ESE | GATE | PSUs

ELECTRONICS & TELECOMMUNICATION ENGINEERING

NETWORK THEORY

Text Book: Theory with worked out Examples and Practice Questions
01. Ans: (c)  
Sol: We know that;  
\[ i(t) = \frac{dq(t)}{dt} \]  
\[ dq(t) = i(t).dt \]  
\[ q = \int_{0}^{\text{5\mu sec}} i(t)dt \]  
Area under \( i(t) \) upto 5 \( \mu \)sec  
\[ q = q_1 + q_2 + q_3 \]  
\[ = \left( \frac{1}{2} \times 3 \times 3 \right) + \left( \frac{1}{2} \times 1 \times 2 + (1 \times 3) \right) + \left( \frac{1}{2} \times 1 \times 1 + (1 \times 3) \right) \]  
\[ q = 15 \mu \text{c} \]  

02. Ans: (a)  
Sol:  
\[ \text{Applying KCL at node 'b'} \]  
\[ I + 4 = 4 \]  
\[ \Rightarrow I = 0A \]  
And  
\[ \frac{8}{R} = 4 \]  
\[ \Rightarrow R = 2\Omega \]  

03. Ans: (a)  
Sol: The energy stored by the inductor (1\( \Omega \), 2H) upto first 6 sec:  
\[ E_{\text{stored upto 6 sec}} = \int P_L dt \]  
\[ = \int \left( L \frac{d}{dt}(i(t)) \right) dt \]  
\[ = \int \left[ \left( \frac{d}{dt}(3t) \right) \times 3t \right] dt + \int \left[ \left( \frac{d}{dt}(6) \right) \times 6 \right] dt \]  
\[ = \left[ \frac{1}{2} \left( \frac{d}{dt}(-3t+18) \right) \times (-3t+18) \right] dt \]  
\[ = \int_{0}^{6} 18t dt + \int_{0}^{6} 0 dt + \int_{0}^{6} \left( -6 \left[ -3t+18 \right] \right) dt \]  
\[ = 36 + 0 - 36 = 0 \text{ J} \]  
(or)  
\[ E_{\text{stored upto 6 sec}} = \frac{1}{2} L \left( i(t) \right|_{t=6}^6)^2 \]  
\[ = \frac{1}{2} \times 2 \times 0^2 = 0 \text{ J} \]  

04. Ans: (d)  
Sol: The energy absorbed by the inductor (1\( \Omega \), 2H) upto first 6 sec:  
\[ E_{\text{absorbed}} = E_{\text{dissipated}} + E_{\text{stored}} \]  
Energy is dissipated in the resistor  
\[ E_{\text{dissipated}} = \int P_R dt = \int (i(t))^2 R dt \]  
\[ = \int_{0}^{6} (3t)^2 \times 1 dt + \int_{0}^{6} (6)^2 \times 1 dt + \int_{0}^{6} (-3t+18)^2 \times 1 dt \]  
\[ = \int_{0}^{6} 9t^2 dt + \int_{0}^{6} 36 dt + \int_{0}^{6} (9t^2 + 324 - 108t) dt \]  
\[ = 24 + 72 + 24 \]  
\[ = 120 \text{ J} \]
05. Ans: (a)  
Sol: Point \((-20, 0) \Rightarrow V = -20V\) and \(I = 0A\)

By KVL \(\Rightarrow I_s R_s - V = 0\)
\(\Rightarrow I_s R_s + 20 = 0\)
\(\Rightarrow I_s R_s = -20V \quad \text{(1)}\)

Point: \((0, -2) \Rightarrow V = 0V\) and \(I = -2A\)

\(\Rightarrow I_s = -2A\)

Substituting \(I_s\) in eq (1)

\(R_s = 10\Omega\)

From the diagram;
\(I = -1A\) and \(V = -10V\)

06. Ans: (a)  
Sol:  
* linear
* Passive
* bilateral

07. Ans: (b)  
Sol:  
* Non linear
* Active
* Unilateral

08. Ans: (e)  
Sol:  
* Non linear
* Passive
* Unilateral

09. Ans: (c)  
Sol:  
* Linear
* Active
* Bilateral

10. Sol:  
(1) By KVL \(\Rightarrow +10 + 8 + E + 4 = 0\)
\(E = -22V\)
(2) By KVL \(\Rightarrow + V_1 - 2 + 4 = 0\)
\(V_1 = -2V\)
(3) By KVL \(\Rightarrow + V_2 + 6 - 8 - 10 = 0\)
\(V_2 = 12V\)
11. Ans: (d)
Sol:

Here the 2V voltage source and 3V voltage source are in parallel which violates the KVL. Hence such circuit does not exist. (But practical voltage sources will have some internal resistance so that when two unequal voltage sources are connected in parallel current can flow and such a circuit may exist).

12. Ans: (d)
Sol:

Applying KVL,
\[-V_1 + 12\left(I_{in} - \frac{V_1}{5}\right) + 2\left(I_{in} - \frac{16V_1}{5}\right) = 0\]
\[-V_1 + 12I_{in} - \frac{12V_1}{5} + 2I_{in} - \frac{32V_1}{5} = 0\]

\[14I_{in} = \frac{49}{5}V_1\]

\[\Rightarrow V_1 = \frac{70I_{in}}{49} \quad \text{......... (1)}\]

\[\therefore V_{out} = 2\left(I_{in} - \frac{16V_1}{5}\right) \quad \text{......... (2)}\]

Substitute equation (1) in equation (2)
\[V_{out} = 2\left(I_{in} - \frac{16\times\frac{70I_{in}}{49}}{5}\right)\]
Applying KVL,
\[ V + 1.5I + 2I = 0 \]
\[ V = -3.5I \]

15. Ans: (c)
Sol:

\[ \frac{V_x + 15}{8} - 2V_x = 0 \Rightarrow V_x = 4V \]

By using nodal Analysis at \( V_z \) node
\[ \frac{V_x + 15}{18} - 2 = 0 \Rightarrow V_x = 21V \]

16. Sol:

By KVL
\[ 1 - i_1 - i_1 = 0 \]
\[ i_1 = 0.5A \]

By KVL
\[ -i_2 + 2 = 0 \]
\[ i_2 = 0.5A \]

By KVL
\[ V_1 - 0.5 + 2 - V_2 = 0 \]
\[ V_2 = V_1 + 2V \]

17. Sol: As the bridge is balanced; voltage across (G) is “0V”.
By KCL at node “A” \(-I_s + 5m + 5m = 0\)
\[ I_s = 10mA \]

18. Sol: Given data:
\[ V_R = 5V \text{ and } V_C = 4\sin 2t \text{ then } V_L = ? \]
\[ i_c = \frac{CdV_c}{dt} = \frac{d}{dt}(4\sin 2t) = 8\cos 2t \]
By KCL:
\[ -1 - 2 + i_L + i_c = 0 \]
\[ i_L = 3 - 8\cos 2t \]

We know that:
\[ V_L = L \frac{di_L}{dt} = 2 \frac{d}{dt}(3 - 8\cos 2t) \]
\[ = 2(-8)(-2)\sin 2t \]
\[ V_L = 32\sin 2t \text{ volt} \]
19.  
Sol:  \( V = ? \) If power dissipated in 6\( \Omega \) resistor is zero.

\[
P_{6\Omega} = 0 \text{ W (Given)}
\]
\[
\Rightarrow i^2_{6\Omega} \cdot 6 = 0
\]
\[
\Rightarrow i_{6\Omega} = 0 \quad (V_{6\Omega} = 0)
\]
\[
\frac{V_1 - V_2}{6 + j8} = 0; \quad V_1 = V_2
\]
By Nodal ⇒
\[
\frac{V_1 - 20\angle 0^0}{1} + \frac{V_1}{j1} + 0 = 0
\]
\[
V_1 = 10 \sqrt{2} \angle 45^0 = V_2
\]
By Nodal ⇒
\[
0 + \frac{V_2}{5} + \frac{V_2 - V}{5} = 0
\]
\[
V = 2V_2 = 2(10 \sqrt{2} \angle 45^0)
\]
\[
\therefore V = 20 \sqrt{2} \angle 45^0
\]

20. Ans: (d)  
Sol:

\[
\text{Note: Since no independent source in the network, the network is said to be unenergised, so called a DEAD network}.
\]

21. Ans: (a)  
Sol:  

The behavior of this network is a load resistor behavior.

By Nodal ⇒
\[
-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0
\]
\[
3V = 8I_1
\]
\[
R_{eq} = \frac{V}{I_1} = \frac{8}{3} \Omega
\]

22. Sol:  

The behavior of the network is to be load resistor behavior.

By Nodal ⇒
\[
-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0
\]
\[
3V = 8I_1
\]
\[
R_{eq} = \frac{V}{I_1} = \frac{8}{3} \Omega
\]
Since $2 \times 4 = 4 \times 2$; the given bridge is balanced one, therefore the current through the middle branch is zero. The bridge acts as below:

\[
Z_{ab} = \frac{4 \times 8}{4 + 8} = \frac{8}{3} \Omega
\]

24. Sol: Redraw the circuit diagram as shown below:

Using $\Delta$ to star transformation:

\[
R_{ab} = 1 + \frac{4}{3} = \frac{7}{3} \Omega
\]
25. Sol: On redrawing the circuit diagram

26. Ans: (b)
Sol: The equivalent capacitance across a, b is calculated by simplifying the bridge circuit as shown in Fig. 1 to Fig. 5. \[ C_{ab} = 0.1 \, \mu F \]

Note: The bridge is balanced and the answer is easy to get.

27. Ans: (a)
Sol: Consider a Δ connected network

28. Ans: (a)
Sol: Network is redrawn as
29. Ans: (c)  
Sol: Applying KCL
\[ I_{0.25\Omega} = 2i + i = 3i \]
\[ I_{0.125\Omega} = (1 - 3i) \text{ A} \]
Applying KVL in upper loop.
\[ -\frac{(1-3i)}{8} + i + \frac{3i}{4} = 0 \]
\[ 5i = \frac{1-3i}{4} \Rightarrow 10i = 1 - 3i \]
\[ i = \frac{1}{13} \text{ A} \]
\[ V = \frac{3i}{4} = \frac{3}{4} \times \frac{1}{13} = \frac{3}{52} \text{ V} \]

30. Ans: (a)  
Sol:
Applying KCL at Node V

V + V - 2i_{x} + i_{x} = 0  
\[ \frac{V}{2} - \frac{V}{4} + i_{x} = 0 \]  
\[ \frac{V}{4} = \frac{V}{2} - i_{x} \]
\[ i_{x} = V + 10 \Rightarrow V = 6i_{x} - 10 \]
Put in equation (1), we get
\[ 3i_{x} + 5i_{x} - 2.5 + i_{x} = 0 \]
\[ 5i_{x} = 7.5 \]
\[ i_{x} = 1.5 \text{ A} \]
\[ V = -1 \text{ V} \]
\[ I_{\text{dependent source}} = \frac{V - 2i_{x}}{4} = \frac{-1 - 3}{4} = -1 \text{ A} \]
\[ \therefore \text{Power absorbed} = (I_{\text{dependent source}}) (2i_{x}) = (-1) (3) = -3 \text{ W} \]

31. Ans: (d)  
Sol: \( V_{0} = ? \)
Applying KCL
\[ +2 + 3 = 0 \]
\[ +5 \neq 0 \]
Since the violation of KCL in the circuit; physical connection is not possible and the circuit does not exist.

32. Ans: (b)  
Sol: Redraw the given circuit as shown below:
Applying KVL
\[ -15 - V_{0} = 0 \]
\[ V_{0} = -15 \text{ V} \]
33. Ans: (d)
Sol: Redraw the circuit diagram as shown below:
Across any element two different voltages at a time is impossible and hence the circuit does not exist.
Another method:
By KVL ⇒
5 + 10 = 0
15 ≠ 0
Since the violation of KVL in the circuit, the physical connection is not possible.

34. Ans: (d)
Sol: Redraw the given circuit as shown below:
By KVL ⇒
−10 − 10 = 0
−20 ≠ 0
Since the violation of KVL in the circuit, the physical connection is not possible.

35. Ans: (b)
Sol: Redraw the given circuit as shown below:
By KVL ⇒
10 − 10 = 0
0 = 0

KVL is satisfied
I_{5Ω} = \frac{10}{5} = 2A
I_{5Ω} = 2A

36. Ans: (d)
Sol:

The diode is forward biased. Assuming that the diode is ideal, the Network is redrawn with node A marked as in Fig. 1.
Apply KCL at node A
\[ \frac{4 - v_a}{2} = \frac{v_a}{2} + \frac{v_0 + 2}{2} \]
\[ 3v_a = 1 \]
\[ v_a = \frac{2}{3} \text{V} \]
(Here polarity is different what we assume so
\[ V_a = -\frac{2}{3} \text{V} \]

37. Sol: The actual circuit is
Voltage across 2A = 10 + 20 + 10 – 5 = 35 V
∴ Power supplied = VI = 35 x 2 = 70 W

39. Ans : (d)
Sol:

Applying KCL at node V
\[ \frac{V_{12}}{6} + \frac{V}{12} = V_0 + V_b = 0 \]
⇒ \[ \frac{V}{6} + \frac{V}{12} = 2 \Rightarrow V = 8V \]
∴ \[ V_0 = 4V \]
Applying KVL in outer loop
⇒ \[ -V + 1(V_0) + V_\text{ab} = 0 \]
⇒ \[ V_\text{ab} = V - V_0 = 8 - 4 = 4V \]

40. Sol: By KVL
⇒ \[ V_i = 6 - 10 = 0 \]
\[ V_i = 16V \]
\[ P_{10} = (8 * 2) = 16 \text{ watts} \text{ – absorbed} \]
\[ P_{2A} = (24 * 2) = 48 \text{ watts delivered} \]
\[ P_{10} = (6 * 2) = 12 \text{ watts} \text{ – absorbed} \]
\[ P_{10V} = (10 * 2) = 20 \text{ watts} \text{ – absorbed} \]

Since; \[ P_{\text{del}} = P_{\text{abs}} = 48 \text{ watts}. \text{ Tellegen’s Theorem is satisfied.} \]
41. 
Sol: By KVL in first mesh
\[ V_x - 6 + 6 - 12 = 0 \]
\[ V_x = 12V \]
\[ P_{12V} = (12 \times 9) = 108 \text{ watts delivered} \]
\[ P_{4\Omega} = (12 \times 3) = 36 \text{ watts absorbed} \]
\[ P_{6V} = (6 \times 6) = 36 \text{ watts absorbed} \]
\[ P_{2\Omega} = (12 \times 6) = 72 \text{ watts delivered} \]
Since \( P_{\text{del}} = P_{\text{abs}} \), Tellegen’s theorem is satisfied.

42. 
Sol:
\[ V - 4 + \frac{V}{2} + 4V_x = 0 \]
\[ \frac{5V}{6} = 4 - 2V_x \]
By KVL
\[ V_x - 2I + 4V_x = 0 \]
\[ 5V_x - 2I = 0 \]
Substitute (3) in (1), we get
\[ V = V_x \]
\[ V_x = \frac{24}{17} \]
\[ V_3 = \frac{24}{17} \text{ Volt and } I = \frac{60}{17} \text{ A} \]
\[ P_{4\Omega} = 0.663 \text{W absorbed} \]
\[ P_{4\Omega} = 64 \text{W absorbed} \]
\[ P_{4A} = 69.64 \text{W delivered} \]
\[ P_{2\Omega} = 24.91 \text{W absorbed} \]
\[ P_{4V3} = 19.92 \text{W delivered} \]
Since \( P_{\text{del}} = P_{\text{abs}} = 89.57 \text{W} \); Tellegen’s Theorem is satisfied.

43. Ans: (c) 
Sol:
\[ V_C = V_0 + \frac{1}{C_0} \int i_c (t)dt \]
\[ 0 < t < 1; \]
\[ i_c(t) = 2t \text{ and } V_0 = 0V \]
\[ \therefore V_C = 0 + \frac{1}{1/2} \int_0^1 2tdt \]
\[ = 2t \bigg|_0^1 \]
\[ = 2V \text{ at } t=1 \]
And \( V_C \) varies as parabolic
Continue to do like this with initial condition.

44. Ans: (c) 
Sol: KCL as well as KVL are applicable to any lumped electric circuit at any time ‘t’. Statement I is True.
The sum of the rms currents at any junction of the circuit is not zero in general. It depends upon the nature of the elements connected at the junction.
Statement II is false.
45. **Ans:** (d)  
**Sol:** $\Delta-Y$ transformations are true for any arbitrary frequency, $\omega$. Statement I is False.  
Impedances in $\Delta-Y$ vary with frequency. Statement II is True.

46. **Ans:** (a)  
**Sol:**  
\[
q = \int_{0}^{0} i(t) \, dt = \int_{0}^{0} \delta(t) \, dt = 1 \text{ Coulomb}
\]
Across capacitor, \( v = \frac{q}{C} = \frac{1}{C} \)

Energy inserted instantly from \( t = 0^- \) to \( t = 0^+ \)
\[
= \frac{1}{2} C v^2 = \frac{1}{2} C \left( \frac{1}{2} C^2 \right) = \frac{1}{2} \frac{1}{2C^2} 1
\]

Statement I is True, Statement II is also True and is the correct explanation.

47. **Ans:** (b)  
**Sol:** If there are \((n+1)\) nodes in a NW, by selecting a datum or reference node.
The node pair voltages of all the other n-nodes wrt this datum node are identified.

By knowing \( \vec{V} - \vec{I} \) relation of the branch KCL is used at each of the \( n \)-nodes to obtain a set of \( n \)-simultaneous independent equations in \( n \)-voltage variables, which when solved will provide information concerning the magnitudes and phase angles of the voltages across each branch.

The ideal generator maintains a constant voltage amplitude and wave-shape regardless of the amount of current it supplies to the circuit.

\( \therefore \) Both Statement I and Statement II are true and statement II is not the correct explanation of Statement I.

48. **Ans:** (a)  
**Sol:** All networks made up of passive, linear time invariant elements are reciprocal. Not only passivity and time-invariance but also linearity of elements is necessary to guarantee the reciprocity of the NW.

\( \therefore \) Statement I is true. Statement II is also true and correctly explains.

49. **Ans:** (b)  
**Sol:** Duals:

A. Mesh $\rightarrow$ Node (4)  
B. Outside mesh $\rightarrow$ Reference node (3)  
C. Mesh current $\rightarrow$ Node voltage (2)  
D. Number of meshes $\rightarrow$ Number of nodes (1)

50. **Ans:** (b)  
**Sol:** In Duality resistance equivalent to conductance  
Inductance equivalent to capacitance  
Loop current equivalent to node pair voltages  
Number of loops equivalent to number of node pairs.

51. **Ans:** (a)  
**Sol:**

(A) \( \frac{R}{L} = \frac{1}{\tau} \rightarrow (\text{Second})^{-1} \) (4)  
(B) \( \frac{1}{LC} = \omega^2 \rightarrow \text{(Radian/second)}^2 \) (3)  
(C) \( CR = \tau \rightarrow \text{Second} \) (1)  
(D) \( \sqrt{\frac{L}{C}} = R \rightarrow \text{Ohm} \) (2)
Conventional Practice Solutions

01. 
Sol: \( C = 30 \text{ mF} \)

For \( 0 \leq t \leq 2, \quad \frac{d\psi(t)}{dt} = 5 \text{ V} / \text{ms} \)

\[
i(t) = C \frac{d\psi(t)}{dt}
\]

\[
\therefore i(0.5 \text{ ms}) = 30 \times 10^{-3} \times 5 \times 10^3
\]

\[
= 150 \text{ A}
\]

\[
\frac{d\psi(t)}{dt} \bigg|_{t=2.5 \text{ ms}} = 0
\]

\[
\therefore i(2.5 \text{ ms}) = 0
\]

\[ E = \text{Energy delivered by the source till 7 ms} \]

\[
= \frac{1}{2} CV^2 (7 \text{ ms})
\]

\[
= \frac{1}{2} \times 30 \times 10^{-3} \times (5)^3 = 0.375 \text{ J}
\]

02. 
Sol: The currents in all the branches are marked as shown in Fig.

\[
I_y = \frac{I_1}{2} + I_2 + I_a,
\]

\[
I_x = \frac{3I_1}{2} + I_2 + I_a
\]

Inner Mesh equation:

\[
I_a \times 1 + 2I_1 + I_a \times 1 = 0
\]

\[
I_a + 2I_1 + \frac{3I_1}{2} + I_2 + I_a = 0
\]

03. 
Sol: Convert \( Y \) in to \( \Delta \) as shown in below figure.

\[
x_1 = 20 + 20 = 60 \Omega = x_2 = x_3
\]

Convert \( \Delta \) in to \( Y \)

\[
R_1 = \frac{12 \times 12}{36} = 4 \Rightarrow R_2 = R_3
\]
05

Sol: (i) Given the mesh equations:

\[
\begin{align*}
8 I_1 - 5 I_2 - I_3 &= 110 \\
-5 I_1 + 10 I_2 + 0 &= 0 \\
-I_1 + 7 I_3 &= 115
\end{align*}
\]

The NW must have 3 meshes with two sources and all possible resistances in general as shown in Fig.1

Write the mesh equations

\[
\begin{align*}
I_1(R_1+R_2+R_3) - I_2R_2 - I_3R_3 &= 110 \\
-I_1R_3 + I_2(R_2+R_4+R_5) - I_1R_2 &= 0 \\
-I_1R_3 - I_2R_5 + I_3(R_3+R_5+R_6) &= 115
\end{align*}
\]

Comparing the above set of equations (I) and (II):

\[
\begin{align*}
R_1+R_2+R_3 &= 8, R_2 = 5 \Omega \\
R_2 &= 5 \Omega \\
R_2+R_4+R_5 &= 10, R_3 = 0 \Omega \\
R_3 &= 1 \Omega \\
R_4 &= R_3 = 0 \Omega \\
R_3+R_5+R_6 &= 7 \\
R_5+5+1 &= 8 \\
R_1+R_4+0 &= 10, 1+0+R_5 &= 7 \\
R_1 &= 2 \Omega, R_4 = 5 \Omega, R_5 &= 6 \Omega \\
R_6 &= 0 \Omega \\
R_1 &= 2 \Omega, R_2 = 5 \Omega, R_3 = 1 \Omega \\
R_4 &= 5 \Omega, R_5 = 0 \Omega, R_6 &= 6 \Omega
\end{align*}
\]
(ii) Current in the 110 V source = \( I_1 = \frac{D_1}{D} \)

\[
\begin{vmatrix}
110 & -5 & -1 \\
0 & 10 & 0 \\
115 & 0 & 7 \\
\end{vmatrix}
\]

= 110(70) -1(-1150) = 8850

\[
\begin{vmatrix}
8 & -5 & -1 \\
-5 & 10 & 0 \\
-1 & 0 & 7 \\
\end{vmatrix}
\]

= 8 \times 70 + 5(-35) -1(10)

= 375

\[
I_1 = \frac{8850}{375} = \frac{118}{5} = 23.6 \text{ A}
\]

06.

Sol:

Apply KVL at loop 1
\[ V_1 - 36 - V_2 = 0 \]
\[ V_1 - V_2 = 36 \] \hspace{1cm} .......... (1)

Apply KCL at Super node
\[ \frac{V_1 + 12}{4} + \frac{V_1}{6} + \frac{V_2}{12} + \frac{V_2 - 24}{3} = 0 \]

\[ V_1 \left(\frac{1}{4} + \frac{1}{6}\right) + V_2 \left(\frac{1}{12} + \frac{1}{3}\right) + 3 - 8 = 0 \]

From Fig.2, 1 Amp is divides equally through the four wires because of the infinite nature of the mesh : \( I_1 = 0.25 \text{ Amp} \).

Similarly from Fig.3, \( I_2 = 0.25 \text{ Amp} \)

\[ \therefore \text{Current in the wire, AC} = I = I_1 + I_2 \]

\[ = 0.5 \text{ Amp} \]

07.

Sol: To find the current, \( I \) in the wire AC, of Fig. 1, assume a ground point D. Feeding 1 amp at point A and taking 1 Amp from point C is equivalent to the superposition of two current sources as shown in Fig. 2 and Fig.3

\[ V_1 \left(\frac{5}{12}\right) + V_2 \left(\frac{5}{12}\right) \]

\[ V_1 + V_2 = 12 \]

By solving equation (1) and (2)

\[ V_1 = 24 \]
\[ V_2 = -12 \]
08. Sol:

\[10 V_2 = 21 V_1 - 52\]
\[= (21 \times 2.483) - 52\]
\[V_2 = 0.8675V\]

\[\text{Ammeter reading} = \frac{V_2}{2} = 0.434 \text{A}\]

\[\text{Voltmeter reading} = 2 + \frac{V_1 - 2}{5} = 2.1178V\]

09. Sol: By applying KCL at \(V_1\):

\[2 = \frac{V_1 - 20}{10} + \frac{V_1 - 0.5V_1 - V_2}{5}\]

\[V_1 - V_2 = 20 \quad \text{(1)}\]

By applying KCL at \(V_2\):

\[\frac{V_2 - 30}{10} + \frac{V_2}{2} + \frac{V_1 - V_2}{5} = 0\]

\[-V_1 + 8V_2 = 80 \quad \text{(2)}\]

From (1) & (2):

\[-V_1 + 8(10 - V_1) = 80\]

\[7V_1 = 240\]
\[V_1 = \frac{240}{7} = 34.28V\]

\[V_2 = V_1 - 20 = 34.28 V - 20 = 14.3V\]
01. Sol: The current “I” = ?

By superposition theorem, treating one independent source at a time.

(a) When 1A current source is acting alone.

Since the bridge is balanced; \( I_1 = 0 \) A

(b) When 1V voltage source is acting alone

\( I_2 = 0 \) A

Since the bridge is balanced.

(c) When 2V voltage source is acting alone

\[ I_3 = \frac{2}{3} = 0.66 \text{A} \]

By superposition theorem; \( I = I_1 + I_2 + I_3 \)

\[ I = 0 + 0 + 0.66 \text{A} \]

\[ I = 0.66 \text{A} \]

02. Sol:

By superposition theorem; treating only one independent source at a time.

(a) When 10V voltage source is acting alone

By KVL

\[ 10 - 2i_x - i_{x1} - 2i_{x1} = 0 \]

\[ i_{x1} = 2 \text{A} \]
(b) When 3A current source is acting alone

\[ V = \frac{2}{3} + \frac{(V - 2i_{x2})}{1} = 0 \]

\[ 3V - 4i_{x2} = 6 \] \hspace{1cm} \text{........... (1)}

And

\[ i_{x2} = \frac{0 - V}{2} \rightarrow V = -2i_{x2} \] \hspace{1cm} \text{........... (2)}

Put (2) in (1), we get

\[ i_{x2} = -\frac{3}{5} \text{A} \]

By SPT;

\[ i_x = i_{x1} + i_{x2} = 2 - \frac{3}{5} = \frac{7}{5} \]

\[ \therefore i_x = 1.4 \text{A} \]

03

Sol:

For 120 V \[ \rightarrow i_1 = 3 \text{A} \]

For 105 V \[ \rightarrow i_1 = \frac{105}{120} \times 3 = 2.625 \text{A} \]

For 120 V \[ \rightarrow V_2 = 50 \text{V} \]

04. Ans: (b)

Sol: It is a liner network

\[ V_x \text{ can be assumed as function of } i_{x1} \text{ and } i_{x2} \]

\[ V_x = A_1 i_{x1} + B_1 i_{x2} \]

\[ 80 = 8A + 12B \] \hspace{1cm} \text{........... (1)}

\[ 0 = -8A + 4B \] \hspace{1cm} \text{........... (2)}

From equation 1 & 2

\[ A = 2.5; \ B = 5 \]

Now, \[ V_x = (2.5)(20) + (5)(20) \]

\[ V_x = 150 \text{V} \]

05. Ans: (c)

Sol:

For finding Norton’s equivalent resistance independent voltage sources to be short circuited and independent current sources to be open circuited, then the above circuit becomes

\[ \Rightarrow R_N = 3 + 4 = 7 \Omega \]
06. Ans: (b)  
Sol:  
\[ \begin{align*} 
\text{Excite with a voltage source 'V' } 
\end{align*} \]

\[ \begin{align*} 
\text{Apply KCL at node } V_1 & \quad -I + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 0 \\
\Rightarrow 2V_1 - V_2 - I &= 0 \quad \text{(1)} \\
\text{Apply KCL at node } V_2 & \quad \frac{V_2 - V_1}{1} + \frac{V_2 + 2V_x}{1} = 0 \\
2V_2 - V_1 + 2V_x &= 0 \quad \text{(2)} \\
\text{But from the circuit, } V_x &= 2I \quad \text{(3)} \\
\text{Substitute (3) in (2) } & \quad \Rightarrow 2V_2 - V_1 + 4I = 0 \\
4V_2 - 2V_1 + 8I &= 0 \\
\text{From (1), } & \quad 2V_1 = V_2 + I \\
\Rightarrow 4V_2 - (V_2 + I) + 8I &= 0 \\
3V_2 + 7I &= 0 \\
\Rightarrow V_2 &= \frac{-7I}{3} \\
\end{align*} \]

\[ \Rightarrow \text{Req } V = \frac{4I}{3} \]

\[ \Rightarrow R_{eq} = \frac{4}{3} \Omega \]

07. Sol:  
\[ \begin{align*} 
\text{Here } j1\Omega \text{ and } -j1\Omega \text{ combination will act as open circuit. } \\
\text{The circuit becomes } \\
\Rightarrow V_{th} &= \frac{100\angle 0^\circ \times j4}{3 + j4} \\
&= 80\angle 36.86^\circ \text{ V} 
\end{align*} \]
08.
Sol: Thevenin’s and Norton’s equivalents across a, b.

\[ I_{SC} = \left( \frac{10}{4} + 5 \right) = \frac{15}{2} \text{A} \]

\[ I_{SC} = \frac{15}{2} \text{A} \]

\[ R_{th} = \frac{V_{th}}{I_{SC}} = \frac{150}{\frac{15}{2}} = 20 \Omega \]

\[ a \]

\[ b \]

By Nodal ⇒

\[ \frac{V}{5} - 10 + \frac{V - V_{th}}{5} = 0 \]

\[ \frac{V_{th}}{5} - \frac{V}{5} - \frac{V_x}{4} = 0 \]

\[ 2V = \left( 10 + \frac{V_{th}}{5} \right) \]

\[ \frac{V_{th}}{5} = \frac{V}{10} + \frac{V}{5} \]

\[ V_x = \frac{2V}{5} \]

\[ V_{th} = 150 \text{V}, V = 100 \text{V} \]

\[ a \]

\[ b \]

\[ a \]

\[ b \]

09.
Sol:

Super nodal equation

\[ i_a - 0.2i_b + i_b - 1 = 0 \]

\[ I = i_a + 0.8i_b \]

\[ V = 80i_b ; i_b = \frac{V}{80} \]

= Inside the supernode, always the KVL is written.

