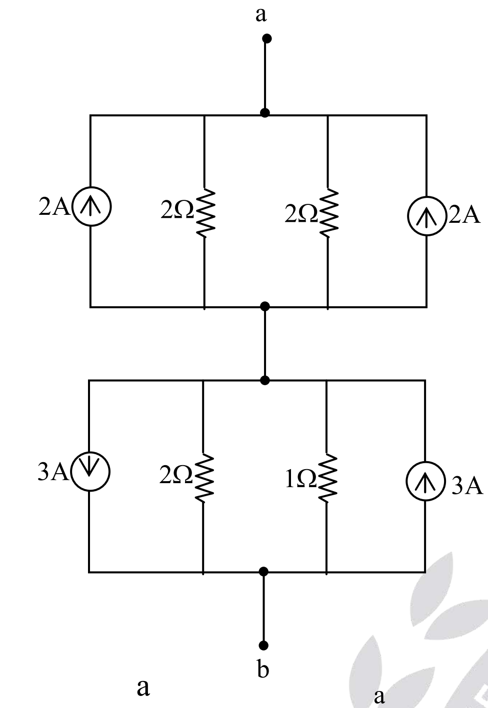
(Here polarity is different what we assume so

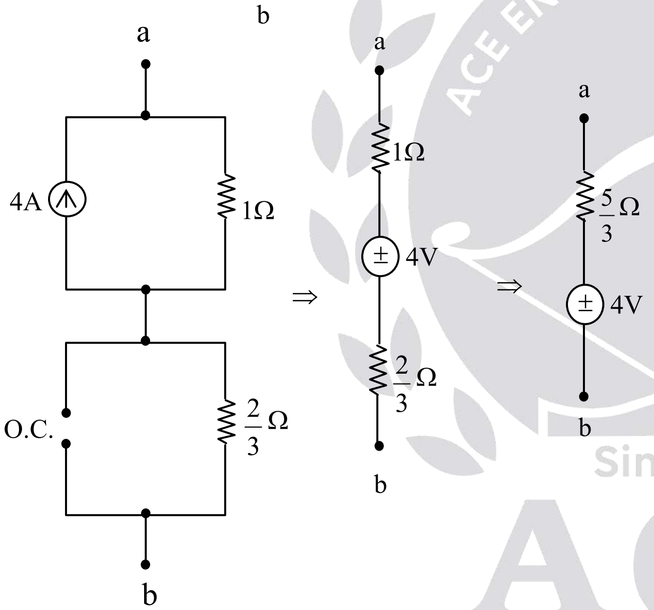
$$V_0 = \frac{-2}{3}V$$

37.

Sol: The actual circuit is

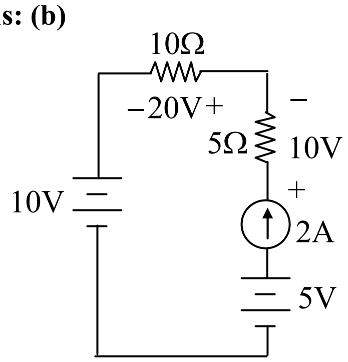






38. Ans: (b)

Sol:

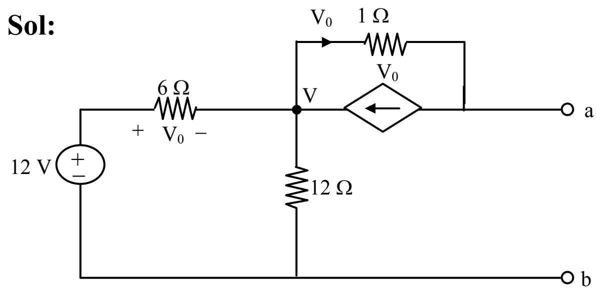


Voltage across 2A = 10 + 20 + 10 – 5 = 35 V

$\therefore$ Power supplied = VI = 35× 2 = 70 W

39. Ans :(d)

Sol:



Applying KCL at node V

$$\frac{V-12}{6}+\frac{V}{12}-V_0+V_0=0$$

$$\Rightarrow \frac{V}{6}+\frac{V}{12}=2 \Rightarrow V = 8V$$

$\therefore$ $V_0 = 4V$

Applying KVL in outer loop

$\Rightarrow -V+1(V_0)+V_{ab}=0$

$\Rightarrow V_{ab} = V - V_0 = 8 - 4 = 4V$

40.

Sol: By KVL

$\Rightarrow V_i - 6 - 10 = 0$

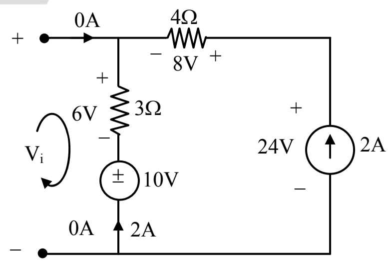
$V_i = 16V$

$P_{4\Omega} = (8 * 2) = 16$watts – absorbed

$P_{2A} = (24 * 2) = 48$ watts delivered

$P_{3\Omega} = (6*2) = 12$ watts – absorbed

$P_{10V} = (10 * 2) = 20$ watts – absorbed



Since; $P_{del} = P_{abs} = 48$ watts. Tellegen's Theorem is satisfied.

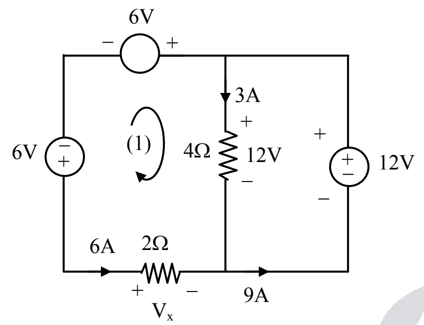
41.

Sol: By KVL in first mesh

$\Rightarrow V_x - 6 + 6 - 12 = 0$

$V_x = 12V$

$P_{12v} = (12 \times 9) = 108$ watts delivered



$P_{4\Omega} = (12 \times 3) = 36$ watts – absorbed

$P_{6V} = (6 \times 6) = 36$ watts – absorbed

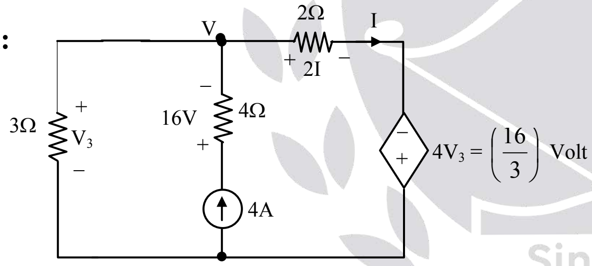
$P_{6V} = (6 \times 6) = 36$ watts – delivered

$P_{2\Omega} = (12\times6) = 72$ watts – absorbed

Since $P_{del} = P_{abs;}$ Tellegen's theorem is satisfied.

42.

Sol:



By Nodal $\Rightarrow$

$$\frac{V}{3} - 4 + \frac{V}{2} + \frac{4V_3}{2} = 0$$

$$\frac{5V}{6} = 4 - 2V_3 \qquad \text{………. (1)}$$

By KVL $\Rightarrow$

$V_3 - 2I + 4V_3 = 0$

$5V_3 - 2I = 0$ ……….. (2)

By KVL $\Rightarrow$

$V = V_3$ ………. (3)

Substitute (3) in (1), we get

$$V_3 = \frac{24}{17}$$

$$V_3 = \frac{24}{17} \text{ Volt and } I = \frac{60}{17} \text{ A}$$

$P_{3\Omega} = 0.663$W absorbed

$P_{4\Omega} = 64$W absorbed

$P_{4A} = 69.64$W delivered

$P_{2\Omega} = 24.91$W absorbed

$P_{4V3} = 19.92$Wdelivered

Since $P_{del} = P_{abs} = 89.57$W; Tellegen's Theorem is satisfied.

43. Ans: (c)

Sol: $V_C = V_0 + \frac{1}{c}\int_0^t i_c(t)dt$

$0 < t < 1$:

$i_c(t) = 2t$ and

$V_0 = 0V$

$$\therefore V_C = 0 + \frac{1}{1/2}\int_0^1 2t dt$$

$$= 2t^2\Big|_0^1$$

$\therefore V_C = 0V$ at t=0

$= 2V$ at t= 1

And V_C varies as parabolic

Continue to do like this with initial condition.

44. Ans: (c)

Sol: KCL as well as KVL are applicable to any lumped electric circuit at any time 't'. Statement I is True.

The sum of the rms currents at any junction of the circuit is not zero in general. It depends upon the nature of the elements connected at the junction.

Statement II is false.

45. Ans: (d)

Sol: Δ–Y transformations are true for any arbitrary frequency, ω. Statement I is False.

Impedances in Δ–Y vary with frequency. Statement II is True.

46. Ans: (a)

Sol: $q = \int_{0_-}^{0_+} i(t)\, dt = \int_{0_-}^{0_+} \delta(t)\, dt = 1$ Coulomb

Across capacitor, $v = \frac{q}{C} = \frac{1}{C}$

Energy inserted instantly from

$t = 0^-$ to $t = 0^+$

$$= \frac{1}{2} C v^2 = \frac{1}{2} C \frac{1}{C^2} = \frac{1}{2C} \text{ J}$$

Statement I is True, Statement II is also True and is the correct explanation.

47. Ans: (b)

Sol: If there are (n + 1) nodes in a NW, by selecting a datum or reference node.

The node pair voltages of all the other n-nodes wrt this datum node are identified.

By knowing $\vec{V} - \vec{I}$ relation of the branch KCL is used at each of the n-nodes to obtain a set of n-simultaneous independent equations in n-voltage variables, which when solved will provide information concerning the magnitudes and phase angles of the voltages across each branch.

The ideal generator maintains a constant voltage amplitude and wave-shape regardless of the amount of current it supplies to the circuit.

$\therefore$ Both Statement I and Statement II are true and statement II is not the correct explanation of Statement I.

48. Ans: (a)

Sol: All networks made up of passive, linear time invariant elements are reciprocal. Not only passivity and time-invariance but also linearity of elements is necessary to guarantee the reciprocity of the NW.

$\therefore$ Statement I is true. Statement II is also true and correctly explains.

49. Ans: (b)

Sol: Duals:

A. Mesh $\rightarrow$ Node (4)

B. Outside mesh $\rightarrow$ Reference node (3)

C. Mesh current $\rightarrow$ Node voltage (2)

D. Number of meshes $\rightarrow$ Number of nodes (1)

50. Ans: (b)

Sol: In Duality resistance equivalent to conductance

Inductance equivalent to capacitance

Loop current equivalent to node pair voltages

Number of loops equivalent to number of node pairs.

51. Ans: (a)

Sol: (A) $\frac{R}{L} = \frac{1}{\tau}$ $\rightarrow$ (Second)$^{-1}$ (4)

(B) $\frac{1}{LC} = \omega^2$ $\rightarrow$ (Radian/second)^2 (3)

(C) $CR = \tau$ $\rightarrow$ Second (1)

(D) $\sqrt{\frac{L}{C}} = R$ $\rightarrow$ Ohm (2)

Conventional Practice Solutions

01.

Sol: C = 30 mF

For $0 \le t \le 2$, $\frac{dv(t)}{dt} = 5\ V/ms$

$i(t) = C\frac{dv(t)}{dt}$

$\therefore$ i (0.5 ms) = $30 \times 10^{-3} \times 5 \times 10^3$

= 150 A

$\left.\frac{dv(t)}{dt}\right|_{t=2.5\ ms} = 0$

$\therefore$ i (2.5 ms) = 0

E = Energy delivered by the source till 7 ms

$= \frac{1}{2}CV^2\ (7\,ms)$

$= \frac{1}{2} \times 30 \times 10^{-3} \times (5)^2 = 0.375$ J

02.

Sol: The currents in all the branches are marked as shown in Fig.

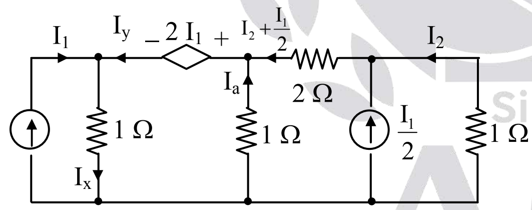


Fig.

$$I_y = \frac{I_1}{2} + I_2 + I_a,$$

$$I_x = \frac{3I_1}{2} + I_2 + I_a$$

Inner Mesh equation:

$I_a \times 1 + 2I_1 + I_x \times 1 = 0$

$I_a + 2I_1 + \frac{3I_1}{2} + I_2 + I_a = 0$

$\frac{7I_1}{2} + I_2 + 2I_a = 0$ ……….. (1)

Right side mesh equation

$I_2 + 2I_2 + I_1 - I_a = 0$

$\Rightarrow I_a = 3I_2 + I_1$ ………. (2)

Substitute (2) in (1)

$\Rightarrow \frac{7I_1}{2} + I_2 + 6I_2 + 2I_1 = 0$

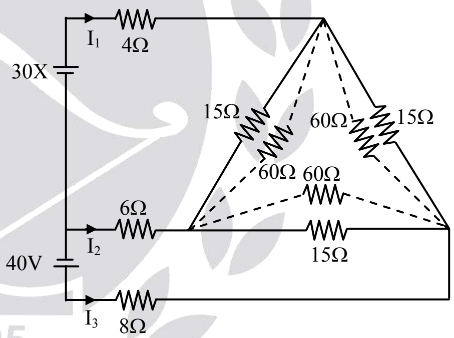
$\Rightarrow 11\frac{I_1}{2} + 7I_2 = 0$

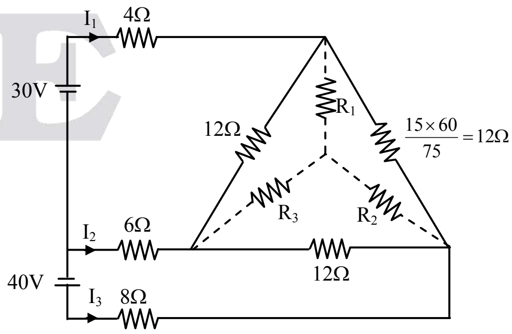
$\Rightarrow \frac{I_2}{I_1} = \frac{-11}{14}$

03.

Sol: Convert Y in to Δ as shown in below figure.

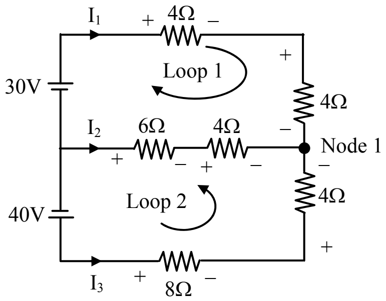
$x_1 = 20 + 20 + \frac{20 \times 20}{20} = 60\Omega = x_2 = x_3$





Convert Δ in to Y

$$R_1 = \frac{12 \times 12}{36} = 4 = R_2 = R_3$$



Apply KCL at Node 1

$I_1 + I_2 + I_3 = 0$ .... (1)

Apply KVL at loop

$30 - 8I_1 + 10I_2 = 0$

$8I_1 - 10I_2 = 30$ .... (2)

Apply KVL at loop 2

$40 - 12I_3 + 10I_2 = 0$

$12I_3 - 10I_2 = 40$ .... (3)

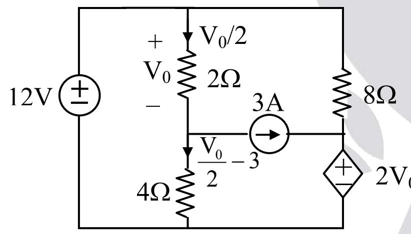
$I_1 = 0.878$ A

$I_2 = -2.29$

$I_3 = 1.418$

04.

Sol:



By applying KVL for V_0

$$12 = V_0 + \left(\frac{V_0}{2} - 3\right)4$$

$12 = 3V_0 - 12 \Rightarrow V_0 = 8V$

$$P = \frac{V_0^2}{2} = \frac{8^2}{2} = \frac{64}{2} = 32\text{Watts}$$

$$W_r = \int_5^{10} 32dt = 32(10-5)$$

$= 160J$

05

Sol: (i) Given the mesh equations:

$$\left.\begin{aligned} 8I_1 - 5I_2 - I_3 &= 110 \\ -5I_1 + 10I_2 + 0 &= 0 \\ -I_1 + 0 + 7I_3 &= 115 \end{aligned}\right\} \text{......... (I)}$$

The NW must have 3 meshes with two sources and all possible resistances in general as shown in Fig.1

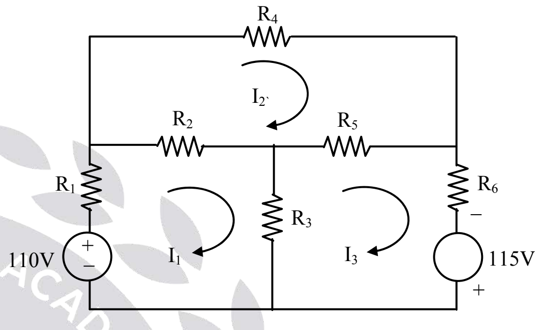


Fig.1

Write the mesh equations

$$\left.\begin{aligned} I_1(R_1+R_2+R_3) - I_2R_2 - I_3R_3 &= 110 \\ -I_1R_2 + I_2(R_2+R_4+R_5) - I_3R_5 &= 0 \\ -I_1R_3 - I_2R_5 + I_3(R_3+R_5+R_6) &= 115 \end{aligned}\right\} \text{....(II)}$$

Comparing the above set of equations (I) and (II):

$R_1+R_2+R_3 = 8$, $R_2 = 5\ \Omega$ $R_3 = 1\ \Omega$

$R_2 = 5\ \Omega$ $R_2+R_4+R_5 = 10$ $R_5 = 0\ \Omega$

$R_3 = 1\ \Omega$ $R_5 = 0\ \Omega$

$R_3+R_5+R_6 = 7$

$\therefore R_1+5+1 = 8$

$\therefore 5+R_4+0 = 10$ $1+0+R_6 = 7$

$R_1 = 2\ \Omega$ $R_4 = 5\ \Omega$ $R_6 = 6\ \Omega$

$\therefore R_1 = 2\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 1\ \Omega$,

$R_4 = 5\ \Omega$, $R_5 = 0\ \Omega$, $R_6 = 6\ \Omega$

(ii) Current in the 110 V source = $I_1 = \frac{D_1}{D}$

$$D_1 = \begin{vmatrix} 110 & -5 & -1 \\ 0 & 10 & 0 \\ 115 & 0 & 7 \end{vmatrix}$$

$= 110\,(70) - 1(-1150)$

$= 8850$

$$D = \begin{vmatrix} 8 & -5 & -1 \\ -5 & 10 & 0 \\ -1 & 0 & 7 \end{vmatrix}$$

$= (8 \times 70) + 5(-35) - 1(10)$

$= 375$

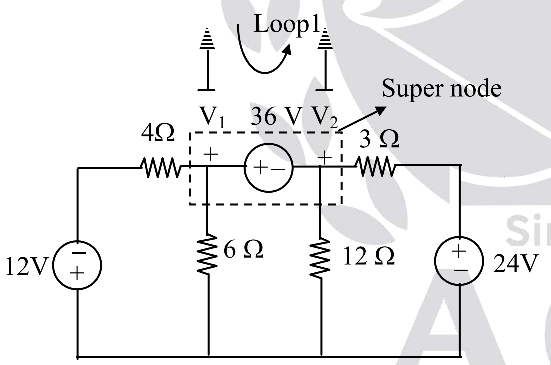
$$I_1 = \frac{8850}{375}$$

$$= \frac{118}{5}$$

$= 23.6$ A

06.

Sol:



Apply KVL at loop 1

$V_1 - 36 - V_2 = 0$

$V_1 - V_2 = 36$ ……… (1)

Apply KCL at Super node

$$\frac{V_1 + 12}{4} + \frac{V_1}{6} + \frac{V_2}{12} + \frac{V_2 - 24}{3} = 0$$

$$V_1\left(\frac{1}{4} + \frac{1}{6}\right) + V_2\left(\frac{1}{12} + \frac{1}{3}\right) + 3 - 8 = 0$$

$$V_1\left(\frac{5}{12}\right) + V_2\left(\frac{5}{12}\right) = 5$$

$V_1 + V_2 = 12$ ……….. (2)

By solving equation (1) and (2)

$V_1 = 24$

$V_2 = -12$

07.

Sol: To find the current, I in the wire AC, of Fig. 1, assume a ground point D. Feeding 1 amp at point A and taking 1 Amp from point C is equivalent to the superposition of two current sources as shown in Fig. 2 and Fig.3

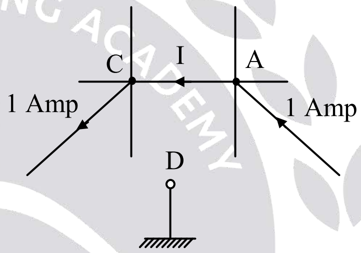


Fig.1

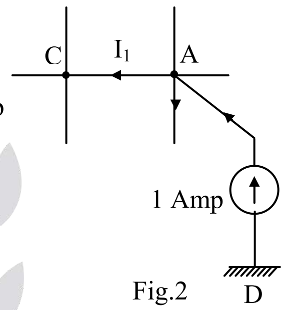


Fig.2

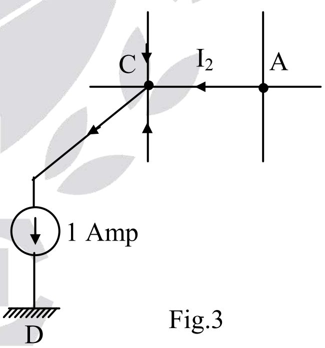


Fig.3

From Fig.2, 1 Amp is divides equally through the four wires because of the infinite nature of the mesh : $I_1 = 0.25$ Amp.

Similarly from Fig.3, $I_2 = 0.25$ Amp

$\therefore$ Current in the wire, AC = $I = I_1 + I_2$

$= 0.5$ Amp

08.

Sol:

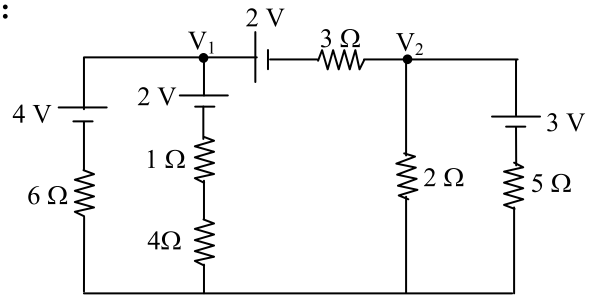


Fig.

Nodal equations:

Node 1:

$$\frac{V_1-4}{6}+\frac{V_1-2}{5}+\frac{V_1-V_2-2}{3}=0 \quad ....(1)$$

$5(V_1–4) + 6 (V_1–2) + 10 (V_1–V_2–2) = 0$

$21 V_1 – 10 V_2 – 20 – 12 – 20 = 0$

$21 V_1 – 10 V_2 = 52$ ……………………(2)

Node 2:

$$\frac{V_2-V_1+2}{3}+\frac{V_2}{2}+\frac{V_2-3}{5}=0 \quad ....(3)$$

$10 (V_2 – V_1 + 2) + 15 V_2 + 6 (V_2 – 3) = 0$

$– 10 V_1 + 31 V_2 + 20 – 18 = 0$

$– 10 V_1 + 31 V_2 = –2$ ………………..(4)

$21 V_1 = 52 + 10 V_2$

$$= 52 + 10\,\frac{(10\,V_1-2)}{31}$$

$$V_1\left(21-\frac{100}{31}\right)=52-\frac{20}{31}$$

$V_1 = 2.889V$

$10 V_2 = 21 V_1 – 52$

$= (21 \times 2.483) – 52$

$V_2 = 0.8675V$

$$\text{Ammeter reading} = \frac{V_2}{2} = 0.434\,A$$

$$\text{Voltmeter reading} = 2+\frac{V_1-2}{5}=2.1178\,V$$

09.

Sol: By applying KCL at V_1

$$2=\frac{V_1-20}{10}+\frac{V_1-0.5V_1-V_2}{5}$$

$V_1 – V_2 = 20$ …….. (1)

By applying KCL at V_2

$$\frac{V_2-30}{10}+\frac{V_2}{2}+\frac{V_2+\frac{V_1}{2}-V_2}{5}=5$$

$–V_1 + 8V_2 = 80$ …….. (2)

From (1) & (2) $–V_1 + 8(V_1 – 20) = 80$

$7V_1 = 240$

$$V_1=\frac{240}{7}=34.28V$$

$V_2 = V_1 – 20 = 34.28\ V – 20$

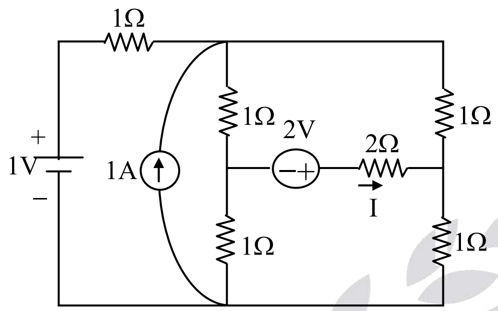
$= 14.3V$

Chapter 2 Circuit Theorems

Objective Practice Solutions

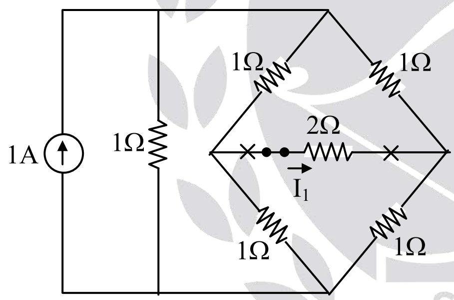
01.

Sol: The current "I" = ?



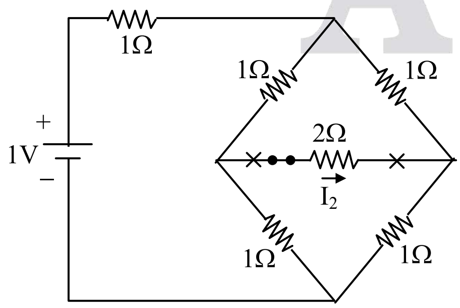
By superposition theorem, treating one independent source at a time.

(a) When 1A current source is acting alone.



Since the bridge is balanced; $I_1 = 0A$

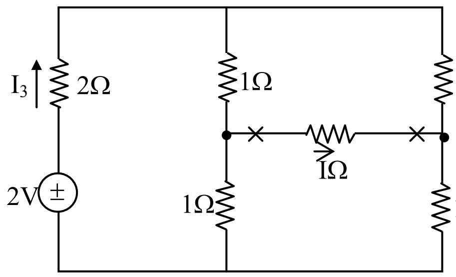
(b) When 1V voltage source is acting alone



$I_2 = 0A$

Since the bridge is balanced.

(c) When 2V voltage source is acting alone



$I_3 = \frac{2}{3} = 0.66A$

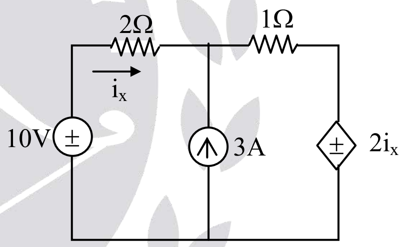
By superposition theorem; $I = I_1 + I_2 + I_3$

$I = 0 + 0 + 0.66A$

$I = 0.66A$

02.

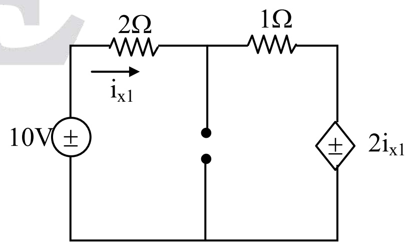
Sol:



$i_x = ?$

By super position theorem; treating only one independent source at a time

(a) When 10V voltage source is acting alone

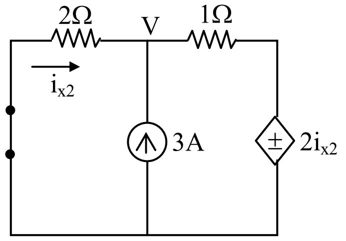


By KVL ⇒

$10 - 2i_{x1} - i_{x1} - 2i_{x1} = 0$

$i_{x1} = 2A$

(b) When 3A current source is acting alone



By Nodal ⇒

$$\frac{V}{2} - 3 + \frac{(V - 2i_{x2})}{1} = 0$$

$3V - 4i_{x2} = 6$ ……….. (1)

And

$i_{x2} = \frac{0 - V}{2} \Rightarrow V = -2i_{x2}$ ………. (2)

Put (2) in (1), we get

$i_{x2} = -\frac{3}{5}$ A

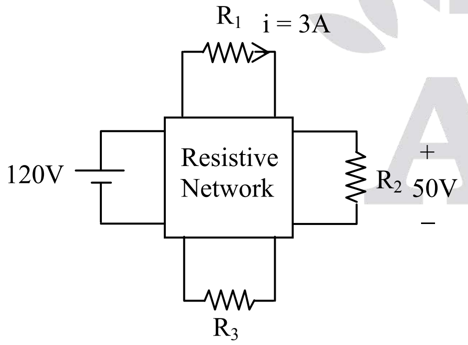
By SPT;

$i_x = i_{x1} + i_{x2} = 2 - \frac{3}{5} = \frac{7}{5}$

$\therefore\ i_x = 1.4$A

03

Sol:



$P_{R_3} = 60\,W$

For 120 V → $i_1 = 3$ A

For 105 V → $i_1 = \frac{105}{120} \times 3 = 2.625$A

For 120 V → $V_2 = 50$ V

For 105 V → $V_2 = \frac{105}{120} \times 50 = 43.75\,V$

$V_2 = 120$ V ⇒ $I^2R_3 = 60$ W⇒ $I = \sqrt{\frac{60}{R_3}}$

For $V_S = 105$ V

$$P_3 = \left(\frac{105}{120}\sqrt{\frac{60}{R_3}}\right)^2 \times R_3 = 45.9\ W$$

04. Ans: (b)

Sol: It is a liner network

$\therefore V_x$ can be assumed as function of i_{s1} and i_{s2}

$V_x = Ai_{s_1} + Bi_{s_2}$

$80 = 8A + 12B$ ……….. (1)

$0 = -8A + 4B$ ……….. (2)

From equation 1 & 2

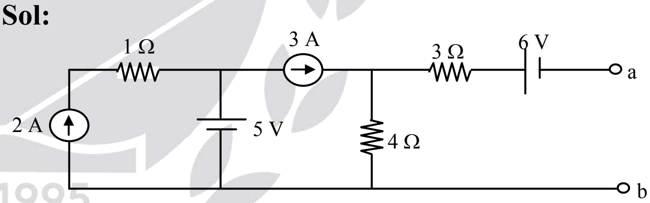
A = 2.5: B = 5

Now, $V_X = (2.5)(20) + (5)(20)$

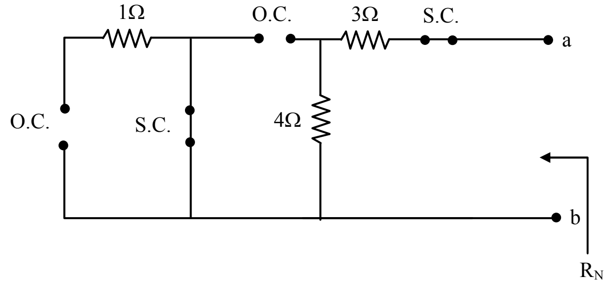
$V_x = 150$V

05. Ans: (c)

Sol:



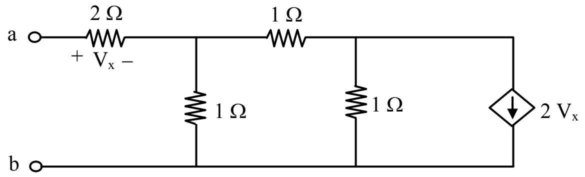
For finding Norton's equivalent resistance independent voltage sources to be short circuited and independent current sources to be open circuited, then the above circuit becomes



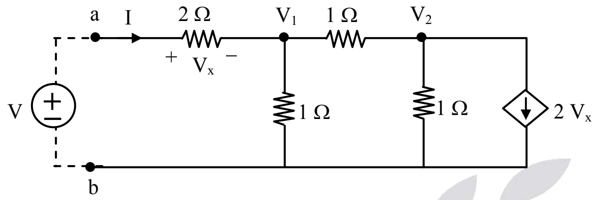
$\Rightarrow R_N = 3 + 4 = 7\Omega$

06. Ans: (b)

Sol:



Excite with a voltage source 'V'



Apply KCL at node V_1

$$-I+\frac{V_1}{1}+\frac{V_1-V_2}{1}$$

$\Rightarrow 2V_1 - V_2 - I = 0$ ………(1)

Apply KCL at node V_2

$$\frac{V_2-V_1}{1}+\frac{V_2}{1}+2V_x=0$$

$2V_2 - V_1 + 2V_x = 0$ ………(2)

But from the circuit,

$V_x = 2I$ ………(3)

Substitute (3) in (2)

$\Rightarrow 2V_2 - V_1 + 4I = 0$

$4V_2 - 2V_1 + 8I = 0$

From (1),

$2V_1 = V_2 + I$

$\therefore 4V_2 - (V_2 + I) + 8I = 0$

$\Rightarrow 3V_2 + 7I = 0$

$$\Rightarrow V_2=-\frac{7I}{3}$$

Substitute (2) in (1)

$$2V_1-\left(-\frac{7I}{3}\right)-I=0$$

$$2V_1+\frac{7}{3}I-I=0\Rightarrow 2V_1=\frac{-4I}{3}$$

$$\Rightarrow V_1=\frac{-2I}{3}$$

$$\therefore V=V_x+V_1=2I+\left(-\frac{2I}{3}\right)=\frac{4I}{3}$$

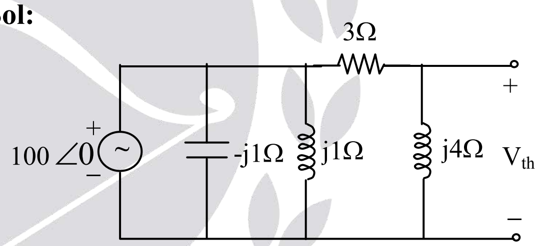
$$\Rightarrow V=\frac{4I}{3}$$

$$\Rightarrow \frac{V}{I}=\frac{4}{3}\Omega$$

$$\Rightarrow R_{eq}=\frac{4}{3}\Omega$$

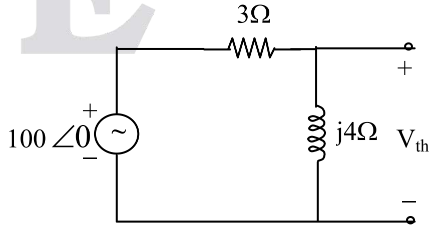
07.

Sol:



Here j1Ω and – j1Ω combination will act as open circuit.

The circuit becomes

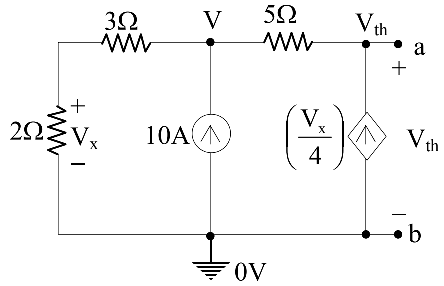


$$\Rightarrow V_{th}=\frac{100\angle 0^\circ \times j4}{3+j4}$$

$$=80\angle 36.86^\circ\ V$$

08.

Sol: Thevenin's and Norton's equivalents across a, b.



By Nodal ⇒

$$\frac{V}{5} - 10 + \frac{V}{5} - \frac{V_{th}}{5} = 0$$

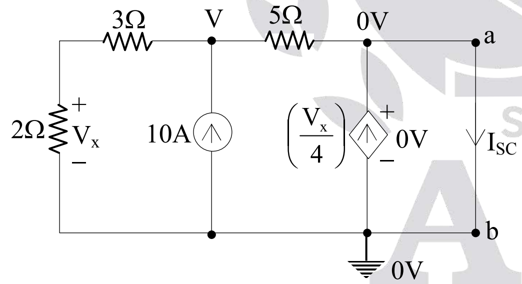
$$\frac{V_{th}}{5} - \frac{V}{5} - \frac{V_x}{4} = 0$$

$$\frac{2V}{5} = \left(10 + \frac{V_{th}}{5}\right)$$

$$\frac{V_{th}}{5} = \left(\frac{V}{10} + \frac{V}{5}\right)$$

$$V_x = \left(\frac{2V}{5}\right)$$

$V_{th} = 150V$, $V = 100$ V



$$\frac{V}{5} - 10 + \frac{V}{5} = 0$$

$$\frac{2V}{5} = 10$$

$V = 25V$

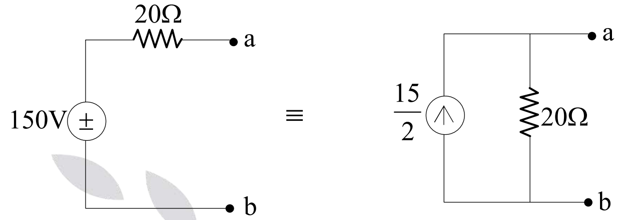
$$V_x = \frac{2V}{5} = \frac{2\times 25}{5}$$

$V_x = 10V$

$$I_{SC} = \left(\frac{10}{4} + 5\right) = \frac{15}{2}A$$

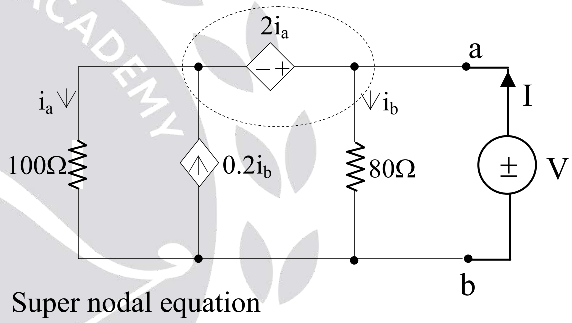
$$I_{SC} = \frac{15}{2}A$$

$$R_{th} = \frac{V_{th}}{I_{SC}} = \frac{150}{\frac{15}{2}} = 20\Omega$$



09.

Sol:



Super nodal equation

$\Rightarrow i_a - 0.2i_b + i_b - I = 0$

$I = i_a + 0.8i_b$

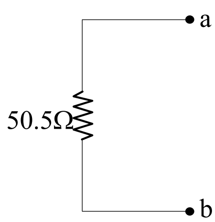
$V = 80i_b$; $i_b = \dfrac{V}{80}$

- Inside the supernode, always the KVL is written.

By KVL ⇒

$100i_a + 2i_a - 80i_b = 0$

$$I = \frac{V}{102} + \frac{0.8\times V}{80}$$



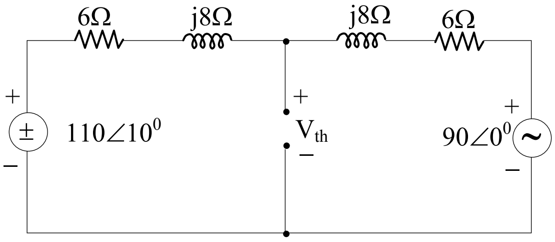
$$\frac{V}{I} = R_L = \frac{1}{\frac{1}{102} + \frac{1}{100}}$$

$= 50.5\Omega.$

$R_L = 50.5\Omega$

10.

Sol: V_{th}:

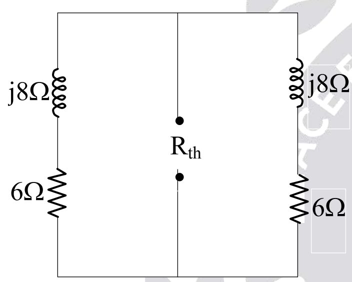


By Nodal ⇒

$$\frac{V_{th}}{(6+j8)} - \frac{110\angle 0^0}{(6+j8)} + \frac{V_{th}}{(6+j8)} - \frac{90\angle 0^0}{(6+j8)} = 0$$

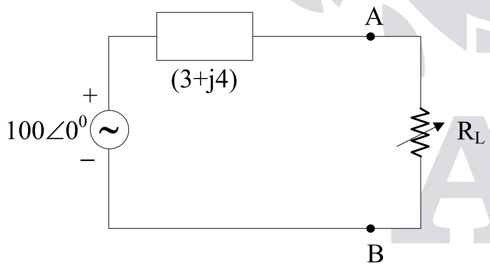
$2V_{th} = 200\angle 0^0 \Rightarrow V_{th} = 100\angle 0^0$.

R_{th}:



$R_{th} = (6 + j8)||(6+j8)$

$\equiv (3 + j4)\Omega$



$R_L = |3+j4| = 5\Omega$

$$I = \frac{100\angle 0^0}{(8+j4)}$$

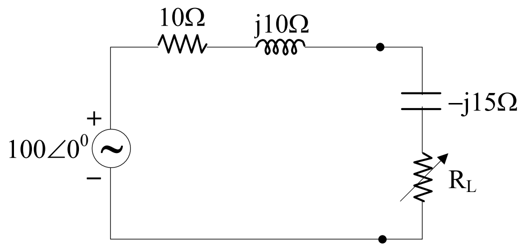
$P = |I|^2 \times R_L$

$P_{max} = 125 \times 5 = 625$ W

$\therefore P_{max} = 625$ watts

11.

Sol:



The maximum power delivered to "R_L" is

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$$

Here $R_S = 10\Omega$; $X_S = 10\Omega$ & $X_L = -15$

$$R_L = \sqrt{10^2 + (10-15)^2}$$

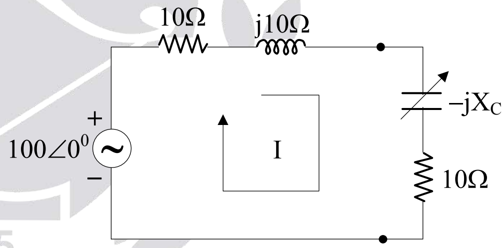
$R_L = 5\sqrt{5}\ \Omega$.

$$I = \frac{100\angle 0^0}{(10 + j10 - j15 + 5\sqrt{5})}$$

$$P_{max} = |I|^2 .5\sqrt{5} = 236\text{W}$$

12.

Sol:



The maximum power delivered to 10Ω load resistor is:

$Z_L = 10 - jX_C = 10 + j(-X_C)$

$X_L = -X_C$

So for MPT; $(X_S + X_L) = 0$

$10 - X_C = 0$;

$X_C = 10$

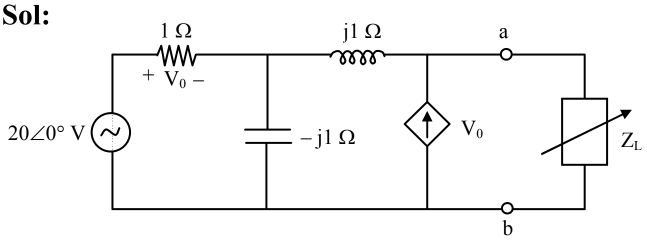
$$I = \frac{100\angle 0^0}{(10 + j10 - j10 + 10)} = 5\angle 0^0$$

$P_{max} = |I|^2 R_L = 5^2(10) = 250$W

$P_{max} = 250$ Watts

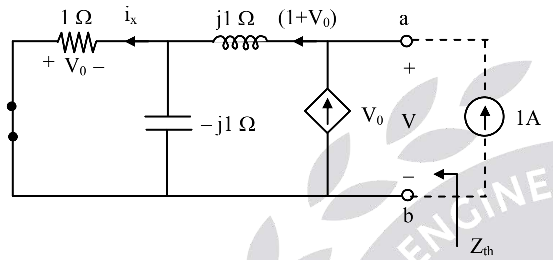
13. Ans: (b)

Sol:



For maximum power delivered to Z_L,

$Z_L = Z_{th}^*$



$$i_x = (1+V_0)\times\frac{-j1}{1-j1} = (1+V_0)\ (0.5-j0.5)$$

But

$V_0 = -i_x$

$= -(1+V_0)\ (0.5 - j0.5)$

$(-1-j)\ V_0 = 1 + V_0$

$\Rightarrow V_0\ (-1-j-1) = 1$

$$V_0 = \frac{1}{-2-j} = -0.4 + j0.2$$

Applying KVL

$$+V_0 - j1(1+V_0) + V = 0$$

$\Rightarrow V = -V_0 + j1(1+V_0)$

$= 0.4 - j0.2 + j1(0.6+j0.2)$

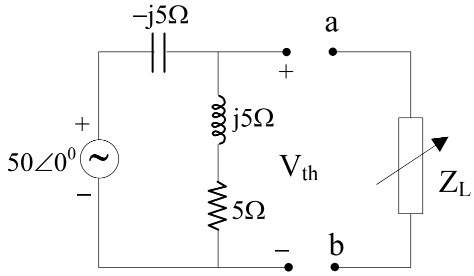
$V = (0.2 + j\ 0.4)V$

$$\therefore Z_{th} = \frac{V}{1} = V = (0.2 + j0.4)\Omega$$

$$\therefore Z_L = Z_{th}^* = (0.2 - j\ 0.4)\ \Omega$$

14.

