

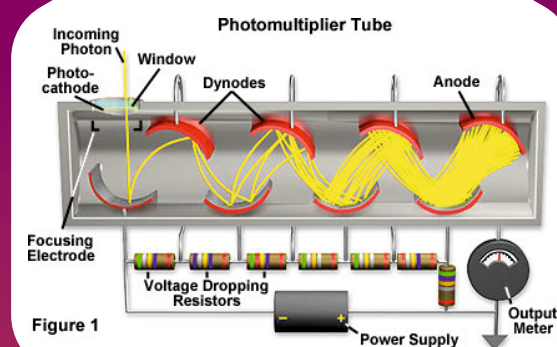
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INSTRUMENTATION ENGINEERING

Process Control & Optical Instrumentation

Text Book : Theory with worked out Examples
and Practice Questions



Chapter 1 Process Dynamics

(Solutions for Text Book Practice Questions)

01. Ans: (c)

Sol: For any first order instrument like thermocouple, the time to reach 98% of steady state value is equal to 4 times Time constant.

$$0.98 T_0 = T_0 (1 - e^{-t/\tau})$$

$$0.98 = (1 - e^{-t/\tau})$$

$$t = 4 \tau$$

02. Ans: (c)

Sol: Error is measurement when a ramp of magnitude 'K' is applied is

$$e_{ss} = K \tau$$

Where τ = time constant = 100 m sec;

$$e_{ss} = \frac{1^\circ\text{C}}{\text{sec}} \times 100 \times 10^{-3} \text{ sec}$$

$$= 0.1^\circ\text{C}$$

$\therefore$ The reading of thermocouple will be
 $= 70^\circ\text{C} - 0.1^\circ\text{C} = 69.9^\circ\text{C}$

03. Ans: (c)

Sol: $96.5 = 100(1 - e^{-t/\tau}) + 30(e^{-t/\tau})$

$$96.5 = 100 + e^{-30/\tau} (30 - 100)$$

$$\tau = 10.01 \text{ sec}$$

$$98 = 100 (1 - e^{-t/10}) + 30 e^{-t/10}$$

$$e^{-t/10} (30 - 100) + 100 = 98$$

$$t = 35.6 \text{ sec}$$

04. Ans: (a)

Sol: $2 \frac{dT_i}{dt} + T_i - T_a = 0$

By applying Laplace Transform on both sides.

$$2sT_i(s) + T_i(s) - T_a(s) = 0.$$

$$T_i(s) [2s+1] = T_a(s)$$

$$\frac{T_i(s)}{T_a(s)} = \frac{1}{2s+1}$$

$$\frac{T_i(s)}{T_a(s)} = \frac{0.5}{s+0.5}$$

Here cut-off frequency

$$\omega_c = 0.5 = \frac{1}{2}$$

$$\text{or } 2\pi f_c = \frac{1}{2}$$

$$f_c = \frac{1}{4\pi} \text{ Hz}$$

Chapter **2** Controller Tuning

01. Ans: (b)

Sol: $K_u = 10$

$P_u = 8\text{Hz}$

In case of Ziegler Nichols tuning for proportional controller

$K_P = 0.5 K_u = 0.5 \times 10 = 5.$

02. Ans: (a)

Sol: Distance between the roll point and the point at which the thickness is measured

$$= d = 0.5 \text{ m}$$

$v = \text{velocity of the sheet} = 0.2 \text{ m/s}$

$$\text{delay} = \frac{d}{v} = \frac{0.5}{0.2} = 2.5 \text{ sec}$$

$$\therefore \text{T.F} = e^{-2.5s}$$

03. Ans: (c) & (d)

Sol: $\text{OLTF} = K e^{-2.5s}$

$$\angle \text{OLTF} = -2.5 \omega, |\text{OLTF}| = K$$

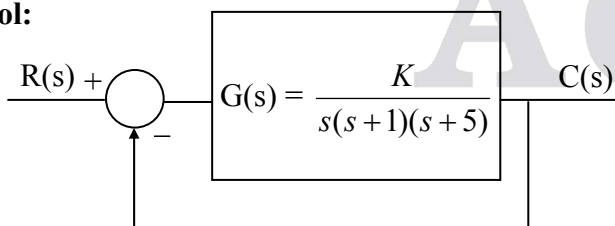
Ultimate value of K for stability = 1

$$-2.5 \omega = -\pi,$$

$$\omega = \frac{\pi}{2.5} = 0.4 \pi \text{ rad/sec}$$

04. Ans: (c)

Sol:



Characteristics equation

$$1 + \frac{k}{s(s+1)(s+5)} = 0$$

$$s^3 + 6s^2 + 5s + K = 0$$

By R-H criteria

$$s^3 \quad 1 \quad 5$$

$$s^2 \quad 6 \quad K$$

$$s^1 \quad \frac{30-K}{6} \quad 0$$

$$s^0 \quad K$$

$$K_u = 30$$

As, $0 < K < 30$ stable

$$6s^2 + k = 0$$

$$6s^2 + 30 = 0$$

$$s^2 = -30/6 = -5$$

$$s = \pm j\sqrt{5}$$

Chapter 3 Control Strategies

01.

Sol: a) For, $U(s) = 0$

By, mason's gain formula

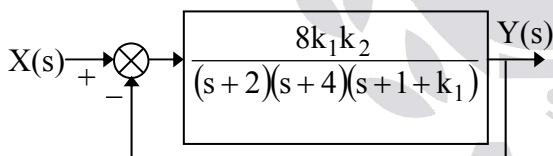
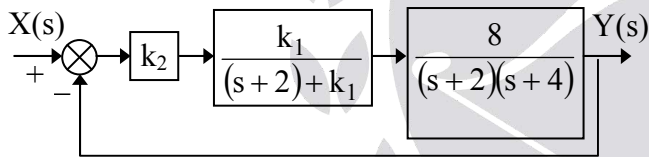
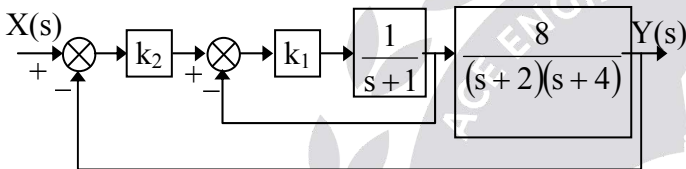
$$\frac{Y(s)}{W} = \frac{G_p(s)[1 + G_f(s)G_c(s)]}{1 + G_p(s).H(s)G_c(s)}$$

b) In order to eliminate the effect of W.

$$G_f(s) = -\frac{1}{G_c(s)}$$

02. **Ans: (d)**

Sol:



$$\frac{Y(s)}{X(s)} = \frac{8k_1k_2}{(s+2)(s+4)(s+1+k_1) + 8k_1k_2}$$

Thus, characteristics equation is

$$(s+2)(s+4)(s+1+k_1) + 8k_1k_2 = 0$$

$$s^3 + (7+k_1)s^2 + (14+6k_1)s + 8 + 8k_1 + 8k_1k_2 = 0$$

According to R-H array

$$\begin{array}{l|l} s^3 & 1 \quad 14+6k_1 \\ s^2 & 7+k_1 \quad 8(1+k_1+k_1k_2) \\ s^1 & \frac{(7+k_1)(14+6k_1) - 8(1+k_1+k_1k_2)}{7+k_1} \quad 0 \\ s^0 & 8(1+k_1+k_1k_2) \quad 0 \end{array}$$

For stable system, all the elements of first column should be greater than zero. By substituting all the given options in coefficient of s^1 will give negative and zero values except option (d).

03.

Sol: Applying Mason's gain formula

$$TF = \frac{Y(S)}{D(S)} = \frac{1 + K \cdot \frac{1}{s+2}}{1 - \left[50 \cdot \frac{1}{s+2} (-1) \right]} = 0$$

$$1 + K \cdot \frac{1}{s+2} = 0$$

$$s+2 + K = 0$$

$$K = -(s+2)$$

At frequency ($\omega = 0$)

$$K = -2$$

01.

Sol: $P = 24 \times 0.433$
 $= 10.392 \text{ Psi}$

We know for valve

$$C_v = Q \sqrt{\frac{G}{\Delta P}}$$

Where, Q = flow rate

ΔP = differential pressure across the valve.

G = Specific gravity of water

So,

$$C_v = 120 \sqrt{\frac{1}{10.392}}$$

$$= 37.22$$

02.

