GATE | PSUs

CIVIL ENGINEERING

Strength of Materials

Text Book : Theory with worked out Examples and Practice Questions

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2 Strength of Materials

(Solutions for Text Book Practice Questions)

01. Simple Stress and Strains

Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

01. Ans: (b)

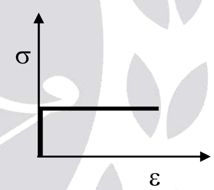
Sol:

- **Ductility:** The property of materials to allow large deformations or large extensions without failure (large plastic zone) is termed as ductility.
- **Brittleness:** A brittle material is one which exhibits a relatively small extensions or deformations prior to fracture. Failure without warning (No plastic zone) i.e. no plastic deformation.
- **Tenacity**: High tensile strength.
- **Creep:** Creep is the gradual increase of plastic strain in a material with time at constant load.
- **Plasticity:** The property by which material undergoes permanent deformation even after removal of load.
- **Endurance limit:** The stress level below which a specimen can withstand cyclic stress indefinitely without failure.
- **Fatigue:** Decreased Resistance of material to repeated reversal of stresses.

02. Ans: (a)

Sol:

- When the material is subjected to stresses, it undergoes to strains. After removal of stress, if the strain is not restored/recovered, then it is called inelastic material.
- For rigid plastic material:



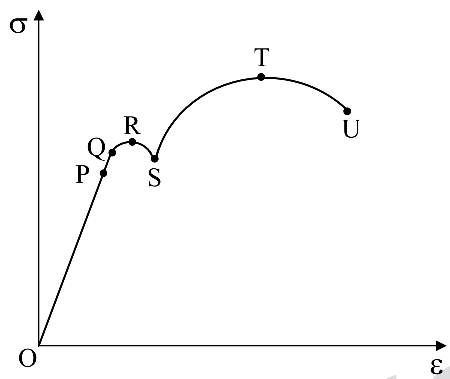
- Any material that can be subjected to large strains before it fractures is called a ductile material. Thus, it has large plastic zone.
- Materials that exhibit little or no yielding before failure are referred as brittle materials. Thus, they have no plastic zone.

03. Ans: (a)

Sol: *Refer to the solution of Q. No. (01).*

04. Ans: (b)

Sol: The stress-strain diagram for ductile material is shown below.



P – Proportionality limit
Q – Elastic limit
R – Upper yield point
S – Lower yield point
T – Ultimate tensile strength
U – Failure

From above,

OP → Stage I
PS → Stage II
ST → Stage III
TU → Stage IV

05. Ans: (b)

Sol:

- If the response of the material is independent of the orientation of the load axis of the sample, then we say that the material is **isotropic** or in other words we can say the isotropy of a material is its characteristics, which gives us the information that the properties are same in the three orthogonal directions x, y and z.
- A material is **homogeneous** if it has the same composition throughout the body. Hence, the elastic properties are the same at every point in the body in a given direction. However, the properties need not to be the same in all the directions for the material. Thus, both A and B are false.

06. Ans: (a)

Sol: **Strain hardening** increase in strength after plastic zone by rearrangement of molecules in material.

- **Visco-elastic material** exhibits a mixture of creep as well as elastic after effects at room temperature. Thus their behavior is time dependant

07. Ans: (a)

Sol: *Refer to the solution of Q. No. (01).*

08. Ans: (a)

Sol: Modulus of elasticity (Young's modulus) of some common materials are as follow:

Material	Young's Modulus (E)
Steel	200 GPa
Cast iron	100 GPa
Aluminum	60 to 70 GPa
Timber	10 GPa
Rubber	0.01 to 0.1 GPa

09. Ans: (a)

Sol: Addition of carbon will increase strength, thereby ductility will decrease.

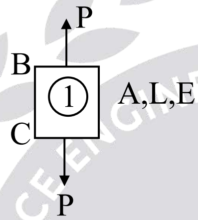
Elastic Constants and Their Relationships

01. Ans (c)

Sol: We know that,

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\therefore \quad \mu = \frac{\Delta D/8}{\dfrac{PL}{AE}/L}$$



$$\therefore \quad \mu = \frac{\Delta D}{8}\,\frac{AE}{P}$$

$$\therefore \quad 0.25 = \frac{\Delta D}{8}\,\frac{\frac{\pi}{4}(8)^2 \times 10^6}{50000}$$

$$\Rightarrow \quad \Delta D = 1.98 \times 10^{-3} \cong 0.002 \text{ cm}$$

02. Ans: (c)

Sol: We know that,

$$\text{Bulk modulus} = \frac{\delta P}{\delta V/V}$$

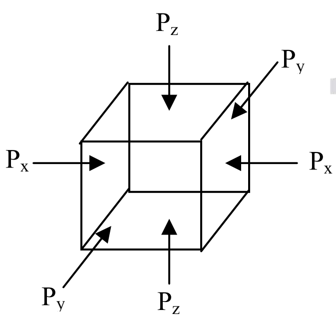
$$\Rightarrow \quad 2.5\times10^5 = \frac{200 \times 20}{\delta V}$$

$$\Rightarrow \quad \delta V = 0.016 \text{ m}^3$$

Linear and Volumetric Changes of Bodies

01. Ans: (d)

Sol:



Let $P_y = P_z = P$

$\varepsilon_y = 0$,

$\varepsilon_z = 0$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu.\frac{\sigma_x}{E}$$

$$\therefore \quad 0 = \frac{(-P)}{E} - \mu\frac{(-P)}{E} - \mu\frac{(P_x)}{E}$$

$$\Rightarrow \quad P = \frac{\mu.P_x}{(1-\mu)}$$

02. Ans: (a)

Sol: Given that, $\sigma_c = 4\tau$

Punching force = Shear resistance of plate

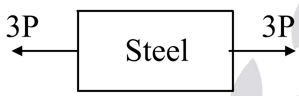
$\therefore\ \sigma\left(\text{Cross section area}\right) = \tau(\text{surface Area})$

$$\therefore\ 4\times\tau\times\frac{\pi.D^2}{4} = \tau(\pi.D.t)$$

$\Rightarrow D = t = 10$ mm

03. Ans: (d)

Sol:



