

GATE | PSUs



CIVIL ENGINEERING

Reinforced Cement Concrete

Text Book : Theory with worked out Examples
and Practice Questions



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Reinforced Cement Concrete

(Solutions for Text Book Practice Questions)

03. Limit State Design- Singly Reinforced Beams

01. Ans: (a)

Sol: For Fe415,

$$\begin{aligned} M_{u \text{ limit}} &= \text{Equation (1) with } x_{u \text{ max}} \\ &= 0.138 f_{ck} b d^2 \\ &= 0.138 \times 15 \times 200 \times (500)^2 \\ &= 103.5 \text{ kN-m} \end{aligned}$$

02. Ans: (c)

Sol: Balanced (or) limiting percentage of steel
(use $x_{u \text{ max}}$)

$$\begin{aligned} C &= T \\ 0.36 f_{ck} b x_{u \text{ max}} &= 0.87 f_y A_{st} \\ 0.36 f_{ck} b (0.48d) &= 0.87 \times 415 A_{st} \\ 0.36 \times 15 \times 200 \times 0.48 \times 300 &= 0.87 \times 415 A_{st} \\ A_{st} &= 430 \text{ mm}^2 \end{aligned}$$

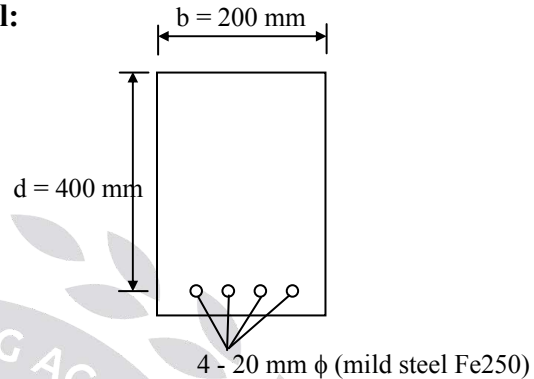
03. Ans: (b)

Sol: $M_u = 138 \times 10^6 \text{ N-mm}$

$$\begin{aligned} M_u &= M_{u \text{ limit}} \\ &= 0.138 \times f_{ck} b d^2 - (\text{design as BS}) \\ 138 \times 10^6 &= 0.138 \times 20 \times 200 \times d^2 \\ d &= 500 \text{ mm} \end{aligned}$$

04. Ans: (b)

Sol:



$$\begin{aligned} \text{i) } x_{u \text{ max}} &= 0.53 \times d \\ &= 0.53 \times 400 \\ &= 212 \text{ mm} \end{aligned}$$

ii) $x_u = ?$ $C = T$

$$\begin{aligned} 0.36 \times f_{ck} \times b \times x_u &= 0.87 \times f_y \times A_{st} \\ 0.36 \times 15 \times 200 \times x_u &= 0.87 \times 250 \times 4 \\ &\times \left(\frac{\pi}{4} \times 20^2 \right) \end{aligned}$$

$$\Rightarrow 1080 x_u = 273318.5$$

$$x_u = 253.1 \text{ mm}$$

$x_u > x_{u \text{ max}} \Rightarrow$ over reinforced section

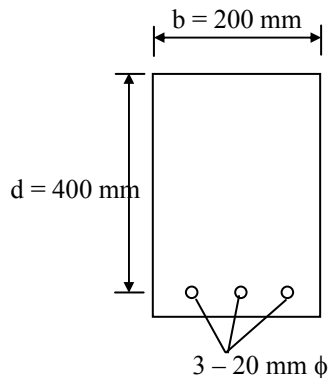
Over reinforcement section fails suddenly

To avoid sudden fail decrease the MR to that of a balanced section

$$\begin{aligned} M_{u \text{ limit}} &= 0.148 \times f_{ck} b d^2 \\ &= 0.148 \times 15 \times 200 \times 400^2 \\ &= 71040000 \text{ N-mm} = 71.04 \text{ kN-m} \\ &\simeq 72 \text{ kN-m} \end{aligned}$$

05. Ans: (d)

Sol:



i) $x_{u \max} = 0.53 \times d$

$$= 0.53 \times 400 = 212 \text{ mm}$$

ii) $C = T$

$$0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$$

$$0.36 \times 15 \times 200 \times x_u = 0.87 \times 250$$

$$\times \left(3 \times \frac{\pi}{4} \times 20^2 \right)$$

$$1080 x_u = 204988.92$$

$$x_u = 190 \text{ mm}$$

$$x_u < x_{\max} \Rightarrow \text{Under reinforced section}$$

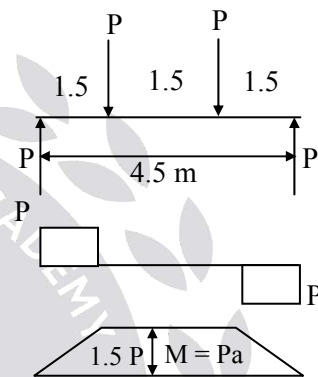
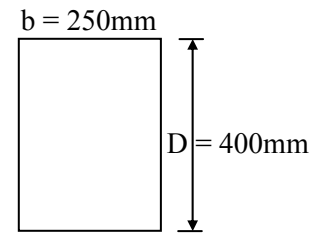
$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 15 \times 200 \times 190 (400 - 0.42 \times 190)$$

$$M_u = 65.7 \text{ kN.m} \approx 66 \text{ kN.m}$$

06. Ans: 8.86 kN

Sol:



Homogenous beam

$$f_{cr} = 2 \text{ MPa}$$

Modulus of rupture/tensile stress of concrete from bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow M = f_{cr} \times Z \quad \left[\because Z = \frac{bD^2}{6} \right]$$

$$= 2 \left[\frac{250 \times 400^2}{6} \right] = 13.33 \times 10^6 \text{ N-mm}$$

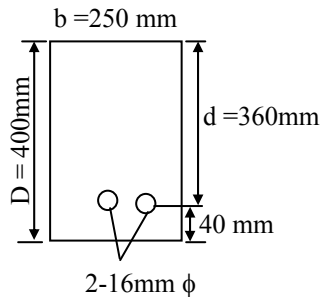
$$M = P.a$$

$$13.3 = P \times 1.5$$

$$P = \frac{13.3}{1.5} = 8.86 \text{ kN}$$

07. Ans: 31.6 kN

Sol:



Reinforced concrete beam

i) $x_{u\max} = 0.48d$

$$= 0.48 \times 360 = 172.8 \text{ mm}$$

$C = T$

$$0.36f_{ck}bx_u = 0.87f_yA_{st}$$

$$0.36 \times 20 \times 250 \times x_u = 0.87 \times 415$$

$$\times \left(2 \times \frac{\pi}{4} \times 16^2 \right)$$

$$1800 x_u = 145186.8$$

$$x_u = 80.65 \text{ mm}$$

$$x_u < x_{\max}$$

∴ Under reinforced section

$$M.R = 0.36f_{ck}bx_u(d - 0.42x_u)$$

$$= 0.36 \times 20 \times 250 \times 80.65$$

$$(360 - 0.42 \times 80.65)$$

$$M_u = 47.5 \text{ kN-m}$$

$$M_u = P \times a$$

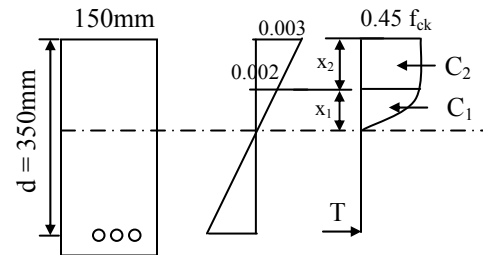
$$47.5 = P \times a$$

$$P = \frac{47.5}{1.5}$$

$$P = 31.6 \text{ kN}$$

08. Ans: 51 kN-m

Sol:



$$x_{u\max} = 0.48 \times d$$

$$= 0.48 \times 350$$

$$= 168 \text{ mm}$$

$$M_{u\text{ limit}} = 0.36f_{ck}bx_{u\max}(d - 0.42x_{u\max})$$

$$= 0.36 \times 20 \times 150 \times 168(350 - 0.42 \times 168)$$

$$= 50.70 \times 10^6 \text{ N-m}$$

$$= 51 \text{ kN-m}$$

09. Ans: 503 mm²

Sol: $C = T$

$$0.36f_{ck}bx_{u\max} = 0.87f_yA_{st}$$

$$A_{st} = \frac{0.36f_{ck}bx_{u\max}}{0.87 \times f_y}$$

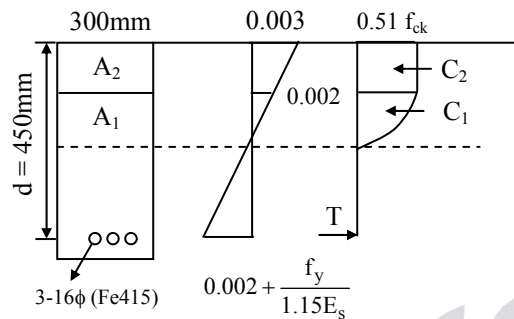
$$= \frac{0.36 \times 20 \times 150 \times 168}{0.87 \times 415}$$

$$= 502.53 \text{ mm}^2$$

$$A_{st} \approx 503 \text{ mm}^2$$

10. Ans: 196 mm

Sol:



$$x_{u\max} = 0.003 \rightarrow (1)$$

$$(d - x_{u\max}) = \left(0.002 + \frac{f_y}{1.1E_s} \right)$$

$$450 - x_{u\max} = \left(0.002 + \frac{415}{1.1 \times 2 \times 10^5} \right) \rightarrow (2)$$

$$\frac{450 - x_{u\max}}{x_{u\max}} = \frac{0.002 + \frac{415}{1.1 \times 2 \times 10^5}}{0.003}$$

On solving

$$x_{u\max} = 196.04 \text{ mm} \\ = 196 \text{ mm}$$

04. Limit State Design - Doubly Reinforced Beams

01. Ans: (c)

Sol: BM = 300 kN-m

Concrete, $M_{15} = f_{ck} = 15$

Steel, $f_y = 415$

$f_{sc} = 353.7 \text{ MPa}$

Effective Cover $d' = 50 \text{ mm}$

In LSM, we have to use

Factored moment

$$M_u = M \times \gamma_f$$

Use $\gamma_f = 1.5$

$$= 300 \times 1.5 = 450 \text{ kN-m}$$

To calculate $M_{u \text{ limit}}$

$$M_{u \text{ limit}} = 0.138 f_{ck} b d^2 \\ = 0.138 \times 15 \times 350 \times (700)^2$$

$$M_{u \text{ limit}} = 355 \text{ kN-m}$$

$$M_u = 450 \text{ kN-m}$$

$$\therefore M_u > M_{u \text{ limit}}$$

So we need to use 'DRB'

$$M_{u \text{ limit}} = 0.87 f_y A_{st} (d - 0.42 x_{u \max})$$

$$355 \times 10^6 = 0.87 \times 415 \times A_{st} (700 - 0.42 \times 0.48 \times 700)$$

$$A_{st} = 1759.31 \text{ mm}^2$$

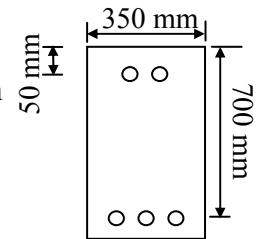
for extra moment we need to provide tensile steel & comp. steel

$$M_u - M_{u \text{ limit}} = 0.87 f_y (d - d') A_{st2}$$

$$(450 - 355) \times 10^6 = 0.87 \times 415 A_{st2} (700 - 50) \\ = 234682.5 A_{st2}$$

$$A_{st2} = 404.8 \approx 405 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 2165 \text{ mm}^2$$



Now our purpose is to calculate 'A_{sc}'

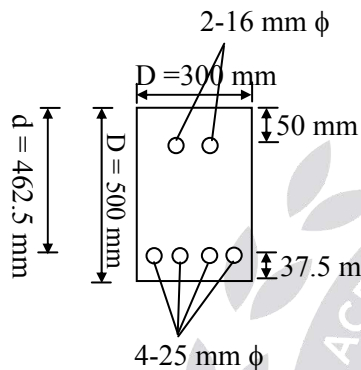
$$M_u - M_{u\text{limit}} = f_{sc} A_{sc} (d - d')$$

$$(\text{or}) f_{sc} A_{sc} = 0.87 f_y A_{st2}$$

$$A_{sc} = 413.2 \text{ mm}^2$$

02. Ans: 271 kN-m

Sol:



$$b = 300 \text{ mm}, D = 500 \text{ mm}, d = 462.5 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2,$$

$$f_{sc} = 0.8566 f_y$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.495 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$\Rightarrow C = T$$

$$\Rightarrow C_1 + C_2 = T$$

$$0.36 \times f_{ck} \times b \times x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 300 \times x_u + (0.8566 \times 415) \times 402.12 = 0.87 \times 415 \times 1963.495$$

$$x_u = 209.618 \text{ mm}$$

$$x_{u\text{max}} = 0.48 \times d$$

$$= 0.48 \times 462.5 = 222 \text{ mm}$$

$$x_u < x_{u\text{max}}$$

$\therefore$ under reinforced section.