Sol: Maximum out flow = maximum inflow
 $= 120 \text{ gpm}$

$$\Delta P = (26 - 14) \times (0.433)$$

$$= 5.196 \text{ Psi}$$

So, $C_v = Q \sqrt{\left(\frac{G}{\Delta P}\right)}$

$$= 120 \sqrt{\left(\frac{1}{5.19}\right)}$$

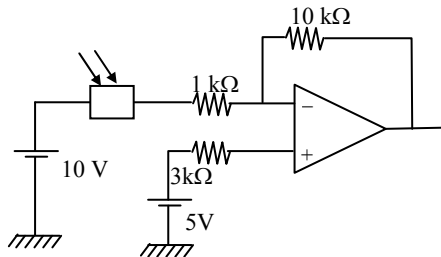
$$= 52.67$$

Chapter 1 Optical Sources & Detectors

(Solutions for Vol-1_Classroom Practice Questions)

01.

Sol:



1st case:

$R_p \rightarrow 1\text{ k}\Omega$, no 10 V source

2nd case:

$R_p \rightarrow 5\text{ k}\Omega \rightarrow 10\text{ V}$ source is present

So, $V = V_{01(\text{Only } 10\text{V})} + V_{02(\text{Only } 5\text{V})}$

$$V = \frac{-10\text{ k}\Omega}{1\text{ k}\Omega + R_p}$$

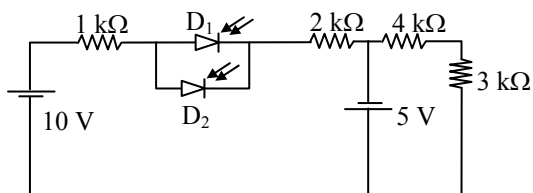
$$= \left(\frac{-10\text{ k}\Omega}{1\text{ k}\Omega + 5\text{ k}\Omega} \right) \times 10 + \left(1 + \frac{10\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ k}\Omega} \right) \times 5$$

$$V = \left(\frac{-10\text{ k}\Omega}{6\text{ k}\Omega} \times 10 \right) + \left(1 + \frac{10\text{ k}\Omega}{2\text{ k}\Omega} \right) \times 5$$

$$V = 13.33\text{ V}$$

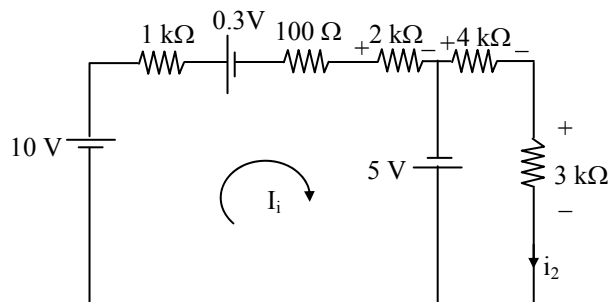
02.

Sol:



D_1, D_2 are in forward bias

D_2 —ON, D_1 —OFF



$$V_{2k} = ?$$

$$V_{3k\Omega} = ?$$

$$i_2 = \frac{-5\text{ V}}{4\text{ k}\Omega + 3\text{ k}\Omega} = \frac{-5\text{ V}}{7\text{ k}\Omega} = -0.714\text{ mA}$$

$$\begin{aligned} V_{3k\Omega} &= i_2 \times 3\text{ k}\Omega \\ &= (-0.714) \times 3 \times 10^3 \\ &= -2.14\text{ V} \end{aligned}$$

From circuit

$$I_i = 1.41\text{ mA}$$

So

$$V_{2k} = 1.41\text{ mA} \times 2\text{ k}\Omega = 2.8\text{ V}$$

03. Ans: (c)

Sol: Given data.

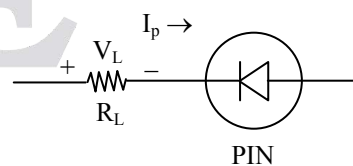
$$C_j = 6\text{ pF}$$

$$A = 10\text{ mm}^2$$

$$R = 0.5\text{ A/W}$$

$$I = 1\text{ mW/cm}^2$$

$$R_L = 100\text{ k}\Omega$$



$$V_L = ?$$

We know

$$V_L = I_p \times R_L$$

$$R = \frac{I_p}{P_0}$$

$$P_0 = A \times I$$

$$I_p = \frac{0.5A}{W} \times A \times I$$

$$I_p = \frac{0.5A}{W} \times 10\text{mm}^2 \times \frac{1\text{mW}}{10\text{mm}^2}$$

$$I_p = \left(0.5 \times \frac{10}{100} \times 1\text{m}\right) \text{Amp}$$

$$I_p = 5 \times 10^{-5} \text{ amp}$$

$$V_L = I_p \times R_L = 5 \times 10^{-5} \times 100 \text{ k}\Omega$$

$$\therefore V_L = 5 \text{ volts}$$

04. Ans: (c)

Sol: Given:

$$\eta = 0.65$$

$$\lambda = 900 \text{ nm}$$

$$P_0 = 0.5 \text{ }\mu\text{W}$$

$$I_m = 10 \text{ }\mu\text{A}$$

$$M = ?$$

$$M = \frac{I_m}{I_p} = \frac{10\mu\text{A}}{I_p}$$

We know

$$\eta = \frac{EI_p}{P_0q}$$

$$0.65 = \frac{hcI_p}{\lambda P_0q}$$

$$\Rightarrow 0.65 = \left(\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{900\text{nm} \times 0.5 \times 10^{-6} \times 1.6 \times 10^{-19}} \right) \times I_p$$

$$\Rightarrow I_p = 2.36 \times 10^{-7}$$

$$M = \frac{10\mu\text{A}}{2.36 \times 10^{-7} \text{ A}}$$

$$= 42.4 \approx 43$$

05. Ans: -1V

Sol: Output is independent V_r

06. Ans: 2

Sol: Given

$$\text{Area} = 10 \text{ mm}^2$$

$$\text{Sensitivity} = 0.5 \text{ A/W}$$

$$\text{Intensity} = 4 \text{ W/m}^2$$

Photodiode current

$$I_p = \text{Area} \times \text{sensitivity} \times \text{Intensity}$$

$$I_p = 10 \text{ mm}^2 \times 0.5 \text{ A/W} \times 4 \text{ W/m}^2$$

$$I_p = 20 \text{ }\mu\text{A}$$

I to V converter sensitivity is $100 \text{ mV}/\mu\text{A}$

$$\text{So, } V_o = \frac{100\text{mV}}{\mu\text{A}} \times 20\mu\text{A} \\ = 2 \text{ Volt}$$

07. Ans: 75.18

$$\text{Sol: } \frac{I}{P} = \frac{\eta e \lambda}{hc}$$

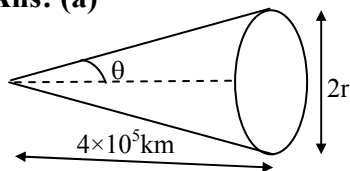
$$I = \frac{\eta e \lambda}{hc} \times P$$

$$= \frac{0.75 \times 1.6 \times 10^{-19} \times 830 \times 10^{-9} \times 100 \times 10^{-6}}{6.624 \times 10^{-34} \times 2 \times 10^8}$$

$$I = 75.18 \mu\text{A}$$

01. Ans: (b)

Sol: $2i = 115^{\circ}.34' = 115.566^{\circ}$,
 $i = 57.783^{\circ}$, $u = \tan i = \tan 57.783^{\circ}$
 $= 1.587$

02. Ans: (a)**Sol:**

$$\theta = 1 \text{ m rad}$$

$$\tan \theta = \frac{r}{4 \times 10^5 \times 1000} = 1 \text{ mrad}$$

(tan $\theta \approx \theta$)

$$r = 4 \times 10^5 \text{ meters}$$

$$= 400 \text{ km}$$

$$\text{Diameter} = 2 \times r$$

$$= 2 \times 400 \text{ km}$$

$$= 800 \text{ km}$$

03. Ans: (b)**Sol:** Given:

$$L = 500 \text{ mm}$$

$$\text{Bandwidth} = 1500 \text{ MHz}$$

$$\Delta f = ?$$

Number of longitudinal oscillating modes

$$= \frac{BW}{\Delta f}$$

We know

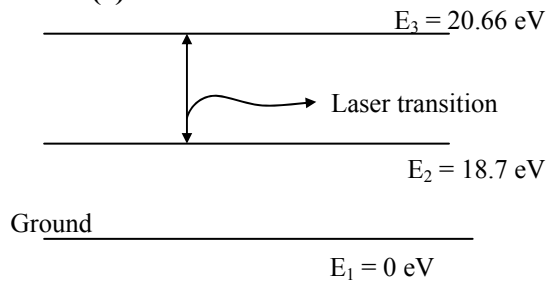
$$\Delta f = \frac{C}{2L}$$

Number of longitudinal oscillating modes

$$= \frac{1500 \text{ MHz}}{\left(\frac{3 \times 10^8}{2 \times 500 \times 10^{-3}} \right)}$$