$$\sigma_s = 140 \text{ MPa } = \frac{P_s}{A_s}$$

$$\Rightarrow P_S = \frac{140\times500}{3} \approx 23{,}300 \text{ N}$$

$$\sigma_{Al} = 90 \text{ MPa } = \frac{P_{A\ell}}{A_{A\ell}}$$

$\Rightarrow P_{Al} = 90 \times 400 = 36{,}000$ N

$$\sigma_B = 100 \text{ MPa } = \frac{P_B}{A_B}$$

$$\Rightarrow P_B = \frac{100\times200}{2} = 10{,}000 \text{ N}$$

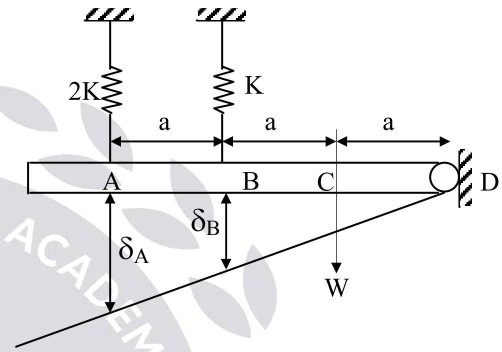
Take minimum value from P_S, A_{Al} and P_B.

$\Rightarrow P = 10{,}000$ N

[Correct answer is (d)]

04. Ans: (c)

Sol:



From similar triangle

$$\frac{3a}{\delta_A} = \frac{2a}{\delta_B}$$

$3\delta_B = 2\delta_A$ ……. (1)

$$\text{Stiffness } K = \frac{W}{\delta}$$

$$\therefore K_A = \frac{W_A}{\delta_A} \Rightarrow \delta_A = \frac{W_A}{2K}$$

$$\text{Similarly} \qquad \delta_B = \frac{W_B}{K}$$

$$\text{From equation (1)} \quad 3\times\frac{W_B}{K} = 2\times\frac{W_A}{2K}$$

$$\Rightarrow \frac{W_A}{W_B} = 3$$

Thermal/Temperature Stresses

01. Ans: (b)

Sol: Free expansion = Expansion prevented

$$[\ell \alpha t]_s + [\ell \alpha t]_{Al} = \left[\frac{P\ell}{AE}\right]_s + \left[\frac{P\ell}{AE}\right]_{AL}$$

$$11\times10^{-6}\times20+24\times10^{-6}\times20$$

$$= \frac{P}{100\times10^3\times200} + \frac{P}{200\times10^3\times70}$$

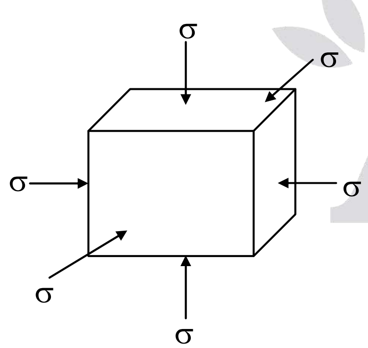
$\Rightarrow$ P = 5.76 kN

$$\sigma_s = \frac{P}{A_s} = \frac{5.76\times10^3}{100} = 57.65\ \text{MPa}$$

$$\sigma_{Al} = \frac{P}{A_{al}} = \frac{5.76\times10^3}{200} = 28.82\ \text{MPa}$$

02. Ans: (a)

Sol:



Strain in X-direction due to temperature,

$$\varepsilon_t = \alpha(\Delta T)$$

Strain in X-direction due to volumetric stress,

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\therefore\quad \varepsilon_x = \frac{-\sigma}{E}(1-2\mu)$$

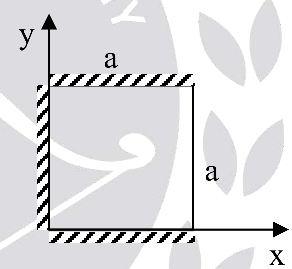
$$\therefore\quad -\sigma = \frac{(\varepsilon_x)(E)}{1-2\mu}$$

$$\therefore\quad -\sigma = \frac{\alpha(\Delta T)E}{(1-2\mu)}$$

$$\Rightarrow\quad \sigma = \frac{-\alpha(\Delta T)E}{1-2\mu}$$

03. Ans: (b)

Sol:



- Free expansion in x direction is aαt.
- Free expansion in y direction is aαt.
- Since there is restriction in y direction expansion doesn't take place. So in lateral direction, increase in expansion due to restriction is **μaαt**.

  Thus, total expansion in x direction is,

  $$= a\,\alpha\,t + \mu\,a\,\alpha\,t$$

  $$= a\,\alpha\,t\,(1+\mu)$$

02. Complex Stresses and Strains

01. Ans: (b)

Sol: Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

Maximum shear stress $= \dfrac{\sigma_1 - \sigma_2}{2} = 13$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{18 + (-8)}{2} = 5$$

02. Ans: (b)

Sol: Radius of Mohr's circle, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

$$\therefore \quad 20 = \frac{\sigma_1 - 10}{2}$$

$$\Rightarrow \quad \sigma_1 = 50 \text{ N/mm}^2$$

03. Ans: (b)

Sol: Given data,

$\sigma_x = 150$ MPa, $\sigma_y = -300$ MPa, $\mu = 0.3$

Long dam $\rightarrow$ plane strain member

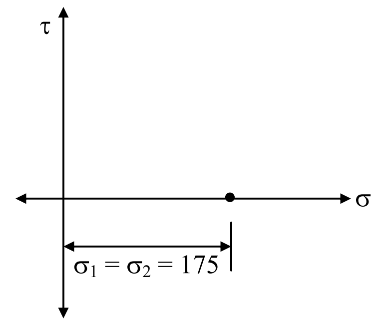
$$\varepsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\therefore 0 = \sigma_z - 0.3 \times 150 + 0.3 \times 300$$

$$\Rightarrow \sigma_z = 45 \text{ MPa}$$

04. Ans: (b)

Sol:



From the bove, we can say that Mohr's circle is a point located at 175 MPa on normal stress axis.