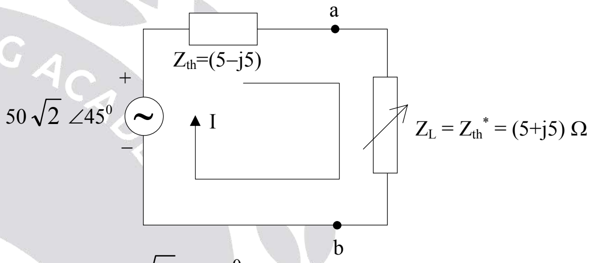
Sol:



The maximum true power delivered to "Z_L" is :

$$V_{th} = \left(\frac{50\angle 0^0}{-j5+j5+5}\right)(j5+5) = 50\sqrt{2}\ \angle 45^0$$

$Z_{th} = (-j5)||(5+j5) = (5-j5)\Omega$



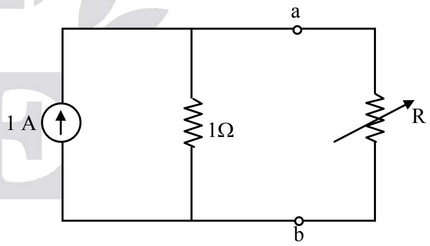
$$I = \frac{50\sqrt{2}\angle 45^0}{(5-j5+5+j5)} = 5\sqrt{2}\angle 45^0$$

$P = |I|^2 5 = |\,5\sqrt{2}\,|^2\ .5 = 250$ Watts

$\therefore\ P_{max} = 250$ watts

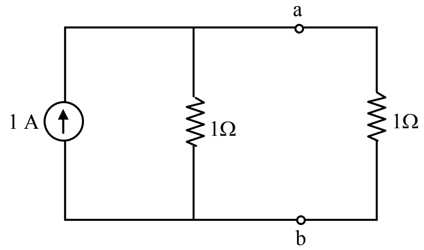
15. Ans: (c)

Sol:



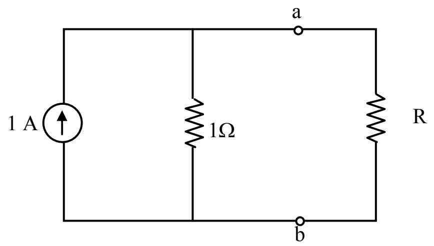
Maximum power will occurs when $R = R_s$

$\Rightarrow R = 1\ \Omega$



$$\therefore P_{max} = \left(\frac{1}{2}\right)^2 \times 1 = \frac{1}{4} W$$

$$25\% \text{ of } P_{max} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} W$$



Current passing through 'R'

$$I = 1 \times \frac{1}{1+R} = \frac{1}{1+R}$$

$$\therefore P = I^2R = \left(\frac{1}{1+R}\right)^2 R = \frac{1}{16}$$

$\Rightarrow (R+1)^2 = 16R$

$\Rightarrow R^2 + 2R + 1 = 16R$

$\Rightarrow R^2 - 14R + 1 = 0$

$R = 13.9282\Omega$ or 0.072Ω

From the given options $72 m\Omega$ is correct

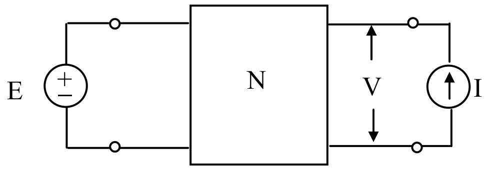
16. The network 'N' shown in figure contains only resistances.

E = 10 V and 0V

I = 0A and 2A

V = 3V and 2V respectively.

If E = 100 V and I is replaced by R = 2Ω, then determine V.



Sol: For, E = 10 V, I= 0A then V =3V

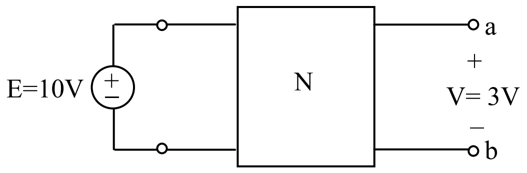


Fig.(b)

V_{oc} = 3V (with respect to terminals a and b)

For, E = 0V, I = 2A then V =2V

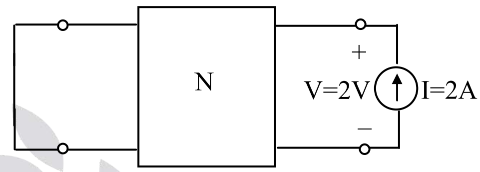
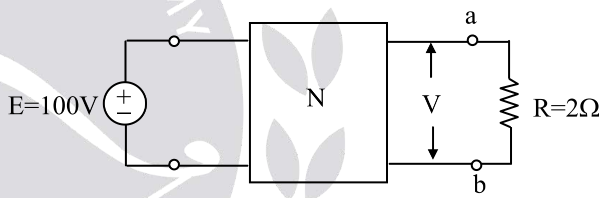


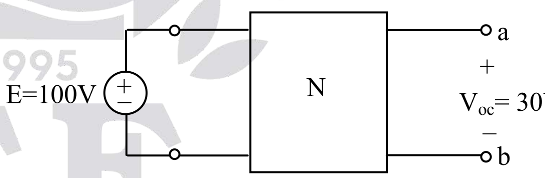
Fig.(c)

Now when E = 100 V, and I is replaced by R = 2Ω then V = ?



When E = 100V,

From Fig.(b) using homogeneity principle



For finding Thevenin's resistance across ab independent voltage sources to be short circuited & independent current sources to be open circuited.

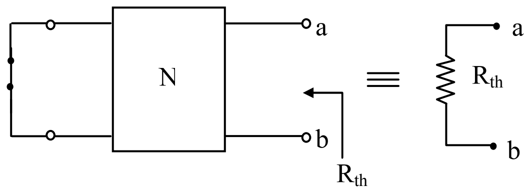
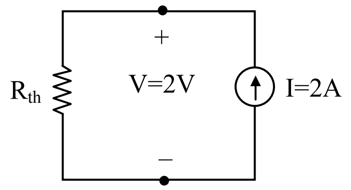


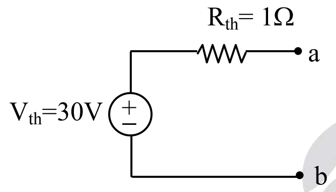
Fig.(d)

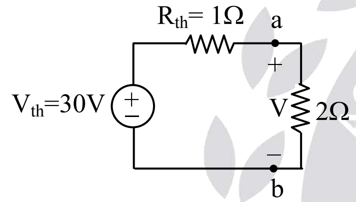
Fig.(c) is the energized version of Fig. (d)



$$\Rightarrow R_{th} = \frac{2}{2} = 1\Omega$$

∴With respect to terminals a and b the Thevenin's equivalent becomes.





$$V = 30\times\frac{2}{2+1} = 20V$$

∴ V = 20V

17.

Sol: Superposition theorem cannot be applied to fig (b)

Since there is only voltage source given:

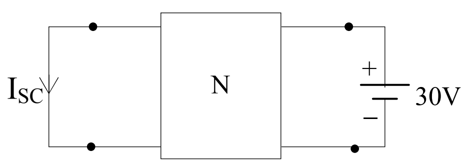
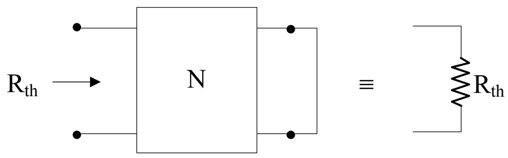


Fig (c)

By homogeneity and Reciprocity principles to fig (a);

I_{SC} = 6A

For R_{th}:



Statement: Fig (a) is the energized version of figure (d)

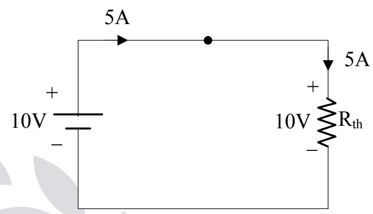


Fig (a)

$10 = R_{th}.\ 5\big|_{\text{by ohm's law}}$

R_{th} = 2Ω.

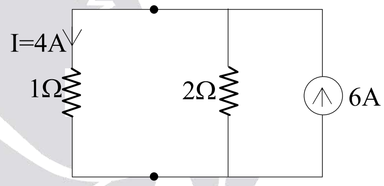


Fig (b)

$$I = \frac{6\times 2}{(2+1)} = 4A$$

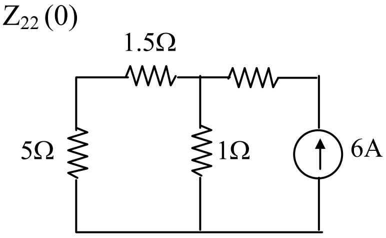
I = 4A

18. Ans: (b)

Sol: $\begin{bmatrix}10\\4\end{bmatrix} = \begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$10 = Z_{11}\,(4) + Z_{12}\,(0)$

$4 = Z_{21}\,(4) + Z_{22}\,(0)$



$$Z_{11} = \frac{10}{4} = 2.5$$

$$Z_{21} = \frac{4}{4} = 1$$

$$I_{5\Omega} = \frac{6\times1}{6.5+1} = \frac{6}{7.5} = 0.8\,A$$

19. Ans: (b)

Sol:

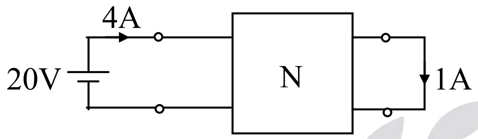


Fig.(a)

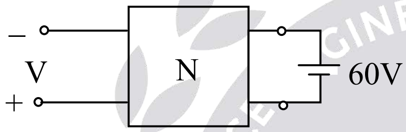
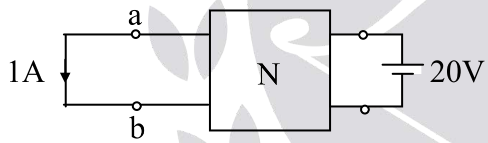


Fig.(b)

Using reciprocity theorem, for Fig.(a)



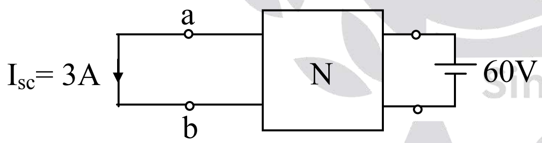


Fig.(c)

Norton's resistance between a and b is

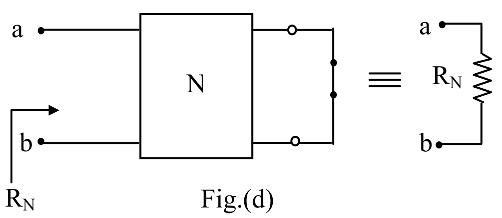
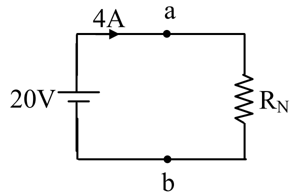


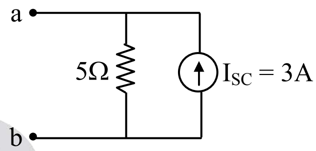
Fig.(d)

Fig.(a) is the energized version of Fig.(d)

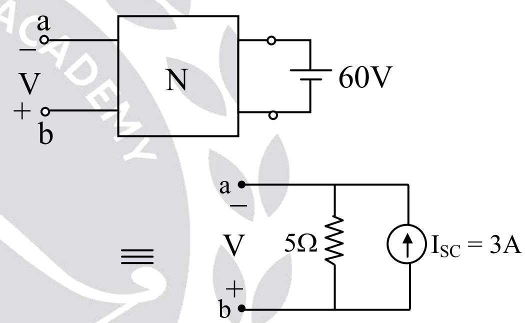


$$\Rightarrow R_N = \frac{20}{4} = 5\Omega$$

With respect to terminals a and b the Norton's equivalent of Fig.(b) is



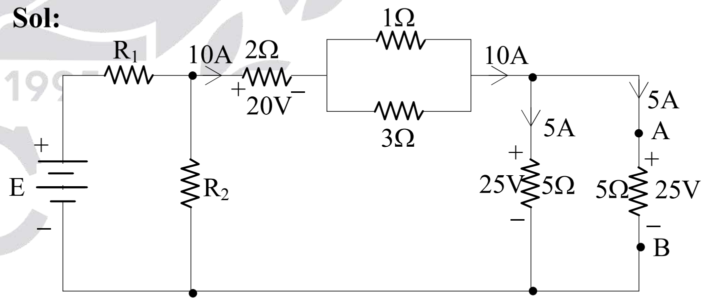
$\therefore$ From Fig.(b)

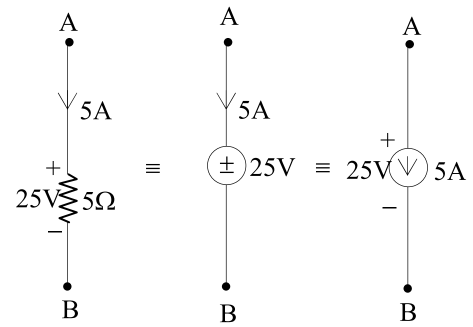


$\Rightarrow V = -15V$

20.

Sol:

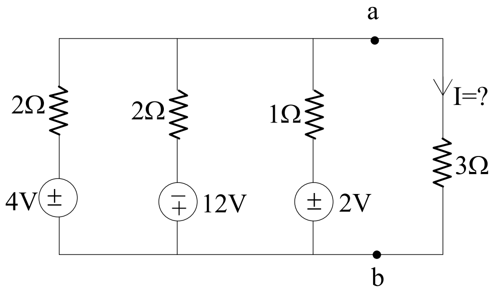




$P_{AB} = P_{5\Omega} = P_{25V} = P_{5A} = 5*25 = 125$ watts (ABSORBED)

21.

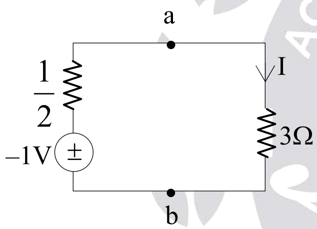
Sol:



By Mill Man's theorem;

$$V' = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$$

$$\equiv \frac{\frac{4}{2} - \frac{12}{2} + \frac{2}{1}}{\left(\frac{1}{2} + \frac{1}{2} + 1\right)} = \frac{4 - 12 + 4}{2*2} \equiv -1V$$



$\therefore V' = -1V$

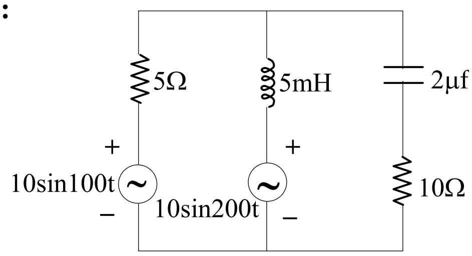
$$\frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{2} + 1 = 2$$

$$\therefore R^1 = \frac{1}{2}\Omega$$

$$I = \frac{-1}{\left(\frac{1}{2} + 3\right)} \Rightarrow I = \frac{-2}{7} A$$

22. Ans: (d)

Sol:



Since the two different frequencies are operating on the network simultaneously; always the super position theorem is used to evaluate the responses since the reactive elements are frequency sensitive

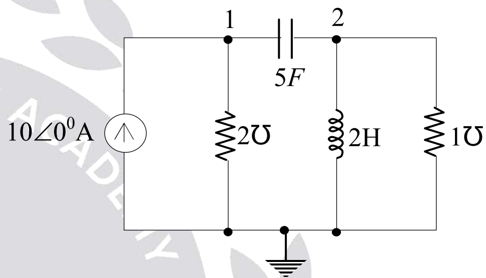
i.e., $Z_L = j\omega L$ and $Z_C = \frac{1}{j\omega c}\ \Omega$.

23.

Sol: In the above case if both the source are100rad/sec, each then Millman's theorem is more conveniently used.

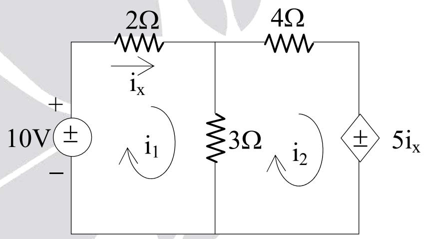
24.

Sol:



25.

Sol:



Nodal equations

$i = GV$

$i_x = i_1$

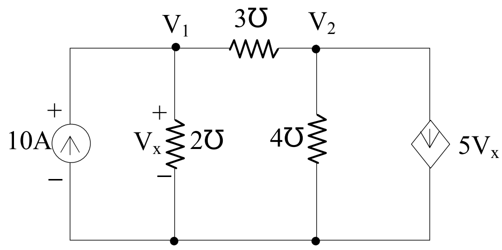
$10 = 2i_1 + 3(i_1 - i_2)$ ………. (1)

$0 = 4i_2 + 2i_x + 3(i_2 - i_1)$ …...…. (2)

$V_x = V_1$

$10 = 2V_1 - 3(V_1 - V_2)$ ……….. (3)

$0 = 4V_2 + 2V_x + 3(V_2 - V_1)$ ……….. (4)



26.

Sol: When only E_1 is acting, $I_1^2 R = 18$

$$\Rightarrow I_1 = \sqrt{\frac{18}{R}} = 3\sqrt{\frac{2}{R}}$$

Similarly, $I_2 = 5\sqrt{\frac{2}{R}}; I_3 = 7\sqrt{\frac{2}{R}}$

When all sources are acting,

$I_{total} = I_1 + I_2 + I_3$

Maximum power consumed by R is

$$P = I_{total}^2 R$$

$$= \left(3\sqrt{\frac{2}{R}} + 5\sqrt{\frac{2}{R}} + 7\sqrt{\frac{2}{R}}\right)^2 R$$

$$= \frac{2}{R}(3+5+7)^2 .R$$

$= 450$ W

Minimum power consumed

$$P = \frac{2}{R}(3+5-7)^2 R = 2 \text{ W}$$

27. Ans: (c)

Sol: $I_L = \frac{100}{R_g + 4 + 10}$, $P_L = I_L^2 R_L$

P_L is maximum, when I_L is maximum.

I_L is maximum, when R_g is minimum $= 3\Omega$

Statement (I) is True.

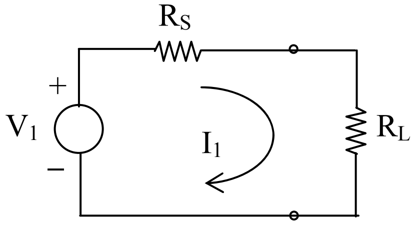
During maximum power transfer, (i.e., when $R_g = 3\Omega$), $|Z_g| = \sqrt{R_g^2 + 4^2} = 5\ \Omega$.

$$\therefore R_L \neq |Z_g|$$

Statement (II) is false.

28. Ans: (b)

Sol:



$V_1 = I_1(R_S + R_L)$

Thevenin and Norton equivalents are derivable for linear NW's only.

29. Ans: (b)

Sol: Conversion to equivalent T – NW and application of Thevenin's Theorem have no relation.

30. Ans: (d)

Sol: Z_L should be equal to Z_S^* and η=50 %

$\therefore$ Statement (I) is false but Statement (II) is true.

31. Ans: (a)

Sol: Diode is a nonlinear and unilateral device. Hence, Thevenin's theorem cannot be applied. Both Statement (I) and Statement (II) are true and Statement (II) is the correct explanation of Statement (I).

32. Ans: (c)

Sol: A. Load impedance $(10 + j\,20)^*$

$= 10 - j\,20$ (5)

B. Total impedance $Z_i + Z_L = 20$ (4)

C. Current $\frac{50}{20} = 2.5$ (3)

D. Maximum power

$(2.5)^2 \times 10 = 62.5$ (1)

33. Ans: (b)

34. Ans: (b)

Sol: A. Superposition theorem is applicable for linear networks only (1)

B. Tellegen's theorem utilizes the structure of the NW irrespective (3) of the nature of the elements

C. The equivalent circuit of a NW at two terminals can be obtained by using Norton's theorem. (2)

D. Reciprocity theorem is applicable to Bilateral networks (4)

35. Ans: (c)

Sol: A. Reciprocity

– Bilateral (2)

B. Tellegen's

$$-\sum_{k=0}^{b} v_{jk}(t_1) i_{jk}(t_2) = 0 \quad (3)$$

C. Superposition

– Linear (4)

D. Maximum power Transfer

– Impedance matching (1)

36. Ans: (d)

Conventional Practice Solutions

01.

Sol: The given circuit is shown in Fig 1 with terminals marked.

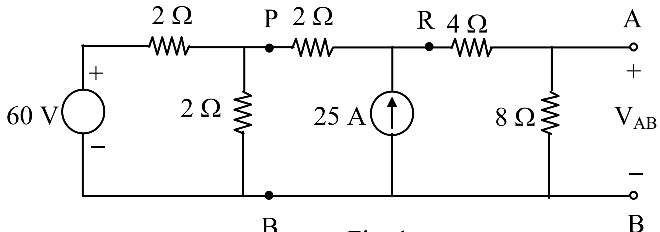


Fig. 1

Source transformation is used successively as shown in Fig. (2) to (9)

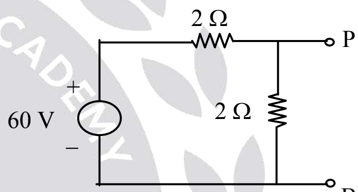


Fig. 2

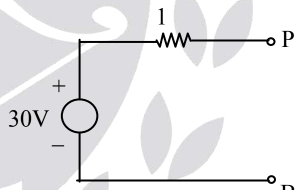


Fig.

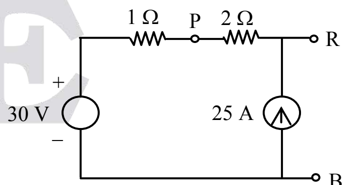


Fig. 4

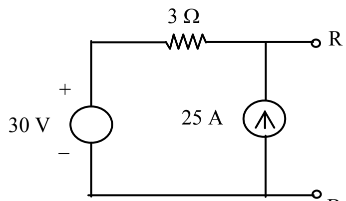


Fig. 5

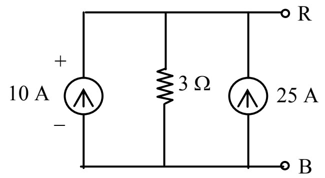


Fig. 6

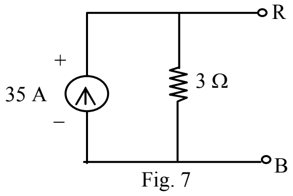


Fig. 7

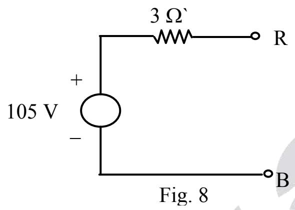


Fig. 8

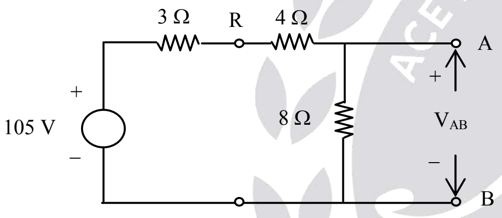


Fig. 9

From fig. (9), $V_{AB} = \dfrac{105 \times 8}{15} = 56\ \text{V}$

02.

Sol: Case (i):

Source 'V' active: $I = I_1$

$I_1 = 0$, as can be seen from Fig.1

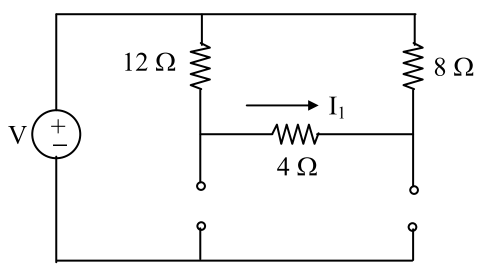


Fig.1

Case (ii):

Source '1A' active: $I = I_2$

$I_2 = 0.5$ A as can be seen from Fig. 2

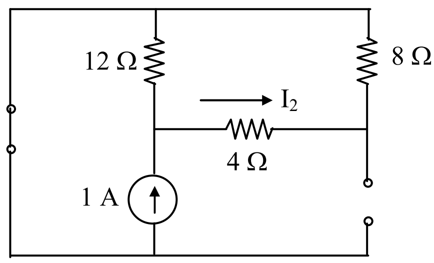


Fig.2

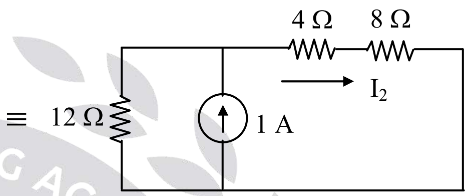


Fig.3

Case (iii):

Source '3A' active: $I = I_3$

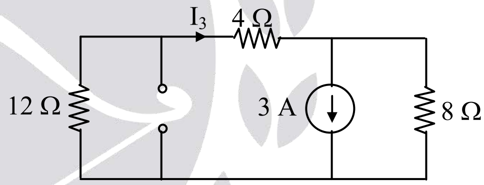


Fig.4

From Fig. 4,

$$I_3 = 3 \times \frac{8}{24} = 1\ \text{A}$$

$\therefore\ I = I_1 + I_2 + I_3 = 0 + 0.5 + 1 = 1.5$ A

Note: The answer can be obtained quickly by writing KVL equation from the given circuit, Fig. 5 directly:

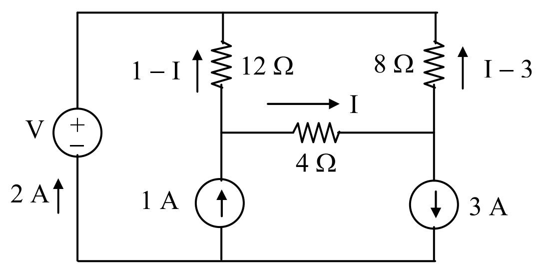


Fig. 5

$4I + 8(I - 3) = (1 - I)\,12$

$24I = 36$

$I = 1.5$ A

03.

Sol: Case (i):

Due to 6 V source, the currents are shown in Fig. 1.

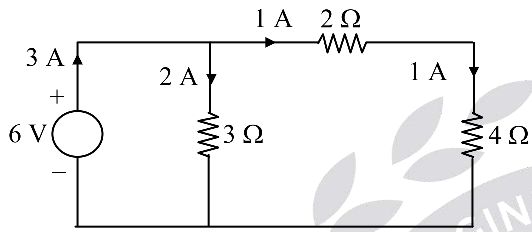


Fig.1

Case (ii):

Due to 3 A - source, the currents are shown in Fig. 2.

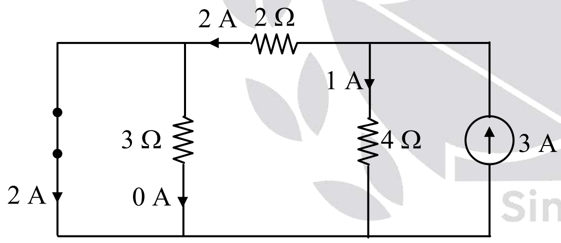


Fig.2

Using the superposition theorem, the currents due to both the sources are shown in Fig. 3.

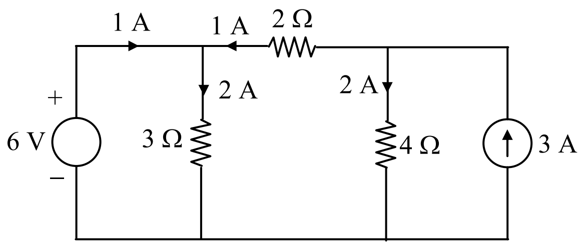
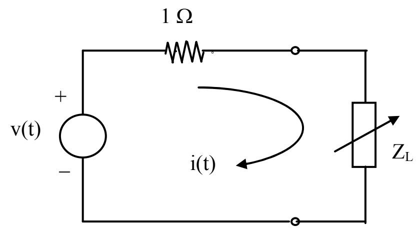


Fig.3

04.

Sol: $v(t) = 10 \sin(2\pi \times 10^6)\, t$



i) Maximum power is generated by the generator when $Z_L = 0$

$$P_{Gen}\,(\max) = \left(\frac{10}{\sqrt{2}}\right)^2 = 50 \text{ W}$$

Maximum power is delivered to the load if $Z_L = 1\ \Omega$.

Under this condition

ii) $P_L\,(\max) = I_{rms}^2 \times 1 = \left(\frac{5}{\sqrt{2}}\right)^2 \times 1 = 12.5$ W

iii) Power generated by generator $= 2 \times 12.5$

$= 25$ W

05.

Sol: Let the Thevenin impedance be $R + jX$

Magnitude of $Z_{th} = \sqrt{R^2 + X^2}$

$$= \frac{\text{open circuit voltage}}{\text{short circuit current}}$$

$$= \frac{125}{5.59} = 22.36$$

With 10 Ω resistive load,

$$\frac{125}{\sqrt{(R+10)^2 + X^2}} = 4.41$$

$$\sqrt{(R+10)^2 + X^2} = \frac{125}{4.41} = 28.34$$

$R^2 + X^2 = 499.97$

$R^2 + X^2 + 100 + 20R = 803.15$

$20R + 100 = 303.18$

$$R = \frac{203.18}{20} = 10.16\ \Omega\ ;\ X = 19.91$$

$V_{th} = 125$ V, $Z_{th} = 10.16 + j\ 19.91\ \Omega$

06.

Sol: V_{th} is calculated from Fig. 1

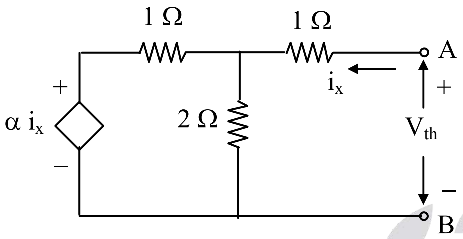


Fig. 1

There is no independent source.

$\therefore V_{th} = 0$

R_{th} is calculated from Fig. 2

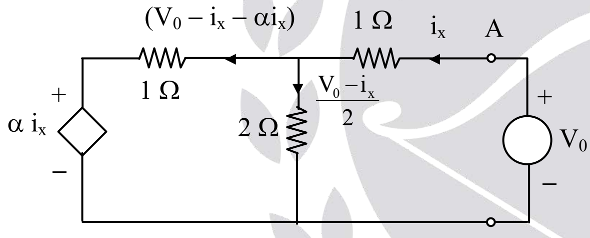


Fig. 2

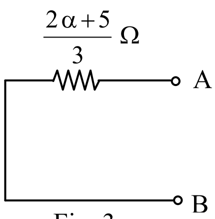


Fig. 3

$$R_{th} = \frac{V_0}{i_x}\ ,\quad i_x = V_0 - i_x - \alpha i_x + \frac{V_0 - i_x}{2}$$

$$i_x = \frac{3}{2}V_0 - i_x\left(\frac{3}{2} + \alpha\right)$$

$$\left(\frac{5}{2} + \alpha\right) i_x = \frac{3}{2} V_0$$

$$\frac{V_0}{i_x} = \frac{2}{3}\ \frac{5+2\alpha}{2} = \frac{2\alpha+5}{3}$$

07.

`Sol: The circuit is simplified as shown in Fig. (1), (2) and (3) by using source transformation.

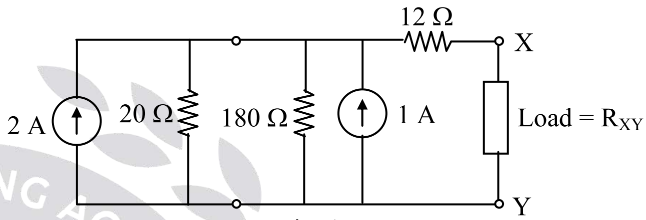


Fig. 1

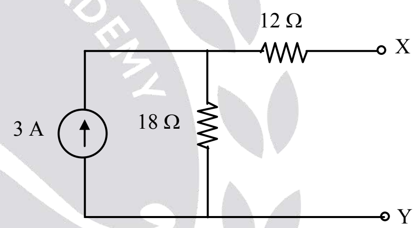


Fig. 2

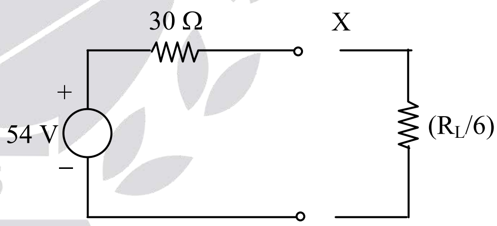


Fig. 3

Load:

$$Y_{XY} = \frac{1}{R_L} + \frac{2}{R_L} + \frac{3}{R_L} = \frac{6}{R_L}$$

$$Z_{XY} = \frac{R_L}{6}$$

For maximum power transfer to the three resistor load, Z_{XY}

$$\frac{R_L}{6} = 30\ ,\qquad R_L = 180\ \Omega$$

$\therefore V_{XY} = 27$ V

Power delivered to $R_L = \frac{(27)^2}{180} = 4.05\ W$

Power delivered to $\frac{R_L}{2} = \frac{(27)^2}{90} = 8.1\ W$

Power delivered to $\frac{R_L}{3} = \frac{(27)^2}{60} = 12.15\ W$

08.

Sol: The given circuit is shown in Fig. 1, where 40 Ω across 50 V can be deleted.

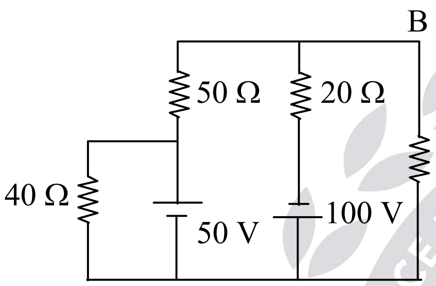


Fig. 1

$\equiv$

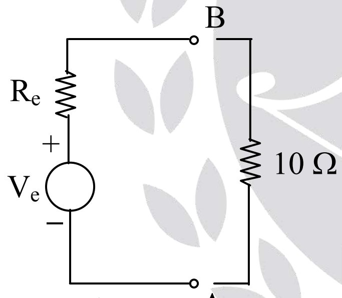


Fig. 2

V_{BA} is found by reducing the given circuit to the left of BA into a single voltage source, V_e and a series resistance, R_e by using Milliman's theorem.

The equivalent circuit is shown in Fig. 2

$$V_e = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2}$$

$$= \frac{\frac{50}{50} - \frac{100}{20}}{\frac{1}{50} + \frac{1}{20}}$$

$$= \frac{-4 \times 100}{7} = -\frac{400}{7}\ V$$

$$R_e = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{50} + \frac{1}{20}} = \frac{100}{7}\ \Omega$$

$$V_{BA} = -\frac{400}{7} \times \frac{10}{\frac{100}{7} + 10}$$

$$= -\frac{400}{7} \times 10 \times \frac{7}{170} = -\frac{400}{17}\ V$$

09.

Sol: Maximum Power transfer theorem:

This is used to find the value of the load impedance Z_L (optimum) that absorbs maximum power from a given network shown in Fig. 1.

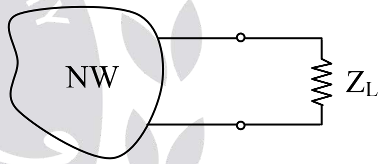


Fig.1

The NW is replaced by its Thevenin's equivalent circuit as shown in Fig. 2.

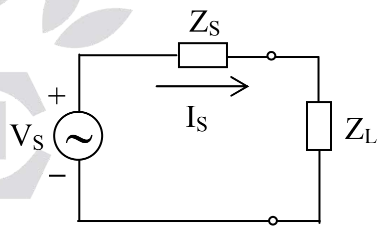


Fig.2

$Z_L = R_L + j\,X_L$ is complex load

V_S is Thevenin's voltage phasor (RMS Value)

Z_S is Thevenin's equivalent impedance

$= R_S + j\,X_S$

V_s and Z_s can be understood as the source voltage and source impedance wrt the load impedance, Z_L.

$$I_S = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + R_L) + j(X_L + X_S)}$$

………. (1)

P = Power delivered to

$$Z_L = |I_S|^2 R_L = \frac{V_S^2 \; R_L}{(R_S + R_L)^2 + (X_L + X_S)^2}$$

………. (2)

For maximum power transfer to Z_L :

Case 1:

When only X_L is variable in the load,

$$\frac{\partial P}{\partial X_L} = 0, 2(X_L + X_S) = 0 \text{ or } X_L = -X_S \text{ ...(3)}$$

Then maximum power transferred to Z_L =

$$\frac{V_S^2 R_L}{(R_S + R_L)^2} \qquad \text{……….(4)}$$

Case 2: When only R_L is variable, $\frac{\partial P}{\partial R_L} = 0$

$(R_S + R_L)^2 + (X_L + X_S)^2 - R_L(R_L + R_S)2 = 0$

$R_S^2 + R_L^2 + 2R_SR_L + (X_L + X_S)^2 - 2R_LR_S - 2R_L^2 = 0$

$R_S^2 - R_L^2 + (X_L + X_L)^2 = 0$ or

$$R_L = \sqrt{R_S^2 + (X_L + X_S)^2} \qquad \text{……….(5)}$$

Then maximum power delivered to

$$Z_L = \frac{V_S^2 R_L}{(R_S + R_L)^2 + R_L^2 - R_S^2}$$

$$= \frac{V_S^2}{2(R_L + R_S)} \qquad \text{.............(6)}$$

Case 3 :

When X_L as well as R_L are variable, then from (3) and (5) $X_L = -X_S$, $R_L = R_S$

Z_L (optimum) = $R_S - j\,X_S$ …..……(7)

= complex conjugate of Z_S

Then maximum power transferred to $Z_L = \frac{V_S^2}{4R_L}$

………. (8)

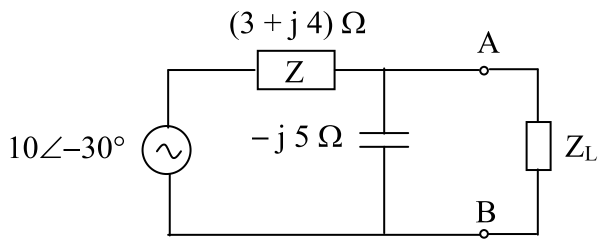


Fig. 1

Let Z_L = Impedance of Loudspeaker across the terminals A, B for maximum power dissipation in it as shown in Fig. 1.

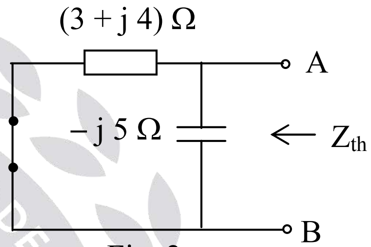


Fig. 2

Then Z_L = Thevenin's impedance across terminals A, B into the network = Z_{th}

Z_{th} is found from Fig. (2)

$$Z_{th} = \frac{(3+j4)\,(-j5)}{(3-j4)-j5}$$

$$= \frac{20-j15}{3-j1} = \frac{5(4-j3)\,(3+j1)}{9+1}$$

$$= \frac{1}{2}(12+3-j9+j4)$$

$$= \frac{1}{2}(15 - j5) = 7.5 - j\,2.5$$

$\therefore Z_L = Z_{th}^*$

$\Rightarrow Z_L = (7.5 - j\,2.5)\,\Omega$

10.

Sol: Duality of Thevenin's and Norton's theorems:

The NW in Fig. 1 can be represented by the equivalent circuit shown in Fig. 2 by Thevenin's theorem and in Fig. 3 by Norton's theorem.

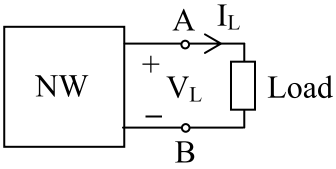


Fig. 1

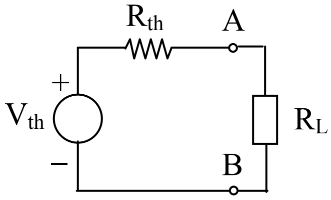


Fig. 2

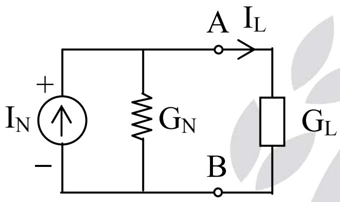


Fig. 3

Load: R_L Ω From Fig. 2

From Fig. 3

$$\text{or } G_L = \frac{1}{R_L}\ ℧ \quad ..(1)$$

$$V_{AB} = V_L = V_{th}\frac{R_L}{R_{th}+R_L} \quad …………(2)$$

$$I_{AB} = I_L = I_N\frac{G_L}{G_N+G_L} \quad ……….. (3)$$

$$\text{Where, } G_N = \frac{1}{R_N} = \frac{1}{R_{th}}$$

Equations (2) and (3) are dual equations where the duality is indicated by the dual quantities given below:

Voltage across load, V_L → Current through load, I_L

Open circuit voltage across→ Short circuit current from A to B = I_N

A, B = V_{th}

Load Resistance, R_L→ Load Conductance,

$$G_L = \frac{1}{R_L}$$

Thevenin Resistance, R_{th} → Norton's Conductance, $G_N = \frac{1}{R_N} = \frac{1}{R_{th}}$

→ The given circuit is shown in Fig.1 with 3 Ω across the terminals A, B.

The current through each element is marked by assuming I as the current through 1 Ω.

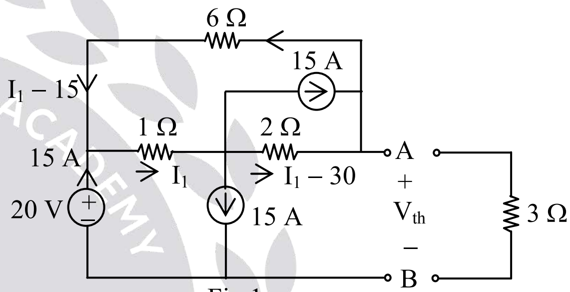


Fig.1

Writing KVL equations:

$1I_1 + 2(I_1 - 30) + V_{th} = 20$

or $3I_1 + V_{th} = 80$ ……..…. (1)

and

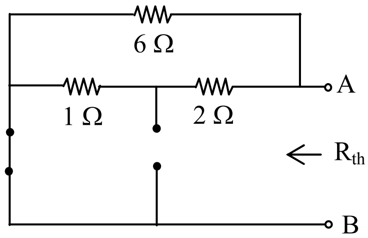
$6(I_1 - 15) + 1\,I_1 + 2(I_1 - 30) = 0$

or $9I_1 - 90 - 60 = 0$

$$I_1 = \frac{150}{9} = \frac{50}{3}\ \text{A} \quad ………. (2)$$

From (1) and (2), $V_{th} = 80 - 3\,I_1 = 30$ V

R_{th} is obtained from the following circuit, Fig. 2



← $R_{th} = 3\ \Omega \parallel 6\ \Omega = 2\ \Omega$

Fig.2

Thevenin's equivalent circuit across the terminals A, B is shown in Fig. 3.

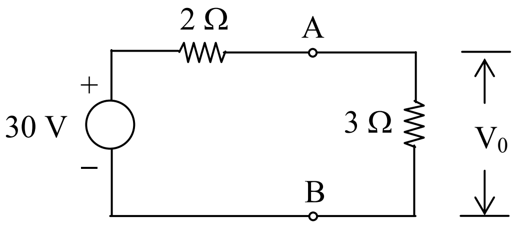
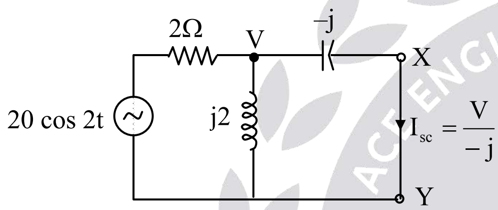


Fig.3

$$V_0 = \frac{30\times3}{5} = 18\ V$$

11.

Sol:



$$\frac{V-20/\sqrt{2}}{2} + \frac{V}{j2} + \frac{V}{-j} = 0$$

$$V\left(\frac{1}{2}+\frac{1}{j2}-\frac{1}{j}\right) = \frac{10}{\sqrt{2}}$$

$$V\left(\frac{1}{2}-\frac{1}{j2}\right) = \frac{10}{\sqrt{2}}$$

$$V\left(\frac{(j2-2)}{j4}\right) = \frac{10}{\sqrt{2}}$$

$$V = \frac{40j}{\sqrt{2}(j2-2)}$$

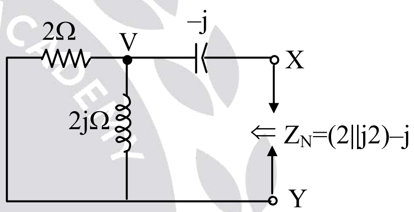
$$I_{sc} = \frac{V}{-j} = \frac{\frac{40j}{\sqrt{2}(j2-2)}}{-j} = \frac{40}{(2-j2)\sqrt{2}}$$

$$I_{sc} = \frac{40(2+j2))}{\sqrt{2}(8)} = \frac{10\sqrt{2}\angle45^\circ}{\sqrt{2}}$$

$I_{sc} = 10\angle45°A$

$$I_{sc} = 5\sqrt{2}\cos(2t+45^\circ)A$$

For Z_N :



$$= \frac{(j2)(2)}{(2+j2)} - j$$

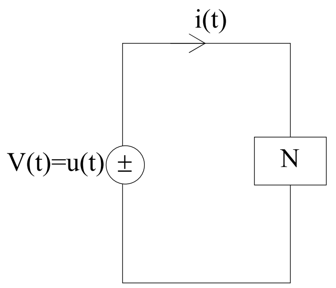
$$Z_N = \frac{j2}{(1+j)} - j$$

$$= \frac{j2(1-j)}{2} - j = 1\Omega$$

Chapter 3 Transient Circuit Analysis

Objective Practice Solutions

01.
Sol:



$i(t) = e^{-3t}$A for t > 0 (given)

Determine the elements & their connection

$$\frac{\text{Response Laplace transform}}{\text{Excitation Laplace transform}} = \text{System transfer function}$$

i.e., $$\frac{I(s)}{V(s)} = H(s) = \frac{\frac{1}{(s+3)}}{\frac{1}{s}}$$

$$= \frac{s}{(s+3)} = y(s) = \frac{1}{Z(s)}$$

$$\therefore Z(s) = \left(\frac{s+3}{s}\right)$$

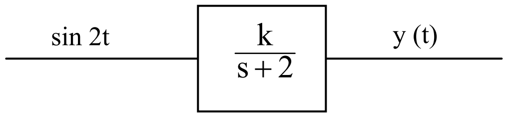
$$= 1 + \frac{1}{s\left(\frac{1}{3}\right)} = R + \frac{1}{SC}$$

$\therefore$ R = 1Ω and C = $\frac{1}{3}$ F are in series

02. Ans: (c)

Sol: The impulse response of first order system is Ke^{-2t}.

