# GATE | PSUs

# INSTRUMENTATION ENGINEERING

## Sensors & Industrial Instrumentation

**Text Book :** Theory with worked out Examples and Practice Questions

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# Chapter 1 Basics of transducer

## *(Solutions for Text Book Practice Questions)*

**01. Ans: (d)**

**Sol:** P: Charge amplifier with very low bias current and high input impedance - Piezoelectric sensor for measurement of static force

Q: Voltage amplifier with low bias current and very high input impedance- Glass electrode pH sensor

R: Voltage amplifier with very high CMRR- Strain gauge in unipolar DC Wheatstone bridge

**02. Ans: (b)**

**Sol:** A. Mc Leod gauge- Pressure

B. Turbine meter- Flow

C. Pyrometer-Temperature

D. Synchros- Displacement

**03. Ans: (b)**

**Sol:**

P. Radiation Pyrometer - Temperature measurement

Q. Dall tube- Flow measurement

R. Pirani gauge-Vacuum pressure measurement

S. Gyroscope-Angular velocity measurement

# Chapter 2 Resistive, Inductive & Capacitive Transducers

**01.**

**Sol:** Given

$R_m = 2.5\ R_P$

$\frac{X_t}{2} = X_i$

$\frac{R_P}{R_m} = \frac{1}{2.5} = 0.4$

$\frac{X_i}{X_t} = \frac{1}{2} = 0.5 = K$

$$E_L = \frac{K}{1 + K(1-K)\times\frac{R_P}{R_m}} \times E_i$$

$$= \frac{0.5}{1 + 0.5\times0.4\times0.5} \times E_i$$

$= 0.4545\times E_i$

True value = $K\times E_i$

$= 0.5\ E_i$

$$\%\ \text{error} = \frac{MV - TV}{TV}\times100$$

$$= \frac{0.4545\times E_i - 0.5E_i}{0.5E_i}\times100$$

$= -0.0917\ \%$

**02. Ans: 1.1234**

**Sol:** Given

Slide wire resistance = 10 Ω for 1000 mm

For 234 mm length $R = \frac{234}{1000}\times10 = 2.34\ \Omega$

When dial is at position 11

total resistance = 110 Ω

$V_x = (110 + 2.34) \times 10$ mA

$V_x = 1.1234$ Volt

**03. Ans: (b)**

**Sol:** We know $R = \frac{\rho L}{A}$

L is increased and becomes α L so, the new resistance is

$$R_{new} = \frac{\rho L\times\alpha}{\frac{A}{\alpha}}$$

$$= \frac{\rho\times L\times\alpha\times\alpha}{A}$$

$$= \frac{\rho\times L\times\alpha^2}{A}$$

$= \alpha^2 R$

So, $R_{new} = \alpha^2 R$

**04. Ans: (a)**

**Sol:** As we always place the strain gauge experiencing same stress in opposite direction to get maximum output voltage.

**05. Ans: 1.16**

**Sol:** Given

$R_g = 120\,\Omega$, $G_F = 2.0$

So, $R_1 = 2\times100\times43.4\Omega/100m$

$= 86.8\,\Omega$

We know the relation

$$\alpha = \frac{R_1}{R_g} = \frac{86.8}{120} = 0.72$$

From formula we know that

$$(G_F)_{eff} = \frac{G_F}{1+\alpha} = \frac{2.0}{1.12} = 1.16$$

**06. Ans: (c)**

**Sol:** $\text{CMRR} = \frac{A_d}{A_c} = \frac{\text{differential gain}}{\text{Common mode gain}}$

CMRR in dB = $20\log\left(\frac{A_d}{A_c}\right)$

$A_d = 10$

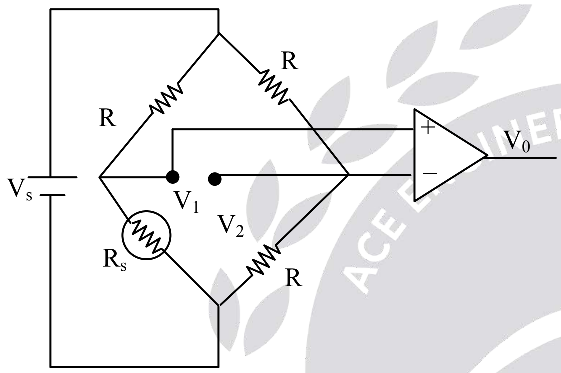
$A_c$ common mode gain

$$=\frac{\Delta V_0}{\Delta V_i}=\frac{(4-3)mV}{(3-2)V}=1\times10^{-3}$$

$$CMRR=\frac{A_d}{A_c}=\frac{10}{10^{-3}}=10\times10^3=10^4$$

**07. Ans: (b)**

**Sol:**




$$V_1-V_2=\frac{V_s}{4}\frac{\Delta R}{R}\Rightarrow\frac{V_s}{4}\frac{\pm R\times\frac{1}{100}}{R}$$

$$\Rightarrow\frac{+V_s}{400}\text{ to }\frac{-V_s}{400}$$

$$\Rightarrow 125\text{ to }-125$$

As the data says the differential amplifier has an additional constant offset voltage at output. Given data for common mode input $V_i = 2V$ and 3V common mode output $V_0$=3mV and $V_0$ = 4mV. Under no load conditions common mode input

$V_1 = V_2 = 2.5$ (Strain Gauge Resistance –R)

So the common mode input signal becomes 2.5V, hence common mode output signal.

For corresponding 2.5V is

$$\frac{V_0+V_{02}}{2}=\frac{3mV+4mV}{2}=3.5mV$$

If we add this offset to the output of differential amplifier for R + ΔR is

$V_0 = 124.3 + 3.5 = 127.8$

For R – ΔR = –125+3.5 =121.5mV

So answer is +128mV to –122mV

**08. Ans: (c)**

**Sol:** given

$\overline{V_1}=1.0V\angle0^\circ = 1e^{j0}$ = (cos 0 + jsin0) = 1

$\overline{V_2}=1.0V\angle10^\circ = 1e^{j0}$ = (cos 10 + jsin10)

= 0.984 + j0.113

$V_1 - V_2$ = (1– 0.984) – j0.173

= 0.016 – j0.173

$$|V_1 - V_2|=\sqrt{(0.016)^2+(0.173)^2}$$

= 0.174 V

**09.**

**Sol:** a) for a displacement of 0.5mm, the output is 2mV, so the Sensitivity (s) = 2/0.5 mV/mm

= 4mV/mm

b) for the whole setup, the sensitivity is

$S = 150\times 4mV/mm$

= 1V/mm

c) Given that,

The output of the voltmeter is 5V with 100 divisions which means that each division = 5V/100 = 0.05V

The minimum voltage that can read is 1/5th of a division, so the minimum voltage is

$$=\frac{0.05V}{5}=0.01V$$

Which corresponds to 0.01mm so Resolution

'R' = 0.01mm

**10.**

**Sol:** Given $\beta=250$

$S_{LVDT}$ = 1mV/0.5mm = 2mV/mm

1mm → 2mV

0.25mm → 0.25×2mV

$E_0 = \beta \times E_{diff}$

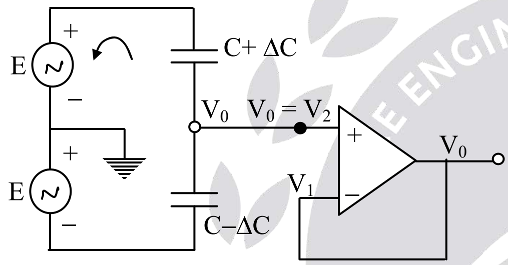
= 250 ×0.5mV = 125mV

$\text{Sensitivity} = \frac{125mV}{1N}$

So, sensitivity of the whole system is $\frac{125\text{mV}}{1\text{N}}$

**11. Ans: (b)**

**Sol:**




Virtual ground concept

$V_1 = V_2, \qquad V_2 = V_0$

$$\frac{V_0 - E}{X_{C+\Delta C}} + \frac{V_0 - (-E)}{X_{C-\Delta C}} = 0$$

$$\frac{V_0 - E}{\frac{1}{j\omega(C+\Delta C)}} + \frac{V_0 + E}{\frac{1}{j\omega(C-\Delta C)}} = 0$$

$$(V_0 - E)j\omega(C+\Delta C) + (V_0 + E)\, j\omega(C-\Delta C) = 0$$

$V_0\,C + V_0\,\Delta C - EC$

$= EC + V_0\,C - V_0\,\Delta C + EC - E\,\Delta C = 0$

$2\,V_0\,C = 2\,E\,\Delta C$

$$V_0 = E\frac{\Delta C}{C}$$

**12. Ans: 3.56**

**Sol:** Given

p = 0, d = 4 mm, c = 300 pF,

p = 500 kN/$m^2$,

Average deflection = 0.28 mm,

Δf = ?