By KVL ⇒

\[ 100i_a + 2i_a - 80i_b = 0 \]

\[ I = \frac{V}{102} + \frac{0.8 \times V}{80} \]

\[ \frac{V}{I} = R_L = \frac{1}{\frac{1}{102} + \frac{1}{100}} \]

\[ = 50.5 \Omega. \]

\[ R_L = 50.5 \Omega \]
10. Sol: $V_{th}$:

\[
\begin{align*}
0^\circ & \quad j8\Omega \\
\downarrow & \quad 110^\circ \downarrow \\
\downarrow & \quad 90^\circ \downarrow \\
6\Omega & \quad j8\Omega \\
6\Omega & \quad j8\Omega \\
\end{align*}
\]

By Nodal ⇒

\[
\frac{V_{th}}{6 + j8} - \frac{110^\circ}{6 + j8} + \frac{90^\circ}{6 + j8} = 0
\]

\[2V_{th} = 200^\circ \Rightarrow V_{th} = 100^\circ.
\]

11. Sol:

The maximum power delivered to “$R_L$” is

\[
R_L = \sqrt{R_S^2 + (X_S + X_L)^2}
\]

Here $R_S = 10\Omega$; $X_S = 10\Omega$ & $X_L = -15$

\[
R_L = \sqrt{10^2 + (10 - 15)^2}
\]

\[
R_L = 5\sqrt{5}\Omega
\]

\[
I = \frac{100^\circ}{10 + j10 - j15 + 5\sqrt{5}}
\]

\[
P_{\text{max}} = |I|^2R_L = (5\sqrt{5})^2 = 236W
\]

12. Sol:

The maximum power delivered to $10\Omega$ load resistor is:

\[
Z_L = 10 - jX_C = 10 + j(-X_C)
\]

\[X_L = -X_C
\]

So for MPT; $(X_S + X_L) = 0$

\[10 - X_C = 0;
\]

\[X_C = 10
\]

\[
I = \frac{100^\circ}{10 + j10 - j10 + 10} = 5^\circ
\]

\[
P_{\text{max}} = |I|^2R_L = 5^2(10) = 250W
\]

\[
P_{\text{max}} = 250\text{Watts}
\]
13. Ans: (b)  
Sol:
\[ \begin{align*}
V_0 &= 20 + j0 \text{ V} \\
Z_L &= Z_{th}^* \\
i_x &= (1 + V_0) \times \frac{-j1}{1-j1} = (1 + V_0) (0.5 - j0.5) \\
V_0 &= -i_x \\
&= -(1+V_0) (0.5 - j0.5) \\
&= -1-j V_0 = 1 + V_0 \\
\Rightarrow V_0 &= -1 - j - 1 = 1 \\
V_0 &= \frac{1}{2-j} = -0.4 + j0.2 \\
\end{align*} \]

Applying KVL
\[ + V_0 - j1(1 + V_0) + V = 0 \]
\[ \Rightarrow V = -V_0 + j1(1+V_0) \]
\[ = 0.4 - j0.2 + j1(0.6+j0.2) \]
\[ V = (0.2 + j0.4)\text{V} \]
\[ \therefore Z_{th} = \frac{V}{I} = V = (0.2 + j0.4)\Omega \]
\[ \therefore Z_L = Z_{th}^* = (0.2 - j0.4) \Omega \]

14. Sol:

The maximum true power delivered to “Z_L” is:
\[ V_{th} = \left( \frac{50\angle0}{-j5+j5+5} \right)(j5+5) = 50\sqrt{2} \angle 45^\circ \]
\[ Z_{th} = (-j5)(5+j5) = (5-j5)\Omega \]
\[ I = \frac{50\sqrt{2} \angle 45^\circ}{(5-j5+5+j5)} = 5\sqrt{2} \angle 45^\circ \]
\[ P = |I|^2 Z_{th} = (5\sqrt{2})^2 \cdot 5 = 250 \text{ Watts} \]
\[ \therefore P_{max} = 250 \text{ watts} \]

15. Ans: (e)  
Sol:

Maximum power will occur when R = R_s 
\[ \Rightarrow R = 1 \Omega \]
\[ P_{\text{max}} = \left(\frac{1}{2}\right)^2 \times 1 = \frac{1}{4} \text{ W} \]

25% of \( P_{\text{max}} \) = \( \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \text{ W} \)

\[
\text{Current passing through ‘R’}
\]

\[ I = 1 \times \frac{1}{1 + R} = \frac{1}{1 + R} \]

\[ \therefore P = I^2 R = \left(\frac{1}{1 + R}\right)^2 R = \frac{1}{16} \]

\[ \Rightarrow (R + 1)^2 = 16R \]

\[ \Rightarrow R^2 + 2R + 1 = 16R \]

\[ \Rightarrow R^2 - 14R + 1 = 0 \]

\[ R = 13.9282 \Omega \text{ or } 0.072 \Omega \]

From the given options 72m\(\Omega\) is correct

16. The network ‘N’ shown in figure contains only resistances.

E = 10 V and 0V
I = 0A and 2A
V = 3V and 2V respectively.

If E = 100 V and I is replaced by R = 2\(\Omega\), then determine V.

\[ \text{Sol: For, } E = 10 \text{ V, } I = 0 \text{A then } V = 3\text{V} \]

\[ E=10V \quad + \quad V=3V \quad - \]

\[ \text{V}_{\text{oc}} = 3\text{V} \text{ (with respect to terminals a and b)} \]

For, E = 0V, I = 2A then V = 2V

\[ E=0V \quad + \quad V=2V \quad I=2A \]

Now when E = 100 V, and I is replaced by \( R = 2\Omega \) then V = ?

\[ E=100V \quad + \quad V_{\text{oc}}=30V \quad - \]

When E = 100V,

From Fig.(b) using homogeneity principle

For finding Thevenin’s resistance across ab independent voltage sources to be short circuit & independent current sources to be open circuited.
Fig. (c) is the energized version of Fig. (d)

\[ R_{th} = \frac{2}{2} = 1 \Omega \]

\[ V = 30 \times \frac{2}{2 + 1} = 20 \text{ V} \]

\[ V = 20 \text{ V} \]

17. **Sol:** Superposition theorem cannot be applied to Fig. (b)

Since there is only voltage source given:

By homogeneity and Reciprocity principles to Fig. (a);

\[ I = 4 \text{ A} \]

18. **Ans:** (b)

\[ \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} \]

19. **Ans:** (b)

\[ \begin{bmatrix} 5 \Omega \\ 1 \Omega \end{bmatrix} \]

\[ 6 \text{ A} \]
\[ Z_{11} = \frac{10}{4} = 2.5 \]
\[ Z_{21} = \frac{4}{4} = 1 \]
\[ I_{5\Omega} = \frac{6 \times 1}{6.5 + 1} = \frac{6}{7.5} = 0.8 \text{ A} \]

19. Ans: (b)
Sol:

Using reciprocity theorem, for Fig.(a)

\[ I_{sc} = 3 \text{ A} \]

Norton’s resistance between a and b is

Fig.(a)

Fig.(b)

\[ V = -15 \text{ V} \]

Fig.(c)

Fig.(d)

Fig.(a) is the energized version of Fig.(d)
Postal Coaching Solutions

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P_{AB} = P_{SO} = P_{25V} = P_{5A} = 5*25 = 125 watts
(ABSORBED)

21. Sol:

By Mill Man’s theorem;

\[ V' = \frac{4V}{2} + \frac{12V}{2} + \frac{2V}{2} \]

\[ = \frac{4 - 12 + 4}{2} \]

\[ = \frac{-1V}{2} \]

\[ \therefore V' = -1V \]

\[ \frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{2} + 1 = 2 \]

\[ \therefore R^1 = \frac{1}{2} \Omega \]

\[ I = \frac{-1}{\left(\frac{1}{2} + 3\right)} \Rightarrow I = \frac{-2}{7} \text{A} \]

22. Ans: (d)

Sol:

Since the two different frequencies are operating on the network simultaneously; always the super position theorem is used to evaluate the responses since the reactive elements are frequency sensitive

i.e., \( Z_L = j\omega L \) and \( Z_C = \frac{1}{j\omega C} \).

23. Sol: In the above case if both the source are100rad/sec, each then Millman’s theorem is more conveniently used.

24. Sol:

Nodal equations

\[ i = G \]

\[ i = i_1 \]

\[ 10 = 2i_1 - i_2 \]

\[ 0 = 2i_1 - i_2 + 3(i_2 - i_1) \]

\[ V_x = V_1 \]

\[ 10 = 2V_1 - 3(V_1 - V_2) \]

\[ 0 = 4V_2 + 2V_x + 3(V_2 - V_1) \]

\[ V_1 \]

\[ V_2 \]

\[ V_x \]
26. **Sol:** When only $E_1$ is acting, $I_1^2R = 18$

\[
I_1 = \frac{1}{\sqrt{R}} \frac{18}{2} = 3\sqrt{2}R
\]

Similarly, $I_2 = 5\sqrt{2}R$; $I_3 = 7\sqrt{2}R$

When all sources are acting,

$I_{\text{total}} = I_1 + I_2 + I_3$

Maximum power consumed by $R$ is

\[
P = I_{\text{total}}^2R
\]

\[
= \left(3\sqrt{2}R + 5\sqrt{2}R + 7\sqrt{2}R\right)^2 R
\]

\[
= \frac{2}{R} (3 + 5 + 7)^2 R
\]

\[
= 450 W
\]

Minimum power consumed

\[
P = \frac{2}{R} (3 + 5 - 7)^2 R = 2 W
\]

27. **Ans: (c)**

**Sol:**

\[
I_L = \frac{100}{R_g + 4 + 10}, \quad P_L = I_L^2 R_L
\]

$P_L$ is maximum, when $I_L$ is maximum.

$I_L$ is maximum, when $R_g$ is minimum

$= 3\Omega$

Statement (I) is True.

During maximum power transfer, (i.e., when $R_g = 3\Omega$),

\[
|Z_s| = \sqrt{R_g^2 + 4^2} = 5 \Omega
\]

\[
\therefore R_L \neq |Z_s|
\]

Statement (II) is false.

28. **Ans: (b)**

**Sol:**

\[
V_1 = I_1 (R_S + R_L)
\]

Thevenin and Norton equivalents are derivable for linear NW’s only.

29. **Ans: (b)**

**Sol:** Conversion to equivalent T – NW and application of Thevenin’s Theorem have no relation.

30. **Ans: (d)**

**Sol:** $Z_L$ should be equal to $Z_s^*$ and $\eta=50\%$

\[
\therefore \text{Statement (I) is false but Statement (II) is true.}
\]

31. **Ans: (a)**

**Sol:** Diode is a nonlinear and unilateral device. Hence, Thevenin’s theorem cannot be applied. Both Statement (I) and Statement (II) are true and Statement (II) is the correct explanation of Statement (I).

32. **Ans: (c)**

**Sol:**

A. Load impedance $(10 + j 20)^\ast$

\[
= 10 - j 20
\]

B. Total impedance $Z_i + Z_L = 20$

C. Current $\frac{50}{20} = 2.5$

D. Maximum power

\[
2.5^2 \times 10 = 62.5
\]

33. **Ans: (b)**

34. **Ans: (b)**

**Sol:**

A. Superposition theorem is applicable for linear networks only

B. Tellegen’s theorem utilizes the structure of the NW irrespective of the nature of the elements
C. The equivalent circuit of a NW at two terminals can be obtained by using Norton’s theorem. (2)

D. Reciprocity theorem is applicable to Bilateral networks (4)

35. **Ans: (c)**

**Sol:**

A. Reciprocity
   - Bilateral (2)

B. Tellegen’s
   - \[ \sum_{k=0}^{b} v_{jk}(t_1) i_{jk}(t_2) = 0 \] (3)

C. Superposition
   - Linear (4)

D. Maximum power Transfer
   - Impedance matching (1)

36. **Ans: (d)**

### Conventional Practice Solutions

01. **Sol:** The given circuit is shown in Fig 1 with terminals marked.

Source transformation is used successively as shown in Fig. (2) to (9)
Case (ii):  
Source ‘1A’ active: $I = I_2$  
$I_2 = 0.5$ A as can be seen from Fig. 2

Case (iii):  
Source ‘3A’ active: $I = I_3$  
$I_3 = 0.33$ A as can be seen from Fig. 4

Note: The answer can be obtained quickly by writing KVL equation from the given circuit, Fig. 5 directly:

02. 
Sol: Case (i):  
Source ‘V’ active: $I = I_1$  
$I_1 = 0$, as can be seen from Fig. 1
03. 
**Sol: Case (i):**

Due to 6 V source, the currents are shown in Fig. 1.

![Fig. 1](image1.png)

**Case (ii):**

Due to 3 A source, the currents are shown in Fig. 2.

![Fig. 2](image2.png)

Using the superposition theorem, the currents due to both the sources are shown in Fig. 3.

![Fig. 3](image3.png)

04. 
**Sol:** \( v(t) = 10 \sin \left(2\pi \times 10^6\right) t \)

i) Maximum power is generated by the generator when \( Z_L = 0 \)

\[ P_{\text{gen}} (\text{max}) = \left(\frac{10}{\sqrt{2}}\right)^2 = 50 \text{ W} \]

Maximum power is delivered to the load if \( Z_L = 1 \Omega \).

Under this condition

ii) \( P_L (\text{max}) = I_{\text{rms}}^2 \times 1 = \left(\frac{5}{\sqrt{2}}\right)^2 \times 1 = 12.5 \text{ W} \)

iii) Power generated by generator = \( 2 \times 12.5 = 25 \text{ W} \)

05. 
**Sol:** Let the Thevenin impedance be \( R + jX \)

Magnitude of \( Z_{\text{th}} = \sqrt{R^2 + X^2} = \frac{\text{open circuit voltage}}{\text{short circuit current}} \)

\[ = \frac{125}{5.59} = 22.36 \]

With 10 \( \Omega \) resistive load,

\[ \frac{125}{\sqrt{(R+10)^2 + X^2}} = 4.41 \]

\[ \frac{125}{\sqrt{(R+10)^2 + X^2}} = \frac{125}{4.41} = 28.34 \]

\( R^2 + X^2 = 499.97 \)

\( R^2 + X^2 + 100 + 20R = 803.15 \)

20 \( R = 100 = 303.18 \)
\[ R = \frac{203.18}{20} = 10.16 \Omega \ ; \ X = 19.91 \]

\[ V_{th} = 125 \text{ V}, \ Z_{th} = 10.16 + j 19.91 \Omega \]

06.

**Sol:** \( V_{th} \) is calculated from Fig. 1

\[ R_{th} = \frac{V_0}{i_x} \]

\[ i_x = V_0 - i_x - \alpha i_x + \frac{V_0 - i_x}{2} \]

\[ i_x = \frac{3}{2} V_0 - i_x \left( \frac{3}{2} + \alpha \right) \]

\[ \left( \frac{5}{2} + \alpha \right) i_x = \frac{3}{2} V_0 \]

\[ \frac{V_0}{i_x} = \frac{2}{3} \frac{5 + 2\alpha}{2} = \frac{2\alpha + 5}{3} \]

07.

**Sol:** The circuit is simplified as shown in Fig. (1), (2) and (3) by using source transformation.

\[ \text{Load:} \]

\[ Y_{XY} = \frac{1}{R_L} + \frac{2}{R_L} + \frac{3}{R_L} = \frac{6}{R_L} \]

\[ Z_{XY} = \frac{R_L}{6} \]

For maximum power transfer to the three resistor load, \( Z_{XY} \)

\[ \frac{R_L}{6} = 30, \quad R_L = 180 \Omega \]

\[ \therefore V_{XY} = 27 \text{ V} \]
Power delivered to $R_L = \frac{(27)^2}{180} = 4.05$ W
Power delivered to $R_L = \frac{(27)^2}{90} = 8.1$ W
Power delivered to $R_L = \frac{(27)^2}{60} = 12.15$ W

08. **Sol:** The given circuit is shown in Fig. 1, where 40 Ω across 50 V can be deleted.

![Fig. 1](image1)

V<sub>BA</sub> is found by reducing the given circuit to the left of BA into a single voltage source, V<sub>e</sub> and a series resistance, R<sub>e</sub> by using Milliman’s theorem.

The equivalent circuit is shown in Fig. 2

![Fig. 2](image2)

\[ V_e = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} \]
\[ = \frac{50}{\frac{50}{1} + \frac{100}{20}} \]
\[ = \frac{50}{1 + \frac{1}{20}} \]

09. **Sol:** Maximum Power transfer theorem:

This is used to find the value of the load impedance $Z_L$ (optimum) that absorbs maximum power from a given network shown in Fig. 1.

The NW is replaced by its Thevenin’s equivalent circuit as shown in Fig. 2.

\[ Z_L = R_L + j X_L \text{ is complex load} \]
\[ V_S \text{ is Thevenin’s voltage phasor (RMS Value)} \]
\[ Z_S \text{ is Thevenin’s equivalent impedance} = R_S + j X_S \]
\[ V_s \text{ and } Z_s \text{ can be understood as the source voltage and source impedance wrt the load impedance, } Z_L. \]
\[ I_s = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{(R_s + R_L) + j(X_L + X_S)} \]
\[ P = \text{Power delivered to} \]
\[ Z_L = |I_s|^2 R_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_L + X_S)^2} \]

For maximum power transfer to \( Z_L \):

**Case 1:**

When only \( X_L \) is variable in the load,
\[ \frac{\partial P}{\partial X_L} = 0, 2(X_L + X_S) = 0 \text{ or } X_L = -X_S \] ...
Then maximum power transferred to \( Z_L = \frac{V_s^2 R_L}{(R_s + R_L)^2} \) ...

**Case 2:** When only \( R_L \) is variable,
\[ \frac{\partial P}{\partial R_L} = 0 \]
\[ (R_s + R_s)^2 + (X_L + X_S)^2 - R_L(R_s + R_S) = 0 \]
\[ R_s^2 + R_s^2 + 2R_sR_s + (X_L + X_S)^2 - 2R_L R_s - 2R_L^2 = 0 \]
\[ R_s^2 - R_s^2 + (X_L + X_S)^2 = 0 \text{ or } R_L = \sqrt[2]{R_s^2 + (X_L + X_S)^2} \] ...
Then maximum power delivered to
\[ Z_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + R_L^2 - R_s^2} \]
\[ = \frac{V_s^2}{2(R_L + R_s)} \] ...

**Case 3:**

When \( X_L \) as well as \( R_L \) are variable, then from (3) and (5)
\[ X_L = -X_S, R_L = R_S \]
\[ Z_L \text{ (optimum)} = R_S - jX_S \] ...

Then maximum power transferred to \( Z_L = \frac{V_s^2}{4R_L} \) ...

---

Let \( Z_L \) = Impedance of Loudspeaker across the terminals A, B for maximum power dissipation in it as shown in Fig. 1.

**Fig. 1**

**Fig. 2**

Then \( Z_L = \) Thevenin’s impedance across terminals A, B into the network = \( Z_{th} \)

\( Z_{th} \) is found from Fig. (2)
\[ Z_{th} = \frac{(3 + j4) - (j5)}{(3 - j4) + j5} \]
\[ = \frac{20 - j15}{3 - j1} = \frac{5(4 - j3)(3 + j1)}{9 + 1} \]
\[ = \frac{1}{2} \left( 12 + 3 - 9j + j4 \right) \]
\[ = \frac{1}{2} (15 - j5) = 7.5 - j2.5 \]

\( \therefore Z_L = Z_{th} \)
\[ \Rightarrow Z_L = (7.5 - j2.5) \Omega \]
From Fig. 2

\[ V_{AB} = V_L = V_{th} \frac{R_L}{R_{th} + R_L} \]  \hspace{1cm} (2)

\[ I_{AB} = I_L = I_N \frac{G_L}{G_N + G_L} \]  \hspace{1cm} (3)

Where, \( G_N = \frac{1}{R_N} = \frac{1}{R_{th}} \)

Equations (2) and (3) are dual equations where the duality is indicated by the dual quantities given below:

Voltage across load, \( V_L \rightarrow \) Current through load, \( I_L \)

Open circuit voltage across → Short circuit current from A to B = \( I_N \)

Writing KVL equations:

\[ I_1 + 2(I_1 - 30) + V_{th} = 20 \]

or 3I_1 + V_{th} = 80 \hspace{1cm} (1)

and

\[ 6(I_1 - 15) + 1 I_1 + 2(I_1 - 30) = 0 \]

or 9I_1 - 90 - 60 = 0

\[ I_1 = \frac{150}{9} = \frac{50}{3} A \] \hspace{1cm} (2)

From (1) and (2), \( V_{th} = 80 - 3 I_1 = 30 V \)

\( R_{th} \) is obtained from the following circuit, Fig. 2

\[ R_{th} = 3 \Omega \parallel 6 \Omega = 2 \Omega \]
Thevenin’s equivalent circuit across the terminals A, B is shown in Fig. 3.

![Thevenin's equivalent circuit](image)

\[ V_0 = \frac{30 \times 3}{5} = 18 \text{ V} \]

### 11. Sol:

\[ V = 20 \cos 2t \]  
\[ j^2 = -1 \]

\[ V - \frac{20}{\sqrt{2}} + \frac{V}{j^2} + \frac{V}{-j} = 0 \]

\[ V \left( \frac{1}{2} + \frac{1}{j^2} - \frac{1}{j} \right) = \frac{10}{\sqrt{2}} \]

\[ V \left( \frac{1}{2} - \frac{1}{j^2} \right) = \frac{10}{\sqrt{2}} \]

For \( Z_N \):

\[ Z_N = \left( \frac{2}{j^2} \right) - j \]

\[ \frac{j^2}{(1+j)^2} - j = 1 \Omega \]
Chapter 3

Transient Circuit Analysis

Objective Practice Solutions

01. Sol:

\[ i(t) = e^{-3t} A \text{ for } t > 0 \text{ (given)} \]

Determine the elements & their connection

\[ \text{Response Laplace transform} \]

\[ \text{Excitation Laplace transform} \]

\[ \text{System transfer function} \]

\[ i.e., \quad \frac{I(s)}{V(s)} = H(s) = \frac{1}{s + 3} \]

\[ \frac{s}{s + 3} = y(s) = \frac{1}{Z(s)} \]

\[ Z(s) = \left( \frac{s + 3}{s} \right) \]

\[ = 1 + \frac{1}{s} \left( \frac{1}{3} \right) = R + \frac{1}{SC} \]

\[ \therefore \quad R = 1 \Omega \text{ and } C = \frac{1}{3} F \text{ are in series} \]

02. Ans: (c)

Sol: The impulse response of first order system is \( Ke^{-2t} \).

So \( T/F = L(I.R) = \frac{K}{s + 2} \)

\[ \sin 2t \]

\[ \frac{k}{s + 2} \]

\[ y(t) \]

\[ G(s) = \frac{K}{s + 2} \]

\[ |G(j\omega)| = \frac{K}{\sqrt{\omega^2 + 2^2}} = \frac{K}{2\sqrt{2}} \]

\[ \angle G(j\omega) = -\tan^{-1} \frac{\omega}{2} = -\tan^{-1} 1 = -\frac{\pi}{4} \]

So steady state response will be \( y(t) = \frac{K}{2\sqrt{2}} \sin \left( 2t - \frac{\pi}{4} \right) \)

03. Sol:

\[ \begin{array}{c}
\text{By KVL } v(t) = (5 + 10\sin t) \text{ volt} \\
\text{Evaluating the system transfer function } H(s). \\
\text{Desired response } L.T \\
\text{Excitation response } L.T \\
\text{System transfer function} \\
\end{array} \]

\[ \frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R + SL + \frac{1}{SC}} \]

\[ H(s) = \frac{S}{(2s^2 + s + 1)} \]

\[ H(j\omega) = \frac{1}{\left( 1 + \frac{1}{j\omega} + 2j\omega \right)} \]
II. Evaluating at corresponding $\omega$ of the input

$H(j\omega)|_{\omega=0} = 0$

$H(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}} - 45^\circ$

III. \[ \frac{I(s)}{V(s)} = H(s) \]

$I(s) = H(s)V(s)$

\[ i(t) = 0 \times 5 + \frac{1}{\sqrt{2}} \times 10 \sin(t - 45^\circ) \]

\[ i(t) = 7.07 \sin(t - 45^\circ) \text{A} \]

OBS: DC is blocked by capacitor in steady state

04.

**Sol:** \[ \frac{V(s)}{I(s)} = H(s) = Z(s) \]

\[ Z(s) = \frac{1}{Y(s)} = \frac{1}{\left( \frac{1}{R} + \frac{1}{sL} + sC \right)} \]

\[ H(s) = \frac{1}{\left( 1 + \frac{1}{s} + s \right)} \]

\[ H(j\omega)|_{\omega=1} = \frac{1}{\left( 1 + \frac{1}{j} + j \right)} = 1 \]

\[ V(s) = I(s)H(s) = \sin t \]

\[ v(t) = \sin t \text{ Volts} \]

05.

**Sol:** \[ \tau = \frac{L_{eq}}{R_{eq}} \]

\[ R_{eq} = (2 \parallel 2) + 9 = 10 \Omega \]

\[ L_{eq} = (2 \parallel 2) + 1 = 2 \text{H} \]

\[ \therefore \tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{10} = 0.2 \text{ sec} \]

06.

**Sol:** \[ \tau = R_{eq}C_{eq} \]

\[ R_{eq} = 3 \Omega \]

\[ C_{eq} = 1 \text{F} \]

\[ \therefore \tau = 3 \times 1 = 3 \text{ sec} \]

07.

**Sol:** \[ \tau = R_{eq}C \]
08.

Sol: Let us assume that switch is closed at \( t = -\infty \), now we are at \( t = 0^- \) instant, still the switch is closed i.e., an infinite amount of time, the independent dc source is connected to the network and hence it is said to be in steady state. In steady state, the inductor acts as short circuit and nature of the circuit is resistive.

\[
\text{At } t = 0^- : \text{ Steady state: A resistive circuit}
\]

Note: The number of initial conditions to be evaluated at just before the switching action is equal to the number of memory elements present in the network.

(i) \( t = 0^- \)

\[
i_L(0^-) = 2 = i_L(0^+)
\]
\[
t = 5 \tau = 5 \times \frac{1}{5} = 1 \text{sec for steady state}
\]
practically i.e., with in 1 sec the total 8 J stored in the inductor will be delivered to the resistor.

