GATE | PSUs

# CIVIL ENGINEERING

## Fluid Mechanics & Hydraulic Machines

**Text Book :** Theory with worked out Examples
and Practice Questions

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# 8 Fluid Mechanics & Hydraulic Machines

**(Solutions for Text Book Practice Questions)**

## 01. Properties of Fluids

**01. Ans: (c)**

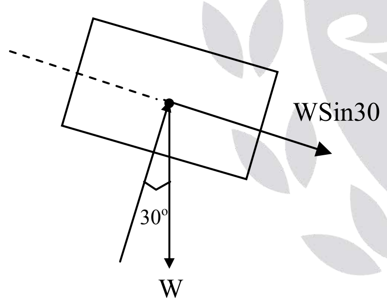
**Sol:** For Newtonian fluid whose velocity profile is linear, the shear stress is constant. This behavior is shown in option (c).

**02. Ans: 100**

**Sol:** $\tau = \dfrac{\mu V}{h} = \dfrac{0.2\times1.5}{3\times10^{-3}} = 100\ \text{N/m}^2$

**03. Ans: 1**

**Sol:**




$$F = \tau\times A$$

$$W\sin 30 = \frac{\mu A V}{h}$$

$$\frac{100}{2} = \frac{1\times0.1\times V}{2\times10^{-3}}$$

$$V = 1\ \text{m/s}$$

**Common data Q. 04 & 05**

**04. Ans: (c)**

**Sol:** $D_1 = 100$ mm, $D_2 = 106$ mm

$$\text{Radial clearance, } h = \frac{D_2 - D_1}{2}$$

$$= \frac{106-100}{2} = 3\,\text{mm}$$

L = 2 m

μ = 0.2 pa.s

N = 240 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times240}{60}$$

$$\omega = 8\pi$$

$$\tau = \frac{\mu\omega r}{h} = \frac{0.2\times8\pi\times50\times10^{-3}}{3\times10^{-3}}$$

$$= 83.77\ \text{N/m}^2$$

**05. Ans: (b)**

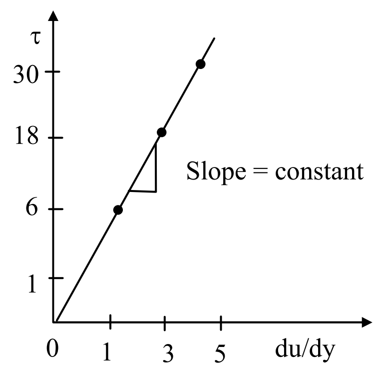
**Sol:** $$\text{Power, } P = \frac{2\pi\omega^2\mu L r^3}{h}$$

$$= \frac{2\pi\times(8\pi)^2\times0.2\times2\times(0.05)^3}{3\times10^{-3}}$$

$$= 66\ \text{Watt}$$

**06. Ans: (c)**

**Sol:**




∴ Newtonian fluid

**07. Ans: (a)**

**Sol:** $\tau = \mu \frac{du}{dy}$

$u = 3\sin(5\pi y)$

$\frac{du}{dy} = 3\cos(5\pi y) \times 5\pi = 15\pi\cos(5\pi y)$

$\tau\big|_{y=0.05} = \mu \frac{du}{dy}\Big|_{y=0.05}$

$= 0.5 \times 15\pi\cos(5\pi \times 0.05)$

$= 0.5 \times 15\pi \times \cos\left(\frac{\pi}{4}\right)$

$= 0.5 \times 15\pi \times \frac{1}{\sqrt{2}}$

$= 7.5\times3.14\times0.707 \approx 16.6$ N/m$^2$

**08. Ans: (d)**

**Sol:**

- Ideal fluid → Shear stress is zero.
- Newtonian fluid → Shear stress varies linearly with the rate of strain.
- Non-Newtonian fluid → Shear stress does not vary linearly with the rate of strain.
- Bingham plastic → Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain.

**09. Ans: (b)**

**Sol:** $V = 0.01$ m$^3$

$\beta = 0.75 \times 10^{-9}$ m$^2$/N

$dP = 2\times10^7$ N/m$^2$

$K = \frac{1}{\beta} = \frac{1}{0.75\times10^{-9}} = \frac{4}{3}\times10^9$

$K = \frac{-dP}{dV/V}$

$dV = \frac{-2\times10^7\times10^{-2}\times3}{4\times10^9} = -1.5\times10^{-4}$

**10. Ans: 320 Pa**

**Sol:** $\Delta P = \frac{8\sigma}{D} = \frac{8\times0.04}{1\times10^{-3}}$

$= \frac{32\times10^{-2}}{10^{-3}}$

$\Delta P = 320$ N/m$^2$

## 02. Pressure Measurement & Fluid Statics

**01. Ans: (a)**

**Sol:** 1 millibar = $10^{-3}\times10^{5}$ = 100 N/m$^2$

One mm of Hg = $13.6\times10^{3}\times9.81\times1\times10^{-3}$

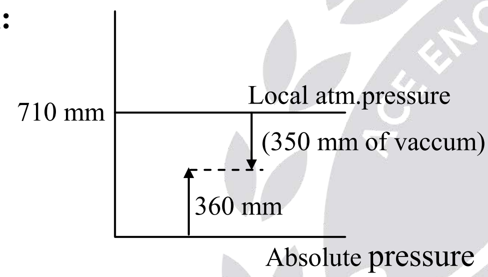
= 133.416 N/m$^2$

1 N/mm$^2$ = $1\times10^{6}$ N/m$^2$

1 kgf/cm$^2$ = $9.81\times10^{4}$ N/m$^2$

**02. Ans: (b)**

**Sol:**




**03. Ans: (c)**

**Sol:** Pressure does not depend upon the volume of liquid in the tank. Since both tanks have the same height, the pressure $P_A$ and $P_B$ are same.

**04. Ans: (b)**

**Sol:**

- The manometer shown in Fig.1 is an open ended manometer for negative pressure measurement.
- The manometer shown in Fig. 2 is for measuring pressure in liquids only.
- The manometer shown in Fig. 3 is for measuring pressure in liquids or gases.
- The manometer shown in Fig. 4 is an open ended manometer for positive pressure measurement.

**05. Ans: 2.2**

**Sol:** $h_p$ in terms of oil

$$s_o\,h_o = s_m h_m$$

$$0.85\times h_0 = 13.6\times0.1$$

$$h_0 = 1.6\text{m}$$

$$h_p = 0.6+1.6$$

$$\Rightarrow\quad h_p = 2.2\text{m of oil}$$

(or) $P_p - \gamma_{oil}\times0.6 - \gamma_{Hg}\times0.1 = P_{atm}$

$$\frac{P_p - P_{atm}}{\gamma_{oil}} = \left(\frac{\gamma_{Hg}}{\gamma_{oil}}\times0.1+0.6\right)$$

$$= \frac{13.6}{0.85}\times0.1+0.6 = 2.2\text{ m of oil}$$

Gauge pressure of P in terms of m of oil

= 2.2 m of oil

**06. Ans: (b)**

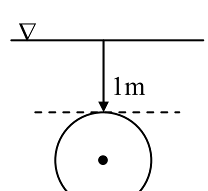
**Sol:** $$h_M - \frac{s_w}{s_0}h_{w_1} = h_N - \frac{s_w h_{w_2}}{s_0} - h_0$$

$$h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3$$

$$h_M - h_N = -13.843\,\text{cm of oil}$$

**07. Ans: 2.125**

**Sol:** $$h_p = \bar{h} + \frac{I}{A\bar{h}}$$



$$= 2 + \frac{\pi D^4 \times 4}{64 \times D^2 \times 2 \times \pi}$$

$$= 2 + \frac{2^2 \times 4}{64 \times 2} = 2.125\text{m}$$

**08. Ans: 10**

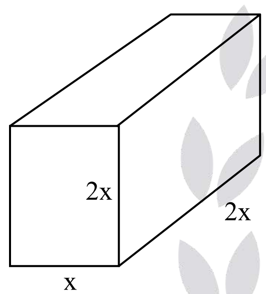
**Sol:** $F = \rho g \bar{h} A$

$$= 9810 \times 1.625 \times \frac{\pi}{4}\left(1.2^2 - 0.8^2\right)$$

$F = 10$ kN

**09. Ans: 1**

**Sol:**



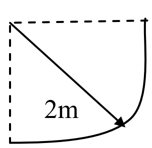

$F_{bottom} = \rho g \times 2x \times 2x \times x$

$F_V = \rho g x \times 2x \times 2x$

$$\frac{F_B}{F_V} = 1$$

**10. Ans: 10**

**Sol:**




$F_V = x \times \pi$

$F_V = \rho G v$

$$= 1000 \times 10 \times \frac{\pi \times 2^2}{4}$$

$F_V = 10\pi$ kN

$\therefore\ x = 10$

**11. Ans: (d)**

**Sol:** $F_{net} = F_{H1} - F_{H2}$

$$F_{H1} = \gamma \times \frac{D}{2} \times D \times 1 = \frac{\gamma D^2}{2}$$

$$F_{H2} = \gamma \times \frac{D}{4} \times \frac{D}{2} \times 1 = \frac{\gamma D^2}{8}$$

$$= \gamma D^2\left(\frac{1}{2} - \frac{1}{8}\right) = \frac{3\gamma D^2}{8}$$

**12. Ans: 2**

**Sol:** Let P be the absolute pressure of fluid f3 at mid-height level of the tank. Starting from the open limb of the manometer (where pressure = $P_{atm}$) we write:

$$P_{atm} + \gamma \times 1.2 - 2\,\gamma \times 0.2 - 0.5\,\gamma \times \left(0.6 + \frac{h}{2}\right) = P$$

or $P - P_{atm} = P_{gauge}$

$$= \gamma(1.2 - 2 \times 0.2 - 0.5 \times 0.6 - 0.5 \times \frac{h}{2})$$

For $P_{gauge}$ to be zero, we have,

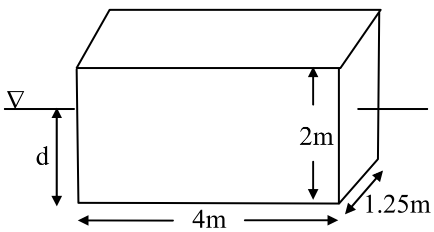
$$\gamma(1.2 - 0.4 - 0.3 - 0.25\,h) = 0$$

or $$h = \frac{0.5}{0.25} = 2$$

## 03. Buoyancy and Metacentric Height

**01. Ans: (d)**

**Sol:**




$$F_B = \text{weight of body}$$

$$\rho_b g V_b = \rho_f g V_f d$$

$$640\times4\times2\times1.25 = 1025\times(4\times1.25\times d)$$

$$d = 1.248\text{m}$$

$$V_{fd} = 1.248\times4\times1.25$$

$$V_{fd} = 6.24\text{m}^3$$

**02. Ans: (c)**

**Sol:** Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

$$4\pi r^2 = 6a^2$$

$$\frac{2\pi}{3} = \left(\frac{a}{r}\right)^2$$

$$F_{b,s} \propto V_s$$

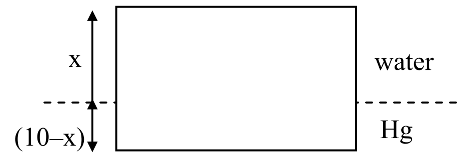
$$= \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\frac{\pi r^3}{\left(r\sqrt{\frac{2\pi}{3}}\right)^3}$$

$$= \frac{4}{3}\frac{\pi r^3}{\left(\sqrt{\frac{2\pi}{3}}\times\sqrt{\frac{2\pi}{3}}r^3\right)} = \sqrt{\frac{6}{\pi}}$$

**03. Ans: 4.76**

**Sol:** $F_B = F_{B,Hg} + F_{B,W}$

$W_B = F_B$




$$\rho_b g\forall_b = \rho_{Hg} g\forall_{Hg} + \rho_w g\forall_w$$

$$\rho_b\forall_b = \rho_{Hg}\forall_{Hg} + \rho_w\forall_w$$

$$S\times\forall_b = S_{Hg}\forall_{Hg} + S_w\forall_w$$

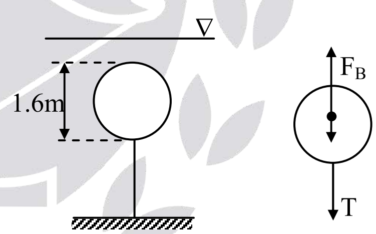
$$7.6\times10^3 = 13.6\times10^2(10-x) + 10^2\times x$$

$$-6000 = -1260x$$

$$x = 4.76\text{ cm}$$

**04. Ans: 11**

**Sol:**




$$F_B = W + T$$

$$W = F_B - T$$

$$= \rho_f g V_{fd} - T$$

$$= 10^3\times9.81\times\frac{4}{3}\pi(0.8)^3 - (10\times10^3)$$

$$= 21 - 10$$

$$W = 11\text{ kN}$$

**05. Ans: 1.375**

**Sol:** $W_{water} = 5N$

$W_{oil} = 7N$

$S = 0.85$

W – Weight in air

$F_{B1} = W - 5$

$F_{B2} = W - 7$

$W - 5 = \rho_1 g V_{fd}$…..(1)