Thus, $\sigma_1 = \sigma_2 = 175$ MPa

05. Ans: (c)

Sol: Given that, $\sigma_2 = 0$

$$\therefore \quad \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\therefore \quad \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\therefore \quad \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}$$

$$\therefore \quad \tau^2_{xy} = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

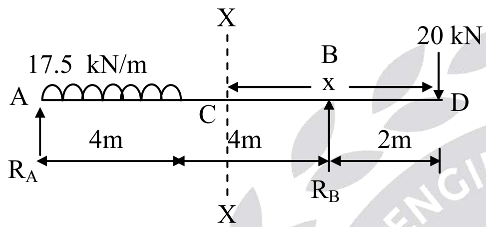
$$\therefore \quad \tau^2_{xy} = \sigma_x.\sigma_y$$

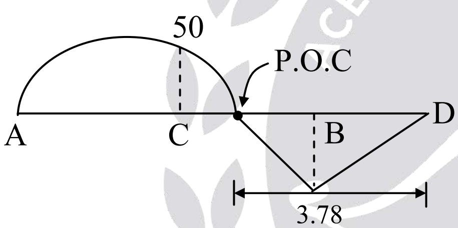
$$\Rightarrow \quad \tau_{xy} = \sqrt{\sigma_x.\sigma_y}$$

03. Shear Force and Bending Moment

01. Ans: (b)

Sol: Contra flexure is the point where BM is becoming zero.





Taking moment about A,

$\Sigma M_A = 0$

$\therefore\ 17.5\times4\times\frac{4}{2}+20\times10-R_B\times8=0$

$\therefore\ R_B = 42.5$ kN

Now, $M_x = -20x + R_B(x-2)$

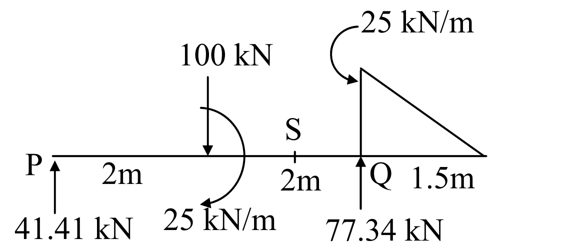
For bending moment be zero $M_x = 0$,

$-20x + 42.5(x-2) = 0$

$\Rightarrow x = 3.78$ m from right i.e. from D.

02. Ans: (b)

Sol:



Take $\Sigma M_P = 0$

$$\frac{1}{2}\times25\times1.5\times\left(\frac{1.5}{3}+4\right)-(R_Q\times4)+100\times2+25=0$$

$\therefore\ R_Q = 77.34$ kN

Also, $\sum V = 0$

$\therefore\ R_P + R_Q = 100+\frac{1}{2}\times25\times1.5 = 118.75$ kN

$\therefore\ R_p = 41.41$ kN

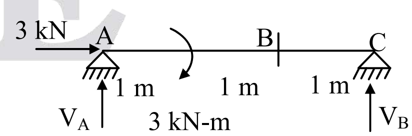
$\Rightarrow$ Shear force at P = 41.41 kN

03. Ans: (c)

Sol: $M_S = R_P(3) + 25 - (100\times1) = 49.2$ kN-m

04. Ans: (c)

Sol:



$-V_B\times3+3=0$

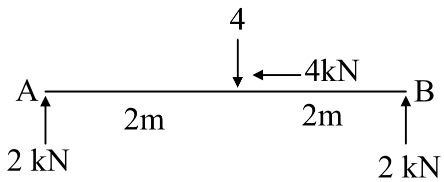
$\therefore\ V_C = 1$ kN

$\therefore$ Bending moment at B,

$\Rightarrow M_B = V_C\times1 = 1$ kN-m

05. Ans: (a)

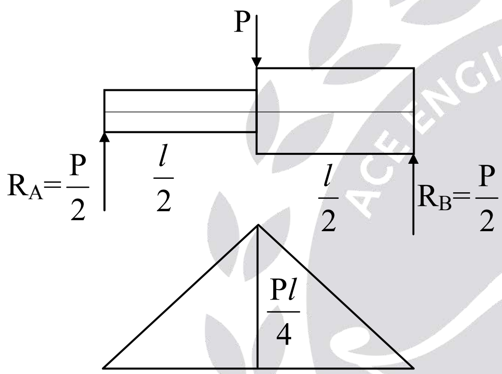
Sol:



Reaction at both the supports are 2 kN and in upward direction.

06. Ans: (c)

Sol:



BMD Diagram

Bending moment at $\frac{l}{2}$ from left is $\frac{Pl}{4}$.

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)

Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

04. Centre of Gravity & Moment of Inertia

01. Ans: (a)

Sol: $\bar{y} = \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2}$

$$\Rightarrow \bar{y} = \frac{2E_2\left(h + \frac{h}{2}\right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \quad (\because E_1 = 2E_2)$$

$$\Rightarrow \bar{y} = 1.167h \text{ from base}$$

02. Ans: (b)

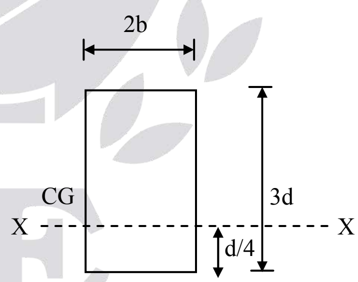
Sol: $\bar{y} = \frac{A_1E_1Y_1 + A_2E_2Y_2}{A_1E_1 + A_2E_2}$

$$= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2E_1 + 6a^2(2E_1)}$$

$$= \frac{22.5a^3E_1}{15a^2E_1} = 1.5a$$

03. Ans: 13.875 bd^3

Sol:



$y = \frac{5}{4}d$

$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3$$

$$\text{M.I about } X - X\,|_{\text{at } d/4 \text{ distance}} = I_G + Ay^2$$

$$= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2$$

$$= \frac{111}{8}bd^3 = 13.875bd^3$$

04. Ans: 6.885×10^6 mm^4

Sol:

$$I_x = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12} + Ah^2\right)$$

$$= \frac{60\times120^3}{12} - 2\left(\frac{30\times30^3}{12} + (30\times30)\times30^2\right)$$

$$= 6.885\times10^6 \text{ mm}^4$$

05. Ans: 152146 mm^4

Sol:

$$I_x = \frac{30\times40^3}{12} - \frac{\pi\times20^4}{64} = 152146\,\text{mm}^4$$