By KCL;
\[-2 + i_L(0^-) = 0\]
\[i_L(0^-) = 2 \text{A}\]
\[V(0^-) = R \cdot i_L(0^-) \text{ by Ohm's law}\]
\[V(0^-) = 20 \times 2 = 40 \text{V}\]
By KVL;
\[V_L(0^-) + V(0^-) = 0\]
\[V_L(0^-) = -40 \text{V} = V_L(t)\bigg|_{t=0^-}\]

Observations:
\[t = 0^- \quad t = 0^+\]
\[i_L(0^-) = 2 \text{A} \quad i_L(0^+) = 2 \text{A}\]
\[i_{20}(0^-) = 0 \text{A} \quad i_{20}(0^+) = 2 \text{A}\]
\[V_{20}(0^-) = 0 \text{V} \quad V_{20}(0^+) = 40 \text{V}\]
\[V_L(0^-) = 0 \text{V} \quad V_L(0^+) = -40 \text{V}\]

Conclusion:
To keep the same energy as \(t = 0^-\) and to protect the KCL and KVL in the circuit (i.e., to ensure the stability of the network), the inductor voltage, the resistor current and its voltage can change instantaneously i.e., within zero time at \(t = 0^+\).

(2)
\[
\begin{align*}
\text{For } t \geq 0 \\
i_L(t) &= 2 e^{-5t} \text{A for } 0 \leq t \leq \infty \\
V_L(t) &= -40 e^{-5t} \text{V for } 0 \leq t \leq \infty
\end{align*}
\]

Conclusion:
For all the source free circuits, \(V_L(t) = -\text{ve}\) for \(t \geq 0\), since the inductor while acting as a temporary source (upto \(5\tau\)), it discharges from positive terminal i.e., the current will flow from negative to positive terminals. (This is the must condition required for delivery, by Tellegen’s theorem)

(3) \(V_L(0^+) = -40 \text{V}\)
\[
L \frac{d}{dt} i_L(t)\bigg|_{t=0^-} = -40
\]
\[
\frac{d}{dt} i_L(t)\bigg|_{t=0^-} = -\frac{40}{4} = -10 \text{A/sec}
\]

Check:
\[
i_L(t) = 2 e^{-5t} \text{A for } 0 \leq t \leq \infty
\]
\[
\frac{d}{dt} i_L(t)\bigg|_{t=0^-} = -10 e^{-5t} \text{A/sec for } 0 \leq t \leq \infty
\]
\[
\frac{d}{dt} i_L(t)\bigg|_{t=0^-} = -10 \text{A/sec}
\]
09. Sol:

\[ i_L(0^+) = 2.4 \text{ A} \]
\[ V(0^+) = -96 \text{ V} \]
\[ i_L(t) = 2.4 e^{-10t} \text{ A for } 0 \leq t \leq \infty \]

10. Sol:

\[ V_C(0^+) = 50 \text{ V} ; \ i(0^+) = 62.5 \text{ mA} \]
\[ V_C(t) = 50 e^{-1.2 \times 10^{-3} t} \text{ V for } t \geq 0 \]
\[ i_C = C \frac{dV_C}{dt} \text{ By Ohm's law} \]

11. Sol: Case(i): \( t < 0 \)

\[ V_C(0^-) = 20 \text{ V} \& \ i(0^-) = 0.1 \text{ A} \]

Case(ii): \( t > 0 \)

To find the time constant \( \tau = R_{eq}C \)

After switch closed
\[ R_{eq} = 50 \Omega \quad C = 20 \mu \text{F} \]
\[ i(0^+) = 0 \text{ A} \]
\[ \tau = 50 \times 20 \mu \text{s} \]
\[ \tau = 1 \text{ msec} \]
\[ V_C(t) = V_0 e^{-t/\tau} = 20 e^{-t/1 \text{msec}} \]
\[ V_C(t) = 20 e^{-t/1 \text{msec}} \text{ V; } 0 \leq t \leq \infty \]

12. Sol: After performing source transformation;

By KVL:
\[ 5i_L - 30i_L - 5 \frac{di_L}{dt} = 0 \]
\[ \frac{di_L}{dt} + 5i_L = 0 \]
\[ (D + 5)i_L = 0 \]
\[ i_L(t) = K e^{-5t} \text{ A for } 0 \leq t \leq \infty \]
\[ \tau = \frac{1}{5} \text{ sec} \]
13. Sol: \( i_{L_1}(0) = 10 \, A \); \( i_{L_2}(0) = 2 \, A \)

\[
i_{L_1}(t) = I_0 e^{-\frac{t}{\tau}}
\]

\( \tau = \frac{L}{R} = \frac{1}{1} = 1 \, \text{sec} \)

\( i_{L_1}(t) = 10 e^{-t} \, A \)

Similarly, \( i_{L_2}(t) = I_0 e^{-\frac{t}{\tau}} \)

\( \tau = \frac{L}{R} = 2 \, \text{sec} \)

\( i_{L_2}(t) = 20 e^{-\frac{t}{2}} \, A \)

14. Sol:

\[
\begin{align*}
\text{At } t = 0^-: & \text{ Steady state: A resistive circuit} \\
\text{By Nodal:} & \\
-6 \, mA + \frac{V_C(0^-)}{4 \, K} + \frac{V_C(0^-)}{2 \, K} &= 0 \\
V_C(0^-) &= 8 \, V = V_C(0^+) \\
\end{align*}
\]

For \( t \geq 0 \): A source free circuit

\[
V_s = 6 \, m \times 4 \, K = 24 \, V
\]

\( \tau = R_{eq} C = (5 \, K)2 \mu = 10 \, m \, \text{sec} \)

15. Sol: By KCL:

\[
\begin{align*}
i(t) &= i_R(t) + i_L(t) \\
&= \frac{V_R(t)}{R} + \frac{1}{L} \int_{-\infty}^{t} V_L(t) \, dt \\
&= \frac{V_S(t)}{10} + i_L(0) + \frac{1}{L} \int_{0}^{t} V_S(t) \, dt \\
i(t) &= 4 t + 4 t^3 \\
i(t) \big|_{t=2} &= 8 + 16 + 5 \\
&= 29 \, A \\
&= 29000 \, mA
\end{align*}
\]

16. Ans: (c)
17. Sol:

\[ V_C = 20 e^{-\frac{t}{\tau_C}} \quad \text{V for } 0 \leq t \leq \infty \]

\[ i_L = 20 e^{-\frac{t}{\tau_L}} \quad \text{mA for } 0 \leq t \leq \infty \]

\[ V_L = L \frac{di_L}{dt} \quad \text{By Ohm's law} \]

\[ i_C = C \frac{dV_C}{dt} \quad \text{By Ohm's law} \]

At \( t = 0^- \): steady state: A resistive circuit.

(i) \( t = 0^- \)

\[ V_C(0^-) = 20 \quad \text{V} = V_C(0^+) \]

\[ i_L(0^-) = \frac{20}{1K} = 20 \text{ mA} = i_L(0^+) \]

For \( t \geq 0 \): A source free RL & RC circuit

\[ \tau = \frac{0.1}{1K} = 0.1 \mu\text{sec} \]

\[ \tau_C = 200 \times 10^{-9} \times 10 \times 10^3 = 2 \text{ m sec} \]

\[ \frac{\tau_C}{\tau_L} = 20 \quad \tau_C = 20 \tau_L \]

**Observation:**

\( \tau_L < \tau_C \); therefore the inductive part of the circuit will achieve steady state quickly i.e., 20 times faster.

\[ i_L(0^-) = \frac{12 \times 8}{8 + 4} = 8 \text{A} \]

At \( t = 0^+ \)

Applying KVL in the loop,

\[ 8(4) + 4(8) + V_L = 0 \]

\[ V_L = -64 \]

\[ L \frac{di_L}{dt} = -64 \]

\[ \frac{di_L}{dt} = -32 \text{ A/sec} \]
19. Ans: (c)  
Sol: 
\[ V_c(s) = \frac{5}{s(2Rs + 3)} \] 
\[ V_c(\infty) - V_c(s) = \frac{5}{s} = 0 \] 
\[ V_c(\infty) = V_c(s) + \frac{5}{s} \] 
\[ V_c(\infty) = L \lim_{s \to 0} \left[ \frac{5}{s(2Rs + 3)} + \frac{5}{s} \right] \] 
\[ = \frac{5}{3} + 5 = \frac{20}{3} \] 

21. Ans: (d)  
Sol:  
At \( t < 0 \) 
Apply KVL in loop 1: \( V_c(0^-) - 100 = 0 \) 
\( \Rightarrow V_c(0^-) = 100 \) 

22. Sol:  
Case -1 at \( t = 0^+ \) 
By redrawing the circuit
Current through the battery at \( t = 0^- \) is \( \frac{10}{3} \) Amp

Case -2 at \( t = \infty \)

Current through the battery at \( t = \infty \) is 10 Amp

23. Sol:

\(\begin{align*}
\text{At } t = 0^- & : \text{ Steady state : A resistive circuit} \\
\text{At } t = 0^+ & : \text{ A resistive circuit : Network is in transient state}
\end{align*}\)

(i) \( t = 0^- \):
\( i_L(0^-) = \frac{60}{3} = 20 \text{ A} = i_L(0^+) \)
\( V_{1\Omega} = 20 \text{ V} = V_c(0^+) = V_c(0^+) \)

24. Sol: Repeat the above problem procedure:
\(\begin{align*}
\frac{di_L(t)}{dt} \bigg|_{t=0^+} &= \frac{V_L(0^+)}{L} = 0 \text{ A/sec} \\
\frac{dV_c(t)}{dt} \bigg|_{t=0^+} &= \frac{i_c(0^+)}{C} = -10^6 \text{ V/sec}
\end{align*}\)

25. Sol: Observation: So, the steady state will occur either at \( t = 0^- \) or at \( t = \infty \), that depends where we started i.e., connected the source to the network.
At $t = \infty$: Steady state: A Resistive circuit

$V_{C_1}(\infty) = \frac{100}{50K} \times 40K = 80\, V$

$V_{C_2}(\infty) = \frac{80 \times 3 \mu F}{(2+3) \mu F} = 48\, V$

$V_{C_3}(\infty) = \frac{80 \times 2 \mu F}{5 \mu F} = 32\, V$

26. Sol:

At $t = 0^- :$ Circuit is in Steady state: Resistive circuit

$i_l(0^+) = 3\, A = i_l(0^-)$

$V_{4\Omega} = 4 \times 3 = 12\, V$

$V_{2C(0^-)} = 12\, V$

$V_{C(0^-)} = 8\, V = V_{C(0^+)}$

$V_{2C(0^+)} = \frac{12 \times C}{2C + C}$

$= 4\, V = V_{2C(0^+)}$

and redrawing the circuit

By Nodal:

$\frac{-6}{2} + \frac{4}{2} + i_{2C}(0^+) = 0$

$i_{2C}(0^+) = 2\, A = i_{2C}(0^-)$

$\frac{8 - 12}{4} - i_{2C}(0^+) + 3 + i_c(0^+) = 0$

$i_c(0^+) = 0\, A = i_c(0^-)$

27. Sol: $t = 0^- \quad t = 0^+ \quad t = 0^+$

$i_l(0^-) = 5\, A \quad i_l(0^+) = 5\, A$

$\frac{di_l(0^+)}{dt} = \frac{V_L(0^+)}{L} = 40$
\[ i_R(0^-) = -5 \, \text{A} \quad i_R(0^+) = -1 \, \text{A} \]

\[ \frac{d}{dt} i_R(0^-) = -40 \, \text{A/sec} \]

\[ i_C(0^-) = 0 \, \text{A} \quad i_C(0^+) = 4 \, \text{A} \]

\[ \frac{d}{dt} i_C(0^-) = -40 \, \text{A/sec} \]

\[ V_L(0^-) = 0 \, \text{V} \quad V_L(0^+) = 120 \, \text{V} \]

\[ \frac{d}{dt} V_L(0^+) = 1098 \, \text{V/sec} \]

\[ V_R(0^-) = -150 \, \text{V} \quad V_R(0^+) = -30 \, \text{V} \]

\[ \frac{d}{dt} V_R(0^+) = -1200 \, \text{V/sec} \]

\[ V_C(0^-) = 150 \, \text{V} \quad V_L(0^+) = 150 \, \text{V} \]

\[ \frac{d}{dt} V_C(0^+) = 108 \, \text{V/sec} \]

(i). \( t = 0^- \)

By KCL \( \Rightarrow i_L(t) + i_R(t) = 0 \)

\[ t = 0^- \Rightarrow i_L(0^-) + i_R(0^-) = 0 \]

\[ i_g(0^-) = -5 \, \text{A} \]

\[ V_R(t) = R \, i_R(t) \, \text{By Ohm's law} \]

\[ V_R(0^-) = R \, i_R(0^-) = 30(-5) = -150 \, \text{V} \]

By KVL \( \Rightarrow V_L(t) - V_R(t) - V_C(t) = 0 \)

\[ V_C(0^-) = V_L(0^-) - V_R(0^-) = 150 \, \text{V} \]

(ii). \( t = 0^+ \)

By KCL at 1st node \( \Rightarrow \)

\[ -4 + i_L(t) + i_R(t) = 0 \]

\[ -4 + i_L(0^+) + i_R(0^+) = 0 \]

\[ i_g(0^+) = -i_L(0^+) + 4 \]

\[ i_L(0^+) = -5 + 4 \]

\[ = -1 \, \text{A} \]

\[ V_R(t) = R \, i_R(t) \, \text{By Ohm's law} \]

\[ V_R(0^+) = -30 \, \text{V} \]

By KVL \( \Rightarrow V_L(t) - V_R(t) - V_C(t) = 0 \)

\[ V_L(0^+) = V_R(0^+) + V_C(0^+) \]

\[ = 150 - 30 \]

\[ = 120 \, \text{V} \]

By KCL at 2nd node;

\[ -5 + i_C(t) - i_R(t) = 0 \]

\[ i_C(0^+) = 4 \, \text{A} \]

(iii). \( t = 0^+ \)

By KCL at 1st node \( \Rightarrow \)

\[ -4 + i_L(t) + i_R(t) = 0 \]

\[ 0 + \frac{d}{dt} i_L(t) + \frac{d}{dt} i_R(t) = 0 \]

\[ V_R(t) = R \, i_R(t) \, \text{By Ohm's law} \]

\[ \frac{d}{dt} V_R(t) = R \, \frac{d}{dt} i_R(t) \]

\[ V_R(0^+) = R \, i_R(0^+) \]

By KVL \( \Rightarrow \)

\[ V_L(t) - V_R(t) - V_C(t) = 0 \]

\[ \frac{d}{dt} V_L(t) - \frac{d}{dt} V_R(t) - \frac{d}{dt} V_C(t) = 0 \]

By KCL at node 2:

\[ -5 + i_C(t) - i_R(t) = 0 \]

\[ 0 + \frac{d}{dt} i_C(t) - \frac{d}{dt} i_R(t) = 0 \]

\[ \frac{d}{dt} i_C(0^+) = -(-40) \]

\[ = 40 \, \text{A/sec} \]

\[ \textbf{28.} \]

**Sol:** Transform the network into Laplace domain

\[ i(t) = \frac{1}{sL} \quad R \quad i(s) = \frac{1}{sL} \]

\[ S \, \text{- domain} \]
29. **Sol:** By Time domain approach;

\[ V_C(0^-) = 5 \times 2 = 10 \text{ V} = V_C(0^+) \]

\[
\begin{align*}
\text{At } t = \infty: \text{ Steady state: A resistive circuit} \\
\text{Nodal } \Rightarrow V_C(\infty) - \frac{25}{10} + \frac{V_C(\infty)}{5} - 2 &= 0 \\
V_C(\infty) &= 15 \text{ V} \\
\tau &= R_{eq} C = (5 \parallel 10) \cdot 1 = (10/3) \text{ sec} \\
V_C(t) &= 15 - 5 \cdot e^{-3t/10} \text{ V for } t \geq 0 \\
i_c &= C \frac{dV_c}{dt} = 1.5 \cdot 3e^{-3t/10} \text{ A for } t \geq 0
\end{align*}
\]

30. **Sol:**

\[
\frac{\mathrm{d}V_c}{\mathrm{d}t} = \frac{V}{R} e^{-t/\tau} \text{ for } t \geq 0 \\
i(t) = (1-e^{-t/\tau}) \text{ for } t \geq 0
\]

That is the response is oscillatory in nature

31. **Sol:**

\[
i(0^-) = 0 \text{ A } = i(0^+)
\]

\[
i(\infty) = \frac{V}{R} \text{ A} \\
\tau = \frac{L}{R} \text{ sec} \\
i(t) = \frac{V}{R} + \left(0 - \frac{V}{R}\right) e^{t/\tau} = \frac{V}{R} (1 - e^{-t/\tau})
\]

\[
V_L = \frac{L}{dt} \frac{d}{dt} V = V e^{-Rt/L} \text{ for } t \geq 0
\]

32. **Sol:**

\[
V_C(0^-) = 0 = V_C(0^+) \\
V_C(\infty) = V \\
\tau = RC \\
V_C = V + (0-V) e^{t/\tau} = V(1-e^{-t/\tau}) \text{ for } t \geq 0 \\
i_C = C \frac{dV_C}{dt} = \frac{V}{R} e^{-t/RC} \text{ for } t \geq 0
\]

\[
i(t) = V_C(t) \\
\tau = RC \text{ sec}
\]
33. Sol: It's an RL circuit with $L = 0 \Rightarrow \tau = 0$ sec

\[ i(t) = \frac{V}{R}, \forall t \geq 0 \text{ So, } 5\tau = 0 \text{ sec} \]

\[ i(t) = \frac{V}{R} \]

i.e. the response is constant

34. Sol: $i = \frac{100u(t) - V_L}{10}$

\[ i_1 = \left(10u(t) - \frac{1}{100} \frac{di_L}{dt}\right)A \]

Nodal \Rightarrow

\[-i_1 + i_L - \frac{V_L - 20i_1}{20} = 0 \]

\[-2i_1 + i_L + \frac{1}{200} \frac{di_L}{dt} = 0 \]

Substitute $i_1$;

\[ \frac{di_L}{dt} + 40i_L = 800u(t) \]

\[ SI_L(s) - i_L(0+) + 40I_L(s) = \frac{800}{s} \]

\[ i_L(0^-) = 0A = i_L(0^-) \]

\[ I_L(s) = \frac{800}{s(s + 40)} = \frac{20}{s} - \frac{20}{s + 40} \]

\[ I_1(t) = 20u(t) - 20e^{-40t}u(t) \]

\[ I_L(t) = 20(1-e^{-40t})u(t) \]

\[ i_1 = 10u(t) - \frac{1}{100} \frac{di_L}{dt} \]

\[ i_1 = (10-8e^{-40t})u(t) \]

35. Sol: By Laplace transform approach:

\[ I_c(s) = \left( \frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} \right) \]

\[ \Rightarrow i_c(t) = \frac{1}{4} e^{-t} A \text{ for } t \geq 0 \]

By KVL \Rightarrow \[ V_C(s) - \frac{1}{2s} - \frac{1}{s} I_C(s) = 0 \]
36.

**Sol:** By Time domain approach;

\[ V_C(0) = 6 \text{ V (given)} \]
\[ V_C(\infty) = 10 \text{ V} \]

\[ \tau = R C = 8 \text{ sec} \]
\[ V_C = 10 + (6 - 10) e^{-t/8} \]
\[ V_C = 10 - 4 e^{-t/8} \]
\[ V_C(0) = 6 \text{ V} \]
\[ i_C = C \frac{dV_C}{dt} = e^{-t/8} = i(t) \]

\[ E_{\Delta t} = \int_0^\infty (e^{-t/8})^2 \times 4 \text{ d}t = 16 \text{ J} \]

37.

**Sol:**

![Circuit Diagram]

At \( t = 0^- \): Network is not in steady state i.e., unenergised

\[ i_L(0^-) = 0 \text{ A} = i_L(0^+) \]
\[ V_L(0^+) = 10 \times 10 = 100 \text{ V} \]

At \( t = 0^- \): Network is in transient state: A resistive circuit

\[ i_L(\infty) = 10 \text{ A (since inductor becomes short)} \]
\[ \tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ sec} \]
\[ i_L(t) = 10 + (0 - 10) e^{-t\tau} \]
\[ = 10 (1 - e^{-10 t}) \text{ A for } 0 \leq t \leq \infty \]

\[ V_L(t) = L \frac{d}{dt} i_L(t) = 100 e^{-2t} \text{ V for } 0 \leq t \leq \infty \]

\[ E_L \bigg|_{t=5 \tau \text{ or } t=\infty} = \frac{1}{2} L i^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ J} \]

38. **Ans:** (b)

**Sol:**

![Circuit Diagram]

At \( t = 0^- \): Steady state: A resistive circuit
By KVL ⇒
\[ V - V_{C1}(0^+) = 0 \]
\[ V_{C1}(0^-) = V = V_{C1}(0^+) \]
\[ V_{C2}(0^-) = 0V = V_{C2}(0^+) \]
\[ i_L(0^-) = 0A = i_L(0^+) \]

\[ \begin{align*}
V(s) &= Z(s) \cdot I(s) \\
\text{By KVL in S-domain} \Rightarrow \\
-RI_1(s) - \frac{V}{s} - \frac{I_1(s)}{SC} - SL(I_1(s) - I_2(s)) &= 0 \\
\text{Similarly:} \\
-RI_2(s) - \frac{I_2(s)}{SC} - SL(I_1(s) - I_2(s)) &= 0
\end{align*} \]

39. **Sol:** (b) Transform the network given in fig. (a) into the S-domain.

At \( t = 0^+ \): A resistive circuit: Network is in transient state.
\[ i_1(0^+) = i_2(0^+) \]

By KVL ⇒
\[ -RI_i(0^+) - V - RI_i(0^+) = 0 \]
\[ i_1(0^+) = \frac{-V}{2R} = i_2(0^+) \]

**OBS:** \( i_1(t) = i_1(t) - i_2(t) \)
At \( t = 0^+ \) ⇒
\[ i_L(0^+) = i_1(0^+) - i_2(0^+) \]
\[ = 0A \]
⇒ Inductor: open circuit

40. **Sol:** Evaluation of \( i_L(t) \) and \( e_1(t) \) for \( t \geq 0 \) by Laplace transform approach.
\[ i_L(0^+) = 6A; i_L(\infty) = 4A \]
\[ e_1(0^+) = 8V; e_1(\infty) = 8V \]

Transform the above network into Laplace domain.
S-domain:

\[ E_1(s) = \frac{2\Omega}{s^2 + 8s + 16} \]

Nodal in S-domain

\[ E_1(s) - \frac{16}{s} + \frac{8}{s} + \frac{E_1(s) + 3}{2 + \frac{s}{2}} = 0 \]

\[ E_1(s) = \frac{8\left(s^2 + 6s + 32\right)}{s^2 + 8s + 32} \]

\[ e_1(t) = 8 - 4e^{-4t} \sin 4t \text{ V for } t \geq 0 \]

\[ i_1(t) = 4 + 2e^{-4t} \cos 4t \text{ A} \]

for \( t \geq 0 \), \( \omega_n = 4 \text{ rad/sec} \)

OBS:

\[ \tau = \frac{1}{4}, \quad \zeta = \frac{1}{\omega_n} \]

\[ \omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}\times\frac{1}{8}}} = 4 \]

\[ \frac{1}{4} \times \omega_n = \frac{1}{\zeta} \]

\[ \zeta = \frac{4}{\omega_n} = \frac{4}{4} = 1 \]

\( \zeta = 1 \) (A critically damped system)

41.

\[ t_o = \tan^{-1}\left(\frac{\omega L}{R}\right) \]

\[ t_o = \tan^{-1}\left(\frac{\omega_0}{\omega\tau}\right) \]

\[ 2\pi(50)t_o = \tan^{-1}\left(\frac{2\pi(50)(0.01)}{5}\right) \]

\[ t_o = 32.14 \times \frac{\pi}{180} \text{ msec.} \]

So, by switching exactly at 1.78 msec from the instant voltage becomes zero, the current is free from Transient.

42.

\[ t_o + \phi = \tan^{-1}(\omega_0\tau) + \frac{\pi}{2} \]

\[ 2t_o + \frac{\pi}{4} = \tan^{-1}\left(\frac{2\left(\frac{1}{2}\right)}{1}\right) + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{2} \]

\[ 2t_o = \frac{\pi}{2} \Rightarrow t_o = 0.785 \text{ sec} \]

43. Ans: (a)

Sol. At \( t=0^+ \) the circuit is
Inductor never allows sudden change in current but if we allow the current to suddenly change then impulse voltage will establish redistributing flux and then current become equal in them.

Now solving using Laplace transform.

\[ I(s) [4s] = 4 - 2 \]
\[ = 2 \]
\[ \Rightarrow I(s) = \frac{1}{2s} \]
\[ i(t) = L^{-1}[I(s)] = \frac{1}{2} A \]

46. **Ans: (b)**
**Sol:**
- A – 1: Linearity property
- B – 6: Shift property
- C – 4: Time differentiation property
- D – 3: Integration property

\[ \int_{-\infty}^{t} f(t) \, dt \rightarrow \frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^{0} f(x) \, dx \]

and \[ \int_{0}^{t} f(t) \, dt \rightarrow \frac{F(s)}{s} \]

47. **Ans: (a)**
**Sol:**
- (A) \[ v(t) = u(t) \]
- V(s) = \[ \frac{1}{s} \]
- I(s) = \[ \frac{1}{s+1} \]

- (B) \[ v(t) = r(t) \]
- V(s) = \[ \frac{1}{s^2} \]
- I(s) = \[ \frac{1}{s(s+1)} \]

- (C) \[ v(t) = \delta(t) \]
- V(s) = \[ 1 \]
- I(s) = \[ \frac{s}{s+1} \]

- (D) \[ v(t) = e^{-t} u(t) \]
- V(s) = \[ \frac{1}{s+1} \]
- I(s) = \[ \frac{s}{(s+1)^2} \]

48. **Ans: (a)**
**Sol:**
- Statement (I): True
- Statement (II): True & correct explanation
48.  Ans: (d)  
Sol: 

<table>
<thead>
<tr>
<th>Value of R</th>
<th>Location of poles</th>
<th>i(t), Fig</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) R &gt;&gt; R_C (Over damping)</td>
<td>p_1 = -\sigma_1, p_2 = -\sigma_2</td>
<td>(4)</td>
</tr>
<tr>
<td>(B) R = R_C (Critical damping)</td>
<td>p_1 = p_2 = -\sigma</td>
<td>(3)</td>
</tr>
<tr>
<td>(C) R &lt; R_C (Under damping)</td>
<td>p_1 = \alpha + j \beta, p_2 = \alpha - j \beta, \alpha &lt; 0</td>
<td>(2)</td>
</tr>
<tr>
<td>(D) R = 0 (No damping)</td>
<td>p_1 = j\beta, p_2 = -j\beta</td>
<td>(1)</td>
</tr>
</tbody>
</table>

49.  Ans: (d)  
Sol:  
A.  The internal impedance of an ideal current source is infinity (7).  
Note that for ideal voltage source, the internal impedance is zero.
B.  Attenuated natural oscillations, the poles of the transfer function must lie on the left-hand part of the complex frequency plane, like s = -\alpha, s = -\alpha + j \beta, \alpha > 0 (5)
C.  Maximum power transferred is  
\[
\left( \frac{E}{2R} \right)^2 \times R = \frac{E^2}{4R} \quad (3)
\]
D.  The roots of the characteristic equation give natural response of the circuit. (2)

So the answer is (d)

---

**Conventional Practice Solutions**

01.  Sol:  
The relevant circuit is shown in Fig. for t>0.

\[
\begin{align*}
C &= 1 \mu F \\
\text{Neon lamp ionizes at 20 sec and glows when } v_C(t) &= 75 \text{ V} \\
\text{For } t > 0, \quad v_C(t) &= 100 \left(1 - e^{-\frac{20}{t}}\right), \quad \tau = RC \\
75 &= 100 \left(1 - e^{-\frac{20}{4}}\right), \\
\left(1 - e^{-\frac{20}{4}}\right) &= \frac{3}{4}, \quad e^{-\frac{20}{4}} = \frac{1}{4} \\
\frac{20}{4} &= 5, \quad \frac{20}{\tau} = \log_e 4 \\
\tau &= RC = \frac{20}{\log_e 4} = 14.42 \\
R &= \frac{14.42}{10^{-6}} = 14.42 \text{ M}\Omega
\end{align*}
\]

02.  Sol:  
The given circuit is shown in Fig. 1

The behaviour of the circuit at t = 0⁻ is shown in Fig. 2
The behaviour of the circuit at \( t = 0^+ \) is shown in Fig. 3.