$W - 7 = \rho_2 g V_{fd}$…..(2)

$V_{fd} = V_b$

$$\begin{aligned} W - 5 &= \rho_1 g V_b \\ W - 7 &= \rho_2 g V_b \\ \hline 2 &= (\rho_1 - \rho_2) g V_b \end{aligned}$$

$$V_b = \frac{2}{(1000-850)9.81}$$

$V_b = 1.3591\times10^{-3} m^3$

$W = 5+(9810\times1.3591\times10^{-3})$

$W = 18.33N$

$W = \rho_b\, g\, V_b$

$$\frac{18.33}{9.81\times1.3591\times10^{-3}} = \rho_b$$

$\rho_b = 1375.05\ kg/m^3$

$S_b = 1.375$

**06. Ans: (d)**

**Sol:** For a floating body to be stable, metacentre should be above its center of gravity. Mathematically GM > 0.

**07. Ans: (b)**

**Sol:** $W = F_B$

$\rho_b g V_b = \rho_f g V_{fd}$

$\rho_b V_b = \rho_f V_{fd}$

$$0.6\times\frac{\pi}{4}d^2\times2d = 1\times\frac{\pi}{4}d^2\times x$$

$x = 1.2d$

GM = BM – BG

$$BM = \frac{I}{V} = \frac{\pi d^4}{64\times\frac{\pi}{4}d^2\times1.2d} = \frac{d}{19.2} = 0.052d$$

BG = d – 0.6d = 0.4d

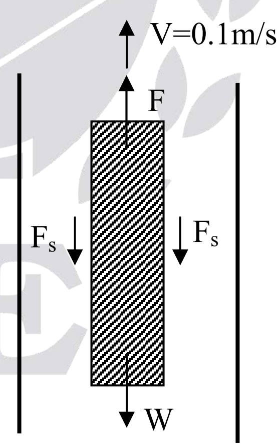
Thus, GM = 0.052d – 0.4d = –0.348 d

GM < 0

$\Rightarrow$ Hence, the cylinder is in unstable condition.

**08. Ans: 122.475**

**Sol:**




The thickness of the oil layer is same on either side of plate

y = thickness of oil layer

$$= \frac{23.5-1.5}{2} = 11mm$$

Shear stress on one side of the plate

$$\tau = \frac{\mu dU}{dy}$$

$F_s$ = total shear force (considering both sides of the plate)

$$= 2A \times \tau = \frac{2A\mu V}{y}$$

$$= \frac{2\times1.5\times1.5\times2.5\times0.1}{11\times10^{-3}}$$

= 102.2727 N

Weight of plate, W = 50 N

Upward force on submerged plate,

$F_v = \rho gV = 900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3}$

= 29.7978 N

Total force required to lift the plate

$= F_s + W - F_v$

= 102.2727 + 50 – 29.7978

= 122.4749 N

## 04. Fluid Kinematics

**01. Ans: (b)**

**Sol:**

- Constant flow rate signifies that the flow is steady.
- For conically tapered pipe, the fluid velocity at different sections will be different. This corresponds to non-uniform flow.

***Common Data for Questions 02 & 03***

**02. Ans: 0.94**

**Sol:** $a_{Local} = \frac{\partial V}{\partial t}$

$$= \frac{\partial}{\partial t}\left(2t\left(1-\frac{x}{2L}\right)^2\right)$$

$$= \left(1-\frac{x}{2L}\right)^2 \times 2$$

$$(a_{Local})_{at\ x = 0.5,\ L = 0.8} = 2\left(1-\frac{0.5}{2\times0.8}\right)^2$$

$= 2(1 - 0.3125)^2$

$= 0.945\ m/sec^2$

**03. Ans: –13.68**

**Sol:** $a_{convective} = v.\frac{\partial v}{\partial x} = \left[2t\left[1-\frac{x}{2L}\right]^2\right]\frac{\partial}{\partial x}\left[2t\left(1-\frac{x}{2L}\right)^2\right]$

$$= \left[2t\left[1-\frac{x}{2L}\right]^2\right]2t\left[2\left(1-\frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$$

At t = 3 sec; x = 0.5 m; L = 0.8 m

$$a_{convective} = 2\times3\left[1-\frac{0.5}{2\times0.8}\right]^2 \times2\times3\left[2\left(1-\frac{0.5}{2\times0.8}\right)\right]\left[\frac{-1}{2\times0.8}\right]$$

$a_{convective} = -14.62\ m/sec^2$

$a_{total} = a_{local} + a_{convective} = 0.94 - 14.62$

$= -13.68\ m/sec^2$

**04. Ans: (d)**

**Sol:** $u = 6xy - 2x^2$

Continuity equation for 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 6y - 4x$$

$$(6y - 4x) + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = (4x - 6y) = 0$$

$$\partial v = (4x-6y)\, dy$$

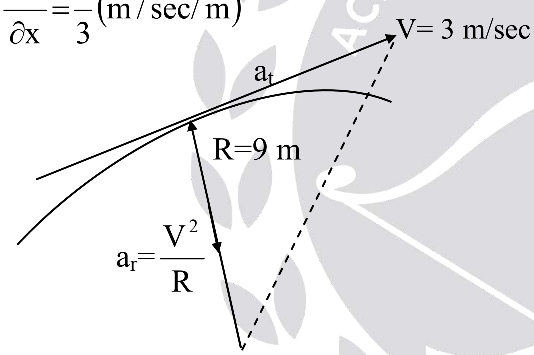
$$v = \int 4x dy - \int 6y dy$$

$$= 4xy - 3y^2 + c$$

$$= 4xy - 3y^2 + f(x)$$

**05. Ans: $\sqrt{2}$ = 1.414**

**Sol:** $\frac{\partial V}{\partial x} = \frac{1}{3}(m/sec/m)$




$$a_r = \frac{V^2}{R} = \frac{(3)^2}{9} = \frac{9}{9} = 1 \text{ m/s}^2$$

$$a_t = V\frac{\partial V}{\partial x} = 3 \times \frac{1}{3} = 1 \text{ m/s}^2$$

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m/sec}^2$$

**06. Ans: 13.75**

**Sol:** $a_{t\,(conv)} = V_{avg} \times \frac{dV}{dx}$

$$a_{t\,(conv)} = \left(\frac{2.5+3}{2}\right)\left(\frac{3-2.5}{0.1}\right) = 2.75 \times 5$$

$$a_{t\,(conv)} = 13.75 \text{ m/s}^2$$

**07. Ans: 0.3**

**Sol:** $Q = Au$

$$a_{Local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

$$a_{local} = \frac{1}{A}\frac{\partial Q}{\partial t}$$

$$a_{Local} = \left(\frac{1}{0.4 - 0.1x}\right)\frac{\partial Q}{\partial t}$$

$$(a_{Local})_{at\ x=0} = \frac{1}{0.4} \times 0.12 \quad \left(\because \frac{\partial Q}{\partial t} = 0.12\right)$$

$$= 0.3 \text{ m/sec}^2$$

**08. Ans: (b)**

**Sol:** $\psi = x^2 - y^2$

$$a_{Total} = (a_x)\,\hat{i} + (a_y)\,\hat{j}$$

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 - y^2) = 2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= (2y)(0) + (2x)(2)$$

$$\therefore a_x = 4x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= (2y)\times(2) + (2x)\times(0)$$

$$a_y = 4y$$

$$\therefore a = (4x)\,\hat{i} + (4y)\,\hat{j}$$

**09. Ans: (b)**

**Sol:** Given, The stream function for a potential flow field is $\psi = x^2 - y^2$

$\phi = ?$

$$u = \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial\left(x^2 - y^2\right)}{\partial y}$$

$u = 2y$

$$u = -\frac{\partial\phi}{\partial x} = 2y$$

$$\int \partial\phi = -\int 2y\partial x$$

$\phi = -2\,xy + c_1$

Given, $\phi$ is zero at (0,0)

$\therefore\ c_1 = 0$

$\therefore\ \phi = -2xy$

**10. Ans: 4**

**Sol:** Given, 2D – flow field

Velocity, V = 3xi + 4xyj

$u = 3x,\ v = 4xy$

$$\omega_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

$$\omega_z = \frac{1}{2}\left(4y - 0\right)$$

$$\left(\omega_Z\right)_{at(2,2)} = \frac{1}{2}\times 4(2) = 4 \text{ rad/sec}$$

**11. Ans: (b)**

**Sol:** Given, $u = 3x$,

$v = Cy$,

$w = 2$

The shear stress, $\tau_{xy}$ is given by

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left[\frac{\partial}{\partial y}(3x) + \frac{\partial}{\partial x}(Cy)\right]$$

$= \mu\,(0 + 0) = 0$

## 05. Energy Equation and its Applications

**01. Ans: (c)**

**Sol:** Applying Bernoulli's equation for ideal fluid

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

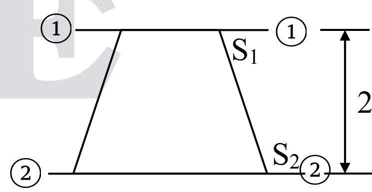
$$\frac{P_1}{\rho g} + \frac{(2)^2}{2g} = \frac{P_2}{\rho g} + \frac{(1)^2}{2g}$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{4}{2g} - \frac{1}{2g}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{3}{2g} = \frac{1.5}{g}$$

**02. Ans: (c)**

**Sol:**




$$\frac{V_1^2}{2g} = 1.27\text{m}\ , \qquad \frac{P_1}{\rho g} = 2.5\text{m}$$

$$\frac{V_2^2}{2g} = 0.203\text{m}\ , \qquad \frac{P_2}{\rho g} = 5.407\text{m}$$

$Z_1 = 2$ m , $Z_2 = 0$ m

Total head at (1) – (1)

$$= \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + Z_1$$

$= 1.27 + 2.5 + 2 = 5.77$ m

Total head at (2) – (2)

$$= \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + Z_2$$

$= 0.203 + 5.407 + 0 = 5.61$ m

Loss of head = 5.77 – 5.61 = 0.16 m

∴ Energy at (1) – (1) > Energy at (2) – (2)

∴ Flow takes from higher energy to lower energy

i.e. from ($S_1$) to ($S_2$)

Flow takes place from top to bottom.

**03. Ans: 1.5**

**Sol:** $A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\ \text{mm}^2$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\ \text{mm}^2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$Z_1 = Z_2$, it is in horizontal position

Since, at outlet, pressure is atmospheric

$P_2 = 0$

Q = 100 lit/sec = 0.1 $m^3$/sec

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{7.85\times10^{-3}} = 12.73\ \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{1.96\times10^{-3}} = 51.02\ \text{m/sec}$$

$$\frac{P_{1\text{gauge}}}{\rho_{air}\times g} + \frac{(12.73)^2}{2\times10} = 0 + \frac{(51.02)^2}{2\times10}$$

$$\frac{P_1}{\rho_{air}.g} = 121.53$$

$$P_1 = 121.53\times\rho_{air}\times g$$

= 1.51 kPa

**04. Ans: 395**

**Sol:** Q = 100 litre/sec = 0.1 $m^3$/sec

$V_1 = 100$ m/sec; $P_1 = 3 \times 10^5$ N/$m^2$

$V_2 = 50$ m/sec; $P_2 = 1 \times 10^5$ N/$m^2$

Power (P) = ?