$$F = \frac{1}{2\pi\sqrt{Lc}}$$

$$f \propto \frac{1}{\sqrt{c}}$$

So

$$\frac{f_1}{f_2} = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{d_1}{d_2}}$$

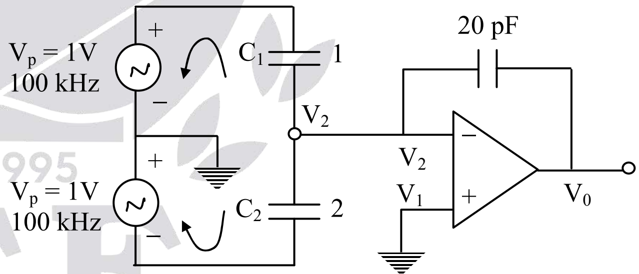
$$f_2 = f_1\sqrt{\frac{d_2}{d_1}}$$

$$= 100\text{ k}\sqrt{\frac{4-0.28}{4}} = 96.43\text{Hz}$$

$\Delta f = f_1 - f_2 = 3.56\text{Hz}$

**13. Ans: (b)**

**Sol:**




$V_1 = V_2$ Virtual ground concept

Apply nodal analysis at $V_2$

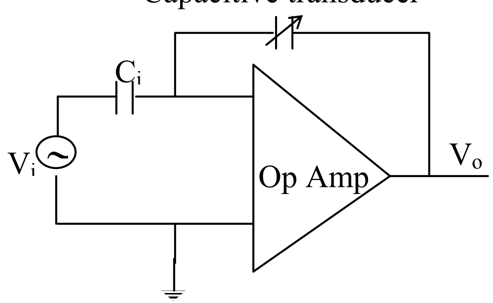
$$\frac{V_2 - 1}{X_{C1}} + \frac{V_2(-1)}{X_{C2}} + \frac{V_2 - V_0}{X_{C3}} = 0$$

$$-1(j\omega_{C1}) + 1(j\omega_{C2}) - V_0(j\omega_{C3}) = 0$$

$$V_0(j\omega_{C3}) = j\omega\left[\frac{C_2 - C_1}{C_3}\right]$$

$V_0 = 0.354$ V

**14.**
**Sol:**




Here $V_o = kx$

$$K = \left(\frac{-C_i}{\varepsilon_0 A} V_i\right) V/m$$

Where K is sensitivity

Given data

$A = 200\ mm^2$, $C_i = 10$ pF

So from the above formula,

The sensitivity

$$K = \left(\frac{10\times10^{-12}}{8.85\times10^{-12}\times200\times10^{-6}}\times10\right) V/m$$

$$= 0.057\times10^{-6}\ V/mm$$

So output voltage for 1μm is

$V_0 = 0.057\times10^{-6}\times10^{-6}$

$= 0.057V$

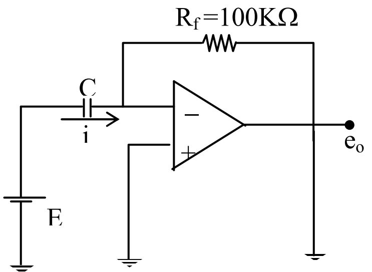
$= 57mV$

**15.**
**Sol:** From the above figure

$$i = E\times\frac{-c}{x}.\frac{dx}{dt}$$

$$e_0 = -R_f\times i$$

$$e_0 = R_f\times\frac{c}{x}\times E\times\frac{dx}{dt}$$




Given data E = 100V

$R_f = 100K\Omega$

C = 50 pF

X = 5mm

$$e_0 = \left\{\frac{100\times10^3\times50\times10^{-12}\times100}{5\times10^{-3}}\right\}\frac{dx}{dt}$$

$$\frac{e_0}{\frac{dx}{dt}} = \frac{100\times10^3\times50\times100}{50\times10^{-3}} = 0.1$$

**16. Ans: (d)**

**Sol:** $V_0 = A_d\times V_d$

$$V_B = \frac{V_s\times\Delta C}{2C}$$

Given $V_0 = 10$ V

$$\Delta C = \frac{5}{100} = 0.05$$

So

$$10\ V = A_d\times\frac{0.05}{2}\times10$$

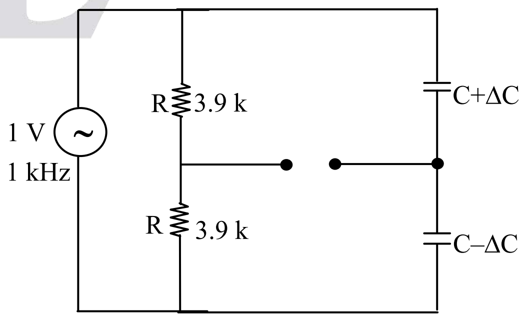
$$A_d = \frac{2}{0.05} = 40$$

**17. Ans: (b)**

**Sol:** Sensitivity of the measurement system never changes.

**18. Ans: (b)**

**Sol:** Given $C_0 = C = 0.01$ μF




$$V_B = V_s\cdot\frac{\Delta C}{2C}$$

Sensitivity

$$\frac{V_B}{\Delta C} = \frac{V_s}{2C} = \frac{1V}{2\times 0.01\mu F}$$

$$= \frac{100}{2} V/\mu F$$

$$= \frac{50V}{\mu F} = 0.05\ mV/pF$$

**19.**

**Sol:** For multiple parallel plate capacitive transducers sensitivity is

$$S = \frac{-(n-1)\varepsilon_0 \omega}{d}$$

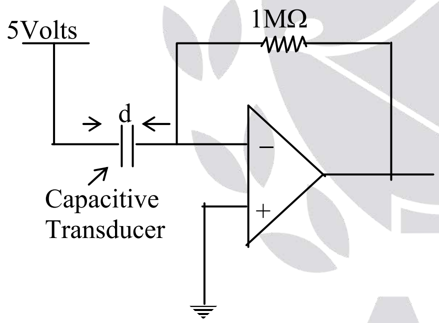
$$= \frac{-(5-1)\times 8.85\times 10^{-12}\times 25\times 10^{-3}}{0.25\times 10^{-3}}$$

$$= -3.54\times 10^{-9} F/m$$

$$= -3.54\ nF/m$$

**20. Ans: (b)**

**Sol:**




$$X(t) = d(t) = 10^{-3}\{1 + 0.1\sin(1000\pi t\}m$$

$$d(t) = 10^{-3}\,m + 0.1mm\ \sin(1000\,\pi t)$$

C = 2pF at t = 0m sec

C = $C_o$ at 2pF

$$i(t) = E.\frac{d}{dt}c(t)$$

$$C = \frac{\in A}{d} = \frac{\in A}{X}$$

$$\frac{dc}{dt} = \frac{-\in A}{X^2}.\frac{dx}{dt} = \frac{-C}{X}.\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{d}{dt}\{1mm + 0.1mm\sin(1000\pi t)\ \}$$

$$= \frac{\pi}{10}\cos\omega t$$

$$i(t) = -E.\frac{d}{dt}(c) = -E\frac{C}{X}.\frac{\pi}{10}\cos\omega t$$

$$i(t) = -5V\times\frac{2pF}{1mm}\times\frac{\pi}{10}\cos\omega t$$

$$i(t) = -\pi\cos\omega t\ n\ amp$$

$V_0(t) = \pi\cos\omega t\ mV \quad (V_0 = -i(t)\times 1M\Omega)$

$V_0(t) = \pi\cos 1000t(mV)$

$V_0(2\text{ m sec}) = \pi\cos(2000\pi)mV$

$V_0 = \pi mV$

**21. Ans: (b)**

**Sol:** $C_1 = \dfrac{\varepsilon_0.\varepsilon_r\times\ell\times\omega}{d}$

$$C_2 = \frac{\varepsilon_0\times\varepsilon_r\times x\times\omega}{d} + \frac{\varepsilon_0\times\varepsilon_r\times(\ell - x)\omega}{d}$$