Given \( i_d(0^+) = 1.2 \text{ A} \)

\[ i_L(0^-) = i_s(0^+) \quad \text{and} \quad v_C(0^+) = v_C(0^-) \]  

Let the current through \( 4 \Omega \) be \( i_1 \).

Apply KCL at P.

\[ i_1 = i_L(0^+) - i_s(0^+) \]  

Apply KVL around the mesh APBCA

\[ 24 - 2i_1R - 4i_1 = v_C(0^+) \]

\[ 24 - i_1(2R + 4) = v_C(0^+) \]

Using equations (1), (2), (3) and (4)

\[ 24 = 16i_d(\infty) + 4i_d(\infty) \]

\[ i_d(\infty) = \frac{24}{20} = 1.2 \text{ A} \]

At \( t = \infty \), the behaviour of the circuit is shown in Fig. 4.

Apply KVL around the mesh APCA:

\[ 24 = 16i_d(\infty) + 4i_d(\infty) \]

\[ i_1(\infty) = \frac{24}{20} = 1.2 \text{ A} \]

The negative resistance is not valid.

\[ \therefore R = 8 \Omega \]

At \( t = \infty \), the behaviour of the circuit is shown in Fig. 4.

\[ i(t) = 5 \left( 1 - e^{-\frac{R}{L}t} \right) u(t) - 5 \left[ 1 - e^{-\frac{R}{L}(t-T)} \right] u(t-T) \]
\[ \frac{R}{L} = \frac{2}{5} \times 10^6 = 0.4 \times 10^6 \text{ sec}^{-1} \]

\[ i(t) = 5(1 - e^{-0.4 \times 10^{-6}t}) \]

\[ -5[1 - e^{-0.4 \times 10^6(t - 10 \times 10^{-6})}] u(t - 10 \times 10^{-6}) \]

\[ i(10 \mu s) = 5(1 - e^{-4}) = 4.91 \text{ A} \]

\[ \text{Sol: } \text{The RC circuit and its input are shown in Fig. 1} \]

\[ v(t) = 10 [u(t) - u(t - t_0)], \quad t_0 = 1 \mu \text{sec} \]

The transform equivalent circuit is shown in Fig. 2.

\[ V(s) = 10 \left[ \frac{1}{s} - \frac{e^{-100s}}{s} \right] \]

\[ Z(s) = 0.5 + \frac{1}{2s} = \frac{s + 1}{2s} \]

\[ I(s) = \frac{10s}{s} (1 - e^{-100s}) \left( \frac{2s}{s + 1} \right) \]

\[ = \frac{20}{s + 1} - \frac{20}{s + 1} e^{-60s} \]

\[ i(t) = 20 e^{-t} u(t) - 20 e^{-(t-10^{-6})} u(t-10^{-6}) \]

\[ t = 0, \quad i(0^+) = 20 \text{ A} \]

\[ t = 1 \mu \text{sec}, \quad i(1^-) = 20 \exp(-10^{-6}) \]

and \[ i(1^+) = 20 \exp(-10^{-6}) - 20 \]

The variation of \( i(t) \) is shown in Fig. 3.

\[ \text{Sol: } \text{Given } v(t) \text{ can be expressed as follows} \]

\[ V(t) = \frac{1}{2} [r(t) - r(t - t_0)] \]

\[ = \frac{1}{2} [r(t) - r(t - 2)] \]

\[ V(s) = \frac{1}{2s^2} (1 - e^{-2s}) \]

Converting everything into Laplace domain
\[ I(s) = \frac{1}{10} \left[ \frac{1}{s} - \frac{1}{s + 5} \right] - \frac{1}{10} \left[ \frac{1}{s} - \frac{1}{s + 5} \right] e^{-2s} \]

Taking inverse Laplace transform

\[ i(t) = \frac{1}{10} \left[ u(t) - e^{-5t} u(t) \right] - \frac{1}{10} \left[ u(t - 2) - e^{-5(t - 2)} u(t - 2) \right] \]

So,

\[ i(t) = \frac{1}{10} \left[ u(t) - u(t - 2) + e^{-5t} \left[ u(t - 2) - u(t) \right] \right] \]

06

Sol: The RLC series circuit is shown in Fig. 1.

\[ I(s) = \frac{V(s)}{Z(s)} \]

\[ Z(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2 + RCs + 1}{Cs} \]

\[ = \frac{L \left( s^2 + \frac{R}{L}s + \frac{1}{LC} \right)}{s} \]

\[ = \frac{s^2 + 3s + 2}{s} = \frac{(s + 1)(s + 2)}{s} \]

(i) For \( v(t) = r(t) \), \( V(s) = 1/s^2 \)

\[ I_r(s) = \frac{1}{s(s + 1)(s + 2)} = \frac{1/2}{s} + \frac{-1}{s + 1} + \frac{1/2}{s + 2} \]

\[ \Rightarrow i_r(t) = \left( \frac{1}{2} - \frac{1}{s}e^{-t} + \frac{1}{2}e^{-2t} \right) u(t) \]

When \( r(t) \) is magnitude scaled by 12 and delayed by 2, i.e., \( v(t) = 12r(t - 2) \)

\[ i(t) = 12 \left[ \frac{1}{2} - \frac{1}{s}e^{-1(t - 2)} + \frac{1}{2}e^{-2(t - 2)} \right] u(t - 2) \]

(ii) For \( v(t) = 1u(t) \)

\[ I_v(s) = \frac{1}{(s + 1)(s + 2)} = \frac{1}{s + 1} + \frac{-1}{s + 2} \]

\[ \Rightarrow i_v(t) = (1 e^{-1t} - 1 e^{-2t}) u(t) \]

When \( v(t) = 2 u(t - 3) \)

\[ i(t) = 2 \left( e^{(t-3)} - e^{-2(t-3)} \right) u(t - 3) \]

(iii) For \( V(t) = \delta(t) \Rightarrow V(s) = 1 \)

\[ \Rightarrow I_\delta(s) = \frac{s}{(s + 1)(s + 2)} \]

\[ = \frac{-1}{s} + \frac{2}{s + 1} + \frac{2}{s + 2} \]

\[ \Rightarrow i_\delta(t) = \frac{-1}{u(t)} + \frac{2}{u(t)} \]

When \( V(t) = 3\delta(t - 1) \)

\[ i(t) = -3e^{(t-1)} u(t-1) + 6e^{-(t-1)} u(t-1) \]

07.

Sol: 

\[ \begin{array}{c}
\text{S-domain} \\
\text{R L C}
\end{array} \]

\[ \begin{array}{c}
\text{Fig.1} \\
\text{R = 3 \Omega, L = 1 H, C = 0.5 F}
\end{array} \]

\[ \frac{200\Omega}{0.1H} \quad \frac{100\mu F}{200V} \]

\[ \frac{200\Omega}{1 \text{CS}} \]

\[ 200V \]

\[ \frac{200}{S} \]

\[ \frac{200}{S} = I(S) \left( R + LS + \frac{1}{CS} \right) \]

\[ \frac{200}{S} = I(S) \left( RCS + LCS^2 + \frac{1}{CS} \right) = L \left( S^2 + \frac{R}{L} S + \frac{1}{LS} \right) \]
\[
I(S) = \frac{200}{L} \left[ \frac{1}{S^2 + \frac{R}{L}S + \frac{1}{LC}} \right]
\]
\[
I(S) = \frac{200}{0.1} \left[ \frac{1}{S^2 + \frac{200}{0.1}S + \frac{1}{0.1 \times 100 \times 10^8}} \right]
\]
\[
I(S) = \frac{2000}{S^2 + 2000S + 10^8}
\]
\[
eq 2000 \left[ \frac{1}{(S - 52)(S + 1948)} \right]
\]
i(t) = 1.055[e^{-52t} - e^{-1948t}]

08.

Sol:

For t < 0, K is opened
At t = 0, L is short circuit,

\[
i_1(0^-) = \frac{10}{2.5} = 4A = I_1(0^-)
\]
\[
i_2(0^+) = \frac{I_1(0)}{2} = 2A
\]

For t > 0 K is closed

\[
I(S) = \frac{10}{S + 4} = \frac{10}{S(S + 2)} + \frac{4}{S + 2}
\]
\[
I(S) = 5 \left( \frac{1}{S} - \frac{1}{S + 2} \right) + \frac{4}{S + 2}
\]
\[ V_1(S) = I(S) \]
\[ V_\infty(S) = V_{th}(S) = 2 + \frac{10}{S(S+2)} + \frac{4}{(S+2)} \]
\[ = \frac{2(S^2 + 4S + 5)}{S(S+2)} \]

For \( R_{th} \):
\[ Z_{th} = \frac{(1+S)+(S+2)S}{(S+2)} = \frac{S^2 + 3S + 1}{(S+2)} \]

Thevenin’s equivalent circuit is

\[ I(S) = \frac{V_{th}}{R_{th} + 1} = \frac{2(S^2 + 4S + 5)}{S(S+2) + 1} \]

\[ I(S) = \frac{2(S^2 + 4S + 5)/S}{S^2 + 3S + 1 + S + 2} \]

\[ I(S) = \left[ \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+3} \right] \]
\[ = \left[ \frac{10/3}{S} + \frac{-2}{S+1} + \frac{2/3}{S+3} \right] \]
\[ i(t) = \left[ \frac{10}{3} - 2e^{-t} + \frac{2}{3}e^{-3t} \right] u(t) \]

**09. Sol:**

**Fig.**

\[ v(t) = L \frac{di(t)}{dt} + R \, i(t) \]
\[ L \frac{di}{dt} + R \, i(t) = E \sin(\omega t + \phi) \]

Phasor voltage, \( \vec{V} = E \, e^{j\phi} \)

Phasor current, \( \vec{I} = \frac{\vec{V}}{Z} = \frac{E \, e^{j\phi}}{R + j\omega L} \]

\[ = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \, e^{j(\phi - \tan^{-1}(\frac{\omega L}{R}))} \]

\[ i_{ss}(t) = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \sin \left[ \omega t + \phi - \tan^{-1}\left( \frac{\omega L}{R} \right) \right] \]

The transient part of current is obtained from the homogeneous equation:

\[ \frac{di(t)}{dt} + \frac{R}{L} \, i(t) = 0, \quad i_{ss}(t) = Ke^{\frac{-R}{L}t} \]

\[ i_{ss}(t) = Ke^{\frac{-R}{L}t} + i_{ss}, \quad i_{ss}(0) = 0 \]

\[ K = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \sin \left[ \phi - \tan^{-1}\left( \frac{\omega L}{R} \right) \right] \]

There is no transient in the current if \( i_{ss}(t) = 0 \) or \( K = 0 \) or \( \phi = \tan^{-1}\left( \frac{\omega L}{R} \right) \)
10. 

Sol: 
\[ \frac{1}{dt} \frac{di_1}{dt} + i_1(t) - \frac{2}{dt} \frac{di_2}{dt} = 5 \]

\[ -2 \frac{di_1}{dt} + 4 \frac{di_2}{dt} + i_2(t) = 0 \]

\[ i_1(t) \rightarrow I_1(s), \ i_2(t) \rightarrow I_2(s) \]

\[ (s+1)I_1 - 2sI_2 = \frac{5}{s} \]

\[ -2sI_1 + (4s + 1)I_2 = 0 \]

\[ I_1(s) = \frac{\Delta_1}{\Delta} \]

\[ \Delta_1 = \begin{vmatrix} 5 & -2s \\ 0 & 4s + 1 \end{vmatrix} = \frac{5(4s+1)}{s} \]

\[ \Delta = \begin{vmatrix} s+1 & -2s \\ -2s & 4s+1 \end{vmatrix} = 5s+1 = 5(s+0.2) \]

\[ I_1(s) = \frac{4(s+0.25)}{s(s+0.2)} = \frac{5}{s} - \frac{1}{s + 0.25} \]

\[ i_1(t) = (5 - e^{-0.2t})u(t) \]
**Objective Practice Solutions**

### 01.

**Sol:**

\[ I_{\text{avg}} = I_{dc} = \frac{1}{T} \int_{0}^{T} i(t) \, dt \]

\[ = 3 + 0 + 0 = 3 \text{A} \]

\[ I_{\text{rms}} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2(t) \, dt} \]

\[ = \sqrt{3^2 + \left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2} = 5\sqrt{2} \text{A} \]

### 02.

**Sol:**

\[ V_{dc} = V_{avg} = \frac{1}{T} \int_{0}^{T} V(t) \, dt = 2 \text{V} \]

Here the frequencies are same, by doing simplification

\[ v(t) = 2 - 3\sqrt{2} \left(\cos 10t \times \frac{1}{\sqrt{2}} - \sin 10t \times \frac{1}{\sqrt{2}}\right) + 3\cos 10t \]

\[ = 2 + 3\sin 10t \text{ V} \]

So \[ V_{\text{rms}} = \sqrt{(2)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{8.5} \text{ V} \]

### 03.

**Sol:**

\[ X_{\text{avg}} = X_{dc} = \frac{1}{T} \int_{0}^{T} x(t) \, dt = 0 \]

\[ X_{\text{rms}} = \sqrt{\frac{1}{T} \int_{0}^{T} x^2(t) \, dt} = \frac{A}{\sqrt{3}} \]

### 04.

**Ans:** (a)

**Sol:** For a symmetrical wave (i.e., area of positive half cycle = area of negative half cycle.) The RMS value of full cycle is same as the RMS value of half cycle.

### 05.

**Sol:** Complex power, \[ S = VI^* \]

\[ \Rightarrow I = \frac{300 \angle 0^\circ}{2 + j12.5 + 4 - j8} \]

\[ \Rightarrow I = 40 \angle 36.86^\circ \]

\[ \therefore \text{Complex power, } S = VI^* = 300 \angle 0^\circ \times 40 \angle 36.86^\circ = 9600 + j7200 \]

\[ \therefore \text{Reactive power delivered by the source } Q = 72000 \text{ VAR} = 7.2 \text{ KVAR} \]

### 06.

**Sol:**

\[ Z = j1 + (1-j1)|(1+j2) = 1.4 + j0.8 \]

\[ I = \frac{E_{1}}{Z} \text{ by ohm's law} = \frac{10 \angle 20^\circ}{1.4 + j8} \]

\[ = 6.2017 \angle -9.744^\circ \text{ A} \]

\[ I_{1} = \frac{I(1+j2)}{1-j1+1+j2} = 6.2017 \angle 27.125^\circ \text{ A} \]
\[I_2 = \frac{I(1 - j)}{1 - j\omega + 1 + j2}
\]
\[= 3.922 \angle -81.31^\circ \text{A}
\]
\[E_2 = (1-j)I_1 = 8.7705 \angle -17.875^\circ \text{V}
\]
\[E_0 = 0.5I_2 = 1.961 \angle -81.31^\circ \text{V}
\]

07. Sol: Since two different frequencies are operating on the network simultaneously always the super position theorem is used to evaluate the response.

By SPT: (i)

\[
\begin{align*}
\text{Network is in steady state, therefore the network is resistive. } I_{R1}(t) &= \frac{10}{2} = 5\text{A} \\
\text{(ii) Network is in steady state}
\end{align*}
\]

As impedances of L and C are present because of \(\omega = 2\). They are physically present.
\[Z_L = j\omega L; Z_C = \frac{1}{j\omega C} |_{\omega = 2}
\]

08. Ans: (c)

\[
\begin{align*}
v_0 &= 0, v_1 = 6.32 \angle 18.44^\circ \\
I(s) &= \frac{1}{s^2 + 1} - I(s) \left(2 + 2s + \frac{1}{s}\right) = 0 \\
I(s) &= \frac{1}{s^2 + 1} \\
i(t) &= \cos t + \frac{2d^2i}{dt^2} + \frac{2di}{dt} = \cos t \\
2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) &= \cos t
\end{align*}
\]

09. Sol: 
\[V = \sqrt{V_R^2 + (V_L - V_C)^2}
\]
\[V = V_R = 1.8 \text{V} \\
100 = 1.20; I = 5\text{A}
\]

Power factor = \(\cos \phi = \frac{V_R}{V} = \frac{V_R}{V_R} = 1
\]

So, unity power factor.

10. Sol: By KCL in phasor – domain
\[ I_1 = I_C = \frac{V}{Z_C} = \frac{V}{X_C} \angle 90^\circ \]
\[ I_2 = \frac{V}{2 + j\omega L} = \frac{V}{2 + j2} = \frac{V}{2\sqrt{2}} \angle 45^\circ \]

Therefore, the phasor \( I_1 \) leads \( I_2 \) by an angle of 135°.

14.
Sol: \( I_2 = \sqrt{I_R^2 + I_C^2} \Rightarrow 10 = \sqrt{I_R^2 + 8^2} \)

\[ I_R = 6A \]
\[ I_I = I = \sqrt{I_R^2 + (I_L - I_C)^2} \]
\[ 10 = \sqrt{6^2 + (I_L - I_C)^2} \]
\[ I_L - I_C = \pm 8A \]
\[ I_L = 8\pm 8 \]
\[ I_L - 8 = -8 \text{(Not acceptable)} \]
Since \( I_L = \frac{V}{Z_L} \neq 0 \).
\[ I_L = 8 \]
\[ I_L = 16A \]
\[ I_L > I_C \]

I_2 leads 120° by \( \tan^{-1}\left(\frac{8}{6}\right) \)

I_1 lags 120° by \( \tan^{-1}\left(\frac{8}{6}\right) \)
Power factor \( \cos \phi = \frac{I_R}{I} = \frac{I_R}{I} \)
\( = \frac{6}{10} = 0.6 \text{ (lag)} \)

15. Sol:

\[ |I_C| = \frac{V}{Z_C} = \frac{300 \angle 0^\circ}{(1/j\omega)} = V_{oc} \]
\( = 300 \times 2\pi \times 50 \times 159.23 \times 10^{-6} \)
\( I_C = 15A \)
\( I = \sqrt{I_R^2 + I_C^2} \)
\( 25 = \sqrt{I_R^2 + 15^2} \)
\( I_R = 20A \)

V_R = R I_R \text{ (By ohm’s law)}
300 = R \times 20
R = 15 \Omega

Network is in steady state
\( I_R = \frac{360}{15} = 24A \)

So the required \( I_C = \sqrt{25^2 - 24^2} \)
\( \text{voc} = 7 \)
\( 360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7 \)
\( f = 19.4\text{Hz} \)

OBS: \( I_C = \frac{V}{Z_C} \)

16. Sol:

\( P_{\Omega} = 10\text{Watts (Given)} \)
\( = P_{\text{avg}} = I_{\text{rms}}^2 R \)
\( 10 = I_{\text{rms}}^2 \times 5 \)
\( I_{\text{rms}} = \sqrt{2} \text{A} \)

Power delivered = Power observed
(By Tellegen’s Theorem)
\( P_T = I_{\text{rms}}^2 (5 + 10) \)
\( V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi = (\sqrt{2})^2 (15) \)
\( \frac{50}{\sqrt{2}} \times \sqrt{2} \cos \phi = 2 \times 15 \)
\( \cos \phi = 0.6 \text{ (lag)} \)

17. Ans: (d)

Sol:

\( V_L = 14V \)
\( V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{3^2 + (14 - 10)^2} \)
\( V = 5\text{V} \)

18. Sol:

\( Y = Y_L + Y_C = \frac{1}{Z_L} + \frac{1}{Z_C} \)
\( = \frac{1}{30 \angle 40^\circ} + \frac{1}{j\omega} \)
= jωc + \frac{1}{30} \angle -40^0
= jωc + \frac{1}{30} (\cos 40^0 - j\sin 40^0)

Unit power factor \Rightarrow j^{-}\text{term} = 0
ωc = \frac{\sin 40^0}{30}
C = \frac{\sin 40^0}{2\pi \times 50 \times 30} = 68.1\mu F

C = 68.1\mu F

19. Ans: (b)
Sol: To increase power factor shunt capacitor is to be placed.
VAR supplied by capacitor
= P (\tan \phi_1 - \tan \phi_2)
= 2 \times 10^3 [\tan (\cos^{-1} 0.65) - \tan (\cos^{-1} 0.95)]
= 1680 VAR

VAR supplied = \frac{V^2}{X_C} - V^2 ωC = 1680

\therefore \quad C = \frac{1680}{(115)^2 \times 2 \pi \times 60} = 337\mu F

20.
Sol: \quad Z = \frac{V}{I} = 160\angle 10^0 - 90^0 = 32\angle 30^0

\phi = 30^0 \text{ (Inductive)}

V_{rms} = \frac{160}{\sqrt{2}} \text{ V}_{j}, I_{rms} = \frac{5}{\sqrt{2}}

Real power (P) = \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos 30^0
= 200 \sqrt{3} \text{ W}

Reactive power (Q) = \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \frac{1}{2}
= 200 \text{ VAR}

Complex power = P + jQ = 200(\sqrt{3} + j1) \text{ VA
I_L > I_C: Inductive nature of the circuit.

\[ I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{4^2 + 3^2} = 5 \text{A} \]

Power factor = \( \frac{I_R}{I} = \frac{4}{5} = 0.8 \) (lagging)

25. **Ans:** (a)

**Sol:**

\[ \begin{align*}
I_1 &= \frac{100 \angle 0^\circ}{(3 + j4) \Omega} \implies |I_1| = 20 = I_{1\text{rms}} \\
I_2 &= \frac{100 \angle 0^\circ}{(1 - j) \Omega} \implies |I_2| = \frac{100}{\sqrt{2}} = I_{2\text{rms}} \\
P &= P_1 + P_2 \\
&= (I_{1\text{rms}})^2 \cdot 3 + (I_{2\text{rms}})^2 \cdot 1 \\
&= 20^2 \cdot 3 + \left(\frac{100}{\sqrt{2}}\right)^2 \cdot 1 \\
P &= 6200 \text{ W} \\
Q &= Q_1 + Q_2 = (I_{1\text{rms}})^2 \cdot 4 + (I_{2\text{rms}})^2 \cdot 1 \\
&= 3400 \text{VAR} \\
S &= P + jQ = (6200 + j3400) \text{VA}
\end{align*} \]

NW is in Steady state.

\[ V = 100 \angle 0^\circ \implies V_{\text{rms}} = 100 \text{V} \]

\[ I_1 = \frac{100 \angle 0^\circ}{(3 + j4) \Omega} \implies I_{1\text{rms}} = 20 \text{A} \]

\[ I_2 = \frac{100 \angle 0^\circ}{(1 - j) \Omega} \implies I_{2\text{rms}} = \frac{100}{\sqrt{2}} \text{A} \]

\[ P = P_1 + P_2 \\
= (I_{1\text{rms}})^2 \cdot 3 + (I_{2\text{rms}})^2 \cdot 1 \\
= 20^2 \cdot 3 + \left(\frac{100}{\sqrt{2}}\right)^2 \cdot 1 \\
P &= 6200 \text{ W} \\
Q &= Q_1 + Q_2 = (I_{1\text{rms}})^2 \cdot 4 + (I_{2\text{rms}})^2 \cdot 1 \\
&= 3400 \text{VAR} \\
S &= P + jQ = (6200 + j3400) \text{VA} \]

26. **Sol:**

\[ \text{Power dissipation} = I_{\text{rms}}^2 \cdot R = 5^2 \times 10 = 250 \text{ W} \]

27. **Ans:** (c)

**Sol:**

\[ I_{\text{rms}} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{25} = 5 \text{A} \]

\[ Q = Q_1 + Q_2 = (I_{1\text{rms}})^2 \cdot 4 + (I_{2\text{rms}})^2 \cdot 1 \\
= 3400 \text{VAR} \\
S = P + jQ = (6200 + j3400) \text{VA} \]

28. **Sol:**

\[ X_C = X_L \implies \omega = \omega_0, \text{ the circuit is at resonance} \]

\[ V_C = QV_S \angle -90^\circ \]

\[ Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2 \\
= \frac{1}{\omega_0 cR} = X_C = 2 \\
\Rightarrow V_C = 200 \angle -90^\circ = -j200 \text{V} \]
29. Sol: Series RLC circuit
f = f_L, PF = cos φ = 0.707 (lead)
f = f_H, PF = cos φ = 0.707 (lag)
f = f_0, PF = cos φ = 1

30. Ans: (b)
Sol: Network is in steady state (since no switch is given)

1×10^{-3} < 0^0 A

Let I = 1mA
ω = ω_0 (Given)
⇒ I_R = 1
I_L = QI < -90^0 = -jQI
I_C = QI < 90^0 = jQI
I_L + I_C = 0
|I_R + I_L| = |I - jQI|
|I_R + I_C| = |I + jQI|

31. Ans: (c)
Sol: Since; “I” leads voltage, therefore capacitive effect and hence the operating frequency (f < f_0)

32. Sol: Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j/\omega C}
= \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{R_C + j/\omega C}{R_C^2 + (1/\omega C)^2}

33. Sol:
The given circuit is shown in Fig.
Z_{AB} = 10 + Z_1
where, Z_1 = \left(\frac{j}{\omega}\right) || \left(\frac{j4\omega - j}{\omega}\right)
= \frac{-j}{\omega} + j4\omega - \frac{j}{\omega}
= 4 - \frac{1}{\omega^2}
\frac{j4\omega - j2}{\omega}

For circuit to be resonant i.e., \frac{1}{\omega^2} = \frac{1}{4}
\omega = \frac{1}{2} = 0.5 \text{ rad/sec}
\therefore \omega_{\text{resonance}} = 0.5 \text{ rad/sec}
34. Sol: (i) \( \frac{L}{C} = R^2 \Rightarrow \) circuit will resonate for all the frequencies, out of infinite number of frequencies we are selecting one frequency.

i.e., \( \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2} \) rad/sec

then \( Z = R = 2\Omega \).

\[
I = \frac{V}{Z} = \frac{10\angle 0^0}{2} = 5\angle 0^0
\]

\[i(t) = 5\cos \frac{t}{2} A\]

\[Z_L = j\Omega_0 L = j2\Omega; \quad Z_C = \frac{1}{j\Omega_0 C} = -j2\Omega\]

\[I_L = \frac{1}{2} I - j\frac{\sqrt{2}}{2} I = \frac{1}{\sqrt{2}} I \angle -45^0\]

\[i_L = \frac{5}{\sqrt{2}} \cos \left( \frac{t}{2} - 45^0 \right) A\]

\[I_c = \frac{1}{2} I + j\frac{\sqrt{2}}{2} I = \frac{1}{\sqrt{2}} I \angle 45^0\]

\[i_c = \frac{5}{\sqrt{2}} \cos \left( \frac{t}{2} + 45^0 \right) A\]

\[P_{avg} = I_{L\text{rms}}^2 R + I_{C\text{rms}}^2 R = \left( \frac{5}{\sqrt{2}} \right)^2 R + \left( \frac{5}{\sqrt{2}} \right)^2 R = 25 \text{ watts}\]

(ii) \( \frac{L}{C} \neq R^2 \) circuit will resonate at only one frequency.

i.e., at \( \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4} \) rad/sec

Then \( Y = \frac{2R}{R^2 + \frac{L}{C}} \) mho

\[Y = \frac{2(2)}{2^2 + \frac{4}{4}} = \frac{4}{5} \text{ mho}\]

\[Z = \frac{5}{4} \Omega\]

\[I = \frac{V}{Z} = \frac{10\angle 0^0}{5/4} = 8\angle 0^0\]

\[i(t) = 8\cos \frac{t}{4} A\]

\[Z_L = j\omega_0 L = j1\Omega; \quad Z_C = \frac{1}{j\omega_0 C} = -j1\Omega\]

\[I_L = \frac{1}{4} I - j\frac{\sqrt{2}}{4} I = \frac{1}{2\sqrt{2}} I \angle -45^0\]

\[i_L = \frac{5}{2\sqrt{2}} \cos \left( \frac{t}{2} - 45^0 \right) A\]

\[I_c = \frac{1}{4} I + j\frac{\sqrt{2}}{4} I = \frac{1}{2\sqrt{2}} I \angle 45^0\]

\[i_c = \frac{5}{2\sqrt{2}} \cos \left( \frac{t}{2} + 45^0 \right) A\]

\[P_{avg} = I_{L\text{rms}}^2 R + I_{C\text{rms}}^2 R = \left( \frac{2\sqrt{5}}{\sqrt{2}} \right)^2 R + \left( \frac{2\sqrt{5}}{\sqrt{2}} \right)^2 R = 40 \text{ watts}\]

35. Sol: (i) \( Z_{ab} = 2 + (Z_L \parallel Z_C \parallel 2) \)

\[= 2 + jX_L - jX_C \parallel 2 = 2 + 2X_L X_C (X_L X_C - j2(X_L - X_C)) \]

\[= \left( \frac{X_L X_C}{X_L - X_C} \right)^2 + 4(X_L - X_C)^2 \]

\[j\text{-term} = 0 \Rightarrow -2(X_L - X_C) = 0\]

\[X_L = X_C\]
\[ \omega_0L = \frac{1}{\omega_0C} \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.4}} = \frac{1}{4} \text{ rad/ sec} \]
At resonance entire current flows through 2Ω only.