$$M_u = 0.36 f_{ck} \times b \times x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 300 \times 209.6$$

$$(462.5 - 0.42 \times 209.6) + (0.8556 \times 415)$$

$$\times 402.12 (462.5 - 50)$$

$$= 270.9 \text{ kN-m}$$

03. Ans: 18.82 kN/m

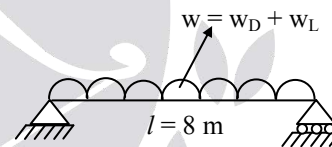
Sol: Working /line moment,

$$M = \frac{270.9}{1.5} = 180.6 \text{ kN-m}$$

$$\text{Self weight of beam, } w_D = (\gamma_c) \times b \times D$$

$$= (25 \text{ kN/m}^3) \times (0.3 \times 0.5)$$

$$W = 3.75 \text{ kN/m}$$



$$M = \frac{(w_D + w_L) \times l^2}{8}$$

$$180.6 = \frac{(3.75 + w_L) \times 8^2}{8}$$

$$w_L = 18.825 \text{ kN/m}$$

05. Limit State Design- Flanged Beams

01. Ans: (c)

Sol:

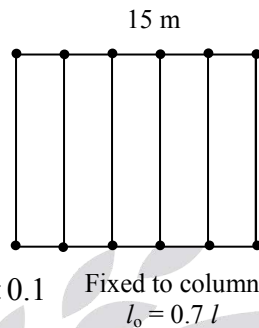
For T-beams,

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

$$= \frac{0.7 \times 10}{6} + 0.25 + 6 \times 0.1$$

$$= 2.01 \text{ m} \not\geq c = 3 \text{ m}$$

$$\therefore b_f = 2.01 \text{ m}$$



02. Ans: (d)

Sol: L – beam

$$B_f = \frac{l_0}{12} + b_w + 3D_f$$

$$= \frac{10}{12} + 0.25 + 3 \times 0.1$$

$$= 1.38 \text{ m} \not\geq c = 3 \text{ m}$$

$$\therefore b_f = 1.38 \text{ m}$$

03. Ans: (d)

Sol: $D_f = 100 \text{ mm}$, $b_w = 300 \text{ mm}$, $d = 500 \text{ mm}$,

$c = 3 \text{ m}$, $l = 6 \text{ m}$, $l_0 = 3.6 \text{ m}$, $b_f = ?$

$$b_f = \frac{l_0}{6} + b_w + 6D_f \not\geq c$$

$$= \frac{3.6}{6} + 0.3 + 6 \times 0.1$$

$$= 1.5 \text{ m} \not\geq c = 3 \text{ m}$$

$$= 1.5 \times 1000 \text{ mm} = 1500 \text{ mm}$$

06. Limit State of Collapse - Shear

01. Ans: (b)

Sol:

$$V_u = 120 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\text{Main steel, } f_y = 415 \text{ N/mm}^2$$

$$\text{Stirrups, } f_y = 250 \text{ N/mm}^2$$

$$\tau_c = 0.48 \text{ N/mm}^2$$

i) 8mm–2 legged

Stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.53 \text{ mm}^2$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{120 \times 10^3}{400 \times 230}$$

$$= 1.3 \text{ N/mm}^2$$

$$\tau_v \leq \tau_{c \text{ max}} - \text{safe in shear}$$

ii) $\tau_v > \tau_c$ – not safe in shear reinforcement

Minimum shear reinforcement is required

$$V_{us} = \frac{(0.87f_y)A_{sv} \times d}{S_v}$$

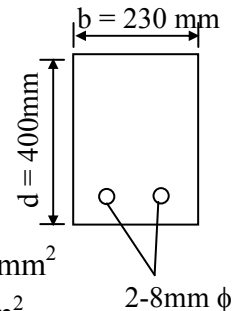
$$V_{us} = V_u - \tau_c b.d$$

$$= 120 \times 10^3 - 0.48 \times 400 \times 230$$

$$= 75840 \text{ N} = 75.84 \text{ kN}$$

$$75.84 \times 10^3 = \frac{0.87 \times 250 \times 100.53 \times 400}{S_v}$$

$$S_v = 115 \text{ mm c/c}$$



02. Ans: (c)

Sol: $T = 10.90 \text{ kN-m}$

$$V_e = V_u + \frac{1.6 T_u}{b}$$

$$= 120 \times 10^3 + \frac{1.6 \times 10.90 \times 10^6}{230}$$

$$V_e = 196 \text{ kN}$$

Design shear force

$$V_{us} = V_e - \tau_c \cdot b \cdot d$$

$$= 196 \times 10^3 - 0.48 \times 230 \times 400$$

$$V_{us} = 151.84 \times 10^3 \text{ N}$$

$$= 151.84 \text{ kN}$$

03. Ans: (d)

Sol: $b = 230 \text{ mm}$, $d = 450 \text{ mm}$

$$V_u = 50 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$\tau_{c \max} = 2.8 \text{ MPa}, \tau_c = 0.75 \text{ MPa}$$

$$\tau_v = \frac{V_u}{bd} = \frac{50 \times 10^3}{230 \times 450} = 0.483 \text{ MPa}$$

$$\tau_v < \tau_{c \max} \text{ safe in shear.}$$

Provide minimum shear reinforcement.

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y}$$

$$A_{sv} = 2 \times \frac{\pi \times 8^2}{4} = 100.53 \text{ mm}^2$$

$$S_v = \frac{100.53 \times 0.87 \times 250}{0.4 \times 230}$$

$$= 237.7 \text{ mm c/c}$$

$$S_v \geq 0.75 d = 0.75 \times 450 = 337.5 \text{ mm}$$

$$S_v \geq 300 \text{ mm}$$

$\therefore$ Provide spacing of 230 mm c/c

04. Ans: (c)

Sol: $V_u = 100 \text{ kN}$

$$\tau_v = \frac{V_u}{b \times d} = \frac{100 \times 10^3}{230 \times 450} = 0.966$$

$$\tau_v < \tau_{c \max} - \text{shear reinforcement safe}$$

$$\tau_v > \tau_c \text{ not safe in shear reinforcement}$$

Shear reinforcement is required.

Design shear force for shear reinforcement

$$V_{us} = V_u - \tau_c b d$$

$$= 100 \times 10^3 - 0.75 \times 230 \times 450$$

$$= 22.375 \text{ kN}$$

For vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 250 \times 100.53 \times 450}{22.375 \times 10^3} = 439.75 \text{ mm}$$

Min spacing:

i. 439.75 mm

ii. $0.75d = 0.75 \times 450 = 337.5 \text{ mm}$

iii. 300 mm

iv. Spacing for min shear reinforcement

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y} \Rightarrow S_v = 237.7 \text{ mm}$$

Provide min spacing of 230 mm c/c.

