GATE | PSUs

ELECTRICAL ENGINEERING

Electromagnetic Fields

Text Book : Theory with worked out Examples and Practice Questions

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Electromagnetic Fields

(Solutions for Volume-1 Class Room Practice Questions)

Static Fields & Maxwell's Equations

01. Ans: 1

Sol: $\vec{V} = x\cos^2 y\hat{i} + x^2e^z\hat{j} + z\sin^2 y\hat{k}$

$= x\cos^2 y\hat{a}_x + x^2e^z\hat{a}_y + z\sin^2 y\hat{a}_z$

From divergence theorem

$$\oiint \overline{V}.\hat{n}\,ds = \int_v \left(\nabla.\overline{D}\right)dv \;.................1$$

$$\nabla.\overline{D} = \frac{\partial}{\partial x}\left(x\cos^2 y\right) + \frac{\partial}{\partial y}\left(x^2e^z\right) + \frac{\partial}{\partial z}\left(z\sin^2 y\right)$$

$$= \cos^2 y + \sin^2 y = 1$$

dv = dxdydz

Putting these value in equation 1 we have

$$\oiint \overline{V}.\hat{n}\,ds = \int_0^1\int_0^1\int_0^1 1\times dx\,dy\,dz$$

$$= \int_0^1 dx\int_0^1 dy\int_0^1 dz = 1$$

02 Ans: (c)

Sol: For the given $\vec{A} = xy\,\vec{a}_x + x^2\,\vec{a}_y$

Let I = $\oint \vec{A}.d\vec{\ell}$, I is evaluated over the path shown in the Fig., as follows

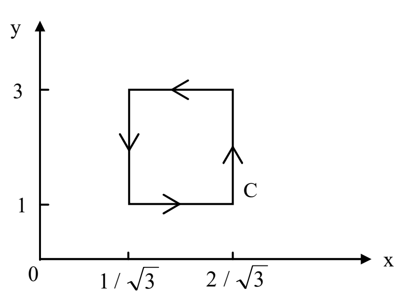


Fig.

$$I = \oint \vec{A}.dx\,\vec{a}_x,\; y = 1,\; x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$+ \int \vec{A}.dy\,\vec{a}_y,\; x = \frac{2}{\sqrt{3}},\; y = \text{from 1 to 3}$$

$$- \int \vec{A}.dx\,\vec{a}_x,\; y = 3, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$- \int \vec{A}.dy\,\vec{a}_y,\; x = 1/\sqrt{3},\; y = \text{from 1 to 3}$$

$$= \int xy\,dx + \int x^2\,dy - \int xy\,dx - \int x^2\,dy$$

$$= y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2 y\Big|_1^3 - y\left.\frac{x^2}{2}\right|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2 y\Big|_1^3$$

at $y = 1 \quad x = 2/\sqrt{3} \quad y = 3 \quad x = 1/\sqrt{3}$

$$= \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) + \frac{4}{3}(3-1) - \frac{3}{2}\left(\frac{4}{3} - \frac{1}{3}\right) - \frac{1}{3}(3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\rho = \rho a_\rho + \rho\sin^2\phi\, a_\phi - za_z$

$= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$

$$\nabla.F = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho F_\rho\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(F_\phi\right) + \frac{\partial}{\partial z}\left(F_Z\right)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho^2\right) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\rho\sin^2\phi\right) + \frac{\partial}{\partial z}(-z)$$

$= 2 + 2\sin\phi\cos\phi - 1$

$= 1 + 2\sin\phi\cos\phi$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2,\; \nabla.F\big|_{\phi=0} = 1$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2\nabla.F\big|_{\phi=0}$$

04. Ans: (c)

Sol: $\overline{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_Z$ $\quad\quad \overline{D} = |\overline{D}|\overline{a}_n$