(ii) \[ Z_{ab} \bigg|_{\omega = \omega_0} = 2 + 2 = 4\Omega \]
\[ X_L = X_C \]

(iii) \[ V_i(t) = V_m \sin \left( \frac{t}{4} \right) \]
\[ Z = 4\Omega \]
\[ i(t) = \frac{V_i(t)}{Z} = \frac{V_m}{2} \sin \left( \frac{t}{4} \right) = i_R \]
\[ V = 2i_R = \frac{V_m}{2} \sin \left( \frac{t}{4} \right) \]
\[ V = V_c = V_L \]

\[ i_C = C \frac{dV_C}{dt} = \frac{V_m}{2} \cos \left( \frac{t}{4} \right) \]
\[ i_C = \frac{V_m}{2} \sin \left( \frac{t}{4} + 90^\circ \right) \]
\[ i_L = \frac{1}{L} \int V_L \, dt = -\frac{V_m}{2} \cos \left( \frac{t}{4} \right) \]
\[ i_L = \frac{V_m}{2} \sin \left( \frac{t}{4} - 90^\circ \right) \]

OBS: Here \( i_L + i_C = 0 \)
\( \Rightarrow \) LC Combination is like an open circuit.

36. Ans: (d)
Sol:
\[ S = V.I \]
\[ = V \cdot \frac{V}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} \]
\[ S = \frac{V^2}{R^2 + (\omega L)^2} - \frac{V^2j\omega L}{R^2 + (\omega L)^2} \]
\[ S = P + jQ \]
Active power \( (P) = \frac{V^2}{R^2 + (\omega L)^2} \)
\[ P = \frac{V^2}{R^2(1 + Q^2)} \]
As \( Q \) is doubled, \( P \) decreases by four times.

37. Sol:
\[ Z_C = \frac{1}{j\omega C} \]
\( \omega = 0, Z_C = \infty \) \( \Rightarrow \) C: open circuit \( \Rightarrow i_2 = 0 \)
\( \omega = \infty, Z_C = 0 \) \( \Rightarrow \) C: Short Circuit \( \Rightarrow i_2 = \frac{E_m}{R_2} \angle 0^\circ \)

Transform the given network into phasor domain.

Network is in phasor domain.
By KCL in P-d \( \Rightarrow I = I_1 + I_2 \)
\[ I_1 = \frac{E_m \angle 0^\circ}{R_1} \]
\[ I_2 = \frac{E_m \angle 0^\circ}{R_2 + \frac{1}{j\omega C}} = \frac{E_m \angle 0^\circ}{R_2 - \frac{1}{\omega C}} \]
\[ I_2 = \frac{E_m \angle 90^\circ}{\sqrt{R^2 + \frac{1}{\omega C}}} \]

\[ \omega = \infty \Rightarrow I_2 = \frac{E_m \angle 0^\circ}{R_2} \]

\[ \omega = 0 \Rightarrow I_2 = 0 \text{A} \]

\( \omega \) (0 and \( \infty \)) the current phasor \( I_2 \) will always lead the voltage \( E_m \angle 0^\circ \).

39.

Sol: \( I = I_1 + I_2 \)

\[ I_1 = \frac{E_m \angle 0^\circ}{R_1} \]

\[ I_2 = \frac{E_m \angle \theta}{R_2 + j\omega L} \]

\[ = \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R_2}\right) \]

(i) If \( \omega \) Varied

R_2 = 0 \Rightarrow I_2 \angle 0^\circ

R_2 = \infty \Rightarrow I_2 = 0 \text{A}
ii. If “R₂” is varied

\[ I_R = 0 \text{ is the bridge is balanced. i.e., } Z_1 Z_4 = R_2 R_3 \]

Where \( Z_i = R_i + j\omega L_i \),

\[ Z_4 = R_4 + \frac{j}{\omega C_4} \]

As \( R_2 R_3 \) is real, imaginary part of \( Z_i; Z_4 = 0 \)

\[ \omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \quad \text{or} \quad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4} \]

or \( Q_i = Q_4 \)

Where \( Q \) is the Quality factor.

40. Ans: (a)
Sol: The given circuit is a bridge.

During negative half cycle \( D_2 \) is forward biased, \( D_1 \) is reverse biased so current does not flow though ammeter.

\[ \text{Half wave rectifier waveform} \]

\[ I_{0_{\text{avg}}} = \frac{I_m}{\pi} = \frac{V_m}{R\pi} = \frac{4}{10k \times \pi} \]

42. Ans: (d)
Sol: For \(-V_0 \sin \omega_0 t \rightarrow I_1 = \frac{V_0}{\omega_0 L} = I_0\)

For \(2V_0 \sin \omega_0 t \rightarrow I_2 = \frac{2V_0}{2\omega_0 L} = I_0\)

For \(3V_0 \sin \omega_0 t \rightarrow I_3 = \frac{3V_0}{3\omega_0 L} = I_0\)

For \(4V_0 \sin \omega_0 t \rightarrow I_4 = \frac{4V_0}{4\omega_0 L} = I_0\)

RMS value \( = \sqrt{4I_0^2} = 2I_0\)

43. Ans: (b)
Sol:

During positive half cycle of supply \( D_1 \) is forward biased, \( D_2 \) is reverse biased so current flows through the ammeter.
\[ V^2 = V_R^2 + V_L^2 \]
\[ \Rightarrow 100 = V_R^2 + 36 \]
\[ \Rightarrow V_R = 8 \text{V} \]
\[ I_R = \frac{V_R}{R} = \frac{8}{2} = 4 \text{A} \]

44. Ans: (b)
Sol: Full wave rectifier
Here each half of secondary winding will received \(2\sin \omega t\)
\[ V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \]
\[ P_{\text{avg}} = \frac{V_{\text{RMS}}^2}{R} = \frac{\left(\sqrt{2}\right)^2}{10} = 0.2 \text{W} \]

45. Ans: (b)
Sol: Complex power,
\[ S = \bar{V} \bar{I}^* = (100 - j50) (3 + j4) \]
\[ = 300 + 200 + j250 = 500 + j250 \]
True power = \(Re[\bar{V} \bar{I}^*] = 500 \text{W} \)
Reactive power
\[ = \text{Im}[\bar{V} \bar{I}^*] = 250 \text{W} \]
So Statement (I) is True, Statement (II) is also True, but Statement (II) is not the correct explanation.

46. Ans: (d)
Sol: In series RLC circuit,
i(t) is maximum at resonance frequency,
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]
\[ I_{\text{max}} = \frac{V_S}{R} \]
\[ V_C = \frac{V_S}{\omega C \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \]
\[ = V_S \text{ for } \omega = 0 \]
\[ = Q V_S \text{ for } \omega = \omega_0 \]
\[ = 0 \text{ for } \omega \to \infty \]

\(V_C\) is maximum at \(\omega = 0\) (i.e., \(\omega < \omega_0\)) provided \(Q < 1\)
Statement (I) is false, but statement (II) is true if \(Q < 1\)

47. Ans: (a)
Sol: When the input impedance is purely resistive, the voltage and current are in phase.
Note that at resonance, power factor is also unity.

48. Ans: (c)
49. Ans: (c)
Sol: At resonance, the power factor of circuit is unity.
Hence statement (II) is false.

50. Ans: (c)
Sol: \(\omega_{\text{res}} = \omega_0 \sqrt{\frac{L - R_1^2 C}{L - R_2^2 C}}\), \(\omega_0 = \frac{1}{\sqrt{LC}}\)
Resonance occurs at all frequencies, if
\[ R_1^2 = R_2^2 = \frac{L}{C} \]
and the resonant impedance
\[ = R_1 = R_2 = \frac{\sqrt{L}}{C} \]
\(\therefore\) Statement (I) is True, Statement (II) is False
51. Ans: (a)
Sol: \( G(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{j\omega C + \frac{1}{j\omega R C}} = \frac{1}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega R C} \)
\[ = 1 - 0 \,, \; \omega = 0 \]
\[ = 0.707 e^{-j45^\circ} \,, \; \omega = \frac{1}{R C} \]
\[ = 0 \angle -90^\circ \,, \; \omega \to \infty \]

52. Ans: (c)
Sol: Curve AA → Current waveform, having maximum value at
\[ \omega = \omega_{\text{max}} \]
Curve BB → |Z|
\[ = R + j \left( \omega L - \frac{1}{\omega C} \right) = -j\infty \,, \; \omega = 0 \]
\[ Z = R \,, \; \omega = \omega_{\text{max}} \]
\[ Z = j\infty \,, \; \omega = \infty \]
Curve CC → X_C = - \frac{j}{\omega C} \,, \; \text{Capacitive reactance}
Curve DD → Net reactance,
\[ X = j \left( \omega L - \frac{1}{\omega C} \right) \]
\[ = -j\infty \,, \; \omega = 0 \]
\[ = 0 \,, \; \omega = \omega_{\text{max}} \]
\[ = j\infty \,, \; \omega = \infty \]
\[ Z = \frac{V}{I} = \frac{120 \angle -20^\circ}{4 \angle 10^\circ} = 30 \angle -30^\circ \]
\[ = 25.98 \angle j15 \Omega \]
\[ P_f \cos (-30^\circ) = 0.866 \text{ (leading)}. \]

The load impedance \( Z \) can be modelled by a 25.98\( \Omega \) resistor in series with a capacitor with \( X_c = -15 = -\frac{1}{\omega C} \)
\[ C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2 \mu F \]

**04.**

**Sol:** Nodes 1 and 2 forms a super node as shown in below figure.

Applying KCL at the super node gives
\[ 3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} \]
\[ 36 = j4V_1 + (1-j2)V_2 \] ............(1)

and
\[ V_1 = V_2 + 10 \angle 45^\circ \]

Substituting equation \( V_1 \) in equation (1) results in
\[ 36 - 40 \angle 135^\circ = (1 + j2) V_2 \]
\[ \Rightarrow V_2 = 31.41 \angle -87.18^\circ \text{ V} \]

From equation
\[ V_1 = V_2 + 10 \angle 45^\circ \]
\[ = 25.78 \angle -70.48^\circ \text{ V} \]

**05.**

**Sol:**

Meshes 3 and 4 form a super mesh due to the current source between the meshes.

For mesh 1, KVL gives
\[ -10+(8-j2)I_1 -(j2)I_2 -8 I_3 = 0 \]
\[ (8-2j)I_1 + j2I_2 -8I_3 = 10 \]  .......(1)

For mesh 2
\[ I_2 = -3 \]  .......(2)

For the super mesh
\[ (8-j4)I_3 - 8I_1 + (6+j5)I_4 - j5I_2 = 0 \]  .......(3)

Due to the current source between meshes 3 and 4, at node A
\[ I_4 = I_3 + 4 \]  .......(4)

Instead of solving the above four equations, we reduce to two by elimination

Combines equation (1) & (2)
\[ (8-j2)I_1 - 8I_3 = 10 + j6 \]  .......(5)

Combines (3) and (4)
\[ -8I_1 + (14+j)I_3 = -24-j35 \]  .......(6)

From equations (5) and (6) we obtain the matrix equation
\[ \begin{bmatrix}
8 - j2 & -8 \\
-8 & 14 + j
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_3
\end{bmatrix} = \begin{bmatrix}
10 + j6 \\
-24 - j35
\end{bmatrix} \]

We obtain the following determinants
\[ \Delta = \begin{vmatrix}
8 - j2 & -8 \\
-8 & 14 + j
\end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20 \]
\[ \Delta_1 = \begin{bmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{bmatrix} \]
\[ = 140 + j10 + j84 - 6 - 192 - j280 \]
\[ = -58 - j186 \]

Current \( \mathbf{I}_1 \) is obtained as
\[ \mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618 \angle 274.5^\circ \text{A} \]

The required voltage \( \mathbf{V}_0 \) is
\[ \mathbf{V}_0 = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618 \angle 274.5^\circ + 3) \]
\[ = -7.2134 - j6.568 = 9.756 \angle 222.32^\circ \text{V} \]

**06.**

**Sol:** The given circuit is shown in figure 1.

\[ z = 8 + (j4 || 4) \]
\[ = 8 + \frac{4 \cdot j4}{4 + j4} \]
\[ = 8 + \frac{j4}{1 + j} \]
\[ z = \frac{8 + 12j}{1 + j} \]
\[ I = \frac{100 \angle 0^\circ}{8 + 12j} \]
\[ = 9.615 - j1.923 \ldots \ldots (1) \]

Apply KCL at node 'V'
\[ \mathbf{V} - 100 \angle 0^\circ + \frac{\mathbf{V}}{j4} + \frac{\mathbf{V}}{4} = 0 \]
\[ \mathbf{V} - j\mathbf{V} + \frac{\mathbf{V}}{4} - \frac{100 \angle 0^\circ}{8} = 0 \]

\[ \mathbf{V} \left[ \frac{1 - j}{8} + \frac{4}{4} \right] = \frac{100 \angle 0^\circ}{8} \]
\[ \mathbf{V} \left[ \frac{1 - 2j + 2}{8} \right] = \frac{100 \angle 0^\circ}{8} \]
\[ \mathbf{V} \angle 0 = \frac{100}{3 - 2j} \]

\[ \mathbf{V} \angle = 27.735 \angle 33.69^\circ \]

The given circuit is shown in figure 2. and Assume \( \mathbf{V} \angle 0 = 27.745 \angle 33.69^\circ \text{V} \]

**07.**

**Sol:** The given circuit is shown in Fig.1

The energy stored in the capacitor varies with time, the stored energy dissipated by the resistor over this interval. These are actually two completely different questions.

The only source of energy in the circuit is the independent voltage source, which has a value of 10sin2\( \pi \)t V. In the time interval of \( 0 < t < 0.5 \text{s} \).
The power dissipated by the resistor in terms of the current $i_R$.

$$i_R = \frac{V}{R} = \frac{10 \sin 2\pi t}{1000}$$

$$i_R = 0.01\sin 2\pi t \text{ A}$$

and so

$$P_R = i_R^2 R = (0.01)^2 \times (1000)\sin^2 2\pi t$$

$$= 0.1\sin^2 2\pi t$$

So that the energy dissipated in the resistor between 0 and 0.5s is

$$\omega_R = \int_{0}^{0.5} P_R dt$$

$$= \int_{0}^{0.5} 0.1\sin^2 2\pi t dt$$

$$= 0.1 \left[ \frac{1 - \cos 4\pi t}{2} \right]_0^{0.5}$$

$$= \frac{1}{20} \left[ (0.5 - 0) - (0 - 0) \right]$$

$$\omega_R = \frac{1}{40} J$$

$$V_c(t) = 10 \sin 2\pi t$$

$$i_c(t) = V_c(t) \times \frac{R}{R + \frac{1}{j\omega_c}}$$

or

$$i_c(t) = \frac{c}{dt} \frac{dV_c(t)}{dt}$$

$$= c. 10 \cos 2\pi t. 2\pi$$

$$i_c(t) = 20\pi \times 10^{-6} \cos 2\pi t$$

The energy stored in capacitor between 0 and 0.5s is

$$\omega_c = \int_{0}^{0.5} C.V \frac{dV}{dt} dt$$

$$= 10^{-6} \int_{0}^{0.5} 10 \sin 2\pi t. 20\pi \times 10^{-6} \cos 2\pi t dt$$

$$= \frac{200\pi \times 10^{-12}}{2} \int_{0}^{0.5} 2 \sin 2\pi t \cos 2\pi t dt$$

$$= 100\pi \times 10^{-12} \int_{0}^{0.5} \sin 4\pi t dt$$

$$= 100\pi \times 10^{-12} \left[ \frac{- \cos 4\pi t}{4\pi} \right]_0^{0.5}$$

$$= \frac{100\pi \times 10^{-12}}{4\pi} \left[ \cos(0) - \cos 4\pi(0.5) \right]$$

So that the energy dissipated in the resistor between 0 and 0.5s is

$$\omega_c = 0 J$$

**08. Sol:** The given parallel circuit is shown in Fig. 1.

The phasor diagram is shown in Fig. 2.
For unity P.F, $\phi = 0$.

\[
\frac{I}{V} = Y = j\omega C + \frac{1}{R + j\omega L} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}
\]

\[
= \frac{R}{R^2 + \omega^2 L^2} + j\omega \left( C - \frac{L}{R^2 + \omega^2 L^2} \right)
\]

For unity P.F, I and V should be in phase

\[
\text{Im } Y = 0
\]

\[
\therefore C = \frac{L}{R^2 + \omega^2 L^2}
\]

09.

**Sol:** At resonance frequency, $\omega = \omega_0$

\[
\frac{\omega_0}{\omega} L = \frac{1}{\omega_0 C}
\]

\[
\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6}}} = 10^3 \text{ rad/sec}
\]

\[
I = \frac{100}{10} = 10 \text{ A}
\]

Power consumed = $I^2 R = 100 \times 10 = 1000 \text{ W}$

\[
Z = 10 + j\left( \omega - \frac{10^6}{\omega} \right)
\]

\[
|Z| = \sqrt{100 + \left( \omega - \frac{10^6}{\omega} \right)^2}
\]

\[
I = \frac{V}{|Z|} = \frac{100}{|Z|}
\]

Power consumed = $I^2 R = \frac{100^2}{|Z|^2} R$

\[
0.1 \times 1000 = 10^3, \quad |Z|^2 = \frac{10^3}{100} = 10^3
\]

\[
100 + \left( \omega - \frac{10^6}{\omega} \right)^2 = 10^3
\]

\[
\omega - \frac{10^6}{\omega} = \pm \sqrt{900} = \pm 30
\]

\[
\omega^2 - 30 \omega - 10^6 = 0
\]

\[
\omega = \frac{\pm 30 \pm \sqrt{900 + 4 \times 10^6}}{2}
\]

\[
= 1000 + 15 \text{ and } 1000 - 15
\]

\[
= 1015 \text{ rad/sec and } 985 \text{ rad/sec}
\]

10.

**Sol:**

\[
Z(j\omega) = \frac{R}{R + j\omega L} - \frac{j}{\omega C}
\]

\[
R = 1 \Omega, L = 1 \text{ H}, C = 2 \text{ F}
\]

\[
Z(j\omega) = \frac{j\omega}{1 + j\omega} - \frac{j}{\omega}
\]

\[
= \frac{j2\omega^2 - j + \omega}{2\omega(1 + j\omega)} = \frac{\omega + j(2\omega^2 - 1)}{2\omega(1 + j\omega)}
\]

\[
= \frac{[\omega + j(2\omega^2 - 1)](1 - j\omega)}{2\omega(1 + \omega^2)}
\]

For resonance, Im $Z(j\omega) = 0$, $\omega^2 = 1$, $\omega = 1 \text{ r/s}$

\[
\therefore \text{ Resonance frequency, } \omega_0 = 1 \text{ r/s}
\]

\[
\text{Re } Z(j\omega) = \frac{1}{2}
\]

If the positions of $L$ and $C$ are interchanged,

\[
Z(j\omega) = \frac{-j \omega}{2\omega - j} + j\omega = \frac{-j}{2\omega - j} + j\omega
\]

\[
= \frac{-j + j2\omega^2 + \omega}{2\omega - j} = \frac{\omega + j(2\omega^2 - 1)}{2\omega - j}
\]

\[
= \frac{[\omega + j(2\omega^2 - 1)](2\omega + j)}{4\omega^2 + 1}
\]
\[ \text{Im} Z(j\omega) = 0, \text{ gives } 2\omega (2\omega^2 - 1) + \omega = 0 \]
\[ 2 (2\omega^2 - 1) + 1 = 0 \]
\[ 4\omega^2 = 1, \quad \omega = \frac{1}{2} \text{ rad/s} \]
\[ \therefore \text{ Resonance frequency, } \omega_0 = \frac{1}{2} \text{ rad/s} \]
and \[ \text{Re } Z(j\omega_0) = \frac{1}{4\omega_0 + 1} = \frac{1}{2} \Omega \]
\[ \omega_0 \text{ changes from 1 rad/s to } \frac{1}{2} \text{ rad/s and resonant impedance is resistive and remains the same as } \frac{1}{2} \Omega. \]

11. \textbf{Sol:} The input admittance is
\[ Y = j\omega(0.1) + \frac{1}{10} + \frac{1}{2 + j\omega} \]
\[ = 0.1 + j\omega(0.1) + \frac{2 - j\omega^2}{4 + 4\omega^2} \]

At resonance \[ \text{Im}(Y) = 0 \text{ and } \omega_0(0.1) - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \]
\[ \frac{2\omega_0}{4 + 4\omega_0^2} = \omega_0(0.1) \]
\[ 1 = 0.2 + 0.2\omega_0^2 \]
\[ \omega_0^2 = 4 \]
\[ \omega_0 = 2 \text{ rad/s} \]

12. \textbf{Sol:} The given bridge circuit is shown in Fig.1

From Fig.1
\[ Z_1(j\omega) = R_L + j0 \]
\[ Z_2(j\omega) = 4 + (-j5) \]
\[ Y_1(j\omega) = \frac{1}{Z_1(j\omega)} = \frac{1}{R_L + j\omega} \]
\[ Y_2(j\omega) = \frac{1}{Z_2(j\omega)} = \frac{1}{4 - j5} \]
\[ \therefore Y(j\omega) = Y_1(j\omega) + Y_2(j\omega) \]
\[ = \frac{1}{R_L + j0} + \frac{1}{4 - j5} \]
\[ Y(j\omega) = \frac{R_L - j10}{R_L^2 + (10)^2} + \frac{4 + j5}{16 + 25} \quad \text{-------- (1)} \]
For finding resonance frequency
\[ \text{Im}[Y(j\omega)] = 0 \]
From (1)
\[ \frac{-j10}{R_L^2 + 100} + \frac{j5}{16 + 25} = 0 \]
\[ \frac{10}{R_L^2 + 100} = 5 \]
\[ \frac{R_L^2}{41} = 82 - 100 \]
\[ R_L = \sqrt{-18} \]
\[ R_L = j\sqrt{18} \]
The resistance which is having displacement angle will not exist. So, the resonant frequency for above network there can be no value of \( R_L. \)
Objective Practice Solutions

01. **Sol:** \( X_C = 12 \) (Given)

- \( X_{eq} = 12 \) (must for series resonance)
- So the dot in the second coil at point “Q”
- \( L_{eq} = L_1 + L_2 - 2M \)
- \( L_{eq} = L_1 + L_2 - 2K\sqrt{L_1L_2} \)
- \( \omega L_{eq} = \omega L_1 + \omega L_2 - 2K\sqrt{L_1L_2} \omega \)
- \( 12 = 8 + 8 - 2K\sqrt{8.8} \)
- \( \Rightarrow K = 0.25 \)

02. **Sol:** \( X_C = 14 \) (Given)

- \( X_{eq} = 14 \) (must for series resonance)
- So the dot in the 2nd coil at “P”
- \( L_{eq} = L_1 + L_2 + 2M \)
- \( L_{eq} = L_1 + L_2 + K\sqrt{L_1L_2} \)
- \( \omega L_{eq} = \omega L_1 + \omega L_2 + 2K\sqrt{L_1L_2} \omega \)
- \( 14 = 2 + 8 + 2K\sqrt{2(8)} \)
- \( \Rightarrow K = 0.5 \)

03. **Sol:** \( L_{ab} = 4H + 2 - 2 + 6H + 2 - 2 + 8H - 2 - 2 \)

- \( L_{ab} = 14H \)

04. **Ans:** (c)

**Sol:** Impedance seen by the source

\[
Z_s = \frac{Z_L}{16} + (4 - j2) = \frac{10 \angle 30^\circ}{16} + (4 - j2) = 4.54 - j1.69
\]

05. **Sol:**

\[
Z_m = \left( \frac{N_1^2}{N_2^2} \right) \cdot Z_L
\]

- \( R_m = n^2.5 \)
- For maximum power transfer; \( R_L = R_s \)
- \( n^5 = 45 \Rightarrow n = 3 \)

06. **Ans:** (b)

**Sol:**

Apply KVL at input loop

\[-6 - 30 \times 10^3 \frac{di_1}{dt} + 5 \times 10^3 \frac{di_1}{dt} - 50i_1 = 0 \quad \cdots (1)\]

Take Laplace transform

\[-\frac{6}{s} + [-30 \times 10^{-3} (s) - 50]i_1(s) + 5 \times 10^{-3} s_1(s) = 0 \quad \cdots (2)\]

Apply KVL at output loop

\[V_2(s) - 30 \times 10^{-3} \frac{di_2}{dt} + 5 \times 10^{-3} \frac{di_1}{dt} = 0\]
Take Laplace transform

\( V_2(s) - 30 \times 10^{-3} s I_2(s) + 5 \times 10^{-3} s I_1(s) = 0 \)

Substitute \( I_2(s) = 0 \) in above equation

\( V_2 + 5 \times 10^{-3} s I_1(s) = 0 \) ……… (3)

From equation (2)

\[
I_1(s) = \frac{-6}{s(30 \times 10^{-3} s + 50)} \quad \text{……… (4)}
\]

Substitute eqn (4) in eqn (3)

\[
V_2(s) = \frac{-5 \times 10^{-3}(s)(-6)}{s(30 \times 10^{-3}(s) + 50)}
\]

Apply Initial value theorem

\[
\lim_{s \to \infty} \frac{-5 \times 10^{-3}(s)(-6)}{s(30 \times 10^{-3}(s) + 50)} = +1
\]

07.

Sol: \( R_{in}' = 8 \times 2 = 2 \Omega \)

\( R_{in} = 3 + R_{in}' = 3 + 2 = 5 \Omega \)

\[
I_1 = \frac{10 \angle 20}{5} = 2 \angle 20^\circ
\]

\[
I_1 = \frac{n = 2}{2} \Rightarrow I_2 = 1 \angle 20^\circ \text{A}
\]

08.

Sol: By the definition of KVL in phasor domain

\[
V_S - V_0 - V_2 = 0
\]

\[
V_0 = V_S - V_2 = V_S \left( 1 - \frac{V_2}{V_S} \right)
\]

\[
V = ZI
\]

09.

Sol: Transform the above network into phasor domain

By KVL

\[
V_S = j\omega L_1 I_1 + j\omega M_1 I_2
\]

\[
V_2 = j\omega L_2(0) + j\omega M_1 I_1
\]

\[
V_0 = V_S \left( 1 - \frac{M}{L_1} \right)
\]

10.