Given $2C_1 = C_2$

$$\frac{2\times\varepsilon_0.\varepsilon_r\times\ell\times\omega}{d} = \frac{\varepsilon_0\times\varepsilon_r\times x\times\omega}{d} + \frac{\varepsilon_0\varepsilon_r(\ell - x)\omega}{d}$$

$$\Rightarrow x = \frac{L}{2}$$

**22. Ans: 0.5**

**Sol:** $R_g = 1000\Omega$

$G_f = 2.5$

Strain = 400μm/m

$V_0 = ?$

$$V_0 = \frac{V_S}{4}.\frac{\Delta R}{R}$$

$$V_0 = \frac{V_s}{4}.G_f.\text{Strain} = \frac{2}{4}\times 2.5\times 400\times 10^{-6}$$

$$= 500\times 10^{-6}V$$

$$= 5\times 10^{-4}$$

$V_0 = 0.5mV$

**23. Ans: (d)**

**Sol:** $\Delta V = V_1 - V_2$

$$\Delta V = \frac{x}{d}E$$

$$= \frac{10mm}{50mm}\times 10$$

$$= 2V$$

**Detailed solution:**

$$C_1 = \frac{\varepsilon A}{d+x}, \qquad C_2 = \frac{\varepsilon A}{d-x}$$

$$E_1 = \frac{C_2}{C_1+C_2}.E \qquad E_2 = \frac{C_1.E}{C_1+C_2}$$

$$E_1 = \frac{\frac{\varepsilon A}{d-x}}{\left(\frac{\varepsilon A}{d-x}\right)+\left(\frac{\varepsilon A}{d+x}\right)}$$

$$E_2 = \frac{\frac{\varepsilon A}{d+x}}{\left(\frac{\varepsilon A}{d-x}\right)+\left(\frac{\varepsilon A}{d+x}\right)}$$

$$E_1 = \frac{d+x}{2d}.E \qquad E_2 = \frac{d-x}{2d}$$

$$\Delta V = E_1 - E_2 = E\frac{x}{d} = 2V$$

**24. Ans: (d)**

**Sol:** $$V_{Bri} = \frac{V_s}{4}\frac{\Delta R}{R}$$

$$V_{Bri} = \frac{V_s}{4}G_f \times \text{strain}$$

$$1\times 10^{-3} = \frac{2}{4}\times 2\times \text{strain}$$

$$\text{Strain} = 1\times 10^{-3} = 1000\mu$$

# Chapter 3 Piezo Electric Transducers

**01.**

**Sol:** Given constant ‘g’ $= 12 \times 10^{-3}\ \dfrac{V/m}{N/m^2}$

Dielectric constant $= 1.250 \times 10^{-8}$ F/m

Young’s modulus $E = 1.2 \times 10^{11}$ N/m$^2$

$d = 8$ mm, $t = 2$ mm and

$R = 10^8\ \Omega$

a) $g = \dfrac{K}{\in}$

K is sensitivity in C/N

$K = g\in = 12 \times 10^{-3} \times 1.25 \times 10^{-8}$

$= 15 \times 10^{-11}$ C/N

$C = \dfrac{\in A}{t}$

$C = 31.41 \times 10^{-11}$

Sensitivity in V/m $= \dfrac{K}{C}$

$= \dfrac{15 \times 10^{-11}}{31.41 \times 10^{-11}}$

$= 0.477$ V/m

b) If force is 10 N

$A = \dfrac{\pi d^2}{4} = \dfrac{\pi \times (8\times 10^{-3})^2}{4}$

$A = 50.26 \times 10^{-6}$ m$^2$

Pressure $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{10}{50.26 \times 10^{-6}}$

$= 0.198 \times 10^6$

$= 19.8 \times 10^4$ N/m$^2$

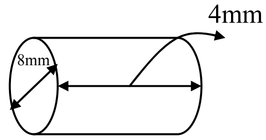
$V_0 = g \times p \times t$

$= 12 \times 10^{-3} \times 19.8 \times 10^4 \times 2 \times 10^{-3}$

$= 4.752$ V

**02.**

**Sol:** $\varepsilon = 4 \times 10^{-11}$ F/m




$y = 8.6 \times 10^{-10}$ N/m$^2$

$\Delta t = x = 10^{-9}$ m

$\dfrac{F}{A} = y\dfrac{\Delta t}{t}$

$F = y.\dfrac{\Delta t}{t}.A$

$F = 8.6 \times 10^{-10} \times 10^{-9} \times \dfrac{\pi (8 \times 10^{-3})^2}{4} = 1.08$N

$C = \dfrac{\varepsilon A}{t} = 0.5$pF

$Q = d.\ f = 2.16$pC

$V = \dfrac{Q}{C_p} = 4.3$V

**03. Ans: (a)**

**Sol:** Given

$d = 2$ pC/N

$C_p = 1600$ pF

$R_p = 1012\Omega$

$F = 0.1 \sin 10t$ N

We know for piezoelectric transducer

$\left|\dfrac{E}{F}\right| = M = \dfrac{k}{\sqrt{1 + \left(\dfrac{1}{\tau\omega}\right)^2}}$

$K = \dfrac{d}{C_p}$

$\tau = R_F C_F$

$$M = \frac{2\times10^{-12}}{\sqrt{1+\frac{1}{(0.1\times10)^2}}} = 1.414\times10\text{-}3\ V/N$$

$$|M| = \frac{E(S)}{F(S)} = 1.141\ \frac{mV}{N}$$

$$E(s) = M\,F(s) = \left(1.414\times\frac{mV}{N}\right)\times0.1N$$

$$= 0.141\ mV$$

**04.**

**Sol:** Given t = 2 mm

$g = 0.05$ Vm/N

$\frac{F}{A} = 15\times10^5\ N/m^2$

We know

$$g = \frac{E_0/t}{F/A}$$

So $E_0 = g\times\frac{F}{A}\times t$

$= 0.05\ Vm/N\times15\times10^5\ N/m^2\times2\times10^{-3}$

$= 150$ V

**05. Ans: 2.512ms**

**Sol:** Charge sensitivity $d_{33} = d = 150\times10^{-12}$ C/N

$C_{pz} = 25\times10^{-12}$ F

$R_{pz} = 10^{10}\,\Omega$

Input force = 2u(t) (step force) (N)

The Voltage generated

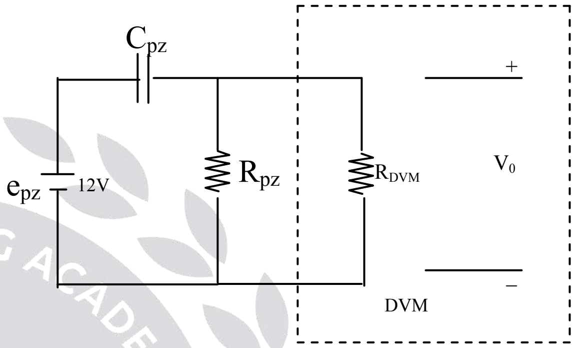
$$= \frac{\text{Charge Generated}}{C_{pz}}$$

$$e_{PZ} = e_0 = \frac{d\times F}{C} = \frac{q}{c}$$

$$e_{PZ} = \frac{150\times10^{-12}C/N\times2N}{25\times10^{-12}}$$

$e_{PZ} = 12V$

The equivalent circuit under the given condition is




$$\frac{V_0(S)}{e_{pz}(S)} = \frac{\tau s}{\tau s+1}$$

$\tau = C_{pz}(R_{pz}\,||\,R_{DVM})$

$\tau = 20pF\ (10^{10}\Omega\ ||\ 10^{13}\Omega)$

$\tau = 25\times10^{-12}\times999\times10^9\,\Omega$

$\tau = 0.25$ sec

$V_0(t) = e_{pz}\times e^{-t/\tau}$

Where $V_0(t)$ is voltage across the DVM .

Drop allowed from peak value i.e, 12V is no more than 0.12V. Time at which $V_0$ falls to 11.88V must be calculated.