Energy equation:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{3\times10^5}{1000\times10} + \frac{100^2}{2\times10} + 0 = \frac{1\times10^5}{1000\times10} + \frac{50^2}{2\times10} + 0 + h_L$$

⇒ $h_L = 395$ m

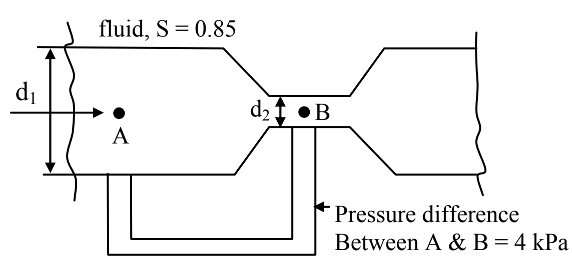
$P = \rho g Q.h_L$

$P = 1000 \times 10 \times 0.10 \times 395$

P = 395 kW

**05. Ans: 35**

**Sol:**




$d_1 = 300$ mm, $d_2 = 120$ mm

$$Q_{Th} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\Delta P}{w}\right)}$$

$$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.30)^2 = 0.07\,m^2$$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.12)^2 = 0.011\,m^2$$

$\Delta P = 4$ kPa,

$$h = \frac{\Delta P}{w} = \frac{\Delta P}{\rho_f . g}$$

$$= \frac{\Delta P}{s_f\rho_w g} = \frac{4\times10^3}{0.85\times1000\times9.81}$$

$$Q_{Th} = \frac{0.07\times0.011}{\sqrt{(0.07)^2 - (0.011)^2}}\sqrt{\frac{2\times9.81\times4\times10^3}{0.85\times1000\times9.81}}$$

$= 0.035\ m^3/sec = 35.15$ ltr/sec

**06. Ans: 65**

**Sol:** $h_{stag} = 0.30$ m

$h_{stat} = 0.24$ m

$$V = c\sqrt{2gh_{dyna}}$$

$$V = 1\sqrt{2g(h_{stag} - h_{stat})}$$

$$= \sqrt{2(9.81)(0.30 - 0.24)} = 1.085\ m/s$$

$= 1.085 \times 60 = 65.1$ m/min

**07. Ans: 81.5**

**Sol:** $x = 30$ mm, $g = 10\ m/s^2$

$\rho_{air} = 1.23\ kg/m^3$; $\rho_{Hg} = 13600\ kg/m^3$

$C = 1$

$$V = \sqrt{2gh_D}$$

$$h_D = x\left(\frac{S_m}{S} - 1\right)$$

$$h_D = 30\times10^{-3}\left(\frac{13600}{1.23} - 1\right)$$

$h_D = 331.67$ m

$$V = 1\times\sqrt{2\times10\times331.67} = 81.5\ m/sec$$

**08. Ans: 140**

**Sol:** $$Q_a = C_d\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

$$C_d \propto \frac{1}{\sqrt{h}}$$

$$\frac{C_{d_{venturi}}}{C_{d_{orifice}}} = \frac{0.95}{0.65} = \sqrt{\frac{h_{orifice}}{h_{venturi}}}$$

$h_{venturi} = 140$ mm

## 06. Momentum Equation and its Applications

**01. Ans: 1600**

**Sol:** S = 0.80

$A = 0.02\ m^2$

V = 10 m/sec

$F = \rho.A.V^2$

$F = 0.80 \times 1000 \times 0.02 \times 10^2$

F = 1600 N

**02. Ans: 6000**

**Sol:** $A = 0.015\ m^2$

V = 15 m/sec (Jet velocity)

U = 5 m/sec (Plate velocity)

$F = \rho A\ (V + U)^2$

$F = 1000 \times 0.015\ (15 + 5)^2$

F = 6000 N

**03. Ans: 19.6**

**Sol:** V = 100 m/sec (Jet velocity)

U = 50 m/sec (Plate velocity)

d = 0.1 m

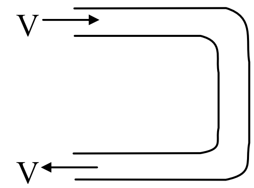
$F = \rho A\ (V - U)^2$

$$F = 1000 \times \frac{\pi}{4} \times 0.1^2 \times (100 - 50)^2$$

F = 19.6 kN

**04. Ans: (a)**

**Sol:**




$F_x = \rho aV(\ V_{1x} - V_{2x})$

$= \rho aV(\ V - (-V))$

$= 2\ \rho a\ V^2$

$= 2\times 1000 \times 10^{-4} \times 5^2 = 5\ N$

**05. Ans: (d)**

**Sol:** Given, V = 20 m/s,

u = 5 m/s

$F_1 = \rho A(V - u)^2$

Power $(P_1) = F_1 \times u = \rho A(V - u)^2 \times u$

$F_2 = \rho.A.V \times V_r$

$= \rho.A(V).(V-u)$

Power $(P_2) = F_2 \times u = \rho AV(V-u)u$

$$\frac{P_1}{P_2} = \frac{\rho A(V-u)^2 \times u}{\rho AV(V-u) \times u}$$

$$= \frac{V-u}{V} = 1 - \frac{u}{V}$$

$$= 1 - \frac{5}{20} = 0.75$$

**06. Ans: 2035**

**Sol:** Given, $\theta = 30°$, $\dot{m} = 14$ kg/s

$(P_i)_g = 200$ kPa,

$(P_e)_g = 0$

$A_i = 113\times10^{-4}\ m^2$ ,

$A_e = 7\times10^{-4}\ m^2$

$\rho = 10^3\ kg/m^3$,

$g = 10\ m/s^2$

From the continuity equation :

$\rho A_i V_i = 14$

or $V_i = \dfrac{14}{10^3\times113\times10^{-4}} = 1.24\,m/s$

Similarly, $V_e = \dfrac{14}{10^3\times7\times10^{-4}} = 20\,m/s$

Let $F_x$ be the force exerted by elbow on water in the +ve x-direction. Applying the linear momentum equation to the C.V. enclosing the elbow, we write :

$(P_i)_g A_i + F_x = \dot{m}(V_e\cos30^o - V_i)$

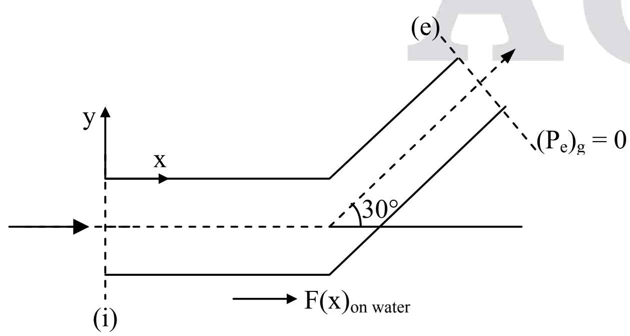
$F_x = \dot{m}(V_e\cos30^o - V_i) - (P_i)_g A_i$

$= 14\,(20\times\cos30° - 1.24) - 200\times10^3\times113\times10^{-4}$

$= 225.13 - 2260$

$= -2034.87\ N \approx -2035\ N$

The x-component of water force on elbow is $-F_x$ (as per Newton's third law), i.e., ≅ 2035 N




## 07. Laminar Flow

**01. Ans: (d)**

**Sol:** In a pipe, the flow changes from laminar flow to transition flow at Re = 2000. Let V be the average velocity of flow. Then

$$2000 = \frac{V\times8\times10^{-2}}{0.4\times10^{-4}} \Rightarrow V = 1\,m/s$$

In laminar flow through a pipe,

$V_{max} = 2\times V = 2\ m/s$

**02. Ans: (d)**

**Sol:** The equation $\tau = \left(-\dfrac{\partial P}{\partial x}\right)\left(\dfrac{r}{2}\right)$ is valid for laminar as well as turbulent flow through a circular tube.

**03. Ans: (d)**

**Sol:** $Q = A.V_{avg}$

$Q = A.\dfrac{V_{max}}{2}$ $(\because V_{max} = 2\,V_{avg})$

$$Q = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2\times\frac{1.5}{2} = \frac{\pi}{4}\times(0.04)^2\times0.75$$

$$= \frac{\pi}{4}\times\frac{4}{100}\times\frac{4}{100}\times\frac{3}{4} = \frac{3\pi}{10000}\ m^3/sec$$

**04. Ans: 1.92**

**Sol:** $\rho = 1000\ kg/m^3$

$Q = 800\ mm^3/sec = 800\times(10^{-3})^3\ m^3/sec$

$L = 2\ m$

$D = 0.5\ mm$

$\Delta P = 2$ MPa $= 2 \times 10^6$ Pa

$\mu = ?$

$$\Delta P = \frac{128.\mu QL}{\pi D^4}$$

$$2\times10^6 = \frac{128\times\mu\times800\times(10^{-3})^3\times2}{\pi(0.5\times10^{-3})^4}$$

$\mu = 1.917$ milli Pa – sec

**05. Ans: 0.75**

**Sol:** $U_r = U_{max}\left(1-\left(\frac{r}{R}\right)^2\right)$

$$\left[\because \frac{U}{U_{max}} = 1-\left(\frac{r}{R}\right)^2\right]$$

$$= 1\left(1-\left(\frac{50}{200}\right)^2\right)$$

$$= 1\left(1-\frac{1}{4}\right) = \frac{3}{4} = 0.75 \text{ m/s}$$

**06. Ans: 0.08**

**Sol:** Given,

$\rho = 0.8 \times 1000 = 800$ kg/m$^3$

$\mu = 1$ Poise $= 10^{-1}$ N-s/m$^2$

d = 50 mm = 0.05 m

Velocity = 2 m/s

Reynolds Number, $Re = \frac{\rho VD}{\mu}$

$$= \frac{800\times2\times0.05}{10^{-1}} = 800$$

$(\because Re < 2000)$

$\therefore$ Flow is laminar,

For laminar, Darcy friction factor

$$f = \frac{64}{Re} = \frac{64}{800} = 0.08$$

**07. Ans: 16**

**Sol:** For fully developed laminar flow,

$$h_f = \frac{32\mu VL}{\rho g D^2} \quad (\therefore Q = AV)$$

$$h_f = \frac{32\mu\left(\frac{Q}{A}\right)L}{\rho g D^2} = \frac{32\mu QL}{AD^2\times\rho g}$$

$$h_f = \frac{32\mu QL}{\frac{\pi}{4}D^2\times D^2\times\rho g}$$

$$h_f \propto \frac{1}{D^4}$$

$$h_{f1} D_1^4 = h_{f_2} D_2^4$$

**Given,** $D_2 = \frac{D_1}{2}$

$$h_{f1}\times D_1^4 = h_{f2}\times\left(\frac{D_1}{2}\right)^4$$

$$h_{f_2} = 16 h_{f_1}$$

$\therefore$ Head loss, increases by 16 times if diameter is halved.

**08. Ans: 5.2**

**Sol:** Oil viscosity, $\mu = 10$ poise $= 10\times0.1$ $= 1$ N-s/m$^2$

$y = 50 \times 10^{-3}$m

L = 120 cm = 1.20 m

$\Delta P = 3 \times 10^3$Pa

Width of plate = 0.2 m

Q = ?

Q = A.$V_{avg}$ = (width of plate×y)V

$$\Delta P = \frac{12\mu VL}{B^2}$$

$$3\times10^3 = \frac{12\times1\times V\times1.20}{\left(50\times10^{-3}\right)^2}$$

V = 0.52 m/sec

Q = A$V_{avg}$ = $(0.2\times50\times10^{-3})$ (0.52)

= 5.2 lit/sec

**09. Ans: (a)**

**Sol:** Wall shear stress for flow in a pipe is given by,

$$\tau_o = -\frac{\partial P}{\partial x}\times\frac{R}{2} = \frac{\Delta P}{L}\times\frac{D}{4}$$

$$= \frac{\Delta P\, D}{4L}$$

**10. Ans: 72**

**Sol:** Given, $\rho = 800$ kg/m$^3$,

$\mu = 0.1$ Pa.s

Flow is through an inclined pipe.

$d = 1 \times 10^{-2}$ m,

$V_{av} = 0.1$ m/s,

$\theta = 30°$

$$Re = \frac{\rho V_{av} d}{\mu} = \frac{800\times0.1\times1\times10^{-2}}{0.1} = 8$$

⇒ flow is laminar.

Applying energy equation for the two sections of the inclined pipe separated by 10 m along the pipe,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_f$$

But $V_1 = V_2$,

$(Z_2 - Z_1) = 10 \sin30° = 5$ m

and $$h_f = \frac{32\mu V_{av} L}{\rho g d^2}$$

$$\frac{(P_1 - P_2)}{\gamma} = (Z_2 - Z_1) + \frac{32\mu V_{av} L}{\rho g d^2}$$

$$(P_1 - P_2) = \rho g(Z_2 - Z_1) + \frac{32\mu V_{av} L}{d^2}$$

$$= 800\times10\times5 + \frac{32\times0.1\times0.1\times10}{\left(1\times10^{-2}\right)^2}$$

$= 40 \times 10^3 + 32\times10^3 = 72$ kPa

## 08. Flow Through Pipes

**01. Ans: (d)**

**Sol:**

- The Darcy-Weisbash equation for head loss is written as:

$$h_f = \frac{f\,L\,V^2}{2g\,d}$$

  where V is the average velocity, f is friction factor, L is the length of pipe and d is the diameter of the pipe.
- This equation is used for laminar as well as turbulent flow through the pipe.
- The friction factor depends on the type of flow (laminar or turbulent) as well as the nature of pipe surface (smooth or rough)
- For laminar flow, friction factor is a function of Reynolds number.