Sol: Evaluation of Initial conditions:

\[
i_1(0^-) = 0 \text{A} \quad ; \quad i_2(0^-) = 0 \text{A}
\]

Evaluation of final conditions:

\[
i_1(\infty) = 5 \text{A} \quad ; \quad i_2(\infty) = 0 \text{A}
\]

By KVL

\[
5 = i_1(t) + \frac{4 di_1(t)}{dt} - \frac{2 di_2(t)}{dt}
\]
By Laplace transform to the above equations.
\[ \frac{5}{s} = i_1(s) + 4[sI_1(s) - i_1(0^+)] - 2[sI_2(s) - i_2(0^+)] \]

By KVL
\[ 0 = 1i_2(t) + 2 \frac{di_2(t)}{dt} - 2 \frac{di_1(t)}{dt} \]
\[ 0 = 1i_2(s) + 2[sI_2(s) - i_2(0^+)] - [sI_1(s) - i_1(0^+)] \]

On solving, we can obtain \( i_1(t) \) and \( i_2(t) \)
\[ i_1(t) = 5 - e^{-\frac{t}{4}} \left[ 5 \cosh \left( \frac{\sqrt{5}}{4} t \right) - \sqrt{5} \sinh \left( \frac{\sqrt{5}}{4} t \right) \right] \]

11. Ans: (c)

Sol:
\[ L_1 = \frac{N_1 \phi_1}{i_1} \Rightarrow \phi_1 = \frac{L_1 i_1}{N_1} \]
\[ \phi_1 = \frac{1}{2} \frac{5 \sin 400 t}{N_1} \]
But
\[ \frac{N_1}{N_2} = \frac{L_1}{L_2} \Rightarrow N_1 = N_2 \frac{L_1}{L_2} \]
\[ N_1 = 1000 \frac{0.5}{0.2} = 1581.13 \]
\[ \phi_1 = \frac{2.5 \sin 400 t}{1581.13} \]
\[ \phi_1 = 1.58 \text{m} \sin 400 t \]
\[ \phi_1 = \phi_{\text{max}} \sin \omega \]
So, \( \phi_{\text{max}} = 1.58 \text{mWb} \)

12. Ans: (a)

Sol:
\[ M = \frac{k \phi_1 N_2}{i_1} = \frac{k \phi_2 N_1}{i_2} \]
Given, \( i_1 = 1 \text{A}; \phi_1 = 0.1 \text{mWb} \)
\[ N_1 = 1000; \quad N_2 = 2000 \]
\[ k = 0.6 \]
\[ M = \frac{(0.6)(0.1m)(2000)}{1} = 0.12 \text{H} \]

\[ \text{Conventional Practice Solutions} \]

01.
Sol: KVL in mesh
\[ R_4 i_1(t) + (L_1 + L_2) \frac{di_2(t)}{dt} - L_2 \frac{di_2(t)}{dt} + M_{12} \frac{d}{dt}(i_1 - i_2) \]
\[ + M_{21} \frac{d}{dt} i_1(t) + M_{13} \frac{d}{dt} i_2(t) + M_{23} \frac{d}{dt} i_2(t) = v_1(t) \ldots (l) \]

KVL in mesh
\[ R_5 i_2(t) + (L_1 + L_3) \frac{di_2(t)}{dt} - L_2 \frac{di_2(t)}{dt} + M_{12} \frac{d}{dt}(i_1 - i_2) \]
\[ + M_{23} \frac{d}{dt} i_2(t) + M_{13} \frac{d}{dt} i_2(t) + M_{23} \frac{d}{dt} i_2(t) = v_2(t) \ldots (2) \]

02.
Sol: \[ I = \frac{20 \angle 20^\circ}{Z_m} \]
\[ Z_m = 8 + j 10 + j 5 - j 12 \]
\[ = 8 + j 3 = 8.54 \angle 20.56 \]
\[ I = \frac{20 \angle 20^\circ}{8.54 \angle 20.56} = 2.34 \angle -0.56 \text{ A} \]
\[ V_{\text{th}} = (4 + j 5 - j 6) I = (4 - j) I \]
\[ = 4.123 \angle -14.03 \times 2.34 \angle -0.56 \]
\[ = 9.65 \angle -14.59 \text{ Volts} \]
\[ Z_{\text{th}} \text{ is calculated from the circuit shown in Fig.} \]

\[ \text{Fig.} \]
\[ (4 + j 5) I_1 + (j 6) I_2 = V_0 \]
\[ (j 6) I_1 + (4 + j 10) I_2 = V_0 \]
\[ I_1 = \frac{V_0}{V_0} j6 \]
\[ \Delta = V_0 \} \]

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\[ I_2 = \begin{bmatrix} 4 + j5 \\ j6 \end{bmatrix} V_0 \Delta = V_0(4 - j1) + \Delta \\
\Delta = \begin{bmatrix} 4 + j5 \\ j6 \end{bmatrix} \begin{bmatrix} j6 \\ 4 + j10 \end{bmatrix} = 16 - 50 + j60 + 36 = 2 + j60 \\
I_0 = I_1 + I_2 = \frac{V_0(8 + j3)}{\Delta} = \frac{8 + j3}{2 + j60} \\
V_0 = 2 + j60 \\
I_0 = \frac{8 + j3}{2 + j60} \\
Z_{eq} = 3 + \frac{2 + j60}{8 + j3} = 26 + j69 \Rightarrow 73.7 \angle 69.35^\circ \\
8.54 \angle 20.55^\circ \\
Z_{eq} = 8.63 \angle 48.8^\circ = 5.68 + j6.49 \\
For maximum power transfer, \\
Z_L = Z_{eq}^* = 8.63 \angle -48.8^\circ = 5.68 - j6.49 \\
\]

03.

\[ jX_{eq} = \begin{bmatrix} j8 \\ j5 \end{bmatrix} \]

\[ jX_{eq} = j \frac{(X_1 X_2 - X_m^2)}{(X_1 + X_2 - 2 X_m)} \]

\[ L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \]

\[ X_1 = \omega L_1, X_2 = \omega L_2 \]

\[ M = k \sqrt{L_1 L_2} \]

\[ X_m = \omega M = \omega k \sqrt{L_1 L_2} \]

\[ = k \sqrt{X_1 X_2} = \sqrt{40} k \]

\[ jX_{eq} = j \frac{40(1-k^2)}{13 - 2 \sqrt{40} k} \]

From Fig. 2

\[ Z = R + j X_{eq} = 10 + j X_{eq} \]

\[ I = \frac{20 \angle 30^\circ}{10 + j X_{eq}} = \frac{20 \angle 30^\circ - \theta_2}{\sqrt{10^2 + X_{eq}^2}} \]

\[ \theta_2 = \tan^{-1}\left(\frac{X_{eq}}{10}\right) \] ; \[ |I| = \frac{20}{\sqrt{100 + X_{eq}^2}} = 32 \]

\[ 100 + X_{eq}^2 = \frac{4000}{32} = 125 \]

\[ X_{eq}^2 = 25 \] ; \[ X_{eq} = 5 \]

\[ 40(1-k^2) = 5 \]

\[ 13 - 2 \sqrt{40} k \]

\[ 8(1-k^2) = 13 - 2 \sqrt{40} k \]

\[ 8k^2 - 2 \sqrt{40} k + 5 = 0 \]

\[ k = 2 \sqrt{40} \pm \sqrt{160 - 160} \]

\[ 16 = 0.79 \]

If the terminals of one of the coils are interchanged,

\[ jX_{eq} = j \frac{40(1-k^2)}{13 + 2 \sqrt{40} k} \]

\[ X_{eq} = \frac{15.07}{22.99} = 0.65 \] ; \[ I = \frac{20 \angle 30^\circ}{10 + j 0.65} \]

\[ P = |I|^2 R = \frac{4000}{100 + (0.65)^2} = 39.83 \text{ W} \]
04. Sol: Refer to Fig. 1, where I₁ and I₂ are mesh currents.

\[
\begin{align*}
&+ j 11 \omega I_1 + (- j 6 \omega - j 2 \omega - j 3 \omega) I_2 = V_1 \\
&or + j 11 \omega I_1 - j 11 \omega I_2 = V_1 \quad \text{........ (1)}
\end{align*}
\]

and

\[
\begin{align*}
&- j 6 \omega - j 2 \omega) I_1 + j 3 \omega (I_2 - I_1) + j 3 \omega I_2 + j 23 \omega I_2 = 0 \\
&or - j 11 \omega I_1 + j 29 \omega I_2 = 0 \quad \text{........ (2)}
\end{align*}
\]

\[
I_1 = \begin{bmatrix} V_1 & -j 11 \omega \\ 0 & j 29 \omega \end{bmatrix} = \begin{bmatrix} j 29 \omega V_1 \\ -j 11 \omega \end{bmatrix}
\]

\[
V_1 = -198 \omega^2, I_1 = 6.83 \omega
\]

\[
\therefore \text{Effective inductance} = 6.83 \text{H}
\]

05. Sol: The given circuit is shown in fig. 1 assume input voltage ‘V’

\[
\begin{align*}
L_1 &= 8 \text{ mH} \quad X_{L_2} = W_{L_1} \Rightarrow 1000 \times 8 \times 10^{-3} \Rightarrow 8 \Omega \\
L_2 &= 6 \text{ mH} \quad X_{L_2} = W_{L_2} \Rightarrow 1000 \times 6 \times 10^{-3} \Rightarrow 6 \Omega \\
M &= 5 \text{ mH} \quad X_m = \text{om} \Rightarrow 1000 \times 5 \times 10^{-3} \Rightarrow 5 \Omega
\end{align*}
\]

\[
C = \frac{1}{6} \mu \text{F} \quad X_C = \frac{1}{\omega C}
\]

\[
\Rightarrow X_C = \frac{1}{1000 \times \frac{1}{6} \times 10^{-6}} = 6000 \Omega
\]

\[
\text{Apply KVL in the above network loop} - 1
\]

\[
- V + j 8 i_1 + j 5 (i_1 - i_2) - j 6 000 i_1 + j 6 (i_1 - i_2) + j 5 i_1 = 0
\]

\[
- V + j 8 i_1 + j 5 (i_1 - i_2) - j 6 000 i_1 + j 6 i_1 - j 6 i_2 + j 5 i_1 = 0
\]

\[
i_1 = j (-5966) i_1 - j 11 i_2 = V \quad --------(1)
\]

In loop - 2

\[

j 6 (i_2 - i_1) - j 5 i_1 + 3 i_2 = 0
\]

\[
i_2 = \frac{\frac{1}{j 11}}{3 + j 6} i_1
\]

\[
\text{Put } 'i_2' \text{ in equation (V)}
\]

\[
- j 5 966 i_1 - j 11 \left[ \frac{1}{3 + j 6} \right] i_1 = V
\]

\[
(8.066 - j 5 982.13) i_1 = V
\]

\[
Z = \frac{V}{i_1} = (8.066 - j 5 982.13) \Omega
\]
Objective Practice Solutions

01.
Sol: The defining equations for open circuit impedance parameters are:
\[ V_1 = Z_{11}I_1 + Z_{12}I_2 \]
\[ V_2 = Z_{21}I_1 + Z_{22}I_2 \]
\[ [Z] = \begin{bmatrix} 10 & 4s+10 \\ s & 3s+10 \end{bmatrix} \Omega \]

02. Ans: (b)
Sol: The matrix given is
\[ \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \]
since \( y_{11} \neq y_{22} \)
⇒ Asymmetrical, and
\( y_{12} \neq y_{21} \)
⇒ Non reciprocal network

03.
Sol: Convert \( Y \) to \( \Delta \):

\[ Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \]
\[ Y_B = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix} \]

04.
Sol:
\[ Y = \begin{bmatrix} S+\frac{2}{3} & -S-\frac{1}{3} \\ -S-\frac{1}{3} & S+\frac{2}{3} \end{bmatrix} \text{mho} \]

05.
Sol: Convert \( Y \) to \( \Delta \):

\[ Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \]
\[ Y_B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} \]
06.

Sol: \( T_1 = T_2 = \begin{bmatrix} 1 + \frac{1}{j} & 1 \\ 1 & -\frac{1}{j} & 1 \end{bmatrix} \)

\( T_3 \Rightarrow Z_1 = 1 \Omega; Z_2 = \infty \)

\( T_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \)

\( T = (T_1)(T_2)(T_3) \)

\( T = \begin{bmatrix} 1 + j & 2 + j4 \\ -1 + j2 & j3 \end{bmatrix} \)

07.

Sol: \( T_1 : Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \)

\( T_1 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \)

\( T_2 : Z_1 = 0; Z_2 = 2 \Omega \)

\( T_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \)

\( T = [T_1][T_2] \)

\( T = \begin{bmatrix} 3.5 & 3 \\ 2 & 2 \end{bmatrix} \)

08. Ans: (a)

Sol: For \( I_2 = 0 \) (O/P open), the Network is shown in Fig.1

\( V_1 = -2 I_1 \) \hspace{1cm} (1)

\( \frac{V_1}{I_1} = -2 \)

\( V_2 = -6 I_1 + V_1 \) \hspace{1cm} (2)

From (1) and (2)

\( V_2 = -6 I_1 - 2 I_1 \)

or \( V_2 = -8 I_1 \)

\( Z_{11} = \frac{V_1}{I_1} = -8 \)

For \( I_1 = 0 \) (I/P open), the network is shown in Fig.2

\( V_1 = 1 I_2 \), \hspace{1cm} \frac{V_1}{I_2} = 1 \)

\( V_2 = 3 I_2 \), \hspace{1cm} \frac{V_2}{I_2} = 3 \)

Note: that the dependent current source with current \( 3 I_1 \) is open circuited.

\( V_1 = 1 I_2 \), \hspace{1cm} \frac{V_1}{I_2} = 1 \)

\( V_2 = 3 I_2 \), \hspace{1cm} \frac{V_2}{I_2} = 3 \)

\( [Z] = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix} \)
09.
Sol: By Nodal
\[-I_1 + V_1 - 3V_2 + V_1 + 2V_1 - V_2 = 0\]
\[-I_2 + V_2 + V_2 - 2V_1 = 0\]
\[Y = \begin{bmatrix} 4 & -4 \\ -3 & 2 \end{bmatrix} \Omega\]
\[[Z] = [Y]^{-1}\]
We can also obtain \([g], [h], [T] and [T]^{-1}\) by re-writing the equations.

10.
Sol: The defining equations for open-circuit impedance parameters are:
\[V_1 = Z_{11}I_1 + Z_{12}I_2\]
\[V_2 = Z_{21}I_1 + Z_{22}I_2\]
In this case, the individual Z-parameter matrices get added.
\[(Z) = (Z_a) + (Z_b)\]
\[[Z] = \begin{bmatrix} 10 & 2 \\ 2 & 7 \end{bmatrix} \Omega\]

11.
Sol: For this case the individual \(y\)-parameter matrices get added to give the \(y\)-parameter matrix of the overall network.
\[Y = Y_a + Y_b\]
The individual \(y\)-parameters also get added
\[Y_{11} = Y_{11a} + Y_{11b} \text{ etc}\]
\[[Y] = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \text{ mho}\]

12. Ans: (c)
Sol: \[Y_{11} = \frac{I_1}{V_1}|_{V_2=0}\]

13. Sol: (i) \([T_a] = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}\]
(ii) \([T_b] = \begin{bmatrix} \frac{1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & \frac{1}{Z_2} \end{bmatrix}\]
\([T_a]\) and \([T_b]\) are obtained by defining equations for transmission parameters.

14. Sol: In this case, the individual T-matrices get multiplied
\[(T) = (T_1) \times (T_{N1})\]
\[(T) = (T_1)(T_{N1}) = \begin{bmatrix} 1 + \frac{s}{4} & \frac{s}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3s + 8 & 3.5s + 4 \\ 6 & 7 \end{bmatrix}\]

15. Sol: \[Z_m = R_m = \frac{V_i}{I_i} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{V_2 - 2I_2}{V_2 - 3I_2}\]
\[V_2 = 10(-I_2)\]
\[Z_m = \frac{12}{13} \Omega\]
16. Sol: \[
\begin{align*}
\left. \frac{V_1}{I_1} \right|_{I_1=0} &= Z_{11} \\
\Rightarrow V_1 &= (4 \parallel 4)I_1 \left|_{I_1=0} \right. \\
\Rightarrow Z_{11} &= 2\Omega \\
V_2 &= (4 \parallel 4)I_2 \left|_{I_2=0} \right. \\
\Rightarrow Z_{22} &= 2\Omega \\
\text{By KVL} \Rightarrow \frac{3I_1}{2} - V_2 - \frac{I_1}{2} &= 0 \\
V_2 &= I_1 \\
\Rightarrow Z_{21} &= 1\Omega = Z_{12} \\
Z &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega \\
Y &= Z^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -1 & \frac{3}{2} \end{bmatrix} \Omega \\
Z &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega \\
\text{Substituting (3) in (1):} \quad V_1 = 2(V_2 - 2I_2) + I_2 = 2V_2 - 3I_2 \quad \ldots \ldots (4) \\
T &= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\
T^t &= T^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \\
\text{Now h parameters} \\
2I_2 &= -I_1 + V_2 \\
I_2 &= \frac{-I_1 + V_2}{2} \quad \ldots \ldots (5)
\end{align*}
\]

17. Ans: (a) Sol: \[
Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_2=0} \\
\text{Just use reciprocity of fig (a)}\]

Now use Homogeneity

\[
\text{So, } Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_2=0} = \left. \frac{5}{5} = 1 \text{ mho} \right.
\]

This has nothing to do with fig (b) since fig (b) also valid for some specific resistance of 2\(\Omega\) at port-1, but \(Y_{22}, V_1=0\). So S.C port-1
18. Sol: \[
\begin{align*}
\frac{V_2}{V_1} &= \frac{N_2}{N_1} = n = -\frac{I_1}{I_2} \\
\frac{V_2}{V_1} &= n \\
\Rightarrow V_1 &= \frac{1}{n} V_2 - (0) I_2 \\
\Rightarrow T &= \begin{bmatrix} 1 & 0 \\ n & 0 \end{bmatrix}
\end{align*}
\]

\[
T^T = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}
\]

\[
T^T = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}
\]

Now h-parameters
\[
V_1 = (0) I_1 + \frac{1}{n} V_2 \\
I_2 = -\frac{I_1}{n} + (0) V_2 \\
g = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\
h = \begin{bmatrix} 0 & -n \\ n & 0 \end{bmatrix}
\]

Note: In an ideal transformer, it is impossible to express \(V_1\) and \(V_2\) in terms of \(I_1\) and \(I_2\), hence the ‘Z’ parameters do not exist. Similarly, the y-parameters.

19. Ans: (c) 
Sol: 
\[
Z_{22} = \frac{V_2}{I_2} \bigg|_{V_i = 0}
\]

\[
V_1 = \frac{1}{n} I_2 \quad \text{and} \quad V_2 = \frac{n}{I_1}
\]

\[
\begin{align*}
V_1 &= \frac{1}{n} I_2 \\
V_2 &= \frac{n}{I_1}
\end{align*}
\]

20. Sol: 
For series parallel connection individual h-parameters can be added.
\[
\therefore \text{For network 1, } h_1 = g_1^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}
\]

\[
V_1 = \frac{1}{n} V_2 \\
V_2 - V_1 = \frac{I_1}{R} \\
I_2 = I_1 + I_1 \\
\frac{1}{n} I_1 = \frac{I_2}{n} - \frac{I_1}{n} = \frac{I_1}{n} - \frac{I_1}{n} \\
I_2 = \frac{1}{n} + 1 = \frac{1+n}{n}
\]

\[
I_2 = \left(\frac{1+n}{n}\right) I_1 \\
I_2 = \left(\frac{1+n}{n}\right) \left(\frac{V_2 - V_1}{R}\right) \\
I_2 = \left(\frac{1+n}{n}\right) \left(\frac{V_2 - \frac{1}{n} V_2}{R}\right) \\
I_2 = \left(\frac{1+n}{n}\right) \left(\frac{n-1}{nR}\right) \\
V_2 = n^2 R \\
\frac{V_2}{I_2} = \frac{n^2 R}{n^2 - 1}
\]

\[
\begin{align*}
\text{For network 1, } h_1 &= g_1^{-1} \\
&= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}
\end{align*}
\]
For network 2, \( h_2 = g_2^{-1} \)
\[
\begin{bmatrix}
1 & 1 \\
0 & 1 \\
-1 & 1 \\
0 & 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -1 \\
0 & 1 \\
2 & -1 \\
0 & 2
\end{bmatrix}
\]
\[
\begin{bmatrix}
2/3 & 1/3 \\
1/3 & 2/3
\end{bmatrix}
\]

21. Ans: (b)
Sol: \([Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \), \([Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \)

For Reciprocal NW, \( Z_{12} = Z_{21}, Y_{12} = Y_{21} \),
\[
[Z] \text{ and } [Y] \text{ matrices are symmetrical.}
\]
\([Y] = [Z]^{-1} \) is true for reciprocal as well as non-reciprocal NW’s.

22. Ans: (a)
Sol: Definition:
\[
\tilde{I}(s) = [Y] \tilde{V}(s) \quad \tilde{V}(s) = [Z] \tilde{I}(s) = [Y]^{-1} \tilde{I}(s),
\]
\[
[Z] = [Y]^{-1}
\]

23. Ans: (a)
Sol: In reciprocal 2-port NW’s, \( y_{12} = y_{21}, z_{12} = z_{21} \),
\( h_{12} = -h_{21} \), \( AD = BC = 1 \)

24. Ans: (d)
Sol: Convert the middle - \( \pi \) of \( 1\Omega \) into a \( T \) - network as shown in Fig.

\[
\begin{bmatrix}
(1/3)\Omega & (1/3)\Omega \\
(1/3)\Omega & (1/3)\Omega \\
(1/3)\Omega & (1/3)\Omega
\end{bmatrix}
\]

25. Ans: (d)
Sol:
\[
\begin{align*}
L_s & = \frac{1}{2} \frac{1}{C_s} \\
L_s & = \frac{1}{2} \frac{L_s}{LC_s^2 + 2} \\
Z_u(s) & = \frac{L_s}{2} + \frac{L_s}{LC_s^2 + 2} \\
& = \frac{L_s(LC_s^2 + 2) + 2L_s}{2L (s^2 + 2)} \\
Z_u(j\omega) & = \frac{j\omega L (4 - \omega^2 LC)}{2L (2 - \omega^2)} = 0 \\
\end{align*}
\]
\[
\text{At } \omega = 0 \text{ and } \frac{1}{\sqrt{LC}} = \infty , \text{ at } \omega = \infty
\]

26. Ans: (c)
Sol:
\[
\begin{align*}
h_{11} & = \frac{V_1}{I_1} \bigg|_{I_1 = 0} \\
h_{12} & = \frac{V_1}{V_2} \bigg|_{I_1 = 0} \\
h_{21} & = \frac{I_2}{V_2} \bigg|_{I_1 = 0} \\
h_{22} & = \frac{I_2}{V_2} \bigg|_{V_2 = 0}
\end{align*}
\]
According to the definitions above, \( h_{11} \) is in ohms (\( \Omega \))
\( h_{12} \) and \( h_{21} \) are dimensionless and \( h_{22} \) is in Siemens.
27. Ans: (b)

Sol: A → 4, \( I_N = \frac{V_{th}}{R_{th}}, R_N = R_{th} \)

\[ B \to 2, \quad h_{22} = \frac{I_2}{V_2} \bigg|_{I_1=0} \]

C → 1, \( Y_{12} = Y_{21}, Z_{21} = Z_{12} \) etc

D → 3, \[ \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} -V_2 \\ I_2 \end{bmatrix} \]

28. Ans: (d)

Sol: \( h_{11} = \frac{V_1}{I_1} \to \text{Impedance (1)} \)

\[ h_{12} = \frac{V_2}{V_1} \to \text{voltage ratio (4)} \]

\[ h_{22} = \frac{I_2}{V_2} \to \text{Admittance (2)} \]

\[ h_{21} = \frac{I_2}{I_1} \to \text{Current ratio (3)} \]

29. Ans: (c)

Sol: \[ V_b = h_{11} I_1 + h_{12} V_c \]

\[ I_2 = h_{21} I_1 + h_{22} V_c \]

\[ V_b = r_c I_1 + r_b (I_1 + I_2) \quad \text{..... (1)} \]

\[ V_c = (I_2 + \alpha I_1) r_c + (I_1 + I_2) r_b \quad \text{..... (2)} \]

or \[ V_c = (\alpha r_c + r_b) I_1 + (r_c + r_b) I_2 \]

or \[ I_2 = \frac{V_c - (\alpha r_c + r_b) I_1}{r_c + r_b} \quad \text{..... (3)} \]

Substitute \( I_2 \) in equation (1)

\[ V_b = r_c I_1 + r_b I_1 + r_b \left[ \frac{V_c - (\alpha r_c + r_b) I_1}{r_c + r_b} \right] \]

\[ = r_c I_1 + r_b I_1 + \frac{r_b V_c - \alpha (r_b r_c) I_1 - r_b^2 I_1}{r_b + r_c} \]

\[ = I_1 \left( r_c + \frac{r_b V_c}{r_b + r_c} \right) \]

30. Ans: (d)

Sol: A → 2, B → 4

C → 1, D → 3
Conventional Practice Solutions

01.
Sol: A two-port circuit can be declared as a reciprocal circuit, if the 2-port parameters satisfy the following relations:

(i) \(Z_{12} = Z_{21}\)

(ii) \(Y_{12} = Y_{21}\)

(iii) \(AD - BC = 1\)

(iv) \(h_{12} = -h_{21}\)

The two-port circuit shown in Fig.1 is reciprocal.

This is justified by taking 15 V voltage source and showing \(Y_{12} = Y_{21}\) as shown below.

First, convert the \((30 \, \Omega, 20 \, \Omega \text{ and } 60 \, \Omega)\) \(T\) network into a \(\Pi\) network and next get the overall \(\Pi\) network (Fig. 3)

\[
\begin{align*}
V_1 &= -I_2 \left( \frac{60}{7} \right) \\
Y_{21} &= \frac{I_2}{V_1} \bigg|_{V_2=0} = \frac{-7}{60} \, \Omega
\end{align*}
\]

Short circuit current response at port 2 with excitation, \(V_1 = 15\) V at port 1

\[
V_2 = -I_1 \left( \frac{60}{7} \right) = \frac{-7}{60} \times 15 = -1.75 \, A
\]

\[
Y_{12} = \frac{I_1}{V_2} \bigg|_{V_1=0} = \frac{-7}{60} \, \Omega
\]

When the excitation and response are interchanged, Short circuit current at port 1 with excitation, \(V_2 = 15\) V at port 2

\[
= \frac{-7}{60} \times 15 = -1.75 \, A
\]

The ratio of response to excitation remains constant for reciprocal network when the response and excitation are interchanged.

02.
Sol: The given circuit is shown in Fig. 1
\[ V_1 = A V_2 - B I_2 \]
\[ I_1 = C V_2 - D I_2 \]
Keep \( I_2 = 0 \) (Fig. 2)

\[
\begin{align*}
V_2 &= V_1 \cdot \frac{10}{12} = \frac{5}{6} V_1 \\
A &= \frac{V_1}{V_2} = \frac{6}{5} = 1.2 \\
V_2 &= 10 I_1 \\
C &= \frac{I_1}{V_2} = 0.1 \Omega \\
\text{Keep } V_2 &= 0 \text{ (Fig. 3)}
\end{align*}
\]

\[
\begin{align*}
B &= -\frac{V_1}{I_2}, \\
D &= -\frac{I_1}{I_2} \\
- I_2 &= I_1 \times \frac{10}{14} = \frac{5}{7} I_1, \\
D &= \frac{7}{5} = 1.4 \\
V_1 &= 2 I_1 - 4 I_2 = 2 I_2 \left( \frac{I_1}{I_2} - 2 \right) \\
V_1 &= 2 I_2 (-3.4) = -6.8 I_2 \\
B &= 6.8 \Omega \\
A &= 1.2, B = 6.8 \Omega, C = 0.1 \Omega, D = 1.4
\end{align*}
\]

03.
Sol:

\[ I_1(s) = Y_{11} V_1(s) + Y_{12} V_2(s) \]
\[ I_2(s) = Y_{21} V_1(s) + Y_{22} V_2(s) \]
For \( v_2(t) = 0, V_2(s) = 0 \), as shown in Fig. 1,
\[ i_1(t) = 1 u(t), \quad I_1(s) = \frac{1}{s} \]
\[ v_1(t) = (1 - e^{-4t}) u(t) \]
\[ V_2(s) = \frac{1}{s} - \frac{1}{s+4} = \frac{4}{s(s+4)} \]
\[ i_2(t) = -e^{-3t} u(t) \]
\[ I_2(s) = -\frac{1}{s+3} \]
\[ Y_{11} = \frac{I_1}{V_1} = \frac{s+4}{4} \]
\[ Y_{21} = \frac{I_2}{V_1} = -s(s+4) \]
\[ Y_{22} = 4(s+3) \]
For \( R_L = 1 \Omega \) as in Fig. 2

\[ i_1(t) = u(t), \quad I_1(s) = \frac{1}{s}, \quad V_2(s) = -I_2(s) \]
\[ v_1(t) = (1 - e^{-4t} + t e^{-4t}) u(t) \]
\[ V_1(s) = \frac{1}{s} - \frac{1}{s+4} + \frac{1}{(s+4)^2} \]
\[ = \frac{5s+16}{s(s+4)^2} \]
\[ i_2(t) = -e^{-7t}u(t), \]
\[ I_2(s) = -\frac{1}{s+7}, \]
\[ \Rightarrow V_2(s) = \frac{1}{s+7} \]
\[ I_1(s) = Y_{11}V_1(s) + Y_{12}V_2(s) \]
\[ \frac{1}{s} = \left( \frac{4+s}{4} \right) \frac{5s+16}{s(s+4)^2} + Y_{12} \left( \frac{1}{s+7} \right) \]
\[ \Rightarrow \frac{1}{s} = \frac{5s+16}{4s(s+4)} + Y_{12} \left( \frac{1}{s+7} \right) \]
\[ \Rightarrow Y_{12} \left( \frac{1}{s+7} \right) = \frac{1}{s} \cdot \frac{5s+16}{4s(s+4)} \]
\[ = \frac{4(s+4) - 5s - 16}{4s(s+4)} = \frac{4s + 16 - 5s - 16}{4s(s+4)} \]
\[ = \frac{-s}{4s(s+4)} = -\frac{1}{4(s+4)} \]
\[ \Rightarrow Y_{12} = -\frac{(s+7)}{4(s+4)} \]
\[ I_2(s) = Y_{21}V_1(s) + Y_{22}V_2(s) \]
\[ \frac{-1}{s+7} = \frac{-s(s+4)(5s+16)}{4(s+3)(s(s+4)^2)} + Y_{22} \left( \frac{1}{s+7} \right) \]
\[ \Rightarrow Y_{22} \left( \frac{1}{s+7} \right) = \frac{-1}{s+7} + \frac{5s+16}{4(s+3)(s+4)} \]
\[ \Rightarrow Y_{22} = \frac{s^2 + 23s + 64}{4(s+3)(s+4)} \]
And \[ Z_m(s) = \frac{V_1(s)}{I_1(s)} = \frac{5s+16}{s(s+4)^2} \]

04. Sol: Given network

![Diagram](image)

(Data given is \( C_1 = C_2 = C_a = 1 \ \Omega \). This can not correct. Assume \( C_1 = C_2 = C_a = 1 \ F \).