Assigning $V_0(t) = 11.88V$

$$11.88 = 12\times e^{\frac{-t}{\tau}}$$

$$\frac{11.8}{12} = e^{\frac{-t}{\tau}}$$

$\Rightarrow \ln\left(\frac{11.88}{12}\right) = \frac{-t}{\tau}$

$\Rightarrow \frac{t}{\tau} = \ln\left(\frac{12}{11.88}\right)$

$\Rightarrow t = \tau \times \ln\left(\frac{12}{11.88}\right)$

$= 0.025 \ln\left(\frac{12}{11.88}\right)\text{sec}$

$\Rightarrow t = 2.512$ ms

**06.**

**Sol:** Given $F = 8$ N

$d = 140 \times 10^{-9}$ F/m,

$\varepsilon = 12\times10^{-9}$F/m

$Y = 11.5\times10^{6}$ N/m$^2$

**i)** We know

$\frac{F}{A} = Y \cdot \text{strain}$

$\text{Strain} = \frac{F/A}{Y}$

$= \frac{8}{36\times10^{-6}\times11.5\times10^{6}}$

$= 0.0193$

**ii)** Capacitance (c) = $\frac{\varepsilon A}{d}$

$= \frac{12\times10^{-9}\,F/m\times36\times10^{-6}}{1.5\times10^{-3}}$

$= 2.88\times10^{-10}$

$= 288$ pF

**iii)** Voltage generated

We know Q = C.V

$V = \frac{Q}{C}$

Given $d = \frac{Q}{F}$

So, $Q = d\times F$

$= 1.12\times10^{-9}$ C

$V = \frac{Q}{C} = \frac{1.12\times10^{-9}}{2.88\times10^{-12}}$

$= 3.88$V

**07. Ans: (a)**

**Sol:** Piezo electric transducers is suitable for dynamic inputs only.

**08. Ans: (b)**

**Sol:** Resolution of encoder = $\frac{V_{ref}}{2^n - 1}$

1kPa $\Rightarrow$ 30mV

100kPa $\Rightarrow$3000 mV = 3V

Noise of readout circuit = 3V + 0.3mV

$V_{ref}$ = 3.0003V

Resolution = $\frac{3.0003}{2^{10}-1} = \frac{3.0003}{1023}$

= 0.00293 V

Smallest readout by system = 0.00293V

1kPa $\rightarrow$ 30mV

$0.00293V\times\frac{1kPa}{30mV} \leftarrow 0.00293V$

Resolution from i/p side

$= 97.666Pa \approx 100Pa$

# Chapter 4 Measurement of Temperature

**01.**

**Sol:** For a first order system

$$T = T_0\left\{1-\exp\left(\frac{-t}{\tau}\right)\right\}$$

Given $T = 30$, $T_0 = 50^0C$, $\tau = 120sec$

So, $30^0C = 50^0C\left\{1-\exp\left(\frac{-t}{120}\right)\right\}$

So $t = 110$ sec.

**02. Ans: (b)**

**Sol:** $R_T = R_0\,[1+0.004T]$

True model

$R_T = R_0\,[1+0.004T+6\times10^{-7}T^2]$

Note: measurement model calculated by being measurement value end approximated equation. But true model calculation to based on true value & accurate expression

$R_T = 100[1+0.004(100-0)+6\times10^{-7}(1000)]$

$R_T = 140.06\Omega$

$R_T = 102[1+0.004(100)]$ ---(1)

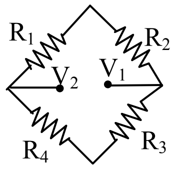
$R_T = 98\,[\,1+0.004(100)]$ ----(2)

$e_1 = 142.8-140.06 = 2.74\Omega$

$e_2 = 137.2-140.06 = -2.86\Omega$

**03. Ans: (b)**

**Sol:** 10 mV/ $^oC$ – Change in $1^0C$ in RTD output of Bridge 10 mV




$R_T = R_0\,[1+\alpha T]$

$= 100[1+0.00392(1-0)]$

$R_T = 100.392\Omega$

$V_{out} = A_d\,[V_1-V_2]$

$10mV = A_d\,[V_B]$

$$V_1 - V_2 = \left(\frac{R_4}{R_2+R_4}\right)V - \left(\frac{R_3}{R_1+R_3}\right)V$$

$$\Rightarrow 10\left[\frac{150.392}{100.392+10k}-\frac{100}{100+10k}\right] = 0.3842 \text{ mV}$$

$$A_d = \frac{10}{0.382v} = 26.02$$

**04. Ans: (b)**

**Sol:** $1^0 \rightarrow 10mV$ (o/p)

$100^0 \rightarrow 100mV$ (o/p)

$$= 10\left[\frac{139.2}{139.2+10k}-\frac{100}{100+10k}\right]$$

$= 0.03827$ V

Op Amp gain

$V_0 = 26.02\,[V_1 - V_2]$

$V_0 = 996$ mV

Error = measured value – True value

$= 996mV - 1000mV$

$= -4mV$

$1^0C \rightarrow 10mV$

$1mV \rightarrow \frac{1^0}{10}C$

$-4mV \rightarrow -0.4^0C$

**05.**

**Sol:** $T_0$ $\quad 0^o\,C \rightarrow 100\,\Omega = R_0$

$T_1$ $\quad 100^oC \rightarrow 150\,\Omega = R_1$

$T_2 \rightarrow$ gas temp $\rightarrow 300\,\Omega = R_2$

$\alpha = 0.0039\,C^{-1}$

As per the relation

$$\frac{R_{T_2} - R_{T_1}}{T_2 - T_1} = \alpha \cdot R_0$$

$$\frac{300 - 150}{T_2 - 100} = 0.0039 \times 100$$

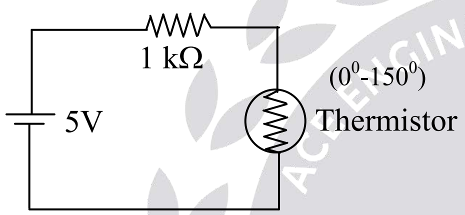
$$\Rightarrow \frac{150}{0.39} = T_2 - 100$$

$$\Rightarrow T_2 = 384.61 + 100$$

$$T_2 = 484.61\ {}^0C$$

**06. Ans: (b)**

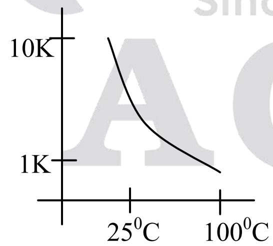
**Sol:**




At $25^o \rightarrow 10k\Omega$

$100^o \rightarrow 1k\Omega$

This thermistor is used in a temperature range of 0-150°C. What is the power dissipated at thermistor when operating at more temperature




$$P = I^2R = \frac{V^2}{R}$$

$$R_T = R_0\ e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

$$\ln\left[\frac{R_T}{R_0}\right] = \beta\left[\frac{1}{T} - \frac{1}{T_0}\right]$$

$T_0 = 25^oC + 273 = 293k \rightarrow 10k = R_0$

$T = 100 + 273 = 373k \rightarrow 1k = R_T$

$$\ln\left[\frac{1}{10}\right] = \beta\left[\frac{1}{373} - \frac{1}{298}\right]$$

$$\beta = 3412.55\ k$$

Now

$T_0 = 100^oC + 273 = 373\ k = R_0 = 1k$

$T = 150^oC + 273 = 473\ k = R_T = ?$

$$\ln\left[\frac{R_T}{1K}\right] = 3412.55\left[\frac{1}{423} - \frac{1}{373}\right]$$

$$R_T = 339.12\ \Omega$$

Thermistor resistor at 150°C

$$I = \frac{5V}{1k + R_{Th}} = \frac{5}{1339.12}$$

$$= 3.733 \times 10^{-3}$$

$$P = I^2 \times R$$

$$P = 4.72\ mV$$

**07.**

**Sol:** $R_t = R_0 a\, e^{\frac{b}{T}}$

$$\frac{R_t}{R_0} = ae^{\frac{b}{T}}$$

$$\log_e\left\{\frac{R_T}{R_0}\right\} = \log_e\left\{ae^{\frac{b}{T}}\right\}$$

$$\log_e\left\{\frac{R_T}{R_0}\right\} = \log_e a + \frac{b}{T}\log_e e$$

$$\log_e\left(\frac{1050}{4000}\right) = \log_e\left(300 \times 10^{-6}\right) + \frac{2850}{T} \times 1$$

$$\Rightarrow -1.337 = -8.11 + \frac{2850}{T}$$

$$\Rightarrow \frac{2850}{T} = 6.78$$

$$\Rightarrow T = \frac{2850}{6.78} = 420.35\ K$$

**08. Ans: $-0.04088K^{-1}$**

**Sol:** $R = R_\theta = R_{\theta 0}.e^{\beta\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)}$ for thermistor

$\theta = 316K \quad \theta_0 = 298k.$

$R_{316} = 465k\Omega \quad R_{298} = 1000\,\Omega$

$$\frac{dR}{d\theta} = R\theta_0\, e^{\beta\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)}\left(-\frac{\beta}{\theta^2}\right)$$

$$\frac{1}{R}.\frac{dR}{d\theta} = \frac{R\theta_0.e^{\beta\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)}}{R\theta_0.e^{\beta\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)}}.(-\beta/\theta^2)$$

Sensitivity is $\frac{-\beta}{\theta^2}$ where $\beta$ must be calculated. Q is given as 316K.