**02. Ans: 481**

**Sol:** Given data,

$\dot{m} = \pi$ kg/s, $\quad d = 5\times10^{-2}$ m,

$\mu = 0.001$ Pa.s , $\quad \rho = 1000$ kg/m$^3$

$$V_{av} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho\pi d^2} = \frac{4\times\pi}{\rho\pi d^2} = \frac{4}{\rho d^2}$$

$$Re = \frac{\rho V_{av} d}{\mu} = \rho\times\frac{4}{\rho d^2}\times\frac{d}{\mu} = \frac{4}{\mu d}$$

$$= \frac{4}{0.001\times5\times10^{-2}} = 8\times10^4$$

⇒ Flow is turbulent

$$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{\left(8\times10^4\right)^{0.25}} = 0.0188$$

$$\Delta P = \rho g\frac{f\,L\,V_{av}^2}{2gd} = f\,\rho\,L\times\left(\frac{4}{\rho d^2}\right)^2\times\frac{1}{2d}$$

$$\frac{\Delta P}{L} = f\times\frac{16}{\rho d^5}\times\frac{1}{2} = \frac{8f}{\rho d^5} = \frac{8\times0.0188}{10^3\times\left(5\times10^{-2}\right)^5}$$

$$= 481.28 \text{ Pa/m}$$

**03. Ans: (a)**

**Sol:** In pipes Net work, series arrangement

$$\therefore\ h_f = \frac{f.lV^2}{2gd} = \frac{f.lQ^2}{12.1\times d^5}$$

$$\frac{h_{f_A}}{h_{f_B}} = \frac{f_A.\ell_A.Q_a^2}{12.1\times d_A^5}\times\frac{12.1\times d_B^5}{f_B.l_B.Q_B^2}$$

Given $l_A = l_B$, $f_A = f_B$, $Q_A = Q_B$

$$\frac{h_{f_A}}{h_{f_B}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2d_B}\right)^5$$

$$= \left(\frac{1}{1.2}\right)^5 = 0.4018 \approx 0.402$$

**04. Ans: (a)**

**Sol:** Given, $d_1 = 10$ cm; $d_2 = 20$ cm

$f_1 = f_2$ ;

$l_1 = l_2 = l$

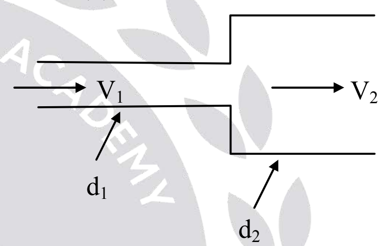
$l_e = l_1 + l_2 = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \Rightarrow \frac{2l}{d_e^5} = \frac{l}{10^5} + \frac{l}{20^5}$$

$\therefore\ d_e = 11.4$ cm

**05. Ans: (c)**

**Sol:**




Given $d_2 = 2d_1$

Losses due to sudden expansion,

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1-\frac{V_2}{V_1}\right)^2$$

By continuity equation,

$$Q = A_1V_1 = A_2V_2$$

$$\therefore\ \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1-\frac{1}{4}\right)^2$$

$$h_L = \frac{9}{16}\times\frac{V_1^2}{2g}$$

$$\frac{h_L}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

**06. Ans: (b)**

**Sol:** Pipes are in parallel

$$Q_e = Q_A + Q_B \quad \text{------- (i)}$$

$$h_{Le} = h_{L_A} = h_{L_B}$$

$$L_e = 175 \text{ m}$$

$$f_e = 0.015$$

$$\frac{f_e L_e Q_e^2}{12.1 D_e^5} = \frac{f_A . L_A Q_A^2}{12.1 D_A^5} = \frac{f_B L_B Q_B^2}{12.1 D_B^5}$$

$$\frac{0.020 \times 150 \times Q_A^2}{12.1 \times (0.1)^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$$Q_A = 1.747\, Q_B \quad \text{-------(ii)}$$

From (i) $Q_e = 1.747\, Q_B + Q_B$

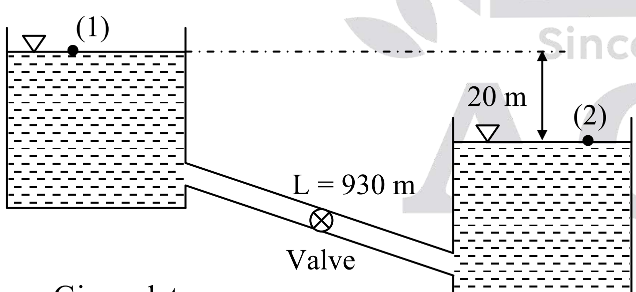
$$Q_e = 2.747\, Q_B \quad \text{-------(iii)}$$

$$\frac{0.015 \times 175 (2.747 Q_B)^2}{12.1 \times D_e^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$D_e = 116.6 \text{ mm} \simeq 117 \text{ mm}$

**07. Ans: 0.141**

**Sol:**




Given data,

$L = 930$ m, $k_{valve} = 5.5$

$k_{entry} = 0.5$, $d = 0.3$ m

$f = 0.03$, $g = 10 \text{ m/s}^2$

Applying energy equation for points (1) and (2), we write:

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2 + h_{L,entry} + h_{L,valve} + h_{L,exit} + h_{f,pipe}$$

But $P_1 = P_2 = P_{atm} = 0$

$V_1 = 0 = V_2$

$Z_1 - Z_2 = 20$ m, $k_{exit} = 1$

$$Z_1 - Z_2 = 0.5\frac{V^2}{2g} + 5.5\frac{V^2}{2g} + 1 \times \frac{V^2}{2g} + \frac{f L V^2}{2gd}$$

$$= 7\frac{V^2}{2g} + \frac{f L V^2}{2gd} = \frac{V^2}{2g}\left(7 + \frac{f L}{d}\right)$$

or $$20 = \frac{V^2}{2g}\left[7 + \frac{0.03 \times 930}{0.3}\right] = 100\frac{V^2}{2g}$$

or $$V^2 = \frac{20 \times 2g}{100} = \frac{20 \times 2 \times 10}{100}$$

$\Rightarrow \quad V = 2$ m/s

Thus, discharge, $Q = \frac{\pi}{4} \times 0.3^2 \times 2 = 0.1414 \text{ m}^3/\text{s}$

**08. Ans: (c)**

**Sol:** Given data :

Fanning friction factor, $f = m\, Re^{-0.2}$

For turbulent flow through a smooth pipe.

$$\Delta P = \frac{\rho f_{Darcy} L V^2}{2d} = \frac{\rho (4f) L V^2}{2d}$$

$$= \frac{2\rho m Re^{-0.2} L V^2}{d}$$

or $\Delta P \propto V^{-0.2} V^2 \propto V^{1.8}$ (as all other parameters remain constant)

We may thus write :

$$\frac{\Delta P_2}{\Delta P_1} = \left(\frac{V_2}{V_1}\right)^{1.8} = \left(\frac{2}{1}\right)^{1.8} = 3.4822$$

or $\Delta P_2 = 3.4822 \times 10 = 34.82$ kPa

**09. Ans: (b)**

**Sol:** Given data :

Rectangular duct, L = 10 m,

X-section of duct = 15 cm × 20 cm

Material of duct - Commercial steel,

$\varepsilon = 0.045$ mm

Fluid is air ($\rho = 1.145$ kg/m$^3$,

$\nu = 1.655\times10^{-5}$ m$^2$/s)

$V_{av} = 7$ m/s

$$Re = \frac{V_{av}\times D_h}{\nu}$$

where, $D_h$ = Hydraulic diameter

$$= \frac{4\times \text{Cross sectional area}}{\text{Perimeter}}$$

$$= \frac{4\times0.15\times0.2}{2(0.15+0.2)} = 0.1714\,\text{m}$$

$$Re = \frac{7\times0.1714}{1.655\times10^{-5}} = 72495.5$$

$\Rightarrow$ Flow is turbulent.

Using Haaland equation to find friction factor,

$$\frac{1}{\sqrt{f}} \simeq -1.8\log\left[\frac{6.9}{Re}+\left(\frac{\varepsilon/D_h}{3.7}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{72495.5}+\left(\frac{0.045\times10^{-3}}{0.1714\times3.7}\right)^{1.11}\right]$$

$= -1.8\log[9.518\times10^{-5} + 2.48\times10^{-5}]$

$= -1.8\log(11.998\times10^{-5})$

$$\frac{1}{\sqrt{f}} = 7.058$$

$f = 0.02$

The pressure drop in the duct is,

$$\Delta P = \frac{\rho f L V^2}{2D_h}$$

$$= \frac{1.145\times0.02\times10\times7^2}{2\times0.1714} = 32.73\text{ Pa}$$

The required pumping power will be

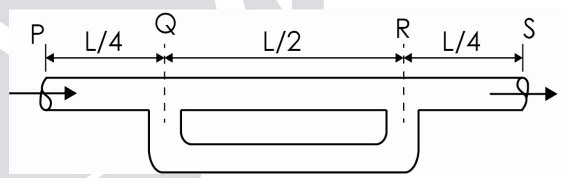
$P_{pumping} = Q\,\Delta P = A\,V_{av}\times\Delta P$

$= (0.15\times0.2)\times7\times(32.73)$

$= 6.87\text{ W} \simeq 7\text{ W}$

**10. Ans: 26.5**

**Sol:**




**Case I:** Without additional pipe,

Let Q be the discharge through the pipe. Then

$$\frac{P_P}{\gamma}+\frac{V_P^2}{2g}+Z_P = \frac{P_S}{\gamma}+\frac{V_S^2}{2g}+Z_S+\frac{fLQ^2}{12.1\,d^5}$$

But $V_P = V_S$ and $Z_P = Z_S$

$P_P$ and $P_S$ are the pressures at sections P and S, respectively.

Thus,

$$\frac{P_P}{\gamma}-\frac{P_S}{\gamma} = \frac{fLQ^2}{12.1d^5} \quad \text{-------(1)}$$

**Case II:** When a pipe (L/2) is connected in parallel.

In this case, let Q′ be the total discharge.

$$Q_{Q-R} = \frac{Q'}{2} \quad \text{and } Q_{R\text{-}S} = Q'$$

Then,

$$\frac{P'_P}{\gamma} + \frac{V'^2_P}{2g} + Z'_P = \frac{P'_S}{\gamma} + \frac{V'^2_S}{2g} + Z'_S + \frac{f(L/4)Q'^2}{12.1\,d^5} + \frac{f(L/2)(Q'/2)^2}{12.1\,d^5} + \frac{f(L/4)Q'^2}{12.1\,d^5}$$

$P_P'$ and $P_S'$ are the pressures at sections P and S in the second case.

But $V_P' = V_S'$ ; $Z_P' = Z_S'$

So, $$\frac{P'_P}{\gamma} - \frac{P'_S}{\gamma} = \frac{f\,LQ'^2}{12.1d^5}\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{4}\right]$$

$$= \frac{5}{8}\times\frac{f\,LQ'^2}{12.1d^5} \quad \text{-------(2)}$$

Given that end conditions remain same.

i.e., $$\frac{P_P}{\gamma} - \frac{P_S}{\gamma} = \frac{P'_P}{\gamma} - \frac{P'_S}{\gamma}$$

Hence, equation (2) becomes,

$$\frac{f\,LQ^2}{12.1d^5} = \frac{5}{8}\frac{f\,LQ'^2}{12.1d^5} \quad \text{from eq.(1)}$$

or $$\left(\frac{Q'}{Q}\right)^2 = \frac{8}{5}$$

or $$\frac{Q'}{Q} = 1.265$$

Hence, percentage increase in discharge is

$$= \frac{Q'-Q}{Q}\times 100 = (1.265 - 1)\times 100$$

$$= 26.5\ \%$$

**11. Ans: 20%**

**Sol:** Since, discharge decrease is associated with increase in friction.

$$\frac{df}{f} = -2\times\frac{dQ}{Q} = 2\left[-\frac{dQ}{Q}\right] = 2\times 10 = 20\%$$

## 09. Elementary Turbulent Flow

**01. Ans: (b)**

**Sol:** The velocity distribution in laminar sublayer of the turbulent boundary layer for flow through a pipe is linear and is given by

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

where V* is the shear velocity.