$$= \frac{1500 \times 10^6}{3 \times 10^8} \times 1000 \times 10^{-3}$$

$$= 5$$

04. Ans: (c)**Sol:****05. Ans: (c)**

$$\text{Sol: } E_3 - E_2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_3 - E_2} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{(20.66 - 18.7) \times 1.6 \times 10^{-19} \text{ J}}$$

$$= 633.8 \text{ nm}$$

06. Ans: (d)**Sol:** Given

$$\lambda = 6328 \text{ \AA}$$

$$\text{Bandwidth} = 1 \text{ MHz}$$

$$C_l = ?$$

We know

$$C = \frac{C_l}{C_t}$$

$$C_l = 3 \times 10^8 \times \frac{1}{1 \text{ MHz}} \left(\because C_t = \frac{1}{f} \right)$$

$$C_l = 300 \text{ m}$$

07. Ans: 40**Sol:** Area for photodiode D_1

$$A_1 = 10 \text{ mm} \times (5 \text{ mm} + 0.1 \text{ mm})$$

$$= 51 \times 10^{-6} \text{ m}^2$$

Area for photodiode D_2

$$A_2 = 10 \text{ mm} \times (5 \text{ mm} - 0.1 \text{ mm})$$

$$= 49 \times 10^{-6} \text{ m}^2$$

$$P_1 = 50 \left(\frac{W}{\text{m}^2} \right) \times 51 \times 10^{-6} (\text{m}^2)$$

$$P_1 = 50 \times 51 \times 10^{-6} \text{ (W)}$$

$$P_2 = 50 \left(\frac{\text{W}}{\text{m}^2} \right) \times 49 \times 10^{-6} \text{ (m}^2\text{)}$$

$$P_2 = 50 \times 49 \times 10^{-6} \text{ (W)}$$

Difference between photo currents

$$\Delta I = I_{D1} - I_{D2}$$

$$= \text{Photodiode sensitivity} \times \Delta P$$

$$= 0.4 \left(\frac{\text{A}}{\text{W}} \right) \times (P_1 - P_2)$$

$$= 0.4 \times 50 (51 - 49) \times 10^{-6}$$

$$= 0.4 \times 50 \times 2 \times 10^{-6}$$

$$= 40 \text{ (}\mu\text{A)}$$

08. Ans: 2

Sol: $E_g = \frac{hC}{\lambda}$

$$E_g = \frac{4.13567 \times 10^{-15} \times 3 \times 10^8}{620 \times 10^{-9}}$$

$$= 2\text{eV}$$



Chapter 3 Interferometers

01.

Sol: Given data:

$$t = 5 \mu\text{m}$$

$$n = 5$$

$$\lambda = 589 \text{ nm}$$

$$\mu_g = ?$$

We know

$$t(\mu_g - 1) = n\lambda$$

$$\Rightarrow 5 \times 10^{-6}(\mu_g - 1) = 5 \times 589 \times 10^{-9}$$

$$\Rightarrow (\mu_g - 1) = \frac{5 \times 589 \times 10^{-9}}{5 \times 10^{-6}}$$

$$\Rightarrow (\mu_g - 1) = 0.589$$

$$\Rightarrow \mu_g = 1.589$$

02.

Sol: Given data:

$$\lambda = 515 \text{ nm}$$

$$\text{Refractive index } (\mu) = 1.6$$

$$\theta_R = 45^\circ$$

$$t = ?$$

we know

$$t(\mu - 1) = n\lambda$$

$$t = \frac{n\lambda}{(\mu - 1)}$$

$$\Rightarrow t = \frac{515 \times 10^{-9}}{1.6 - 1} = 8.58 \times 10^{-7}$$

$$\Rightarrow t = 0.85 \mu\text{m}$$

03.

Sol: Given data

$$t = 1.5 \mu\text{m}$$

$$\lambda = 0.5 \mu\text{m}$$

$$n = ?$$

We know

$$t = \frac{n\lambda}{2}$$

$$\Rightarrow 1.5 \times 10^{-6} = \frac{n \times 0.5 \times 10^{-6}}{2}$$

$$\Rightarrow \frac{1.5 \times 10^{-6} \times 2}{0.5 \times 10^{-6}} = n$$

$$\Rightarrow n = 6$$

04.