Use Operational impedance, \( \frac{1}{r} \).

The network is redrawn as follows:

In this form, it is easy to see that two passive linear 2 – port a and b have been put in parallel to form one large two point.

For the 2 – port labeled ‘a’; we have

\[ I_{i_a} = y_{i_{i_a}} V_1 + y_{i_{12_a}} V_2 \]
\[ I_{2_a} = y_{21_a} V_1 + y_{22_a} V_2 \] ------ (1)
For the 2-port labeled ‘b’, we have
\[ I_{b} = y_{1b} V_1 + y_{1b} V_2 \] and
\[ I_{2b} = y_{2b} V_1 + y_{2b} V_2 \] ------- (2)

The overall 2–port has voltage \( V_1 \) and current \( (I_{1a} + I_{1b}) \) at port1 and voltage \( V_2 \) and current \( (I_{2a} + I_{2b}) \) at port2

But from (1) and (2),
\[ \left( y_{1a} + y_{1b} \right) V_1 + \left( y_{1a} + y_{1b} \right) V_2 \]
\[ \left( y_{2a} + y_{2b} \right) V_1 + \left( y_{2a} + y_{2b} \right) V_2 \]

Thus, the overall admittance parameter are,
\[ y_{11} = \left( y_{1a} + y_{1b} \right); \quad y_{12} = \left( y_{1a} + y_{1b} \right); \]
\[ y_{21} = \left( y_{2a} + y_{2b} \right); \quad y_{22} = \left( y_{2a} + y_{2b} \right); \]

It now remains to determine the ‘a’–parameters & ‘b’–parameters.

Addition will give the overall y–parameters.

\[ y \text{ – parameters of the 2–port ‘a’}: \]

\[ y_{11} : \text{ short-circuit port-2} \]
\[ y_{11} = \frac{V_1}{s + \left( \frac{1}{1/s} \right)} = \frac{V_1 (s+1)s}{(2s+1)} \]
\[ y_{11} = \frac{I_{1a}}{V_1} = \frac{s(s+1)}{2s+1} \]

\[ y_{12} : \text{ short–circuit port-1} \]

With a little algebraic work; we get
\[ y_{12} = \frac{I_{1b}}{2s^2} \]

\[ y_{21} : \text{ Because the network is reciprocal, we get} \]
\[ y_{21} = y_{12} = \frac{s^2}{2s+1} \]

\[ y_{22} : \text{ From the symmetry of the network, we get} \]
\[ y_{22} = y_{11} = \frac{s(s+1)}{2s+1} \]

\[ y \text{ – parameters of the 2–part ‘b’}: \]

\[ y_{11b} = y_{22b} = \frac{1 + s}{2 + s} \]
\[ y_{12b} = y_{21b} = \frac{-1}{s + 2} \]

Now the overall 2-part parameters and be found.
(The network given is called a twin-T network and it is parallel connection of two T-networks)

05.

Sol:
\[ V_1 = I_1 (R_1 + R_3) + I_2 R_3 \]
\[ V_2 = I_1 (R_3 + dR_2) + I_2 (R_2 + R_3) \]

\[ \begin{align*}
  \begin{bmatrix}
    V_1 \\
    V_2 \\
  \end{bmatrix} &= 
  \begin{bmatrix}
    R_1 + R_3 & R_3 \\
    R_3 + dR_2 & R_2 + R_3 \\
  \end{bmatrix}
  \begin{bmatrix}
    I_1 \\
    I_2 \\
  \end{bmatrix}
\end{align*} \]

\[ Z_{12} \neq Z_{21} \]

\[ \therefore \text{ The given two port network is not a reciprocal network} \]
06. 
Sol: Z- parameters of network:

\[ V_1 = Z_{11}I_1 + Z_{12}I_2 \]
\[ V_2 = Z_{21}I_1 + Z_{22}I_2 \]

\[ Z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0} \]
\[ Z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} \]

KVL

\[ V_1 = 10I_1 + 10I_1 \]
\[ V_1 = 20I_1 \Rightarrow Z_{11} = 20\Omega \]
\[ Z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} = 20\Omega \]

KVL, in above circuit,

\[ -10I_1 - 10I_1 + V_2 = 0 \]
\[ V_2 = 20I_1 \]
\[ Z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} = 20\Omega \]

\[ Z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0} \]

KVL, \( V_2 = 15I_2 \Rightarrow Z_{22} = 15 \Omega \)

\[ Z_{12} = \frac{V_1}{I_2} \bigg|_{I_1=0} \Rightarrow V_1 = 5I_2 \Rightarrow Z_{12} = 5 \Omega \]

\[ [Z] = \begin{bmatrix} 20 & 5 \\ 20 & 15 \end{bmatrix} \Omega \]

07. 
Sol:

\[ V_1 = 10I_1 + 0.2V_2 + (I_1 + 0.6I_2) 20 \]
\[ V_1 = 10I_1 + 0.2V_2 + 20I_1 + 12I_2 \]
\[ V_1 = 30I_1 + 12I_2 + 0.2V_2 \]
\[ V_2 = 20(I_1 + 0.6I_2) = 20I_1 + 12I_2 \]
\[ V_1 = 34I_1 + 14.4I_2 \]
\[ V_2 = 20I_1 + 12I_2 \]

\[ [V] = [Z]^{-1} \begin{bmatrix} 12 & -14.4 \\ -20 & 37 \end{bmatrix} \]

\[ [Y] = [Z]^{-1} \begin{bmatrix} 0.1 & -0.112 \\ -0.167 & 0.283 \end{bmatrix} \]

08. 
Sol:
\[ Y_{12} = \frac{I_1}{V_2} \bigg|_{v_1 = 0} \]
\[ I_1(s) + I_2(s) = \frac{V_2(s)}{2s} + \frac{V_1(s)}{6} \]
\[ = V_2(s)(2S + 6) \]
\[ V_2(s) = I_1(s) \left( \frac{3/2 \times 3/2S}{3 + 3/2 + 2S} \right) + 1 \right) \div \frac{1}{2S} \div \frac{1}{6} \]
\[ V_2(s) = -I_1(s) \left( \frac{9/4S}{3 \left( \frac{1}{2} + \frac{1}{S} \right)} \right) + 1 \]
\[ \frac{V_2(s)}{I_1(s)} = \frac{3}{2(s + 1)} + 1 \]
\[ = \frac{3 + 2S + 2}{2(s + 1)} = \frac{-(2S + 5)}{2(s + 1)} \]
\[ Y_{12} = \frac{I_1(S)}{V_2(s)} = -\frac{2(s + 1)}{(2s + 5)} \]

**09.**
**Sol:** The given circuit is shown in Fig.1

\[ V_2 = -I_2R_L \quad V_2 = -2000I_2 \]
\[ \Rightarrow I_2 = -\frac{V_2}{2000} \quad \text{-----------(1)} \]

h-parameters:
consider the 2-port NW shown in Fig.2

\[ V_1 = h_{11}I_1 + h_{12}V_2 \]
\[ I_2 = h_{21}I_1 + h_{22}V_2 \]
\[ V_1 = 100I_1 + 0.0025V_2 \quad \text{-----}(2) \]
\[ I_2 = 20I_1 + 10^{-3}V_2 \quad \text{-----}(3) \]

Put equation (1) in equation (3)
\[ -\frac{V_2}{2000} = 20I_1 + 10^{-3}V_2 \]
\[ I_1 = -\frac{3V_2 \times 10^{-3}}{40} \quad \text{---------}(4) \]

Put equation (4) in equation (2)
\[ V_1 = 100 \left[ -\frac{3V_2}{40 \times 10^3} \right] + 0.0025V_2 \]
\[ V_2 = V_2 \left[ \frac{-300 + 100}{40 \times 10^3} \right] \]

\[
\begin{align*}
V_1 & = \frac{40 \times 10^3}{200} \\
V_2 & = -200 \\
V_1 & = -200
\end{align*}
\]
### Objective Practice Solutions

<table>
<thead>
<tr>
<th>No.</th>
<th>Ans</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 01. | (c) | \( n > \frac{b}{2} + 1 \)  
**Note:** Mesh analysis simple when the nodes are more than the meshes. |
| 02. | (c) | \( \text{Loops} = b - (n-1) \Rightarrow \text{loops} = 5 \)  
\( n = 7 \)  
\( \therefore b = 11 \) |
| 03. | (a) | |
| 04. | | \( \text{Nodal equations required} = \text{f-cut sets} \)  
\( = (n-1) = (10 - 1) = 9 \)  
\( \text{Mesh equations required} = \text{f-loops} \)  
\( = b - n + 1 = 17 - 10 + 1 = 8 \)  
So, the number of equations required  
\( = \text{Minimum (Nodal, mesh)} = \text{Min}(9,8) = 8 \) |
| 05. | (c) | Not a tree (Because trees are not in closed path) |
| 06. | (a) | |
| 07. | | For a complete graph;  
\( b = n_{c_2} \Rightarrow \frac{n(n-1)}{2} = 66 \)  
\( n = 12 \)  
\( \text{f-cut sets} = (n-1) = 11 \)  
\( \text{f-loops} = (b - n + 1) = 55 \)  
\( \text{f-loop} = \text{f-cut set matrices} = n^{(n-2)} \)  
\( = 12^{12-2} = 12^{10} \)  
\( \text{Ans: (a)} \)  
\( \text{Sol:} \)  
Nodes = 1, Branches = 0; \( \text{f-loops} = 0 \)  
Let \( N = 2 \)  
Nodes = 2; Branches = 1; \( \text{f-loop} = 0 \)  
Let \( N = 3 \)  
Nodes = 3; Branches = 3; \( \text{f-loop} = 1 \)  
\( \Rightarrow \text{Links} = 1 \)  
Let \( N = 4 \)  
Nodes = 4; Branches = 4; \( \text{f-loops} = \text{Links} = 1 \)  
Still \( N = 4 \)  
Nodes = 5; Branches = 6; \( \text{f-loops} = \text{Links} = 3 \)  
Let \( N = 5 \)  
Nodes = 5; Branches = 8; \( \text{f-loops} = \text{Links} = 4 \)  
etc  
Therefore, the graph of this network can have at least “N” branches with one or more closed paths to exist. |
09. Ans: (b)  
Sol:

10. Ans: (d)  
Sol: 
(a) 1, 2, 3, 4 →  
(b) 2, 3, 4, 6 →  
(c) 1, 4, 5, 6 →  
(d) 1, 3, 4, 5 →  

11. Ans: (b)  
Sol:  

12. Ans: (d)  

13. Ans: (d)  
Sol:  

14. Ans: (b)  
Sol:

15. Ans: (d)  
Sol: 

Fundamental loop should consist only one link, therefore option (d) is correct.

16. Ans: (d)  

17. Ans: (a)  
Sol:  

18. Ans: (d)  
Sol:  

A link with one or more of the twigs forms a closed loop.  
∴ Statement (II) is True.

19. Ans: (b)  
Sol:

The graph has  
No. of nodes = n = 4,  
No. of branches = b = 6  
No. of twigs = No. of tree branches  
= n – 1 = 3  
No. of independent loops = No. of links = 1  
= b – (n – 1) = 3  

Order of B matrix or Fundamental loop matrix = 1 × b = 3 × 6  
Correct answer is A = 6, B = 3,  
C = 3 × 6, D = 3
20. Ans: (a)
Sol: If 1, 2, 3 and 8 are the co-tree branches or chords or links, and then 4, 5, 6 and 7 should be Tree branches or twigs.
f - cutset (1, 2, 3, 4) is defined by 4 and f - loop (6, 7, 8) is defined by 8.

21. Ans: (a)
Sol: The Tree (1, 2, 3, 4, 5) is shown with thick lines.
The dotted lines (6, 7, 8) are links or chords.
f – circuit or f – loops are
Edge set : L₁ (1, 2, 4, 6) defined by chord 6
Edge set : L₂ (2, 4, 5, 7) defined by chord 7
Edge set : L₃ (2, 3, 5, 8) defined by chord 8
Note that the twigs or tree branches can be drawn so that they do not cross each other.

Conventional Practice Solutions

01.
Sol: A tree is a connected sub-graph of a connected graph containing all the nodes of the graph but containing no loops.

Properties of trees:
1. A connected sub-graph of a connected graph is a tree if there exists only one path between any pair of nodes in it.
2. Every connected graph has at least one tree.
3. The number of terminal nodes of every tree are two.
4. A connected sub-graph of a connected graph is a tree if there exists all the nodes of the graph.
5. Each tree has (n – 1) branches, where n is the number of nodes of the tree.
6. The rank of a tree is (n – 1).
The given connected graph has 7 nodes and 12 branches. Five different Trees are shown below from Fig. 1 to Fig. 5.
02. Sol: Let the fundamental loop matrix be $B$ and the fundamental cut-set matrix be $Q$ of the same oriented $G$, and let both matrices pertain to the same tree $T$; then

$$B Q^T = 0 \quad \text{and} \quad Q B^T = 0 \quad \ldots \ldots \ldots \ldots (1)$$

If we number the links from 1 to $l$ and number the tree branches from $l + 1$ to $b$, then

$$B = [I_j : F] \quad \text{and} \quad Q = [-F^T : I_n] \quad \ldots \ldots \ldots \ldots (2)$$

03. Sol: The eleven branches $a, b, c, \ldots, k$ are marked on the graph (Fig.). The incidence matrix is written with the usual convention: example: branch ‘a’ leaving node 1(taken as 1) and entering node 2 (taken as $-1$).

$$\begin{bmatrix} a & b & c & d & e & f & g & h & i & j & k \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig.

Dimensions of the incidence matrix is $7 \times 12$.

The oriented graph with 7 nodes and 12 branches is shown in Fig. 1.

To obtain the fundamental circuit matrix for these graph, a tree with 7 nodes and 6 branches is chosen (Fig. 2).

Number of f-circuits = Number of links

$$= b - (n - 1) = 6$$

The links are the branches 1, 3, 4, 6, 9 and 11.
f – circuit with 1: [1, 10, 12, 8, 2]
\[ v_1 - v_2 + v_8 + v_{10} - v_{12} = 0 \]
f – circuit with 3: [3, 2, 8, 7, 5]

\[ v_2 + v_3 - v_5 - v_7 - v_8 = 0 \]
f – circuit with 4: [4, 8, 7, 5]

\[ v_4 - v_5 - v_7 - v_8 = 0 \]
f – circuit with 6: [6, 7, 8]
\[ v_6 + v_7 + v_8 = 0 \]
\[ f\text{-circuit with 9: [9, 10, 12, 8]} \]

\[ v_8 + v_9 + v_{10} - v_{12} = 0 \]
\[ f\text{-circuit with 11: [11, 8, 12]} \]

According to the above \( f\text{-circuit} \) (tie-set) equations, the tie-set matrix, \( B_{6\times12} \) is constructed as shown in Table.

<table>
<thead>
<tr>
<th>( f\text{-circuit} )</th>
<th>Branches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8 9 10 11 12</td>
</tr>
<tr>
<td>1</td>
<td>1 -1 0 0 0 0 0 1 0 1 0 -1</td>
</tr>
<tr>
<td>3</td>
<td>0 1 1 0 -1 0 -1 -1 0 0 0 0</td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 1 -1 0 -1 -1 0 0 0 0</td>
</tr>
<tr>
<td>6</td>
<td>0 0 0 0 0 1 1 1 0 0 0 0</td>
</tr>
<tr>
<td>9</td>
<td>0 0 0 0 0 0 0 1 1 1 0 -1</td>
</tr>
<tr>
<td>11</td>
<td>0 0 0 0 0 0 0 -1 0 0 1 1</td>
</tr>
</tbody>
</table>

\( B_{6\times12} = \)

05.
Sol:

[Diagram of a circuit with labels and values]
\[ Q | Y_b | Q^T | P_{twigs} = (Q|I_s)-[Q|Y_b|[V_s] \]

\[
\begin{bmatrix}
\frac{1}{x} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ Y_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \]

\[ e_{twigs} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} \]

\[ [V_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \]

\[ LHS = [Q|Y_b|Q^T|e_{twigs}] = \begin{bmatrix} \frac{1}{x+y} & \frac{1}{x+y} & \frac{1}{x+y} \\
\frac{1}{2+y} & \frac{1}{2+y} & \frac{1}{2+y} \\
\frac{1}{2+y} & \frac{1}{2+y} & \frac{1}{2+y} \\
\frac{1}{2+y} & \frac{1}{2+y} & \frac{1}{2+y} \\
\frac{1}{2+y} & \frac{1}{2+y} & \frac{1}{2+y} \\
\frac{1}{2+y} & \frac{1}{2+y} & \frac{1}{2+y} \end{bmatrix}_{3 \times 1} \]

\[ [Q][I_s] = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \]

\[ [Q][Y_b][V_s] = \begin{bmatrix} 8 \\ 0 \end{bmatrix} \]

\[ RHS = [Q][I_s] - [Q][Y_b][V_s] = \begin{bmatrix} -8 \\ -1 \end{bmatrix} \]

Apply limits (x \to 0 \& y \to \infty) on both sides.

\[ \Rightarrow \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -8 \\ -1 \end{bmatrix} \]

\[ e_1 = -8 \] \hspace{1cm} (1)

\[ e_1 + \frac{3e_2}{2} = -1 \] \hspace{1cm} (2)

\[ e_1 + \frac{3e_3}{2} = -1 \] \hspace{1cm} (3)
By solving equations (1), (2) and (3)
e_1 = -8, e_2 = 2 and e_3 = 2

Now branch voltages are

\[ [Q]^T [e_{twig}] = [V_b] \]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & -1 & 0 \\
-1 & 0 & -1 \\
-1 & -1 & -1
\end{bmatrix}_{6x3}
\begin{bmatrix}
-8 \\
2 \\
2 \\
3x1
\end{bmatrix}
= 
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6_{6x1}
\end{bmatrix}
\]

So \( V_2 = 2 \) volts
\( V_3 = 2 \) volts
**Chapter 8** Passive Filters

### Objective Practice Solutions

01. **Sol:**
   \[
   \begin{align*}
   \omega = 0 \Rightarrow V_0 &= V_i \\
   \omega = \infty \Rightarrow V_0 &= 0
   \end{align*}
   \]
   \(\Rightarrow\) Low pass filter

02. **Sol:**
   \[
   \begin{align*}
   \omega = 0 \Rightarrow V_0 &= \frac{V_i R_2}{R_1 + R_2} \\
   \end{align*}
   \]
   “\(V_0\)” is attenuated \(\Rightarrow V_0 = 0\)
   \[
   \begin{align*}
   \omega = \infty \Rightarrow V_0 &= V_i \\
   \end{align*}
   \]
   It represents a high pass filter characteristics.

03. **Sol:**
   \[
   H(s) = \frac{V_i(s)}{I(s)} = \frac{S^2 LC + SRC + 1}{SC}
   \]
   Put \(s = j\omega\)
   \[
   \begin{align*}
   \omega = 0 \Rightarrow H(s) &= 0 \\
   \omega = \infty \Rightarrow H(s) &= 0
   \end{align*}
   \]
   It represents band pass filter characteristics

04. **Sol:**
   \[
   \begin{align*}
   \omega &= 0 \Rightarrow V_0 = 0 \\
   \omega &= \infty \Rightarrow V_0 = 0
   \end{align*}
   \]
   It represents Band pass filter characteristics

05. **Sol:**
   \[
   \begin{align*}
   \omega &= 0 \Rightarrow V_0 = 0 \\
   \omega &= \infty \Rightarrow V_0 = V_i
   \end{align*}
   \]
   It represents High Pass filter characteristics.

06. **Sol:**
   \[
   H(s) = \frac{1}{s^2 + s + 1}
   \]
   \[
   \begin{align*}
   \omega = 0 & \Rightarrow S = 0 \Rightarrow H(s) = 1 \\
   \omega = \infty & \Rightarrow S = \infty \Rightarrow H(s) = 0
   \end{align*}
   \]
   It represents a Low pass filter characteristics

07. **Sol:**
   \[
   H(s) = \frac{s^2}{s^2 + s + 1}
   \]
   \[
   \begin{align*}
   \omega = 0 & \Rightarrow S = 0 \Rightarrow H(s) = 0 \\
   \omega = \infty & \Rightarrow S = \infty \Rightarrow H(s) = 1
   \end{align*}
   \]
   It represents a High pass filter characteristics

08. **Sol:**
   \[
   \begin{align*}
   \omega &= 0; V_0 = V_i \\
   \omega &= \infty; V_0 = 0
   \end{align*}
   \]
   It represents a low pass filter characteristics.

09. **Sol:**
   \[
   \begin{align*}
   \omega &= 0 \Rightarrow V_0 = V_{in} \\
   \omega &= \infty \Rightarrow V_0 = V_{in}
   \end{align*}
   \]
   It represents a Band stop filter or notch filter.

10. **Sol:**
    \[
    H(s) = \frac{S}{s^2 + s + 1}
    \]
    \[
    \begin{align*}
    \omega &= 0 \Rightarrow S = 0 \Rightarrow H(s) = 0 \\
    \omega &= \infty \Rightarrow S = \infty \Rightarrow H(s) = 0
    \end{align*}
    \]
    It represents a Band pass filter characteristics

11. **Sol:**
    \[
    H(s) = \frac{S^2 + 1}{s^2 + s + 1}
    \]
    \[
    \begin{align*}
    \omega &= 0 \Rightarrow S = 0 \Rightarrow H(s) = 1 \\
    \omega &= \infty \Rightarrow S = \infty \Rightarrow H(s) = 1
    \end{align*}
    \]
    It represents a Band stop filter
12. 
Sol: \[ H(s) = \frac{1 - s}{1 + s} \]
\[ \omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1 \]
\[ \omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = -1 = 1 \angle 180^0 \]
It represents an All pass filter

13. Ans: (c) 
Sol.
\[ H(j\omega) = -\tan^{-1}\left(\frac{f}{f_L}\right) \]
\[ f = 0 \Rightarrow \phi = 0^0 = \phi_{\text{min}} \]
\[ f = f_L \Rightarrow \phi = -45^0 = \phi_{\text{max}} \]

14. Ans: (b) 
Sol:

First order high pass filter = \[ \frac{s}{1 + sT} \]
Phase shift = 90 – tan\(\omega T\)
Max. phase shift is at corner frequency
\[ \omega = \frac{1}{T} \]
Max. phase shift = 90 – tan\(^{-1}\omega T\) = 90 – tan\(^{-1}\left(\frac{1}{T} \times T\right)\) = 90 – 45 = 45°

15. Ans: (d)

16. Ans: (a) 
Sol: Half power of series RC circuit is at \( t = T \) (Time constant)
\( T = RC \)
Frequency = \( \frac{1}{RC} \)

17. Ans: (c) 
Sol: Magnitude of voltage gain 0.707 is at half power frequency
\[ \omega = \frac{1}{RC} \]
Conventional Practice Solutions

01. Sol:

The frequency range for AM broadcasting is 540 to 1600 kHz. We consider the low and high ends of the band. Since the resonant circuit in figure is a parallel type.

\[ \omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \]
\[ C = \frac{1}{\omega_0^2 L} \]

For Low end of the AM band \( f_0 = 40 \) kHz and the corresponding \( C \) is

\[ C_1 = \frac{1}{4\pi^2 \times 540^2 \times 10^6 \times 10^{-6}} = 86.9 \text{nF} \]

For High end of the AM band \( f_0 = 1600 \) kHz corresponding \( C \) value is

\[ C_2 = \frac{1}{4\pi^2 \times 1600^2 \times 10^6 \times 10^{-6}} = 9.9 \text{nF} \]

Thus \( C \) must be adjustable in the range (9.9 nF to 86.9 nF)

02. Sol:

RLC circuit band pass filter with \( R = 10\Omega, \)
\( L = 25\text{mH}, \) & \( C = 0.4\mu\text{F} \)

\[ Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{25}{0.4 \times 10^{-6}}} = 10 \sqrt{\frac{25 \times 10^{-3}}{4 \times 10^{-7}}} \]

\[ Q_0 = \frac{500}{20} = 25 \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]
\[ = \frac{1}{\sqrt{(25 \times 10^{-3})(0.4 \times 10^{-6})}} \]
\[ = 10 \text{krad/sec} \]

\[ \omega_2 = \omega_0 \pm \frac{1}{2} (\text{band width}) \]

\[ \omega_1 = \omega_0 - \frac{1}{2} (\text{band width}) \]

\[ B.W = \frac{\omega_0}{Q} = \frac{10}{25} \]
\[ = \frac{10000}{25} = 400 \text{ rad/sec} \]

\[ \omega_1 = 10 - 0.2 = 9.8 \text{ krad/sec} \]
\[ f_1 = \frac{9.8}{2\pi} = 1.56 \text{ kHz} \]

\[ \omega_2 = 10 + 0.2 = 10.2 \text{ krad/sec} \]
\[ f_2 = \frac{10.2}{2\pi} = 1.62 \text{kHz} \]

Frequency range is 1.56 kHz < \( f < 1.62 \) kHz

03. Sol:

\[ Z(S) = \left( \frac{1}{1 + \frac{1}{S}} \right) \left( \frac{1}{S} \right) + 1 \]
\[ = \frac{1}{S + 1 + \frac{1}{S}} + 1 \]
\[
Z(S) = \frac{S^2 + 3S + 1}{S(S + 2)} \\
I = \frac{V(S)}{Z(S)} \\
I_1 = \frac{1}{S} + \frac{1}{S} + \frac{V(S)}{Z(S)(S + 2)} \\
V_0(S) = I_1(S) = \frac{V(S)}{\left(\frac{S + 3S + 1}{5(S + 2)}\right)(S + 2)} = \frac{V(S)}{\left(\frac{S^2 + 3S + 1}{5S + 2}\right)} \\
H(S) = \frac{V_0}{V_s} = \frac{S}{(S^2 + 3S + 1)} \\
2\delta\omega_0 = 3 \\
2\delta = 3 \Rightarrow \delta = \frac{3}{2} \\
\omega_0^2 = 1 \Rightarrow \omega_0 = 1 \text{ rad/sec} \\
Q = \frac{1}{2\delta} = \frac{1}{2} \left(\frac{3}{2}\right) \Rightarrow Q = \frac{1}{3} \\
\text{Bandwidth} = \frac{\omega_0}{Q} = \frac{1}{1/3} = 3 \text{ rad/sec} \\
\]

04. Sol:
The circuit parameters for series R-L-C band stop filter are R = 2K, L = 0.1H, C = 40PF

\[
\begin{align*}
\omega_0 &= \text{centre frequency} \\
&= \frac{1}{\sqrt{LC}} = 0.5 \text{ Mrad} / \text{sec} \\
\text{(b) bandwidth} &= \frac{R}{L} = \frac{2 \times 10^3}{0.1} = 2 \times 10^4 \\
\text{Quality factor} &= Q = \frac{\omega_0}{B} = 25 \\
\text{As } Q > 10, \\
\omega_1 &= \omega_0 - \frac{1}{2} \text{B.W.} = 5 \times 10^{-5} - \frac{1}{2} \left(2 \times 10^4\right) \\
&= (50-1) \times 10^4 = 490 \text{ k rad/sec} \\
\omega_2 &= \omega_0 + \frac{1}{2} \text{B.W.} = 510 \text{ rad/sec} \\
\end{align*}
\]

05. Sol:

\[
\begin{align*}
\text{V}_0 &= \frac{V_s R_p}{R_g + R_s + R_p} \\
&= \frac{R_p}{R_g + R_s + R_p} \\
&= \frac{R_p}{\alpha} \\
R_{eq} &= R_p \left|R_s + R_g\right| = R_{eq} \ldots \ldots \ldots (1) \\
R_p\left(R_s + R_g\right) &= R_g \\
R_p + R_s + R_g &= R_g \\
R_g^2 + R_g R_p + R_g R_s &= R_p R_s + R_p R_g \\
\end{align*}
\]
\[ R_g^2 + R_g R_s - R_p R_s = 0 \]
\[ R_g (R_g + R_s) = R_p R_s \]

From (1):
\[ R_g + R_s = R_p \left( \frac{1}{\alpha} - 1 \right) \]
\[ R_g R_p \left( \frac{1 - \alpha}{\alpha} \right) = R_s R_p \]
\[ R_g = R_s \left( \frac{\alpha}{1 - \alpha} \right) \]
\[ R_s = \frac{(1 - \alpha) R_g}{\alpha} = \left( \frac{1 - 0.128}{0.128} \right) 100 \]
\[ = 700 \Omega \]

From (1):
\[ R_g + R_s + R_p = \frac{R_p}{\alpha} \]
\[ 100 + 700 + R_p \left( \frac{1 - \alpha}{\alpha} \right) = R_p \left( \frac{1 - 0.125}{0.125} \right) \]
\[ R_p = 114.29 \Omega \]

(b) \[ V_{th} = V_{oc} = V_0 = 0.125 V_g \]
\[ = (0.125 \times 12) \]
\[ = 1.5V \]

\[ I = \frac{1.5}{100 + 50} = \frac{1.5}{150} = \frac{150 \times 10^{-2}}{150} \]
\[ I = 10 Ma \]
Objective Practice Solutions

01. Ans: (c)
Sol: \( F(s) = \frac{(s + 2)}{(s + 1)(s + 3)} \)

The given \( F(s) \) has pole-zero structure as P-Z-P-Z alternating on the negative real axis of the \( s \)-plane, with a pole nearest the origin at \( s = -1 \) and a zero at \( s = \infty \). This \( F(s) \) corresponds to RC impedance or RL admittance.