**02. Ans: (d)**

**Sol:** $$\Delta P = \rho g h_f$$

$$= \frac{\rho f L V^2}{2D} = \frac{\rho g f L Q^2}{12.1D^5}$$

For Q = constant

$$\Delta P \propto \frac{1}{D^5}$$

or $$\frac{\Delta P_2}{\Delta P_1} = \frac{D_1^5}{D_2^5} = \left(\frac{D_1}{2D_1}\right)^5 = \frac{1}{32}$$

**03. Ans: 2.4**

**Sol:** Given: V = 2 m/s

f = 0.02

$V_{max}$ = ?

$V_{max} = V(1 + 1.43\sqrt{f})$

$= 2\left(1+1.43\sqrt{0.02}\right)$

$= 2 \times 1.2 = 2.4$ m/s

**04. Ans: (c)**

**Sol:** Given data:

D = 30 cm = 0.3 m

$Re = 10^6$

f = 0.025

Thickness of laminar sub layer, $\delta' = ?$

$$\delta' = \frac{11.6\nu}{V^*}$$

where $V^*$ = shear velocity = $V\sqrt{\frac{f}{8}}$

$\nu$ = Kinematic viscosity

$$Re = \frac{V.D}{\nu}$$

$$\therefore \nu = \frac{V.D}{Re}$$

$$\delta' = \frac{11.6 \times \frac{VD}{Re}}{V\sqrt{\frac{f}{8}}}$$

$$\delta' = \frac{11.6 \times D}{Re\sqrt{\frac{f}{8}}}$$

$$= \frac{11.6 \times 0.3}{10^6 \times \sqrt{\frac{0.025}{8}}}$$

$= 6.22 \times 10^{-5}$ m = 0.0622 mm

**05. Ans: 25**

**Sol: Given:**

L = 100 m

D = 0.1 m

$h_L$ = 10 m

$\tau = ?$

For any type of flow, the shear stress at wall/surface $\tau = \frac{-dP}{dx} \times \frac{R}{2}$

$$\tau = \frac{\rho g h_L}{L} \times \frac{R}{2}$$

$$\tau = \frac{\rho g h_L}{L} \times \frac{D}{4}$$

$$= \frac{1000 \times 9.81 \times 10}{100} \times \frac{0.1}{4}$$

$= 24.525$ N/m$^2$ = 25 Pa

**06. Ans: 0.905**

**Sol:** k = 0.15 mm

$\tau = 4.9$ N/m$^2$

$\nu$ = 1 centi-stoke

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/sec}$$

$\nu$ = 1 centi-stoke

$$= \frac{1}{100}\text{stoke} = \frac{10^{-4}}{100} = 10^{-6}\,m^2/sec$$

$$\frac{k}{\delta'} = \frac{0.15 \times 10^{-3}}{\left(\frac{11.6 \times \nu}{V^*}\right)}$$

$$= \frac{0.15 \times 10^{-3}}{\frac{11.6 \times 10^{-6}}{0.07}} = 0.905$$

**07. Ans: (a)**

**Sol:** The velocity profile in the laminar sublayer is given as

$$\frac{u}{V^*} = \frac{yV^*}{\nu}$$

or $\nu = \frac{y(V^*)^2}{u}$

where, V* is the shear velocity.

Thus, $\nu = \frac{0.5\times10^{-3}\times(0.05)^2}{1.25}$

$= 1\times10^{-6}\ m^2/s$

$= 1\times10^{-2}\ cm^2/s$

**08. Ans: 47.74 N/m²**

**Sol:** Given data:

$d = 100\ mm = 0.1\ m$

$u_{r=0} = u_{max} = 2\ m/s$

Velocity at r = 30 mm = 1.5 m/s

Flow is turbulent.

The velocity profile in turbulent flow is

$$\frac{u_{max}-u}{V^*} = 5.75\log\left(\frac{R}{y}\right)$$

where u is the velocity at y and V* is the shear velocity.

For pipe, $y = R - r$

$= (50 - 30)\ mm = 20\ mm$

Thus,

$$\frac{2-1.5}{V^*} = 5.75\log\left(\frac{50}{20}\right) = 2.288$$

or $V^* = \frac{0.5}{2.288} = 0.2185\ m/s$

Using the relation,

$$V^* = \sqrt{\frac{\tau_w}{\rho}} \quad \text{or } \tau_w = \rho\,(V^*)^2$$

$$\tau_w = 10^3\times(0.2185)^2 = 47.74\ N/m^2$$

## 10. Boundary Layer Theory

**01. Ans: (c)**

**Sol:** $Re_{Critical} = \frac{U_\infty x_{critical}}{\nu}$

Assume water properties

$$5\times10^5 = \frac{6\times x_{critical}}{1\times10^{-6}}$$

$x_{critical} = 0.08333\ m = 83.33\ mm$

**02. Ans: 1.6**

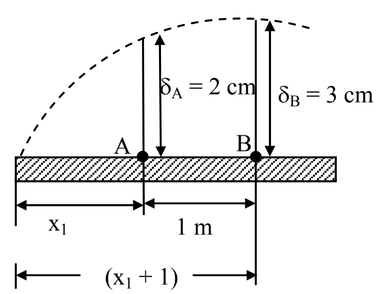
**Sol:** $\delta \propto \frac{1}{\sqrt{Re}}$ (At given distance 'x')

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{Re_2}{Re_1}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6$$

**03. Ans: 80**

**Sol:**



$\delta \propto \sqrt{x}$

$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1+1)}}$

$x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1+1}}$

$\frac{4}{9} = \frac{x_1}{x_1+1}$

$5x_1 = 4 \Rightarrow x_1 = 80$ cm

**04. Ans: 2**

**Sol:** $\tau \propto \frac{1}{\delta}$

$\tau \propto \frac{1}{\sqrt{x}} \quad \because \delta \propto \sqrt{x}$

$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}$

$\frac{\tau_1}{\tau_2} = \sqrt{4} = 2$

**05. Ans: 3**

**Sol:** $\frac{U}{U_\infty} = \frac{y}{\delta}$

$\frac{\delta^*}{\theta} =$ Shape factor = ?

$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

$= \int_0^\delta \left(1 - \frac{y}{8}\right) dy$

$= y - \frac{y^2}{2\delta}\Big|_0^\delta$

$= \delta - \frac{\delta}{2} = \frac{\delta}{2}$

$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$

$= \int_0^\delta \frac{y}{8}\left(1 - \frac{y}{\delta}\right) dy$

$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta}\Big|_0^\delta$

$= \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$

Shape factor $= \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

**06. Ans: 22.6**

**Sol:** Drag force,

$F_D = \frac{1}{2} C_D.\rho.A_{Proj}. U_\infty^2$

B = 1.5 m, $\rho = 1.2$ kg/m$^3$

L = 3.0 m, $\nu = 0.15$ stokes

$U_\infty = 2$ m/sec

$Re = \frac{U_\infty L}{\nu} = \frac{2 \times 3}{0.15 \times 10^{-4}} = 4 \times 10^5$

$C_D = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{4 \times 10^5}} = 2.09 \times 10^{-3}$

Drag force,

$F_D = \frac{1}{2} \times 2.09 \times 10^{-3} \times 1.2 \times (1.5 \times 3) \times 2^2$

= 22.57 milli-Newton

**07. Ans: 1.62**

**Sol:** Given data,

$U_\infty = 30$ m/s,

$\rho = 1.2$ kg/m$^3$

Velocity profile at a distance x from leading edge,

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

$\delta = 1.5$ mm

Mass flow rate of air entering section ab,

$$(\dot{m}_{in})_{ab} = \rho U_\infty(\delta \times 1) = \rho U_\infty \delta \text{ kg/s}$$

Mass flow rate of air leaving section cd,

$$(\dot{m}_{out})_{cd} = \rho\int_0^\delta u(dy\times 1) = \rho\int_0^\delta U_\infty\left(\frac{y}{\delta}\right)dy$$

$$= \frac{\rho U_\infty}{\delta}\left[\frac{y^2}{2}\right]_0^\delta = \frac{\rho U_\infty \delta}{2}$$

From the law of conservation of mass :

$$(\dot{m}_{in})_{ab} = (\dot{m}_{out})_{cd} + (\dot{m}_{out})_{bc}$$

Hence, $(\dot{m}_{out})_{bc} = (\dot{m}_{in})_{ab} - (\dot{m}_{out})_{cd}$

$$= \rho U_\infty \delta - \frac{\rho U_\infty \delta}{2}$$

$$= \frac{\rho U_\infty \delta}{2}$$

$$= \frac{1.2\times 30\times 1.5\times 10^{-3}}{2}$$

$= 27\times10^{-3}$ kg/s

$= 27\times10^{-3}\times60$ kg/min

$= 1.62$ kg/min

**08. Ans: (b)**

**Sol:** For 2-D, steady, fully developed laminar boundary layer over a flat plate, there is velocity gradient in y-direction, $\frac{\partial u}{\partial y}$ only.

The correct option is (b).

**09. Ans: 28.5**

**Sol:** Given data,

Flow is over a flat plate.

$L = 1$ m,

$U_\infty = 6$ m/s

$\nu = 0.15$ stoke $= 0.15\times10^{-4}$ m$^2$/s

$\rho = 1.226$ kg/m$^3$

$$\delta(x) = \frac{3.46x}{\sqrt{Re_x}}$$

Velocity profile is linear.

Using von-Karman momentum integral equation for flat plate.

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2} \text{ --------(1)}$$

we can find out $\tau_w$ .

From linear velocity profile, $\frac{u}{U_\infty} = \frac{y}{\delta}$, we evaluate first θ, momentum thickness as

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy = \int_0^\delta\left(\frac{y}{\delta}-\frac{y^2}{\delta^2}\right)dy$$

$$= \left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right)_0^{\delta} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\Rightarrow \theta = \frac{\delta}{6} = \frac{1}{6} \times \frac{3.46\,x}{\sqrt{Re_x}}$$

$$= \frac{3.46}{6} \frac{x^{1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}}$$

Differentiating θ w.r.t x, we get :

$$\frac{d\theta}{dx} = \frac{3.46}{6 \times 2} \frac{x^{-1/2}}{\left(\frac{U_\infty}{\nu}\right)^{1/2}} = 0.2883 \frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0.5\,m} = 0.2883 \times \frac{1}{\sqrt{\frac{6 \times 0.5}{0.15 \times 10^{-4}}}} = \frac{0.2883}{447.2}$$

-----(2)

From equation (1)

$$\tau_w\Big|_{x=0.5m} = \left.\frac{d\theta}{dx}\right|_{x=0.5\,m} \times \rho U_\infty^2$$

$$= \frac{0.2883}{447.2} \times 1.226 \times 6^2$$

$$= 0.02845 \text{ N/m}^2$$

$$\simeq 28.5 \text{ mN/m}^2$$

## 11. Forces on Submerged Bodies

**01. Ans: 8**

**Sol:** Drag power = Drag Force × Velocity

$$P = F_D \times V$$

$$P = C_D \times \frac{\rho A V^2}{2} \times V$$

$$P \propto V^3$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^3$$

$$\frac{P_1}{P_2} = \left(\frac{V}{2V}\right)^3$$

$$P_2 = 8P_1$$

Comparing the above relation with XP,

We get, X = 8

**02. Ans: 4.56 m**

**Sol:** $F_D = C_D . \frac{\rho A V^2}{2}$

$$W = 0.8 \times 1.2 \times \frac{\frac{\pi}{4}(D)^2 \times V^2}{2}$$

(Note: A = Normal (or)

projected Area = $\frac{\pi}{4} D^2$)

$$784.8 = 0.8 \times 1.2 \times \frac{\pi}{4}(D)^2 \times \frac{10^2}{2}$$

$\therefore$ D = 4.56 m

**03. Ans: 4**

**Sol: Given data:**

$l$ = 0.5 km = 500 m

d = 1.25 cm

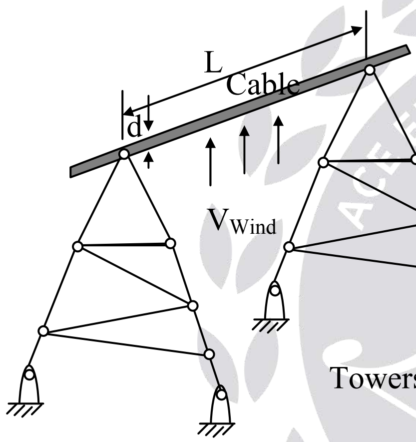
$V_{Wind}$ = 100 km/hr

$\gamma_{Air} = 1.36\times9.81 = 13.4$ N/m$^3$

$\nu = 1.4\times10^{-5}$ m$^2$/s

$C_D$ = 1.2 for Re > 10000

$C_D$ = 1.3 for Re < 10000




$$Re = \frac{V.L}{\nu} = \frac{\left(\frac{100\times5}{18}\right)\times0.0125}{1.4\times10^{-5}} = 24800$$