$|\overline{D}| = \sqrt{16} = 4 \quad\quad = \rho_s \hat{a}_n$

$$\therefore \overline{D} = 4\left\{\frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_Z}{4}\right\}$$

$$= \rho_s \hat{a}_n \quad\quad \therefore \rho_s = 4\, C/m^2$$

05. Ans: (d)

Sol: Poisson's equation is

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0} \quad \text{(or)}$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right](10y^4 + 20x^3) = \frac{-\rho}{\varepsilon_0}$$

$$\overline{E} = -\nabla V,\ \nabla.E = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

$$20 \times 3 \times 2 \times x + 10 \times 4 \times 3y^2 = \frac{-\rho_v}{\varepsilon_0}$$

$$= -60x^2\,\hat{a}_x - 40y^3\hat{a}_y$$

at point (2,0) $\quad\quad \overline{D} = \varepsilon_0 \overline{E}$

$$20 \times 3 \times 2 \times 2 = \frac{-\rho_v}{\varepsilon_0}$$

$$= -60\varepsilon_0 x^2\,\hat{a}_x - 40\varepsilon_0 y^3 \hat{a}_y$$

$$\nabla.\overline{D} = \rho_v = -120\varepsilon_0 x - 120\varepsilon_0 y$$

$$\rho_V = -240\varepsilon_0$$

06. Ans: (d)

Sol: Given

$V(x, y, z) = 50\,x^2 + 50\,y^2 + 50\,z^2$

$\vec{E}\,(x, y, z)$ in free space $= -\text{grad}\,(V)$

$$= -\nabla V$$

$$= -\left[\frac{\partial}{\partial x} V\overrightarrow{a_x} + \frac{\partial}{\partial y} V\overrightarrow{a_y} + \frac{\partial}{\partial z} V\overrightarrow{a_z}\right]$$

$$= -\left[100x\,\overrightarrow{a_x} + 100y\,\overrightarrow{a_y} + 100z\,\overrightarrow{a_z}\right] V/m$$

$\vec{E}\,(1, -1, 1) =$

$$-\left[100\,\overrightarrow{a_x} - 100\,\overrightarrow{a_y} + 100\,\overrightarrow{a_z}\right] V/m$$

$$E(1,-1,1) = 100\sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of $\vec{E}$.

$$\vec{a}_E = \frac{\vec{E}(1,-1,1)}{|E(1,-1,1)|} = \frac{1}{\sqrt{3}}\left[-\overrightarrow{a_X} + \overrightarrow{a_Y} - \overrightarrow{a_Z}\right]$$

or in i , j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}}[-i + j - k]$

07. Ans: (b)

Sol: For valid B, $\nabla.B = 0$

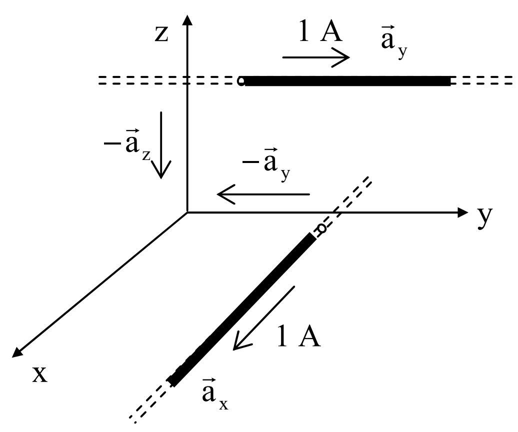
$$\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)(x^2 a_x - xya_y - Kxza_z) = 0$$

$2x - x - Kx = 0 \Rightarrow 2 - 1 - K = 0$

$\therefore\ K = 1$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the $\vec{a}_y$ direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the $\vec{a}_x$ direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors along the 'x', 'y' and 'z' axes respectively.

$\therefore$ x and z components of magnetic field are non-zero at the origin.