$R_\theta = R_{\theta 0}\,.\, e^{\beta\left(\frac{1}{\theta}-\frac{1}{\theta_0}\right)}$

$R_{316} = R_{298}.e^{\beta\left(\frac{1}{316}-\frac{1}{298}\right)}$ to find $\beta$

$465\,\Omega = 1000\,\Omega\, e^{\beta(316^{-1}-298^{-1})}$

$\ln(0.465) = \beta(316^{-1} - 298^{-1})$

$\beta \cong 4006k$

Therefore Sensitivity

$$= -\frac{4006k}{(316)^2} = -0.0408K^{-1}$$

**09. Ans: (a)**

**Sol:** $2\frac{dT_i}{dt} + T_i - T_a = 0$

$T_i$ = Indicated temperature

$T_a$ = ambient temperature

The -3 DB cut-off frequency in the frequency response of the thermometer is as For any problem frequency response allows take Laplace Transform


$2sT_i(s) + T_i(s) - T_a(s) = 0$

$[2s+1]T_i(s) = T_a(s)$

$$\frac{T_i(s)}{T_a(s)} = \frac{1}{2s+1} = \frac{1}{1+2j\omega}$$

$$|M| = \frac{1}{\sqrt{1+4\omega^2}} = \frac{1}{\sqrt{2}}$$

$$F = \frac{1}{4\pi}$$

**10. Ans: (b)**

**Sol:** Cn – Pt = –35 μV/k

a) Pt – Cn = 35 μV/k

b) Nichrome – Constantan

= 25 – (–35) = 60

c) Nickel – constantan

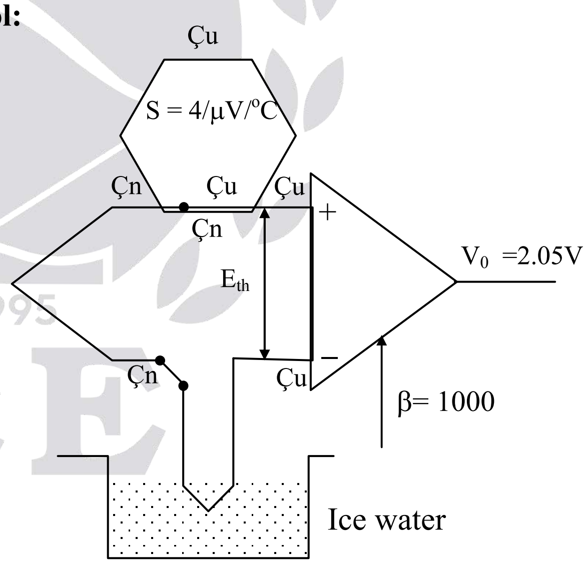
= –25 – (–35) = 10

d) Cu – Ni = 6 + 25 = 31

So maximum sensitivity around 273 k is Given by (b) Nichrome – constantan

**11. Ans: (d)**

**Sol:**




$\therefore V_0 = \beta V_{in}$

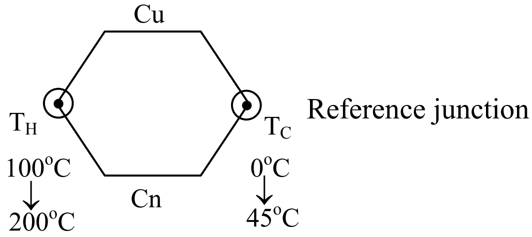
$= \beta\, E_{TH}$

$V_0 = \beta S_{TH}\,[T_H - T_C]$

$T_H = 50^oC$

**12. Ans: (b)**

**Sol:**




$E_{Tk} = S_{TH}\,[T_H - T_c]$

$T.V = E_{Tk} = 53\ \mu V\,[100 - 0]^oC$

$= 53000\ \mu V$

$MV = E_{Tk} = 53\ \mu k\,[100 - 45] = 2915\ \mu V$

$e_1 = \dfrac{M.V - T.V}{T.V} \times 100$

$= \dfrac{2915 - 5300}{5300} \times 100$

$e_1 = -45\%$

$T.V = E_{TH} = 10600$

$M.V = E_{TH} = 8215$

$e_2 = -22.5\%$

**13. Ans: (a)**

**Sol:** Given

$E_{1G} = 53T\,\mu V$

$Ec_1c_2 = 43T\,\mu V$

$Ec_2I = (43T - 53T)\mu V = -10\ T\mu V$

$Ec_2I = -10 \times 70\mu V = -700\ \mu V$

**14. Ans: (d)**

**Sol:** From the table

Sensitivity of thermocouple is

$S_{TH} = \dfrac{125 - 35\ \mu V}{90\,C}$

$= 1\ \mu V/^oC$

We know the relation

$E_{TH} = S_{TH}\ \{T_2 - T_1\}$

$48\ \mu V = 1 \times \{T_2 - 2^oC\}$

$\Rightarrow\ T_2 = 48 + 2$

$T_2 = 50\ °C$

**15.**

**Sol:** For the given thermocouple

Emf

$Emf_{Chrom - copper}|_{at\ 30^0} = E_{chr - const} + E_{const- copper}$

$= E_{chr - const} - E_{copper - copper}$

$= 1.801 - 1.196$

$= 0.605mV$

$E_{Hot} = V_0 + E_{chrom - copper}$

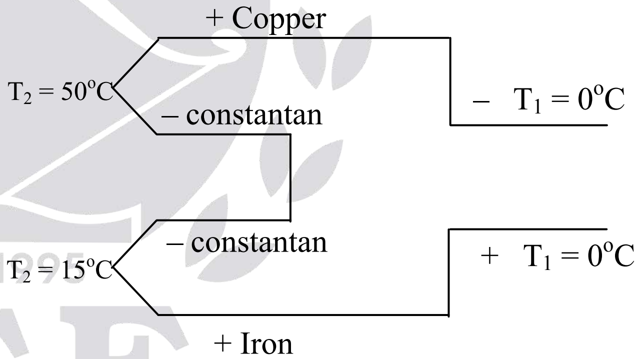
$= 26.74mV + 0.605mV$

$= 27.345mV$

Temperature corresponding to 27.345mV is $380^0C$.

**16. Ans: 1.2 to 1.3 mV**

**Sol:**




A = copper, B = constantan, C = iron

$e_A = 1.9\ \mu V/^oC$ with respect to platinum

$e_B = -38.3\mu V/^oC$ with respect to platinum

$e_C = 13.3\mu V/\ ^oC$ with respect to platinum

$E_{AB} = e_{A/B}.T_2{}^oC$

$= 40.2\mu V/^oC \times 50^oC$

$E_{AB} = 2.01mV$

Similarly

$E_{CB} = e_{C/B}\times T_3{}^oC = 51.6\mu V/^oC\times 15^oC$

$= 0.774mV.$

$\therefore E_0 = E_{CB} - E_{AB} = -1.236mV.$

$|E_0| = 1.236mV.$

**17. Ans: 77**

**Sol:** $V_0\left(\frac{1k}{2k+1k}\right) = \frac{V_0}{3} = \frac{2.1}{3} = 0.7 = V_+$ (virtual and concept)

$I = \frac{V_+}{2k} = 0.35mA = T\mu A$

$T = 350\,K$

$T = 77°C$

**18. Ans: 1612**

**Sol:** Given data:

Sensitivity $= 10\left(\frac{\mu V}{^0C}\right)$

Thermocouple measures 10mV at t = τ = 1sec

As given system is first order we use first order system equation

$y(t) = AK\left(1 - e^{-t/\tau}\right)$

A = amplitude

= final temperature – initial temperature

$A = \theta_f - \theta_i = (\theta_f - 30)^0C$

y(t) = temperature measured at time t

$10\times 10^{-3} = (\theta_f - 30)\times 10\times 10^{-6} \times (1-e^{-1})$

$\theta_f = 1612^0C$

**19. Ans: 0.2**

**Sol:** KCL at inverting terminal:-

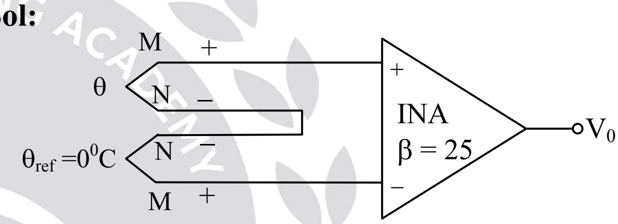
$$\frac{0-12}{R(1+x)} + \frac{0+12}{R} + \frac{0-V_0}{R} = 0$$

{Given $V_0 = +2V$}

$x = 0.2$

**20. Ans: $96^0C$**

**Sol:**




$S = 40\mu V/^0C$

β = Gain of amplifier = 25

$V_0 = 96mV$

θ = ?