**Note:** The characteristic dimension for electric power transmission tower wire is "d"

Re = 24800 > 10,000

$\therefore$ $C_D$ = 1.2

$$F_D = C_D\times\frac{\rho AV^2}{2}$$

$$= 1.2\times\frac{\left(\frac{13.4}{9.81}\right)(L\times d)V^2}{2}$$

$$= \frac{1.2\times\left(\frac{13.4}{9.81}\right)(500\times0.0125)\left(100\times\frac{5}{18}\right)^2}{2}$$

= 3952.4 N

= 4 kN

**04. Ans: 0.144 & 0.126**

**Sol: Given data:**

$W_{Kite}$ = 2.5 N

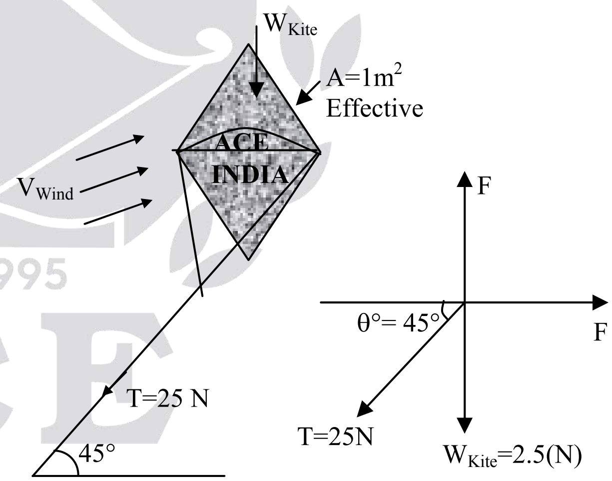
A = 1 m$^2$

$\theta = 45^o$

T = 25 N

$V_{Wind}$ = 54 km/hr

$= 54\times\frac{5}{18} = 15$ m/s




Resolving forces horizontally

$F_D = T\cos45^o$

$$C_D\times\frac{\rho AV^2}{2} = 25\times\cos45^o$$

$$\frac{C_D \times \left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 25 \times \frac{1}{\sqrt{2}}$$

$\therefore \quad C_D = 0.126$

Resolving forces vertically

$F_L = W_{Kite} + T\sin45^o$

$$\frac{C_L \rho A V^2}{2} = 2.5 + 25\sin45^o$$

$$\frac{C_L\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 2.5 + \frac{25}{\sqrt{2}}$$

$\therefore C_L = 0.144$

**05. Ans: (a)**

**Sol: Given data:**

$C_{D_2} = 0.75\, C_{D_1}$ (25% reduced)

Drag power = Drag force × Velocity

$$P = F_D \times V = \frac{C_D \rho A V^2}{2} \times V$$

$$P = C_D \times \frac{\rho A V^3}{2}$$

Keeping ρ, A and power constant

$C_D V^3$ = constant = C

$$\frac{C_{D_1}}{C_{D_2}} = \left(\frac{V_2}{V_1}\right)^3$$

$$\left(\frac{C_{D_1}}{0.75 C_{D_1}}\right)^{1/3} = \frac{V_2}{V_1}$$

$\therefore V_2 = 1.10064 V_1$

% Increase in speed = 10.064%

**06. Ans: (c)**

**Sol:** When a solid sphere falls under gravity at its terminal velocity in a fluid, the following relation is valid :

Weight of sphere = Buoyant force + Drag force

**07. Ans: 0.62**

**Sol:** Given data,

Diameter of dust particle, d = 0.1 mm

Density of dust particle,

$\rho = 2.1\ g/cm^3 = 2100\ kg/m^3$

$\mu_{air} = 1.849 \times 10^{-5}$ Pa.s,

At suspended position of the dust particle,

$W_{particle} = F_D + F_B$

where $F_D$ is the drag force on the particle and $F_B$ is the buoyancy force.

From Stokes law:

$F_D = 3\pi\mu\ V\ d$

Thus,

$$\frac{4}{3} \times \pi r^3 \times \rho \times g = 3\pi\mu V d + \frac{4}{3}\pi r^3 \rho_{air} g$$

$$\text{or, } \frac{4}{3}\pi r^3 g(\rho - \rho_{air}) = 3\pi\mu_{air} V(2r)$$

$$\text{or } V = \frac{2}{9} r^2 g\left(\frac{\rho - \rho_{air}}{\mu_{air}}\right)$$

$$= \frac{2}{9} \times \left(0.05 \times 10^{-3}\right)^2 \times 9.81 \times \frac{(2100 - 1.2)}{1.849 \times 10^{-5}}$$

$= 0.619$ m/s $\approx 0.62$ m/s

**08. Ans: (b)**

**Sol:** Since the two models $M_1$ and $M_2$ have equal volumes and are made of the same material, their weights will be equal and the buoyancy forces acting on them will also be equal. However, the drag forces acting on them will be different.

From their shapes, we can say that $M_2$ reaches the bottom earlier than $M_1$.

**09. Ans: (a)**

**Sol:**

- Drag of object $A_1$ will be less than that on $A_2$. There are chances of flow separation on $A_2$ due to which drag will increase as compared to that on $A_1$.
- Drag of object $B_1$ will be more than that of object $B_2$. Because of rough surface of $B_2$, the boundary layer becomes turbulent, the separation of boundary layer will be delayed that results in reduction in drag.
- Both the objects are streamlined but $C_2$ is rough as well. There will be no pressure drag on both the objects. However, the skin friction drag on $C_2$ will be more than that on $C_1$ because of flow becoming turbulent due to roughness. Hence, drag of object $C_2$ will be more than that of object $C_1$.
- Thus, the correct answer is option (a).

## 12. Open Channel Flow

**02. Ans: (b)**

**Sol:** $Q_1 = 15\text{m}^3/\text{sec}$, $y = 1.5$ m

$$S_1 = \frac{1}{1690}, \text{if } S_2 = \frac{1}{1000}$$

Then $Q_2 = ?$

$$Q \propto \sqrt{S}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{\frac{1}{1000}}{\frac{1}{1690}}}$$

$Q_2 = 1.3 \times 15 = 19.5\ \text{m}^3/\text{s}$

**03. Ans: (d)**

**Sol:** $Q = AV$

$$= B \times y \times \frac{1}{n} R^{2/3} S^{1/2}$$

$$= B \times y \times \frac{1}{n} y^{2/3} S^{1/2}$$

$= R \approx y \rightarrow$ For wide rectangular channel $Q \propto y^{5/3}$

$$\frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{\frac{5}{3}}$$

$$\frac{Q_2}{Q_1} = \left(\frac{1.25 y_1}{y_1}\right)^{\frac{5}{3}}$$

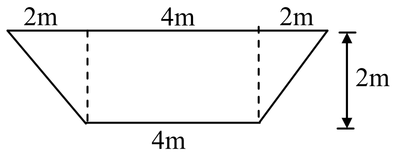
$$\frac{Q_2}{Q_1} = 1.45$$

$Q_2 = 1.45\ Q_1$

It is increased by 45%

**05. Ans: 24.33**

**Sol:**




$\tau_{avg} = \gamma_w RS$

$R = \dfrac{A}{P}$

$A = 2\times\left(\dfrac{1}{2}\times 2\times 2\right) + 4\times 2$

$= 2\times 2 + 4\times 2 = 12\ m^2$

$P = 4 + 2\sqrt{2^2+2^2}$

$= 9.66$ m

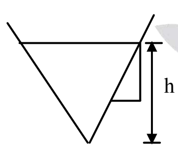
$R = \dfrac{12}{9.66} = 1.24\,m$

$\tau_{avg} = 9810 \times 1.24 \times 0.002$

$= 24.33\ N/m^2$

**06. Ans: (d)**

**Sol: Triangular:**




Triangle

P = 2 (Inclined portion)

$P = 2I = 2h\sqrt{1+m^2} \quad \left(\because I = h\sqrt{1+m^2}\right)$

$= 2h\sqrt{1+1^2}$

$= 2h\sqrt{2}$

$\dfrac{P}{h} = 2\sqrt{2} = 2.83$

**Trapezoidal:** Efficient trapezoidal section is half of the Hexagon for which all sides are equal


Trapezoidal

$I = h\sqrt{1+m^2}$

$P = I = h\sqrt{(1)+\left(\dfrac{1}{\sqrt{3}}\right)^2} = h(1.15)$

$\dfrac{P}{h} = 1.15\times 3 = 3.46$ (3 sides are equal)

**Rectangular:**

$P = b + 2h = 2h + 2h = 4h$ ( $b = 2y$ )

$\dfrac{P}{h} = 4$

**07. Ans: 0.37**

**Sol:** A = y (b + my)

$A = \dfrac{Q}{V} = 4\,m^2$

$4 = \left(b + \dfrac{y}{\sqrt{3}}\right)y$ .......(I) $\left(\because m = \dfrac{1}{\sqrt{3}}\right)$

But b = I ($\because$ Efficient trapezoidal section)

$b = y\sqrt{1+m^2}$

$b = \dfrac{2y}{\sqrt{3}}$ ...............(II)

From (I) & (II)

y= 1.519 m

$\therefore\ D = \dfrac{(b+my)y}{b+2my} = 1.14\,m$

$\therefore F_r = \dfrac{V}{\sqrt{gD}}$

$F_r = 0.37$

**08. Ans: (a)**

**Sol:** Alternate depths

$y_1 = 0.4$ m

$y_2 = 1.6$ m

Specific energy at section =?

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2g\,y_2^2}$$

$$0.4 + \frac{q^2}{2\times9.81\times0.4^2} = 1.6 + \frac{q^2}{2\times9.81\times1.6^2}$$

$$q^2\left(\frac{1}{3.1392} - \frac{1}{50.22}\right) = 1.6 - 0.4$$

$$q^2(0.298) = 1.2$$

$$q^2 = 4.02$$

$$q = 2\ m^3/s/m$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2}$$

$$E_1 = 0.4 + \frac{2^2}{2\times9.81\times0.4^2} = 1.68\ m$$

**09. Ans: (b)**

**Sol:** Depth = 1.6 m

Specific energy = 2.8 m

$$E_1 = \left[y_1 + \frac{v^2}{2g}\right] \Rightarrow 2.8 = 1.6 + \frac{V^2}{2\times9.81}$$

$V = 4.85$ m/s

$$F_r = \frac{V}{\sqrt{gy}}$$

$$F_r = \frac{4.85}{\sqrt{9.81\times1.6}} = 1.22 > 1\,(\text{Supercritical})$$

**10. Ans: (c)**

**Sol:** $F_r = 5.2$ (uniform flow)

The ratio of critical depth to normal depth $\frac{y_c}{y_n} = ?$

**Note:** The given two depths $y_c$ & $y_n$ are not alternate depths as they will have different specific energies.

$$F_r = \frac{V}{\sqrt{gy}} \Rightarrow F_r^2 = \frac{V^2}{gy} = \frac{q^2}{gy^3}\left(\because v = \frac{q}{y}\right)$$

$$\frac{(F_{rn})^2}{(F_{rc})^2} = \frac{q^2}{gy_n^3} \times \frac{gy_c^3}{q^2} = \frac{y_c^3}{y_n^3}$$

$$\frac{y_c^3}{y_n^3} = \frac{(F_{rn})^2}{(F_{rc})^2} \Rightarrow \frac{y_c}{y_n} = \frac{(F_{rn})^{2/3}}{(F_{rc})^{2/3}}$$

$$\frac{y_c}{y_n} = (5.2)^{2/3} = 3$$

**11. Ans: (c)**

**Sol:** Rectangular channel

Alternate depths $y_1 = 0.2$, $y_2 = 4$m

$E_1 = E_2$ ($\because$ alternate depths), $F_r = \frac{V}{\sqrt{gD}}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left[1 + \frac{Fr_2^2}{2}\right]$$

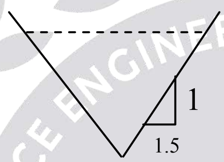
$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{Fr_2^2}{2}}{1 + \frac{Fr_1^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1+\frac{4^2}{2}}{1+\frac{0.2^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left(\frac{2+16}{2+0.04}\right) = 8.8$$

**12. Ans: (d)**

**Sol:** Triangular channel




H:V = 1.5:1

Specific energy = 2.5 m

$$E_C = \frac{5}{4}y_c$$

$$\frac{4}{5}E_C = y_c$$

$y_c = 2$m

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} \Rightarrow 2 = \left(\frac{2\times Q^2}{9.81\times 1.5^2}\right)^{1/5}$$

Q = 18.79 $m^3$/sec

**13. Ans: 0.47**

**Sol:** $E_1 = E_2 + (\Delta z)$

$$V_1 = \frac{Q}{A_1} = \frac{12}{2.4\times 2} = 2.5\,m/sec$$

$A_2 = (b_2 + my_2)y_2 = (1.8 + 1\times 1.6)\ 1.6$

$= 5.44\ m^2$

$$V_2 = \frac{Q}{A_2} = \frac{12}{5.44} = 2.2m/sec$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{(2.5)^2}{2\times 9.81} = 2.318m$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 1.6 + \frac{2.2^2}{2\times 9.81} = 1.846\ m$$

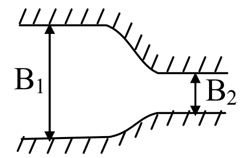
$2.318 = 1.846 + \Delta Z \Rightarrow \Delta Z = 0.47$ m

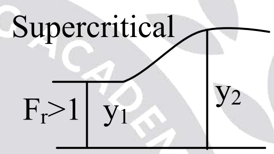
**14. Ans: (c)**

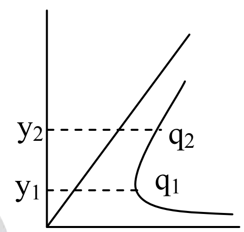
**Sol:** $F_r > 1$

$B_2 < B_1$

$q_2 > q_1$



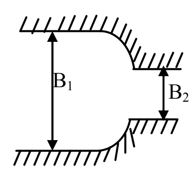

As Potential energy (y) increases then kinetic energy (v) decreases

∴ 'y' increases and 'v' decreases.