So T/F = L(I.R) = $\frac{K}{s+2}$



$$G(s) = \frac{K}{s+2}$$

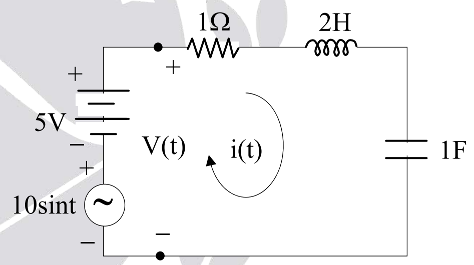
$$|G(j\omega)| = \frac{K}{\sqrt{\omega^2+2^2}} = \frac{K}{2\sqrt{2}}$$

$$\angle G(j\omega) = -\tan^{-1}\frac{\omega}{2} = -\tan^{-1}1 = -\frac{\pi}{4}$$

So steady state response will be

$$y(t) = \frac{K}{2\sqrt{2}}\sin\left(2t - \frac{\pi}{4}\right)$$

03.
Sol:



By KVL ⇒ v(t) = (5 + 10sint)volt

Evaluating the system transfer function H(s).

$$\frac{\text{Desired response L.T}}{\text{Excitation response L.T}} = \text{System transfer function}$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{\left(R + SL + \frac{1}{SC}\right)}$$

$$H(s) = \frac{S}{\left(2s^2+s+1\right)}$$

$$H(j\omega) = \frac{1}{\left(1 + \frac{1}{j\omega} + 2j\omega\right)}$$

II. Evaluating at corresponding ω_s of the input

$$H(j\omega)|_{\omega=0} = 0$$

$$H(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}}\angle{-45^{\circ}}$$

III. $$\frac{I(s)}{V(s)} = H(s)$$

$$I(s) = H(s)V(s)$$

$$i(t) = 0\times 5 + \frac{1}{\sqrt{2}}\times 10\sin(t-45^{\circ})$$

$$i(t) = 7.07\sin(t-45^{\circ})\text{A}$$

OBS: DC is blocked by capacitor in steady state

04.

Sol: $$\frac{V(s)}{I(s)} = H(s) = Z(s)$$

$$= \frac{1}{Y(s)} = \frac{1}{\left(\frac{1}{R}+\frac{1}{sL}+sC\right)}$$

$$H(s) = \frac{1}{\left(1+\frac{1}{s}+s\right)}$$

$$H(j\omega)\Big|_{\omega=1} = \frac{1}{\left(1+\frac{1}{j}+j\right)} = 1$$

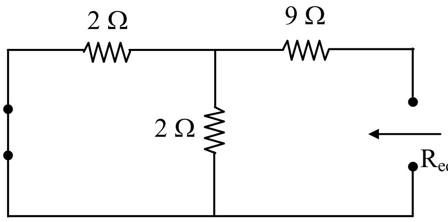
$V(s) = I(s)\,H(s) = \sin t$

$v(t) = \sin t$ Volts

05.

Sol: $$\tau = \frac{L_{eq}}{R_{eq}}$$

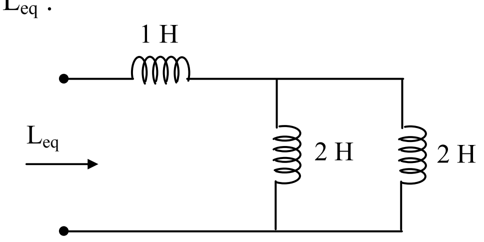
R_{eq} :



$R_{eq} = (2\|2) + 9 = 10\ \Omega$

$R_{eq} = (2 \| 2) + 9 = 10\ \Omega$

L_{eq} :



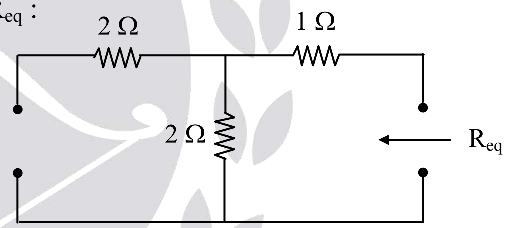
$$L_{eq} = (2 \| 2) + 1 = 2\text{ H}$$

$$\therefore\ \tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{10} = 0.2\text{ sec}$$

06.

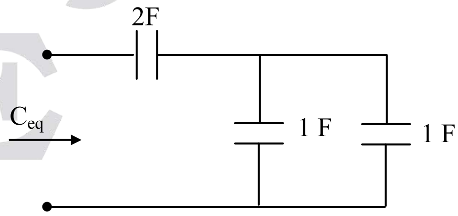
Sol: $\tau = R_{eq}\,C_{eq}$

R_{eq} :



$R_{eq} = 3\ \Omega$

C_{eq} :



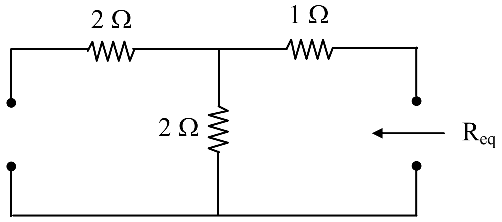
$C_{eq} = 1$ F

$\therefore\ \tau = 3\times 1 = 3$ sec

07.

Sol: $\tau = R_{eq}\,C$

R_{eq} :

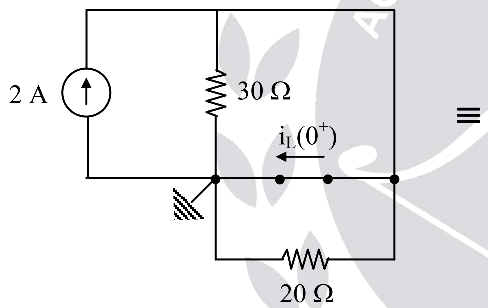


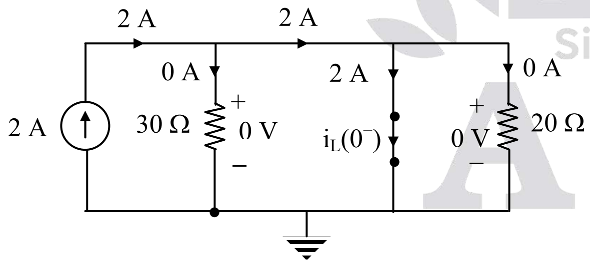
$R_{eq} = 3\ \Omega$

$\therefore\ \tau = 3 \times 1 = 3$ sec

08.

Sol: Let us assume that switch is closed at $t = -\infty$, now we are at $t = 0^-$ instant, still the switch is closed i.e., an infinite amount of time, the independent dc source is connected to the network and hence it is said to be in steady state. In steady state, the inductor acts as short circuit and nature of the circuit is resistive.





At $t = 0^-$: Steady state: A resistive circuit

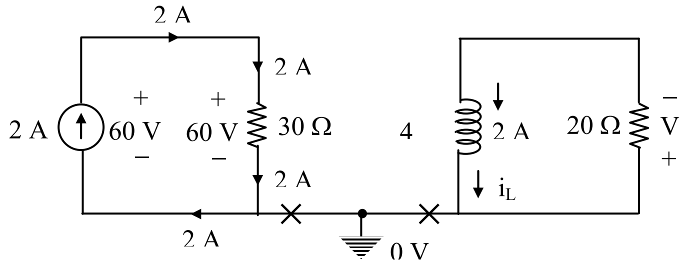
Note: The number of initial conditions to be evaluated at just before the switching action is equal to the number of memory elements present in the network.

(i) $t = 0^-$

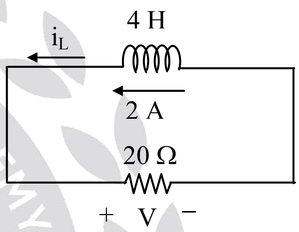
$i_L(0^-) = 2 = i_L(0^+)$

$$E_L(0^-) = \frac{1}{2} L\, i_L^2(0^-)$$

$$= \frac{1}{2} \times 4 \times 2^2 = 8\,\text{J} = E_L(0^+)$$



For $t \geq 0$



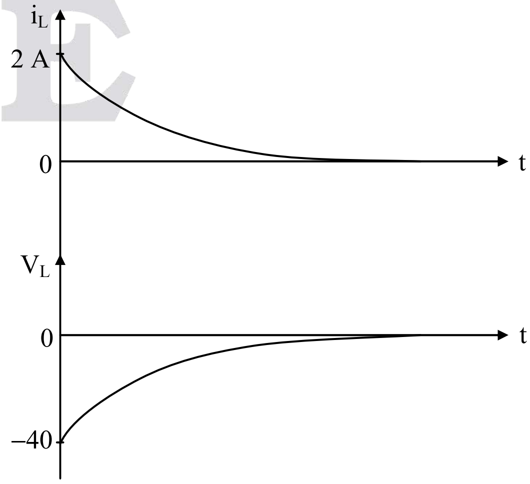
For $t \geq 0$: Source free circuit

$$I_0 = 2\text{ A}\ ;\ \tau = \frac{L}{R} = \frac{4}{20} = \frac{1}{5}\ \text{sec}$$

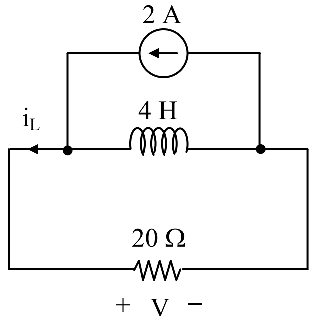
$$i_L = I_0\, e^{-\frac{t}{\tau}}$$

$$i_L = 2\, e^{-5t} \quad \text{for } 0 \leq t \leq \infty$$

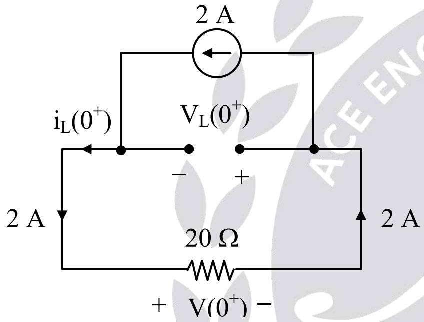
$$V_L = L\frac{di_L}{dt} = -40\, e^{-5t}\ \text{V} \quad \text{for } 0 \leq t \leq \infty$$



$t = 5\tau = 5 \times \frac{1}{5} = 1$ sec for steady state practically i.e., with in 1 sec the total 8 J stored in the inductor will be delivered to the resistor.



For $t \geq 0$



At $t = 0^+$: Resistive circuit :
Network is in transient state

By KCL;

$-2 + i_L(0^+) = 0$

$i_L(0^+) = 2$ A

$V(0^+) = R\, i_L(0^+)\,|_{\text{By Ohm's law}}$

$V(0^+) = 20\,(2) = 40$ V

By KVL ;

$V_L(0^+) + V(0^+) = 0$

$$V_L(0^+) = -V(0^+) = -40 \text{ V} = V_L(t)\big|_{t=0^+}$$

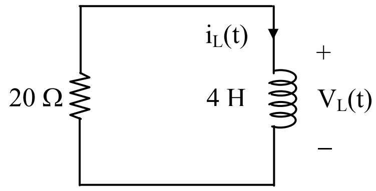
Observations:

$t = 0^-$	$t = 0^+$
$i_L(0^-) = 2$ A	$i_L(0^+) = 2$ A
$i_{20\Omega}(0^-) = 0$ A	$i_{20\Omega}(0^+) = 2$ A
$V_{20\Omega}(0^-) = 0$ V	$V_{20\Omega}(0^+) = 40$ V
$V_L(0^-) = 0$ V	$V_L(0^+) = -40$ V

Conclusion:

To keep the same energy as $t = 0^-$ and to protect the KCL and KVL in the circuit (i.e., to ensure the stability of the network), the inductor voltage, the resistor current and its voltage can change instantaneously i.e., within zero time at $t = 0^+$.

(2)



For $t \geq 0$

$i_L(t) = 2\,e^{-5t}$ A for $0 \leq t \leq \infty$

$V_L(t) = -40\,e^{-5t}$ V for $0 \leq t \leq \infty$

Conclusion:

For all the source free circuits, $V_L(t) = -$ve for $t \geq 0$, since the inductor while acting as a temporary source (upto 5τ), it discharges from positive terminal i.e., the current will flow from negative to positive terminals. (This is the must condition required for delivery, by Tellegan's theorem)

(3) $V_L(0^+) = -40$ V

$$V_L(t)\big|_{t=0^+} = -40 \text{ V}$$

$$L\,\frac{d\,i_L(t)}{dt}\bigg|_{t=0^+} = -40$$

$$\frac{d\,i_L(t)}{dt}\bigg|_{t=0^+} = -\frac{40}{L} = -\frac{40}{4} = -10 \text{ A/sec}$$

Check :

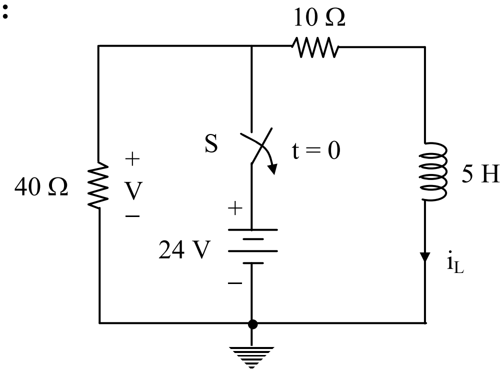
$i_L(t) = 2\,e^{-5t}$ A for $0 \leq t \leq \infty$

$$\frac{d\,i_L(t)}{dt} = -10\,e^{-5t} \text{ A/sec for } 0 \leq t \leq \infty$$

$$\frac{d\,i_L(t)}{dt}\bigg|_{t=0^+} = -10 \text{ A/sec}$$

09.

Sol:



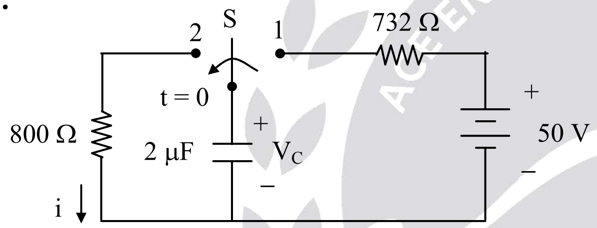
$i_L(0^+) = 2.4$ A

$V(0^+) = -96$ V

$i_L(t) = 2.4\ e^{-10t}$ A for $0 \le t \le \infty$

10.

Sol:



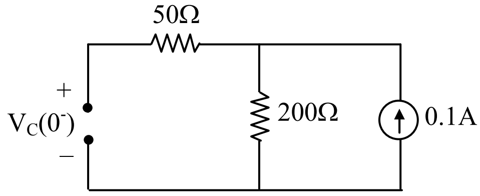
$V_C(0^+) = 50$ V ; $i(0^+) = 62.5$ mA

$$V_C(t) = 50\ e^{-\frac{t}{1.6\times10^{-3}}}\ \text{V for } t \ge 0$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}}$$

11.

Sol: Case(i): $t < 0$

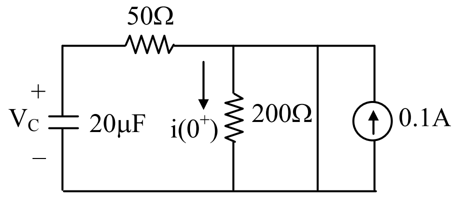


$V_C(0^-) = 20V$ & $i(0^-) = 0.1A$

∵ capacitor never allows sudden changes in voltages

$V_C(0^-) = V_C(0) = V_C(0^+) = 20V$

Case(ii): $t > 0$



To find the time constant $\tau = R_{eq}C$

After switch closed

$R_{eq} = 50\Omega$ $C = 20\mu F$

$i(0^+) = 0A$

$\tau = 50\times20\mu$

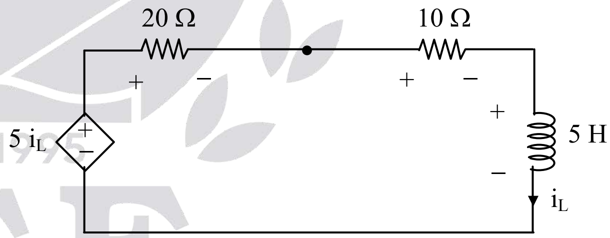
$\tau = 1$msec

$V_C(t) = V_0e^{-t/\tau} = 20e^{-t/1m}$

$V_C(t) = 20e^{-t/1m}V$; $0\le t\le\infty$

12.

Sol: After performing source transformation ;



By KVL;

$$5\ i_L - 30\ i_L - 5\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 5\ i_L = 0$$

$(D + 5)\ i_L = 0$

$$i_L(t) = K\ e^{-5t}\ \text{A for } 0 \le t \le \infty$$

$$\tau = \frac{1}{5}\ \text{sec}$$

13.

Sol: $i_{L_1}(0) = 10\text{ A}\ ;\ i_{L_2}(0) = 2\text{ A}$

$$i_{L_1}(t) = I_0\, e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{1}{1} = 1\text{ sec}$$

$$i_{L_1}(t) = 10\, e^{-t}\text{ A}$$

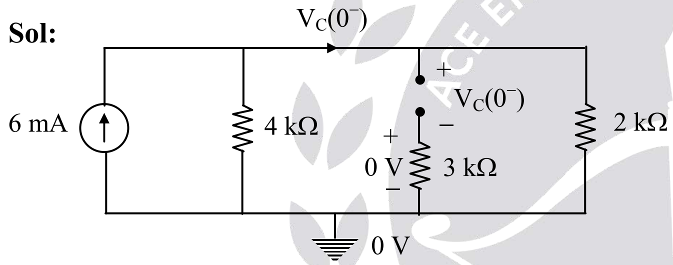
Similarly, $i_{L_2}(t) = I_0\, e^{-\frac{t}{\tau}}$

$$\tau = \frac{L}{R} = 2\text{ sec}$$

$$i_{L_2}(t) = 20\, e^{-\frac{t}{2}}\text{ A}$$

14.

Sol:

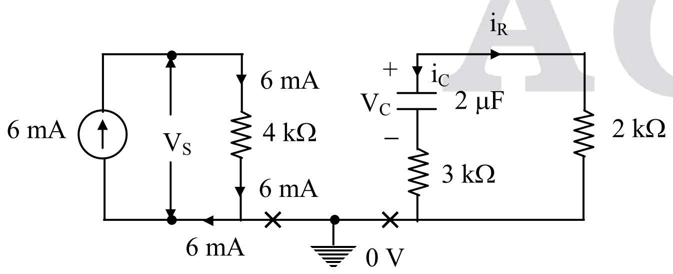


At $t = 0^-$: Steady state : A resistive circuit

By Nodal :

$$-6\text{ mA} + \frac{V_C(0^-)}{4\text{K}} + \frac{V_C(0^-)}{2\text{K}} = 0$$

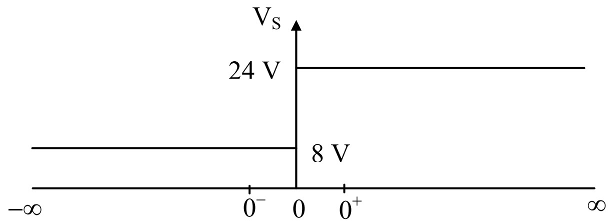
$V_C(0^-) = 8\text{ V} = V_C(0^+)$



For $t \geq 0$: A source free circuit

$V_s = 6\text{ m} \times 4\text{ K} = 24\text{ V}$

$\tau = R_{eq}C = (5\text{ K})\, 2\,\mu = 10\text{ m sec}$



$$V_C = 8\, e^{-\frac{t}{10\text{m}}} = 8\, e^{-100t}\text{ V}\quad \text{for } 0 \leq t \leq \infty$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}} = -1.6\, e^{-100\,t}\text{ mA}\quad \text{for } 0 \leq t \leq \infty$$

By KCL:

$i_C + i_R = 0$

$i_R = -\,i_C = 1.6\, e^{-100\,t}\text{ mA}\quad \text{for } 0 \leq t \leq \infty$

Observation:

In all the source free circuit, $i_C(t) = -\text{ve}$ for $t \geq 0$ because the capacitor while acting as a temporary source it discharges from the +ve terminal i.e., current will flow from −ve to +ve terminals.

15.

Sol: By KCL :

$i(t) = i_R(t) + i_L(t)$

$$= \frac{V_R(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} V_L(t)\, dt$$

$$= \frac{V_S(t)}{10} + i_L(0) + \frac{1}{L}\int_{0}^{t} V_S(t)\, dt$$

$i(t) = 4\,t + 5 + 4\,t^2$

$i(t)\,|_{t = 2\text{ sec}} = 8 + 16 + 5$

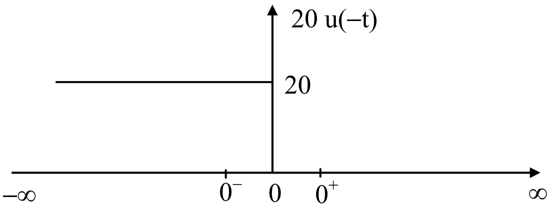
$= 29\text{ A}$

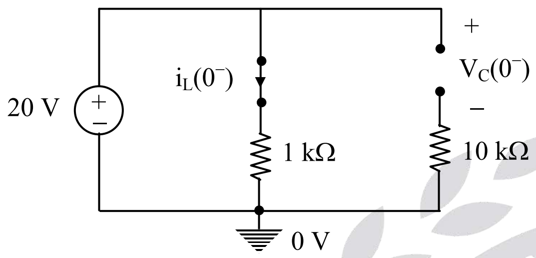
$= 29000\text{ mA}$

16. Ans: (c)

17.

Sol:



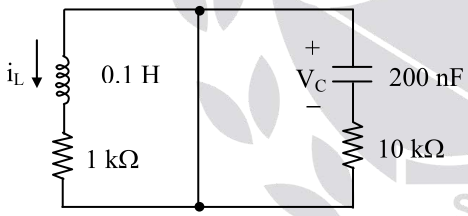


At $t = 0^-$: steady state: A resistive circuit.

(i) $t = 0^-$

$V_C(0^-) = 20\text{ V} = V_C(0^+)$

$$i_L(0^-) = \frac{20}{1K} = 20\text{ mA} = i_L(0^+)$$



For $t \ge 0$: A source free RL & RC circuit

$$\tau = \frac{0.1}{1K} = 100\ \mu\text{sec}$$

$\tau_C = 200 \times 10^{-9} \times 10 \times 10^3 = 2$ m sec

$$\frac{\tau_C}{\tau_L} = 20\ ;\quad \tau_C = 20\,\tau_L$$

Observation:

$\tau_L < \tau_C$; therefore the inductive part of the circuit will achieve steady state quickly i.e., 20 times faster.

$$V_C = 20\,e^{-\frac{t}{\tau_C}}\ \text{V} \quad \text{for } 0 \le t \le \infty$$

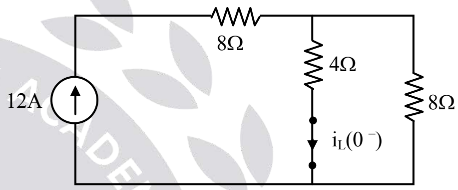
$$i_L = 20\,e^{-\frac{t}{\tau_L}}\ \text{mA} \quad \text{for } 0 \le t \le \infty$$

$$V_L = L\left.\frac{di_L}{dt}\right|_{\text{By Ohm's law}}$$

$$i_C = C\left.\frac{dV_C}{dt}\right|_{\text{By Ohm's law}}$$

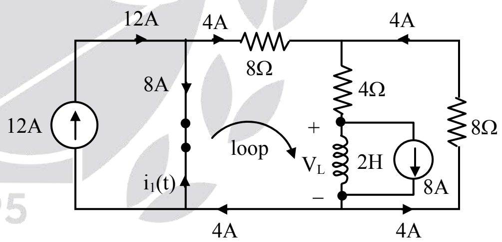
18.

Sol: At $t = 0^-$



$$\Rightarrow i_L(0^-) = \frac{12 \times 8}{8+4} = 8\text{A}$$

At $t = 0^+$



$\therefore\ i_l(0^+) = -8\text{A}$

Applying KVL in the loop,

$\Rightarrow 8(4) + 4(8) + V_L = 0$

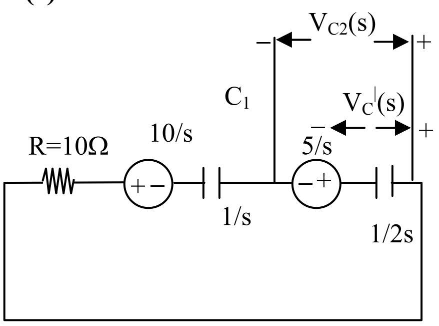
$\Rightarrow V_L = -64$

$$\Rightarrow L\frac{di_L}{dt} = -64$$

$$\Rightarrow \frac{di_L}{dt} = -32\ \text{A/sec}$$

19. Ans: (c)

Sol:



$$V_c^{|}(s) = \frac{\frac{5}{s}\left(\frac{1}{2s}\right)}{R + \frac{1}{s} + \frac{1}{2s}}$$

$$= \frac{\frac{5}{2s^2}}{\frac{2Rs+2+1}{2s}} = \frac{5}{s(2Rs+3)}$$

$$V_{c_2}(\infty) - V_{c^|}{}^{|}(s) - \frac{5}{s} = 0$$

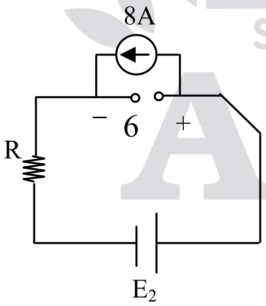
$$V_c(\infty) = V_c^{|}(s) + \frac{5}{s}$$

$$V_c(\infty) = \underset{s\to 0}{\text{Lt}}\, s.\left[\frac{5}{s(2Rs+3)} + \frac{5}{s}\right]$$

$$= \frac{5}{3} + 5 = \frac{20}{3}$$

20. Ans: (d)

Sol: at t = 0



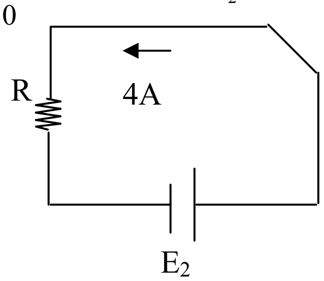
$$L\frac{di(0)}{dt} = V_L(0)$$

$$V_L = 2\times 3 = 6$$

$$V_L = 6V$$

$E_2 + 6 - 8R = 0$

$E_2 = 8R - 6$



$E_2 - 4R = 0$

$E_2 = 4R$

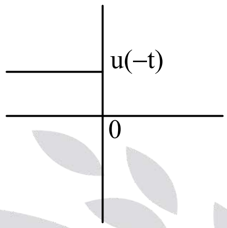
8R–6=4R

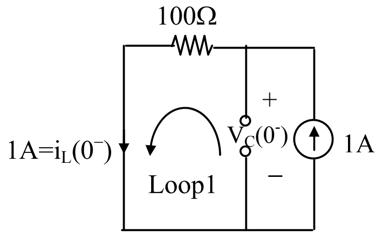
4R = 6

R = 1.5Ω

21. Ans: (d)

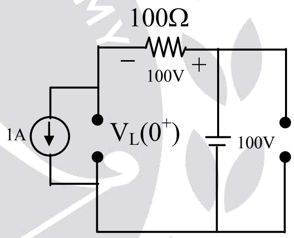
Sol: at t<0





Apply KVL in loop1 ⇒ $V_C(0^-)$–100 = 0

⇒ $V_C(0^-)$ = 100V



At t = 0^+

$$V_L(0^+) = 0$$

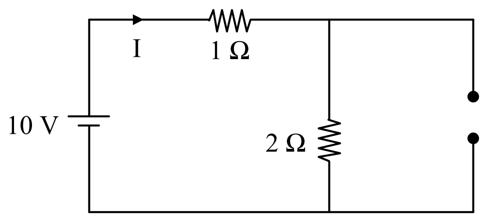
$$L\frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = 0$$

22.

Sol: Case -1 at $t = 0^+$

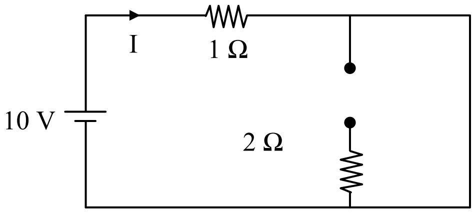
By redrawing the circuit



Current through the battery at $t = 0^+$ is

$$\frac{10}{3}\ \text{Amp}$$

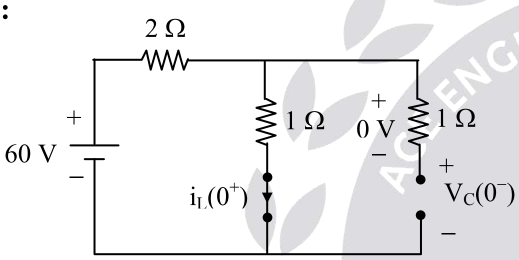
Case -2 at $t = \infty$



Current through the battery at $t = \infty$ is10 Amp

23.

Sol:

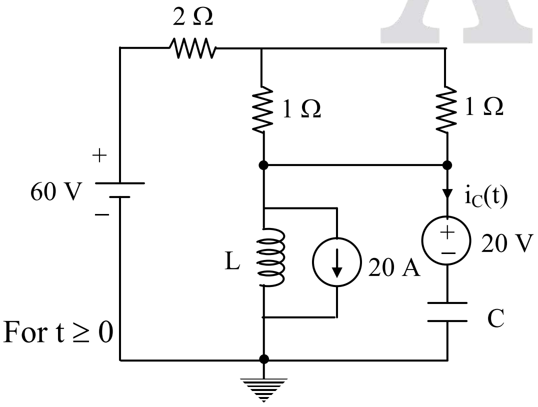


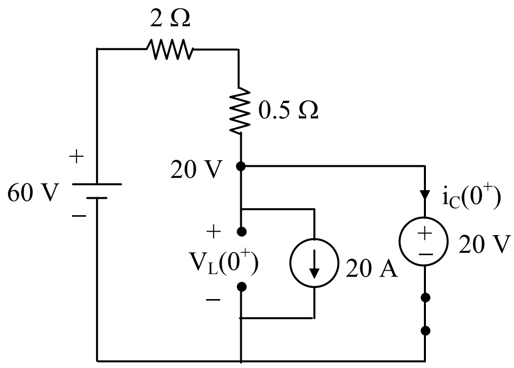
At $t = 0^-$: Steady state : A resistive circuit

(i) $t = 0^-$:

$$i_L(0^-) = \frac{60}{3} = 20\ \text{A} = i_L(0^+)$$

$V_{1\Omega} = 20\ \text{V} = V_C(0^-) = V_C(0^+)$





At $t = 0^+$: A resistive circuit :
Network is in transient state

$V_L(0^+) = 20$ V

Nodal :

$$\frac{20-60}{2.5} + 20 + i_C(0^+) = 0$$

$i_C(0^+) = -4$ A

24.

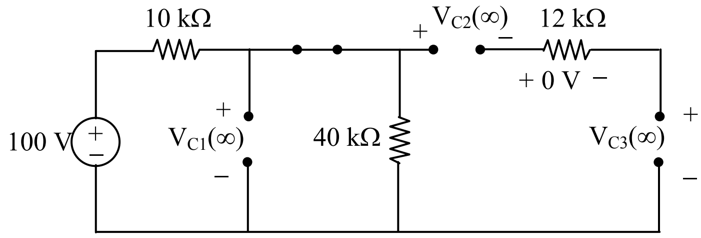
Sol: Repeat the above problem procedure :

$$\left.\frac{d\,i_L(t)}{dt}\right|_{t=0^+} = \frac{V_L(0^+)}{L} = 0\ \text{A/sec}$$

$$\left.\frac{d\,V_C(t)}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = -10^6\ \text{V/sec}$$

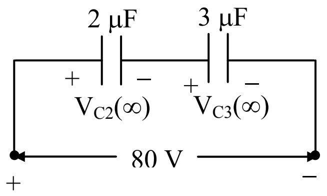
25.

Sol: **Observation:** So, the steady state will occur either at $t = 0^-$ or at $t = \infty$, that depends where we started i.e., connected the source to the network.



At t = ∞ : Steady state: A Resistive circuit

$$V_{C_1}(\infty) = \frac{100}{50K} \times 40K = 80\ V$$

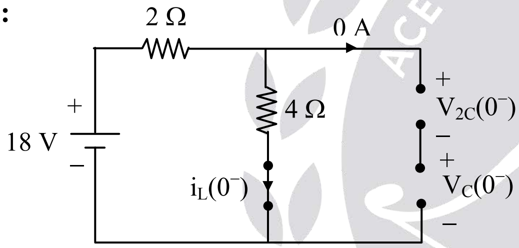


$$V_{C_2}(\infty) = \frac{80 \times 3\mu F}{(2+3)\mu F} = 48\ V$$

$$V_{C_3}(\infty) = \frac{80 \times 2\mu F}{5\mu F} = 32\ V$$

26.

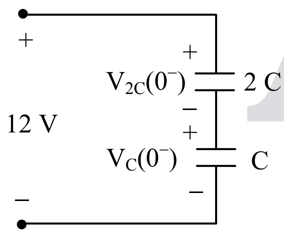
Sol:



At t = 0^- : Circuit is in Steady state: Resistive circuit

$i_L(0^-) = 3\ A = i_L(0^+)$

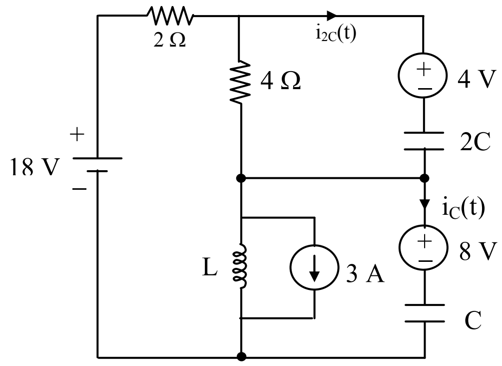
$V_{4\Omega} = 4 \times 3 = 12\ V$



$$V_{2C}(0^-) = \frac{12 \times C}{2C + C}$$

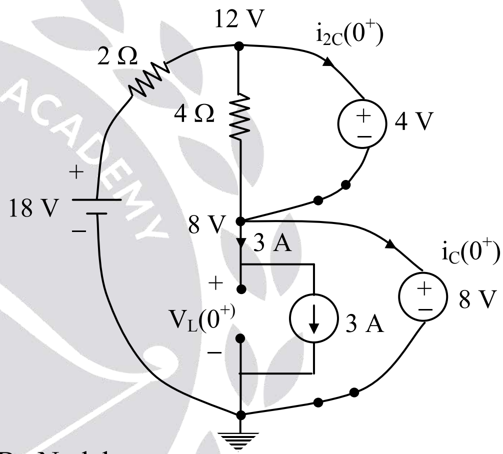
$$= 4\ V = V_{2C}(0^+)$$

$V_C(0^-) = 8\ V = V_C(0^+)$



For t ≥ 0

and redrawing the circuit



By Nodal;

$$\frac{12-18}{2} + \frac{12-8}{4} + i_{2C}(0^+) = 0$$

$$\frac{-6}{2} + \frac{4}{4} + i_{2C}(0^+) = 0$$

$i_{2C}(0^+) = 2\ A = i_{2C}(0^-)$

$$\frac{8-12}{4} - i_{2C}(0^+) + 3 + i_C(0^+) = 0$$

$i_C(0^+) = 0\ A = i_C(0^-)$

27.

Sol: t = 0^- t = 0^+ t = 0^+

$i_L(0^-) = 5\ A$ $i_L(0^+) = 5\ A$

$$\frac{d i_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = 40$$

$i_R(0^-) = -5$ A $\qquad$ $i_R(0^+) = -1$A

$$\frac{d\,i_R(0^+)}{dt} = -40 \text{ A/sec}$$

$i_C(0^-) = 0$ A $\qquad$ $i_C(0^+) = 4$A

$$\frac{d\,i_C(0^+)}{dt} = -40 \text{ A/sec}$$

$V_L(0^-) = 0$ V

$V_L(0^+) = 120$ V

$$\frac{d\,V_L(0^+)}{dt} = 1098 \text{ V/sec}$$

$V_R(0^-) = -150$ V

$V_R(0^+) = -30$ V

$$\frac{d\,V_R(0^+)}{dt} = -1200 \text{ V/sec}$$

$V_C(0^-) = 150$ V

$V_L(0^+) = 150$ V

$$\frac{d\,V_C(0^+)}{dt} = 108 \text{ V/sec}$$

(i). $t = 0^-$

By KCL $\Rightarrow i_L(t) + i_R(t) = 0$

$t = 0^- \;\Rightarrow\; i_L(0^-) + i_R(0^-) = 0$

$i_R(0^-) = -5$ A

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$V_R(0^-) = R\, i_R(0^-) = 30(-5) = -150$ V

By KVL $\Rightarrow V_L(t) - V_R(t) - V_C(t) = 0$

$V_C(0^-) = V_L(0^-) - V_R(0^-) = 150$ V

(ii). At $t = 0^+$

By KCL at 1st node $\Rightarrow$

$-4 + i_L(t) + i_R(t) = 0$

$-4 + i_L(0^+) + i_R(0^+) = 0$

$i_R(0^+) = -\,i_L(0^+) + 4$

$i_R(0^+) = -\,5 + 4$

$= -\,1$ A

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$V_R(0^+) = R\, i_R(0^+)$

$V_R(0^+) = -\,30$ V

By KVL $\Rightarrow V_L(t) - V_R(t) - V_C(t) = 0$

$V_L(0^+) = V_R(0^+) + V_C(0^+)$

$= 150 - 30$

$= 120$ V

By KCL at 2nd node;

$-\,5 + i_C(t) - i_R(t) = 0$

$i_C(0^+) = 4$ A

(iii). $t = 0^+$

By KCL at 1st node $\Rightarrow$

$-\,4 + i_L(t) + i_R(t) = 0$

$$0 + \frac{d\,i_L(t)}{dt} + \frac{d}{dt}\,i_R(t) = 0$$

$V_R(t) = R\, i_R(t)\,|_{\text{By Ohm's law}}$

$$\frac{d}{dt}V_R(t) = R\,\frac{d}{dt}\,i_R(t)$$

By KVL $\Rightarrow$

$V_L(t) - V_R(t) - V_C(t) = 0$

$$\frac{d\,V_L(t)}{dt} - \frac{d\,V_R(t)}{dt} - \frac{d\,V_C(t)}{dt} = 0$$

By KCL at node 2:

$-\,5 + i_C(t) - i_R(t) = 0$

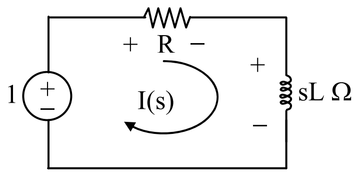
$$0 + \frac{d}{dt}\,i_C(t) - \frac{d}{dt}\,i_R(t) = 0$$

$$\frac{d}{dt}\,i_C(0^+) = -\,(-40)$$

$$= 40 \text{ A/sec}$$

28.

Sol: Transform the network into Laplace domain



S - domain

$V(s) = Z(s)\, I(s)$

By KVL in S-domain ⇒

$1 - R\, I(s) - s\, L\, I(s) = 0$

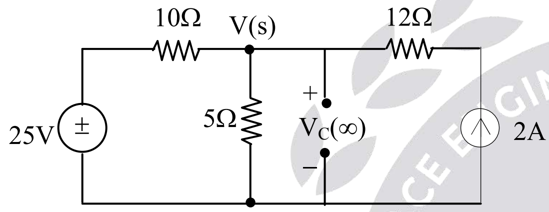
$$I(s) = \frac{1}{L}\,\frac{1}{\left(s + \frac{R}{L}\right)}$$

$$i(t) = \frac{1}{L}\, e^{-\frac{R}{L}t}\ \text{A} \quad \text{for} \quad t \ge 0$$

29.

Sol: By Time domain approach ;

$V_C(0^-) = 5 \times 2 = 10\ \text{V} = V_C(0^+)$



At t =∞: Steady state: A resistive circuit

$$\text{Nodal} \Rightarrow \frac{V_C(\infty) - 25}{10} + \frac{V_C(\infty)}{5} - 2 = 0$$

$V_C(\infty) = 15\ \text{V}$

$\tau = R_{eq}\, C = (5 \parallel 10)\ .\ 1 = (10/3)\ \text{sec}$

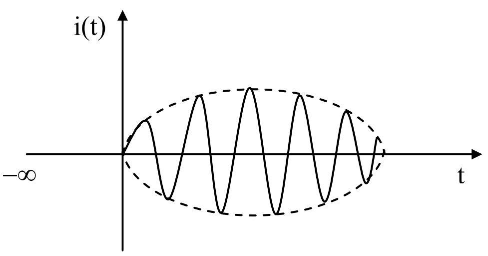
$$V_C = 15 + (10 - 15)\, e^{-\frac{t}{(10/3)}}$$

$V_C = 15 - 5\, e^{-3t/10}\ \text{V} \ \text{ for } \ t \ge 0$

$$i_C = C\,\frac{dV_C}{dt} = 1.5\, e^{-3t/10}\ \text{A} \quad \text{for} \quad t \ge 0$$

30.

Sol:



That is the response is oscillatory in nature

31.

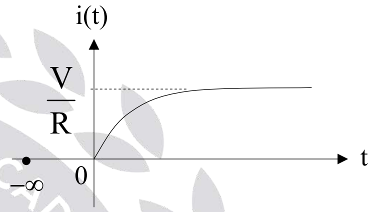
Sol: $i(0^-) = 0\ \text{A} = i(0^+)$

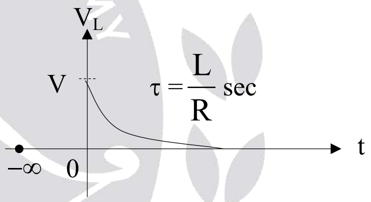
$$i(\infty) = \frac{V}{R}\ \text{A}$$

$$\tau = \frac{L}{R}\ \text{sec}$$

$$i(t) = \frac{V}{R} + \left(0 - \frac{V}{R}\right) e^{-t/\tau} = \frac{V}{R}\,(1 - e^{-t/\tau})$$

$$V_L = \frac{L\,di(t)}{dt} = V\, e^{-Rt/L} \ \text{for}\ t \ge 0$$





Expontionaly Increasing Response

32.

Sol: $V_C(0^-) = 0 = V_C(0^+)$

$V_C(\infty) = V$

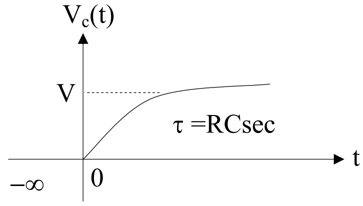
$\tau = RC$

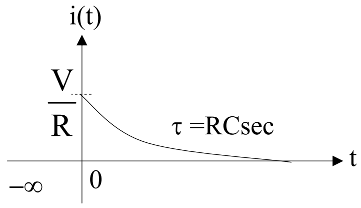
$V_C = V + (0-V)e^{-t/\tau}$

$= V(1-e^{-t/RC})$ for $t \ge 0$

$$ic = C\frac{dv_c}{dt} = \frac{V}{R}e^{-t/RC} \ \text{ for } t \ge 0$$

$= i(t)$



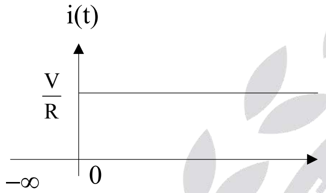


Expontionaly Decreasing Response

33.

Sol: It's an RL circuit with L = 0 ⇒ τ = 0 sec

$i(t) = \frac{V}{R}$, ∀t ≥ 0 So, 5τ = 0 sec



i.e. the response is constant

34.

Sol: $i_1 = \frac{100u(t) - V_L}{10}$

$$i1 = \left(10u(t) - \frac{1}{100}\frac{di_L}{dt}\right)A$$

Nodal ⇒

$$-i_1 + i_L + \frac{V_L - 20i_1}{20} = 0$$

$$-2i_1 + i_L + \frac{1}{200}\frac{di_L}{dt} = 0$$

Substitute i_1;

$$\frac{di_L}{dt} + 40i_L = 800u(t)$$

$$SI_L(s) - i_L(0+) + 40I_L(s) = \frac{800}{s}$$

$i_L(0^-) = 0A = i_L(0^+)$

$$I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

$I_Lt) = 20u(t) - 20e^{-40t}\,u(t)$

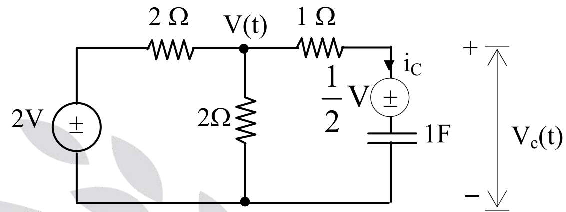
$I_L(t) = 20(1\text{-}e^{-40t})\,u(t)$

$$i_l = 10u(t) - \frac{1}{100}d\frac{i_L}{dt}$$

$i_l = (10\text{-}8e^{-40t})\,u(t)$

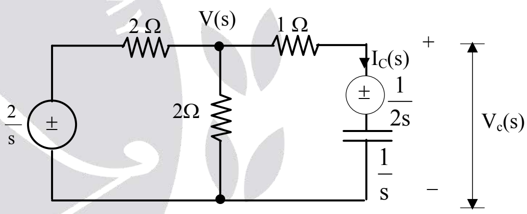
35.

Sol: By Laplace transform approach:



For t≥ 0

Transform the above network into the Laplace domain



For t≥ 0

Nodal ⇒

$$\frac{V(s) - \frac{2}{s}}{2} + \frac{V(s)}{2} + \frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} = 0$$

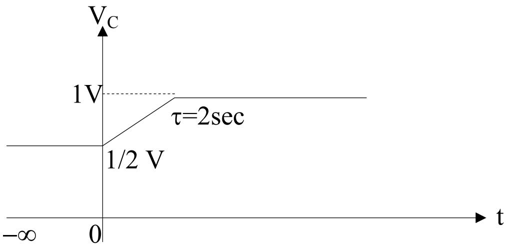
$$I_c(s) = \left(\frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}}\right)$$

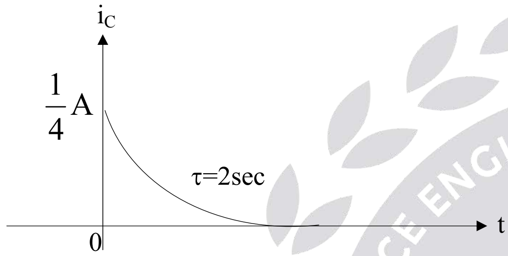
$$\Rightarrow i_c(t) = \frac{1}{4}e^{-\frac{t}{2}}\ \text{A for } t \geq 0$$

By KVL ⇒ $V_C(s) - \frac{1}{2s} - \frac{1}{s}I_C(s) = 0$

$$V_C(s) = \frac{1}{2s} + \frac{1}{s} I_C(s)$$

$$v_C(t) = 1 - \frac{1}{2} e^{-\frac{t}{2}} \text{ V} \quad \text{for } t \geq 0$$



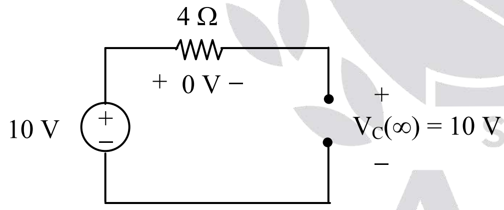


36.