05. Ans: (c)

Sol: $V_u = 150 \text{ kN}$

$$\tau_v = \frac{150 \times 10^3}{230 \times 450} = 1.449 \text{ MPa}$$

$\tau_v < \tau_{c, \max}$ – safe in shear reinforcement

$\tau_v > \tau_c \rightarrow$ Shear reinforcement is required.

Design shear force,

$$\begin{aligned} V_{us} &= V_u - \tau_c bd \\ &= 150 \times 10^3 - 0.75 \times 230 \times 450 \\ &= 72.375 \text{ kN} \end{aligned}$$

Shear force taken by bent-up bars.

$$\begin{aligned} V_{us1} &= 0.87 f_y A_{sv} \sin \alpha \\ &= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^\circ \\ &= 102.66 \text{ kN} \end{aligned}$$

$$\nless 0.5 V_{us} = 36.18 \text{ kN}$$

$$\therefore V_{us1} > 0.5 V_{us}$$

As per IS: 456 ; $V_{us1} \nless 0.5 V_{us}$. In this case

V_{us1} is exceeding $0.5 V_{us}$. Therefore limit V_{us1} as 36.18 kN, the remaining S.F i.e 36.195 kN should be resisted by vertical stirrups.

Vertical stirrups:

For $V_{us2} = 36.195 \text{ kN}$

$$\begin{aligned} 36.195 \times 10^3 &= \frac{0.87 f_y A_{sv} \cdot d}{S_v} \\ S_v &= \frac{0.87 \times 250 \times \left(2 \times \frac{\pi}{4} \times 8^2 \right) \times 450}{36.195 \times 10^3} \\ &= 271.708 \text{ mm} \end{aligned}$$

Provide minimum center to center spacing of 230 mm c/c

06. Ans: (a)

Sol: Beam -P

$$\tau_{c \max} = 2.1 \text{ MPa}$$

$$f_{ck} = 30 \text{ N/mm}^2$$

$$\tau_c = 0.75 \text{ MPa}$$

$$V_u = 400 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{400 \times 10^3}{750 \times 400}$$

$$\tau_v = 1.33 \text{ N/mm}^2$$

i) $\tau_v < \tau_{c \max}$ –shear reinforcement safe

ii) $\tau_v > \tau_c$ Minimum shear reinforcement is required

$$\begin{aligned} V_{us} &= V_u - \tau_c bd \\ &= 400 \times 10^3 - 0.75 \times 400 \times 750 \\ V_{us} &= 175 \text{ kN} \end{aligned}$$

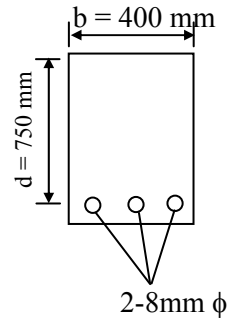
Beam -Q

$$V_u = 750 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{750 \times 10^3}{750 \times 400} = 2.5 \text{ N/mm}^2$$

$$\tau_v > \tau_{c \max}$$

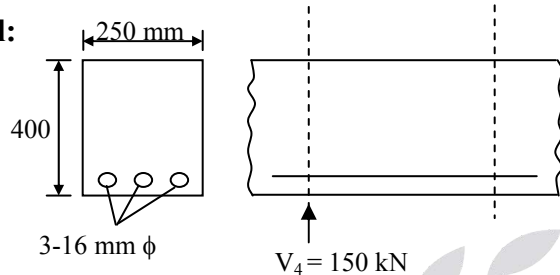
The beam is not safe in shear. It should be revised.



07. Bond

01. Ans: (c)

Sol:



Flexural bond:

Steel in tension (sagging moment)

$$L_d \neq \frac{M_1}{V_u} + l_0 \rightarrow \text{continuous beam}$$

$$l_0 = 12\phi = 12 \times 16 = 192\text{mm}$$

$$d = 400\text{mm}$$

Which is greater

Take $l_0 = 400\text{ mm}$

$$L_d = \frac{0.87f_y\phi}{4\tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1} = 870\text{ mm}$$

$$x_{u, \max} = 0.53 \times 400 = 212\text{ mm}$$

$$x_u = \frac{0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 15 \times 250}$$

$$= 97.18\text{ mm}$$

$x_u < x_{u, \max} \rightarrow$ Under reinforcement section.

$$M_1 = 0.36 \times 15 \times 250 \times 97.18 (400 - 0.42 \times 97.18)$$

$$= 47.12 \times 10^6 \text{ N-mm}$$

$$L_d \neq \frac{47.12 \times 10^6}{150 \times 10^3} + 400 = 714.15\text{ mm}$$

$$L_d > 714.15$$

not safe in bond.

02. Ans: (d)

Sol: $\phi = 12\text{mm}$

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 30 \text{ N/mm}^2, \tau_{bd} = 2.4 \text{ MPa}$$

$$L_d = \frac{\phi\sigma_s}{\tau_{bd} \times 4}$$

$$= \frac{12 \times 0.87 \times 415}{(1.6 \times \tau_{bd}) \times 4} = 282.0703$$

$$L_d = 282.0703\text{ mm}$$

$$L_d \text{ with } 90^\circ \text{ bend} = 282.0703 - 8\phi$$

$$= 282.0703 - 8 \times 12$$

$$= 186.1\text{ mm}$$

03. Ans: (d)

Sol: Axially loaded short column

$$\phi = d = 20\text{mm}, \text{ spliced} = 16\text{ mm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \text{ MPa}$$

$$\left. \begin{array}{l} \text{lap} \leq l_d \\ \leq 24\phi \end{array} \right\} \text{max}$$

Use smaller diameter $\Rightarrow \phi = 16\text{ mm}$

$$L_d = \frac{\phi\sigma_s}{4 \times \tau_{bd}} = \frac{16 \times 0.87 \times 415}{1.25 \times 4 \times 1.2 \times 1.6}$$

$$= 601.75\text{ mm}$$

$$\text{Lap length} \leq L_d = 601.75\text{ mm}$$

$$\leq 24\phi = 384\text{ mm}$$

Use maximum, i.e., 601.75 mm

04. Ans: (d)

Sol: 1) Pull out (bond fail)

$$P_1 = \tau_{bd}[\pi D l]$$

2) Breaking of steel bar

$$P_2 = \sigma_{st} \left[\frac{\pi}{4} \times D^2 \right]$$

} minimum

05. Ans: 46.8

Sol: $f_{ck} = 20 \text{ N/mm}^2$,

$\tau_{bd} = 1.2 \text{ MPa}$ $\uparrow 60\%$ - HYSD bars

Steel bar is in tension

$$L_d = \frac{\phi \sigma_s}{4 \times \tau_{bd}} = \frac{\phi \times 360}{4 \times 1.6 \times 1.2} = 46.8 \phi$$