**15. Ans: (a)**

**Sol:** Q = 3$m^3$/s

$B_1$ = 2m, D = 1.2 m




Width reduce d to 1.5 m ($B_2$)

Assume channel bottom as horizontal

$\therefore E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{B_1 y_1} = \frac{3}{2\times 1.2} = 1.25\,m/sec$$

$$V_2 = \frac{Q}{B_2 y_2} = \frac{3}{1.5 \times y_2} = \frac{2}{y_2}$$

$$1.2 + \frac{(1.25)^2}{2 \times 9.81} = y_2 + \frac{\left(\frac{2}{y_2}\right)^2}{2 \times 9.81}$$

$$1.27 = y_2 + \frac{4}{y_2^2 \times 19.62}$$

$$1.27 = y_2 + \frac{0.2}{y_2^2}$$

$$y_2^2(1.27) = y_2^3 + 0.2$$

$$y_2^3 - 1.27y_2^2 + 0.2 = 0$$

$$y_2 = 1.12 \text{ m}$$

$$F_{r_1} = \frac{1.25}{\sqrt{9.81 \times 1.2}}\left[\frac{V}{\sqrt{gD}} < 1\right] = 0.364 < 1$$

Approaching flow is sub critical. If approaching flow is sub critical the level at water falls in the throat portion.

**16. Ans: (d)**

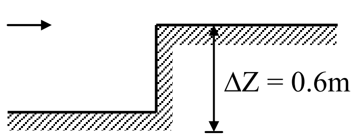
**Sol: Rectangular Channel**

$y_1 = 1.2$m

$V_1 = 2.4$m/s

$\Delta Z = 0.6$m

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{(2.4)^2}{2 \times 9.81} = 1.49\text{m}$$




$Q = 2.4 \times 1.2 = 2.88$ m$^3$/s/m

Assuming channel width as constant, the critical depth

$$y_c = \left[\frac{Q^2}{gB^2}\right]^{\frac{1}{3}} = 0.94\text{m}$$

Critical specific energy for rectangular channel $E_C = \frac{3}{2}y_c$

$$E_c = \frac{3}{2}(0.94) = 1.41$$

We know for critical flow in the hump portion $E_1 = E_2 + (\Delta Z) = E_C + (\Delta Z)_C$

$\Rightarrow 1.49 = 1.41 + (\Delta Z)_C$

$\therefore (\Delta Z)_C = 0.08$m

If the hump provided is more than the critical hump height the u/s flow gets affected.

(or)

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{2.4}{\sqrt{9.81 \times 1.2}} = 0.69 < 1$$

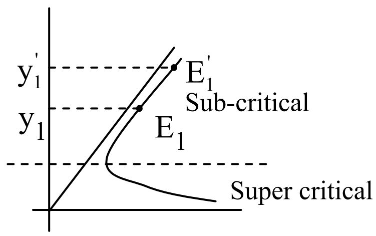
$\Rightarrow$ Hence sub-critical.

If the approaching flow is sub-critical the level of water will fall in the hump portion.

Option (b) is correct if the hump height provided is less than critical hump height.

As the hump height provided is more than critical, the u/s flow gets affected with the increase of the specific energy from $E_1$ to $E_1^1$.

In the sub-critical region as the specific energy increases, the level of water rises from $y_1$ to $y_1^1$ in the form of a surge.




$$E_1^1 = y_1^1 + \frac{v^{1^1}}{2g}$$

$$E_1^1 = y_1^1 + \frac{q^2}{2g{y_1^1}^2} \quad \dots (1)$$

Also $E_1^1 = E_c + (\Delta Z)$ provided.

$= 1.41 + 0.6$

$= 2.01\text{m}$

$$\therefore 2.01 = y_1^1 + \frac{2.88^2}{2\times 9.81\times y_1^2}$$

Solve by trial & error

for $y_1^1 > 1.2\text{m}$

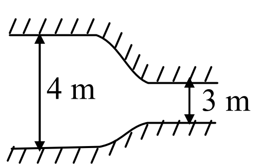
**17. Ans: (c)**

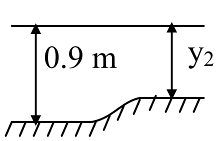
**Sol:** $B_1 = 4$ m

$B_2 = 3$ m

(U/S) $y_1 = 0.9$ m

$E_1 = E_2 + \Delta Z$




$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta Z$$

$V_1 = V_2$

According to continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2 V_2$

$A_1 = A_2$

$B_2y_1 = B_2 y_2$

$4\times 0.9 = 3\times y_2$

$y_2 = 1.2$ m

$y_1 = y_2 + \Delta Z$

$0.9 = 1.2 + \Delta Z$

$\Delta Z = -0.3$ m

Negative indicates that the hump assumed is wrong infact it is a drop.

**18. Ans: (a)**

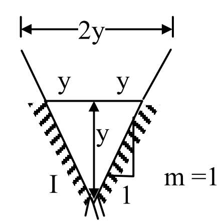
**Sol: Given :**

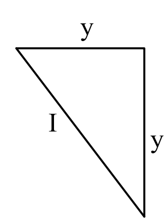
Top width = 2y

$$\text{Area} = \frac{1}{2}\times b\times h$$

$$= \frac{1}{2}\times 2y\times y$$

$A = y^2$




Wetted perimeter

$$I^2 = \sqrt{y^2 + y^2} = y\sqrt{2}$$

(Both sides) total wetted perimeter

$(P) = \sqrt{2}.y + \sqrt{2}.y = 2\sqrt{2}.y$

Hydraulic mean depth

$$(R) = \frac{A}{P} = \frac{y^2}{2\sqrt{2}y} = \frac{y}{2\sqrt{2}}$$

$y = y_n$(say)

**Using Mannings formula**

$$Q = A.\frac{1}{n}.(R)^{2/3}.(S)^{1/2}$$

$$0.2 = y_n^2\,\frac{1}{0.015}\left[\frac{y_n}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$\frac{1}{y_n^{8/3}} = \frac{1}{0.015\times 0.2}\times\left[\frac{1}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$y_n^{8/3} = 0.2\times 0.015\times\left(2\sqrt{2}\right)^{2/3}\left[\frac{1}{0.001}\right]^{1/2}$$

$$(y_n)^{8/3} = 0.189$$

$$y_n = (0.189)^{3/8}$$

$y_n = 0.54$ m

$$\text{critical depth}(y_c) = \left[\frac{2Q^2}{g}\right]^{1/5}$$

(for triangle)

$$y_c = \left[\frac{2\times 0.2^2}{9.81}\right]^{1/5} = 0.382\,\text{m}$$

$y_n > y_c$   (0.54>0.38)

$\therefore$ mild slope

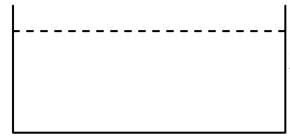
If (actual) depth at flow = 0.4m = y

$Y_n > y > y_c$ [0.54 > 0.4 > 0.38]

$\therefore$ Profile is $M_2$

**19. Ans: $4.36\times 10^{-5}$**

**Sol:**




$\therefore$ Discharge, Q = 29 $m^3$/sec

Area of rectangular channel, A = 15 × 3 = 45 $m^2$

Perimeter, P = 15 + 2 × 3 = 21 m

Hydraulic radius, $R = \frac{A}{P} = \frac{45}{21} = 2.142$ m

$\therefore$ The basic differential equation governing the gradually varied flow is

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2T}{gA^3}}$$

$\frac{dy}{dx}$ = Slope of free water surface w.r.t to channel bottom

Velocity of flow $V = \frac{Q}{A} = \frac{29}{45}$

= 0.644 m/sec

$\therefore$ By Chezy's equation

Velocity, $V = C\sqrt{RS_f}$

$$0.644 = 65\sqrt{2.142\times S_f}$$

$S_f = 4.589\times 10^{-5}$

$$S_o = \frac{1}{5000} = 2\times 10^{-4}$$

$$\frac{Q^2T}{gA^3} = \frac{29^2 \times 15}{9.81 \times 4^3} = 0.0141$$

$$\therefore \frac{dy}{dx} = \frac{2\times10^{-4} - 4.589\times10^{-5}}{1-0.0141}$$

$= 1.5631 \times 10^{-4}$

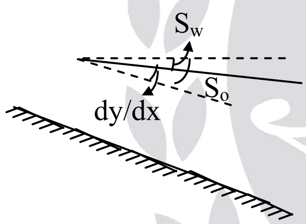
$$\therefore S_o = S_w + \frac{dy}{dx}$$

$S_W$ water surface slope with respect to horizontal

$$S_w = S_o - \frac{dy}{dx}$$

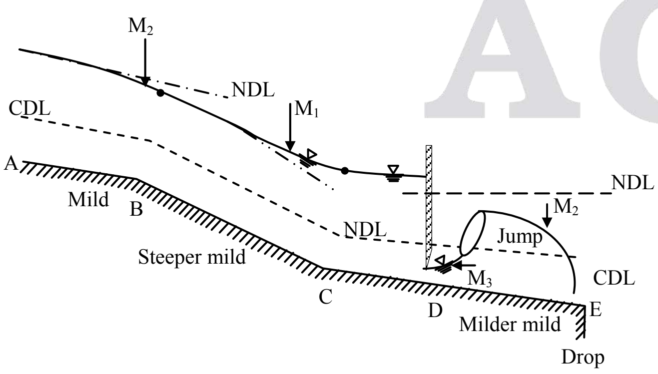
$= 2 \times 10^{-4} - 1.563 \times 10^{-4}$

$S_W = 4.36 \times 10^{-5}$




**20. Ans: (a)**

**Sol:**




**22. Ans: 0.74**

**Sol:** Free fall →2nd profile

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

$$y_c = \left(\frac{2^2}{9.81}\right)^{\frac{1}{3}} = 0.74 \text{ m}$$

$$V = \frac{q}{y_n}$$

$$\frac{2}{y_n} = \frac{1}{n} y_n^{2/3} S^{1/2}$$

$$\frac{2}{y_n} = \frac{1}{0.012} \times y_n^{2/3} (0.0004)^{1/2}$$

$y_n = 1.11$ m

$y_n > y_c$

Hence the water surface will have a depth equal to $y_c$

$y_c = 0.74$ m

**23. Ans: (d)**

**Sol:** q= 2 m$^2$/sec

$y_A = 1.5$ m; $y_B = 1.6$ m

$\Delta E = 0.09$

$$S_o = \frac{1}{2000}$$

$\overline{S}_f = 0.003$

$$\Delta x = \frac{\Delta E}{S_o - \overline{S}_f} = \frac{0.09}{\frac{1}{2000} - 0.003} = -36 \text{ m}$$

**24. Ans: (d)**

**Sol:** Given $q_1 = Q/B = 10\ m^3/s$

$v_1 = 20$ m/s

$$\therefore y_1 = \frac{q_1}{v_1} = \frac{10}{20} = 0.5\,m$$

We know that relation between $y_1$ and $y_2$ for hydraulic jump is

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.81\times0.5}} = 9.03$$

$$\therefore \frac{y_2}{0.5} = \frac{1}{2}\left[-1+\sqrt{1+8\times(9.03)^2}\right]$$

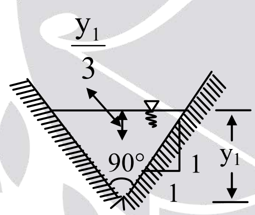
$y_2 = 6.14$ m

**25. Ans: (c)**

**Sol:** $Q = 1\ m^3/s$

$y_1 = 0.5$ m

$y_2 = ?$




As it is not a rectangular channel, let us work out from fundamentals by equating specific force at the two sections.

$$\left[\frac{Q^2}{gA}+A\bar{z}\right]_1 = \left[\frac{Q^2}{gA}+A\bar{z}\right]_2$$

$$\frac{1^2}{9.81\times y_1^2} + y_1^2\times\frac{y_1}{3} = \frac{1^2}{9.81y_2^2} + y_2^2\times\frac{y_2}{3}$$

$$0.449 = \frac{1}{9.81y_2^2} + \frac{y_2^3}{3}$$

$y_2 = 1.02$ m

**26. Ans: (b)**

**Sol: Given:**

Head = 5 m = ($\Delta E$)

Froud number = 8.5

Approximate sequent depths =?