Sol: By Time domain approach;

$V_C(0) = 6$ V (given)

$V_C(\infty) = 10$ V



At t = ∞ : Steady state : Resistive circuit

$\tau = R C = 8$ sec

$V_C = 10 + (6 - 10) e^{-t/8}$

$V_C = 10 - 4 e^{-t/8}$

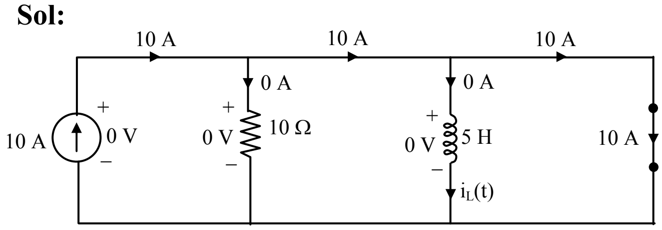
$V_C(0) = 6$ V

$$i_C = C \frac{dV_C}{dt} = e^{-t/8} = i(t)$$

$$E_{4\Omega} = \int_0^{\infty} (e^{-t/8})^2 \; 4 \; dt = 16 \text{ J}$$

37.

Sol:

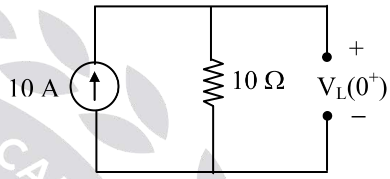


At $t = 0^-$: Network is not in steady state i.e., unenergised

$t = 0^-$:

$i_L(0^-) = 0 \text{ A} = i_L(0^+)$

$V_L(0^+) = 10 \times 10 = 100$ V



At $t = 0^+$: Network is in transient state : A resistive circuit

$i_L(\infty) = 10$ A (since inductor becomes short)

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ sec}$$

$i_L(t) = 10 + (0 - 10) e^{-t/\tau}$

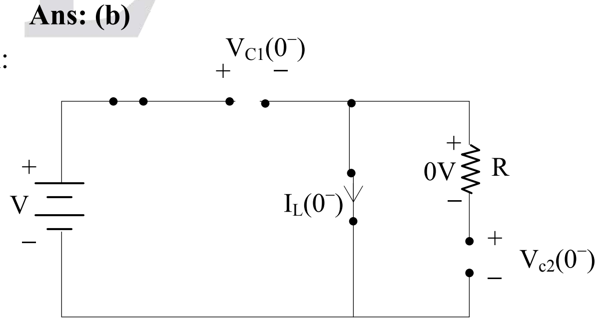
$\quad = 10 (1 - e^{-t/0.5})$ A for $0 \leq t \leq \infty$

$$V_L(t) = L \frac{d}{dt} i_L(t) = 100 e^{-2t} \text{ V} \quad \text{for } 0 \leq t \leq \infty$$

$$E_L \Big|_{t=5\tau \text{ or } t=\infty} = \frac{1}{2} L i^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{J}$$

38. Ans: (b)

Sol:



At $t = 0^-$: Steady state: A resistive circuit

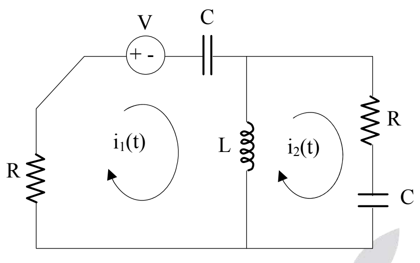
By KVL $\Rightarrow$

$V - V_{c1}(0^-) = 0$

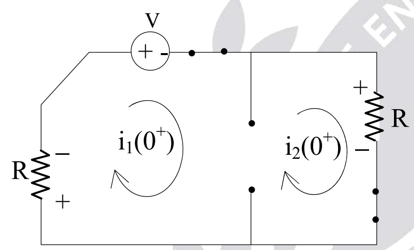
$V_{C1}(0\text{-}) = V = V_{C1}(0^+)$

$V_{C2}(0^-) = 0V = V_{C2}(0^+)$

$i_L(0^-) = 0A = i_L(0^+)$



For $t \geq 0$ Fig (a)



At $t = 0^+$: A resistive circuit: Network is in transient state.

$i_l(0^+) = i_2(0^+)$

By KVL $\Rightarrow$

$-Ri_1(0^+)\text{-}V\text{-}Ri_1(0^+) = 0$

$$i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$$

OBS: $i_L(t) = i_1(t) \sim i_2(t)$

At $t = 0^+ \Rightarrow$

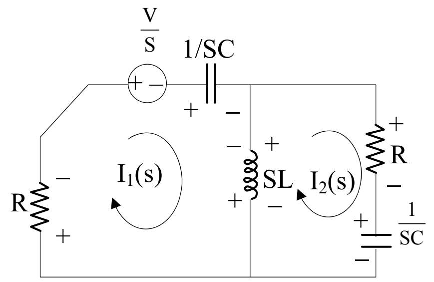
$i_L(0+) = i_1(0+) \sim i_2(0+)$

$= 0A$

$\Rightarrow$ Inductor: open circuit

39.

Sol: (b) Transform the network given in fig. (a) into the S-domain.



$V(s) = Z(s) \cdot I(s)$

By KVL in S-domain $\Rightarrow$

$$-RI_1(s) - \frac{V}{s} - \frac{I_1(s)}{SC} - SL(I_1(s) - I_2(s)) = 0$$

Similarly:

$$-RI_2(s) - \frac{I_2(s)}{SC} - SL(I_2(s) - I_1(s)) = 0$$

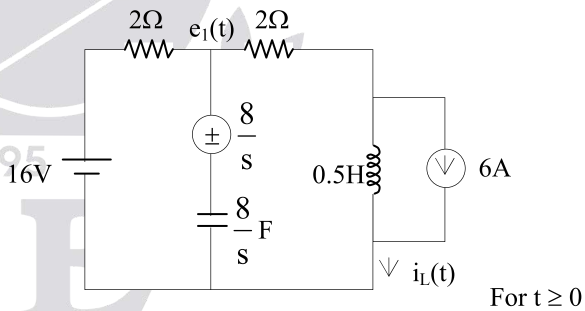
$$\begin{bmatrix} R + SL + \frac{1}{SC} & -SL \\ -SL & R + SL + \frac{1}{SC} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -V/S \\ 0 \end{bmatrix}$$

40.

Sol: Evaluation of $i_L(t)$ and $e_1(t)$ for $t \geq 0$ by Laplace transform approach.

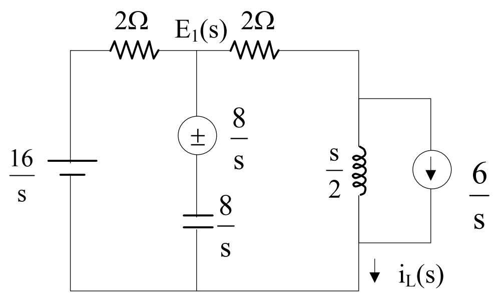
$i_L(0^+) = 6A$; $i_L(\infty) = 4A$

$e_1(0^+) = 8V$; $e_1(\infty) = 8V$

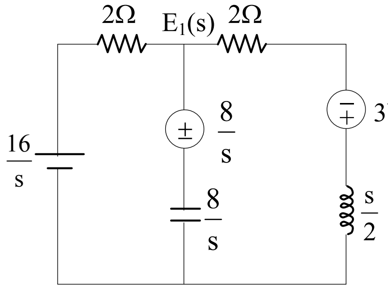


For $t \geq 0$

Transform the above network into Laplace domain.



S-domain :



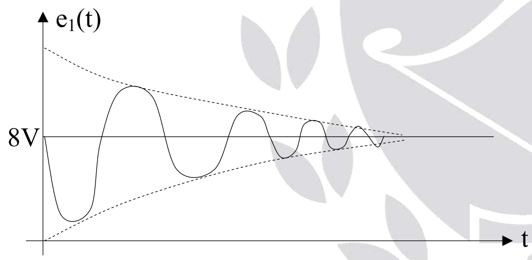
Nodal in S-domain

$$\frac{E_1(s)-16/s}{2}+\frac{E_1(s)-\frac{8}{s}}{\frac{8}{s}}+\frac{E_1(s)+3}{2+\frac{s}{2}}=0$$

$$E_1(s)=\frac{8}{s}\left(\frac{s^2+6s+32}{s^2+8s+32}\right)$$

$$E_1(s)=\frac{8}{s}\left(1-\frac{2s}{(s+4)^2+4^2}\right)$$

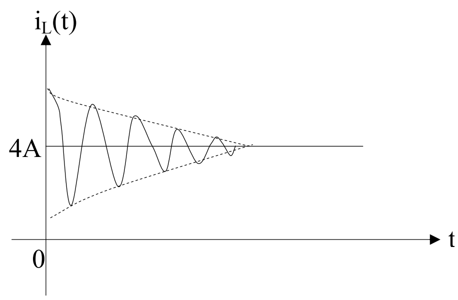
$e_1(t) = 8 - 4e^{-4t}\sin 4t$ V for $t \geq 0$



$$I_L(s)=\frac{E_1(s)+3}{2+\frac{s}{2}}$$

$i_L(t) = 4+2e^{-4t}\cos 4t$ A

for $t \geq 0$ $\omega_n = 4$ rad/sec



OBS: $\tau=\frac{1}{4}\sec=\frac{1}{\xi\omega_n}$ $\Bigg|$ $\omega_n=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{\frac{1}{2}\times\frac{1}{8}}}=4$

$$\frac{1}{4}\times\omega_n=\frac{1}{\xi}$$

$$\xi=\frac{4}{\omega_n}=\frac{4}{4}=1$$

$\xi = 1$ (A critically damped system)

41.

Sol: $\omega t\big|_{t=t_0}=\tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\omega t_o=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$2\pi(50)t_o=\tan^{-1}\left(\frac{2\pi(50)(0.01)}{5}\right)$$

$$t_o=32.14\times\frac{\pi}{180^o}$$

$t_o = 1.78$ msec.

So, by switching exactly at 1.78msec from the instant voltage becomes zero, the current is free from Transient.

42.

Sol: $\omega t_o + \phi = \tan^{-1}(\omega CR) + \frac{\pi}{2}$

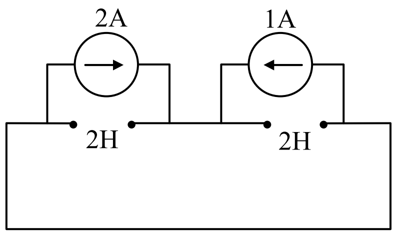
$$2t_o+\frac{\pi}{4}=\tan^{-1}(\omega CR)+\frac{\pi}{2}$$

$$2t_o+\frac{\pi}{4}=\tan^{-1}\left(2\left(\frac{1}{2}\right)(1)\right)+\frac{\pi}{2}=\frac{\pi}{4}+\frac{\pi}{2}$$

$$2t_o=\frac{\pi}{2}\Rightarrow t_o=0.785\sec$$

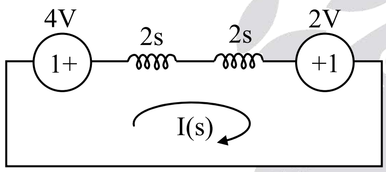
43. **Ans: (a)**

Sol. At $t=0^+$ the circuit is



Inductor never allows sudden change in current but if we allow the current to suddenly change then impulse voltage will establish redistributing flux and then current become equal in them.

Now solving using Laplace transform.



$$I(s)\,[4s] = 4 - 2$$
$$= 2$$
$$\Rightarrow I(s) = \frac{1}{2s}$$
$$i(t) = L^{-1}[I(s)] = \frac{1}{2}A$$

44. Ans: (b)

Sol: For an LTI network:

$y(t) = h(t) * x(t)$, $Y(s) = H(s)\,X(s)$

Statement (I) is True.

$\delta(t) \xrightarrow{LT} 1$

Statement (II) is True and is not the correct explanation of Statement (I).

45. Ans: (a)

Sol: Statement (I): True

Statement (II): True & correct explanation

46. Ans: (b)

Sol: A – 1 : Linearity property

B – 6 : Shift property

C – 4 : Time differentiation property

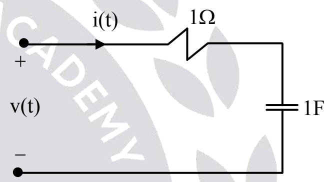
D – 3 : Integration property

$$\int_{-\infty}^{t} f(t)\,dt \to \frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0} f(x)\,dx$$

and $\int_{0}^{t} f(t)\,dt \to \frac{F(s)}{s}$

47. Ans: (a)

Sol:



$$I(s) = \frac{V(s)}{1+\frac{1}{s}} = \frac{s\,V(s)}{s+1}$$

(A) $v(t) = u(t)$,

$V(s) = \frac{1}{s}$, $I(s) = \frac{1}{s+1}$ ………. (2)

(B) $v(t) = r(t)$,

$V(s) = \frac{1}{s^2}$, $I(s) = \frac{1}{s(s+1)}$ ………. (4)

(C) $v(t) = \delta(t)$, $V(s) = 1$,

$I(s) = \frac{s}{(s+1)}$ ……….(1)

(D) $v(t) = e^{-t}u(t)$, $V(s) = \frac{1}{s+1}$,

$I(s) = \frac{s}{(s+1)^2}$ ……….. (3)

48. Ans: (d)

Sol:

	Value of R	Location of poles	i(t), Fig
(A)	$R >> R_C$ (Over damping)	$p_1 = -\sigma_1$, $p_2 = -\sigma_2$	(4)
(B)	$R = R_C$ (Critical damping)	$p_1 = p_2 = -\sigma$	(3)
(C)	$R < R_C$ (Under damping)	$p_1 = \alpha + j\beta$ $p_2 = \alpha - j\beta$ $\alpha < 0$	(2) Sinusoid Decaying
(D)	$R = 0$ (No damping)	$p_1 = j\beta$ $p_2 = -j\beta$	(1) Sustained (constant amplitude) oscillations

49. Ans: (d)

Sol: A. The internal impedance of an ideal current source is infinity (7).

Note that for ideal voltage source, the internal impedance is zero.

B. Attenuated natural oscillations, the poles of the transfer function must lie on the left hand part of the complex frequency plane, like $s = -\alpha$, $s = -\alpha + j\beta$, $\alpha > 0$ (5)

C. Maximum power transferred is

$$\left(\frac{E}{2R}\right)^2 \times R = \frac{E^2}{4R} \quad (3)$$

D. The roots of the characteristic equation give natural response of the circuit. (2)

So the answer is (d)

Conventional Practice Solutions

01.

Sol: The relevant circuit is shown in Fig. for t>0.

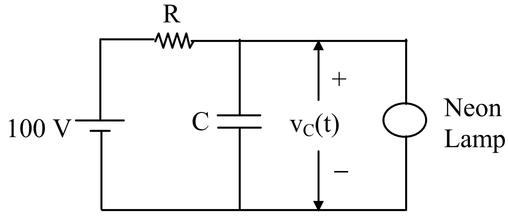


Fig.

$C = 1\ \mu F$

Neon lamp ionizes at 20 sec and glows when v_C-(20) = 75 V

For t > 0, $v_C(t) = 100\left(1 - e^{-\frac{20}{\tau}}\right)$, $\tau = RC$

$$75 = 100\left(1 - e^{-\frac{20}{\tau}}\right),$$

$$\left(1 - e^{-\frac{20}{\tau}}\right) = \frac{3}{4},\ e^{-\frac{20}{\tau}} = \frac{1}{4}$$

$$e^{\frac{20}{\tau}} = 4,\ \frac{20}{\tau} = \log_e 4$$

$$\tau = RC = \frac{20}{\log_e 4} = 14.42$$

$$R = \frac{14.42}{10^{-6}} = 14.42\ M\Omega$$

02.

Sol: The given circuit is shown in Fig. 1

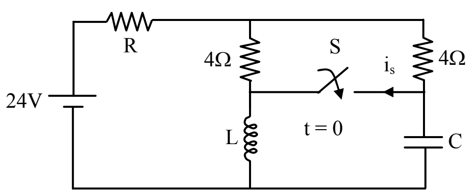


Fig.1

The behaviour of the circuit at $t = 0^-$ is shown in Fig. 2

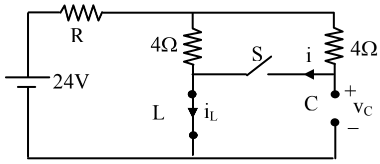


Fig.2

$$i_L(0^-) = \frac{24}{R+4} \quad \text{.......... (1)}$$

$$v_C(0^-) = \frac{24 \times 4}{R+4} = \frac{96}{R+4} \quad \text{.......... (2)}$$

$i_s(0^-) = 0$

The behaviour of the circuit at $t = 0^+$ is shown in Fig. 3

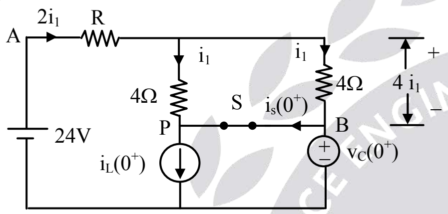


Fig. 3

Given $i_s(0^+) = 1.2$ A

$i_L(0^+) = i_L(0^-)$, $v_C(0^+) = v_C(0^-)$ ....(3)

Let the current through 4 Ω be i_1.

Apply KCL at P.

$i_1 = i_L(0^+) - i_s(0^+)$ .... (4)

Apply KVL around the mesh APBCA

$24 - 2\,i_1\,R - 4\,i_1 = v_C(0^+)$

$24 - i_1(2R + 4) = v_C(0^+)$

Using equations (1), (2), (3) and (4)

$$24 - (2R + 4)\left[\frac{24}{R+4} - 1.2\right] = \frac{96}{R+4}$$

$$24 - (2R + 4)\left[\frac{24 - 1.2R - 4.8}{R+4}\right] = \frac{96}{R+4}$$

$24\,(R + 4) - (2\,R + 4)\,(-1.2\,R + 19.2) = 96$

$24\,R - (-2.4\,R^2 + 38.4\,R - 4.8\,R + 76.8) = 0$

$R^2 - 4\,R - 32 = 0$

$(R - 8)\,(R + 4) = 0$

$R = 8\ \Omega, -4\ \Omega$

The negative resistance is not valid.

$\therefore$ R = 8 Ω

At $t = \infty$, the behaviour of the circuit is shown in Fig. 4.

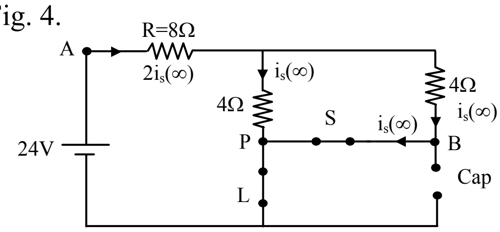


Fig. 4

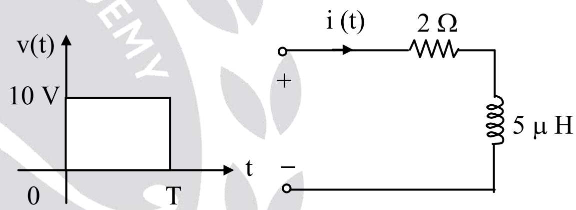
Apply KVL around the mesh APCA:

$24 = 16\,i_s(\infty) + 4\,i_s(\infty)$

$$i_s(\infty) = \frac{24}{20} = 1.2\ \text{A}$$

03.

Sol:



T = 10 μ s

$$I(s) = \frac{10}{s}(1 - e^{-Ts})\,\frac{1}{Ls + R}$$

$$= \frac{10}{L}\,\frac{1}{s\left(s + \frac{R}{L}\right)}(1 - e^{-Ts})$$

$$= \frac{10}{L}\left[\frac{L}{R}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right)\right](1 - e^{-Ts})$$

$$= \frac{10}{R}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right)(1 - e^{-Ts})$$

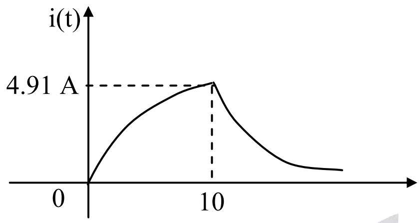
$$i(t) = 5\left(1 - e^{-\frac{R}{L}t}\right)u(t) - 5\left[1 - e^{-\frac{R}{L}(t-T)}\right]u(t - T)$$

$$\frac{R}{L} = \frac{2}{5} \times 10^6 = 0.4 \times 10^6 \ \text{sec}^{-1}$$

$$i(t) = 5\left(1 - e^{-0.4 \times 10^6 t}\right)$$

$$-5\left[1 - e^{-0.4 \times 10^6 (t - 10 \times 10^{-6})}\, u(t - 10 \times 10^{-6})\right]$$

$$i(10\ \mu s) = 5\left(1 - e^{-4}\right) = 4.91\ A$$



04.

Sol: The RC circuit and its input are shown in Fig. 1

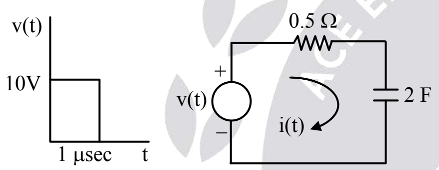


Fig. 1

$v(t) = 10\,[\,u(t) - u(t - t_0)\,]$, $t_0 = 1\ \mu\text{sec}$

The transform equivalent circuit is shown in Fig. 2.

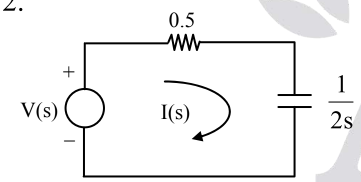


Fig. 2

$$V(s) = 10\left[\frac{1}{s} - \frac{e^{-st_0}}{s}\right]$$

$$Z(s) = 0.5 + \frac{1}{2s} = \frac{s+1}{2s}$$

$$I(s) = \frac{10}{s}\left(1 - e^{-t_0 s}\right)\frac{2s}{(s+1)}$$

$$= \frac{20}{s+1} - \frac{20}{s+1}e^{-t_0 s}$$

$$i(t) = 20\,e^{-1t}\,u(t) - 20\,e^{-(t - 10^{-6})}\,u(t - 10^{-6})$$

$t = 0$, $\quad i(0^+) = 20$ A

$t = 1\ \mu s$, $\ i(1^-) = 20\exp(-10^{-6})$

and $\quad i(1^+) = 20\exp(-10^{-6}) - 20$

The variation of i(t) in shown in Fig. 3.

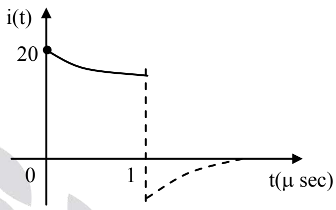


Fig. 3

05.

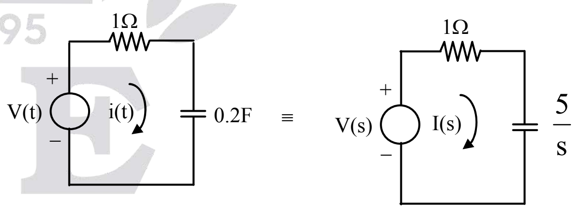
Sol: Given v(t) can be expressed as follows

$$V(t) = \frac{1}{2}[r(t) - r(t - t_0)]$$

$$= \frac{1}{2}[r(t) - r(t-2)]$$

$$V(s) = \frac{1}{2s^2}\left(1 - e^{-2s}\right)$$

Converting everything into Laplace domain



$$I(s) = \frac{V(s)}{1 + \frac{5}{s}} = \frac{V(s).s}{s+5}$$

$$I(s) = \frac{1 - e^{-2s}}{2s(s+5)}$$

$$\frac{1 - e^{-2s}}{2s(s+5)} = \frac{1}{2}\left[\frac{1}{s(s+5)} - \frac{e^{-2s}}{s(s+5)}\right]$$

$$I(s)=\frac{1}{10}\left[\frac{1}{s}-\frac{1}{(s+5)}\right]-\frac{1}{10}\left[\frac{1}{s}-\frac{1}{s+5}\right]e^{-2s}$$

Taking inverse Laplace transform

$$i(t)=\frac{1}{10}\left[u(t)-e^{-5t}u(t)\right]-$$

$$\frac{1}{10}\left[u(t-2)-e^{-5t}u(t-2)\right]$$

So,

$$i(t)=\frac{1}{10}\left\{u(t)-u(t-2)+e^{-5t}\left[u(t-2)-u(t)\right]\right\}$$

06

Sol: The RLC series circuit is shown in Fig. 1.

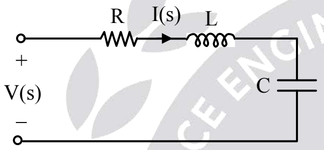


Fig.1

R = 3 Ω, L = 1 H, C = 0.5 F

$$I(s) = \frac{V(s)}{Z(s)}$$

$$Z(s) = R+Ls+\frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$$

$$= \frac{L\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}{s}$$

$$= \frac{s^2+3s+2}{s} = \frac{(s+1)\ (s+2)}{s}$$

(i) For v(t) = r(t), V(s) = $1/s^2$

$$I_r(s) = \frac{1}{s(s+1)(s+2)} = \frac{1/2}{s}+\frac{(-1)}{s+1}+\frac{1/2}{s+2}$$

$$\Rightarrow\ i_r(t) = \left(\frac{1}{2}-1e^{-1t}+\frac{1}{2}e^{-2t}\right)u(t)$$

When r(t) is magnitude scaled by 12 and delayed by 2, i.e., v(t) = 12 r(t – 2)

$$i(t) = 12\left[\frac{1}{2}-1e^{-1(t-2)}+\frac{1}{2}e^{-2(t-2)}\right]u(t-2)$$

(ii) For v(t) = 1 u(t)

$$I_u(s) = \frac{1}{(s+1)\ (s+2)} = \frac{1}{s+1}+\frac{-1}{s+2}$$

$\Rightarrow i_u(t) = (1\ e^{-1\,t} - 1\ e^{-2\,t})\ u(t)$

When v(t) = 2 u(t – 3)

$i(t) = 2\ [\ e^{-(t-3)} - e^{-2\,(t-3)}\ ]\ u(t-3)$

(iii) For V(t) = δ(t) ⇒V(s) = 1

$$\Rightarrow I_\delta(s) = \frac{s}{(s+1)(s+2)}$$

$$= \frac{-1}{s+1}+\frac{2}{s+2}$$

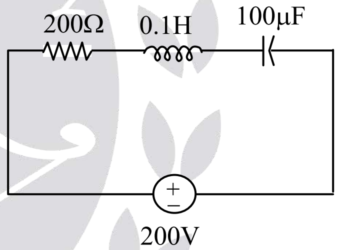
$\Rightarrow i_\delta(t) = -e^{-t}u(t) + 2e^{2t}u(t)$

When V(t) = 3δ(t –1)

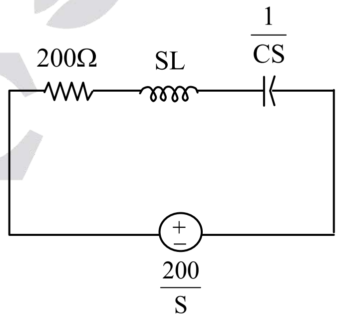
$i(t) = -3e^{-(t-1)}u(t-1) + 6e^{-2(t-1)}u(t-1)$

07.

Sol:



S-domain



$$\frac{200}{S} = I(S)\left(R+LS+\frac{1}{CS}\right)$$

$$\frac{200}{S} = I(S)\left(\frac{RCS+LCS^2+1}{CS}\right) = L\left(S^2+\frac{R}{L}S+\frac{1}{LS}\right)$$

$$I(S) = \frac{200}{L}\frac{1}{\left(S^2 + \frac{R}{L}S + \frac{1}{LC}\right)}$$

$$I(S) = \frac{200}{L}\left[\frac{1}{S^2 + \frac{R}{L}S + \frac{1}{LC}}\right]$$

$$I(S) = \frac{200}{0.1}\left[\frac{1}{S^2 + \frac{200}{0.1}S + \frac{1}{0.1\times 100\times 10^{-6}}}\right]$$

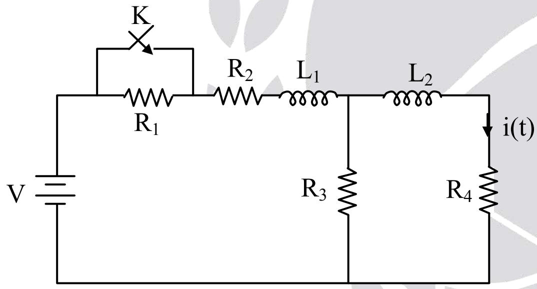
$$I(S) = 2000\left[\frac{1}{S^2 + 2000s + 10^5}\right]$$

$$= 2000\left[\frac{1}{(S - 52)(S + 1948)}\right]$$

$$i(t) = 1.055[e^{-52t} - e^{-1948t}]$$

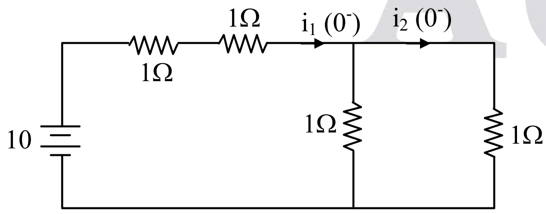
08.

Sol:



For t < 0, K is opened

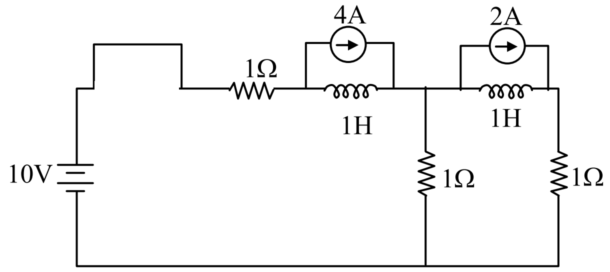
At $t = 0^-$, L is short circuit,



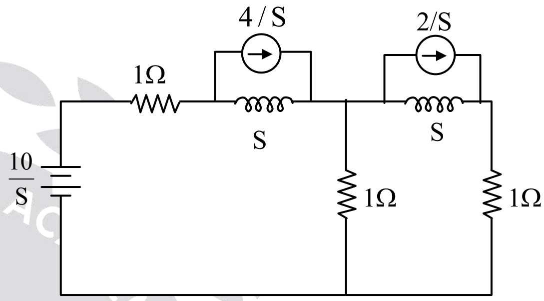
$$i_1(0^-) = \frac{10}{2.5} = 4A = I_1(0^-)$$

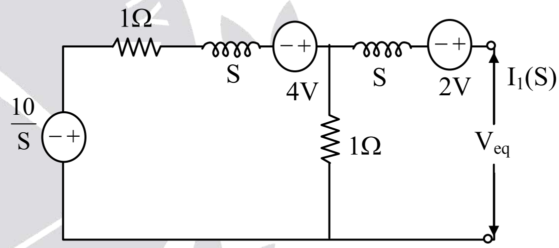
$$i_2\left(0^-\right) = \frac{I_1(0)}{2} = 2A$$

For t > 0 K is closed

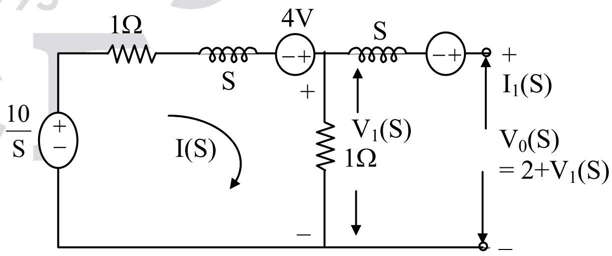


S- domain





For $V_{th} = V_{0C}$



$$I(S) = \frac{\frac{10}{S} + 4}{(2+S)} = \frac{10}{S(S+2)} + \frac{4}{(S+2)}$$

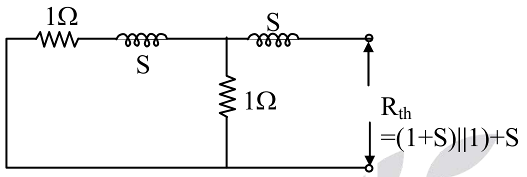
$$I(S) = 5\left(\frac{1}{S} - \frac{1}{S+2}\right) + \frac{4}{S+2}$$

$V_1(S) = I(S)$

$$V_{oc}(S) = V_{th}(S) = 2 + \frac{10}{S(S+2)} + \frac{4}{(S+2)}$$

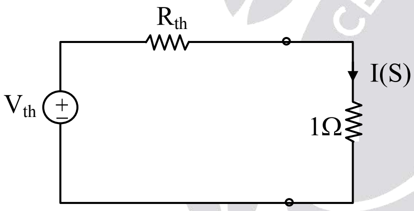
$$= \frac{2(S^2+4S+5)}{S(S+2)}$$

For R_{th}:



$$Z_{th} = \frac{(1+S)+(S+2)S}{(S+2)} = \frac{S^2+3S+1}{(S+2)}$$

Thevenin's equivalent circuit is



$$I(S) = \frac{V_{th}}{R_{th}+1} = \frac{\dfrac{2(S^2+4S+5)}{S(S+2)}}{\dfrac{S^2+3S+1}{(S+2)}+1}$$

$$I(S) = \frac{2(S^2+4S+5)/S}{(S^2+3S+1+S+2)}$$

$$I(S) = \frac{2(S^2+4S+5)}{S(S^2+4S+3)} = \frac{2(S^2+4S+5)}{S(S+1)(S+3)}$$

$$I(S) = \left[\frac{A}{S} + \frac{B}{(S+1)} + \frac{C}{(S+3)}\right]$$

$$= \left[\frac{10/3}{S} + \frac{(-2)}{(S+1)} + \frac{(2/3)}{(S+3)}\right]$$

$$i(t) = \left[\frac{10}{3} - 2e^{-t} + \frac{2}{3}e^{-3t}\right]u(t)$$

09.

Sol:

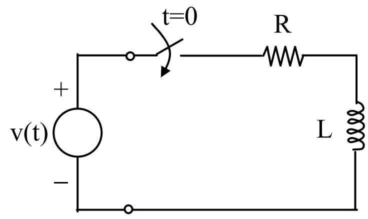


Fig.

$$v(t) = L\,\frac{di(t)}{dt} + R\,i(t)$$

$$L\,\frac{di}{dt} + R\,i(t) = E\sin(\omega t + \phi)$$

Phasor voltage, $\vec{V} = E\,e^{j\phi}$

Phasor current, $\vec{I} = \dfrac{\vec{V}}{Z} = \dfrac{E\,e^{j\phi}}{R+j\omega L}$

$$= \frac{E}{\sqrt{R^2+\omega^2L^2}}\,e^{j\left(\phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}$$

$$i_{ss}(t) = \frac{E}{\sqrt{R^2+\omega^2L^2}}\sin\left[\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

The transient part of current is obtained from the homogeneous equation:

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0, \qquad i_{tr}(t) = K\,e^{-\frac{R}{L}t}$$

$$i_{tot}(t) = K\,e^{-\frac{R}{L}t} + i_{ss}, \quad i_{tot}(0) = 0$$

$$K = -\frac{E}{\sqrt{R^2+\omega^2L^2}}\sin\left[\phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

There is no transient in the current if $i_{tr}(t) = 0$ or $K = 0$ or $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

10.

Sol: $\frac{1\,di_1}{dt} + 1i_1(t) - \frac{2\,di_2}{dt} = 5$

$$-2\frac{di_i}{dt} + 4\frac{di_2}{dt} + 1i_2(t) = 0$$

$$i_1(t) \to I_1(s),\ i_2(t) \to I_2(s)$$

$$(s+1)I_1 - 2sI_2 = \frac{5}{s}$$

$$-2\,s\,I_1 + (4\,s+1)I_2 = 0$$

$$I_1(s) = \frac{\Delta_1}{\Delta},$$

$$\Delta_1 = \begin{vmatrix} \frac{5}{s} & -2s \\ 0 & 4s+1 \end{vmatrix} = \frac{5(4s+1)}{s}$$

$$= \frac{20(s+0.25)}{s}$$

$$\Delta = \begin{vmatrix} s+1 & -2s \\ -2s & 4s+1 \end{vmatrix} = 5s+1 = 5(s+0.2)$$

$$I_1(s) = \frac{4(s+0.25)}{s(s+0.2)} = \frac{5}{s} - \frac{1}{s+0.25}$$

$$i_1(t) = (5 - e^{-0.2t})u(t)$$

Chapter 4 AC Circuit Analysis

Objective Practice Solutions

01.

Sol: $I_{avg} = I_{dc} = \frac{1}{T}\int_0^T i(t)dt$

$= 3 + 0 + 0 = 3A$

$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$

$= \sqrt{3^2 + \left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + 0 + 0 + 0}$

$= 5\sqrt{2}A$

02.

Sol: $V_{dc} = V_{avg} = \frac{1}{T}\int_0^T V(t)dt = 2V$

Here the frequencies are same, by doing simplification

$v(t) = 2 - 3\sqrt{2}\,(\cos 10t \times \frac{1}{\sqrt{2}} - \sin 10t \times \frac{1}{\sqrt{2}}) + 3\cos 10t$

$= 2 + 3\sin 10t$ V

So $V_{rms} = \sqrt{(2)^2 + (\frac{3}{\sqrt{2}})^2} = \sqrt{8.5}$ V

03.

Sol: $X_{avg} = X_{dc} = \frac{1}{T}\int_0^T x(t)dt = 0$

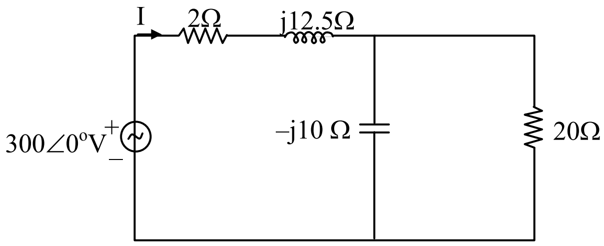
$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \frac{A}{\sqrt{3}}$

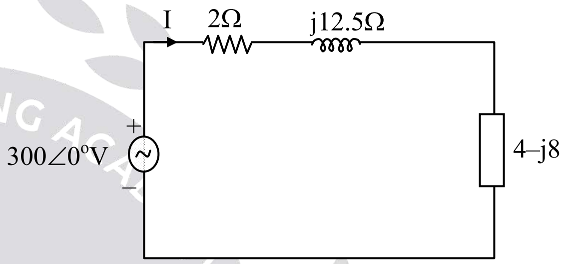
04. Ans: (a)

Sol: For a symmetrical wave (i.e., area of positive half cycle = area of negative half cycle.) The RMS value of full cycle is same as the RMS value of half cycle.

05.

Sol: Complex power, $S = VI^*$





$\Rightarrow I = \frac{300\angle 0°}{2 + j12.5 + 4 - j8}$

$\Rightarrow I = 40\angle -36.86°$

$\therefore$ Complex power, $S = VI^*$

$= 300\angle 0° \times 40\angle 36.86°$

$= 9600 + j7200$

$\therefore$ Reactive power delivered by the source

Q = 72000 VAR

= 7.2 KVAR

06.

Sol: $Z = j1 + (1 - j1)\|(1 + j2) = 1.4 + j\,0.8$

$I = \frac{E_1}{Z}\Big|_{\text{By ohm's law}} = \frac{10\angle 20}{1.4 + j8}$

$= 6.2017\angle -9.744°$ A

$I_1 = \frac{I(1 + j2)}{1 - j1 + 1 + j2}$

$= 6.2017\angle 27.125°$ A

$$I_2 = \frac{I(1-j1)}{1-j1+1+j2}$$

$$= 3.922\angle -81.31^\circ \text{ A}$$

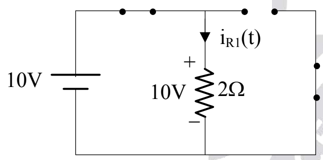
$E_2 = (1-j1)I_1 = 8.7705\angle -17.875^\circ$ V

$E_0 = 0.5I_2 = 1.961\angle -81.31^\circ$ V

07.

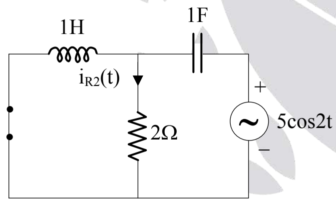
Sol: Since two different frequencies are operating on the network simultaneously always the super position theorem is used to evaluate the response.

By SPT: (i)



Network is in steady state, therefore the network is resistive. $I_{R1}(t) = \frac{10}{2} = 5A$

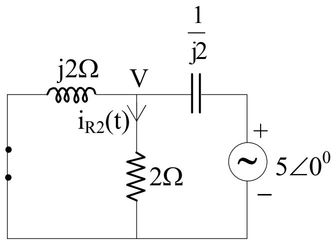
(ii)



Network is in steady state

As impedances of L and C are present because of ω = 2. They are physically present.

$Z_L = j\omega L$; $Z_c = \frac{1}{j\omega C}\Big|_{\omega=2}$



Network is in phasor domain

Nodal ⇒

$$\frac{V}{j2} + \frac{V}{2} + \frac{V - 5\angle 0^0}{-j0.5} = 0$$

$V = 6.32\angle 18.44^0$

$I_{R2} = \frac{V}{2} = 3.16\angle 18.44^0 = 3.16\,e^{j18.14^0}$

$i_{R2}(t) = \text{R.P}[I_{R2}e^{j2t}]$A

$= 3.16\cos(2t + 18.44^0)$

By super position theorem,

$i_R(t) = i_{R1}(t) + i_{R2}(t)$

$= 5 + 3.16\cos(2t + 18.44^0)$A

08. Ans: (c)

Sol: $\frac{1}{s^2+1} - I(s)\left(2 + 2s + \frac{1}{s}\right) = 0$

$$I(s)\left(\frac{2s + 2s^2 + 1}{s}\right) = \frac{1}{s^2+1}$$

$$I(s) + 2s^2I(s) + 2sI(s) = \frac{s}{s^2+1}$$

$$i(t) + \frac{2d^2i}{dt^2} + 2\frac{di}{dt} = \cos t$$

$$2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \cos t$$

09.

Sol: $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$V = V_R = I.R$

$100 = I.20$; $I = 5A$

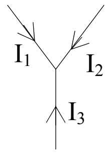
Power factor $= \cos\phi = \frac{V_R}{V} = \frac{V_R}{V_R} = 1$

So, unity power factor.

10.

Sol: By KCL in phasor – domain

$\Rightarrow -I_1 - I_2 - I_3 = 0$



$I_3 = -(I_1 + I_2)$

$i_1(t) = \cos(\omega t + 90^0)$

$I_1 = 1\angle 90^0 = j1$

$I_2 = 1\angle 0^0 = (1 + j0)$

$I_3 = \sqrt{2}\angle \pi + 45^0 = \sqrt{2}\, e^{j(\pi + 45)}$

$i_3(t) = \text{Real part}[I_3.e^{j\omega t}]\text{mA}$

$= -\sqrt{2}\cos(\omega t + 45^0 + \pi)\text{mA}$

$i_3(t) = -\sqrt{2}\cos(\omega t + 45^0)\text{mA}$

11.

Sol: $I = \dfrac{V}{R} + \dfrac{V}{Z_L} + \dfrac{V}{Z_C} = 8 - j12 + j18$

$I = 8 + 6j$

$|I| = \sqrt{100} = 10\text{A}$

12.

Sol: By KCL $\Rightarrow$

$-I + I_L + I_C = 0$

$I = I_L + I_C$

$I_L = \dfrac{V}{Z_L} = \dfrac{V}{j\omega L} = \dfrac{3\angle 0^o}{j(3).\left(\dfrac{1}{3}\right)}$

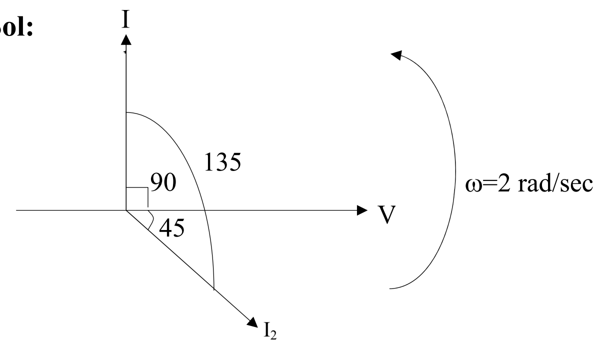
$I_L = \dfrac{3\angle 0^0}{j} = \dfrac{3\angle 0^0}{\angle 90^0} = 3\angle -90^0$

$I = 3\angle -90^0 + 4\angle 90^0$

$= -j3 + j4 = j1 = 1\angle 90^0$

13. Ans: (d)

Sol:



$I_1 = I_C = \dfrac{V}{Z_C} = \dfrac{V}{X_C}\angle 90^0$

$I_2 = \dfrac{V}{2 + j\omega L} = \dfrac{V}{2 + j2} = \dfrac{V}{2\sqrt{2}}\angle 45^0$

Therefore, the phasor I_1 leads I_2 by an angle of 135°.

14.