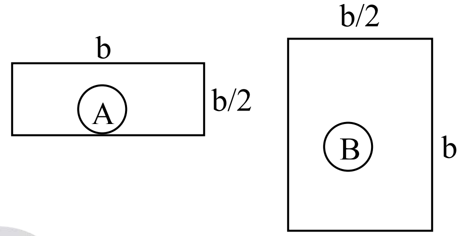
$$I_y = \frac{40\times30^3}{12} - \left(\frac{\pi\times20^4}{64} + 2\left(\frac{\pi}{2}\times10^2\times\left(15-\frac{4\times10}{3\pi}\right)^2\right)\right)$$

$$= 45801.34 \text{ mm}^4$$

05. Theory of Simple Bending

01. Ans: (b)

Sol:



By using flexural formula, $\sigma = \frac{M}{Z}$

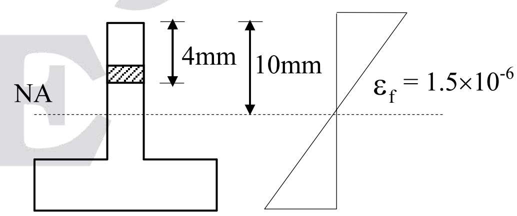
$$\therefore\ \sigma \propto \frac{1}{Z} \qquad (\because M \text{ is constant})$$

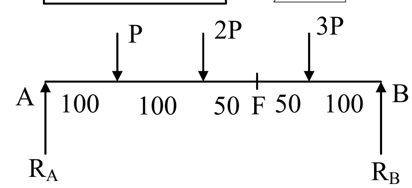
$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\dfrac{\left(\frac{b}{2}\times b^2\right)}{6}}{\dfrac{b\times\left(\frac{b}{2}\right)^2}{6}} = 2$$

$$\Rightarrow \sigma_A = 2\sigma_B$$

02. Ans: (b)

Sol:





$$\therefore \sum M_A = 0$$

$$\therefore\ P\times100 + 2P\times200 + 3P\times300 = R_B\times400$$

$\therefore R_B = 3.5P, R_A = 2.5P$

Take moments about F and moment at F

$M_F = R_B \times 150 - 3P \times 50 = 375P$

Also, $\frac{M_F}{I} = \frac{\sigma_b}{y_F}$

$$\therefore \frac{375P}{2176} = \frac{(1.5\times10^{-6}\times200\times10^{3})}{6}$$

$\Rightarrow \quad P = 0.29$ N

03. Ans: (b)

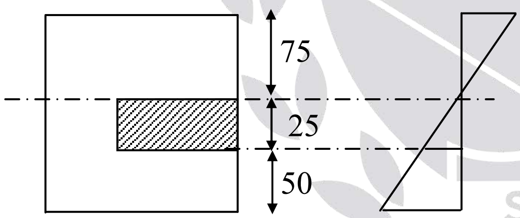
Sol: By using Flexural formula,

$$\frac{E}{R} = \frac{\sigma_b}{y_{max}} \Rightarrow \frac{2\times10^5}{250} = \frac{\sigma_b}{(0.5/2)}$$

$\Rightarrow \sigma_b = 200$ N/mm^2

04. Ans: (c)

Sol:



By using flexural formula,

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \frac{16\times10^6}{\frac{100\times150^3}{12}} = \frac{f}{25} \Rightarrow f = 14.22 \text{ MPa}$$

Now, Force on hatched area

= Average stress × Hatched area

$$= \left(\frac{0+14.22}{2}\right)(25\times50) = 8.9 \text{ kN}$$

05. Ans: (b)

Sol: By using flexural formula, $\frac{f_{Tensile}}{y_{top}} = \frac{M}{I}$

$$\Rightarrow f_{Tensile} = \frac{0.3\times3\times10^6}{3\times10^6}\times70$$

(maximum bending stress will be at top fibre so $y_1 = 70$ mm)

$\Rightarrow f_{Tensile} = 21$ N/mm^2 = 21 MN/m^2

06. Ans: (c)

Sol: Given data:

P = 200 N, M = 200 N.m

A = 0.1 m^2, I = 1.33×10^{-3} m^4

y = 20 mm

Due to direct tensile force P,

$$\sigma_d = \frac{P}{A} = \frac{200}{0.1}$$

= 2000 N/m^2 (Tensile)

Due to the moment M,

$$\sigma_b = \frac{M}{I}\times y$$

$$= \frac{200}{1.33\times10^{-3}}\times20\times10^{-3}$$

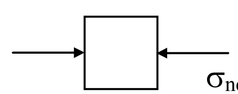
= 3007.52 N/m^2 (Compressive)

$\sigma_{net} = \sigma_d - \sigma_b$

= 2000 – 3007.52

= – 1007.52 N/m^2

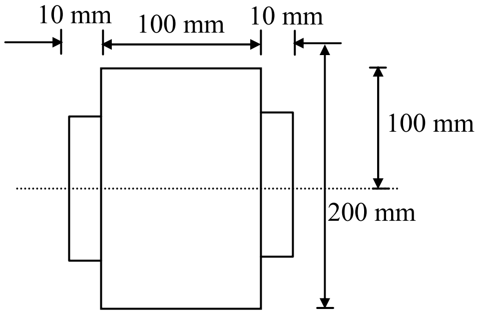
Negative sign indicates compressive stress.



$\sigma_{net} = 1007.52$ N/m^2

07. Ans: 80 MPa

Sol:



Maximum stress in timber = 8 MPa

Modular ratio, m = 20

Stress in timber in steel level,

$100 \rightarrow 8$

$50 \rightarrow f_w$

$\Rightarrow f_w = 4$ MPa

Maximum stress developed in steel is $= m \cdot f_w$

$= 20 \times 4 = 80$ MPa

Convert whole structure as a steel structure by using modular ratio.