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_{r1}^2}\right]$$

$$= \frac{1}{2}\left[-1+\sqrt{1+8(8.5)^2}\right]$$

$= 11.5$ m

$y_2 = 11.5\ y_1$

$$\left.\begin{array}{l}\text{(a) } y_2 = 11.5(0.3) = 3.45\\ \text{(b) } y_2 = 11.5\,(0.2) = 2.3\text{ m}\end{array}\right\}\text{from options}$$

$y_1 = 0.2$, $y_2 = 2.3$ m

**(or)**

$\Delta E = 5$ m

$$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$$

$$\frac{(11.5y_1-y_1)^3}{4(11.5y_1)y_1} = 5$$

$$(10.5y_1)^3 = 230y_1^2$$

$$1157.625\ y_1 = 230$$

$y_1 = 0.2$ m

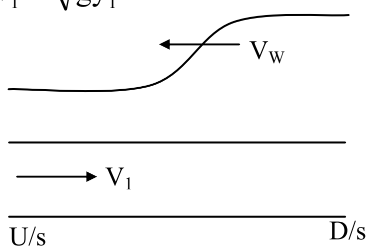
$y_2 = 11.5(0.2)$

$y_2 = 2.3$ m

**27. Ans: 1.43**

**Sol:** $y_1 = 1.2$ m

$$V_w + V_1 = \sqrt{gy_1}$$




$$V_1 = \sqrt{9.81\times1.2} - 2$$

$V_1 = 1.43$ m/s

In this problem if the wave moves downstream the velocity of wave is

$$V_w - V_1 = \sqrt{gy_1}$$

$$V_w = \sqrt{gy_1} + V_1$$

$$= \sqrt{9.81\times1.2} + 2$$

$= 5.43$ m/s

## 13. Dimensional Analysis

**01. Ans: (c)**

**Sol:** Total number of variables,

n = 8 and m = 3 (M, L & T)

Therefore, number of π's are = 8 – 3 = 5

**02. Ans: (b)**

**Sol:** 1. $\dfrac{T}{\rho D^2V^2} = \dfrac{MLT^2}{ML^{-3}\times L^2\times L^2\times T^{-2}} = 1$.

→ It is a non-dimensional parameter.

2. $\dfrac{VD}{\mu} = \dfrac{LT^{-1}\times L}{ML^{-1}T^{-1}} \neq 1$.

→ It is a dimensional parameter.

3. $\dfrac{D\omega}{V} = 1$.

→ It is a non-dimensional parameter.

4. $\dfrac{\rho VD}{\mu} = Re$.

→ It is a non-dimensional parameter.

**03. Ans: (b)**

**Sol:** T = f (*l*, g)

Total number of variable,

n = 3, m = 2 (L & T only)

Hence, no. of π terms = 3 – 2 = 1

**04. Ans: (c)**

**Sol:**

- Mach Number → Launching of rockets
- Thomas Number → Cavitation flow in soil
- Reynolds Number → Motion of a submarine
- Weber Number → Capillary flow in soil

**05. Ans: (b)**

**Sol:** According to Froude's law

$$T_r = \sqrt{L_r}$$

$$\frac{t_m}{t_p} = \sqrt{L_r}$$

$$t_p = \frac{t_m}{\sqrt{L_r}} = \frac{10}{\sqrt{1/25}}$$

$t_p = 50$ min

**06. Ans: (a)**

**Sol:** L = 100 m

$V_P = 10\,m/s$,

$L_r = \frac{1}{25}$

As viscous parameters are not discussed, follow Froude's law.

According to Froude,

$V_r = \sqrt{L_r}$

$\frac{V_m}{V_p} = \sqrt{\frac{1}{25}}$

$V_m = \frac{1}{5} \times 10 = 2$ m/s

**07. Ans: (d)**

**Sol:** Froude number = Reynolds number.

$\nu_r = 0.0894$

If both gravity & viscous forces are important then

$\nu_r = (L_r)^{3/2}$

$\sqrt[3]{(\nu_r)^2} = L_r$

$L_r = 1:5$

**08. Ans: (c)**

**Sol:** For distorted model according to Froude's law

$Q_r = L_H L_V^{3/2}$

$L_H = 1:1000$,

$L_V = 1:100$

$Q_m = 0.1\ m^3/s$

$Q_r = \frac{1}{1000} \times \left(\frac{1}{100}\right)^{3/2} = \frac{0.1}{Q_p}$

$Q_P = 10^5\ m^3/s$

**09. Ans: (c)**

**Sol:** For dynamic similarity, Reynolds number should be same for model testing in water and the prototype testing in air. Thus,

$$\frac{\rho_w \times V_w \times d_w}{\mu_w} = \frac{\rho_a \times V_a \times d_a}{\mu_a}$$

or $V_w = \frac{\rho_a}{\rho_w} \times \frac{d_a}{d_w} \times \frac{\mu_w}{\mu_a} \times V_a$

(where suffixes w and a stand for water and air respectively)

Substituting the values given, we get

$$V_w = \frac{1.2}{10^3} \times \frac{4}{0.1} \times \frac{10^{-3}}{1.8 \times 10^{-5}} \times 1 = \frac{8}{3} m/s$$

To calculate the drag force on prototype, we equate the drag coefficient of model to that of prototype.

i.e, $\left(\frac{F_D}{\rho A V^2}\right)_P = \left(\frac{F_D}{\rho A V^2}\right)_m$

Hence, $(F_D)_p = (F_D)_m \times \frac{\rho_a}{\rho_w} \times \frac{A_a}{A_w} \times \left(\frac{V_a}{V_w}\right)^2$

$$= 4 \times \frac{1.2}{10^3} \times \left(\frac{4}{0.1}\right)^2 \times \left(\frac{1}{8/3}\right)^2$$

= 1.08 N

**10. Ans: 47.9**

**Sol:** Given data,

| | Sea water (Prototype testing) | Fresh water (model testing) |
|---|---|---|
| V | 0.5 | ? |
| ρ | 1025 kg/m$^3$ | $10^3$ kg/m$^3$ |
| μ | $1.07 \times 10^{-3}$ Pa.s | $1\times10^{-3}$ Pa.s |

For dynamic similarity, Re should be same in both testing.

i.e., $\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho_p V_p d_p}{\mu_p}$

$$V_m = V_P \times \frac{\rho_p}{\rho_m} \times \frac{d_p}{d_m} \times \frac{\mu_m}{\mu_p}$$

$$= 0.5 \times \frac{1025}{10^3} \times 100 \times \frac{10^{-3}}{1.07\times10^{-3}}$$

= 47.9 m/s

## 14. Hydraulic Machines

**01. Ans: 1000**

**Sol:** T = Moment of momentum of water in a turbine = Torque developed = 15915 N-m

Speed (N) = 600 rpm

Power developed = $\frac{2\pi NT}{60}$

$$= \frac{2\times\pi\times600\times15915}{60}$$

= $1000 \times 10^3$ W = 1000 kW

**02. Ans: 4000**

**Sol:** Q = 50 m$^3$/sec

H = 7.5 m

$\eta_{\text{Turbine}} = 0.8$

$$\eta_{\text{Turbine}} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{\rho g Q(H - h_f)}$$

$$0.8 = \frac{P_{shaft}}{1000\times9.81\times50(7.5-0)}$$

$P_{shaft} = 2943\times10^3$ W = 2943 kW

$$= \frac{2943}{0.736} \text{ HP} = 4000 \text{ HP}$$

**03. Ans: 1**

**Sol:** We know that

$$U = \frac{\pi DN}{60} = k_u.\sqrt{2gH}$$

where D = diameter of wheel

N = speed of turbine = 600 rpm

H = Head available of Pelton wheel turbine = 300 m

$$\therefore \frac{\pi\times D\times600}{60} = 0.41\sqrt{2\times9.81\times300}$$

D = 1.0 m

**04. Ans: (b)**

**Sol:** Specific speed of turbine is expressed as :

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{T^{-1}\sqrt{FLT^{-1}}}{L^{5/4}}$$

$$= F^{\frac{1}{2}} L^{\frac{1}{2}-\frac{5}{4}} T^{-1-\frac{1}{2}}$$

$$= F^{1/2} L^{-3/4} T^{-3/2}$$

**05. Ans: (b)**

**Sol:** P = 8.1 MW = 8100 kW

H = 81 m

N = 540 rpm

Specific speed $N_S = \frac{N\sqrt{P}}{(H)^{5/4}}$

$= \frac{540\times\sqrt{8100}}{(81)^{5/4}}$

$= \frac{540\times 90}{243} = 200$

$60 < N_S < 300$ (Francis Turbine)

**06. Ans: (a)**

**Sol:** The specific speed is lowest for Pelton wheel and highest for Kaplan turbine. $N_s$ for Francis turbine lies between those of Pelton wheel and Kaplan turbine.

**07. Ans: (b)**

**Sol:**

- Only the tangential component of absolute velocity is considered into the estimation of theoretical head of a turbo machine. Hence, statement (a) is correct.
- A high head turbine has a low value of specific speed. Hence, statement (b) is wrong.
- For the same power, a turbo machine running at high specific speed will be small in size. Hence, statement (c) is correct.
- Pelton wheel is the tangential flow turbine whereas the Propeller and Kaplan turbines are axial flow units. Hence, statement (d) is correct.

**08. Ans: (a)**

**Sol:** $u = \frac{\pi DN}{60}$,

But $u \propto \sqrt{H}$

Hence, for a given scale ratio.

$N \propto H^{1/2}$

**09. Ans: (d)**

**Sol:** Cavitation in any flow passage will occur, if the local pressure at any point in the flow passage falls below the vapour pressure corresponding to the operating temperature.

**10. Ans: (d)**

**Sol:** Cavitation in a reaction turbine may occur at inlet to draft tube. It is expected that the pressure at inlet to draft tube may fall below the vapour pressure.

**11. Ans: 1000**

**Sol:** Given $N_p$ = 500 rpm

$\frac{D_m}{D_p} = \frac{1}{2}$

We know that

$\left(\frac{ND}{\sqrt{H}}\right)_m = \left(\frac{ND}{\sqrt{H}}\right)_P$

Given H is constant

$$\therefore \frac{N_m}{N_p} = \frac{D_p}{D_m}$$

$$\therefore \frac{N_m}{500} = 2$$

$\Rightarrow N_m = 1000$ rpm

**12. Ans: 73**

**Sol:** Given $P_1 = 100$ kW

$H_1 = 100$ m and $H_2 = 81$ m

We know that

$$\left(\frac{P}{(H)^{3/2}}\right)_1 = \left(\frac{P}{(H)^{3/2}}\right)_2$$

$$\therefore \frac{100}{(100)^{3/2}} = \frac{P_2}{(81)^{3/2}}$$

$P_2 = 72.9$ kW $\simeq 73$ kW

$\therefore$ New power developed by same turbine

= 73 kW

**13. Ans: (b)**

**Sol:** Given data :

$D_{runer} = D_{tip} = 3$ m ,

$D_{hub} = \frac{1}{3} \times D_{runner} = 1$ m,

Velocity of flow, $V_f = 5$ m/s,

u = 40 m/s

Discharge through the runner is,

$$Q = \frac{\pi}{4}\left(D_{tip}^2 - D_{hub}^2\right) \times V_f$$

$$= \frac{\pi}{4}\left(3^2 - 1^2\right) \times 5$$

$= 31.4$ m$^3$/s