Sol: $I_2 = \sqrt{I_R^2 + I_C^2} \Rightarrow 10 = \sqrt{I_R^2 + 8^2}$

$I_R = 6\text{A}$

$I_1 = I = \sqrt{I_R^2 + (I_L - I_C)^2}$

$10 = \sqrt{6^2 + (I_L - I_C)^2}$

$I_L - I_C = \pm 8\text{A}$

$I_L - 8 = \pm 8$

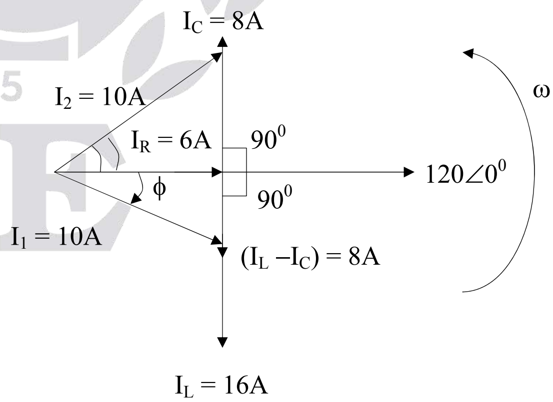
$I_L - 8 = -8$(Not acceptable)

Since $I_L = \dfrac{V}{Z_L} \neq 0$.

$I_L - 8 = 8$

$I_L = 16\text{A}$

$I_L > I_C$



I_2 leads $120\angle 0^0$ by $\tan^{-1}\left(\dfrac{8}{6}\right)$

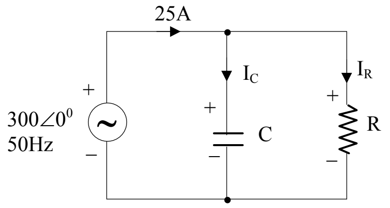
I_1 lags $120\angle 0^0$ by $\tan^{-1}\left(\dfrac{8}{6}\right)$

Power factor $\cos\phi = \dfrac{I_R}{I} = \dfrac{I_R}{I}$

$= \dfrac{6}{10} = 0.6$ (lag)

15.

Sol:



Network is in steady state.

$|I_C| = \left|\dfrac{V}{Z_C}\right| = \left|\dfrac{300\angle 0^0}{(1/j\omega c)}\right| = v\omega c$

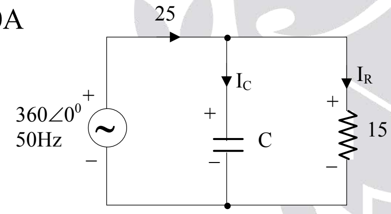
$= 300 \times 2\pi \times 50 \times 159.23 \times 10^{-6}$

$I_C = 15A$

$I = \sqrt{I_R^2 + I_C^2}$

$25 = \sqrt{I_R^2 + 15^2}$

$I_R = 20A$



$V_R = RI_R$|By ohm's law

$300 = R.20$

$R = 15\Omega$

Network is in steady state

$I_R = \dfrac{360}{15} = 24A$

So the required $I_C = \sqrt{25^2 - 24^2}$

$v\omega c = 7$

$360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7$

$f = 19.4Hz$

OBS: $I_C = \dfrac{V}{Z_C}$

$Z_C = \dfrac{1}{j\omega c}\,\Omega$

As $f\downarrow \Rightarrow Z_C\uparrow \Rightarrow I_C\downarrow$

16.

Sol: $P_{5\Omega} = 10$Watts (Given)

$= P_{avg} = I_{rms}^2 R$

$10 = I_{rms}^2.5$

$I_{rms} = \sqrt{2}$ A

Power delivered = Power observed

(By Tellegen's Theorem)

$P_T = I_{rms}^2(5 + 10)$

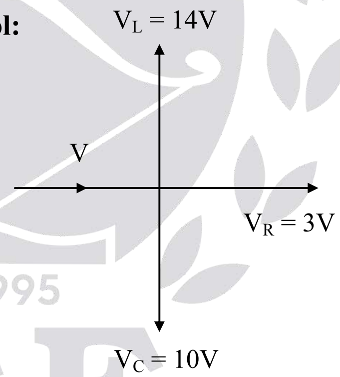
$V_{rms} I_{rms} \cos\phi = \left(\sqrt{2}\right)^2 (15)$

$\dfrac{50}{\sqrt{2}} \times \sqrt{2} \cos\phi = 2 \times 15$

$\cos\phi = 0.6$ (lag)

17. Ans: (d)

Sol:



$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(3)^2 + (14-10)^2}$

$V = 5$ V

18.

Sol: $Y = Y_1 + Y_c = \dfrac{1}{Z_L} + \dfrac{1}{Z_C}$

$= \dfrac{1}{30\angle 40^0} + \dfrac{1}{\left(\dfrac{1}{j\omega c}\right)}$

$= j\omega c + \frac{1}{30}\angle -40^0$

$= j\omega c + \frac{1}{30}(\cos 40^0 - j\sin 40^0)$

Unit power factor $\Rightarrow$ j –term = 0

$\omega c = \frac{\sin 40^0}{30}$

$C = \frac{\sin 40^0}{2\pi \times 50 \times 30} = 68.1\mu F$

$C = 68.1\mu F$

19. Ans: (b)

Sol: To increase power factor shunt capacitor is to be placed.

VAR supplied by capacitor

$= P(\tan\phi_1 - \tan\phi_2)$

$= 2\times 10^3[\tan(\cos^{-1} 0.65) - \tan(\cos^{-1} 0.95)]$

$= 1680$ VAR

VAR supplied $= \frac{V^2}{X_C} = V^2\omega C = 1680$

$\therefore C = \frac{1680}{(115)^2 \times 2\pi \times 60} = 337\mu F$

20.

Sol: $Z = \frac{V}{I} = \frac{160\angle 10^o - 90^o}{5\angle -20^o - 90^o} = 32\angle 30^o$

$\phi = 30^o$ (Inductive)

$V_{rms} = \frac{160}{\sqrt{2}} Vj, I_{rms} = \frac{5}{\sqrt{2}}$

Real power (P) $= \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos 30^0$

$= 200\sqrt{3}$ W

Reactive power (Q) $= \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \frac{1}{2}$

$= 200$ VAR

Complex power = P + jQ = $200(\sqrt{3} + j1)$ VA

21.

Sol: $V = 4\angle 10^o$ and $I = 2\angle -20^o$

Note: When directly phasors are given the magnitudes are taken as rms values since they are measured using rms meters.

$V_{rms} = 4V$ and $I_{rms} = 2A$

$Z = \frac{V}{I} = 2\angle 30^o;\ \phi = 30^o$ (Inductive)

$P = 10\sqrt{3}$ W, Q = 10VAR

$S = 10(\sqrt{3} + j1)$ VA

22. Ans: (a)

Sol: $S = VI^*$

$= (10\angle 15°)(2\angle 45°)$

$= 10 + j17.32$

$S = P + jQ$

P = 10 W Q = 17.32 VAR

23. Ans: (c)

Sol: $P_R = (I_{rms})^2 \times R$

$I_{rms} = \frac{10}{\sqrt{2}}$

$P_R = \left(\frac{10}{\sqrt{2}}\right)^2 \times 100$

24.

Sol: $P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{240}{\sqrt{2}}\right)^2}{60} = 480$ watts

$V = 240\angle 0^0$

$I_R = \frac{V}{R} = \frac{240}{60} = 4A$

$I_L = \frac{V}{Z_L} = \frac{V}{X_L} = \frac{240}{40} = 6A$

$I_C = \frac{V}{Z_C} = \frac{V}{X_C} = \frac{240}{80} = 3A$

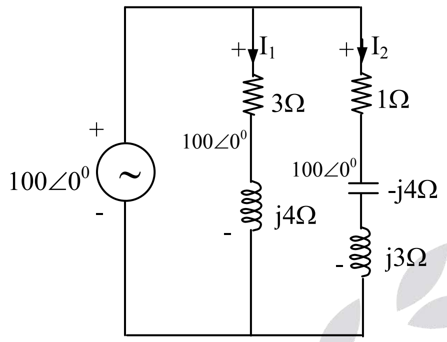
$I_L > I_C$: Inductive nature of the circuit.

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{4^2 + 3^2} = 5A$$

$$\text{Power factor} = \frac{I_R}{I} = \frac{4}{5} = 0.8 \text{ (lagging)}$$

25. Ans: (a)

Sol:



NW is in Steady state.

$V = 100\angle 0^0 \Rightarrow V_{rms} = 100V$

$$I_1 = \frac{100\angle 0^0}{(3+j4)\Omega} \Rightarrow |I_1| = 20 = I_{1rms}$$

$$I_2 = \frac{100\angle 0^0}{(1-j1)\Omega} \Rightarrow |I_2| = \frac{100}{\sqrt{2}} A = I_{2rms}$$

$P = P_1 + P_2$

$= (I_{1rms})^2.3 + (I_{2rms})^2.1$

$= 20^2.3 + \left(\frac{100}{\sqrt{2}}\right)^2.1$

$P = 6200$ W

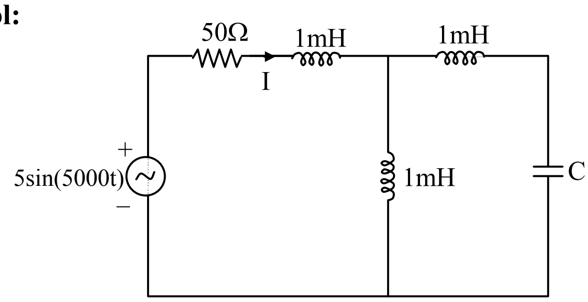
$Q = Q_1 + Q_2 = (I_{1rms})^2.4 + (I_{2rms})^2.(1)$

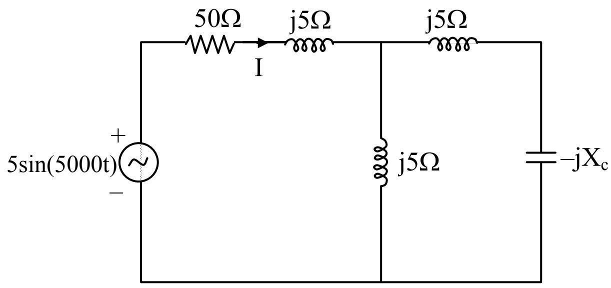
$= 3400$VAR

So, $S = P + jQ = (6200 + j3400)$ VA

26.

Sol:





When I = 0,

⇒ impedance seen by the source should be infinite

$\Rightarrow Z = \infty$

$\therefore Z = (50+j5) + (j5) \,||\, j(5 - X_c)$

$$= 50 + j5 + \frac{j5 \times j(5 - X_c)}{j5 + j(5 - X_c)} = \infty$$

$\Rightarrow j(10 - X_c) = 0$

$$\Rightarrow X_c = 10 \Rightarrow \frac{1}{\omega c} = 10$$

$$\Rightarrow C = \frac{1}{5000 \times 10} = 20\ \mu F$$

27. Ans: (c)

Sol: $$I_{rms} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$$

$$= \sqrt{25} = 5A$$

Power dissipation $= I_{rms}^2\ R$

$= 5^2 \times 10$

$= 250$ W

28.

Sol: $X_C = X_L$

$\Rightarrow \omega = \omega_0$, the circuit is at resonance

$V_C = QV_S\angle -90^0$

$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2$$

$$= \frac{1}{\omega_0 cR} = \frac{X_C}{R} = 2$$

$\Rightarrow V_C = 200\angle -90^0 = -j200V$

29.

Sol: Series RLC circuit

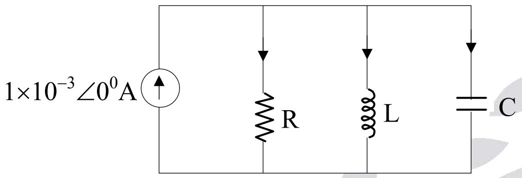
$f = f_L$, PF = $\cos\phi = 0.707$(lead)

$f = f_H$, PF = $\cos\phi = 0.707$(lag)

$f = f_o$, PF = $\cos\phi = 1$

30. Ans: (b)

Sol: Network is in steady state (since no switch is given)



Let I = 1mA

$\omega = \omega_0$(Given)

$\Rightarrow I_R = I$

$I_L = QI\angle{-90^0} = -jQI$

$I_C = QI\angle 90^0 = jQI$

$I_L + I_C = 0$

$|I_R + I_L| = |I - jQI|$

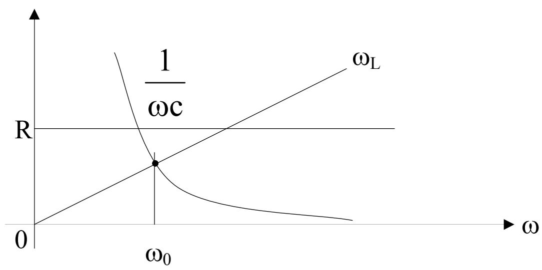
$= I\sqrt{1+Q^2} > I$

$|I_R + I_C| = |I + jQI|$

$= I\sqrt{1+Q^2} > I$

31. Ans: (c)

Sol: Since; "I" leads voltage, therefore capacitive effect and hence the operating frequency ($f < f_0$)



32.

Sol: $Y = \dfrac{1}{R_L + j\omega L} + \dfrac{1}{R_C - \dfrac{j}{\omega C}}$

$$= \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{R_C + j/\omega c}{R_C^2 + (1/\omega C)^2}$$

j – term $\Rightarrow 0$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}} \text{ rad/sec}$$

33.

Sol:

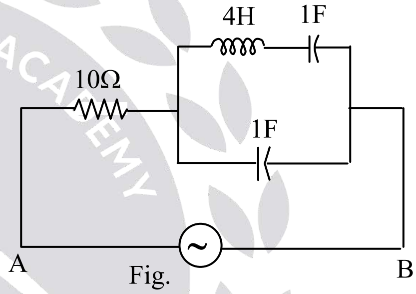


Fig.

The given circuit is shown in Fig.

$Z_{AB} = 10 + Z_1$

where, $Z_1 = \left(\dfrac{-j}{\omega}\right) \| \left(j4\omega - \dfrac{j}{\omega}\right)$

$$= \frac{\left(\frac{-j}{\omega}\right)\left(j4\omega - \frac{j}{\omega}\right)}{\frac{-j}{\omega} + j4\omega - \frac{j}{\omega}}$$

$$= \frac{4 - \frac{1}{\omega^2}}{j4\omega - \frac{j2}{\omega}}$$

For circuit to be resonant i.e., $\omega^2 = \dfrac{1}{4}$

$\omega = \dfrac{1}{2} = 0.5$ rad/sec

$\therefore\ \omega_{resonance} = 0.5$ rad/sec

34.

Sol: (i) $\frac{L}{C}=R^2 \Rightarrow$ circuit will resonate for all the frequencies, out of infinite number of frequencies we are selecting one frequency.

i.e., $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2}$ rad/sec

then $Z = R = 2\Omega$.

$$I = \frac{V}{Z} = \frac{10\angle 0^0}{2} = 5\angle 0^0$$

$$i(t) = 5\cos\frac{t}{2}\ A$$

$$Z_L = j\omega_0 L = j2\Omega\ ;\ Z_C = \frac{1}{j\omega_0 c} = -j2\Omega.$$

$$I_L = \frac{I(2-j2)}{2+j2+2-j2} = \frac{I}{\sqrt{2}}\angle -45^0$$

$$i_L = \frac{5}{\sqrt{2}}\cos\left(\frac{t}{2}-45^0\right)A$$

$$i_c = \frac{I(2+j2)}{2+j2+2-j2} = \frac{I}{\sqrt{2}}\angle 45^0$$

$$i_c = \frac{5}{\sqrt{2}}\cos\left(\frac{t}{2}+45^o\right)A$$

$$P_{avg} = I^2_{L(rms)}.R + I^2_{c(rms)}.R$$

$$= \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2.2 + \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2.2$$

$= 25$ watts

(ii) $\frac{L}{C} \neq R^2$ circuit will resonate at only one frequency.

i.e., at $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4}$ rad/sec

$$\text{Then } Y = \frac{2R}{R^2+\frac{L}{C}}\ \text{mho}$$

$$Y = \frac{2(2)}{2^2+\frac{4}{4}} = \frac{4}{5}\ \text{mho}$$

$$Z = \frac{5}{4}\Omega$$

$$I = \frac{V}{Z} = \frac{10\angle 0^0}{5/4} = 8\angle 0^0$$

$$i(t) = 8\cos\frac{t}{4}\ A$$

$$Z_L = j\omega_0 L = j1\Omega$$

$$Z_c = \frac{1}{j\omega_0 C} = -j1\Omega$$

$$I_L = \frac{I(2-j1)}{2+j1+2-j1} = \frac{\sqrt{5}}{4}I.\angle\tan^{-1}\left(\frac{1}{2}\right)$$

$$i_L = \frac{8\sqrt{5}}{4}\cos\left(\frac{t}{4}-\tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$I_c = \frac{I(2+j1)}{2+j1+2-j1} = \frac{\sqrt{5}}{4}I\angle\tan^{-1}\left(\frac{1}{2}\right)$$

$$i_c = \frac{8\sqrt{5}}{4}\cos\left(\frac{t}{4}+\tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$P_{avg} = I^2_{Lrms}.R + I^2_{Crms}R$$

$$= \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2 + \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2$$

$= 40$ watts

35.

Sol: (i) $Z_{ab} = 2 + (Z_L \parallel Z_C \parallel 2)$

$= 2 + jX_L \| -jX_C \| 2$

$$= \frac{2+2X_LX_C\left(X_LX_C - j2(X_L - X_C)\right)}{(X_LX_C)^2 + 4(X_L - X_C)^2}$$

j-term = 0

$\Rightarrow -2(X_L - X_C) = 0$

$X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.4}} = \frac{1}{4}\,\text{rad/sec}$$

At resonance entire current flows through 2Ω only.

(ii) $Z_{ab}\big|_{\omega=\omega_0} = 2 + 2 = 4\Omega$

$$X_L = X_C$$

(iii) $V_i(t) = V_m \sin\left(\frac{t}{4}\right)V$

$$Z = 4\Omega$$

$$i(t) = \frac{V_i(t)}{Z} = \frac{V_m}{4}\sin\left(\frac{t}{4}\right) = i_R$$

$$V = 2i_R = \frac{V_m}{2}\sin\left(\frac{t}{4}\right)V = V_C = V_L$$

$$i_C = C\frac{dV_C}{dt} = \frac{V_m}{2}\cos\left(\frac{t}{4}\right)$$

$$i_c = \frac{V_m}{2}\sin\left(\frac{t}{4} + 90^0\right)A$$

$$i_L = \frac{1}{L}\int V_L.dt = \frac{-V_m}{2}\cos\left(\frac{t}{4}\right)$$

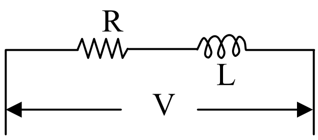
$$i_L = \frac{V_m}{2}\sin\left(\frac{t}{4} - 90^0\right)A$$

OBS: Here $i_L + i_C = 0$

$\Rightarrow$ LC Combination is like an open circuit.

36. Ans: (d)

Sol:



$$Q = \frac{\omega L}{R}$$

$$Q = \frac{2\omega L}{R} = 2 \times \text{orginal} \rightarrow Q - \text{doubled}$$

$$S = V.I$$

$$= V.\frac{V}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L}$$

$$S = \frac{V^2}{R^2 + (\omega L)^2} - \frac{V^2.j\omega L}{R^2 + (\omega L)^2}$$

$$S = P + jQ$$

$$\text{Active power }(P) = \frac{V^2}{R^2 + (\omega L)^2}$$

$$P = \frac{V^2}{R^2(1 + Q^2)}$$

$$P \approx \frac{V^2}{R^2 Q^2}$$

As Q is doubled, P decreases by four times.

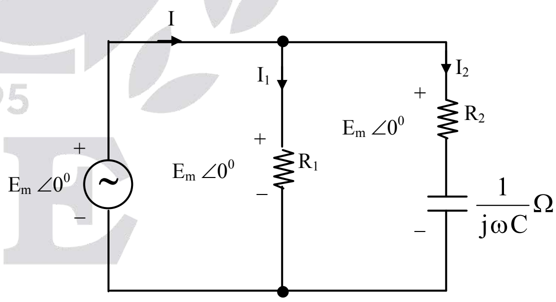
37.

Sol: $Z_C = \frac{1}{j\omega C}$

$\omega = 0; Z_C = \infty \Rightarrow C$: open circuit $\Rightarrow i_2 = 0$

$\omega = \infty; Z_C = 0 \Rightarrow C$: Short Circuit $\Rightarrow i_2 = \frac{E_m}{R_2}\angle 0^o$

Transform the given network into phasor domain.



Network is in phasor domain.

By KCL in P-d $\Rightarrow$ $I = I_1 + I_2$

$$I_1 = \frac{E_m\angle 0^o}{R_1}$$

$$I_2 = \frac{E_m\angle 0^o}{R_2 + \frac{1}{j\omega C}} = \frac{E_m\angle 0^o}{R_2 - \frac{j}{\omega C}}$$

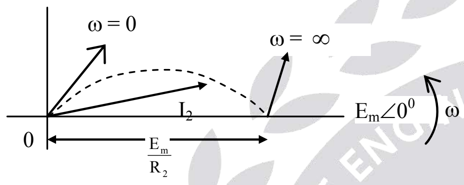
$$I_2 = \frac{E_m \angle \tan^{-1}\left(\frac{1}{\omega C R_2}\right)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)}}$$

$$\omega = \infty \Rightarrow I_2 = \frac{E_m \angle 0^o}{R_2}$$

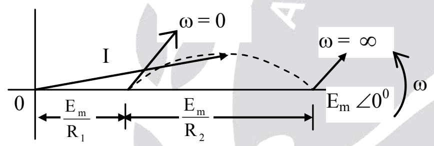
$$\omega = 0 \Rightarrow I_2 = 0A$$

$\omega : (0 \text{ and } \infty)$ j the current phasor I_2 will always lead the voltage $E_m \angle 0^o$.

(a)



(b)

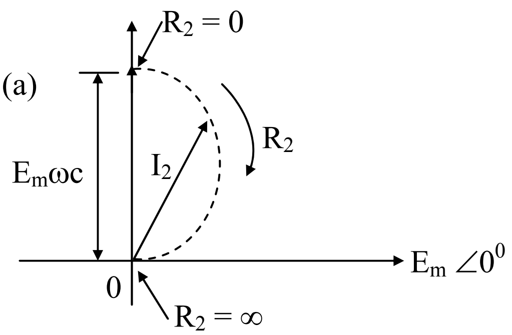


38.

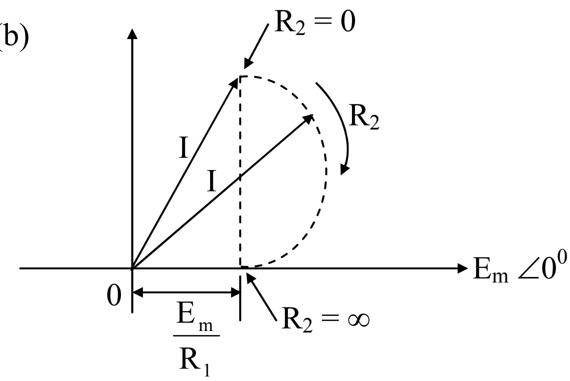
Sol: $R_2 = 0 \Rightarrow I_2 = \frac{E_m \angle 0^o}{0 + \frac{1}{j\omega C}} = E_m \omega C \angle 90^o$

$R_2 = \infty \Rightarrow I_2 = 0\,A$

(a)



(b)



39.

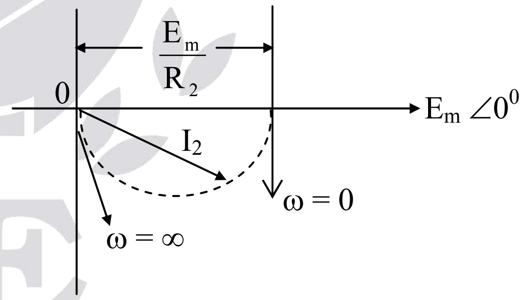
Sol: $I = I_1 + I_2;\ I_1 = \frac{E_m \angle 0^o}{R_1}$

$$I_2 = \frac{E_m \angle 0^o}{R_2 + j\omega L}$$

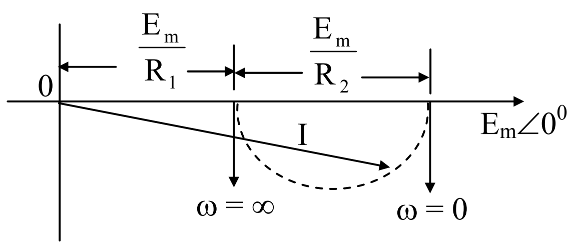
$$= \frac{E_m}{\sqrt{R_2^2 + (WL)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R_2}\right)$$

(i) If "ω" Varied

(a)

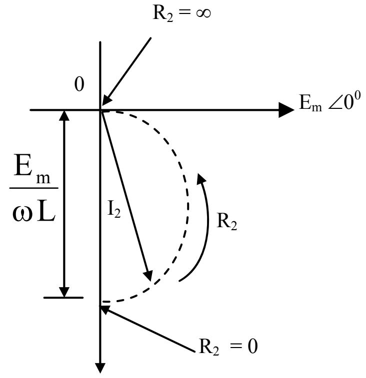


(b)

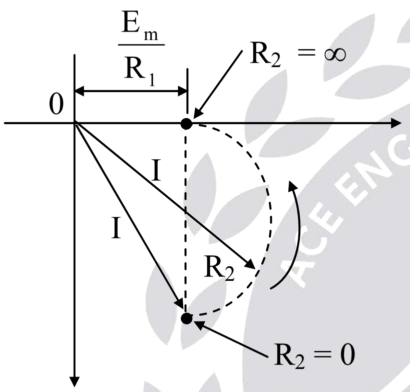


ii. If "R_2" is varied

(a)



(b)



40. Ans: (a)

Sol: The given circuit is a bridge.

$I_R = 0$ is the bridge is balanced. i.e., $Z_1 Z_4 = R_2 R_3$

Where $Z_1 = R_1 + j\omega L_1$,

$$Z_4 = R_4 - \frac{j}{\omega C_4}$$

As $R_2 R_3$ is real, imaginary part of $Z_1 Z_4 = 0$

$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \quad \text{or} \quad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

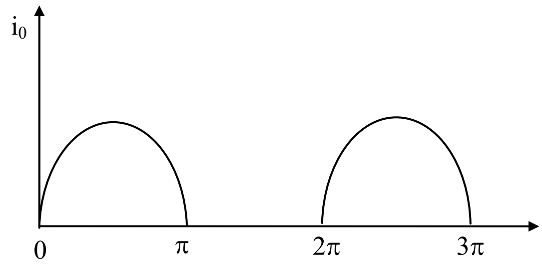
or $Q_1 = Q_4$

Where Q is the Quality factor.

41. Ans: (d)

Sol. During positive half cycle of supply D_1 is forward biased, D_2 is reverse biased so current flows through the ammeter.

During negative half cycle D_2 is forward biased, D_1 is reverse biased so current does not flow though ammeter.



Half wave rectifier waveform

$$I_{0\,avg} = \frac{I_m}{\pi} = \frac{V_m}{R\pi} = \frac{4}{10k \times \pi}$$

$$I_{0\,avg} = \frac{0.4}{\pi}\,\text{mA}$$

42. Ans: (d)

Sol: For $-V_0 \sin \omega_0 t \rightarrow I_1 = \frac{V_0}{\omega_0 L} = I_0$

For $2V_0 \sin \omega_0 t \rightarrow I_2 = \frac{2V_0}{2\omega_0 L} = I_0$

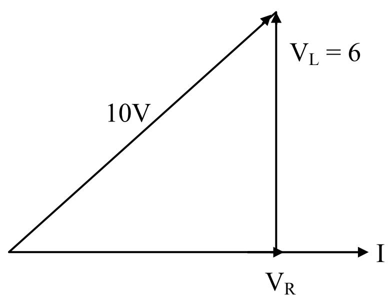
For $3V_0 \sin \omega_0 t \rightarrow I_3 = \frac{3V_0}{3\omega_0 L} = I_0$

For $4V_0 \sin \omega_0 t \rightarrow I_4 = \frac{4V_0}{4\omega_0 L} = I_0$

$$\text{RMS value} = \sqrt{4I_0^2} = 2I_0$$

43. Ans: (b)

Sol:



$$V^2 = V_R^2 + V_L^2$$

$\Rightarrow 100 = V_R^2 + 36$

$\Rightarrow V_R = 8V$

$$I_R = \frac{V_R}{R} = \frac{8}{2} = 4A$$

44. Ans: (b)

Sol: Full wave rectifier

Here each half of secondary winding will received 2sinωt

$$V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$P_{avg} = \frac{{V_{RMS}}^2}{R} = \frac{(\sqrt{2})^2}{10} = 0.2W$$

45. Ans: (b)

Sol: Complex power,

$$S = \overline{V}\,\overline{I}^* = (100 - j50)\,(3 + j4)$$

$$= 300 + 200 + j250 = 500 + j\,250$$

True power $= \text{Re}[\overline{V}\,\overline{I}^*] = 500\text{ W}$

Reactive power

$= \text{Im}[\overline{V}\,\overline{I}^*] = 250\text{ W}$

So Statement (I) is True, Statement (II) is also True, but Statement (II) is not the correct explanation.

46. Ans: (d)

Sol: In series RLC circuit,

i(t) is maximum at resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

$$I_{max} = \frac{V_S}{R}$$

$$V_C = \frac{V_S}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$= V_S \quad \text{for} \quad \omega = 0$

$= Q\,V_S \quad \text{for} \quad \omega = \omega_0$

$= 0 \quad \text{for} \quad \omega \to \infty$

V_C is maximum at ω = 0 (i.e., $\omega < \omega_0$) provided $Q < 1$

Statement (I) is false, but statement (II) is true if $Q < 1$

47. Ans: (a)

Sol: When the input impedance is purely resistive, the voltage and current are in phase.

Note that at resonance, power factor is also unity.

48. Ans: (c)

49. Ans: (c)

Sol: At resonance, the power factor of circuit is unity.

Hence statement (II) is false.

50. Ans: (c)

Sol: $\omega_{res} = \omega_0\sqrt{\dfrac{L - R_1^2 C}{L - R_2^2 C}}$, $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Resonance occurs at all frequencies, if

$$R_1^2 = R_2^2 = \frac{L}{C}$$

and the resonant impedance

$$= R_1 = R_2 = \sqrt{\frac{L}{C}}$$

$\therefore$ Statement (I) is True, Statement (II) is False

51. Ans: (a)

Sol: $G(j\omega)=\dfrac{V_0(j\omega)}{V_i(j\omega)}$

$$=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{1}{1+j\omega RC}$$

$= 1\angle 0$, $\omega = 0$

$= 0.707\ e^{-j45^\circ}$, $\omega = \dfrac{1}{RC}$

$= 0\angle -90^\circ$, $\omega \to \infty$

52. Ans: (c)

Sol: Curve AA → Current waveform, having maximum value at

$\omega = \omega_{res}$ ………. (1)

Curve BB → | Z | ……….. (2)

$$Z=R+j\left(\omega L-\frac{1}{\omega C}\right)=-j\infty,\ \omega=0$$

$Z= R$, $\omega = \omega_0$

$Z = j\infty$, $\omega = \infty$

Curve CC→ $X_C = -\dfrac{j}{\omega C}$, Capacitive reactance ………. (3)

Curve DD → Net reactance,

$$X = j\left(\omega L-\frac{1}{\omega C}\right) \quad ……….\ (4)$$

$= -j\infty$, $\omega = 0$

$= 0$, $\omega = \omega_0$

$= j\infty$, $\omega = \infty$

Conventional Practice Solutions

01.

Sol: $V_1 = V_2 = 120$ V,

$R_1 = 15\ \Omega$, $R_2 = 7\ \Omega$, $P = 550$ W

$P = I^2 (R_1 + R_2)$, $I^2 = \dfrac{550}{22} = 25$, $I = 5$ A

Let X_1 and X_2 be the reactances of the coils.

$5\sqrt{R_1^2 + X_1^2} = 120$, $\sqrt{15^2+X_1^2} = 24$

$X_1^2 = 24^2 - 15^2$, $X_1 = \sqrt{24^2-15^2} = 18.7\ \Omega$

$5\sqrt{7^2+X_2^2} = 120$, $X_2 = \sqrt{24^2-7^2} = 22.96\ \Omega$

02.

Sol: The instantaneous power is given by

$P = vi = 1200\cos(377t + 45^\circ)\cos(377t - 10^\circ)$

$P = 600\,[\cos(754t + 35^\circ) + \cos 55^\circ]$

$$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$P(t) = 344.2 + 600\cos(754t+35^\circ)$ W

The average power is

$$P=\frac{1}{2}V_m I_m\cos(\theta_u-\theta_i)$$

$$=\frac{1}{2}(120)(10)\cos[45^\circ-(-10^\circ)]$$

$= 600\cos 55^\circ = 344.2$ W

03.

Sol: The apparent power is

$$S = V_{rms}I_{rms} = \frac{120}{\sqrt{2}}\times\frac{4}{\sqrt{2}} = 240\,VA$$

The power factor is

$Pf = \cos(\theta_v - \theta_i) = \cos(-20^\circ-10^\circ)$

$= 0.866$ (leading)

The pf is leading because the current leads the voltage.

The pf may also be obtained from the load impedance.

$$Z = \frac{V}{I} = \frac{120\angle -20^\circ}{4\angle 10^\circ} = 30\angle -30^\circ$$

$$= 25.98 - j15\,\Omega$$

Pf cos (–30°) = 0.866 (leading).

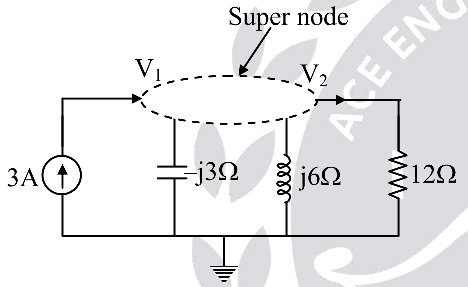
The load impedance Z can be modelled by a 25.98Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

$$C = \frac{1}{15\omega} = \frac{1}{15\times 100\pi} = 212.2\,\mu F$$

04.

Sol: Nodes 1 and 2 forms a super node as shown in below figure



Applying KCL at the super node gives

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

$36 = j4V_1 + (1–j2)V_2$ …………(1)

and

$V_1 = V_2 + 10\angle 45°$

Substituting equation V_1 in equation (1) results in

$36 – 40\angle 135° = (1 + j2)\ V_2$

$\Rightarrow V_2 = 31.41\angle –87.18°$ V

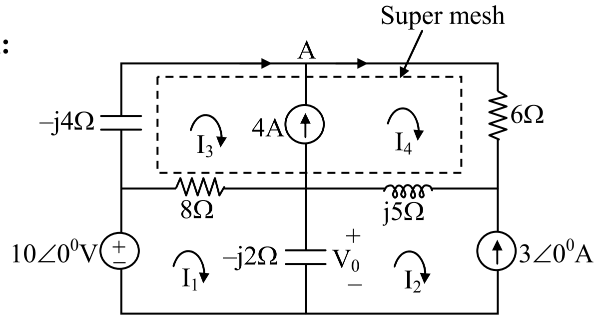
From equation

$V_1 = V_2 + 10\angle 45°$

$= 25.78\ \angle –70.48°$ V

05.

Sol:



Meshes 3 and 4 form a super mesh due to the current source between the meshes.

For mesh 1, KVL gives

$–10+(8–j2)I_1 –(–j2)I_2 –8\ I_3\ = 0$

$(8–2j)I_1 + j2I_2 –8I_3 = 10$ -------(1)

For mesh 2

$I_2 = –3$ ----- (2)

For the super mesh

$(8–j4)I_3 – 8I_1 + (6+j5)I_4 – j5I_2 = 0$ ---- (3)

Due to the current source between meshes 3 and 4, at node A

$I_4 = I_3 + 4$ ----- (4)

Instead of solving the above four equations, we reduce to two by elimination

Combines equation (1) & (2)

$(8–j2)I_1 – 8I_3 = 10 + j6$ ------(5)

Combines (3) and (4)

$–8I_1 + (14+j)I_3 = –24–j35$ ------(6)

From equations (5) and (6) we obtain the matrix equation

$$\begin{bmatrix} 8-j2 & -8 \\ -8 & 14+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10+j6 \\ -24-j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8-j2 & -8 \\ -8 & 14+j \end{vmatrix}$$

$$= 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix}$$

$$= 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

current I_1 is obtained as

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-58-j186}{50-j20} = 3.618\angle 274.5^0\,A$$

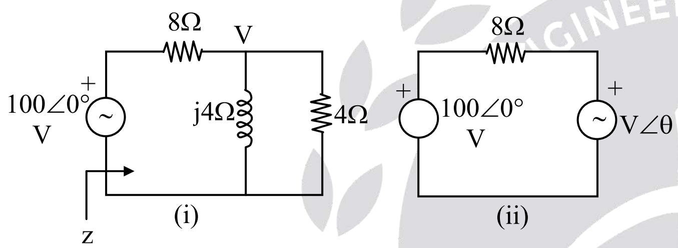
The required voltage V_0 is

$V_0 = -j2(I_1 - I_2) = -j2(3.618\angle 274.5^0 + 3)$

$= -7.2134 - j6.568 = 9.756\angle 222.32^0$ V

06.

Sol: The given circuit is shown in figure 1.



$$z = 8 + (j4 \,||\, 4)$$

$$= 8 + \frac{4.j4}{4+j4}$$

$$= 8 + \frac{j4}{1+j}$$

$$z = \frac{8+12j}{1+j}$$

$$I = \frac{100\angle 0^o}{\left(\dfrac{8+12j}{1+j}\right)}$$

$I = 9.615 - j\,1.923$ .... (1)

Apply KCL at node 'V'

$$\frac{V - 100\angle 0^o}{8} + \frac{V}{j4} + \frac{V}{4} = 0$$

$$\frac{V}{8} - \frac{jV}{4} + \frac{V}{4} - \frac{100\angle 0^o}{8} = 0$$

$$V\left[\frac{1}{8} - \frac{j}{4} + \frac{1}{4}\right] = \frac{100\angle 0^o}{8}$$

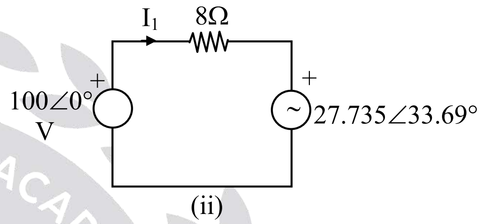
$$V\left[\frac{1-2j+2}{8}\right] = \frac{100\angle 0^o}{8}$$

$$V\angle\theta = \frac{100}{3-2j}$$

$V\angle\theta = 27.735\angle 33.69°$

The given circuit is shown in figure 2. and

Assume $V\angle\theta = 27.745\angle 33.69°$V



$$I_1 = \frac{100\angle 27.735\angle 33.69^o}{8}$$

$I_1 = 9.615 - j1.923$ ....(2)

It is clear that from equations (1) and (2) to become circuit (i) and circuit (ii) $V\angle\theta = 27.735\angle 33.69°$

07.

Sol: The given circuit is shown in Fig.1

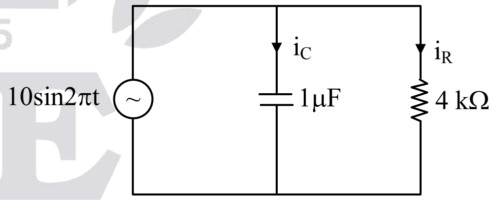


Fig.1

The energy stored in the capacitor varies with time, the stored energy dissipated by the resistor over this interval. These are actually two completely different questions.

The only source of energy in the circuit is the independent voltage source, which has a value of 10sin2πt V. In the time interval of $0 < t < 0.5$s.

The power dissipated by the resistor in terms of the current i_R.

$$i_R = \frac{V}{R} = \frac{10\sin 2\pi t}{1000}$$

$$i_R = 0.01\sin 2\pi t \text{ A}$$

and so

$$P_R = i_R^2 R = (0.01)^2 \times (1000)\sin^2 2\pi t$$

$$= 0.1\sin^2 2\pi t$$

So that the energy dissipated in the resistor between 0 and 0.5s is

$$\omega_R = \int_0^{0.5} P_R\,dt$$

$$= \int_0^{0.5} 0.1\sin^2 2\pi t dt$$

$$= 0.1\int_0^{0.5}\left[\frac{1-\cos 4\pi t}{2}\right]dt$$

$$= \frac{1}{20}\left[t - \frac{\sin 4\pi t}{4\pi}\right]_0^{0.5}$$

$$= \frac{1}{20}\left[(0.5-0)-(0-0)\right]$$

$$\omega_R = \frac{1}{40}\text{J}$$

$$V_c(t) = 10\sin 2\pi t$$

$$i_c(t) = V_c(t) \times \frac{R}{R + \frac{1}{j\omega_c}}$$

or

$$i_c(t) = c\frac{dV_c(t)}{dt}$$

$$= c.10\cos 2\pi t.2\pi$$

$$i_c(t) = 20\pi \times 10^{-6}\cos 2\pi t$$

The energy stored in capacitor between 0 and 0.5s is

$$\omega_c = \int_0^{0.5} C.V\frac{dv}{dt}dt$$

$$= 10^{-6}\int_0^{0.5} 10\sin 2\pi t.20\pi \times 10^{-6}\cos 2\pi t\, dt$$

$$= \frac{200\pi \times 10^{-12}}{2}\int_0^{0.5} 2\sin 2\pi t \cos 2\pi t dt$$

$$= 100\pi \times 10^{-12}\int_0^{0.5}\sin 4\pi t dt$$

$$= 100\pi \times 10^{-12}\left[\frac{-\cos 4\pi t}{4\pi}\right]_0^{0.5}$$

$$= \frac{100\pi \times 10^{-12}}{4\pi}\left[\cos(0) - \cos 4\pi(0.5)\right]$$

$$\omega_c = 0 \text{ J}$$

08.

Sol: The given parallel circuit is shown in Fig. 1

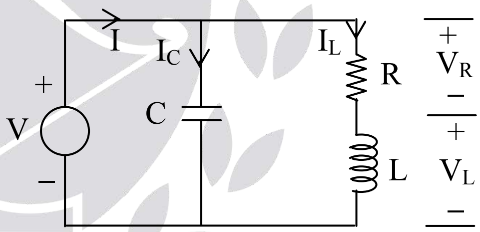


Fig.1

The phasor diagram is shown in Fig. 2

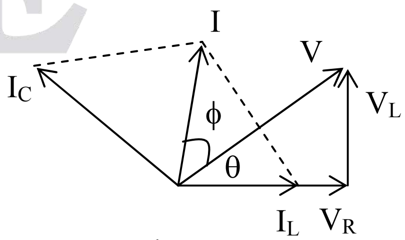


Fig. 2

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

For unity P.F, $\phi = 0$.

$$\frac{I}{V} = Y = j\omega C + \frac{1}{R + j\omega L}$$

$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)$$

For unity P.F, I and V should be in phase

Im Y = 0

$$\therefore C = \frac{L}{R^2 + \omega^2 L^2}$$

09.

Sol: At resonance frequency, $\omega = \omega_0$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-6}}} = 10^3 \text{ rad/sec}$$

$$I = \frac{100}{10} = 10 \text{ A}$$

Power consumed $= I^2 R = 100 \times 10$

$= 1000$ W

$$Z = 10 + j\left(\omega - \frac{10^6}{\omega}\right)$$

$$|Z| = \sqrt{100 + \left(\omega - \frac{10^6}{\omega}\right)^2},$$

$$I = \frac{V}{|Z|} = \frac{100}{|Z|}$$

Power consumed $= I^2 R = \dfrac{100^2}{|Z|^2} R$

$$0.1 \times 1000 = \frac{10^5}{|Z|^2}, \quad |Z|^2 = \frac{10^5}{100} = 10^3$$

$$100 + \left(\omega - \frac{10^6}{\omega}\right)^2 = 10^3$$

$$\omega - \frac{10^6}{\omega} = \pm\sqrt{900} = \pm 30$$

$$\omega^2 \pm 30\omega - 10^6 = 0$$

$$\omega = \frac{\pm 30 \pm \sqrt{900 + 4 \times 10^6}}{2} = \pm 15 \pm 10^3$$

$= 1000 + 15$ and $1000 - 15$

$= 1015$ rad/sec and 985 rad/sec

10.

Sol: $Z(j\omega) = \dfrac{R\ j\omega L}{R + j\omega L} - \dfrac{j}{\omega C}$

$R = 1\ \Omega$, $L = 1$ H, $C = 2$ F

$$Z(j\omega) = \frac{j\omega}{1 + j\omega} - \frac{j}{2\omega}$$

$$= \frac{j2\omega^2 - j + \omega}{2\omega(1 + j\omega)} = \frac{\omega + j(2\omega^2 - 1)}{2\omega(1 + j\omega)}$$

$$= \frac{[\omega + j(2\omega^2 - 1)]\,(1 - j\omega)}{2\omega(1 + \omega^2)}$$

$$= \frac{\omega + \omega(2\omega^2 - 1)}{2\omega(1 + \omega^2)} + j\,\frac{(2\omega^2 - 1 - \omega^2)}{2\omega(1 + \omega^2)}$$

$$= \frac{\omega}{1 + \omega^2} + j\,\frac{(\omega^2 - 1)}{2\omega(1 + \omega^2)}$$

For resonance, Im $Z(j\omega) = 0$, $\omega^2 = 1$, $\omega = 1$ r/s

$\therefore$ Resonance frequency, $\omega_0 = 1$ r/s

$$\text{Re}\,Z(j\omega) = \frac{1}{2}\,\Omega$$

If the positions of L and C are interchanged,

$$Z(j\omega) = \frac{-\frac{j}{2\omega} \times 1}{1 - \frac{j}{2\omega}} + j\omega = \frac{-j}{2\omega - j} + j\omega$$

$$= \frac{-j + j2\omega^2 + \omega}{2\omega - j} = \frac{\omega + j(2\omega^2 - 1)}{(2\omega - j)}$$

$$= \frac{[\omega + j(2\omega^2 - 1)]\ (2\omega + j)}{(4\omega^2 + 1)}$$

Im Z(jω) = 0, gives $2\omega(2\omega^2 - 1) + \omega = 0$

$$2(2\omega^2 - 1) + 1 = 0$$

$$4\omega^2 = 1, \quad \omega = \frac{1}{2}\ \text{r/s}$$

$\therefore$ Resonance frequency, $\omega_0 = \frac{1}{2}\ \text{r/s}$

and $\text{Re } Z(j\omega_0) = \frac{1}{4\omega_0^2 + 1} = \frac{1}{2}\ \Omega$

ω_0 changes from 1 r/s to $\frac{1}{2}$ r/s and resonant impedance is resistive and remains the same as $\frac{1}{2}\ \Omega$.