08. Ans: 2.43 mm

Sol: From figure $A_1B_1 = l = 3$ m (given)

$$AB = \left(R - \frac{h}{2}\right)\alpha = l - l\alpha t_1 \text{ ------ (1)}$$

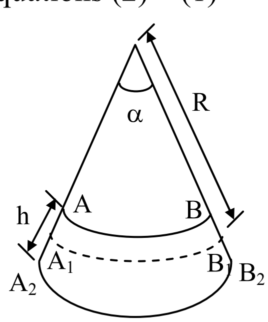
$$A_2B_2 = \left(R + \frac{h}{2}\right)\alpha = l + l\alpha t_2 \text{ ------ (2)}$$

Subtracting above two equations (2) – (1)

$h(\alpha) = l\alpha(t_2 - t_1)$

but $A_1B_1 = l = R\alpha$

$$\Rightarrow \alpha = \frac{l}{R}$$



$$\therefore h\left(\frac{l}{R}\right) = l\alpha(\Delta T)$$

$$R = \frac{h}{\alpha(\Delta T)}$$

$$= \frac{250}{(1.5\times10^{-5})(72-36)}$$

$R = 462.9$ m

From geometry of circles

$$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2} \quad \text{\{ref. figure in Q.No.02\}}$$

$$2R\cdot\delta - \delta^2 = \frac{L^2}{4} \text{ (neglect } \delta^2\text{)}$$

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8\times462.9} = 2.43 \text{ mm}$$

Shortcut:

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^o - 36^o = 36^o$). Bottom temperature being more, the beam deflects down.

$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h} = \frac{1.5\times10^{-5}\times36\times3000^2}{8\times250}$$

$= 2.43$ mm (downward)

06. Shear Stress Distribution in Beams

01. Ans: (a)

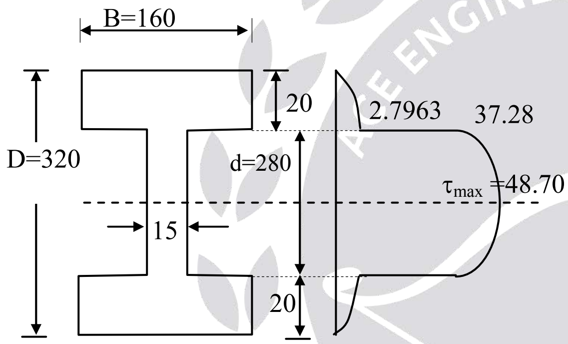
Sol: $\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{f}{b.d}$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$\therefore d = 250 \text{ mm} = 25 \text{ cm}$

02. Ans: 37.3 MPa

Sol:



All dimensions are in mm

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12}$$

$= 171.65 \times 10^6 \text{ mm}^4$

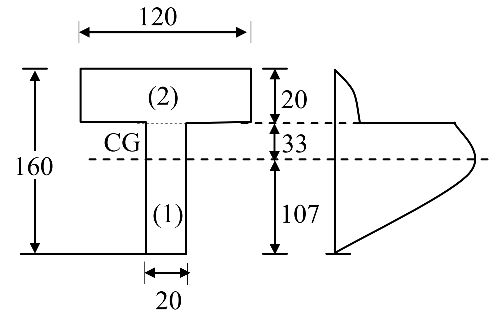
$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (160 \times 20 \times 150)$$

$= 37.28$ MPa

03. Ans: 61.43 MPa

Sol:



All dimensions are in mm

$I_{NA} = 13 \times 10^6 \text{ mm}^4$

$y_{CG} = 107$ mm from base

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

$A\bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5)$

$= 114090 \text{ mm}^3$

$$\tau_{max} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20}$$

$= 61.43$ MPa

07. Torsion

01. Ans: (c)

Sol: Twisting moment = 2 × 0.5 + 1 × 0.5

= 1.5 kN-m

Correct answer is (c).

02. Ans: (d)

Sol: $$\frac{(\text{Strength})_{\text{solid}}}{(\text{Strength})_{\text{hollow}}} = \frac{1}{1-K^4}$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)^4} = \frac{16}{15}$$

03. Ans: 43.27 MPa & 37.5 MPa

Sol: Given D_o = 30 mm, t = 2 mm

∴ D_i = 30 – 4 = 26 mm

We know that $\frac{\tau}{J} = \frac{q}{R}$

$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{max}}{\left(\frac{30}{2}\right)}$$

q_{max} = 43.279 N/mm^2

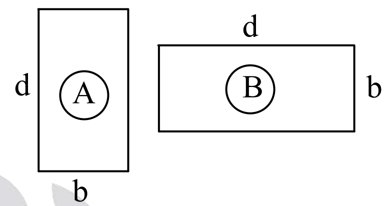
$$\frac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \frac{q_{min}}{\left(\frac{26}{2}\right)}$$

q_{min} = 37.5 N/mm^2

08. Slopes and Deflections

01. Ans: (c)

Sol:



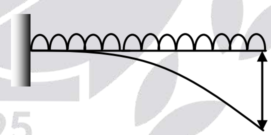
$$y_{max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

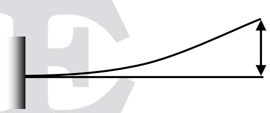
$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

02. Ans: (b)

Sol: Total load W= w*l*



$$y_{max} = \frac{W\ell^3}{8EI} \text{ (Downward)}$$



$$y_{max} = \frac{W\ell^3}{3EI} \text{(Upward)}$$

$$y_{net} = \downarrow y_{udl} - \uparrow y_w$$

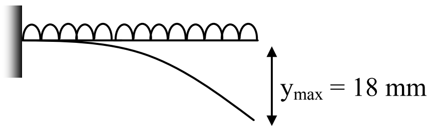
$$\text{Total Net deflection} = \frac{WL^3}{8EI} - \frac{WL^3}{3EI}$$

$$= \frac{-5WL^3}{24EI}$$

(Negative sign indicates upward deflection)

03. Ans: (c)

Sol:



$$\theta_{max} = \frac{wl^3}{6EI} = 0.02 \quad \text{-------(i)}$$

$$y_{max} = \frac{wL^4}{8EI}$$

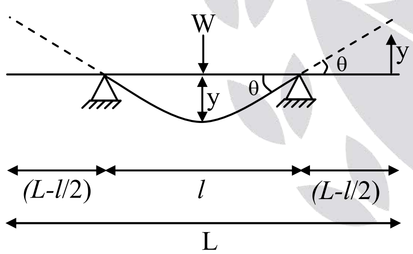
$$\therefore\ 0.018 = \left(\frac{WL^3}{6EI}\right) \times \frac{L \times 6}{8}$$

$$\therefore\ 0.018 = \frac{0.02 \times L \times 6}{8} \qquad [\because \text{ Equation (i)}]$$

$$\Rightarrow \quad L = 1.2 \text{ m}$$

04. Ans: (a)

Sol:



Conditions given

$$\downarrow y = \frac{wl^3}{48EI}$$

$$\theta = \frac{wl^2}{16EI}$$

$$\tan\theta = \frac{y}{(L-\ell)/2}$$

θ is small ⇒ tan θ = θ

$$\therefore\ \theta = \frac{y}{(L-\ell)/2}$$

$$\therefore y = \theta\left(\frac{L-\ell}{2}\right)$$

$$\uparrow y = \theta\left(\frac{L-\ell}{2}\right)$$

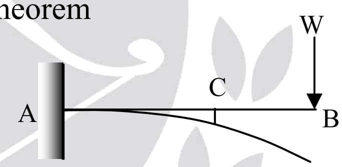
Thus y↓ = y↑

$$\therefore\ \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-\ell}{2}\right)$$

$$\Rightarrow \frac{L}{\ell} = \frac{5}{3}$$

05. Ans: (c)

Sol: By using Maxwell's law of reciprocals theorem



$$\delta_{C/B} = \delta_{B/C}$$

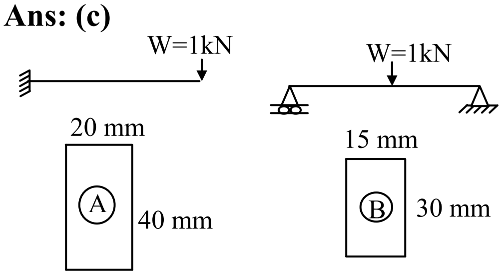
Deflection at 'C' due to unit load at 'B'

= Deflection at 'B' due to unit load at 'C'

As the load becomes half deflection becomes half.

06. Ans: (c)

Sol:

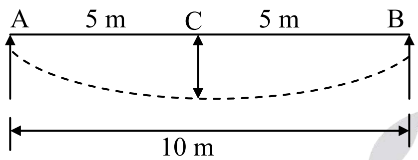


$$y_A = y_B \Rightarrow \left(\frac{wL^3}{3EI}\right)_A = \left(\frac{wL^3}{48EI}\right)_B$$

$\therefore L_B = 400$ mm

07. Ans: 0.05

Sol:



$\therefore$ Curvature, $\frac{d^2y}{dx^2} = 0.004$

Integrating with respect to x,

We get, $\frac{dy}{dx} = 0.004x$

$$y = \frac{0.004x^2}{2}$$

$$y = 0.002x^2$$

At mid span, x = 5 m

$\therefore$ $y = 0.002\,x^2$

$y = 0.05$ m

09. Thin Pressure Vessels

01. Ans: (b)

Sol: $\tau_{max} = \sigma_l = \frac{\sigma_h - 0}{2} = \frac{PD}{4t}$

$$\therefore \tau_{max} = \frac{1.6\times 900}{4\times 12} = 30\ \text{MPa}$$

02. Ans: 2.5 MPa & 2.5 MPa

Sol: Given data:

R = 0.5 m, D = 1m, t = 1mm,

H = 1 m, $\gamma = 10$ kN/m^3, h = 0.5 m

At mid-depth of cylindrical wall (h = 0.5m):

Circumferential (hoop) stress,

$$\sigma_c = \frac{P_{at\ h=0.5m}\times D}{4t} = \frac{\gamma h\times D}{4t}$$

$$= \frac{10\times 10^3\times(2\times 0.5)}{4\times 1\times 10^{-3}}$$

$= 2.5\times 10^6$ N/m^2 = 2.5 MPa

Longitudinal stress at mid-height,

$$\sigma_\ell = \frac{\text{Net weight of the water}}{\text{Cross-section area}}$$

$$= \frac{\gamma\times\text{Volume}}{\pi D\times t}$$

$$= \frac{\gamma\times\frac{\pi}{4}D^2L}{\pi D\times t} = \frac{\gamma\times DL}{4t}$$

$$= \frac{10\times 10^3\times 1\times 1}{4\times 10^{-3}}$$

$= 2.5\times 10^6$ N/m^2 = 2.5 MPa

10. Columns

01. Ans: (c)

Sol: By using Euler's formula, $P_e = \dfrac{\pi^2 \times EI}{l_e^2}$

For a given system, $l_e = \dfrac{l}{2}$

$\therefore \quad P_e = \dfrac{4\pi^2 \times EI}{l^2}$

02. Ans: (b)

Sol: We know that, $P_{cr} = \dfrac{\pi^2 EI}{\ell_e^2}$

$\therefore\ P_{cr} \propto \dfrac{1}{\ell_e^2}$

$\therefore\ \dfrac{P_1}{P_2} = \dfrac{l_{2e}^2}{l_{1e}^2}$

$\therefore\ \dfrac{P_1}{P_2} = \dfrac{l^2}{(2l)^2} \Rightarrow P_1 : P_2 = 1 : 4$

03. Ans: 4

Sol: Euler's crippling load,

$$P = \frac{\pi^2}{l^2}\ EI$$

$\therefore\ P \propto I$

$$\Rightarrow \frac{P}{P_o} = \frac{I_{bonded}}{I_{loose}} = \frac{\left[\dfrac{b(2t)^3}{12}\right]}{2\left[\dfrac{bt^3}{12}\right]} = 4$$

04. Ans: (c)

Sol: Euler's theory is applicable for axially loaded columns.

Force in member AB, $P_{AB} = \dfrac{F}{\cos 45^o} = \sqrt{2}F$

$$P_{AB} = \frac{\pi^2 EI}{L_e^2}$$

$$\therefore\ \sqrt{2}\ F = \frac{\pi^2 EI}{L_e^2}$$

$\Rightarrow F = \dfrac{\pi^2 EI}{\sqrt{2} L^2}$ **Correct answer is (c).**

05. Ans: (a)

Sol: Given data:

$L_e = L = 3$ m,

$\alpha = 12 \times 10^{-6}$ /°C,

d = 50 mm = 0.05 m

Buckling load, $P_e = \dfrac{\pi^2 EI}{L_C^2}$

$$\therefore \quad \frac{P_e L}{AE} = L\alpha\Delta T$$

$$\therefore \quad \frac{\pi^2 EI \times L}{L^2 \times AE} = L\alpha\Delta T$$

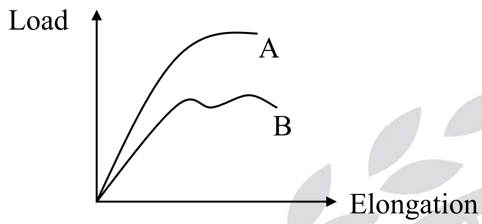
$$\therefore \quad \frac{\pi^2 \times E \times \dfrac{\pi}{64} \times d^4 \times L}{L^2 \times \dfrac{\pi}{4} d^2 \times E} = L\alpha\Delta T$$

$$\therefore \quad \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}}$$

$\Rightarrow \quad \Delta T = 14.3$°C

11. Strain Energy

01. Load-elongation curves for two rods A & B of same dimensions are shown. If E and D are elasticity and ductility respectively, then



(a) $E_A = E_B$ (b) $E_A < E_B$

(c) $D_A = D_B$ (d) $D_B > D_A$

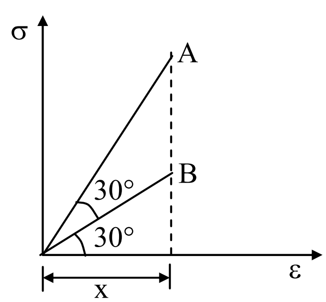
01. Ans: (d)

Sol:

- Slope of the stress-strain curve in the elastic region is called modulus of elasticity.
  For the given curves,
  (Modulus of elasticity)$_A$ > (Modulus of elasticity)$_B$
  $\therefore$ $\mathbf{E_A > E_B}$
- The material for which plastic region is more is stress-strain curve is possessed high ductility. Thus, $\mathbf{D_B > D_A}$.

02. Ans: (b)

Sol:

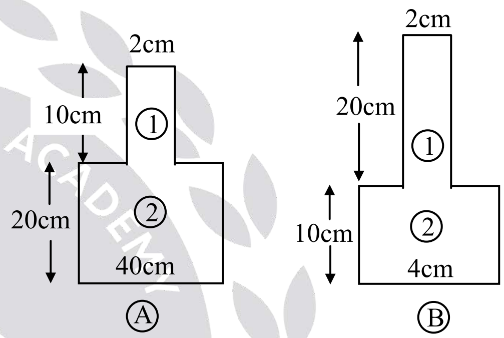


$$\frac{(SE)_A}{(SE)_B} = \frac{\text{Area under curve A}}{\text{Area under curve B}}$$

$$= \frac{\frac{1}{2}\times x \times x \tan 60^o}{\frac{1}{2}\times x \times x \tan 30^o} = \frac{3}{1}$$

03. Ans: (a)

Sol:



$$\frac{U_B}{U_A} = \frac{(V_1 + V_2)_B}{(V_1 + V_2)_A}$$

$$\therefore \frac{U_B}{U_A} = \frac{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_B}{\left[\frac{\sigma_1^2}{2E}\times V_1 + \frac{\sigma_2^2}{2E}\times V_2\right]_A}$$

$$= \frac{\left[\frac{P^2}{A_1^2}\times A_1 \times L_1 + \frac{P^2 \times A_2 \times L_2}{A_2^2}\right]}{\left[\frac{P^2 \times A_1 \times L_1}{A_1^2} + \frac{P^2 \times A_2 \times L_2}{A_2^2}\right]_A}$$

$$\Rightarrow \frac{U_B}{U_A} = \frac{\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]_B}{\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]_A} = \frac{7.165}{4.77} = \frac{3}{2}$$

04. Ans: (c)

Sol: A_1 = Modulus of resilience

$A_1 + A_2$ = Modulus of toughness

$A_1 = \frac{1}{2} \times 0.004 \times 70 \times 10^6 = 14 \times 10^4$

$A_2 = \frac{1}{2} \times \left(0.008 \times 50 \times 10^6\right) + \left(0.008 \times 70 \times 10^6\right)$

$= 76 \times 10^4$

$A_1 + A_2 = (14 + 76) \times 10^4 = 90 \times 10^4$

05. Ans: (d)

Sol: Strain energy, $U = \frac{P^2}{2A^2E}.V$

$\therefore\ U \propto P^2$

Due to the application of P_1 and P_2 one after the other

$(U_1 + U_2) \propto P_1^2 + P_2^2$ …….. (1)

Due to the application of P_1 and P_2 together at the same time.

$U \propto (P_1 + P_2)^2$ ……...........(2)

It is obvious that,

$(P_1^2 + P_2^2) < (P_1 + P_2)^2$

$\Rightarrow\ (U_1 + U_2) < U$

06. Ans: 1.5

Sol: Given data:

$L = 100$ mm

$G = 80 \times 10^3\ \ N/mm^2$

$J_1 = \frac{\pi}{32}(50)^4; J_2 = \frac{\pi}{32}(26)^4$

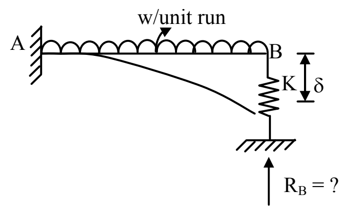
$U = U_1 + U_2 = \frac{T^2L}{2GJ_1} + \frac{T^2L}{2GJ_2}$

$\Rightarrow U = 1.5$ N-mm

12. Propped and Fixed Beams

01. Ans: (d)

Sol:



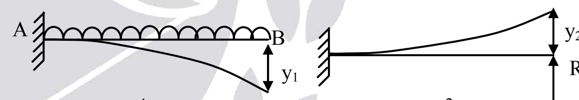
$K = \text{Stiffness} = \frac{\text{Load}}{\text{deflection}}$

$\therefore K = \frac{R_B}{\delta}$

$\therefore$ Compatibility condition

Deflection @ B = δ

$\because K = \frac{R_B}{\delta} \Rightarrow \delta = \frac{R_B}{K}$



$y_1 = \frac{w\ell^4}{8EI}$ $\qquad y_2 = \frac{R_B\ell^3}{3EI}$ $\qquad \downarrow(+)$ $\quad \uparrow(-)$