06. Ans: 290 mm

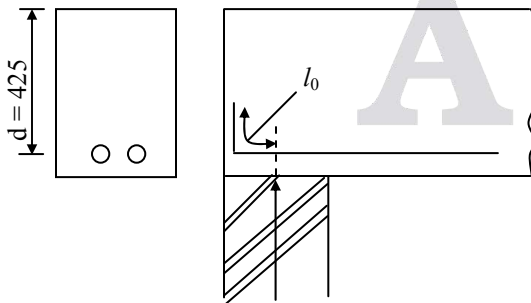
Sol: Given, $V_u = 220 \text{ kN}$

$$A_{st} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$b = 250 \text{ mm}$, $d = 425 \text{ mm}$

Fe 415, M_{20} , $\tau_{bd} = 1.2 \text{ MPa}$

$l_0 = ?$ for 90° bond



$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 16}{4 \times 1.6 \times 1.2} = 752.1875 \text{ mm}$$

$$L_d (\text{req}) = 752.1875 - 8 \times 16 = 624.1875 \text{ mm}$$

$$x_{u \max} = 0.48 \times 425 = 204$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 402.12}{0.36 \times 20 \times 250} = 80.65 \text{ mm}$$

$$= 80.65 \text{ mm}$$

$x_u < x_{u \max} \rightarrow$ Under reinforced section

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u) = 0.87 \times 415 \times 402.12 (425 - 0.42 \times 80.65) = 56.78 \times 10^6 \text{ N-mm}$$

$$L_d = \frac{1.3 M_1}{V} + l_0$$

$$624.1875 = 1.3 \times \frac{56.78 \times 10^6}{220 \times 10^3} + l_0$$

$$l_0 = 288.66 \text{ mm}$$

Minimum extension beyond centre of support = 290 mm

08. Limit State of Collapse - Torsion
01. Ans: (d)
Sol: i) size – 300 × 1000 mm

$$V_u = 150 \text{ kN}; \quad M_u = 150 \text{ kN}$$

$$T_u = 30 \text{ kN-m}$$

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 150 \times 10^3 + \frac{1.6 \times 30 \times 10^6}{300} = 310 \text{ kN}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[1 + \frac{D}{b} \right]}{1.7}$$

$$= 150 + \frac{30 \left[1 + \frac{1000}{300} \right]}{1.7}$$

$$= 226.47 \text{ kN-m}$$

02. Ans: (d)

$$b = 300 \text{ mm}, \quad D = 600 \text{ mm}$$

$$V = 100 \text{ kN}, \quad M = 100 \text{ kN-m}$$

$$T = 34 \text{ kN-m}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[1 + \frac{D}{b} \right]}{1.7}$$

$$= 100 + \frac{34 \left[1 + \frac{600}{300} \right]}{1.7}$$

$$= 160 \text{ kN-m}$$

03. Ans: (a)

Sol: $T = 68 \text{ kN-m}$

$$M_{e2} = M_T - M_u$$

 If $M_T < M_u$ then no need of A_{sc}

$$M_T = \frac{T_u \left(1 + \frac{D}{b} \right)}{1.7} = \frac{68 \left(1 + \frac{600}{300} \right)}{1.7} = 120 \text{ kN-m}$$

$M_T > M_u$ – additional compression steel is required for M_{e2} i.e $M_{e2} = M_T - M_u$
 $= 120 - 100$
 $= 20 \text{ kN-m}$

04. Ans: (a)

Sol: $b = 500, \quad D = 700 \text{ mm}$

$$d = 35 \text{ mm}, \quad V = 15 \text{ kN}$$

$$M = 100 \text{ kN-m}, \quad T = 10 \text{ kN-m}$$

$$\tau_c = 1.5 \text{ MPa}$$

 If $\tau_{ve} \neq \tau_c$ ignore torsion

 If $\tau_{ve} > \tau_c$ consider torsion for A_{st}

$$V_e = V_u + V_T$$

$$= V_u + 1.6 \frac{T_u}{b}$$

$$= 15 + 1.6 \left(\frac{10}{0.5} \right)$$

$$= 47 \text{ kN}$$

$$\tau_{ve} = \frac{V_e}{b.d} = \frac{47 \times 10^3}{500 \times (700 - 35)} \approx \frac{47}{0.5 \times 0.7}$$

$$= 0.14 \text{ MPa}$$

$$\tau_{ve} < \tau_c$$

 $\therefore$ Design BM for A_{st} is M_u only

$$M_u = 100 \text{ kN-m}$$

05. Ans: (d)

Sol: $V = 20 \text{ kN}$, $T = 9 \text{ kN-m}$
 $b = 300 \text{ mm}$, $M = 200 \text{ kN-m}$
 gross depth = 425 mm
 cover = 25 mm
 $V_e = V_u + V_T$

$$= V_u + 1.6 \frac{T_u}{b} = 20 + 1.6 \left(\frac{9}{0.3} \right)$$

$$= 68 \text{ kN}$$

06. Ans: (b)

Sol: $A_s \tau_{ve} < \tau_c$
 $T_u = 0$
 $M_{e1} = M_u = 200 \text{ kN-m}$
 A_{st} based on M_u only

10. Limit State of Collapse - Compression

01. Ans: (c)

Sol: $b = 300 \text{ mm}$
 $d = 600 \text{ mm}$
 $f_y = 415 \text{ MPa}$
 $f_{ck} = 20 \text{ MPa}$

$$P_u = 0.40 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_{sc} = 0.8\% A_g$$

$$= \frac{0.8}{100} (300 \times 600) = 1440 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= 300 \times 600 - 1440$$

$$= 178560 \text{ mm}^2$$

$$P_u = 0.4 \times 20 \times 178560 + 0.67 \times 415 \times 1440$$

$$P_u = 1829 \text{ kN}$$

02. Ans: (d)

Sol: $d = 300 \text{ mm}$; $f_{ck} = 20 \text{ N/mm}^2$
 $f_y = 415 \text{ N/mm}^2$

$$P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$A_{sc} = \left(\frac{\pi}{4} \times 300^2 \right) \times \frac{1}{100} = 706.85 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= \left(\frac{\pi}{4} \times 300^2 \right) - 706.85$$

$$= 69978.98 \text{ mm}^2$$

$$P_u = 1.05 (0.4 \times 20 \times 69978.98 + 0.67 \times 415 \times 706.85)$$

$$= 794.19 \text{ kN}$$

03. Ans: (d)

Sol: $A_g = 300 \times 300 \text{ mm}$

$$f_{ck} = 20 \text{ N/mm}^2,$$

$$A_c = A_g \text{ (neglecting } A_{sc})$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63$$

$$P_u = 0.4 \times 20 \times 300 \times 300 + 0.67 \times 415 \times 1256.63$$

$$= 1069 \text{ kN}$$

04. Ans: (d)

Sol: $m = \frac{E_{\text{strong}}}{E_{\text{weak}}} = \frac{E_{\text{steel}}}{E_{\text{conc}}}$

compatibility condition for composite (RCC) members

$$\delta_s = \delta_c$$

$$\frac{P_s l}{A_s E_s} = \frac{P_c l}{A_c E_c}$$

$$\frac{P_s}{P_c} = \frac{A_s}{A_c} \left(\frac{E_s}{E_c} \right) = \frac{1\% A_c}{A_c} \times 10 = 10\%$$