11.

Sol: The input admittance is

$$Y = j\omega(0.1) + \frac{1}{10} + \frac{1}{2 + j\omega 2}$$

$$= 0.1 + j\omega(0.1) + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance Im(Y) = 0 and

$$\omega_0(0.1) - \frac{2\omega_0}{4 + 4\omega_0^2} = 0$$

$$\frac{2\omega_0}{4 + 4\omega_0^2} = \omega_0(0.1)$$

$$1 = 0.2 + 0.2\ \omega_0^2$$

$$\omega_0^2 = 4$$

$$\omega_0 = 2\text{rad/s}$$

12.

Sol: The given bridge circuit is shown in Fig.1

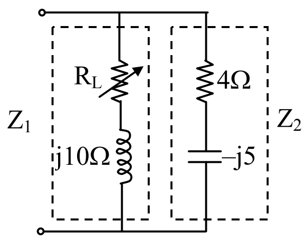


Fig.1

From Fig.1

$$Z_1(j\omega) = R_L + j10$$

$$Z_2(j\omega) = 4 + (-j5)$$

$$Y_1(j\omega) = \frac{1}{Z_1(j\omega)} = \frac{1}{R_L + j\omega}$$

$$Y_2(j\omega) = \frac{1}{Z_2(j\omega)} = \frac{1}{4 - j5}$$

$$\therefore Y(j\omega) = Y_1(j\omega) + Y_2(j\omega)$$

$$= \frac{1}{R_L + j10} + \frac{1}{4 - j5}$$

$$Y(j\omega) = \frac{R_L - j10}{R_L^2 + (10)^2} + \frac{4 + j5}{16 + 25} \quad \text{……….. (1)}$$

For finding resonance frequency

$$\text{Im}[Y(j\omega)] = 0$$

From (1) $\frac{-j10}{R_L^2 + 100} + \frac{j5}{16 + 25} = 0$

$$\frac{10}{R_L^2 + 100} = \frac{5}{41}$$

$$R_L^2 = 82 - 100$$

$$R_L = \sqrt{-18}$$

$$R_L = j\sqrt{18}$$

The resistance which is having displacement angle will not exist. So, the resonant frequency for above network there can be no value of R_L.

Chapter 5 Magnetic Circuits

Objective Practice Solutions

01.

Sol: $X_C = 12$ (Given)

$X_{eq} = 12$ (must for series resonance)

So the dot in the second coil at point "Q"

$L_{eq} = L_1 + L_2 - 2M$

$$L_{eq} = L_1 + L_2 - 2K\sqrt{L_1 L_2}$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 - 2K\sqrt{L_1 L_2 \omega.\omega}$$

$$12 = 8 + 8 - 2K\sqrt{8.8}$$

$\Rightarrow K = 0.25$

02.

Sol: $X_C = 14$ (Given)

$X_{Leq} = 14$ (must for series resonance)

So the dot in the 2nd coil at "P"

$L_{eq} = L_1 + L_2 + 2M$

$$L_{eq} = L_1 + L_2 + K\sqrt{L_1 L_2}$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 + 2K\sqrt{\omega L_1 L_2 \omega}$$

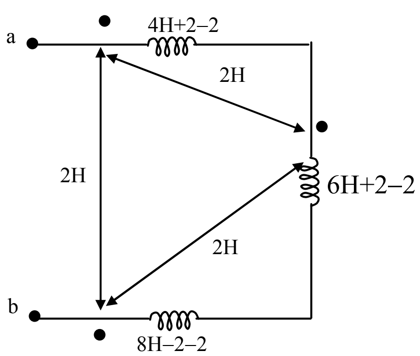
$$14 = 2 + 8 + 2K\sqrt{2(8)}$$

$\Rightarrow K = 0.5$

03.

Sol: $L_{ab} = 4H + 2 - 2 + 6H + 2 - 2 + 8H - 2 - 2$

$L_{ab} = 14H$



04. Ans: (c)

Sol: Impedance seen by the source

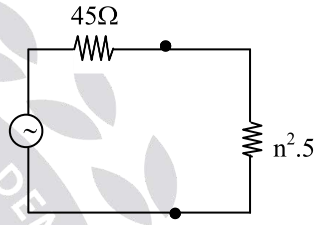
$$Z_s = \frac{Z_L}{16} + (4 - j2)$$

$$= \frac{10\angle 30^\circ}{16} + (4 - j2)$$

$$= 4.54 - j1.69$$

05.

Sol:



$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 .Z_L$$

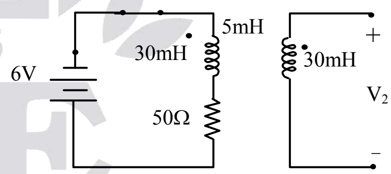
$$R'_{in} = n^2.5$$

For maximum power transfer; $R_L = R_s$

$n^2 5 = 45 \Rightarrow n = 3$

06. Ans: (b)

Sol:



Apply KVL at input loop

$$-6 - 30\times 10^{-3}\frac{di_1}{dt} + 5\times 10^{-3}\frac{di_2}{dt} - 50i_1 = 0 \ldots(1)$$

Take Laplace transform

$$-\frac{6}{s} + [-30\times 10^{-3}(s) - 50]I_1(s) + 5\times 10^{-3} s I_2(s) = 0 \ldots(2)$$

Apply KVL at output loop

$$V_2(s) - 30\times 10^{-3}\frac{di_2}{dt} + 5\times 10^{-3}\frac{di_1}{dt} = 0$$

Take Laplace transform

$$V_2(s) - 30\times10^{-3}\, s\, I_2(s) + 5\times10^{-3}\, s\, I_1(s) = 0$$

Substitute $I_2(s) = 0$ in above equation

$V_2 + 5 \times 10^{-3}\, sI_1(s) = 0$ ……… (3)

From equation (2)

$$-\frac{6}{s} + (-30\times10^{-3}(s) + 50)\, I_1(s) = 0$$

$$I_1(s) = \frac{-6}{s\,(30\times10^{-3}(s)+50)} \quad \text{……… (4)}$$

Substitute eqn (4) in eqn (3)

$$V_2(s) = \frac{-5\times10^{-3}\,(s)\,(-6)}{s(30\times10^{-3}\,(s)+50)}$$

Apply Initial value theorem

$$\underset{s\to\infty}{\text{Lt}}\; s\; \frac{-5\times10^{-3}\,(s)(-6)}{s\,(30\times10^{-3}\,(s)+50)}$$

$$v_2(t) = \frac{-5\times10^{-3}\times(-6)}{30\times10^{-3}} = +1$$

07.

Sol: $R_{in}' = \frac{8}{2^2} = 2\Omega$

$R_{in} = 3 + R_{in}' = 3 + 2 = 5\Omega$

$I_1 = \frac{10\angle 20}{5} = 2\angle 20^0$

$\frac{I_1}{I_2} = n = 2 \Rightarrow I_2 = 1\angle 20°\text{A}$

08.

Sol: By the definition of KVL in phasor domain

$V_S - V_0 - V_2 = 0$

$$V_0 = V_S - V_2 = V_S\left(1 - \frac{V_2}{V_S}\right)$$

V = ZI

By KVL

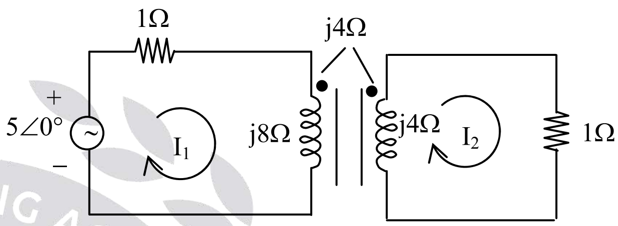
$V_S = j\omega L_1.I_1 + j\omega M\,(0)$

$V_2 = j\omega L_2(0) + j\omega M I_1$

$$V_0 = V_S\left(1 - \frac{M}{L_1}\right)$$

09.

Sol: Transform the above network into phasor domain



Network is in Phasor -domain

V = Z.I

By KVL in p-d ⇒

$5\angle 0° = I_1 + j8.I_1 - j4.I_2$

$0 = I_2 + j4I_2 - j4I_1$

$$I_1 = \frac{\Delta_1}{\Delta};\, i_1(t) = \text{Real part}\left[I_1 e^{j2t}\right]\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta};\, i_2(t) = \text{Real part}\left[I_2 .e^{j2t}\right]\text{A}$$

$I_1(t) = 1.072\cos\,(2t + 114.61^0)\text{A}$

$I_2(t) = 1.416\cos\,(2t + 128.65^0)\text{A}$

10.

Sol: Evaluation of Initial conditions:

$i_1\,(0^-) = 0\text{A} = i_1(0^+)$

$i_2\,(0^-) = 0\text{A} = i_2\,(0^+)$

Evaluation of final conditions:

$i_1(\infty) = 5\text{A}$; $i_Z\,(\infty) = 0\text{A}$

By KVL ⇒

$$5 = i_1(t) + \frac{4di_1(t)}{dt} - 2\frac{di_2(t)}{dt}$$

By Laplace transform to the above equations.

$$\frac{5}{s}=I_1(s)+4\left[sI_1(s)-i_1(0^+)\right]-2\left(s.I_2(s)-i_2(0^+)\right)$$

By KVL ⇒

$$0=1.i_2(t)+2\frac{di_2(t)}{dt}-2\frac{di_1(t)}{dt}$$

$$0=1.I_2(s)+2\left[sI_2(s)-i_2(0^+)\right]-\left[sI_1(s)-i_1(0^+)\right]$$

On solving, we can obtain $i_1(t)$ and $i_2(t)$

$$i_1(t)=5-e^{-\frac{3t}{4}}\left[5\cosh\left(\frac{\sqrt{5}}{4}t\right)-\sqrt{5}\sinh\left(\frac{\sqrt{5}}{4}t\right)\right]A$$

11. Ans: (c)

Sol: $L_1=\frac{N_1\phi_1}{i_1}\Rightarrow\phi_1=\frac{L_1i_1}{N_1}$

$$\phi_1=\frac{1}{2}\frac{5\sin 400t}{N_1}$$

But $\frac{N_1}{N_2}=\sqrt{\frac{L_1}{L_2}}\Rightarrow N_1=N_2\sqrt{\frac{L_1}{L_2}}$

$$N_1=1000\sqrt{\frac{0.5}{0.2}}=1581.13$$

$$\phi_1=\frac{2.5\sin 400t}{1581.13}$$

ϕ_1 = 1.58m sin400t

$\phi_1=\phi_{max}\sin\omega t$

So, ϕ_{max} = 1.58mWb

12. Ans: (a)

Sol: $M=\frac{k\phi_1N_2}{i_1}=\frac{k\phi_2N_1}{i_2}$

Given, i_1 = 1A; ϕ_1 = 0.1mWb

N_1 = 1000; N_2 = 2000

k = 0.6

$$M=\frac{(0.6)(0.1m)(2000)}{1}=0.12\,H$$

Conventional Practice Solutions

01.

Sol: KVL in mesh 1

$$R_4 i_1(t)+(L_1+L_2)\frac{di_1}{dt}-L_2\frac{di_2}{dt}+M_{12}\frac{d}{dt}(i_1-i_2)$$
$$+M_{12}\frac{d}{dt}i_1(t)+M_{13}\frac{d}{dt}i_2(t)+M_{23}\frac{d}{dt}i_2(t)=v_1(t)\ \ldots(1)$$

KVL in mesh 2

$$R_5 i_2(t)+(L_2+L_3)\frac{di_2}{dt}-L_2\frac{di_1}{dt}+M_{23}\frac{d}{dt}(i_2-i_1)$$
$$+M_{23}\frac{di_2}{dt}+M_{13}\frac{di_1}{dt}+M_{12}\frac{di_1}{dt}=v_2(t)\ \ldots\ldots(2)$$

02.

Sol: $I=\frac{20\angle 20^\circ}{Z_{in}}$

Z_{in} = 8 + j 10 + j 5 – j 12

= 8 + j3 = 8.54∠20.56

$$I=\frac{20\angle 20^\circ}{8.54\angle 20.56}=2.34\angle -0.56\text{ A}$$

V_{th} = (4 + j 5 – j 6) I = (4 – j) I

= 4.123∠–14.03 × 2.34∠–0.56

= 9.65∠–14.59 Volts

Z_{th} is calculated from the circuit shown in Fig.

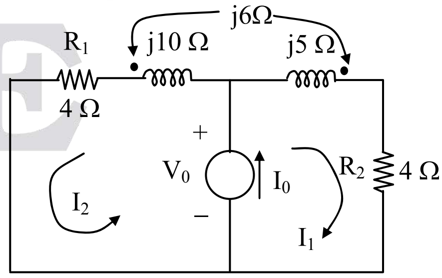


Fig.

$(4+j5)I_1+(j6)I_2=V_0$

$(j6)I_1+(4+j10)I_2=V_0$

$$I_1=\begin{vmatrix}V_0 & j6\\ V_0 & 4+j10\end{vmatrix}\div\Delta=V_0(4+j4)\div\Delta$$

$$I_2 = \begin{vmatrix} 4+j5 & V_0 \\ j6 & V_0 \end{vmatrix} \div \Delta = V_0(4-j1) \div \Delta$$

$$\Delta = \begin{vmatrix} 4+j5 & j6 \\ j6 & 4+j10 \end{vmatrix}$$

$$= 16 - 50 + j60 + 36 = 2+j60$$

$$I_0 = I_1 + I_2 = \frac{V_0\ (8+j3)}{\Delta}$$

$$= V_0\ \frac{8+j3}{2+j60}$$

$$\frac{V_0}{I_0} = \frac{2+j60}{8+j3}$$

$$Z_{th} = 3 + \frac{2+j60}{8+j3} = \frac{26+j69}{8+j3} = \frac{73.7\angle 69.35^\circ}{8.54\angle 20.55^\circ}$$

$$= 8.63\ \angle 48.8^\circ\ \Omega = 5.68 + j6.49$$

For maximum power transfer,

$Z_L = Z_{th}^* = 8.63\angle{-48.8^\circ} = 5.68 - j\,6.49$

03.

Sol:

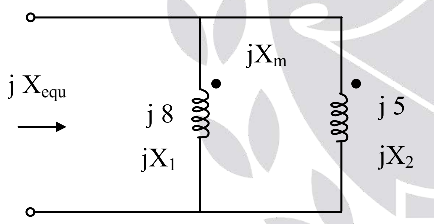


Fig. 1

The equivalent reactance between the terminals A, B of coupled inductors (Fig. 1) is given by

$$jX_{equ} = j\ \frac{(X_1 X_2 - X_m^2)}{(X_1 + X_2 - 2X_m)}$$

$$L_{equ} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$X_1 = \omega L_1,\ X_2 = \omega L_2$

$$M = k\sqrt{L_1 L_2}$$

$$X_m = \omega M = \omega k\sqrt{L_1 L_2}$$

$$= k\sqrt{X_1 X_2} = \sqrt{40}\ k$$

$$jX_{equ} = j\ \frac{40(1-k^2)}{13 - 2\sqrt{40}\,k}$$

From Fig. 2

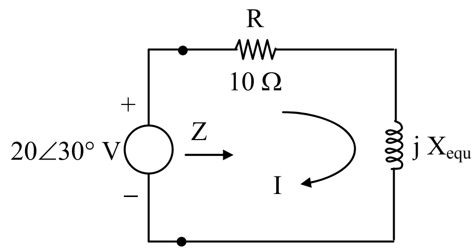


Fig. 2

$Z = R + j\,X_{equ} = 10 + j\,X_{equ}$

$$I = \frac{20\angle 30^\circ}{10 + jX_{equ}} = \frac{20\ \angle 30^\circ - \theta_2}{\sqrt{10^2 + X_{equ}^2}}$$

$$\theta_2 = \tan^{-1}\left(\frac{X_{equ}}{10}\right);\ |I| = \frac{20}{\sqrt{100 + X_{equ}^2}}$$

$$P = |I|^2\ R = \frac{4000}{100 + X_{equ}^2} = 32$$

$$100 + X_{equ}^2 = \frac{4000}{32} = 125$$

$$X_{equ}^2 = 25\ ,\ X_{equ} = 5$$

$$\frac{40(1-k^2)}{13 - 2\sqrt{40}\,k} = 5$$

$$8(1-k^2) = 13 - 2\sqrt{40}\ k$$

$$8k^2 - 2\sqrt{40}\,k + 5 = 0$$

$$k = \frac{2\sqrt{40} \pm \sqrt{160 - 160}}{16} = 0.79$$

If the terminals of one of the coils are interchanged,

$$jX_{equ} = j\ \frac{40(1-k^2)}{13 + 2\sqrt{40}\,k}$$

$$X_{equ} = \frac{15.07}{22.99} = 0.65\ ,\ I = \frac{20\angle 30^\circ}{10 + j0.65}$$

$$P = |I|^2\ R = \frac{4000}{100 + (0.65)^2} = 39.83\ \text{W}$$

04.

Sol: Refer to Fig. 1, where I_1 and I_2 are mesh currents.

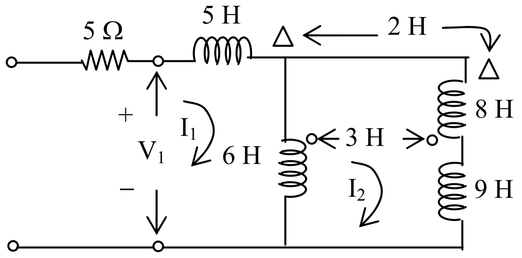


Fig. 1

Mesh equations:

$+j\,11\omega\, I_1 + (-j\,6\omega - j\,2\omega - j\,3\omega)\, I_2 = V_1$

or $+j\,11\omega\, I_1 - j\,11\omega\, I_2 = V_1$ ……… (1)

and $(-j\,6\omega - j\,2\omega)\, I_1 + j\,3\omega(I_2 - I_1) + j\,3\omega\, I_2 + j\,23\,\omega\, I_2 = 0$

or $-j\,11\omega\, I_1 + j\,29\omega\, I_2 = 0$ ……….. (2)

$$I_1 = \frac{\begin{vmatrix} V_1 & -j11\omega \\ 0 & j29\omega \end{vmatrix}}{\begin{vmatrix} j11\omega & -j11\omega \\ -j11\omega & j29\omega \end{vmatrix}} = \frac{j29\omega V_1}{-198\omega^2}$$

$$\frac{V_1}{I_1} = \frac{-198\omega^2}{j29\omega} = j\,6.83\,\omega$$

$\therefore$ Effective inductance = 6.83 H.

05.

Sol: The given circuit is shown in fig.1 assume input voltage 'V'

$L_1 = 8$ mH $\quad X_{L2} = WL_1 \Rightarrow 1000\times 8\times 10^{-3} \Rightarrow 8\ \Omega$

$L_2 = 6$ mH $\quad X_{L2} = WL_2 \Rightarrow 1000\times 6\times 10^{-3} \Rightarrow 6\ \Omega$

$M = 5$ mH $\quad X_m = \omega m \Rightarrow 1000\times 5\times 10^{-3} \Rightarrow 5\Omega$

$$C_1 = \frac{1}{6}\mu F \qquad X_C = \frac{1}{\omega C}$$

$$\Rightarrow X_C = \frac{1}{1000\times\frac{1}{6}\times 10^{-6}} = 6000\ \Omega$$

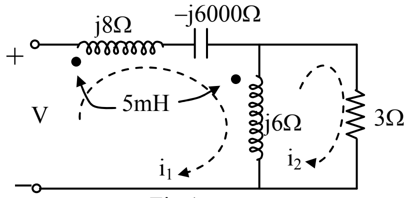


Fig.1

Apply KVL in the above network

loop – 1

$-V + j8i_1 + j5(i_1-i_2) - j6000i_1 + j6(i_1-i_2) + j5i_1 = 0$

$-V + j8i_1 + j5i_1 - j5i_2 - j6000i_1 + j6i_1 - j6i_2 + j5i_1 = 0$

$j(-5966)i_1 - j11i_2 = V$--------(1)

In loop – 2

$j6(i_2-i_1) - j5i_1 + 3i_2 = 0$

$j6i_2 - j6i_1 - j5i_1 + 3i_2 = 0$

$$i_2 = \left[\frac{j11}{3+j6}\right] i_1$$

Put 'i_2' in equation (V)

$$-j5966i_1 - j11\left[\frac{j_{11}}{3+j6}\right] i_1 = V$$

$(8.066 - j5982.13)i_1 = V$

$$Z = \frac{V}{i_1} = (8.066 - j5982.13)\Omega$$

Chapter 6 Two Port Networks

Objective Practice Solutions

01.

Sol: The defining equations for open circuit impedance parameters are:

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$[Z] = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix} \Omega$$

02. Ans: (b)

Sol: The matrix given is $\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

since $y_{11} \neq y_{22}$

$\Rightarrow$ Asymmetrical, and

$Y_{12} \neq y_{21}$ ‘

$\Rightarrow$ Non reciprocal network

03.

Sol: Convert Y to Δ :

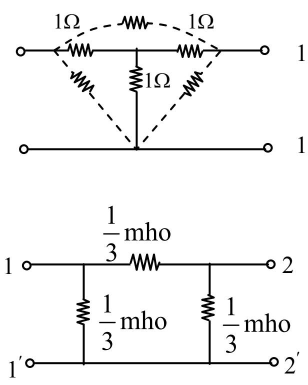


Fig:A

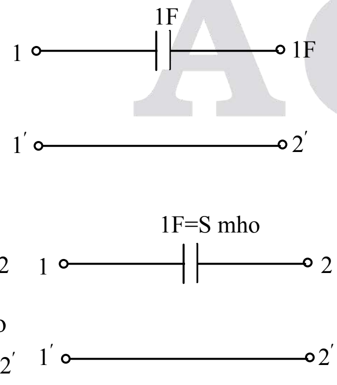


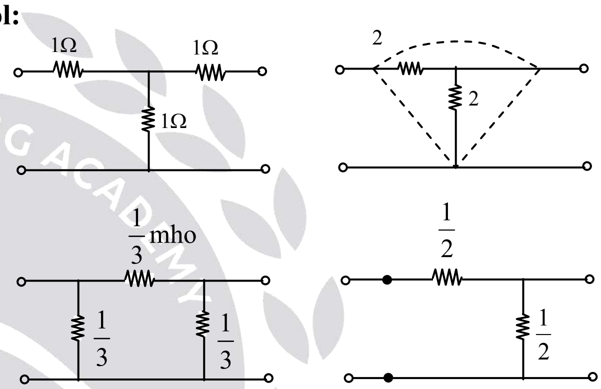
Fig:B

$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad Y_B = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix}$$

$$Y = \begin{bmatrix} S+\frac{2}{3} & -S-\frac{1}{3} \\ -S-\frac{1}{3} & S+\frac{2}{3} \end{bmatrix} \text{mho}$$

04.

Sol:

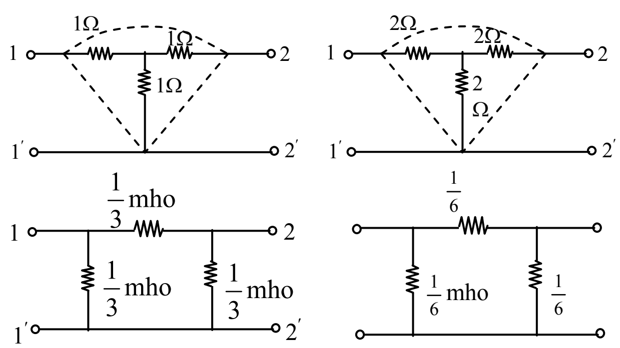


$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad Y_B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{7}{6} & -\frac{5}{6} \\ -\frac{5}{6} & \frac{5}{3} \end{bmatrix}$$

05.

Sol: Convert Y to Δ : Convert Y to Δ :



$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \text{mho} \quad Y_B = \begin{bmatrix} \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{2}{6} \end{bmatrix} \text{mho}$$

$$Y = \begin{bmatrix} \frac{6}{6} & -\frac{3}{6} \\ -\frac{3}{6} & \frac{6}{6} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

06.

Sol: $T_1 = T_2 = \begin{bmatrix} 1 + \frac{1}{-j1} & 1 \\ \frac{1}{-j1} & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+j & 1 \\ j & 1 \end{bmatrix}$$

$T_3 \Rightarrow Z_1 = 1\Omega$; $Z_2 = \infty$

$$T_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$T = (T_1)(T_2)(T_3)$

$$T = \begin{bmatrix} j3 & 2 + j4 \\ -1 + j2 & j3 \end{bmatrix}$$

07.

Sol: $T_1 : Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$T_1 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$T_2 : Z_1 = 0$; $Z_2 = 2\ \Omega$

$$T_2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$T = [T_1]\ [T_2]$

$$T = \begin{bmatrix} 3.5 & 3 \\ 2 & 2 \end{bmatrix}$$

08. Ans: (a)

Sol: For $I_2 = 0$ (O/P open), the Network is shown in Fig.1

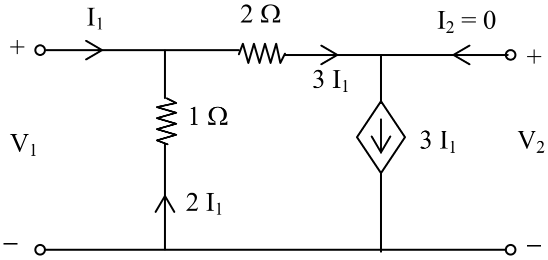


Fig. 1

$V_1 = -2\ I_1$ ………… (1)

$$Z_{11} = \frac{V_1}{I_1} = -2$$

$V_2 = -6\ I_1 + V_1$ ………… (2)

From (1) and (2)

$V_2 = -6\ I_1 - 2\ I_1$

or $V_2 = -8\ I_1$

$$Z_{21} = \frac{V_2}{I_1} = -8$$

For $I_1 = 0$ (I/P open), the network is shown in Fig.2

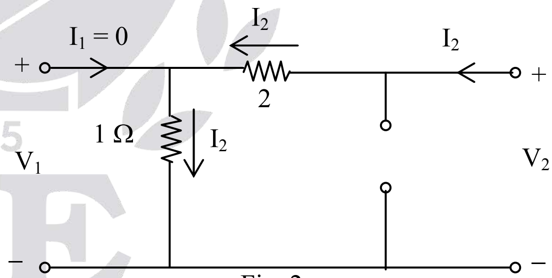


Fig. 2

Note: that the dependent current source with current $3\ I_1$ is open circuited.

$V_1 = 1\ I_2$, $\quad Z_{12} = \frac{V_1}{I_2} = 1$

$V_2 = 3\ I_2$, $\quad Z_{22} = \frac{V_2}{I_2} = 3$

$$[Z] = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$$

09.

Sol: By Nodal

$$-I_1 + V_1 - 3V_2 + V_1 + 2V_1 - V_2 = 0$$

$$-I_2 + V_2 + V_2 - 2V_1 = 0$$

$$Y = \begin{bmatrix} 4 & -4 \\ -3 & 2 \end{bmatrix} \mho$$

$[Z] = Y^{-1}$

We can also obtain [g], [h], [T] and $[T]^{-1}$ by re-writing the equations.

10.

Sol: The defining equations for open-circuit impedance parameters are:

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

In this case, the individual Z-parameter matrices get added.

$(Z) = (Z_a) + (Z_b)$

$$[Z] = \begin{bmatrix} 10 & 2 \\ 2 & 7 \end{bmatrix} \Omega$$

11.

Sol: For this case the individual y-parameter matrices get added to give the y-parameter matrix of the overall network.

$Y = Y_a + Y_b$

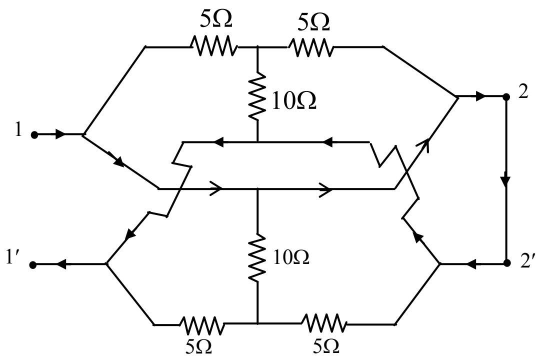
The individual y-parameters also get added

$Y_{11} = Y_{11a} + Y_{11b}$ etc

$$[Y] = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \text{mho}$$

12. **Ans: (c)**

Sol: $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$



$$Y_{11} = \frac{I_1}{0} = \infty$$

13.

Sol: (i). $[T_a] = \begin{bmatrix} 1 + \dfrac{Z_1}{Z_2} & Z_1 \\ \dfrac{1}{Z_2} & 1 \end{bmatrix}$

(ii). $[T_a] = \begin{bmatrix} 1 & Z_1 \\ \dfrac{1}{Z_2} & 1 + \dfrac{Z_1}{Z_2} \end{bmatrix}$

$[T_a]$ and $[T_b]$ are obtained by defining equations for transmission parameters.

14.

Sol: In this case, the individual T-matrices get multiplied

$(T) = (T_1) \times (T_{N1})$

$$(T) = (T_1)(T_{N1}) = \begin{pmatrix} 1 + s/4 & s/2 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 8 & 4 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3s + 8 & 3.5s + 4 \\ 6 & 7 \end{pmatrix}$$

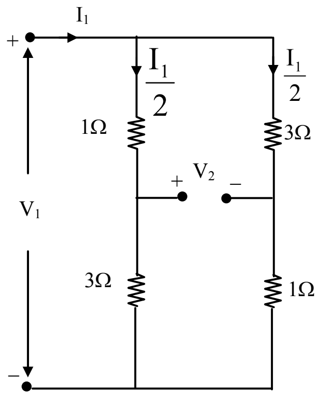
15.

Sol: $Z_{in} = R_{in} = \dfrac{V_1}{I_1} = \dfrac{AV_2 - BI_2}{CV_2 - DI_2} = \dfrac{V_2 - 2I_2}{V_2 - 3I_2}$,

$V_2 = 10(-I_2)$

$Z_{in} = R_{in} = \dfrac{12}{13} \Omega$

16.



Sol: $\left.\dfrac{V_1}{I_1}\right|_{I_2=0} = Z_{11}$

$\Rightarrow V_1 = (4 \parallel 4) I_1 \,|_{I_2=0}$

$\Rightarrow Z_{11} = 2\Omega$

$V_2 = (4 \parallel 4) I_2 |_{I_1=0}$

$\Rightarrow Z_{22} = 2\Omega$

By KVL $\Rightarrow$

$\dfrac{3I_1}{2} - V_2 - \dfrac{I_1}{2} = 0$

$V_2 = I_1$

$\Rightarrow Z_{21} = 1\Omega = Z_{12}$

$Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$

$Y = Z^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} ℧$

Now [T] parameters;

$V_1 = 2I_1 + I_2$ …….. (1)

$V_2 = I_1 + 2I_2$ ………. (2)

$\Rightarrow I_1 = V_2 - 2I_2$ ……… (3)

Substituting (3) in (1):

$V_1 = 2(V_2 - 2I_2) + I_2 = 2V_2 - 3I_2$ ……(4)

$T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$T^1 = T^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Now h parameters

$2I_2 = -I_1 + V_2$

$I_2 = \dfrac{-I_1}{I_2} + \dfrac{V_2}{2}$ ……… (5)

Substitute (5) in (1)

$V_1 = 2I_1 \dfrac{-I_1}{2} + \dfrac{V_2}{2}$

$V_1 = \dfrac{3}{2} I_1 + \dfrac{1}{2} V_2$ ………. (6)

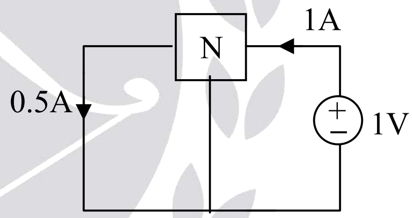
$h = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$

$g = [h]^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

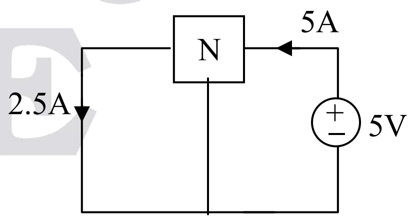
17. Ans: (a)

Sol: $Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0}$

Just use reciprocity of fig (a)



Now use Homogeneity



So, $Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1=0} = \dfrac{5}{5} = 1\,\text{mho}$

This has noting to do with fig (b) since fig (b) also valid for some specific resistance of 2 Ω at port-1, but Y_{22} , V_1= 0. So S.C port-1

18.

Sol: $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = n = \dfrac{-I_1}{I_2}$

$$\frac{V_2}{V_1} = n$$

$$\Rightarrow V_1 = \frac{1}{n}V_2 - (0)I_2$$

$$\Rightarrow T = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

$$T^1 = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$T^1 = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

Now h-parameters

$$V_1 = (0)I_1 + \frac{1}{n}V_2$$

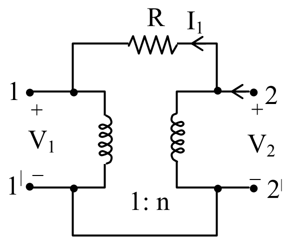
$$I_2 = \frac{-I_1}{n} + (0)V_2$$

$$g = \begin{bmatrix} 0 & \frac{1}{n} \\ \frac{-1}{n} & 0 \end{bmatrix}$$

$$h = \begin{bmatrix} 0 & -n \\ n & 0 \end{bmatrix}$$

Note: In an ideal transformer, it is impossible to express V_1 and V_2 interms of I_2 and I_2, hence the 'Z' parameters do not exist. Similarly, the y-parameters.

19. Ans: (c)



Sol: $Z_{22} = \dfrac{V_2}{I_2^1}\Bigg|_{V_1=0}$

$$\frac{V_1}{V_2} = \frac{1}{n} = \frac{I_2}{I_1}$$

$$V_1 = \frac{1}{n}V_2$$

$$\frac{V_2 - V_1}{R} = I_1$$

$$I_2^1 = I_2 + I_1$$

$$\frac{1}{n} = \frac{I_2}{I_1} = \frac{I_2^1 - I_1}{I_1} = \frac{I_2^1}{I_1} - 1$$

$$\frac{I_2^1}{I_1} = \frac{1}{n} + 1 = \frac{1+n}{n}$$

$$I_2^1 = \left(\frac{1+n}{n}\right)I_1$$

$$I_2^1 = \left(\frac{1+n}{n}\right)\left(\frac{V_2 - V_1}{R}\right)$$

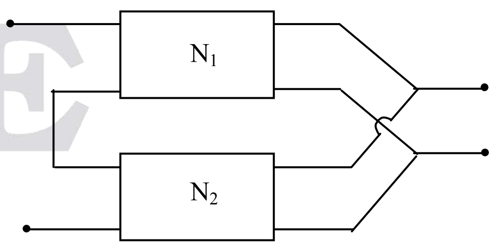
$$I_2^1 = \left(\frac{1+n}{n}\right)\left(\frac{V_2 - \frac{1}{n}V_2}{R}\right)$$

$$\frac{I_2^1}{V_2} = \left(\frac{1+n}{n}\right)\left(\frac{n-1}{nR}\right)$$

$$\frac{V_2}{I_2^1} = \frac{n^2R}{n^2 - 1}$$

20.

Sol:



For series parallel connection individual h-parameters can be added.

$\therefore$ For network 1, $h_1 = g_1^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

For network 2, $h_2 = g_2^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore h = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$\therefore$ overall g-parameters,

$$g = h^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$g = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

21. Ans: (b)

Sol: $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, $[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

For Reciprocal NW, $Z_{12} = Z_{21}$, $Y_{12} = Y_{21}$,

$\therefore$ [Z] and [Y] matrices are symmetrical.

$[Y] = [Z]^{-1}$ is true for reciprocal as well as non-reciprocal NW's.

22. Ans: (a)

Sol: Definition :

$\vec{I}(s) = [Y]\,\vec{V}(s)\;\vec{V}(s) = [Z]\,\vec{I}(s) = [Y]^{-1}\,\vec{I}(s)$,

$\therefore [Z] = [Y^{-1}]$

23. Ans: (a)

Sol: In reciprocal 2-port NW's, $y_{12} = y_{21}$, $z_{12} = z_{21}$, $h_{12} = -h_{21}$, AD – BC = 1

24. Ans: (d)

Sol: Convert the middle - π of 1Ω into a T - network as shown in Fig.

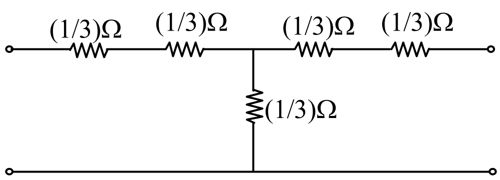


Fig.

$$z_{11} = \frac{2}{3} + \frac{1}{3} = 1 = z_{22} \qquad \text{……….} (4)$$

$$z_{12} = z_{21} = \frac{1}{3}\,\Omega \qquad \text{………..} (1)$$

$$z = \begin{bmatrix} 1 & (1/3) \\ (1/3) & 1 \end{bmatrix} \quad y = z^{-1} = \frac{9}{8}\begin{bmatrix} 1 & -(1/3) \\ -(1/3) & 1 \end{bmatrix}$$

$$y_{12} = y_{21} = -\frac{9}{8} \times \frac{1}{3} = -\frac{3}{8} \text{ mho } (3)$$

$$y_{11} = y_{22} = \frac{9}{8} \text{ mho} \qquad \text{……….} (2)$$

25. Ans: (d)

Sol: $$\frac{\frac{Ls}{2}\cdot\frac{1}{Cs}}{\frac{Ls}{2} + \frac{1}{Cs}} = \frac{Ls}{LCs^2 + 2}$$

$$Z_{is}(s) = \frac{Ls}{2} + \frac{Ls}{LCs^2 + 2}$$

$$= \frac{Ls(LCs^2 + 2) + 2Ls}{2L(s^2 + 2)}$$

$$Z_{is}(j\omega) = \frac{j\omega L(4 - \omega^2 LC)}{2L(2 - \omega^2)} = 0$$

$$\text{At } \omega = 0 \text{ and } \frac{2}{\sqrt{LC}} = \infty, \text{ at } \omega = \infty$$

26. Ans: (c)

Sol: $h_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$, $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$

$h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$, $h_{22} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$

According to the definitions above, h_{11} is in ohms (Ω)

h_{12} and h_{21} are dimensionless and h_{22} is in Siemens.

27. Ans: (b)

Sol: $A \rightarrow 4, \quad I_N = \frac{V_{th}}{R_{th}}, R_N = R_{th}$

$B \rightarrow 2, \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

$C \rightarrow 1, \quad Y_{12} = Y_{21},\ Z_{21} = Z_{12}$ etc

$D \rightarrow 3, \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} -V_2 \\ I_2 \end{bmatrix}$

28. Ans: (d)

Sol: $h_{11} = \frac{V_1}{I_1} \rightarrow$ Impedance (1)

$h_{12} = \frac{V_1}{V_2} \rightarrow$ voltage ratio (4)

$h_{22} = \frac{I_2}{V_2} \rightarrow$ Admittance (2)

$h_{21} = \frac{I_2}{I_1} \rightarrow$ Current ratio (3)

29. Ans: (c)

Sol: $V_b = h_{11} I_1 + h_{12} V_c$

$I_2 = h_{21} I_1 + h_{22} V_c$

$V_b = r_e I_1 + r_b(I_1 + I_2)$ ………..... (1)

$V_c = (I_2 + \alpha I_1) r_c + (I_1 + I_2) r_b$ ………….. (2)

or $V_c = (\alpha r_c + r_b) I_1 + (r_c + r_b) I_2$

or $I_2 = \frac{V_c - (\alpha r_c + r_b) I_1}{r_c + r_b}$ ………… (3)

Substitute I_2 in equation (1)

$$V_b = r_e I_1 + r_b I_1 + r_b\left[\frac{V_c - (\alpha r_c + r_b) I_1}{r_c + r_b}\right]$$

$$= r_e I_1 + r_b I_1 + \frac{r_b V_c - \alpha (r_b r_c) I_1 - r_b^2 I_1}{r_b + r_c}$$

$$= I_1\left(r_e + r_b - \frac{\alpha r_b r_c - r_b^2}{r_b + r_c}\right) + \frac{r_b V_C}{r_b + r_c}$$

$$= I_1\left(\frac{r_e r_b + r_e r_c + r_b^2 + r_b r_c - \alpha r_b r_c - r_b^2}{r_b + r_c}\right)$$

$$= I_1\left[r_e + \frac{r_b (r_c - \alpha r_c)}{r_b + r_c}\right]$$

$$V_b = \left[r_e + r_b - \frac{r_b}{r_e + r_b}(\alpha r_c + r_b)\right] I_1$$

$$+ \left[\frac{r_b}{r_c + r_b}\right] V_c \quad \text{………. (5)}$$

From equation (3)

$$I_2 = \frac{-(\alpha r_c + r_b) I_1}{r_b + r_c} + \frac{1}{r_b + r_c} V_c \quad \text{………… (4)}$$

Comparing (5) & (4) with (1) & (2) the matching in $A \rightarrow 1, B \rightarrow 4, C \rightarrow 2, D \rightarrow 3$.

30. Ans: (d)

Sol: $A \rightarrow 2, B \rightarrow 4$

$C \rightarrow 1, D \rightarrow 3$

Conventional Practice Solutions

01.

Sol: A two-port circuit can be declared as a reciprocal circuit, if the 2-port parameters satisfy the following relations:

(i) $Z_{12} = Z_{21}$

(ii) $Y_{12} = Y_{21}$

(iii) $AD - BC = 1$

(iv) $h_{12} = -h_{21}$

The two-port circuit shown in Fig.1 is reciprocal.

This is justified by taking 15 V voltage source and showing $Y_{12} = Y_{21}$ as shown below.

First, convert the (30 Ω, 20 Ω and 60 Ω) T network into a Π network and next get the overall Π network (Fig. 3)

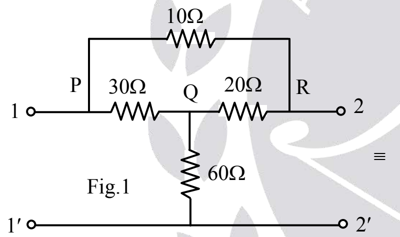


Fig.1

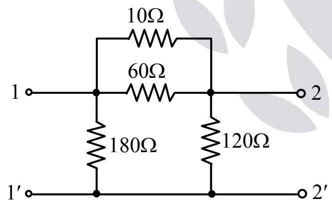


Fig. 2

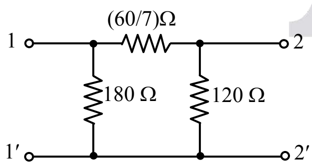


Fig. 3

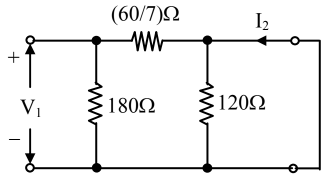


Fig. 4

$$V_1 = -I_2\left(\frac{60}{7}\right)$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{7}{60}\ \Omega$$

Short circuit current response at port 2 with excitation, $V_1 = 15$ V at port 1

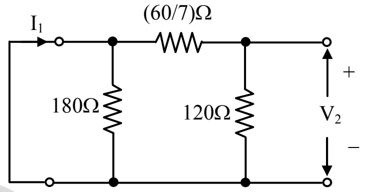


Fig. 5

$$V_2 = -I_1\left(\frac{60}{7}\right)$$

$$= -\frac{7}{60}\times 15 = -1.75\ \text{A}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{7}{60}\ \Omega$$

When the excitation and response are interchanged, Short circuit current at port 1 with excitation, $V_2 = 15$ V at port 2

$$= -\frac{7}{60}\times 15 = -1.75\ \text{A}$$

The ratio of response to excitation remains constant for reciprocal network when the response and excitation are interchanged.

02.

Sol: The given circuit is shown in Fig. 1

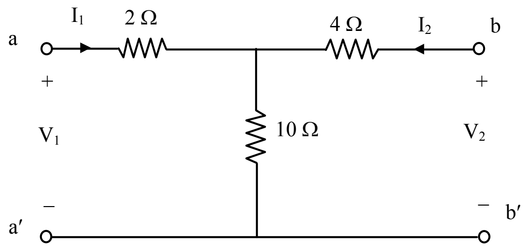


Fig. 1

$V_1 = A\,V_2 - B\,I_2$

$I_1 = C\,V_2 - D\,I_2$

Keep $I_2 = 0$ (Fig. 2)

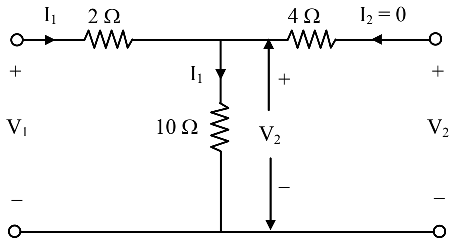


Fig. 2

$$V_2 = V_1\,\frac{10}{12} = \frac{5}{6}V_1$$

$$A = \frac{V_1}{V_2} = \frac{6}{5} = 1.2$$

$V_2 = 10\,I_1$

$$C = \frac{I_1}{V_2} = 0.1\ \Omega$$

Keep $V_2 = 0$ (Fig. 3)

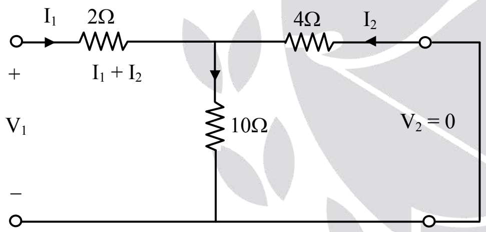


Fig. 3

$$B = -\frac{V_1}{I_2}, \quad D = -\frac{I_1}{I_2}$$

$$-I_2 = I_1 \times \frac{10}{14} = \frac{5}{7}I_1\,,$$

$$D = \frac{7}{5} = 1.4$$

$$V_1 = 2\,I_1 - 4\,I_2 = 2I_2\left(\frac{I_1}{I_2} - 2\right)$$

$V_1 = 2\,I_2\,(-3.4) = -6.8\,I_2$

$B = 6.8\ \Omega$

$A = 1.2$, $B = 6.8\ \Omega$, $C = 0.1\ \mho$, $D = 1.4$

03.