Sol: Given data:

$$n = 100$$

$$\lambda = 6328 \text{ \AA}$$

$$t = 20 \text{ cm}$$

$$\mu = ?$$

We know

$$2t(\mu - 1) = n\lambda$$

$$\Rightarrow 2 \times 20 \times 10^{-2}(\mu - 1) = 100 \times 6328 \times 10^{-10}$$

$$\mu = 1.0001582 \approx 1$$

05.**Sol:** Given data

$$R.I = \mu_g = 1.53$$

$$\mu_{\text{air}} = 1.0$$

$$R = \left(\frac{\mu_g - \mu_{\text{air}}}{\mu_g + \mu_{\text{air}}} \right)^2$$

$$R = 0.044$$

$$R = 4.4 \% \text{ of loss}$$



01. Ans: (d)

$$\begin{aligned}\text{Sol: } NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{(1.44)^2 - (1.4)^2} \\ &= 0.34\end{aligned}$$

02. Ans: (c)**Sol:** Given data

$$\epsilon_r = 2.5$$

$$n = ?$$

 $n = \text{refractive index}$

We know,

$$n = \sqrt{\epsilon_r \mu_r}$$

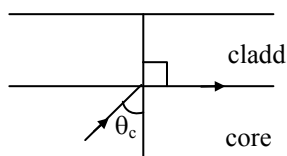
 $\epsilon_r = \text{relative permittivity}$
 $\mu_r = \text{relative permeability}$

$$\begin{aligned}n &= \sqrt{2.5} \quad (\because \mu_r = 1) \\ &= 1.58\end{aligned}$$

03. Ans: (d)**Sol:** $n_1 = 1.6$

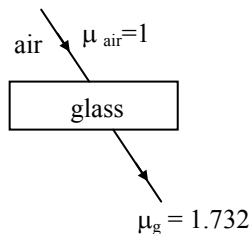
$$n_2 = 1.422$$

$$\theta_c = ?$$



$$\frac{\sin \theta_c}{\sin 90^\circ} = \frac{n_2}{n_1}$$

$$\begin{aligned}\theta_c &= \sin^{-1} \left(\frac{1.422}{1.64} \right) \\ &= 60.12^\circ \\ &\approx 60^\circ\end{aligned}$$

04. Ans: (c)**Sol:** $\mu_g = 1.732$ 

$$\tan \theta_B = \frac{\mu_g}{\mu_{\text{air}} = 1}$$

$$\theta_B = \tan^{-1}(1.732)$$

$$\theta_B = 60^\circ$$

05. Ans: (a)**Sol:** Given $\mu_{\text{glass}} = 1.720$

$$R = \left(\frac{\mu_{\text{air}} - \mu_{\text{glass}}}{\mu_{\text{air}} + \mu_{\text{glass}}} \right)^2 \times 100$$

$$\begin{aligned}R &= \left(\frac{1 - 1.72}{1 + 1.72} \right)^2 \times 100 \\ &= 7\%\end{aligned}$$

06. Ans: (d)

$$\text{Sol: } \theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

Given

$$n_1 = 1.641$$

$$n_2 = 1.422$$

$$\begin{aligned}\theta_c &= \sin^{-1} \left(\frac{1.422}{1.641} \right) \\ &= 60^\circ\end{aligned}$$

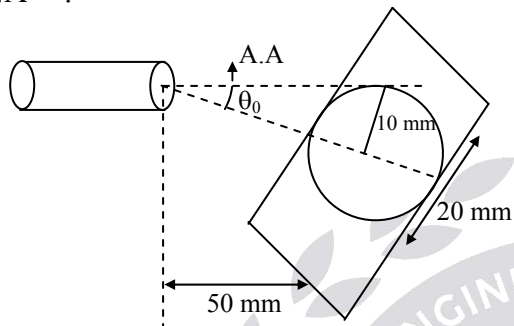
07. Ans: (d)

$$\text{Sol: } \frac{\mu_t}{\mu_g} = \frac{1.33}{1.5} = \frac{C}{V_t} \times \frac{V_g}{C}$$

$$\frac{V_t}{V_g} = \frac{1.55}{1.33}$$

08. Ans: (b)

Sol: NA = ?



$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \mu_0 \sin \theta_0$$

$$NA = \sin \theta_0$$

$$NA = \frac{10}{\sqrt{10^2 + 50^2}}$$

$$= 0.196$$

$$\approx 0.2$$

09. Ans: 0.75

$$\text{Sol: } \frac{n_1}{n_2} = \frac{t_1}{t_2} = 0.75 \quad (n \propto t)$$