11. Footings

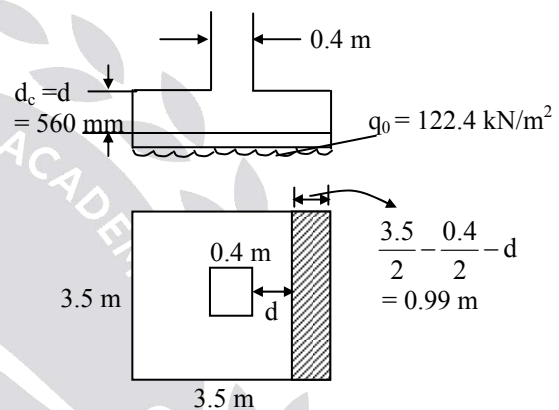
01. Ans: (b)

Sol: $B = 3.5 \text{ m}$

column size = 400 mm

$$d = 560 \text{ mm}$$

$$q_0 = 122.4 \text{ kN/m}^2$$



For one way shear

$$V_u = q_0 [\text{hatched area}]$$

$$= 122.4 [0.99 \times 3.5]$$

$$= 425 \text{ kN}$$

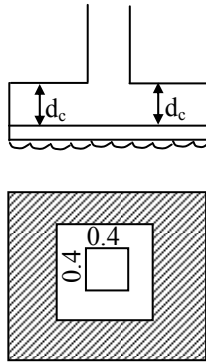
$$\tau_v = \frac{V_u}{b.d_c} = \frac{425 \times 10^3}{3500 \times 560}$$

$$= 0.22 \text{ N/mm}^2$$

$$= 0.22 \text{ MPa}$$

02. Ans: (c)

Sol:



$$B = 0.4 + \frac{0.56}{2} + \frac{0.56}{2} = 0.96$$

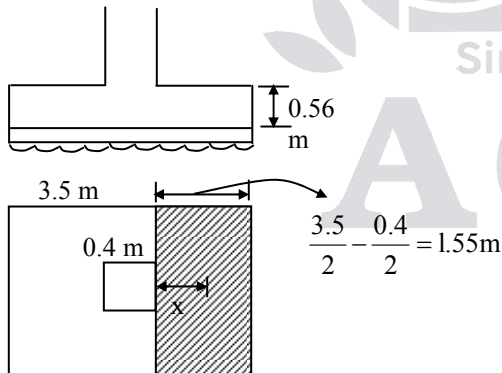
$$\begin{aligned} V_u &= q_0 [\text{hatched area}] \\ &= 122.4 \times [3.5^2 - 0.96^2] \\ &= 1386 \text{ kN} \end{aligned}$$

$$\begin{aligned} \tau_v &= \frac{V_u}{pd} = \frac{1386 \times 10^3}{(4 \times 960)(560)} \\ &= 0.64 \text{ MPa} \end{aligned}$$

V_u is more for 2-way
2-way shear is critical

03. Ans: (a)

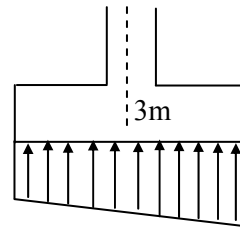
Sol:



$$\begin{aligned} M_u &= q_0 [\text{hatched area} \times \bar{x}] \\ &= 122.4 \left[3.5 \times 1.55 \times \frac{1.55}{2} \right] = 515 \text{ kN} \end{aligned}$$

04. Ans: (a)

Sol:



$$\begin{aligned} \left. \begin{aligned} \sigma_{\max} \\ \sigma_{\min} \end{aligned} \right\} &= \frac{P}{A} \pm \frac{M}{Z} \\ &= \frac{450}{3 \times 2} \pm \frac{60}{\left(\frac{2 \times 3^2}{6} \right)} \end{aligned}$$

$$\sigma_{\max} = 95 \text{ kN/m}^2 \text{ compression}$$

$$\sigma_{\min} = 55 \text{ kN/m}^2 \text{ compression}$$

As per IS 456 -2000 the assumed pressure distribution below the footing is uniform

05. Ans: (a)

Sol: $l = 2\text{ m}$; $d = 200 \text{ mm}$

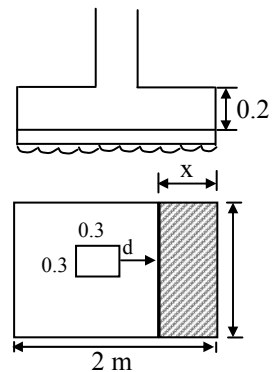
column size = $300 \times 300 \text{ mm}$

$q_0 = 320 \text{ kN}$

$\tau_v = ?$

$$q_0 = \frac{320}{2 \times 2} = 80 \text{ kN/m}^2$$

$$\begin{aligned} x &= \frac{2}{2} - \frac{0.3}{2} - 0.2 \\ &= 1 - 0.15 - 0.2 \\ &= 0.65 \end{aligned}$$



$$\begin{aligned} \text{One way shear } V_u &= q_0 [\text{hatched area}] \\ &= 80 [0.65 \times 2] = 104 \text{ kN} \end{aligned}$$

$$\tau_v = \frac{V_u}{bd_c} = \frac{104 \times 10^3}{2000 \times 200} = 0.26$$

14. Analysis of Prestressed Concrete Members

01. Ans: (b)

Sol: Prestressing force, $P = 2500 \text{ kN}$

Effective span, $l = 10 \text{ m}$

udl on the beam, $w = 40 \text{ kN/m}$

For load balancing

$$P \cdot e = \frac{w \ell^2}{8}$$

$$(2500)(e) = \frac{(40)(10)^2}{8}$$

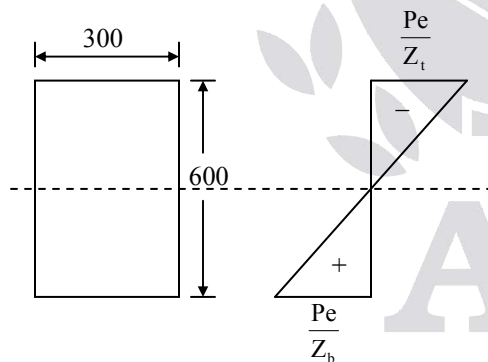
$$e = 0.2 \text{ m} = 200 \text{ mm}$$

02. Ans: (b)

Sol: $\gamma_c = 24 \text{ kN/m}^3$

$$\sigma_t = 2 \text{ MPa}$$

$$\sigma_b = 20 \text{ MPa}$$



$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z} \quad \text{----- (1)}$$