Sol:

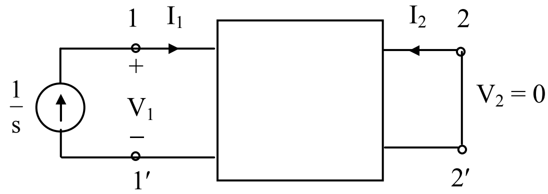


Fig.1

$I_1(s) = Y_{11}\,V_1(s) + Y_{12}\,V_2(s)$

$I_2(s) = Y_{21}\,V_1(s) + Y_{22}\,V_2(s)$

For $v_2(t) = 0$, $V_2(s) = 0$, as shown in Fig. 1,

$$i_1(t) = 1\,u(t)\ ,\quad I_1(s) = \frac{1}{s}$$

$v_1(t) = (1 - e^{-4t})\,u(t)$

$$V_1(s) = \frac{1}{s} - \frac{1}{s+4} = \frac{4}{s(s+4)}$$

$i_2(t) = -e^{-3t}\,u(t)$

$$I_2(s) = -\frac{1}{s+3}$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{(s+4)}{4}$$

$$Y_{21} = \frac{I_2}{V_1} = -\frac{s(s+4)}{4(s+3)}$$

For $R_L = 1\ \Omega$ as in Fig. 2

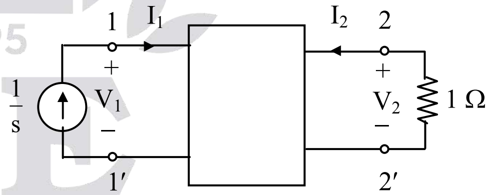


Fig.2

$$i_1(t) = u(t),\quad I_1(s) = \frac{1}{s},\ V_2(s) = -I_2(s)$$

$v_1(t) = (1 - e^{-4t} + t\,e^{-4t})\,u(t)$

$$V_1(s) = \frac{1}{s} - \frac{1}{s+4} + \frac{1}{(s+4)^2}$$

$$= \frac{(s+4)^2 - s(s+4) + s}{s(s+4)^2} = \frac{5s+16}{s(s+4)^2}$$

$i_2(t) = -e^{-7t}u(t)$,

$$I_2(s) = -\frac{1}{(s+7)}$$

$$\Rightarrow V_2(s) = \frac{1}{s+7}$$

$I_1(s) = Y_{11}V_1(s) + Y_{12}V_2(s)$

$$\frac{1}{s} = \left(\frac{4+s}{4}\right)\frac{5s+16}{s(s+4)^2} + Y_{12}\left(\frac{1}{s+7}\right)$$

$$\Rightarrow \frac{1}{s} = \frac{5s+16}{4s(s+4)} + Y_{12}\left(\frac{1}{s+7}\right)$$

$$\Rightarrow Y_{12}\left(\frac{1}{s+7}\right) = \frac{1}{s} - \frac{5s+16}{4s(s+4)}$$

$$= \frac{4(s+4)-5s-16}{4s(s+4)}$$

$$= \frac{4s+16-5s-16}{4s(s+4)}$$

$$= \frac{-s}{4s(s+4)} = \frac{-1}{4(s+4)}$$

$$\Rightarrow Y_{12} = \frac{-(s+7)}{4(s+4)}$$

$I_2(s) = Y_{21}V_1(s) + Y_{22}V_2(s)$

$$\frac{-1}{s+7} = \frac{-s(s+4)}{4(s+3)}\frac{(5s+16)}{s(s+4)^2} + Y_{22}\left(\frac{1}{s+7}\right)$$

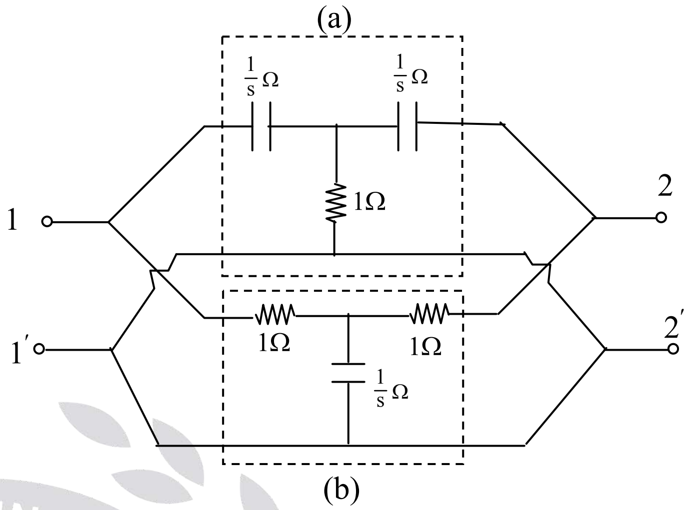
$$\Rightarrow Y_{22}\left(\frac{1}{s+7}\right) = \frac{-1}{s+7} + \frac{5s+16}{4(s+3)(s+4)}$$

$$\Rightarrow Y_{22} = \frac{s^2+23s+64}{4(s+3)(s+4)}$$

And $Z_{in}(s) = \dfrac{V_1(s)}{I_1(s)} = \dfrac{5s+16}{s(s+4)^2}$

04.

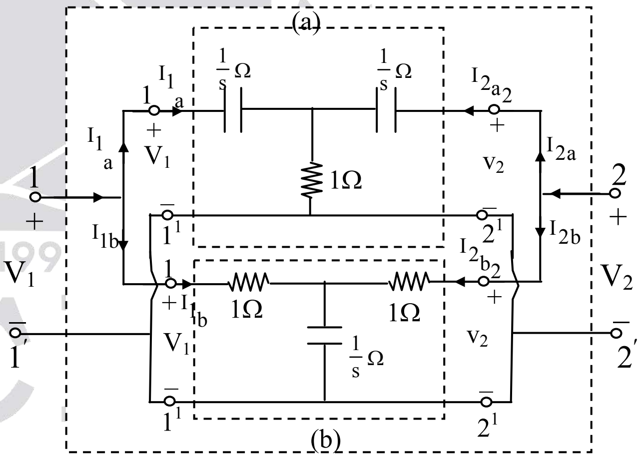
Sol: Given network



(Data given is $C_1 = C_2 = C_a = 1\ \Omega$. This can not correct. Assume $C_1 = C_2 = C_a = 1$ F.

Use Operational impedance, $\frac{1}{r}$)

The network is redrawn as follows:



In this form, it is easy to see that two passive linear 2 – port a and b have been put in parallel to form one large two point.

For the 2 – port labeled 'a'; we have

$I_{1_a} = y_{11_a}V_1 + y_{12_a} + V_2$ and

$I_{2a} = y_{21_a V_1} + y_{22_a}V_2$ ------ (1)

For the 2-port labeled ‘b’, we have

$I_{1_b} = y_{11_b}V_1 + y_{12_b}V_2$ and

$I_{2_b} = y_{21_b}V_1 + y_{22_b}V_2$ ------ (2)

The overall 2–port has voltage V_1 and current $(I_{1a} + I_{1b})$ at port1 and voltage V_2 and current $(I_{2a} + I_{2b})$ at port2

But from (1) and (2),

$$I_{1_a} + I_{1_b} = \left(y_{11_a} + y_{11_b}\right)V_1 + \left(y_{12_a} + y_{12_b}\right)V_2$$

$$I_{2a} + I_{2b} = \left(y_{21_a} + y_{21_b}\right)V_1 + \left(y_{22_a} + y_{22_b}\right)V_2$$

Thus, the overall admittance parameter are,

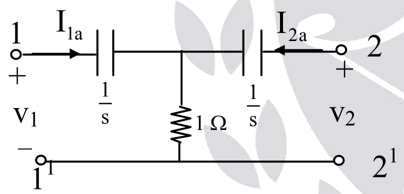
$y_{11} = \left(y_{11_a} + y_{11_b}\right)$; $\quad y_{12} = \left(y_{12_a} + y_{12_b}\right)$;

$y_{21} = \left(y_{21_a} + y_{21_b}\right)$; and $y_{22} = \left(y_{22_a} + y_{22_b}\right)$

It now remains to determine the ‘a’–parameters & ‘b’– parameters.

Addition will give the overall y–parameters.

y – parameters of the 2–port ‘a’:



y_{11_a} : **short-circuit port-2**

$$I_{1_a} = \frac{V_1}{\frac{1}{s} + \frac{(1/s)}{\left(1 + \frac{1}{s}\right)}} = \frac{V_1(s+1)s}{(2s+1)}$$

$$y_{11_a} = \left.\frac{I_{1_a}}{V_1}\right|_{V_2 = 0} = \frac{s(s+1)}{(2s+1)}$$

y_{12_a} : **short–circuit port-1**

With a little algebraic work; we get

$$y_{12_a} = \left.\frac{I_{1_a}}{V_2}\right|_{V_1 = 0} = \frac{-s^2}{2s+1}$$

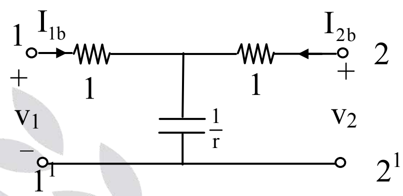
y_{21_a} : Because the network is reciprocal, we get

$$y_{21_a} = y_{12_a} = \frac{-s^2}{2s+1}$$

y_{22_a} : From the symmetry of the network, we get

$$y_{22_a} = y_{11_a} = \frac{s(s+1)}{2s+1}$$

y – parameters of the 2–part ‘b’:



By a procedure similar to the above, we get

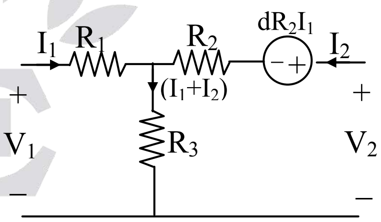
$$y_{11_b} = y_{22_b} = \left(\frac{1+s}{2+s}\right) \quad \text{and}$$

$$y_{12_b} = y_{21_b} = -\frac{1}{s+2}$$

Now the overall 2-part parameters and be found. (The network given is called a twin-T network and it is parallel connection of two T-networks)

05.

Sol:



$V_1 = I_1(R_1+R_3) + I_2 R_3$

$V_2 = I_1 (R_3 + dR_2) + I_2 (R_2 + R_3)$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_1 + R_3 & R_3 \\ R_3 + dR_2 & R_2 + R_3 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$Z_{12} \neq Z_{21}$

$\therefore$ The given two port network is not a reciprocal network

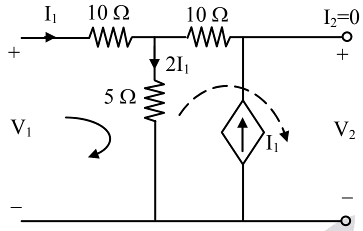
06.

Sol: Z- parameters of network:

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$



KVL

$V_1 = 10I_1 + 10I_1$

$V_1 = 20I_1 \Rightarrow Z_{11} = 20\Omega$

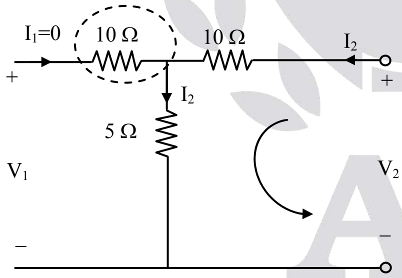
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

KVL, in above circuit,

$-10I_1 - 10I_1 + V_2 = 0$

$V_2 = 20I_1$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 20\Omega$$



$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

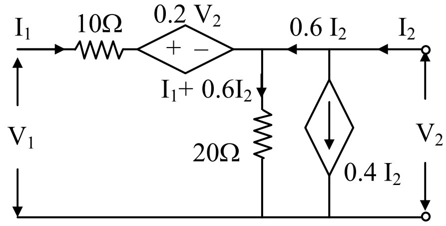
KVL, $V_2 = 15I_2 \Rightarrow Z_{22} = 15\ \Omega$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Rightarrow V_1 = 5\ I_2 \Rightarrow Z_{12} = 5\ \Omega$$

$$[Z] = \begin{bmatrix} 20 & 5 \\ 20 & 15 \end{bmatrix}\Omega$$

07.

Sol:



$V_1 = 10\ I_1 + 0.2\ V_2 + (I_1 + 0.6\ I_2)\ 20$

$V_1 = 10I_1 + 0.2\ V_2 + 20\ I_1 + 12\ I_2$

$V_1 = 30I_1 + 12I_2 + 0.2\ V_2$

$V_2 = 20\ (I_1 + 0.6\ I_2) = 20\ I_1 + 12\ I_2$

$V_1 = 30I_1 + 12\ I_2 + 0.2(20I_1 + 12\ I_2)$

$V_1 = 34I_1 + 14.4\ I_2$

$V_2 = 20\ I_1 + 12I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 34 & 14.4 \\ 20 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V = Z\ I$

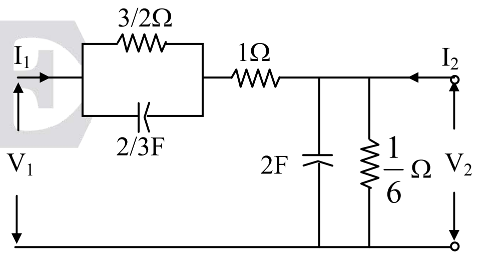
$$[Y] = [Z]^{-1} = \frac{1}{(34)(12)-(20)(14.4)}\begin{bmatrix} 12 & -14.4 \\ -20 & 34 \end{bmatrix}$$

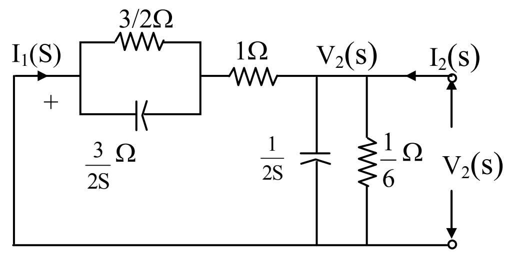
$$= \frac{1}{120}\begin{bmatrix} 12 & -14.4 \\ -20 & 37 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & -0.112 \\ -0.167 & 0.283 \end{bmatrix}$$

08.

Sol:





$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$I_1(s) + I_2(s) = \frac{V_2(s)}{\frac{1}{2s}} + \frac{V_2(s)}{\frac{1}{6}}$$

$$= V_2(s)(2S+6)$$

$$V_2(s) = I_1(s)\left(\left[\frac{3/2\times\frac{3}{2S}}{\frac{3}{2}+\frac{3}{2S}}\right]+1\right)//\frac{1}{2S}//\frac{1}{6}$$

$$V_2(s) = -I_1(s)\left[\left[\frac{9/4S}{\frac{3}{2}\left(1+\frac{1}{S}\right)}\right]+1\right]$$

$$\frac{V_2(s)}{I_{(s)}} = \left[\frac{3}{2}\frac{1}{(s+1)}+1\right]$$

$$= \frac{3+2S+2}{2(s+1)}+1 = \frac{-(2S+5)}{2(s+1)}$$

$$Y_{12} = \frac{I_1(S)}{V_2(s)} = \frac{-2(s+1)}{(2s+5)}\ \mho$$

09.

Sol: The given circuit is shown in Fig.1

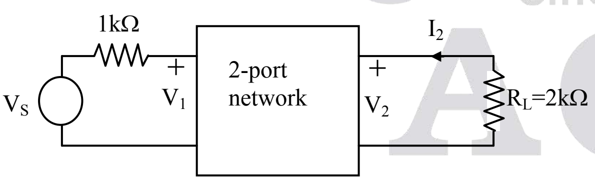


Fig.1

Apply KVL in above circuit

$V_2 = -I_2R_L$ $\quad\quad$ $V_2 = -2000I_2$

$$\Rightarrow I_2 = -\frac{V_2}{2000} \text{ ----------(1)}$$

h-parameters:

consider the 2-port NW shown in Fig.2

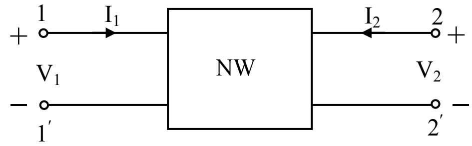


Fig.2

h-parameters relate the input and output port currents, I_1 and I_2 as a linear combination of the input and output port voltage, V_1 and V_2.

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$V_1 = 100I_1 + 0.0025V_2$-----(2)

$I_2 = 20I_1 + 10^{-3}V_2$------(3)

Put equation (1) in equation (3)

$$-\frac{V_2}{2000} = 20I_1 + 10^{-3}V_2$$

$$I_1 = -\frac{3V_2\times10^{-3}}{40} \text{ ---------(4)}$$

Put equation (4) in equation (2)

$$V_1 = 100\left[-\frac{3V_2}{40\times10^3}\right] + 0.0025V_2$$

$$V_1 = V_2\left[\frac{-300+100}{40\times10^3}\right]$$

$$\frac{V_2}{V_1} = -\frac{40\times10^3}{200}$$

$$\frac{V_2}{V_1} = -200$$

Chapter 7 Graph Theory

Objective Practice Solutions

01. Ans: (c)

Sol: $n > \frac{b}{2} + 1$

Note: Mesh analysis simple when the nodes are more than the meshes.

02. Ans: (c)

Sol: Loops = b – (n–1) $\Rightarrow$ loops = 5

n = 7 $\quad \therefore$ b = 11

03. Ans: (a)

04.

Sol: Nodal equations required = f-cut sets

= (n–1) = (10 – 1) = 9

Mesh equations required = f-loops

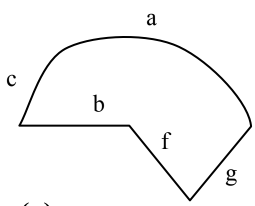
= b – n + 1 = 17 – 10 + 1 = 8

So, the number of equations required

= Minimum (Nodal, mesh)=Min(9,8)=8

05. Ans: (c)

Sol: Not a tree (Because trees are not in closed path)



06. Ans: (a)

07.

Sol: For a complete graph;

$$b = n_{C_2} \Rightarrow \frac{n(n-1)}{2} = 66$$

n = 12

f-cut sets = (n–1) = 11

f-loops = (b – n + 1) = 55

f-loop = f-cutset matrices = $n^{(n-2)}$

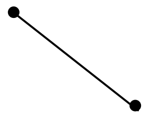
$= 12^{12-2} = 12^{10}$

08. Ans: (a)

Sol: Let N = 1

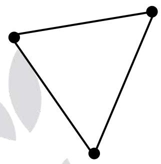
Nodes = 1, Branches = 0 ; f-loops = 0

Let N = 2



Nodes = 2; Branches = 1; f-loop = 0

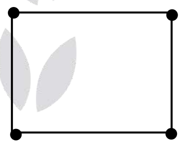
Let N=3



Nodes = 3; Branches = 3; f-loop = 1

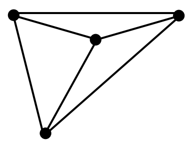
$\Rightarrow$ Links = 1

Let N = 4



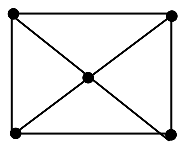
Nodes = 4; Branches = 4; f-loops = Links = 1

Still N = 4



Branches = 6; f-loops = Links = 3

Let N = 5

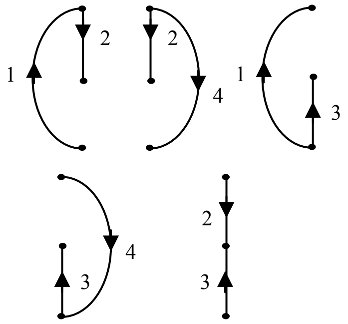


Nodes = 5; Branches = 8; f –loops = Links = 4 etc

Therefore, the graph of this network can have at least "N" branches with one or more closed paths to exist.

09. Ans: (b)

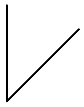
Sol:



10. Ans: (d)

Sol:

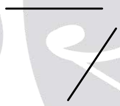
(a) 1,2,3,4 →



(b) 2,3,4,6 →

(c) 1,4,5,6 →

(d)1,3,4,5 →



11. Ans: (b)

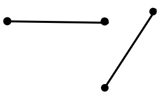
Sol: m = b – n + 1 = 8 – 5 + 1 = 4

12. Ans: (d)

13. Ans: (d)

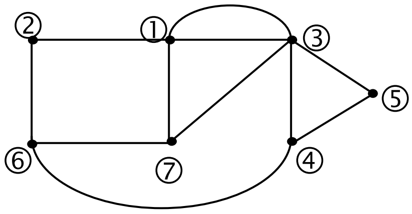
Sol: The valid cut –set is

(1,3,4,6)



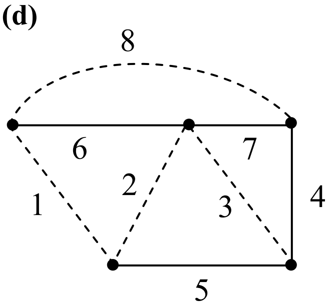
14. Ans: (b)

Sol:



15. Ans: (d)

Sol:



Fundamental loop should consist only one link, therefore option (d) is correct.

16. Ans: (d)

17. Ans: (a)

Sol: Statement (I) – True, Statement (II) – True and is correct explanation.

18. Ans: (d)

Sol: f – loop should contain only one link.

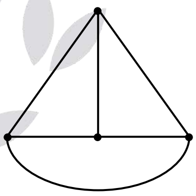
$\therefore$ Statement (I) is False.

A link with one or more of the twigs forms a closed loop.

$\therefore$ Statement (II) is True.

19. Ans: (b)

Sol:



The graph has

No. of nodes = n = 4,

No. of branches = b = 6

No. of twigs = No. of tree branches

= n – 1 = 3

No. of independent loops = No. of links = l

= b – (n –1) = 3

Order of B matrix or Fundamental loop matrix = l × b = 3 × 6

Correct answer is A = 6, B = 3,

C = 3 × 6, D = 3

20. Ans: (a)

Sol: If 1, 2, 3 and 8 are the co-tree branches or chords or links, and then 4, 5, 6 and 7 should be Tree branches or twigs.

f - cutset (1, 2, 3, 4) is defined by 4 and f - loop (6, 7, 8) is defined by 8.

21. Ans: (a)

Sol: The Tree (1, 2, 3, 4, 5) is shown with thick lines.

The dotted lines (6, 7, 8) are links or chords.

f – circuit or f – loops are

Edge set : L_1 (1, 2, 4, 6) defined by chord 6

Edge set : L_2 (2, 4, 5, 7) defined by chord 7

Edge set : L_3 (2, 3, 5, 8) defined by chord 8

Note that the twigs or tree branches can be drawn so that they do not cross each other

Conventional Practice Solutions

01.

Sol: A tree is a connected sub-graph of a connected graph containing all the nodes of the graph but containing no loops.

Properties of trees:

1. A connected sub-graph of a connected graph is a tree if there exists only one path between any pair of nodes in it.
2. Every connected graph has atleast one tree.
3. The number of terminal nodes of every tree are two.
4. A connected sub-graph of a connected graph is a tree if there exists all the nodes of the graph.
5. Each tree has (n – 1) branches, where n is the number of nodes of the tree.
6. The rank of a tree is (n – 1).

The given connected graph has 7 nodes and 12 branches. Five different Trees are shown below from Fig. 1 to Fig. 5.

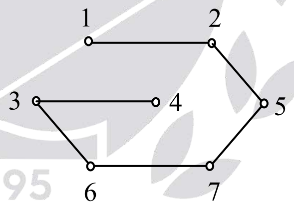


Fig.1

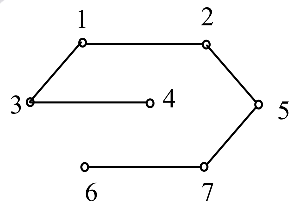


Fig.2

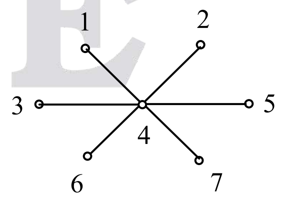


Fig.3

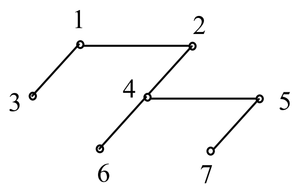


Fig.4

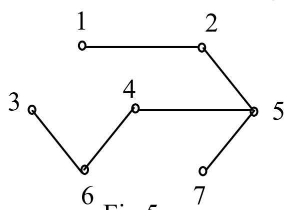


Fig.5

02.

Sol: Let the fundamental loop matrix be B and the fundamental cut-set matrix be Q of the same oriented G, and let both matrices pertain to the same tree T ; then

$B\,Q^T = 0$ and $Q\,B^T = 0$ …………..(1)

If we number the links from 1 to l and number the tree branches from $l + 1$ to b, then

$B = [1_l \vdots F]$ and $Q = [-F^T \vdots l_n]$ ……(2)

$$B_f = \left[\begin{array}{cccccc:cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$= [1_l \vdots F]$

$Q = [-F^T \vdots l_n]$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

03.

Sol: The eleven branches a, b, c,.., k are marked on the graph (Fig.). The incidence matrix is written with the usual convention: example: branch 'a' leaving node 1(taken as 1) and entering node 2 (taken as –1).

branches

$$\begin{array}{ll} & \begin{array}{ccccccccccc} a & b & c & d & e & f & g & h & i & j & k \end{array} \\ \begin{array}{c} \text{n}\;1 \\ \text{o}\;2 \\ 3 \\ \text{d}\;4 \\ \text{e}\;5 \\ 6 \\ \text{s}\;7 \end{array} \; A_{7\times 11} = & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}_{7\times 11} \end{array}$$

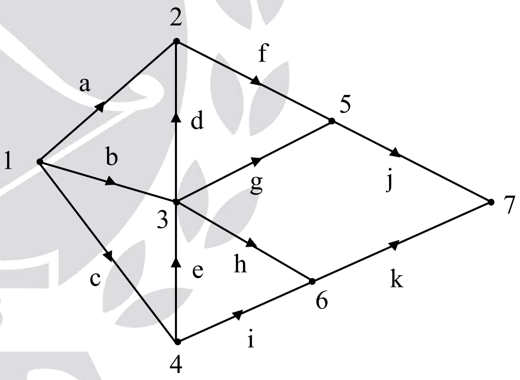


Fig.

04.

Sol:

branches

$$\begin{array}{ll} & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ A_a = \begin{array}{c} \text{n}\;1 \\ 2 \\ \text{o}\;3 \\ \text{d}\;4 \\ \text{e}\;5 \\ \text{s}\;6 \\ 7 \end{array} & \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

Dimensions of the incidence matrix is 7 × 12.

The oriented graph with 7 nodes and 12 branches is shown in Fig. 1.

To obtain the fundamental circuit matrix for these graph, a tree with 7 nodes and 6 branches is chosen (Fig. 2) .

Number of f-circuits = Number of links

$= b - (n - 1) = 6$

The links are the branches 1, 3, 4, 6, 9 and 11.

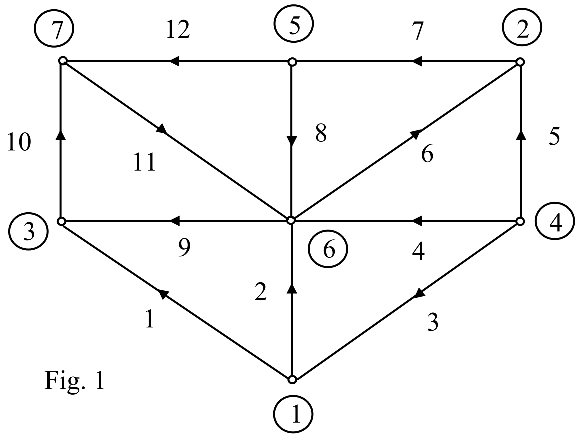


Fig. 1

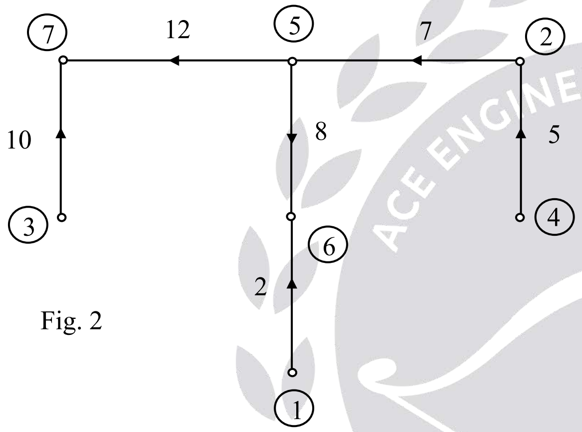


Fig. 2

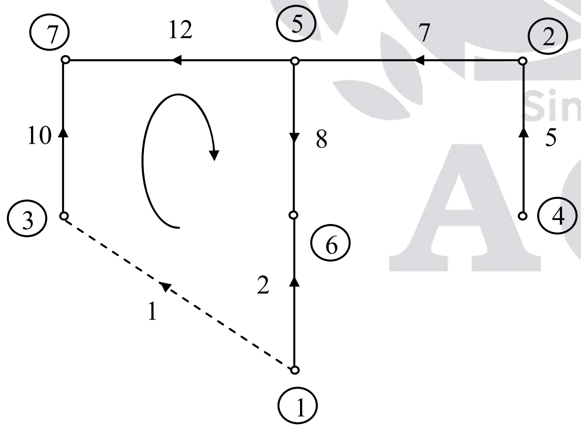


Fig. 3

f – circuit with 1 : [1, 10, 12, 8, 2]

$v_1 - v_2 + v_8 + v_{10} - v_{12} = 0$

f – circuit with 3 : [3, 2, 8, 7, 5]

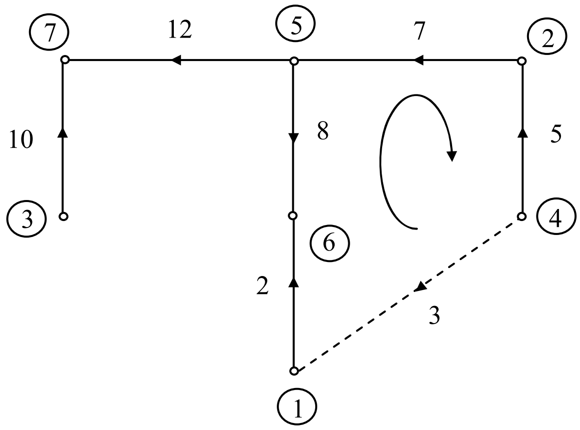


Fig. 4

$v_2 + v_3 - v_5 - v_7 - v_8 = 0$

f – circuit with 4 : [4, 8, 7, 5]

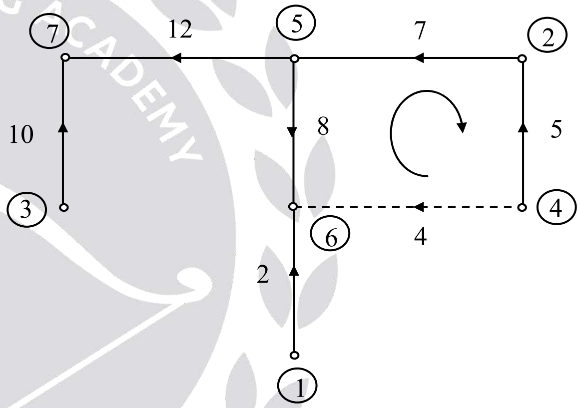


Fig. 5

$v_4 - v_5 - v_7 - v_8 = 0$

f – circuit with 6 : [6, 7, 8]

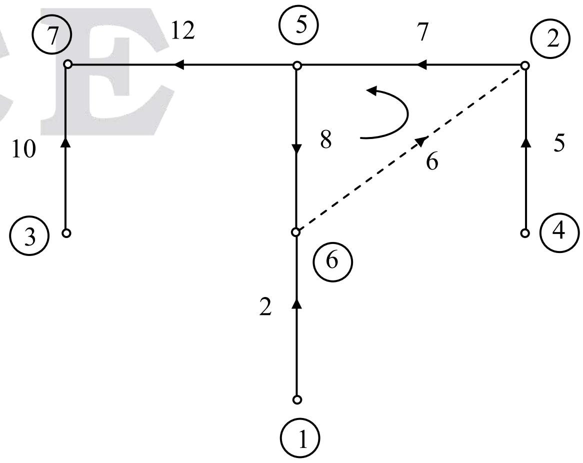


Fig. 6

$v_6 + v_7 + v_8 = 0$

f – circuit with 9 : [9, 10, 12, 8]

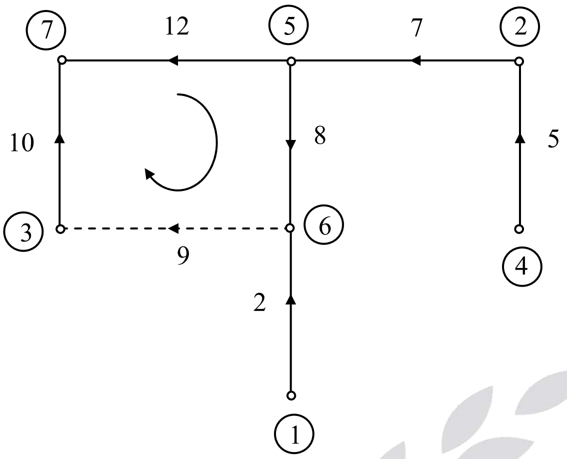


Fig. 7

$v_8 + v_9 + v_{10} - v_{12} = 0$

f – circuit with 11 : [11, 8, 12]

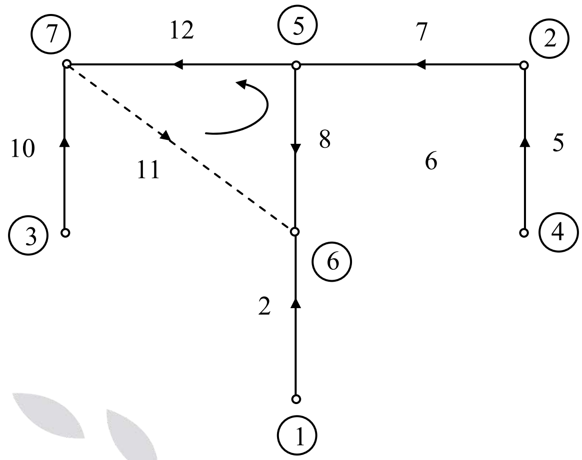


Fig. 8

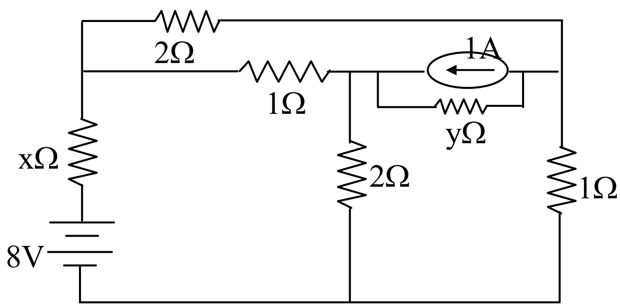
$-v_8 + v_{11} + v_{12} = 0$

According to the above f – circuit (tie-set) equations, the tie-set matrix, $B_{6\times12}$ is constructed as shown in Table.

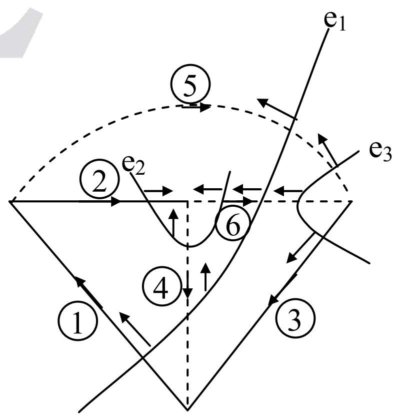
	f -circuit	Branches 1	2	3	4	5	6	7	8	9	10	11	12
$B_{6\times12}$=	1	1	–1	0	0	0	0	0	1	0	1	0	–1
	3	0	1	1	0	–1	0	–1	–1	0	0	0	0
	4	0	0	0	1	–1	0	–1	–1	0	0	0	0
	6	0	0	0	0	0	1	1	1	0	0	0	0
	9	0	0	0	0	0	0	0	1	1	1	0	–1
	11	0	0	0	0	0	0	0	–1	0	0	1	1

05.

Sol:



TREE



$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$[Q] = \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array}\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}_{3\times 6}$$

$$[Q][Y_b]\,[Q]^T\,[\rho_{twing}] = [Q]\,[I_s] - [Q]\,[Y_b][V_s]$$

$$[Y_b] = \begin{bmatrix} \frac{1}{x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{y} \end{bmatrix}_{6\times 6}$$

$$[e_{twigs}] = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3\times 1}$$

$$[V_s] = \begin{bmatrix} +8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1} \qquad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{6\times 1}$$

$$[Q]\,[Y_b] = \begin{bmatrix} \frac{1}{x} & 0 & 0 & \frac{-1}{2} & \frac{-1}{2} & -\frac{1}{y} \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{-1}{y} \\ 0 & 0 & 1 & 0 & \frac{-1}{2} & -\frac{1}{y} \end{bmatrix}_{3\times 6}$$

$$[Q][Y_b][Q^T] = \begin{bmatrix} \frac{1}{x} & 0 & 0 & \frac{-1}{2} & \frac{-1}{2} & -\frac{1}{y} \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{-1}{y} \\ 0 & 0 & 1 & 0 & \frac{-1}{2} & -\frac{1}{y} \end{bmatrix}_{3\times 6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix}_{6\times 3}$$

LHS:

$$[Q][Y_b][Q^T][e_{twigs}] = \begin{bmatrix} \left(1+\frac{1}{x}+\frac{1}{y}\right) & \left(\frac{1}{2}+\frac{1}{y}\right) & \left(\frac{1}{2}+\frac{1}{y}\right) \\ \left(\frac{1}{2}+\frac{1}{y}\right) & \left(\frac{3}{2}+\frac{1}{y}\right) & \frac{1}{y} \\ \left(\frac{1}{2}+\frac{1}{y}\right) & \frac{1}{y} & \left(\frac{3}{2}+\frac{1}{y}\right) \end{bmatrix}_{3\times 3} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3\times 1}$$

$$[Q][Yb][Q]T[e_{twig}] = \begin{bmatrix} \left(1+\frac{1}{x}+\frac{1}{y}\right)e_1 + \left(\frac{1}{2}+\frac{1}{y}\right)e_2 + \left(\frac{1}{2}+\frac{1}{y}\right)e_3 \\ \left(\frac{1}{2}+\frac{1}{y}\right)e_1 + \left(\frac{3}{2}+\frac{1}{y}\right)e_2 + \frac{1}{y}e_3 \\ \left(\frac{1}{2}+\frac{1}{y}\right)e_1 + \frac{1}{y}e_2 + \left(\frac{3}{2}+\frac{1}{y}\right)e_3 \end{bmatrix}_{3\times 1}$$

RHS:

$$[Q]\,[I_s] = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}_{3\times 1} \qquad [Q][Y_b][V_s] = \begin{bmatrix} \frac{8}{x} \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

$$[Q]\,[I_s] - [Q]\,[Y_b]\,[V_s] = \begin{bmatrix} -\left(1+\frac{8}{x}\right) \\ -1 \\ -1 \end{bmatrix}_{3\times 1}$$

Final LHS = Final RHS

Apply limits ($x \to 0$ & $y \to \infty$) on both sides.

$$\Rightarrow \begin{bmatrix} e_1 \\ \frac{e_1}{2}+\frac{3}{2}e_2 \\ \frac{e_1}{2}+\frac{3}{2}e_3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} -8 \\ -1 \\ -1 \end{bmatrix}_{3\times 1}$$

$$e_1 = -8 \qquad \text{------ (1)}$$

$$\frac{e_1}{2} + \frac{3e_2}{2} = -1 \qquad \text{------ (2)}$$

$$\frac{e_1}{2} + \frac{3e_3}{2} = -1 \qquad \text{------ (3)}$$

By solving equations (1), (2) and (3)

$e_1 = -8$, $e_2 = 2$ and $e_3 = 2$

Now branch voltages are

$[Q]^T [e_{twig}] = [V_b]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix}_{6\times3} \begin{bmatrix} -8 \\ 2 \\ 2 \end{bmatrix}_{3\times1} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}_{6\times1}$$

So $V_2 = 2$ volts

$V_3 = 2$ volts

Chapter 8 Passive Filters

Objective Practice Solutions

01.

Sol:

$$\left.\begin{array}{l}\omega=0\Rightarrow V_0=V_i\\ \omega=\infty\Rightarrow V_0=0\end{array}\right\}\Rightarrow \text{Low pass filter}$$

02.

Sol: $\omega=0\Rightarrow V_0=\dfrac{V_iR_2}{R_1+R_2}$

"V_0" is attenuated $\Rightarrow V_0=0$

$\omega=\infty\Rightarrow V_0=V_i$

It represents a high pass filter characteristics.

03.

Sol: $H(s)=\dfrac{V_i(s)}{I(s)}=\dfrac{S^2LC+SRC+1}{SC}$

Put s = $j\omega i=-\dfrac{\omega^2LC+j\omega RC+1}{j\omega C}$

$\omega=0\Rightarrow H(s)=0$

$\omega=\infty\Rightarrow H(s)=0$

It represents band pass filter characteristics

04.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = 0$

It represents Band pass filter characteristics

05.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = V_i$

It represents High Pass filter characteristics.

06.

Sol: $H(s)=\dfrac{1}{s^2+s+1}$

$\omega = 0 : S = 0 \Rightarrow H(s)= 1$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Low pass filter characteristics

07.

Sol: $H(s)=\dfrac{s^2}{s^2+s+1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 1$

It represents a High pass filter characteristics

08.

Sol: $\omega=0; V_0=V_i$

$\omega=\infty; V_0=0$

It represents a low pass filter characteristics.

09.

Sol: $\omega = 0 \Rightarrow V_0 = V_{in}$

$\omega = \infty \Rightarrow V_0 = V_{in}$

It represents a Band stop filter or notch filter.

10.

Sol: $H(s)=\dfrac{S}{s^2+s+1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Band pass filter characteristics

11.

Sol: $H(s)=\dfrac{S^2+1}{s^2+s+1}$

$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = 1$

It represents a Band stop filter

12.

Sol: $H(s)=\dfrac{1-s}{1+s}$

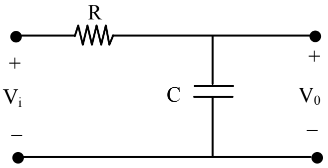
$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = -1 = 1\ \angle 180^0$

It represents an All pass filter

13. Ans: (c)

Sol.



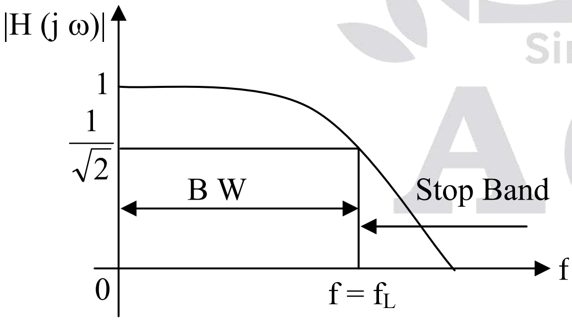
$\omega = 0 \Rightarrow V_0 = V_i$

$\omega = \infty \Rightarrow V_0 = 0$

$$V_0(s)=\left(\frac{V_i(s)}{R+\dfrac{1}{sc}}\right)\left(\frac{1}{sc}\right)$$

$$\frac{V_0(s)}{V_i(s)}=H(s)=\frac{1}{SscR+1}$$

$$H(j\omega)=\frac{1}{1+j\omega c R}=\frac{1}{1+j\dfrac{f}{f_L}}$$



Where $f_L = \dfrac{1}{2\pi RC}$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\dfrac{f}{f_L}\right)^2}}$$

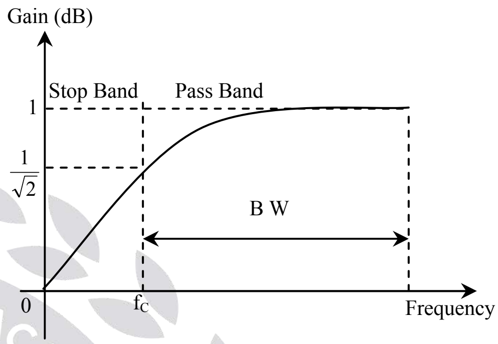
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{f}{f_L}\right)$$

$f = 0 \Rightarrow \phi = 0^0 = \phi_{min}$

$f = f_L \Rightarrow \phi = -45^0 = \phi_{max}$

14. Ans: (b)

Sol:



First order high pass filter = $\dfrac{s}{1+sT}$

Phase shift = 90 –tanωT

Max. phase shift is at corner frequency

$\omega = \dfrac{1}{T}$

Max. phase shift = $90 - \tan^{-1}\omega T$

$= 90 - \tan^{-1}\left(\dfrac{1}{T}\times T\right)$

= 90 – 45

= 45°

15. Ans: (d)

16. Ans: (a)

Sol: Half power of series RC circuit is at t = T (Time constant)

T = RC

Frequency = $\dfrac{1}{RC}$

17. Ans: (c)

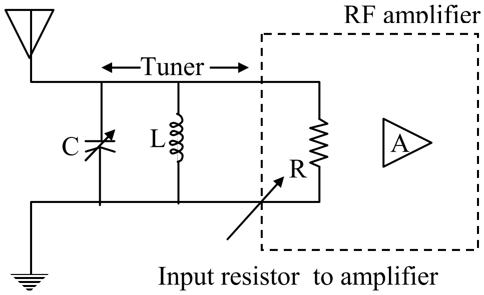
Sol: Magnitude of voltage gain 0.707 is at half power frequency

$\omega = \dfrac{1}{RC}$

Conventional Practice Solutions

01.

Sol:



The frequency range for AM broadcasting is 540 to 1600 kHz. We consider the low and high ends of the band. Since the resonant circuit in figure is a parallel type.