$V_0 = \beta.V_{in}$

$V_0 = \beta E_{TC}$

$= \beta S_{TC}[\theta - \theta_{ref}]$

$96mV = (25)\ \frac{40\mu V}{^0C}[\theta - 0]\ ^0C$

$96mV = 100\mu V .\theta$

$96mV = 10^{-3}V\ \theta$

$\theta = 96^0C$

# Chapter 5 Measurement of Flow & Viscosity

**01. Ans: (b)**

**Sol:** $P_1 - P_2 = 30\text{kPa}$

$Q = 50$ lts

$P_1 - P_2 = 20$ kPa

By Bernoullis equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0.4$$

$$\Rightarrow \frac{30}{1000 \times 9.8} = \frac{V_2^2 - V_1^2}{2g} + 0.4$$

(By assuming $V_1 = 0$) $V_2 = 7.2$ m/sec

$$\frac{20}{1000 \times 9.8} = \frac{(V_2')^2}{2g} + 0.4$$

$$V_2' = 5.67 \text{m/sec}$$

$$\frac{Q_1}{Q_2} = \frac{V_2}{V_2'}$$

$$\Rightarrow \frac{50}{Q_2} = \frac{7.22}{5.67}$$

Q = 39.27 lit/sec

**02. Ans: 4.45**

**Sol:** $Re = \rho \frac{dv}{\mu}$

$Q = Av$

$V = Q/A = \frac{\text{mass rate}}{\rho \times A}$

$\rho = \frac{\text{mass}}{\text{volume rate}}$

Volume rate $= \frac{\text{mass}}{\rho}$

$Re = \frac{\rho \times d \times \text{massrate}}{\mu \times \pi \times d}$

d = 4.45 cm

**03. Ans: (d)**

**Sol:** Pressure and volume have an inverse relation.

**04. Ans: (c)**

**Sol:** $P_b - P_a = \frac{\overline{W}_{Float}}{A_{Float}}$

$$P_b - P_a = \frac{W_{Float} - B_{Float}}{A_{Float}}$$

$$P_b - P_a = \frac{gVd_1 - gVd_2}{A}$$

$$(P_b - P_a)A = gV(d_1 - d_2)$$

$$(P_a - P_b)A = Vg(d_2 - d_1)$$

**05. Ans: (d)**

**Sol:** Using Pitot-static tube, flow velocity of fluid is given by

$$V = \sqrt{\frac{2(p_{stag} - p_{stat})}{\rho}}$$

Given density, $\rho = 1000 \text{ kgm}^{-3}$

$p_{stag} - p_{stat} = 10 \text{ kPa} = 10^4 \text{ N/m}^2$

$\therefore V = \sqrt{20} \text{ m/sec} = V_1$

Pipe dia, d = 0.05 m

Area, $A_1 = \frac{\pi d^2}{4}$

Cylindrical drum dia, D = 1 m

Area, $A_2 = \frac{\pi D^2}{4}$

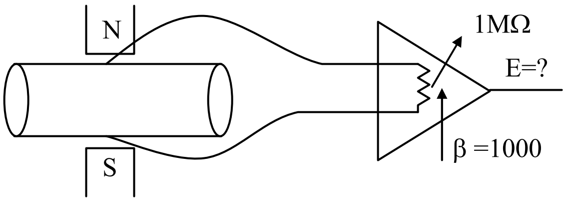
If the rate of reduction in water level in the drum is $V_2$,

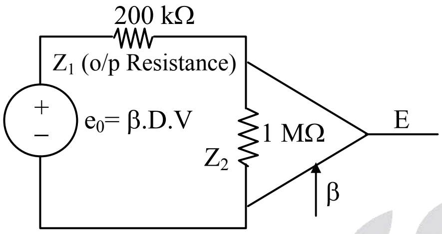
$V_1 A_1 = V_2 A_2$

$$V_2 = V_1 \frac{A_1}{A_2} = \frac{\sqrt{20}\ (0.05)^2}{(1)^2} = \frac{1}{40\sqrt{5}} \text{ m/s}$$

**06. Ans: (d)**

**Sol:**




$E = (\beta).\ E_{in}$

$$= \beta\left[\frac{1M\Omega}{200k\Omega + 1M\Omega}\right]e_0 = 0.0833V$$

**07. Ans: (c)**

**Sol:** Given $\Delta t = 10\times10^{-9}$ sec

$V_s = 1000$m/sec

$d = 25$mm

$\theta = 60^0$

So, $V_f = \frac{\Delta t \times V_s^{\ 2}}{29\cos\theta}$

$$= \frac{10\times10^{-9}\times(1000)^2}{2\times25\times10^{-3}\times\cos(60^0)}$$

$= 0.4$ m/sec

**08.**

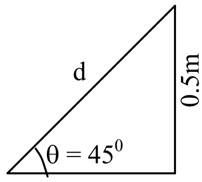
**Sol:** $\Delta t = \frac{1}{5\,MHz} = 0.2\ \mu sec$

$V_s = 1500$ m/sec

$V_f = ?$

d = distance between crystals

From the questions we can draw the figure




$\sin 45^0 = \frac{0.5}{d}$

$$d = \frac{0.5}{\sin 45^0} = 0.7m$$

$$\Delta t = \frac{2d\,V_f\cos\theta}{V_s^{\ 2}}$$

$$V_f = \frac{\Delta t\times V_s^{\ 2}}{2d\cos\theta}$$

$$V_f = \frac{0.2\times10^{-6}\times(1500)^2}{2\times0.7\times\cos45^0}$$

$V_f = 0.45$ m/sec

**09. Ans: (c)**

**Sol:** Induced voltage of turbine flow meter is

$E = \beta.\ n.\omega$

where, $\beta$ = amplitude of time varying flux.

$\alpha$ = mean flux

n = no. of teeth on wheel

given, speed N = 72 rpm.

$\alpha = 3$, $\beta = 1$ and $n = 4$

$\omega = \frac{2\pi N}{60} = 7.536,$

$f = \frac{n\omega}{2\pi} = \frac{4\times7.5}{2\times3.14} = 4.8Hz$

now, $E = \omega\beta n = 7.5\times1\times4 = 30.144$

**10.**

**Sol:** Given

$\rho_{oil} = 900\ kg/m^3$

$\mu = 0.006\ Ns/m^2$

$L = 30$ cm

$\Delta P = s \times g \times h$

$= 900 \times 9.8 \times 20 \times 10^{-2}$

$= 1764$

$$Q = \frac{\pi D^4}{128L} \cdot \frac{\Delta p}{\mu}$$

$$= \frac{\pi \times (2\times 10^{-2})^4 \times 1764}{128 \times 30 \times 10^{-2} \times 0.006}$$

$= 38.5\ cm^3$

**11. Ans: (a)**

**Sol:** For U type manometer

$$f = \frac{1}{2\pi}\sqrt{\frac{2g}{L}}$$

$V = a_m \times L$

$$= \frac{\pi D^2}{4} \times L$$

$$\Rightarrow L = \frac{4V}{\pi D^2}$$

$$\Rightarrow f_n = \frac{2g}{\sqrt{\frac{4V}{\pi D^2}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2g \cdot \pi D^2}{4V}}$$

$$= \frac{D}{2\sqrt{2}\pi}\sqrt{\frac{g\pi}{V}}$$

$$= \frac{D}{2\sqrt{2\pi}}\sqrt{\frac{g}{V}}$$

**12. Ans: 1.9634**

**Sol:** $e = Bl V = 100 \times 10^{-3} \times 0.25 \times V$

$l = 0.25 \times 10^{-1} V$

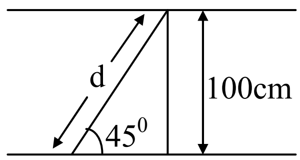
$V = 40$ (m/sec)

$$Q = AV = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times (0.25)^2 \times 40$$