$$\sigma_t = \frac{P}{A} - \frac{Pe}{Z} \quad \text{----- (2)}$$

Adding (1) & (2)

$$20 = \frac{P}{A} + \frac{Pe}{Z}$$

$$-2 = \frac{P}{A} - \frac{Pe}{Z}$$

$$18 = \frac{2P}{A}$$

$$P = 1620 \text{ kN}$$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{Z}$$

$$20 = \frac{1620 \times 10^3}{300 \times 600} + \frac{1620 \times 10^3 \times 6 \times e}{300 \times 600^2}$$

$$e = 122 \text{ mm}$$

$$e \approx 135 \text{ mm}$$

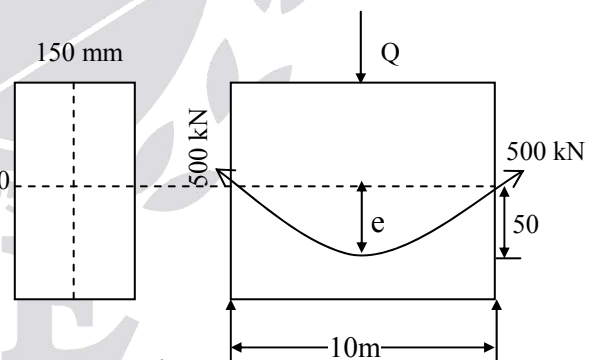
03. Ans: (a)

Sol: $150 \times 300 \text{ mm}$

$l = 10 \text{ m}$, e at support = 0 mm

$e = 50 \text{ mm}$ (center), $P = 500 \text{ kN}$

$Q = ?$ (at center of span)



$$Pe = \frac{Q \times l}{4}$$

$$500 \times \frac{50}{1000} = \frac{Q \times 10}{4}$$

$$100 = Q \times 10$$

$$Q = 10 \text{ kN}$$

04. Ans: (b)

Sol: Self weight

$$\begin{aligned} w_D &= \gamma_c \times b \times D \\ &= (24 \text{ kN/m}^3) \times 0.15 \times 0.3 \\ &= 1.08 \text{ kN/m} \end{aligned}$$

P – line at upper kern point ($\sigma_b = 0$)

$$M_D = \frac{w_D l^2}{8} = \frac{1.08 \times 10^2}{8} = 13.5$$

$$\begin{aligned} \sigma_b = 0 &= \frac{P}{A} + \frac{Pe}{z} - \frac{M_D}{z} - \frac{M_L}{z} \\ &= \frac{500 \times 10^3}{300 \times 150} + \frac{500 \times 10^3 \times 50}{\left(\frac{150 \times 300^2}{6}\right)} - \frac{13.5 \times 10^6}{\left(\frac{150 \times 300^2}{6}\right)} \end{aligned}$$

$$0 = 11.11 + 11.11 - 6 - \frac{M_L}{225 \times 10^4}$$

$$M_L = 16.22 \times 225 \times 10^4$$

$$M_L = 36.5 \text{ kN-m,}$$

$$M_L = \frac{Ql}{4}$$

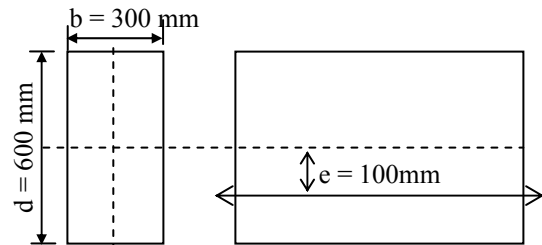
$$36.5 = \frac{Q \times 10}{4}$$

$$146 = Q \times 10$$

$$Q = 14.6 \text{ kN}$$

05. Ans: (c)

Sol: $l = 6 \text{ m, } b = 300 \text{ mm, } d = 600 \text{ mm}$
 $e = 100 \text{ mm, } P = 1000 \text{ kN,}$



Neglecting self weight of the beam

$$\begin{aligned} \sigma_b &= \frac{P}{A} + \frac{Pe}{z} \\ &= \frac{1000 \times 10^3}{300 \times 600} + \frac{1000 \times 10^3 \times 100}{\left(\frac{300 \times (600)^2}{6}\right)} \\ &= 5.55 + 5.55 = 11.11 \text{ MPa} \end{aligned}$$

06. Ans: (b)

Sol: $b = 200 \text{ mm, } D = 250 \text{ mm}$

$A = 500 \text{ mm}^2, P = 1000 \text{ MPa}$

$m = 10$

$\epsilon_s = \epsilon_c$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_c = \sigma_s \left(\frac{\epsilon_c}{\epsilon_s} \right) = \frac{\sigma_s}{m} = \frac{1000}{10}$$

$$\sigma_c = 100 \text{ MPa}$$

Prestressing force on steel $= \sigma_s \cdot A_s$

$$= 1000 \times 500 = 500 \times 10^3 \text{ N}$$

Compression force in concrete $= 500 \text{ kN}$

$$= \sigma_c \cdot A_c$$

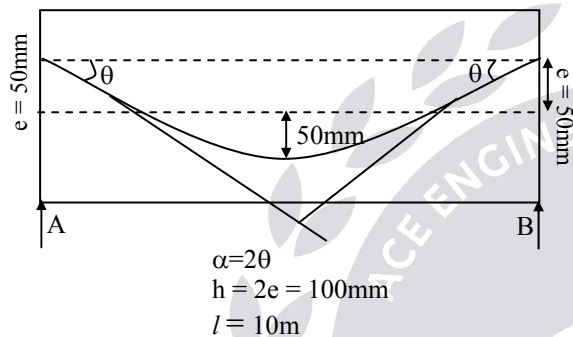
Compression stress in concrete $\sigma_c = \frac{P_c}{A_c}$

$$= \frac{500 \times 10^3}{200 \times 250} = 10 \text{ MPa}$$

15. Losses of Prestress

01. Ans: (b)

Sol: $l = 10 \text{ m}$, $b = 100 \text{ mm}$,
 $D = 300 \text{ mm}$ $A = 200 \text{ sq-mm}$,
 $e = 50 \text{ mm}$, $\mu = 0.35$;
 $k = 0.0015 \text{ per m}$