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \qquad C = \frac{1}{\omega_0^2 L}$$

For Low end of the AM band f_0 = 40 kHz and the corresponding C is

$$C_1 = \frac{1}{4\pi^2 \times 540^2 \times 10^6 \times 10^{-6}} = 86.9\text{nF}$$

For High end of the AM band f_0 = 1600 kHz corresponding C value is

$$C_2 = \frac{1}{4\pi^2 \times 1600^2 \times 10^6 \times 10^{-6}} = 9.9\text{nF}$$

Thus C must be adjustable in the range (9.9 nF to 86.9 nF)

02.

Sol: RLC circuit band pass filter with R = 10Ω, L = 25mH, & C =0.4μF

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{25\text{m}}{0.4\mu}} = \frac{1}{10}\sqrt{\frac{25\times10^{-3}}{4\times10^{-7}}}$$

$$Q_0 = \frac{500}{20} = 25$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{(25\times10^{-3})(0.4\times10^{-6})}}$$

$$= 10\text{krad/sec}$$

$$\omega_2 = \omega_0 + \frac{1}{2}(\text{band width})$$

$$\omega_1 = \omega_0 - \frac{1}{2}(\text{band width})$$

$$\text{B.W} = \frac{\omega_0}{Q} = \frac{10}{25}\text{k}$$

$$= \frac{10000}{25} = 400 \text{ rad/sec}$$

ω_1 = 10 – 0.2 = 9.8 krad/sec

$$f_1 = \frac{9.8}{2\pi} = 1.56 \text{ kHz}$$

ω_2 = 10 + 0.2 = 10.2 krad/sec

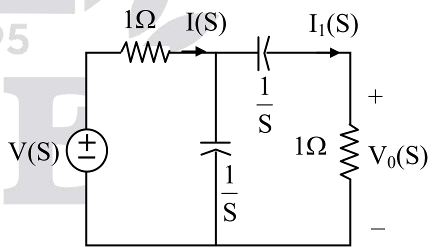
$$f_2 = \frac{10.2}{2\pi} = 1.62\text{kHz}$$

Frequency range is

1.56 kHz < f <1.62 kHz

03.

Sol:



$$Z(S) = \left(\left(1+\frac{1}{S}\right)\Big\|\frac{1}{S}\right) + 1$$

$$= \frac{\left(\frac{S+1}{S}\right)\frac{1}{S}}{\frac{1}{S}+\frac{1+S}{S}} + 1$$

$$= \frac{(S+1)}{S(S+2)} + 1 = \frac{(S+1)+S^2+2S}{S(S+2)}$$

$$Z(S) = \frac{S^2+3S+1}{S(S+2)}$$

$$I = \frac{V(S)}{Z(S)}$$

$$I_1 = \frac{\left(\frac{V(S)}{Z(S)}\right)\left(\frac{1}{S}\right)}{\frac{1}{S}+\frac{1}{S}+1}$$

$$I_1 = \frac{(V(S))}{Z(S)(S+2)}$$

$$V_0(S) = I_1(S) = \frac{V(S)}{\frac{(S^2+3S+1)}{5(S+2)}(S+2)} = \frac{V(S)S}{(S^2+3S+1)}$$

$$H(S) = \frac{V_0}{V_s} = \frac{S}{(S^2+3S+1)}$$

$2\delta\omega_0 = 3$

$$2\delta = 3 \Rightarrow \delta = \frac{3}{2}$$

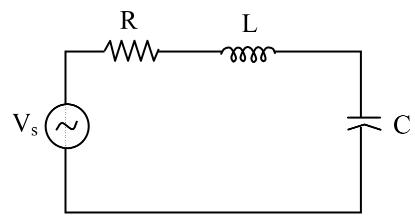
$\omega_0^2 = 1 \Rightarrow \omega_0 = 1$ rad/sec

$$Q = \frac{1}{2\delta} = \frac{1}{2\left(\frac{3}{2}\right)} \Rightarrow Q = \frac{1}{3}$$

$$\text{Bandwidth} = \frac{\omega_0}{Q} = \frac{1}{1/3} = 3\,\text{rad/sec}$$

04.

Sol: The circuit parameters for series R-L-C band stop filter are R = 2K, L = 0.1H, C = 40PF



(a) ω_0 = centre frequency

$$= \frac{1}{\sqrt{LC}} = 0.5\text{Mrad/sec}$$

(b) $$\text{bandwidth} = \frac{R}{L} = \frac{2\times10^3}{0.1} = 2\times10^4$$

Quality factor $Q = \frac{\omega_0}{B} = 25$

As Q > 10,

$$\omega_1 = \omega_0 - \frac{1}{2}\text{B.W} = 5\times10^5 - \frac{1}{2}(2\times10^4)$$

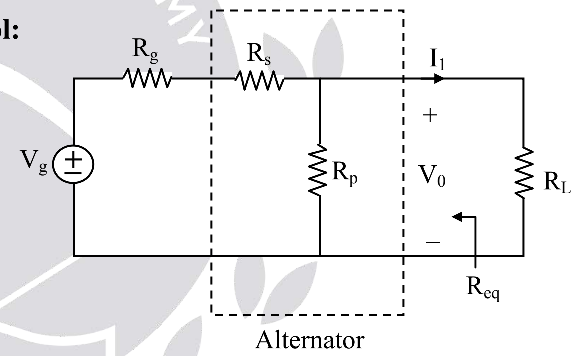
$= (50-1)\times10^4 = 490$ k rad/sec

$$\omega_2 = \omega_0 + \frac{1}{2}\text{B.W} = 510\,\text{rad/sec}$$

(c) Q = 25

05.

Sol:



$$V_0 = \frac{V_g R_p}{(R_g+R_s+R_p)}$$

$$\Rightarrow \alpha = \frac{V_0}{V_g} = \frac{R_P}{R_g+R_s+R_p}$$

$$= \left(R_g+R_s+R_p = \frac{R_p}{\alpha}\right)$$

$$R_{eq} = R_p \,||\, (R_s+R_g) = R_{eg} \;.......(1)$$

$$\frac{R_p(R_s+R_g)}{R_p+R_s+R_g} = R_g$$

$$R_g^2 + R_gR_p + R_gR_s = R_pR_s + R_pR_g$$

$$R_g^2 + R_g R_s - R_p R_s = 0$$

$R_g (R_g + R_s) = R_p R_s$

From (1) $R_g + R_s = R_p\left(\frac{1}{\alpha} - 1\right)$

$$R_g R_p\left(\frac{1-\alpha}{\alpha}\right) = R_s R_p$$

$$R_g = R_s\left(\frac{\alpha}{1\alpha}\right)$$

$$R_s = \frac{(1-\alpha)}{\alpha} R_g = \left(\frac{1-0.128}{0.128}\right)100$$

$= 700\Omega$

From (1)

$$R_g + R_s + R_p = \frac{R_p}{\alpha}$$

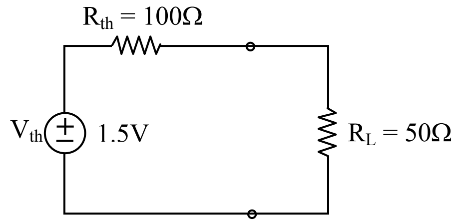
$$100 + 700 + R_p\left(\frac{1-\alpha}{\alpha}\right) = R_p\left(\frac{1-0.125}{0.125}\right)$$

$R_p = 114.29\Omega$

(b) $V_{th} = V_{oc} = V_0 = 0.125\ V_g$

$= (0.125\times12)$

$= 1.5V$



$$I = \frac{1.5}{100+50} = \frac{1.5}{150} = \frac{150\times10^{-2}}{150}$$

I = 10 Ma

Chapter 9 Network Functions & Synthesis

Objective Practice Solutions

01. Ans: (c)

Sol: $F(s) = \dfrac{(s+2)}{(s+1)\,(s+3)}$

The given F(s) has pole-zero structure as P-Z-P-Z alternating on the negative real axis of the s-plane, with a pole nearest the origin at s = –1 and a zero at s = ∞ . This F(s) corresponds to RC impedance or RL admittance.

02. Ans: (b)

Sol: For RC and RL driving point functions, the poles and zeros should alternate on the negative real axis, where as for LC driving point functions the poles and zeros should alternate the imaginary axis.

03. Ans: (c)

04. Ans: (b)

Sol: Remember that parallel LC networks in cascade is Foster -I form and series LC networks in shunt is Foster -II form. Ladder NW with series elements as inductors and shunt elements as capacitors is Cauer-I form and the ladder NW with capacitors as series elements and inductors as shunt elements is Cauer - I form. The given circuit in this question is Foster-I form.

05. Ans: (c)

Sol: Given: $Z(s) = \dfrac{s\left(s^2+1\right)}{s^2+4}$

Location of Poles : s = ±j2

Location of Zeros : s = 0, ±j1

Poles and Zeros are simple and lie on the imaginary axis, but they do not alternate.

Hence the given Z(s) is not realizable.

06. Ans: (b)

Sol: Poles and zeros of driving point function [Z(s) or Y(s)] of LC network are simple and alternate on the jω axis.

07. Ans: (c)

Sol: V = I Z(s)

$$V = I\sqrt{\frac{\omega^2+\alpha^2}{\omega^2+\beta^2}}\angle \tan^{-1}\left(\frac{\omega}{\alpha}\right) - \tan^{-1}\left(\frac{\omega}{\beta}\right)$$

voltage load the current

$$\tan^{-1}\left(\frac{\omega}{\beta}\right) < \tan^{-1}\left(\frac{\omega}{\alpha}\right) < \frac{\omega}{\alpha}(\alpha<\beta)(\beta>\alpha)$$

08. Ans: (d)

09. Ans: (b)

Sol: $s = -1 \pm j$

$(s+1)\,((s+1)+j)\,((s+1)-j)$

$(s+1)^2 + (1)^2 = s^2 + 2s + 2$

$$Z(s) = \frac{K(s+3)}{s^2+2s+2}$$

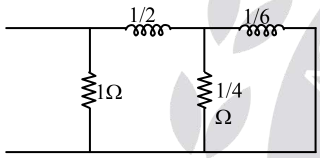
$$Z(0) = \frac{K(3)}{2} = 3 \Rightarrow K = 2$$

$$\therefore Z(s) = \frac{2(s+3)}{s^2+2s+2}$$

10. Ans: (d)

Sol:

$$s^2+2s)\,s^2+4s+3\,(1=\frac{1}{R}$$
$$\underline{s^2+2s}$$
$$2s+3)\,s^2+2s\,(\frac{s}{2}=sL$$
$$\underline{s^2+\frac{3s}{2}}$$
$$\frac{s}{2})\,2s+3\,(4=\frac{1}{R}$$
$$\underline{2s+}$$
$$3)\,\frac{s}{2}\,(\frac{s}{6}=sL$$
$$\underline{\frac{s}{2}}$$
$$0$$



$$y(s)=\frac{s^2+4s+3}{s^2+2s}$$

No. of elements = 4

11. Ans: (b)

Sol:

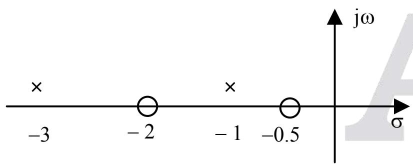


Fig.

Given $Y(s) = \frac{s^2+2.5s+1}{s^2+4s+3}$

$$Y(s) = \frac{(s+0.5)\,(s+2)}{(s+1)\,(s+3)}$$

Its pole-zero pattern is shown in Fig.

From the pattern it can be observed that

→ Poles and zeros alternate on the negative real axis of s-plane.

→ The lowest critical frequency is a zero.

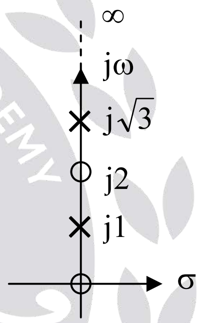
→ From the given Y(s), Y(0) = 1/3 and Y(∞) = 1, Y(0) < Y(∞), Y(σ) has +ve slope.

It is an admittance of the RC network, as the above properties are true for RC admittance.

12. Ans: (b)

13. Ans: (a)

Sol:



$F(s) = \frac{s(s^2+4)}{(s^2+1)(s^2+6)}$ represents an LC immittance function with pole-zero pattern as shown in Fig. Hence it is p.r.

$F(s) = \frac{s(s^2-4)}{(s^2+1)(s^2+6)}$ is not p.r as it has a zero in the RH at s = 2

$F(s) = \frac{s^3+3s^2+2s+1}{4s}$ is not p.r

as the difference in degrees of highest degree terms in N(s) and D(s) is more than 1. For this F(s), difference is 2.

$$F(s) = \frac{s(s^4+3s^2+1)}{(s+1)\,(s+2)\,(s+3)\,(s+4)}$$

14. Ans: (a)

Sol: Given $Z(s) = \dfrac{(s^2+4)(s^2+16)}{s(s^2+9)}$

Out of the given figs., Foster – I form should be either (1) or (4) and Foster –II form should be either (2) or (3). Foster–I form can be confirmed as Fig. 1 by seeing the behavior of Z(s) at s = ∞ and s = 0.

Z(s) = 1 at s = ∞, L = 1 H

$Z(s) = \dfrac{64}{9s}$ at s = ∞, $C = \dfrac{9}{64}$F

Foster – II form can be confirmed as fig. (3) as

$L = \dfrac{12}{7} \,||\, \dfrac{12}{5} = 1\text{H}$, at s = ∞

and $C = \dfrac{7}{192} + \dfrac{5}{48} = \dfrac{9}{64}$F at s = 0.

The exact realization can be done as shown below. Foster–I form is obtained by expanding the given Z(s) in partial fractions.

$$Z(s) = k_1 s + \frac{k_2}{s} + \frac{k_3 s}{s^2+9}$$

$$= 1s + \frac{64}{9s} + \frac{35}{9}\frac{s}{s^2+9} \quad \ldots\ldots\ldots\ldots..(1)$$

As $k_1 = \lim\limits_{s\to\infty} \dfrac{Z(s)}{s} = 1$

$$k_2 = s\,Z(s)\big|_{s=0} = \frac{64}{9}$$

$$k_3 = \frac{(s^2+9)}{s} Z(s)\bigg|_{s^2=-9}$$

$$= \frac{(-9+4)(-9+16)}{-9} = \frac{35}{9}$$

It can be seen from equation (1), the first Foster form corresponds to Fig. I (not Fig. IV) Foster - II form is obtained by taking partial fractions of

$$Y(s) = \frac{s(s^2+9)}{(s^2+4)(s^2+16)}$$

$$= \frac{k_1 s}{(s^2+4)} + \frac{k_2 s}{(s^2+16)} = Y_1(s) + Y_2(s)$$

$$k_1 = \frac{(s^2+4)}{s} Y(s)\bigg|_{s^2=-4} = \frac{-4+9}{-4+16} = \frac{5}{12}$$

$$k_2 = \frac{(s^2+16)}{s} Y(s)\bigg|_{s^2=-16} = \frac{-16+9}{-16+4} = \frac{7}{12}$$

$$Y_1(s) = \frac{\frac{5}{12}s}{s^2+4}$$

$$= \frac{1}{\frac{12}{5}s + \frac{48}{5s}} = \frac{1}{Ls + \frac{1}{Cs}}$$

$$L = \frac{12}{5}\text{ H},\ C = \frac{5}{48}\text{ F}$$

∴ It can be seen that Foster – II form corresponds to Fig. III (not Fig. II)It is instructive to find out the remaining elements in Fig. I and III.

15. Ans: (a)

16. Ans: (d)

Sol: Given:

$$Z_D(s) = \frac{2(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$= \frac{2s^4+8s^2+6}{s^3+2s}$$

Out of the figs. given (d) is in the form of Cauer-I network and (a) is in the form of Cauer-II. The Cauer network can be confirmed as (d) by seeing the behaviour of

Z(s) at s = ∞ and at s = 0

Z(s) = 2, at s = ∞, giving L = 2 H

$Z(s) = \dfrac{3}{s}$, at $s \to \infty$, giving

$$C = \frac{1}{3}F = \left(\frac{1}{4}+\frac{1}{12}\right)F$$

Exact realizations of Cauer – I and Cauer – II forms can be obtained as shown below:

Cauer – I Network is obtained by successive removal of poles at s = ∞. As the given $Z_D(s)$ has a pole at s = ∞, removal of it gives the first element as L=2H. Follow the Continued Fraction (CF) expansion given below, which confirms to the Network in (d).

$$s^3+2s\,\big]\,2s^4+8s^2+6\,\big[\,2s,$$
$$\underline{2s^4+4s^2}$$
$$4s^2+6\,\big]\,s^3+2s\,\big[\,\frac{1}{4}s,$$
$$\underline{s^3+\frac{3}{2}s}$$
$$\frac{s}{2}\,\big]\,4s^2+6\,\big[\,8s,$$
$$\underline{4s^2}$$
$$6\,\big]\,\frac{s}{2}\,\big[\,\frac{s}{12},$$
$$\underline{\frac{s}{2}}$$
$$\underline{0}$$

Quotient values

$$L = 2H,\ C = \frac{1}{4}F,\ L = 8H,\ C = \frac{1}{12}F$$

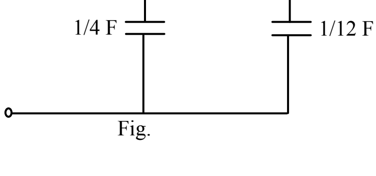


Fig.

Cauer - II NW is obtained by successive removal of poles at s = 0.

$Z_D(s)$ also has a pole at s = 0, removal of it gives the first element as $C = \dfrac{1}{3}$ F.

Follow the CF expansion below.

$$2s+s^3\,\big]\,6+8s^2+2s^4\,\big[\,\frac{3}{s},\ C = \frac{1}{3}F$$
$$\underline{6+3s^2}$$
$$5s^2+2s^4\,\big]\,2s+s^3\,\big[\,\frac{2}{5s},\ L = \frac{5}{2}H$$
$$\underline{2s+\frac{4}{5}s^3}$$
$$\frac{1}{5}s^3\,\big]\,5s^2+2s^4\,\big[\,\frac{25}{s},\ C = \frac{1}{25}F$$
$$\underline{5s^2}$$
$$2s^4\,\big]\,\frac{1}{5}s^3\,\big[\,\frac{1}{10s},\ L = 10H$$
$$\underline{\frac{1}{5}s^3}$$
$$\underline{0}$$

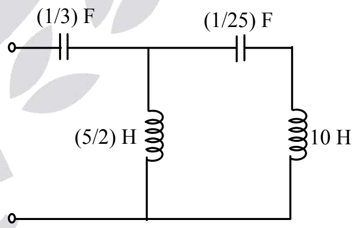


Fig.

So the answer must be the Cauer - I NW in (d). It is instructive to find the Cauer - I and Cauer–II structures by completing the CF expansions above.

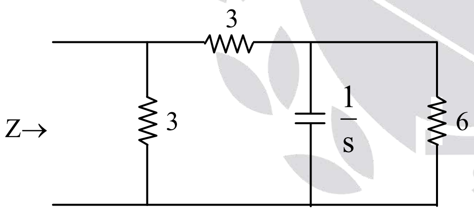
17. Ans: (c)

Sol:

F(s)		Type of F(s)
A. $\frac{(s^2-s+4)}{s^2+s+4}$	zeros in the right half plane	Non-minimum phase (2)
B. $\frac{(s+4)}{s^2+3s-4}$	poles in the right half plane	Unstable (4)
C. $\frac{s+4}{s^2+6s+5}$	Poles and zeros alternate on the negative real axis with first critical frequency near the origin as a pole.	RC impedance (3)
D. $\frac{s^3+3s}{s^4+2s^2+1}$	multiple poles on the imaginary axis	Non-positive real (1)

18. Ans: (c)

Sol:



$$=\frac{\frac{6}{s}}{6+\frac{1}{s}}+3=3+\frac{\frac{6}{s}}{\frac{6s+1}{s}}=\frac{6}{6s+1}+3$$

$$=\frac{18s+3+6}{6s+1}=\frac{9+18s}{6s+1}$$

$$=\frac{3\left(\frac{18s+9}{6s+1}\right)}{3+\frac{18s+9}{6s+1}}=\frac{\frac{3(18s+9)}{6s+1}}{\frac{18s+3+18s+9}{6s+1}}$$

$$=\frac{3\times18\left(s+\frac{1}{2}\right)}{36\left(s+\frac{1}{3}\right)}=\frac{\left(s+\frac{1}{2}\right)}{s+\frac{1}{3}}$$

19. Ans: (b)

Sol: $p(s) = s^4 + s^3 +2s^2 + 4s + 3$

$$y(s)=\frac{\text{even part}}{\text{odd part}}=\frac{s^4+2s^2+3}{s^3+4s}$$

$$s^3+4s)s^4+2s^2+3(s$$
$$\underline{s^4+4s^2}$$
$$-2s^2+3)s^3+4s(-\frac{s}{2}\Rightarrow -\text{ve quotients}$$
$$\underline{s^3+\frac{3s}{2}}$$

p(s) is not Hurwitz

$Q(s) = s^5 + 3s^2 +s$ missing terms

Q(s) is not Hurwitz

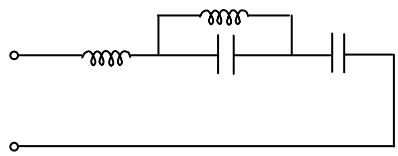
20. Ans: (a)

21. Ans: (d)

22. Ans: (b)

Sol: Foster – I form consists of LC tank circuits in series to realize $Z_{LC}(s)$.
This form is obtained by taking partial fractions of Z(s).

$$Z(s) = 4\left[1s+\frac{A}{s}+\frac{Bs}{s^2+4}\right]$$



n = 1 with an inductance and capacitance in series.

23. Ans: (a)

Sol: Assertion given is the necessary condition for Y(s) to be positive real because the definition of positive real function includes the statement that Y(s) is real for real s.

24. Ans: (d)

Sol: The function $10\dfrac{(s^2+1)(s^2+9)}{s(s^2+4)}$ is a valid reactance function as poles and zeros alternate on the jω-axis.

Statement (I) is false, statement (II) is true.

25. Ans: (c)

Sol: The existence of two poles or two zeroes in successive on the real frequency axis of the s-plane requires that the slope be negative over part of the frequency range. So the slope of reactance curve may be negative.

∴ Statement (II) is false.

26. Ans: (a)

Sol: The poles and zeros of driving point function should be in the left half of the s-plane. A is True.

Only PR function can be realized as the driving point function of a network and PR function has its poles and zeros in the left half of the s-plane. R is True and is the correct explanation of A.

27. Ans: (c)

Sol: For a system to be stable, all coefficients of the characteristic polynomial must be positive. This is a necessary condition for stability, but not a sufficient condition.

A is true, R is false.

28. Ans: (a)

Sol: $Z(s) = \dfrac{k\,(s^2+1)(s^2+5)}{(s^2+2)(s^2+10)}$

For Z(s) to be an LC function, the highest powers of numerator and denominator should differ by 1. For the given Z(s), the highest powers of numerator and denominator are not differing by one. They are same equal to 2.

29. Ans: (a)

Sol: $Q \propto \dfrac{1}{\xi}$

For circuits with high Q, ξ is less. If damping is less, the real part of the poles are close to the jω-axis in the left-half plane.

30. Ans: (a)

Sol:

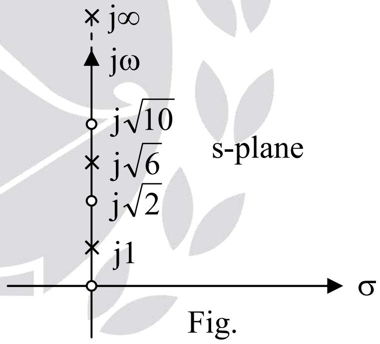


Fig.

Given: $Z(s) = \dfrac{Ks(s^2+2)(s^2+10)}{(s^2+1)(s^2+6)}$

It represents an LC driving point impedance function because it satisfies the property: Poles and zeros interlace on the imaginary axis of the complex s – plane as shown in Fig.

31. Ans: (b)

Conventional Practice Solutions

01.

Sol: The realizable function used for driving point synthesis is known as Positive Real (P.R) function.

PR function:

Positive real function, F(s) is defined as function satisfying the following two requirements:

Re F(s) $\geq$ 0 for Re s $\geq$ 0

or |Arg F(s)| $\leq$ |Arg s| for |Arg s| $\leq$ (π/2)

and F(s) is real when 's' is real.

It is easier to test the P-R character of a function, F(s) by means of the following equivalent necessary and sufficient conditions:

a) Y(s) must be real when s is real.

b) If Y(s) = p(s)/q(s), then p(s) + q(s) must be Hurwitz.
   This requires that:
   i) The continued fraction expansion of the Hurwitz test give only real and positive α's, and
   ii) The continued fraction not end prematurely.

c) In order that Re Y(jω) $\geq$ 0 for all ω, it is necessary and sufficient that $A(\omega^2) = m_1 m_2 - n_1 n_2 \big|_{s=j\omega}$ have no real positive roots of odd multiplicity. This may be determined by factoring $A(\omega^2)$ or by the use of Sturm's theorem.

02.

Sol: i) The pole-zero plot of impedance is shown in Fig.

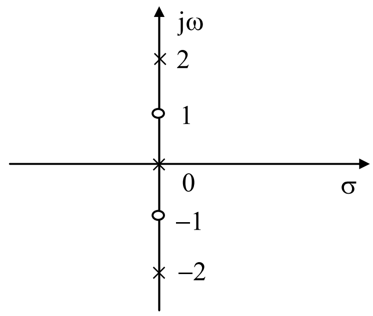


Fig.

$$Z(s) = \frac{(s^2+1)}{s\,(s^2+4)} = \frac{(s^2+1)}{s^3+4s}$$

Foster – I form:

$$Z(s) = \frac{k_1}{s} + \frac{k_2 s}{s^2+4}$$

$$k_1 = s\,Z(s)\big|_{s\to 0} = \frac{1}{4}$$

$$k_2 = \frac{s^2+4}{s}\,Z(s)\Big|_{s^2=-4} = \frac{-4+1}{-4} = \frac{3}{4}$$

$$Z(s) = \frac{1}{4s} + \frac{3s}{4(s^2+4)} = \frac{1}{4s} + \frac{1}{\frac{4}{3}s + \frac{16}{3s}}$$

The realization is shown in Fig. 1.

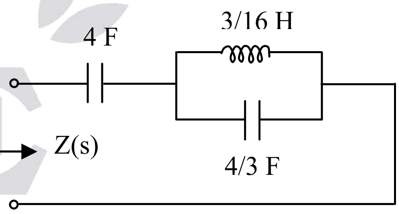


Fig. 1

\

Foster – II form:

$$Y(s) = \frac{s(s^2+4)}{(s^2+1)} = k_1 s + \frac{k_2 s}{s^2+1}$$

$$k_1 = \frac{Y(s)}{s}\Big|_{s\to\infty} = 1$$

$$k_2 = \frac{s^2+1}{s}\,Y(s)\Big|_{s^2=-1} = (-1+4) = 3$$

$$Y(s) = 1s + \frac{3s}{(s^2+1)} = 1s + \cfrac{1}{\frac{1}{3}s + \frac{1}{3s}}$$

The realization is shown in Fig.2.

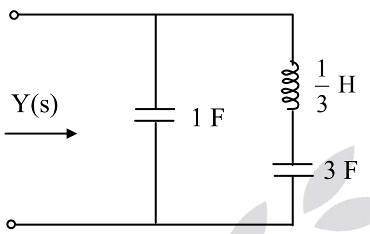


Fig. 2

Cauer - I form:

Start the continued fraction expansion from Y(s)

$$Y(s) = \frac{s^3+4s}{s^2+1}$$

$$\begin{array}{l}
s^2+1\,\big]\,s^3+4s\,\big[\,1s \\
\quad\;\; \underline{s^3+1s} \\
\qquad\qquad 3s\,\big]\,s^2+1\,\big[\,(1/3)s \\
\qquad\qquad\quad\;\; \underline{s^2} \\
\qquad\qquad\qquad\quad 1\,\big]\,3s\,\big[\,3s \\
\qquad\qquad\qquad\qquad \underline{3s} \\
\qquad\qquad\qquad\qquad\;\; 0
\end{array}$$

The realization is shown in Fig. 3.

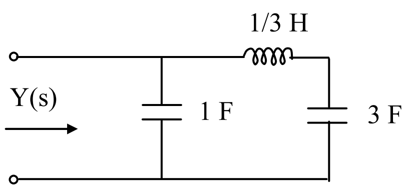


Fig. 3

Cauer-II form:

$$Z(s) = \frac{1+s^2}{4s+s^3}$$

$$\begin{array}{l}
4s+s^3\,\big]\,1+s^2\,\big[\,(1/4s) \\
\quad \underline{1+\frac{1}{4}s^2} \\
\qquad\quad \frac{3}{4}s^2\,\big]\,4s+s^3\,\big[\,(16/3s) \\
\qquad\qquad \underline{4s} \\
\qquad\qquad\qquad s^3\,\big]\,\frac{3}{4}s^2\,\big[\,(3/4s) \\
\qquad\qquad\qquad\quad \underline{\frac{3}{4}s^2} \\
\qquad\qquad\qquad\qquad 0
\end{array}$$

The realization is shown in Fig. 4

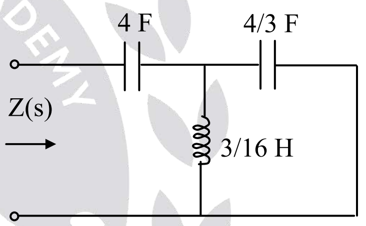


Fig. 4

ii) The pole-zero plot of impedance is shown in Fig. 1

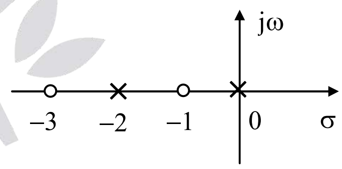


Fig.1

This represents an RC NW.

$$Z(s) = \frac{(s+1)\,(s+3)}{s\,(s+2)} = \frac{s^2+4s+3}{s^2+2s}$$

Foster - I form:

Taking partial fractions of Z(s):

$$Z(s) = k_1 + \frac{k_2}{s+2} + \frac{k_3}{s}$$

$$k_1 = Z(s)\big|_{s\to\infty} = 1$$

$$k_2 = (s+2)\,Z(s)\big|_{s=-2}$$

$$= \frac{(-2+1)\,(-2+3)}{-2} = \frac{1}{2}$$

$$k_3 = s\,Z(s)\big|_{s\to 0} = \frac{3}{2}$$

$$Z(s) = 1 + \frac{1}{2(s+2)} + \frac{3}{2s}$$

$$= 1 + \frac{1}{2s+4} + \frac{3}{2s}$$

The realization is shown in Fig. 2

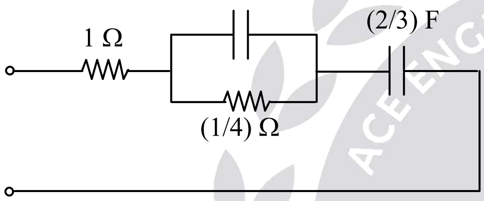


Fig. 2

Foster – II form:

This is obtained from the partial fractions of not Y(s), but $\frac{Y(s)}{s}$.

$$\frac{Y(s)}{s} = \frac{(s+2)}{(s+1)\,(s+3)}$$

$$= \frac{k_1}{s+1} + \frac{k_2}{s+3}$$

$$k_1 = (s+1)\,\frac{Y(s)}{s}\bigg|_{s=-1} = \frac{-1+2}{-1+3} = \frac{1}{2}$$

$$k_2 = (s+3)\,\frac{Y(s)}{s}\bigg|_{s=-3} = \frac{-3+2}{-3+1} = \frac{1}{2}$$

$$\frac{Y(s)}{s} = \frac{1}{2(s+1)} + \frac{1}{2(s+3)}$$

$$\therefore Y(s) = \frac{s}{2s+2} + \frac{s}{2s+6} = \frac{1}{2+\frac{2}{s}} + \frac{1}{2+\frac{6}{s}}$$

The realization is shown in Fig.3.

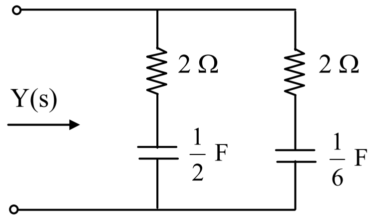


Fig. 3

Cauer – I form:

This is obtained from continued fraction expansion of Z(s).

$$\begin{array}{l}
s^2+2s\,\big]\,s^2+4s+3\,\big[\,1 \\
\qquad\quad \underline{s^2+2s} \\
\qquad\qquad 2s+3\,\big]\,s^2+2s\,\big[\,(1/2)s \\
\qquad\qquad\qquad \underline{s^2+\frac{3}{2}s} \\
\qquad\qquad\qquad\quad \frac{1}{2}s\,\big]\,2s+3\,\big[\,4 \\
\qquad\qquad\qquad\qquad \underline{2s} \\
\qquad\qquad\qquad\qquad\quad 3\,\big]\,\frac{1}{2}s\,\big[\,(1/6)s \\
\qquad\qquad\qquad\qquad\qquad \underline{\frac{1}{2}s} \\
\qquad\qquad\qquad\qquad\qquad\;\; \underline{0}
\end{array}$$

The realization is shown in Fig.4.

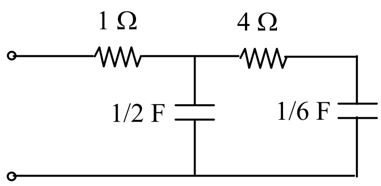


Fig. 4

Cauer – II form:

$$Z(s) = \frac{3+4s+s^2}{2s+s^2}$$

$$2s+s^2\,\big]\,3+4s+s^2\,\big[\,(3/2s)$$
$$\underline{3+\tfrac{3}{2}s}$$
$$\tfrac{5}{2}s+s^2\,\big]\,2s+s^2\,\big[\,(4/5)$$
$$\underline{2s+\tfrac{4}{5}s^2}$$
$$\tfrac{1}{5}s^2\,\big]\,\tfrac{5}{2}s+s^2\,\big[\,(25/2s)$$
$$\underline{\tfrac{5}{2}s}$$
$$s^2\,\big]\,\tfrac{1}{5}s^2\,\big[\,(1/5)$$
$$\underline{\tfrac{1}{5}s^2}$$
$$\underline{0}$$

The realization is shown in Fig. 5.

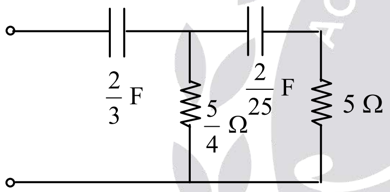


Fig. 5

03.

Sol: $Z_D(s) = \dfrac{s^2+12s+35}{s^3+15s^2+62s+48}$

Factoring the numerator and denominator,

$$Z_D(s) = \frac{(s+5)\,(s+7)}{(s+1)\,(s+6)\,(s+8)}$$

This is an RC impedance function.

Obtain partial fractions of $Z_D(s)$ to get Foster-I form

$$Z_D(s) = \frac{k_1}{s+1} + \frac{k_2}{s+6} + \frac{k_3}{s+8}$$

$$k_1 = \frac{(-1+5)\,(-1+7)}{(-1+6)\,(-1+8)} = \frac{4\times 6}{5\times 7} = \frac{24}{35}$$

$$k_2 = \frac{(-6+5)\,(-6+7)}{(-6+1)\,(-6+8)} = \frac{(-1)\,(1)}{(-5)\,(2)} = \frac{1}{10}$$

$$k_3 = \frac{(-8+5)\,(-8+7)}{(-8+1)\,(-8+6)} = \frac{(-3)\,(-1)}{(-7)\,(-2)} = \frac{3}{14}$$

$$Z_D(s) = \frac{(24/35)}{s+1} + \frac{(1/10)}{s+6} + \frac{(3/14)}{s+8}$$

$$= \frac{1}{\frac{35}{24}s + \frac{35}{24}} + \frac{1}{10s+60} + \frac{1}{\frac{14}{3}s + \frac{112}{3}}$$

Identifying the elements from the denominators, Foster – I form is shown in Fig. 1.

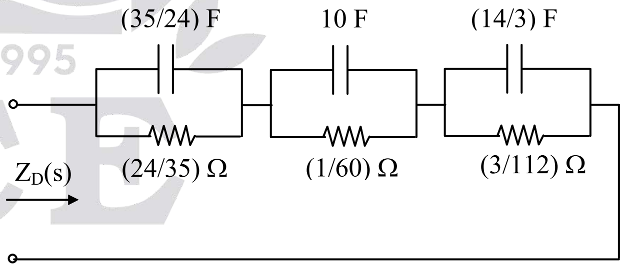


Fig. 1

Cauer – I form is obtained from the Continued fraction expansion starting from $Y_D(s)$, which has pole at $s = \infty$.

$$Y_D(s) = \frac{s^3+15s^2+62s+48}{s^2+12s+35}$$

$$s^2+12s+35\,\big]\,s^3+15s^2+62s+48\,\big[\,1s$$
$$\underline{s^3+12s^2+35s}$$
$$3s^2+27s+48\,\big]\,s^2+12s+35\,\big[\,(1/3)$$
$$\underline{s^2+\ 9s+16}$$
$$3s+19\,\big]\,3s^2+27s+48\,\big[\,1s$$
$$\underline{3s^2+19s}$$
$$8s+48\,\big]\,3s+19\,\big[\,(3/8)$$
$$\underline{3s+18}$$
$$1\,\big]\,8s+48\,\big[\,8s$$
$$\underline{8s}$$
$$48\,\big]\,1\,\big[\,(1/48)$$
$$\underline{1}$$
$$\underline{0}$$

Identifying the elements from the quotients, Cauer – I form is obtained as shown in Fig.2.

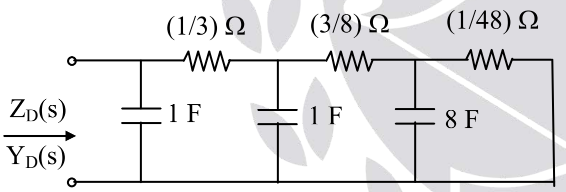


Fig. 2

04.

Sol: PR Function:

Positive real function, F(s) is defined as function satisfying the following two requirements:

Re F(s) ≥ 0 for Re s ≥ 0

or $|\text{Arg } F(s)| \leq |\text{Arg } s|$ for $|\text{Arg } s| \leq (\pi/2)$

and F(s) is real when 's' is real.

It is easier to test the P-R character of a function, F(s) by means of the following equivalent necessary and sufficient conditions:

(a) Y(s) must be real when s is real.

(b) If Y(s) = p(s)/q(s), then p(s) + q(s) must be Hurwitz.

This requires that:

i) the continued fraction expansion of the Hurwitz test give only real and positive α's, and

ii) the continued fraction not end prematurely.

(c) In order that Re Y(jω) $\geq$ 0 for all ω, it is necessary and sufficient that

$A(\omega^2) = m_1 m_2 - n_1 n_2\big|_{s=j\omega}$ have no real positive roots of odd multiplicity. This may be determined by factoring $A(\omega^2)$ or by the use of Sturm's theorem.

It can be shown that the function,

$$F(s) = \frac{s^2+\frac{3}{4}s+\frac{3}{4}}{s^2+s+4}$$ is not P-R as it does not satisfy requirement (c) above.

$A(\omega^2) = (\omega^2 - 3)\,(\omega^2 - 1)$ is negative

for $1 < \omega < \sqrt{3}$

The given circuit is shown in Fig.1

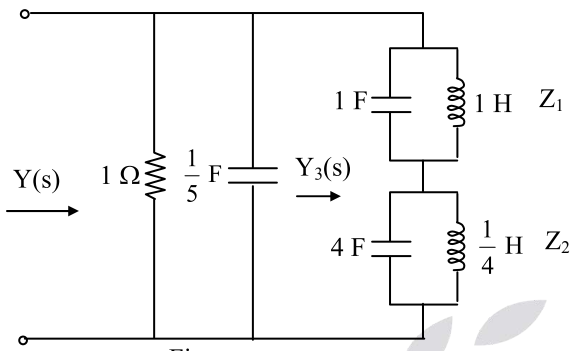


Fig.

First, Y(s) is obtained:
then its pole-zero diagram can be drawn easily.
Next, its Canonic realization (with minimum number of elements) can be obtained.

$$Y_1(s) = \frac{1}{Z_1(s)} = 1s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$Z_1(s) = \frac{s}{s^2+1}$$

$$Y_2(s) = \frac{1}{Z_2(s)} = 4s + \frac{4}{s} = 4\,\frac{(s^2+1)}{s}$$

$$Z_2(s) = \frac{s}{4(s^2+1)}$$

$$Z_3(s) = Z_1(s) + Z_2(s) = \frac{5}{4}\,\frac{s}{s^2+1}$$

$$Y_3(s) = \frac{4(s^2+1)}{5s}$$

$$\therefore\ Y(s) = 1 + \frac{1}{5}s + \frac{4(s^2+1)}{5s}$$

$$= 1 + \frac{1}{5}\,\frac{5s^2+4}{s} = 1 + \frac{s^2+0.8}{s}$$

$$= 1 + s + \frac{0.8}{s} = \frac{1}{R} + Cs + \frac{1}{Ls} \quad \text{.......(I)}$$

$$= \frac{s^2+s+0.8}{s} \quad \text{.........(II)}$$

From eqn. (I), the Canonic realization is obtained as parallel RLC network shown in Fig. 2.

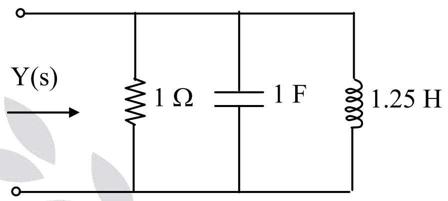


Fig. 2

Considering eqn.(II), Zeros of Y(s) are obtained from $s^2 + s + 0.8 = 0$

$$s = \frac{-1 \pm \sqrt{1-3.2}}{2}$$

$$= \frac{-1 \pm j\sqrt{2.2}}{2}$$

$$= -0.5 \pm j0.74$$

Poles of Y(s) are at $s = 0$ and $s = \infty$.

The pole-zero diagram is shown in Fig.3.

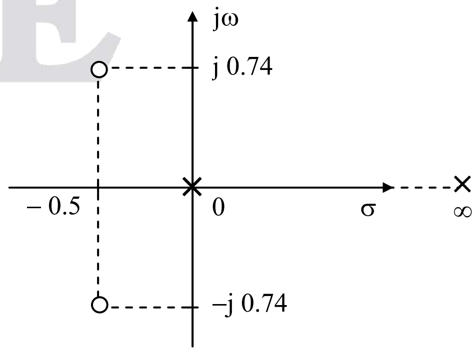


Fig.3

05.

Sol: $Z(s) = \dfrac{s(s^2+10)}{(s^2+4)(s^2+16)}$

This is clearly a L-C driving point impedance function.

Fosters – I: (i.e,) series impedance form obtained by partial fraction expansion.

$$Z(s) = \frac{s(s^2+10)}{(s^2+4)(s^2+16)}$$

The standard form of representing a L-C driving point impedance in fosters-I form is,

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\sigma_i^2} + k_\alpha s$$

But in the given function, There is no pole at origin and there is no pole at infinity.

So, doing partial fraction expansion

$$Z(s) = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+16}$$

Comparing coefficients

4A + 4C = 4 …….. (1)

16A + 4C = 10 …. .. (2)

By solving, A = $\frac{1}{2}$, B = $\frac{1}{2}$

B+D = 0 ……. (3)

16B + 4D = 0 ……… (4)

By solving, B = 0, D = 0

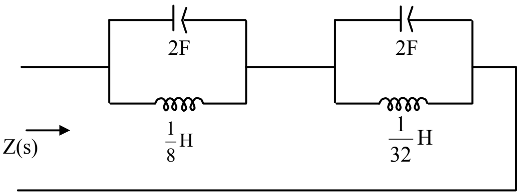
$$\text{So, } \frac{\frac{s}{2}}{s^2+4} + \frac{\frac{s}{2}}{s^2+16}$$

$$Z(s) = \frac{1}{\frac{s^2}{s/2} + \frac{4}{s/2}} + \frac{1}{\frac{s^2}{s/2} + \frac{16}{s/2}}$$

$$Z(s) = \frac{1}{2s + \frac{8}{s}} + \frac{1}{2s + \frac{32}{s}}$$

$$= \frac{1}{Y_1(s)} + \frac{1}{Y_2(s)}$$

So, Fasters – I form of realization is,



06.

Sol: Given $Z(s) = \dfrac{(s+1)\,(s+3)\,(s+5)}{s\,(s+2)\,(s+4)\,(s+6)}$

This impedance corresponds to RC NW

Foster - I form is realized by taking the partial fractions of Z(s).

$$Z(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{D}{s+6}$$

$$A = \frac{15}{48} = \frac{5}{16}$$

$$B = \frac{(-2+1)\,(-2+3)\,(-2+5)}{(-2)\,(-2+4)\,(-2+6)} = \frac{3}{16}$$

$$C = \frac{(-4+1)\,(-4+3)\,(-4+5)}{(-4)\,(-4+2)(-4+6)} = \frac{3}{16}$$

$$D = \frac{(-6+1)\,(-6+3)\,(-6+5)}{(-6)\,(-6+2)(-6+4)} = \frac{5}{16}$$

$$Z(s) = \frac{5}{16}\frac{1}{s} + \frac{3}{16}\frac{1}{(s+2)} + \frac{3}{16}\frac{1}{(s+4)} + \frac{5}{16}\frac{1}{(s+6)}$$

$$= \frac{1}{(16/5)\,s} + \frac{1}{(16/3)s+(32/3)} + \frac{1}{(16/3)s+(64/3)} + \frac{1}{(16/5)s+(96/5)}$$

The realization is shown in Fig. 1.

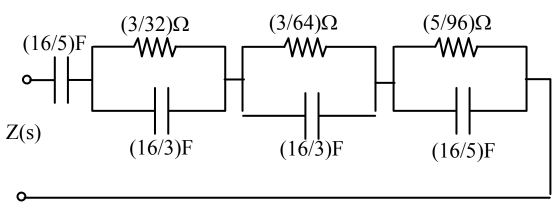


Fig. 1