09. Ans: (a)

Sol: $\nabla.\overline{B} = 0$

A divergence less vector may be a curl of some other vector

$\overline{B} = \nabla \times \overline{A}$

$\nabla \times \overline{A} = \overline{B}$

$\oint_l \overline{A}\,.\overline{dl} = \int_s \overline{B}\,.\,\overline{ds}$

$\int_s \overline{B}\,.\,\overline{ds}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

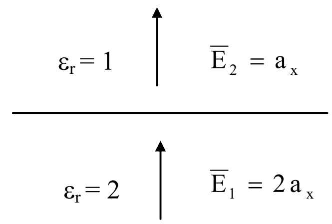
Sol: In general, for an infinite sheet of current density k A/m

$H = \frac{1}{2} k \times a_n = \frac{1}{2}\left(8\overline{a}_x \times \overline{a}_z\right)$

$= -4\ \overline{a}_y \quad \left(\because \overline{a}_x \times \overline{a}_z = -\overline{a}_y\right)$

11. Ans: (b)

Sol:



$D_{n_2} - D_{n_1} = \rho_S$

$D_{n_2} = \varepsilon\, E_{n_2} = \varepsilon_0\, a_x$

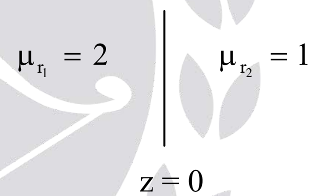
$D_{n_1} = \varepsilon_0\, 2 \times 2\, a_x = 4\varepsilon_0\, a_x$

From (a)

$(\varepsilon_0 - 4\varepsilon_0)\, a_x = \rho_s \Rightarrow \rho_s = -3\varepsilon_0$

12. Ans: (a)

Sol:



$B_1 = 1.2\vec{a}_x + 0.8\vec{a}_y + 0.4\vec{a}_z$

$B_{n_1} = 0.4\ \overline{a}_z$

(Since z = 0 has normal component a_x)

$B_{t_1} = 1.2\ \overline{a}_x + 0.8\ \overline{a}_y$

We know magnetic flux density is continuous

$B_{n_1} = B_{n_2}$

$B_{n_2} = 0.4\ \overline{a}_z$

Surface charge, $\overline{k} = 0$

$H_{t_2} - H_{t_1} = 0$

$H_{t_2} = H_{t_1}$

$\mu_1 B_{t_2} = \mu_2 B_{t_1}$

$B_{t_2} = \frac{1}{2}(1.2a_x + 0.8a_y)$

$B_2 = B_{t_2} + B_{n_2}$

$= 0.6\,\bar{a}_x + 0.\bar{4}\,a_y + 0.4\bar{a}_z$

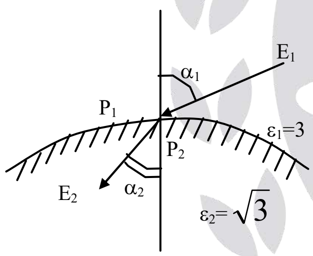
$\mu_0 \mu_{r_2} H_2 = 0.6\,\bar{a}_x + 0.\bar{4}\,a_y + 0.4\bar{a}_z$

$H_2 = \frac{1}{\mu_0}[0.6\,\bar{a}_x + 0.\bar{4}\,a_y + 0.4\bar{a}_z]\ A/m$

13. Ans: (b)

Sol: Tangential components of electric fields are continuous $\left(E_{t_1} = E_{t_2}\right)$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$...........(1)



Normal component of electric flux densities are continuous across a charge free interface

$D_{n_1} = D_{n_2}$

$3E_1 \cos\alpha_1 = \sqrt{3}E_2 \cos\alpha_2$............(2)

$\alpha_1 = 60^0$

$\frac{(1)}{(2)} \Rightarrow \frac{\tan\alpha_1}{3} = \frac{\tan\alpha_2}{\sqrt{3}} \Rightarrow \tan\alpha_2 = 1$

$\alpha_2 = 45^o$

14. Ans: (c)

Sol: N = 100

$\phi = t^3 - 2t$ mWb

According to Faraday's law

$E = N\frac{d\phi}{dt}\Big|_{t=4sec}$

$= 100 \times (3t^2 - 2)$ mV = 4.6 V