Initial stress in wires = 1200 MPa

$$\text{Loss of stress in wires} = \sigma(\mu\alpha + kx)$$

$$= 1200[0.35 \times \alpha + 0.0015 \times 10]$$

From equation of parabola

$$\theta = \frac{4 \times 0.1}{10} = 0.04 \text{ radians}$$

$$\alpha = 2 \times \theta = 0.08$$

$$\text{Loss} = 1200[0.35 \times 0.08 + 0.0015 \times 10]$$

$$\text{Loss of stress} = 51.6 \text{ MPa}$$

$$\% \text{ loss of stress} = \frac{51.6}{1200} \times 100$$

$$= 4.28 \approx 4.3\%$$

02. Ans: (b)

Sol:

Tensioning from both the ends % loss of stress

$$= \frac{\% \text{ loss of stress}}{2} = \frac{4.28}{2} = 2.15$$

03. Ans: (b)

Sol: Straight tendon tensioned from one end

$$\text{Loss of stress in wires} = \sigma[\mu\alpha + kx]$$

$$(\because \alpha = 0)$$

$$1200(0.35 \times (0) + 0.0015 \times 10) = 18 \text{ MPa}$$

$$\% \text{ of loss} = \frac{18}{1200} \times 100$$

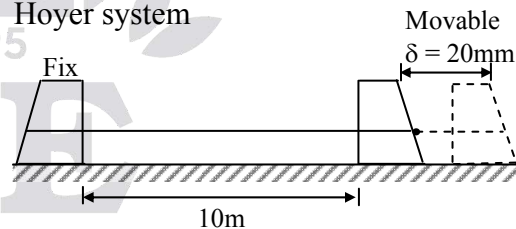
$$= 1.5\%$$

If tensioned from two ends

$$\frac{\% \text{ of loss}}{2} = \frac{1.5}{2} = 0.75\%$$

04. Ans: (c)

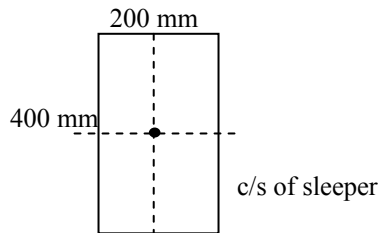
Sol: Hoyer system



$$\delta = \frac{PL}{AE} \quad \left(\text{as } \sigma = \frac{P}{A} \right)$$

$$\text{Prestress induced in steel wire, } \sigma = \frac{\delta E}{L}$$

$$\sigma = \frac{20 \times 2 \times 10^5}{10,000} = 400 \text{ MPa}$$



Eccentricity of Prestress, $e = 0$

$$\begin{aligned} \text{Prestressing force in steel wire} &= P = \sigma_s \cdot A_s \\ &= 400 \times 500 \text{ mm}^2 \\ &= 200 \text{ kN} \end{aligned}$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} (e) = \frac{200 \times 10^3}{200 \times 400} = 2.5 \text{ MPa}$$

Loss due to elastic shortening

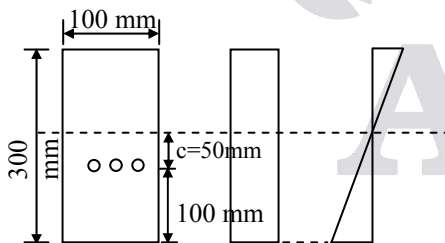
$$= m \times f_c = \left(\frac{E_s}{E_c} \right) f_c$$

$$\sigma = \left(\frac{200,000}{20,000} \right) \times 2.5 = 25 \text{ MPa}$$

$$\% \text{ loss of Prestress} = \frac{25}{400} \times 100 = 6.25\%$$

05. Ans: (d)

Sol:



$$f_c = \frac{P}{A} + \frac{P}{I} (e)^2$$

Initial stress in steel wire = 1200 MPa

Prestressing force in each steel wire

$$P = \sigma_s \cdot A_s$$

$$P = 1200 \times 50 = 60 \text{ kN}$$

$$f_c = \frac{60 \times 10^3}{100 \times 300} + \frac{60 \times 10^3}{\left(\frac{100 \times 300^3}{12} \right)} \times (50)^2$$

$$f_c = 2.66 \text{ MPa}$$

Simultaneous tensioning = loss of prestress is zero

06. Ans: (a)

Sol: Successive tensioning of the 3 cables

$$= \frac{n(n-1)}{2} (m \cdot f_c)$$

$$= \frac{3(3-1)}{2} (6 \times 2.66)$$

$$= 48.0 \text{ MPa}$$

$$\% \text{ of loss} = \frac{48.0}{1200} \times 100 = 4\%$$

(or) For pretensioning system

$$\text{Loss} = n(m \times f_c)$$

$$= 3(6 \times 2.66) = 48.0 \text{ MPa}$$

07. Ans: (c)

Sol: Anchorage slip = 3 mm

$$l = 30 \text{ m}, \sigma = 1200 \text{ MPa}$$

$$E = 2.1 \times 10^5 \text{ MPa}$$

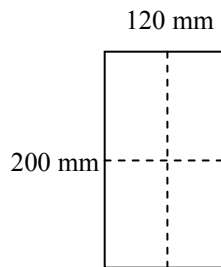
$$E = \frac{\delta E}{l} = \frac{3 \times 2.1 \times 10^5}{30 \times 10^3}$$

$$\sigma = 21 \text{ MPa}$$

$$\% \text{ of loss} = \frac{21}{1200} \times 100 = 1.73\%$$

08. Ans: (b)

Sol:



$$P = 150 \text{ kN}, e = 20 \text{ mm}$$

$$A = 187.5 \text{ mm}^2$$

$$E_s = 2.1 \times 10^5 \text{ MPa}$$

$$E_c = 3.0 \times 10^4 \text{ MPa}$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} \cdot e$$

$$= \frac{150 \times 10^3}{187.5} + \frac{150 \times 10^3 \times 20^2}{\left(\frac{120 \times 200^3}{12} \right)}$$

$$= 800 + 0.75 \text{ MPa}$$

$$f_c = 800.75 \text{ MPa}$$

$$\text{loss due to elastic shortening} = m \cdot f_c$$

$$= \left(\frac{E_s}{E_c} \right) f_c$$

$$= \frac{2.1 \times 10^5}{3.0 \times 10^4} \times 7 = 4.9$$

Percentage loss in the prestressing steel due to elastic deformation

$$= \frac{4.9}{800.75} \times 100$$

$$= 6.12\%$$

09. Ans: (c)

$$\text{Sol: } \epsilon = \epsilon_{\text{shrink}} + \epsilon_{\text{creep}}$$

$$= 0.0008$$

$$\text{Loss of prestress on steel} = \epsilon \times E_s$$

$$= 0.0008 \times 200 \times 10^3$$

$$= 160 \text{ MPa}$$

$$\text{Stress remaining after loss} = \text{Initial stress} - \text{Loss}$$

$$= 200 - 160$$

$$= 40 \text{ MPa}$$