$= 1.9634 (m^3/sec)$

**13. Ans: 5.025**

**Sol:**




From diagram $\sin 45^0 = \frac{100}{d}$

$d = 100\sqrt{2}$ cm

$t_1 = 0.9950$ms; $t_2 = 1.0000$ msec

$$f_1 = \frac{1}{0.9950ms} = 1.005025 \times 10^3 Hz$$

$$f_2 = \frac{1}{1m\sec} = 1 \times 10^3\ Hz$$

$\Delta f = f_1 - f_2 = [1.005025 - 1]10^3$ Hz

$= 5.025$

$$\Delta f = \frac{2v_f \cos\theta}{d}$$

$$v_f = \frac{\Delta f . d}{2\cos\theta} = \frac{(5.025) \times 100\sqrt{2} cm}{2 \times \frac{1}{\sqrt{2}}}$$

$= 502.5$ cm/sec

$= 5.025$ m/sec

**14. Ans: 20**

**Sol:** $S_{fluid} = 1 kg/m^3$

$\Delta p = 200 N/m^2$

$V_f = ?$

$C_V = 1.0$

$$V_f = C_V \sqrt{2\frac{\Delta p}{\rho}}$$

$$= 1\sqrt{\frac{2 \times 200}{1}}$$

$= 20$m/s

# Chapter 6 Measurement of force & torque

**01. Ans: (b)**

**Sol:** given

$g = 50\times10^{-3}$

$A = 4\ cm^2$

$\tau = 20$ Nm (force × length)

We know

$$g = \frac{E_0 / t}{\frac{F}{A}}$$

$$\tau = \vec{F}\times\vec{L}$$

L= 0.5 m given

$$F = \frac{\tau}{L} = 40\ N$$

$$E_0 = g\times\frac{F}{A}\times t$$

$$E_0 = 50\times10^{-3}\times\frac{40}{4\times(10^{-2})^2}\times1\times10^{-3}$$

$= 5$ V

**02. Ans: 848 Nm**

**Sol:** We know

Angle of shear $\theta = \frac{2T}{\pi G r^3}$

Where G is shaft shear mode

r is the radius of shaft

T is the applied torque

An area of shaft surface, originally square with the sides of unit length and deformed to parallelogram due to the application of strain.

The original length of diameter is $T_2$ and if $\theta$ is small then change in length is $\frac{\theta}{\sqrt{2}}$

So

$$\varepsilon = \frac{\Delta L}{L} = \frac{\frac{\theta}{\sqrt{2}}}{\sqrt{2}} = \frac{\theta}{2}$$

but, $\frac{\Delta R}{R} = Gf \in = 2\times\frac{\theta}{2} = \theta$

Or $\theta = \frac{\Delta R}{R} = \frac{0.24}{120} = 2\times10^{-3}$ rad

$$T = \frac{\pi G r^3}{2}\times\theta$$

$$\theta = \frac{\pi\times80\times10^9\times(15\times10^{-3})^3\times2\times10^{-3}}{2}$$

= 848Nm.

# Chapter 7 Intermediate Quantity Measurement

**01.**

**Sol:** Input range = 0 to 5 g

Damping ratio = 0.8

Output range = 0 to 10 V

m = 0.005 kg

k = 20 N/m

a) input displacement range

$$\frac{m}{k} = \frac{x}{acc}$$

$$\Rightarrow \frac{0.005\,kg}{20\,N/m} = \frac{x}{5\times10^{-3}\times9.81\,m/sec^2}$$

$$\Rightarrow \quad x = 12.2625 \text{ mm}$$

**b)** 1 kΩ POT

$$X_i = \frac{0.005}{20}\times2\times10^{-3}\times9.81$$

$= 4.905\times10^{-3}$ for 2g acceleration

$= 0.0049$ m

$$k = \frac{x_i}{x_t} = \frac{0.0049}{0.0126} = 0.388$$

$$\frac{R_p}{R_m} = \frac{1k}{10k} = 0.1$$

$$\%\ LE = \frac{MV - TV}{TV}\times100$$

$$MV = \frac{0.38E_i}{1+0.38(1-0.38)\times0.1}$$

$TV = 0.38\ E_i$

$$\%\ error = \frac{\frac{0.38E_i}{1+0.38(1-0.38)\times0.1} - 0.4E_i}{0.4E_i}\times100$$

$= -2.36\ \%$

**02.**

**Sol:** For accelerometer ratio of amplitude of output and input

$$\frac{(x_2 - x_1)0}{x_{m1}} = \frac{u^2}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

$$= \frac{u^2}{\sqrt{(1-u^2)^2 + (2\times0.7u)^2}} = 0.99$$

u = normalized frequency $= \frac{\omega_1}{\omega_2} = 2.45$

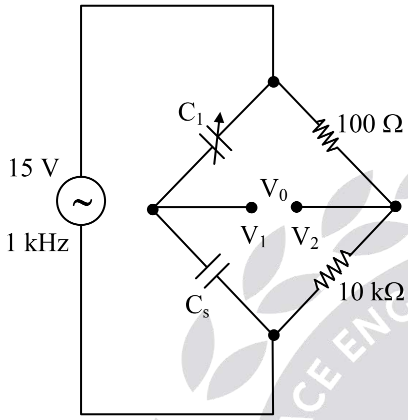
# Chapter 8 Measurement of Level

**01.**

**Sol: a)** Given $l = 8$ m

$h = 7$ m

$\in_{r\,liq} = 2.4$




tank is empty

$$C_s = \frac{2\pi\varepsilon_o \ell}{\ln\left(\frac{D_1}{D_2}\right)}$$

$$= \frac{2\pi\times 8.85\times 10^{-12}\times 8}{\ln(2)}$$

$= 642$ pF

For bridge balance condition

$Z_1Z_4 = Z_2Z_3$

$(XC_1)\times(10k\Omega) = (100\Omega)\times(XCs)$

$$\frac{1}{j\omega C_1}\times(10\ k\Omega) = 100\times\frac{1}{j\omega C_s}$$

$$C_1 = \frac{C_s\times 10k}{100}$$

$C_1 = 64.2$ NF

**b)** When tank is full

$$C_s = \frac{2\pi\in_0}{\ln\left(\frac{D_1}{D_2}\right)}\left(\varepsilon r_1 h_1 + \varepsilon r_2 h_2\right)$$

$$= \frac{2\pi\times 8.85\times 10^{-12}}{\ln(2)}\times\{(2.4)\times 7+1\}$$

$\Rightarrow C_s = 1425$ pF

$V_0 = V_1 - V_2$

$$= 15V\left(\frac{64.2nF}{64.2nF+1425pF} - \frac{10k}{10k+100\Omega}\right)$$

$= 0.177$ V.

**02. Ans: (c)**

**Sol:** $A_w$ = Area of water

$P_1 = P_2$

$$\sin\theta = \frac{h_d}{\ell + y}$$

$\ell\sin\theta + y\sin\theta = h_d$ ----- (1)

$$\sin\theta = \frac{AB}{y}$$

$AB = y\sin\theta$ ---- (2)