02. Ans: (b)
Sol: For RC and RL driving point functions, the poles and zeros should alternate on the negative real axis, where as for LC driving point functions the poles and zeros should alternate the imaginary axis.

03. Ans: (c)

04. Ans: (b)
Sol: Remember that parallel LC networks in cascade is Foster-I form and series LC networks in shunt is Foster-II form. Ladder NW with series elements as inductors and shunt elements as capacitors is Cauer-I form and the ladder NW with capacitors as series elements and inductors as shunt elements is Cauer-I form. The given circuit in this question is Foster-I form.

05. Ans: (c)
Sol: Given: \( Z(s) = \frac{s(s^2 + 1)}{s^2 + 4} \)

Location of Poles: \( s = \pm j2 \)

Location of Zeros: \( s = 0, \pm j1 \)

Poles and Zeros are simple and lie on the imaginary axis, but they do not alternate.

Hence the given \( Z(s) \) is not realizable.

06. Ans: (b)
Sol: Poles and zeros of driving point function \([Z(s) \text{ or } Y(s)]\) of LC network are simple and alternate on the jo axis.

07. Ans: (c)
Sol: \[ V = I Z(s) \]
\[ V = I \left( \frac{\omega^2 + \alpha^2}{\omega^2 + \beta^2} \right) \tan^{-1} \left( \frac{\omega}{\alpha} \right) - \tan^{-1} \left( \frac{\omega}{\beta} \right) \]

Voltage load the current
\[ \tan^{-1} \left( \frac{\omega}{\beta} \right) < \tan^{-1} \left( \frac{\omega}{\alpha} \right) < \frac{\omega}{\alpha} (\alpha < \beta) \]

08. Ans: (d)

09. Ans: (b)
Sol: \( Z(s) = \frac{K(s + 3)}{s^2 + 2s + 2} \)

\[ Z(0) = \frac{K(3)}{2} = 3 \implies K = 2 \]

\[ \therefore Z(s) = \frac{2(s + 3)}{s^2 + 2s + 2} \]
10. Ans: (d)
Sol:
\[ s^2 + 2s ) s^2 + 4s + 3 = \frac{1}{R} \]
\[ s^2 + 2s \]
\[ 2s + 3s^2 + 2s(\frac{s}{2} = sL) \]
\[ s^2 + 3s \]
\[ \frac{s}{2})2s + 3(4 = \frac{1}{R} \]
\[ 2s + \]
\[ 3) \frac{s}{2} (\frac{s}{6} = sL \]
\[ \frac{s}{2} \]
\[ y(s) = \frac{s^2 + 4s + 3}{s^2 + 2s} \]
No. of elements = 4

11. Ans: (b)
Sol:
\[ F(s) = \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 6)} \]
represents an LC immittance function with pole-zero pattern as shown in Fig. Hence it is p.r.
\[ F(s) = \frac{s(s^2 - 4)}{(s^2 + 1)(s^2 + 6)} \]
is not p.r as it has a zero in the RH at \( s = 2 \)
\[ F(s) = \frac{s^3 + 3s^2 + 2s + 1}{4s} \]
is not p.r as the difference in degrees of highest degree terms in \( N(s) \) and \( D(s) \) is more than 1. For this \( F(s) \), difference is 2.

Its pole-zero pattern is shown in Fig. From the pattern it can be observed that:
→ Poles and zeros alternate on the negative real axis of s-plane.
→ The lowest critical frequency is a zero.
→ From the given \( Y(s) \), \( Y(0) = 1/3 \) and \( Y(\infty) = 1 \), \( Y(0) < Y(\infty) \), \( Y(\sigma) \) has +ve slope.
It is an admittance of the RC network, as the above properties are true for RC admittance.

12. Ans: (b)

13. Ans: (a)
Sol:
\[ Y(s) = \frac{s^2 + 2.5s + 1}{s^2 + 4s + 3} \]
\[ Y(s) = \frac{(s + 0.5)(s + 2)}{(s + 1)(s + 3)} \]
14. Ans: (a)
Sol: Given \( Z(s) = \frac{(s^2 + 4)(s^2 + 16)}{s(s^2 + 9)} \)

Out of the given figs., Foster – I form should be either (1) or (4) and Foster – II form should be either (2) or (3). Foster – I form can be confirmed as Fig. 1 by seeing the behavior of \( Z(s) \) at \( s = \infty \) and \( s = 0 \).

\( Z(s) = 1 \) at \( s = \infty \), \( L = 1 \) H

Foster – II form can be confirmed as Fig. (3) as

\[
L = \frac{12}{7} \quad \| \quad \frac{12}{5} = 1 \text{ H, at } s = \infty
\]

and \( C = \frac{7}{192} + \frac{5}{48} = \frac{9}{64} \) F at \( s = 0 \).

The exact realization can be done as shown below. Foster – I form is obtained by expanding the given \( Z(s) \) in partial fractions.

\[
Z(s) = k_1 \frac{s}{s^2 + 9} + k_2 \frac{s}{s^2 + 9} + k_3 \frac{s}{s^2 + 9} = 1 + \frac{64}{9s} + \frac{35}{9} \frac{s}{s^2 + 9} \quad \text{………(1)}
\]

As \( k_1 = \lim_{s \to \infty} \frac{Z(s)}{s} = 1 \)

\[
k_2 = s \left. Z(s) \right|_{s=0} = \frac{64}{9}
\]

\[
k_3 = \left. \frac{(s^2 + 9)}{s} Z(s) \right|_{s=-9} = \frac{(-9 + 4)(-9 + 16)}{-9} = \frac{35}{9}
\]

It can be seen from equation (1), the first Foster form corresponds to Fig. I (not Fig. IV) Foster - II form is obtained by taking partial fractions of

\[
Y(s) = \frac{s(s^2 + 9)}{(s^2 + 4)(s^2 + 16)} = \frac{k_1 s}{(s^2 + 4)} + \frac{k_2 s}{(s^2 + 16)} = Y_1(s) + Y_2(s)
\]

\[
k_1 = \left. \frac{Y(s)}{s} \right|_{s^2 = -4} = \frac{-4 + 9}{-4 + 16} = \frac{5}{12}
\]

\[
k_2 = \left. \frac{Y(s)}{s} \right|_{s^2 = -16} = \frac{-16 + 9}{-16 + 4} = \frac{7}{12}
\]

\[
Y_1(s) = \frac{\frac{5}{s}}{s^2 + 4} = \frac{1}{Ls + \frac{1}{Cs}}
\]

\[
L = \frac{12}{5} \quad \| \quad \frac{5}{48} = \frac{9}{64} \quad \text{F}
\]

It can be seen that Foster – II form corresponds to Fig. III (not Fig. II).
It is instructive to find out the remaining elements in Fig. I and III.

15. Ans: (a)

16. Ans: (d)
Sol: Given:

\[
Z_0(s) = \frac{2(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)} = \frac{2s^4 + 8s^2 + 6}{s^3 + 2s}
\]

Out of the figs. given (d) is in the form of Cauer-I network and (a) is in the form of Cauer-II. The Cauer network can be confirmed as (d) by seeing the behaviour of

\( Z(s) \) at \( s = \infty \) and at \( s = 0 \)

\( Z(s) = 2, \text{ at } s = \infty, \text{ giving } L = 2 \text{ H} \)
\[ Z(s) = \frac{3}{s}, \text{ at } s \to \infty, \text{ giving } \]
\[ C = \frac{1}{3} F = \left( \frac{1}{4} + \frac{1}{12} \right) F \]

Exact realizations of Cauer – I and Cauer – II forms can be obtained as shown below:
Cauer – I Network is obtained by successive removal of poles at \( s = \infty \). As the given \( Z_D(s) \) has a pole at \( s = \infty \), removal of it gives the first element as \( L = 2H \). Follow the Continued Fraction (CF) expansion given below, which confirms to the Network in (d).

\[
s^3 + 2s \left( 2s^4 + 8s^2 + 6 \right) \left( 2s, \frac{2s^4 + 4s^2}{4s^2 + 6} \right) s^3 + 2s \left( \frac{1}{4} s \right) \]
\[
\left( s^3 + \frac{3}{2} s \right) s^2 + 2s \left( \frac{1}{4} s \right),
\]
\[
\left( s^2 + \frac{3}{2} s \right) \frac{s}{2} 4s^2 + 6 \left( 8s, \frac{4s^2}{4s^2} \right)
\]
\[
\frac{s}{2} \left( \frac{s}{2} \right) 12s, \frac{s}{2} \frac{1}{12}, \frac{s}{2} 0
\]

Quotient values
\( L = 2H, \ C = \frac{1}{4} F, \ L = 8H, \ C = \frac{1}{12} F \)

Cauer – II NW is obtained by successive removal of poles at \( s = 0 \).
\( Z_D(s) \) also has a pole at \( s = 0 \), removal of it gives the first element as \( C = \frac{1}{3} F \).

Follow the CF expansion below.
\[
2s + s^3 \left( 6 + 3s^2 \right) \left( \frac{3}{8}, \ C = \frac{1}{3} F \right)
\]
\[
6 + 3s^2 \left( 5s^2 + 2s^4 \right) \left( \frac{2}{5s}, \ L = \frac{5}{2} H \right)
\]
\[
2s + 4s^3 \left( \frac{1}{5} s^3, \ C = \frac{1}{25} F \right)
\]
\[
5s^2 \left( \frac{2}{5s}, \ L = 10 H \right)
\]
\[
\frac{1}{5} s^3 \left( \frac{1}{10s}, \ L = 10 H \right)
\]
\[
0
\]

So the answer must be the Cauer - I NW in (d).
It is instructive to find the Cauer - I and Cauer–II structures by completing the CF expansions above.
17. Ans: (c)  
Sol:  
<table>
<thead>
<tr>
<th>F(s)</th>
<th>Type of F(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{(s^2 - s + 4)}{s^2 + s + 4} )</td>
<td>zeros in the right half plane</td>
</tr>
<tr>
<td>( \frac{s + 4}{s^2 + 6s + 5} )</td>
<td>poles in the right half plane</td>
</tr>
<tr>
<td>( \frac{s^3 + 3s}{s^4 + 2s^2 + 1} )</td>
<td>poles and zeros alternate on the negative real axis with first critical frequency near the origin as a pole.</td>
</tr>
</tbody>
</table>

18. Ans: (c)  
Sol:  
\[
Z(s) = \frac{3}{s} + \frac{1}{s} + \frac{6}{6s + 1} + \frac{18s + 3 + 6}{6s + 1} + \frac{3(18s + 9)}{6s + 1} + \frac{3 + 18s + 9}{6s + 1}
\]

19. Ans: (b)  
Sol:  
\[
p(s) = s^4 + s^3 + 2s^2 + 4s + 3
\]
\[
y(s) = \frac{even \ part}{odd \ part} = \frac{s^4 + 2s^2 + 3(s)}{s^3 + 4s}
\]
\[
= \frac{s^3 + 3s}{s^2 + 3s + 4s(-\frac{s}{2} \Rightarrow -ve \ quotients)}
\]
\[
Q(s) = s^5 + 3s^2 + s \text{ missing terms}
\]
Q(s) is not Hurwitz

20. Ans: (a)

21. Ans: (d)

22. Ans: (b)  
Sol:  
Foster – I form consists of LC tank circuits in series to realize \( Z_{LC}(s) \).  
This form is obtained by taking partial fractions of \( Z(s) \).  
\[
Z(s) = 4 \left[ \frac{1s + A}{s} + \frac{B}{s^2 + 4} \right]
\]
\[
n = 1 \text{ with an inductance and capacitance in series.}
\]
23. Ans: (a)
Sol: Assertion given is the necessary condition for 
Y(s) to be positive real because the definition of 
positive real function includes the statement that 
Y(s) is real for real s.

24. Ans: (d)
Sol: The function \( \frac{10(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \) is a valid 
reactance function as poles and zeros alternate 
on the jω-axis.
Statement (I) is false, statement (II) is true.

25. Ans: (c)
Sol: The existence of two poles or two zeroes in 
successive on the real frequency axis of the s-
plane requires that the slope be negative over 
part of the frequency range. So the slope of 
reactance curve may be negative.
∴ Statement (II) is false.

26. Ans: (a)
Sol: The poles and zeros of driving point function 
should be in the left half of the s-plane. A is 
True.
Only PR function can be realized as the driving 
point function of a network and PR function has 
its poles and zeros in the left half of the s-plane.
R is True and is the correct explanation of A.

27. Ans: (c)
Sol: For a system to be stable, all coefficients of the 
characteristic polynomial must be positive. This 
is a necessary condition for stability, but not a 
sufficient condition.
A is true, R is false.

28. Ans: (a)
Sol: 
\[ Z(s) = \frac{k(s^2 + 1)(s^2 + 5)}{(s^2 + 2)(s^2 + 10)} \]

For Z(s) to be an LC function, the highest 
powers of numerator and denominator should 
differ by 1. For the given Z(s), the highest 
powers of numerator and denominator are not 
differing by one. They are same equal to 2.

29. Ans: (a)
Sol: 
\[ Q \propto \frac{1}{\xi} \]

For circuits with high Q, ξ is less. If damping is 
less, the real part of the poles are close to the jω-
axis in the left-half plane.

30. Ans: (a)
Sol: 
\[ Z(s) = \frac{Ks(s^2 + 2)(s^2 + 10)}{(s^2 + 1)(s^2 + 6)} \]

It represents an LC driving point impedance 
function because it satisfies the property: Poles 
and zeros interlace on the imaginary axis of the 
complex s – plane as shown in Fig.

31. Ans: (b)
01. Sol: The realizable function used for driving point synthesis is known as Positive Real (P.R) function.

PR function:
Positive real function, F(s) is defined as function satisfying the following two requirements:
\[ \text{Re } F(s) \geq 0 \quad \text{for } \text{Re } s \geq 0 \]
or \[ |\text{Arg } F(s)| \leq |\text{Arg } s| \quad \text{for } |\text{Arg } s| \leq (\pi/2) \]
and \( F(s) \) is real when ‘s’ is real.

It is easier to test the P-R character of a function, \( F(s) \) by means of the following equivalent necessary and sufficient conditions:

\[ \text{a) } Y(s) \text{ must be real when } s \text{ is real.} \]

\[ \text{b) } \text{If } Y(s) = p(s)/q(s), \text{ then } p(s) + q(s) \text{ must be Hurwitz.} \]

This requires that:
\[ \text{i) The continued fraction expansion of the Hurwitz test give only real and positive } \alpha \text{'s, and} \]
\[ \text{ii) The continued fraction not end prematurely.} \]

\[ \text{c) In order that } \text{Re } Y(j\omega) \geq 0 \text{ for all } \omega, \text{ it is necessary and sufficient that} \]
\[ A(\omega^2) = m_1 m_2 - n_1 n_2 \bigg|_{s=j\omega} \text{ have no real positive roots of odd multiplicity. This may} \]
\[ \text{be determined by factoring } A(\omega^2) \text{ or by the use of Sturm’s theorem.} \]

02. Sol: i) The pole-zero plot of impedance is shown in Fig.

\[ Z(s) = \frac{(s^2 + 1)}{s(s^2 + 4)} = \frac{(s^2 + 1)}{s^3 + 4s} \]

Foster – I form:
\[ Z(s) = \frac{k_1}{s} + \frac{k_2 s}{s^2 + 4} \]
\[ k_1 = s Z(s) \bigg|_{s \to 0} = \frac{1}{4} \]
\[ k_2 = \frac{s^2 + 4}{s} Z(s) \bigg|_{s^2 = -4} = \frac{-4 + 1}{-4} = \frac{3}{4} \]
\[ Z(s) = \frac{1}{4s} + \frac{3s}{4(s^2 + 4)} = \frac{1}{4s} + \frac{1}{4s + \frac{16}{3s}} \]

The realization is shown in Fig. 1.

\[ \text{Foster – II form:} \]
\[ Y(s) = \frac{s(s^2 + 4)}{(s^2 + 1)} = k_1 s + \frac{k_2 s}{s^2 + 1} \]
\[ k_1 = \frac{Y(s)}{s} \bigg|_{s \to \infty} = 1 \]
The realization is shown in Fig. 2.

\[ Y(s) = \frac{s^2 + 1}{s} \quad Y(s) \bigg|_{s^2 = -1} = (-1 + 4) = 3 \]

The realization is shown in Fig. 3.

\[ Y(s) = 1s + \frac{3s}{s^2 + 1} = 1s + \frac{1}{\frac{1}{3}s} + \frac{1}{\frac{1}{3}s} \]

The realization is shown in Fig. 4.

\[ Z(s) = \frac{1 + s^2}{4s + s^3} \]

\[ Z(s) = \frac{1 + s^2}{4s + s^3} \left( \frac{1}{4s} \right) \]

\[ \frac{1 + \frac{1}{4}s^2}{\frac{3}{4}s^2} \left( \frac{16}{3s} \right) \]

\[ \frac{4s}{s^3 - \frac{3}{4}s^3} \left( \frac{3}{4s} \right) \]

\[ \frac{\frac{3}{4}s^2}{0} \]

Cauer-II form:

This represents an RC NW.

\[ Z(s) = \frac{(s+1)(s+3)}{s(s+2)} = \frac{s^2 + 4s + 3}{s^2 + 2s} \]

**Foster - I form:**

Taking partial fractions of \( Z(s) \):

\[ Z(s) = k_1 + \frac{k_2}{s+2} + \frac{k_3}{s} \]
\[ k_1 = Z(s) \big|_{s \to \infty} = 1 \]
\[ k_2 = (s+2)Z(s) \big|_{s=-2} = \frac{(-2+1)(-2+3)}{-2} = \frac{1}{2} \]
\[ k_3 = s Z(s) \big|_{s=0} = \frac{3}{2} \]
\[ Z(s) = 1 + \frac{1}{2(s+2)} + \frac{3}{2s} = 1 + \frac{1}{2s+4} + \frac{3}{2s} \]

The realization is shown in Fig. 2.

**Foster – II form:**
This is obtained from the partial fractions of not \( Y(s) \), but \( \frac{Y(s)}{s} \).

\[ Y(s) = \frac{(s+2)}{(s+1)(s+3)} \]
\[ = \frac{k_1}{s+1} + \frac{k_2}{s+3} \]
\[ k_1 = (s+1) \frac{Y(s)}{s} \big|_{s=-1} = \frac{-1+2}{-1+3} = \frac{1}{2} \]
\[ k_2 = (s+3) \frac{Y(s)}{s} \big|_{s=-3} = \frac{-3+2}{-3+1} = \frac{1}{2} \]

\[ \frac{Y(s)}{s} = \frac{1}{2(s+1)} + \frac{1}{2(s+3)} \]

\[ \therefore Y(s) = \frac{s}{2s+2} + \frac{s}{2s+6} = \frac{1}{2} \frac{2}{s} + \frac{1}{2} \frac{6}{s} \]

The realization is shown in Fig. 4.

**Cauer – I form:**
This is obtained from continued fraction expansion of \( Z(s) \).

\[ \frac{s^2 + 2s}{s^2 + 4s + 3} \left[ 1 + \frac{1}{2s+3} \right] \]

The realization is shown in Fig. 3.
Cauer – II form:

\[
Z(s) = \frac{3+4s+s^2}{2s+s^2} \frac{3+\frac{3}{2}s}{2s+s^2} \frac{\frac{5}{2}s+s^2}{2s+s^2} \frac{(4/5)}{\frac{5}{2}s+s^2} \frac{2s+\frac{4}{5}s^2}{\frac{1}{5}s^2} \frac{\frac{5}{2}s+s^2}{\frac{2}{5}s} \frac{(25/2s)}{s^2} \frac{\frac{1}{5}s^2}{(0/5)} \frac{\frac{1}{5}s^2}{0}
\]

The realization is shown in Fig. 5.

03.

Sol: \(Z_D(s) = \frac{s^2+12s+35}{s^3+15s^2+62s+48}\)

Factoring the numerator and denominator,

\[
Z_D(s) = \frac{(s+5)(s+7)}{(s+1)(s+6)(s+8)}
\]

This is an RC impedance function.

Obtain partial fractions of \(Z_D(s)\) to get Foster-I form

\[
Z_D(s) = \frac{k_1}{s+1} + \frac{k_2}{s+6} + \frac{k_3}{s+8}
\]

\[
k_1 = \frac{(-1+5)(-1+7)}{(-1+6)(-1+8)} = \frac{4 \times 6}{5 \times 7} = \frac{24}{35}
\]

\[
k_2 = \frac{(-6+5)(-6+7)}{(-6+1)(-6+8)} = \frac{(-1)(1)}{(-5)(2)} = \frac{1}{10}
\]

\[
k_3 = \frac{(-8+5)(-8+7)}{(-8+1)(-8+6)} = \frac{(-3)(-1)}{(-7)(-2)} = \frac{3}{14}
\]

\[
Z_D(s) = \frac{(24/35)}{s+1} + \frac{(1/10)}{s+6} + \frac{(3/14)}{s+8}
\]

Identifying the elements from the denominators, Foster – I form is shown in Fig. 1.

Cauer – I form is obtained from the Continued fraction expansion starting from \(Y_D(s)\), which has pole at

\[
s = \infty, \quad Y_D(s) = \frac{s^3+15s^2+62s+48}{s^2+12s+35}
\]
Identifying the elements from the quotients, Cauer – I form is obtained as shown in Fig. 2.

### Sol: PR Function

Positive real function, \( F(s) \) is defined as function satisfying the following two requirements:

- \( \text{Re} \ F(s) \geq 0 \) for \( \text{Re} \ s \geq 0 \)
- \( |\text{Arg} \ F(s)| \leq |\text{Arg} \ s| \leq (\pi/2) \)

It is easier to test the P-R character of a function, \( F(s) \) by means of the following equivalent necessary and sufficient conditions:

- (a) \( Y(s) \) must be real when \( s \) is real.
- (b) If \( Y(s) = \frac{p(s)}{q(s)} \), then \( p(s) + q(s) \) must be Hurwitz.
  - This requires that:
    - i) the continued fraction expansion of the Hurwitz test give only real and positive \( \alpha \)'s, and
    - ii) the continued fraction not end prematurely.
- (c) In order that \( \text{Re} \ Y(j\omega) \geq 0 \) for all \( \omega \), it is necessary and sufficient that
  \[ A(\omega^2) = m_1 m_2 - n_1 n_2 |_{s=j\omega} \] have no real positive roots of odd multiplicity. This may be determined by factoring \( A(\omega^2) \) or by the use of Sturm’s theorem. It can be shown that the function, \( \frac{s^2 + 12s + 35}{s^3 + 15s^2 + 62s + 48} \), is not P-R as it does not satisfy requirement (c) above.
$A(\omega^2) = (\omega^2 - 3)(\omega^2 - 1)$ is negative for $1 < \omega < \sqrt{3}$.

The given circuit is shown in Fig. 1.

\[ \frac{1}{Z_1(s)} = \frac{1}{s^2 + 1} \]

\[ Z_1(s) = \frac{s}{s^2 + 1} \]

\[ Y_1(s) = \frac{1}{Z_1(s)} + 1s + \frac{1}{s} = \frac{s^2 + 1}{s} \]

\[ Z_2(s) = \frac{s}{4(s^2 + 1)} \]

\[ Z_3(s) = Z_1(s) + Z_2(s) = \frac{5}{4} \frac{s}{s^2 + 1} \]

\[ Y_3(s) = \frac{4(s^2 + 1)}{5s} \]

\[ Y(s) = 1 + \frac{1}{5} s + \frac{4(s^2 + 1)}{5s} \]

\[ = 1 + \frac{1}{5} s + \frac{4s^2 + 4}{5s} = 1 + \frac{s^2 + 0.8}{s} \]

\[ = 1 + s + \frac{0.8}{s} = \frac{1}{R} + Cs + \frac{1}{Ls} \]

\[ = \frac{s^3 + s + 0.8}{s} \]

From eqn. (I), the Canonic realization is obtained as parallel RLC network shown in Fig. 2.

Considering eqn. (II), Zeros of $Y(s)$ are obtained from $s^2 + s + 0.8 = 0$

\[ s = -1 \pm \sqrt{1 - 3.2} \]

\[ = -1 \pm j\sqrt{2.2} \]

\[ = -0.5 \pm j0.74 \]

Poles of $Y(s)$ are at $s = 0$ and $s = \infty$.

The pole-zero diagram is shown in Fig. 3.
05.

Sol: \[ Z(s) = \frac{s(s^2 + 10)}{(s^2 + 4)(s^2 + 16)} \]

This is clearly a L-C driving point impedance function.

**Fosters – I:** (i.e.,) series impedance form obtained by partial fraction expansion.

\[ Z(s) = \frac{s}{s^2 + 4} + \frac{2k_1s}{s^2 + \sigma_1^2} + \ldots + \frac{k_n}{s^2 + \sigma_n^2} \]

But in the given function, there is no pole at origin and there is no pole at infinity.

So, doing partial fraction expansion,

\[ Z(s) = \frac{As + B}{s^2 + 4} + \frac{Cs + D}{s^2 + 16} \]

Comparing coefficients

\[ 4A + 4C = 4 \quad \ldots \ldots \quad (1) \]

\[ 16A + 4C = 10 \quad \ldots \ldots \quad (2) \]

By solving, \( A = \frac{1}{2}, B = \frac{1}{2} \)

\[ B + D = 0 \quad \ldots \ldots \quad (3) \]

\[ 16B + 4D = 0 \quad \ldots \ldots \quad (4) \]

By solving, \( B = 0, D = 0 \)

So, \( \frac{s}{s^2 + 4} + \frac{s}{s^2 + 16} \)

\[ Z(s) = \frac{1}{2} \left( \frac{s}{s^2 + 4} + \frac{1}{s^2 + 16} \right) \]

So, Fosters – I form of realization is,

\[ \frac{1}{8} \quad \frac{1}{32} \]

06.

Sol: Given \[ Z(s) = \frac{(s + 1)(s + 3)(s + 5)}{s(s + 2)(s + 4)(s + 6)} \]

This impedance corresponds to RC NW Fosters-I form is realized by taking the partial fractions of \( Z(s) \).

\[ A = \frac{15}{48} = \frac{5}{16} \]

\[ B = \frac{3}{16} \]

\[ C = \frac{3}{16} \]

\[ D = \frac{5}{16} \]

\[ Z(s) = \frac{1}{16s} + \frac{1}{16(s + 2)} + \frac{3}{16(s + 4)} + \frac{3}{16(s + 6)} + \frac{1}{(16/5)s + (32/3)} + \frac{1}{(16/3)s + (64/3)} + \frac{1}{(16/5)s + (96/5)} \]

The realization is shown in Fig. 1.