$y_1 - y_2 = \delta$

$\therefore \frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \delta$

$\frac{w\ell^4}{8EI} - \frac{R_B\ell^3}{3EI} = \frac{R_B}{K}$

$\frac{w\ell^4}{8EI} = \frac{R_B}{K} + \frac{R_B\ell^3}{3EI}$

$\frac{w\ell^4}{8EI} = R_B\ell^3\left[\frac{1}{K\ell^3} + \frac{1}{3EI}\right]$

$$\frac{w\ell^4}{8EI} = R_B\left[\frac{3EI + K\ell^3}{3EI \times K\ell^3}\right] \times \ell^3$$

$$\frac{w\ell}{8EI} = \frac{R_B}{3EI}\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

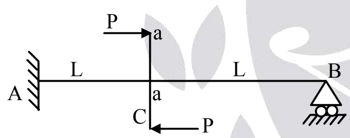
$$\frac{3w\ell}{8} = R_B\left[\frac{3EI + K\ell^3}{K\ell^3}\right]$$

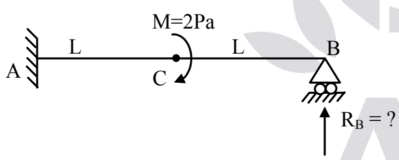
$$\frac{3w\ell}{8} = R_B\left[1 + \frac{3EI}{K\ell^3}\right]$$

$$R_B = \frac{3w\ell/8}{1 + \dfrac{3EI}{K\ell^3}}$$

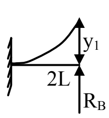
02. Ans: $\frac{9pa}{8L}$

Sol:

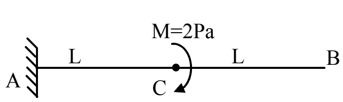




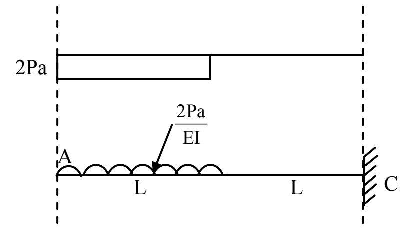
Applying, superposition principle



$$y_1 = \frac{R_B(2L)^3}{3EI} = \frac{8R_BL^3}{3EI}$$



By conjugate beam method



$\therefore y_c$ = deflection @ C

= B.M.D. @ C by conjugate beam

$$y_c = \frac{2Pa}{EI} \times L \times \left[L + \frac{L}{2}\right]$$

$$= \frac{2Pa}{EI} \times L \times \frac{3L}{2}$$

$$= \frac{3PaL^2}{EI}$$

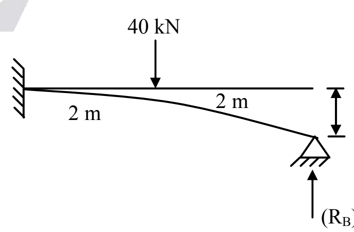
Compatibility Condition (y_B) = 0

$\therefore y_1 = y_c$

$$\frac{8R_BL^3}{3EI} = \frac{3PaL^2}{EI}$$

$$R_B = \frac{9Pa}{8L}(\uparrow)$$

03. Ans: 12.51 kN



E = 200 GPa

$I = 2 \times 10^{+6}\ mm^4$

As per compatablity

$$\frac{(R_B)(4000)^3}{3EI} = \frac{(40\times10^3)(2000)^3}{3\times EI} + \frac{40\times10^3\times(2000)^2}{2EI}\times2000 + 1mm$$

$$\frac{R_B(2\ell)^3}{3EI} = \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(b) + 1mm\left[use\, a = b = \frac{L}{2} = 2000\,mm\right]$$

where EI = 4×10^{11} N/mm^2

$$\therefore \frac{R_B)(4000)^3}{3\times4\times10^{11}} = \frac{40\times10^3\times(2000)^3}{3\times4\times10^{11}} + \frac{40\times10^3\times(2000)^3}{2\times4\times10^{11}} + 1$$

R_B = 12.51 kN

13. Shear Centre

01. Ans: (a)

Sol:

- Shear centre is related to torsion
- On principal plane shear stress is zero
- At fixed end slope is zero.
- Middle third rule is to avoid tension in columns.

02. Ans: (b)

Sol: If the resultant force is acting through shear centre torsion developed in the c/s is zero.

14. Theories of Failure

01. Ans: (d)

Sol: $\sigma = \sigma_y$ = 2500 kg/cm^2

σ_1 = 2000 kg/cm^2

σ_3 = ?

Maximum shear stress theory

$$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2} \not> \frac{\sigma_y}{2}$$

$$= \frac{2000 - \sigma_3}{2} = \frac{2500}{2}$$

σ_3 = –500 (comp)

02. Ans: (b)

Sol: D = 100 cm

P = 10 kg/cm^2

$\sigma = \sigma_y$ = 2000 kg/cm^2

FOS = 4 t = ?

Maximum Principal stress theory

$$\sigma_1 = \sigma_h = \frac{PD}{2t} \not> \sigma_y$$

$$\frac{10\times100}{2\times t} = 2000$$

t = 2.5 mm

Safe thickness of plate = 2.5 × F.O.S

= 2.5 ×4

= 10 mm

03. Ans: (b)

Sol: $\sigma_1 = 1.5$ (T)

$\sigma_2 = \sigma$ (T)

$\sigma_3 = -\sigma/2$ (C)

$\sigma_y = 2000$ kg/cm^2

$\mu = 0.3$

In which theory of failure $\sigma = 1000$ kg/cm^2
Check

(a) Maximum principal stress theory

$\sigma_1 = \sigma_y$

$1.5\sigma_1 = 2000$

$\sigma_1 = 1333$ kg/cm^2

(b) Maximum shear stress theory

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \frac{\sigma_y}{2}$$

$$\left(\frac{1.5\sigma + \frac{\sigma}{2}}{2}\right) = \frac{2000}{2}$$

$$\frac{4}{2}\sigma = 2000$$

$$\sigma = 1000 \text{ kg/cm}^2$$

04. Ans: (c)

Sol: $\sigma_1 = 800$ kg/cm^2

$\sigma_2 = 400$ kg/cm^2

$\mu = 0.25$

$$\varepsilon_1 \le \frac{\sigma_y}{E}$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \frac{\mu\sigma_3}{E} = \frac{\sigma_y}{E}$$

$$\frac{800}{E} - 0.25\frac{(400)}{E} = \frac{\sigma_y}{E}$$

$\sigma_y = 800 - 100 = 700$ kg/cm^2