By low of volume

Volume of mercury lost in well

= volume of mercury gain in cap

$a_w\times A_B = a_t\times\ell$

$a_w\times y\sin\theta = a_t\times\ell$

$$y\sin\theta = \frac{a_t}{a_w}\times\ell \text{ ---- (3)}$$

from (2) & (3)

$$\sin\theta = \frac{h_d}{\ell} - \frac{a_t}{a_w}$$

$$\theta = \sin^{-1}\left[\frac{h_d}{\ell} - \frac{a_t}{a_w}\right]$$

# Chapter 9 Measurement of Pressure

**01.**

**Sol:** We know for diaphragm pressure gauge

$$Y_{max} = t/3 = \frac{3\Delta P}{16Et^3} r^4\left(1-u^2\right)$$

$$t^4 = \frac{9\Delta Pr^4\left(1-\mu^2\right)}{16Et^3}$$

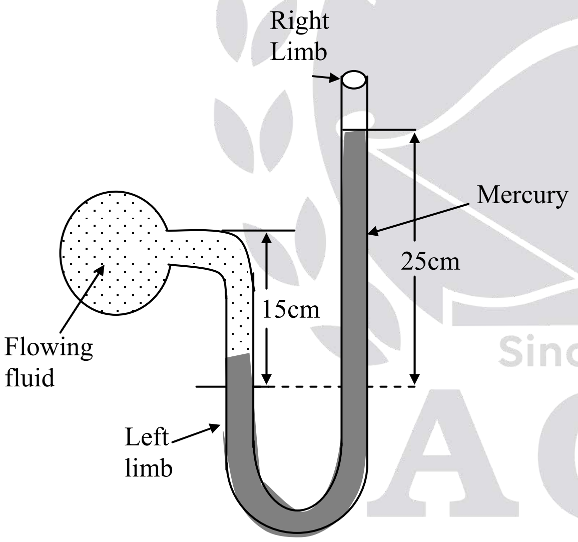
$$= \frac{9\times1.5\times10^6\times2.5^4\left\{1-(0.3)^2\right\}}{16\times200\times10^9}$$

$$= 149.96\times10^{-6}\text{cm}^4$$

$$t = 0.1106\text{cm}$$

**02. Ans: 32103**

**Sol:**




P = pressure of the fluid in the pipe

$h_1 = 15$ cm

$h_2 = 25$ cm

$\rho_1 = 850\ \text{kg/m}^3$

$\rho_2 = 13600\ \text{kg/m}^3$

$g = 9.81\ \text{m/sec}^2$

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P = (13600\times9.81\times25\times10^{-2}) - (850\times9.81\times15\times10^{-2})$$

$$= 33354 - 1250.775$$

$$= 32103.225\ \text{N/m}^2$$

# Chapter 10 Miscellaneous

**01.**

**Sol:** Let X dB and Y dB be the sound pressure levels from the sound sources.

Then the noise level with both sources operating together is given by

Combined spl

$$= 10\log_{10}\left[\text{anti}\log\left(\frac{X}{10}\right) + \text{anti}\log\left(\frac{Y}{10}\right)\right]$$

$$= 10\log_{10}\left[\text{anti}\log\left(\frac{78}{10}\right) + \text{anti}\log\left(\frac{82}{10}\right)\right]$$

$$= 10\log_{10}\left[6.31\times10^{7} + 15.85\times10^{7}\right]$$

$$= 10\log_{10}\left[22.16\times10^{7}\right] = 10\times8.3456$$

$$= 83.456\,\text{dB}$$

**02.**

**Sol:** $\text{Resultant spl} = 10\log_{10}\left[\text{anti}\log\left(\frac{X}{10}\right) + \text{anti}\log\left(\frac{Y}{10}\right)\right]$

Substituting the numerical values from the given data,

$$85 = 10\log_{10}\left[\text{anti}\log\left(\frac{X}{10}\right) + \text{anti}\log\left(\frac{73}{10}\right)\right]$$

Where X dB is the sound pressure level of the machine.

$$\therefore \text{anti}\log\left(\frac{X}{10}\right) = \text{anti}\log\left(\frac{85}{10}\right) - \text{anti}\log\left(\frac{73}{10}\right)$$

Take $\log_{10}$ of both sides

$$X = 10\log_{10}\left[\text{anti}\log 8.5 - \text{anti}\log 7.3\right]$$

$$X = 10\log_{10}\left[3.612\times10^{8} - 0.1995\times10^{8}\right]$$

$$X = 10\log_{10}\left[2.9625\times10^{8}\right] = 10\times8.4717$$

$$= 84.717\,\text{dB}$$

Thus the sound pressure level of the machine alone is 84.717 dB.

**03. Ans: –23**

**Sol:** $T_{true} = T_{measured}\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{0.25}$

$T_{measured} = 820^{o}C = 1093^{o}C$

$\varepsilon_1 = 0.75$

$\varepsilon_2 = 0.69$

$$T_{time} = 1093\left(\frac{0.75}{0.69}\right)^{0.25}$$

$$= 1093(1.087)^{0.25}$$

$$= 1116^{o}K$$

Error in temperature = $T_{measured} - T_{true}$

= 1093 – 1116

= $-23^{o}K$

**04. Ans: 8**

**Sol:** Area target factor

$$= \frac{\text{distance or receiver from target}}{\text{useful diameter of target}}$$

$$\therefore \text{Diameter of target} = \frac{160}{20} = 8\,\text{cm}$$

**05.**

**Sol:** Absolute temperature with emissivity of 0.82 is

$= 1065 + 273 = 1338^{0}k$

$\therefore$ Apparent absolute temperature is,

$T_a = (0.82)^{-1/4} (1338) = 1273^0 k$

Actual absolute temperature when the emissivity is 0.75

$= (0.75)^{-1/4} (1273) = 1368^0 k$

Actual temperature

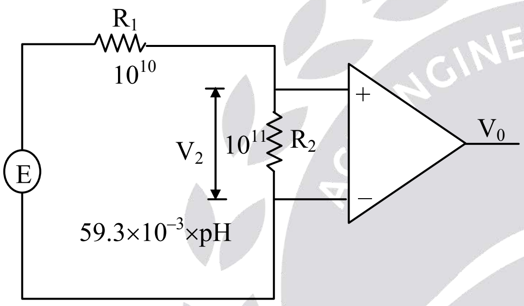
$= 1368 - 273 = 1095^0 C$

Hence error in temperature measure

$= 1065 - 1095 = -30^0 C.$

**06. Ans: (b)**

**Sol:**




$V_0 = \beta \times V_2$

$= \beta \times \frac{R_2}{R_1 + R_2} \times E$

$V_{0_1} = \beta \times \left(\frac{R_2}{R_1 + R_2}\right) \times 59.3 \times 10^{-3} \times 6.5$

$= 100 \times \frac{10^{11}}{10^{10} + 10^{11}} \times 59.3 \times 10^{-3} \times 6.5$

$= 35.04$ V

$V_{0_2} = \beta \times \left(\frac{R_2}{R_1 + R_2}\right) \times 59.3 \times 10^{-3} \times 7.8$

$= 100 \times \frac{10^{11}}{10^{10} + 10^{11}} \times 59.3 \times 10^{-3} \times 7.8$

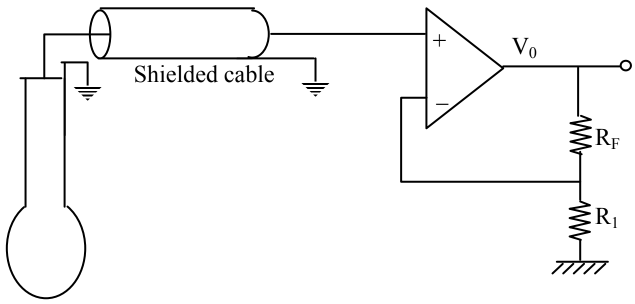
$= 42.04$V

$\Delta V = 42.04 - 35.04$

$= 7$V

**07.**

**Sol:**




pH electrode

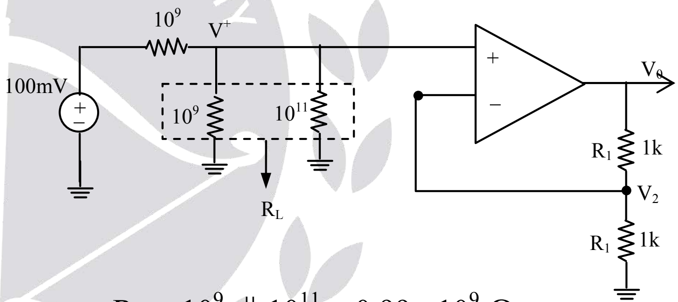
Given

$R = R_i \left(1 + A_0 \frac{R_F}{R_1}\right)$

$R_i = 10^6 \Omega$, $R_{electrode} = 10^9 \Omega$

$A_0 = 10^5 \Omega$, $R_{cable} = 10^9 \Omega$

$R = 10^6 \left(1 + 10^5 \times \frac{1}{1}\right) \approx 10^{11} \Omega$




$R_L = 10^9 \parallel 10^{11} = 0.99 \times 10^9 \ \Omega$

$V^+ = 100\,mV \times \frac{R_L}{R_L + 10^9}$

$V^+ = 49.75$mV

$V_0 = \left(1 + \frac{R_F}{R_1}\right) V^+ = 99.5$mV

**08. Ans: 118.4mV**

**Sol:** Sensitivity of pH electrode is approximately is around 59.2mV/pH at $25^0$C ambient Temperature.

Therefore output of the pH electrode at 6 pH is 355.2 mV and at 8 pH it is 473.6mV

The change in voltage is 118.4mV.